

**BIPLANES  $(56, 11, 2)$  WITH A FIXED-POINT-FREE  
INVOLUTORY AUTOMORPHISM**

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ABSTRACT. The aim of this article is to prove that exactly four biplanes with parameters  $(56, 11, 2)$  admit a fixed-point-free action of an involutory automorphism. These are: Hall's biplane  $B_{20}$ , Salwach and Mezzaroba's biplane  $B_{22}$ , Denniston's biplane  $B_{24}$  and Denniston's biplane  $B_{26}$ .

## 1. INTRODUCTION AND PRELIMINARIES

A biplane with parameters  $(56, 11, 2)$  is a symmetric design having 56 points and lines, each line consisting of 11 points, each point lying on 11 lines, each two lines intersecting in 2 points and each pair of points lying on 2 lines. Five different biplanes with parameters  $(56, 11, 2)$  were discovered until 1985. These biplanes are known in literature and denoted by  $B_{20}$ ,  $B_{22}$ ,  $B_{24}$ ,  $B_{26}$  and the Janko-Tran van Trung biplane, which we shall denote by  $J-T$  (see [8] and [5]). The existence of them all was proved using the computer and assuming additionally an action of an automorphism group  $G \leq \text{Aut}\mathcal{D}$ . Until now, it has been unknown whether there exists a biplane for  $(56, 11, 2)$  allowing an action of a nontrivial automorphism group and not isomorphic to one of the already listed ones. The only remaining possibility is the case of an involution acting fixed-point-freely on such a biplane. The reason why this case has not been solved before lies in the combinatorial expansion that arises during the construction, which is by a multiple larger than in any other case.

If  $G \leq \text{Aut}\mathcal{D}$  is an automorphism group of the biplane  $\mathcal{D}$  for  $(56, 11, 2)$  and if  $p$  is a prime divisor of  $|G|$ , then it holds:

$$p \in \{2, 3, 5, 7, 11\}.$$

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All the cases for  $p > 2$  can easily be solved using the program by V. Čepulić described in [2]. The following results came out, which V. Čepulić hasn't published yet:

$$\begin{aligned} p = 11, & \quad \text{no biplane;} \\ p = 7, & \quad \text{the biplane } B_{20}; \\ p = 5, & \quad \text{the biplane } B_{20}; \\ p = 3, & \quad \text{the biplanes } B_{20}, B_{22}, B_{26}, J\text{-}T. \end{aligned}$$

For  $p = 2$ , all possibilities for which an involution acts with  $F > 0$  fixed points have already been examined. Using the known bounds for the number of fixed points and lines

$$1 + \frac{k-1}{\lambda} \leq F \leq k + \sqrt{k-\lambda}, \quad F \neq 0,$$

by putting  $k = 11$ ,  $\lambda = 2$  in it, one gets

$$F \in \{6, 8, 10, 12, 14\}.$$

The case  $F = 6$  produces the biplanes  $B_{20}$ ,  $B_{22}$ ,  $B_{24}$  and  $J\text{-}T$ , what can be found in [7]. Other cases have been solved again by the program by V. Čepulić with the following outcome. The case  $F = 8$  gives all the 5 known biplanes, the cases  $F = 10$  and  $F = 12$  fail, while the case  $F = 14$  gives biplanes  $B_{20}$ ,  $B_{22}$  and  $B_{24}$ . The last case has been published in [3], and the other cases haven't been published yet.

From the short summary presented above, it is clear that the solution of the case  $F = 0$ , together with a publication of the cases solved by V. Čepulić, completes the classification of all biplanes  $\mathcal{D}(56, 11, 2)$  with nontrivial automorphism groups. For all the details considering the constructions and classifications mentioned above the reader is referred to the forthcoming paper [4].

Hence, it is our intention now to construct all designs  $\mathcal{D}$  with the parameter triple  $(56, 11, 2)$ , which admit an involutory automorphism  $\rho$  acting without any fixed points and lines. We shall use the fundamental construction idea introduced by Janko and Tran van Trung in [6]. At first we shall build all possible orbit structures  $\mathcal{S}$  of  $\mathcal{D}$  admitting such a  $\rho$ . After that we shall build the biplanes themselves by "indexing" the "big points" of  $\mathcal{S}$ .

## 2. CONSTRUCTION OF ORBIT STRUCTURES FOR A FIXED-POINT-FREE INVOLUTION

First we introduce some notation. The automorphism group  $\langle \rho \rangle$  has 28 orbits on the set of points of  $\mathcal{D}$ . We denote these orbits and their points by:

$$\mathcal{P}_1 \equiv \{1_0, 1_1\}, \dots, \mathcal{P}_j \equiv \{j_0, j_1\}, \dots, \mathcal{P}_{28} \equiv \{28_0, 28_1\},$$

and  $\rho$  maps  $j_s$  onto  $j_{(s+1)(\text{mod } 2)}$  for every  $1 \leq j \leq 28$ . The  $\langle \rho \rangle$ -orbits of lines of  $\mathcal{D}$  we denote by:

$$\mathcal{B}_1 \equiv \{x_1, x_1\rho\}, \dots, \mathcal{B}_i \equiv \{x_i, x_i\rho\}, \dots, \mathcal{B}_{28} \equiv \{x_{28}, x_{28}\rho\}.$$

Let us consider the form of any line of  $\mathcal{D}$  in terms of the number of occurrences of symbols for  $\langle \rho \rangle$ -orbits. We refer to the well-known formulae for the multiplicities of orbit symbols when a group of prime order  $p$  acts fixed-point-freely. If  $\mu_{ij}$  is the number of occurrences of an orbit symbol  $j$  on any line from  $\mathcal{B}_i$ , we have

$$(1) \quad \sum_{j=1}^{28} \mu_{ij}(\mu_{ij} - 1) = \lambda(p - 1) = 2 \cdot (2 - 1) = 2, \quad 1 \leq i \leq 28.$$

Since  $k = 11$ , this means that on each line there is one orbit symbol which occurs twice, and nine orbit symbols which occur once.

If  $\mu_{ij}$  and  $\mu_{rj}$  are the numbers of occurrences of an orbit symbol  $j$  on two representatives of distinct  $\langle \rho \rangle$ -orbits  $\mathcal{B}_i$  and  $\mathcal{B}_r$  then it holds

$$(2) \quad \sum_{j=1}^{28} \mu_{ij}\mu_{rj} = \lambda p = 2 \cdot 2 = 4, \quad 1 \leq i, r \leq 28, \quad i \neq r.$$

We say briefly that the "game product" of two lines from distinct  $\langle \rho \rangle$ -orbits is equal to 4.

DEFINITION 1. We call the  $28 \times 28$  matrix  $\mathcal{S} = [\mu_{ij}]$ , satisfying conditions (1) and (2), the multiplicity matrix (or orbit structure) of  $\mathcal{D}$  for  $\langle \rho \rangle$ .

Dualizing our arguments we obtain:

LEMMA 2. A multiplicity matrix  $\mathcal{S} = [\mu_{ij}]$  of  $\mathcal{D}(56, 11, 2)$  for a fixed-point-free involution  $\rho$  consists of 28 rows

$$\hat{x}_1, \hat{x}_2, \dots, \hat{x}_{28}$$

and 28 columns  $1, 2, \dots, 28$ . Every of them consists of one entry equal to 2, nine 1's and eighteen 0's, satisfying:

$$(3) \quad \sum_{r=1}^{28} \mu_{ir}\mu_{jr} = \sum_{t=1}^{28} \mu_{ti}\mu_{tj} = 4, \quad 1 \leq i, j \leq 28, \quad i \neq j.$$

For two orbit structures  $\mathcal{S}_1 = [\mu'_{ij}]$  and  $\mathcal{S}_2 = [\mu''_{ij}]$ , an isomorphism  $\sigma$  from  $\mathcal{S}_1$  onto  $\mathcal{S}_2$  is a bijection which maps rows of  $\mathcal{S}_1$  onto rows of  $\mathcal{S}_2$ , and columns of  $\mathcal{S}_1$  onto columns of  $\mathcal{S}_2$ , preserving the entries:  $\mu''_{\sigma(i)\sigma(j)} = \mu'_{ij}$ . Next we define a precedence relation for rows of orbit structures, and then for orbit structures themselves.

DEFINITION 3. Suppose that there is given an order among the columns of  $\mathcal{S}$ . For two rows  $\hat{x} = [\mu_{ij}]_i$  and  $\hat{y} = [\mu'_{ij}]_i$  we define that  $\hat{x}$  precedes  $\hat{y}$ ,  $\hat{x} \preceq \hat{y}$ , if there is some  $r$ ,  $1 \leq r < 28$ , such that  $\mu_{ij} = \mu'_{ij}$  for  $j < r$  and  $\mu_{ir} > \mu'_{ir}$ . As usual,  $\hat{x} \prec \hat{y}$  will stand for  $\hat{x} \preceq \hat{y}$  and  $\hat{x} \neq \hat{y}$ .

DEFINITION 4. Let  $\mathcal{S}_1$  and  $\mathcal{S}_2$  be two orbit structures of  $\mathcal{D}$  for  $\langle \rho \rangle$ . We define that  $\mathcal{S}_1$  precedes  $\mathcal{S}_2$ ,  $\mathcal{S}_1 \preceq \mathcal{S}_2$ , if  $\mathcal{S}_1$  precedes  $\mathcal{S}_2$  in terms of rows precedence.  $\mathcal{S}_1 \prec \mathcal{S}_2$  will stand for  $\mathcal{S}_1 \preceq \mathcal{S}_2$  and  $\mathcal{S}_1 \neq \mathcal{S}_2$ .

Now we sketch our algorithm for constructing all orbit structures  $\mathcal{S}$  of  $\mathcal{D}$  for  $\langle \rho \rangle$  (see also [1]). We produce the structures by building up the rectangular schemes level by level. The  $i$ -th layer of  $\mathcal{S}$ , denoted  $\hat{x}^{(i)}$ , consists of all possible rows with one entry equal to 2, nine entries equal to 1, and the remaining eighteen entries being 0. We can easily compute the number of possibilities for  $\hat{x}^{(i)}$ :

$$N_i = |\hat{x}^{(i)}| = \frac{28!}{18!9!} = 131\,231\,000, \quad i = 1, 2, \dots, 28.$$

A partial orbit structure of  $l$ -th level, denoted by  $\mathcal{S}(l)$ , is any matrix with  $l$  rows from  $\hat{x}^{(i)}$ ,  $i = 1, 2, \dots, l$ , satisfying the consistence condition (3) for rows, and not violating the consistence condition (3) for columns. Let  $\mathcal{S}^{(l)}$  be the set of all possible partial structures  $\mathcal{S}(l)$ ,  $\mathcal{S}^{(1)} = [2111111111100000000000000000]$  being obviously the only member of  $\mathcal{S}^{(1)}$ . We construct  $\mathcal{S}^{(l)}$  from  $\mathcal{S}^{(l-1)}$ ,  $2 \leq l \leq 28$ , in the following way. For each partial orbit structure  $\mathcal{S}(l-1) \in \mathcal{S}^{(l-1)}$  we exhaust all 131 231 000 possibilities for the  $l$ -th level, by generating the corresponding rows  $[\mu_{lr}]_l$  in the lexicographical order defined above. For a particular  $[\mu_{lr}]_l$ , after testing the condition (3), we include

$$\mathcal{S}(l) = \mathcal{S}(l-1) \cup [\mu_{lr}]_l$$

into  $\mathcal{S}^{(l)}$ , if it cannot be eliminated by finding some automorphism  $\sigma$  such that a scheme  $\mathcal{S}(l)\sigma$  precedes  $\mathcal{S}(l)$ . If  $\mathcal{S}(l)\sigma \prec \mathcal{S}(l)$  in terms of the precedence of partial schemes considered as parts of the whole orbit structures  $\mathcal{S}$ ,  $\mathcal{S}(l)$  is omitted. In this way, we ensure the elimination of a lot of isomorphic orbit structures, retaining only those among them which are the first in terms of the defined precedence. At the end of this procedure  $\mathcal{S}^{(28)}$  will be the set of all possible orbit structures for our particular problem.

Applying the algorithm we have obtained as the only solutions (up to isomorphism) ten orbit structures:  $\mathcal{S}_1 - \mathcal{S}_{10}$ . This result has been achieved after nearly 4000 hours of continuous computing on a computer "DynatechDCS - 1/320". The greatest number of schemes we have gotten on level 12, where we have counted approximately 80 000 000 (not necessarily non-isomorphic) schemes. Below we enclose all the 10 solutions.

STRUCTURE  $S_1$   
 21111111110000000000000000000000  
 12000000001111111100000000000000  
 1020000000111000001111100000  
 100200000010100001000011110  
 1000200000100011000110011001  
 1000020000010011000001100111  
 1000002000001100100101010101  
 1000002000011000100101010101  
 100000020000010111100101100  
 100000002000001111011010010  
 011110000000000200010100111  
 011101000000000020101011001  
 011000110000002001100001110  
 0101001100000020001011110000  
 0100110010000200001111000010  
 0100110001002000001000111100  
 010000101120000000001101011  
 0100000111020000000110010101  
 0011000011001111001000000002  
 00101010011010010000010020  
 0010100101100110010001000200  
 0010010110100101100000120000  
 0001101001010101010100200000  
 0001100110011001100002001000  
 0001011010101001010020000100  
 000101010110101010020000010  
 0000111100110000112000000001

STRUCTURE  $S_3$   
 21111111110000000000000000000000  
 12000000001111111100000000000000  
 1020000000111000001111100000  
 1002000000100110001100011110  
 1000200000100001100011011010  
 1000020000010101001010000111  
 1000002000010010010001110110  
 100000200001010101000101011  
 100000020001001010101001101  
 1000000020001001010110110001  
 0111100000100000010000100112  
 0110011000010000100100012001  
 0110000110001100000010020110  
 0101010001001100000002101010  
 0101001100000011000121000001  
 0100110010000011001100210000  
 0100100101010000101101000200  
 0100001011100000012010001010  
 0011010100000001121001010000  
 0011000011010011100100000020  
 0010110001001020010010001100  
 0010101010000210101001000001  
 0010001101100102000000101100  
 0001101001012001001000010001  
 0001100110020100010010101000  
 0001011010101000200010100100  
 0000111100101100010200000010  
 000010111210010000001010001

STRUCTURE  $S_2$   
 21111111110000000000000000000000  
 12000000001111111100000000000000  
 1020000000111000001111100000  
 1002000000100110001100011100  
 1000200000100001100011011010  
 1000020000010101001010000111  
 10000020000100100100001110110  
 1000002000010101010000101011  
 100000020001001010101001101  
 100000002000100110110110001  
 0111100000100000010000100112  
 0110011000010000100100012001  
 0110000110001100000010020110  
 0101001100000011000121000001  
 0101000011010000101101000020  
 0100110010000011001100210000  
 0100101001001000012010001100  
 0100010101100100000002101100  
 0011001001001100000001010001  
 0100010101100100000002101100  
 0011010100000001121001010000  
 0011001001001102000000101010  
 0010101010000210101001000001  
 0010100101010011100100000200  
 0010010011100020010010001010  
 0001110001012010000001010001  
 0001100110020100010010101000  
 0001100110020100010010101000  
 0000111100101100010200000010  
 000001111210001001000010001

STRUCTURE  $S_4$   
 21111111110000000000000000000000  
 12000000001111111100000000000000  
 1020000000111000001111100000  
 1002000000110100001000011110  
 1000200000010110001100011100  
 100002000001000110001111010  
 1000001100100011000101000021  
 1000001100010000110010100201  
 1000000011000210000011110001  
 1000000011000001112100001001  
 0111001000000010100011002001  
 0111000100000001010100120001  
 0110110000001100001000000112  
 0101000020001000010111000110  
 0100100110010010001000201010  
 0100100101100000101002010100  
 0100011001100100000200101100  
 0100011001010001001020010010  
 0011000002001011100000100110  
 0010101010010100200100010010  
 0010100101100100020010001010  
 0010011010100020011000010100  
 0010010110010102000001001100  
 0001110010200001100010100001  
 0001110001020010010101000001  
 0001102000001101011001100000  
 0001010200001110101110000000  
 000000111112100000000011001

STRUCTURE $S_5$	STRUCTURE $S_6$
21111111110000000000000000000000	21111111110000000000000000000000
1200000000111111110000000000	1200000000111111110000000000
1020000000111000001111100000	1020000000111000001111100000
1002000000100110001100011100	1002000000100110001100011100
1000200000100001100011011010	1000110000110100000010010021
1000020000010100010010110110	1000110000000001112001001001
1000001100010001011000102001	10000011000100011001010000210
1000001100002000100100010111	1000001100000020010011110001
1000000011000100111002000101	1000000011002001000010011101
1000000011000021000110100011	1000000011000100110100201010
0111100000100000010000100112	0111100000000002000101110101
0110011000000011001001020001	0110101000000011000110002010
0110000200000110000011001110	011000020000110001001000111
0101010010001000101020001001	0101100001001000010102010010
0101000002011000000001111010	0101000200011000001000101011
010010100010001001110000200	0100011010010001001100120000
0100100110000100101100210000	010001100120000000000101101
0100011010100000010201001010	0100010101000100011120000100
0011001010010101101000000020	0011020000001011010000100110
0011000101000001120110010000	0011001001010101010100000002
0010110001001201000100001001	0010100110110000020000011100
00101000200011010010000011100	0010011100001200100001011000
0010011001100010200000101100	0010000102100011101000010010
0001110100020010100101000001	0001100110100102000011100000
000102000001110010011100000	0001010011020010100011001000
0001010110101002000001100100	0001002010101000111010000010
0000110101101010012000000010	000020100101110001000100100
000000111210100000010010001	00001101101010100100200000001
STRUCTURE $S_7$	STRUCTURE $S_8$
211111111100000000000000000000	211111111100000000000000000000
1200000000111111110000000000	1200000000111111110000000000
1020000000111000001111100000	1020000000111000001111100000
100200000000111001110011000	1001100000200100001000011110
1000110000110000101000020110	1001010000000120000111010001
100011000000110010102000101	1000101000000002101110001001
1000001100101001000001002110	1000010100010000021001001101
1000001100000100200110100011	1000001100001010100100100210
1000000011010000021010001011	1000000011010100100000211001
1000000011001011000000210101	1000000011001001010011010020
0110101000100010000010011002	0111001000000010010010102010
0110100010000200001000101110	0110110000000001010100120100
0110001001000001010200010110	0110001100100000100001010012
0101010100020000000100101101	0101001010000101001002100100
0101010001100010000011100020	0101000101010000101020010100
0101001001001000102001000101	0100110001001010002000100011
0100100200000001011011110000	01001001010100000201001010
0100010020001000100111011000	0100010020101000000110001101
0011100010010002100001000011	0011100010020011000000000111
0011010100002100010000010011	0011010010000100211100000010
0011000110100010110010000200	0010101001001200010010000101
0010021000000011111000101000	0010010002100011100001001100
0010000102010110100001011000	0010000210001111001000011000
0001200001101000110100101000	0002000101101001010100100001
0001002010110100010001110000	0001111000012000100001011000
0000111001011101000020000100	0000200110100010110011100000
0000101110011020001100000010	0000021100110101000010100010
0000010111200101001100000001	0000002011110010011100010000

STRUCTURE $S_9$	STRUCTURE $S_{10}$
21111111110000000000000000000000	21111111110000000000000000000000
12000000001111111100000000000000	12000000001111111100000000000000
1020000000111000001111100000	1011000000210000001111100000
1001100000200100001000011110	1011000000001110001000021100
100101000000120000111010001	1000110000110001000000011021
100010100000002101110001001	1000110000001000201110000101
100001010001000101010101000210	1000001100010001010101001200
1000001100001100100000210101	1000001100000210000110100011
1000000011010010100010101020	1000000011001010100002101010
1000000011001000021001011001	1000000011000001021010110001
0111001000000010010100102100	0120100000000010010011000111
0110110000000001010010120010	0110101000000001100100211000
0110010100100000100001001012	0101010010000010011200001010
0101001010000101001002100010	0101000200000001101011010010
0101000101001000101200010010	0101000110101000000000101102
0100110001010010002000100101	01001001001010100002001001001
0100100101001100000021001100	01000200100101000000011110100
0100001020110000000110010101	0100001002101000000110010110
0011100010002011000000000111	0011010001001102000101000001
0011010010000100211010000100	0010011100012000011000100010
0010101001010200010100000011	0010010101100100110010002000
0010001002100011100001010100	0010001020100101101000000110
0010000210010111001000011000	0010000111020010100100010001
0002000101110001010010100001	0002100001010100110000100110
0001111000021000100001011000	0001101010011011000020001000
0000200110100010110101100000	0001012000100010110001010001
0000020011101101000100101000	0000200110101100010101010000
0000012100101010011010000010	0000110101100021001000100100

### 3. FINAL RESULTS

Applying the first step of our algorithm we have found all possible solutions for  $\mathcal{S}$ . In the following, we shall write every orbit structure  $\mathcal{S} = [\mu_{jr}]$  as a set of 28 orbit lines  $\hat{x}_j = [\mu_{jr}]_j$ ,  $j = 1, \dots, 28$ , represented as sequences of their  $k = 11$  "big points". If  $\mu_{jr_0} = 2$  and  $\mu_{jr_i} = 1$  for  $i = 1, 2, \dots, 9$ , then we write

$$\hat{x}_j \equiv r_0 r_0 r_1 r_2 r_3 r_4 r_5 r_6 r_7 r_8 r_9,$$

the numbers  $r_0, r_1, \dots, r_9 \in \{1, 2, \dots, 28\}$  being the "big points" of  $\mathcal{S}$ . Now we try to construct the biplanes by "indexing"  $\mathcal{S}$ , e.g. supporting its big points with appropriate indices from  $\{0, 1\}$ . We give a brief description of our algorithm.

Let  $\mathcal{S} = [\mu_{jr}]$  be the orbit structure under consideration. For the  $j$ -th row of  $\mathcal{S}$  we construct lines  $x_j$  from the line orbit  $\mathcal{B}_j$ , by supplying the orbit numbers of  $\hat{x}_j$  with indices from  $\{0, 1\}$ . For  $x'_j, x''_j$  corresponding to the same  $\hat{x}_j$  we define:  $x'_j$  precedes  $x''_j$ ,  $x'_j \prec x''_j$ , if the sequence of indices of big points corresponding to  $x'_j$  precedes that of  $x''_j$  lexicographically. Among two lines of the orbit  $\mathcal{B}_j$  we take out as its representative the first in terms of the defined precedence, thus obtaining  $\hat{x}_j$  - the canonical form of  $x_j$ . In the following we identify  $\hat{x}_j$  with  $x_j$  and call it the *canonical line*. The set of all  $j$ -th level canonical lines we denote  $x^{(j)}$ .

After finding  $x^{(j)}$ , we build the partial designs. A *partial design of  $j$ -th level*, denoted by  $\Delta_j$ , consists of  $j$  canonical lines which satisfy the design

conditions. By  $\mathcal{D}^{(j)}$  we denote the set of  $j$ -th level partial designs  $\Delta_j$  which we construct in our procedure,  $\Delta_1 \equiv 1_0 1_1 2_0 3_0 4_0 5_0 6_0 7_0 8_0 9_0 10_0$  being obviously the only member of  $\mathcal{D}^{(1)}$ . For two partial designs  $\Delta'_j$  and  $\Delta''_j$  we say that  $\Delta'_j$  *precedes*  $\Delta''_j$ ,  $\Delta'_j \prec \Delta''_j$ , if there exists some  $q$ ,  $q \leq j$ , such that: (i) corresponding  $i$ -th level canonical lines of  $\Delta'_j$  and  $\Delta''_j$  coincide for  $1 \leq i < q$ , and (ii)  $q$ -th level canonical line of  $\Delta'_j$  precedes that of  $\Delta''_j$ .

We construct  $\mathcal{D}^{(j)}$  from  $\mathcal{D}^{(j-1)}$ ,  $2 \leq j \leq 28$ , in the following way. To each partial design  $\Delta_{j-1} \in \mathcal{D}^{(j-1)}$  we join all possible  $j$ -th level canonical lines  $x_j$  which intersect each line of  $\Delta_{j-1}$  in exactly two points. In such a way we obtain one by one potential partial designs  $\Delta_j = \Delta_{j-1} \cup x_j$  of the  $j$ -th level. Now, we include  $\Delta_j$  into  $\mathcal{D}^{(j)}$  if it cannot be eliminated by finding some automorphism  $\sigma$  such that  $\Delta_j \sigma \prec \Delta_j$  in terms of the precedence of partial designs. At the end of this procedure,  $\mathcal{D}^{(28)}$  will be the set of all possible biplanes with the orbit structure  $\mathcal{S}$ , admitting the action of the given involution  $\rho$ .

The described procedure has been carried out by computer as well, the computing time being only 20 minutes. It has turned out that among 10 orbit structures  $\mathcal{S}_1$ - $\mathcal{S}_{10}$  only five of them can be supplied by indices. Namely, we have obtained the following biplanes:

( $\mathcal{S}_1$ ) none	( $\mathcal{S}_6$ ) none
( $\mathcal{S}_2$ ) $B_{22}$ (2 copies), $B_{24}$ (2 copies)	( $\mathcal{S}_7$ ) $B_{22}$ (1 copy), $B_{26}$ (3 copies)
( $\mathcal{S}_3$ ) $B_{20}$ (2 copies), $B_{22}$ (2 copies)	( $\mathcal{S}_8$ ) none
( $\mathcal{S}_4$ ) none	( $\mathcal{S}_9$ ) none
( $\mathcal{S}_5$ ) $B_{24}$ (2 copies)	( $\mathcal{S}_{10}$ ) $B_{24}$ (16 copies)

The first appearances of the obtained biplanes are enclosed below. We shall write down only the 28 line orbit representatives. The remaining 28 lines one can get by changing all the indices of these representatives modulo 2. We were able to identify the resulting biplanes as  $B_{20}$ ,  $B_{22}$ ,  $B_{24}$  or  $B_{26}$  using their chain representations (see [8] for details).



BIPLANE  $B_{20}$

1 <sub>0</sub>	1 <sub>1</sub>	2 <sub>0</sub>	3 <sub>0</sub>	4 <sub>0</sub>	5 <sub>0</sub>	6 <sub>0</sub>	7 <sub>0</sub>	8 <sub>0</sub>	9 <sub>0</sub>	10 <sub>0</sub>
1 <sub>0</sub>	2 <sub>0</sub>	2 <sub>1</sub>	11 <sub>0</sub>	12 <sub>0</sub>	13 <sub>0</sub>	14 <sub>0</sub>	15 <sub>0</sub>	16 <sub>0</sub>	17 <sub>0</sub>	18 <sub>0</sub>
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BIPLANE  $B_{22}$

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BIPLANE  $B_{24}$ 

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7 <sub>0</sub>	8 <sub>1</sub>	9 <sub>1</sub>	10 <sub>0</sub>	11 <sub>0</sub>	11 <sub>1</sub>	12 <sub>0</sub>	16 <sub>1</sub>	19 <sub>1</sub>	24 <sub>1</sub>	28 <sub>1</sub>

BIPLANE  $B_{26}$ 

1 <sub>0</sub>	1 <sub>1</sub>	2 <sub>0</sub>	3 <sub>0</sub>	4 <sub>0</sub>	5 <sub>0</sub>	6 <sub>0</sub>	7 <sub>0</sub>	8 <sub>0</sub>	9 <sub>0</sub>	10 <sub>0</sub>
1 <sub>0</sub>	2 <sub>0</sub>	2 <sub>1</sub>	11 <sub>0</sub>	12 <sub>0</sub>	13 <sub>0</sub>	14 <sub>0</sub>	15 <sub>0</sub>	16 <sub>0</sub>	17 <sub>0</sub>	18 <sub>0</sub>
1 <sub>0</sub>	3 <sub>0</sub>	3 <sub>1</sub>	11 <sub>0</sub>	12 <sub>1</sub>	13 <sub>1</sub>	19 <sub>0</sub>	20 <sub>0</sub>	21 <sub>1</sub>	22 <sub>0</sub>	23 <sub>0</sub>
1 <sub>0</sub>	4 <sub>0</sub>	4 <sub>1</sub>	14 <sub>1</sub>	15 <sub>0</sub>	16 <sub>1</sub>	19 <sub>1</sub>	20 <sub>1</sub>	21 <sub>0</sub>	24 <sub>0</sub>	25 <sub>0</sub>
1 <sub>0</sub>	5 <sub>1</sub>	6 <sub>0</sub>	11 <sub>1</sub>	12 <sub>0</sub>	17 <sub>1</sub>	19 <sub>0</sub>	24 <sub>0</sub>	24 <sub>1</sub>	26 <sub>1</sub>	27 <sub>0</sub>
1 <sub>0</sub>	5 <sub>0</sub>	6 <sub>1</sub>	14 <sub>0</sub>	15 <sub>1</sub>	18 <sub>1</sub>	20 <sub>1</sub>	22 <sub>0</sub>	22 <sub>1</sub>	26 <sub>1</sub>	28 <sub>0</sub>
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2 <sub>0</sub>	3 <sub>0</sub>	5 <sub>1</sub>	7 <sub>1</sub>	11 <sub>0</sub>	15 <sub>1</sub>	21 <sub>1</sub>	24 <sub>0</sub>	25 <sub>0</sub>	28 <sub>0</sub>	28 <sub>1</sub>
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3 <sub>0</sub>	4 <sub>0</sub>	8 <sub>1</sub>	9 <sub>1</sub>	11 <sub>1</sub>	15 <sub>1</sub>	17 <sub>0</sub>	18 <sub>0</sub>	21 <sub>0</sub>	26 <sub>0</sub>	26 <sub>1</sub>
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Hence the following theorem has been proved.

**THEOREM 1.** *Let  $\mathcal{D}$  be a biplane with parameters (56, 11, 2) admitting an involutory automorphism acting fixed-point-freely. Then  $\mathcal{D}$  is isomorphic to one of the following four known biplanes: Hall's biplane  $B_{20}$ , Salwach and Mezzaroba's biplane  $B_{22}$ , Denniston's biplane  $B_{24}$ , or Denniston's biplane  $B_{26}$ .*

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