2004

Status Quo Analysis of the Flathead River Conflict

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Received 15 August 2003; revised 6 November 2003; accepted 5 December 2003; published 2 April 2004.

[1] Status quo analysis algorithms developed within the paradigm of the graph model for conflict resolution are applied to an international river basin conflict involving the United States and Canada to assess the likeliness of various compromise resolutions. The conflict arose because the state of Montana feared that further expansion of the Sage Creek Coal Company facilities in Canada would pollute the Flathead River, which flows from British Columbia to Montana. Significant insights not generally available from a static stability analysis are obtained about potential resolutions of the conflict under study and about how decision makers' interactions may direct the conflict to distinct resolutions. Analyses also show how political considerations may affect a particular decision maker's choice, thereby influencing the evolution of the conflict. *INDEX TERMS:* 1899 Hydrology: General or miscellaneous; 1871 Hydrology: Surface water quality; 6309 Policy Sciences: Decision making under uncertainty; *KEYWORDS:* conflict analysis, graph model for conflict resolution, status quo analysis, water resources conflict

Citation: Li, K. W., D. M. Kilgour, and K. W. Hipel (2004), Status quo analysis of the Flathead River conflict, *Water Resour. Res.*, *40*, W05S03, doi:10.1029/2003WR002596.

1. Introduction

[2] Continuing growth in demand for freshwater around the globe raises concerns about the sustainability of water resources utilization [*Gleick*, 2002]. Competing uses of relatively scarce freshwater resources create conflicts at all levels of governance [*Dimitrov*, 2002; *Froukh*, 2003; *Kuylenstierna et al.*, 1997; *Mostert*, 1998]. Transboundary water resources conflicts have received substantial attention across a wide array of research traditions, such as geography [*Giordano et al.*, 2002; *Wolf*, 1997, 2002], environmental studies [*Conca and Dabelko*, 2003; *Dimitrov*, 2002], law [*International Bureau of the Permanent Court of Arbitration*, 2003], policy and political science [*Just and Netanyahu*, 1998], water resources management [*Kilgour and Dinar*, 2001; *Nandalal and Simonovic*, 2002], and systems engineering [*Hipel et al.*, 1997].

[3] This paper takes an engineering perspective and treats a transboundary water dispute as a strategic conflict among different interest groups. In a strategic conflict, different interest groups can be modeled as decision makers (DMs), where each DM is able to make its choice unilaterally and the combinations of choices of all DMs together determine the possible outcomes of the conflict. In general, DMs' interests and objectives conflict, and they are reflected in DMs' preferences over possible outcomes. A variety of methodologies has been proposed to handle strategic conflicts, including metagame analysis [*Howard*, 1971], hypergame analysis [*Bennett*, 1980; *Wang et al.*, 1988], conflict analysis [*Fraser and Hipel*, 1984], the graph model for conflict resolution [*Kilgour et al.*, 1987; *Fang et al.*, 1993], drama theory [*Howard*, 1999], and the theory of moves [*Brams*, 1994] and its fuzzy counterpart, the theory of fuzzy moves [*Zhang and Kandel*, 1997; *Li et al.*, 2001]. As *Kilgour* [1995] points out, an essential commonality of these approaches is their ‘‘game-theoretic roots.’’ K. Hipel (Conflict resolution: Theme overview paper in conflict resolution in Encyclopedia of Life Support Systems (EOLSS), 2002, http://www.eolss.net) provides an overview of approaches to conflict resolution developed in a range of fields such as sociology, psychology, political science, business, management sciences, operational research, and systems engineering.

[4] Among these approaches to conflict analysis the graph model for conflict resolution is conceived to be a simple but efficient decision aid tool for analyzing conflict and providing structural insights [*Fang et al.*, 1993]. The graph model takes states (possible outcomes of the conflict), instead of individual DM's choices, as basic components and tracks DMs' moves and countermoves among feasible states. Preference information is taken into account when a stability analysis is conducted. Stability of a state is assessed at both individual and aggregate levels: a state is deemed stable for a particular DM if this DM has no incentive to deviate from the state; a state constitutes an equilibrium if all DMs find it to be stable. The notion of stability here is general; there is a range of ways to define it, including Nash stability [*Nash*, 1950a, 1950b, 1951], general metarationality (GMR)[*Howard*, 1971], symmetric metarationality

0043-1397/04/2003WR002596

(SMR) [*Howard*, 1971], sequential stability [*Fraser and Hipel*, 1979, 1984], limited move stability (L_h, $h > 1$) [*Kilgour*, 1985; *Zagare*, 1984], and nonmyopic stability [*Brams and Wittman*, 1981; *Kilgour*, 1984]. These different stability definitions are referred to as solution concepts in the graph model [*Fang et al.*, 1993]. Solution concepts are designed to describe human behavior patterns in conflict, and *Fang et al.* [1989] examine the interrelationships of the graph model solution concepts. Naturally, the more solution concepts under which a state is an equilibrium, the more likely this state is to point to a resolution of the conflict under study. The graph model has been implemented in the decision support system (DSS) GMCR II [*Peng*, 1999; *Hipel et al.*, 1997; *Fang et al.*, 2003a, 2003b], which can carry out stability analyses expeditiously when necessary inputs are provided and hence dramatically enhance the applicability of this decision tool [*Hipel et al.*, 2001; *Kilgour et al.*, 2001].

[5] When a strategic conflict is modeled, a point in time has to be specified. In the graph model the initial state of the conflict is referred to as the status quo. Although a static stability analysis can predict individual stability and overall equilibrium information, thereby suggesting where the conflict may settle (presumably, one of the predicted equilibria), the accessibility of these predicted resolutions remains unexplored. Status quo analysis [*Peng*, 1999; *Li et al.*, 2002; *Li*, 2003; *Li et al.*, 2003] addresses concerns about the reachability of any specified state from the status quo and examines evolutionary aspects of conflict. Status quo analysis can expeditiously determine whether a state is attainable from the status quo and yield significant insights on how a DM should act or interact with other DMs to direct a conflict to a desired achievable resolution. Since the graph model introduces the notion of irreversible moves (DMs can cause the conflict to move in one direction but not the other) to reflect no-return decisions, the reachability of a state from a status quo is not automatically guaranteed. If some of the moves are specified as irreversible at the modeling stage, it is possible that some of the predicted resolutions cannot be attained from the status quo state, and an analyst may safely eliminate such states from the potential resolution list and concentrate on a refined list. Therefore it is important to carry out status quo analysis and to investigate the dynamic evolution of a conflict.

[6] Section 2 describes our approach to status quo analysis. In section 3 the methodology is applied to the Flathead River conflict, a transboundary water resources management dispute between the United States and Canada. The paper concludes with some comments in section 4.

2. Status Quo Analysis

[7] A conflict model in the graph form consists of a collection of sets: DMs set N, feasible states set S, and directed graphs set $\{D_i = (S, A_i),\ i \in N\}$, where D_i is a directed graph to depict available moves in one step among states in S for DM i and A_i includes all directed arcs in D_i characterizing such moves, and relative preference rankings on S, which are characterized by a pair of binary relations, strict preference, and indifference, $\{\succ_i, \sim_i\}$. Algebraically, D_i can be described by the notion of a reachable list, which includes all states reachable in one step from a specific state by DM i, and the strict p nce relations can be represented by the concept of unilateral improvement (UI) list, which incorporates preference information into the immediate decision possibilities at any state by DM i. Using set notation, reachable and UI lists are defined as

$$R_i(s) = \{s' \in S : (s, s') \in A_i\} \quad (1)$$

$$R_i^+(s) = \{s' \in R_i(s) : s' \succ_i s\}. \quad (2)$$

[8] Relations (1) and (2) are used to determine the stability of states. Stability analysis is able to provide a list of potential resolutions for the conflict under consideration, indicating where the conflict may settle. However, stability analysis essentially takes a static approach: it treats each state separately and examines whether a DM has the incentive to deviate from a state or not, but it is not concerned with how the state is achieved or even whether the conflict may evolve to it. In addition, a status quo state is treated exactly the same as any other state at this stage. Therefore stability analysis cannot address concerns about the accessibility of a state from a given status quo or about evolutionary aspects of the conflict. Status quo analysis is designed to add a dynamic dimension to the graph model methodology and to investigate these problems systematically.

[9] To apply the status quo analysis approach given below, a graph model must be calibrated first. The DSS GMCR II provides a convenient way to build a graph model. The analyst identifies the stakeholders holding decision powers in a conflict, the options available to them, and the feasible combinations of options, which enables GMCR II to generate a feasible state list. Relative preference rankings and restrictions on transitions between feasible states for each DM are also a required input. In practice, the analyst must have the assistance of a domain expert or must conduct research into the details of the dispute. If possible, interviews with the stakeholders are preferred so that first-hand information is incorporated into the model. When an analyst works with a client to build a model of a dispute in which the client is involved, ~3 hours of discussions are required to calibrate the conflict model. Subsequently, stability results can be instantly obtained using GMCR II, and the client can reflect upon strategic insights that are gained in order to decide how he or she can interact with his or her competitors in the best possible way.

[10] Data collection for the graph model is relatively easy because the data requirements are minimal. For example, relative preferences can be conveniently elicited by furnishing some simple yes/no preference statements in terms of options. However, of course, the quality of the model rests on the quality of the data. If the analyst is unsure of some information included in the model, sensitivity analysis is recommended to assess the robustness of predictions. GMCR II can do this well. In this paper our main purpose is to introduce status quo analysis and to illustrate it using a transboundary water conflict. The graph model of the case study (section 3) was developed earlier, so the modeling process is described only briefly here. For more details, readers are referred to *Hipel et al.* [1997].

2.1. Status Quo Analysis Diagram

[11] The graph model for conflict resolution and its associated DSS GMCR II constitute an analysis paradigm for strategic conflicts that has up to now relied mainly on

stability analysis for its conclusions. In this section, algorithms are proposed to apply another analysis technique, status quo analysis, to a graph model. Status quo analysis is dynamic and forward looking, in contrast to stability analysis, which is static and contingent.

[12] *Li et al.* [2002] proposed a procedure for status quo analysis in which an evolution tree from the status quo state is generated. The strength of this approach is that evolution paths are directly given in the "tree," but a particular state may appear more than once along different paths. *Li et al.* [2003] provided another methodology to carry out status quo analysis, producing a status quo analysis diagram starting from the status quo and creating a table based on the diagram; they illustrated the approach by applying it to an aquifer contamination conflict that took place in Elmira, Ontario, Canada. Both procedures only permit UIs (all moves must bring immediate benefit to the movers, in terms of preferences) when the tree or diagram is generated. *Li* [2003] relaxes this constraint, allowing any moves as the diagram expands from the status quo state. Furthermore, variations of algorithms are also investigated when the graph model and/or preferences are transitive. This paper furnishes a more in-depth description of the algorithms reported by *Li et al.* [2003] and applies the algorithms for the first time to the Flathead River conflict.

[13] A status quo analysis diagram is, in fact, a directed graph rooted at the status quo state. This diagram takes states as its basic component and then tracks DMs' moves and countermoves via directed arcs joining pairs of states. In the diagram each vertex stands for a feasible state of the conflict model, and each arc specifies a legal one-step move between two states by a given DM. Since a general rule in the graph model is that consecutive moves are not allowed [*Fang et al.*, 1993], a move by a DM is regarded as illegal if it immediately follows another move by the same DM.

[14] The generation of the status quo analysis diagram starts from the status quo state. At each iterative step the algorithm determines which states can be reached at this stage by examining the UI lists of states that are attainable at the immediate previous step for all DMs. Information regarding the last mover(s) to a state is also tracked so that consecutive moves can be screened out and arcs joining ordered pairs of states can be specified. This iterative process stops when no more states or arcs are eligible to be added to the diagram. Algorithm 2.1, which generates the status quo analysis diagram, is formulated as follows, where SQ is status quo state, $S_i^{(h)}$ is the set of states that can be reached from SQ in exactly h UIs, with DM i as the last mover, $V^{(h)}$ is the set of states that can be reached from SQ in at most h UIs, and $A_i^{(h)}$ is the set of arcs for DM i in the status quo analysis diagram, where DM i participates in a legal sequence of UIs starting from the status quo state and the total number of moves is at most h.

[15] 1. Start from SQ. Let $h = 0$, $S_i^{(0)} = \{\mathrm{SQ}\}$, $V^{(0)} = \{\mathrm{SQ}\}$, $A_i^{(0)} = \emptyset$, and $i \in N$.

[16] 2. Let $h \leftarrow h + 1$. Update $S_i^{(h)}$, $V^{(h)}$, and $A_i^{(h)}$ as follows:

$$S_i^{(h)} = \left\{ R_i^+(s) : s \in \cup_{j \in N-i} S_j^{(h-1)} \right\} \tag{3}$$

$$V^{(h)} = \qquad {}_i^{(h)} \cup V^{(h-1)} \tag{4}$$

$$A_i^{(h)} = \begin{cases} A_i^{(h-1)}, \text{if } S_i^{(h)} = \emptyset; \\ A_i^{(h-1)} \cup \left\{ (s, s') : s \in \cup_{j \in N-i} S_j^{(h-1)} \text{ and } s' \in R_i^+(s) \right\}, \text{otherwise.} \end{cases} \tag{5}$$

[17] 3. If $V^{(h)} = V^{(h-1)}$ and $\cup_{i \in N} A_i^{(h)} = \cup_{i \in N} A_i^{(h-1)}$, stop. Otherwise, return to step 2.

[18] When the termination conditions in step 3 are satisfied, let $V = V^{(h-1)}$ and $A = \cup_{i \in N} A_i^{(h-1)}$; then the status quo analysis diagram is (V, A). Since the diagram includes more than one DM's moves, it is recommended that each arc be labeled with a DM's name as appropriate. The labels make it easy to identify evolution path(s).

[19] From the status quo analysis diagram the analyst can assess the reachability of any state by examining whether it appears in the diagram: any state in the diagram is attainable from SQ, and conversely, a state not in the diagram is not achievable from SQ. In practice, equilibria are of special interest to the analyst since they usually correspond to potential resolutions of the conflict, and this feature of the status quo analysis diagram enables the analyst to evaluate the accessibility of equilibria, which is generally unavailable from the stability analysis. Another significant feature of the diagram is that often states have incoming arcs but no outgoing arcs. Such states are called attractors and are likely to correspond to strong equilibria in the conflict model, which are equilibria satisfying several solution concepts as mentioned in section 1. Moreover, if the analyst is interested in a particular equilibrium, it is possible to identify all evolution paths to it by tracing arcs in the diagram, thereby investigating and evaluating DMs' actions and interactions along the paths.

2.2. Status Quo Analysis Table

[20] When a model is relatively large with many DMs and feasible states (for instance, the case study presented in this paper), the status quo diagram can be quite complicated, and some key information is not so easy to identify, such as which states are accessible (i.e., in the diagram) and what the minimum number of moves to reach a particular equilibrium is. In addition, sometimes the analyst is interested only in the accessibility of specific states and the shortest paths to them. To present some key information more clearly, an algorithm to create a table based on the status quo analysis diagram has been formulated.

[21] The status quo analysis table is developed starting from the status quo state, SQ, and is expanded iteratively. At step h ($h = 0, 1, 2, \ldots$), the head row gives $V^{(h)}$, the states that are achievable within h moves from SQ. A cell at the intersection of row h and a column headed by a particular state stores the reachability information for this state: if it is attainable within h moves and the last mover is unique, the cell contains the last mover's name or label; if the last mover can be two or more DMs, a $\surd$ is placed in the cell; otherwise, the state cannot be reached in h moves, and the cell is left blank. Let $V^{(h)}(s)$ be the value of the cell in the intersection of row h and column s, $s \in V^{(h)}$. Then

$$V^{(h)}(s) = \begin{cases} \surd, & \text{if } s \in V^{(h)} \text{ and two or more DMs may be last mover;} \\ i, & \text{if } s \in V^{(h)} \text{ and DM } i \text{ is the unique last mover;} \\ \emptyset, & \text{if } s \notin V^{(h)}. \end{cases} \tag{6}$$

[22] For convenience, let $\Delta V^{(h)} = V^{(h)} - V^{(h-1)}$, $\Delta A_i^{(h)} = A_i^{(h)} - A_i^{(h-1)}$, $i \in N$, and $V^{(0)}(\mathrm{SQ}) = \surd$. The value for

any new cell added to the table is set to Ø, and algorithm 2.2 provides a mechanism to update the newly added row if applicable. A cell with Ø is left blank in the table. Algorithm 2.2, which creates the status quo analysis table, is as follows.

[23] 1. Initiate the table

$V^{(h)}$	SQ
$V^{(0)}$	$\surd$

[24] 2. Let $h \leftarrow h + 1$. If $\Delta V^{(h)} = \emptyset$, go to step 3; otherwise, insert one column for each $s \in \Delta V^{(h)}$, then go to step 4.

[25] 3. If $\cup_{i \in N} \Delta A_i^{(h)} = \emptyset$, stop; otherwise, go to step 4.

[26] 4. Insert a new row $V^{(h)}$. For each $s \in V^{(h)}$, update cell $V^{(h)}(s)$ as follows:

[27] 4.1. If $V^{(h-1)}(s) = \surd$, then $V^{(h)}(s) = \surd$.

[28] 4.2. If $V^{(h-1)}(s) = i$, $i \in N$, and $\exists\, j \in N$, $j \neq i$, and $s' \in V^{(h-1)}$, such that $(s', s) \in \Delta A_j^{(h)}$, then $V^{(h)}(s) = \surd$; otherwise, $V^{(h)}(s) = i$.

[29] 4.3. If $V^{(h-1)}(s) = \emptyset$ and $\exists\, i_1, i_2 \in N$, $i_1 \neq i_2$, and s', $s'' \in V^{(h-1)}$, such that $(s', s) \in \Delta A_{i_1}^{(h)}$ and $(s'', s) \in \Delta A_{i_2}^{(h)}$, then $V^{(h)}(s) = \surd$; otherwise, $V^{(h)}(s) = i$, where $i \in N$ and $s' \in V^{(h-1)}$ are such that $(s', s) \in \Delta A_i^{(h)}$, which must exist uniquely according to the definitions of $\Delta V^{(h)}$ and $\Delta A_i^{(h)}$.

[30] 5. Go to step 2.

[31] A status quo analysis table furnishes the analyst with a convenient tool to examine the reachability of any state from the status quo. If a state is present in the head row, it is attainable; otherwise, it cannot be attained from the status quo state. The table also reveals the number of moves in the shortest path(s) from the status quo to any specified state: if $V^{(h)}(s)$ is the first nonempty cell in column s, then the shortest path(s) from the status quo to state s contain(s) exactly h moves. This information may be of help when the analyst traces the moves and countermoves of DMs in the status quo analysis diagram to identify the shortest path(s).

[32] Note that the algorithms described here allow UIs only, and hence the interpretations apply to UIs only. In particular, if our analysis reveals that an equilibrium is not reachable from a status quo, then this equilibrium cannot be attained through any sequence of UIs. If the restriction to UIs is dropped and any unilateral move (UM) is allowed, conclusions about reachability may be different. For algorithms permitting any UM, see *Li* [2003].

3. Flathead River Water Resources Management Conflict

[33] The Flathead River flows from southeastern British Columbia, Canada, into Flathead Lake in Montana, United States, and eventually into the Columbia River. In 1970 the Sage Creek Coal Company Limited was established to develop coal resources along the Flathead River. In 1984, British Columbia issued an approval-in-principle license for Sage Creek's proposal for further development. The conflict arose because environmental activists and governments on the U.S. side feared that the proposed expansion of Sage Creek's operations would cause significant environmental degradation and eventually loss of use of the Flathead River, Flathead Lake, and Glacier National Park. The International Joint Commission (IJC), plementation body of the Boundary Waters Treaty of 1909 between the United States and Canada, commissioned a board of experts to examine the Flathead River water resources dispute beginning in 1986 [*International Joint Commission*, 1988]. Meanwhile, Sage Creek was forming plans for an expanded development, and British Columbia was considering whether it should accept the expansion, stand firm, or withdraw its original approval.

Table 1. DMs and Options of the Flathead River Conflict[a]

DMs and Options	Status Quo
Sage Creek Coal Company Limited (SC)	
1. Continue: Continue original development	Y
2. Modify: Modify to reduce environmental impacts	N
British Columbia Provincial Government (BC)	
3. Original: Support original project	Y
4. Modification: Require modification	N
Montana (MT)	
5. Oppose: Oppose any development	Y
International Joint Commission (IJC)	
6. Original: Recommend original project	N
7. Modification: Recommend modification	N
8. No: Recommend no project	N

[a]Source is the work of *Hipel et al.* [1997]. "Y" indicates that this option is selected by the DM who controls it, and "N" means that it is not.

[34] A graph model of the Flathead River water resources dispute, at a point in time just prior to the IJC's announcement of its recommendations in December 1988, was constructed by *Hipel et al.* [1997]. There the DSS GMCR II was employed to carry out a static stability analysis and to extract useful equilibrium information conveniently and expeditiously. Here a new feature of the graph model methodology, status quo analysis, is applied to the Flathead River conflict to assess the reachability of the equilibria and to examine the dynamics of the conflict.

[35] This conflict involves four DMs: British Columbia government (BC), Sage Creek Coal Company Ltd. (SC), Montana (MT), and IJC. Table 1 displays the four DMs, their available options, and the status quo state. A "Y" opposite an option indicates that this option is selected by the DM who controls it, and an "N" means it is not.

[36] After removing infeasible states, *Hipel et al.* [1997] identify the 55 feasible states listed in Table 2. As usual in the graph model, one can easily identify the pattern of the feasible states from Table 2. Except for the status quo, state 1, the feasible states can be placed into three groups; each group corresponds to one of the IJC's recommendations and contains 18 states. Although the 55 feasible states listed in Table 2 are exactly the same as those by *Hipel et al.* [1997], the numbering does not completely agree due to the upgrade of the software package GMCR II.

[37] Once the feasible states and allowable transitions are identified, the reachable list $R_i(s)$ for each feasible state for every DM is immediately available from GMCR II. Preference rankings over feasible states are required to calibrate a graph model. The preference information assumed by *Hipel et al.* [1997] is shown in Table 3, where states are ranked from most to least preferred (top to bottom of column, respectively), with states 2–55 being equally preferred by IJC. (This would be highlighted in a single color within the GMCR II environment.) Hence as shown in column 4 of Table 3, all states are equally preferred for the IJC except state 1; the IJC prefers to make a decision rather than remain at the status quo. From SC's viewpoint the most preferred

Table 2. Feasible State List for the Flathead River Conflict

	State Numbers																											
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Sage Creek																												
1. Continue	Y	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N
2. Modify	N	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y
C																												
3. Original	Y	N	N	N	Y	Y	Y	N	N	N	N	N	N	Y	Y	Y	N	N	N	N	N	N	Y	Y	Y	N	N	N
4. Modification	N	N	N	N	N	N	N	Y	Y	Y	N	N	N	N	N	N	Y	Y	Y	N	N	N	N	N	N	Y	Y	Y
Montana																												
5. Oppose	Y	N	N	N	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	N	N	N	N	N	N	N	N
IJC																												
6. Original	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	N	N	N	N	N	N	N	N
7. Modification	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y	Y	Y	Y
8. No	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N

	State Numbers																										
	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55
Sage Creek																											
1. Continue	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N
2. Modify	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y	N	N	Y
C																											
3. Original	N	N	N	Y	Y	Y	N	N	N	N	N	N	Y	Y	Y	N	N	N	N	N	N	Y	Y	Y	N	N	N
4. Modification	N	N	N	N	N	N	Y	Y	Y	N	N	N	N	N	N	Y	Y	Y	N	N	N	N	N	N	Y	Y	Y
Montana																											
5. Oppose	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	N	N	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y	Y	Y	Y
IJC																											
6. Original	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
7. Modification	Y	Y	Y	Y	Y	Y	Y	Y	Y	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N	N
8. No	N	N	N	N	N	N	N	N	N	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y	Y

outcomes involve proceeding with its original project (select option 1); next SC prefers a modified project (option 2). It is in SC's interest that BC approve its original proposal (option 3) or at least a modified version (option 4) and that IJC then recommend the original project (option 6) or a modification (option 7); least important to SC is whether MT opposes the project (it prefers that option 5 not be selected). By examining the "Y/N" representation of states in Table 2, one can identify the pattern of SC's preference ranking in Table 3. For a justification of preferences of BC and MT in the model, see *Hipel et al.* [1997]. GMCR II [*Fang et al.*, 2003a] provides two option-based approaches, option weighting and option prioritization, for eliciting relative preference ranking over states and contains a direct ranking tool for fine-tuning preferences. It is worth noting that the graph model can generally handle both transitive and intransitive preferences, although the DSS GMCR II requires preferences to be transitive. Transitivity of preferences is thus assumed for this illustration.

[38] By combining a DM's reachable list, $R_i(s)$, with the preferences given in Table 3, DM i's UI list, $R_i^+(s)$, can be readily obtained, as shown in Table 4. Note that a dash indicates that the corresponding list is empty, and subscripts 1, 2, 3, and 4 designate DMs Sage Creek, BC, Montana, and IJC, respectively.

[39] GMCR II expeditiously calculates the wide range of stability and equilibrium information shown in Figure 1. From this output we can see that three states, 15, 37, and 38, are equilibria under all solution concepts incorporated into GMCR II. These strong equilibria constitute the most likely resolutions for the conflict. Some other states, such as 5, 7, 14, 16, 26, 28, 35, and 47, are equilibria but only under the GMR and SMR solution definitions.

[40] As a follow-up to stability analysis, status quo analysis is designed to look further into this equilibrium information by determining which equilibria are reachable from the status quo state. Algorithms 2.1 and 2.2 are applied to this conflict model next to demonstrate how the algorithms developed in section 2 can be applied in practice to extract further insights into a conflict model.

[41] Calculations indicate that $V^{(8)} = V^{(7)}$ and $A_i^{(8)} = A_i^{(7)}$ for $i = 1, 2, 3, 4$. According to the termination conditions in step 3 of algorithm 2.1, the status quo analysis diagram is given by $\{V^{(7)}, \cup_{i\in N} A_i^{(7)}\}$, which is shown in Figure 2. The corresponding table is Table 5. Significant insights can be gleaned by examining Figure 2 and Table 5. Note that only UIs are allowed when the status quo analysis diagram is generated, so the following interpretations are based on this assumption.

[42] By examining the vertices in Figure 2 or the head row (column headings) of Table 5, one can easily observe that only 23 of the 55 feasible states, including the status quo state 1, may be reached from the status quo. Many of the weak (GMR and SMR only) equilibria shown in Figure 1, such as states 5, 7, 14, and 16, are unattainable from the status quo state. Therefore, although these states are conceptually stable for all DMs under these definitions, they could not arise in this model provided that all DMs take only UIs. One can also easily identify three attractors (with only incoming arcs but no outgoing arcs) given by states 15, 37, and 38 in Figure 2; these states are indeed equilibria under all solution concepts incorporated into

Table 3. Preference Rankings for DMs in the Flathead River Conflict

Sage Creek	BC	Montana	IJC[a]
6	6	38	2
15	28	47	3
24	38	44	4
33	15	53	5
1	37	41	6
42	47	50	7
51	26	20	8
7	40	29	9
16	35	26	10
25	49	35	11
34	5	23	12
43	39	32	13
52	14	11	14
10	48	2	15
19	7	8	16
28	27	17	17
37	16	5	18
46	36	14	19
55	22	49	20
5	44	40	21
14	31	55	22
23	53	46	23
32	20	52	24
41	46	43	25
50	29	31	26
8	55	22	27
17	45	37	28
26	54	28	29
35	21	34	30
44	30	25	31
53	1	13	32
2	3	4	33
11	41	19	34
20	12	10	35
29	50	16	36
38	43	7	37
47	52	48	38
9	2	39	39
4	42	54	40
3	11	45	41
18	51	51	42
13	4	42	43
12	13	30	44
27	9	21	45
22	25	36	46
21	18	27	47
36	34	33	48
31	23	1	49
30	32	24	50
45	8	12	51
40	17	3	52
39	10	18	53
54	24	9	54
49	19	15	55
48	33	6	1

[a]States are ranked from most preferred at the top of the column to least preferred at the bottom, with states 2–55 being equally preferred by IJC.

GMCR II, as disclosed in Figure 1, and hence are most likely to occur in practice. Our status quo analysis indicates that all of these potential resolutions are attainable from the status quo state. Furthermore, when any of these states is attained, no DM would have the incentive to deviate from it unilaterally under the implemented graph model solution concepts.

[43] Theoretically, all possible evolution paths from the status quo state to any achievable state can be identified by examining Figure 2. However, the analyst is usually more interested in the short list of potential resolutions, consisting of states 15, 37, and 38 in this case study. Furthermore, among the different paths to a possible resolution, the ones of greatest interest are those that contain the fewest moves and hence require less attention from DMs and consume fewer resources. The status quo analysis diagram and the associated table provide the analyst with a convenient vehicle to identify the shortest path(s) to states of interest.

[44] An analyst who is interested in a particular state s and would like to know the shortest path to it should consult Table 5. If s does not appear in the head row, then it cannot be reached from the status quo. Otherwise, look down the corresponding column and find the first nonempty cell $V^{(h)}(s)$; the shortest path to state s consists of h moves. In addition, the last mover i may be identified at the same time if the entry in the cell is i, $i \in N$. Otherwise, at least two distinct DMs can be the last mover. If the last mover is unique, this information can help us trace back to the status quo state and identify the shortest path in the status quo analysis diagram. For instance, look at state 38 in Table 5. The first nonempty cell is $V^{(4)}(38) = 3$, and hence the shortest path involves four moves, and the last mover is MT (note that 3 = Montana). By examining Figure 2, the shortest path, SQ $\overset{\text{IJC}}{\rightarrow}$ 51 $\overset{\text{BC}}{\rightarrow}$ 48 $\overset{\text{SC}}{\rightarrow}$ 47 $\overset{\text{MT}}{\rightarrow}$ 38, can then be identified. Similarly, one can easily find in Table 5 that the shortest paths to equilibria 15 and 37 contain one and three moves, respectively. Figure 2 reveals that the shortest paths are SQ $\overset{\text{IJC}}{\rightarrow}$ 15 and SQ $\overset{\text{IJC}}{\rightarrow}$ 33 $\overset{\text{BC}}{\rightarrow}$ 36 $\overset{\text{SC}}{\rightarrow}$ 37, respectively.

[45] If $V^{(h)}(s) = \surd$, then at least two DMs can take the conflict to state s in h moves or fewer, and therefore there always exists at least one exit if there are outgoing arcs from state s. For example, state $V^{(3)}(47) = 1$, so the shortest path to 47 takes three moves, where SC (DM 1) is the last mover, while $V^{(4)}(47) = \surd$ indicates that at least two DMs may take the conflict to state 47 in a path of four or fewer moves. Hence if 47 is not an attractor, as in this case, there always exists some way out of this state. For example, the status quo analysis diagram makes clear that MT can always direct the conflict from 47 to 38, whether it is SC or BC that causes the conflict to move to 47.

[46] Some more specific features of this conflict model can also be deduced from the diagram. For instance, there are three relatively independent branches in Figure 2; each branch corresponds to one of the three possible recommendations from the IJC and possesses a unique attractor. The diagram clearly indicates that IJC is the only DM with the capability to shift the conflict away from the status quo; its move essentially dictates the resolution of the conflict. If the IJC supports the original development plan, the conflict will settle at state 15; if the IJC recommends a modification, the final resolution is likely to be at state 37; if the IJC's choice is no project at all, state 38 will be the final outcome of this conflict.

[47] Before the IJC discloses its recommendation, the other three DMs have no incentive to act. Furthermore, except the branch SQ $\overset{\text{IJC}}{\rightarrow}$ 15, which immediately brings the conflict to a possible resolution, the next DMs who will move following IJC's recommendation are BC and then SC. After IJC makes its recommendation, BC will immediately respond by modifying its license if necessary, and then SC will have to update its development plan, thereby conforming to the available license from BC. This finding is quite consistent with the real-world situation: although IJC's

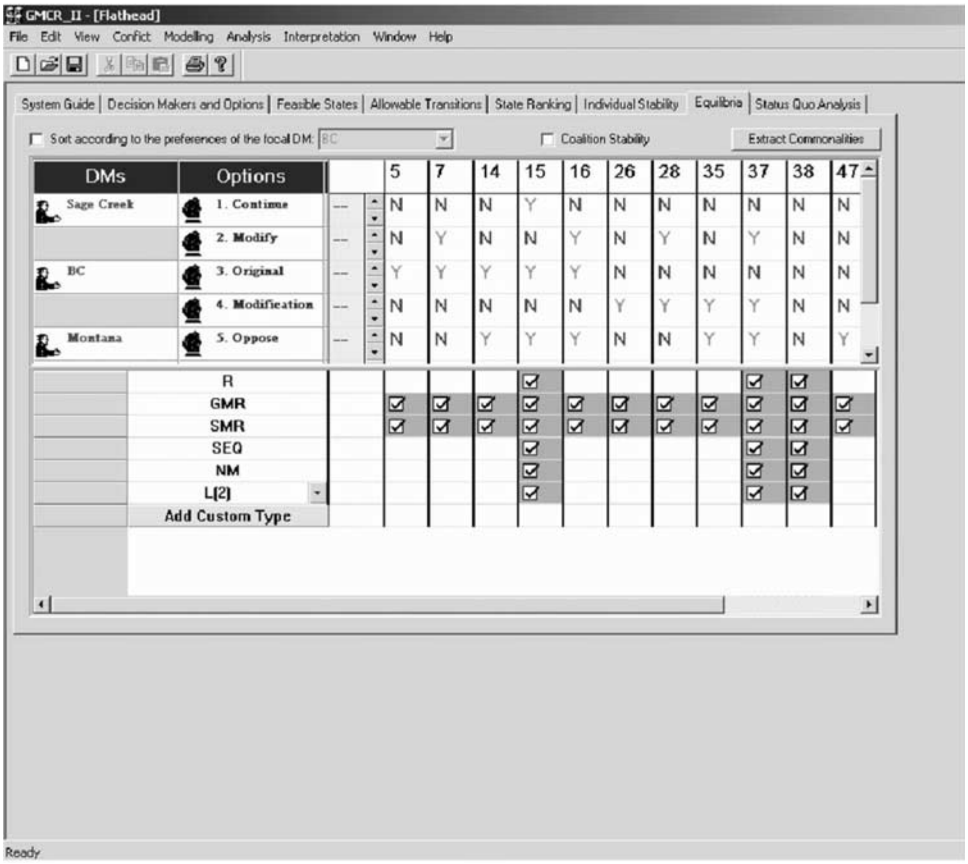
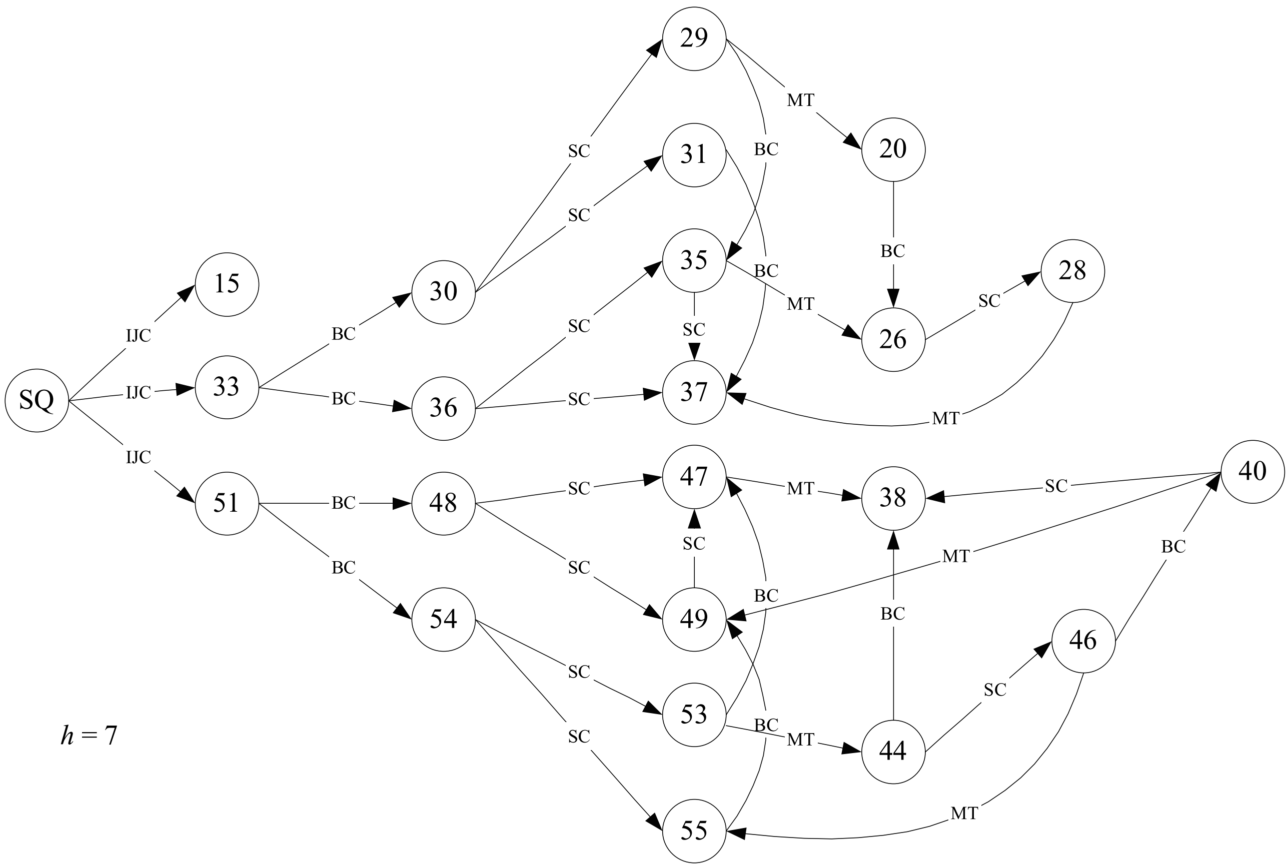


Figure 1. Equilibria for the Flathead River conflict.



e 2. Status quo analysis diagram for the Flathead River conflict.

Table 4. Reachable and UI Lists for the Flathead River Conflict

State	Sage Creek		BC		Montana		IJC	
s	$R_1(s)$	$R_1^+(s)$	$R_2(s)$	$R_2^+(s)$	$R_3(s)$	$R_3^+(s)$	$R_4(s)$	$R_4^+(s)$
1	–	–	–	–	–	–	15, 33, 51	15, 33, 51
2	3, 4	–	5, 8	5	11	11	–	–
3	2, 4	2, 4	6, 9	6	12	12	–	–
4	2, 3	2	7, 10	7	13	13	–	–
5	6, 7	6, 7	2, 8	–	14	–	–	–
6	5, 7	–	3, 9	–	15	15	–	–
7	5, 6	6	4, 10	–	16	16	–	–
8	9, 10	10	2, 5	2, 5	17	–	–	–
9	8, 10	10, 8	3, 6	3, 6	18	18	–	–
10	8, 9	–	4, 7	4, 7	19	19	–	–
11	12, 13	–	14, 17	14	2	–	–	–
12	11, 13	11, 13	15, 18	15	3	–	–	–
13	11, 12	11	16, 19	16	4	–	–	–
14	15, 16	15, 16	11, 17	–	5	5	–	–
15	14, 16	–	12, 18	–	6	–	–	–
16	14, 15	15	13, 19	–	7	–	–	–
17	18, 19	19	11, 14	11, 14	8	8	–	–
18	17, 19	17, 19	12, 15	12, 15	9	–	–	–
19	17, 18	–	13, 16	13, 16	10	–	–	–
20	21, 22	–	23, 26	26	29	–	–	–
21	20, 22	20, 22	24, 27	27	30	30	–	–
22	20, 21	20	25, 28	28	31	31	–	–
23	24, 25	24, 25	20, 26	20, 26	32	–	–	–
24	23, 25	–	21, 27	21, 27	33	33	–	–
25	23, 24	24	22, 28	22, 28	34	34	–	–
26	27, 28	28	20, 23	–	35	–	–	–
27	26, 28	26, 28	21, 24	–	36	36	–	–
28	26, 27	–	22, 25	–	37	37	–	–
29	30, 31	–	32, 35	35	20	20	–	–
30	29, 31	29, 31	33, 36	36	21	–	–	–
31	29, 30	29	34, 37	37	22	–	–	–
32	33, 34	33, 34	29, 35	29, 35	23	23	–	–
33	32, 34	–	30, 36	30, 36	24	–	–	–
34	32, 33	33	31, 37	31, 37	25	–	–	–
35	36, 37	37	29, 32	–	26	26	–	–
36	35, 37	35, 37	30, 33	–	27	–	–	–
37	35, 36	–	31, 34	–	28	–	–	–
38	39, 40	–	41, 44	–	47	–	–	–
39	38, 40	38, 40	42, 45	–	48	48	–	–
40	38, 39	38	43, 46	–	49	49	–	–
41	42, 43	42, 43	38, 44	38, 44	50	–	–	–
42	41, 43	–	39, 45	39, 45	51	51	–	–
43	41, 42	42	40, 46	40, 46	52	52	–	–
44	45, 46	46	38, 41	38	53	–	–	–
45	44, 46	44, 46	39, 42	39	54	54	–	–
46	44, 45	–	40, 43	40	55	55	–	–
47	48, 49	–	50, 53	–	38	38	–	–
48	47, 49	47, 49	51, 54	–	39	–	–	–
49	47, 48	47	52, 55	–	40	–	–	–
50	51, 52	51, 52	47, 53	47, 53	41	41	–	–
51	50, 52	–	48, 54	48, 54	42	–	–	–
52	50, 51	51	49, 55	49, 55	43	–	–	–
53	54, 55	55	47, 50	47	44	44	–	–
54	53, 55	53, 55	48, 51	48	45	–	–	–
55	53, 54	–	49, 52	49	46	–	–	–

recommendation does not bind the DMs, BC will face substantial political pressure if it does not follow the IJC's recommendation. As long as BC modifies its original license, SC has no choice but to change its plan accordingly. The response from MT depends on other DMs' choices. If any version of the project proceeds, it is in MT's interest to maintain its opposition. However, MT's "decision power" in this model is minimal because in some likely resolutions, such as 15 and 37, its opposition does not have much impact as long as BC can tolerate the pressure applied by the opposition.

Table 5. Status Quo Analysis Table for the Flathead River Conflict

$V^{(h)}$	SQ	15	33	51	30	36	48	54	29	31	35	37	47	49	53	55	20	26	38	44	28	46	40
$V^{(0)}$	√																						
$V^{(1)}$	√	4	4	4																			
$V^{(2)}$	√	4	4	4	2	2	2	2															
$V^{(3)}$	√	4	4	4	2	2	2	2	1	1	1	1	1	1	1	1							
$V^{(4)}$	√	4	4	4	2	2	2	2	1	1	√	√	√	√	1	1	3	3	3	3			
$V^{(5)}$	√	4	4	4	2	2	2	2	1	1	√	√	√	√	1	1	3	√	√	3	1	1	
$V^{(6)}$	√	4	4	4	2	2	2	2	1	1	√	√	√	√	1	√	3	√	√	3	1	1	2
$V^{(7)}$	√	4	4	4	2	2	2	2	1	1	√	√	√	√	1	√	3	√	√	3	1	1	2

[48] As described by *Hipel et al.* [1997], the historical outcome is state 38, in which the IJC recommends no project, and BC and SC are forced to terminate their license and project; Montana, completely satisfied by this development, withdraws its opposition. An ad hoc analysis revealed a path of transitions from the status quo state to the final outcome as shown in Figure 3. It is easy to verify that this transition process is consistent with one of the three shortest paths in the status quo analysis diagram, SQ $\overset{IJC}{\to}$ 51 $\overset{BC}{\to}$ 48 $\overset{SC}{\to}$ 47 $\overset{MT}{\to}$ 38. Note that in this path and one other, SQ $\overset{IJC}{\to}$ 33 $\overset{BC}{\to}$ 36 $\overset{SC}{\to}$ 37, BC's decision to follow the IJC's recommendation is largely due to the expected political pressure from the opposition group because the IJC's recommendation is not binding. If BC and Montana could somehow cooperate and reach an agreement, the resolution would be quite different from the historical one since the potential political pressure would be significantly reduced and BC might be able to withstand other pressures and support either a partial or even a full project by SC. Therefore SC may exert its influence to encourage BC to cooperate with Montana, thereby leading to a more favorable resolution (for SC).

[49] In a conflict environment, resolution requires the participation of all DMs. Here each DM should maintain some dialogue channels with other DMs to keep up to date on other DMs' concerns and interests. Doing so may eliminate or reduce costs. For instance, had SC more quickly understood IJC's objection to its proposal and BC's willingness to follow IJC's suit (as shown in the shortest paths to states 37 and 38), SC might have scaled back its development plans or even halted investment in this

Sage Creek								
1. *Continue*	Y		Y		Y	→	N	N
2. *Modify*	N		N		N		N	N
BC								
3. *Original*	Y		Y	→	N		N	N
4. *Modification*	N		N		N		N	N
Montana								
5. *Oppose*	Y		Y		Y		Y →	N
IJC								
6. *Original*	N		N		N		N	N
7. *Modification*	N		N		N		N	N
8. *No*	N	→	Y		Y		Y	Y
State Number	1		51		48		47	38

Figure 3. State transition from the status quo to the final outcome. "Y" means yes and "N" means no.

project, which would have reduced its loss due to later license cancellation. On the other hand, SC might have lobbied the British Columbia government, arguing that it is in BC's interest to withstand political pressure on this issue. Had SC done so, BC's preferences might have been different, and a different resolution might have emerged.

4. Conclusions

[50] A transboundary river basin conflict involving the United States and Canada is modeled as a graph model, and a stability analysis is then carried out by using the DSS GMCR II. To extract more structural insights, a status quo analysis is conducted using two recently developed algorithms. This application demonstrates how the reachability of a state can be easily assessed by using this new approach and how distinct resolutions may be achieved through different interactions among DMs. Examination of possible evolutionary paths leads to a better understanding of how BC's reaction to the threat of political pressure from the United States is a major determinant of the resolution.

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