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Theoretical and empirical examinations of spatial scale and aggregation effects on the principal axis factoring technique when the observations are areal units

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Theoretical and Empirical Examinations of Spatial Scale and Aggregation Effects on the Principal Axis Factoring Technique when the Observations are Areal Units.

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Abstract

Previous assessments of factor analytic invariance to scale and aggregation effects have led to discrepant results. To determine the true effects, this study comprehensively examines the influence of scale and aggregation on factorial ecologies. This investigation is completed for three data sets, four scales, and thirty aggregations at each scale. Of these three data sets, two are artificial. These two data sets differ only by levels of spatial autocorrelations as one data set contains independent areal unit observations while the other set includes modest positive spatial autocorrelations. The third data set consists of variables from the 1986 Saskatoon enumeration areas. Several prominent themes emerge from these findings. When areal unit observations are independent, scale effects are trivial and aggregation effects are substantial. However, introduction of positive spatial autocorrelations among variables generates sizable scale effects and reduced aggregation effects. The theoretical data results are also moderately predictable from basic spatial unit data characteristics. Empirical results display considerable scale effects and modest aggregation effects. When increasing scale with the empirical data, communalities, eigenvalues, percentage of explainable data set variation, factor scores, and factor loadings are altered. These exact variations include increasing explanatory power of factor models with fewer significant factors and increasing generality of the largest unrotated factors. These findings along with several other modifiable results, attest to the substantial effects of scale and aggregation on factorial ecologies. With modifiable results from factorial ecologies, one must question the completion of contemporary factorial ecologies in geography.

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Chapter 1: Introduction and Framework

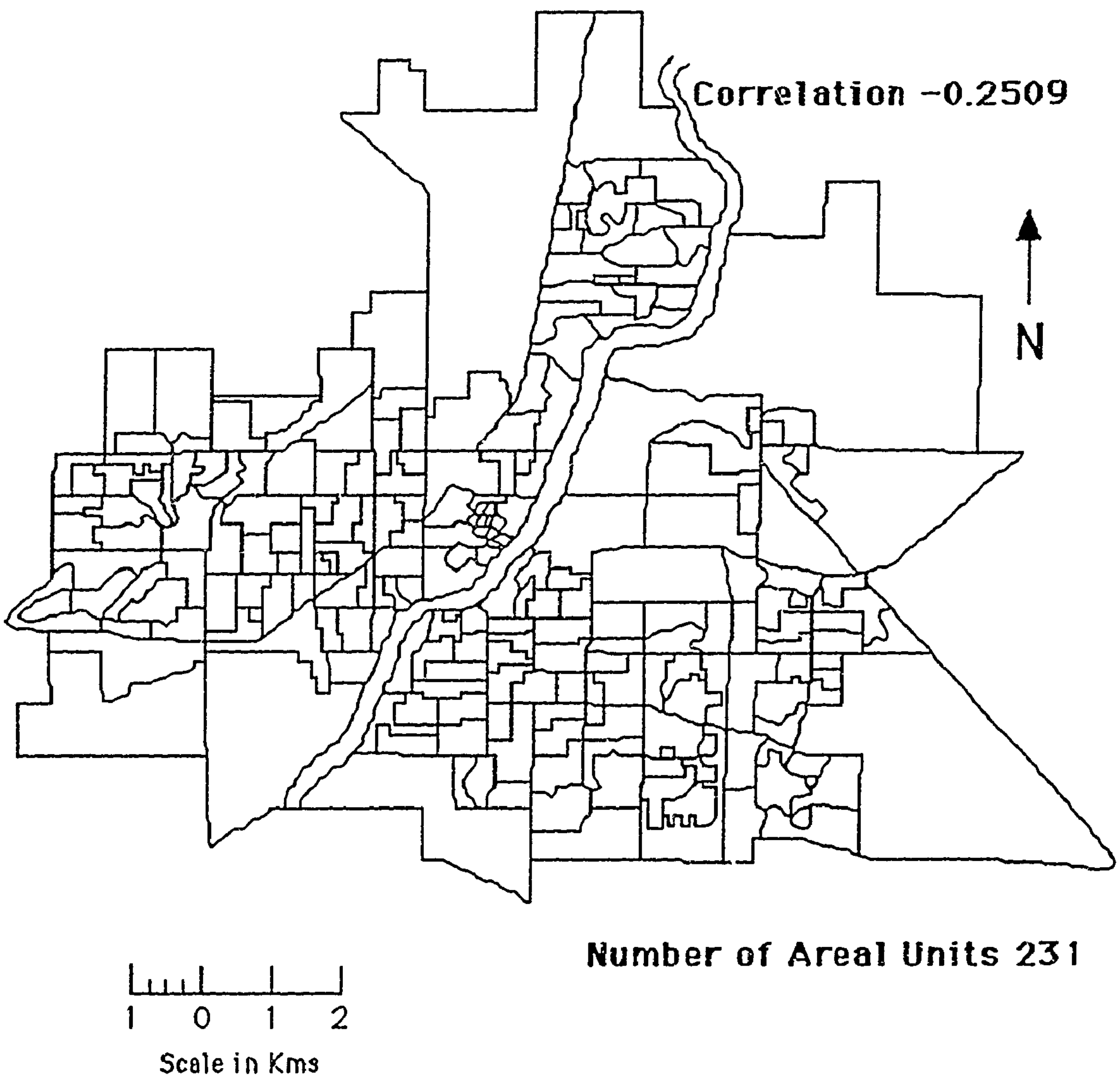
This chapter outlines the problems associated with scale and aggregation on factor models employing areal unit observations. First, the issues of scale and aggregation are discussed. Detailed below are the purpose, framework, and hypotheses of this research. As well, an outline of the entire thesis is provided.

1.1: Scale and Aggregation Effects

Scale and aggregation effects are serious obstacles in geography that influence areal unit data analyses. These problems stem from the modifiable nature of aggregated areal unit data (Yule and Kendall, 1950). The following sentences clarify the terms of scale and aggregation effects. Scale effects arise when measurement of a statistic by areal unit observations varies by altering the number of areal units partitioning the study area. Historically, the Product Moment correlation coefficient has exhibited an increase with scale ((Gehlke and Biehl, 1934) and (Yule and Kendall, 1950) among others). Aggregation effects transpire when the number of observations are fixed, and alternate divisions of the study area yield inconsistent results. Research on the correlation coefficient's variability has suggested aggregation effects are not systematic but are large due to the myriad of permutations available (Openshaw and Taylor, 1979). These two related problems of scale and aggregation were labelled the modifiable areal unit problem or MAUP by Openshaw and Taylor (1979). MAUP is also related to the well known ecological fallacy issue popularized by Robinson (1950). An ecological fallacy develops when ecological, i.e., aggregated, results are applied to individuals. Ecological fallacies are specific scale effects since one shifts the scale of analysis from individuals to aggregated data.

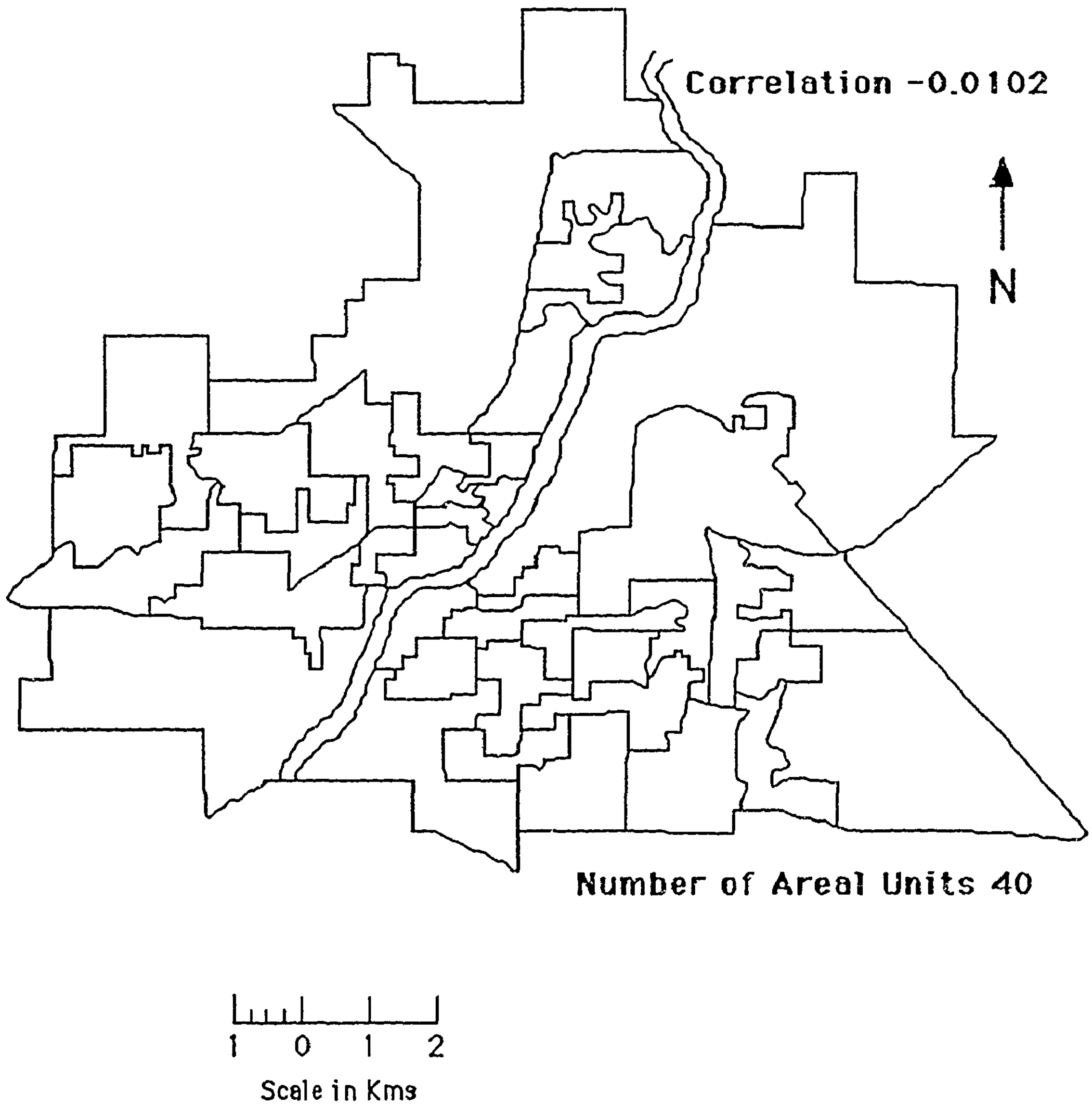
Since the terms of scale and aggregation effects may still be obscure, a visual example is provided. This example inspects the Product Moment correlation coefficient for the average family income and rate of migrant population variables from the Saskatoon data in 1986. Map 1.1.1 displays the correlation of these variables at the enumeration area level. With a correlation of -0.25, one might be tempted to say that the two variables are negatively related. However, when the scale of analysis is increased by reducing the number of areal units to forty, the associations between the variables disappears, see Map 1.1.2. Furthermore, using an alternate zoning system with forty observations could produce a correlation of 0.40, see Map 1.1.3. Two important properties

Map 1.1.1: Correlations Between Average Family Income and Rate of Migrant Population for the Saskatoon Enumeration Areas.



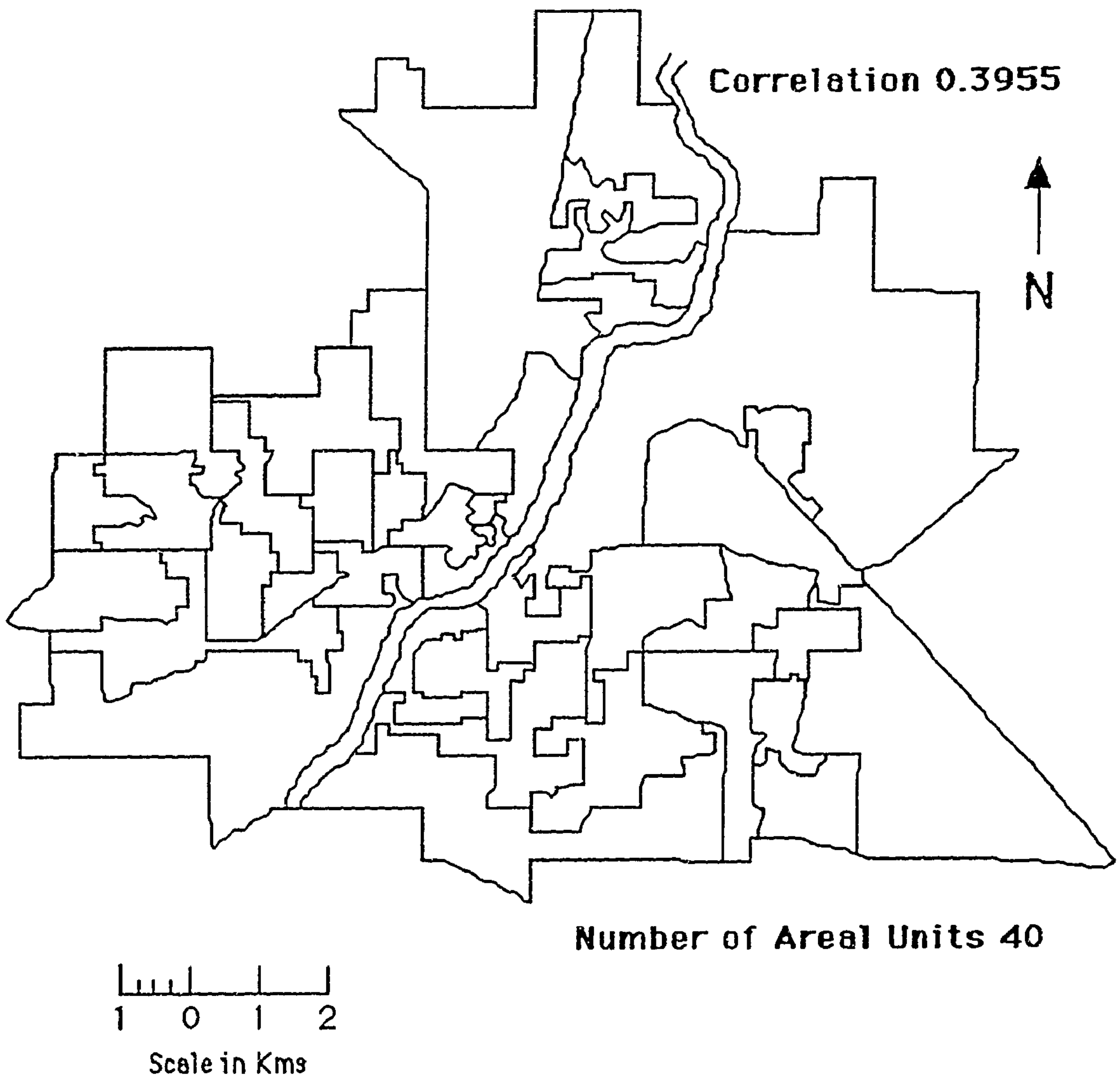
Source: Statistics Canada, 1986a

Map 1.1.2: Correlations Between Average Family Income and Rate of Migrant Population for Forty Observation Level, Saskatoon, 1986.



Source: Statistics Canada, 1986a

Map 1.1.3: Correlations Between Average Family Income and Rate of Migrant Population for Forty Observation Level, Saskatoon, 1986.



Source: Statistics Canada, 1986a

are displayed from these maps. First, aggregation effects are large as they range from correlations of -0.01 to 0.40 at the forty observation level for the variables of this example. Additionally, it should be apparent that any attempt to assess scale effects must employ a sample of aggregations and not one case. If only one aggregation was employed to measure scale effects from the enumeration to forty observation level, the results would be highly modifiable to aggregation effects. The focus will now return to the MAUP issue.

Researchers inadvertently exacerbate the MAUP issue by employing arbitrary areal units for their exercises. Clearly, MAUP is absent when one analyzes data to determine underlying traits for the areal units employed, e.g., an analysis to determine differences in average incomes across provinces. Since areal units employed for most analyses are arbitrary, many distinct areal unit partitions and/or scales are possible for any study. An example of employing arbitrary areal units could be the determination of income differences within a city by analyzing the average incomes of census tracts. The results, in this situation, are suspicious since the observations are extraneous to the study and are modifiable at choice. It is conceded that census tracts are created with several objectives in mind (Statistics Canada, 1981), but there is no denying that many alternate partitions of a city could be completed with the same objectives. In the example above, attention should focus on the spatial patterns of the city and not on the differences between units partitioning the city. Further intensifying the MAUP issue is the scarcity of individual socioeconomic data. Census data are only accessible in predefined aggregated areal units. By using previously aggregated data, any results acquire biases from the first aggregation. Accordingly, the MAUP issue will persist in geography as most research undertaken will employ areal unit observations.

Of the work completed on the MAUP issue, most has examined its effect on the correlation coefficient. Here the evidence (Gehlke and Biehl, 1934; Yule and Kendall, 1950; and Openshaw and Taylor, 1979) reveals that scale and aggregation effects question the results of correlation analyses using areal units. Furthermore, since the Product Moment correlation coefficient is the foundation for many multivariate statistical techniques including factor analysis, it is inevitable that scale and aggregation effects extend to such techniques.

The purpose of this study is to inspect the effects of MAUP on factor analysis in geography. The technique of factor analysis is examined with four scales and thirty aggregations at each scale for three data sets. To present situations typical of geographical analyses, this study preserves contiguity in every aggregated group. An inspection of the agreement of factor analytic results at alternate scales and aggregations assists in judging the robustness of the technique to MAUP effects. To facilitate formation of a comprehensive discernment of scale and aggregation effects on factor analysis, the techniques described above are performed for three data sets. Since the level of spatial dependency among areal units is influential in biasing the correlation coefficient with changes in scale (Arbia, 1989), two artificial data sets are created. One data set contains positive spatial autocorrelations between variables and is thus contaminated. The other data set has variables free of spatial autocorrelations, i.e., it is uncontaminated. The third data set is an empirical data base, from the enumeration areas of the Saskatoon C.M.A. in 1986, and is used to evaluate those findings acquired from the artificial data results.

Because factor analysis begins with the intervariable correlation matrix, it is essential to discern the correlation coefficients' variability before assessing the effects of MAUP on factor analysis. To attain some insight on the variability of the correlation coefficient, this study employs results from Arbia (1989). Arbia (1989) has completed the derivation of the aggregated Product Moment correlation coefficient from six elements measured at a lower scale. These elements are described in Chapter two. It is expected that MAUP effects on factor analysis may be predictable from Arbia's (1989) work on the correlation coefficient.

1.2: Pertinence of Research

Factor analysis has been employed to examine areal unit observations for some time in geography, e.g., factorial ecologies. Besides validating social area analysis themes, Johnston (1978) has suggested additional applications for factor analysis in geography. With increasing factor analytic applications in geography, enigmas associated with the modifiable nature of spatial data (Yule and Kendall, 1950) may become more prevalent. As well, the foundation of most factor analyses, i.e., the Product Moment correlation coefficient, is biased by scale changes (Gehlke and Biehl, 1934) and is highly variable under different aggregations (Openshaw and Taylor, 1979). Despite knowledge of the correlation coefficient's variability and bias, few attempts have evaluated the effects of scale and aggregation on factor analyses. Disregard or inadequate examinations have been paid to the variability of aggregation effects by Berry and Spodek (1971), Romsa *et al.* (1972), Perle (1977), Davies (1983), Openshaw (1984b), and Dudley (1991). These above studies also centred upon the resemblance of factor loadings across scales, and not on any other possible indicators of scale effects. Even with similar factor loadings, changes in other components of the procedure, such as eigenvalues or the Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy may profoundly affect the interpretation and confidence placed upon results. If scale and aggregation effects occur in factor analysis, any hypotheses or policies generated from factor analytic results are debatable.

1.3: Objectives

This study will be thorough in its treatment and cognizance of scale and aggregation effects on factor analysis. From the evidence provided by analyses of the three data bases, the question of factor analytic invariance to scale and/or aggregation effects should be resolved. If results are dependent upon the scale and aggregation employed, research completed by factor analyzing variables collected within areal units is suspect. Unless a theory is afforded to estimate scale and aggregation effects, continued use of factor analysis on spatial data may be unacceptable. Drawing heavily from the work of Arbia (1989) on the correlation coefficient, this study will provide evidence and progress towards such a theory given basic spatial unit data attributes.

1.4: Techniques of Analysis

1) Analyze data sets through R-mode principal axis factoring technique with varimax rotation at four scales and thirty aggregations at each scale.

2) Analyze descriptively various statistics of factor analysis, e.g., communalities and eigenvalues.

3) Assess factor loading structures across scales and aggregations with RELATE program by Veldman (1967).

4) Identify approximate limits of aggregation effects and thus the probability of scale effects through Arbia's (1989) group correlation formulas by adhering to common themes found in urban analysis, e.g., social area analysis.

1.5: Hypotheses of Exercises

1) Uncontaminated artificial data

Scale effects

- The results should remain unbiased. This situation should be analogous to the effect of random aggregations without contiguity constraints on the correlation coefficient as completed by Gehlke and Biehl (1934) and Blalock (1964).

Aggregation effects

- The variability of aggregation effects should be large and increase with scale.

2) Contaminated artificial data

Scale effects

i) There should be an increasing upward bias with scale for most of the factor analytic statistics. These are described in greater detail in Chapter Three.

ii) The factor loadings should increase in magnitude with scale for the first few unrotated factors extracted, and these factors should eventually resemble general factors.

iii) There should be fewer factors extracted from the data set with increasing scale, but the explanatory power of the factors should increase with scale.

Aggregation effects

- The variability of the results should be less than the analysis with uncontaminated data. As well, the variability of aggregation effects should increase with scale.

3) **Empirical data**

- The empirical data results should parallel the results of the contaminated data set.

This paragraph reveals the format of the remainder of this study. The first chapter deals with the introduction and background of the problem in some detail. Additionally, the research areas, methodologies, and hypotheses are all briefly disclosed. The second chapter reviews research relevant to this study. Of the sections that partition the second chapter, the most important ones review studies based on scale and aggregation effects on the correlation coefficient and factor analysis. The third chapter introduces the data sets and variables employed. As well, this chapter reviews the technique of factor analysis and forwards a rationale for choosing specific factor model options. Chapter four discloses results from the theoretical data sets. From the theoretical data set results, development of accurate hypotheses for empirical data results is provided. Chapter five tests these hypotheses on the empirical data. The final chapter summarizes the results of this study. Additionally, the final chapter suggests areas for future research.

Chapter 2: A Review of Past Investigations into Scale and Aggregation Effects

Studies using areal unit observations for statistical tests are prevalent in geography. This is partially attributable to the provision of data in areal aggregations by census agencies. Since areal units are modifiable entities, results may vary when completing analyses of the same variables and study area with different areal unit observations. Areal units are modifiable in their total number covering a geographic area and the units' shape, size, and/or orientation. These issues are the basis behind the modifiable areal unit problem or MAUP (Openshaw, 1977a). As explained before, MAUP consists of scale and aggregation effects. Despite knowledge by politicians and political geographers since the 1800's[1], only recently have aggregation effects been examined for non political issues. Aggregation effects are also greater than scale effects (Openshaw and Taylor, 1979). This is due to the myriad of different aggregations available for any scale (Keane, 1975; Cliff and Haggett, 1970), while geographers analyze a significantly smaller range of spatial scales (Haggett, 1983; 1990). Since

[1] The problem of aggregation was known to exist long ago by politicians and political geographers in terms of gerrymandering (Johnston and Taylor, 1978). Gerrymandering is the deliberate alteration of electoral ridings to bias election results.

aggregation effects are apparent and large (Openshaw and Taylor, 1979), a researcher should evaluate a sample of different aggregations for scales to determine scale effects. This would eliminate the possibility of inferring scale effects from an extreme instance (Fotheringham, 1989). The following sections illustrate the research accomplishments on this topic.

2.1: Introduction

While traditionally receiving scant attention in social science research, recently considerable achievements have been obtained on MAUP. The contemporary research is generally more specialized than past research that was performed almost exclusively on the correlation coefficient. Some anomalies of this rule do exist, as Duncan, Cuzzort, and Duncan (1961) evaluated scale effects upon simple indices, and others have examined the regression coefficient, e.g., Blalock (1964) and Clark and Avery (1976).

Five sections divide this chapter with this introduction first. The studies examining the effects of scale and aggregation on the correlation coefficient's robustness comprise the second section of this review. It is impossible to overstate the importance of the correlation coefficient as the usual index of association for factor analysis is the intervariable correlation matrix. Consequently, the impact of MAUP on any correlation coefficient will be magnified throughout the factor analytic model. The third section examines MAUP on factor analysis. This section reviews the invariance of factor analysis across alternate scales and aggregations. A fourth minor section succinctly reveals the known effects of MAUP on other multivariate statistical techniques. The final section examines the problem in its entirety.

2.2: Review of Correlation Coefficient

The correlation coefficient's systematic increase with scale has been known before 1934 (Neprash, 1934; Gehlke and Biehl, 1934). However, Gehlke and Biehl provided the first narrative of scale effects, known to the author. Their study of Cleveland census tracts examined the correlation of male juvenile delinquencies and median monthly rental variables. They successively aggregated the census tracts into seven scales from 252 census tracts to twenty-five observations. They dramatically showed an increasing correlation coefficient with scale for both absolute and rated variables. The increase in the correlation coefficient was, however, less pronounced with rated variables. They also noted that randomly aggregating data, i.e., without contiguity constraints, led to an unbiased correlation coefficient. From their results, the increasing correlation coefficients found with geographic aggregations were produced by empirical spatial data attributes, and not because of sample size effects. Although not tacitly declared by Gehlke and Biehl, their findings support this tenet.

Yule and Kendall (1950) expanded upon the earlier results of Gehlke and Biehl. Yule and Kendall examined the correlation between wheat and potato yields for successive areal aggregations of forty-eight English agricultural counties in 1936. Although the newly aggregated groups were not entirely contiguous, the results exhibited an increasing correlation coefficient from 0.2189 at forty-eight groups to 0.9902 at three groups. From these changes, Yule and Kendall inferred that areal units could be aggregated or disaggregated to generate a correlation from zero to one. They also inquired whether any real correlation between wheat and potato yields exists. They forwarded this question since the correlation coefficient is influenced by associations between the two variables and the arrangement of the areal units. Later, Openshaw (1977a) would take their phrase of modifiable areal units to place the label of modifiable areal unit problem to both scale and aggregation effects. Finally, Yule and Kendall stated this about modifiable units.

> They (correlations) measure as it were, not only the variation of the quantities under consideration, but the properties of the unit mesh which we have imposed on the system in order to measure it (Yule and Kendall 1950, p. 312).

In an often quoted paper, Robinson (1950) displayed a unique scale effect known as an ecological fallacy. An ecological fallacy results when one makes individual level inferences from results obtained with ecological, i.e., aggregated, data. The fallacy only develops when the results obtained with aggregated data differ from the results of individual data. Ecological fallacies were, however, known long before Robinson's work by both Thorndike (1939) and Gehlke and Biehl (1934). Gehlke and Biehl stated the following striking remark.

> A relatively high correlation might conceivably occur by census tracts when the traits so studied were completely dissociated in the individuals or families of those traits. (Gehlke and Biehl 1934, p. 170).

From this statement, apparently these researchers had an appreciation of the ecological fallacy issue. Thorndike (1939) also noted the affinity for individual level correlation coefficients to be closer to zero than ecological correlations. This was illustrated by a theoretical example of intelligence quotient and the number of rooms per person. In the example, the aggregated data had a higher correlation, 0.90, than the disaggregated data, 0.45. Robinson's results were similar to Thorndike's results except Robinson derived his results with empirical data.

Robinson (1950) examined the correlation coefficient for illiteracy and black nativity in the United States. He discovered larger ecological correlations than individual correlations; the correlations were determined from the Pearsonian fourfold point measures. The individual correlations were determined from cross tabulations of the two variables. He cautioned that individual level results do not support conclusions reached from ecological data. From this notable difference between ecological and individual correlations, many researchers placed a rigid taboo on the use of ecological correlations. To clarify the potential caveats of making an ecological fallacy, a humorous example is provided from Taylor (1977). Taylor stated how it is conceivable to find a positive ecological association between the rate of the population that is Jewish and the rate of population that is anti-Semitic. Obviously, to conclude that Jewish people are anti-Semitic would be asinine, but if one takes the ecological relationships as a surrogate of individual level associations this would be the case.

Besides his well known work on the ecological fallacy issue, Robinson (1956) also provided research on the causality of the correlation coefficient's bias across scales. Robinson theorized that weighting observations would eliminate any bias between ecological and individual correlations. To establish the need for weighting observations, the following hypothetical situation of two centres A and B with populations of one thousand and one hundred, respectively is presented. Assume variable X is rated by population and has values of 0.5 for A and 0.1 for B. If the two centres are considered equal in the computation of basic statistics, dissimilarities may arise from individual results. In the example above, the mean value of the two centres would be 0.3, i.e., (0.5 + 0.1) / 2. To conclude that 30% of the population has this trait would be deceiving since the individual mean is 0.456, i.e., (1000 * 0.5 + 100 * 0.1) / (1000 + 100). These possible disparities on the mean would also distort the computation of variances and covariances which determine the Product Moment correlation coefficient. Robinson recommended using a weighting formula by area or any other variable to replicate the individual correlations. Nonetheless, Thomas and Anderson (1965) proved that Robinson's solution was only exact in specific circumstances. Arbia (1989) would later reveal that the weighting formula fails to reproduce individual results when positive spatial autocorrelation affects a variable. When positive spatial autocorrelation is present, it alters the new group variance (Arbia, 1989), and again variance influences the correlation coefficient. Arbia also displayed a recursive approach to solve group variance and spatial autocorrelations since both are circularly related. Despite some defects, when analyses attempt to reproduce individual level results, execution of Robinson's idea of weighting observations makes intuitive sense.

Blalock (1964) completed an early attempt to investigate scale effects with different aggregations. Blalock examined the correlation and regression coefficients for successive aggregations of 150 southern American Counties. The research variables employed were differences in income between whites and non whites (dependent variable) and the percentage of the population non white (independent variable). His results, displayed in Table 2.2.1, were completed through aggregating by proximity, by randomization, and by maximizing the variance in the independent and dependent variables, respectively.

Congruent with the results from Gehlke and Biehl (1934), the coefficients from the randomly created aggregations were unbiased. However, increasing biases transpired with the correlation coefficient when aggregations maximized the variance of the independent and dependent variables. The case of maximizing the variance of the dependent variable biased the regression and correlation coefficient, but the regression coefficient was stable when aggregating by maximizing the independent variable. The results of the final grouping, proximity, fell between the two extremes above. Discovery of

Table 2.2.1: Scale Effects on Regression and Correlation Coefficients between Differences of Non-White and White Incomes (y) and Percentage of Population Non-White (x).

Aggregating Procedure	Number of Groups 150	75	30	15	10
Random					
rxy	0.54	0.67	0.61	0.62	0.26
byx	0.26	0.36	0.31	0.27	0.18
bxy	1.10	1.23	1.23	1.39	0.37
Maximize Variance of Independent Variable (x)					
rxy	0.54	0.67	0.84	0.88	0.95
byx	0.26	0.26	0.26	0.26	0.26
bxy	1.10	1.70	2.69	2.97	3.44
Maximize Variance of Dependent Variable (y)					
rxy	0.54	0.67	0.87	0.91	0.95
byx	0.26	0.41	0.68	0.75	0.84
bxy	1.10	1.11	1.10	1.11	1.07
Proximity					
rxy	0.54	0.63	0.70	0.84	0.81
byx	0.26	0.27	0.28	0.28	0.34
bxy	1.10	1.48	1.77	2.52	1.91

Source: Blalock (1964, p. 103)

bias in the proximity aggregation was induced as Hannan (1970) would note implicitly and Taylor (1977) explicitly by positive spatial autocorrelation in the independent variable. Consequently, any scale increase by contiguous aggregations would be analogous to aggregating groups to maximize the variance in the independent variable. Blalock explained all of the results from the confounding effects of other variables. These other variables were related to the dependent variable and were expected to behave quite differently under alternate zonings. When grouping by maximizing the variance of the independent variable, the explanatory power of the confounding variables should decline. With relatively declining importance in the confounding variables, the independent variable should explain more variation in the dependent variable. Since a regression coefficient relates the changes in one variable from another and not the variability of the variables, the regression coefficient should be less influenced than correlation coefficients (Taylor, 1977, p. 221). Hannan (1970), Taylor (1977), and Williams (1977) all pursued this issue of confounding variables on the correlation between two variables. This approach is no longer supported as the research by Arbia (1989) on spatial data configuration and dependencies appears more plausible. Despite this, Williams has provided a statement which geographers should acquaint themselves with.

> No self respecting statistician would take just any selection of individuals as his sample in a study and give it no further thought. Likewise we would hope the days are numbered for urban and regional scientists who produce zoning systems as it were, out of a hat and proceed to use them, blissfully unaware of the effects the grouping might have on any subsequent empirical investigation they carry out (Williams 1977, p. 64).

Openshaw (1977a; 1978; 1984a) and Openshaw and Taylor (1979; 1981) have immensely contributed to the cognizance of the modifiable areal unit problem and specifically the effects of alternate aggregations. In one example, Openshaw and Taylor (1979) examined the correlation between the percentage vote for Republican candidates in the Iowa congressional election of 1968 and the percentage of population over sixty years old from the 1970 U.S. census. Table 2.2.2 displays their results for maximizing and minimizing the correlation coefficient for alternate aggregations. It is apparent that the correlation's range encompasses the entire spectrum very quickly, i.e., +1.00 to -1.00, by simply modifying the aggregations. After examining the problem of aggregation effects, they concluded, ". . . We have been able to find a wide range of correlations. We simply do not know why we have found them . . . . ." (Openshaw and Taylor 1979, p. 142).

Table 2.2.2: Correlation Range of Vote and Age Variables for Different Scales of Iowa Data, 1970.

	Zoning Systems (geographic)		Grouping Systems (random)	
Number of Groups	**Min rxy**	**Max rxy**	**Min rxy**	**Max rxy**
6	-0.999	0.999	-0.999	0.999
12	-0.984	0.999	-0.999	0.999
18	-0.936	0.996	-0.977	0.999
24	-0.811	0.979	-0.994	0.999
30	-0.770	0.968	-0.989	0.999
36	-0.745	0.949	-0.987	0.998
42	-0.613	0.891	-0.980	0.996
48	-0.548	0.886	-0.967	0.995
54	-0.405	0.823	-0.892	0.983
60	-0.379	0.777	-0.787	0.983
66	-0.180	0.709	-0.698	0.953
72	-0.059	0.703	-0.579	0.927

Source: Openshaw and Taylor (1979, p. 130)

One reason Openshaw and Taylor could not resolve the range of the correlation coefficient was due to their automatic zoning procedure from Openshaw (1977b). This random procedure constructs aggregated areal units without any regard for the internal size of groups. For example, if the objective is to produce two aggregates from fifty basic spatial units (BSUs), i.e., original areal unit observations, the results could be as extreme as forty-nine BSUs in one aggregate and one BSU in the other. Consequently, the variability Openshaw and Taylor found in correlations was not only attributable to aggregation effects, but also, as Robinson (1956) had forewarned, because of problems associated with weighting observations. With samples of ten thousand alternate aggregations at each scale, it is highly probable that several aggregations were extreme, and thus extended the normal range for the coefficient.

Openshaw (1978; 1984a) did appraise the possibilities of exploiting zoning systems that satisfy specific objectives such as equal area, equal population, and zonal homogeneity to mention a few. Although declaring no additional benefit arises from alternate zone definitions, Openshaw did only examine one aggregation for each objective. Since this aggregation could be extreme, Openshaw's dismissal of these aggregating procedures may be premature and warrant further investigation.

Openshaw (1978; 1984a) and Openshaw and Taylor (1979) also observed the effect of maximum, minimum, and absent spatial autocorrelations for the voting and age variables previously mentioned. Their findings are illustrated in Table 2.2.3 for spatial autocorrelations, measured by Moran's I statistic, of maximum, 0.82 and 0.92, minimum, -0.71 and -0.57, and zero for the voting and age variables, respectively.

Table 2.2.3: Aggregation Effects on the Correlation Coefficient for Various Levels of Spatial Autocorrelations.

	Max Negative		**Absent**		**Max Positive**	
# of Groups	**Max rxy**	**Min rxy**	**Max rxy**	**Min rxy**	**Max rxy**	**Min rxy**
6	-0.99	0.99	-0.99	0.99	-0.99	0.99
12	-0.97	0.99	-0.99	0.99	-0.98	0.99
18	-0.97	0.99	-0.97	0.99	-0.92	0.99
24	-0.98	0.99	-0.90	0.99	-0.89	0.98
30	-0.93	0.98	-0.86	0.98	-0.78	0.95
36	-0.93	0.98	-0.80	0.98	-0.61	0.93
42	-0.92	0.97	-0.79	0.96	-0.52	0.93
48	-0.87	0.96	-0.66	0.95	-0.39	0.89
54	-0.85	0.95	-0.52	0.91	-0.32	0.88

Source: Openshaw (1984a, p. 22)

Surprisingly, Openshaw and Taylor detected scale effects on the correlation coefficient when no spatial autocorrelation was present in the data set. This finding could arise from, as Arbia (1989) has noted, not controlling for lagged cross correlations; this topic will be elaborated upon later. The ranges of the correlations do substantiate the expectation of lower ranges with positive spatial autocorrelations. This expectation arises since the range of variances is at a minimum when forming groups homogeneously (Arbia, 1989).

Finally, Openshaw (1984a; 1977a) has explored MAUP from a unique viewpoint of relaxing the idea of fixed observations in statistical analyses. Furthermore, he postulated a radical change of fixing model parameters and then calibrating a zoning scheme reproducing this value. This procedure has been called spatial engineering or applied gerrymandering (Openshaw, 1984a), among other things. As well, since it is feasible to produce these zoning systems, Openshaw distrusts the normal science paradigm in geography. Another less radical idea proposed by Openshaw is creation of zoning systems that meet required assumptions of analytical techniques, e.g., normality. Although these suggestions have been clearly ignored and may be extremist in view, Openshaw's work on the modifiable areal unit problem has brought tremendous insight to the caveats related with using areal unit observations.

Cliff and Ord (1981) also inspected the scale effect on areal units using various floor space uses from Jones and Sinclair's (1968) Atlas of London. Cliff and Ord examined the variance and correlations between office, commerce, and industry floor spaces upon a twenty-four by twenty-four square lattice. Through consecutive aggregations of four, four,

four, three, and three cells from the previous aggregations, they found the ordinary increase in the correlation coefficient with scale. Since the internal sizes of all newly aggregated groups were constant, this analysis was unique from previous examinations. Therefore, any tenet that scale effects are the result of not weighting observations was dispelled. Haining (1990) also recommends creating aggregated groups with equal internal sizes, i.e., the same number of basic spatial units comprise each aggregated group, to assure the same scale processes are behaving in each group. This aggregating procedure could also confront the issue of correlation range under alternate aggregations that Openshaw and Taylor (1979) could not resolve. However, since Cliff and Ord based their results from only one aggregation at a given scale, it is not possible to determine the true variability of aggregation effects on the correlation coefficient.

Perhaps the most significant research on the modifiable areal unit problem has been performed by Arbia (1989a; 1986). The work of Arbia demonstrates a cognizance of MAUP far beyond anyone previously (Fotheringham and Wong, 1991). Arbia examined both issues of spatial configuration and spatial dependencies of data and their relationships to scale and aggregation effects.

One objective of Arbia's research was to derive the group processes of the univariate; mean, variance, and spatial autocorrelation and bivariate; correlation and lagged cross correlation from basic spatial unit data. The areal units were based upon square lattices and were subject to the assumption of stationarity[2] in two dimensions. Stationarity is important since it assures that there are no fundamental changes in the structure of a process that would make prediction difficult or impossible (Judge *et al.*, 1982, p. 671). The idea of local stationarity was also used to determine spatial autocorrelations and lagged (cross) correlations from first order contiguities only. Before forging ahead with Arbia's results, there is a warrant for a review of the different types of spatial dependencies.

As asserted by Arbia (1989), there are two major dependencies influencing spatial data, reaction and interaction. Reaction is the effect one variable has upon another, which is equivalent to regression (Arbia, 1989). Interaction is the effect induced on a site by its geographic neighbours, and it is subdivided into two further components. The first component is the well known spatial autocorrelation

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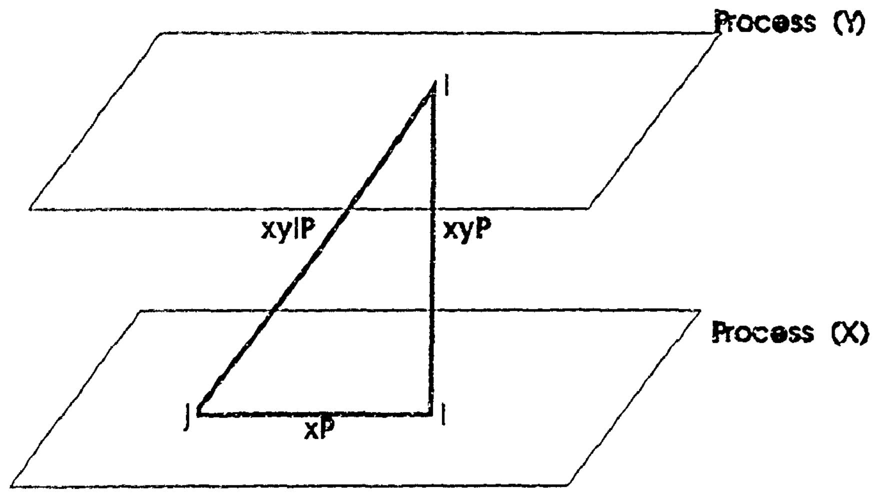
[2] Stationarity is a concept most frequently used in time-series analyses. In time series analysis, stationarity implies that the covariance between two time periods depends only upon the time interval and not on time itself (Granger, 1989). In a spatial sense, stationarity implies that statistical properties of a spatial process do not change over space (Arbia, 1989).

effect (Cliff and Ord, 1973) where one variable is correlated among geographic neighbours. The second interaction effect is far less recognized and is a combination of the two dependencies already mentioned. In this situation, two different variables are correlated across neighbouring spatial units. The term for this dependency is lagged cross correlation or for convenience lagged correlation. Figure 2.2.1 illustrates these dependencies.

Only the principal conclusions are presented since calculating the group process correlation is involved. Those who are interested in the actual derivations of all formulas are referred to the original work of Arbia (1989). All statistics and measurements are completed at the basic spatial unit level, i.e., the lowest level of aggregation, unless otherwise stated. The qualifiers between group or aggregated are used interchangeably to represent measurements made from the aggregated data at the increased scale. When computing values by averaging for aggregated groups containing equal number of basic spatial units, not surprisingly, one finds an unbiased mean value. The between group variance is affected by the variance, by the level of spatial autocorrelation, by average within group connectedness, and by the number of BSUs forming the new aggregation. If no spatial autocorrelation was present in a variable, the between group variance would equal the variance divided by the number of BSUs forming an



Definition of terms

xyP - correlation between processes X and Y

xyIP - correlation of processes X and Y between Neighbouring units

xP - spatial autocorrelation of a process (correlation between neighbouring units of the same process).

After: Arbia (1989, p.156).

Figure 2.2.1: Typologies of Spatial Dependencies in the Bivariate Case.

aggregate (Arbia, 1989). Robinson's weighting formula could also be employed in this instance. When positive spatial autocorrelation is present, the between group variance is dependent on the spatial autocorrelation and the average within group connectedness. Simply put, if the groups are more compact, i.e., high average within group connectedness, internally the groups will be homogeneous. This within group homogeneity assures maximizing of the between group variance. To calculate the group covariance, Arbia revised the correlation coefficient formula to solve for covariance. Consequently, the group covariance equals the square root of the two between group variances multiplied by the group correlation value. Since it is expected that aggregated and individual correlations will be different, the group correlation formula is altered to add the lagged correlation multiplied by the average within group connectedness. The following formulas are taken from and sometimes adapted from Arbia's work (Arbia 1989, pp. 69-70 and 157-160).

To determine the group correlation coefficient from individual data, six elements must be known, i.e., the BSU correlation, lagged correlation, both spatial autocorrelations, the average within group connectedness, and the size of the new group (Arbia, 1989). Rearrangement of this final formula can solve any parameter's value required to increase the correlation coefficient with knowledge of the other five parameters. The formula provided can also judge the range of aggregation effects by substituting the maximum and minimum average connectedness values for any scale (Arbia, 1989). Arbia checks these formulas with reasonable accuracy,

$$VG_x = \frac{V_x * (1 + WC * xp)}{r}$$

$$CVG_{xy} = \frac{(xp * yp)^{0.5} * (xyp + WC * xylp)}{r}$$

where

VG_x	- *grouped variance of x*
$V_x(V_y)$	- *basic spatial unit (BSU) variance of x (y)*
CVG_{xy}	- *grouped covariance of x and y*
WC	- *within average group connectedness*
$xp(yp)$	- *BSU level of spatial autocorrelation in x (y)*
r	- *the number of BSUs aggregated into a new group*
CRG_{xy}	- *grouped level correlation coefficient*
xyp	- *BSU level correlation coefficient*
$xylp$	- *BSU lagged correlation coeffcient*

it follows that the grouped correlation coefficients equa.

$$CRG_{xy} = \frac{CVG_{xy}}{(VG_x * VG_y)^{0.5}}$$

$$CRG_{xy} = \frac{(VG_x * VG_y)^{0.5} * (xyp + WC * xylp)}{(r * (V_x * (1 + WC * xp) * (1 + WC * yp))^{0.5}}$$

which simplifies to

$$CRG_{xy} = \frac{(xyp + WC * xylp)}{((1 + WC * xp) * (1 + WC * yp))^{0.5}}$$

and this author has found that the formulas work properly for at least lattices.

Arbia's findings are not without criticisms, even beyond the assumption of stationarity. Fotheringham and Wong (1991) see little opportunity to apply this bivariate framework to multivariate statistics because of the complexity of determining partial correlation coefficients. Furthermore, dependencies based on first order neighbours may be unrealistic and consequently the general results are debatable. Finally, these results depend upon groups with the same interconnectedness and size, and this is a rigid assumption for empirical data.

2.3: Review of Factor Analytic Robustness

After substantiating urban social area analysis theories postulated by Shevky and Williams (1949), Shevky and Bell (1955), and Bell (1955), factor analysis became vogue in the social sciences. After these initial factorial ecologies, almost every urban centre with available socioeconomic data has been factor analyzed. The findings from these factor analyses yielded indications of underlying themes or constructs of a city. Factor analysis is also employable in less well known geographical applications. These alternatives include reducing a data set into more workable terms or concocting a set of independent factors for subsequent analyses (Johnston, 1978). Johnston (1978) provides a practical example for data set reduction. When data collection cost is high in time or expense, a sample of observations could be factor analyzed to reveal variable redundancy. By revealing redundant variables, there is a reduction in the time and cost of data collection (Johnston, 1978). Defining variables into a set of independent constructs is particularly valuable when a technique such as multiple regression is subsequently completed. Since orthogonal rotations produce independent factor scores, employing factors as independent variables averts the problem of multicollinearity. Although factor analysis has substantial relevance on geographic studies, the effects of scale and particularly aggregation upon this technique are unknown.

Several studies have attempted to assess scale effects on factorial ecologies. Even as early as 1969, Murdie (1969) warned of possible scale effects on the factor analytic model. Within the MAUP studies completed on factor analysis, several common themes emerge, despite the contradictory opinions each researcher has advanced. If one only inspected the conclusions provided by each researcher, one would erroneously infer that scale effects on factor analysis are unresolved.

Berry and Spodek (1971) performed one of the earliest studies of scale effects on factor analysis. They factor analyzed the Indian city of Bombay at three separate scales of fifteen, eighty-eight, and 437 observations with fourteen socioeconomic variables. Although concluding upon the consistency of their results across scales, a review of their data does not substantiate this assertion. Since Berry and

Spodek did not provide the unrotated factor loading matrix, only the rotated factor loading matrix can be inspected for the strengthening factor notion. The number of factor loadings greater than ABS(0.30) is presented in Table 2.3.1.

Table 2.3.1: Strength of Rotated Factor Loadings Among Alternate Scales, Bombay, 1961.

	Scale of Analysis		
Rotated Factor Loadings (ABS)	**15**	**88**	**437**
(0.9 - 1.0)	4	3	5
(0.8 - 0.9)	4	3	3
(0.7 - 0.8)	4	3	2
(0.6 - 0.7)	2	2	1
(0.5 - 0.6)	1	4	2
(0.4 - 0.5)	4	6	2
(0.3 - 0.4)	6	4	5
Total	25	25	20

After: Berry and Spodek (1971, pp. 275-276)

These results display increasing magnitudes of large factor loadings with scale. It should be noted that the scales with eighty-eight and 437 observations both had five significant factors not four, and thus a varimax rotation was more likely to detect higher loadings. This was especially true with 437 observations where several factors had only two variables with high loadings. The few variables loading on a factor suggest an absence of general factors, and this lowest scale was able to identify the largest number of highest loadings. However, only twenty rotated factor loadings were greater than ABS(0.3) for the smallest scale compared to twenty-five for the other scales. Table 2.3.2 displays the cumulative percentage of variation explained by the factors for all scales.

Table 2.3.2: Cumulative Percentage of Data Set Variation Explained For Different Scales, Bombay, 1961.

	Scale of Analysis		
Factor	**15**	**88**	**437**
1	29.8	21.4	19.3
2	51.1	41.0	35.7
3	71.4	58.7	50.5
4	88.3	69.9	62.8
5	N/A	80.2	71.7

Source: Berry and Spodek (1971, pp. 275-276)

Other measures also point to the pronounced effects of scale on the Bombay analysis. The number of significant factors decreases as scale is increased, and these factors explain more of the variation in the data set. The first factor extracted exhibits increasing generality since the percentage of explained data set variation increases from 19.3 at 437 observations to 29.8 with fifteen observations. Finally, the communalities of the variables also increased with scale. Berry and Spodek remarked about the stability of their analysis across scales. However, with a better awareness of how scale effects manifest themselves in factor analysis, the importance of scale effects to this study is evident.

Romsa *et al.* (1974) completed an assessment of scale effects on factorial ecologies. They performed factorial ecologies on Windsor with thirty socioeconomic variables for both 343 enumeration districts and forty-three census tracts. These results, as the results from Berry and Spodek (1971), may be suspect since the ratio of observations to variables is very low, i.e., forty-three : thirty. The major objective of their study was to provide a quantitative test for assessing the scale invariance of factor loadings. The test selected was the coefficient of congruence (Burt, 1948; Tucker, 1951), since it acts like a correlation coefficient. Because the sign of loadings highly influences this coefficient (Pinneau and Newhouse, 1964), it has produced odd results (Gorsuch, 1983). In turn, it has been suggested that a congruence coefficient should be 0.90 for considering two factors to be identical (Cureton and A'gostino, 1983). The range of the coefficient of congruence analysis for the two scales of the Windsor analysis varied from 0.77 to -0.32. From the relatively small size of the coefficients, it appears that the two studies are sufficiently different in terms of their factor loadings.

Despite publishing only rotated factor loadings and congruence coefficients, the results of Romsa *et al.* do augment the expectations of scale effects. For the two data sets, sixteen of the rotated loadings are greater than

ABS(0.70) for the census tracts, while only eleven were greater than ABS(0.70) for the enumeration districts. Romsa *et al.* (1974) were first to state how scale effects significantly influence factor analysis.

In a study of Detroit, Perle (1977) disclosed scale effects while contending for the need of higher order factor analyses. He examined forty-three variables at two scales of 444 census tracts and sixty-two subcommunities. The results confirmed some typical scale effects. The percentage of data set variation explained was 86.5 for the subcommunities and 70.2 for the census tracts with six significant factors for each. These results again confirm that increases in scale lead to increases in the extraction of variance. Perle (1977) maintained that scale alters factor analytic results, and alternate aggregations can vary results.

Also contributing to the study of scale effects on factor analysis was research forwarded by Openshaw (1984b). Openshaw examined ecological fallacies on several multivariate statistical techniques including factor analysis, multiple regression, and cluster analysis. Openshaw's paper marked the first effort to use knowledge of correlation variability to scale and extrapolate these effects onto factor analysis. He accurately suggested that the first few eigenvalues should increase with scale. In turn, these factors accounted for larger percentages of the data set variation. His first example evaluated socioeconomic data of Sunderland determined from a 10% sample. These data were subsequently aggregated to form different scales including individual data, 500 m lattice aggregations, one km lattice aggregations, and polling districts. Table 2.2.3 displays the findings.

Table 2.3.3: Number of Significant Eigenvalues and Percentage of Data Set Variation Explained for Alternate Scales, Sunderland, 1976.

	Scale of Analysis			
Statistic	**Individual Data**	**500 m Lattice**	**1 km Lattice**	**Polling Districts**
Significant Eigenvalues	18	15	14	8
Percentage of Variance	62.5	73.4	82.5	85.0

Source: Openshaw (1984b, p. 71)

From the above results the number of significant eigenvalues, i.e., those principal component eigenvalues greater than one, decreases with increasing scale, while the percentage of explained variation increases with scale. Openshaw also examined Florence socioeconomic data by individual and enumeration district level data. The individual data contained fourteen significant eigenvalues while the enumeration district level had eight eigenvalues accounting for 57.8 and 73.7 percent of the variation, respectively. The decreasing number of significant factors with increasing scale was concluded as the result of increasing areal associations among the variables. Additional tests using the coefficient of congruence also revealed the differences between factors at different scales.

Davies (1983) performed a fourth study of scale effects on the Welsh city of Cardiff in 1971. The principal component analyses were conducted on 541 enumeration districts and twenty wards with twenty-seven variables. Since there are more variables than observations in the second data set, these results are probably debatable. Following corresponding results of previous studies, six factors were extracted at the enumeration district level with only five at the ward level. As well, these significant factors accounted for sixty-seven and ninety-two percent of the overall variation for the enumeration district and ward level, respectively. The generality of factors was also stated as far more minor loadings were found on each vector (Davies, 1983, p. 97).

Davies further analyzed this data set by employing the coefficient of congruence. For the five common factors he found coefficients of 0.88, -0.81, 0.94, 0.63, -0.56. Both views of emerging general factors with scale and of similarity among the first few axes across scales were concluded by Davies (1984). This latter statement arose from the similarity of the coefficients of congruence among these factors. However, the coefficient of congruence is suspect, and thus only those coefficients greater than 0.90 (Cureton and A'gostino, 1983) should be considered identical. Davies' (1983) congruence coefficients may also be greater than anticipated since a principal components and not a common factor model was employed.

The final review focuses on several factor analyses completed by Dudley (1991) on Toronto, 1986. Dudley explored factor analysis at two scales and five alternate aggregations. This study, unlike those alluded to previously, was first to employ a sample of aggregations to ascertain scale effects. Of these five aggregations, three aggregations were spatially assembled by census tracts, by grid allocation of enumeration areas, and by clustering observations by proximity. The two other aggregations were formulated from cluster analyses on ethnic and occupational variables. Since only two aggregations strictly followed contiguously formed groups, there is doubt whether this small number of aggregations is sufficient to examine scale and aggregation effects.

The results found are certainly interesting. Dudley attains results which are not apparent elsewhere in the literature. At the smallest scale he finds six significant eigenvalues, i.e., principal component eigenvalues greater than one, accounting for 80.57% of the variation in the data set. When increasing the scale of analysis, the number of significant eigenvalues increases to eight for all five alternate aggregations. This increase in the number of significant eigenvalues with scale is unexpected. Additionally, the percentage of variance increases only slightly with scale, and this increase is almost solely produced by the larger number of factors.

At first, the results from Dudley cast doubt upon the hypotheses of this study. Nevertheless, unless other studies validate these findings by Dudley, the results should only be considered as an anomaly. Despite Dudley's contradictory results, the hypotheses of this study will not be altered.

Several attempts to determine the impact of scale effects on factor analysis were accomplished, while very few endeavours at discerning aggregation effects were attempted. Since the above results were normally established by one aggregation for each scale, the effectiveness of these studies must be pondered. These findings may come from extreme cases and could be unlike average effects. Additionally, some studies may have employed factor analysis with the inappropriate number of observations to variables. Cattell advises using a ratio of four observations for every variable (Cattell, 1952). Although employment of this ratio for purely descriptive analyses is optional (Rummel, 1970), these studies assess the robustness of factor analysis to MAUP. With attempts to assess robustness, the suggested observation to variable ratio may be necessary to make such inferences. Despite widely contradictory opinions forwarded by each author, reviews of the studies illustrate resemblance of many findings, except Dudley (1991). For those authors who concluded falsely about scale effects, these errors were partially attributable to their myopic examination of only the factor pattern/structure matrix. As well, not one study examined the initial correlation matrix for increasing correlation coefficients with scale. Finally, there is a void of research on scale and aggregation effects upon factor analysis. This study is expected to help develop an informed base for determining and estimating MAUP effects on factor analysis.

2.4: Other Relevant Work on MAUP

Factor analysis is not the only multivariate statistical technique evaluated for scale and aggregation effects. The following section provides a sample of the other multivariate techniques inspected. First, is a review of the effects of MAUP on the multiple regression family. The next section evaluates the invariance of marketing and economic models such as spatial interaction models and input-output analysis. From these brief reviews, it should be apparent how MAUP can exist in many distinct multivariate models.

This section reviews the effects of MAUP on the limited regression family. Linear multiple regression, logit, and Poisson regression models are all briefly examined. Although the uses of the various models may differ, they are grouped in this section for convenience.

Fotheringham and Wong (1991) assessed the impact of scale and aggregation on the linear multiple regression and logit models. They analyzed 871 block groups in the Buffalo Metropolitan Area in 1980. From analyses completed by aggregating the basic spatial units into 800, 400, 200, 100, fifty, and twenty-five units with twenty-five alternate aggregations for each, scale effects were found. To analyze aggregation effects, 150 alternate aggregations were completed with 218 observations, and as with scale effects aggregation effects were found.

Amrhein and Flowerdrew (1989; 1992) and Amrhein (1992) assessed the impact of scale and aggregation on Poisson regression modelling of Canadian migration flows. They began with the 260 census divisions of Canada, excluding territories, and subsequently aggregated this data set to 130, sixty-five, and ten areal units. Although noting some minor effects of aggregation, they posit the Poisson model may be invariant to the modifiable areal unit problem.

The effects of MAUP on statistical analyses are apparent in several disciplines of geography. In this section, the effects of scale and aggregation on techniques used in marketing and economic geography are displayed. It is also worth noting that the effects of scale and aggregation are apparent in location allocation models by Goodchild (1979) and Bach (1981). The reader is referred to these articles for the proofs.

Openshaw (1977c) examined the impact of scale and aggregation upon spatial interaction models. The examinations were completed with four SIMs (spatial interaction models) for 261 separate twenty-two and 87 separate forty-two zone partitions of the original 73 Durham counties. Openshaw established that SIMs are very sensitive to the zoning systems employed.

Putman and Chung (1989) examined several aggregation procedures on a spatial interaction model of the 108 basic spatial units of Minneapolis. Since the model they employed had many parameters, this study was unlike the Openshaw example. They tested aggregations based upon five separate criteria and thirty alternate aggregations for each. Although aggregation effects were apparent, the aggregating procedures ensuring equal population and equal basic spatial units were more stable. This finding also indicates that Openshaw and Taylor's (1979) range for the correlation was likely overstated.

To study the effects of spatially aggregated regions in input-output models, Blair and Miller (1983) examined theoretical and 1963 U.S. multiregional data. They concluded, " . . . spatial aggregation in multiregional input-output models produces 'acceptable' not large errors . . . . ." (Blair and Miller 1983, p.196). However, their sample used to assess scale and aggregation effects may have been too small to draw this inference.

As shown, MAUP affects many different models used in geography today. There are cases where invariance to these effects may exist, e.g., Poisson regression models (Amrhein and Flowerdrew, 1989; 1992 and Amrhein, 1992), but clearly more research must be completed upon this tenet. This section has been provided to afford the reader the opportunity to examine the general effect of MAUP in geography.

2.5: Summary

There is no quick and painless solution to the problem of arbitrary areal units. Instead, research should concentrate on the conditions generating the problems. Arbia (1989) has determined the parameters that change ecological correlations, and these findings should be employed for predicting effects upon other analyses. As well, Fotheringham (1989, p.222) lists several possible solutions to the MAUP issue including (i) derivation of optimal zoning systems, (ii) the identification of basic entities, (iii) sensitivity analysis, (iv) abandonment of traditional statistical analysis, and (v) shifting the emphasis of spatial analysis towards relationships that focus on rates of change. It is doubtful that creating optimal zoning systems satisfying the large number of variables analyzed in multivariate statistical analysis is possible (Fotheringham, 1989). Even what constitutes optimal is debatable as both Moellering and Tobler (1972) and Openshaw (1978) express different views. The other possible solutions are also unobtainable with certain data sets. There are several other avenues research on MAUP has been following as several are revealed below.

Besides the above list of possible solutions, increasing attention has also been devoted to fractals (Fotheringham, 1989), spatial entropy (Batty and Sikdar, 1982), and hierarchy of needs concepts (Dudley, 1991). These attempts offer some prosp ct for finding solutions independent of the modifiable nature of areal unit data. As well, Tobler (1989) has suggested the statistic employed, e.g., the Product Moment correlation coefficient, may be the source of the modifiable results and not the nature of areal data. He advocates the search for scale invariant statistics for employment in analyses. Along a similar vein, Carlin and Bendel (1989) state that ecological fallacies transpire because of the Product Moment correlation coefficient. They argue that a Bayesian estimate of the ecological correlation coefficient would provide more desired results. Finally, Wrigley (1993) has suggested that ecological fallacies could be removed by publishing individual characteristics of a small number of key variables. The technique involves estimation of individual level statistics for a group of variables from the key variables, and when the differences between the individual and group level statistics are removed the ecological fallacy should be removed. Although this technique offers some promise, it is doubtful that any manageable set of variables could be found at the individual level that accounts for a large portion of variation in a selected data set.

Despite the latest wave of attempts to eliminate MAUP from geography, it is unlikely that any of these will make significant inroads. The problem will persist due to the accessibility of areal unit data supplied by statistical agencies and the increasing use of statistical packages. Although it may be a cynical view, most research in geography will continue to report results based on areal unit observations without the slightest pretence for the effects of alternate scales and aggregations. Hopefully, this opinion is inaccurate and the MAUP issue will deserve the attention, as Openshaw (1984a, p. 6) stated, as the most serious unresolved issue in the discipline of geography today.

Chapter 3: Data Sets and Methodologies

This chapter addresses many conceptual problems arising during the development of this thesis. First, a section explores data set issues where the selection of variables and problems associated with the data sets are forwarded. The next section examines the spatial scales and aggregations this evaluation employs. The following section reviews the technique of factor analysis. Additionally, addressed are the factor analytic options available. Finally, the different means applied to examine factor analytic robustness are explored.

3.1: Data Sets

To ascertain the causality of scale and aggregation effects upon factor analyses employing areal unit observations, two theoretical data sets were developed. These data sets, created from the IMSL statistical library[3], are from a multivariate normal distribution and set on a twenty by

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[3] IMSL is a statistical and mathematical library of Fortran, and C, programs which may be employed by any researcher. The random variables from a multivariate normal distribution employed here, allows specification by the user for the number of variables and the covariances between each variables.

twelve rectangular lattice. A multivariate normal distribution is an assumption of factor analysis (Rummel, 1970), and it also helps to meet other suppositions. A multivariate normal distribution increases the likelihood of linear relationships between variables, reduces the probability of heteroscedasticity, and decreases the chance of limiting the correlation's range (Rummel, 1970). For these reasons, a multivariate normal distribution was chosen. The intervariable associations selected for the data sets were determined from a sample of empirical correlations. By selecting correlations in this fashion, these results should clone empirical data set results. It should also be acknowledged that the theoretical data correlations are different from the empirical correlations. The correlations deviate vastly, with some variables meagrely associated with others. With some poorly associated variables, the conviction of having strong communalities to complete a proper factor analysis is violated. For two distinct reasons, this study departs from this view. First, in purely exploratory factor analyses, some low communalities will be present in a data set. Furthermore, determining changes associated with variables having different relationships is an objective of this research. For these reasons, it was deemed worthwhile to include these correlations and data sets. There are differences in the two theoretical data sets and they will be explained below.

The two theoretical data sets differ only by the spatial dependencies among areal unit observations. To emulate data free of scale effects, the random variables were repositioned on the lattice[4], to remove spatial autocorrelations. Here, the summation of the ten absolute value spatial autocorrelations, measured by Moran's I with the below weighting scheme, was held to 0.05, or each spatial autocorrelation could average ABS(0.005).

$$I = \frac{\sum_i \sum_j (X_i - \bar{X}) \frac{1}{cc_i} (X_j - \bar{X})}{\sum (X_i - \bar{X})^2}$$

where

X_i - *is the ith observation*
X_j - *is observation j that neighbours i*
cc_i - *is the number of contiguous neighbours of i*

Although no constraints were placed on lagged cross correlations, they all gravitated around the zero mark, see Appendix C. There could be an objection of the constraints placed on theoretical data since the expected value for Moran's I statistic is (-1/(240-1)) and not zero (Cliff and Ord, 1973). Regardless, such differences are trivial and do not warrant any changes. Following Arbia (1989), this study uses rook's case contiguities to determine spatial

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[4] A Fortran program was created to reposition the random variables' values on the lattice. Depending on the objective set for this program, spatial autocorrelation could be maximized, minimized, removed, or placed anywhere in between.

autocorrelations. The weights for the I statistic are equal to the inverse of the total number of contiguous neighbours for a given observation which is equal to the inverse of the row summation in the contiguity matrix. This weighting adjustment avoided inflating the power of empirical observations that had many neighbours.

The second theoretical data set was created by repositioning the same random variables and bivariate correlations to introduce positive spatial autocorrelations, see Appendix C. The values of spatial autocorrelations for these variables are similar to those autocorrelations from the Saskatoon data. From the examinations of empirical spatial autocorrelations, the range of spatial autocorrelations was allowed to vary from 0.4931 to 0.2643. The lagged spatial autocorrelations were again unconstrained. The next question that needs to be addressed is why were two data sets created.

At this point, it is worth emphasizing the rationale for creating two similar theoretical data sets. Arbia (1989) has shown the importance of spatial dependencies on ecological correlations. Since correlations are scale invariant when aggregations are completed randomly without contiguity (Gehlke and Biehl, 1934 and Blalock, 1964), the characteristics unique to empirical spatial data are solely responsible for altering correlations. The results from data absent of spatial dependencies should, therefore, be analogous to aggregating without contiguity. As well, with absence of spatial autocorrelations, the uncontaminated data set does not violate any factor analytic assumptions. The data set containing positive spatial autocorrelations should provide results analogous to empirical data. From these two data sets, MAUP effects on empirical spatial data sets should be determined. At this point the empirical data set will be introduced.

The enumeration areas of the urbanized Saskatoon C.M.A. as displayed in Map 3.1.1 comprise the empirical data set. Saskatoon is a medium sized Canadian metropolitan area with populations of 210 023 in 1991 and 200 665 in 1986 (Statistics Canada, 1991). Saskatoon was chosen because of familiarity with this data set, and its size is manageable yet not restrictive. Absence and suppression[5] of data for several enumeration areas led to deletion of observations leaving 231 usable observations. To enable a properly functioning contiguity matrix, the geographic space consumed by these deleted enumeration areas had to be reallocated. The geographic space was allocated by a consistent allocation precept that appended this space to the largest neighbouring enumeration area within the same census tract boundary. Although the census tracts of Saskatoon are not entirely

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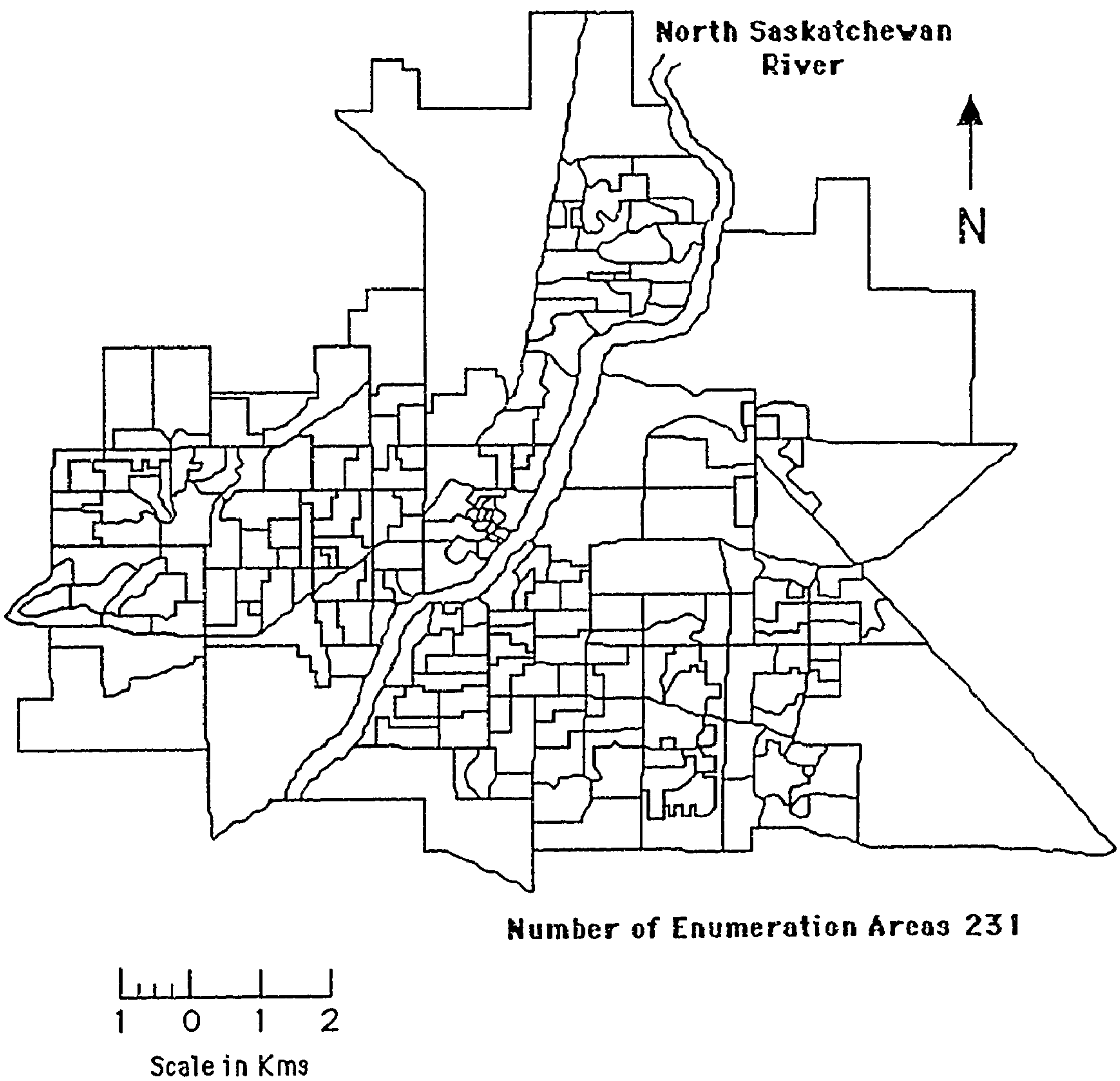
[5] Statistics Canada may suppress data in cases where the geographical areas are small or there are small cells in tables (Statistics Canada, 1986c).

Map 3.1.1: Study Area for Empirical Data: Saskatoon Enumeration Areas, Saskatoon, 1986.



Source: Statistics Canada, 1986a

homogeneous (Hunt, 1992), on average, the properties of the missing areal units should be analogous to the neighbouring observations in the same census tract. The Saskatoon data set employed here should be typical to those used in geographical analyses today.

Below are listed a number of problems associated with using enumeration area data. First, a condition known as the small number problem (Kennedy, 1989) affects the empirical data set. This problem stems from rating variables with either small numerators or denominators. With many small values found in variables at the enumeration area scale, large relative changes in measures can arise even when absolute changes are small. When this problem is combined with random rounding and several variables based on a 20% sample, the validity of enumeration area data analyses is questionable (Statistics Canada, 1981, p.25). Despite these serious concerns, there are several reasons why this study employs enumeration area data. First, and most important, the purpose of subsequent analyses with different scales and aggregations is to replicate enumeration area results. There is no attempt made to reproduce individual level results. Whether enumeration area level results drastically differ from the individual level results is extraneous. If the purpose here was to decipher urban themes, the validity of these results would be debatable. Moreover, with each subsequent scale of analysis the successively aggregated areal units should reduce many of these problems. Since variable selection for empirical data may profoundly affect results (Gorsuch, 1983), they were selected cautiously.

The variable selection for the empirical data follows the social area analysis themes found from past research (Shevky and Bell, 1955; Bell, 1955; Shevky and Williams, 1949). These social area analysis themes include economic status, family status, and segregation. These are not the only underlying constructs found in urban analyses, e.g., those recent themes found by Bourne (1987) and Davies and Murdie (1991). The research here did not include other urban constructs because of the small number of variables. To prove or disprove new urban hypotheses, it would be necessary to increase the number of variables from ten. However, the intent of this research is to examine scale and aggregation effects in factor analysis and not to decipher urban themes. Even when using traditional social area analysis factors, use of such a small data set is controversial. Provided below, is a list of the variables employed for this analysis. For more accurate variable interpretations, the reader should consult Appendix A. The variables portraying economic status are average family income, male unemployment rate, rate of highly educated population, and rate of workforce in blue collar occupations. For social status, the variables include ratio of young children to females in child bearing years, average household size, and the female labour participation rate. The rate of population with non official mother tongue language and rate of population born as an immigrant describes the ethnicity or segregation theme. Finally, one further variable was added, i.e., rate of migrant population. Although most of these variables are similar to those from traditional social area analysis studies, e.g., (Bell, 1955), addition of a housing stock variable was not possible because of insufficient data. There was also a conscious avoidance of count variables. If count variables were employed, enumeration areas differing greatly in population size would contain all extreme cases. The resulting size effect factor is typically useless as it is basically the population variable. Since variables were selected prudently with expected themes, the results from the empirical analysis should be alike other factorial ecologies.

3.2: Methodology

This next section describes in detail the aggregation procedures and number of variables employed for the data sets. First, the aggregation procedures are detailed below illustrating the differences between empirical and theoretical aggregations. As well, a scope of the number of analyses completed is also provided.

The aggregating procedures for the theoretical and empirical data differ and are completed with alternate objectives in mind. The empirical data set contains 231 enumeration areas while the theoretical data sets each have 240 observations. These initial observations are then aggregated through contiguity constraints to produce three additional scales of 120, sixty, and forty observations. The number of basic spatial units used for the theoretical data was 240 since it is divisible into all scales and approximates the 231 enumeration areas. Because at the largest scale there are only forty observations, selection of ten variables maintained a ratio of observations to variables of four : one (Cattell, 1952). For each scale, examination of thirty alternate aggregations should demonstrate the effects of aggregation on factor analysis. Furthermore, the same geographical aggregations were completed for both theoretical data sets to ensure consistent comparisons. All groups aggregated from the theoretical data sets contain an equal number of basic spatial units, e.g., at the forty group scale every group consists of six BSUs. To imitate spatial databases, this equal BSU criterion was relaxed for the empirical data set. The range of BSUs for the empirical groups are displayed in Table 3.2.1. As can be seen, the range each group can take becomes larger as scale is increased. The largest scale, forty observations, also approximates the 37 census tracts in Saskatoon (Statistics

Canada, 1986b). In all, 272 separate factor analyses are completed for these three data sets and four scales of analysis. A factor model program by Veldman (1967) was modified for this specific study so that the large number of factor analyses was manageable. Such a large sample should expose scale and aggregation effects upon factor analysis.

Table 3.2.1: Ranges of Empirical Groups.

Scale	**120 Groups**	**60 Groups**	**40 Groups**
Minimum BSUs	one	two	three
Maximum BSUs	four	seven	ten

3.3: Factor Analysis

This study is opposite of most geographic research completed. Rather than using a statistical technique to draw inferences about a spatial process, this study examines the variability of spatial processes to judge a technique's robustness. The method examined here is factor analysis, which is a popular method used in geography to substantiate the themes postulated about social area analysis. Within the factor model, there are many different options available. These options include extraction techniques, determining the number of significant factors, and factor rotation, among others. Although most factor analytic options lead to robust results, it is conceivable that some options may behave differently under aggregation procedures. Throughout the paragraphs below, the options selected for this study are provided along with the appropriate rationale. In almost every case, the selected options are the same as those options traditionally used by geographers, even when such a decision may be undesirable. By creating a model that is comparable to those used by other geographers, these results will be applicable to those of contemporary geographical factor analyses. First, before discussing the factor analytic options, the factor model employed for this analysis is illustrated.

Figure 3.3.1 displays the factor model selected for this study. Since most of the factor analytic options are dealt with in greater detail below, only cursory attention is addressed to the model here. There are two separate steps in completing the factor model for this analysis. The first step is concerned with identifying the number of significant factors and determining whether the data are adequate for the factor analytic model. The Kaiser-Meyer-Olkin (Kaiser, 1970; Kaiser and Rice, 1974) and individual measures of sampling adequacies are employed to assess the suitability of the data for the factor analytic model. After determining the number of significant factors, the next step is to complete the factor model with all the options shown in Figure 3.3.1. The following paragraphs deal with the various factor analytic

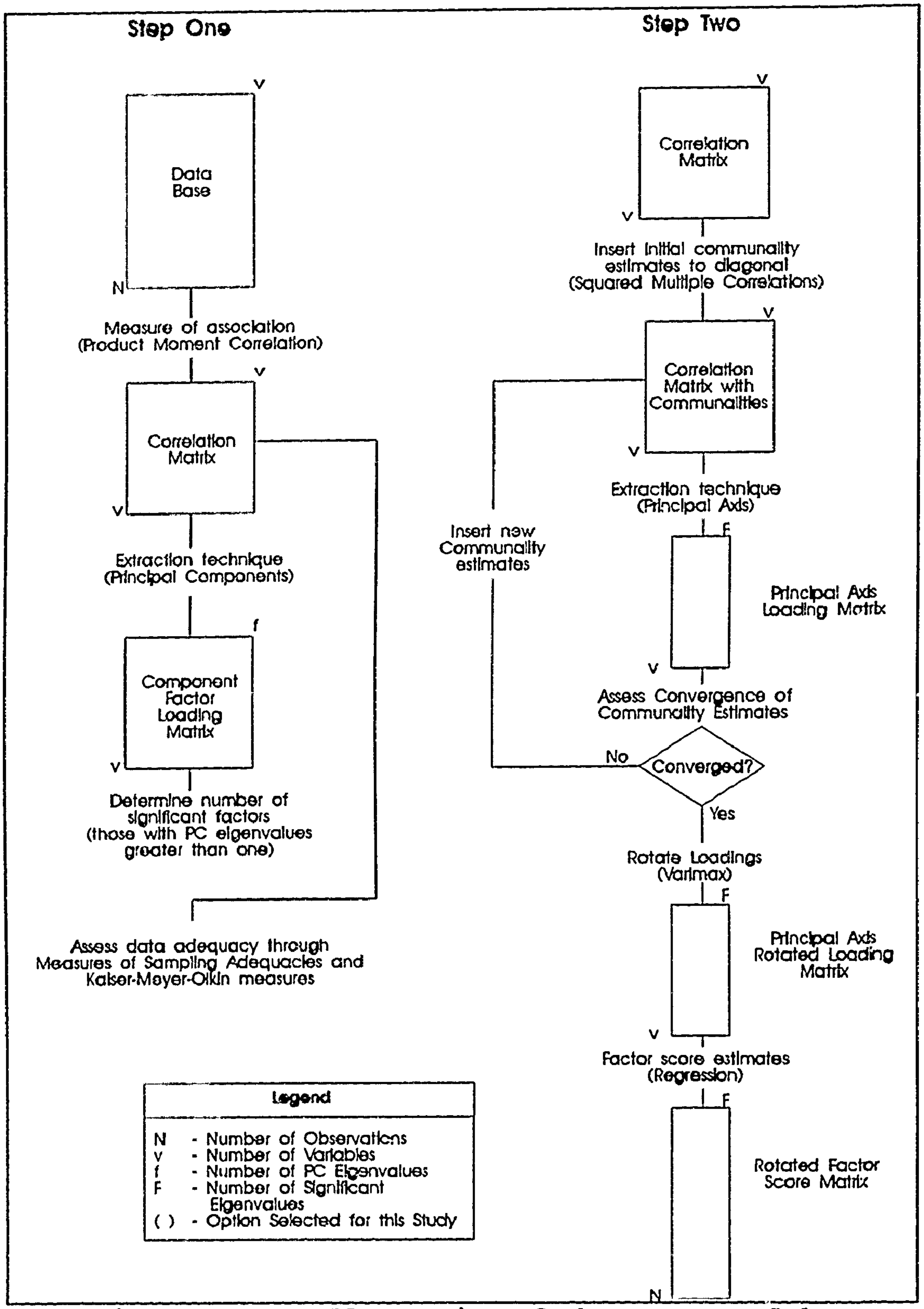


Figure 3.3.1: Illustration of the Factor Model.

options available to a researcher and the rationale for the options selected here.

The first factor analytic option available is the index of association. Many different measurements are available, depending in part on the measurement scales of the variables used, including intervariable correlation matrix, covariance matrix, and cross products and distance measures (Gorsuch, 1983). Geographic research normally uses the Product Moment correlation coefficient to produce the intervariable correlation matrix. This study will also employ Product Moment correlation coefficients, and the research here will draw heavily on Arbia's research on grouped correlations.

The next selection available is the type of extraction procedure used to derive the factor pattern and structure matrices. There are two general choices: a component or a common factor model. If one uses a component model, it is assumed that variation of any variable is explainable by all other variables. Since this assumption usually is incorrect, a common factor model with expected unique variances is used. An entire family of extraction procedures exists within the common factor model including principal axis, image, alpha, maximum likelihood, least squares, and generalized least squares. Of the common model possibilities, geographical research employs the principal axis factoring method most often (Davies, 1984). Principal axis factoring was selected, since it extracts the most data set variation with the fewest factors and reproduces the correlation matrix with the greatest accuracy (Gorsuch, 1983). Squared multiple correlations are inserted as initial communality estimates of the variables. This technique is again the most popular although others do exist. Communalities are measures, or initially, estimates, of how well the factor model explains the variation of a particular variable. The squared multiple correlations are the coefficients of determination for regressions using all other variables. It should be apparent that the extraction procedure and communality estimate used for this study are based on a strong contemporary tradition in geography.

After selection of an extraction procedure and a communality estimator, the selection of the number of significant factors from the data set is essential. There should be fewer factors than variables to ensure the principle of parsimony (Harman, 1976). If there are almost as many factors as variables, one must question the utility of conducting a factor analysis. Rules of thumb are available for selecting the number of significant factors. These include taking all factors having principal component eigenvalues greater than unity, explaining at least 5% of the data set variation, and lying above an appropriate break in the eigenvalue slope (Cattell, 1966). It should be sufficient to note that eigenvalues are indicators of the strength and size of extracted factors. The convention in geography is to select the number of factors with principal component eigenvalues greater than one. By taking only factors with principal component eigenvalues greater than one, it assures that every factor accounts for more variation than that associated with one variable. With only ten variables, it would be ludicrous to select those factors explaining 5% of the variation as the average variable accounts for 10% of the variation itself. The greater than 5% criterion rule of thumb should only be considered when the number of variables is greater than twenty. Scree plots are typically used in combination with other criteria, and despite their subjectivity they are powerful techniques. In fact, the eigenvalue slope break, as referenced by Gorsuch (1983), has been translated into an objective test labelled the CNG test. Selection of the principal component eigenvalue greater than one criterion allows comparisons of this study to many in geography.

After the extraction of eigenvalues and eigenvectors, i.e., the association of variables to a factor, through iterations of estimating communalities from the eigenvectors, there is a potential for a nonsense case to develop. This case, known as a heywood case from Heywood (1931), arises when the final communality of a variable exceeds one. Only a few options are available in this situation including altering the convergence criterion, selecting more factors (Harman, 1976), and halting the iteration procedure before this case arises. These cases are handled by terminating the iterations before a heywood case is reached and it is quite typical, although never mentioned, way to avert these cases. This does mean that the convergence criterion is not met, and this may cause minor problems in the final communality estimates.

Once the final iterated communalities become stable, it is often useful to rotate factor loadings to a simple structure (Thurstone, 1935). Rotation is available because the reference axes can be altered in space. The interpretation of factors is enhanced through rotation and is normally completed for any research. Two general rotations available are orthogonal and oblique. Orthogonal rotations assure independence between all factors. Oblique rotations allow correlations among factors, and thus factor scores can be used to reveal higher order factors. Orthogonal rotations are by far the most favoured technique in geography. However, popularity does not always correspond with appropriateness. Far too many geographers use orthogonal rotations only because factor loadings are easy to interpret, i.e., because the factor pattern matrix equals the factor structure matrix. As Perle (1977) stated, spatial processes are not required to be and usually are not independent of each other. As well, it is believed that an oblique rotation may be more scale invariant than an orthogonal rotation. This belief is spawned because correlations usually increase between all variables when scale is increased. Therefore, even relatively independent factors extracted from the enumeration area analysis should become more related with increasing scale. Obviously, orthogonal rotations cannot continue to identify the same factors at different scales if relationships between factors increase. This study employs the varimax orthogonal rotation (Kaiser, 1958) to correspond with traditional geographic factor analyses.

The final factor analytic choice available involves estimating factor scores. Factor scores are estimates, when using the common factor model, of the score a particular observation has for a given factor. Factor scores are derived from many techniques that reproduce scores for every observation and every factor. Depending upon the rotation employed, these factor scores are available for subsequent analyses including higher order factor analysis or multiple regression. The techniques for estimating factor scores include regression estimates, minimizing unique factors, and uncorrelated scores minimizing unique factors (Gorsuch, 1983). The analysis here uses the regression method.

One hypothesis outlined in Chapter One was not clearly defined. Now with an introduction to the factor model, the hypothesis is explained in greater detail. When increasing the scale of analysis with contaminated or empirical data, certain factor analytic statistics are expected to increase in size. These statistics include eigenvalues, communalities, and measures of sampling adequacies. Listed below are the techniques utilized to test all the hypotheses put forth.

Examinations of scale and aggregation effects upon factor analysis use several simple statistics. Factor analytic statistics including initial and final communalities and eigenvalues, rotated factor scores, and Kaiser-Meyer-Olkin (Kaiser, 1970; Kaiser and Rice, 1974) and individual measures of sampling adequacy are examined by the moments about the mean. Scale effects may be ascertained by viewing changes in the mean value of thirty aggregations for different scales. If any statistic increases or decreases systematically with changing scales, scale effects are present. Aggregation effects are noted by the dispersion about the mean the thirty aggregations produce. As well, comparisons of the deviations and ranges should indicate if aggregation effects increase with scale, as they should. If the distribution of aggregations for any statistic has a small variance, aggregation effects are trivial, whereas large deviations may question any analysis at a particular scale. All of the above assessments depend upon near normal distributions of the sample of a statistic's value. If distributions depart greatly from normality, the examinations of mean and variance are suspect. A RELATE program (Veldman, 1967) examines the invariance of factor structures. The program measures whether two separate factor structures are similar through correlation estimates. This procedure is discussed in more detail in the following chapter. Finally, from Arbia's (1989) work on the correlation coefficient, predictions of factor analytic results for the theoretical data sets are undertaken. The evaluation of factor analysis uses techniques that are far simpler than the technique being analyzed.

Chapter 4: Evaluation of Theoretical Data Sets

This chapter presents the results from the theoretical data sets. It is divided into six sections with the first section outlining the remainder of the chapter. The following section reviews the attributes of the two theoretical data sets employed for these analyses. Particularly, attention is forwarded to the contrasts and similarities between the data sets. Section three illustrates the effects of alternate scales on the factor analytic model. In this division, the averages of the thirty scale specific aggregations solely estimate scale effects. The fourth section examines aggregation effects on these theoretical data sets. The standard deviations of the thirty scale specific aggregations almost exclusively assess aggregation effects. Section five attempts to predict scale and aggregation effects. These estimates are based from the formula Arbia (1989) derived for the grouped correlation coefficient. The final section comments upon the hypotheses developed for the empirical data set results.

4.1: Theoretical Data Set Properties

Since Chapter Three outlines the theoretical data sets, this section only provides a brief review. The two theoretical data sets, which are laid on a twenty by twelve lattice, only differ in one respect. To change the spatial dependency between areal units, the positions of the values of random variables upon the lattice are altered. Positioning of one data set assures independence of areal unit observations. Rearrangement of the other data set allows dependency of areal unit observations, or in other words permits positive spatial autocorrelations. Table 4.1.1 displays the spatial autocorrelations, measured by Moran's I, for both data sets and all variables.

Table 4.1.1: Spatial Autocorrelations, Weighted Moran's I, for the Theoretical Data Sets.

Variable	Uncontaminated Data	Contaminated Data
1	0.0011	0.4085
2	0.0068	0.3593
3	0.0036	0.4054
4	-0.0075	0.4931
5	-0.0027	0.3158
6	-0.0042	0.2643
7	-0.0036	0.2752
8	-0.0076	0.2861
9	0.0152	0.3385
10	0.0087	0.3518
Expected Value	-0.0042	-0.0042

Two reasons exist for constructing theoretical data sets with different spatial dependencies. First, and most important, the positive spatial autocorrelations found in the contaminated data are similar to those found in empirical geographical data sets. The similarity between these two data sets allows development of comparable hypotheses for empirical data. Since uncontaminated data results should be analogous to randomly grouping observations without contiguity, these results should indicate what if any effects scale and aggregation have on data sets with any attributes. Furthermore, factor analysis requires implicit data assumptions. The assumptions include multivariate normality, homoskedacticity, no multicollinearity, and independence of observations (Davies, 1984, pp. 112-118). Since the uncontaminated data set satisfies these assumptions, the results will be uncontaminated. Because of the nature of areal unit data, geographic research commonly violates the independent observation assumption. This assumption, however, may not be essential for exploratory analyses.

The theoretical data sets also contain different properties, which are important. First, the theoretical data sets are arranged with specific bivariate associations among variables. The theoretical correlations were chosen after examining empirical relationships at the Saskatoon enumeration area level. It must be stressed that the relationships among theoretical variables are not identical to those in the empirical data set. If the same relationships were chosen, the peculiarities of a specific correlation matrix could obscure any general results. The results of these analyses, if similar among all the empirical and theoretical data sets, should be more plausible with different correlation matrices. Furthermore, the theoretical variables were selected without consideration of their relationship to potential factors. Since no specific factors are expected, e.g., no economic status factor, etc., the factors that are extracted from the theoretical variables will likely be more variable than those from the empirical data. In the empirical data where variables are chosen to represent certain factors, the variability of factor extractions should be considerably smaller. As well, some theoretical variables are poorly associated with many others because there are no expected factors. Finally, the aggregation algorithms that increase the scale of analysis contain important properties. When aggregation is undertaken, the same number of basic spatial units comprises every newly created group. This equality is necessary for several various reasons. First, with equal, internal sized, aggregated groups, these results may be predictable from the correlation coefficient analysis of Arbia (1989). These results also satisfy the suggestion by Haining (1990) of possessing matching scale processes operating on each group. The assurance of group equality also dispels any tenet that scale effects arise from unequal weights. The values for variables in aggregated groups are found by summing the basic spatial unit values and dividing them by the number of BSUs in a group. By aggregating in this fashion, the BSU level results can be reproduced. These aggregation procedures are very important in determining the effects that are found in this chapter.

4.2: Scale Effects on the Factor Model

This section assesses scale effects on the factor analytic model for both theoretical data sets. Means from the sample of thirty scale specific aggregations judge the effects of scale on statistics such as eigenvalues, communalities, and measures of sampling adequacies. Since changes in factor loadings and factor scores can also indicate potential scale effects, they too are examined. Finally, the rotated factor loadings are examined through a RELATE procedure. A summary paragraph reviews the general scale effects on these analyses.

The mean values of initial eigenvalues for all scales are illustrated in Figure 4.2.1 and Figure 4.2.2. These charts display the initial, i.e., principal component, eigenvalues for both the contaminated and uncontaminated, also called pure, data sets. If principal component eigenvalues alter with scale, a number of consequences can result. By far the most serious effect scale specific eigenvalues can produce, is to vary the number of significant factors. Traditionally, the number of significant factors is determined from those principal component eigenvalues greater than unity, above breaks in the eigenvalue slope, or greater than five percent of data set variation. If initial eigenvalues' sizes or slope varies with different scales, the number of significant factors selected from the data set may be modifiable.

As theorized earlier, Figure 4.2.1 unveils an increase in the prominent eigenvalues with scale for the contaminated data source. The increasing size of the major eigenvalues with scale suggests increasing generality among the principal factors since eigenvalues are equal to the column sum of squared unrotated factor loadings. The first eigenvalue extracted from the contaminated data ranges from a mean of 3.30 at the basic spatial unit level to a mean of 4.29 at the forty group level. With a range of almost one for this eigenvalue, the percentage of data set variation explained by this component increases ten percent from 33 to 42.9. The slope of the eigenvalues also exhibits some interesting trends. Around a value of one, the sizes of eigenvalues decrease with increasing scale. If eigenvalue slope breaks determine the number of significant factors, the task of selecting the appropriate number of factors becomes easier as

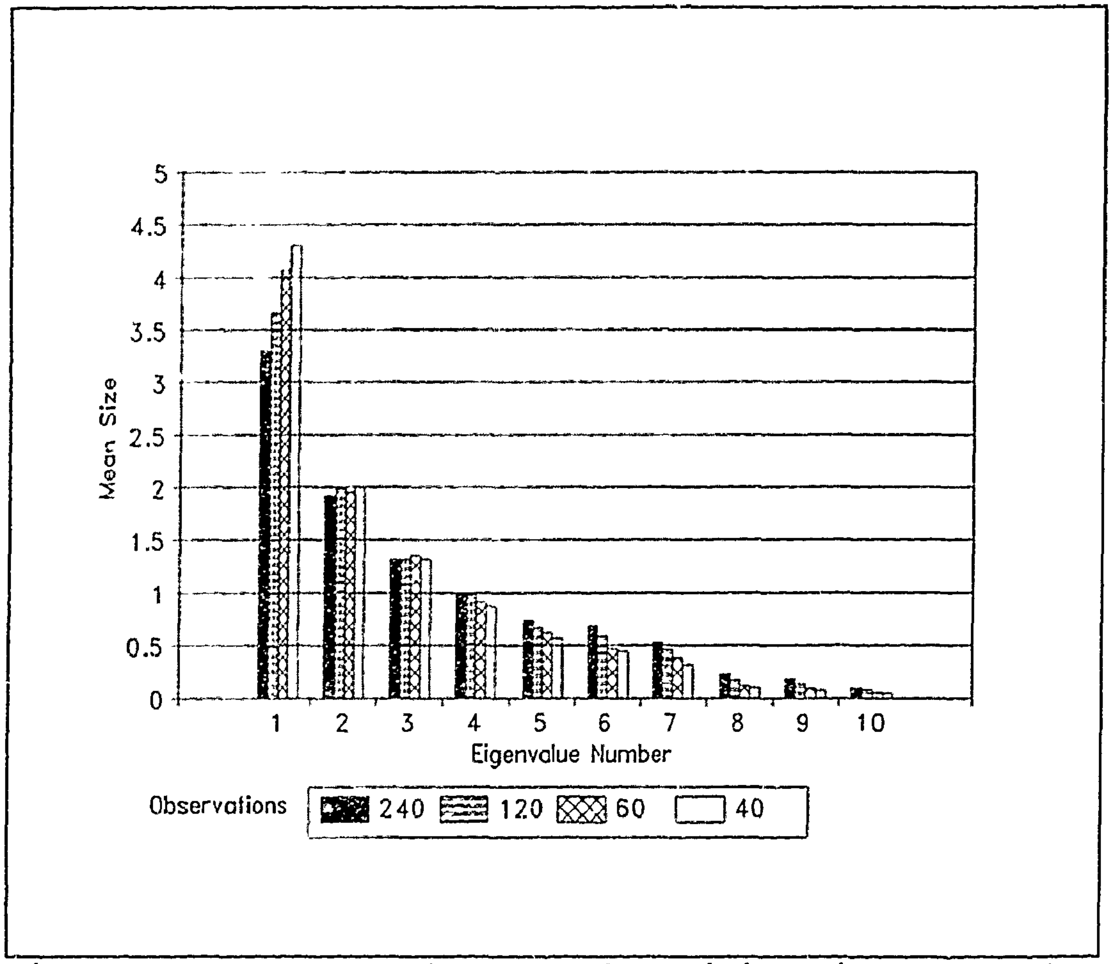


Figure 4.2.1: Scale Effects on the Initial Eigenvalues for the Contaminated Data Set.

scale increases. In terms of the actual number of significant factors, there will likely be fewer factors extracted as scale is increased. Therefore, comparisons of the same processes at higher scales may lead to fewer, but stronger, factors. The above results clearly illustrate that large scale effects exist on initial eigenvalues when variables contain positive spatial autocorrelations.

Figure 4.2.2. displays the means of initial eigenvalues, i.e., principal component eigenvalues, for the uncontaminated, pure, data set. The uncontaminated data results are presented to assure that the contaminated data results are not simply functions of sample size. As the figure displays, there is

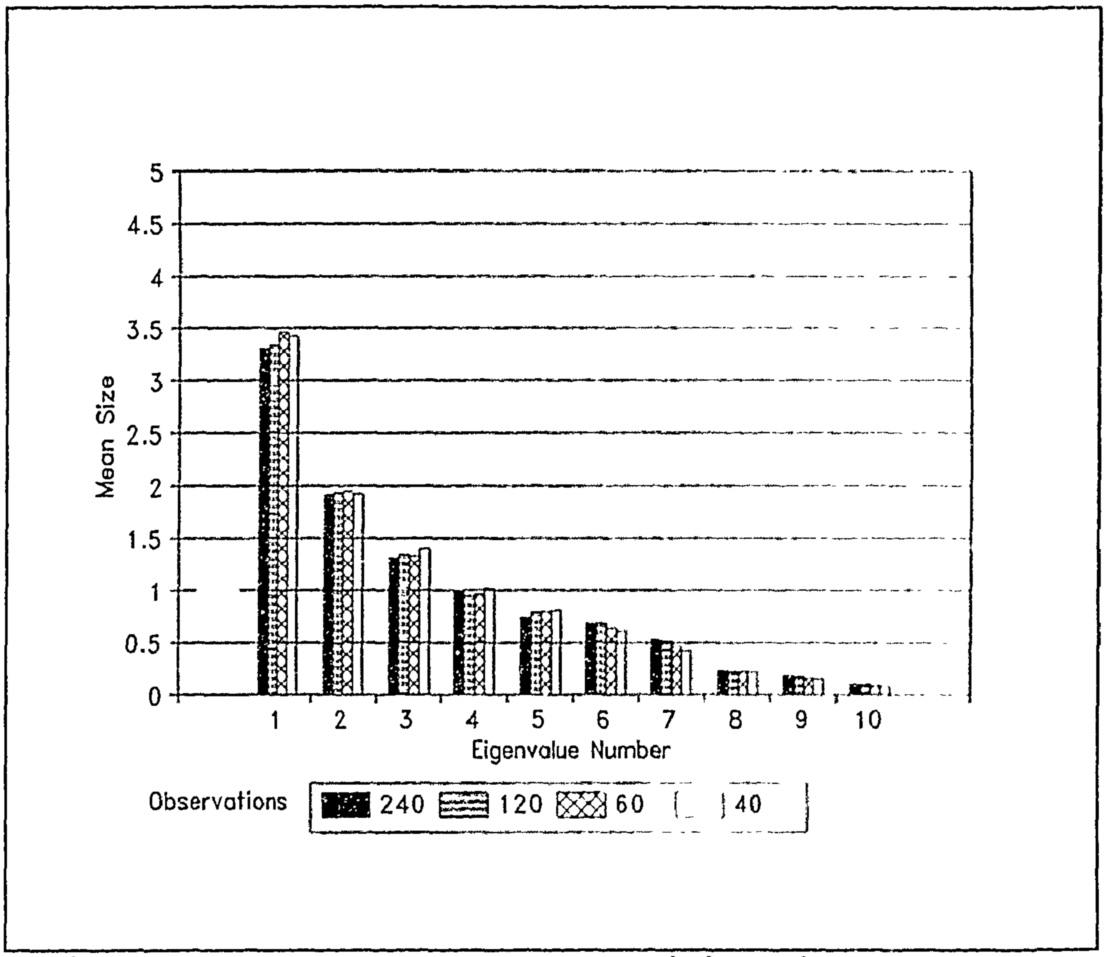


Figure 4.2.2: Scale Effects on Initial Eigenvalues for Uncontaminated Data Set.

little variation between these mean eigenvalues across scales. The first eigenvalue extracted only increases from 3.30 at the basic spatial unit level to 3.42 at the forty observation level, hardly a symptom of scale effects. The apparent absence of scale effects with pure data suggest scale effects arise from interactions of spatial dependencies and spatial configuration of the data as Arbia (1989) suggested. As well, the absence of scale effects implies that aggregating groups without contiguity constraints or with contiguity and variables free of areal unit dependencies leads to unbiased estimates for any scale. It should be stressed that unbiased estimates do not suggest that data void of spatial autocorrelations will always provide unbiased results. Unbiased results are only found when employing the mean of a sample of aggregations. For any individual aggregation, there could be disparities from the mean due to aggregation effects. It is obvious from the above results that scale does not alter initial eigenvalues when the variables are void of spatial dependencies.

Displayed in Table 4.2.1 are the mean values for the final eigenvalues extracted, i.e., before rotation. The results are similar to the findings for principal component eigenvalues above. Final eigenvalues are determined from the convergence of factor loadings and final communalities through iteration procedures. Final eigenvalues and eigenvectors reproduce a correlation matrix by fewer themes, i.e., the principle of parsimony. As well, the size of final eigenvalues relates to the size of unrotated factor loadings, and if eigenvalues become larger with scale the loadings should become general in appearance. Although there are fewer final than initial eigenvalues, the largest eigenvalues increase with scale for the contaminated data. The larger eigenvalues also are revealed by the average percentage of variation explained by the first three factors. The first three factors on average account from 53.3% to 66.5% for contaminated data compared to 53.3% to 57.4% for the pure data. It is obvious from these differences that increasing scale with contaminated data leads to greater explanatory power of factors. Only the first three eigenvalues were totalled to determine percent of variation explained since the number of significant factors is modifiable. From inspection of mean sizes of eigenvalues, scale effects are large and present when variables have positive spatial autocorrelations. The data set void of spatial dependencies is, to much extent, void of scale effects. Since the contaminated data was created to mimic empirical data, scale effects on the empirical eigenvalues should be present and large.

The mean values of the initial and final communalities display some scale effects. Initial communalities, which are squared multiple correlations, always increase with scale for both data sets although the increase is more pronounced for the contaminated data set, see Figure 4.2.3. The average increases in communality size across extreme scales are 0.20

Table 4.2.1: Scale Effects, Means, on Final Eigenvalues.

Scale	Eig 1	Eig 2	Eig 3	Eig 4
Contaminated Data				
240 Obs	3.05	1.49	0.79	NA
120 Obs	3.45	1.63	0.82	0.44
60 Obs	3.87	1.60	0.88	0.44
40 Obs	4.11	1.68	0.86	0.49
Uncontaminated Data				
240 Obs	3.05	1.49	0.79	NA
120 Obs	3.11	1.54	0.85	0.56
60 Obs	3.24	1.56	0.89	0.52
40 Obs	3.21	1.57	0.96	0.58

from the contaminated data and 0.09 from the pure data. Even the smallest communalities became greater as variable seven increases with scale from 0.10 to 0.35 for the contaminated data set. With modifiability of initial communalities, several variables could be deleted from the model depending on the scale chosen. If a variable has a low communality estimate, it may be excluded from factor analysis or other explanatory variables may be added to increase its communality. As well, in situations where all communalities are high, a principal components not a common factor model may be employed. With the variability of initial communality estimates, employing initial communality sizes to evaluate the appropriateness of variables is questionable.

The pure data set results also display larger communalities with scale, albeit at a slower rate. At first, increases in these communality estimates seem to contradict

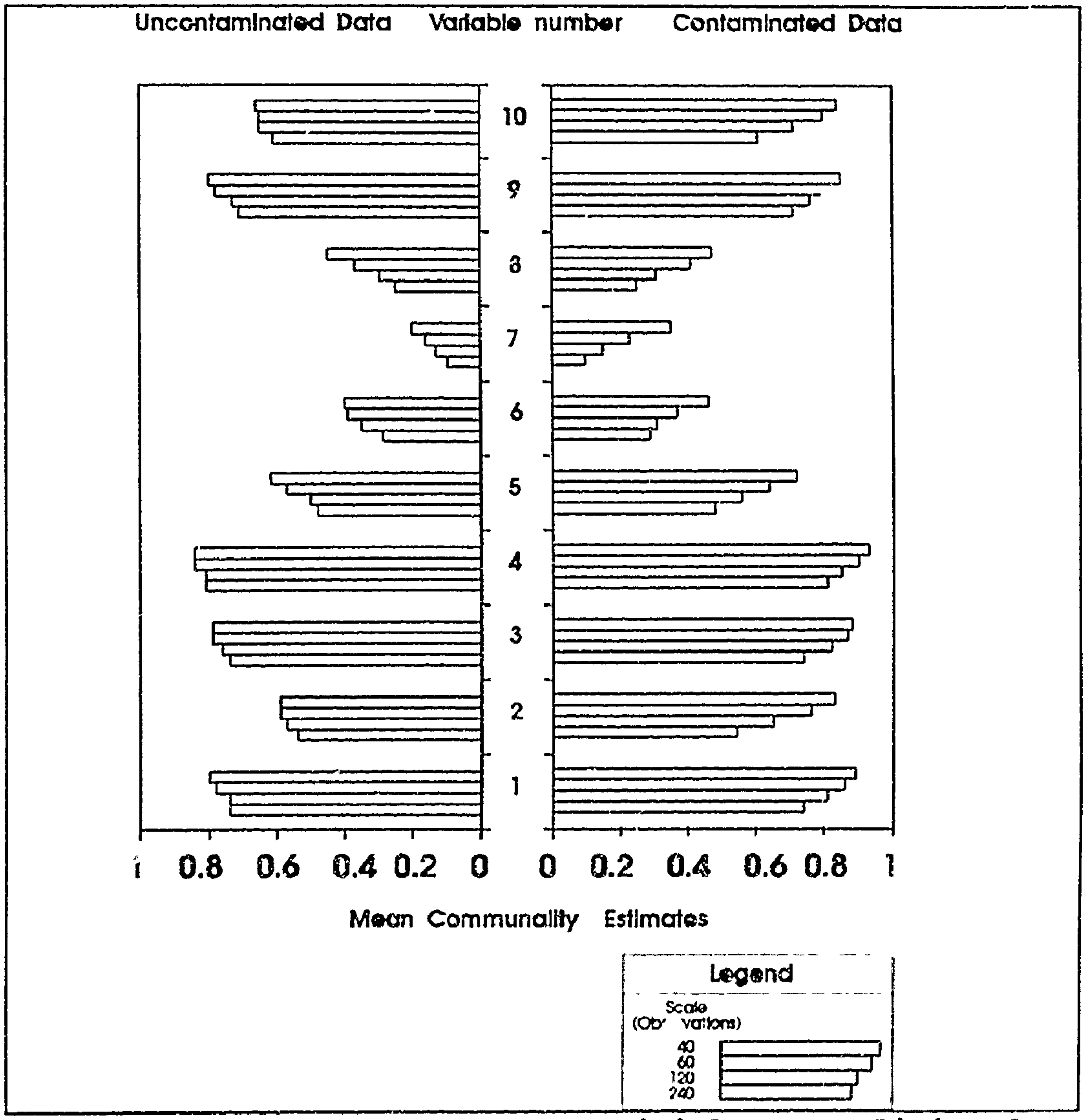


Figure 4.2.3: Scale Effects on Initial Communalities for Both Theoretical Data Sets.

the hypothesis of unbiased results with pure data. However, as Section 4.4 explains, the label of uncontaminated data is a misnomer. The interplay of small spatial autocorrelations and lagged cross correlations in these data leads to slight scale effects. With some scale effects present, the increasing communalities are simply artifacts of residual dependencies left in the pure data. The effects of scale on initial communalities clearly indicate increasing biases in estimated values.

Scale effects upon final communalities are much more difficult to clarify, see Figure 4.2.4. Before discussing the results, it should suffice to say that final communalities are found by iterating the eigenvectors and communality estimates. Since communalities symbolize the size of squared row loadings, i.e., the summation of all squared factor loadings on a given variable, they display the model's ability to replicate the variance of a variable. With contaminated data, final communalities generally increase with scale. The increase is as large as 0.28 for variable two of this data set. For the pure data, there is less evidence of increasing communalities with scale. Because the estimating procedure for final communalities is iterative, scale effects on final communalities are less than hypothesized. Constraint problems can arise during iteration since communalities cannot exceed one. With the contaminated data, three of the ten final communalities cannot vastly increase since their values are all around one. For the variables with lower communalities, the trend of increasing size with scale appears more reliable. Despite weaker scale effects with final communalities, sufficient evidence exists to confirm that changing scale alters communality estimates.

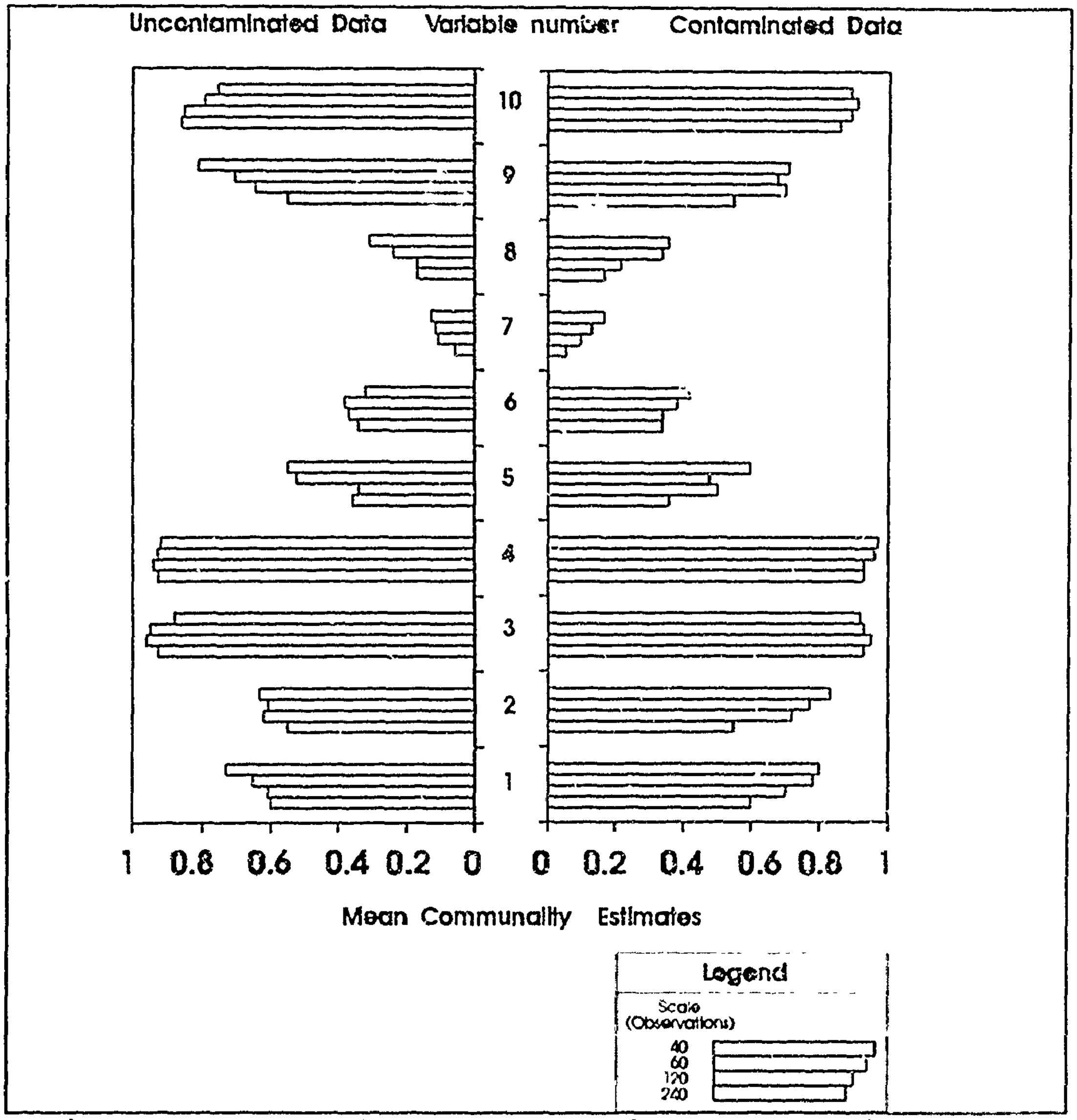


Figure 4.2.4: Scale Effects on Final Communalities for Both Theoretical Data Sets.

Larger communalities with scale seriously influence the interpretation of factor analysis. First, since there is a close relationship between final communalities and factor loadings, increasing final communalities implies increasing loadings. Larger factor loadings can have substantial impact on the identification of factors. If factor loadings greater

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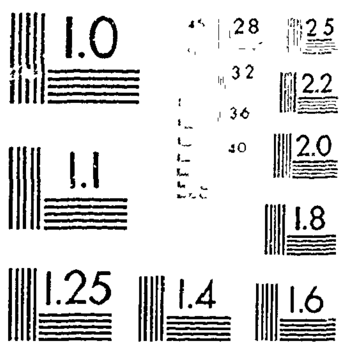

than some arbitrary value, e.g., ABS(0.70), are only interpreted, the number of interpretable variables for a factor is modifiable. With changes to the number of interpretable variables, the identification of factors may also be variable. As well, Davies (1983) has suggested a different way to determine the number of significant factors. Davies suggests that when adding or deleting a factor from the analysis does not alter final communality estimates the correct number of factors to extract has been found. With unstable communality estimates with scale, using Davies' approach to find the significant number of factors is also likely defective. Since communalities change with scale, interpretation and confidence placed upon a factor model may be modifiable.

Individual measures of sampling adequacy (MSA) and Kaiser-Meyer-Olkin (KMO) behave differently under contrasting data characteristics as shown by Table 4.2.2. Values of the MSA and KMO identify whether a correlation matrix or individual variables are adequate to be factor analyzed. MSA and KMO behave in systematic ways depending upon several data set characteristics. Measures of sampling adequacy and KMO are known to increase as the number of variables increases, as the effective number of factors decreases, as the number of observations increases, and as the general level of correlations increases (Kaiser, 1970, p.405). With contaminated data, forces working in opposite directions leave the statistic almost constant. The reduction in number of observations with scale reduces the KMO, but the decreasing effective number of factors and increasing overall correlations raise the KMO value. As stated above, in this example the overall effects on the KMO and MSAs are scale invariant values. The range of the contaminated data means for the KMO is from 0.58 at the basic spatial unit level to 0.60 at the forty observation scale. Pure data results decrease from 0.58 to 0.51 through the extreme scales. The decreasing KMO with pure data is undoubtedly the effect of reduced observations with scale since the general level of correlations and number of significant factors are relatively constant. From these results, the KMO statistic may be both scale invariant and a suitable statistic to employ for sampling adequacy with empirical data. However, a word of caution is provided since different data set attributes and scales may cause predictable changes in the KMO.

To assess the invariance of factor structures across scales and aggregations, a RELATE program by Veldman (1967) was used[6]. The RELATE procedure measures the similarity of rotated factors, and the coefficients from the tests, i.e.,

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[6] When using the RELATE program provided by Veldman (1967), all variables should be converted into double precision to avoid potentially nonsense results. These nonsense results can occur because of rounding problems associated with very small eigenvalues.

Table 4.2.2: Scale Effects on KMO and Individual Measures of Sampling Adequacies.

	Contaminated Data				Uncontaminated Data			
STAT	**240**	**120**	**60**	**40**	**240**	**120**	**60**	**40**
KMO	0.58	0.60	0.61	0.60	0.58	0.56	0.53	0.51
MSA1	0.55	0.59	0.62	0.63	0.55	0.53	0.52	0.49
MSA2	0.61	0.60	0.60	0.59	0.61	0.62	0.61	0.60
MSA3	0.65	0.67	0.70	0.72	0.65	0.60	0.58	0.55
MSA4	0.64	0.68	0.69	0.66	0.64	0.64	0.61	0.58
MSA5	0.36	0.40	0.40	0.43	0.36	0.33	0.30	0.31
MSA6	0.49	0.58	0.57	0.57	0.49	0.44	0.39	0.41
MSA7	0.64	0.61	0.63	0.54	0.64	0.67	0.58	0.48
MSA8	0.35	0.33	0.32	0.23	0.35	0.29	0.29	0.30
MSA9	0.52	0.56	0.60	0.60	0.52	0.52	0.51	0.51
MSA10	0.71	0.70	0.69	0.67	0.71	0.70	0.72	0.70

cosines, are interpretable as correlation coefficients. Therefore, if factor analytic invariance exists these correlations should be close to one when comparing factors. It is important to mention that the RELATE program standardizes the sum of squared factor loadings on each variable to one. Standardization of factor loadings facilitates comparisons between two factor structures. The cost of facilitating comparisons is that if the unique variances of variables systematically change among scales, the standardization will remove these differences. Provided below, is an example illustrating the potential caveats of standardization. Here two factor structures are compared for two factors and one variable. If the factor loadings are 0.8 and 0.6 for the first factor structure and 0.57 and 0.42 for the second factor structure, the RELATE procedure would identify a perfect positive correlation. Since scale effects are present in final communalities, this procedure overestimates the similarity of factor structures across scales. As well, the interpretation of factors across two scales may be significantly different though their structures are identical. The procedure is, however, used to display the similarity among factor structures although the results are obviously exaggerated.

The following analyses are completed by assessing every aggregation at one scale by every aggregation at another for both data sets. There are thirty tests for comparisons of the BSU level with any other scale and 900 for any other comparisons of scales. Since the order of rotated factors is unimportant, results are aggregated for replication of any two factors. Results are displayed by the rate of replication of any factor for correlations of ABS(0.90) or greater, ABS(0.80-0.90), and ABS(0.70-0.80). Absolute correlation values are selected since it is thought to be inconsequential whether factors are negatively related. Additionally, no factors correlating less than ABS(0.70) are shown since more than two factors can correlate with a factor below ABS(0.70). From the subsequent analyses, the effects of factor replication should be ascertained.

The results from the contaminated data are similar to those of the pure data as displayed in Figure 4.2.5 and Figure 4.2.6. Generally, the rate of factor structure replication is

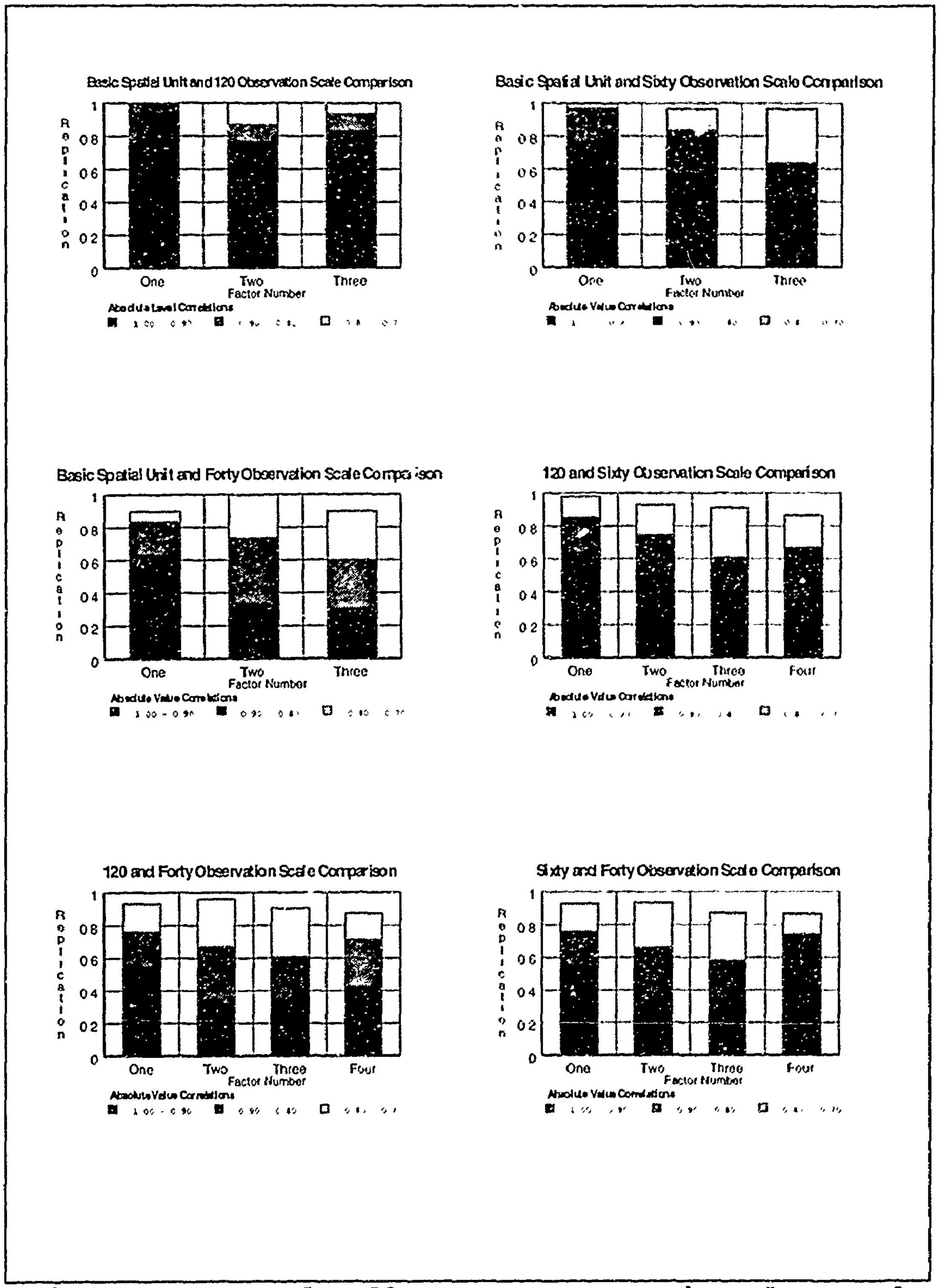

Figure 4.2.5: Scale Effects Upon Uncontaminated Rotated Factor Loadings as Measured by RELATE Correlations.

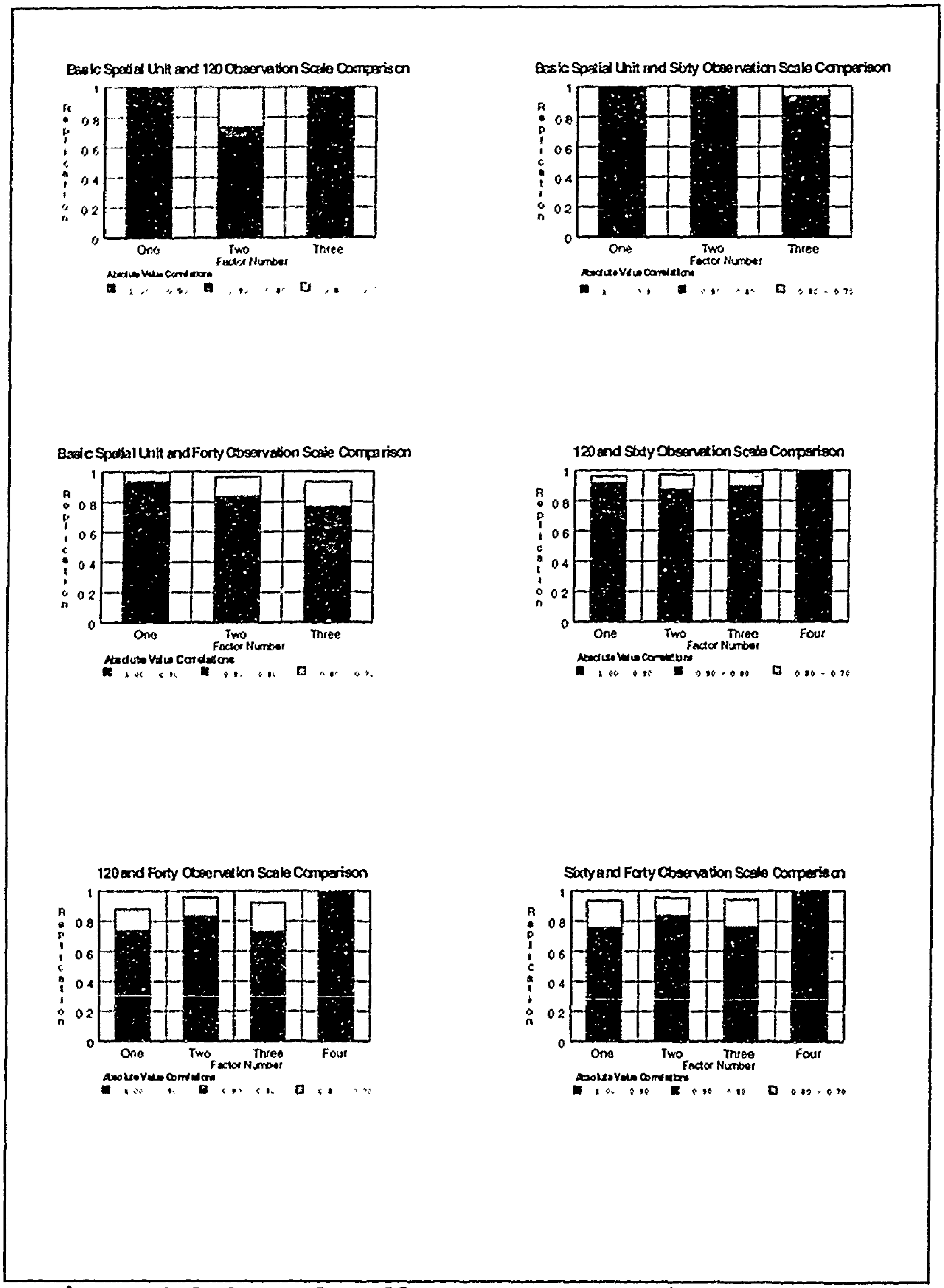

Figure 4.2.6: Scale Effects Upon Contaminated Rotated Factor Loadings as Measured by RELATE Correlations.

excellent as only four of the forty-five factors examined for all scales and data sets are below 90% when compared at greater than ABS(0.70). Within the range of correlations some trends do appear. The uncontaminated data results display some heterogeneity among factor structures with increasing scale when measured at the ABS(0.90). When comparing scales from BSU to 120 observations and BSU to forty observations the rate of factors replicating ABS(0.90) decreases from about 0.80 to 0.40. The same also holds when comparing results from the 120 observation level to both the sixty and forty observation scale. Figure 4.2.5 indicates that as scale is increased from the BSU level, the rate of factor structure replication decreases. As well, when aggregations are further from the BSU level scale, e.g., comparison of the sixty to forty observation scale, the overall replication of factors is reduced. Despite some indications of scale effects, the results from the uncontaminated data here show some semblance of invariance between factor structures of different scales.

The RELATE results for the contaminated data are mainly similar to the pure data results. Again an increasing difference between the two scales analyzed displays subtle contrasts in the replication rates of factor structures. Comparisons of results previously aggregated from the BSU level are also less well replicated than results compared to BSU level results. The most notable point of the contaminated data results lies in its contrast from the pure data results. Factor structure reproduction rates are greater for the contaminated data than the uncontaminated data. At the ABS(0.90) correlation level, the contaminated data results are slightly better reproduced across scales than with the pure data. The reason scale invariance is larger for contaminated data factor structures than pure data is due to the spatial dependencies of the data sets. With positive spatial autocorrelations, contiguously aggregating BSUs should maintain the similarities of relationships at the BSU level. When these groups are compared with groups of another scale the same structures should be identified. If aggregated groups are formed contiguously with no spatial autocorrelations, there is a greater opportunity for several groups to be quite distinct from other aggregates. In essence, the reason scale effects are larger with uncontaminated data than contaminated data is that all aggregates are compared to each other and not only the mean values. It is satisfying for empirical analyses that scale effects are slightly reduced when variables contain positive spatial autocorrelations. However, since scale effects exist on final communalities with contaminated data, it is likely that scale effects are present when identifying factors.

Examination of the first unrotated factor loadings helps substantiate the tenet of increasing generality of factors.

Only the first unrotated factor loadings are examined, since principal axis factoring extracts the greatest amount of data set variation in the first factor. Therefore, if evidence of generality exists, it should be most evident in this first factor. From the basic spatial unit level results derived for both theoretical data sets, i.e., because at the BSU level both factor analyses are identical, there are three factor loadings greater than ABS(0.70). Table 4.2.3 displays the average[7] factor loadings for both data sets and all aggregated scales. When employing the contaminated data set and reducing the number of observations to forty, the average number of factor loadings greater than ABS(0.70) increases to five. The larger number of factor loadings greater than ABS(0.70) illustrates the increasing generality of this factor. Results from the uncontaminated data show no similar trend. The number of factor loadings greater than ABS(0.70) decreases to two at the forty observation level. The increasing generality with scale found in the contaminated data can influence interpretation of unrotated factors. Usually factors are only interpreted after rotation, but it is often useful to inspect the largest theme of a data set. It is possible that varimax rotation eliminates some generality, although it is doubtful all would disappear.

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[7] In some cases, the unrotated factor loadings were reflected so they would all be in general agreement. Without this reflection, calculation of the average factor loading would be misleading.

Table 4.2.3: Average Factor Loadings on the First Unrotated Factor.

	Contaminated Data			Uncontaminated Data		
	Number of Areal Unit Observations					
Var	120	Sixty	Forty	120	Sixty	Forty
1	0.76	0.81	0.84	0.68	0.70	0.69
2	0.54	0.61	0.64	0.52	0.53	0.55
3	0.79	0.81	0.92	0.75	0.75	0.69
4	-0.92	-0.92	-0.93	-0.89	-0.90	-0.88
5	-0.31	-0.32	-0.38	-0.24	-0.28	-0.27
6	0.12	0.12	0.16	0.00	-0.02	-0.08
7	0.20	0.29	0.32	0.21	0.18	0.18
8	-0.00	-0.07	-0.07	0.00	-0.02	-0.05
9	0.70	0.75	0.77	0.68	0.72	0.75
10	0.68	0.74	0.75	0.66	0.65	0.61

The final examination of scale effects on the theoretical data sources involves examining the distributions of the rotated factor scores from the first factor. Only the first rotated factor scores were examined for the sake of brevity. The summary statistics associated with the factor scores are displayed in Appendix B. Factor scores are important in geography for identifying and interpreting factors. The identification of observations with extreme scores typically aids in labelling and identifying a particular factor. From the analysis, the range of scores decreases and the kurtosis becomes increasingly negative, i.e., platykurtic, with scale. The contaminated data set shows a larger reduction of extreme scores than the pure data. This is unexpected as positive spatial autocorrelations were thought to maintain the extreme cases. Whereas, aggregating a data set with no autocorrelations was thought to reduce outliers through averaging unlike cases. Since scale effects restrict the range of values factor scores can take, the interpretations of factors and spatial structures are much more difficult.

Apparently, scale effects manifest themselves in factor analyses when variables have positive spatial autocorrelations. Even in data void of autocorrelations some scale effects exist, but they are significantly less pronounced. Scale effects are serious and may lead to many problems for any researcher. It is possible for researchers to place tremendous confidence in results completed at a highly aggregated scale because of high explanatory power and associated high factor loadings and communalities. These same researchers, however, may question the utility of conducting the same factor analysis at lower scales. It is cautioned here that altering the number of observations can substantially influence the factor model.

Scale effects were determined from comparisons of a statistic's mean value from thirty aggregations. Drawing results from means eliminated the effects of aggregation on these results. It is now time to decipher whether aggregation effects are important in the factor analytic model.

4.3: Aggregation Effects

Examinations of the theoretical data sets now change to assess the effects of alternate aggregations. Since at the basic spatial unit level results are void of aggregation effects, i.e., only one aggregation is available, this section excludes the BSU scale. For each other scale, thirty separate aggregations were completed. From these different aggregations, the actual effect of aggregation on factor analysis should be resolved. The standard deviations of a statistic from the thirty aggregations at a scale almost solely determine aggregation effects. Employing standard deviations to assess aggregation effects would be disputable if these distributions are not normal. Since most of these distributions do not depart drastically from normality, the standard deviation represents aggregation effects. The distributional characteristics, i.e., kurtosis, skewness, range, maximum, and minimum, are all displayed in Appendix B. Although this information is relevant and is referenced, there is no feasible way to incorporate it into text. As with the section on scale effects, the initial and final eigenvalues are first examined. Examination of initial and final communalities and measures of sampling adequacies including the KMO follows the eigenvalues. Finally, the RELATE procedure outlined earlier examines whether factor structures are invariant to aggregations.

Figure 4.3.1 displays standard deviations from the sample

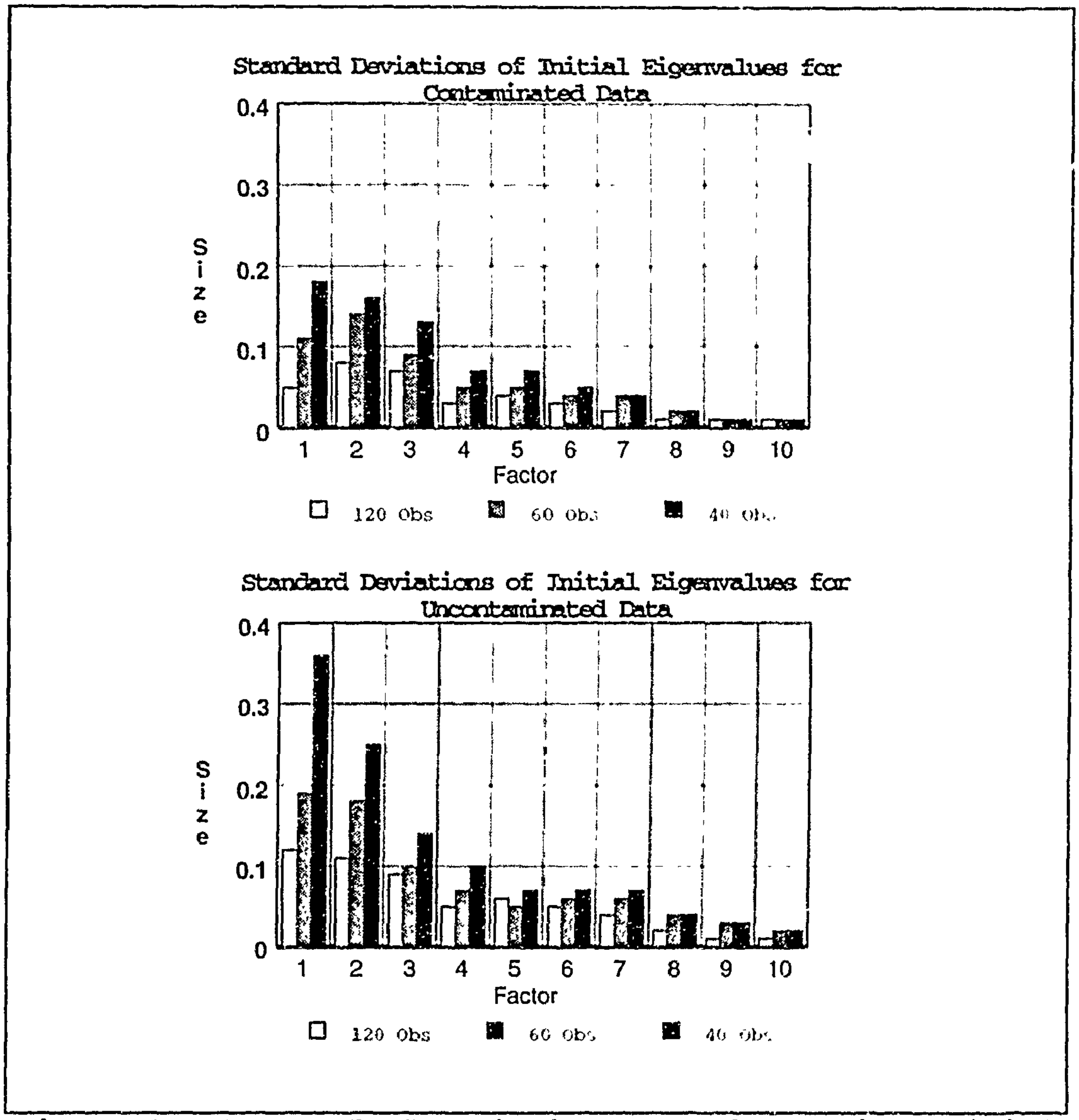

Figure 4.3.1: Standard Deviations Associated with Initial Eigenvalues.

of initial eigenvalues for both data sets and all available scales. The diagram illustrates two striking points. First, standard deviations increase with scale. This increase suggests, as was expected, that it may be misleading to obtain results from only one aggregation. The increasing deviations with scale induce many problems most notably concerning the selection of the appropriate number of factors. Indeed, in the pure data example at forty observations, the fourth eigenvalue ranges from 0.81 to 1.24. If significant factors are determined from those having principal component eigenvalues greater than unity, analyses holding the scale constant can provide dissimilar numbers of significant factors by rearranging the study area partitions. Modifiability in the selection of significant factors produces difficulties in replicating rotated factors. There are at least three significant factors for both data sets and all scales, but some scales contain as many as four significant factors. Four significant factors are found in five, twelve, and sixteen of the thirty aggregations for the 120, sixty, and forty observation levels with uncontaminated data. The contaminated data has nine, one, and one, aggregations with four significant factors for the same scales as above. The second important point from Figure 4.3.1 is the deviations surrounding the uncontaminated data are larger than their counterparts in the contaminated data. This finding substantiates the hypothesis and statements by Arbia (1989) that aggregation effects are larger when data are free of spatial autocorrelations. In terms of aggregation effects, when variables have positive spatial autocorrelations, results are less sensitive to aggregation effects than those from pure data. Since positive spatial dependencies exist in most empirical data, this finding is encouraging. Despite more invariance with contaminated data, aggregation effects are large when the number of areal unit observations is sufficiently decreased.

Table 4.3.1 shows the standard deviations for the final eigenvalues for both data sets and all scales. The same trends of increasing deviations with scale and significant differences between the data set results are found. The first eigenvalue extracted has deviations increasing with scale from 0.05 to 0.18 for the contaminated data compared to 0.13 to 0.37 for the pure data. At the forty observation level with contaminated data, the first eigenvalue ranges from 3.77 to 4.47 a difference of 0.70. When converted into percent of variation explained, the differences in eigenvalue size is equivalent to 7%. Consequently, by simply altering the shape, size, and orientation of areal units, a factor can account for considerably varying amounts of data set variation. The changing percentage of explainable data set variation, in turn, influences the confidence and interpretation placed on results. As well, the overall extraction of variance varies dramatically with the contaminated data and forty observation scale from 62.73% to 72.28%. This range is, however, partially attributable to the different number of factors. The deviations of final eigenvalues also are indicators of factor loading size which may elevate or reduce the generality of factors.

Table 4.3.1: Aggregation Effects, Standard Deviations, of Final Eigenvalues.

Scale	Eig 1	Eig 2	Eig 3	Eig 4
Contaminated Data				
120 Obs	0.05	0.09	0.08	0.05
60 Obs	0.11	0.15	0.12	NA
40 Obs	0.18	0.19	0.18	NA
Uncontaminated Data				
120 Obs	0.13	0.11	0.12	0.09
60 Obs	0.20	0.19	0.12	0.11
40 Obs	0.37	0.28	0.16	0.11

Standard deviations from the distribution of initial communalities, i.e., squared multiple correlations, are displayed in Figure 4.3.2 for both data sets. The diagram generally displays the trends of increasing deviations with scale and larger aggregation effects upon pure data than contaminated data. For example, at the forty observation level all variables' deviations are larger for pure than contaminated data. The deviations across communalities also influence the interpretation of results. As communalities increase, the confidence in conducting a factor, i.e., common, analysis increases. Communality size is another possible avenue to verify whether a variable belongs in a factor model. At the forty observation scale with contaminated data the communality of variable six ranges from 0.26 to 0.63. With a communality of 0.26, inclusion of this variable is

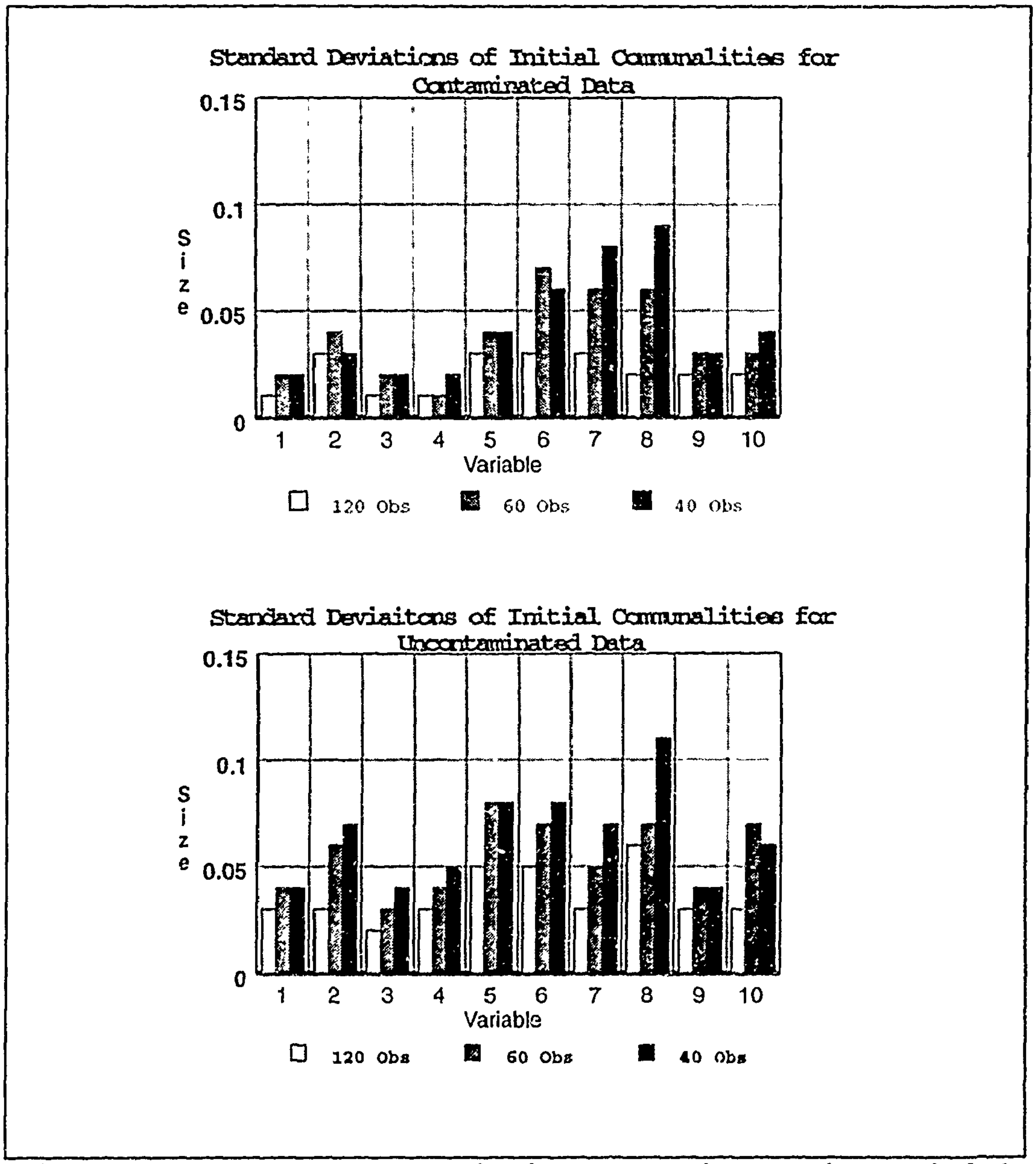

Figure 4.3.2: Standard Deviations Associated with Initial Communalities for Theoretical Data Sets.

questionable. However, with a communality of 0.63 there is little thought given to removing this variable. Obviously, inclusion or removal of variables can profoundly affect factor results.

The standard deviations associated with the final

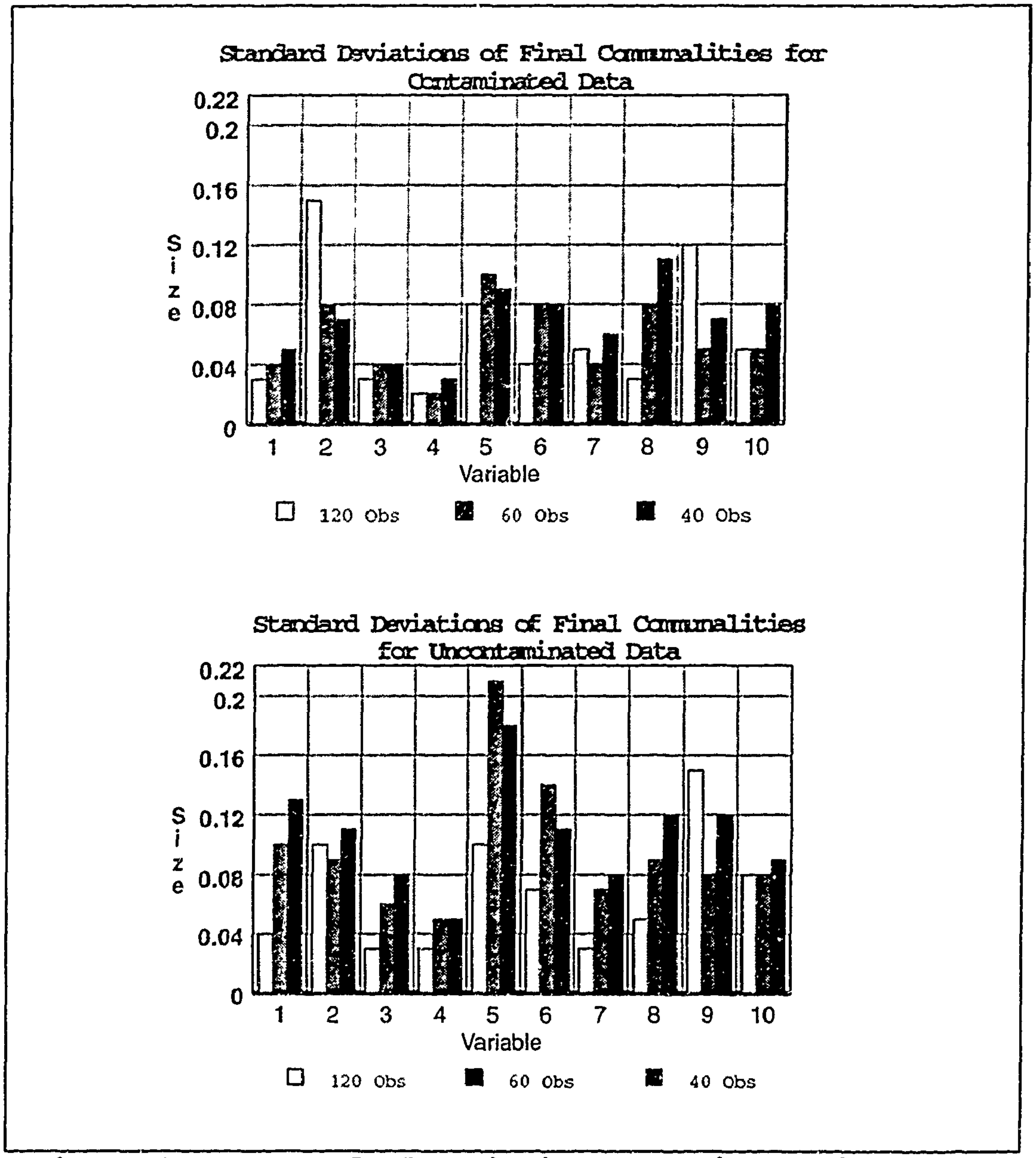

Figure 4.3.3: Standard Deviations Associated with Final Communalities for Both Theoretical Data Sets.

iterated communalities are displayed in Figure 4.3.3. These results illustrate similar trends to those from the initial communality examination, but several standard deviations decrease with increasing scale for uncontaminated data. The effect of several decreasing communality estimates with increasing scale can be explained from several viewpoints. First, only thirty aggregations are selected at each scale. It should not be expected that all standard deviations will be larger when the number of observations is reduced. Furthermore, it bodes well for the increasing deviations with scale hypothesis that more anomalies do not exist. Other explanations include the potential for skewed distributions and the occurrence of heywood cases. Although there is not an exact correspondence with the tenet of larger deviations with scale and these particular results, generally, these findings do support this general hypothesis.

Examinations of standard deviations for measures of sampling adequacy and KMO substantiate the above trends. Table 4.3.2 displays the standard deviations for measures of sampling adequacies. The KMO and MSAs are somewhat insensitive to alternate aggregations. For example, at the forty observation level with contaminated data, the KMO ranges from 0.53 to 0.66 with a standard deviation of 0.03. A range of 0.13 is not very significant at this highly aggregated scale. Despite initially hypothesizing large aggregation effects on these statistics, the results do not support this tenet. The findings again are contradictory to the hypotheses set out initially for these measures. Generally, measures of sampling adequacies and KMO may be robust under alternate aggregations and possibly under scale with empirical like data. Despite minor deviations, the aggregation effects are larger for pure than contaminated data.

Table 4.3.2: Aggregation Effects measured by Standard Deviations upon KMO and Individual Measures of Sampling Adequacies.

	Contaminated Data			Uncontaminated Data		
STAT	**120**	**60**	**40**	**120**	**60**	**40**
KMO	0.02	0.03	0.03	0.03	0.05	0.06
MSA1	0.02	0.05	0.06	0.03	0.07	0.08
MSA2	0.04	0.06	0.06	0.04	0.05	0.10
MSA3	0.02	0.05	0.06	0.02	0.06	0.08
MSA4	0.02	0.04	0.05	0.04	0.06	0.07
MSA5	0.04	0.06	0.07	0.06	0.07	0.08
MSA6	0.07	0.12	0.15	0.09	0.13	0.15
MSA7	0.08	0.11	0.15	0.09	0.14	0.17
MSA8	0.05	0.11	0.08	0.07	0.09	0.11
MSA9	0.03	0.05	0.05	0.05	0.07	0.09
MSA10	0.02	0.05	0.05	0.03	0.05	0.06

A RELATE program by Veldman (1967) was used to assess aggregation effects for a particular scale. The results, those factors correlating ABS(0.70) or better, are reproduced in Figure 4.3.4. To determine aggregation effects on factor structures, all factor analyses completed for one scale, i.e., thirty, were compared with each other for a total of 435 at each scale. As with the results from the scale analysis section, it is cautioned that results here will likely overestimate factor replication.

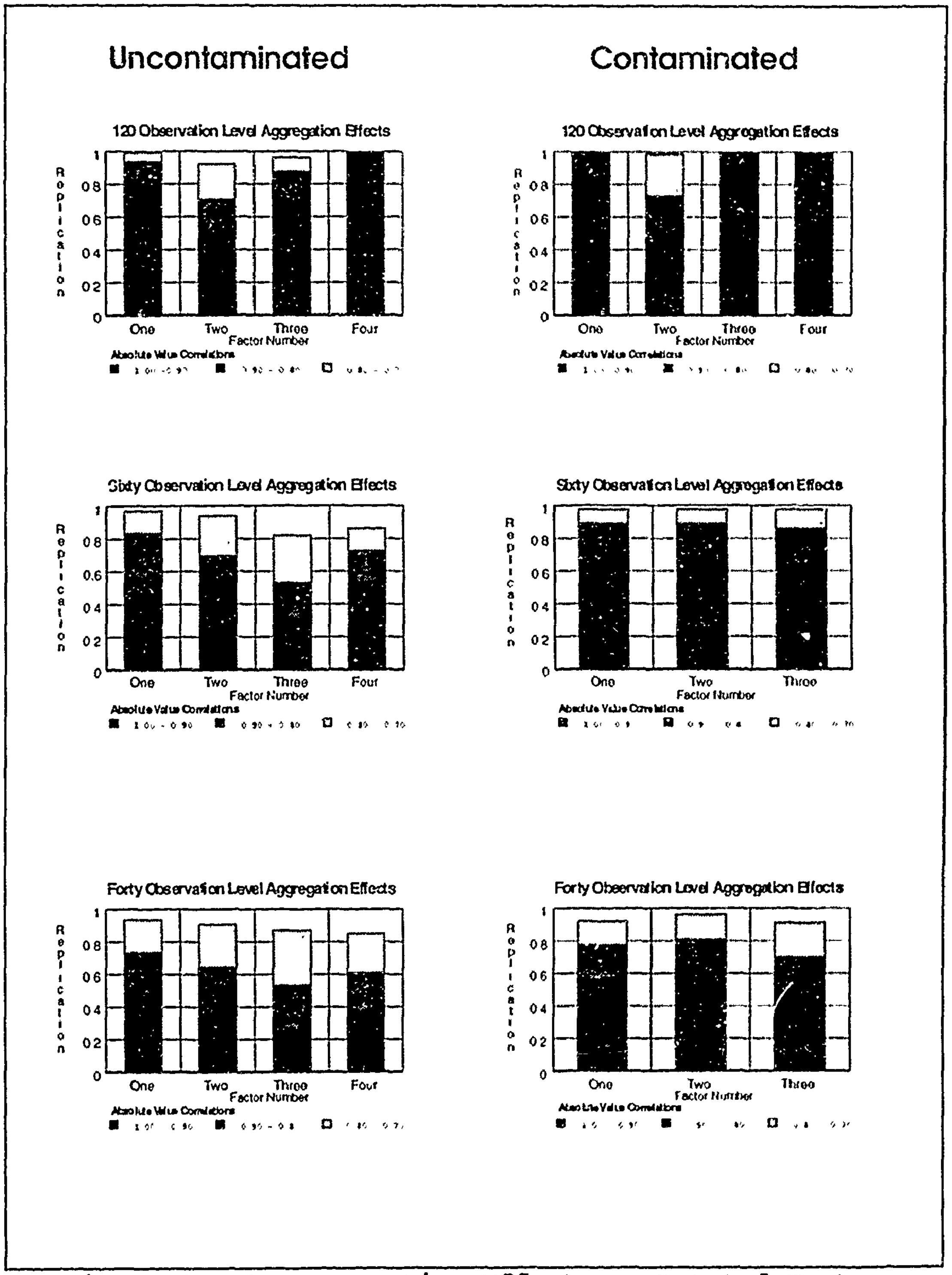

Figure 4.3.4: Aggregation Effects on Rotated Factor Loadings as Measured by RELATE Correlations for Theoretical Data.

The results suggest reproductions of contaminated data factors at a given scale are more likely than factors from the pure data. Overall, factor replications correlating ABS(0.70) or better are more frequent when data are contaminated. Of those factors replicating ABS(0.70) or greater, only eight of the twelve pure data factors were above 90% replication. With the contaminated data, all ten factors were reproduced 90% of the time. When comparing factor structures relating at ABS(0.90) or better, the same differences between data sets are found. At the forty observation level the pure data results are replicated at 0.35 compared to 0.50 for the contaminated data. The smaller aggregation effects with contaminated data are attributable, as before, to the spatial autocorrelations and lagged correlations. Aggregating the same number of groups in different ways should lead to relative stability in values because of internal homogeneity of groups due to positive spatial autocorrelations. Therefore, contaminated results should display signs of invariance to alternate aggregations. Again it must be reiterated that the RELATE results are misleading because of standardization of factor loadings across variables. The general effects of different aggregations are less pronounced with empirical like data than with data that has independent areal unit observations.

It is obvious that aggregation effects behave differently under unlike attributes of spatial data. The higher the level of spatial autocorrelations in the data the more stable results become under alternate aggregations. Aggregation effects always increase with scale no matter what characteristics data have. Since aggregation effects increase with scale, there is probably no empirical data set that could trivialize aggregation effects when scale is increased substantially. The RELATE program results illustrate the higher degree of factor replication with contaminated than uncontaminated data. Adding this result to those from eigenvalue and communality analyses, apparently contaminated data may be a sanctification in terms of aggregation effects. However, contaminated data alters results quite drastically across scales. Now that the results of MAUP on the theoretical data have been revealed, the next question is whether these results can be estimated.

4.4: Predictions Based on Arbia's Formula

Estimation of scale and aggregation effects on the factor analytic model is an important step in attempting to control MAUP effects. Drawing heavily upon the work of Arbia (1989) on the correlation coefficient, this study attempts to initiate the process of estimating MAUP effects on factor analysis given the basic spatial unit attributes of a data set. If the results are predictable, an important step in containing MAUP will be ascertained. With accurate predictions, the transmission mechanism that causes scale and aggregation problems will be understood. Clearly, the MAUP issue becomes diminished when one understands the effects that are transpiring. As well, it is likely that any future solution of the MAUP issue will be facilitated by knowledge of the parameters that create the modifiability of results. Provided below are the estimated scale effects for two scales and both theoretical data sets.

This section employs Arbia's (1989) formula, as shown in Chapter two, for the grouped correlation coefficient to predict the theoretical results. Before displaying these predictions, it is imperative to note that Arbia aggregates groups differently from this study. Arbia aggregates groups to have the same average within group connectedness of basic spatial units. Since this condition is not met in this research, Arbia's formula can only be employed to estimate the range of aggregation effects and not exact changes. Nevertheless, the predicted results should substantiate the trends found in the previous two sections.

An intervariable correlation matrix can be created with knowledge of the six parameters known to change the group level correlation coefficient (Arbia, 1989). By inserting the maximum and minimum average within group connectedness values at a particular scale in the formula, it is possible to predict the maximum and minimum correlation coefficients for that scale. These correlation coefficients can be produced for a number of variables, and when the correlations are arranged together, both a maximum and minimum correlation matrix is created. The maximum and minimum intervariable correlation matrices may then be factor analyzed to estimate the effects of aggregation.

From Arbia's group correlation formula, the results for both data sets and two scales are predicted. Results from the sixty observation scale are not shown since these predictions should fall between the two scales chosen. The results display only the KMO, initial eigenvalues, final eigenvalues, and final communalities. It should be sufficient to note that measures of sampling adequacies (MSAs) are well estimated by this formula. The replication of the MSA statistic suggests the hypothesis of increasing measures of sampling adequacy with scale was incorrect. The values of the KMO and MSA were in contrast to the initial hypothesis set out. Since the estimates also contrast the initial hypothesis, the hypothesis is incorrect. Initial communalities were deleted from this section since their predictability should be similar to those from the final communalities. The first examination predicts the results at the 120 group level. Since with an internal group size of two the average within group connectedness is always one, i.e., a BSU in a group has only one contiguous neighbour from the same group, there is only one set of predicted results; therefore, there are no predicted aggregation effects. In reality, some aggregation effects exist because this model differs from the model created by Arbia. The data in Table 4.4.1 display the predicted values and actual mean values of the selected statistics for both data sets.

The predicted results are remarkably similar to the actual results for both data sets. Clearly, the parameters changing the correlation coefficient with scale also alter factor analytic results in predictable ways. With well produced results at this scale, the importance of spatial autocorrelations and lagged correlations in terms of scale effects is substantiated. Despite large differences between mean values for the theoretical data set results, the predicted values for both data sets are very accurate. It appears when forming groups at very low levels of aggregations and specific criterion, scale effects are entirely predictable from the basic spatial unit level data characteristics.

The subsequent table shows both the means and standard deviation of a statistic for the forty observation scale. As

Table 4.4.1: Predicted Results at the 120 Observation Scale.

	Contaminated Data		Uncontaminated Data	
Statistic	**Mean**	**Prediction**	**Mean**	**Prediction**
KMO	0.60	0.61	0.56	0.56
Final CM1	0.70	0.69	0.61	0.59
Final CM2	0.72	0.64	0.62	0.57
Final CM3	0.95	0.94	0.96	0.97
Final CM4	0.93	0.94	0.94	0.95
Final CM5	0.50	0.43	0.34	0.29
Final CM6	0.34	0.38	0.37	0.34
Final CM7	0.10	0.07	0.11	0.09
Final CM8	0.22	0.21	0.17	0.17
Final CM9	0.70	0.61	0.64	0.56
Final CM10	0.89	0.92	0.85	0.87
Init E1	3.65	3.64	3.34	3.34
Init E2	1.98	1.96	1.93	1.89
Init E3	1.31	1.32	1.34	1.34
Init E4	0.98	0.98	0.94	0.93
Init E5	0.66	0.66	0.78	0.77
Init E6	0.59	0.59	0.68	0.70
Init E7	0.46	0.46	0.51	0.54
Init E8	0.17	0.18	0.22	0.22
Init E9	0.13	0.13	0.17	0.17
Init E10	0.08	0.08	0.10	0.10
Final E1	3.45	3.44	3.11	3.11
Final E2	1.63	1.57	1.54	1.43
Final E3	0.82	0.83	0.85	0.82
Final E4	0.49	NA	0.56	NA

where Final CM(X) - is the final communality of variable X
Init E(F) - is the component eigenvalue of factor F
Final E(F) - is the unrotated principal axis factor eigenvalue of factor F

well, two values of average within group connectedness are employed to determine the predicted range of scale effects. These two predictors are necessary because as scale increases the solutions for average within group connectedness becomes multiple. If groups are set out in a chain, the line transect case, the within average group connectedness for groups of size six will equal 1.67, i.e., four BSUs have two contiguous neighbours and the other two have only one contiguous neighbour. When the group is shaped as a rectangle on the lattice, either three by two or two by three, the average within group connectedness equals 2.67, i.e., four BSUs have three contiguous neighbours while two BSUs have only two neighbours. Since there is a range of possible values for average within group connectedness with groups of size six, the maximum, 2.67, and minimum, 1.67, values act as surrogates for range.

Results from the predictions at the highest scale are shown in Table 4.4.2. When increasing the scale of analysis, the predicted and actual results diverge. However, the general trends established with changes to scale in the previous sections still stand. From the estimated values, results with uncontaminated data should be more invariant to scale than those from contaminated data. The predicted uncontaminated data results also suggest some scale effects should exist, e.g., the first extracted initial eigenvalue is predicted to fall between 3.37 and 3.40 at the forty observation level while the basic spatial unit value for the same eigenvalue is 3.30. The prediction that scale effects should exist in the pure data results, suggest there are some residual dependencies and interactions remaining in this data. The hypothesis of larger deviations associated with uncontaminated data is not supported. This may be because scale is not increased sufficiently in these estimates to display these expected differences in deviations. The contaminated data set predictions consistently underestimate the actual values. This consistent underestimation is thought to be the result of second order, i.e., second neighbour, spatial autocorrelations and lagged correlations. Arbia (1989) assumes a condition of local stationarity for the group correlation formula, and this assumption requires second order relationships to be zero. This assumption is unlikely to exist in empirical data, and the effect of second order relationships also accentuates the effects of scale on an analysis. With this tenet, it is possible to explain why the predicted values of the first aggregated scale are excellent and the predictability of results decreases with higher scales. At the 120 observation level only first order relationships influence scale and aggregation because the groups are only of size two. When increasing the scale to forty observations, second order relationships in spatial autocorrelations and lagged correlations influence scale and aggregation effects. Because first and second order dependencies should alter results in the same fashion, the formula to predict scale effects should increasingly underestimate the actual results. Nevertheless, the formula facilitates predictions of general trends with alternate scales.

Table 4.4.2: Predicted Results at the 40 Observation Scale.

	Contaminated Data				Uncontaminated Data			
Stat	**Mean**	**Dev**	**Pred1**	**Pred2**	**Mean**	**Dev**	**Pred1**	**Pred2**
KMO	0.60	0.03	0.62	0.62	0.51	0.06	0.55	0.54
CM1	0.80	0.05	0.73	0.76	0.73	0.13	0.57	0.57
CM2	0.83	0.07	0.68	0.71	0.63	0.11	0.59	0.60
CM3	0.92	0.04	0.95	0.95	0.88	0.08	0.99	0.99
CM4	0.97	0.03	0.95	0.96	0.92	0.05	0.97	0.99
CM5	0.60	0.09	0.46	0.49	0.55	0.18	0.27	0.25
CM6	0.42	0.08	0.39	0.40	0.32	0.11	0.34	0.33
CM7	0.17	0.06	0.07	0.08	0.13	0.08	0.10	0.12
CM8	0.36	0.11	0.22	0.24	0.31	0.12	0.18	0.20
CM9	0.71	0.07	0.64	0.66	0.81	0.12	0.58	0.60
CM10	0.89	0.08	0.94	0.96	0.75	0.09	0.86	0.85
E1	4.29	0.18	3.79	3.89	3.42	0.36	3.37	3.40
E2	2.00	0.16	1.97	1.99	1.92	0.25	1.87	1.85
E3	1.31	0.13	1.32	1.32	1.39	0.14	1.37	1.40
E4	0.87	0.07	0.97	0.97	1.01	0.10	0.90	0.86
E5	0.56	0.07	0.62	0.59	0.80	0.07	0.80	0.83
E6	0.44	0.05	0.56	0.55	0.61	0.07	0.69	0.69
E7	0.31	0.04	0.43	0.39	0.41	0.07	0.54	0.53
E8	0.10	0.02	0.15	0.13	0.22	0.04	0.22	0.21
E9	0.07	0.01	0.12	0.10	0.14	0.03	0.16	0.15
E10	0.04	0.01	0.07	0.06	0.07	0.02	0.09	0.09
FE1	4.11	0.18	3.60	3.71	3.21	0.37	3.14	3.18
FE2	1.68	0.19	1.61	1.63	1.57	0.28	1.47	1.46
FE3	0.86	0.18	0.84	0.85	0.96	0.16	0.85	0.88
FE4	0.49	NA	NA	NA	0.58	0.11	NA	NA

where CM(X) - is the final communality of variable X
E(F) - is the component eigenvalue of factor F
FE(F) - is the unrotated principal axis factor eigenvalue of factor F

The predictions from Arbia's work on the correlation coefficient are excellent at the lowest levels of aggregation. With increasing scale these predictions become increasingly inaccurate due to the disturbances of second order spatial dependencies. The empirical results from the next chapter also contain varying internal sizes of aggregated groups. When combining the variability of group size with the likelihood of second order spatial autocorrelations, this formula should not be employed to predict any empirical results.

4.5: Summary of Theoretical Data Results

Scale and aggregation effects can transpire in factor analytic studies. Depending upon the attributes of areal unit data, these effects can be significantly different. The general results of assessing MAUP on the theoretical data are displayed in Table 4.5.1. Although scale severely affects results when areal units contain positive spatial autocorrelations, the effects of aggregation are somewhat limited. Uncontaminated data results show little or no scale effects but are greatly influenced by alternate aggregations. Since variables with positive spatial autocorrelations frequent empirical data, hypotheses for empirical data can be drawn from contaminated data results.

Table 4.5.1: General Results of Theoretical Data Analyses.

Data	Scale Effects	Aggregation Effects
Uncontaminated	Negligible	Severe
Contaminated	Severe	Moderate

Changes in scale induce changes in factor results, some of which are important in affecting researchers' confidence in the factor model. Increases in initial and final eigenvalues and communalities with scale seriously influence the interpretation and confidence placed on factor analytic results. It was found that increasing scale generates increasing explanation of data set variation. The increasing explanation of data set variation with scale prompts researchers working at different scales to extract different numbers of significant factors and differing levels of correlation matrix reproduction. The generality of the largest factors also increases with scale, and there is doubt that all of this generality can be removed by rotation. The increasing communalities with scale also alter the confidence and interpretation of factor model results. If final communalities increase with scale, larger factor loadings are apparent. Any change in factor loading sizes alters the identification of factors. The results from the factor structure replication across scales show that factor structures are well reproduced across scales. It is also encouraging that the RELATE contaminated data results are more invariant to scale than uncontaminated results. Overall, there are serious questions about the factor analytic stability under various scales.

Although aggregation effects are considerably smaller for contaminated data than uncontaminated data, there are still large effects. First, as the scale increases so does the variability of the results among different aggregations.

Therefore, a scale can be increased to the point where aggregation effects are always present and large. The variability of communalities and eigenvalues to different aggregations influences the interpretation and confidence of factor analytic results. As well, accepting results from one aggregation and one scale is precarious. As Fotheringham (1989) has suggested, sensitivity analysis may be necessary when using areal units as observations. Although the RELATE procedures produce factor structures remarkably well across aggregations, the effect of standardizing loadings across variables stains this result. Since communality sizes can indicate aggregation effects, this potential variability should be considered before concluding on factor loading stability. Nevertheless, it is encouraging that factor structures are alike across aggregations.

Arbia's (1989) formula accurately predicts the effects of scale and aggregation from the characteristics of basic spatial unit data. When increasing the scale of analysis beyond the 120 group level, the estimates are less reliable and usually under predict the actual values found. These differences between predicted and actual results are due to the effects of second order spatial autocorrelations and lagged correlations. The grouped correlation formula suggests that differences between two data sets transpire because of dissimilar spatial autocorrelations and lagged correlations.

Since it would be imprudent to expect this formula to work adequately in predicting empirical results, no attempt at predicting the empirical results is forwarded.

With the basic understanding of factor analysis on areal units complete, the next inspection is completed upon the empirical data. It is expected that the empirical data results will behave similarly to the contaminated data results. The results in Chapter Five test this supposition.

Chapter 5: Empirical Results

Chapter five reveals the results of empirical factor analyses conducted on the Saskatoon Census Metropolitan Area for 1986 data. The empirical results are expected to mirror the findings from the contaminated theoretical data. Four distinct sections partition this chapter. The first section emphasizes the important attributes of variables and aggregating procedures. Section two examines scale effects on the empirical data set. The following division illustrates the aggregation effects on the results. The final section reviews scale and aggregation effects on factor analysis.

5.1: Data Set and Variables

Since Chapter three detailed most empirical data set characteristics, only scant attention is paid to this data set. The basic spatial units employed for the empirical investigation are from the enumeration areas of the Saskatoon C.M.A.. In all there are 231 usable enumeration areas with several other enumeration areas containing missing or suppressed data. Although enumeration areas vary considerably in population size from 115 to 1,525 (Statistics Canada, 1986b), they are given equal weight in determining results.

No attempt is made to weight these observations since the objective of this study is to reproduce basic spatial unit results and not individual level results. Because areal units differ in population size, any biases associated with enumeration area results from individual level results are preserved for all scales and aggregations. Unlike the theoretical aggregation procedures, these aggregated groups are not constrained to equal internal sizes. Leaving groups unconstrained by size is analogous to other empirical data as Saskatoon's census tracts contain between three and fifteen enumeration areas (Statistics Canada, 1986b) compared to the three to ten range used for the largest groups of this analysis.

As described earlier, the empirical variables for this study were selected according to social area analysis themes, i.e., economic status, family status, and segregation. However, variables are only referenced by a number and not a descriptive label. By labelling variables in this fashion, readers can only focus on the results of scale and aggregation and not on urban patterns. For those who wish to decipher the urban implications of these factor analyses, Appendix A provides the variable definitions and corresponding numbers. As seen in Table 5.1.1 the spatial autocorrelations for these variables are comparable to those from the artificially produced contaminated data. The similarity of spatial autocorrelations permits the results from the previous chapter on contaminated data to hypothesize the empirical results.

Table 5.1.1: Spatial Autocorrelations (I) for Empirical Variables.

Variable Number

1	2	3	4	5	6	7	8	9	10
0.28	0.20	0.64	0.36	0.61	0.36	0.57	0.59	0.25	0.25

-all measures are significant at 99%

5.2: Scale Effects on Empirical Data

Following a similar vein to Chapter four, scale effects are displayed for specific statistics. First, the initial and final eigenvalues are examined by the four scales of analysis. After the eigenvalue analysis, sections inspecting initial and final communalities and measures of sampling adequacies follow. The RELATE procedure inspects scale effects on factor loadings. The hypotheses of increasing generality with scale for the unrotated factors are examined in the next section. Finally, the distributions of the first unrotated factor scores are inspected.

The means from the sample of initial, i.e., principal component, eigenvalues are displayed in Figure 5.2.1. As expected, the largest eigenvalues increase in size when reducing the number of areal unit observations. The first

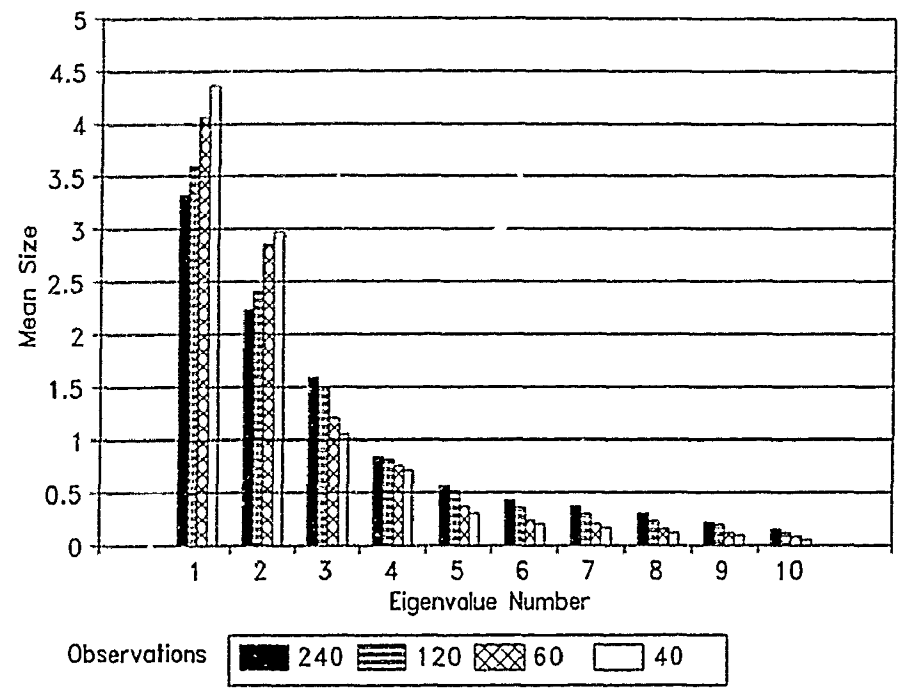

Figure 5.2.1: Scale Effects on Initial Eigenvalues for Saskatoon Data.

eigenvalue extracted increases from 3.32 at the basic spatial unit level to 4.36 at the forty observation scale. From the final eigenvalues, the first eigenvalue extracted also increases from 2.96 to 4.13 across the range of scales. When converted into percentage of variation explained, the difference between the final eigenvalues across the scales is 11.7%. The changing value of explained variation attests to the increasing generality of unrotated factors with scale. After extracting the second factor, the mean eigenvalues decline with a reduction in the number of observations. As stated earlier, larger eigenvalues with scale can alter the reliability, strength, and total explanatory power of factors. Extraction of the number of significant factors is also modifiable, and breaks in the eigenvalue slope becomes more distinct with larger scales. The elevating generality of factors as indicated by stronger principal final eigenvalues also leads to problems with interpretation. The number of significant factors extracted alters from three for the BSU, 120, and sixty observation levels to 40% of the aggregations at the forty observation level reporting only two. With modifications to the number of significant factors, any comments on the spatial processes operating on Saskatoon are also modifiable. The final eigenvalues display the same trends as the initial eigenvalue inspection.

Scale effects on initial and final communalities are displayed in Figure 5.2.2. As with the results from the contaminated data in Chapter four, increasing scale leads to higher communality estimates. All initial communality estimates for variables enlarge with scale. This trend is not as vibrant for the final communalities where only six of the ten communalities increase for all scales. The implications of larger communalities with scale are important in factor analytic results. For example, only one initial communality estimate is greater than 0.70 at the basic spatial unit level.

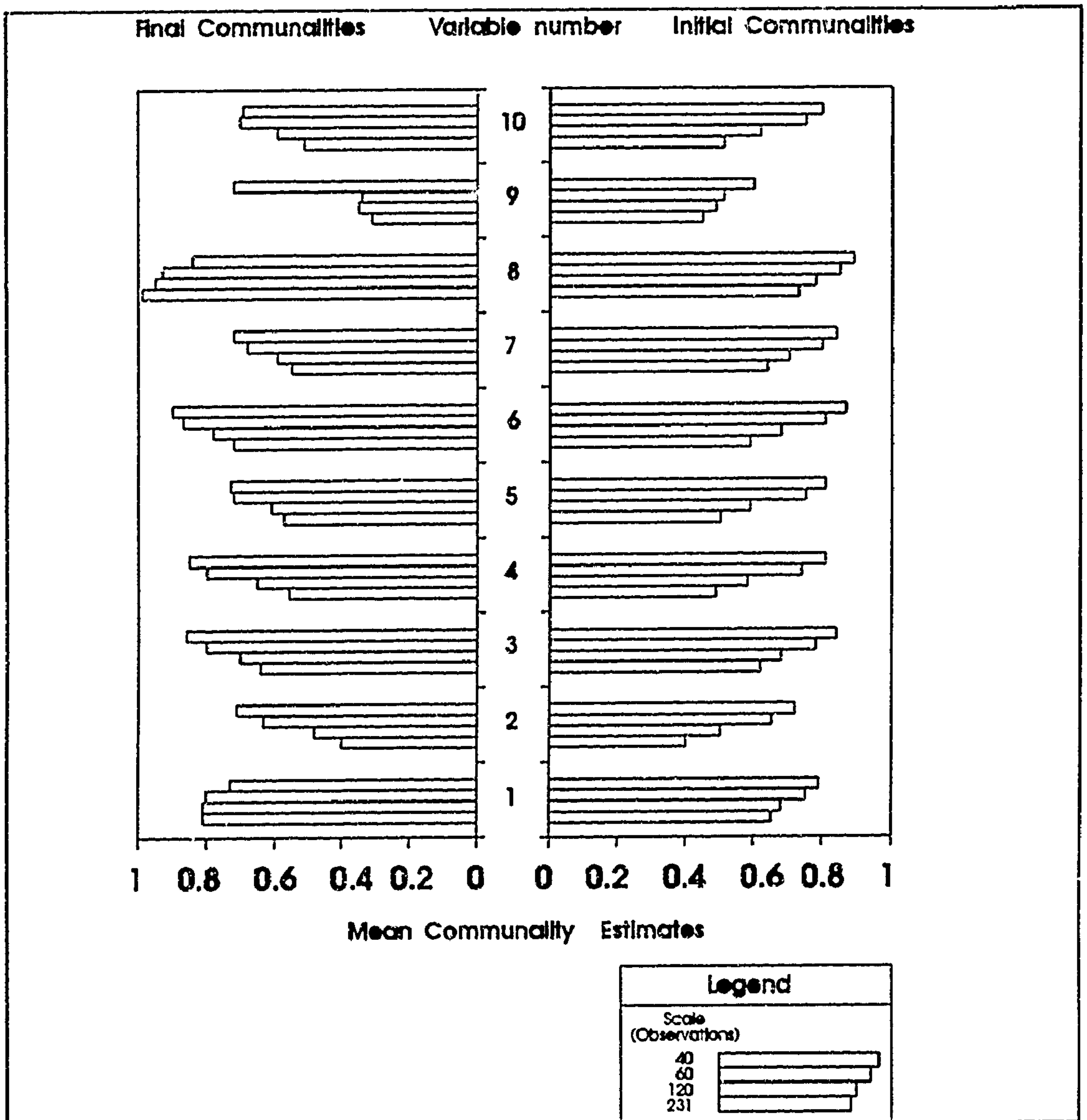

Figure 5.2.2: Scale Effects on Initial and Final Communalities for Saskatoon Data.

When reducing the number of areal unit observations to forty, only one estimate is not greater than 0.70. The final communalities also increase across the range of scales from three to nine variables having communality estimates exceeding 0.70. Higher communalities are a clear indication that factor loadings are stronger with scale. Since significant eigenvalues increase with scale, unrotated factors must move towards generality. As well, it is possible for scale to influence communality estimates to the point where a researcher decides that a component not factor model is suitable for a data set.

The measures of sampling adequacies are for the most part unbiased across scales, see Appendix B. The KMO does exhibit a slight bias with values of 0.68, 0.69, 0.72, and 0.71 for all scales. This slight increase cannot be regarded as significantly different from associated expected deviations of taking a sample of thirty aggregations; therefore the statistic has to be considered unbiased. As the previous chapter exposed, measures of sampling adequacies are altered in predictable ways by the data set employed (Kaiser, 1970; Kaiser and Rice, 1974). The general effect of the mechanisms that change the measures of sampling adequacy for this empirical data set largely cancel out. It is not known from this study whether the KMO statistic is always scale invariant with empirical data or whether certain empirical data sets may produce unique reactions to the KMO.

The RELATE procedure is employed again to decipher the effects of scale on factor structures. The factor structure comparison involves examining the thirty aggregations at one scale by the thirty aggregations at another scale. Again this

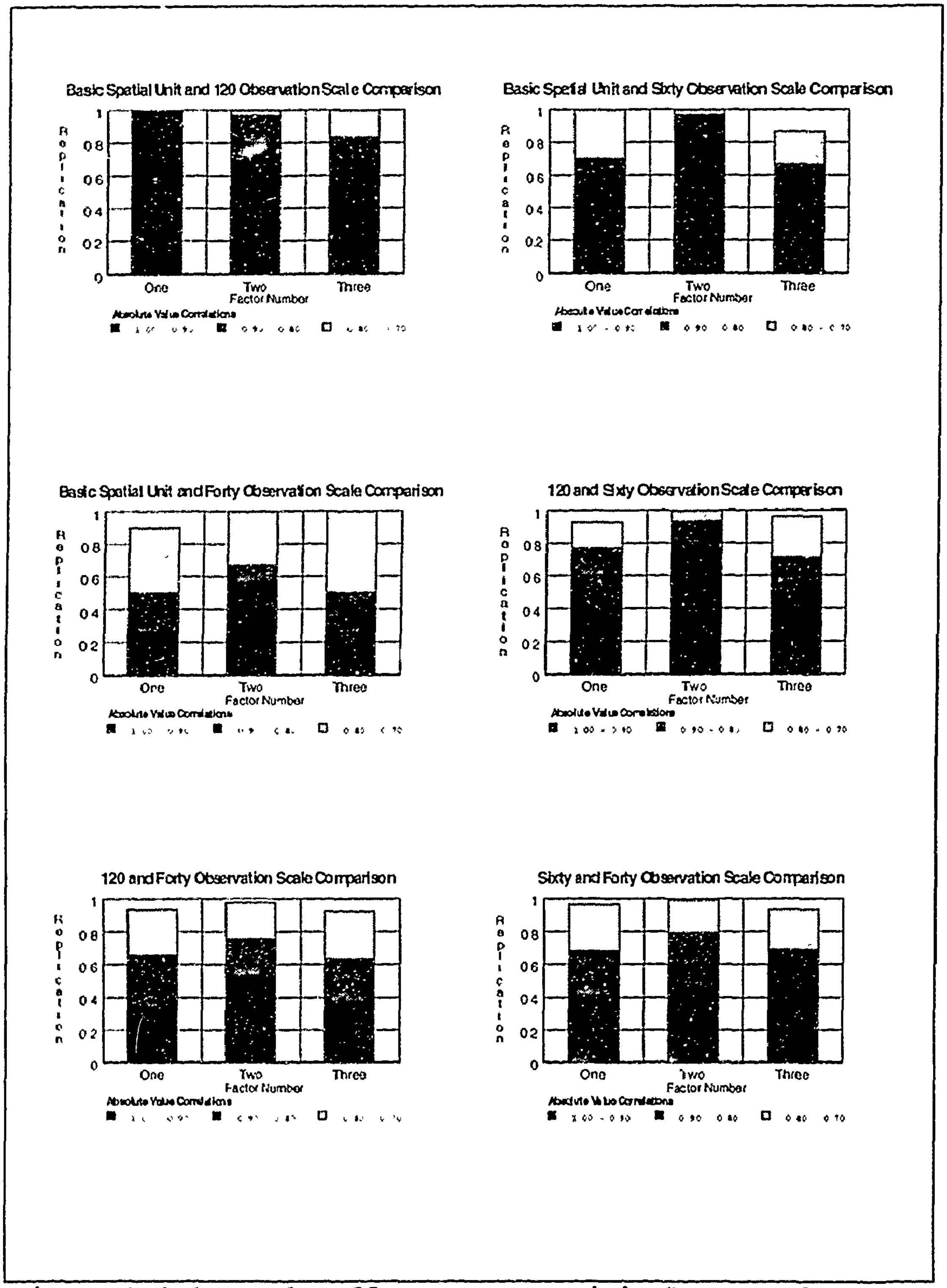

Figure 5.2.3: Scale Effects Upon Empirical Rotated Factor Loadings as Measured by RELATE Correlations.

procedure does not account for the differences of final communality sizes and thus the different sizes of factor loadings. Results shown in Figure 5.2.3 display the rate of factor relationships correlating greater than ABS(0.70). Following similar trends to the examination of theoretical contaminated data, the overall replication of factors across scales is quite high. Almost all scale comparisons illustrate that the same factors, as measured by correlations of ABS(0.70) or greater, are being replicated 90% of the time. When the analysis is examined for those factors correlating ABS(0.90) or greater, different findings transpire. Generally, as the difference of the two scales being compared increases the level of replication decreases. As well, if both scales compared are aggregated further from the BSU level, the reproduction of factors also declines. The examinations from the RELATE analyses establish that factor structures are somewhat invariant to scale effects. It must be remembered that a high correlation among factor structures does not equate to high correlations among factor interpretations. If factor loadings greater than some threshold level aid in factor interpretation, there is little doubt scale effects are present as suggested by increasing final communality estimates with scale.

The following section tests the hypothesis of emerging general factors with larger scales. Table 5.2.1 displays the average, measured in the same fashion as in Section 4.2.2, factor loadings for the scales beyond the BSU level. By examining the average loadings of variables on the first unrotated factor, the expectation of generality can be substantiated. At the basic spatial unit level only one unrotated factor loading is greater than ABS(0.70). When increasing the scale of analysis to forty observations, the average number of factor loadings exceeding ABS(0.70) is four. In all nine of the ten factor loadings associated with the first unrotated factor strengthen as scale is increased from the BSU to the forty observation level. Therefore, the largest factors do increase in generality, although rotations may alter this effect.

Table 5.2.1: Average Factor Loadings on the First Unrotated Factor.

	Number of Areal Unit Observations		
Variable	**120**	**Sixty**	**Forty**
1	0.55	0.71	0.73
2	-0.53	-0.76	-0.82
3	0.46	0.59	0.62
4	-0.48	-0.50	-0.51
5	-0.57	-0.81	-0.84
6	0.63	0.86	0.90
7	0.11	0.10	0.13
8	0.43	0.48	0.47
9	0.10	0.23	0.34
10	-0.48	-0.63	-0.63

Examinations of the empirical distributions of first rotated factor scores comply with the same themes found with theoretical data. Of the thirty factor score distributions created for each scale, results are shown in Appendix B, there is a reduction in the ranges as the number of observations decreases. For those factor score distributions with ranges less than four, there are zero, three, and fifteen for the 120, sixty, and forty observation levels, respectively. If extreme factor scores are employed to facilitate factor interpretations, problems arise when factor score ranges are reduced. For example, if extreme factor scores, those greater than ABS(2.00), were mapped at the forty group level, four of the thirty aggregations contain no extreme scores at all. The factor score distributions, display increasingly negative kurtosis, i.e., a platykurtic or flat distribution, with scale. The decrease in kurtosis with scale again suggests factor scores estimated from the regression technique are subject to scale effects. In turn, it is highly probable that factorial ecologies employing outlying factor scores to help identify factors are susceptible to scale effects.

The effects of scale on the Saskatoon data are apparent on all but the measures of sampling adequacy and to some extent the RELATE procedure. The associated scale effects with empirical data are also larger than those found when analyzing contaminated theoretical data. The larger scale effects with empirical observations could be attributable to the allowance of variable internal sizes of groups. As well, the large scale effects on empirical data may occur from second order spatial autocorrelations and lagged correlations. It is hypothesized that second order spatial autocorrelations and lagged correlations should be higher with empirical data than theoretical data. The reason this tenet is forwarded is that theoretical data were only created with constraints on first order autocorrelations and not second order. Consequently, when increasing scale, empirical data set results should increase faster than their contaminated data counterparts. This is also one reason Arbia's grouped correlation formula was not used to estimate the Saskatoon results. To use Arbia's formula correctly, an assumption of local stationarity with no relationships of areal units beyond contiguous neighbours must be forwarded. The spatial dependencies of variables contained within the enumeration areas will not support the local stationarity assumption, and therefore prediction of scale effects would largely underestimate the actual results. With knowledge that scale effects are large, the next section tests aggregation effects on the factor model.

5.3: Aggregation Effects on Empirical Data

The effects of aggregation on empirical factor analytic results are presented in this section. These effects are examined upon final and initial eigenvalues and communalities, measures of sampling adequacies, and factor loadings. Aggregation effects are almost exclusively determined by the standard deviation of a statistic for any scale. The results here should follow the same patterns found with contaminated theoretical data results. First, aggregation effects should increase with scale. As well, the effects of aggregation should be larger with empirical data than with the contaminated data. Large aggregation effects should arise due to the variability of the internal size of groups for the empirical data. The paragraphs below elaborate upon factor analytic aggregation effects.

Standard deviations for the initial eigenvalues are displayed in Figure 5.3.1. It is both astonishing and encouraging that aggregation effects do not appear to increase noticeably with scale. Possibly, true aggregation effects at a particular scale may become obscured by the varying internal sizes of groups. Although it is promising that aggregation effects are constant with changes to scale, the effects are, nevertheless, still present. Furthermore, the empirical deviations are larger than those derived for contaminated data. The larger deviations are obviously caused by the internal variability of groups. Since aggregation effects are larger in the empirical than contaminated data, the stability of aggregation effects with different scales is insignificant. The ranges are still great as at the forty observation level,

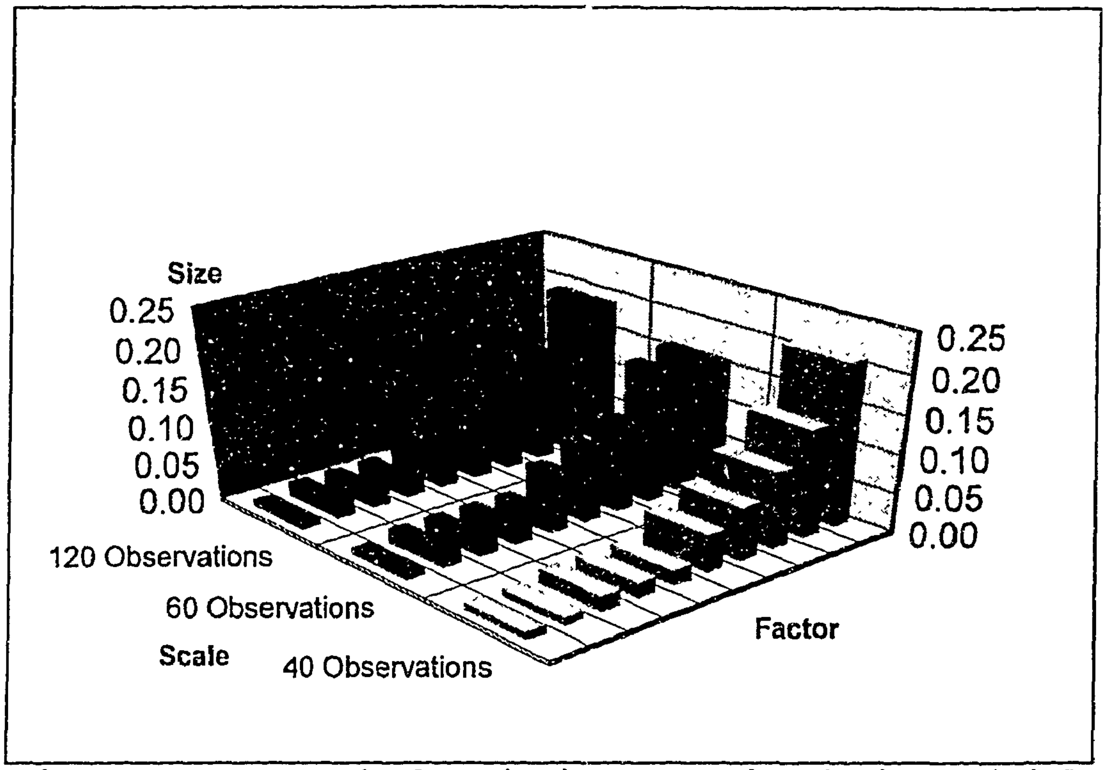

Figure 5.3.1: Standard Deviations Associated with Initial Eigenvalues for the Empirical Data Set.

the first extracted initial eigenvalue ranges from 4.00 to 4.76. Despite apparent stability of aggregation effects to changes in scale, the effects are still very large and would surely complicate any factor model.

The results, in Figure 5.3.2, from the standard deviations of the initial and final communalities corroborate the eigenvalue analysis above. The standard deviations of communalities do not appear to increase markedly with scale. Again the relative stability of aggregation effects is believed to be a product of the range available for internal group sizes at all scales. Although aggregation effects do

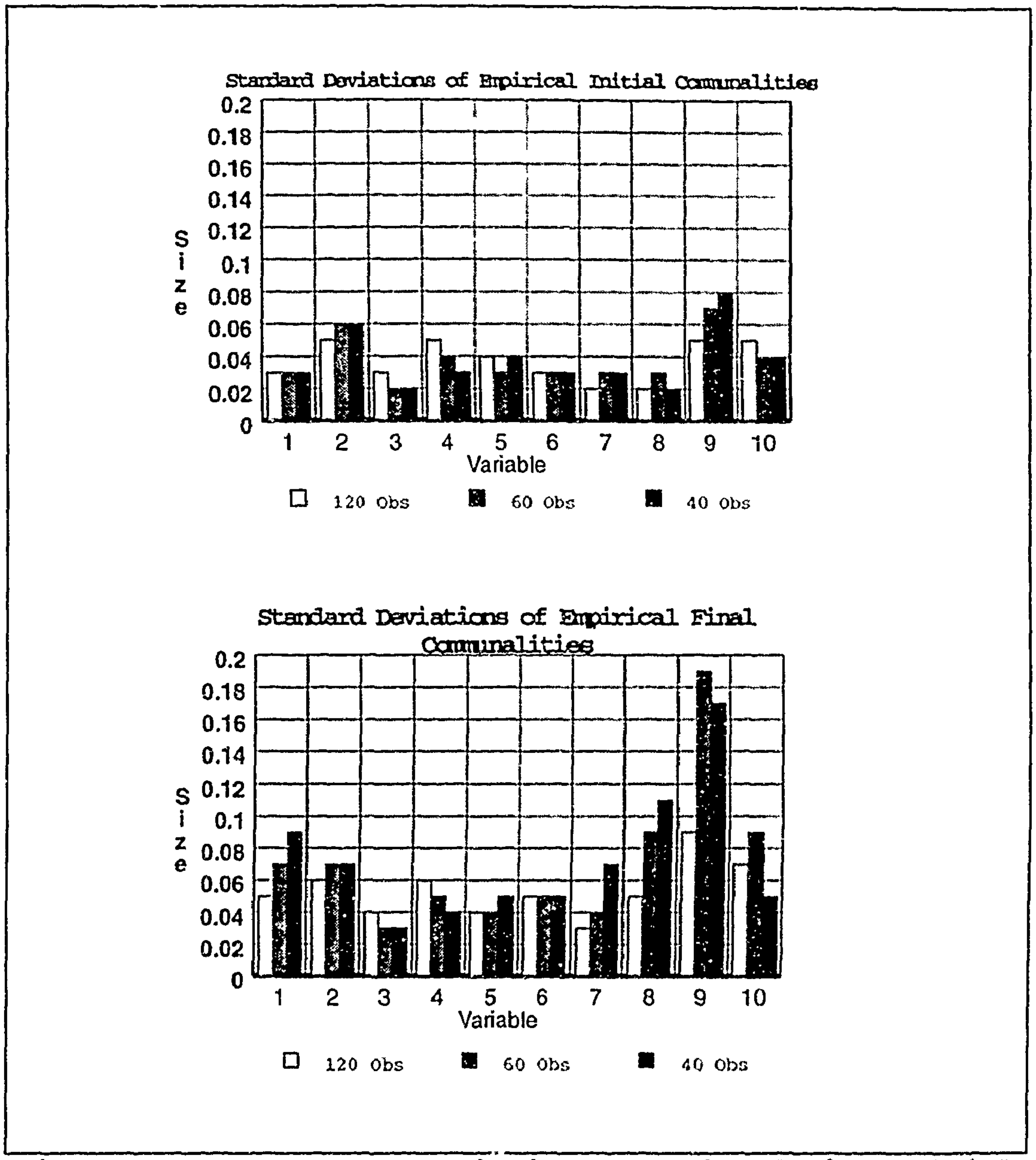

Figure 5.3.2: Standard Deviations Associated with Initial and Final Communalities for the Empirical Data Set.

not increase drastically with changes in scale, they are still large. These ranges are extreme as 0.63 for the final communality of variable nine at the forty observation level. With large ranges of communalities, it is obvious that the factor model will be modified with different aggregations. Changes to final communalities are also directly related to the strength of factor loadings and are masked by the RELATE procedure.

The standard deviations for the distribution of alternate aggregations of sampling adequacies and KMO are presented below in Table 5.3.1. As with the results from the contaminated data this statistic may be invariant to aggregation effects. As well, the trend of increasing deviations with scale may be more plausible for these statistics than the empirical communalities or eigenvalues. Although the KMO is somewhat invariant to aggregation effects, it is possible for large ranges to exist, e.g., from 0.79 to 0.64 at the forty observation scale. With a large range in the KMO, there is potential for researchers employing the same scale, study area, and variables to find contrasting evidence suggesting whether a correlation matrix should be factor analyzed. However, the range of a statistic is not as important as its deviation. With a small standard deviation of 0.03, the KMO and individual measures of sampling adequacies display some invariance under aggregation effects.

Table 5.3.1: Aggregation Effects measured by Standard Deviations upon the KMO and Individual Measures of Sampling Adequacies.

Statistic	120 Observations	Sixty Observations	Forty Observations
KMO	0.02	0.03	0.03
MSA1	0.02	0.03	0.06
MSA2	0.04	0.07	0.05
MSA3	0.04	0.03	0.05
MSA4	0.03	0.03	0.03
MSA5	0.04	0.04	0.06
MSA6	0.04	0.05	0.06
MSA7	0.04	0.06	0.05
MSA8	0.04	0.07	0.05
MSA9	0.04	0.07	0.07
MSA10	0.03	0.04	0.04

The RELATE procedure is employed to assess whether differences in factor structures at a given scale exist. Figure 5.3.3 displays the results of this analysis for each scale. Generally, the level of factor replication correlating at least ABS(0.70) is superb for all scales. Almost all factors are reproduced 90% of the time when measured by correlations of ABS(0.70) or greater. When analyzing only those factors correlating ABS(0.90) or better, the results show internal differences. As scale is increased, the rate of factor reproduction decreases. Therefore, some evidence exists to suggest aggregation effects increase with scale. Again the comparisons here only examine factor structures and not the actual interpretation of factors or the sizes of unique variance. When compared to the theoretical data set results, the empirical factors are better replicated than the uncontaminated factors with a high scale. From this assertion, it is promising that data infected with positive

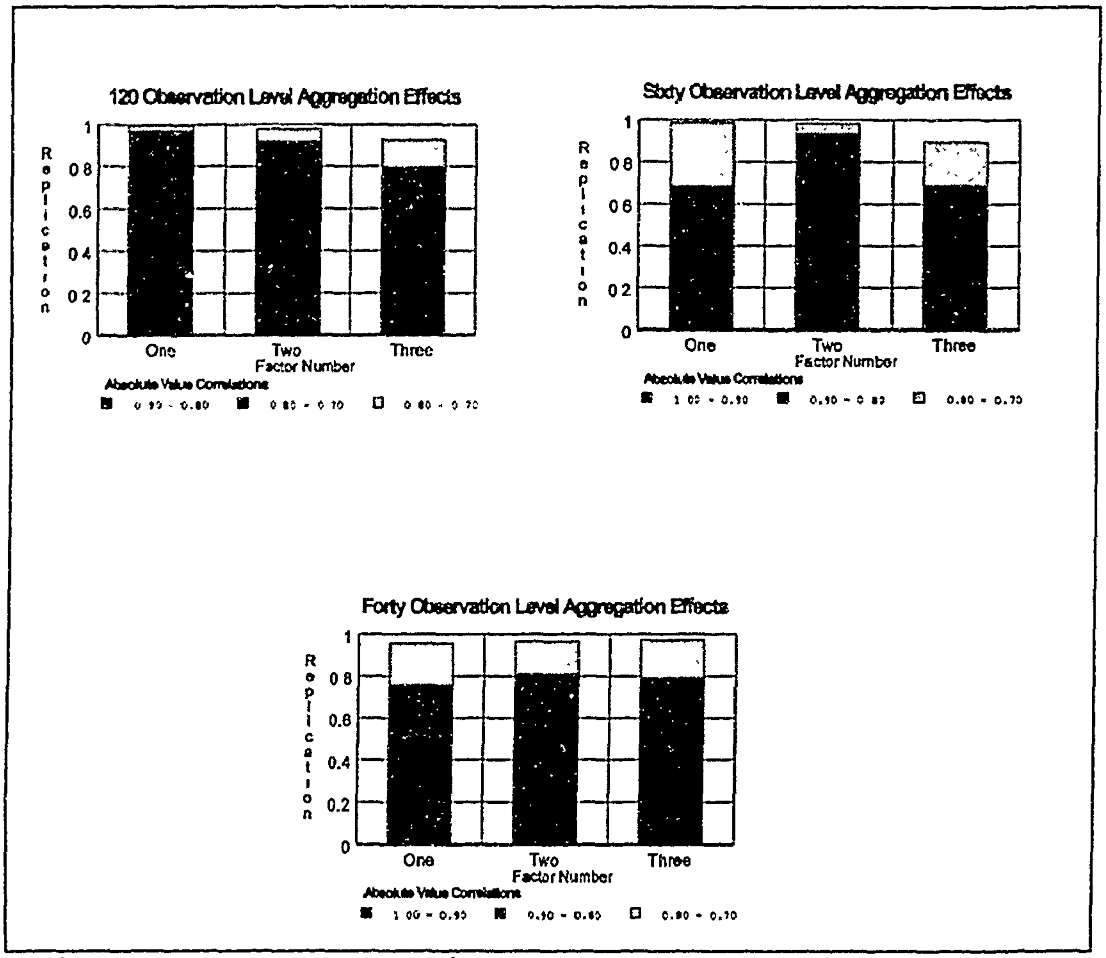

Figure 5.3.3: Aggregation Effects upon Levels of Factor Replication among Factor Structures as Measured by RELATE Correlations.

spatial autocorrelations increases the likelihood of factor structure invariance to aggregation effects.

Aggregation effects on empirical data are unique in many ways. The expected increasing variability of results with scale is not apparent as it was with contaminated data results. The absence of increasing aggregation effects with scale may be due to several possible reasons some of which have been explored. The effects of aggregation, nevertheless, are not removed from this data set, and they can affect results and interpretation of factor analysis.

5.4: Summary of Empirical Data Results

Scale and aggregation effects are present and strong in empirical data. There are some anomalies to the statement above, e.g., the KMO may be invariant to scale and aggregation effects, and thus might be a suitable measure to employ in geographic studies. Factor structures also display the same signs of invariance, but there are still scale and aggregation effects visible when communality size is accounted for. For the most part, MAUP effects are potent and cannot be assumed away. Factor analyses employing areal unit observations including factorial ecologies are subject to MAUP. Unless the variables are free of positive spatial autocorrelations at the basic spatial unit level, subsequent alteration of scales generates pronounced biases in results. Aggregation effects are largest when areal units are independent. The typical positive spatial autocorrelations in empirical data reduce the effects of aggregation. However encouraging the above statement may be, it is doubtful that any level of spatial auotocorrelation will completely negate aggregation effects. With knowledge of scale and aggregation effects, factor analysis is a technique that demands careful consideration when completed with areal unit observations.

The hypotheses generated from the contaminated theoretical data set results generally held well when compared to empirical results. Some discrepancies did develop, but many of these were easily explained. Scale effects were strongest for the empirical data set most likely due to the interaction of second order spatial dependencies. These results also did not substantiate the hypothesis of increasing aggregation effects with scale. The reasons why this hypothesis could not be verified have all been elaborated upon earlier. Overall, the contaminated data set proved quite useful in predicting the results from the empirical data set.

Chapter 6: Conclusions and Impetus for Future Research

It should now be apparent that scale and aggregation effects are present in the principal axis factoring technique when observations are areal units. MAUP effects on factor analysis are serious and deserve much more attention than is presently afforded by researchers employing factor analysis. The results here, unlike many others, prove that alternate scales and aggregations can alter many important factor analytic attributes besides rotated factor loadings. This chapter is set out to review the major findings of this research. Additionally, areas for future research are also discussed at some depth. Finally, the presentation of how one should complete future factor analyses considering the findings here is discussed.

6.1: Overview of Results

The following text reviews several general themes and their relationships to the initial hypotheses. The results from all data sets are compared to the original hypotheses. The first subsection evaluates the hypotheses based upon aggregation effects. After determining the effectiveness of the aggregation hypotheses, hypotheses associated with scale effects are reviewed.

There is sufficient evidence to conclude that aggregation effects increase with scale in the theoretical data results. This theme is not as lucid with the empirical data, where the variable nternal size of groups may influence results. Aggregation effects are also reduced when variables have positive spatial autocorrelations, because the values aggregated groups may take are limited. Table 6.1.1 demonstrates the overall effects of aggregation for all statistics and data sets. From these summary effects, apparently positive spatial auotocorrelations reduce effects of alternate aggregations, but as scale continues to be increased aggregation effects will invariably become large.

When viewing the effects of scale, positive spatial autocorrelations of empirical data accentuate the differences between factor analytic results. The summary effects of scale are displayed in Table 6.1.2 for all scales and data sets. Scale effects are, for the most part, trivial in the uncontaminated data results. However, results based with variables containing positive spatial autocorrelations display serious effects under alternate scales. The empirical data results are also more modifiable with scale than contaminated data results because the general level of spatial

Table 6.1.1 Results of Aggregation Effects.

Statistic	Data Set: Uncontaminated	Contaminated	Empirical
PC Eigenvalues	large and increasing	moderate and increasing	moderate but stable
Final Eigenvalues	large and increasing	moderate and increasing	moderate but stable
Communalities (SMC)	moderate and increasing	small and increasing	small but stable
Final Communalities	moderate and generally increasing	small and generally increasing	small but stable
KMO and MSA	moderate and increasing	small and increasing	small and generally increasing
First Rotated Factor Scores	variance declines and kurtosis for all becomes more negative (platykurtic distribution)		
Factor Loadings	better replication of factors with contaminated and empirical data than uncontaminated data as the data continues to be aggregated further from the BSU level, replication rates decline		

Table 6.1.2: Results of Scale Effects.

	Data Set		
Statistic	**Uncontaminated**	**Contaminated**	**Empirical**
PC Eigenvalues	stable	large increase in significant eigenvalues	large increase in significant eigenvalues
Final Eigenvalues	slightly increasing	all increasing	all increasing
No. of factors	stable	decreasing	decreasing
Percent of variation Explained	slightly increasing	increasing (despite fewer factors)	mainly increasing (despite fewer factors)
Communalities (SMC)	moderately increasing	greatly increasing	greatly increasing
Final Communalities	slightly increasing	generally increasing	generally increasing
KMO and MSA	slightly declining	stable	stable
Generality of First Unrotated Factor	stable	increasing	increasing
Replication of Factor Loadings	good (but declines as scales are increased)	excellent (but declines as scales are increased)	excellent (but declines as scales are increased)

autocorrelations is higher and the interaction of second order spatial dependencies is larger for empirical data.

Most hypotheses set out initially were verified by the examinations. Scale effects were almost entirely absent from the pure data but were very strong for contaminated data. The effects of increasing scale on the contaminated and empirical data results revealed increasing sizes of eigenvalues for the significant factors. Both views that aggregation effects should be largest with pure data and should increase with scale held for the theoretical data results. The empirical results exhibited some discord with respect to increasing aggregation effects with scale. The KMO and MSAs relative invariance to both scale and aggregation effects opposed the initial hypotheses. Since predictions were achieved reasonably well for the theoretical data sets, there may be a possibility to develop accurate predictions of scale effects. The examinations of factor structures by the RELATE correlations submitted some interesting results. First, when variables are influenced by positive spatial autocorrelations, the results are more invariant to aggregation effects as suspected. However, the positive spatial autocorrelated variables also led to more stability among results across different scales. Despite this encouraging result, it must be remembered that RELATE standardizes factor loadings across variables. It is also known that factor loadings will be invariably affected by alternate scales. From all of the above results, scale and aggregation effects are prominent enigmas that transpire in empirical analyses.

6.2: Areas for Future Research

There are many areas for future research on scale and aggregation effects on the factor analytic model. First, this analysis examined MAUP with fixed options for the model. It would be interesting to note whether assorted extraction techniques, communality estimates, significant factor criterion, and factor score estimates alter the sensitivity of the model. If several factor analytic options are less sensitive to MAUP than others, these options should be popularized by present day researchers. However, it would be startling if different factor analytic options would change the effects of scale and aggregation. This study, hopefully, has contributed a foundation from which future research could elaborate upon specific aspects of the model.

Apparently absent from the suggestions above, are the effects of various rotation techniques. This absence was premeditated and this option is expanded in greater detail below. Different orthogonal rotations are not expected to produce noticeable invariance to scale or aggregation effects

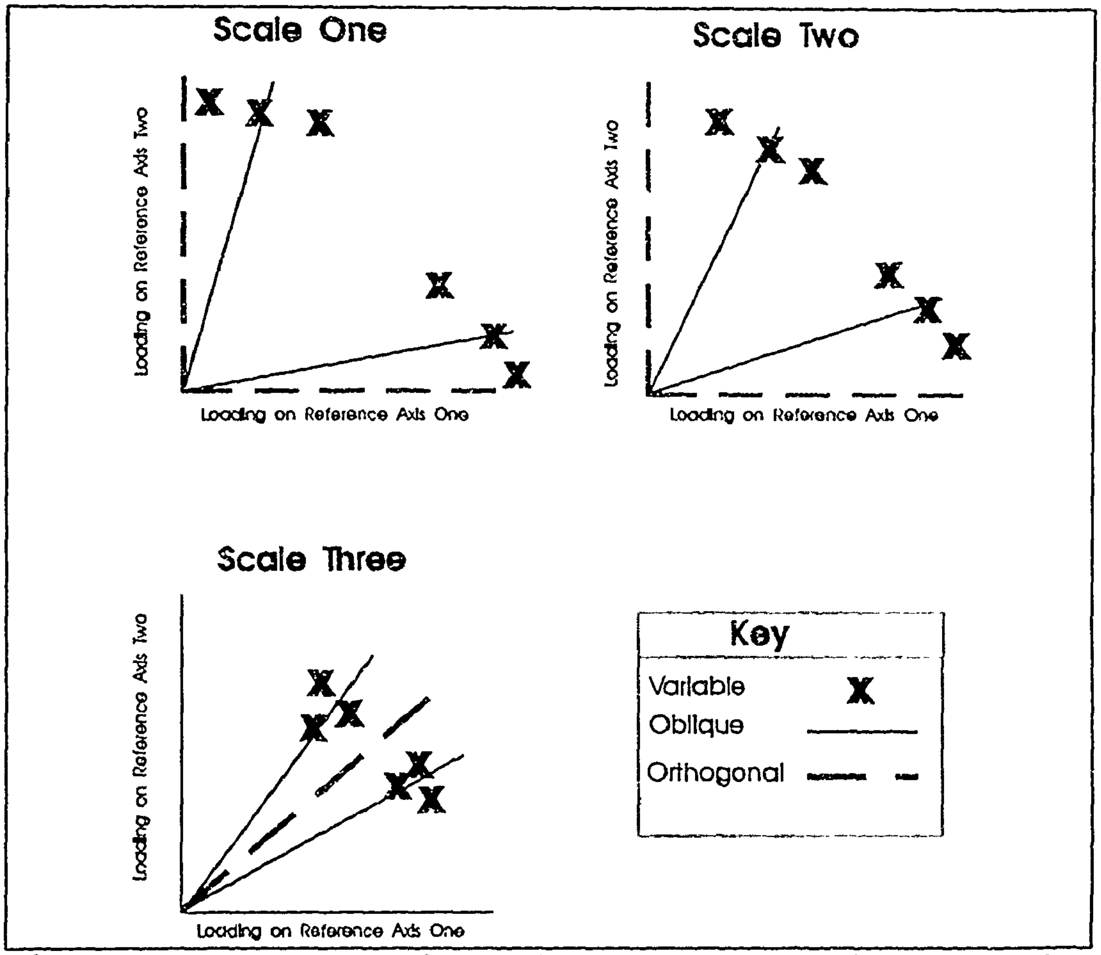

Figure 6.2.1: Hypothesized Effects of Increasing Scale for Oblique and Orthogonal Rotations.

although this avenue could be analyzed. Use of oblique rotations may display signs of invariance particularly to scale. As seen by the results, even the lowest associations between variables increase with scale in terms of communality sizes. The increase of small communalities with scale suggests almost independent variables increase in association with scale. If factors at the basic spatial unit level are even remotely related, this association will increase with reductions to the number of areal unit observations.

Therefore, an oblique rather than orthogonal rotation may prove to be more scale invariant. Figure 6.2.1 displays a hypothetical case with only two factors. As seen in Figure 6.2.1, it is possible for factor relationships to increase to the point where an orthogonal rotation would merge the two processes together in a weak general form. If scale invariance is a goal of factorial ecologies, an oblique rotation may be required. Clearly, this is an area where investigation of scale effects on factor analysis should concentrate.

Other particular aims of this thesis are also available for substantiation by other research. First, the apparent contradictory finding of constant aggregation effects with increasing scale from the Saskatoon data set could be explored. Although aggregation effects were thought to increase with scale for all data sets, with the empirical data this hypothesis did not hold. As well, future research could attempt to predict scale and aggregation effects from Arbia's (1989) correlation formula. Particularly, the formula could be modified to consider second order relationships that would involve only an addition of a second term to the formula. If scale and aggregation effects become predictable from the basic spatial unit data attributes, these effects will not be so omnipotent in the future. It should be apparent that while this study provides an introduction to this topic, many opportunities exist for future research on MAUP effects in factor analysis.

6.3: Conducting Future Factor Analyses

Researchers should consider some findings from this thesis when conducting a factor analysis in the future. First, one has to justify the scale of analysis employed. If the objective is to decipher themes evolving from neighbourhoods, it is imperative to detail explicitly why the areal units employed epitomize neighbourhoods. Interpretation of research must also avoid ecological fallacies and cross-level fallacies (Davies, 1983) when interpreting results. If observations are not based upon meaningful areal units, a sample of alternate aggregations should be employed to judge the sensitivity of the model to aggregation effects. When sensitivity analysis is not feasible the researcher should explain the caveats of completing research with only one arrangement of areal unit observations. With all the advances in region building algorithms and GIS technology, there is little reason for geographers to accept the constraints of areal unit data collected by census agencies. After stating this, there are probably limits upon the basic spatial unit level as socioeconomic data in Canada cannot accurately be found below the enumeration area scale. Any further attempt to disaggregate data below BSU levels would involve rigid assumptions.

Factor analysis is a technique adopted by geographers from other disciplines, most notably psychology. The technique was not developed to handle the intricacies of areal unit observations, and therefore the results are modifiable. Apparently geographic research alone will bear the effects associated with areal unit observations. The results found by factor analysts are specific to the areal units employed in the study. Due to the modifiability of results, factor analytic results are not generally comparable across studies. The similarity of past themes like social area analysis at first appears encouraging, but it may be a symptom of another problem with geographical factor analyses. For example, if one completes a factorial ecology with most variables measuring income, it should be hardly surprising that a significant factor representing income appears. As well, if one only enters socioeconomic variables in a factor analysis, there should be little surprise in the factors that are extracted.

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Appendix A

Appendix A - Definitions of Empirical Variables and Corresponding Number used in Text

Economic Status

1) - Average Family Income

2) - Male Unemployment Rate

3) - Rate of Population Older than twenty-five years with a University Degree

4) - Rate of Employed Labourforce in Blue Collar Occupations

Social Status

6) - Female Labour Participation Rate

7) - Ratio of Children Aged (0-4) to Females Aged (20-44)

8) - Average Household Size

Ethnicity

5) - Rate of Population with Mother Tongue that is a Non-Official Language

10) - Rate of Population Aged 5+ years born as an Immigrant

Other

9) - Rate of Population Aged 5+ years who are Migrants

Appendix B

Table of Contents for Appendix B

Appendix B - Key to Appendix

PCT1	- % of data set variation explained by components
PCT2	- % of data set variation explained by factors
KMO	- Kaiser-Meyer-Olkin measure of sampling adequacy
MS(x)	- Measure of sampling adequacy for variable x
C(x)	- Initial Communality for variable x
E(y)	- Initial Eigenvalue associated with factor y
CM(x)	- Final Communality for variable x
EE(y)	- Final Eigenvalue for factor y
FS(z)	- Factor Score of sample z
St.dev.	- Standard Deviation
Kurt	- Kurtosis
Skew	- Skewness
Min	- Minimum
Max	- Maximum
**	- Based on less than the thirty aggregations

Appendix B.1

Basic Spatial Unit Results for all Data Sets

	Empirical	Theoretical
PCT1	71.51	65.38
PCT2	60.60	53.50
KMO	0.64	0.58
MS1	0.68	0.55
MS2	0.85	0.61
MS3	0.70	0.65
MS4	0.82	0.64
MS5	0.83	0.36
MS6	0.73	0.49
MS7	0.54	0.64
MS8	0.55	0.35
MS9	0.38	0.52
MS10	0.70	0.71
C1	0.65	0.74
C2	0.40	0.54
C3	0.62	0.74
C4	0.49	0.81
C5	0.50	0.48
C6	0.59	0.29
C7	0.64	0.10
C8	0.73	0.25
C9	0.45	0.70
C10	0.51	0.61
E1	3.32	3.30
E2	2.23	1.92
E3	1.59	1.32
E4	0.84	0.99
E5	0.56	0.74
E6	0.42	0.68
E7	0.37	0.52
E8	0.30	0.24
E9	0.22	0.18
E10	0.15	0.10
CM1	0.81	0.60
CM2	0.40	0.55
CM3	0.64	0.93
CM4	0.56	0.93
CM5	0.57	0.36
CM6	0.72	0.34
CM7	0.55	0.06
CM8	0.99	0.17
CM9	0.31	0.55
CM10	0.51	0.86
EE1	2.96	3.05
EE2	1.91	1.50
EE3	1.19	0.80

Appendix B.2

120 Observation Level Results for Uncontaminated Data

STAT	Mean	St.dev.	Kurt	Skew	Range	Min	Max
PCT1	67.83	3.37	1.58	1.57	12.04	64.24	76.28
PCT2	55.98	2.65	1.19	0.90	10.94	51.95	62.89
KMO	0.56	0.03	-0.90	0.08	0.10	0.51	0.61
MS1	0.53	0.03	0.17	-0.11	0.15	0.45	0.60
MS2	0.62	0.04	0.05	-0.49	0.15	0.45	0.60
MS3	0.60	0.02	-0.81	-0.05	0.06	0.57	0.63
MS4	0.64	0.04	-0.76	-0.49	0.14	0.56	0.70
MS5	0.33	0.06	-0.11	0.23	0.24	0.22	0.45
MS6	0.44	0.09	-1.55	0.24	0.26	0.33	0.59
MS7	0.67	0.09	0.95	-0.70	0.39	0.42	0.82
MS8	0.29	0.07	1.48	0.83	0.34	0.16	0.49
MS9	0.52	0.05	-0.36	-0.07	0.19	0.43	0.63
MS10	0.70	0.03	-0.81	0.41	0.12	0.65	0.77
C1	0.74	0.03	0.45	-0.77	0.11	0.68	0.79
C2	0.57	0.03	-0.88	-0.27	0.11	0.51	0.62
C3	0.76	0.02	0.38	-0.56	0.09	0.71	0.80
C4	0.81	0.03	-0.73	-0.16	0.12	0.74	0.86
C5	0.50	0.05	0.71	-0.24	0.21	0.38	0.59
C6	0.35	0.05	-0.52	0.10	0.19	0.26	0.45
C7	0.13	0.03	0.11	-0.13	0.12	0.07	0.20
C8	0.30	0.06	-0.86	-0.18	0.21	0.19	0.41
C9	0.73	0.03	0.64	-0.25	0.15	0.66	0.81
C10	0.65	0.03	-0.21	-0.05	0.14	0.59	0.72
E1	3.34	0.12	-0.53	-0.33	0.46	3.11	3.57
E2	1.93	0.11	-0.14	0.07	0.45	1.72	2.17
E3	1.34	0.09	-0.30	0.55	0.34	1.20	1.54
E4	0.94	0.05	-0.84	0.23	0.19	0.85	1.04
E5	0.78	0.06	0.30	0.25	0.24	0.67	0.90
E6	0.68	0.05	-0.96	-0.05	0.19	0.58	0.77
E7	0.51	0.04	0.61	0.11	0.17	0.44	0.61
E8	0.22	0.02	-0.67	-0.16	0.07	0.18	0.25
E9	0.17	0.01	0.33	0.14	0.06	0.14	0.20
E10	0.10	0.01	-0.35	0.28	0.06	0.07	0.13
CM1	0.61	0.04	-0.02	-0.11	0.17	0.51	0.68
CM2	0.62	0.10	1.39	1.25	0.39	0.49	0.88
CM3	0.96	0.03	-1.11	-0.38	0.11	0.89	1.00
CM4	0.94	0.03	-0.16	-0.31	0.14	0.86	1.00
CM5	0.34	0.10	3.51	1.38	0.50	0.19	0.69
CM6	0.37	0.07	0.67	0.52	0.34	0.23	0.57
CM7	0.11	0.03	-0.65	-0.03	0.12	0.04	0.16
CM8	0.17	0.05	-0.45	0.31	0.22	0.06	0.28
CM9	0.64	0.15	2.24	1.87	0.49	0.50	0.99
CM10	0.85	0.08	-0.64	-0.10	0.31	0.69	1.00

Appendix B.2 (continued)

STAT	Mean	St.dev.	Kurt	Skew	Range	Min	Max
EE1	3.11	0.13	-0.50	-0.14	0.51	2.85	3.36
EE2	1.54	0.11	0.25	-0.40	0.47	1.28	1.75
EE3	0.85	0.12	-0.20	-0.15	0.46	0.62	1.08
EE4**	0.56	0.09	-3.24	-0.59	0.18	0.46	0.64

STAT	Mean	Var.	Kurt	Skew	Range	Min	Max
F1	0.00	1.00	0.38	-0.03	5.30	-2.84	2.46
F2	0.00	1.03	0.04	-0.02	5.84	-2.93	2.91
F3	0.00	0.90	-0.44	0.15	4.45	-2.28	2.16
F4	0.00	1.02	0.86	-0.15	5.81	-3.42	2.39
F5	0.00	1.00	0.28	-0.15	5.63	-3.23	2.41
F6	0.00	0.97	1.07	-0.25	6.04	-3.53	2.52
F7	0.00	1.03	0.54	0.04	6.06	-3.42	2.64
F8	0.00	0.97	0.43	-0.23	5.48	-3.32	2.16
F9	0.00	1.08	0.18	-0.30	5.43	-3.18	2.25
F10	0.00	0.97	-0.29	-0.18	5.07	-2.87	2.20
F11	0.00	1.02	0.10	-0.02	5.09	-2.59	2.50
F12	0.00	1.02	0.94	0.02	6.15	-3.62	2.54
F13	0.00	1.03	0.04	-0.14	5.24	-3.18	2.05
F14	0.00	1.07	0.25	-0.31	5.31	-2.94	2.37
F15	0.00	1.00	-0.37	-0.07	4.71	-2.74	1.98
F16	0.00	0.97	0.30	0.07	5.46	-2.79	2.67
F17	0.00	1.08	0.34	-0.04	5.85	-3.47	2.38
F18	0.00	1.00	0.47	-0.16	5.33	-2.97	2.36
F19	0.00	1.02	0.35	-0.13	5.40	-3.33	2.08
F20	0.00	1.02	0.29	0.00	5.27	-2.71	2.56
F21	0.00	0.97	-0.01	-0.15	4.99	-2.90	2.09
F22	0.00	0.96	0.50	-0.21	5.33	-3.06	2.27
F23	0.00	1.10	0.35	-0.41	5.34	-2.94	2.41
F24	0.00	0.98	0.83	-0.35	5.80	-3.62	2.18
F25	0.00	1.01	0.67	-0.38	5.66	-3.37	2.29
F26	0.00	1.11	0.80	-0.25	6.42	-3.75	2.67
F27	0.00	1.07	0.07	-0.19	5.63	-3.13	2.50
F28	0.00	1.07	0.18	-0.18	5.98	-3.33	2.66
F29	0.00	1.00	-0.09	0.02	4.85	-2.39	2.46
F30	0.00	1.01	-0.11	-0.14	5.18	-2.80	2.38

** - based on five of thirty aggregations

Appendix B.3

120 Observation Level Results for Contaminated Data

STAT	Mean	St.dev.	Kurt	Skew	Range	Max	Min
PCT1	72.38	4.66	-1.20	0.89	12.31	68.18	80.49
PCT2	60.49	3.23	-1.22	0.81	8.69	57.26	65.95
KMO	0.60	0.02	1.67	-0.55	0.08	0.55	0.63
MS1	0.59	0.02	1.01	-0.47	0.12	0.52	0.63
MS2	0.60	0.04	-0.85	-0.10	0.13	0.53	0.66
MS3	0.67	0.02	-0.16	0.26	0.08	0.63	0.71
MS4	0.68	0.02	1.45	-0.67	0.11	0.61	0.73
MS5	0.40	0.04	-0.82	-0.18	0.16	0.32	0.48
MS6	0.58	0.07	-0.72	-0.05	0.28	0.45	0.72
MS7	0.61	0.08	-0.23	-0.27	0.33	0.43	0.76
MS8	0.33	0.05	1.01	-0.79	0.21	0.20	0.41
MS9	0.56	0.03	0.71	-0.20	0.12	0.49	0.61
MS10	0.70	0.02	-0.23	-0.43	0.09	0.65	0.74
C1	0.81	0.01	-0.13	0.49	0.05	0.79	0.83
C2	0.65	0.03	-0.92	-0.42	0.09	0.60	0.69
C3	0.82	0.01	-0.15	0.03	0.05	0.79	0.85
C4	0.85	0.01	0.89	0.21	0.06	0.82	0.88
C5	0.56	0.03	1.10	1.03	0.10	0.53	0.63
C6	0.31	0.03	0.87	-0.25	0.15	0.22	0.37
C7	0.15	0.03	1.27	0.28	0.15	0.08	0.23
C8	0.31	0.02	0.41	0.33	0.10	0.27	0.37
C9	0.76	0.02	0.15	-0.70	0.08	0.72	0.80
C10	0.71	0.02	-0.19	-0.02	0.08	0.67	0.75
E1	3.65	0.05	0.47	-0.12	0.23	3.54	3.77
E2	1.98	0.08	0.32	-0.19	0.36	1.79	2.15
E3	1.31	0.07	-0.70	0.17	0.26	1.20	1.45
E4	0.98	0.03	-0.48	-0.43	0.11	0.92	1.03
E5	0.66	0.04	-0.76	-0.30	0.14	0.58	0.72
E6	0.59	0.03	-0.41	0.60	0.12	0.54	0.66
E7	0.46	0.02	-0.78	-0.18	0.07	0.42	0.49
E8	0.17	0.01	-0.41	0.29	0.05	0.15	0.20
E9	0.13	0.01	0.86	-0.20	0.05	0.10	0.15
E10	0.08	0.01	-0.19	-0.23	0.02	0.06	0.09
CM1	0.70	0.03	0.37	0.02	0.11	0.64	0.75
CM2	0.72	0.15	-0.87	0.86	0.46	0.54	1.00
CM3	0.95	0.03	-0.52	-0.26	0.10	0.89	1.00
CM4	0.93	0.02	-0.42	0.38	0.07	0.89	0.97
CM5	0.50	0.08	0.01	0.52	0.35	0.36	0.70
CM6	0.34	0.04	0.43	-0.50	0.20	0.22	0.42
CM7	0.10	0.05	4.76	2.00	0.23	0.04	0.27
CM8	0.22	0.03	-0.30	-0.45	0.12	0.14	0.26
CM9	0.70	0.12	0.95	1.45	0.43	0.56	1.00
CM10	0.89	0.05	-0.06	0.17	0.22	0.78	1.00

Appendix B.3 (continued)

STAT	Mean	St.dev.	Kurt	Skew	Range	Max	Min
EE1	3.45	0.05	0.37	-0.44	0.22	3.33	3.55
EE2	1.63	0.09	0.66	-0.15	0.40	1.39	1.80
EE3	0.82	0.08	-0.51	0.34	0.31	0.68	0.98
EE4 **	0.49	0.05	0.01	-0.31	0.18	0.39	0.57

STAT	Mean	Var.	Kurt	Skew	Range	Max	Min
F1	0.00	0.99	0.21	-0.01	5.06	-2.62	2.43
F2	0.00	1.04	-0.19	-0.08	4.72	-2.44	2.29
F3	0.00	0.98	-0.19	-0.09	5.00	-2.56	2.43
F4	0.00	1.03	0.09	-0.32	4.87	-2.70	2.17
F5	0.00	0.97	-0.38	-0.17	4.65	-2.74	1.91
F6	0.00	0.93	0.06	-0.17	5.02	-2.64	2.38
F7	0.00	1.00	-0.05	-0.05	4.87	-2.36	2.51
F8	0.00	0.97	-0.51	-0.07	4.58	-2.3[illegible]	2.19
F9	0.00	1.05	-0.15	-0.10	4.60	-2.29	2.30
F10	0.00	0.97	-0.14	-0.21	4.71	-2.67	2.04
F11	0.00	0.97	-0.41	0.10	4.65	-2.32	2.33
F12	0.00	0.97	0.05	-0.13	5.09	-2.58	2.51
F13	0.00	0.97	-0.59	-0.01	4.30	-1.98	2.32
F14	0.00	0.95	-0.12	-0.12	4.68	-2.49	2.19
F15	0.00	0.96	-0.21	-0.06	5.01	-2.57	2.44
F16	0.00	1.01	-0.50	-0.09	5.01	-2.75	2.26
F17	0.00	0.94	-0.33	-0.09	4.54	-2.20	2.34
F18	0.00	1.03	-0.23	-0.02	4.93	-2.57	2.36
F19	0.00	1.00	-0.03	-0.02	4.70	-2.38	2.32
F20	0.00	0.99	0.05	-0.25	4.84	-2.63	2.21
F21	0.00	0.97	-0.29	-0.04	4.89	-2.56	2.33
F22	0.00	1.02	-0.44	-0.07	4.54	-2.29	2.25
F23	0.00	0.99	-0.25	-0.09	5.07	-2.78	2.29
F24	0.00	0.95	-0.20	-0.16	4.71	-2.51	2.20
F25	0.00	1.02	0.04	-0.10	4.98	-2.66	2.32
F26	0.00	1.04	-0.02	-0.34	4.96	-2.70	2.26
F27	0.00	0.98	-0.27	-0.19	4.58	-2.35	2.22
F28	0.00	1.00	-0.40	-0.19	4.92	-2.75	2.17
F29	0.00	0.98	-0.32	-0.21	4.52	-2.40	2.12
F30	0.00	0.98	-0.08	-0.24	5.14	-2.70	2.44

** - based upon nine of thirty aggregations

Appendix B.4

Sixty Observation Level Results for Uncontaminated Data

STAT	Mean	St.dev.	Kurt	Skew	Range	Min	Max
PCT1	71.30	4.76	-1.36	0.38	14.95	64.99	79.95
PCT2	58.92	3.86	-0.44	0.28	16.16	51.15	67.31
KMO	0.53	0.05	-1.13	-0.12	0.17	0.45	0.62
MSA1	0.52	0.07	0.05	0.04	0.28	0.38	0.66
MSA2	0.61	0.05	-0.77	-0.24	0.19	0.51	0.70
MSA3	0.58	0.06	-0.46	-0.42	0.22	0.46	0.68
MSA4	0.61	0.06	-0.77	-0.02	0.23	0.50	0.73
MSA5	0.30	0.07	-0.63	0.41	0.26	0.19	0.45
MSA6	0.39	0.13	0.17	0.99	0.48	0.22	0.70
MSA7	0.58	0.14	0.76	-0.41	0.65	0.21	0.86
MSA8	0.29	0.09	-0.23	0.40	0.37	0.13	0.50
MSA9	0.51	0.07	0.27	-0.13	0.34	0.32	0.66
MSA10	0.72	0.05	-0.32	-0.14	0.21	0.61	0.83
C1	0.78	0.04	-0.46	0.00	0.18	0.70	0.88
C2	0.59	0.06	0.02	-0.57	0.27	0.42	0.70
C3	0.79	0.03	0.30	-0.19	0.14	0.72	0.87
C4	0.84	0.04	-0.59	-0.17	0.15	0.77	0.92
C5	0.57	0.08	1.14	-0.94	0.33	0.36	0.69
C6	0.39	0.07	-0.64	-0.18	0.27	0.26	0.52
C7	0.16	0.05	3.29	1.45	0.25	0.07	0.32
C8	0.37	0.07	-0.29	-0.18	0.30	0.24	0.53
C9	0.78	0.04	-0.52	-0.53	0.16	0.69	0.85
C10	0.65	0.07	1.32	-0.63	0.35	0.45	0.80
E1	3.46	0.19	-0.88	-0.06	0.68	3.13	3.81
E2	1.94	0.18	0.49	0.50	0.84	1.59	2.44
E3	1.32	0.10	-0.38	0.13	0.40	1.14	1.53
E4	0.96	0.07	-0.74	-0.48	0.25	0.81	1.06
E5	0.79	0.05	-1.09	0.00	0.17	0.70	0.87
E6	0.63	0.06	-0.41	-0.44	0.25	0.48	0.73
E7	0.46	0.06	-0.74	0.17	0.24	0.35	0.59
E8	0.22	0.04	1.20	0.80	0.19	0.13	0.33
E9	0.15	0.03	0.82	0.82	0.11	0.10	0.21
E10	0.08	0.02	-0.38	0.41	0.07	0.04	0.11
CM1	0.65	0.10	0.17	0.39	0.45	0.44	0.89
CM2	0.61	0.09	0.90	-0.24	0.46	0.37	0.83
CM3	0.95	0.06	0.33	-1.12	0.18	0.82	1.00
CM4	0.93	0.05	2.73	-1.50	0.23	0.76	1.00
CM5	0.52	0.21	-1.08	0.48	0.68	0.25	0.94
CM6	0.38	0.14	0.32	0.89	0.56	0.18	0.74
CM7	0.12	0.07	4.08	1.83	0.30	0.05	0.34
CM8	0.24	0.09	2.98	0.99	0.45	0.07	0.52
CM9	[illegible]	0.08	0.83	0.04	0.37	0.52	0.90
CM10	[illegible]	0.08	2.96	-1.28	0.38	0.52	0.90

Appendix B.4 (continued)

STAT	Mean	St.dev.	Kurt	Skew	Range	Min	Max
EE1	3.24	0.20	-0.76	-0.25	0.72	2.88	3.59
EE2	1.56	0.19	0.03	0.15	0.87	1.17	2.04
EE3	0.89	0.12	-0.25	-0.19	0.47	0.63	1.10
EE4 **	0.52	0.11	-0.26	0.13	0.36	0.36	0.72

STAT	Mean	Var.	Kurt	Skew	Range	Min	Max
F1	0.00	0.97	0.69	-0.58	4.94	-3.18	1.76
F2	0.00	0.96	-0.32	-0.36	4.37	-2.25	2.12
F3	0.00	1.06	-0.31	0.12	4.55	-2.03	2.52
F4	0.00	1.04	0.32	-0.12	4.77	-2.57	2.19
F5	0.00	1.05	-0.05	-0.46	4.52	-2.72	1.80
F6	0.00	1.02	1.25	0.20	5.09	-2.27	2.81
F7	0.00	1.12	0.36	-0.53	4.76	-2.93	1.83
F8	0.00	0.98	0.91	-0.57	5.39	-3.28	2.11
F9	0.00	1.00	-0.05	-0.21	4.80	-2.35	2.45
F10	0.00	1.09	0.10	0.03	4.86	-2.63	2.22
F11	0.00	0.93	-0.45	0.05	4.49	-2.10	2.39
F12	0.00	1.03	-0.03	-0.08	4.58	-2.47	2.11
F13	0.00	0.81	0.37	-0.43	4.28	-2.64	1.64
F14	0.00	1.06	-0.31	0.12	4.55	-2.03	2.52
F15	0.00	1.08	-0.26	-0.22	4.73	-2.82	1.91
F16	0.00	0.94	-0.47	-0.33	4.01	-2.09	1.92
F17	0.00	1.11	0.64	-0.38	5.72	-3.30	2.42
F18	0.00	1.00	-0.74	-0.10	4.36	-2.43	1.94
F19	0.00	0.93	-0.08	-0.01	4.75	-2.60	2.15
F20	0.00	1.06	-0.36	0.14	4.56	-2.10	2.46
F21	0.00	0.93	-0.41	-0.02	4.15	-2.14	2.01
F22	0.00	1.06	0.79	-0.36	5.82	-3.25	2.58
F23	0.00	0.97	0.32	-0.30	4.65	-2.60	2.05
F24	0.00	1.00	0.94	-0.42	5.38	-2.70	2.68
F25	0.00	0.92	0.18	0.08	4.76	-2.51	2.25
F26	0.00	1.01	3.57	-1.06	6.31	-3.76	2.55
F27	0.00	1.05	-0.68	0.04	4.30	-1.82	2.48
F28	0.00	0.93	0.81	-0.19	4.99	-2.63	2.36
F29	0.00	1.11	1.00	-0.48	5.46	-3.06	2.40
F30	0.00	1.03	-0.01	0.08	5.07	-2.29	2.78

** - based on twelve of thirty aggregations

Appendix B.5

Sixty Observation Level Results for Contaminated Data

STAT	Mean	St.dev.	Kurt	Skew	Range	Min	Max
PCT1	73.90	2.00	17.59	3.77	11.36	71.94	83.30
PCT2	63.69	1.64	4.80	1.68	8.39	61.21	69.60
KMO	0.61	0.03	-0.21	0.08	0.11	0.56	0.67
MSA1	0.62	0.05	0.72	0.45	0.20	0.53	0.74
MSA2	0.60	0.06	-0.16	-0.84	0.20	0.48	0.69
MSA3	0.70	0.05	-0.40	-0.41	0.21	0.59	0.80
MSA4	0.69	0.04	-0.39	-0.12	0.14	0.62	0.76
MSA5	0.40	0.06	-0.06	0.55	0.24	0.28	0.52
MSA6	0.57	0.12	-0.75	0.06	0.43	0.37	0.80
MSA7	0.63	0.11	0.41	0.21	0.52	0.38	0.90
MSA8	0.32	0.11	-0.73	0.56	0.45	0.13	0.55
MSA9	0.60	0.05	-0.83	0.27	0.18	0.52	0.70
MSA10	0.69	0.05	1.14	-0.93	0.21	0.54	0.76
C1	0.86	0.02	-1.44	0.01	0.06	0.83	0.89
C2	0.76	0.04	2.87	1.28	0.17	0.71	0.88
C3	0.87	0.02	0.76	0.54	0.08	0.83	0.91
C4	0.90	0.01	-1.11	-0.08	0.05	0.87	0.92
C5	0.64	0.04	-0.21	0.39	0.17	0.57	0.74
C6	0.37	0.07	2.47	1.22	0.33	0.27	0.60
C7	0.23	0.06	-0.47	0.59	0.22	0.14	0.36
C8	0.41	0.06	-0.04	-0.15	0.26	0.27	0.53
C9	0.82	0.03	0.51	-0.22	0.13	0.75	0.88
C10	0.80	0.03	-0.01	-0.24	0.13	0.74	0.86
E1	4.06	0.11	-1.08	0.16	0.37	3.88	4.25
E2	1.95	0.14	-0.63	-0.09	0.54	1.66	2.21
E3	1.35	0.09	-0.46	-0.24	0.35	1.16	1.51
E4	0.91	0.05	0.11	-0.32	0.21	0.79	1.00
E5	0.62	0.05	-0.90	0.24	0.18	0.53	0.72
E6	0.47	0.04	-1.01	-0.18	0.13	0.40	0.53
E7	0.38	0.04	1.06	-0.38	0.19	0.29	0.47
E8	0.12	0.02	0.69	0.43	0.07	0.09	0.16
E9	0.09	0.01	-0.39	-0.01	0.04	0.07	0.11
E10	0.05	0.01	0.35	0.86	0.03	0.04	0.07
CM1	0.78	0.04	-0.67	0.18	0.16	0.72	0.88
CM2	0.77	0.08	0.95	0.66	0.40	0.60	1.00
CM3	0.93	0.04	-0.44	-0.04	0.15	0.85	1.00
CM4	0.96	0.02	0.20	0.36	0.07	0.92	1.00
CM5	0.48	0.10	0.89	0.98	0.42	0.32	0.74
CM6	0.38	0.08	1.49	1.11	0.33	0.28	0.61
CM7	0.13	0.04	-0.61	0.27	0.16	0.06	0.22
CM8	0.34	0.08	-0.04	-0.44	0.33	0.16	0.49
CM9	0.68	0.05	1.29	0.80	0.23	0.60	0.83
CM10	0.91	0.05	-0.03	-0.56	0.21	0.79	1.00

Appendix B.5 (Continued)

STAT	Mean	St.dev.	Kurt	Skew	Range	Min	Max
EE1	3.87	0.11	-1.05	0.06	0.37	3.69	4.06
EE2	1.60	0.15	0.04	-0.33	0.65	1.23	1.88
EE3	0.88	0.12	-0.66	-0.31	0.44	0.63	1.07
EE4 **	0.44	NA	NA	NA	NA	NA	NA

STAT	Mean	Var.	Kurt	Skew	Range	Min	Max
F1	0.00	0.96	-0.41	-0.01	4.21	-2.22	1.98
F2	0.00	0.95	-0.98	0.01	3.87	-1.95	1.92
F3	0.00	1.00	-0.45	-0.21	4.08	-2.37	1.70
F4	0.00	0.97	0.06	-0.02	4.69	-2.62	2.07
F5	0.00	0.97	-0.54	0.08	4 18	-1.93	2.25
F6	0.00	1.02	0.06	-0.36	4.88	-2.78	2.10
F7	0.00	1.03	-0.44	-0.35	4.34	-2.50	1.83
F8	0.00	0.88	0.00	-0.40	4.61	-2.73	1.88
F9	0.00	1.03	-0.29	-0.28	4.52	-2.55	1.97
F10	0.00	1.04	-0.10	-0.37	4.60	-2.46	2.14
F11	0.00	1.02	-0.27	-0.38	4.13	-2.23	1.91
F12	0.00	0.95	-0.40	0.16	4.61	-1.94	2.67
F13	0.00	0.93	-0.49	-0.19	4.06	-2.06	2.00
F14	0.00	1.01	-0.33	-0.28	4.41	-2.45	1.96
F15	0.00	0.95	-0.49	-0.06	4.01	-2.02	1.99
F16	0.00	0.92	-0.65	-0.10	3.82	-2.12	1.70
F17	0.00	0.92	-0.46	-0.02	3.97	-1.88	2.09
F18	0.00	1.11	0.50	-0.62	5.01	-2.76	2.25
F19	0.00	0.98	-0.44	-0.04	4.08	-2.16	1.92
F20	0.00	1.03	0.09	0.01	4.96	-2.30	2.65
F21	0.00	1.02	-0.05	-0.09	4.92	-2.30	2.61
F22	0.00	1.02	-0.72	-0.01	3.99	-1.91	2.09
F23	0.00	1.01	0.42	-0.52	4.81	-2.96	1.85
F24	0.00	0.97	-0.68	0.04	3.94	-1.98	1.96
F25	0.00	1.00	-0.08	-0.15	4.51	-2.16	2.35
F26	0.00	0.93	-0.52	0.06	4.11	-2.13	1.97
F27	0.00	0.98	-0.73	-0.01	3.92	-1.97	1.95
F28	0.00	0.92	-0.44	-0.16	4.05	-2.08	1.97
F29	0.00	0.96	0.15	0.01	4.61	-2.51	2.10
F30	0.00	0.86	-0.81	0.13	3.62	-1.76	1.86

** - based on 1 of thirty aggregations

Appendix B.6

Forty Observation Level Results for Uncontaminated data

STAT	Mean	St.dev.	Kurt	Skew	Range	Min	Max
PCT1	73.16	4.88	-1.66	-0.03	14.44	65.87	80.31
PCT2	60.47	3.40	-0.50	-0.04	14.02	54.20	68.21
KMO	0.51	0.06	-0.83	0.22	0.21	0.41	0.62
MSA1	0.49	0.08	-0.86	-0.09	0.29	0.32	0.62
MSA2	0.60	0.10	0.08	-0.20	0.42	0.37	0.79
MSA3	0.55	0.08	-0.77	0.22	0.31	0.41	0.72
MSA4	0.58	0.07	-0.64	0.08	0.27	0.45	0.72
MSA5	0.31	0.08	-0.81	0.36	0.31	0.17	0.48
MSA6	0.41	0.15	-0.40	0.36	0.56	0.17	0.73
MSA7	0.48	0.17	-0.10	-0.37	0.69	0.06	0.75
MSA8	0.30	0.11	1.28	1.16	0.44	0.14	0.58
MSA9	0.51	0.09	-0.23	0.18	0.37	0.33	0.71
MSA10	0.70	0.06	0.32	0.75	0.25	0.59	0.85
C1	0.80	0.04	-0.26	0.05	0.16	0.72	0.88
C2	0.59	0.07	-0.11	-0.49	0.27	0.43	0.71
C3	0.79	0.04	4.18	-1.00	0.25	0.64	0.88
C4	0.84	0.05	-0.32	-0.66	0.17	0.73	0.90
C5	0.62	0.08	0.64	-0.62	0.35	0.39	0.74
C6	0.40	0.08	1.74	0.88	0.38	0.26	0.63
C7	0.20	0.07	-0.23	0.51	0.29	0.08	0.37
C8	0.45	0.11	-0.17	0.26	0.43	0.24	0.67
C9	0.80	0.04	1.90	-0.95	0.20	0.66	0.86
C10	0.66	0.06	-0.66	-0.13	0.24	0.53	0.77
E1	3.42	0.36	-0.15	-0.50	1.43	2.64	4.07
E2	1.92	0.25	0.47	0.30	1.19	1.39	2.58
E3	1.39	0.14	1.25	0.27	0.70	1.08	1.78
E4	1.01	0.10	0.24	0.12	0.43	0.81	1.24
E5	0.80	0.07	-0.48	0.52	0.29	0.69	0.98
E6	0.61	0.07	3.14	1.20	0.30	0.50	0.80
E7	0.41	0.07	0.11	0.56	0.28	0.28	0.56
E8	0.22	0.04	0.12	0.30	0.16	0.15	0.31
E9	0.14	0.03	0.14	0.57	0.11	0.10	0.21
E10	0.07	0.02	-0.51	0.52	0.06	0.04	0.10
CM1	0.73	0.13	0.33	0.16	0.52	0.48	1.00
CM2	0.63	0.11	0.02	-0.09	0.48	0.43	0.91
CM3	0.88	0.08	-1.07	0.31	0.25	0.75	1.00
CM4	0.92	0.05	-0.65	-0.29	0.20	0.80	1.00
CM5	0.55	0.18	0.45	0.15	0.85	0.15	1.00
CM6	0.32	0.11	-0.42	0.15	0.44	0.11	0.55
CM7	0.13	0.08	-0.78	0.51	0.29	0.01	0.30
CM8	0.31	0.12	-0.56	0.23	0.46	0.07	0.52
CM9	0.81	0.12	-0.04	-0.44	0.50	0.50	0.99
CM10	0.75	0.09	-0.54	0.08	0.39	0.57	0.97

Appendix B.6 (continued)

STAT	Mean	St.dev.	Kurt	Skew	Range	Min	Max
EE1	3.21	0.37	0.13	-0.57	1.49	2.35	3.85
EE2	1.57	0.28	0.27	0.34	1.25	1.05	2.30
EE3	0.96	0.16	2.06	-0.98	0.78	0.49	1.27
EE4 **	0.58	0.11	-1.18	1.09	0.34	0.41	0.75

STAT	Mean	Var.	Kurt	Skew	Range	Min	Max
F1	0.00	1.00	2.66	-0.90	5.33	-3.45	1.88
F2	0.00	0.82	0.01	0.51	3.99	-1.62	2.37
F3	0.00	1.04	-0.39	-0.12	4.09	-2.26	1.82
F4	0.00	0.85	0.52	-0.12	4.50	-2.52	1.98
F5	0.00	0.99	-0.38	-0.08	4.08	-2.02	2.06
F6	0.00	0.95	0.02	0.25	4.17	-2.17	2.00
F7	0.00	1.08	1.25	-0.43	5.43	-2.79	2.64
F8	0.00	0.94	0.68	-0.56	4.35	-2.61	1.74
F9	0.00	0.90	-0.24	0.28	3.93	-1.87	2.06
F10	0.00	0.93	0.26	-0.24	4.33	-2.21	2.11
F11	0.00	0.95	0.11	0.51	4.12	-1.65	2.47
F12	0.00	1.07	0.32	-0.21	4.63	-2.23	2.40
F13	0.00	0.95	-0.55	-0.08	4.02	-2.02	2.01
F14	0.00	0.91	-0.75	0.16	3.60	-1.63	1.97
F15	0.00	1.01	-0.14	-0.30	4.09	-2.01	2.07
F16	0.00	0.96	1.07	0.11	5.25	-2.49	2.76
F17	0.00	0.93	-0.54	-0.18	4.04	-1.98	2.06
F18	0.00	0.95	-0.13	-0.28	4.08	-2.17	1.91
F19	0.00	0.96	-0.14	-0.11	4.20	-2.32	1.87
F20	0.00	1.06	-0.15	-0.24	4.37	-2.38	1.99
F21	0.00	0.97	1.67	-1.18	4.42	-2.99	1.43
F22	0.00	0.89	-0.74	-0.50	3.23	-1.85	1.39
F23	0.00	1.23	0.01	0.03	4.78	-2.25	2.53
F24	0.00	1.00	-0.38	-0.04	4.31	-2.39	1.92
F25	0.00	0.97	0.48	-0.80	4.45	-2.78	1.66
F26	0.00	0.88	-0.45	-0.24	3.89	-1.99	1.90
F27	0.00	0.87	-0.46	0.21	3.67	-1.66	2.01
F28	0.00	1.13	0.04	-0.41	4.65	-2.49	2.15
F29	0.00	1.07	0.84	-0.42	5.12	-2.91	2.20
F30	0.00	0.97	0.10	0.00	4.40	-2.37	2.03

** - based on sixteen of thirty aggregations

Appendix B.7

Forty Observation Level Results for Contaminated Data

STAT	Mean	St.dev.	Kurt	Skew	Range	Min	Max
PCT1	76.35	2.21	11.80	2.61	13.28	72.58	85.86
PCT2	66.68	2.12	0.54	0.36	9.55	62.73	72.28
KMO	0.60	0.03	-0.54	-0.16	0.13	0.53	0.66
MSA1	0.63	0.06	-0.46	-0.34	0.24	0.49	0.73
MSA2	0.59	0.06	-0.57	-0.07	0.25	0.46	0.71
MSA3	0.72	0.06	0.14	-0.82	0.25	0.57	0.82
MSA4	0.66	0.05	-0.28	0.01	0.21	0.56	0.77
MSA5	0.43	0.07	-0.40	0.43	0.27	0.32	0.59
MSA6	0.57	0.15	-0.82	-0.06	0.57	0.26	0.83
MSA7	0.54	0.15	-0.12	0.17	0.63	0.26	0.90
MSA8	0.23	0.08	-0.75	0.35	0.28	0.10	0.38
MSA9	0.60	0.05	4.44	1.23	0.27	0.51	0.78
MSA10	0.67	0.05	-0.27	0.11	0.19	0.58	0.77
C1	0.89	0.02	-0.21	-0.38	0.09	0.84	0.93
C2	0.83	0.03	0.19	0.35	0.14	0.76	0.90
C3	0.88	0.02	1.64	1.00	0.08	0.85	0.93
C4	0.93	0.02	0.29	-0.78	0.08	0.87	0.96
C5	0.72	0.04	0.48	-0.75	0.19	0.60	0.79
C6	0.46	0.06	-0.29	0.04	0.27	0.32	0.59
C7	0.35	0.08	0.35	0.06	0.35	0.17	0.52
C8	0.47	0.09	-1.02	0.04	0.31	0.33	0.64
C9	0.85	0.03	-0.22	-0.09	0.11	0.80	0.91
C10	0.84	0.04	2.18	-1.12	0.20	0.72	0.92
E1	4.29	0.18	-0.60	-0.26	0.73	3.93	4.66
E2	2.00	0.16	-0.34	-0.10	0.66	1.66	2.32
E3	1.31	0.13	0.02	0.10	0.55	1.01	1.56
E4	0.87	0.07	0.05	-0.42	0.29	0.71	1.00
E5	0.56	0.07	-0.07	-0.26	0.29	0.40	0.69
E6	0.44	0.05	0.57	0.77	0.23	0.36	0.58
E7	0.31	0.04	0.59	-0.19	0.21	0.20	0.41
E8	0.10	0.02	4.34	1.40	0.10	0.07	0.17
E9	0.07	0.01	0.02	-0.29	0.04	0.05	0.09
E10	0.04	0.01	-0.50	0.54	0.03	0.03	0.06
CM1	0.80	0.05	0.26	0.00	0.22	0.69	0.91
CM2	0.83	0.07	-0.10	0.37	0.29	0.70	0.99
CM3	0.92	0.04	-1.00	0.27	0.15	0.85	0.99
CM4	0.97	0.03	-1.23	-0.51	0.08	0.92	1.00
CM5	0.60	0.09	-0.37	-0.48	0.35	0.40	0.75
CM6	0.42	0.08	-0.26	0.44	0.32	0.29	0.60
CM7	0.17	0.06	-0.67	0.42	0.22	0.07	0.29
CM8	0.36	0.11	-0.59	0.61	0.40	0.21	0.61
CM9	0.71	0.07	0.03	-0.59	0.28	0.54	0.82
CM10	0.89	0.08	-0.88	-0.43	0.27	0.73	1.00

Appendix B.7 (continued)

STAT	Mean	St.dev.	Kurt	Skew	Range	Min	Max
EE1	4.11	0.18	-0.78	-0.20	0.69	3.78	4.46
EE2	1.68	0.19	-0.26	-0.25	0.75	1.30	2.05
EE3	0.86	0.18	-0.14	-0.25	0.73	0.41	1.14
EE4 **	0.49	NA	NA	NA	NA	NA	NA

STAT	Mean	Var.	Kurt	Skew	Range	Min	Max
F1	0.00	0.92	-0.56	-0.13	3.90	-2.12	1.78
F2	0.00	0.91	-0.55	0.08	3.86	-1.99	1.88
F3	0.00	1.00	-0.31	-0.51	4.00	-2.22	1.79
F4	0.00	1.03	-0.03	-0.51	4.24	-2.56	1.68
F5	0.00	1.02	-0.20	-0.42	4.06	-2.20	1.87
F6	0.00	0.96	0.01	0.21	4.40	-1.82	2.58
F7	0.00	1.06	-0.88	-0.29	4.00	-2.14	1.87
F8	0.00	1.05	0.25	-0.75	4.18	-2.44	1.74
F9	0.00	0.98	-0.88	0.02	3.95	-2.01	1.94
F10	0.00	1.05	-0.91	0.10	3.64	-1.60	2.04
F11	0.00	1.00	-0.30	-0.21	4.26	-2.55	1.71
F12	0.00	1.10	-0.68	0.29	3.98	-1.92	2.07
F13	0.00	1.04	-1.06	-0.11	3.57	-1.79	1.78
F14	0.00	0.97	-1.06	-0.06	3.69	-1.83	1.86
F15	0.00	1.02	-0.47	-0.29	4.21	-2.14	2.06
F16	0.00	0.97	-10.07	-0.10	3.39	-1.71	1.68
F17	0.00	0.91	-0.67	-0.32	3.56	-1.87	1.69
F18	0.00	1.09	-0.02	0.01	4.46	-2.27	2.19
F19	0.00	0.97	-0.92	-0.37	3.44	-2.05	1.39
F20	0.00	1.02	-0.81	-0.15	3.73	-2.12	1.61
F21	0.00	0.95	-1.03	-0.10	3.40	-1.83	1.58
F22	0.00	1.08	-0.62	0.12	4.18	-1.96	2.23
F23	0.00	0.97	-0.74	0.23	3.98	-1.87	2.11
F24	0.00	0.97	-1.01	0.00	3.90	-1.83	2.07
F25	0.00	0.84	-0.65	-0.15	3.71	-2.10	1.61
F26	0.00	0.91	-0.70	0.12	4.02	-2.03	1.99
F27	0.00	0.99	-0.50	0.14	4.14	-1.88	2.25
F28	0.00	0.97	-10.08	-0.01	3.60	-1.71	1.90
F29	0.00	0.96	-0.11	-0.11	4.15	-2.21	1.94
F30	0.00	0.98	-0.97	-0.16	3.94	-2.08	1.85

** - based on one of thirty aggregations

Appendix B.8

120 Observation Level Results for Empirical Data

STAT	Mean	St.dev.	Kurt	Skew	Range	Min	Max
PCT1	75.00	1.04	-0.67	0.22	4.01	73.18	77.19
PCT2	65.10	1.41	-0.78	0.24	4.97	62.92	67.88
KMO	0.69	0.02	-0.02	-0.07	0.10	0.64	0.75
MSA1	0.71	0.02	-0.90	-0.38	0.08	0.67	0.75
MSA2	0.83	0.04	4.31	-1.61	0.22	0.68	0.90
MSA3	0.72	0.04	-0.39	-0.27	0.14	0.64	0.79
MSA4	0.81	0.03	-0.82	-0.14	0.11	0.75	0.86
MSA5	0.82	0.04	0.95	-0.79	0.16	0.72	0.87
MSA6	0.74	0.04	-0.40	-0.66	0.15	0.65	0.81
MSA7	0.56	0.04	-0.58	0.37	0.17	0.48	0.65
MSA8	0.56	0.04	-0.65	0.06	0.16	0.48	0.64
MSA9	0.39	0.04	-0.60	-0.15	0.16	0.29	0.46
MSA10	0.71	0.03	-0.32	-0.30	0.14	0.62	0.76
C1	0.68	0.03	-0.61	0.04	0.13	0.61	0.74
C2	0.50	0.05	-0.50	0.39	0.20	0.40	0.61
C3	0.68	0.03	1.24	-0.86	0.14	0.61	0.75
C4	0.58	0.05	-0.55	0.16	0.19	0.49	0.68
C5	0.59	0.04	-0.45	0.41	0.16	0.51	0.67
C6	0.68	0.03	-0.70	0.08	0.11	0.63	0.74
C7	0.70	0.02	-0.06	-0.06	0.10	0.66	0.75
C8	0.78	0.02	-0.24	0.06	0.07	0.75	0.82
C9	0.49	0.05	1.56	-0.65	0.26	0.32	0.58
C10	0.62	0.05	-0.23	-0.20	0.19	0.52	0.71
E1	3.59	0.19	-0.44	-0.24	0.76	3.19	3.95
E2	2.40	0.12	0.37	0.13	0.53	2.11	2.63
E3	1.50	0.12	0.00	-0.03	0.54	1.22	1.76
E4	0.81	0.05	0.35	-0.63	0.23	0.67	0.90
E5	0.51	0.05	0.47	-0.25	0.25	0.38	0.63
E6	0.36	0.04	-0.78	-0.31	0.13	0.29	0.42
E7	0.29	0.02	-0.87	-0.42	0.09	0.24	0.33
E8	0.23	0.02	-0.18	-0.04	0.09	0.18	0.27
E9	0.19	0.02	-0.88	0.14	0.06	0.15	0.22
E10	0.12	0.01	-0.62	0.11	0.04	0.10	0.14
CM1	0.81	0.05	0.08	-0.10	0.22	0.69	0.91
CM2	0.48	0.06	0.61	0.50	0.28	0.36	0.64
CM3	0.70	0.04	0.20	-0.57	0.16	0.60	0.76
CM4	0.65	0.06	-0.15	-0.04	0.25	0.53	0.77
CM5	0.61	0.04	3.25	0.46	0.25	0.50	0.74
CM6	0.78	0.05	0.94	-0.29	0.25	0.63	0.88
CM7	0.59	0.03	-0.90	0.03	0.11	0.54	0.64
CM8	0.95	0.05	-0.34	-0.92	0.17	0.83	1.00
CM9	0.35	0.09	0.16	0.34	0.39	0.17	0.55
CM10	0.59	0.07	-0.94	-0.01	0.26	0.47	0.73

Appendix B.8 (continued)

STAT	Mean	St.dev.	Kurt	Skew	Range	Min	Max
EE1	3.28	0.20	-0.34	-0.20	0.83	2.85	3.68
EE2	2.11	0.12	0.52	0.18	0.53	1.80	2.33
EE3	1.12	0.14	-0.17	0.08	0.63	0.80	1.42

STAT	Mean	Var.	Kurt	Skew	Range	Min	Max
F1	0.00	0.91	0.86	-0.16	6.09	-3.46	2.62
F2	0.00	0.87	2.27	-0.75	5.88	-3.48	2.40
F3	0.00	0.93	2.89	-1.42	5.26	-3.77	1.48
F4	0.00	0.93	1.49	-0.92	5.52	-3.76	1.76
F5	0.00	0.85	2.76	-1.08	5.66	-3.61	2.05
F6	0.00	0.85	2.86	-1.18	5.47	-3.26	2.21
F7	0.00	0.93	1.09	-0.34	6.26	-3.53	2.73
F8	0.00	0.87	0.47	-0.62	5.09	-3.04	2.05
F9	0.00	0.92	3.29	-0.97	6.55	-3.93	2.63
F10	0.00	0.89	1.88	0.91	5.57	-1.99	3.58
F11	0.00	0.88	1.29	-0.45	5.86	-3.02	2.84
F12	0.00	0.88	3.03	-1.20	5.63	-3.64	1.99
F13	0.00	0.89	3.03	-0.83	6.55	-3.92	2.62
F14	0.00	0.91	2.03	-0.34	5.82	-3.23	2.59
F15	0.00	0.86	0.77	-0.93	4.56	-3.15	1.41
F16	0.00	0.87	2.06	-0.80	5.53	-3.44	2.09
F17	0.00	0.86	1.39	-0.68	5.68	-3.76	1.92
F18	0.00	0.89	1.99	-1.13	5.24	-3.54	1.69
F19	0.00	0.90	0.85	0.96	4.43	-1.54	2.90
F20	0.00	0.88	2.48	-1.20	5.24	-3.58	1.66
F21	0.00	0.87	0.44	-0.18	5.49	-2.79	2.70
F22	0.00	0.89	2.35	-0.84	5.93	-3.62	2.31
F23	0.00	0.93	0.87	-0.83	5.10	-3.34	1.76
F24	0.00	0.87	3.16	-1.20	6.02	-3.64	2.38
F25	0.00	0.89	2.13	-1.04	5.39	-3.38	2.02
F26	0.00	0.85	1.44	-0.39	5.67	-3.16	2.51
F27	0.00	0.90	1.62	1.02	5.26	-1.45	3.81
F28	0.00	0.90	2.99	-1.30	5.45	-3.79	1.67
F29	0.00	0.88	1.71	-0.51	5.74	-3.28	2.46
F30	0.00	0.91	2.33	-1.28	5.24	-3.60	1.65

Appendix B.9

Sixty Observation Level Results for Empirical Data

STAT	Mean	St.dev.	Kurt	Skew	Range	Min	Max
PCT1	80.88	1.15	-0.54	0.14	4.42	78.78	83.20
PCT2	72.67	1.26	0.30	-0.27	5.65	69.51	75.16
KMO	0.72	0.03	0.15	-0.51	0.12	0.65	0.77
MSA1	0.75	0.03	-0.55	0.01	0.13	0.68	0.81
MSA2	0.84	0.07	1.27	-1.41	0.23	0.68	0.91
MSA3	0.77	0.03	0.17	0.07	0.14	0.70	0.85
MSA4	0.81	0.03	-0.05	-0.73	0.12	0.74	0.86
MSA5	0.81	0.04	1.06	-1.03	0.17	0.69	0.86
MSA6	0.75	0.05	-0.53	-0.15	0.18	0.67	0.84
MSA7	0.60	0.06	-0.17	-0.46	0.23	0.46	0.69
MSA8	0.58	0.07	0.74	-0.61	0.30	0.40	0.70
MSA9	0.37	0.07	0.26	-0.10	0.33	0.20	0.54
MSA10	0.73	0.04	0.29	-0.11	0.18	0.63	0.82
C1	0.75	0.03	-0.08	-0.29	0.12	0.68	0.80
C2	0.65	0.06	-0.47	0.00	0.21	0.54	0.75
C3	0.78	0.02	-0.90	-0.04	0.08	0.74	0.83
C4	0.74	0.04	-0.08	0.72	0.16	0.68	0.84
C5	0.75	0.03	-0.76	-0.38	0.10	0.69	0.79
C6	0.81	0.03	-0.32	0.16	0.11	0.76	0.86
C7	0.80	0.03	-0.10	-0.49	0.11	0.74	0.84
C8	0.85	0.03	0.05	-0.65	0.10	0.79	0.89
C9	0.51	0.07	-0.71	-0.27	0.29	0.36	0.65
C10	0.75	0.04	-0.54	-0.19	0.16	0.67	0.83
E1	4.05	0.16	-1.21	-0.30	0.49	3.78	4.27
E2	2.84	0.15	-0.06	-0.51	0.63	2.48	3.11
E3	1.20	0.09	-0.82	0.12	0.33	1.03	1.36
E4	0.75	0.07	-1.25	-0.01	0.22	0.64	0.87
E5	0.37	0.05	0.90	0.42	0.23	0.27	0.51
E6	0.24	0.02	-0.94	0.08	0.07	0.20	0.27
E7	0.20	0.02	2.35	-0.91	0.09	0.15	0.23
E8	0.15	0.02	-0.86	-0.10	0.06	0.12	0.19
E9	0.12	0.02	-0.93	0.23	0.06	0.09	0.15
E10	0.08	0.01	-0.21	0.67	0.06	0.06	0.11
CM1	0.80	0.07	-0.48	-0.53	0.24	0.66	0.91
CM2	0.63	0.07	1.16	0.78	0.32	0.52	0.84
CM3	0.80	0.03	-0.35	-0.33	0.11	0.74	0.85
CM4	0.80	0.05	0.20	0.96	0.17	0.73	0.91
CM5	0.72	0.04	0.45	-0.16	0.18	0.63	0.81
CM6	0.87	0.05	-0.25	-0.10	0.21	0.76	0.97
CM7	0.68	0.04	-0.64	0.42	0.15	0.62	0.77
CM8	0.93	0.09	1.86	-1.52	0.37	0.63	1.00
CM9	0.34	0.19	1.01	1.17	0.80	0.07	0.86
CM10	0.70	0.09	-0.72	-0.30	0.32	0.51	0.83

Appendix B.9 (continued)

STAT	Mean	St.dev.	Kurt	Skew	Range	Min	Max
EE1	3.81	0.16	-1.30	-0.24	0.50	3.55	4.05
EE2	2.62	0.16	-0.15	-0.46	0.68	2.25	2.94
EE3	0.84	0.10	-0.67	0.20	0.40	0.66	1.06

STAT	Mean	Var.	Kurt	Skew	Range	Min	Max
F1	0.00	0.91	-0.66	-0.21	3.84	-2.13	1.72
F2	0.00	0.94	0.56	-0.26	4.88	-2.85	2.04
F3	0.00	0.93	0.34	-0.61	4.72	-2.95	1.78
F4	0.00	0.94	0.61	-0.58	4.49	-2.72	1.77
F5	0.00	0.96	0.62	-0.34	4.34	-2.39	1.95
F6	0.00	0.93	0.93	-0.76	4.82	-3.06	1.77
F7	0.00	1.05	-0.51	-0.02	4.28	-2.18	2.10
F8	0.00	0.93	0.00	-0.41	4.37	-2.43	1.94
F9	0.00	1.05	-0.29	-0.19	4.50	-2.34	2.16
F10	0.00	0.97	-0.03	-0.46	4.28	-2.64	1.63
F11	0.00	0.93	1.86	-1.12	4.67	-3.01	1.66
F12	0.00	0.97	-0.16	-0.43	4.29	-2.57	1.73
F13	0.00	0.95	0.13	-0.64	4.76	-2.78	1.98
F14	0.00	0.91	2.35	-1.29	4.72	-3.18	1.54
F15	0.00	0.93	0.64	-0.03	5.01	-2.28	2.73
F16	0.00	0.96	-0.19	-0.68	4.04	-2.71	1.33
F17	0.00	0.94	2.61	-1.18	4.91	-3.20	1.71
F18	0.00	1.00	-0.47	-0.13	4.31	-2.00	2.31
F19	0.00	0.95	0.04	-0.58	4.30	-2.63	1.67
F20	0.00	0.96	-0.37	-0.55	4.14	-2.69	1.45
F21	0.00	0.97	0.40	-0.28	4.91	-2.94	1.97
F22	0.00	0.94	0.32	-0.74	4.50	-2.75	1.75
F23	0.00	0.92	0.16	-0.51	4.48	-2.52	1.96
F24	0.00	0.97	0.15	-0.98	3.93	-2.57	1.36
F25	0.00	0.94	0.97	-0.67	4.91	-2.77	2.14
F26	0.00	0.92	0.93	-0.81	4.58	-2.76	1.82
F27	0.00	0.96	0.21	-0.83	4.25	-2.69	1.56
F28	0.00	0.96	-0.10	-0.73	3.89	-2.48	1.41
F29	0.00	0.96	0.83	-1.06	4.34	-2.74	1.60
F30	0.00	1.00	-0.16	-0.54	4.05	-2.41	1.64

Appendix B.10

Forty Observation Level Results for Empirical Data

STAT	Mean	St.dev.	Kurt	Skew	Range	Min	Max
PCT1	79.95	4.85	-1.77	-0.36	13.43	72.68	86.11
PCT2	73.48	3.90	-1.41	-0.18	12.82	67.12	79.94
KMO	0.71	0.03	-0.18	-0.04	0.15	0.64	0.79
MSA1	0.74	0.06	1.46	-0.89	0.27	0.57	0.84
MSA2	0.85	0.05	0.69	-0.74	0.23	0.71	0.93
MSA3	0.76	0.05	0.12	-0.68	0.22	0.63	0.86
MSA4	0.80	0.03	-0.52	-0.44	0.12	0.73	0.85
MSA5	0.79	0.06	0.51	-1.02	0.25	0.64	0.89
MSA6	0.73	0.06	-0.02	0.01	0.27	0.60	0.87
MSA7	0.60	0.05	0.07	0.59	0.18	0.54	0.71
MSA8	0.56	0.05	0.25	0.23	0.22	0.47	0.69
MSA9	0.42	0.07	-0.58	-0.02	0.26	0.29	0.55
MSA10	0.74	0.04	0.78	-0.88	0.18	0.62	0.80
C1	0.79	0.03	-0.76	-0.26	0.13	0.73	0.86
C2	0.72	0.06	1.36	0.88	0.30	0.61	0.91
C3	0.84	0.02	-0.64	-0.09	0.08	0.79	0.87
C4	0.81	0.03	0.95	-0.80	0.16	0.72	0.88
C5	0.81	0.04	-0.08	0.05	0.18	0.71	0.89
C6	0.87	0.03	-0.02	0.27	0.13	0.81	0.94
C7	0.84	0.03	-0.99	-0.02	0.10	0.79	0.89
C8	0.89	0.02	-0.32	-0.37	0.09	0.84	0.93
C9	0.60	0.08	-0.17	-0.14	0.32	0.42	0.74
C10	0.80	0.04	-0.16	0.42	0.15	0.74	0.90
E1	4.36	0.21	-0.77	0.02	0.76	4.00	4.76
E2	2.97	0.14	-0.29	0.01	0.58	2.64	3.22
E3	1.04	0.10	-0.37	-0.03	0.42	0.82	1.25
E4	0.71	0.07	-0.48	0.13	0.29	0.57	0.86
E5	0.30	0.05	-0.52	-0.19	0.18	0.19	0.37
E6	0.20	0.02	0.46	-0.21	0.10	0.15	0.24
E7	0.16	0.02	-0.34	-0.31	0.07	0.12	0.19
E8	0.12	0.02	0.07	0.15	0.08	0.08	0.16
E9	0.09	0.01	-0.43	-0.07	0.05	0.07	0.12
E10	0.05	0.01	0.05	0.60	0.05	0.03	0.08
CM1	0.73	0.09	-1.10	0.01	0.31	0.58	0.89
CM2	0.71	0.07	1.25	0.92	0.32	0.58	0.90
CM3	0.86	0.03	1.57	-0.64	0.14	0.78	0.92
CM4	0.85	0.04	-0.79	-0.18	0.14	0.78	0.92
CM5	0.73	0.05	0.86	0.90	0.20	0.65	0.86
CM6	0.90	0.05	-0.92	0.41	0.17	0.82	0.99
CM7	0.72	0.07	5.18	1.64	0.37	0.62	1.00
CM8	0.84	0.11	-1.26	0.07	0.35	0.65	1.00
CM9	0.32	0.17	0.36	1.08	0.63	0.13	0.76
CM10	0.69	0.05	-0.90	0.35	0.19	0.61	0.80

Appendix B.10 (continued)

STAT	Mean	St.dev.	Kurt	Skew	Range	Min	Max
EE1	4.13	0.20	-0.66	0.11	0.77	3.80	4.57
EE2	2.75	0.15	0.31	-0.36	0.63	2.38	3.00
EE3 **	0.77	0.08	-1.15	0.09	0.26	0.64	0.91

STAT	Mean	Var.	Kurt	Skew	Range	Min	Max
F1	0.00	0.98	0.39	-0.58	4.34	-2.75	1.59
F2	0.00	0.98	6.69	-2.13	5.46	-4.07	1.38
F3	0.00	0.98	-0.06	-0.44	4.23	-2.55	1.68
F4	0.00	0.98	-1.03	-0.30	3.71	-2.10	1.61
F5	0.00	1.08	-0.45	0.08	4.26	-2.01	2.25
F6	0.00	0.97	-0.91	0.11	3.65	-1.75	1.90
F7	0.00	0.98	0.29	-0.81	3.94	-2.66	1.28
F8	0.00	0.95	0.74	-0.78	4.48	-2.75	1.73
F9	0.00	0.96	0.26	-0.70	4.13	-2.54	1.58
F10	0.00	0.98	0.94	-1.07	4.28	-2.82	1.46
F11	0.00	0.99	-0.15	-0.57	4.12	-2.58	1.54
F12	0.00	0.96	-0.23	-0.53	3.78	-2.29	1.50
F13	0.00	1.00	-1.37	-0.20	3.07	-1.64	1.43
F14	0.00	0.96	-0.43	-0.49	3.75	-2.24	1.51
F15	0.00	1.01	-0.14	-0.43	4.36	-2.63	1.73
F16	0.00	0.97	0.12	-0.58	4.24	-2.75	1.49
F17	0.00	0.99	-1.11	-0.10	3.52	-1.62	1.90
F18	0.00	1.00	-0.85	-0.26	3.92	-2.16	1.75
F19	0.00	0.96	0.20	-0.74	3.79	-2.33	1.46
F20	0.00	1.12	0.29	0.58	4.71	-1.91	2.80
F21	0.00	0.97	-0.52	-0.43	3.98	-2.38	1.60
F22	0.00	1.00	0.50	0.61	4.44	-1.67	2.77
F23	0.00	0.98	0.02	-0.70	3.71	-2.31	1.40
F24	0.00	0.96	-0.73	-0.48	3.44	-1.96	1.48
F25	0.00	1.08	0.64	0.21	5.04	-2.13	2.91
F26	0.00	0.95	-0.17	-0.51	3.91	-2.32	1.59
F27	0.00	1.03	-0.02	0.01	4.48	-2.15	2.33
F28	0.00	0.99	-1.41	-0.12	3.20	-1.58	1.62
F29	0.00	1.12	-0.14	-0.40	4.42	-2.62	1.80
F30	0.00	0.99	-0.94	-0.39	3.51	-2.04	1.47

** - based on eighteen of thirty aggregations

Appendix C

Table of Contents for Appendix C

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PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT

1.0 1.1 1.25 1.4 1.6 1.8 2.0 2.2 2.5 28 32 36 40

Appendix C.1

Matrix of Spatial Dependencies for Uncontaminated Data

Variable	1	2	3	4	5	6	7
1	0.001						
2	0.014	0.007					
3	0.007	0.031	0.004				
4	0.200	-0.024	0.019	-0.008			
5	-0.001	0.008	-0.016	0.007	-0.003		
6	-0.003	-0.034	-0.027	0.008	0.050	-0.004	
7	-0.017	0.003	0.033	-0.009	0.027	-0.020	-0.004
8	0.001	0.055	0.022	0.026	0.008	-0.022	0.021
9	-0.005	0.020	-0.006	-0.015	-0.020	-0.008	0.055
10	0.008	-0.006	0.002	-0.037	-0.004	-0.017	0.015

	8	9	10
8	-0.008		
9	-0.005	0.015	
10	0.006	0.040	0.009

- spatial autocorrelations are printed on the diagonal
- lagged correlations are printed on the off diagonal

Appendix C.2

Matrix of Spatial Dependencies for Contaminated Data

Variable	1	2	3	4	5	6	7
1	0.409						
2	0.191	0.359					
3	0.385	0.103	0.405				
4	-0.390	-0.175	-0.402	0.493			
5	-0.123	0.057	-0.165	0.239	0.316		
6	0.050	-0.085	0.092	-0.115	-0.165	0.264	
7	0.088	0.142	0.035	-0.105	-0.040	-0.023	0.275
8	0.062	-0.039	0.080	-0.018	0.014	-0.018	0.005
9	0.256	0.253	0.233	-0.301	-0.031	-0.023	0.071
10	0.[illegible]93	0.300	0.140	-0.266	-0.080	0.002	0.055

	8	9	10
8	0.286		
9	-0.077	0.339	
10	-0.129	0.288	0.352

- spatial autocorrelations are printed on the diagonal
- lagged correlations are printed on the off diagonal

Appendix C.3

Empirical Data Set

Variable	1	2	3	4	5	6	7
1	0.280						
2	-0.185	0.196					
3	0.329	-0.175	0.638				
4	-0.276	0.133	-0.445	0.360			
5	-0.267	0.344	-0.262	0.185	0.612		
6	0.216	-0.264	0.258	-0.100	-0.414	0.356	
7	-0.072	-0.045	-0.297	0.311	-0.063	0.159	0.569
8	0.082	-0.217	-0.148	0.159	-0.240	0.296	0.470
9	0.134	-0.074	0.223	-0.134	-0.087	0.132	0.037
10	-0.065	0.191	0.022	-0.051	0.357	-0.251	-0.203

	8	9	10
8	0.594		
9	-0.017	0.253	
10	-0.300	-0.021	0.255

- spatial autocorrelations are printed on the diagonal
- lagged correlations are printed on the off diagonal