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Short Weak Links with Distinct Chemical Potentials at the Boundary

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The difference between the chemical potentials for pairs (μ_p) and quasiparticles (μ) at the boundaries of a short superconducting weak link, which is usually disregarded, is shown to be important when solving for the behavior of the link using the time-dependent Ginzburg-Landau equation. Selected computational results are presented to illustrate the effects of the improved boundary conditions on the space (time) dependence of μ, μ_p, and normal current J_n.

I. INTRODUCTION

The phenomenological time-dependent Ginzburg-Landau (TDGL) equation has been extremely useful for understanting the space (time) behavior of unidimensional superconducting weak links operating above their critical current.[1–5] In this regime, where there is a conversion of supercurrent into normal current and vice versa, the pair and quasiparticle electrochemical potentials must be different from each other.[1,6] This difference, however, has generally been disregarded when studying various characteristics of short weak links using the TDGL equation.[2–5,7,8]

In this paper we numerically solve the TDGL equation for a short weak link under current bias. In contrast to the usual procedure we take account of differences between chemical potentials when imposing the boundary conditions on the phase. We investigate the regular TDGL limit for a superconductor with ordinary impurities in the dirty limit. We also investigate the TDGL equation using parameters which lead to a larger range for the variation of the normal current and a smaller divergence for currents inside the link. In both cases, treating the chemical potentials as distinct at the boundaries leads to significant modifications in the space (time) behavior of various parameters within the link. The periods of oscillation for example can be substantially reduced below the values previously found,[3,4] thus bringing about changes in the current-voltage characteristics. We also present the first detailed calculation of the space (time) behavior of the pair chemical potential in the phase-slip regime using the TDGL equation, and find agreement with predictions based on more qualitative arguments.[6] Finally, we calculate the time-averaged values for μ and μ_p inside the link. Our results, in contrast to those obtained previously, show that the potential drop across the link can be smaller than the *total* potential drop for the phase-slip region, in accordance with recent experimental results.[9]

II. THEORY

We assume applicability of the TDGL equations with standard normalizations[2,7] for the variables, and write

$$u_0\left(\frac{d}{dt}+i\mu\right)\psi=\frac{d^2\psi}{dx^2}\ , \tag{1}$$

$$J=\mathrm{Im}\left(\psi^*\frac{d\psi}{dx}\right)-\frac{d\mu}{dx}\ . \tag{2}$$

In addition, following Rieger *et al.*,[1] we use the Josephson condition $d\phi/dt=-\mu_p$, where ϕ is the phase of ψ, and write

$$\mu_p=\mu-\frac{\nabla\cdot\vec{J}_s}{u_0|\psi|^2}\ . \tag{3}$$

In the equations, J is the total current (which is fixed because current bias is assumed), J_s is the supercurrent, u_0 is the ratio of the relaxation times of order parameter and current, μ is the quasiparticle

chemical potential, and μ_p is the pair chemical potential. The other quantities have their usual meanings. We use Likarev's boundary conditions[2] modified to take Eq. (3) into account,

$$\psi(0,t)=e^{i\phi(0,t)}\ ,\quad \frac{d\phi}{dt}(0,t)=-\mu_p(0,t)\ ,$$
$$\psi(L,t)=e^{i\phi(L,t)}\ ,\quad \frac{d\phi}{dt}(L,t)=-\mu_p(L,t)\ , \tag{4}$$

where we assume the order parameter to be normalized to its value at the banks. Considering the symmetry of the problem, we choose to measure the potentials and phase relative to the center of the link.

Equations (1)–(4) together with $J=J_n+J_s$ form a complete problem specification. As Likharev originally pointed out,[2] singularities in the time dependence are avoided if the differential equations are reconstituted in terms of the real and imaginary components of ψ. Our computer program follows this procedure. In our calculations we consider not only $u_0=5.79$ (the usual theoretical dirty limit value), but also $u_0=0.3$. This latter value was chosen to simulate spatical variations of the normal current over distances greater than the coherence length, on the order of the quasiparticle diffusion length.[6] In all the calculations discussed here, the length of the link in reduced units is $L=1$ (one coherence length), and the total current is $J=1.5$. We were able to check our computer program by making $\mu=\mu_p$ at the boundary [ignoring Eq. (3)], and comparing our results for the time dependence of several quantities in the center of the link with the values presented in Ref. 3.

III. RESULTS AND DISCUSSION

In Fig. 1 we show the spatial behavior of the normal current J_n for $u_0=5.79$ (the usual dirty limit value) at two points of the cycle: when J_n is maximum, and when $|\psi|$ is maximum at the center of the link. The results obtained with $\mu=\mu_p$ at the boundaries are indicated by dotted lines. For this one-dimensional link we anticipate a spatial "healing" of currents to the equilibrium values over a distance of the order of the coherence length. However the dotted lines show a flattening of the normal current as it reaches the boundary. This is due to the inaccurate boundary conditions which neglect $\nabla\cdot\vec{J}_s$. The solid lines result from Eq. (3) and exhibit the expected behavior. When we take smaller values of u_0 ($u_0=0.3$) a similar but less pronounced difference between the two situations is observed, since in this case the normal current decays over a larger length and $\nabla\cdot\vec{J}_n$ at the boundary of the link is less pronounced. It is interesting to note that because u_0 appears in the denominator of Eq. (3), the chemical potential differences for pair and quasiparticles can still differ significantly even when $\nabla\cdot\vec{J}_n$ (and therefore, with current bias, $\nabla\cdot\vec{J}_s$) is small.

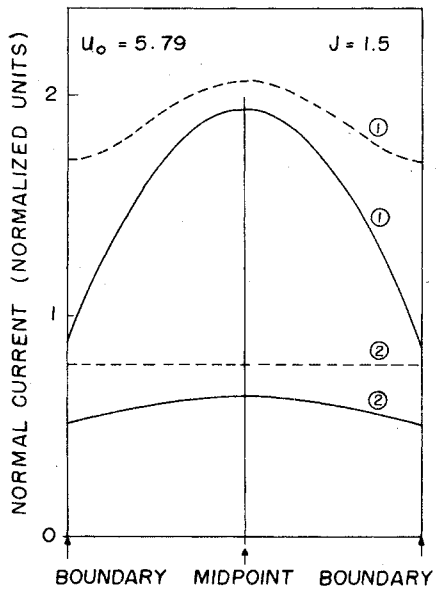

FIG. 1. Space dependence of the normal currents of two different times of the cycle: (i) J_n maximum at the midpoint and (ii) $|\psi|$ maximum at the midpoint. Dotted curves correspond to $\mu=\mu_p$ at boundaries, whereas solid curves were calculated using Eq. (3).

Figure 2 shows the time-averaged behavior for pair (dotted lines) and quasiparticle (full lines) chemical potentials inside the link using $u_0=5.79$. The time-averaged pair potential $\bar{\mu}_p$ is easily obtained from the period (T) of the temporal oscillations, since from Josephson's relation we have $d\phi/dt=-\mu_p$ at any point within the link. We integrate over a period to find the time-average value of μ_p. From the center of the junction to the left we have $\bar{\mu}_p=\pi/T$, and from the center to the right we have $\bar{\mu}_p=-\pi/T$, giving an average drop $\Delta\bar{\mu}_p=2\pi/T$ across the link. We note that since $\mu=\mu_p$ well beyond the boundaries, the total averaged voltage drop $\Delta\bar{\mu}$ across the entire phase-slip region is equal to $\Delta\bar{\mu}_p$, which is in turn determined by solving the TDGL equation inside the link and evaluating T.

Figure 2(a) shows $\bar{\mu}$ and $\bar{\mu}_p$ as calculated assuming $\mu=\mu_p$ at the boundary while Fig. 2(b) shows our results for $\mu\neq\mu_p$ at the boundary. It is clear from Fig. 2 that the first approach underestimates the value for the total potential drop across the phase-slip region (given by the discontinuity in $\bar{\mu}_p$ at the center of the link). It is easy to understand why this happens. Since $\bar{\mu}_p$ is constant outside the phase-slip region and $\mu=\mu_p$ far from the link, the previous approach in which μ equals μ_p at the boundary allows for no variations of $\bar{\mu}$ outside the link. Actually some voltage gradient must be present if the normal currents, which are large at the boundary, are to de-

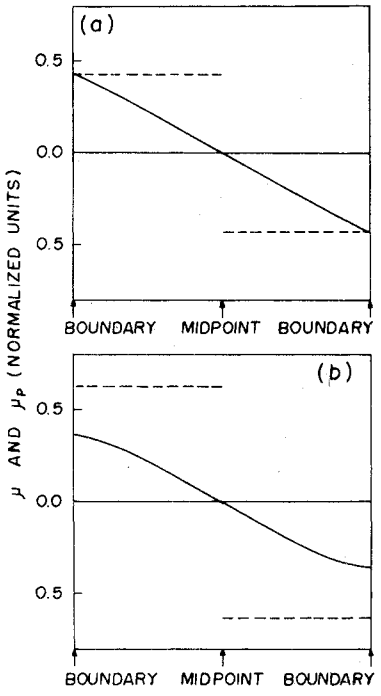

FIG. 2. Spatial behavior for the time-averaged chemical potentials $\bar{\mu}_p$ (dotted lines) and $\bar{\mu}$ (full lines) for $u_0 = 5.79$ and $J = 1.5$. We show results calculated with (a) $\mu = \mu_p$ at the boundary and (b) $\mu \neq \mu_p$ at the boundary.

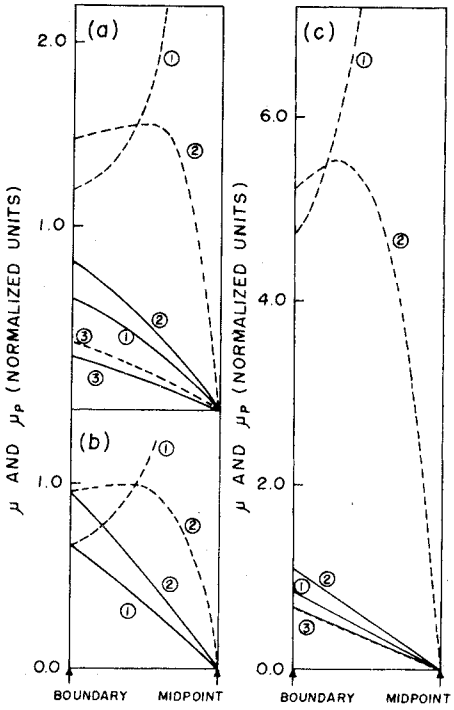

FIG. 3. Spatial behavior of the chemical potentials μ_p (dotted lines) and μ (full lines) for $J = 1.5$ at three different stages of the cycle: (i) near the time at which $|\psi|$ vanishes at the midpoint, (ii) near J_n maximum and (iii) near $|\psi|$ maximum. We show results for $u_0 = 5.79$ with (a) $\mu \neq \mu_p$ at the boundary and (b) $\mu = \mu_p$ at the boundary. In (c), $u_0 = 0.3$ with $\mu \neq \mu_p$ at the boundary.

cay to zero far from the link.

Finally in Fig. 3 we show the space dependence of μ (full lines) and μ_p (dotted lines) for three different stages of a cycle. Figure 3(a) illustrates the behavior for $u_0 = 5.79$, while Fig. 3(c) illustrates the situation when $u_0 = 0.3$. For comparison we also show results [Fig. 3(b)] using the $\mu = \mu_p$ at the boundaries and $u_0 = 5.79$.

The main features of our results agree with Ref. 6. The potential μ decays gradually across the junction as does μ_p when the order parameter is not too small. However when $|\psi| \to 0$ at the center of the link, the pair chemical potential μ_p diverges corresponding to the snap back time of Ref. 6. In our case, though, this divergence sets in gradually as we approach the phase-slip time, since the correct solution of the TDGL equation has been shown to have no discontinuities for the time behavior of the order parameter in the phase-slip regime.[2] Our predictions for the average values of μ and μ_p, presented in Fig. 2(b), also have the same features as those of Ref. 6.

In summary, we have shown that neglecting the differences between pair and quasiparticle chemical potentials at boundaries may lead to erroneous results when treating a short weak link under current bias. This can be true even when $\nabla \cdot \vec{J}_n$ is small inside the link. In particular, the period of oscillation is significantly modified thereby affecting the I-V curves. We further note that the average voltage drop across the phase-slip region can be easily calculated from the oscillation period T. This period is obtained by solving the TDGL equation inside the link with appropriate boundary conditions and does not require detailed information on the behavior of the currents outside the link.

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