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# ON THE COMPLEXITY OF FINDING A SUN IN A GRAPH* 

CHÍNH T. HOÀNG ${ }^{\dagger}$


#### Abstract

The sun is the graph obtained from a cycle of length even and at least six by adding edges to make the even-indexed vertices pairwise adjacent. Suns play an important role in the study of strongly chordal graphs. A graph is chordal if it does not contain an induced cycle of length at least four. A graph is strongly chordal if it is chordal and every even cycle has a chord joining vertices whose distance on the cycle is odd. Farber proved that a graph is strongly chordal if and only if it is chordal and contains no induced suns. There are well known polynomial-time algorithms for recognizing a sun in a chordal graph. Recently, polynomial-time algorithms for finding a sun for a larger class of graphs, the so-called HHD-free graphs (graphs containing no house, hole, or domino), have been discovered. In this paper, we prove the problem of deciding whether an arbitrary graph contains a sun is NP-complete.


Key words. chordal graph, strongly chordal graph, sun
AMS subject classification. 68Q25
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1. Introduction. A hole is an induced cycle with at least four vertices. A graph is chordal if it does not contain a hole as an induced subgraph. Farber [6] defined a graph to be strongly chordal if it is chordal and every cycle in the graph on $2 k$ vertices, $k \geq 3$, has a chord $u v$ such that each segment of the cycle from $u$ to $v$ has an odd number of edges. We denote by $k$-sun the graph obtained from a cycle of length $2 k(k \geq 3)$ by adding edges to make the even-indexed vertices pairwise adjacent. Figure 1 shows a 5 -sun. A sun is simply a $k$-sun for some $k \geq 3$. Farber showed [6] that a graph is strongly chordal if and only if it is chordal and does not contain a sun as induced subgraph. Farber's motivation was a polynomial-time algorithm for the minimum weighted dominating set problem for strongly chordal graphs. The problem is NP-hard for chordal graphs [1]. In this paper, we prove that it is NP-hard to find a sun in an arbitrary graph. This result is motivated by the following discussion on chordal and strongly chordal graphs. For more information on this topic, see [3, 7].

We use $N(x)$ to denote the set of vertices adjacent to vertex $x$ in a graph $G$. Define $N[x]=N(x) \cup\{x\}$. A vertex $x$ in a graph is simplicial if $N(x)$ induces a complete graph. It is well known [4] that graph $G$ is chordal if and only if every induced subgraph $H$ of $G$ contains a simplicial vertex of $H$. Farber proved [6] an analogous characterization for strongly chordal graphs. A vertex $x$ in a graph is simple if the vertices in $N(x)$ can be ordered as $x_{1}, x_{2}, \ldots, x_{k}$ such that $N\left[x_{1}\right] \subseteq N\left[x_{2}\right] \subseteq \cdots \subseteq$ $N\left[x_{k}\right]$. Thus, every simple vertex is simplicial. For a graph $G$, let $\mathcal{R}=v_{1}, v_{2}, \ldots, v_{n}$ be an ordering of vertices of $G$. Let $G(i)=G\left[\left\{v_{i}, v_{i+1}, \ldots, v_{n}\right\}\right]$, i.e., the subgraph induced in $G$ by the set $v_{i}$ through $v_{n}$ of vertices. $\mathcal{R}$ is a simple elimination ordering for $G$ if $v_{i}$ is simple in $G(i), 1 \leq i \leq n$. The following is due to Farber [6].

Theorem 1 (see [6]). The following are equivalent for any graph $G$ :

- $G$ is strongly chordal.
- $G$ is chordal and does not contain a sun.
- Vertices of $G$ admit a simple elimination ordering.

[^0]

Fig. 1. The 5-sun.


FIG. 2. The house, the hole, and the domino.

Thus, suns play an important role in the studies of chordal and strongly chordal graphs. There are well known algorithms $[16,12]$ to test whether a chordal graph is strongly chordal and thus whether it contains a sun. It is natural to investigate the problem of sun testing for larger classes of graphs. A graph is HHD-free if it does not contain a house, a hole, or a domino (see Figure 2). Every chordal graph is an HHD-free graph. HHD-free graphs [10] have several properties analogous to those of chordal graphs. Brandstädt [2] proposed the problem of finding a sun in an HHD-free graph. This problem was proved to be polynomial-time solvable in [13] and [5]. The absence of a sun in a graph seems to suggest that the graph has a certain structure. The author has thought, but has not been able to prove, that a sun-free HHD-free graph contains a homogeneous set or a simple vertex (a set $H$ of vertices of a graph $G=(V, E)$ is homogeneous if $2 \leq|H|<|V|$ and every vertex outside $H$ is adjacent to all, or to no vertices of $H$; homogeneous sets are also known as nontrivial modules). One may wonder whether the existence of the algorithms in $[16,12,13,5]$ is due to the property of being sun-free or of being chordal (or HHD-free). This has led several researchers to ask for the complexity of finding a sun in a graph. In this paper, we will prove the following.

Theorem 2. It is NP-complete to decide whether a graph contains a sun.
The above theorem suggests that it is the property of being chordal (or HHD-free) that allows us to test for a sun efficiently and it is unlikely there is a polynomial-time algorithm for finding a sun in an arbitrary graph. Denote by $k$-hole the hole on $k$ vertices. A $k$-antihole is the complement of a $k$-hole. A graph is weakly chordal [8] if it does not contain a $k$-hole or $k$-antihole with $k \geq 5$. Weakly chordal graphs generalize chordal graphs in a natural way, and they are known to be perfect and have many interesting algorithmic properties (see [9]). In spite of Theorem 2, it is conceivable there are polynomial-time algorithms to solve the sun recognition problem for weakly chordal graphs or even perfect graphs [15]. In this spirit, we will refine Theorem 2 to obtain a stronger result.

Theorem 3. It is $N P$-complete to decide whether a graph $G$ contains a sun, even when $G$ does not contain a $k$-antihole with $k \geq 7$.

Let $k$-CLIQUE (respectively, $k$-SUN) be the problem whose instance is a graph $G$ and an integer $k$, for which the question to be answered is whether $G$ contains a clique on $k$ vertices (respectively, $k$-SUN). It is well known [11] that $k$-CLIQUE is NP-complete. It is not difficult to prove but perhaps interesting to note that $k$-SUN is also NP-complete. Observe that if $k$ is a constant (not part of the input), then the two problems can obviously be solved in polynomial time.

Theorem 4. $k-S U N$ is NP-complete.
Note that Theorem 2 implies Theorem 4: To decide whether a graph contains a sun, we need only to solve $O(n)$ instances of $k$-SUN with $k$ running from 3 to $n / 2$, where $n$ is the number of vertices of the graph. However, we have a short and direct proof of Theorem 4. We will give the proofs of Theorems 2,3 , and 4 in the remainder of the paper.
2. The proofs. First, we need to introduce some definitions. For simplicity, we will say a vertex $x$ sees a vertex $y$ if $x$ is adjacent to $y$; otherwise, we will say $x$ misses $y$. Let $G, F$ be two vertex-disjoint graphs, and let $x$ be a vertex of $G$. We say that a graph $H$ is obtained from $G$ by substituting $F$ for $x$ if $H$ is obtained by replacing $x$ by $F$ in $G$ and adding the edge $a b$ for any $a \in V(G)-\{x\}$ and any $b \in F$ whenever $a x$ is an edge of $G$. In the proofs, we will often use the observation that every vertex in $H-F$ either sees all, or misses all, vertices of $F$.

By $\left(c_{1}, c_{2}, \ldots, c_{k}, r_{1}, r_{2}, \ldots, r_{k}\right)$ we denote the $k$-sun with vertices $c_{1}, c_{2}, \ldots, c_{k}$, $r_{1}, r_{2}, \ldots, r_{k}$ such that $c_{1}, c_{2}, \ldots, c_{k}$ induce a clique and $r_{1}, r_{2}, \ldots, r_{k}$ induce a stable set; each $r_{i}$ has degree two and sees $c_{i}, c_{i+1}$ with the subscripts taken modulo $k$. The vertices $r_{i}$ will be called the rays of the $k$-sun. A triangle is a clique on three vertices.

We will rely on the following NP-complete problem due to Poljak [14].
Stable set in triangle-free graphs.
Instance: A triangle-free graph $G$, an integer $k$.
Question: Does $G$ contain a stable set with $k$ vertices?
Proof of Theorem 2. We will reduce stable set in triangle-free graphs to the problem of finding a sun in a graph.

Let $G=(V, E)$ be a triangle-free graph with $V=\left\{v_{1}, v_{2}, \ldots, v_{n}\right\}$, and without loss of generality assume $k \geq 4$. Define a graph $f(G, k)$ from $G$ as follows. Substitute for each vertex $v_{i}$ a clique $V_{i}=\left\{v_{i}^{1}, v_{i}^{2}, \ldots, v_{i}^{k}\right\}$; add a clique $W$ with vertices $u_{1}, w_{1}, \ldots, u_{k}, w_{k}$; add a stable set $X$ with vertices $x_{1}, \ldots, x_{k}$; for $i=1,2, \ldots, k$, add edges $x_{i} w_{i}$ and $x_{i} u_{i+1}$ (the subscripts are taken module $k$ ); for $i=1,2, \ldots, n$ and $j=1,2, \ldots k$, add edges $v_{i}^{j} u_{j}, v_{i}^{j} w_{j}$. Figure 3 shows a graph $G$ whose graph $f(G, 4)$ is shown in Figure 4 (for clarity, we do not show all edges of $f(G, 4)$; all adjacency between $V_{1}$ and $W$ and between $V_{2}$ and $W$ are shown, and adjacency between $V_{3}$ and $W$ are not shown; the thick line between $V_{1}$ and $V_{2}$ (and between $V_{2}$ and $V_{3}$ ) represents all possible edges between the two sets; there are no edges between $V_{1}$ and $V_{3}$; each of the sets $V_{i}, W$ induces a clique; the set $X$ induces a stable set). We will often rely on the following observations.

Observation 1. Suppose $G$ is triangle-free. Then $f(G, k)$ does not contain a triangle each of whose vertices belongs to a distinct $V_{i}$.

Observation 2. Let $x$ be a vertex in $V_{i}$, and $y$ be a vertex in $V_{j}$ with $i \neq j$. If $x$ and $y$ have a common neighbor $z$ in $W$, then $N(x) \cap W=N(y) \cap W$.

The theorem follows from the following claim.


Fig. 3. The graph $G$.


Fig. 4. The graph $f(G, 4)$.

Claim 1. $G$ has a stable set with $k$ vertices if and only if $f(G, k)$ contains a sun.
Proof of Claim 1. Suppose $G$ has a stable set with vertices $v_{1}, v_{2}, \ldots, v_{k}$. Then $f(G, k)$ has a $2 k$-sun $\left(c_{1}, c_{2}, \ldots, c_{2 k}, r_{1}, r_{2}, \ldots, r_{2 k}\right)$ with $r_{2 i-1}=v_{i}^{i}, r_{2 i}=x_{i}, c_{2 i-1}=$ $u_{i}$, and $c_{2 i}=w_{i}$ for $i=1,2, \ldots, k$.

Now, suppose $f(G, k)$ contains a sun. Write $T=V_{1} \cup V_{2} \cup \cdots \cup V_{n}$. We will establish that
any sun $S$ of $f(G, k)$ is a $2 k$-sun with $k$ rays in $T$.
Consider a sun $S=\left(c_{1}, c_{2}, \ldots, c_{t}, r_{1}, r_{2}, \ldots, r_{t}\right)$ of $f(G, k)$. First, we claim that (with the subscript taken modulo $k$ )

$$
\begin{equation*}
\text { if a ray } r_{j} \text { lies in } X \text {, then } r_{j-1}, r_{j+1} \text { lie in } T \text {. } \tag{2}
\end{equation*}
$$

Let $x_{i}$ be a vertex in $X$ that is a ray $r_{j}$ of $S$. We may assume that $c_{j}=w_{i}$ and $c_{j+1}=u_{i+1}$. Since $r_{j-1}$ sees $w_{i}$ and misses $u_{i+1}$, we have $r_{j-1} \in V_{s}$ for some $s$. Similarly, we have $r_{j+1} \in V_{r}$ for some $r$. Note that $r \neq s$. So, (2) holds.

Since $W$ is a clique, $S$ must have a ray in $T \cup X$. (2) implies that
$T$ contains a ray of $S$.
Next, we will prove

$$
\begin{equation*}
\text { if } r_{i} \in V_{j} \text {, then } c_{i}, c_{i+1} \in W \text {. } \tag{4}
\end{equation*}
$$

Suppose (4) is false. For simplicity, we may assume $i=1$ and $j=1$ (we can always rename the vertices of $f(G, k)$ and $S$ so that this is the case). We will often implicitly use the fact that a vertex in $V_{a}$ either sees all, or misses all, vertices of $V_{b}$ whenever $a \neq b$. Note that $c_{1}, c_{2}$ cannot be in $X$. We will distinguish among several cases.

Case 1. $c_{1}, c_{2} \in V_{1}$. Since $c_{3}$ sees $c_{1}, c_{2}$ and misses $r_{1}, c_{3}$ cannot be in $T$. Thus, $c_{3}$ is in $W$. But no vertex in $W$ can see two vertices in $V_{1}$, a contradiction.

Case 2. $c_{1} \in V_{1}, c_{2} \in V_{j}$ for some $j \neq 1$. We may write $j=2$. Since $c_{3}$ (respectively, $r_{t}$ ) sees $c_{1}$ and misses $r_{1}, c_{3}$ (respectively, $r_{t}$ ) cannot be in $T$. Thus, $c_{3}$ and $r_{t}$ are in $W$. Observation 2, with $z=c_{3}, x=c_{1}$, and $y=c_{2}$, implies $r_{t}$ sees $c_{2}$, a contradiction to the definition of $S$.

Case 3. $c_{1} \in V_{1}, c_{2} \in W$. This case is not possible since a vertex in $W$ can have at most one neighbor in any $V_{j}$.

Case 4. $c_{1}, c_{2} \in V_{j}$ for some $j \neq 1$. We may write $j=2$. Since $r_{2}$ sees $c_{2}$ and misses $c_{1}, r_{2}$ is in $W$. Since $r_{t}$ sees $c_{1}$ and misses $c_{2}, r_{t}$ is in $W$. But then $r_{2}$ sees $r_{t}$, a contradiction.

Case 5. $c_{1} \in V_{j}, c_{2} \in V_{r}$ with $j \neq r, j \neq 1$, and $r \neq 1$. In this case, $r_{1}, c_{1}$, and $c_{2}$ contradict Observation 1.

Case 6. $c_{1} \in V_{j}$ for some $j \neq 1$ and $c_{2} \in W$. We may let $j=2$. If $c_{3} \in$ $W$, then Observation 2, with $z=c_{2}, x=r_{1}$, and $y=c_{1}$, implies $c_{3}$ sees $r_{1}$, a contradiction to the definition of $S$. So, we have $c_{3} \in T$. Since $c_{3}$ misses $r_{1}$, we have $c_{3} \notin V_{1} \cup V_{2}$. So, we may assume $c_{3} \in V_{3}$. We have $r_{2} \notin W$; otherwise Observation 2, with $z=c_{2}, x=c_{1}$, and $y=c_{3}$, implies $r_{2}$ sees $c_{1}$, a contradiction to the definition of $S$. We have $r_{2} \notin V_{1} \cup V_{2} \cup V_{3}$ since $r_{2}$ misses $r_{1}$ and $c_{1}$. So, we may assume $r_{2} \in V_{4}$. Since $r_{3}$ (respectively, $c_{4}$ if it exists) sees $c_{3}$ and misses $r_{1}$, Observation 2 , with $z=c_{2}, x=r_{1}$, and $y=c_{3}$, implies $r_{3} \notin W$ (respectively, $c_{4} \notin W$ ). Since $r_{3}$ (respectively, $c_{4}$ if it exists) misses $r_{1}$ and $r_{2}$, we have $r_{3} \notin V_{1} \cup V_{2} \cup V_{3} \cup V_{4}$ (respectively, $c_{4} \notin V_{1} \cup V_{2} \cup V_{3} \cup V_{4}$ ). Now, if $t=3$, then the three vertices $r_{3}, c_{1}$, and $c_{3}$ contradict Observation 1. But if $t>3$, then the three vertices $c_{4}, c_{1}$, and $c_{3}$ contradict Observation 1.

So, (4) holds. Next, we will establish two more assertions (where the subscripts are taken modulo $k$ ) below.

$$
\begin{equation*}
\text { If a ray } r_{j} \text { lies in } T \text {, then } r_{j-1}, r_{j+1} \text { lie in } X \text {. } \tag{5}
\end{equation*}
$$

By (4) and the definition of $f(G, k)$, we may assume $c_{j}=u_{i}, c_{j+1}=w_{i}$. Since $x_{i}$ is the only vertex of $f(G, k)$ that sees $w_{i}$ and misses $u_{i}$, we have $x_{i}=r_{j+1}$. Similarly, we have $x_{i-1}=r_{j-1}$. So, (5) holds.
(6) If some vertex $x_{i} \in X$ is a ray of $S$, then $x_{i+1}$ is also a ray of $S$.

Let $x_{i}$ be a vertex in $X$ that is a ray $r_{j}$ of $S$. We may assume that $c_{j}=w_{i}$ and $c_{j+1}=u_{i+1}$. By (2), we have $r_{j+1} \in V_{a}$ for some $a$. By (4), we have $c_{j+2}=w_{i+1}$. By (5), $r_{j+2}$ lies in $X$, and so we have $r_{j+2}=x_{i+1}$. Thus, (6) holds.

We are now in a position to prove (1). From (3), we may assume $r_{1}$ lies in $T$. By (5), we have $r_{2} \in X$. By (6), all $x_{j}$ 's are rays of $S$ for $j=1,2, \ldots, k$. It follows from (2) that $S$ has exactly $k$ rays in $T$. Therefore, $S$ is a $2 k$-sun. We have proved (1).

We continue with the proof of Claim 1 (and Theorem 2). Consider the $k$ rays of $S$ that belong to $T$. Since each $V_{i}$ is a clique, it contains at most one ray. So, there are $k$ sets $V_{i}$ containing a ray of $S$. Let these sets be $V_{1}, V_{2}, \ldots, V_{k}$. Clearly, in $G$, the vertices $v_{1}, v_{2}, \ldots, v_{k}$ form a stable set.

Proof of Theorem 3. We will use the notation defined in the proof of Theorem 2 with $G$ being a triangle-free graph. We need only to prove the graph $f(G, k)$ does not contain a $t$-antihole with $t \geq 7$. We will prove by contradiction. Suppose $f(G, k)$ contains a $t$-antihole $A$ with vertices $a_{1}, a_{2}, \ldots, a_{t}$ with $t \geq 7$ such that $a_{i}$ misses $a_{i+1}$ with the subscripts taken modulo $k$. Since the vertices in $X$ have degree two, none of them can belong to $A$. Since each $V_{i}$ is a clique,
no two consecutive vertices of $A$ can belong to the same $V_{i}$.
Similarly,
no two consecutive vertices of $A$ can belong to $W$.
Now, we claim that

$$
\begin{equation*}
\text { one of } a_{i}, a_{i+1} \text { must lie in } W \text { for all } i . \tag{9}
\end{equation*}
$$

Suppose (9) is false for $a_{i}$. For simplicity, we may assume $i=1$, and so we have $a_{1}, a_{2} \in T$. By (7), we may assume $a_{1} \in V_{1}, a_{2} \in V_{2}$. Clearly, we have $a_{t} \notin V_{1}$.

Suppose $a_{t} \in V_{2}$. Then $a_{3}$ has to be in $W$; otherwise $a_{3}$ lies in some $V_{j}$ and so it misses $a_{t}$ (since it misses $a_{2}$ ) implying $t=4$, a contradiction. By symmetry, we have $a_{t-1} \in W$. Since $a_{1}$ sees $a_{3}$, and $a_{t-1}$ is a common neighbor of $a_{1}$ and $a_{2}$, Observation 2 implies that $a_{2}$ sees $a_{3}$, a contradiction to the definition of $A$. So, we have $a_{t} \notin V_{2}$.

Suppose $a_{t} \in W$. By (8), we have $a_{t-1} \in V_{j}$. If $j=2$, then $a_{1}$ misses $a_{t-1}$, a contradiction to the definition of $A$. If $j=1$, then $a_{2}$ misses $a_{t-1}$ implying $t=4$, a contradiction. So, we may assume $a_{t-1} \in V_{3}$. Let $j \in\{t-2, t-3\}$. If $a_{j} \in W$, then since $a_{2}$ sees $a_{t}$, Observation 2 with $z=a_{j}, x=a_{2}$, and $y=a_{1}$ implies $a_{1}$ sees $a_{t}$, a contradiction to the definition of $A$. So, we have $a_{t-2} \in V_{m}$ for some $m$, and $a_{t-3} \in V_{p}$ for some $p$. Since $a_{t-2}$ misses $a_{t-1}$, we have $a_{t-2} \notin V_{1} \cup V_{2} \cup V_{3}$. So, we may assume $m=4$. We have $a_{t-3} \in V_{2} \cup V_{3}$; otherwise the three vertices $a_{t-3}, a_{t-1}$, and $a_{2}$ contradict Observation 1. Since $a_{t-2}$ sees $a_{2}, a_{t-2}$ sees all of $V_{2}$. Thus, we have $a_{t-3} \notin V_{2}$, and so $a_{t-3} \in V_{3}$. Since $t \geq 7$, the vertex $a_{t-4}$ exists. Since $a_{t-4}$ misses $a_{t-3}$ but sees $a_{t-1}, a_{t-4}$ is not in $T$; so we have $a_{t-4} \in W$. Observation 2 with $z=$ $a_{t-4}, x=a_{t-1}$, and $y=a_{t-2}$ implies $a_{t-1}$ sees $a_{t}$, a contradiction to the definition of $A$.

Thus, $a_{t}$ belongs to some $V_{j}$ which is distinct from $V_{1}, V_{2}$. It follows from symmetry and the definition of $f(G, k)$ that $a_{3}, a_{t-1}$ also belong to distinct $V_{i}$. Now, the three vertices $a_{t-1}, a_{1}$, and $a_{3}$ contradict Observation 1. So, (9) holds.

From (8) and (9), we may assume without loss of generality that $a_{i} \in T$ whenever $i$ is odd, and $a_{i} \in W$ whenever $i$ is even. In particular, $t$ is even and at least eight. The definition of $A$ implies that $a_{1}$ sees $a_{4}, a_{6}$. Thus, we have $\left\{a_{4}, a_{6}\right\}=\left\{u_{i}, w_{i}\right\}$ for some $i$. The definition of $f(G, k)$ means that every vertex of $T$ either sees both $a_{4}, a_{6}$ or misses both of them. But $a_{3}$ misses $a_{4}$ and sees $a_{6}$, a contradiction.

Proof of Theorem 4. We will reduce $k$-CLIQUE to $k$-SUN. Let $G, k$ be an instance of $k$-CLIQUE. We may assume $k \geq 4$. Construct a graph $h(G)$ from $G$ by adding a
vertex $v(a, b)$ for each edge $a b$ of $G$ and joining $v(a, b)$ to $a$ and $b$ by an edge of $h(G)$. Let $Y$ be the set of vertices $v(a, b)$. It is easy to see that if $G$ has a clique $K$ on $k$ vertices, then $h(G)$ has a $k$-sun induced by $K$ and some $k$ vertices in $Y$. If $h(G)$ has a $k$-sun $\left(c_{1}, \ldots, c_{k}, r_{1}, \ldots, r_{k}\right)$, then since the vertices in $Y$ have degree two, none of them can be a vertex $c_{i}$; thus, the vertices $c_{1}, \ldots, c_{k}$ induce a clique on $k$ vertices in $G$.

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