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STOCHASTIC MODELS IN CIRCUIT NETWORK GROWTH

BY

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Submitted in partial fulfillment
of the requirements for
the Master of Arts Degree in Geography

DEPARTMENT OF GEOGRAPHY
WILFRID LAURIER UNIVERSITY
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1977

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John D. Radke

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CHAPTER ONE

INTRODUCTION

To describe the spatial pattern of objects or events, and to explain that pattern by way of the causal mechanisms which have generated it, has been one of the traditional aims of geographical research.(Harvey,1967) One method that can be employed for such descriptions and explanations is network analysis.

A network is a meshed fabric of intersecting lines. (Kansky,1963). A more appropriate definition for geographers would be, a set of geographic elements interconnected into a system by a number of relationships.(Kansky,1963). Network analysis is an examination of a complete network, its elements, and their relationships. Networks can be represented in two major ways. The first is graphically, as a map. However, although such a representation can summarize many network characteristics, it often proves too inflexible to permit further analysis. For this reason the second form of network representation is often resorted to. This involves representing the network as a matrix in which the rows and columns represent individual elements, and the entries in the body of the matrix represent the relationships between the elements.

Definitions of Basic Terms used in the Study:

Graph theory is a mathematical technique which concentrates on the topological properties of a network, emphasizing the connectivity of its elements rather than its physical properties. Thus, map representations of a network may take the form of graphs.

A graph is composed of vertices , sometimes known as nodes, which are specific points in space, and linkages which are linear routes (direct connection between two points) which join the nodes. An edge is another term frequently used for a link. (see figure 1.1)

Figure 1.1

Representation of a Network as a Graph

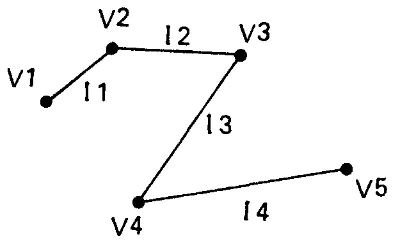
original map
of a network

graph representation
of a network

The term path represents a collection of edges linking a series of different vertices. (see figure 1.2)

Figure 1.2

Identification of a Path Between Two Points in a Graph



the path between V1 and V5 consists of l1+l2+l3+l4.

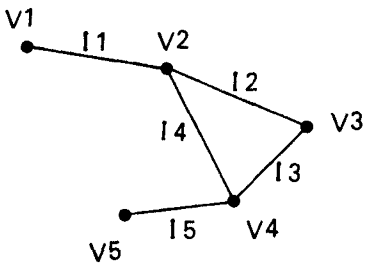
length of the path is 4

The length of a path is, in topological terms, the number of links within it.

The topological distance between two places is the length of the shortest path joining them. This would be measured in number of links. (see figure 1.3).

Figure 1.3

Measurement of Topologic Distance



topological distance between V1 and V5 is l1+l4+l5 = 3

As already stated, a network may also be represented as a matrix. In particular, a connectivity matrix can be constructed which illustrates the degree of linkage each vertex has with the rest of the network. (see table 1.1).

Table 1.1

Representation of a Network as a Connectivity Matrix

	V1	V2	V3	V4	V5
V1	1	1	0	0	0
V2	1	1	1	1	0
V3	0	1	1	1	0
V4	0	1	1	1	1
V5	0	0	0	1	1

matrix representation
of figure 1.3

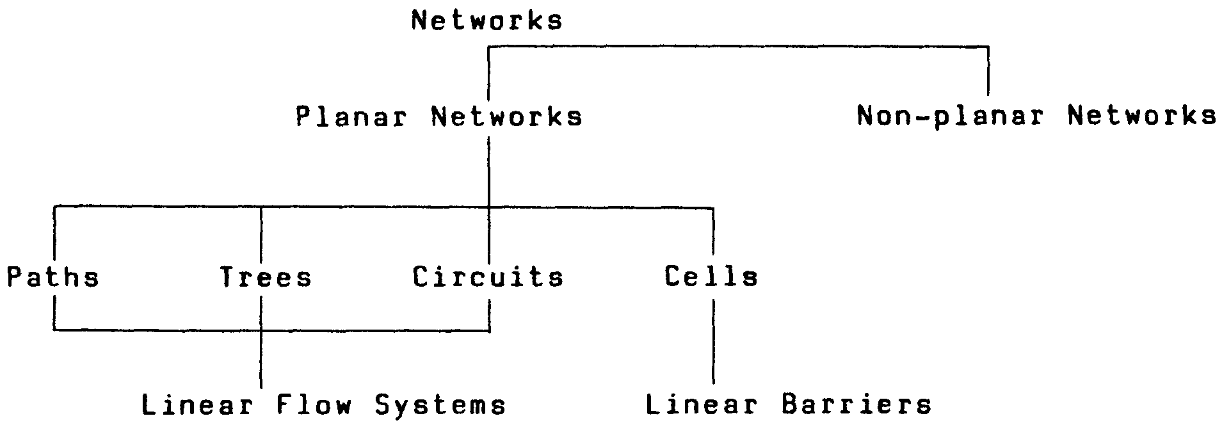
The shortest path matrix is the matrix representation which shows the length of the shortest paths between all vertices in a network. It can be obtained by powering the original binary matrix until all the zeros are eliminated.

Networks have been topologically classified into two major categories in the past; i) planar, located in two-dimensional space where links only intersect at vertices, and ii) non-planar networks located in three or more dimensional space where the intersection of links does not always produce vertices. Within each class, subgraphs can be recognized. (see figure 1.4 for an illustration of the categories). Although circuit networks (transportation networks are usually circuits) have been traditionally considered to be part of the planar network category, they

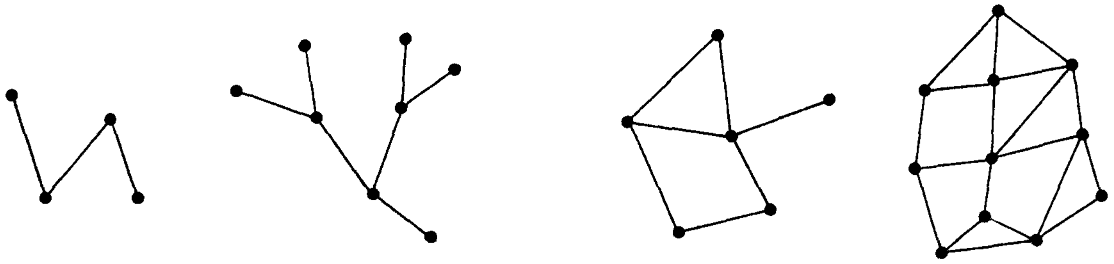
Figure 1.4

Network Classification

Topological Classification of Networks



Graphic Representation of Network Classes



Path Tree Circuit Cell

Source: Modified after Haggett and Chorley (1969) do exist in non-planar form. For example, airline networks and communication networks exist in three-dimensional space.

It was not until 1936 that the first comprehensive treatment of network topology was published by Konig. It dealt mainly with simple elementary structures which were later developed into a more extensive graph theory (Haggett,1965).

Network analysis has been applied in several disciplines besides geography. For example, it has been applied in sociology where vertices represent people and links represent interpersonal contacts, in communications where vertices represent transmissions and links represent signals, and in business administration where vertices represent departments and links represent transactions. In geography it has become a widely used tool in the description and explanation of spatial patterns. In particular, "the representation of the topological characteristics of any network in graph form has become a widely accepted procedure in the analysis of transportation networks" (Tapiero & Boots,1974).

Existing Related Studies:

Recent studies of networks in geography have been, for the most part, concerned with the application of graph theory to existing networks. In particular, two types of studies may be recognized. These are morphometric studies and network growth studies. The former are typically descriptive studies of form, that is to say the structure of a given network. Following Eichenbaum and Gale (1971), form is the visible aspect of a thing, usually taken in the narrow sense of shape or configuration as distinguished from such properties as colour. Form, in the abstract, thus implies something geometrical, detailing the temporally cross-sectional measurable properties of phenomena. For transportation networks analysis is often restricted to just the topologic form of a given network. Consequently, any subsequent evaluation or comparison of the network is by means of norms defined in terms of form (e.g., trees, chains, grids). In contrast, growth studies focus on the processes responsible for the development of the network under study. Again following Eichenbaum and Gale (1971), process can be defined as a continuous or regular action or succession of actions, taking place or carried on in a definite manner, and leading to the accomplishment of some result; a continuous operation or series of operations. Work of this nature concerning circuit networks in geography has concentrated on creating procedures which replicate specific empirical networks.

Studies of both form and process in transportation networks will be discussed in the literature review presented below. The studies were selected to include those which introduced pioneering ideas and established new contributions to circuit network study in geography. Rather than offer an exhaustive review of each study, only their innovative contributions will be mentioned.

Turning first to studies which concentrate on network form, William L Garrison in 1960, using graph theory as an analytical technique, measured the structure of a newly developed interstate highway network in the United States. His work dealt mainly with the analysis of the position of particular places on the route system indicating their relative accessibility. Garrison's work was important for two reasons. It introduced graph theory to geography and it also illustrated how this analytical technique allowed examination of the system both as a unit and in separate components.

Five years later, Garrison co-authored a paper with Duane F Marble entitled, "Graph Theoretic Concepts". This article can be considered a classic since it has not only become widely referenced, but also contains the basic definitions and explanations of graph theory. The article attempts to reveal the relationships of network structure to the physical and socioeconomic features within the network's delineated area. Dependent variables, which are indices measuring network form, were correlated with independent variables (features of the area) to determine if a definite relationship existed. The true value of the paper lies in its pioneering attempt to illustrate a network's form by using mathematical indices and correlating those indices (structural features) with other measurements of features of the network's environment.

K.J.Kansky, another frequently quoted author of graph topology, created his most famous work (his doctorial dissertation) in 1963. The "Structure of Transportation Networks" was a paper which stressed that the structure of the transportation network of any area cannot be divorced from the geographic characteristics of that area. As in the study by Garrison and Marble, Kansky demonstrated that aggregate measures (topological indices) could be used to investigate the relationship between the transportation network of an area and the geographic features of the area.

Kansky's research contained a larger sampling of nations and a greater number of structural measurements than had been seen previously in the literature. It was about time, according to Kansky, that a decrease in past ambiguities so common in written language occurred in geography. He thus devoted a complete chapter in his dissertation, one that is most valuable to geographers today, to the explanation of measures of network structure expressed in the symbolic language of graph theory.

In 1968 Howard Gauthier, like his predecessors in network geography, described the structural form of a network; in this case a Brazilian highway network. In his analysis he found a high degree of relationship between the development of highway accessibility and the growth of manufacturing in subsequent time periods. Gauthier used graph theory to abstract the real network into a form in which the connections between the centres (vertices) were weighted according to transport cost per unit distance. These cost values replaced the simple topological measures to provide, after powering and summing the connectivity matrix, accessibility values for individual vertices.

The article "Linkage Importance in Regional Highway Network" by C.C. Kissling (1969) attempts to define, like Gauthier's study, how accessible places are to each other. Kissling goes one step further and tries to define highway linkage importance in Nova Scotia, so that when it is seen in relation to actual link characteristics, the impact of subsequent improvements to the system may be predicted. After his representation of the network in graph form, he concludes that the "analysis of the network structure is thus likely to reveal probable growth points in the system" (Kissling,1969).

James et al (1970) have suggested that the commonly used measures of graph structure are not adequate. Their main concern is that some of the indices used by their predecessors to measure graph structure, have been poor discriminators among graphs with different structures. They assert that the indices, because of their origin, fail to discriminate among graphs with identical parameters and dissimilar patterns of linkages.

Turning now to research that deals with network growth, we find that attempts to replicate existing spatial networks, is not a recent procedure and has been occurring since the early 1960's.

K.J.Kansky, besides his graphic description of network form in his doctorial dissertation, created simulation models which generated networks. He presented a workable predictive model of network structure based on empirical evidence obtained via a study of various regions. The model contained a probabilistic concept incorporated as a chance mechanism which allowed a range of possible network forms to be generated from a data base of regional characteristics. Kansky summarized his reasons for model simulation when he concluded that the empirical model was derived "not to demonstrate its validity, but to illustrate its practical applicability".

L.A.Brown (1965) repeated earlier experiments instituted by the mathematician Gilbert(1961). Brown produced what has been called a random graph model. In this model Brown used a random number generator to locate 50 vertices in a 50 x 40 unit rectangular grid, which were then linked into subgraphs using a critical distance procedure. Although Brown did not directly relate his model to any empirical evidence, he did consider it to be a predictive model in epidemiology and compared it to the spread of an infectious disease over space.

Kolars and Malin(1970), on the other hand, developed a post-dictive model which simulated the Turkish Railway System. The network was based upon population and topographic features, of which the former had the greatest impact on route construction. The model identified ridge lines of population between major centres which would identify optimum rail linkages giving greatest benefits to rural farmers. A gravity model was used to compute potential interaction between centres, while taking into consideration physical features. Kolars and Malin expressed the significance of their paper in their statement: "In addition to supporting current theory concerning the growth of transportation networks, this study identifies exogenous political and military conditions as important additional factors".

Utilizing data on the development of the Maine railway network, Black (1971) created a simulation model which incorporated distance, potential traffic and angle of linkage in the prediction of edge construction. Black calculated discriminant scores consisting of location and population, as well as other prediction variables, of each node to create potential linkages between the largest nodal scores. The greatest value of the paper lay in the fact that the model it proposed did not depend on complete knowledge of an economic system to function, thus making the model operational at a local level.

Leinbach (1974), in an analysis of the already existing transport system in West Malaysia, also implemented a type of diffusion process. The network growth was modelled as a process of contagious diffusion, comparable to Brown's attempt to illustrate infectious diffusion, where predictor variables consist of road network densities. A regression approach was implemented to provide measures of network orientation over time. The results indicated the importance of the simulation model, incorporating a diffusion process, in transport forecasting.

MacKinnon and Barber (1972) developed a model using a technique somewhat analagous to regression analysis. Their heuristic alogorithm generated a series of line segments, such that the total distance from each of n points to its nearest line segment was minimized. They applied their method to the distribution of fifty-five cities and towns, in Ontario and Quebec, evaluating line segment representations of point patterns in the light of three criteria: "(1) the goodness of fit measured by the mean of orthogonal deviations from every point to its nearest line segment; (2) the total length of all of the line segments, and (3) the complexity of the network as indexed by the total number of line segments.

Objectives:

From this review of the literature it is apparent that while there have been studies of both form and process, few of them have incorporated both procedures. The only exception is the work of Tinkler (1974,1976).

In this study it is proposed that the morphology of any network cannot be divorced from the generative processes involved. Thus, this study will attempt to contribute to geographical knowledge by determining the relationship of certain selected morphological characteristics to changes in generative processes. It is hoped that this will give futher understanding of the development of existing empirical circuit networks, which, in turn, would enable better prediction of the impact of subsequent growth.

Consequently, the present study neither describes the form of existing networks in space, nor attempts to replicate empirical networks. Instead, the goal of this research is to incorporate both process and form into a single theoretical, rather than an empirical approach, to simulate aspects of the growth of circuit networks. More specifically, the approach involves the creation and examination of a model which produces non-planar circuit networks which are examined to determine the effect of a generative process on the morphology of the resultant networks.

This approach has already been successfully used in other areas of network geography. Werner (1972) used this approach in his evaluation of drainage patterns (tree networks) and Crain and Miles (1976) used a similar method while studying polygons determined by random lines in a plane (cell networks). Elsewhere in geography, this approach has become widely accepted in the study of point and area patterns in spatial analysis (Boots and Getis,1977).

To summarize, the main contribution of this research is to illustrate how an approach synthesizing both process and form can be implemented in circuit network analysis. In the course of doing this, a model is introduced which provides a good basic structure from which other models can be built. However, it will not be the intention to discuss this model exhaustively.

In addition, the structural characteristics of networks produced by the model will be examined by indices currently in use in transportation geography. This will provide an indication of the usefulness of these indices in explaining network structure, particularily the indices' sensitivity to changes in process.

Outline of The Study:

Chapter One has reviewed existing related studies and presented the rationale for the thesis. Chapter Two will be devoted to the description of the generation of the simulated networks and analytical processes undertaken in their examination. Chapter Three will analyze the results obtained from the execution of the model and Chapter Four will both summarize the study's findings and discuss possible future research in this area.

CHAPTER TWO

Scope:

As indicated in Chapter One, the study examines the use of only one model which will be described below. This is because the intent of the research is not primarily to explore network generative processes, but rather to illustrate an approach which links generative process to resultant network form. The networks produced by the model are examined using summary characteristics of network form. These characteristics are measured using indices developed from the work of Kansky, Garrison and Marble. These are the indices commonly used in transportation geography. (Hurst,1974).

The Indices:

This section begins by describing the indices used. This is followed by a discussion of the model.

The indices which measure network structure are obtained from two different types of information in the graph. The first type is comprised of indices which are all functions of the number of vertices (v), links (ℓ) and subgraphs (p).

The Mean Local Degree (β) is the average number of links leading to each node ($\beta=2\ell/v$). The larger the value of β the more developed or complex the network. (expression 2.1

illustrates the range of β).

$0 \leq \beta \leq (v-1)$ (Kansky,1963) (2.1)

The Cyclomatic or first Betti number (μ) repesents the number of circuits or fundamental loops within a graph ($\mu=\ell-v+p$). A large value of μ corresponds to a highly connected or "Delta" network, while a small value (approaching u=0) would reflect a less developed or "Spinal" network (Taaffe & Gauthier, 1973). For a graphic representation of these two extremes see figure 2.1. (expression 2.2 represents the range of the Cyclomatic Number)

$0 \leq \mu \leq (v-1)(v-2)/2$ (Kansky,1963) (2.2)

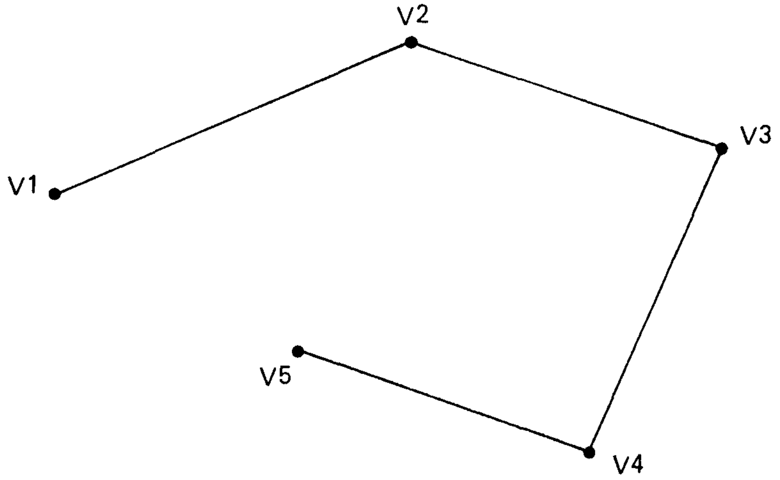
The Alpha Index ($\propto$) is the ratio of the cyclomatic number to the maximum number of fundamental circuits possible in the network ($\propto=2\mu/((v-1)(v-2))$). The result indicates the redundancy or repetitiveness of the graph (the duplication of paths).(expression 2.3 represents the range of Alpha)

$0 \leq \propto \leq 1$ (Kansky,1963) (2.3)

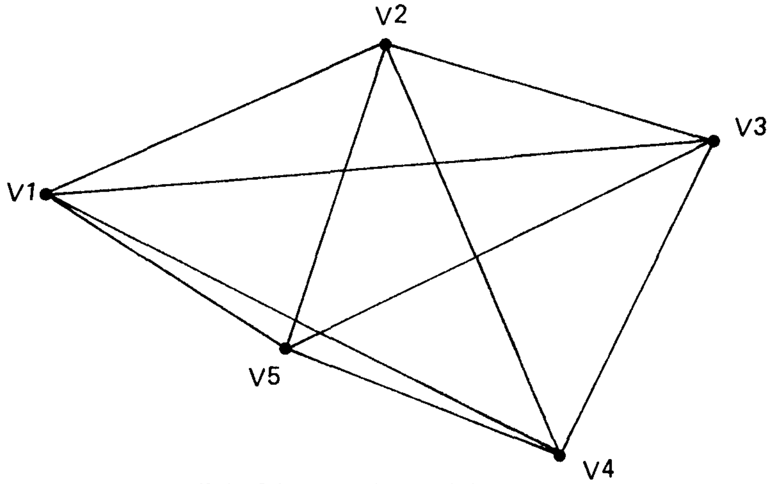
The Gamma Index (γ) illustrates the graph's degree of connectivity and is described as the ratio of actual number of links to the maximum possible links. ($\gamma=2\ell/(v(-1))$).

Figure 2.1

Network Connectivity Classification



"spinal network"



"delta network"

Source: Modified after Taaffe and Gauthier (1973) (expression 2.4 represents the range of Gamma).

$$0 \leq \gamma \leq 1$$ (Kansky,1963) (2.4)

The second category or set of measures is related to information concerning path lengths (measured in terms of number of linkages) in the graph.

Although the cyclomatic number, alpha and gamma indices have had a widespread usage in the literature as measures of connectivity, a Redundancy Ratio was introduced by Alfonso Shimbel in 1953. This was an alternative measure of connectivity. The <u>Redundancy Ratio</u> is defined as the number of elements in the shortest path matrix over the sum of the elements in the shortest path matrix. It is given in expression 2.5 .

$$\frac{V^2}{\sum_{i=1}^{V} \sum_{j=1}^{V} d\,ij}$$ (Marble,1960) (2.5)

(where dij is the topological distance
between two points (i,j) in the matrix)

The <u>Diameter</u> (δ) of a graph is a measure of connectivity and is also referred to as the Maximum Associated Number. The Associated Number of a network is the maximum shortest path distance between any pair of points (ij) in the matrix for all i and j . (topological distance being measured in links). (expression 2.6 illustrates the diameter).

$$\delta = \max d_{ij} \qquad \text{(Kansky,1963) (2.6)}$$

The System Dispersion Index, a measure of dispersion of a network (D(NET)), is defined as the sum of all the links between all pairs of nodes in the system. (expression 2.7 represents the System Dispersion index).

$$D(NET) = \sum_{i=1}^{V} \sum_{j=1}^{V} d_{ij} \qquad \text{(Shimbel,1953)} \qquad (2.7)$$

Although these are not all the indices used in graph theory, they do represent the measures most commonly used to describe basic graph structure.

Methodology:

The design of the study is organized into two phases: i) the simulation of a number of circuit networks, and ii) the analysis of the resultant networks. Although understanding can be ascertained, only upon the examination of the entire study, it is imperative that these two phases of analysis be elaborated upon.

i) The Model (nodes & linkages):

Following the work of Gilbert (1961) and Brown (1965), the model used in this study is termed a random plane model. The process described by the model is a two step one. In the first step a set of vertices (or nodes, or settlements) are generated. The second step involves the creation of edges (or linkages, or routes) between these points.

In step one a planar Poisson process was used to generate the points in a square grid. The Poisson process was chosen because many empirical settlement patterns can be considered the realization of a Poisson process. (King,1962 ; Dacey,1962). In addition, this process forms the building block for many more complex processes used in geography (Getis and Boots,1977). The assumptions of the Poisson process are:

1) Each possible location in a sample space has an equal chance of being chosen as a location for a point.

2) The location of each point chosen is independent of the location of any other point, (Getis and Boots,1977, Chapter 2, Section 2.1).

To select the coordinates of the vertices, via a Poisson process, a random number generator consisting of a computer program, in this case, was used.(see Appendix A) As in Brown's model (1965), a fixed number of vertices, n, are located in a plane. In each run of the model n=50, as this was believed to be the size which is reflective of many empirical networks (e.g. Gauthier, 1968) and which is of sufficient size to minimize boundary constraints. (Dacey,1975). A 100 by 100 grid was used as the region in which the point pattern would be born, producing a constant density of 0.005. (see figure 2.2).

In the second step of the model a critical distance procedure is used to create the linkages between the points. Under this procedure two points become linked if the distance separating them is less than or equal to a critical distance (Dc). The use of a critical distance procedure to generate the linkages was thought to be realistic in network development as one would expect the closer points in a network to have a greater likelihood of becoming linked. It has also been shown empirically that distance decay plays an important role in network growth. (Black,1971 ; MacKinnon and Barber,1972).

In each case, Dc was determined by using the maximum first-order nearest neighbour distance between any pair of points in the network. Figure 2.3 is an illustration of how the maximum nearest neighbour was determined. The maximum distance was employed as a critical distance so that each

Figure 2.2

Point Pattern Created by a Poisson Process

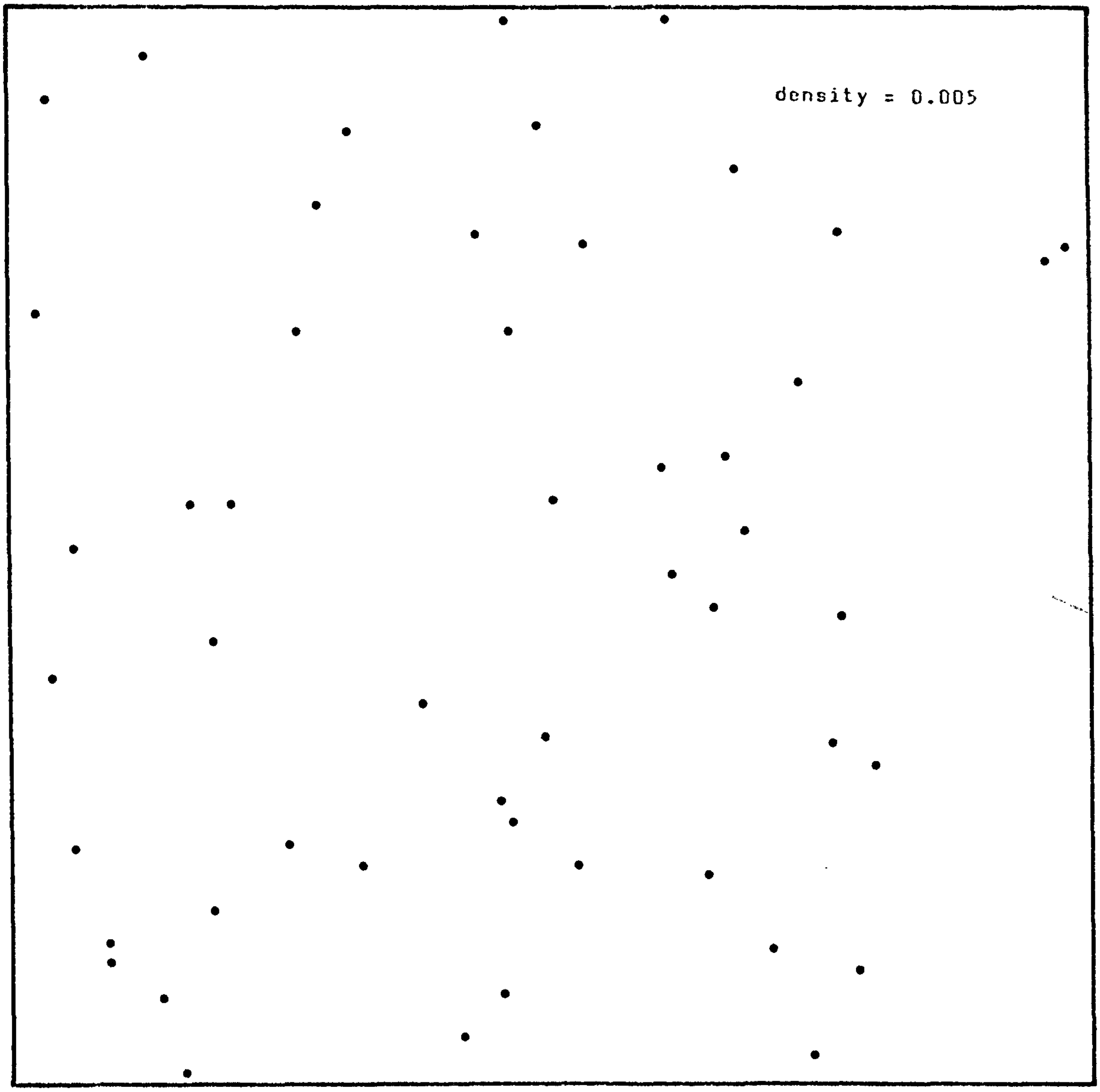
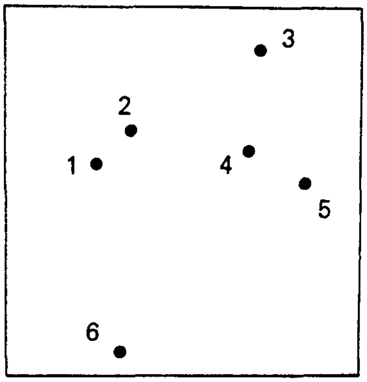


Figure 2.3

Determination of Critical Distance

Point Pattern



Nearest Neighbour

point	nearest neighbour
1	2
2	1
3	4
4	5
5	4
6	1

Distance Matrix

		points					
		1	2	3	4	5	6
points	1	0	1	4	3.5	4.5	4
	2	1	0	3	3	4	4.5
	3	4	3	0	2	3	7
	4	3.5	3	2	0	1	5
	5	4.5	4	3	1	0	5
	6	4	4.5	7	5	5	0

critical distance

point in the distribution would be connected to at least one other point. The advantage of a critical distance generated by this method is that its value varies directly as scale and inversely as density changes. In this way the technique is relatively independent of both the number of points generated and the size of the grid.

This use of the maximum nearest neighbour distance as Dc does not mean that a connected network ,where p=1 (one subgraph exists), will always be produced. It is possible to get two points connected to each other because their nearest neighbour distance was less than Dc, yet remaining isolated from the other points of the network. (see figure 2.4). This problem was solved by incorporating a multiple of the critical distance to create a greater critical distance, thus allowing more points to be connected.

A multiple (M) of the critical distance (Dc) was used to determine a new distance (contact distance between points), which would produce a connected network with p=1 (see figure 2.5). The lowest multiple of Dc, which consistently generated a connected network, was found to be M=1.5 (after fifty tests of the model). At the other end of this range, M=4.5 produced a critical distance (R) which generated a network approaching maximal connectivity.

Figure 2.4

Generated Network (M=1.0)

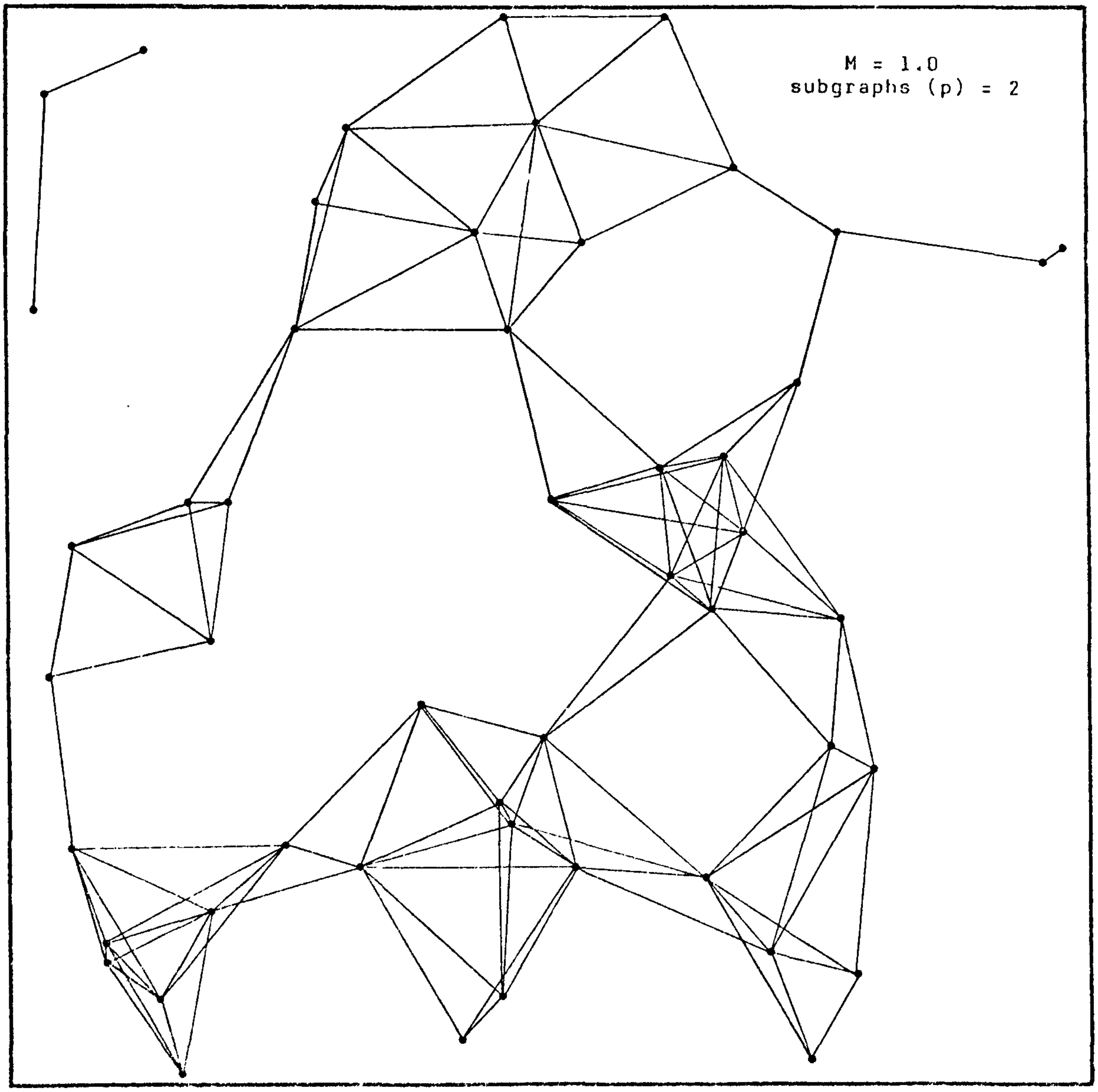
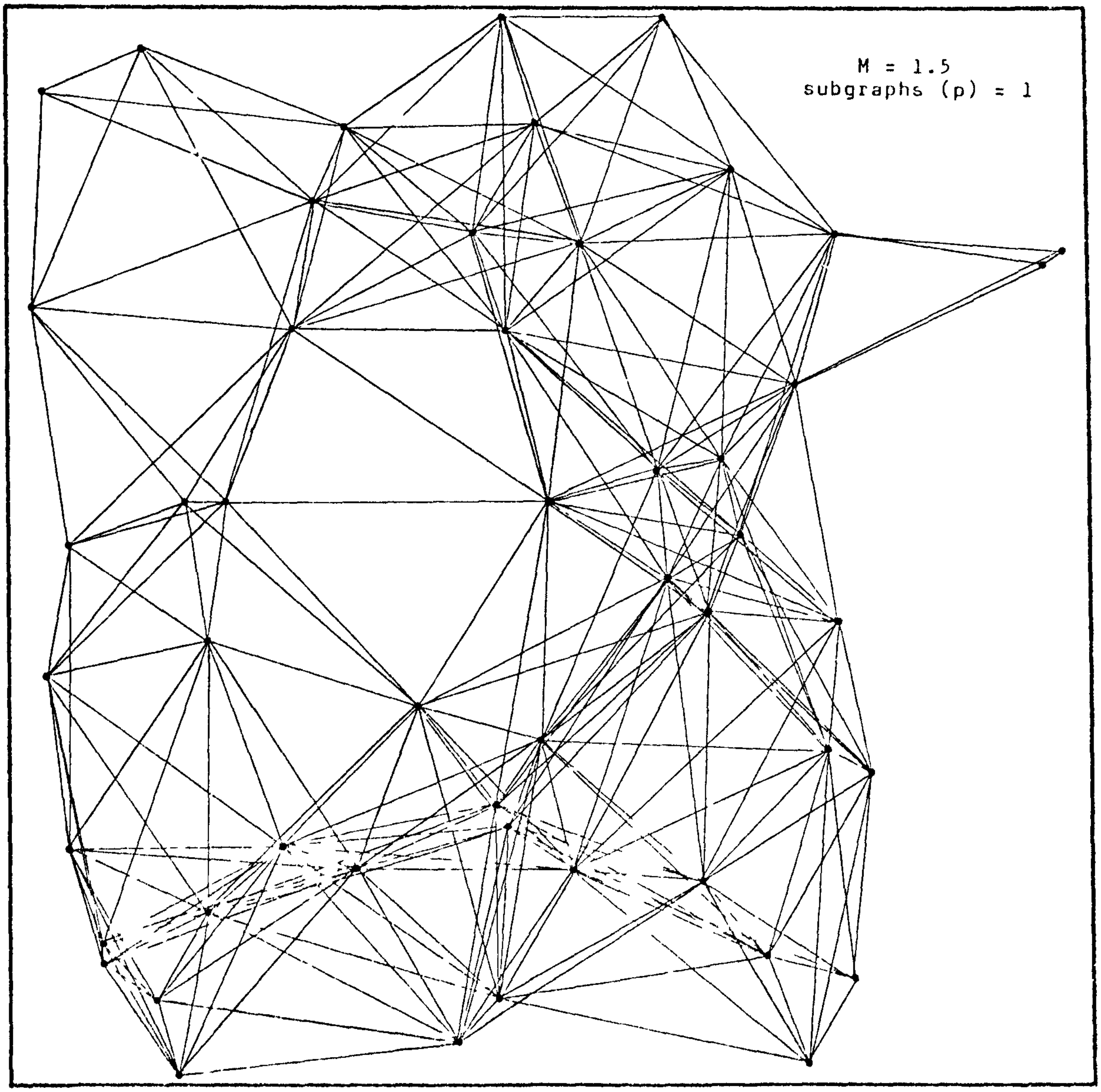


Figure 2.5

Gennerated Network (M=1.5)



The generation process was simulated thirty times (a sample of thirty is the minimum size useful in parametric tests employed subsequently) for each value of M to produce a representative sample. Seven different values of M were chosen (M increasing by .5 within a range of 1.5 to 4.5) so that structural changes could be observed as the network's complexity changed in response to a change in the critical distance (see figure 2.6).

ii) Evaluation of Model Generated Patterns:

For analytical purposes, the graphs that were generated were represented as binary matrices. In these binary matrices, if a pair of vertices (i and j) are directly linked, their corresponding cell (Cij) is given a value of 1 . Otherwise, a value of 0 is entered.(see table 2.1 for a matrix representation of figure 2.5). Conventionally, a point is considered to be connected to itself, therefore a 1 rather than a 0 is entered in the diagonal of the matrix. The matrix is symmetrical because it is considered that the route between two nodes may be travelled in both directions.

The summation of the rows and columns produces the number of points which can be reached directly (with one link) from the node represented by that row or column. However, this measure of connectivity only measures connections of one link length . Therefore, the matrix cells

Table 2.1

Matrix Representation of Figure 2. 5

Vertices	Vertices 1 5 10 15 20 25 30 35 40 45 50
1	11001110001000000010000100100000011000000000001000
2	11001110001000000010010100001100011000000010011000
3	00100000100010110000001000010001000011010000100000
4	00010001000000000000000000100000000100000000001110
5	11001110001000000010010000001100111000000010010000
6	11001110001000000000010000001000000000000010000000
7	11001110001000000000010000001000000000000010000000
8	00010001010100000000000110100010000100100000001110
9	00100000100011110000100000000001000011010000100001
10	00000001010100001000100011100010000000100000000010
11	11001110001000000010010100001100111000000010011000
12	00000001010100001010100111100010010001100000011010
13	00100000100010110100100001010001000001000100000000
14	00000000100001000000000000001100100000010000100001
15	00100000100010110101000000010001000001000100000000
16	00100000100010111101100011010001000001000100000000
17	00000000010100011101100011000010000001101001000000
18	00000000000010111101000011010000000001000100000000
19	11001000001100000010000100100010011100000000011000
20	00000000000000111101000011000000000000101101000000
21	00000000110110011000100111000010010001100000110000
22	01001110001000000000010000001100100000000010000000
23	00100000000000000000001000000001000010010000000000
24	11000001001100000010100101100010011100100000011000
25	00000001010100011101100011000010000001100000000000
26	00000000010110011101100111000010000001100000000000
27	10010001010100000010000100100010011100000000001110
28	00100000000010110100000000010001000001000100000000
29	01001110001001000000010000001100100000000010000000
30	01001000001001000000010000001100100000000000110001
31	00000001010100001010100111100010010000100000011010
32	00100000100010110000001000010001000011000000000000
33	00001000001001000000010000001100100000000010100001
34	11001000001100000010100100100010011100000000011000
35	11001000001000000010000100100000011100000000011100
36	00010001000000000010000100100000011100000000001110
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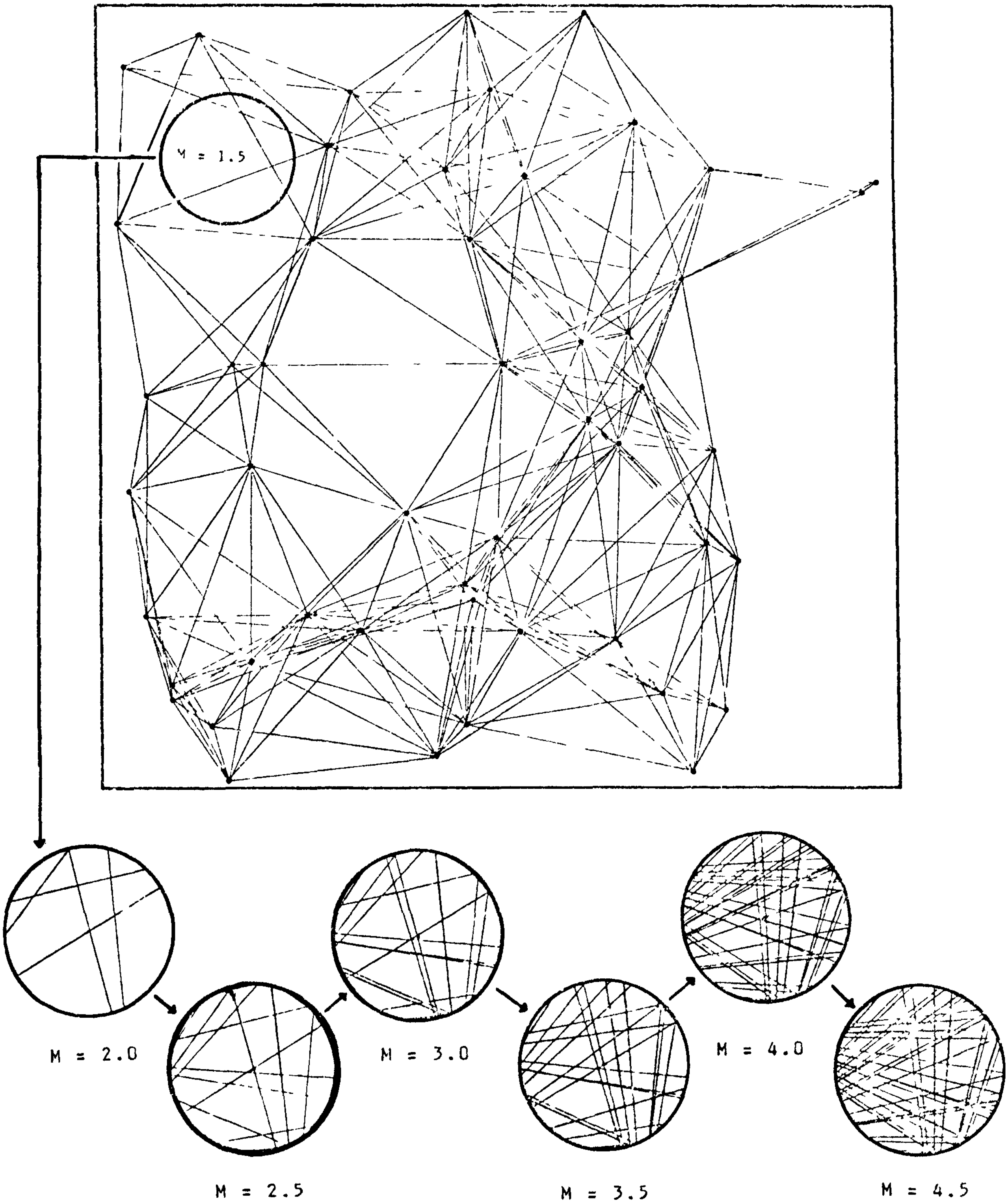
39	00000001010100001001100111000010000001100000000010
40	00100000100001000000001000000000000010010000100001
41	00000000000000001001000000000000000000001001000000
42	00000000000010110101000000010000000000000100000000
43	01001110001000000000010000001000100000000010000000
44	00000000000000001001000000000000000000001001000000
45	00100000100001000000100000000100100000010000110001
46	01001000001100000010100100000110011000000000111001
47	11010001001100000010000100100010011100000000011110
48	00010001000000000000000000100000001100000000001110
49	00010001010100000000000000100010000100100000001110
50	00000000100001000000000000000100100000010000110001

Figure 2.6

Generated Network (M=1.5 to M=4.5)



with a zero value only specify that no direct link is present. They do not suggest whether or not a linkage through a neighbouring node is possible.

When a matrix is multiplied by itself, it is raised to the second power and referred to as a "powered matrix". If a column in a "powered matrix" is summed, it represents the number of different ways in which that node can be reached from all other nodes by using two link moves (travelling over two links before reaching a desired node). By raising the matrix (MAT) to the power of the graph's diameter (δ), it is possible to produce a matrix, $(MAT)^{\delta}$, where all cells contain a value greater than zero. From this is revealed an association between all nodes allowing the connectivity, within the graph, to be determined. This matrix is called the Shortest Path Matrix.(Shimbel,1953) (see page 5) .

In order to determine the structural characteristics of the graph, a number of indices were calculated through the use of a computer program (NODAC) originally developed by Duane F Marble. A number of minor modifications to the original program were necessary to make it compatable with the Xerox Sigma 7 computer at Wilfrid Laurier University. Appendix B contains the modified version of this program as it was employed in this thesis.

The results from NODAC are represented as data matrices, each matrix representing a particular index which measured network structure. Table 2.2 represents one of these matrices. The rows of the matrix represent the thirty different networks generated for each value of M, while the columns represent the different values of M (changing critical distance) used in the model.

To analyze variations in resultant networks that were generated using an identical process, descriptive statistical tests were performed on each column in each data matrix. The mean and the standard deviation of each column in each data matrix would reflect the variation and consistency of a given index at different levels of the critical distance multiplier (M).

Lastly, a one-way analysis of variance was run to examine simultaneously the behaviour of the indices both within the levels of the critical distance multiplier M and between M.

Conclusions:

In this chapter, the approach of the study, the creation and examination of computer models to determine the effect of a generative process on the morphology of the resultant networks, were presented. The limitations of the research, along with a discussion of both the generative process and

Table 2.2

Data Matrix of The Number of Edges Index

	M values						
No. of cases	1.5	2.0	2.5	3.0	3.5	4.0	4.5
1	341	294	615	683	899	1003	1174
2	147	345	538	934	988	819	863
3	211	304	499	782	788	1179	1186
4	181	681	445	857	815	1173	1115
5	310	445	484	535	976	876	1193
6	192	363	409	507	670	983	1221
7	318	379	729	292	825	915	1072
8	273	694	586	530	805	811	1159
9	288	286	222	677	815	1048	1011
10	231	419	775	783	838	700	1151
11	584	747	333	1070	1079	1056	1182
12	204	627	304	505	1013	859	1160
13	179	366	572	523	905	1156	1036
14	227	490	505	841	1068	477	856
15	218	358	426	744	667	1060	1220
16	207	409	898	510	588	1150	1176
17	159	631	821	841	1077	1195	1199
18	244	466	802	671	1031	1191	1009
19	320	399	723	561	882	807	1222
20	218	336	504	1210	912	1188	795
21	402	691	571	798	720	1082	1140
22	238	383	470	482	1065	1195	937
23	270	493	632	1121	896	784	1153
24	232	502	968	866	940	1078	1198
25	186	318	423	797	1068	1153	1064
26	479	496	696	775	889	816	897
27	495	247	492	700	883	1140	1174
28	160	494	478	506	928	874	1016
29	148	477	395	932	908	856	1046
30	236	554	790	573	754	1206	1166

analytical procedures, constitute the remaining sections of the thesis .

CHAPTER THREE

Introduction:

The last chapter concerned the development of the model and described the analytical procedures which were necessary to determine the relationship between the network's structure and the process which generated it. This chapter reports the results of running the model and implementing the analytical procedures.

The Analysis:

After each network had been generated and expressed as a binary matrix, NODAC was used to determine the values of the indices describing the network's form.

The values of the indices, as previously explained, were set up in matrix form and statistically analyzed to determine consistency and sensitivity amongst them. Table 2.2 represents the first matrix with values obtained from the index "the number of edges". The remaining values for the seven remaining indices, stored in matrix form, can be found in Appendix C. Again, each value in this matrix was obtained from a completely different network, although the processes involved in their generation were similar.

The columns of each matrix were analyzed first, to determine the homogeneity of the thirty values of which they were composed. If consistency exists within a column (that is to say the index values are not statistically significantly different), then at least two assertions can be made. Either the generative process had a direct affect on that characteristic of the morphology of the network; or the index used to describe structure may have just been insensitive, regardless of process. Similarily, the greater the diversity within each column, the weaker the relationship between the network's structure and its propogation. In addition, a good index is considered to be one which minimizes variation within each column (the same generative process produces similar network form) and maximizes variation between the columns (different generative processes or different parameters of the same process produce dissimilar network form).

The homogeneity of the index values were assessed by obtaining a coefficient, $\bar{V}$, of variation. This coefficient measures the size of the standard deviation, s.d. , relative to that of the mean, $\bar{X}$. A measure of the relative variability can thus be calculated by dividing the standard deviation by the mean of the sample. Expression 3.1 represents coefficient of variation.

$$\bar{V}= s.d./\bar{x} \qquad \text{(Blalock,1972,p88) (3.1)}$$

If the standard deviation of the sample is small, relative to the mean, then the homogeneity of the index values (in the columns) is high, resulting in a value of $\bar{V}$ approaching zero. Conversely, if the standard deviation is large, relative to the mean, then a lack of consistency exists and the value of $\bar{V}$ is large.

The coefficient of variation is thus be very useful in comparing the relative homogeneity of groups which have differing means such as the index values in this study. One would expect that with a very large mean, one would find a fairly large standard deviation. The primary interest was therefore in the size of the standard deviation relative to that of the mean. (Blalock,1972). When evaluating the differences between the columns (the index values at different values of M), to determine the sensitivity of each index toward a change in critical distance, a coefficient of variation was again implemented.

The columns of the matrices yielded values of $\bar{V}$ which ranged from 0.09 to 0.50 (see table 3.1). A level of $\bar{V}$, which is approximately equal to 0.75 would indicate that the values in question could have been the result of some random process and little consistency in the data would exist (Boots,1977). On the other hand, a level of $\bar{V} \simeq 0.35$, is considered to be a level at which little variation in the values (data within each column) exist.

Table 3.1

Summary of Morphometric Indices

Index		M Value			
		1.5	2.0	2.5	3.0
Number of Edges	Mean	263.27	456.47	570.17	720.20
	Variance	11368.75	18503.63	32874.79	44504.12
	$\bar{v}$	0.41	0.30	0.32	0.29
Mean Local Degree	Mean	10.45	18.26	22.81	28.81
	Variance	18.18	29.61	52.61	71.26
	$\bar{v}$	0.41	0.30	0.32	0.29
Gamma	Mean	0.21	0.37	0.47	0.59
	Variance	0.01	0.01	0.02	0.03
	$\bar{v}$	0.41	0.30	0.32	0.29
Cyclomatic Number	Mean	212.27	407.47	521.17	671.20
	Variance	11400.22	18521.84	32894.05	44418.43
	$\bar{v}$	0.51	0.33	0.35	0.13
Alpha	Mean	0.18	0.35	0.44	0.57
	Variance	0.01	0.01	0.02	0.03
	$\bar{v}$	0.50	0.33	0.35	0.31
System Dispersion	Mean	6749.07	4648.27	4132.33	3605.80
	Variance	2347143.16	602581.28	767470.54	430671.41
	$\bar{v}$	0.23	0.17	0.21	0.18
Redundancy Ratio	Mean	0.39	0.55	0.63	0.71
	Variance	0.01	0.01	0.01	0.01
	$\bar{v}$	0.25	0.17	0.18	0.18
Diameter	Mean	6.33	4.10	3.40	2.87
	Variance	3.12	0.70	0.73	0.47
	$\bar{v}$	0.28	0.21	0.25	0.24

Table 3.1 continued

Summary of Morphometric Indices

Index		M Value		
		3.5	4.0	4.5
Number of Edges	Mean	889.73	994.33	1093.03
	Variance	17106.11	34204.79	14720.08
	$\bar{V}$	0.15	0.19	0.11
Mean Local Degree	Mean	35.59	39.77	43.72
	Variance	27.37	54.72	23.55
	$\bar{V}$	0.15	0.19	0.11
Gamma	Mean	0.73	0.81	0.89
	Variance	0.01	0.02	0.01
	$\bar{V}$	0.15	0.19	0.11
Cyclomatic Number	Mean	840.73	945.33	1093.03
	Variance	16981.52	34330.41	16076.08
	$\bar{V}$	0.16	0.20	0.12
Alpha	Mean	0.71	0.80	0.89
	Variance	0.01	0.02	0.01
	$\bar{V}$	0.16	0.20	0.12
System Dispersion	Mean	3130.20	2927.07	2714.33
	Variance	77611.16	175201.64	59677.46
	$\bar{V}$	0.09	0.14	0.09
Redundancy Ratio	Mean	0.80	0.87	0.93
	Variance	0.01	0.01	0.01
	$\bar{V}$	0.09	0.13	0.09
Diameter	Mean	2.40	2.20	2.03
	Variance	0.25	0.23	0.03
	$\bar{V}$	0.21	0.22	0.09

Although there were values as high as 0.50 and 0.41, which would indicate considerable variation, the majority of the values of $\bar{V}$, for all eight indices, were in a range of approximately 0.35 and lower. It should be noted that the larger values (0.50 - 0.41) existed only when the multiple M was equal to 1.5, indicating a diversity in network structure when the networks, which were generated, lacked complexity.

The columns of each matrix were examined secondly to determine the variation in the measures over different values of M (between the columns). The reason for this was the belief that structural measures, if they were good measures, would minimize variation for similar generative processes (within each M), but maximize variation for different processes (between M).

Further evaluation of table 3.1 at this time, reveals that some indices possess similar values of $\bar{V}$. The eight measures of network structure used can at this time be reduced to five categories, since the variance behaviours of the Number of Edges and Mean Local Degree are identical to that of Gamma, and similarily, the variance behaviours of the Cyclomatic Number and Alpha are the same. These results are expected, as the indices in common are composed of the same basic characteristics. The System Dispersion index and Redundancy Ratio, although they too have similar variance behaviours, will be examined separately because of their slight differences in $\bar{V}$.

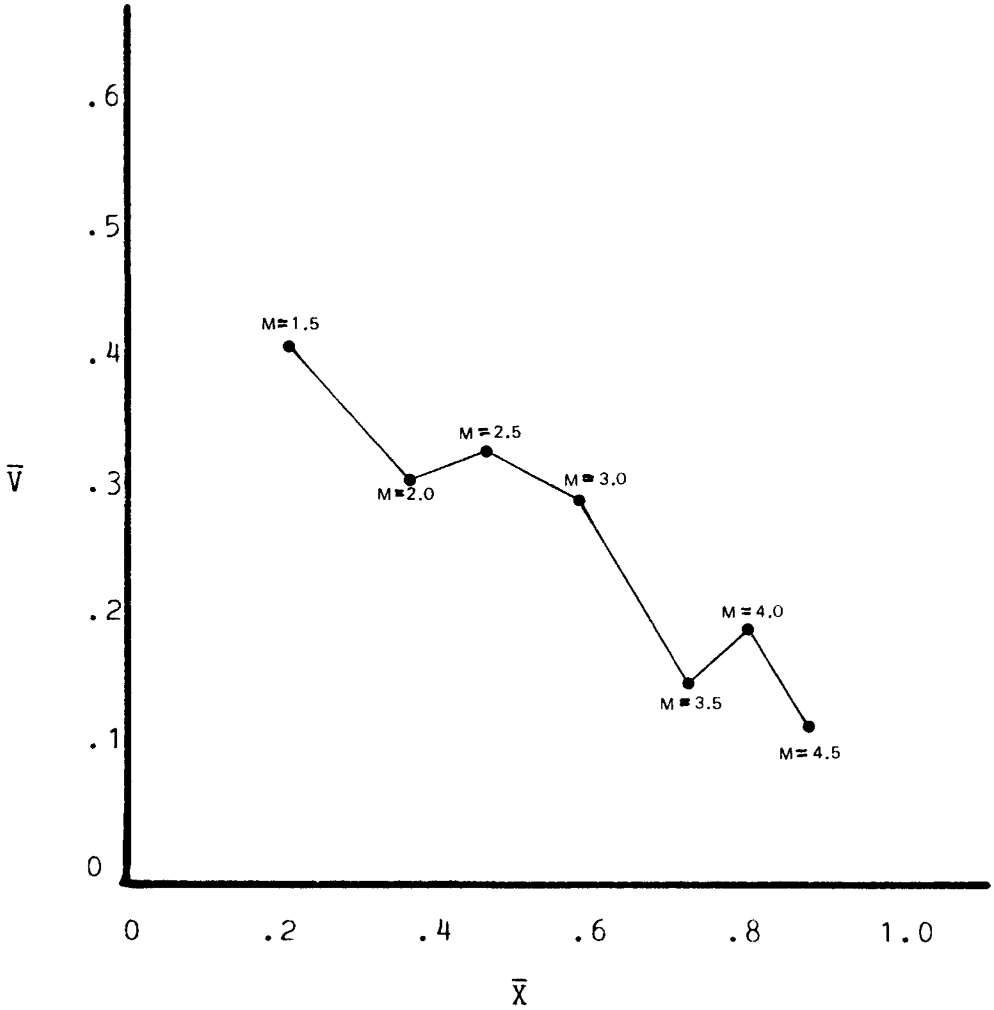
To amplify the findings above, the variance behaviour for each index, values of $\bar{V}$, were plotted, on a graph, against the means of their respective columns for each of the five remaining structural measures. For comparison purposes, all the values were converted to a standardized scale with values between 0 and 1. Appendix D contains the method of conversion for each index.

The first of the five indices examined graphically is the Gamma index, which also represented the Number of Edges and Mean Local Degree at this time. Figure 3.1 represents the coefficients of variation plotted against the means for each value of M. Although $\bar{V}$=0.41 questions the consistency of the data in the first column, it can clearly be seen that the other values of $\bar{V}$ indicate little variation within the columns. However, when comparing the values of $\bar{V}$ between the columns, one discovers that a variation does exist. Such an observation signifies that the parameter in question is sensitive to the generative process and any change in this process would definitely be noted by this measurement of the network's structure.

Further observation of figure 3.1 reveals a plateau or "leveling off" of $\bar{V}$ (when M=2.0 to M=3.0) and then a sharp

Figure 3.1

Gamma Index



decline in $\bar{V}$ between M=3.0 and M=3.5. It seems at this point that as the network's complexity increases, as shown by this parameter, the consistency within the columns also increases.

The Alpha index, also representing the Cyclomatic Number, contains the greatest range of coefficient of variation values. (see figure 3.2). The high value of coefficient variation for M=1.5, along with the drop to the plateau for M=2.0 to M=3.0 and the final decrease in $\bar{V}$, were all previously exemplified by Gamma. As before, a sensitivity to structural alterations in the networks can be detected, along with the move toward increased consistency within the columns as complexity increased.

The Diameter of the networks was the next index examined. Unlike the first two parameters, all the values of $\bar{V}$ fell far below $\bar{V}$=0.35, which indicated a great amount of consistency within the columns. (see figure 3.3) However, it was observed that little variation existed when the values of $\bar{V}$ between the columns were compared.

Like the Diameter, the System Dispersion Index generated coefficient of variation values which fell within a range of 0.0 to 0.35. (Figure 3.4 illustrates this). The consistency within the columns is even greater than that in figure 3.3, which illustrates an even stronger relationship between the

Figure 3.2

Alpha Index

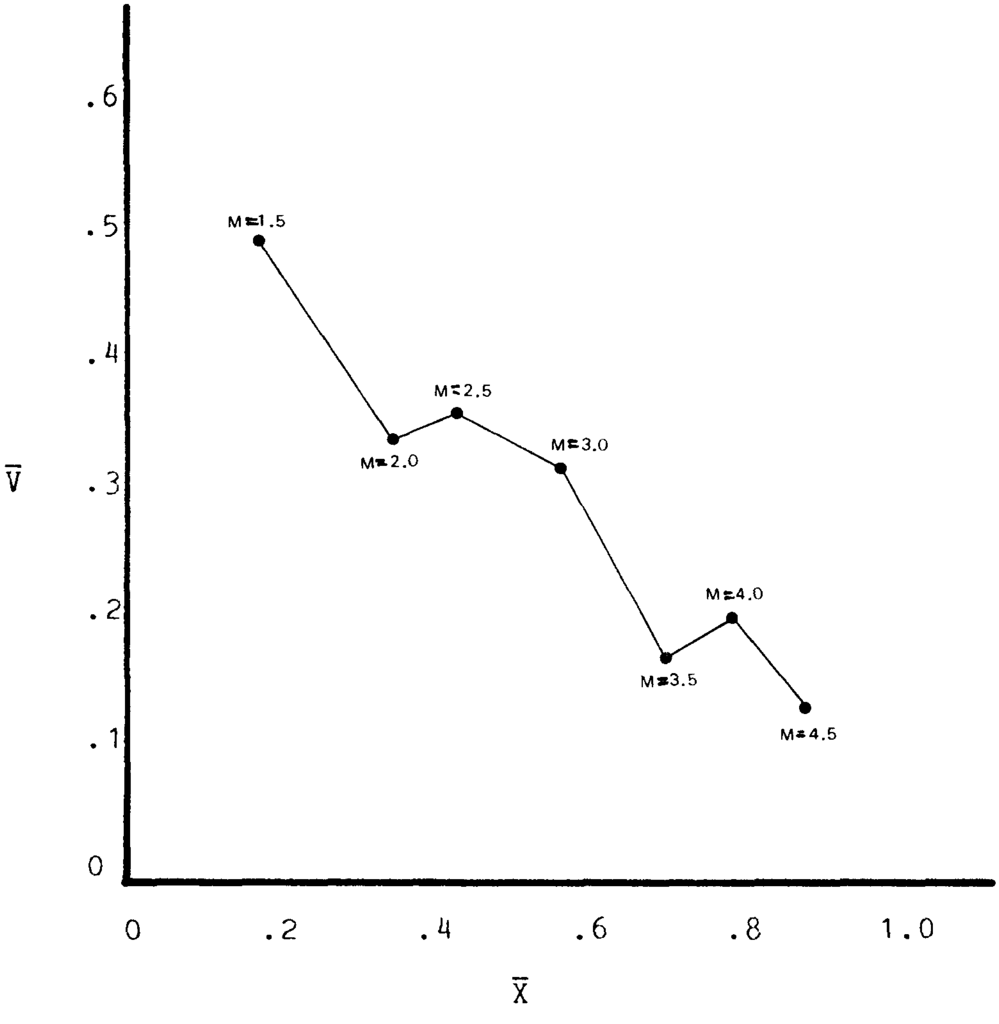


Figure 3.3

Diameter Index

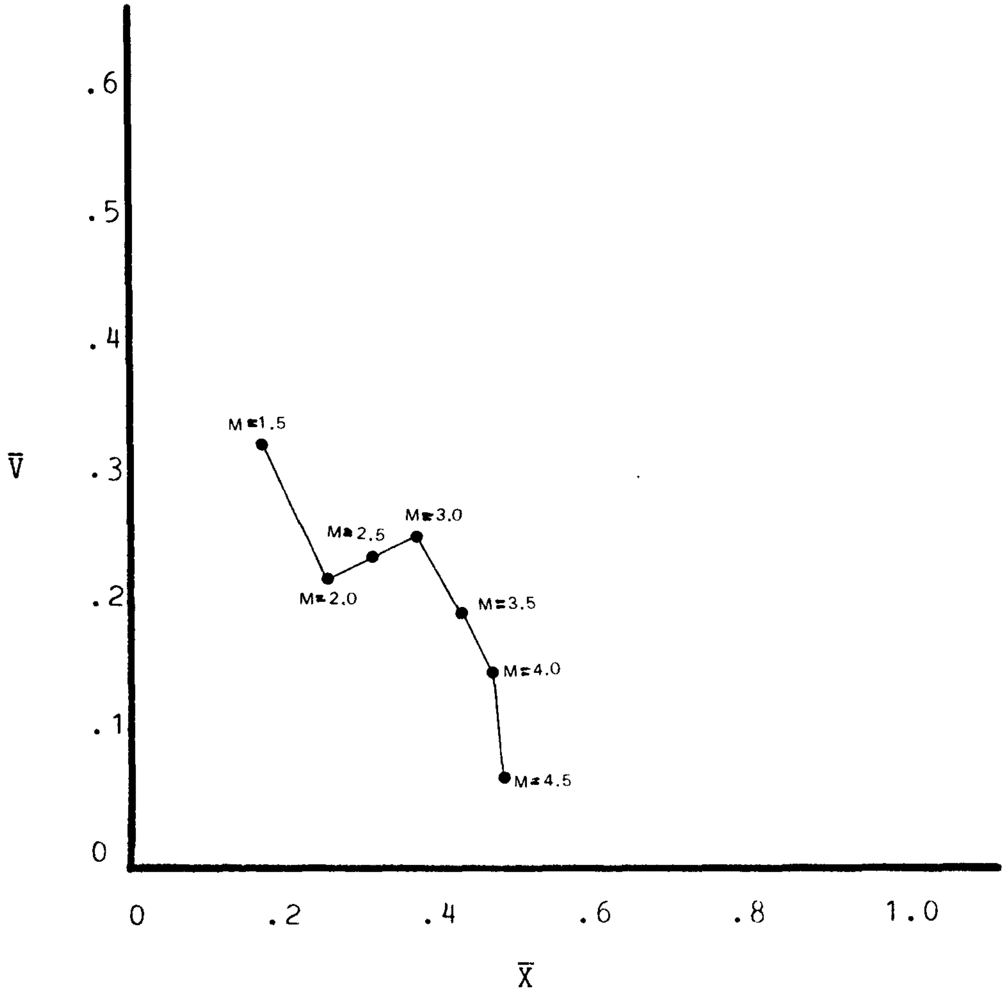
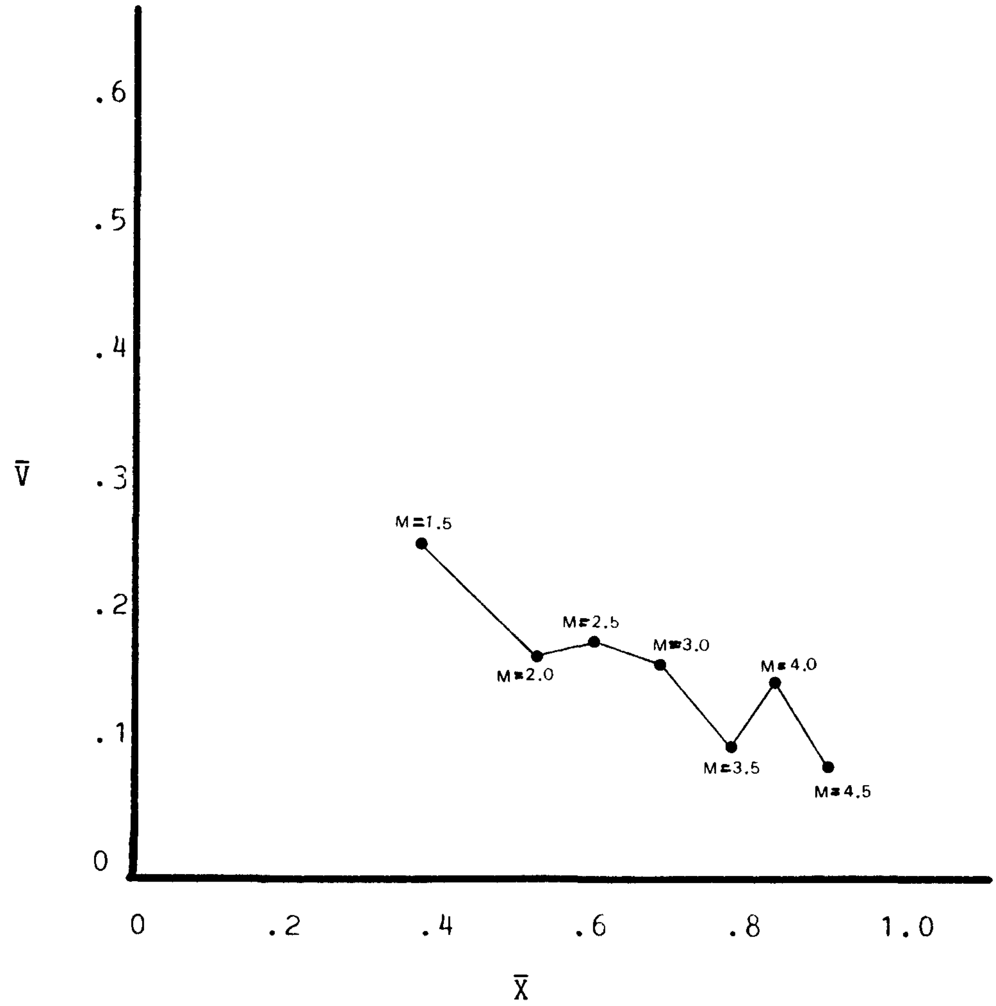


Figure 3.4

System Dispersion Index



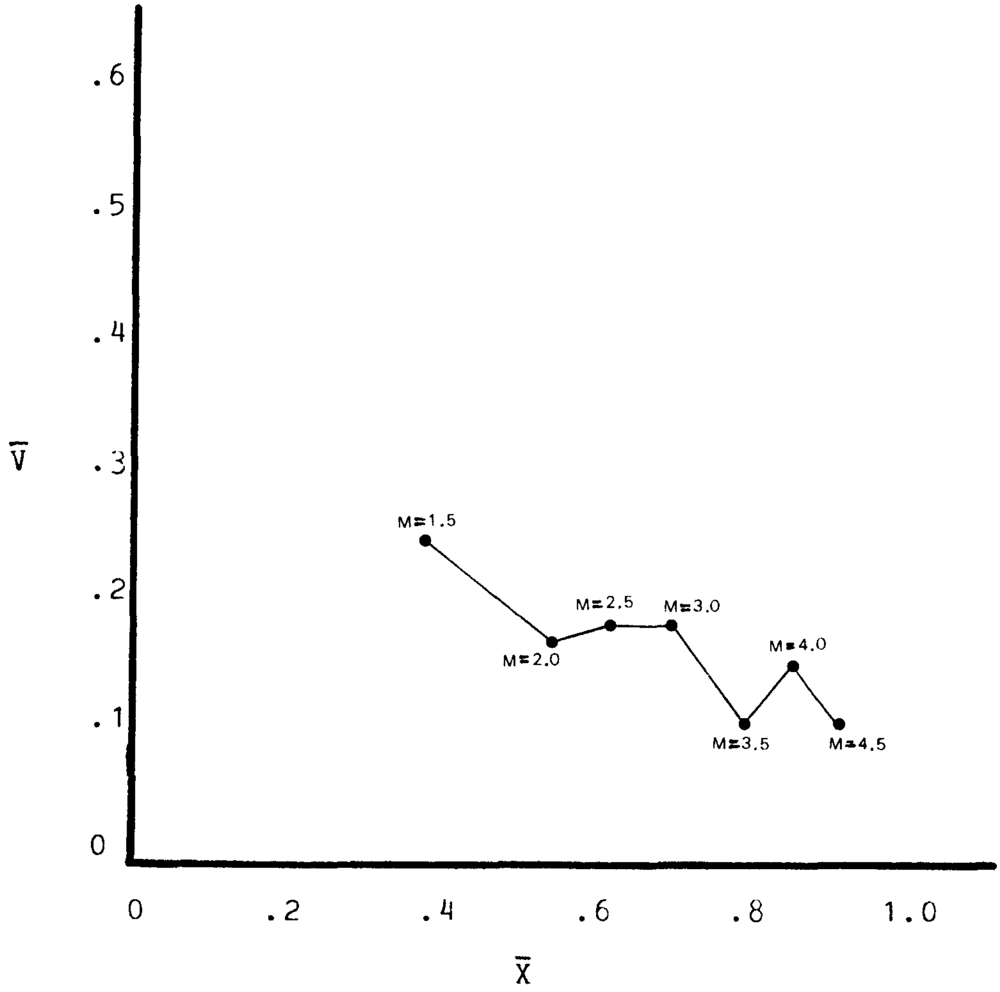
generative process and the resultant structure. Unlike the diameter, however, this index reveals a smaller range of $\bar{V}$ for the different values of M . Although the trend is not as dominant as that found in the three previous graphs, there seems to be a movement toward greater consistency in the columns as M increased.

Figure 3.5 illustrates the Redundancy Ratio which, as previously revealed, closely resembles the System Dispersion index. Even though the coefficient variations are numerically similar, they do present distinct variations. From a high of 0.25 the values of $\bar{V}$ dropped to a plateau of approximately 0.17 (when M equaled 2.0 to 3.0), and then dropped further to a low of 0.09.

The variation of $\bar{V}$ between the columns (for each value of M) is much less pronounced for the Diameter, the System Dispersion index, and the Redundancy Ratio. As with the variation within columns such consistency could arise for one of two reasons; the behaviour of the index is strongly related to the process, or the index is simply a weak measure of structure. To pursue this question, further statistical analysis of each index is undertaken. A one-way analysis of variance is used because this enables the researcher to simultaneously examine the behaviour, both within and between the columns, of the indices. Keeping in mind the criterion that a good index is one which minimizes

Figure 3.5

Redundancy Ratio Index



within column variance and maximizes between column variance, the best indices will be those with the highest F-ratios (since in every instance the degrees of freedom are same).

Table 3.2 contains the results of these analysis of variance procedures. The 95 percent significance level for values of F with 6 and 203 degress of freedom is F=2.10. This means, if there was no significant difference between the columns, the F-ratio would be less than or equal to 2.10. However, the resultant F-ratios, in table 3.2 , indicate that in all cases there was a significant difference in structural measures obtained from different generative processes (changing values of M). Since the F-ratios were all similar in value, ranging from the Diameter at 86.089 to the Redundancy Ratio at 110.695, a considerable amount of difference in the behaviour of the indices is not suggested. The Gamma, Alpha, and Redundancy Ratio, however, seem to be the most sensitive of the indices examined.

Conclusion:

The aim of this chapter was to investigate and determine the sensitivity of the morphological characteristics to changes in the parameters of generative process. Two relationships were examined; one to determine if a

Table 3.2

Analysis of Variance for Morphometric Indices

Index	Source	Degrees of Freedom	F-Ratio
GAMMA	Between	6	110.129
	Within	203	
ALPHA	Between	6	110.127
	Within	203	
SYSTEM DISPERSION	Between	6	91.841
	Within	203	
REDUNDANCY RATIO	Between	6	110.695
	Within	203	
DIAMETER	Between	6	86.089
	Within	203	

relationship did exist between the generative process and the network's structure, and another to reveal if the indices used to detect structural change were sensitive to a change in the values of the parameters used to generate the network.

Analysis of the "data matrices" revealed strong relationships between generative process and resultant morphological structures. This was demonstrated by the behaviour of coeffients of variation which measure the consistency of the structural measures.

The second part of the analysis, determining index sensitivity, revealed that all the indices were sensitive to a change in the parameters of the generative process. This suggested that they were all good measures of network structure, at least for the particular generative process used in this study.

CHAPTER FOUR

Summary of Results:

The primary objective of this research was to demonstrate an approach to transportation network study which links process and form. The secondary objective was to provide information about the sensitivity and behaviour of selected structural measures in current use in network geography. Through the development and analysis of a stochastic model, both objectives were completed.

The model, although simple in form, generated a circuit network in two steps. The first step generated points or vertices in a plane using a Poisson process. The second step of the model linked these points to finalize the generating operation and complete the network. The output of the model was conveniently stored in binary matrix form which allowed easy access to obtain structural measures of the network through mathematical calculations.

Through the use of coefficients of variation, derived from the means and standard deviations of the structural indices, it was determined that some of the measures were duplicated. This duplication was a result of the similarity of basic characteristics used to derive the indices, particularly the fact that in this study the number of vertices was a constant. The variance behaviours of the Number of Edges and the Mean Local Degree are identical to that of Gamma, and similarily, the variance behaviours of the Cyclomatic Number and Alpha are the same. This suggests that the joint use of these structural measures has been redundant in the past since they reveal similar information concerning network structure.

The coefficient of variation also illustrated the variation of measures within a particular process and allowed for a comparison of $\bar{V}$ values between different generative processes. Within a given process it was shown that networks with similar structural measures were generated. The coefficients of variability did indicate that, in general, the internal variation (within a given process) was an inverse function of the value of M (the critical distance multiplier). This was expected because as M increased, the networks that were produced approached an upper morphological limit, that of a fully connected network usually known as a "Delta" network. This relationship was much less pronounced for the Diameter, System Dispersion index and the Redundancy Ratio indicating their weakness as a structural measure or their strong relationship to the generative process (critical distance).

A one-way analysis of variance suggested that all the indices were fairly good measures of structure. The Redundancy Ratio having the highest F-ratio value, indicates that it is a good structural measurement and suggests that it is related to the process of generation. Both the Diameter and System Dispersion index, although their F-ratios are the lowest and they appear to be the weakest measures of structure, are also suggested as being related to the process.

Future Research:

In this study, this approach (process model approach) has been shown to be a valuable approach to circuit networks. Obviously more work is needed along these lines before any definitive statements can be made. The author believes that the basic model presented here offers one means of developing this additional work since the model provides an appropriate base from which to develop more sophisticated models. This is because the model is a two step one in which the first step creates the points and the second creates the linkages. For example, modification of the first step can lead to the examination of different patterns of vertices or the points can be weighted in some appropriate manner or even born to the pattern at different time intervals. The second step can also be modified to change the linkage procedure. For example a nearest neighbour technique could be instituted where, instead of multiples of a critical distance as used in this study, first, second and third nearest neighbours could be implimented. Such proposals, to make the model one which more closely resembles empirical models, have already been suggested by Haggett and Chorley (1969:298-301).

Finally, since the present model produces non-planar circuit networks. Another modification would be to produce planar circuit networks. This would make the model more representative of many empirical railway, road and shipping networks.

APPENDIX A

```
-----------------------------------------------------------
THIS IS A COMPUTER MODEL WHICH GENERATES A CIRCUIT NETWORK
WITHIN A 100 BY 100 GRID. THE NETWORK CONSISTS OF FIFTY
NODES GENERATED USING A POISSON PROCESS. THE LINKAGES
CONNECTING THE NODES, ARE GENERATED USING A CRITICAL
DISTANCE TECHNIQUE. THE FINAL OUTPUT OF THE NETWORK
IS STORED IN BINARY MATRIX FORM.  ORIGINALLY PROGRAMMED
BY JOHN D. RADKE , WILFRID LAURIER UNIVERSITY.
-----------------------------------------------------------

   DIMENSION IX2(100),IY2(100),D(50,50),ICON(50,50)
   COMMON RAND
   RAND=RND(X)
   CALL RANDOM(IX,IY,N,IX2,IY2)
   CALL CRITDIST(IX,IY,N,IX2,IY2)
   STOP
   END

------------------------------
       SUBROUTINE RANDOM
------------------------------

   SUBROUTINE    RANDOM(IX,IY,NNUC,IX2,IY2)
   DIMENSION IX2(100),IY2(100)
   NNUC=50
   NL=100
   NW=100
   DO 6 I=1,NNUC
   RAN=RND(1)
   INT=NL*NW*RAN+1
   IX=(INT-1)/NW+1
   IY=INT-NW*(IX-1)
   IX2(I)=IX
   IY2(I)=IY
 6 CONTINUE
   RETURN
   END
```

SUBROUTINE CRITDIST

```
      SUBROUTINE CRITDIST(IX,IY,N,IX2,IY2)
      DIMENSION IX2(100),IY2(100),D(50,50),ICON(50,50)
      REAL    MAX,MIN,MAX2
      WRITE(6,5)N
      DO 10 I=1,N
      WRITE(6,4)I,IX2(I),IY2(I)
   10 CONTINUE
      DO 20 I=1,N
      DO 30 J=1,N
      D(I,J)=SQRT((IX2(I)-IX2(J))**2+(IY2(I)-IY2(J))**2)
   30 CONTINUE
   20 CONTINUE
      DO 40 I=1,N
      DO 50 J=1,N
      IF(I.EQ.J)D(I,J)=9999.
   50 CONTINUE
   40 CONTINUE
    4 FORMAT(I5,2F10.5)
    5 FORMAT(' NO. OF DATA POINTS = ',I5)
```

DETERMINE CRITICAL DISTANCE

```
      MAX=0.0
      DO 96 I=1,N
      MIN=D(I,1)
      DO 97 J=2,N
   97 IF(D(I,J).LT.MIN) MIN=D(I,J)
      IF(MIN.GT.MAX) MAX=MIN
      WRITE(6,777) MAX
  777 FORMAT('MAX=',F10.5)
   96 CONTINUE
      WRITE(6,657) MAX
  657 FORMAT('MAXIMUM VALUE=',F10.5)
```

CRITICAL DISTANCE ROUTINE

```
      MAX2=MAX*4.5
      DO 22 I=1,N
      DO 32 J=1,N
      IF(D(I,J).GT.MAX2)GO TO 222
      IF(D(I,J).LE.MAX2)D(I,J)=1.0
      GO TO 32
```

```
222 IF(D(I,J).EQ.9999.) GO TO 223
    D(I,J)=0.0
    GO TO 32
223 D(I,J)=1.0
 32 CONTINUE
 22 CONTINUE

------------------------------
 PREPARE INPUT FOR NODAC
------------------------------

    DO 42 I=1,N
    DO 52 J=1,N
    ICON(I,J)=IFIX(D(I,J))
 52 CONTINUE
 42 CONTINUE

--------------------------------
   PREPARE FOR NODAC
--------------------------------

    WRITE(6,155)
155 FORMAT('1 1.00 50   1'///'(50F1.0)')
    WRITE(6,156)
156 FORMAT('  A   B   C   D   E   F   G   H   I   J   K   L   M   N
   @O   P   Q   R'/'  S   T   U   V   W   X   Y   Z  AA  BB  CC  DD  E
   @E  FF  GG  HH  II  JJ'/' KK  LL  MM  NN  OO  PP  QQ  RR  SS  TT  U
   @U  VV  WW  XX')

------------------------------
 WRITE OUT NODAC VALUES
------------------------------

    DO 62 I=1,N
    WRITE(6,99)(ICON(I,J),J=1,N)
 62 CONTINUE
 99 FORMAT(50I1)

--------------------------------
    PREPARE FOR NODAC
--------------------------------

    WRITE(6,157)
157 FORMAT('       00')
    RETURN
    END
```

APPENDIX B

```
C**************************************************************
C THIS IS NODAC**A PROGRAM TO COMPUTE CERTAIN NODE
C ACCESSIBILITY INDICES. ORIGINALLY PROGRAMMED BY
C DUANE F. MARBLE, NORTHWESTERN UNIVERSITY, LATER
C MODIFIED BY JOHN D RADKE, WILFRID LAURIER UNIVERSITY,
C TO RUN ON THE CENTRES  XEROX SIGMA 7.
C**************************************************************
      DIMENSION C(64,64), TEMP(64,64), CP(64,64), CTRA(64,64), CTRB(64,
     14), TITLE(19), NAME(64), FMT(18), DEG(64), ROW(64), COL(64), IROW
     264), ICOL(64), RPERCN(64), CPERCN(64)
      INTEGER DEG,CTRB,SOLTM,SAFETY
      EQUIVALENCE (CTRA,CTRB), (ROW,IROW), (COL,ICOL)
C     READ CONTROL AND TITLE CARDS.
   10 READ (5,650) SWITCH,A,N,NCOPY
      READ (5,660) TITLE
      READ (5,670) FMT
C     CLEAR AND SET SYSTEM.
      DO 20 I=1,N
      DO 20 J=1,N
      C(I,J)=+0.
      CP(I,J)=0.
      TEMP(I,J)=0.
   20 CTRA(I,J)=0.
      DO 30 I=1,N
      DEG(I)=0
      ROW(I)=0.
   30 COL(I)=0.
      ITOTAL=0
      TOTAL=0.
      SUMDEG=0.
      SOLTM=1
      REALN=N
      IF (REALN.GT.25.) SAFETY=REALN/1.4
      IF (REALN.LE.25.) SAFETY=N
C     READ DATA CARDS AND DUPLICATE ORIGINAL MATRIX.
      READ (5,670) (NAME(I),I=1,N)
      READ (5,FMT,END=640) ((C(I,J),J=1,N),I=1,N)
      WRITE (6,FMT)((C(I,J),J=1,N),I=1,N)
   40 DO 70 I=1,N
      DO 70 J=1,N
      IF (SWITCH) 60,60,50
   50 CTRB(I,J)=2.-C(I,J)
   60 TEMP(I,J)=C(I,J)
   70 CONTINUE
```

```
C     DO MATRIX MULTIPLICATION UNTIL SOLUTION TIME IS REACHED.
      IF (SWITCH) 80,80,100
   80 DO 90 I=1,N
      DO 90 J=1,N
   90 CTRA(I,J)=A*C(I,J)
  100 DO 110 I=1,N
      DO 110 K=1,N
      DO 110 J=1,N
  110 CP(I,K)=(C(I,J)*TEMP(J,K))+CP(I,K)
      SOLTM=SOLTM+1
      IF (SOLTM.GE.SAFETY) GO TO 630
      IF (SWITCH) 140,140,120
  120 IT=0
      DO 130 I=1,N
      DO 130 J=1,N
      IF (CP(I,J).GT.0.0005) GO TO 130
      IT=IT+1
      CTRB(I,J)=CTRB(I,J)+1
  130 CONTINUE
      IF (IT) 190,190,170
  140 DO 150 I=1,N
      DO 150 J=1,N
  150 CTRA(I,J)=(A**SOLTM)*CP(I,J)+CTRA(I,J)
      DO 160 I=1,N
      DO 160 J=1,N
      IF (CP(I,J).LT.0.05) GO TO 170
  160 CONTINUE
      GO TO 190
  170 DO 180 I=1,N
      DO 180 J=1,N
      TEMP(I,J)=CP(I,J)
  180 CP(I,J)=0.
      GO TO 100
C     COMPUTE INDICES.
  190 DO 200 I=1,N
      CTRB(I,I)=0
  200 C(I,I)=0.
      DO 210 I=1,N
      DO 210 J=1,N
      IF (C(I,J).GT.0.005) ROW(I)=ROW(I)+1.
  210 CONTINUE
      DO 220 I=1,N
      SUMDEG=SUMDEG+ROW(I)
      DEG(I)=ROW(I)+.5
  220 ROW(I)=0.
      AVEDEG=SUMDEG/REALN
      SUMDEG=SUMDEG/2.
      MUTT=SUMDEG
      CYLNO=SUMDEG-REALN+1.
      ALPHA=(CYLNO/((REALN*REALN-REALN)/2.-REALN+1.))*100.
      GAMMA=(SUMDEG/(REALN*(REALN-1.)))*100.
      IF (SWITCH) 260,260,230
```

```
 230 DO 240 I=1,N
     DO 240 J=1,N
     ITOTAL=ITOTAL+CTRB(I,J)
     IROW(I)=IROW(I)+CTRB(I,J)
 240 ICOL(I)=ICOL(I)+CTRB(J,I)
     TOTAL=ITOTAL
     REDUN=(REALN*REALN)/TOTAL
     DO 250 I=1,N
     TEX=IROW(I)
     TEXC=ICOL(I)
     RPERCN(I)=(TEX/TOTAL)*100.
 250 CPERCN(I)=(TEXC/TOTAL)*100.
     GO TO 290
 260 DO 270 I=1,N
     DO 270 J=1,N
     TOTAL=TOTAL+CTRA(I,J)
     ROW(I)=ROW(I)+CTRA(I,J)
 270 COL(I)=COL(I)+CTRA(J,I)
     DO 280 I=1,N
     RPERCN(I)=(ROW(I)/TOTAL)*100.
 280 CPERCN(I)=(COL(I)/TOTAL)*100.
C    OUTPUT SEQUENCES.
 290 IF (N.GT.60) KK=4
     IF (N.GT.40.AND.N.LE.60) KK=3
     IF (N.GT.20.AND.N.LE.40) KK=2
     IF (N.LE.20) KK=1
     DO 620 NO=1,NCOPY
     WRITE (6,680) TITLE
     IF (SWITCH) 310,310,300
 300 WRITE (6,690)
     GO TO 320
 310 WRITE (6,700) A
 320 WRITE (6,720) N,MUTT
     WRITE (6,880) SOLTM
     IF (SWITCH) 340,340,330
 330 WRITE (6,770) ITOTAL
     WRITE (6,890) REDUN
 340 WRITE (6,710) AVEDEG,CYLNO,ALPHA,GAMMA
     WRITE (6,680) TITLE
     WRITE (6,730)
     GO TO (350,370,390,410), KK
 350 WRITE (6,740) (I,I=1,N)
     DO 360 I=1,N
 360 WRITE (6,750) (I,NAME(I),(C(I,J),J=1,N))
     GO TO 430
 370 WRITE (6,740) (I,I=1,20)
     WRITE (6,750) (I,NAME(I),(C(I,J),J=1,20),I=1,N)
     WRITE (6,680) TITLE
     WRITE (6,740) (I,I=21,N)
     DO 380 I=1,N
 380 WRITE (6,750) (I,NAME(I),(C(I,J),J=21,N))
     GO TO 430
```

```
390 WRITE (6,740) (I,I=1,20)
    WRITE (6,750) (I,NAME(I),(C(I,J),J=1,20),I=1,N)
    WRITE (6,680) TITLE
    WRITE (6,740) (I,I=21,40)
    WRITE (6,750) (I,NAME(I),(C(I,J),J=21,40),I=1,N)
    WRITE (6,680) TITLE
    WRITE (6,740) (I,I=41,N)
    DO 400 I=1,N
400 WRITE (6,750) (I,NAME(I),(C(I,J),J=41,N))
    GO TO 430
410 WRITE (6,740) (I,I=1,20)
    WRITE (6,750) (I,NAME(I),(C(I,J),J=1,20),I=1,N)
    WRITE (6,680) TITLE
    WRITE (6,740) (I,I=21,40)
    WRITE (6,750) (I,NAME(I),(C(I,J),J=21,40),I=1,N)
    WRITE (6,680) TITLE
    WRITE (6,740) (I,I=41,60)
    WRITE (6,750) (I,NAME(I),(C(I,J),J=41,60),I=1,N)
    WRITE (6,680) TITLE
    WRITE (6,740) (I,I=61,N)
    DO 420 I=1,N
420 WRITE (6,750) (I,NAME(I),(C(I,J),J=61,N))
430 WRITE (6,680) TITLE
    IF (SWITCH) 530,530,440
440 WRITE (6,780)
    GO TO (450,470,490,510), KK
450 WRITE (6,740) (I,I=1,N)
    DO 460 I=1,N
460 WRITE (6,760) (I,NAME(I),(CTRB(I,J),J=1,N))
    GO TO 550
470 WRITE (6,740) (I,I=1,20)
    WRITE (6,760) (I,NAME(I),(CTRB(I,J),J=1,20),I=1,N)
    WRITE (6,680) TITLE
    WRITE (6,740) (I,I=21,N)
    DO 480 I=1,N
480 WRITE (6,760) (I,NAME(I),(CTRB(I,J),J=21,N))
    GO TO 550
490 WRITE (6,740) (I,I=1,20)
    WRITE (6,760) (I,NAME(I),(CTRB(I,J),J=1,20),I=1,N)
    WRITE (6,680) TITLE
    WRITE (6,740) (I,I=21,40)
    WRITE (6,760) (I,NAME(I),(CTRB(I,J),J=21,40),I=1,N)
    WRITE (6,680) TITLE
    WRITE (6,740) (I,I=41,N)
    DO 500 I=1,N
500 WRITE (6,760) (I,NAME(I),(CTRB(I,J),J=41,N))
    GO TO 550
510 WRITE (6,740) (I,I=1,20)
    WRITE (6,760) (I,NAME(I),(CTRB(I,J),J=1,20),I=1,N)
    WRITE (6,680) TITLE
```

```
      WRITE (6,740) (I,I=21,40)
      WRITE (6,760) (I,NAME(I),(CTRB(I,J),J=21,40),I=1,N)
      WRITE (6,680) TITLE
      WRITE (6,740) (I,I=41,60)
      WRITE (6,760) (I,NAME(I),(CTRB(I,J),J=41,60),I=1,N)
      WRITE (6,680) TITLE
      WRITE (6,740) (I,I=61,N)
      DO 520 I=1,N
  520 WRITE (6,760) (I,NAME(I),(CTRB(I,J),J=61,N))
      GO TO 550
  530 WRITE (6,790) SOLTM
      WRITE (6,800)
      WRITE (6,810) (I,I=1,N)
      DO 540 I=1,N
  540 WRITE (6,820) (I,NAME(I),(CTRA(I,J),J=1,N))
      GO TO 590
  550 NN=N-1
      DO 580 M=1,N
      DO 570 I=1,N
      K=I+1
      DO 570 J=K,N
      IF (CTRB(M,I)-CTRB(M,J)) 560,570,570
  560 T=CTRB(M,I)
      CTRB(M,I)=CTRB(M,J)
      CTRB(M,J)=T
  570 CONTINUE
  580 CONTINUE
  590 WRITE (6,680) TITLE
      WRITE (6,830)
      IF (SWITCH) 600,600,610
  600 WRITE (6,840)
      WRITE (6,850) (I,NAME(I),DEG(I),ROW(I),RPERCN(I),COL(I),CPERCN(I),
     1I=1,N)
      GO TO 620
  610 WRITE (6,860)
      WRITE (6,870) (I,NAME(I),DEG(I),IROW(I),RPERCN(I),ICOL(I),CPERCN(I
     1),CTRB(I,1),I=1,N)
  620 CONTINUE
      WRITE (6,910)
      GO TO 10
C     ERROR RETURN.
  630 WRITE (6,900) TITLE
      GO TO 10
C     FORMAT STATEMENTS.
  640 PRINT 920
```

```
C
  650 FORMAT (I1,1X,F4.2,1X,I2,1X,I3)
  660 FORMAT (12A6/7A6)
  670 FORMAT (18A4)
  680 FORMAT (1H1,19A6)
  690 FORMAT (1H0,/1H0,74HTHE OPTION INVOLVING COMPUTATION OF THE SHORTE
     1ST PATH MATRIX WAS SELECTED.)
  700 FORMAT (1H0/1H0,59HTHE OPTION INVOLVING WEIGHTED MATRIX POWERING W
     1AS SELECTED./1H0,31HTHE VALUE OF A WAS SET EQUAL TO,F6.2)
  710 FORMAT (1H0,30X,23HTHE MEAN LOCAL DEGREE =,F6.2/1H0,30X,23HTHE CYC
     1LOMATIC NUMBER =,F6.2/1H0,30X,17HTHE ALPHA INDEX =,F6.2/1H0,30X,17
     2HTHE GAMMA INDEX =,F6.2)
  720 FORMAT (1H0,30X,21HTHE NUMBER OF NODES =,I3/1H0,30X,21HTHE NUMBER
     1OF EDGES =,I4)
  730 FORMAT (1H0,50X,17HCONNECTION MATRIX)
  740 FORMAT (1H0,11X,20I5)
  750 FORMAT (1H0,I2,1H.,1A6,2X,20F5.0)
  760 FORMAT (1H0,I2,1H.,1X,1A6,1X,20I5)
  770 FORMAT (1H0,30X,29HTHE SYSTEM DISPERSION INDEX =,I8)
  780 FORMAT (1H0,49X,20HSHORTEST PATH MATRIX)
  790 FORMAT (1H0,52X,14HPOWERED MATRIX,30X,10HDIAMETER =,I3)
  800 FORMAT (1H0,11HELEMENT MAP)
  810 FORMAT (1H0,11X,7I15/10(12X,7I15/))
  820 FORMAT (1H0,I2,1H.,1X,1A6,1X,7E15.7/10(12X,7E15.7/))
  830 FORMAT (1H0,43X,35HTABLE OF NODE ACCESSIBILITY INDICES)
  840 FORMAT (1H0,10X,4HNAME,10X,6HDEGREE,6X,13HPOWER ROW SUM,6X,7HPERCE
     1NT,6X,16HPOWER COLUMN SUM,6X,7HPERCENT)
  850 FORMAT (1H0,3X,I2,1H.,2X,1A6,11X,I2,7X,E15.7,6X,F5.2,7X,E15.7,8X,F
     15.2)
  860 FORMAT (1H0,4X,4HNAME,8X,6HDEGREE,10X,11HSHIMBEL ONE,6X,7HPERCENT,
     110X,11HSHIMBEL TWO,6X,7HPERCENT,5X,17HASSOCIATED NUMBER)
  870 FORMAT (1H0,I2,1H.,1X,1A6,8X,I2,14X,I6,10X,F5.2,14X,I6,10X,F5.2,11
     1X,I4)
  880 FORMAT (1H0,30X,10HDIAMETER =,I4)
  890 FORMAT (1H0,30X,22HTHE REDUNDANCY RATIO =,F7.4)
  900 FORMAT (1H1,3X,19A6/1H3,35X,48HWARNING *** THIS NETWORK IS NOT FUL
     1LY CONNECTED./1H-,35X,50HPROBLEM SKIPPED ** PROCEEDING TO NEXT PRO
     2BLEM SET./1H1)
  910 FORMAT (1H1/1H3,33X,49HTHERE IS SOMETHING FASCINATING ABOUT SCIENC
     1E. ONE/1H0,28X,55HGETS SUCH WHOLESALE RETURNS OF CONJECTURE OUT OF
     2 SUCH A/1H0,28X,28HTRIFLING INVESTMENT OF FACT./1H ,60X,13H-- MARK
     3 TWAIN/1H1)
  920 FORMAT (1H1,'ENCOUNTERED END OF FILE--PROGRAM TERMINATED')
      END
```

APPENDIX C

Data Matrix of Diameter Index

No. of cases	M values						
	1.5	2.0	2.5	3.0	3.5	4.0	4.5
1	4	5	3	3	3	2	2
2	9	5	3	2	2	2	2
3	7	5	3	3	3	2	2
4	10	3	4	3	2	2	2
5	5	4	4	3	2	2	2
6	10	4	4	4	3	2	2
7	5	4	3	5	3	2	2
8	5	3	3	3	3	2	2
9	7	5	6	3	3	2	2
10	6	4	3	3	3	3	2
11	3	3	5	2	2	2	2
12	7	3	5	3	2	3	2
13	8	5	3	3	2	2	2
14	6	4	3	2	2	4	2
15	6	4	4	3	3	2	2
16	7	4	3	3	3	2	2
17	8	3	2	3	2	2	2
18	6	4	3	3	2	2	2
19	5	5	3	3	3	3	2
20	7	5	3	2	2	2	3
21	5	3	3	3	3	2	2
22	6	5	3	4	2	2	2
23	5	3	3	2	2	2	2
24	6	4	2	2	2	2	2
25	8	5	4	3	2	2	2
26	4	4	3	2	2	3	2
27	4	6	3	3	2	2	2
28	8	4	4	3	2	2	2
29	8	4	4	2	2	2	2
30	5	3	3	3	3	2	2

Data Matrix of System Dispersion Index

No. of cases	M values						
	1.5	2.0	2.5	3.0	3.5	4.0	4.5
1	5192	5600	3798	3552	3104	2894	2552
2	9410	5380	4020	3032	2924	3262	3174
3	6924	5784	4188	3338	3326	2542	2528
4	9110	3612	4482	3188	3270	2554	2670
5	5596	4566	4302	4028	2948	3148	2514
6	8880	5074	4686	4180	3608	2934	2458
7	5474	5074	3464	5864	3254	3070	2756
8	5922	3532	3850	4058	3298	3278	2582
9	7436	5716	7446	3566	3282	2804	2878
10	6694	4712	3354	3346	3228	3536	2598
11	3890	3452	5310	2760	2742	2788	2536
12	7102	3690	5374	4270	2874	3186	2580
13	8040	5078	3890	4146	3090	2588	2828
14	6382	4242	4178	3218	2764	4360	3188
15	6912	5086	4670	3420	3614	2780	2460
16	7128	4750	3108	4108	3860	2600	2548
17	8872	3716	3258	3224	2746	2510	2502
18	6506	4390	3298	3596	2838	2518	2882
19	5464	4822	3470	4052	3138	3290	2456
20	6762	5434	4182	2480	3076	2524	3322
21	4872	3586	3944	3308	3472	2736	2620
22	6374	4854	4324	4274	2770	2510	3026
23	6008	4210	3728	2658	3108	3332	2594
24	6574	4236	2964	3168	3020	2744	2504
25	8376	5348	4586	3310	2764	2594	2772
26	4320	4208	3518	3350	3122	3282	3106
27	4256	6570	4188	3532	3134	2620	2552
28	8400	4288	4332	4166	3044	3152	2868
29	9216	4376	4736	3036	3084	3188	2808
30	6380	4062	3322	3946	3404	2488	2568

Data Matrix of Redundancy Ratio Index

	M values						
No. of cases	1.5	2.0	2.5	3.0	3.5	4.0	4.5
1	4815	4464	6582	7038	8054	8639	9796
2	2657	4647	6219	8245	8550	7664	7876
3	3611	4322	5969	7490	7517	9835	9889
4	2744	6921	5578	7842	7645	9789	9363
5	4467	5475	5811	6207	8480	7942	9944
6	2815	4927	5335	5981	6929	8521	10171
7	4567	4927	7217	4263	7683	8143	9071
8	4222	7078	6494	6161	7580	7627	9682
9	3362	4374	3358	7011	7617	8916	8687
10	3735	5306	7454	7472	7745	7070	9623
11	6427	7242	4708	9058	9117	8967	9858
12	3520	6775	4652	5855	8699	7847	9690
13	3109	4923	6427	6030	8091	9660	8840
14	3917	5893	5984	7769	9045	5734	7842
15	3617	4915	5353	7310	6918	8993	10163
16	3507	5263	8044	6086	6477	9615	9812
17	2818	6728	7673	7754	9104	9960	9992
18	3843	5695	7580	6952	8809	9929	8675
19	4575	5185	7205	6170	7967	7599	10179
20	3697	4601	5978	10081	8127	9905	7526
21	5131	6972	6339	7557	7200	9137	9542
22	3922	5150	5782	5849	9025	9960	8262
23	4161	5938	6706	9406	8044	7503	9638
24	3803	5902	8435	7891	8278	9111	9984
25	2985	4675	5451	7553	9045	9638	9019
26	5787	5941	7106	7463	8008	7617	8049
27	5874	3805	5969	7078	7977	9542	9796
28	2976	5830	5771	6001	8213	7931	8717
29	2713	5713	5279	8235	8106	7842	8903
30	3918	6155	7526	6336	7344	10048	9735

Data Matrix of Mean Local Degree Index

No. of cases	M values						
	1.5	2.0	2.5	3.0	3.5	4.0	4.5
1	1364	1176	2460	2732	3596	4012	4696
2	588	1380	2152	3736	3952	3276	3452
3	844	1216	1996	3128	3152	4716	4744
4	724	2724	1780	3428	3260	4692	4460
5	1240	1780	1936	2140	3904	3504	4772
6	768	1452	1636	2028	2680	3932	4884
7	1272	1516	2916	1168	3300	3660	4288
8	1092	2776	2344	2120	3220	3244	4636
9	912	1144	888	2708	3260	4192	4044
10	924	1676	3100	3132	3352	2800	4604
11	2336	2988	1332	4280	4316	4224	4728
12	816	2508	1216	2020	4052	3436	4640
13	716	1464	2288	2092	3620	4624	4144
14	908	1960	2020	3364	4272	1908	3424
15	872	1432	1704	2976	2568	4240	4880
16	828	1636	3592	2040	2352	4600	4704
17	636	2524	3284	3364	4308	4780	4796
18	976	1864	3208	2684	4124	4764	4036
19	1280	1596	2892	2244	3528	3228	4888
20	872	1344	2016	4840	3648	4752	3180
21	1608	2764	2284	3192	2880	4328	4560
22	952	1532	1880	1928	4260	4780	3748
23	1080	1972	2528	4484	3584	3136	4612
24	928	2008	3872	3464	3760	4312	4792
25	744	1272	1692	3188	4272	4612	4256
26	1916	1984	2784	3100	3556	3264	3588
27	1980	988	1968	2800	3532	4560	4696
28	640	1976	1912	2024	3712	3496	4064
29	592	1908	1580	3728	3632	3424	4184
30	944	2216	3160	2292	3016	4824	4664

Data Matrix of Cyclomatic Number Index

	M values						
No. of cases	1.5	2.0	2.5	3.0	3.5	4.0	4.5
1	292	245	566	634	850	954	1125
2	98	296	489	885	939	770	814
3	162	255	450	733	739	1130	1137
4	132	632	396	808	766	1124	1066
5	261	396	435	486	927	827	1144
6	143	314	360	458	621	934	1172
7	269	330	680	243	776	866	1023
8	224	645	537	481	756	762	1110
9	179	237	173	628	766	999	962
10	182	370	726	734	789	651	1102
11	535	698	284	1021	1030	1007	1133
12	155	578	255	456	964	810	1111
13	130	317	523	474	856	1107	987
14	178	441	456	792	1019	428	807
15	169	309	377	695	618	1011	1171
16	158	360	849	461	539	1101	1127
17	110	582	772	792	1028	1146	1150
18	195	417	753	622	982	1142	960
19	271	350	674	512	833	758	1173
20	169	287	455	1161	863	1139	746
21	353	642	522	749	671	1033	1091
22	189	334	421	433	1016	1146	888
23	221	444	583	1072	847	735	1104
24	183	453	919	817	891	1029	1149
25	137	269	374	748	1019	1104	1015
26	430	447	647	726	840	767	848
27	446	198	443	651	834	1091	1125
28	111	445	429	457	879	825	967
29	99	428	346	883	859	807	997
30	187	505	741	524	705	1157	1117

Data Matrix of Alpha Index

No. of cases	M values						
	1.5	2.0	2.5	3.0	3.5	4.0	4.5
1	2483	2083	4813	5391	7228	8112	9566
2	833	2517	4158	7526	7985	6548	6922
3	1378	2168	3827	6233	6284	9609	9668
4	1122	5374	3367	6871	6514	9558	9065
5	2219	3367	3699	4133	7883	7032	9728
6	1216	2670	3061	3895	5281	7942	9966
7	2287	2806	5782	2066	6599	7364	8699
8	1905	5485	4566	4090	6429	6480	9439
9	1522	2015	1471	5340	6514	8495	8180
10	1548	3146	6173	6241	6709	5536	9371
11	4549	5935	2415	8682	8759	8563	9634
12	1318	4915	2168	3878	8197	6888	9447
13	1105	2696	4447	4031	7279	9413	8393
14	1514	3750	3878	6735	8665	3639	6862
15	1437	2628	3206	5910	5255	8597	9957
16	1344	3061	7219	3920	4583	9362	9583
17	935	4949	6565	6735	8741	9745	9779
18	1658	3546	6403	5289	8350	9711	8163
19	2304	2976	5731	4354	7083	6446	9974
20	1437	2440	3869	9872	7338	9685	6344
21	3002	5459	4439	6369	5706	8784	9277
22	1607	2840	3580	3682	8639	9745	7551
23	1879	3776	4957	9116	7202	6250	9388
24	1556	3852	7815	6947	7577	8750	9770
25	1165	2287	3180	6331	8665	9388	8631
26	3656	3801	5502	6173	7143	6522	7211
27	3793	1684	3767	5536	7092	9277	9566
28	944	3784	3648	3886	7474	7015	8223
29	842	3639	2942	7509	7304	6862	8478
30	1590	4294	6301	4456	5995	9838	9498

Data Matrix of Gamma Index

	M values						
No. of cases	1.5	2.0	2.5	3.0	3.5	4.0	4.5
1	1392	1200	2510	2788	3669	4094	4792
2	600	1408	2196	3812	4033	3343	3522
3	861	1241	2037	3192	3216	4812	4841
4	739	2780	1816	3498	3327	4788	4551
5	1265	1816	1976	2184	3984	3576	4869
6	784	1482	1669	2069	2735	4012	4984
7	1298	1547	2976	1192	3367	3735	4376
8	1114	2833	2392	2163	3286	3310	4731
9	931	1167	906	2763	3327	4278	4127
10	943	1710	3163	3196	3420	2857	4698
11	2384	3049	1359	4367	4404	4310	4824
12	833	2559	1241	2061	4135	3506	4735
13	731	1494	2335	2135	3694	4718	4229
14	927	2000	2061	3433	4359	1947	3494
15	890	1461	1739	3037	2722	4327	4980
16	845	1669	3665	2082	2400	4694	4800
17	649	2576	3351	3433	4396	4878	4894
18	996	1902	3273	2739	4208	4861	4118
19	1306	1629	2951	2290	3600	3294	4988
20	890	1371	2057	4939	3722	4849	3245
21	1641	2820	2331	3257	2939	4416	4653
22	971	1563	1918	1967	4347	4878	3824
23	1102	2012	2580	4576	3657	3200	4706
24	947	2049	3951	3535	3837	4400	4890
25	759	1298	1727	3253	4359	4706	4343
26	1955	2024	2841	3163	3629	3331	3661
27	2020	1008	2008	2857	3604	4653	4792
28	653	2016	1951	2065	3788	3567	4147
29	604	1947	1612	3804	3706	3494	4269
30	963	2261	3224	2339	3078	4922	4759

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