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Equivalent-layer method for optical waveguides with a multiple-quantum-well structure

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An equivalent-layer method for analyzing multiple-quantum-well (MQW) waveguides is presented. This method not only allows the whole waveguide to be treated as a three-layer waveguide but offers exact solutions for the propagation constant of MQW waveguides and the power confinement factor within the MQW structure as well. A comparison between this method and two other three-layer models is given.

Thanks to the development of molecular-beam epitaxy and metal-organic chemical-vapor deposition growth techniques, optical waveguides with a multiple-quantum-well (MQW) structure have been applied to various optical devices, such as semiconductor lasers, modulators, and optical switches.[1] For a single quantum well, the optical properties can be analyzed through the standard three-layer dielectric optical waveguide problem found in many textbooks.[2] The analysis of waveguides with MQW structure, however, is substantially more complex in practice and usually involves 2 × 2 matrix calculations.[3,4] In previous papers[5–8] we studied the guiding properties, such as dispersion relation, field distribution, and power flow, for multilayer waveguides. By using these general formulas with a MQW structure, we found that we can use one equivalent layer (EL) to replace whole MQW structure to obtain the propagation constant of the waveguide. In this Letter this EL method is presented.

In addition to the two outermost layers with refractive indices n_a and n_s, the MQW waveguide has $2m - 1$ layers, including m quantum-well layers with indices n_2 and thickness d_2 and $m - 1$ barrier layers with n_1 and d_1. The total thickness of the MQW structure d_t is given by $(m - 1)d_1 + md_2$. Although the quantum structure exhibits quantum size effects, we assume that the refractive indices in the barrier and well layers in a MQW waveguide are expressed by their bulk values n_1 and n_2, respectively, and all interfaces are treated classically. This treatment was used by many researchers.[9–11]

Approximate waveguide characteristics can be obtained by replacing the MQW structure in the waveguide by a planar layer with thickness d_t. The refractive index of that layer is usually approximated by one of two different expressions. One is simply equal to the mean value of the refractive indices over the MQW structure, which is given by[9]

$$\bar{n} = \frac{md_2n_2 + (m - 1)d_1n_1}{d_t}. \tag{1}$$

We call this method the mean-value (MV) approximation. Another is given by[10,11]

$$\bar{n}' = \left(\frac{n_1^2d_1 + n_2^2d_2}{d_1 + d_2}\right)^{1/2} \tag{2}$$

for the TE modes and

$$\bar{n}' = \left(\frac{d_1 + d_2}{d_1/n_1^2 + d_2/n_2^2}\right)^{1/2} \tag{3}$$

for the TM modes. We call this method the root-mean-square-value (RMSV) approximation. By using one layer to replace the whole MQW structure, the problem becomes simple: the whole structure can be treated as a three-layer waveguide. These two approximations, however, cannot give the exact solution for the propagation constant of the waveguide and suffer from large errors in some cases.[3]

Our EL method not only allows the whole waveguide to be treated as a three-layer waveguide but offers exact solutions for the propagation constant of the MQW waveguide as well.

In order to obtain the propagation constant β, the finite MQW structure in a multilayer waveguide is replaced by one layer with an equivalent refractive index n_q and equivalent thickness d_q. The equivalent refractive index has the form[12]

$$n_q^2 = N^2 - (N^2 - n_1^2)R_{m-1}/S_{m-1} \tag{4}$$

for the TE modes and[12]

$$n_q^2 = \frac{n_1^2\{[n_1^4 + 4N^2(N^2 - n_1^2)R_{m-1}/S_{m-1}]^{1/2} - n_1^2\}}{2(N^2 - n_1^2)R_{m-1}/S_{m-1}} \tag{5}$$

for the TM modes, where N is the effective index, which is related to β as $N = \beta/k$, and

$$R_{m-1} = \sinh(p_1d_1)U_{m-2} - \zeta_{12}\sin(h_2d_2)U_{m-1}, \tag{6a}$$

$$S_{m-1} = \sinh(p_1d_1)U_{m-2} + \zeta_{21}\sin(h_2d_2)U_{m-1}. \tag{6b}$$

U_i is the Chebyshev polynomial of the second kind,[13]

$$U_i = \sin(i + 1)K\Lambda/\sin K\Lambda, \tag{7}$$

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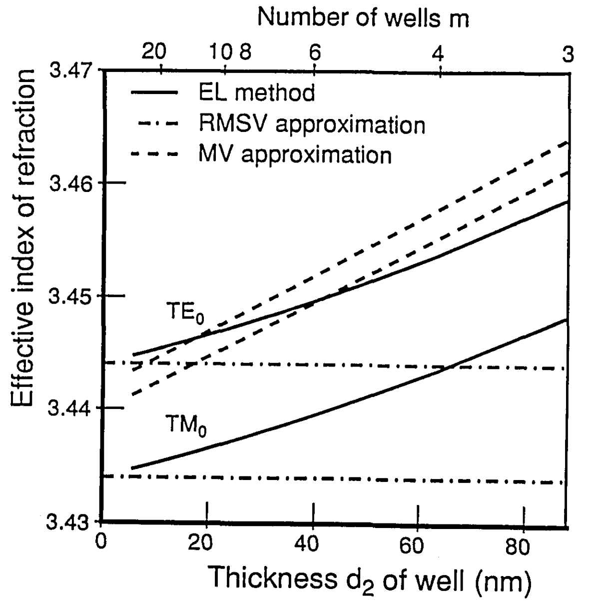

Fig. 1. Effective index N as a function of well thickness d_2 and the number of wells m. The other data are $d_1 = d_2$, $n_a = n_s = n_1 = 3.36$, $n_2 = 3.62$, $\lambda = 875$ nm, and $d_t = 440.7$ nm.

with

$$\cos K\Lambda = \cosh(p_1 d_1)\cos(h_2 d_2) - \frac{1}{2}(\zeta_{12} - \zeta_{21})\sinh(p_1 d_1)\sin(h_2 d_2). \quad (8)$$

In the above equations, i is an integer, $\Lambda = d_1 + d_2$, K is a transverse propagation constant related to the envelope wave in the MQW structure, and p_1 and h_2 are, respectively, the transverse decay constant and the transverse propagation constant in the individual barrier layer and well layer within the MQW structure, given by

$$p_r^2 = k^2(N^2 - n_r^2), \quad (9a)$$

$$h_r^2 = k^2(n_r^2 - N^2), \quad (9b)$$

where r is equal to 1 or 2, k is the wave number in a vacuum, and

$$\zeta_{rs} = \eta_{rs}\frac{p_s}{p_r}, \quad (10)$$

with

$$\eta_{rs} = \begin{cases} 1 & \text{TE modes} \\ (n_r/n_s)^2 & \text{TM modes} \end{cases}, \quad (11)$$

with r and s equal to 1 or 2. The equivalent thickness is given by

$$d_q = \frac{\cos^{-1} T_{m-1}}{h_q}, \quad (12)$$

where h_q is the equivalent transverse propagation constant given by Eq. (9b) with $r = q$ and

$$T_{m-1} = -\cosh(p_1 d_1)U_{m-2} + \cos(h_2 d_2)U_{m-1}. \quad (13)$$

Thus we can replace the MQW waveguide by a three-layer waveguide that includes a central layer with refractive index n_q and thickness d_q, a cladding with refractive index n_a, and a substrate with index n_s. The eigenvalue equation for the whole MQW waveguide is then simply given by[2]

$$\tan(h_q d_q) = \frac{h_q(\eta_{qa}p_a + \eta_{qs}p_s)}{h_q^2 - \eta_{qa}\eta_{qs}p_a p_s}, \quad (14)$$

with η_{qa} and η_{qs} given by Eq. (11). The transcendental equation (14) can be solved either graphically or numerically on a computer. Starting with a trial value for N, we calculate the values of n_q and d_q from Eqs. (4) or (5) and (12). If the eigenvalue equation (14) is satisfied, then the value of N is the solution for a mode. Otherwise, we should try other values of N until a solution is found. After we obtain N for one mode, it is a simple matter to find other guiding properties, such as power confinement and field distributions, for the mode.[9,10]

The results obtained by our EL method were compared with those obtained by Miyoshi *et al.*,[3] who used a more complicated matrix method that treats a $(2m + 1)$-layer problem with $2m$ boundaries and involves many steps of mathematical manipulations. The results obtained by these two different methods were in agreement. The EL method, however, deals only with a simple three-layer problem with two boundaries and substantially simplifies the calculation. By using the EL method, we can calculate the value of N with much less computing time than by using the matrix method. For example, in our case, the CPU time required by the EL method is only approximately 3% of that required by the matrix method.

Figure 1 shows the effective index N as functions of the well thickness d_2 and the number of the wells m at a free-space wavelength of 875 nm. Here we assume that $d_1 = d_2$, $n_a = n_s = n_1 = 3.36$, and $n_2 = 3.62$. The whole thickness d_t is kept as a constant, which is equal to 440.7 nm. The solid curves are obtained by our EL method (also by the matrix

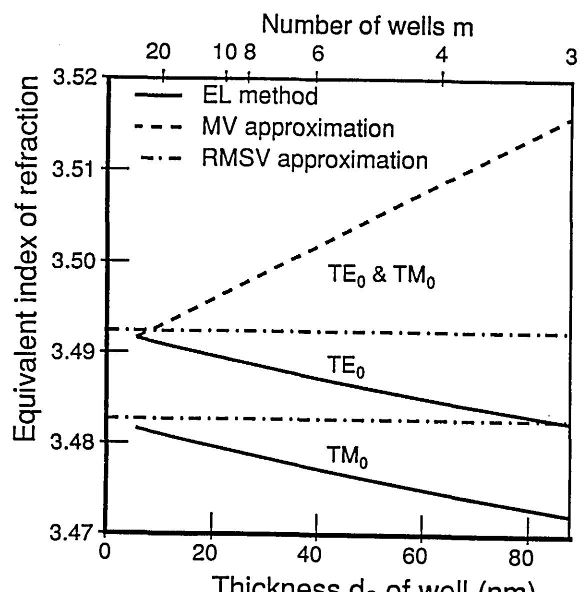

Fig. 2. Refractive index of the equivalent layer of the MQW structure as a function of well thickness d_2 and the number of wells m. The data are the same as in Fig. 1.

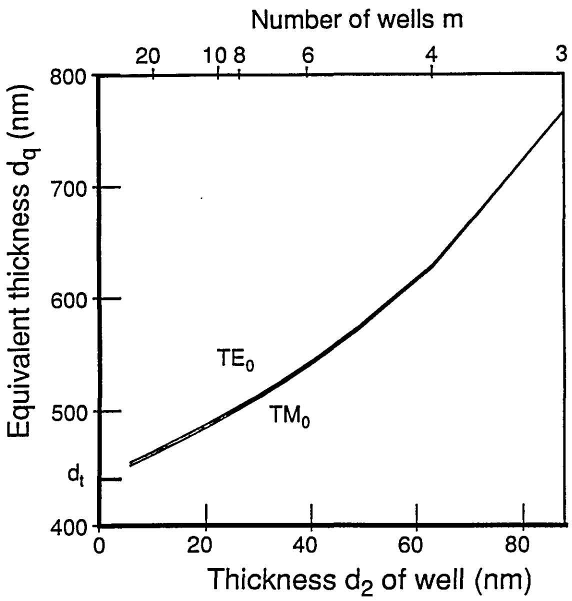

Fig. 3. Equivalent thickness d_q as a function of d_2 and m. The total thickness d_t of the MQW structure is included as a reference. The data are the same as in Fig. 1.

method), the dashed curves by the MV approximation, and the dashed–dotted curves by the RMSV approximation. The results show that the value of N obtained by the RMSV approximation is not dependent on the value of d_2 or m. In this case, the RMSV approximation seems valid for both the TE_0 and TM_0 modes only when m is larger than 20. The MV approximation is valid for the TE_0 mode only when m is large. When m is small, say, less than 10, both MV and RMSV approximations suffer from a large error.

Figure 2 summarizes the equivalent refractive indices of the central layer assumed for the three different three-layer models for the MQW waveguide as functions of d_2 and m for the case shown in Fig. 1. Three observations can be made: (i) $\bar{n}$ obtained by the MV approximation is the same for both the TE_0 and TM_0 modes. (ii) $\bar{n}'$ obtained by the RMSV approximation for both TE_0 and TM_0 modes is independent of the change of the thickness of the well d_2 or the number of the wells m, as Eqs. (2) and (3) show when $d_1 = d_2$. (iii) When m increases, or d_2 decreases, the values of n_q approach the values of $\bar{n}'$.

In Fig. 3, the equivalent thicknesses d_q of the MQW structure is shown as a function of the well thickness d_2 or the number of the wells m for the case shown in Fig. 1. The total thickness d_t of the structure is also put in the figure as reference. It turns out that the difference of d_q for the TE_0 and TM_0 modes is extremely small, and two lines almost overlap. When m increases or d_2 decreases, the equivalent thicknesses for both the TE_0 and TM_0 modes decrease as they approach d_t, the physical thickness of the MQW structure.

In conclusion, in this Letter we have shown that the MQW structure in a multilayer waveguide can be replaced by an equivalent layer to study the guiding properties of the guide. Both the equivalent index and the equivalent thickness of the layer are functions of the effective index N. By using the method, the MQW waveguide was treated as a three-layer waveguide, and the exact solution of N was obtained without using a complicated matrix method. The advantages of the EL method are obvious: (i) It is almost as simple as the MV and RMSV approximations, and (ii) it offers the exact solutions for the effective index N. Also we should point out that our EL method is not only useful for the waveguide with MQW structure surrounded by two semi-infinite materials (cladding and substrate) but can also be employed in MQW waveguides with more complicated structures.[12]

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