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Voronoi diagrams as a visualization technique for spatial autocorrelation in point data sets

Martin A. Cassel
Wilfrid Laurier University

Voronoi Diagrams as a Visualization Technique for Spatial Autocorrelation in Point Data Sets

by

Martin A. Cassel

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ABSTRACT

This thesis explores a new way of visualizing spatial autocorrelation in a GIS environment. It explores some relationships between spatial autocorrelation models, spatial interaction models and weighted Voronoi diagrams. Since the weighted Voronoi diagram is equivalent to a form of spatial interaction models, any GIS with the ability to generate a gravity model can be utilized to perform this new technique of exploratory spatial data analysis. This thesis demonstrates how the cross product form of spatial autocorrelation models like the Geary and the Moran statistics is equivalent to the form of a multiplicatively weighted distance utilized in the definition of weighted Voronoi diagrams. A transformation of the multiplicatively weighted distance into a representation of the Geary or the Moran statistic can be used to generate different weighted Voronoi diagrams. Since such a representation incorporates the spatial variation of data points and the spatial variation of the attribute values assigned to the data points, it provides a more appropriate visual representation than do existing representations that only operate on the spatial distribution of the data points. Data sets with a known degree of spatial autocorrelation are created with a simultaneous autoregressive model. The behaviour of the visual representations of the Geary and the Moran statistics for spatial autocorrelation varying from a high positive to a high negative degree is examined.

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Glossary

Apollonius Circle: The circle consisting of the locus of points the ratio of the distances of which from two given points is a fixed number; the locus of the apexes of all triangles on a given base the other two sides of which are in a fixed proportion. It is a circle the diametral points of which on the extensions of the base are harmonic points (Collins Reference Dictionary Mathematics).

CDA confirmatory data analysis

EDA exploratory data analysis

GIS Geographical (or Geographic) Information System.

SAR simultaneous autoregressive model

SDA spatial data analysis

a_i attractiveness of point i

b_j vector of destination attributes

b_2 $$b_2 = \frac{\sum z_i^4}{\left(\sum z_i^2\right)^2}$$

C $\mathbf{C} = \{C_{ij}\}$, a measure of the spatial proximity between points i and j; connectivity matrix

c Geary statistic

E expected value

e random vector from a normal distribution; $\mathbf{e} = (\varepsilon_1, \ldots, \varepsilon_n)^T$.

d_{ij} distance between points i and j

G	$\mathbf{G} = \{G_{ij}\}$, a measure of the proximity of the attribute values at points i and j
H	connectivity or contiguity matrix $\mathbf{H} = \{h_{ij}\}$
h_{ij}	connection between points i and j
$h_{i.}$	$h_{i.} = \sum_j h_{ij}$
$h_{.j}$	$h_{.j} = \sum_i h_{ji}$
I	Moran statistic
I	identity matrix
N	Normalization
o_i	vector of origin attributes
P	set of data points $P = \{p_1, \ldots, p_n\}$
R	Randomization
$\Re^2$	two-dimensional space
$\Re^m$	m-dimensional space
s_{ij}	vector of segregation attributes
S_1	$S_1 = \frac{1}{2}\sum_{(2)}\left(h_{ij} + h_{ji}\right)^2$
S_2	$S_2 = \sum_i \left(h_{i.} + h_{.j}\right)$

T_{ij} the amount of interaction between i and j due to the relative influence exerted by i on j

Var Variance

w_i attribute value at point i

$\overline{w}$ the mean of the attribute values $\overline{w} = \frac{\sum_i w_i}{n}$

$\mathbf{x_i}$ location vector $\mathbf{x}_i = (x_{i1}, x_{i2})$

$\mathbf{Y}$ autocorrelated vector (SAR); $\mathbf{Y} = (Y_1, ..., Y_n)^T$

Y_i attribute value at point i; used in the SAR model

z_i $z_i = (w_i - \overline{w})$

α distance exponent in a gravity model

ρ factor determinating the degree of autocorrelation in the data set

μ mean value of a distribution

λ_{min} minimum eigenvalue of C.

λ_{max} maximum eigenvalue of C.

Γ general cross-product statistics

$\sum_{(2)}$ $\sum\sum$

$V(p_i)$ the Voronoi polygon associated with p_i;

$V(p_i) = \{\mathbf{x} \mid \|\mathbf{x} - \mathbf{x}_i\| \le \|\mathbf{x} - \mathbf{x}_j\| \text{ for } j \ne i, j \in I_n\}$

ϑ the planar ordinary Voronoi diagram; $\vartheta = \{V(p_1), ..., V(p_n)\}$

$b(p_i, p_j)$ the bisector for the planar ordinary Voronoi diagram;

$$b(p_i, p_j) = \left\{ \mathrm{x} \,\middle|\, \|\mathrm{x} - \mathrm{x}_i\| = \|\mathrm{x} - \mathrm{x}_j\| \right\}, \qquad \text{for } j \neq i.$$

$Dom(p_i, p_j)$ the dominance region for the planar ordinary Voronoi diagram;

$$Dom(p_i, p_j) = \left\{ p \,\middle|\, d(p, p_i) \leq d(p, p_j) \right\}, \qquad \text{for } j \neq i.$$

$d_{mw}(p, p_i)$ MW-distance; $d_{mw}(p, p_i) = w_i^{-1} \|\mathrm{x} - \mathrm{x}_i\|$, $w_i \phi 0$.

$Dom_w(p_i, p_j)$ the dominance region for the MW-Voronoi diagram;

$$Dom_w(p_i, p_j) = \left\{ p \,\middle|\, d_{mw}(p, p_i) \leq d_{mw}(p, p_j) \right\}, \text{ (for } i \neq j).$$

$V_W(p_i)$ the multiplicatively weighted Voronoi region; $V_W(p_i) = \bigcap_{i=1}^{n} Dom_w(p_i, p_j)$.

ϑ_W the MW-Voronoi diagram; $\vartheta_W = \{V_w(p_1), \ldots, V_W(p_n)\}$

1. Introduction

1.1 Spatial Analysis and GIS

The developments in information and computer sciences have changed the face of many disciplines. The introduction of specially developed computer hard- and soft-ware for geography and geographical analysis modified the area of spatial data analysis (SDA). The automatization of calculations and the ability to graphically display spatial data and its properties with the help of desktop computers and workstations made SDA more accessible. In particular, the introduction of Geographical Information Systems (GIS) created a boom in this field. A current issue in geography is to generate a stronger link between SDA and GIS. Openshaw (1990) expresses the view that SDA technology in GIS today is extremely limited and he identifies a "need for developing more relevant and appropriate spatial analysis technology for inclusion in tomorrow's GIS". This need is currently being addressed as indicated by the many articles, workshops, research initiatives and specialized conferences focusing on the issue.

Half of the research initiatives of the US National Center for Geographic Information and Analysis, (NCGIA, Santa Barbara, Buffalo, Maine) concentrated on topics related to the integration of GIS and SDA (Table 1).

Table 1: US NCGIA research initiatives 1987-92

1987	1*	Accuracy of Spatial Databases
	2	Language of Spatial Relations
	3	Multiple Representations
	4	Use and Value of Geographic Information in Decision Making
	5	Architectures of Very Large GIS Databases
	6*	Spatial Decision Support Systems
1988	7*	Visualization of Spatial Data Quality
	8	Expert Systems for Cartographic Design
	9	Institutions Sharing Spatial Information
	10*	Temporal Relations in GIS
	11*	Space-Time Statistical Models in GIS
	12*	Remote Sensing and GIS

Source: NCGIA; adapted from Openshaw (1990)

* topic related to the integration of GIS and SDA

At a workshop on *Spatial Analysis for GIS* organized by R. Haining, Department of Geography, Sheffield, UK, Goodchild et al. (1992), defined SDA as "a set of techniques devised to support a spatial perspective on data". By utilizing the locations and the attributes of objects and combining them in an analysis, SDA is distinct from other forms of analysis that focus only on the attribute values of objects. The incorporation of SDA in GIS will enhance the abilities of GIS to a higher degree and allow more efficient data storage and retrieval for SDA. A main improvement is clearly the enhancement in the area of visualization for the evaluation of models and for preliminary and exploratory data analysis, where the data can be viewed in its arrangement in the geographical space. The potential role of GIS in the context of SDA is often compared to the invention of the microscope and telescope to science (Goodchild et al., 1992).

Researchers in the field of SDA and GIS (Anselin and Getis 1991; Goodchild et al. 1992) agree that there is a strong need for the improvement of GIS abilities to incorporate more spatial statistical techniques to the spatial analysis capabilities. The GIS community seems to agree that GIS is currently not used to its full potential and realizes that there is room for improvement. Openshaw describes the situation as follows:

> "Such Systems are basically concerned with describing the Earth's surface rather than analyzing it. Or if you prefer, traditional 19th-century geography reinvented and clothed in 20th-century digital technology." (Openshaw 1987, p.431)

Clearly, GIS is a new technology with a high potential for SDA, which is up to now mainly used for simple forms of analysis like, for example, the overlay of different map layers and as a sophisticated data storage and retrieval system for spatially referenced data. GIS is often utilized as a tool for computer cartography and as such, stays below its capabilities. Anselin and Getis (1991) exclude only two commercially available GIS from these kinds of GIS systems. IDRISI, a spatial analysis system developed at Clark University, and SPANS, an advanced spatial analysis system by Tydac. The new version of ARC/INFO 6, by ESRI, shows improvement in its ability to perform more sophisticated SDA. There are also projects underway to generate a link between ARC/INFO and a statistics package. Goodchild et al. (1992) describe the reality of commercially available GIS today as:

> "A database containing a discrete representation of geographical reality in the form of static two-dimensional geometrical objects and associated attributes, with a functionality largely limited to primitive geometrical operations to create new objects or to compute relationships between objects, and to simple query and summary descriptions."(Goodchild et al., 1992, p.408)

They further point out, that the GIS market is driven by the needs of governmental agencies and big companies for GIS modules for data management and storage. The academic and scientific need for analytical tools in a GIS to enhance the data analysis capabilities are commercially not lucrative for big GIS vendors. Analytical tools for GIS have been mainly created in the academic world to fulfill its own needs. GRASS, a GIS developed by the US Army Corps of Engineers, gives a good example of this. Because of its public domain status and the free availability of the source code, it is widely distributed in the academic world. Many scientists created customized modules to perform different types of analysis and linked them to the GRASS system. Another example of this is IDRISI. It consists mainly of independent modules, created by different researchers, that are linked via a joined data structure and menu system. A different approach, from the SDA side, is SPACESTAT. It is a system of SDA modules written in GAUSS and provided with mapping capabilities, that was developed by Luc Anselin (1990) and is available from the NCGIA (S-92-1). Possible other ways of increasing the analytical abilities in GIS could be the linkage to already existing statistics packages. Openshaw (1990) rejects this and proposes a different course of action:

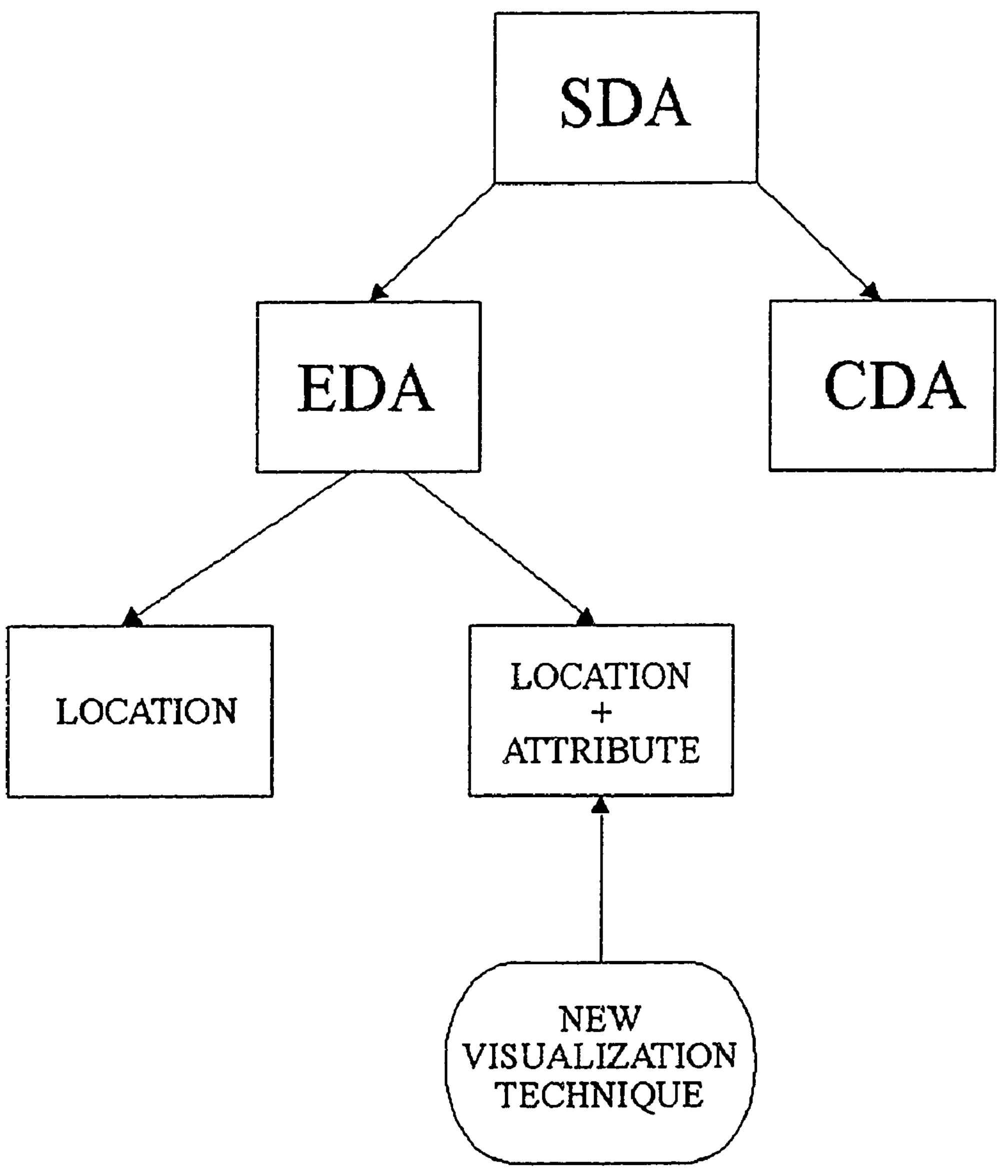
> "Attention needs to be focused on defining a small set of generic spatial analysis functions that can be built-in as standard GIS operations and, also, define another set of analysis tools that would seek to provide new analytical functions which are only possible within a GIS environment." (Openshaw, 1990, p.155)

This thesis makes use of theoretically already existing analytical capabilities in a GIS, modifying them and providing an appropriate interpretation in a SDA context, and thus, follows an approach favored by Openshaw and others.

1.2 Visualizing Spatial Autocorrelation

SDA can theoretically be divided into exploratory data analysis (EDA) and confirmatory data analysis (CDA). Two classes of statistical methods can be utilized in EDA, dealing with the initial exploration, sampling, assessment and rectification of spatial data.

Fig. 1: Spatial data analysis.



EDA can further be divided into two classes. The first class operates on locational data only. It is concerned with the analysis of the spatial distribution of the objects. Examples are point pattern analysis or nearest neighbour analysis. The second class operates on locational and attribute data of objects at the same time. Its focus is on the spatial variation of attribute values.

The visualization technique described in this thesis is part of the group of descriptive statistics in EDA that operate on the spatial variation of the data and can be used as a preliminary analysis of the spatial autocorrelation structure in a point data set. It is an example of an intuitive technique for exploring data in a spatial context. Goodchild et al. (1992, p.411) assume that, "the key to integrating GIS and SDA may lie in an emphasis, at least initially, on the more intuitive, exploratory techniques in the SDA tool kit".

The investigation of the organization and structuring of phenomena in space is one of the central themes in geography (Morrill, 1983). Tobler's First Law of Geography (Tobler, 1970 p.236; 1979) states that, "everything is related to everything else, but near things are more related than distant things", thus it is not appropriate to analyze spatial data in the same way as data without a spatial component is treated. A visual representation of the interdependencies between the objects in a data set helps in the identification of the structure in a spatial data set. It can enhance the understanding of the spatial autocorrelation in the sample structure better than an overall index for the complete data set. For example the preliminary identification of different autocorrelative structures in a study area can lead to the identification of possible subsets for further analysis in separated subareas.

An interpretation of weighted Voronoi diagrams as a form of a spatial autocorrelation measurement can be used as an improvement of the preliminary descriptive statistics for spatial data. The technique described in this thesis allows a visual representation of spatial autocorrelation measurements by utilizing the concept of multiplicatively weighted Voronoi diagrams. It also explores the link between the concepts of spatial association, spatial interaction and spatial autocorrelation. Many GIS have the ability to generate gravity models and thus can be utilized to generate the visualization technique described here. Spatial interaction models can function, under certain conditions described in the following chapters, as a substitute for the construction of multiplicatively weighted Voronoi diagrams. With an appropriate interpretation method they can be used to identify spatial autocorrelation structures in data sets. This visualization technique is an example of the successful integration of SDA in a GIS, by modifying and utilizing one of many GIS already existent capabilities and providing a new interpretation for its results. The outcome is an enhancement in the analytical capabilities of the GIS environment.

Visualization of spatial autocorrelation in point data sets has up to now been done by generating the normal planar Voronoi diagrams for the set of data points and then creating a choropleth map displaying the attribute values of the spatial objects in different classes (Goodchild, 1986, p.18). For a set of n distinct points $P = \{p_1,\ldots,p_n\}$ in $\Re^2$ (two-dimensional space) the location of each point p_i is defined by $\mathbf{x}_i = (x_{i1}, x_{i2})$ (Boots, 1993). The interval-ratio scale value at data site p_i is denoted with w_i. In this thesis w_i is a single attribute but it could also be a vector combining different attributes at data site p_i. A way to identify a measure of spatial association between all data points is accomplished by defining the Voronoi diagram of P, as a point to area interpolation. The resulting areal pattern displays the connectivity between the regions. It is now possible to generate a choropleth map to display the attribute values w_i. An example for this technique is displayed in Figures 2 and 3, for the case of a positive spatial autocorrelation, and in Figures 4 and 5, for negative spatial autocorrelation (Fig. 2 to 5; Boots, 1993).

This technique does not incorporate the interactions between attribute values into the generated areas. The spatial component is only based on the location of the data points in the study area. The influences of the attribute values attached to the data points are not incorporated in the assignment of the areas representing them. Regardless of their attribute values, all data points are treated as equal. The spatial associations for the two data sets displayed in Fig. 3. and Fig. 5. are the same, even though the data set have opposing degrees of spatial autocorrelation (Boots, 1993).

This study proposes a different way of incorporating the locations and the attribute values of the spatial objects into the assignment of the areal representation by means of the weighted Voronoi diagram. It thereby exceeds the limitation of the formerly used method, and maintains the ability to display the attribute values of the data points in a choropleth map.

Fig. 2. Regular triangular grid with positive spatial autocorrelation.

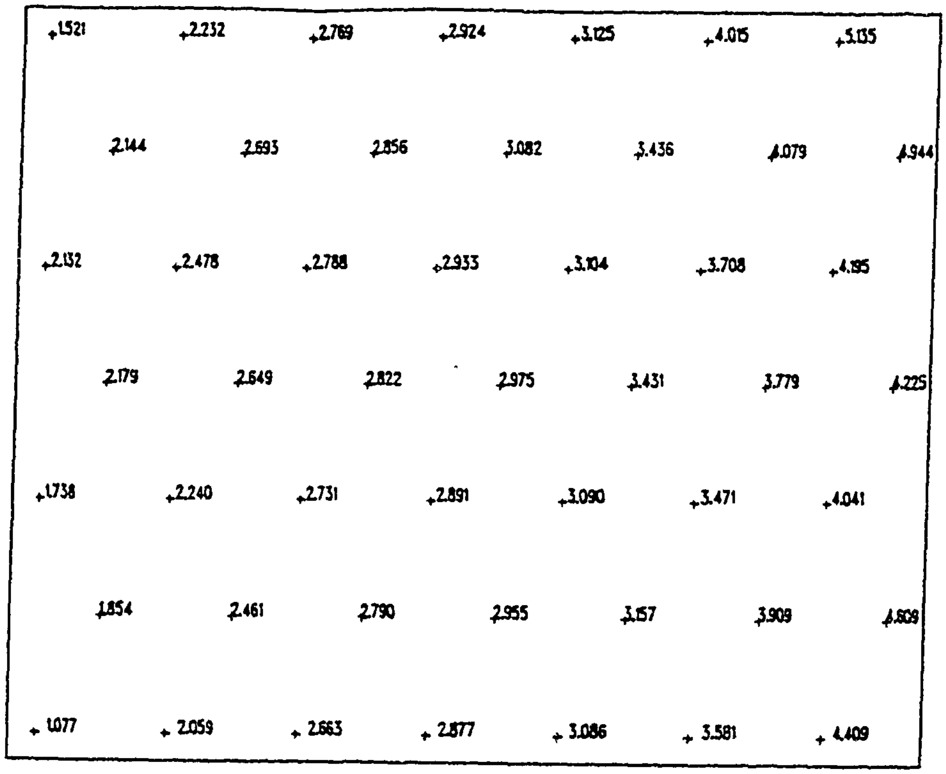
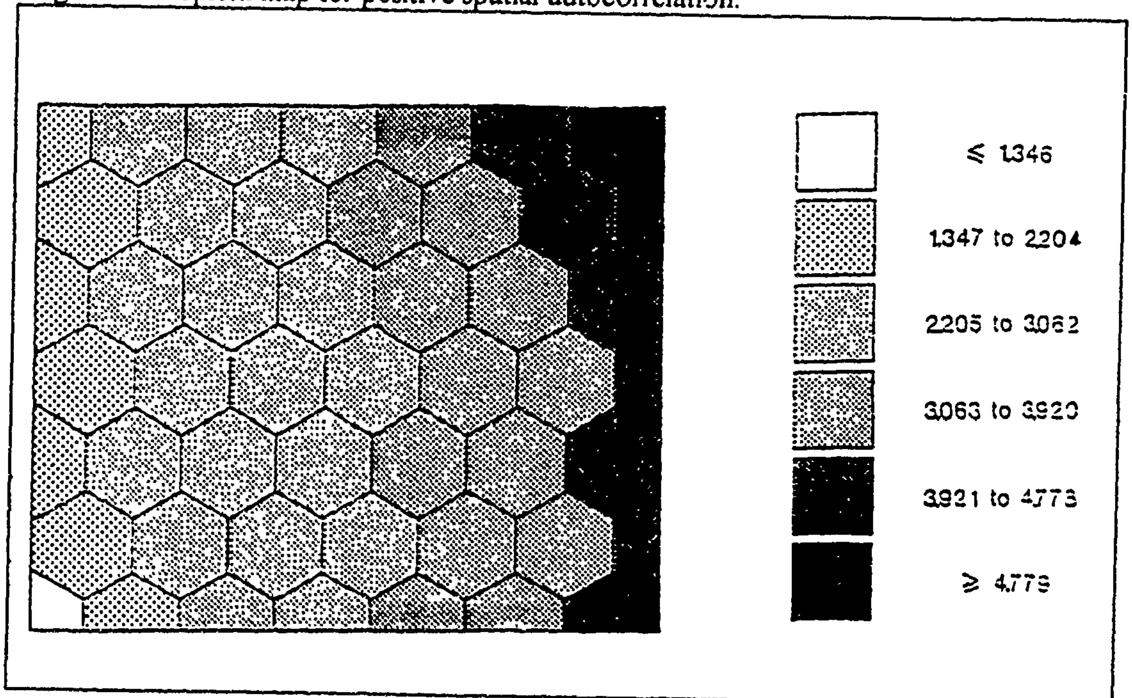


Fig. 3. Choropleth map for positive spatial autocorrelation.



Source: Boots (1993)

Fig. 4. Regular triangular grid with negative spatial autocorrelation.

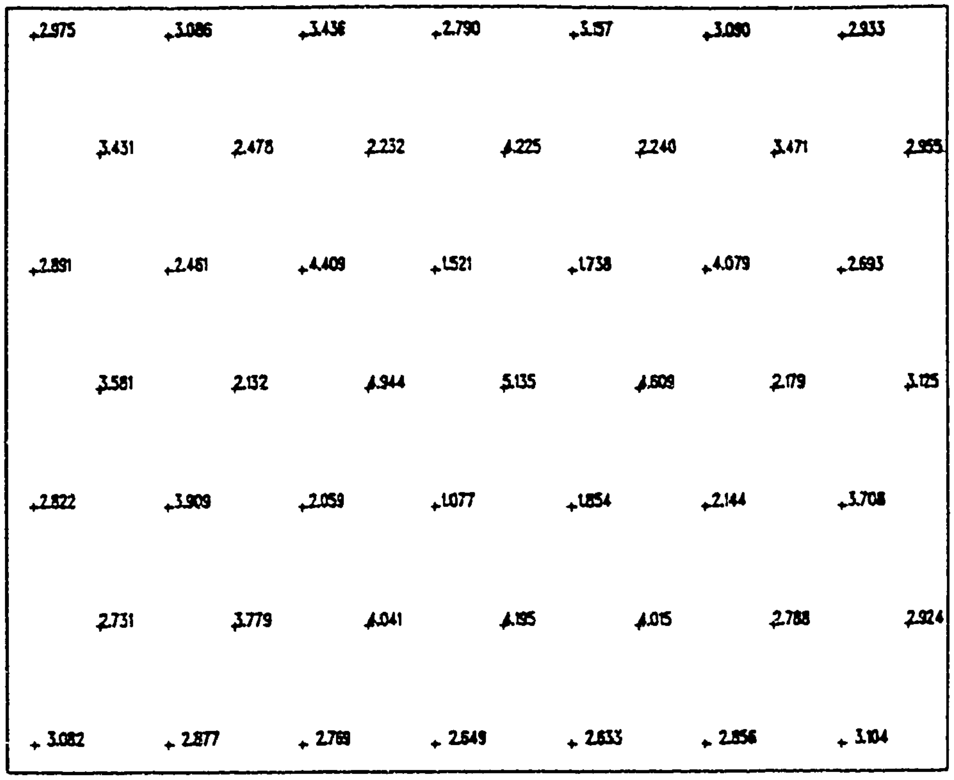
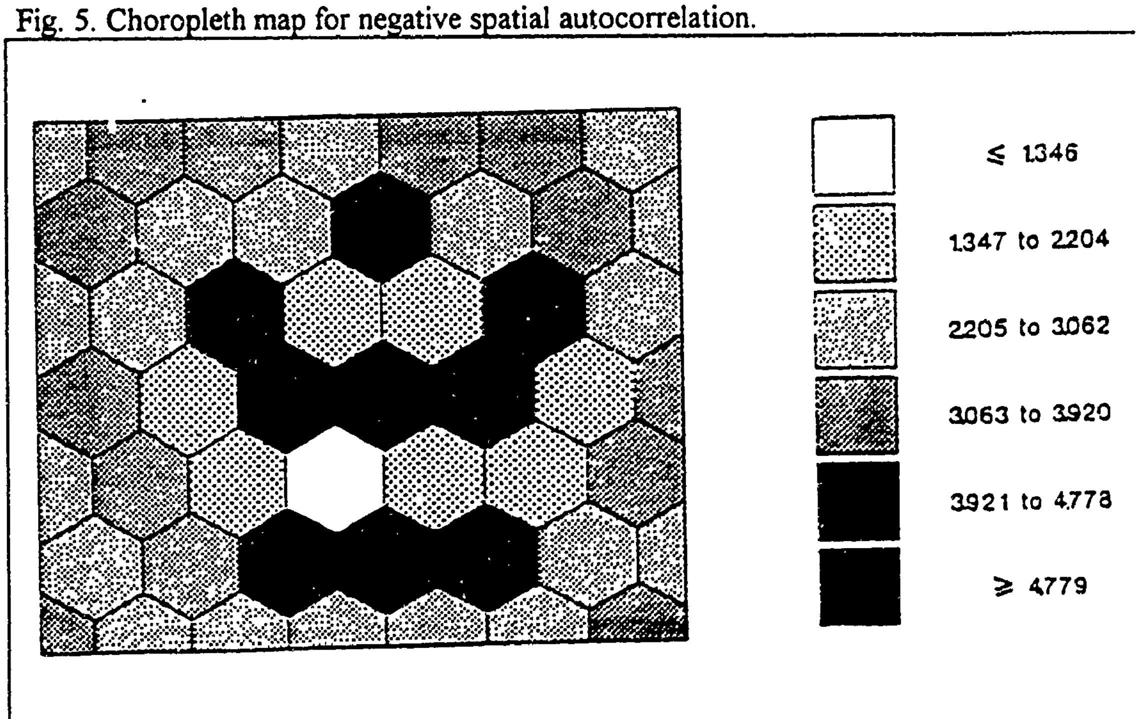


Fig. 5. Choropleth map for negative spatial autocorrelation.



Source: Boots (1993)

1.3 Organization and Limitations of the Study

This thesis can be divided into two conceptual parts. The first part contains the technical aspects of the project, the incorporation of a SDA technique into a GIS environment. It includes the creation of a GIS module for the generation of multiplicatively weighted Voronoi diagrams, that allows for a modification of the attribute values assigned to the data points, so that they might be interpreted in a spatial autocorrelation context. This program then has to be interfaced with a GIS package. The GIS chosen was IDRISI Version 4.0, from Clark University, Worcester, Massachusetts, USA. The reasoning behind this decision was the relatively easily accessible data structure of IDRISI, its wide distribution in the academic world, its accessibility and low price. A FORTRAN77[1] program, based on an algorithm by Gambini (1965), was developed and linked to the IDRISI Version 4 module structure. The implemented modifications allow changes to the attribute values of the data points in a way that they represent a form of weights for the spatial association in the data set. These weights can be altered to represent a Moran or a Geary spatial autocorrelation statistic as will be demonstrated in Chapter 3.

The program has two forms of data input. The first is in the form of an ASCII[2] table, consisting of an identification number for the data point, the X- and Y-coordinates and an attribute value for the data point. The second possibility for data input is the utilization of vector point and attribute values files in the IDRISI file structure. In the first case the program generates all files needed for the IDRISI structure so that a further analysis of the data set utilizing the other IDRISI modules is possible. As with all interpolation programs, the calculation time needed by this program increases dramatically

---

[1]The program is written in standard FORTRAN77 and was compiled with a Microsoft FORTRAN 5.0 compiler.

[2]American Standard Code for Information Interchange.

for an increase in the number of points and the precision of the calculation. An alternative method for the generation of weighted Voronoi Diagrams can be found in any GIS with the ability to generate a gravity model and a way to modify the attribute values of the data points. The second, more substantial part of the study, is the description of the theoretical background of the proposed technique and the development of a valid interpretation method.

This study is limited to give a first insight into a new way of EDA. It is aimed to give a qualitative visual impression of the Moran and Geary statistics for spatial autocorrelation. This first approach is limited to a relatively small sample size of 49 points arranged in a regular triangular grid. As will be demonstrated spatial autocorrelation measurements in a cross-product form, spatial interaction models and weighted Voronoi diagrams can all be expressed in a similar form. Thus, the proposed technique allows for the identification of the underlying structure of spatial autocorrelation in a point data set, its different magnitudes and directions, by visualizing weighted Voronoi Diagrams. This visual analysis is aimed to enhance the EDA functions in a GIS environment. The identification of autocorrelated structures in a data set prevents possible inappropriate conclusions about the data structure, that might lead to a misuse of spatial analysis techniques.

This technique should be used in connection with other preliminary SDA techniques like box plots or the detection of outliers in the data set (Haining 1990, Chapter 6). It can not replace thorough investigation of the data, its distribution in values and in space, but it will enhance the understanding of the predominant spatial structure in a geographical data set.

1.4 Outline

In the second part of the thesis an introduction to the concepts of spatial autocorrelation, spatial interaction models and Voronoi diagrams is provided. Chapter 3 examines the relationships among the three fields and introduces the theoretical background behind the new proposed technique. The main focus in that section is on the exploration of the cross-product form of most spatial autocorrelation measurements (especially Geary's c and Moran's I statistics) and the multiplicatively weighted form of the planar Voronoi diagram. Chapter 4 describes the simulation of autocorrelated data sets with a simultaneous autoregressive method. Chapter 5 discusses the results of the visualization for autoregressive structures ranging from positive to negative spatial autocorrelation.

2. Concepts

2.1 Spatial Autocorrelation

2.1.1 Introduction

Spatial autocorrelation is a measure of the spatial variation of attribute values in a spatial data set. It describes whether there is an interdependency in the spatial arrangement of attribute values of objects or if the realizations are independent of their locations in the geographical space. It identifies if the resulting pattern of the realizations is random, or whether there is a clustering of values with similar magnitude or a clustering of values with opposing magnitude.

The concept of spatial autocorrelation is an important and active field of geographical research. Odland (1988) identifies spatial autocorrelation as "the central theme that unifies geography as a discipline and distinguishes it from other fields of study". Goodchild (1986, p.3) defines spatial autocorrelation as the "degree to which objects or activities at some place on the earth surface are similar to other objects located nearby"

Spatial autocorrelation can either be defined as positive, if spatially close entities are positively related (attraction), and negative if spatially close entities have a reverse relation (repulsion). The absence of spatial autocorrelation implies the spatial independence of the data. The first case, of positive spatial autocorrelation, is the most often found in spatial data sets (Goodchild, 1986). This fact supports Tobler's first law of geography.

Spatial autocorrelation in a data set can lead to complex problems in geographical analysis. Haining (1980) describes problems that can be encountered in an analysis due to spatial autocorrelation. As an example of the impact of spatial autocorrelation in techniques that utilize correlation matrices, he states (p.24) that " ...all statistical procedures such as factor analysis and principal component, that use correlation matrices must be subject to the same inaccuracy and problem of interpretation.". He describes consequences arising out of positive spatial autocorrelation in a regression analysis as (p.25) "The estimate of residual variance will be too small, so that the squared correlation coefficient (R^2) is inflated".

Anselin and Getis (1991) focus on two other problems related to spatial autocorrelation. The first is the expectation that the relations between observations that are near to each other, are stronger than that between observations further apart. The second, and in their view more difficult to handle, is that because of size and configuration of the spatial units the relationships between the variables are due as much to the nature of the units as to the nature of the variables itself.

Out of these problems originates the need to test for the absence or presence of spatial autocorrelation and to quantify its degree of interaction between the attribute values of objects in a data set due to their relative locations. The importance of spatial autocorrelation in a data set, due to the analytical difficulties implied, show clearly the need for spatial descriptive statistics quantifying the degree of autocorrelation in the data The next section introduces the most common ways to test for spatial autocorrelation and to how to quantify its degree.

2.1.2 Spatial Autocorrelation Statistics

The area of spatial analysis that investigates the phenomenon of spatial autocorrelation is an active field of geographical research. New ways of describing and quantifying the degree of spatial autocorrelation in data sets are presently examined (Getis 1991, Getis and Ord 1992, [the introduction of the G statistic]; Boots 1993). Odland (1988, p.9) summarizes the role of spatial autocorrelation statistics:

> Autocorrelation statistics are basic descriptive statistics for any data that are ordered in a sequence because they provide basic information about ordering of the data that is not available from other descriptive statistics such as the mean and variance.

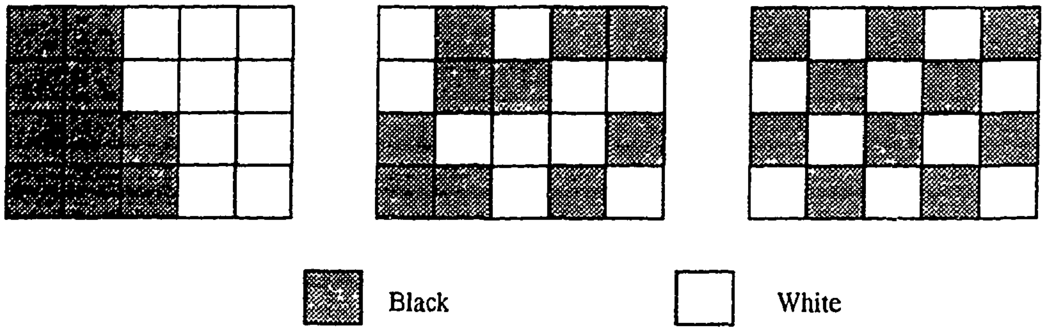
In general the test for spatial autocorrelation assumes spatial independence in the data set (null hypothesis; H_0). The alternative hypothesis (H_1) is that there is an interdependence of the attribute values in a data set due to their spatial locations. The most common measures in geographical analysis are Moran's I and Geary's c, different forms of a general cross product statistic. Ways to define, test for and calculate spatial autocorrelation statistics are described in: Cliff and Ord (1973; 1981), Goodchild (1986), Griffith (1987), Odland (1988), Haining (1990), Getis (1991) and Getis and Ord (1992).

The Join Count Statistic

The join-count statistic describes the simplest case of a spatial autocorrelation test for nominal data in a binary classification scheme of black and white cells. Positive spatial autocorrelation is given if cells of the same kind cluster together on the map. The numbers of BB and WW joins are large relative to the number of BW joins. Respectively, negative spatial autocorrelation is given if the map displays an alternation of cells with different values. The number of BB and WW join is, in this case, small relative to the number of

BW joins. Figure 6 displays clustered, random and alternating patterns for a binary classification scheme of black and white cells.

Fig. 6. Clustered, random and alternating patterns.



Joins	
BB	13
WW	13
BW	5

Joins	
BB	5
WW	6
BW	20

Joins	
BB	0
WW	0
BW	31

Let

$$
\begin{aligned}
y_i &= 1 \quad \text{if the ith county is B} \\
&= 0 \quad \text{if the ith county is W} \\
\delta_{ij} &= 1 \quad \text{if i and j are contiguous and } i \neq j \\
&= 0 \quad \text{otherwise .}
\end{aligned}
$$

Observed count of joins:

$$
\begin{aligned}
\text{BB} &= \frac{1}{2}\sum\nolimits_{(2)} \delta_{ij}\, y_i\, y_j \\
\text{BW} &= \frac{1}{2}\sum\nolimits_{(2)} \delta_{ij}\, \left(y_i - y_j\right)^2 \\
\text{WW} &= A - (BB + BW)
\end{aligned}
$$

where $A = \frac{1}{2}\sum \delta_{ij}$, the total number of joins in the county system

for $i \neq j$, and

$$\sum\nolimits_{(2)} = \sum_i \sum_j .$$

This statistic can be generalized from the binary to an unordered k-colour ($k \succ 2$) situation (Dacey, 1965). Another form of generalization can be performed by modifying the proximity matrix to a connectivity or contiguity matrix $\mathbf{H} = \{h_{ij}\}$. It is now possible to define the connectivity or contiguity on different scales. A first lag situation defines only the links of immediate neighbours as 1 and all other links as 0.

$$h_{ij} = 1 \quad \text{for i and j contiguous}$$
$$= 0 \quad \text{otherwise} .$$

In a second lag situation the links between the immediate neighbours, one cell apart, are assigned $h_{ij} = 0$, and only the links to objects two cells apart are assigned $h_{ij} = 1$. The form of the connectivity reflects the structure of interdependence in the data set that is assumed in the alternative hypothesis.

The Geary Index: c

Geary's c is a spatial autocorrelation statistic for ordinal and interval scale data. It can be defined as the sum of squared differences between pairs of data values as a measure of covariance (Odland, 1988, p.11).

There are two models for the null hypothesis (H_0) for the Geary and the Moran statistics. The first is the assumption of normality (N), that the $\{w_i\}$ are the result of n independent drawings from a normally distributed population. The second is the assumption of randomization (R). It states, that whatever the underlying distribution of the population, the observed value of the Geary (c) or Moran (I) statistic has to be considered relative to the set of all possible **n!** values that c or I could take on, if the $\{w_i\}$were repeatedly randomly permuted around the county system (Cliff and Ord, 1981, p.14).

Geary's c is asymptotically normally distributed for n approaching infinity (Cliff and Ord, 1981) and can be calculated as

$$c = \frac{(n-1)}{2H} \frac{\sum\sum h_{ij}(w_i - w_j)^2}{\sum z_i^2} \quad \text{for } (i \neq j) \tag{2.1.1}$$

where $n =$ number of observations

$H = \sum_{(2)} h_{ij}$, denoting the spatial connectivity

$z_i = (w_i - \overline{w})$

$\overline{w} = \frac{\sum_i w_i}{n}$.

For a clustered situation, the sum of the squared differences over all contiguous pairs would be small relative to a random pattern. This case describes positive spatial autocorrelation. Negative spatial autocorrelation is described by an alternating pattern where the sum of the squared differences over all contiguous pairs would be large relative to a random pattern.

The expected value for c under the two assumptions is defined as:

$$E_N(c) = E_R(c) = 1$$

where $N =$ under normalisation

$R =$ under randomization .

The variance of c under the two assumptions is defined as:

$$Var_N(c) = \frac{\left[(2S_1 + S_2)(n-1) - 4H^2\right]}{2(n+1)H^2}$$

$$Var_R(c) = \frac{\left[(n-1)S_1\left\{n^2 - 3n + 3 - (n-1)b_2\right\}\frac{1}{4}(n-1)S_2\left\{n^2 + 3n - 6 - \left(n^2 - n + 2\right)b_2\right\}H^2\left\{n^2 - 3 - (n-1)^2 b_2\right\}\right]}{n(n-2)^2 H^2}$$

$$\text{where } b_2 = \frac{\sum z_i^4}{\left(\sum z_i^2\right)^2}$$

$$S_1 = \frac{1}{2}\sum_{(2)}\left(h_{ij} + h_{ji}\right)^2$$

$$S_2 = \sum_i \left(h_i + h_j\right)$$

$$\text{and } h_i = \sum_j h_{ij}$$

$$h_j = \sum_i h_{ji} \ .$$

Interpretation of Geary's c:

- Positive spatial autocorrelation is defined for the case that $c \prec E(c) = 1$.
- Negative spatial autocorrelation is given when c is bigger than 1 $\left(c \succ E(c) = 1\right)$.
- if c equals the expected value the entities are independent and not spatially autocorrelated.

The Moran Index: I

Moran's I is a second measure for spatial autocorrelation for ordinal and interval scale data. It can be defined as a spatial autocovariation standardized by two terms.

$$I = \frac{n}{H} \quad \frac{\sum_{(2)} h_{ij} \, z_i z_j}{\sum z_i^2} \ . \qquad (2.1.2)$$

All symbols as defined in equation (2.1.1).

This represents essentially a Pearson correlation coefficient combined with a spatial connectivity or contiguity matrix as a measure of the spatial connectivity for the set of regions. In the situation of positive spatial autocorrelation, denoted by a spatial clustering of similar attribute values in the data set, the cross-product $z_i \, z_j$ will be positive. For an alternating pattern the cross-product will be negative. In the Moran statistic i and j are indirectly compared through the mean. In the Geary statistic i and j are being directly compared. Cliff and Ord (1981) state that the model is asymptotically normally distributed for an increasing number of observations.

The expected value for I:

$$E_N(I) = E_R(I) = (n-1)^{-1} = \mu$$

where $E_N(I) =$ expected value of I under normalisation
$E_R(I) =$ expected value of I under randomization.

The expected value E(I) approaches zero for n approaching infinity. The variance of I under the assumptions of normalization and randomization is defined as:

$$Var_N(I) = \frac{\left(n^2 S_1 - nS_2 + 3H^2\right)}{H^2\left(n^2 - 1\right) - \mu^2}$$

$$Var_R(I) = \frac{\left[n\left\{\left(n^2 - 3n + 3\right)S_1 - nS_2 + 3H^2\right\} - b^2\left\{\left(n^2 - n\right)S_1 - 2nS_2 + 6H^2\right\}\right]}{(n-1)^3 H^2 - \mu^2}$$

All symbols as defined for the variance for c.

Interpretation of I:

- Positive spatial autocorrelation is defined for the case that $I \succ E(I)$. Which means that the neighbouring values are similar.
- Negative spatial autocorrelation is defined by $I \prec E(I)$. The neighboring values are dissimilar and the relation between the spatial entities is inverse.
- if $I = E(I)$, the entities are spatially independent, random and uncorrelated. The spatial autocorrelation equals zero.

Goodchild (1986) states that for a small sample size Geary's c is the more appropriate measure of spatial independence because a value of zero for the Moran statistic is not equivalent with spatial independence. Both scales have no fixed extreme points. The asymmetrical behaviour of the Geary statistic can make the interpretation of the results more complicated than in the case of the Moran statistic.

The General Cross-Product Statistic

The previously described spatial autocorrelation statistics are special cases of a more general form, the cross-product statistic. It can be defined by:

$$\Gamma = \sum_{(2)} C_{ij} G_{ij} \qquad (2.1.3)$$

where C_{ij} = a measure of spatial proximity of locations i and j

G_{ij} = a measure of the proximity of i and j in terms of variate values .

The problem of measuring spatial autocorrelation can be described as the comparison of the two matrices $\mathbf{C} = \{C_{ij}\}$ and $\mathbf{G} = \{G_{ij}\}$. In both the Geary and the Moran statistics, $C_{ij} = h_{ij}$. For Geary's c, the $G_{ij} = (w_i - w_j)^2$ and for the Moran's I, the $G_{ij} = (w_i - \overline{w})(w_j - \overline{w})$. The measure for the spatial proximity **C** is not limited to a binary form where the connection to a natural neighbour is denoted a one and all other connections zero. The inverse of the Euclidean distance between the data points or other measures of proximity, like the Manhattan metric or a measurement for the cognitive perception of space could be utilized.

Other methods to determine spatial autocorrelation are:
Getis' new G statistic (Getis, 1991; 1992), for the detection of localized changes in the autocorrelation structure; the semi-variance; variograms (a plot of the semi-variance against the lag or distance), correlograms (the autocorrelation at different spatial lags plotted against the lag). Detailed descriptions of these techniques can be found in: Cliff and Ord, 1981; Davis 1986; and Odland, 1988.

2.2 Spatial Interaction Models

Spatial interaction models or gravity models are more commonly used in geography than weighted Voronoi diagrams. A short definition of their properties will be given in this section.

In a geographical context, gravity models are mainly used in market area analysis, the estimation of migration movements and the forecasting of traffic. They describe the attraction of places in the geographical space and have their analogy in physics, where they are used to describe the interaction of the forces between objects.

The Newtonian Model (Newton's Law of Universal Gravitation, 1687):

$$F = \frac{GM_1M_2}{d_{1,2}^2}$$

where F = the force with which mass one, M_1 pulls mass two, M_2

G = gravitational constant $(6.67259E-11\ \text{Nm}^2\text{kg}^{-2})$

$d_{1,2}$ = the distance between the two masses M_1 and M_2.

Haggett et al.(1977, p.30ff) trace the first use of the Newtonian model in the social sciences to the English demographer E.G.Ravenstein in the 1880s. Ravenstein observed the trend that workers in England and Wales migrate towards larger cities and that the volume of this migration decreased with increasing distance between the source and the destination areas. He translated this observation into a gravitational context replacing the masses with the populations of the source and destination places and the gravitational constant G with a constant k, that is used to calibrate the model.

The general form of a gravity model, as used in the context of this study, can be defined as a set of three vectors

$$T_{ij} = f(o_i, b_j, s_{ij}) \tag{2.2.1}$$

where T_{ij} = the amount of interaction between i and j due to the relative influence exerted by i on j

o_i = vector of origin attributes

b_j = vector of destination attributes

s_{ij} = vector of segregation attributes.

The attraction between two places can be identified by

$$T_{ij} = a_i a_j d_{ij}^{-\alpha} \qquad (a_i, a_j \succ 0) \qquad (2.2.2)$$

where a_i = attractiveness of point i
a_j = attractiveness of point j
d_{ij} = distance between i and j
α = distance exponent .

The a_i and a_j in equation (2.2.2) resemble the attribute values w_i and w_j from the previous sections. Respectively the potential T_i of a centre p_i to attract from an intermediate centre p at the distance d_i is defined by (Boots, 1993)

$$T_i = a_i d_i^{-\alpha} \ . \qquad (2.2.3)$$

The attractiveness of an object in the geographical space can be understood as an individual weight, or an attribute value, assigned to the location and resembles therefore the w_i used in the spatial autocorrelation context. The distance component with the Euclidean distance between locations and the exponent α, which is used to calibrate the model, defines the connectivity between the locations in space and resembles the **C** used in equation (2.1.3) for the cross-product statistics for the calculation of spatial autocorrelation. Usually the value of the exponent α is between 1.5 and 2. A high distance exponent increases the importance of the friction of distance by reducing the expected level of interaction between centres (Haynes and Fotheringham, 1984). An interpretation of this in the context of market areas is, that in an undeveloped country the distance exponent is high, and therefore, distance is a strong hindrance for market activities. In a developed country the exponent would be low. Due to the sophisticated transportation system and the good infrastructure, distance is less of a hindrance than in a undeveloped nation.

2.3 Voronoi Diagrams

2.3.1 The Ordinary Voronoi Diagram

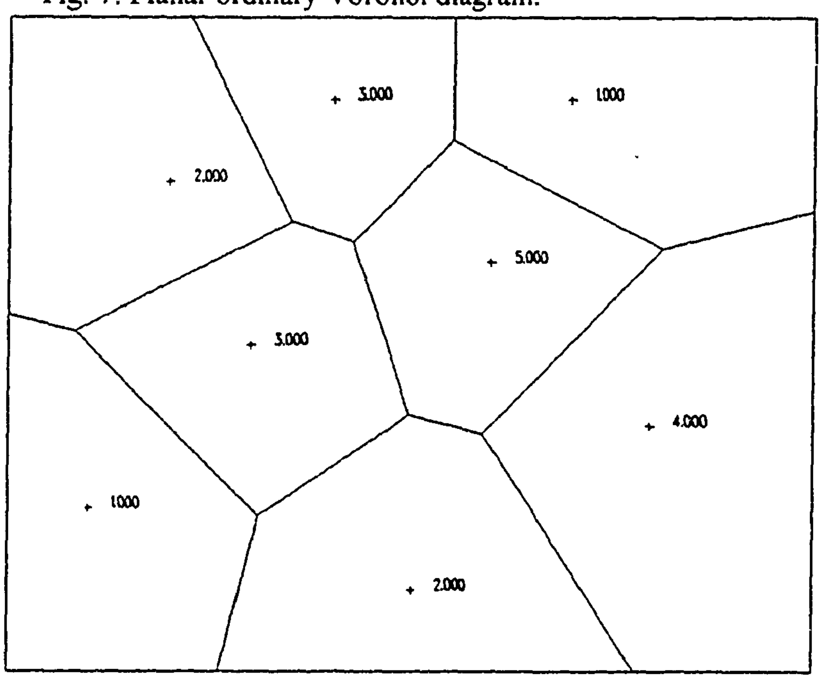
Voronoi diagrams are a form of spatial tessellations. Based on a set of generator points, they partition space into a set of non overlapping regions assigned to the generator points. They can also be seen as a technique for point to area interpolation or to define the areas of influence of points. The following definitions are limited to the planar Voronoi diagram. An exhaustive description of the subject can be found in Okabe et al, (1992, Chapter 2). Definitions of the properties of Voronoi diagram and mathematical proofs, like the expansion of the concepts to $\Re^m$, are described there. The terminology and definitions used in the following sections resemble that of Okabe et al, (1992) and Boots (1993). The mathematical proofs for the properties described can be found there.

General Concepts

Okabe et al. (1992, p.2) point to the interdisciplinarity of the concept of Voronoi diagrams or their equivalents and list a variety of disciplines that make use of this form of spatial tessellation or related concepts like the Delaunay triangulation. The various fields mentioned are: anthropology, archeology, astronomy, biology, cartography, computational geometry, crystallography, ecology, forestry, geography, geology, marketing, metallography, operations research, physics, physiology, statistics and urban and regional planning. This diversity of scientific disciplines explains the variety of applications and names associated with the concept. In geography the terms of Voronoi polygons, Dirichlet Domains or Thiessen polygons are mainly used.

Okabe et al., (1992, p.6ff) trace a possible utilization of the concept of Voronoi diagrams to Descartes and his books *"Le Monde de Mr Descartes, ou le Traité de la Lumière"*(1644) and to Part III of *"Principia Philosophiae"* (1644), where the disposition of matter in the solar system is displayed in a Voronoi-like diagram. Boots (1980) also connects Voronoi diagrams with Descartes. He states that the edges of the Voronoi diagram are part of the family of Descartes' Ovals. The works of Dirichlet (1850) and Voronoi (1908), regarding the distribution of points with integer coordinates that give minima of the values of a given quadric[3] form, lead to the connection of their names with this form of spatial tessellation. For two-dimensional applications in geography and meteorology the term Thiessen polygons is most widely used. This refers to the work of Thiessen (1911), utilizing the Voronoi regions as an aid in estimating regional rainfall averages. An example of a planar ordinary Voronoi diagram for a data set of 8 points is given in Fig. 7.

Fig. 7. Planar ordinary Voronoi diagram.



[3]algebraic equation of second degree

Definitions

A Voronoi diagram represents a spatial tessellation which is defined as the set of all Voronoi polygons. The polygons describe the dominance regions of a set of points $P = \{p_i\}$, where the number of points is n ($2 \leq n \leq \infty$; the generator points) and all points are distinct $\left(x_i \neq x_j \text{ for } i \neq j,\ i,j\ \in I_n = \{1,\ldots,n\}\right)$. For an arbitrary point p with coordinates $\mathbf{x} = (x_1, x_2)$, the Euclidean distance to p_i is defined by

$$d(p, p_i) = \|\mathbf{x} - \mathbf{x}_i\| = \sqrt{(x_1 - x_{i1})^2 + (x_2 - x_{i2})^2} \quad . \tag{2.3.1}$$

The Voronoi polygon of p_i contains all locations that are closer or equal distant to p_i than to p_j. thus the Voronoi polygon associated with p_i is given by

$$V(p_i) = \left\{\mathbf{x} \middle| \ \|\mathbf{x} - \mathbf{x}_i\| \leq \|\mathbf{x} - \mathbf{x}_j\| \quad \text{for } j \neq i, j \in I_n \right\}. \tag{2.3.2}$$

The planar ordinary Voronoi diagram of P is defined by

$$\vartheta = \{V(p_1), \ldots, V(p_n)\}. \tag{2.3.3}$$

The set of loci describing equal distance to generator points defines the boundary of adjacent polygons. This line can also be defined as the Voronoi edge or the bisector (the perpendicular bisecting line segment joining two adjacent generator points p_i and p_j ,for $i \neq j$) and is given by

$$b(p_i, p_j) = \left\{\mathbf{x} \middle| \ \|\mathbf{x} - \mathbf{x}_i\| = \|\mathbf{x} - \mathbf{x}_j\| \ \right\}, \qquad \text{for } j \neq i. \tag{2.3.4}$$

If a location is equidistant from three or more generators, the point is called a Voronoi vertex. The dominance region of a point p_i over a point p_j can be defined as

$$Dom(p_i, p_j) = \{p \mid d(p, p_i) \le d(p, p_j)\}, \quad \text{for } j \ne i. \tag{2.3.5}$$

The Voronoi polygon associated with p_i can alternatively be defined as

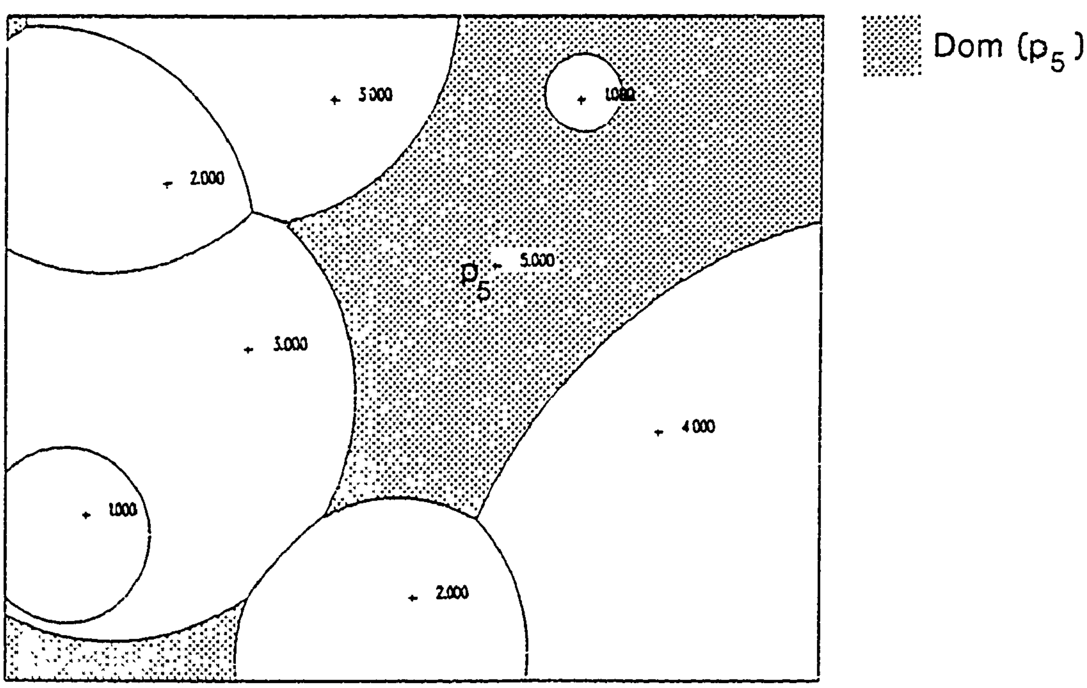
$$V(p_i) = \bigcap_{j \in I_n \setminus \{i\}} Dom(p_i, p_j) \tag{2.3.6}$$

which is an equivalent representation to the definition in equation (2.3.2).

2.3.2 The Weighted Voronoi Diagram

A more general form of the Voronoi diagram is the weighted Voronoi diagram. This thesis concentrates on the special case of a multiplicatively weighted Voronoi diagram (MW-Voronoi diagram), other weighting schemes are described in Okabe et al (1992, Chapter 3). As indicated by the literature (Boots, 1980; 1993)), this case is the one to be considered in the attempt to identify the relationship between spatial association, spatial interaction models and spatial tessellations.

Fig. 8. Planar multiplicatively weighted Voronoi diagram.



In the MW-Voronoi diagrams each generator point p_i is assigned a weight w_i. This weight w_i is equal to the attribute value at point p_i, as described in the previous sections An example for a MW-Voronoi diagram for the same 8 points as in the previous Fig. 7 is given in Fig. 8.

The multiplicatively weighted distance (MW-distance) between two points can now be defined as

$$d_{mw}(p, p_i) = w_i^{-1}\|\mathbf{x} - \mathbf{x}_i\|, \; w_i \succ 0. \tag{2.3.7}$$

where w_i = attribute value or weight at point i
$\mathbf{x}$ = location vector of p in $\Re^m$
$\mathbf{x}_i$ = location vector of p_i in $\Re^m$.

The dominance region of p_i over p_j for the multiplicatively weighted case is now described as the set of locations that satisfy the following condition

$$Dom_w(p_i, p_j) = \{p | \; d_{mw}(p, p_i) \leq d_{mw}(p, p_j)\}, \text{ (for i} \neq \text{j).} \tag{2.3.8}$$

The multiplicatively weighted Voronoi region can be defined as

$$V_W(p_i) = \bigcap_{i=1}^{n} Dom_w(p_i, p_j). \tag{2.3.9}$$

The set of all MW-Voronoi regions , $V_W(p_i)$, is called the MW-Voronoi diagram of P with $d_{mw}(p, p_i)$ and weights $W = \{w_1, \ldots, w_n\}$. It is given by

$$\vartheta_W = \{V_W(p_1), \ldots, V_W(p_n)\}. \tag{2.3.10}$$

The dominance region of p_i over p_j with the MW-distance can alternatively to equation (2.3.8) be defined with the weighted distance as the set of locations satisfying the following equation:

$$Dom_w(p_i, p_j) = \left\{ \mathbf{x} \middle| \; w_i^{-1} \|\mathbf{x} - \mathbf{x}_i\| \le w_j^{-1} \|\mathbf{x} - \mathbf{x}_j\| \right\}, \; i \ne j. \tag{2.3.11}$$

The bisector for the MW-Voronoi diagram can now be defined as

$$b_w(p_i, p_j) = \left\{ \mathbf{x} \middle| \left\| \mathbf{x} - \frac{w_i^2}{w_i^2 - w_j^2}\mathbf{x}_j + \frac{w_j^2}{w_i^2 - w_j^2}\mathbf{x}_i \right\| = \frac{w_i w_j}{w_i^2 - w_j^2} \; \|\mathbf{x}_j - \mathbf{x}_i\| \right\} \tag{2.3.12}$$

for $w_i \ne w_j$; $i \ne j$.

This is "the locus of a point p satisfying the condition that the distance from p to the fixed points, $w_i^2 \, \mathbf{x}_j / (w_i^2 - w_j^2) - w_j^2 \, \mathbf{x}_i / (w_i^2 - w_j^2)$, is constant" (Boots, 1993, p.6). This equation describes a circle in $\Re^2$ that "passes through the interior and exterior division points (denoted by $p_{ij1} p_{ij2}$) of $\overline{p_i p_j}$ with ratio w_i, w_j and its diameter is given by $\overline{p_{ij1} p_{ij2}}$ " (Okabe et al, 1992 , p.130). This circle is known as an Apollonius circle (see Coxeter, 1961 and Boots 1980), that divides $\Re^2$ into two disjoint regions. Apollonius circles are members of the family of Descartes' ovals. An Apollonius circle can be defined as the circle consisting of the locus of points of which the ratio of the distances from two given points is a fixed number.

The MW-Voronoi diagram is reduced to the ordinary Voronoi diagram in the case when all weights w_i are the same constant for all $i \in I_n$. As in the gravity models it is possible that the dominance region of the lower weighted generator point is completely entrapped inside the dominance region of the higher weighted generator point. The dominance region of the lower weighted point can even completely collapse into the generator point itself.

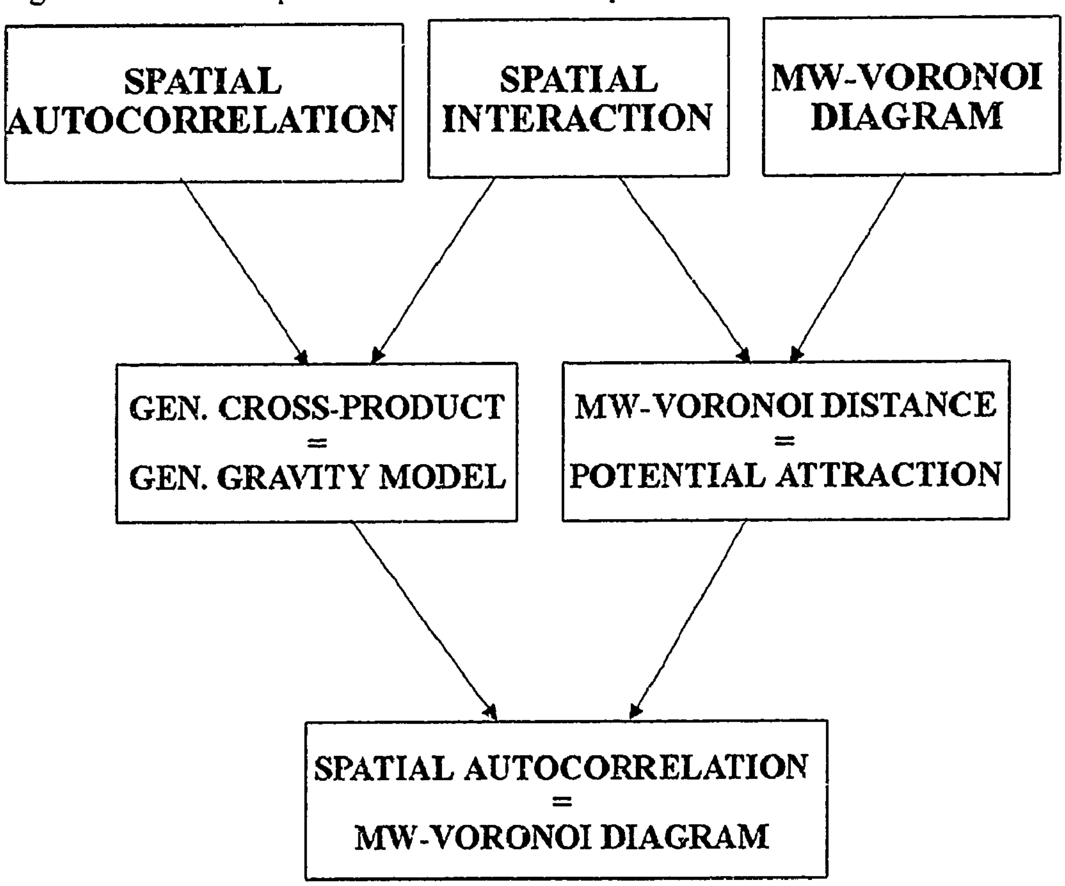
Weighted Voronoi diagrams can be generated with a gravity model. Hanjoul et al. (1984) declare that this fact was first identified by Thuominen (1949), who established that if the distance exponent function in a spatial interaction or gravity model equals one, the resulting area defines an Apollonius circle relative to points p_i and p_j. The link between Apollonius circles and their usage for the generation of weighted Voronoi diagrams was proved by Boots (1980). He confirmed that a gravity model with a distance exponent of one is equivalent to a MW-Voronoi diagram. Both, the MW-Voronoi diagram and the gravity model with a distance exponent of one, can be expressed as Apollonius' circles or as Descartes' ovals.

Furthermore it was identified by Boots (1980) that equation (2.3.12), defining the bisector in the MW-Voronoi diagram, and equation (2.2.2), defining the breakpoint between two centres in a gravity model where $a_i = w_i$ and $a_j = w_j$ and the distance exponent $\alpha = 1$, are equivalent. He also identified that the form of the MW-distance, given in equation (2.3.1) is identical to the potential T_i, of a centre p_i, to attract from an intermediate centre p at distance $d(p,p_i)$ as given by equation (2.2.3). Thus the link of spatial interaction models and MW-Voronoi diagrams was mathematically proved. Therefore, the interpretations of spatial patterns created as market areas in a gravity model or as dominance regions of a MW-Voronoi diagram are interchangeable.

3. The Link of Spatial Association and Spatial Interaction

In the next section the connection between the general cross-product form of a spatial autocorrelation statistic and the spatial interaction model will be established. As seen in the previous section, the MW-Voronoi diagram and the gravity model approach are interchangeable for a distance exponent of one. This property will be used to demonstrate the connection between the Moran and the Geary statistics, as special forms of a general cross-product approach, and the visual representation with the MW-Voronoi diagram with modified weights. Figure 9 gives a schematic overview of the link between spatial autocorrelation, spatial interaction and spatial association.

Fig. 9: Link between spatial autocorrelation and spatial association.



3.1 Spatial Interaction and Spatial Autocorrelation

The main problem is to identify an appropriate link between spatial autocorrelation and spatial interaction models. Getis (1991, 1992) suggested a link of spatial interaction and correlation in a cross-product approach. Boots (1993, p.8) transforms equation (2.2.2) into a form of the general unconstrained gravity model

$$T_{ij} = k a_i^{\delta} a_j^{\beta} d(p_i, p_j)^{-\alpha} \tag{3.1.1}$$

where $k =$ a scalar constant of proportion .

Getis (1991) identifies the fact that this form can be considered equivalent to a cross-product measure as introduced earlier. The $d(p_i, p_j)^{-\alpha}$ in this form resembles a single c_{ij} of equation (2.1.3) ($\Gamma = \sum_{(2)} C_{ij} G_{ij}$, where $C_{ij} = \{c_{ij}\}$ and $G_{ij} = \{g_{ij}\}$), where c_{ij} is a measure of the spatial proximity of the points p_i and p_j. Respectively, the form $a_i^{\delta} a_j^{\beta}$ resembles the g_{ij} of equation (2.1.3), where g_{ij} is defined as a measure of the proximity of the points p_i and p_j in terms of their attribute values w_i and w_j. The relationships in equations (2.1.3) and (3.1.1) are defined between individual pairs of data points.

As shown in the previous section, spatial interaction models can be expressed in an equivalent form to the MW-Voronoi diagram. With the link between spatial interaction models and the cross-product form for the measurement of spatial autocorrelation established, we can consider the MW-distance an equivalent to a measure of spatial autocorrelation between a pair of data points. Boots (1993) points out, that "the individual terms contained in the numerators of most spatial autocorrelation models can be considered weighted distances". A basic condition for all the models is that there is no self-association ($i \neq j$). In the next sections the modified forms of the weighted distances for the Geary and the Moran statistic will be introduced.

3.1.1 The Geary Statistic

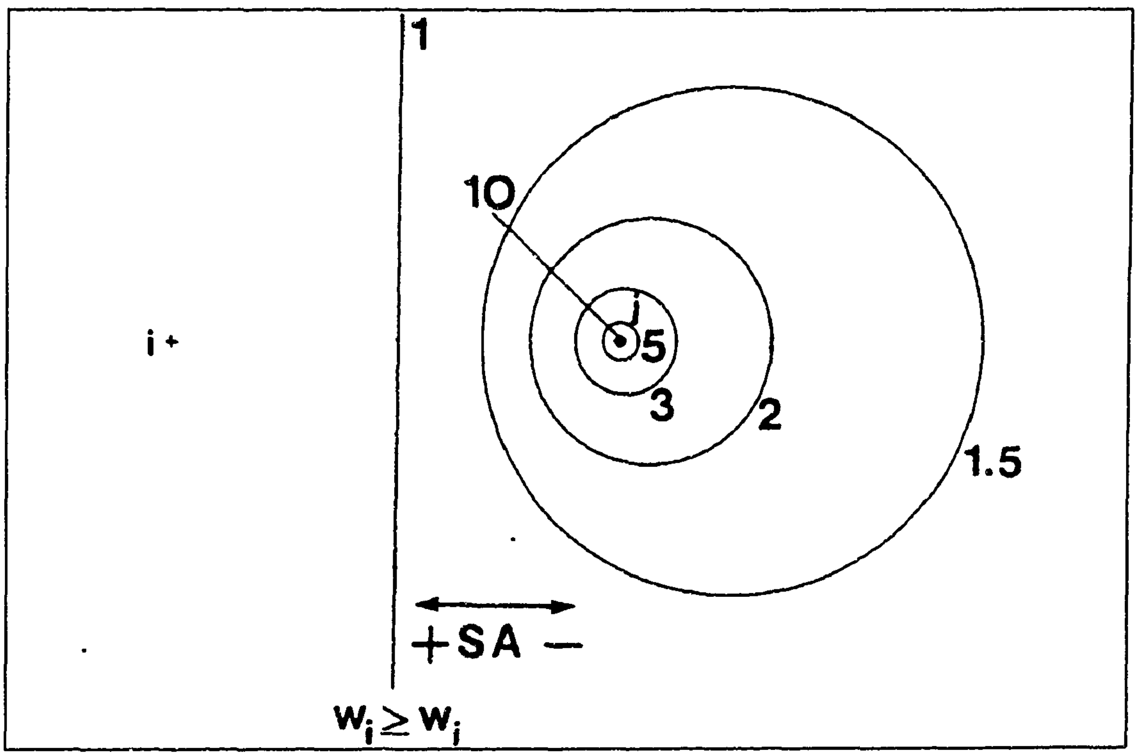
As demonstrated in the previous section, the cross-product form of the spatial autocorrelation measurements is equivalent to a representation in which the individual regions are defined with MW-distances as described in equation (2.3.1). The term for the attribute value w_i at the data point p_i, has to be replaced with a form of a weighted distance representing the specific spatial autocorrelation model.

Since the Geary and Moran statistics are special forms of the cross-product form it is possible to establish a direct relation between the Moran and Geary statistics and the MW-Voronoi diagram. In the case of a Geary representation, the w_i in equation (2.3.7) can be replaced by w_i^2 . By examination of the behaviour of the bisector and the radius and centre of the Apollonius circle for two data points p_i and p_j, it was possible to identify a relationship between the attribute values at the data points and the behaviour of the bisectors.

Interpretation for the Behaviour of the Bisector in the Geary Statistic

Under the condition $w_i \geq w_j \geq 1$,[4] a measure of the relationship of point p_i and point p_j can be defined as $\tau = w_i / w_j$ (Boots, 1993, p.9). For τ approaching its lower limit of one, the bisector approaches a straight line bisecting $\overline{p_i p_j}$. For $w_i = w_j$ $(\tau = 1)$, a circle with a radius of infinity is defined and the bisector is a straight line. For τ approaching infinity, the bisector is reduced to the point p_j and describes a circle with radius zero (Fig. 8.). The case of τ approaching infinity describes an increasing negative spatial autocorrelation.

Fig. 10. Behaviour of the bisector between p_i and p_j for different τ.



SA = spatial autocorrelation

[4] $w_i \geq w_j \geq 1$, to ensure monotone behaviour of the transformation. For $0 \prec w_i, w_j \prec 1$, the square of the attribute values will be lower than the value itself, thus the transformation is locally monotone decreasing (antitone). For $w_i \geq w_j \geq 1$, the transformation is monotone increasing (isotone).

3.1.2 The Moran Statistic

Similar to the approach of the Geary representation, the w_i in the weighted distance of equation (2.3.1) can be replaced by $w_i - \overline{w}$ for the Moran representation. The behaviour of this case is more complicated than the previously described Geary transformation.

The weights associated with p_i and p_j are defined as $(w_i - \overline{w})$, $(w_j - \overline{w})$. For the case of only two points, $|w_i - \overline{w}| = |w_j - \overline{w}|$ always. Therefore the minimum number of points for a Moran transformation must be higher than two $(n \succ 2)$.

A second problem with the Moran transformation is the possibility of $(w_i - \overline{w})$ and / or $(w_j - \overline{w})$ being negative. Equation (2.3.1) for the MW-distance requires that the weights are positive. Boots (1993, p.10) suggests the introduction of a constant e to prevent the weights from being negative. He defines:

$$e = (m + |\max dev|) \tag{3.1.2}$$

where $\max dev =$ the maximum negative deviation from the mean for any member of P and

$P = \{p_1, \ldots, p_n\}$

$m =$ 0.0001; a constant to ensure that all values are greater than zero.

Interpretation for the Behaviour of the Bisector in the Moran Statistic

Under the condition that $(w_i - \overline{w} + e) \geq (w_j - \overline{w} + e)$, τ can now be defined as

$$\tau = (w_i - \overline{w} + e)/(w_j - \overline{w} + e).$$

The behaviour of the bisector in the Moran transformation is now similar to the behaviour for the Geary transformation described previously. For the case of equal weights, $w_i = w_j$ $(\tau = 1)$, the result is a straight line bisecting $\overline{p_i p_j}$, which represents a circle with an infinite radius. Equivalent to the Geary transformation, for τ approaching infinity, the bisector is reduced to the point p_j and describes a circle with radius zero. The interpretation of the spatial autocorrelation is in both cases similar. The case of τ approaching infinity describes an increasing negative spatial autocorrelation, and the case of τ approaching one displays positive spatial autocorrelation. With the theoretical behavior for the two transformations established, an interpretation of the behaviour under simulated situations of different degrees of spatial autocorrelation will be performed in the next section.

As demonstrated in this section, any GIS with the ability to generate a gravity model with a distance exponent of one can be utilized to display the spatial autocorrelation structure in a data set by modifying the attribute values of the data points and generating the MW-Voronoi diagram for the data set.

4. Simulation of Spatially Autocorrelated Data

In this thesis the focus will be on a regular spatial distribution, a triangular grid with $n = 49$ data points. With the help of subroutines from the IMSL Stats and Maths Libraries, several different random samples from a normal distribution have been generated. The spatial autocorrelative structure underlying this sample was changed, to generate different degrees of spatial autocorrelation. The result has then been displayed for a Moran and Geary transformation. The resulting images allow a qualitative interpretation of the spatial autocorrelation structure in a data set and display the change in the visual representation from the case of a high positive to a high negative spatial autocorrelation.

Detailed discussions of simulation processes for the generation of autocorrelated data sets can be found in Anselin (1980), Cliff and Ord (1981), Goodchild (1980), Griffith (1987) and Haining (1980; 1991). Alternative models to the simultaneous autoregressive model that is utilized in this study can be found in the same sources.

4.1 The Simultaneous Autoregressive Model

A simple model for the generation of the autoregressive data sets was used in this thesis. The procedure was first introduced by Whittle (1954). It is described in detail in Cliff and Ord (1981, p.146ff) and Haining (1990, p.81ff). The simultaneous autoregressive scheme for this process can be described by

$$Y_i = \sum_{i \neq j} c_{ij} Y_j + \varepsilon_i \tag{4.1}$$

where Y_i = attribute value at point p_i

ε_i = uncorrelated error terms with $E(\varepsilon_i) = 0$ and $Var = 1$.

This can be expressed in matrix notation as

$$\mathbf{Y} = \mathbf{CY} + \mathbf{e} \tag{4.2}$$

where $\mathbf{Y} = (Y_1, \ldots, Y_n)^{\mathrm{T}}$

$\mathbf{C} = \{c_{ij}\}$, a $n \times n$ matrix

$\mathbf{e} = (\varepsilon_1, \ldots, \varepsilon_n)^{\mathrm{T}}$.

Equation (4.2) can be transformed to

$$\mathbf{Y} = (\mathbf{I} - \mathbf{C})^{-1} \mathbf{e} \tag{4.3}$$

where $\mathbf{I}$ = the identity matrix.

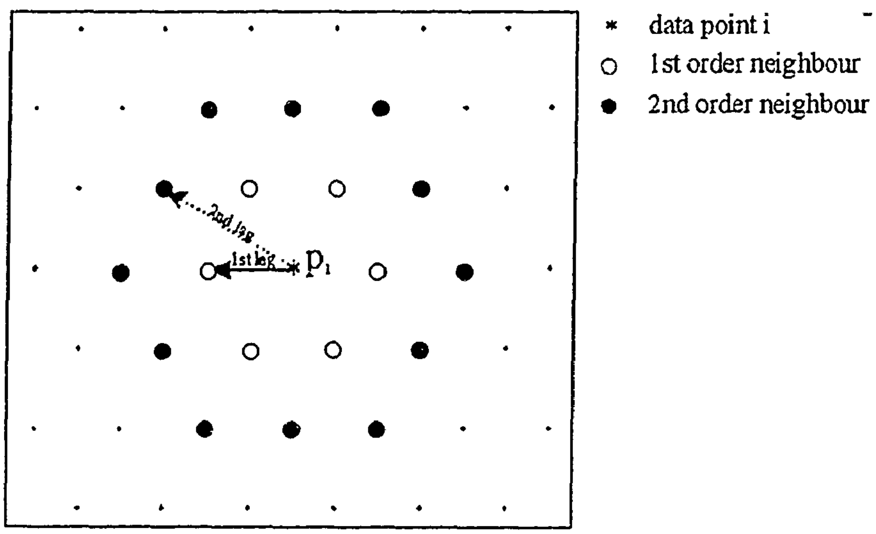
In this equation the term **Y**, the vector of the attribute values at the data points, only occurs on the left side of the equation. The model used in this thesis, for the generation of the spatially autocorrelated data set is based on this equation (see Fig. 12 and Program 1, Appendix I).

The Connectivity Matrix C:

There are several possible forms for the contents of the connectivity matrix **C** in equation (4.3). For a first order process **C** will be a binary matrix, assigning ones to the immediate neighbours of a point p_i and zeros to the connection to all other points (Fig.

1.). For a second lag, the connections to the immediate neighbours of a data point would be assigned zeros and the connections to the neighbouring data points of the immediate neighbours would be assigned ones. Other possibilities for the make up of the connectivity matrix C would be a distance controlled matrix where $c_{ij} = d_{ij}^{-1}$. In this case all data points would influence the values of all other data points according to the inverse of there relative distances to each other.

Fig. 1. First and second lag in a triangular grid.



In the case of a regular, triangular grid of data points, as used here, a data point has six nearest neighbours influencing its attribute value. It is assumed that boundary effects can be excluded if the data point is not a member of the boundary of the data set, where the number of its neighbours would be less than six. To ensure spatial stationarity, entries in C have been divided by six. Thus, the connectivity matrix C consists of zeros and of $(1/6)$ for the six nearest neighbours of a point p_i, to ensure that the row sums are one. Cliff and Ord (1981, p.143) give the following definition for a spatially stationary process:

(For a finite set of sites or for a spatially discrete process) Consider a spatial process Y_i defined at sites i $(i = 1,\ldots,n)$ and let the coordinates of site i be $\mathbf{x}_i = (x_{1i}, x_{2i})^T$. The process is *spatially stationary* if, for all i,

$$E(Y_i) = \mu, \quad Var(Y_i) = \sigma_Y^2,$$

and

$$Cov(Y_i, Y_j) = \sigma_Y^2 \, c(\mathbf{x}_i - \mathbf{x}_j)$$

where c is a correlation function, depending on the relative locations of the two sites.

Haggett et al. (1977, p.342) explain stationarity as "an assumption that the relationship between values of the process (generating the data) is the same for every pair of points whose relative positions are the same." They state that for a stationary process, the relationship between values depends only on the distance between the data points

To introduce spatial autocorrelation into the data set, the connectivity matrix **C** is multiplied by a factor ρ. This factor ρ defines the magnitude of the autocorrelation in the data set. Haining (1990, p.82) defines the range of ρ as:

$$\frac{1}{\lambda_{\min}} \prec \rho \prec \frac{1}{\lambda_{\max}},$$

where $\lambda_{\min}$ = the minimum and

$\lambda_{\max}$ = the maximum eigenvalue of **C**.

For the triangular grid used in this thesis, the connections to nearest neighbours are denoted by $c_{ij} = 1/6$ and $c_{ij} = 0$ otherwise. The minimum and maximum eigenvalues for the connectivity matrix **C** are: $\lambda_{\min} = -0.4804$ and $\lambda_{\max} = 0.9596$. Therefore the range of ρ is: $-2.08\bar{3} \prec \rho \prec 1.041\bar{6}$. The behaviour of ρ is asymmetric. Its range in the negative is approxim. 'y double the range in the positive.

To introduce autocorrelation, equation (4.3) can now be written as:

$$\mathbf{Y} = (\mathbf{I} - \rho\,\mathbf{C})^{-1}\mathbf{e}\ . \tag{4.4}$$

Cliff and Ord (1981, p.147) state, that up to this point no distributional assumptions are necessary. They point out, that if normality is assumed for $\mathbf{e}$, $\mathbf{Y}$ is multivariate normally (MVN) distributed and that a non zero mean ($\mathbf{Y}^* = \mathbf{Y} + \mu$) can be introduced, so that $\mathbf{Y}^* = \text{MVN}(\mu, \text{Var})$. Since the definition of the MW-distance does not allow for negative values and a multiplication of a negative ρ with negative values of **e** would create a positive result, the values of the random vector **e** have to be positive. Therefore **e** is shifted to a positive mean where all ε_i are positive ($\mathbf{e}^* = \mathbf{e} + \mu$) and $\mathbf{e}^*$is normally distributed with (μ, Var). After the multiplication with a negative ρ negative results for **Y** will again be introduced. Therefore **Y** has to be shifted to a positive mean, so that all Y_i will be positive($\mathbf{Y}^* = \mathbf{Y} + \mu$).

Simulation of the Autocorrelated Data Sets

For the generation of the autocorrelated data set, a random vector $\mathbf{e} = (\varepsilon_1, \ldots, \varepsilon_n)^{\mathrm{T}}$, with $n = 121$, was produced out of a normal distribution with the help of the IMSL Stats library (see Fig 12. and Program 1, Appendix I). The vector **e** was used to generate $\mathbf{Y} = (Y_1, \ldots, Y_n)^{\mathrm{T}}$, following equation (4.4). **Y** was then assigned to the regular triangular grid (11×11) that was used to define the connectivity matrix **C**. To ensure that no boundary effects might influence the model, this spatially larger (11×11) grid was only used for the generation of the autocorrelated data sets.

Fig.12: Simplified flow chart for Program 1: SAR

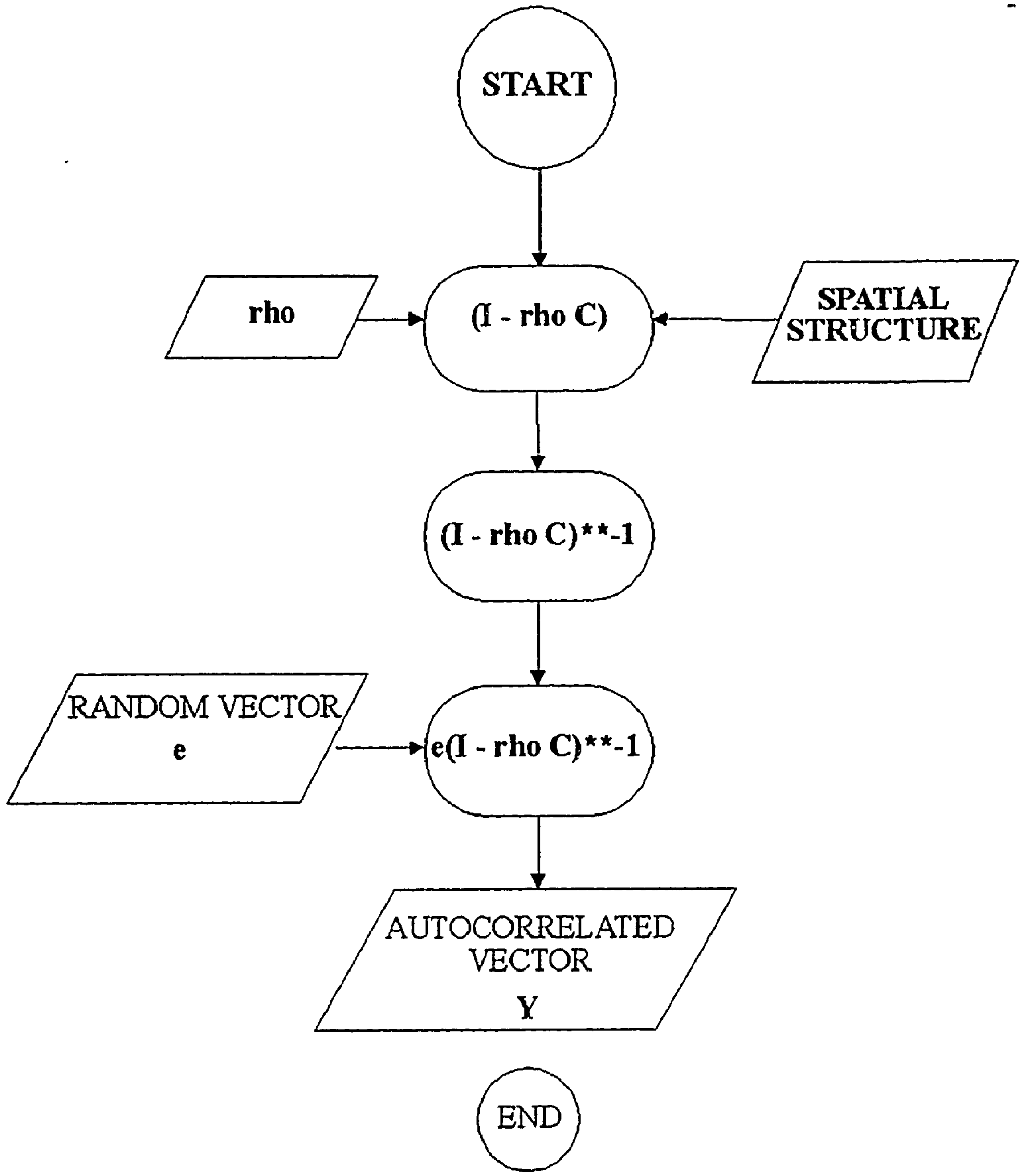
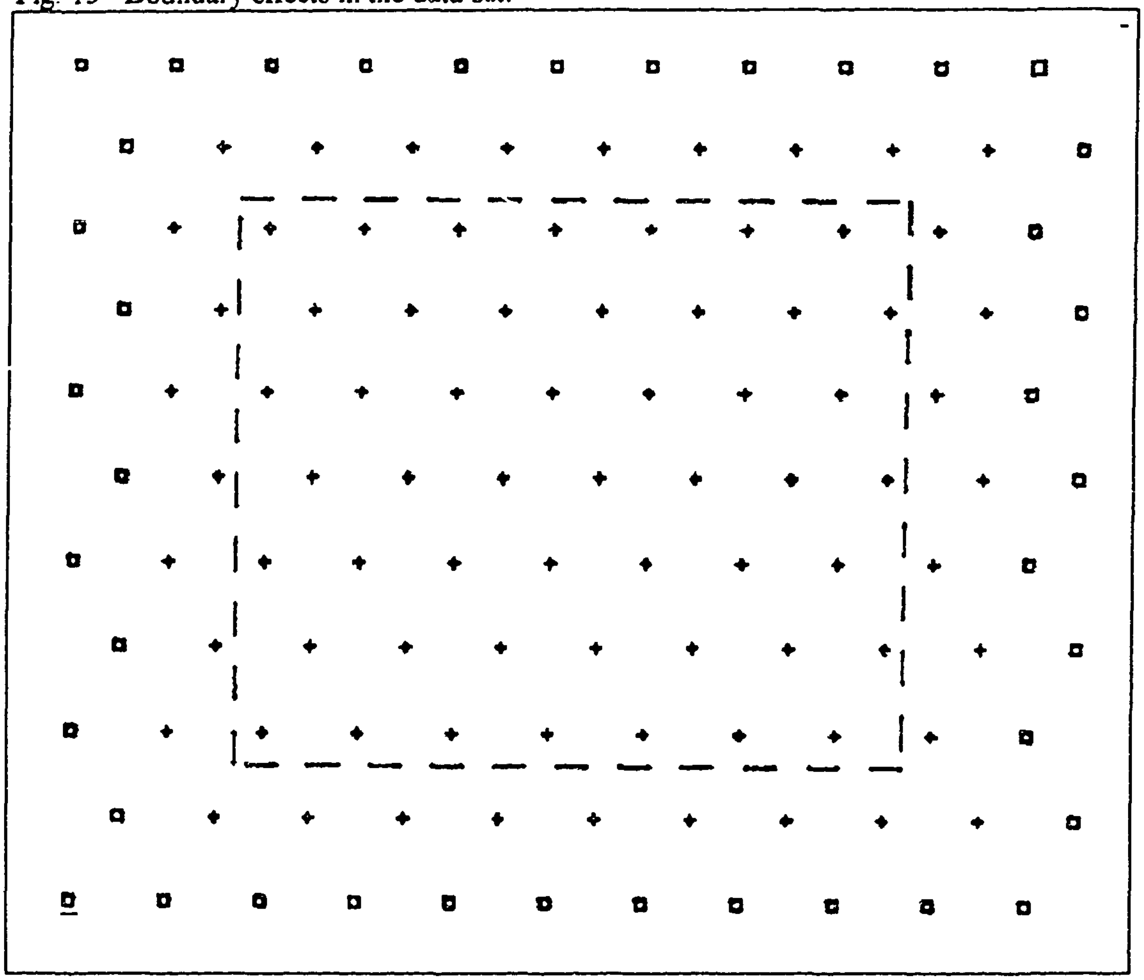


Fig. 13 Boundary effects in the data set.



□ boundary points, removed after the simulation.

\+ points used for the generation of the MW-Voronoi diagram.

--- boundary defining the inner 49 points not affected by boundary effects.

Only the inner 81 points of the triangular grid, denoted with a '+' in Fig. 13, were used to generate the MW-Voronoi diagrams. This (9×9) grid has to be reduced to the inner (7×7) grid, framed by the dashed line in Fig. 13, that can be applied to the visual representation of the spatial autocorrelation in the data sets. This technique of fitting a spatially larger model to the study region excludes the boundary effects resulting from the generation of the simultaneously autoregressive model and from the MW-Voronoi diagrams.

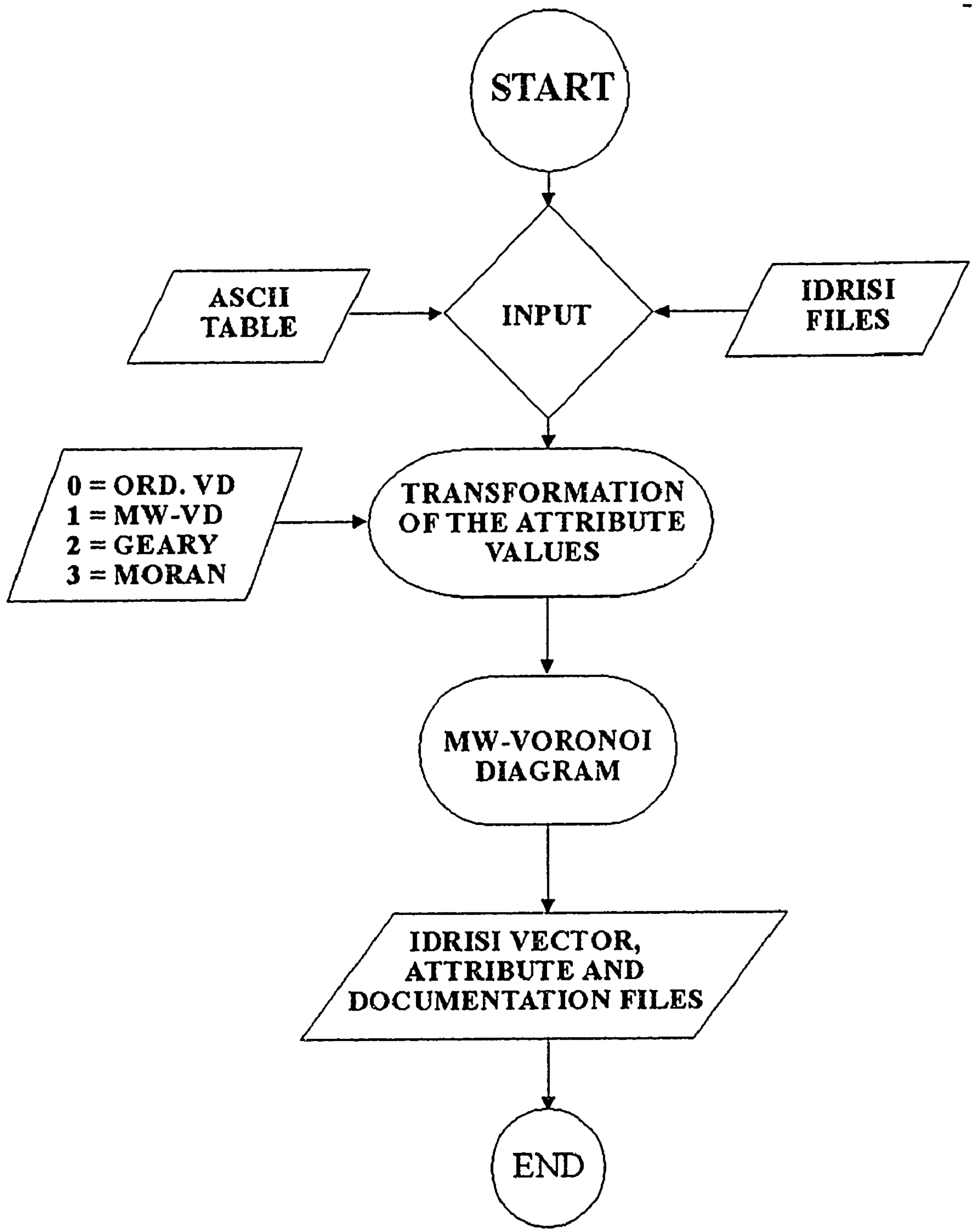
5. Discussion of the Visualization Results

In this part of the thesis, the resulting data sets out of the autocorrelative process described in the previous chapter have been assigned to the triangular grid displayed in Figure 12. In the next step the MW-Voronoi diagrams for these data sets were generated. The attribute values w_i, resulting from the vector **Y** out of the SAR model, were modified according to the transformations for the Geary and the Moran statistics described in Chapter 3.

Description of the Generating Process

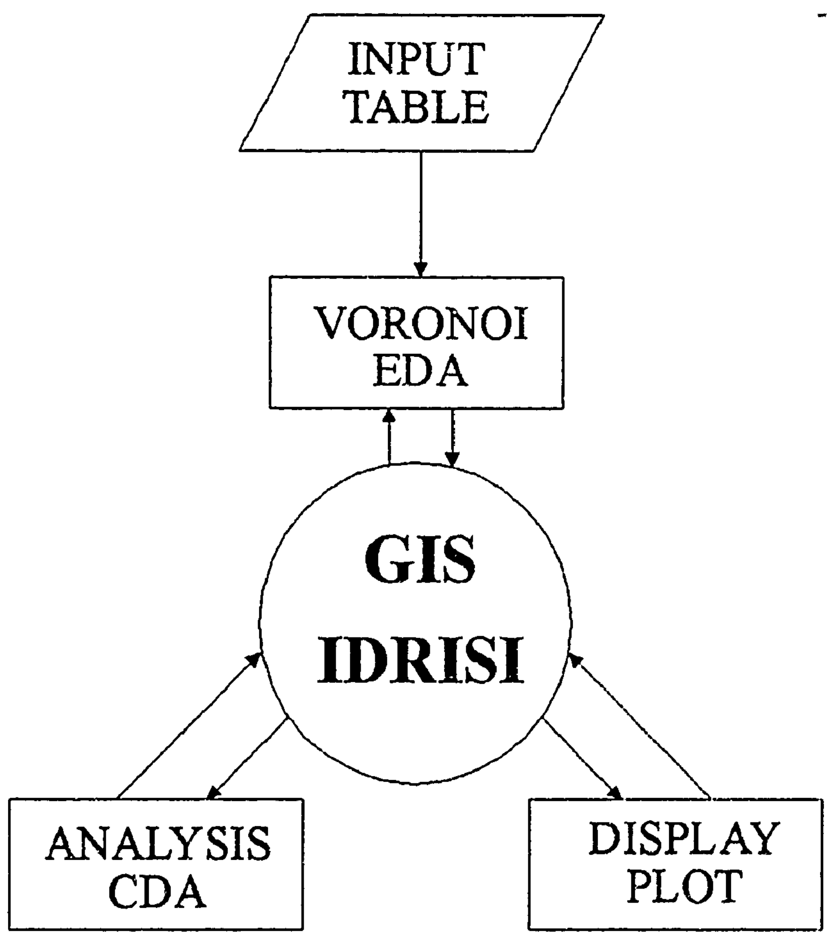
The computer program that was used to perform these transformations and to generate the MW-Voronoi diagrams (see Fig.14 and Program 2; Appendix II) is based on an algorithm by Gambini (1965). The original algorithm generates market spheres for a set of points in a spatial interaction model. It was modified to compute the MW-Voronoi diagrams and to perform the changes in the attribute values w_i of the data points that were needed to generate the Geary and Moran transformations. The program was written in FORTRAN77 and interfaced with the data structure and the plotting facilities in IDRISI Version 4 (GIS). The program offers two options of data input. First an ASCII table consisting of a header line, columns for point identification, X- and Y-coordinates and the attribute value assigned to the point. The second option utilizes IDRISI point, value and documentation files for the data input. The attribute values of the data points can be transformed to generate an ordinary Voronoi diagram, a MW-Voronoi diagram, or the Geary and the Moran transformations for the visualization of the spatial autocorrelation in the data set. Further descriptions of the program and the input and output options available are provided in the "readme" file in Appendix III.

Fig. 14: Simplified flow chart for Program 2: VORONOI.



The program generates a complete set of IDRISI point and line vector files, an attribute values file, a script file for the display or plotting of the results, and all necessary documentation files. The program can be used as a new module for the generation of planar ordinary and multiplicatively weighted Voronoi diagrams and as a preliminary form of SDA before other forms of analysis are performed in IDRISI. It is completely integrated into the IDRISI GIS program structure (Fig. 15).

Fig. 15: Integration of the program VORONOI into the GIS structure.



As shown in the previous figure, the program is an integral part of the system. It offers a new form of data input into the IDRISI GIS structure. It can be used for data from inside or outside of the IDRISI structure.

Visual Results

Figures 16 to 23 show a summary of the results of a visualization process for the Geary and the Moran transformations for four different data sets. The variances for the random vectors **e** from the SAR model that were used to generate these data sets vary from 1 to 4. The MW-Voronoi diagrams for strong, medium and weak positive and negative spatial autocorrelation for the same initial **e** are displayed on one page to allow a comparative view. The complete series for ρ ranging from 0.9 to -1.8 can be found in the Appendix IV.

An increase of the variance in the random vector **e**, used in the SAR model, enhances the sensibility of the transformations to display a change in the autocorrelative structure in a data set. The visual results seem to indicate, that the Moran transformation is more sensitive to the occurrence of negative spatial autocorrelation than the Geary transformation which identifies positive spatial autocorrelation better. This behaviour suggests, that both transformations should be considered together to identify the spatial autocorrelation in a data set. The visual displays for the Moran transformation for ρ ranging from medium positive to medium negative spatial autocorrelation are very similar. They should be compared with the results for the Geary transformation to determine whether the pattern indicates a medium or weak positive or negative spatial autocorrelation.

Fig. 16. Geary transformation for a Variance of 1.

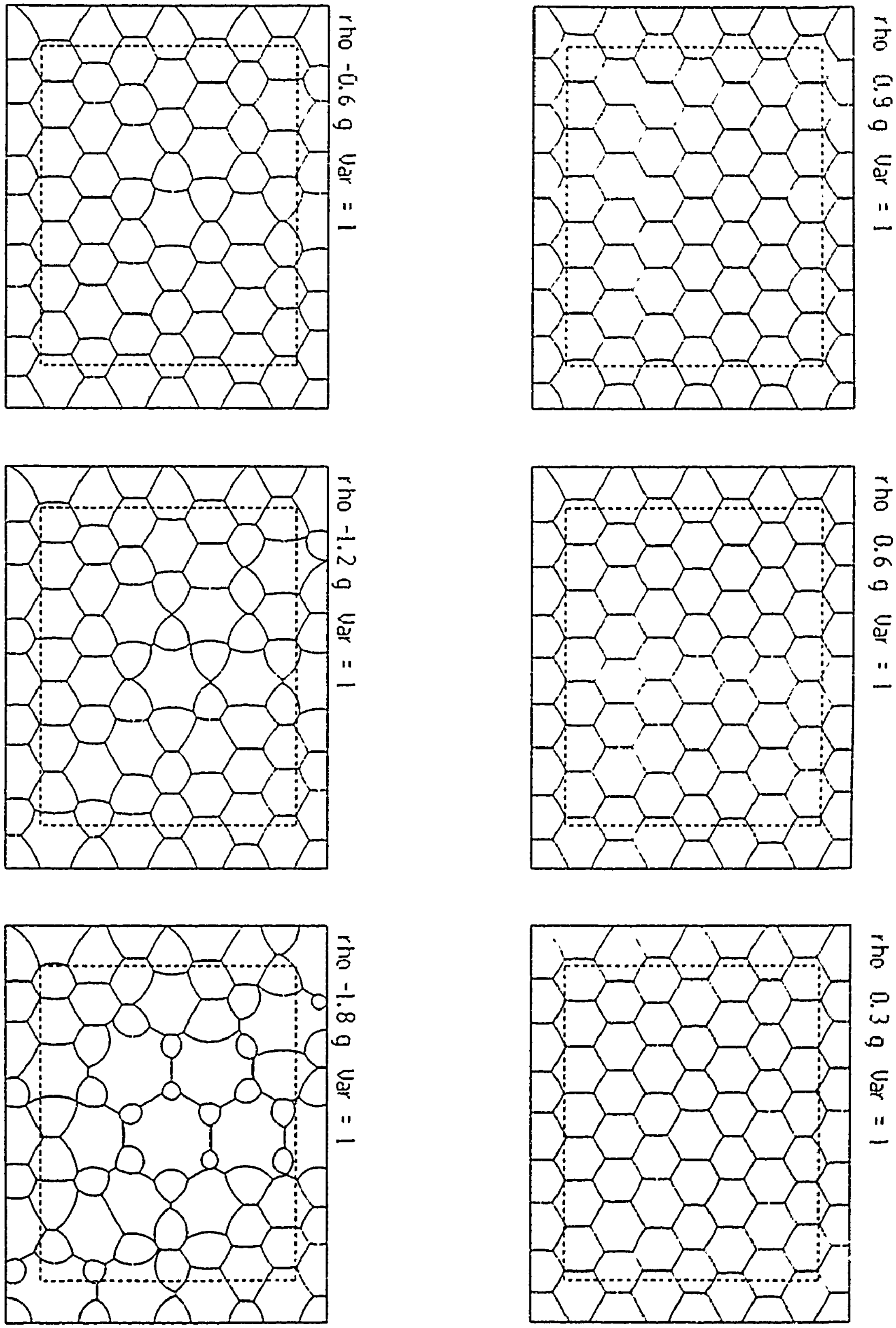


Fig. 17. Moran transformation for a Variance of 1.

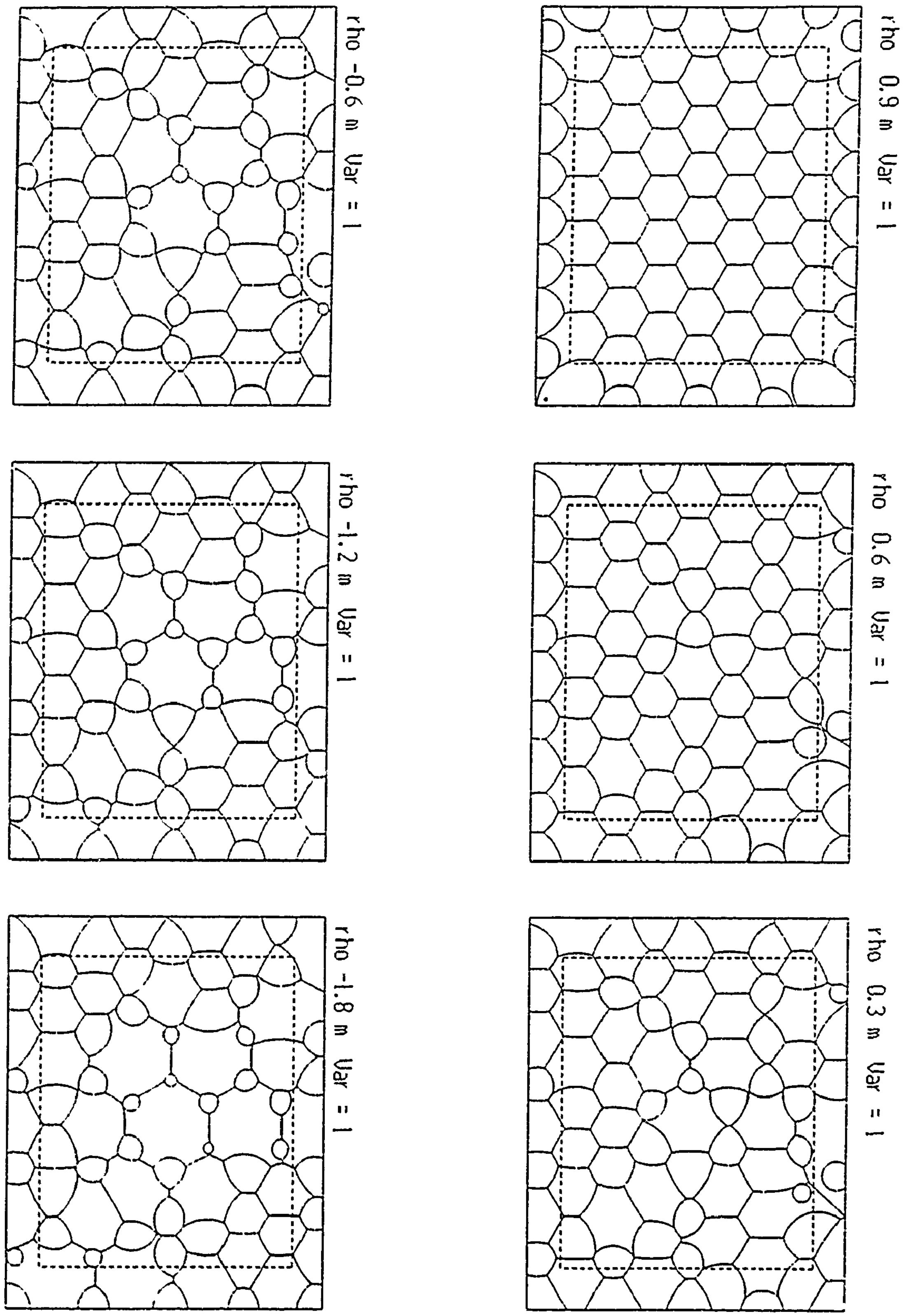


Fig. 18. Geary transformation for a Variance of 2.

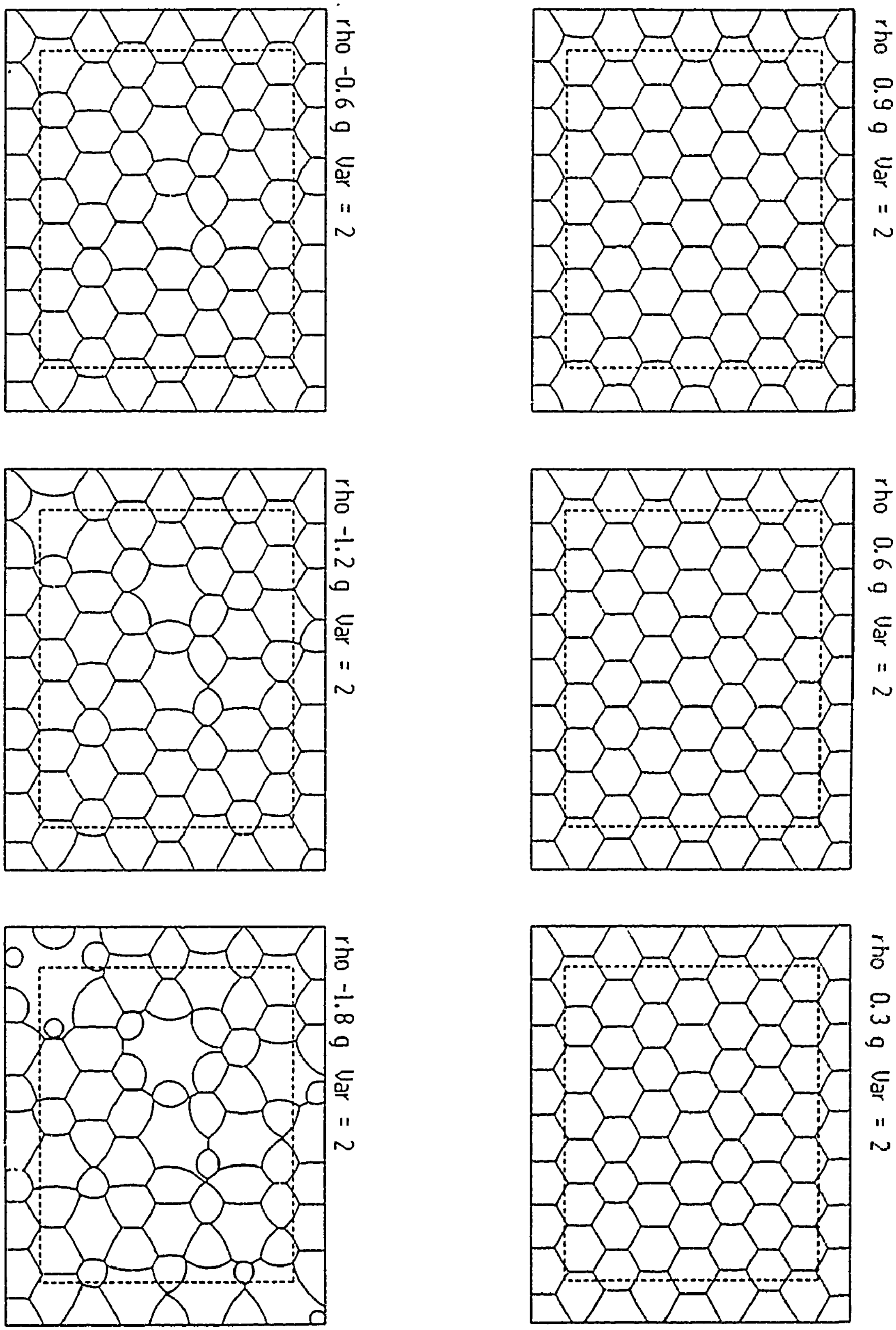


Fig. 19. Moran transformation for a Variance of 2.

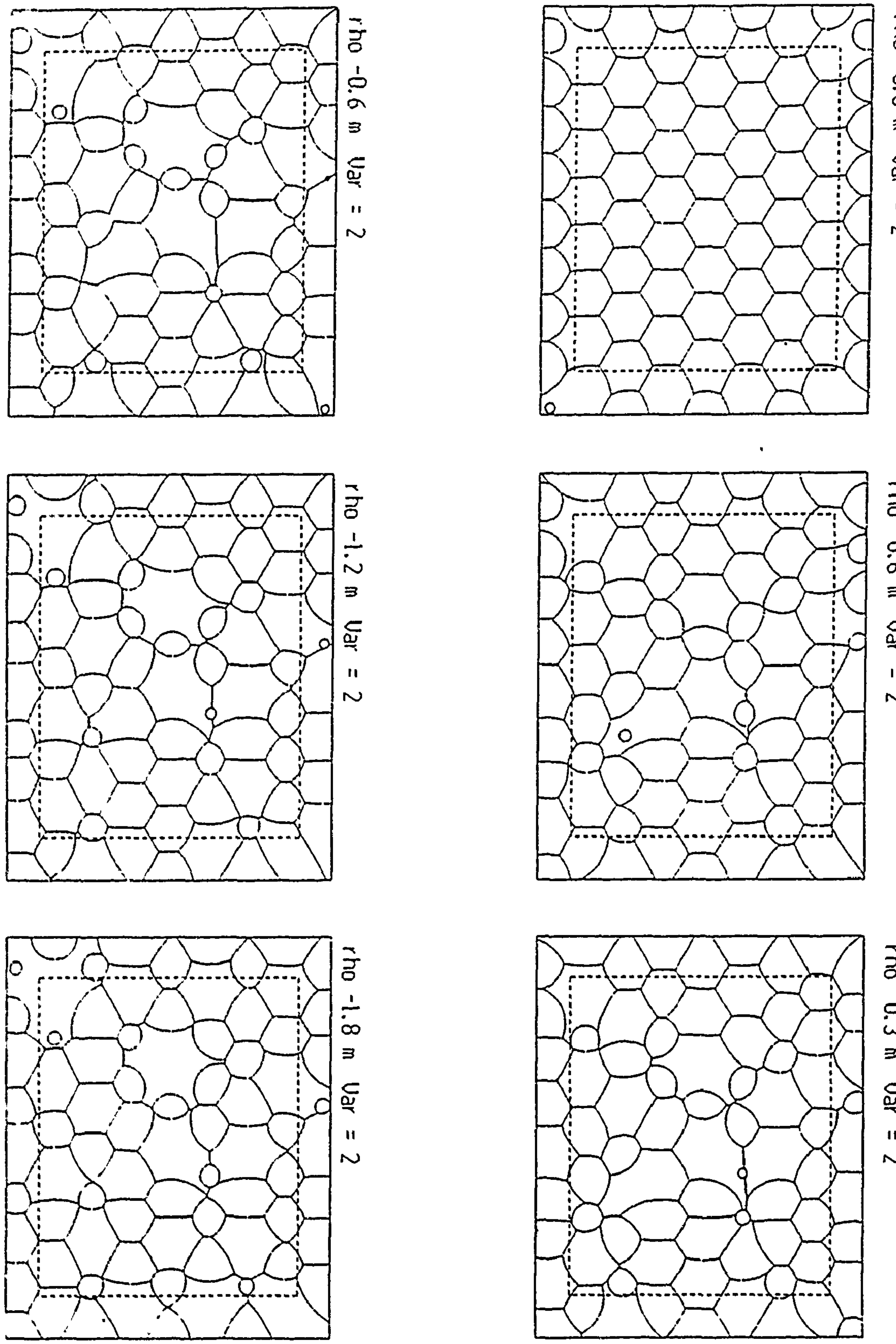


Fig. 20. Geary transformation for a Variance of 3.

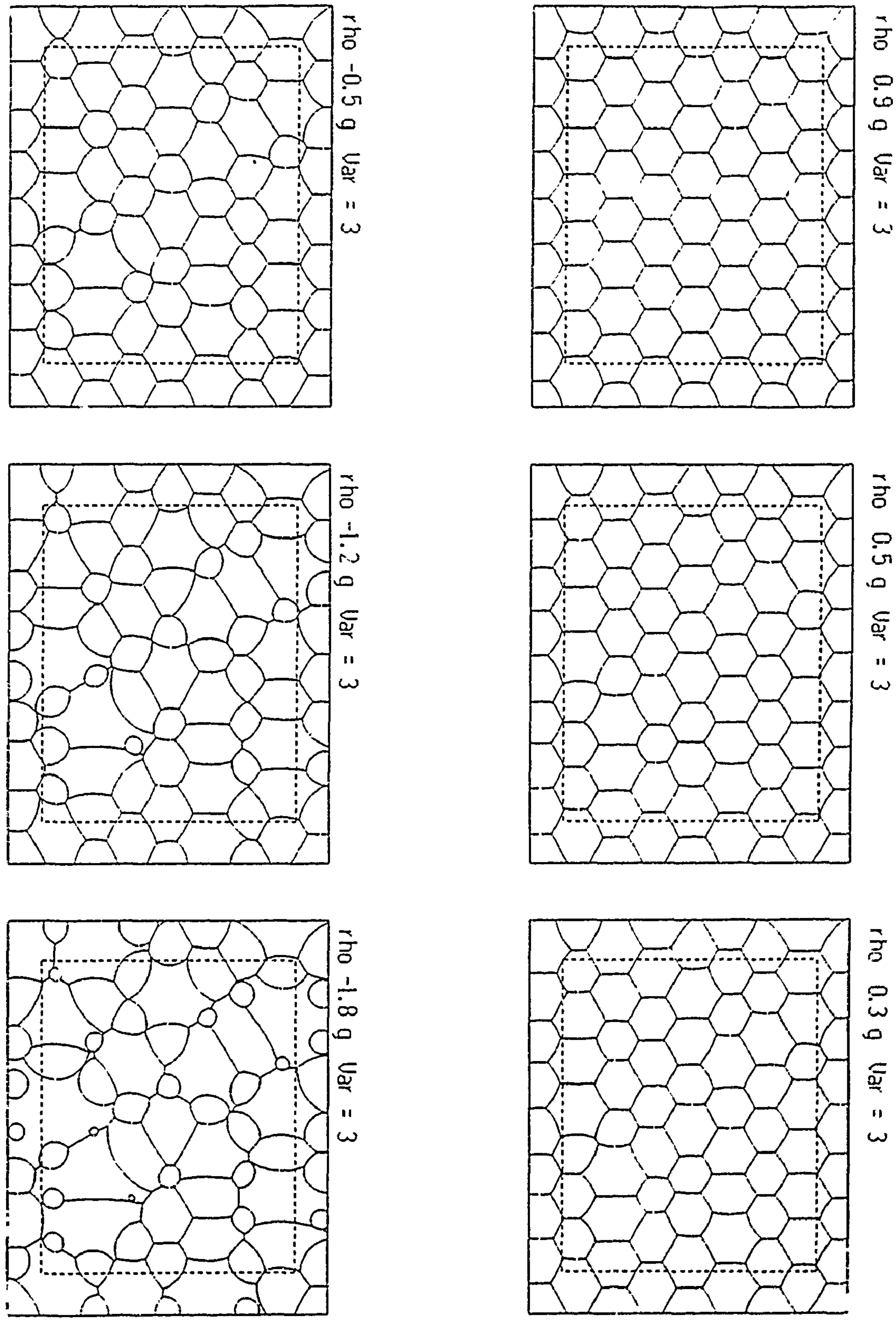


Fig. 21. Moran transformation for a Variance of 3.

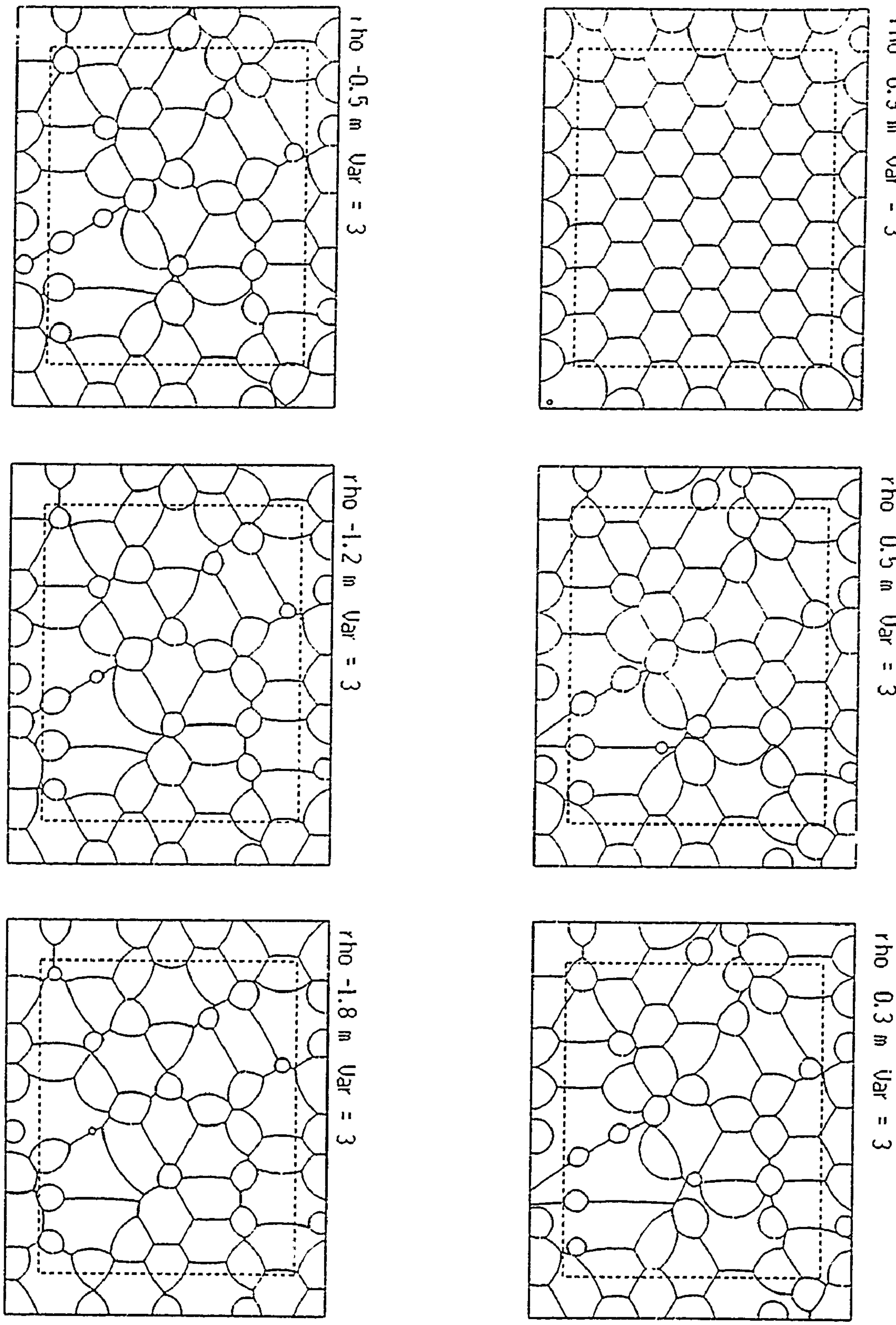


Fig. 22. Geary transformation for a Variance of 4.

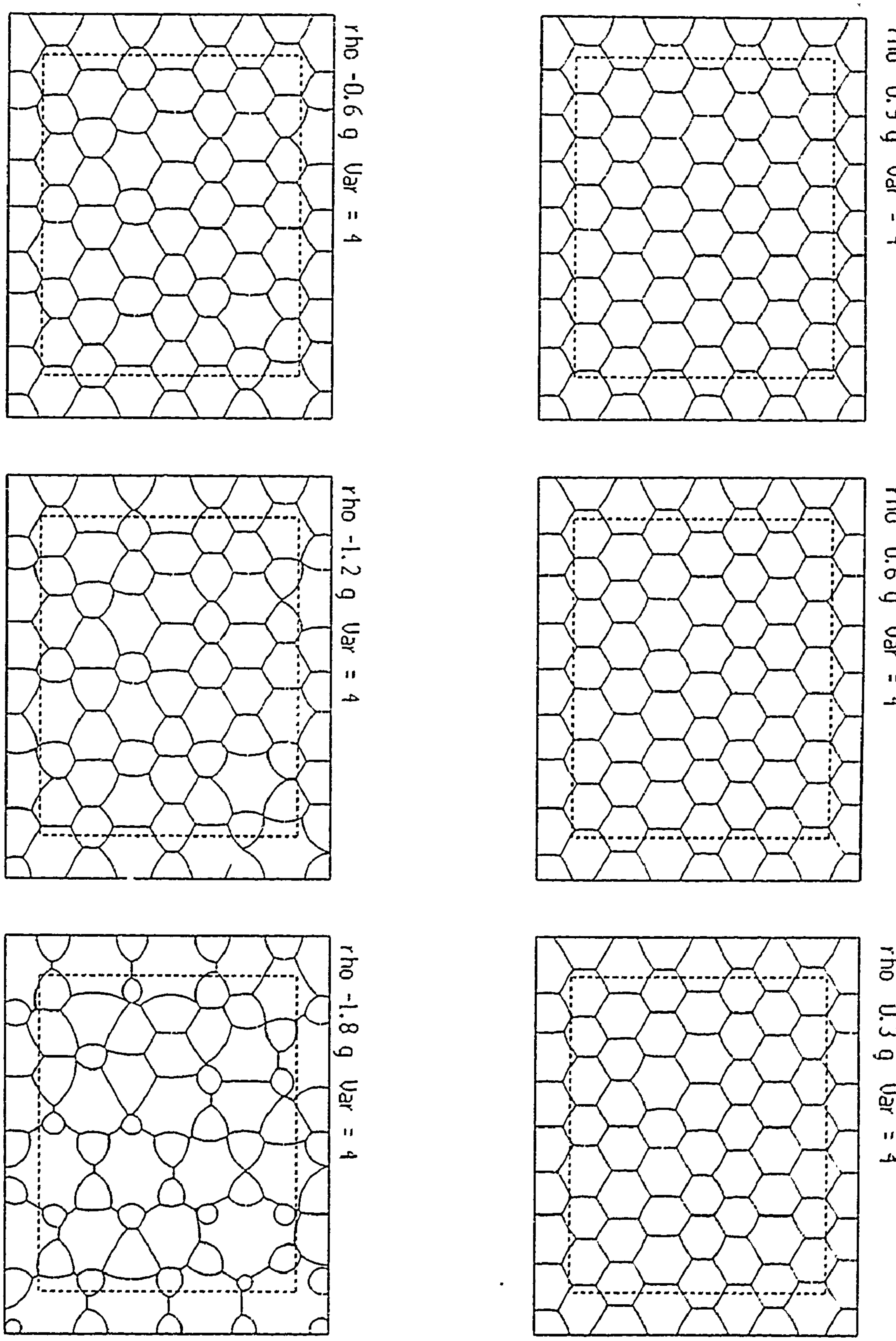
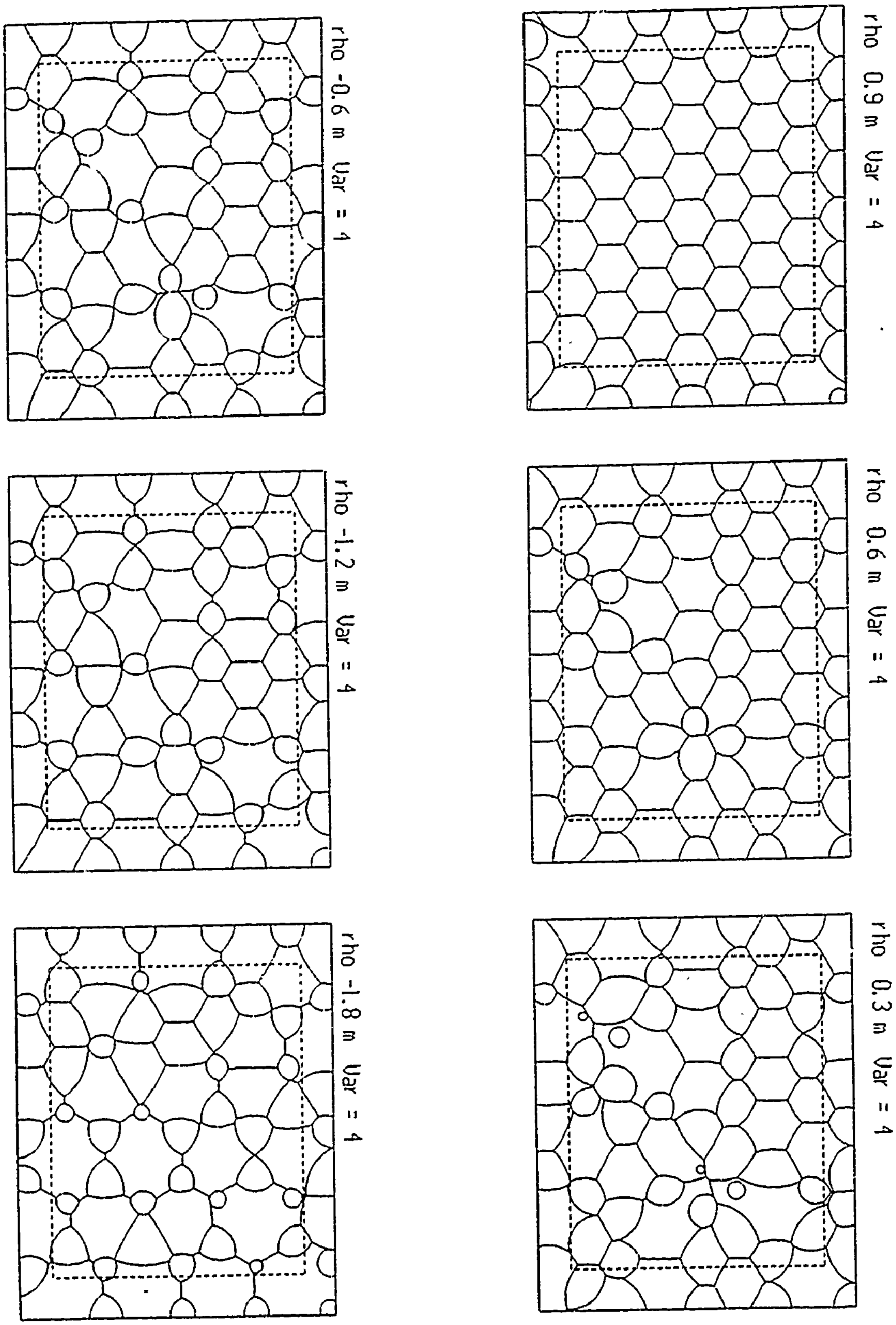


Fig. 23. Moran transformation for a Variance of 4.



As expected, for high positive autocorrelation, the Voronoi polygons approach equal sized hexagons. This pattern resembles the ordinary Voronoi diagram (all attribute values w_i are treated as equal) for a regular triangular grid. For ρ approaching zero, the hexagonal polygons change size. Larger sized hexagons appear and the clusters of equal sized polygons dissolve. For the case of small negative spatial autocorrelation, clusters of larger sized polygons that are surrounded by smaller sized polygons emerge. The hexagonal structure of the polygons tends to change into more circular shapes. The bisectors that previously resembled almost straight lines shift into more circular arcs with an increasing curvature. For ρ approaching its lower limit of negative two, the shape of the small polygons tends towards small circles that are completely entrapped in much larger unintersected areas. At this stage some of the Voronoi polygons even disappear into circles with a zero radius at the generator points. This process is enhanced for the Moran transformation. A reason for this can be found in the transformation described in equation (3.1.2), where the attribute values of the points are shifted to the positive in regard to the mean and the influence of the lowest weighted point practically disappears.

6. Conclusions

It was possible to detect the relationship between spatial autocorrelation measurements in a cross-product form, spatial interaction models and MW-Voronoi diagrams. They were identified as similar concepts. The cross-product form of the numerators of the Geary and the Moran statistic was identified as being equivalent to the MW-distance (Boots, 1993, p.11). The visual representations allows for the detection of the autocorrelative structure in a point data set. The previously utilized procedure of displaying spatial autocorrelation by generating the ordinary Voronoi diagram and creating a choropleth map for the attribute values (described in the introduction) proved to be appropriate only for the case of high positive spatial autocorrelation. The previously utilized technique does not include relationships between data values at the spatial locations into the visual representation. The technique introduced by Boots (1993) and further examined in this thesis considers both the relative spatial location of the data sites and the attribute values associated with them to generate a visual representation of the autocorrelative structure of the data set. Boots (1993) points toward more similarities between spatial autocorrelation models and Voronoi diagrams.

> Each of the individual terms in the summation in the numerator of the spatial autocorrelation models represents an association between a pair of data sites which, taken collectively, summarize the degree of spatial autocorrelation in the data. Similarly, the individual edges of the Voronoi diagram represent associations between individual pairs of data sites which, when considered simultaneously in the form of the complete Voronoi diagram, provide an overall view of the spatial associations. (Boots, 1993, p.12ff)

All the practical results are at the moment limited to an interpretation of a regular triangular grid and a first order spatial lag situation. The mathematical results imply that theoretically the interpretation of the results can be extended to other situations. In the future the behaviour of data set with irregular point distributions and different spatial lags for the connectivity matrix has to be examined. The limitation to the euclidean distance for the MW-distance is not required. Other forms of 'distances' can be used.

A direct comparison of the visual representation of spatial autocorrelation in data sets seems to be possible only for data sets with a similar distribution of the data points. Nevertheless the results allow for a first formulation of the hypothetical general behaviour of the visualization technique for a first order spatial lag situation..

In an irregular grid, a data point will have an average of 6 nearest neighbours in the sense of the Voronoi diagram. This suggests, that for an irregular data point distribution the results of the visual transformation of the spatial autocorrelation will approximate the results of a regular triangular distribution of data points Exemptions could be data sets with an extreme clustering of data points or other regular non-triangular patterns.

For a square grid, the planar ordinary Voronoi diagrams are squares. The hypothetical behaviour will resemble the observed behaviour for the triangular grid. Positive spatial distribution would be denoted by a pattern of square cells, the form of the ordinary Voronoi diagram. For an increase of negative spatial autocorrelation, the squares will dissolve and circles surrounding the data points with low attribute values will appear

A simple test for the spatial autocorrelation in a data set seems to be the comparison of the planar ordinary Voronoi diagram with the MW-Voronoi diagram of a point data set. The closer the resemblance of the two is, the higher the degree of positive spatial autocorrelation in the data set. Respectively, the increase in the number of small, complete circles and of large, unintersected areas, denotes an increase of negative spatial autocorrelation in a point data set. The curvature of all bisectors in the MW-Voronoi diagram is another indicator for the spatial autocorrelation in a data set. The dominance of straight or almost straight lines indicates a high degree of spatial autocorrelation. For an increase in the curvature of the bisectors, up to the point where the bisector is a complete circle, negative spatial autocorrelation can be diagnosed.

Future work with irregular data point distributions has to evaluate these hypotheses. Also the behaviour for a second or higher order spatial lag has to be examined in the next research activities on this topic.

This thesis scratched only the surface of a new and exciting field in the work on spatial autocorrelation. New ideas and questions arise that will hopefully stimulate more work in this area. It is the hope of the author that the new technique described in this thesis might increase the understanding of spatial autocorrelation and the nature of spatial data. This new visual representation of spatial autocorrelation should introduce more geographers to SDA due to its link to GIS and therefore its availability. It should simplify the detection of spatial autocorrelation in a data set. The identification of the autocorrelative structure in a data sets might then help to prevent researchers from violating basic assumptions of the statistical techniques used for analyzing spatially referenced data, such as the assumption of independence of the data as given in regression analysis.

Glossary

Apollonius Circle: The circle consisting of the locus of points the ratio of the distances of which from two given points is a fixed number; the locus of the apexes of all triangles on a given base the other two sides of which are in a fixed proportion. It is a circle the diametral points of which on the extensions of the base are harmonic points (Collins Reference Dictionary Mathematics).

CDA confirmatory data analysis

EDA exploratory data analysis

GIS Geographical (or Geographic) Information System.

SAR simultaneous autoregressive model

SDA spatial data analysis

a_i attractiveness of point i

b_j vector of destination attributes

b_2 $$b_2 = \frac{\sum z_i^4}{\left(\sum z_i^2\right)^2}$$

C $\mathbf{C} = \{C_{ij}\}$, a measure of the spatial proximity between points i and j; connectivity matrix

c Geary statistic

E expected value

e random vector from a normal distribution; $\mathbf{e} = (\varepsilon_1, \ldots, \varepsilon_n)^T$.

d_{ij} distance between points i and j

G	$\mathbf{G} = \{G_{ij}\}$, a measure of the proximity of the attribute values at points i and j
H	connectivity or contiguity matrix $\mathbf{H} = \{h_{ij}\}$
h_{ij}	connection between points i and j
$h_{i.}$	$h_{i.} = \sum_j h_{ij}$
$h_{.j}$	$h_{.j} = \sum_i h_{ji}$
I	Moran statistic
I	identity matrix
N	Normalization
o_i	vector of origin attributes
P	set of data points $P = \{p_1, \ldots, p_n\}$
R	Randomization
$\Re^2$	two-dimensional space
$\Re^m$	m-dimensional space
s_{ij}	vector of segregation attributes
S_1	$S_1 = \frac{1}{2}\sum_{(2)} \left(h_{ij} + h_{ji}\right)^2$
S_2	$S_2 = \sum_i \left(h_{i.} + h_{.j}\right)$

T_{ij}	the amount of interaction between i and j due to the relative influence exerted by i on j
Var	Variance
w_i	attribute value at point i
$\overline{w}$	the mean of the attribute values $\overline{w} = \frac{\sum_i w_i}{n}$
$\mathbf{x_i}$	location vector $\mathbf{x}_i = (x_{i1}, x_{i2})$
$\mathbf{Y}$	autocorrelated vector (SAR); $\mathbf{Y} = (Y_1, \ldots, Y_n)^T$
Y_i	attribute value at point i; used in the SAR model
z_i	$z_i = (w_i - \overline{w})$
α	distance exponent in a gravity model
ρ	factor determinating the degree of autocorrelation in the data set
μ	mean value of a distribution
λ_{min}	minimum eigenvalue of C.
λ_{max}	maximum eigenvalue of C.
Γ	general cross-product statistics
$\sum_{(2)}$	$\sum\sum$
$V(p_i)$	the Voronoi polygon associated with p_i; $V(p_i) = \{\mathbf{x} \mid \lVert \mathbf{x} - \mathbf{x}_i \rVert \le \lVert \mathbf{x} - \mathbf{x}_j \rVert \ \text{ for } j \ne i, j \in I_n\}$
ϑ	the planar ordinary Voronoi diagram; $\vartheta = \{V(p_1), \ldots, V(p_n)\}$

$b(p_i, p_j)$ the bisector for the planar ordinary Voronoi diagram;

$$b(p_i, p_j) = \{\mathbf{x} \mid \|\mathbf{x} - \mathbf{x}_i\| = \|\mathbf{x} - \mathbf{x}_j\|\}, \quad \text{for } j \neq i.$$

$Dom(p_i, p_j)$ the dominance region for the planar ordinary Voronoi diagram;

$$Dom(p_i, p_j) = \{p \mid d(p, p_i) \leq d(p, p_j)\}, \quad \text{for } j \neq i.$$

$d_{mw}(p, p_i)$ MW-distance; $d_{mw}(p, p_i) = w_i^{-1}\|\mathbf{x} - \mathbf{x}_i\|$, $w_i \phi 0$.

$Dom_w(p_i, p_j)$ the dominance region for the MW-Voronoi diagram;

$$Dom_w(p_i, p_j) = \{p \mid d_{mw}(p, p_i) \leq d_{mw}(p, p_j)\}, \text{ (for } i \neq j).$$

$V_W(p_i)$ the multiplicatively weighted Voronoi region; $V_W(p_i) = \mathop{\mathrm{I}}_{i=1}^{n} Dom_w(p_i, p_j)$.

ϑ_W the MW-Voronoi diagram; $\vartheta_W = \{V_W(p_1), \ldots, V_W(p_n)\}$

Appendix I: Program SAR.

Program 1. SAR

This program is a program for a simultaneously autoregressive model (SAR). It was used to generate the data sets with the autocorrelative structures for the simulation process. The program was compiled under a FORTRAN77 compiler on UNIX. The IMSL Math and Stats libraries were used to generate the random vector **E** out of a normal distribution and for the matrix manipulations.

The resulting vector **V** is assigned to a triangular grid.

```
      PROGRAM SAR
      DIMENSION W(121,121), X(121), Y(121), E(121), V(121),
     +          INV(121,121), VDEV(121), ID(121)
      INTEGER ID, N, I, J, IDENT, ACC, SEED, EPS
      REAL W, X, Y, E, RHO, G, EBAR, EVAR, VAR, V, VDEV, VMAXDEV,
     +      ERR, SUMV, VBAR, SA, IA, VARI, MINDIST
      COMMON /WORKSP /RWKSP
      REAL RWKSP(44134)
      CALL IWKIN(44134)

      OPEN (UNIT=20, FILE='big121', STATUS='OLD')
      OPEN (UNIT=25, FILE='dataout')
      OPEN (UNIT=26, FILE='epsilon')

      N = 121
      READ (20,*) RHO
        print *, RHO
      READ (20,*) (ID(K), X(K), Y(K), K=1,N)

      PRINT *,('If a new E should be calculated, hit 0, otherwise 1.')
          READ *,EPS
```

```
   10  FORMAT (F12.9)
       IF (EPS.GT.0) THEN
         DO 103 K=1,N
              READ (26,10) E(K)
  103    CONTINUE
       ELSE

  100    CONTINUE

C        Initialize a random number as a seed for the generation of
C        a series of random numbers out of a normal distribution (0,1).
C        The random sample will be shifted to a mean defined by the
C        variable IA and a Variance defined as the square of variable SA.
C        The result is a vector E.

         VARI = 3

         SA = SQRT (VARI)
         IA = 10
         CALL RNNOF (SEED)
         CALL RNSET (SEED)
         CALL RNNOA (N,E)
         CALL SSCAL (N,SA,E,1)
         CALL SADD  (N,IA,E,1)

C        Check for mean and varianz of E.  *******************

         EBAR = 0.0
         DO 60 K=1,N
             EBAR = EBAR + E(K)
  60     CONTINUE

         EBAR = EBAR / N
         SUME = 0.0
         EMAXDEV = 0.0
```

```
          EVAR = 0
          DO 61 K=1,N
              VAR = (E(K) - EBAR)**2
              EVAR = EVAR + VAR
61        CONTINUE
          EVAR = EVAR / N
          PRINT *, EBAR, EVAR
          PRINT *, 'accept E? (y=1/n=0)'
            READ *, ACC
          IF (ACC.EQ.0) GOTO 100

          WRITE (26,10) (E(K), K=1,N)

        ENDIF



        IF (RHO.EQ.0) THEN

            DO 77 K=1,N
                  V(K) = E(K)
77          CONTINUE

        ELSE

            I = 1
            J = 1
            MINDIST = 10000000
C         Distances between the points. Generation of MINDIST, the
C         distance to the first order neighbours plus an error term.

            DO 31 I=1,N
                  DO 41 J=1,N
                        IF (I.EQ.J) GOTO 41
                        DIST = (X(I)-X(J))**2 + (Y(I' Y(J))**2
                        DIST = SQRT (DIST)
                        IF (DIST.LT.MINDIST) MINDIST = DIST
```

```
 41          CONTINUE
 31      CONTINUE

         MINDIST = (1.5) * MINDIST

C       Binary weight matrix. W(I,J) = 1/6, for the six imediate
C       neighbours, 0 otherwise. For i=j, W(I,J) =0. The matrix W(I,J)
C       is multiplied by RHO (constant denoting the spatial
C       autocorrelation in the vector V.

         DO 11 I=1,N
               DO 21 J=1,N
                     IF (I.EQ.J) THEN
                         W(I,J) = 1
                     ELSE
                         IF (DIST.LT.MINDIST) THEN
                               W(I,J) = (0 - RHO/6)
                         ELSE
                               W(I,J) = 0
                         ENDIF
                     ENDIF
 21            CONTINUE
 11      CONTINUE

         CALL LINRG (121,W,121,INV,121)

         CALL MURRV (121,121,INV,121,121,E,1,121,V)

     ENDIF


C     Addition of an error-term ERR to shift the resulting vector to
C     the positive range and greater than one. If RHO is negative,
C     negative values will be introduced. For values lower than one,
C     the squared form of the Geary transformation might not be valid.
C     All values of V will be shifted to the positive by the maximum
```

```
C     negative deviation of the mean of V + 1.1 .

        IA = 10
        CALL SADD (N,IA,V,1)

      SUMV = 0.0
      VMAXDEV = 0.0
      DO 708 K=1,N
              SUMV = SUMV + V(K)
 708  CONTINUE

      VBAR = SUMV / N

      DO 706 K=1,N
             VDEV(K) = (V(K) - VBAR)
             IF (VMAXDEV.GT.VDEV(K)) VMAXDEV  = VDEV(K)

706   CONTINUE
      print *,VBAR
      print *, VMAXDEV
  8   FORMAT (X,'rho',F4.1)
      WRITE (25,8) RHO

      DO 70 K=1,N
             WRITE (25,*) ID(K), X(K), Y(K), V(K)
 70   CONTINUE

      STOP
      END
```

Appendix II: Program VORONOI.

Program 2. VORONOI

This program creates the planar, normal Voronoi diagram (as a function of distance between points; all weights are equal) and the multiplicatively weighted Voronoi diagram from a set of data points in R^2.
The input file must be an ASCII file. It can either be a table (ID-number; X-Coordinate; Y-Coordinate; Attribute value/weight) or IDRISI Version 4.0 point and values files.
The program is currently set to a maximum number of 100 points. Output will be in IDRISI Vers. 4.0 files (line, point, value, and script files).
The attribute values/weights can be transformed to represent a planar, normal Voronoi diagram, or a planar, multiplicatively weighted Voronoi diagram, which corresponds to a gravity model with a distance exponent of 1. A transformation of the weights can be performed, to create a visual representation for the Moran I, or the Geary c Statistics, as an initial, visual interpretation (exploratory spatial data analysis) of the underlying spatial autocorrelation structure in a point data set.
Most of the source code in capitals is based on an algorithm by Gambini for the calculation and display of the spatial interaction between data points (gravity model). This algorithm was available in form of a FORTRAN IV program, "EQUAL", that was provided by Dr.B.Boots, WLU.
I'm responsible for the incorporation of the initial algorithm into the IDRISI GIS environment (subroutines, data in- and out-put), the modification of the weights (normal Voronoi diagram, Moran and Geary transformation), the transfer of the code from a main frame environment (CP6) to the PC environment, and the transfer to FORTRAN77.
The code is written in standard FORTRAN77, to maintain the compatibility with other compilers and computing environments.

```
      PROGRAM VORONOI
      DIMENSION XC(100), YC(100), AOLD(100), A(100), ADEV(100)
      DIMENSION ID(100), GAM(16)
      REAL8 NAME(13)
      REAL SUMA, ABAR, AOLD, A, PRECI, DMAX, AMAXDEV, ADEV, e,
     +     XC, YC, X0, X1, Y0, Y1, A0, A1, height, xn, yn, zn
      INTEGER NC, counter, weight, FirstLine, col, cname, trash1,
     +     trash2, asc_in, font, ID, para
      CHARACTER24 title
      CHARACTER12 output_file, input_file, idrisi_vec,
     +             idrisi_dvc, idrpnt_vec, idrpnt_dvc, idrisi_val,
     +             idrisi_dvl, idrisi_scr, idr_pntdoc, input_value,
     +             idr_value, idr_valdoc, idr_point, input_point,
     +             new_dvl
      COMMON  A, XC, YC, X0, X1, Y0, Y1, I, J, NC
      COMMON counter, FirstLine, title


   11 FORMAT (A5)
   12 FORMAT (A8)
   13 FORMAT (13A4)
   15 FORMAT (20A4)
   16 FORMAT (I4,' 1')
   17 FORMAT (F8.3,1X,F8.3)
   18 FORMAT (I4,1X,F8.3)
   60 FORMAT (A3,1X,I2,1X,A5)
   62 FORMAT (A3,1X,F10.2,1X,F10.2,A4,I2)
   63 FORMAT (A1,1X,I2)
   71 FORMAT (A3,1x,I1,1X,I2,1X,F5.3,A4,1X,F8.3,1X,F8.3,1X,F9.3)
   81 FORMAT (A31)
   82 FORMAT (A34)

      print ,'Please define the input file structure.'
      print ,'If your input files are in IDRISI structure, type "0"!'
```

```
      print ,'If you import a table(ID; X-Co; Y-Co; Weight),type "1"!'
         read , asc_in

      if (asc_in .eq. 1) then
C        INPUT/OUTPUT FILE NAMES
        print , 'Please define the input and output files.'
        print , 'All file names must be 5 characters long!'
        print , 'Please, do not provide an extention!'
        print , 'input file:'
        print , '(The first line of the file must contain a title!)'
            read (,11) input_file

        open (unit=30, file=input_file, status='old')
            rewind (unit=30)

        print , 'Please, enter the name of the output files!'
        print , 'The IDRISI vector and documentation files will be'
        print , 'generated under the same name; OUTPUT FILES:'
            read (,11) output_file

        idrisi_vec=output_file(1:5)//".vec"
        idrisi_dvc=output_file(1:5)//".dvc"
        idrisi_val=output_file(1:5)//".val"
        idrisi_dvl=output_file(1:5)//".dvl"
        idrpnt_vec=output_file(1:5)//"pnt.vec"
        idrpnt_dvc=output_file(1:5)//"pnt.dvc"
        idrisi_scr=output_file(1:5)//".scr"
        print , 'The following files will be generated:'
            print , idrisi_vec
            print , idrisi_dvc
            print , idrisi_val
            print , idrisi_dvl
            print , idrpnt_vec
            print , idrpnt_dvc
            print , idrisi_scr
        open (unit=32, file=idrisi_vec, status='new')
```

```
      open (unit=34, file=idrpnt_vec, status='new')
      open (unit=35, file=idrisi_dvc, status='new')
      open (unit=36, file=idrpnt_dvc, status='new')
      open (unit=37, file=idrisi_val, status='new')
      open (unit=38, file=idrisi_dvl, status='new')
      open (unit=39, file=idrisi_scr, status='new')
      open (unit=33, status='scratch')

c      Definition of min. and max. values..................
      print , 'Please define the boundary of the plot:'
      print , 'minimum X: '
          read , X0
      print , 'maximum X: '
          read , X1
      print , 'minimum Y: '
          read , Y0
      print , 'maximum Y: '
          read , Y1
c      Definition of NC (number of centers)................
      print , 'Please enter the number of centres!'
          read , NC
      if (NC .lt. 101) goto 9992
          print , 'NC exceeds the current maximum of 100 points!'
          print , 'The program will be terminated!'
c            close files and delete scratch file............
          close (unit=33, status='delete')
          close (unit=30, status='keep')
          close (unit=32, status='delete')
          close (unit=34, status='delete')
          close (unit=35, status='delete')
          close (unit=36, status='delete')
          close (unit=37, status='delete')
          close (unit=38, status='delete')
          close (unit=39, status='delete')
c
          goto 8888
```

```
 9992   continue
       else
C          INPUT/OUTPUT FILE NAMES ............................

         print , 'Please define input and output files. Do not provide'
         print , 'extentions!'
         print , 'The file name must be 8 charaters long!'
         print , 'input point file:'
             read (,12) input_point
         print , 'input values file:'
         print , 'The file name must be 5 charaters long!'
             read (,11) input_value
         print , 'output file:'
         print , 'The file name must be 5 charaters long!'
         print , 'The IDRISI vector and documentation files will be'
         print , 'generated under the same name as the output file.'
             read (,11) output_file

         idrisi_vec=output_file(1:5)//".vec"
         idrisi_dvc=output_file(1:5)//".dvc"
         idr_point=input_point(1:8)//".vec"
         idr_pntdoc=input_point(1:5)//".dvc"
         idr_value=input_value(1:5)//".val"
         idr_valdoc=input_value(1:5)//".dvl"
         idrisi_scr=output_file(1:5)//".scr"
         new_dvl=output_file(1:5)//".dvl"
         print ,'The following files will be generated by the program:'
               print , idrisi_vec
               print , idrisi_dvc
               print , idrisi_scr
               print , new_dvl

         open (unit=40,file=idr_point,status='old')
             rewind (unit=40)
         open (unit=37,file=idr_value,status='old')
             rewind (unit=37)
```

```
      open (unit=39, file=idrisi_scr, status='new')
      open (unit=32, file=idrisi_vec, status='new')
      open (unit=35, file=idrisi_dvc, status='new')
      open (unit=41, file=new_dvl, status='new')
      open (unit=36, file=idr_pntdoc, status='old')
          rewind (unit=36)
      open (unit=38, file=idr_valdoc, status='old')
          rewind (unit=38)
      open (unit=33, status='scratch')
      open (unit=30, status='scratch')
c

      CALL read_doc (NC, X0, X1, Y0, Y1)
      if (NC .lt. 101) goto 9991
        print , 'NC exceeds the current maximum of 100 points'
        print , 'The program will be terminated!'
c         close files and delete scratch file...................
        close (unit=33, status='delete')
        close (unit=30, status='delete')
        close (unit=40, status='keep')
        close (unit=32, status='delete')
        close (unit=35, status='delete')
        close (unit=36, status='keep')
        close (unit=37, status='keep')
        close (unit=38, status='keep')
        close (unit=39, status='delete')
        close (unit=41, status='delete')
c
        goto 8888
 9991 continue
c      read IDRISI point and value files ................
      write (30,11) 'start'
      do 120 I=1,NC
          read (40,) trash1, trash2
          read (40,) xn, yn
          read (37,18) cname, zn
```

```
        open (unit=39, file=idrisi_scr, status='new')
        open (unit=32, file=idrisi_vec, status='new')
        open (unit=35, file=idrisi_dvc, status='new')
        open (unit=41, file=new_dvl, status='new')
        open (unit=36, file=idr_pntdoc, status='old')
            rewind (unit=36)
        open (unit=38, file=idr_valdoc, status='old')
            rewind (unit=38)
        open (unit=33, status='scratch')
        open (unit=30, status='scratch')
c

        CALL read_doc (NC, X0, X1, Y0, Y1)
        if (NC .lt. 201) goto 9991
          print , 'NC exceeds the current maximum of 200 points'
          print , 'The program will be terminated!'
c             close files and delete scratch file...................
          close (unit=33, status='delete')
          close (unit=30, status='delete')
          close (unit=40, status='keep')
          close (unit=32, status='delete')
          close (unit=35, status='delete')
          close (unit=36, status='keep')
          close (unit=37, status='keep')
          close (unit=38, status='keep')
          close (unit=39, status='delete')
          close (unit=41, status='delete')
c
          goto 8888
 9991   continue
c        read IDRISI point and value files ................
        write (30,11) 'start'
        do 120 I=1,NC
            read (40,) trash1, trash2
            read (40,) xn, yn
            read (37,18) cname, zn
```

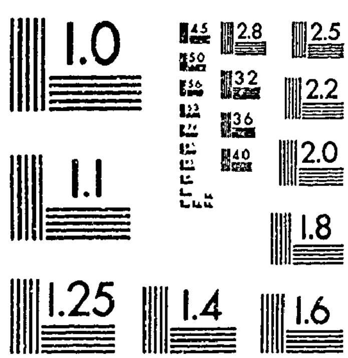
```
          write (30,) cname, xn, yn, zn
  120   continue
        rewind (unit=30)
        rewind (unit=40)
      endif

      FirstLine = 1

C     ......................................................
C       Main part of program EQUAL -
C       only minor changes from original Gambini algorithm
C     ......................................................

 9000 READ (30,13,END=9999) NAME
      READ (30,) (ID(I),XC(I),YC(I),AOLD(I),I=1,NC)
      DO 109 K=1,16
 109        GAM(K)=0.
      GAM(I)=1.0
C       parameters controlling the plot....................
      print,'Please define the precission and the maximal increment'
      print,'for the calculation.'
      print,'Type "0" for the default settings(PRECI=0.08; DMAX=0.2'
      print,'of the X range) or "1" for user defined parameters.'
      print,'A lower value for (=increase in) precission and maximal'
      print,'increment will lengthen the execution time dramaticaly!!'
          read , para
      if (para .eq. 0) then
          PRECI = 0.08
          DMAX = 0.2
      else
          print ,'Please enter the precission (PRECI).'
              read , PRECI
          print ,'Please enter the maximum increment (DMAX).'
          read , DMAX
      endif
```

```
C      ....................................................
C         Weights for Geary or Moran Statistics
C      ....................................................

      print ,'Please, select a transformation for the weights:'
      print ,'   Planar, normal Voronoi Diagram                  =  0'
      print ,'   Weighted Voronoi Diagram:  No Tranformation  =  1'
      print ,'   Weighted Voronoi Diagram:  Geary Statistics  =  2'
      print ,'   Weighted Voronoi Diagram:  Moran Statistics  =  3'
           read , weight

      if (weight.eq.0) then
           do 133 I=1,NC
                  A(I) = 10
 133       continue
           title = 'Planar Voronoi Diagram'
      elseif (weight.eq.1) then
           do 147 I=1,NC
                  A(I) = AOLD(I)
 147       continue
           title = 'Weighted Voronoi Diagram'
      elseif (weight.eq.2) then
           title = 'Geary Transformation'
           do 701 I=1,NC
                  A(I)= AOLD(I)  AOLD(I)
 701       continue
      else
 711       continue
C      ..................................................
C       e = constant = 1 + absoulte of AMAXDEV
C       ADEV = deviation from the mean
C       AMAXDEV = value of the maximum negative
C                 deviation of the mean
C      ..................................................
           title='Moran Transformation'
           SUMA=0.0
```

```
          do 704 I=1,NC
                SUMA=SUMA+AOLD(I)
 704      continue
          ABAR=SUMA/NC
          AMAXDEV = 0.0
          do 706 I=1,NC
                ADEV(I) = (AOLD(I)-ABAR)
                AMAXDEV = MIN (AMAXDEV,ADEV(I))
 706      continue
          AMAXDEV = ABS (AMAXDEV)
          e = 0.001 + AMAXDEV
          do 713 I=1,NC
                A(I) = e + AOLD(I) - ABAR
 713      continue
      endif
C
      FAC=3.1415927/180.
      KK=1
 7000 GAMMA=GAM(KK)
      II=0
      XMAP=(X0+X1)/2.
      YMAP=(Y0+Y1)/2.
      RHO=SQRT((X1-XMAP)2+(Y1-YMAP)2)
      IF(GAMMA.EQ.1)GOTO 90
   90 NS=NC-1
      DO 1000 I=1,NS
        NFC=I+1
        DO 900 J=NFC,NC
          IF(A(I).NE.A(J)) GOTO 300
C.... ....................................STRAIGHT LINE
          AM=XC(J)-XC(I)
          BM=YC(J)-YC(I,
          DEN=SQRT(AM2+BM2)
          AX=(XC(I)+XC(J))/2.
          AY=(YC(I)+YC(J))/2.
          BX=-BM/DEN
```

2 of/de 2

PM-1 3½"x4" PHOTOGRAPHIC MICROCOPY TARGET
NBS 1010a ANSI/ISO #2 EQUIVALENT



```
      BY=AM/DEN
      NSTEP=IFIX(4.RHO/DMAX)
      USTEP=DMAX
      ITEST=2
      U=-2.RHO
C.... .....................  ............STEP PROCESS
      DO 600 ISL=1,NSTEP
            U=U+USTEP
            X=AX+BXU
            Y=AY+BYU
            IP=ITEST
            ITEST=IFUNC(X,Y)
            IF(ITEST.NE.IP)GOTO 211
            IF(ITEST.EQ.2)GOTO 600
            GOTO 216
C.... ..................................RESEARCH END LINE
  211       UEXC=U
            UDEF=U-USTEP
  212       UR=(UDEF+UEXC)/2.
            XR=AX+BXUR
            YR=AY+BYUR
            ERROR=ABS(UDEF-UEXC)
            IF(ERROR.LT.PRECI) GOTO 214
            IROOT=IFUNC(XR,YR)
            IF(IROOT.EQ.IP)GOTO 213
            UEXC=UR
            GOTO 212
  213       UDEF=UR
            GOTO 212
  214       NX=ITEST
            call line_file (NX, XR, YR, FirstLine, counter)
            II=II+1
            IF(ITEST.EQ.2)GOTO 600
  216       NX=2
            call line_file (NX, X, Y, FirstLine, counter)
            II=II+1
```

```
 600      CONTINUE
          GOTO 900
C.... ....................................CIRCLE
 300      XDE=(A(I)XC(J)-A(J)XC(I))/(A(I)-A(J))
          XDI=(A(I)XC(J)+A(J)XC(I))/(A(I)+A(J))
          YDE=(A(I)YC(J)-A(J)YC(I))/(A(I)-A(J))
          YDI=(A(I)YC(J)+A(J)YC(I))/(A(I)+A(J))
          XCENT=(XDE+XDI)/2.
          YCENT=(YDE+YDI)/2.
          R=SQRT((XDE-XCENT)2+(YDE-YCENT)2)
          DISTC=SQRT((XCENT-XMAP)2+(YCENT-YMAP)2)
          IF(DISTC.LT.RHO)GOTO 98
C.... ..........................................BIG CIRCLE
          RATIO=RHO/SQRT(DISTC2-RHO2)
          HALFR=ATAN(RATIO)
          DIFFX=XMAP-XCENT
          DIFFY=YMAP-YCENT
          IF(DIFFX.EQ.0)GOTO 50
          RATIO=DIFFY/DIFFX
          TMEAN=ATAN(RATIO)
          IF(DIFFX.LT.O.)TMEAN=TMEAN+3.1415927
  49      THETA=TMEAN-HALFR
          STEP=DMAX/R
          NSTEP=IFIX(2.HALFR/STEP)+1
          GOTO 99
  50      IF((YMAP-YCENT).GT.0)TMEAN=3.1415927/2.
          IF((YMAP-YCENT).LT.0)TMEAN=-3.1415927/2.
          GOTO 49
C.... .......................................NORMAL CIRCLE
  98      THETA=0.
          STEP=5.FAC
          IF(RSTEP.GT.DMAX)STEP=DMAX/R
          NSTEP=IFIX(360.FAC/STEP)
  99      ITEST=2
C.... .......................................STEP PROCESS
          DO 800 K=1,NSTEP
```

```
            IP=ITEST
            THETA=THETA+STEP
            X=XCENT+RCOS(THETA)
            Y=YCENT+RSIN(THETA)
            ITEST=IFUNC(X,Y)
            IF(ITEST.NE.IP)GOTO 111
            IF(ITEST.EQ.2)GOTO 800
            GOTO 116
C.... .......................................RESEARCH END LINE
  111       TEXC=THETA
            TDEF=THETA-STEP
  112       ROOT=(TEXC+TDEF)/2.
            XR=XCENT+RCOS(ROOT)
            YR=YCENT+RSIN(ROOT)
            ERROR=R(TEXC-TDEF)
            IF(ERROR.LT.PRECI)GOTO 114
            IROOT=IFUNC(XR,YR)
            IF(IROOT.EQ.IP)GOTO 113
            TEXC=ROOT
            GOTO 112
  113       TDEF=ROOT
            GOTO 112
  114       NX=ITEST
            call line_file (NX, XR, YR, FirstLine, counter)
            II=II+1
            IF(ITEST.EQ.2)GOTO 800
  116       NX=2
            call line_file (NX, X, Y, FirstLine, counter)
            II=II+1
  800     CONTINUE
  900   CONTINUE
 1000 CONTINUE
C       II - number of coordinates calculated  ..................

 2000 IF(KK.EQ.16)GOTO 9000
      KK=KK+1
```

```
      IF(GAM(KK).EQ.0)GOTO 9000
      GOTO 7000
 9999 continue

c     ......................................................
c       Writing the last line segment out of the scratch file
c       to the IDRISI vector line file
c     ......................................................
      write (32,) '6   ',NINT( counter)
      rewind( unit=33)
 9987 read(33,,end=9988) x,y
             write(32,) x,y
             goto 9987
 9988 continue

c     ......................................................
c      IDRISI script file for plotting
c
c       unit 39 = IDRISI Version 4.0 script file;
c       meta command file for PLOT module; displays the weighted
c       Voronoi Diagrams and the location and names of the
c       generating centers.
c       col = pen color, eg. col=1 - pen one, etc.
c       height = charater height in percent of the page height
c       font = 1 = Triplex Stroke
c              2 = SansSerif Stroke
c              3 = Small font Stroke
c            - For the use with a plotter, font=3 is recommended
c     ......................................................
c
      height = 0.025
      col=2
      font=3
      write (39,60) 'f u',col,output_file
      col=1
      do 68 I=1,NC
```

```
         write(39,71)'t w',font,col,height,' 0 0',XC(I),YC(I),AOLD(I)
   68 continue

      do 69 I=1,NC
         write (39,62) 'm w',XC(I),YC(I),' 2 1',col
   69 continue
C       draw outline .........................................
      col = 2
      write (39,63) 'b',col
C

C       min. and max. weights ..................................
      A0 = 10000000
      A1 = -10000000
      do 166 I=1,NC
         A0 = MIN (A0,AOLD(I))
         A1 = MAX (A1,AOLD(I))
  166 continue
      if (asc_in .eq. 1) then
C             IDRISI point and value file .......................
         do 121 I=1,NC
              write (34,16) ID(I)
              write (34,17) XC(I), YC(I)
              write (37,18) ID(I), AOLD(I)
  121    continue

         CALL doc0 (title, NC, X0, X1, Y0, Y1, A0, A1)

C             end mark for the IDRISI vector files ..............
         write (32,) '0   0'
         write (34,) '0   0'
C             close files and delete scratch files  .............
         close (unit=33, status='delete')
         close (unit=30, status='keep')
         close (unit=32, status='keep')
         close (unit=34, status='keep')
```

```
           close (unit=35, status='keep')
           close (unit=36, status='keep')
           close (unit=37, status='keep')
           close (unit=38, status='keep')
           close (unit=39, status='keep')
c
      else

           CALL doc1 (title, NC, X0, X1, Y0, Y1, A0, A1)

c           end mark for the IDRISI vector file .................
           write (32,) '0   0'
c            close files and delete scratch files ...............
           close (unit=33, status='delete')
           close (unit=30, status='delete')
           close (unit=32, status='keep')
           close (unit=35, status='keep')
           close (unit=36, status='keep')
           close (unit=37, status='keep')
           close (unit=38, status='keep')
           close (unit=39, status='keep')
           close (unit=40, status='keep')
           close (unit=41, status='keep')
c
      endif
 8888 continue
      STOP
      END

      FUNCTION IFUNC(X,Y)
      DIMENSION XC(100),YC(100),A(100),ATTR(100)
      COMMON A,XC,YC,X0,X1,Y0,Y1,I,J,NC
      IF((X.GT.X1).OR.(X.LT.X0))GOTO 300
      IF((Y.GT.Y1).OR.(Y.LT.Y0))GOTO 300
      DO 100 L=1,NC
      IF((X.EQ.XC(L)).AND.(Y.EQ.YC(L)))GOTO 300
```

```
      ATTR(L)=A(L)/SQRT((X-XC(L))2+(Y-YC(L))2)
  100 CONTINUE
      DO 200 M=1,NC
      IF((M.EQ.I).OR.(M.EQ.J))GOTO 200
      IF(ATTR(I).LT.ATTR(M))GOTO 300
  200 CONTINUE
      IFUNC=3
      GOTO 400
  300 IFUNC=2
  400 RETURN
      END

C        .........................................................
C        This subroutine produces an IDRISI Version 4.0 Vector
C        line file. Together with the documentation file it
C        can be used to display and print the multiplicativly
C        weighted Voronoi Diagram to a HPGL capable plotter
C        or a HP laser printer.
C        ........................................................

      SUBROUTINE line_file (z, x, y, FirstLine, counter)

      INTEGER z, FirstLine, counter
      REAL x, y, xfirst, yfirst


      if (z .eq. 3) then
            if (FirstLine .eq. 1) then
                 counter = 1
                 write (33,) x,y
                 FirstLine=0
            else
                 xfirst = x
                 yfirst = y
                 write (32,) '6    ',NINT( counter)
                 rewind (unit=33)
```

```
9990            do 10000 I=1,counter
                   read(33,) x,y
                   write(32,) x,y
10000           continue
                rewind (unit=33)
                counter = 1
                write (33,) xfirst, yfirst
             endif
      else
             counter = counter + 1
             write(33,) x,y
      endif
      end


C       ..................................................
C       This subroutine reads values out of the IDRISI
C       point- and value-documentation files (units 36 and
C       38) and returns them to the main program.
C       ..................................................

      SUBROUTINE read_doc (NCENT, XMINI, XMAXI, YMINI, YMAXI)

      REAL XMINI, XMAXI, YMINI, YMAXI
      INTEGER NCENT
      CHARACTER14 junk

   75 FORMAT (A13,1X,F12.7)
   77 FORMAT (A13,1X,I5)
   81 FORMAT (A34)

      do 142 I=1,7
          read (36,81)
  142 continue
      read (36,75) junk, XMINI
      read (36,75) junk, XMAXI
```

```
      read (36,75) junk, YMINI
      read (36,75) junk, YMAXI
      rewind (unit=36)

      do 143 I=1,2
          read (38,81)
 143  continue
      read (38,77) junk, NCENT
      rewind (unit=38)
      return
      end

c       .....................................................
c       Definition of the IDRISI vector documentation files
c        unit 35 = vector line file documentation
c        unit 36 = vector point file documentation
c        unit 38 = values file documentation
c       .....................................................

      SUBROUTINE doc0 (title,NCENT,XMINI,XMAXI,YMINI,YMAXI,AMIN,AMAX)

      REAL dist, AMIN, AMAX
      CHARACTER8 ref_sy, units
      CHARACTER24 title
      INTEGER NCENT

      dist = 1.
      print , 'Select the reference system!'
      print , '(type either plane, utm or lat/long)'
           read (,70) ref_sy
      print , 'Select the reference unit dimension!'
      print , '(m for metres; km for kilometres; ft for feet)'
           read (,71) units

   70 FORMAT (A8)
   71 FORMAT (A2)
```

```
73 FORMAT (A14,A8)
75 FORMAT (A14,F11.7)
76 FORMAT (A14,F9.7)
77 FORMAT (A14,I5)
82 FORMAT (A13,1X,A24)
83 FORMAT (A23)
84 FORMAT (A19)
85 FORMAT (A18)
86 FORMAT (A21)
87 FORMAT (A15)
88 FORMAT (A25)
     write (35,82) 'file title  : ',title
     write (35,86) 'id type     : integer'
     write (35,84) 'file type   : ascii'
     write (35,85) 'object type : line'
     write (35,73) 'ref. system : ',ref_sy
     write (35,73) 'ref. units  : ',units
     write (35,76) 'unit dist.  : ',dist
     write (35,75) 'min. X      : ',XMINI
     write (35,75) 'max. X      : ',XMAXI
     write (35,75) 'min. Y      : ',YMINI
     write (35,75) 'max. Y      : ',YMAXI
     write (35,86) 'pos_n error : unknown'
     write (35,86) 'resolution  : unknown'
   rewind (unit=35)
     write (36,82) 'file title  : ',title
     write (36,86) 'id type     : integer'
     write (36,84) 'file type   : ascii'
     write (36,85) 'object type : line'
     write (36,73) 'ref. system : ',ref_sy
     write (36,73) 'ref. units  : ',units
     write (36,76) 'unit dist.  : ',dist
     write (36,75) 'min. X      : ',XMINI
     write (36,75) 'max. X      : ',XMAXI
     write (36,75) 'min. Y      : ',YMINI
     write (36,75) 'max. Y      : ',YMAXI
```

```
          write (36,86) 'pos_n error : unknown'
          write (36,86) 'resolution  : unknown'
        rewind (unit=36)
          write (38,82) 'file title  : ',title
          write (38,84) 'file type   : ascii'
          write (38,77) 'records     : ',NCENT
          write (38,87) 'fields      : 2'
          write (38,88) 'field 0     : identifiers'
          write (38,86) 'data type   : integer'
          write (38,87) 'format      : 0'
          write (38,86) 'field 1     : weights'
          write (38,85) 'data type   : real'
          write (38,87) 'format      : 0'
          write (38,75) 'min. value  : ',AMIN
          write (38,75) 'max. value  : ',AMAX
          write (38,88) 'value units : unspecified'
          write (38,86) 'value error : unknown'
          write (38,85) 'flag value  : none'
          write (38,85) 'flag def_n  : none'
          write (38,87) 'legend cats : 0'
        rewind (unit=38)
        end

c         .......................................................
c         Definition of the IDRISI vector documentation file
c         unit 35 = vector line file documentation
c         unit 41 = new values file documentation
c         .......................................................

        SUBROUTINE docl (title,NCENT,XMINI,XMAXI,YMINI,YMAXI,AMIN,AMAX)

        REAL dist, XMINI, XMAXI, YMINI, YMAXI, AMIN, AMAX
        INTEGER NCENT
        CHARACTER24 title
        CHARACTER14 junk
        CHARACTER8 ref_sys
```

```
      CHARACTER4 units

   73 FORMAT (A13,1X,A8)
   74 FORMAT (A13,1X,A4)
   75 FORMAT (A13,1X,F12.7)
   76 FORMAT (A13,1X,F9.7)
   77 FORMAT (A13,I5)
   81 FORMAT (A34)
   82 FORMAT (A13,1X,A24)
   83 FORMAT (A23)
   84 FORMAT (A19)
   85 FORMAT (A18)
   86 FORMAT (A21)
   87 FORMAT (A15)
   88 FORMAT (A25)

      dist = 1.0
      do 144 I=1,4
           read (36,81)
144   continue
      read (36,73) junk, ref_sys
      read (36,74) junk, units
      rewind (unit=36)
        write (35,82) 'file title  : ',title
        write (35,86) 'id type     : integer'
        write (35,84) 'file type   : ascii'
        write (35,85) 'object type : line'
        write (35,73) 'ref. system : ',ref_sys
        write (35,73) 'ref. units  : ',units
        write (35,76) 'unit dist.  : ',dist
        write (35,75) 'min. X      : ',XMINI
        write (35,75) 'max. X      : ',XMAXI
        write (35,75) 'min. Y      : ',YMINI
        write (35,75) 'max. Y      : ',YMAXI
        write (35,86) 'pos_n error : unknown'
        write (35,86) 'resolution  : unknown'
```

```
rewind (unit=35)
  write (41,82) 'file title  : ',title
  write (41,84) 'file type   : ascii'
  write (41,77) 'records     : ',NCENT
  write (41,87) 'fields      : 2'
  write (41,88) 'field 0     : identifiers'
  write (41,86) 'data type   : integer'
  write (41,87) 'format      : 0'
  write (41,86) 'field 1     : weights'
  write (41,85) 'data type   : real'
  write (41,87) 'format      : 0'
  write (41,75) 'min. value  : ',AMIN
  write (41,75) 'max. value  : ',AMAX
  write (41,88) 'value units : unspecified'
  write (41,86) 'value error : unknown'
  write (41,85) 'flag value  : none'
  write (41,85) 'flag def_n  : none'
  write (41,87) 'legend cats : 0'
rewind (unit=41)
end
```

Appendix III: Readme file; VORONOI.

README FILE - VORONOI

Program description:

This program creates the planar, normal Voronoi diagram (as a function of distance between points; all weights are equal), the multiplicatively weighted Voronoi diagram and Geary and Moran transformations for the visualization of spatial autocorrelation for a set of data points in R^2. Due to limitations of the compiler (64K-barrier) the maximum number of data points is currently set to 100.

To run type: 'voronoi'.

Input Options:

The input file can either be an ASCII file of 4 columns where,

column 1 = point ID
column 2 = X-Coordinate
column 3 = Y-Coordinate
column 4 = point attribute value,

or IDRISI vector and value files in ASCII format.

Calculation options:

This program allows four options for the calculation. To generate:

- the ordinary Voronoi diagram	type '0'
- the multiplicatively weighted Voronoi diagram	type '1'
- the Geary transformation or	type '2'
- the Moran transformation	type '3'

Output options:

Make sure that your IDRISI environment is set correctly!
To display type:

'plot x 1 <filename> idrisi 1 0' - for screen output
'plot x 2 <filename> idrisi 1 0' - for HPGL-plotter output
'plot x 3 <filename> idrisi 1 0' - for HP-LaserJet output

for immediate display, or type:

'plot' and follow the interactive menu.

If option 2 is used, make sure that the com port is set correctly.

Example to set com1, type:

'mode com1:9600,n,8,1,'.

It is highly recommended to become familiar with the IDRISI "PLOT" command and the structure of IDRISI script files.

Interpretation of the Geary and Moran transformations:

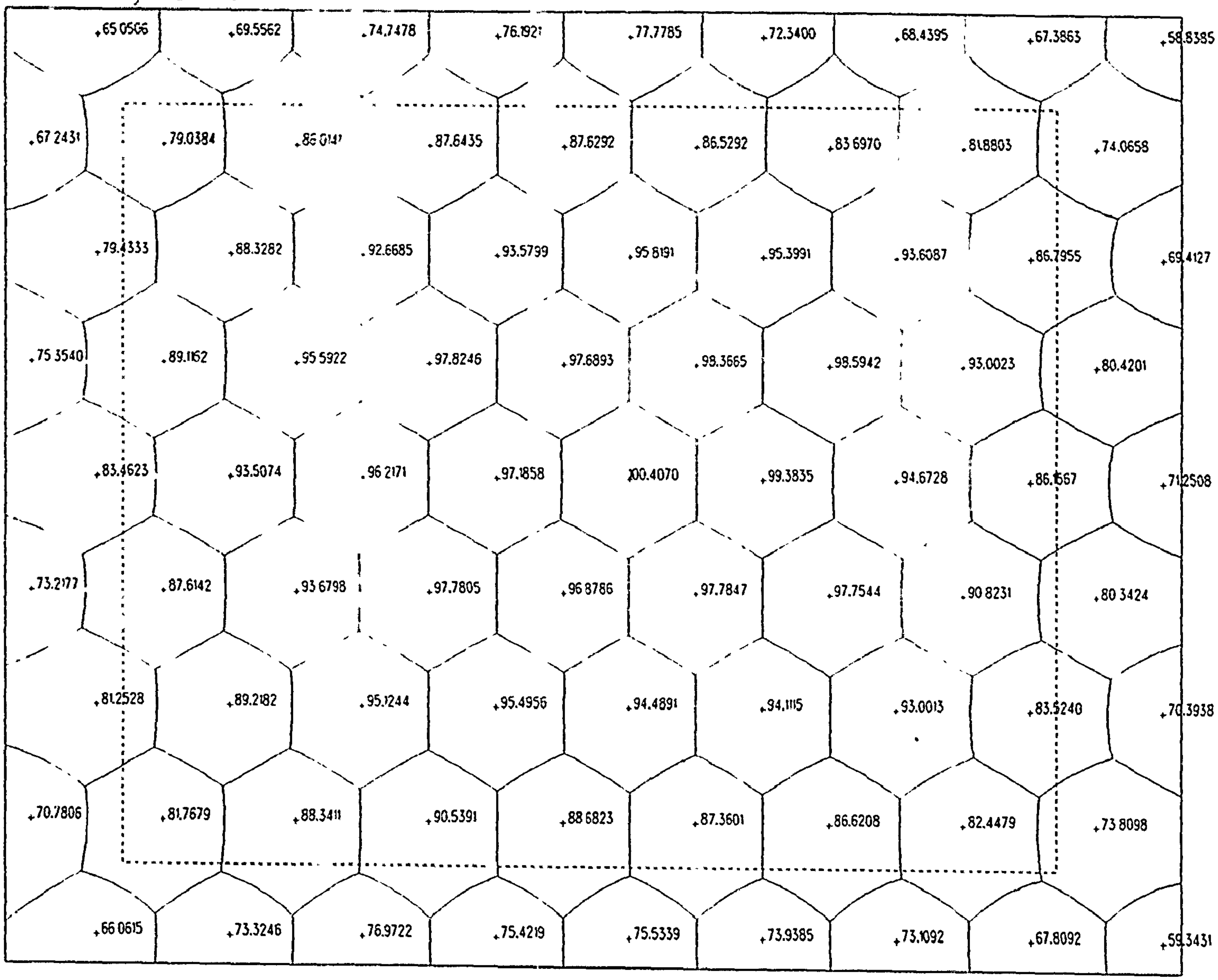
Plot the visualizations for the Geary and the Moran transformation against the ordinary Voronoi diagram. The smaller the differences from the ordinary Voronoi diagram are, the higher the degree of positive spatial autocorrelation in the point data set. With an increase in the curvature of the bisectors, negative spatial autocorrelation increases.

Example for script file with ord. VD, Geary and Moran transformations:

```
f u 1 <ord. VD>
f u 2 <Geary>
f u 3 <Moran>
b 1
```

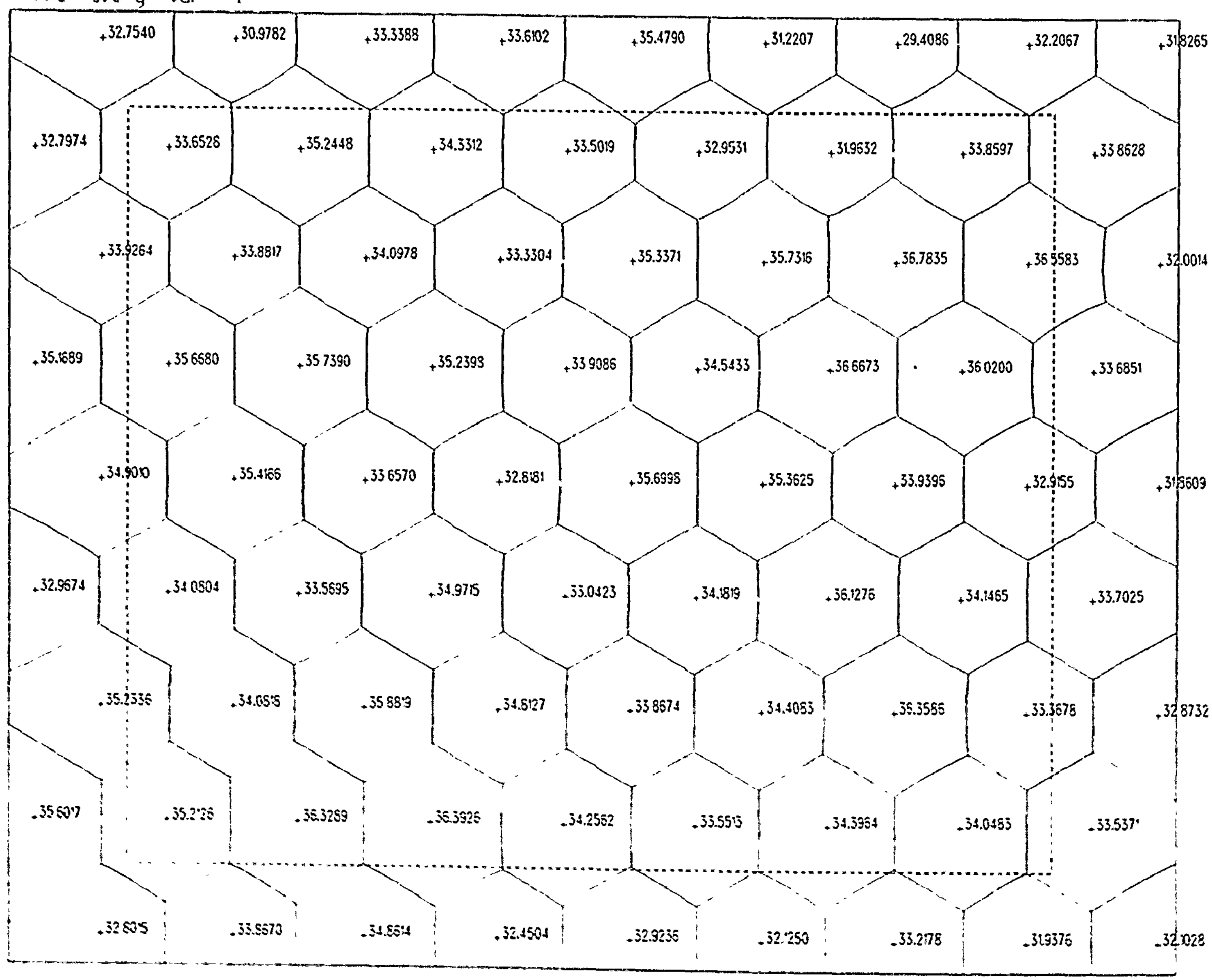
Appendix IV: Visualization results.

rho 0.9 g Var = 1



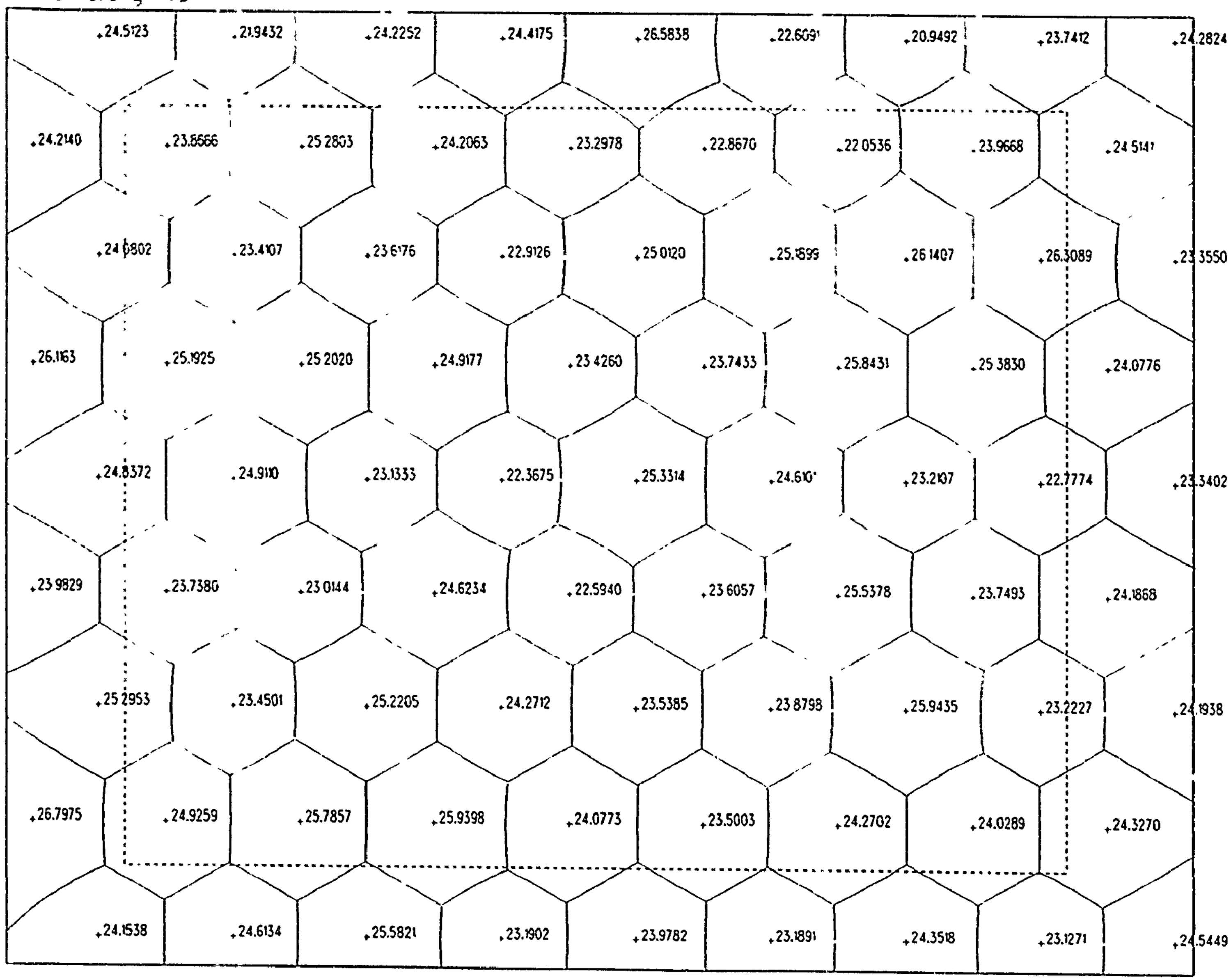
65.0506
69.5562
74.7478
76.1921
77.7785
72.3400
68.4395
67.3863
58.8385
67.2431
79.0384
86.0147
87.6435
87.6292
86.5292
83.6970
81.8803
74.0658
79.4333
88.3282
92.6685
93.5799
95.6191
95.3991
93.6087
86.7955
69.4127
75.3540
89.1162
95.5922
97.8246
97.6893
98.3665
98.5942
93.0023
80.4201
83.4623
93.5074
96.2171
97.1858
100.4070
99.3835
94.6728
86.1567
71.2508
73.2177
87.6142
93.6798
97.7805
96.8786
97.7847
97.7544
90.8231
80.3424
81.2528
89.2182
95.1244
95.4956
94.4891
94.1115
93.0013
83.5240
70.3938
70.7806
81.7679
88.3411
90.5391
88.6823
87.3601
86.6208
82.4479
73.8098
66.0615
73.3246
76.9722
75.4219
75.5339
73.9385
73.1092
67.8092
59.3431

rho 0.6 g Var = 1

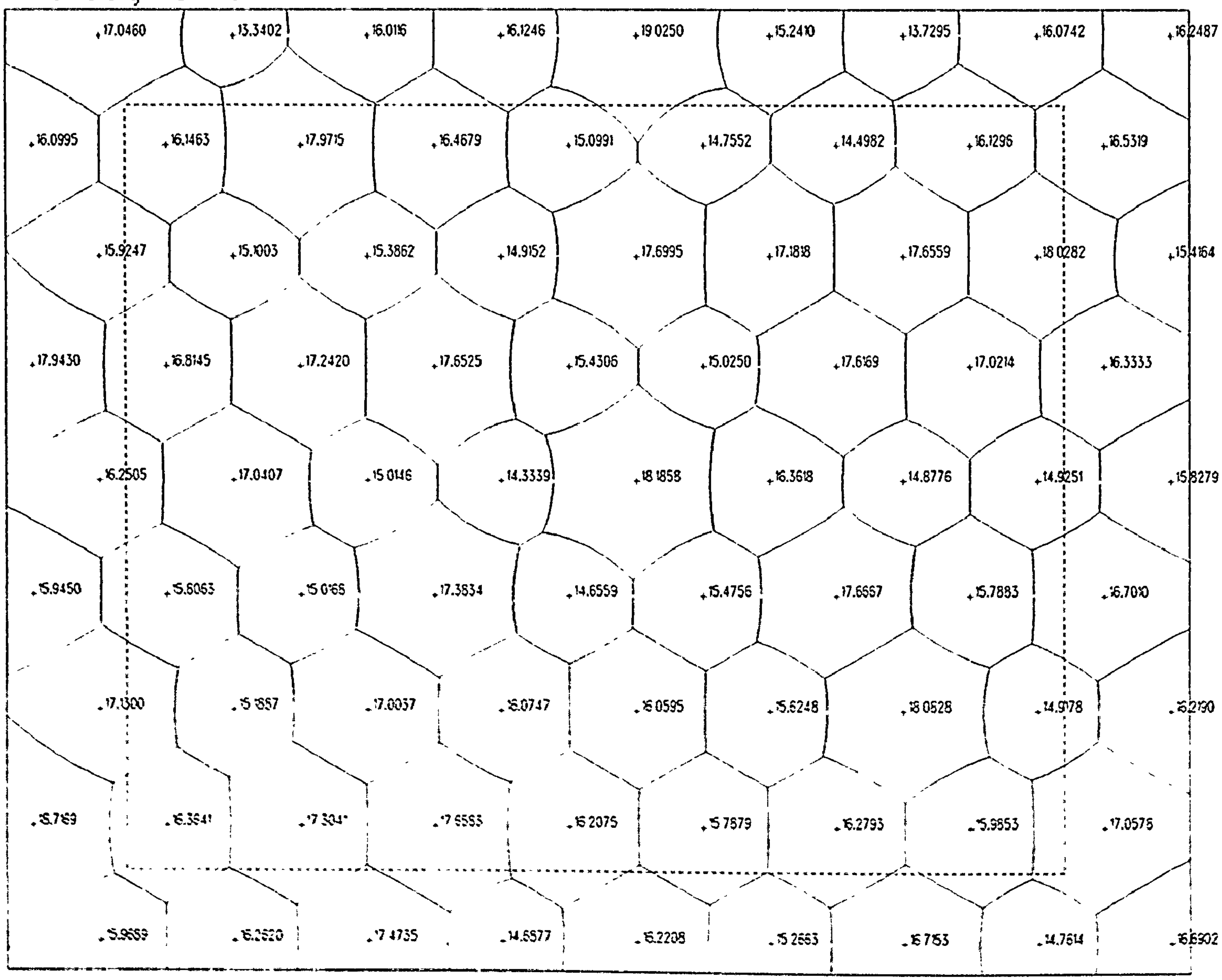


32.7540
30.9782
33.3388
33.6102
35.4790
31.2207
29.4086
32.2067
32.7974
33.6526
35.2448
34.3312
33.5019
32.9531
31.9632
33.8597
33.8628
33.9264
33.8817
34.0978
33.3304
35.3371
35.7316
36.7835
32.0014
35.1689
35.6680
35.7390
35.2393
33.9086
34.5433
36.6673
36.0200
33.6851
34.9010
35.4166
33.6570
32.8181
35.6998
35.3625
33.9396
32.9155
32.9674
34.0504
33.5695
34.9715
33.0423
34.1819
36.1276
34.1465
33.7025
35.2336
34.8127
33.8674
34.4083
35.3586
33.3678
32.8732
36.3289
36.3926
34.2562
33.5513
34.3964
34.0483
33.5570
34.8614
32.4504
32.9236
33.2178
31.9376

rho 0.3 g Var

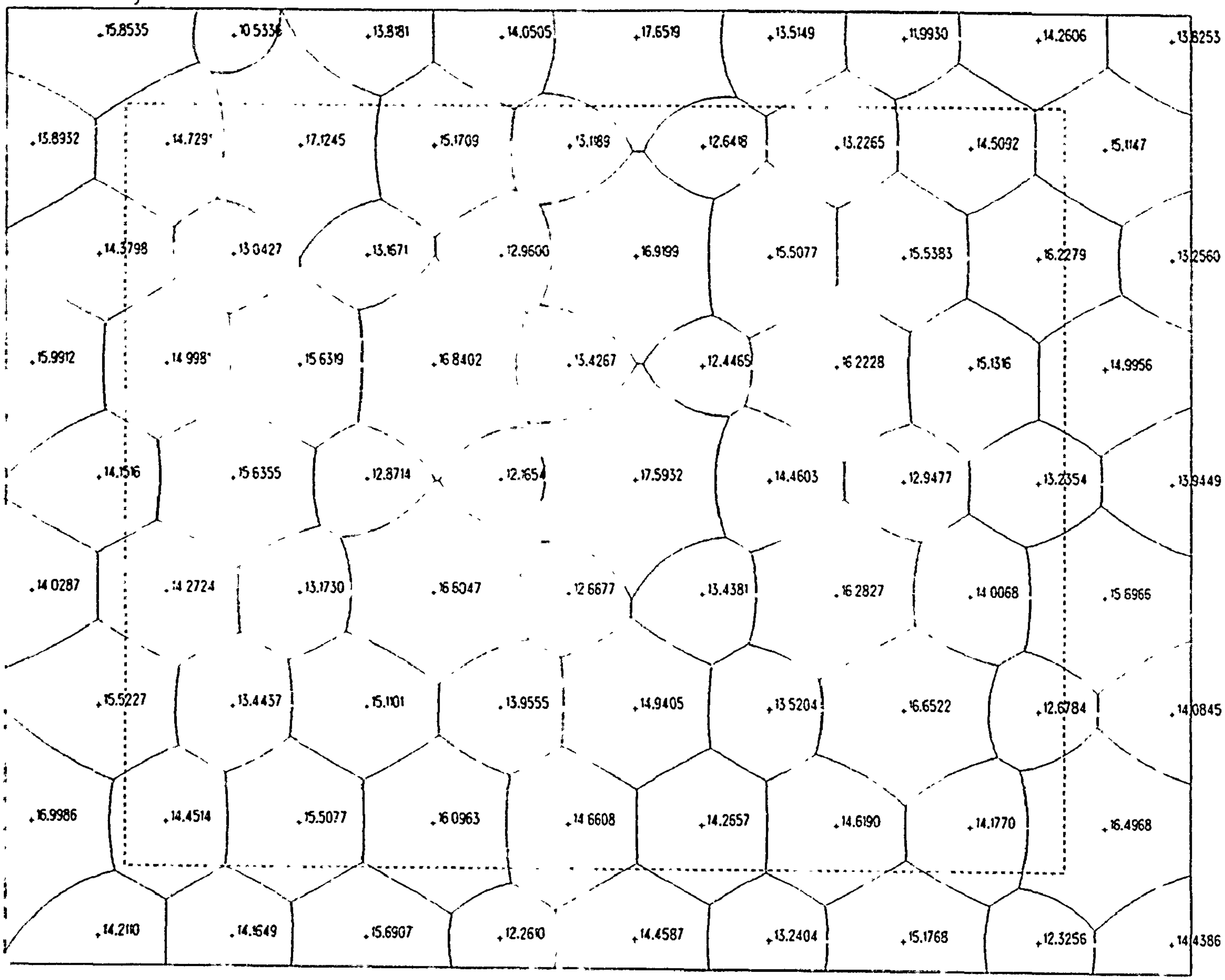

24.5123 21.9432 24.2252 24.4175 26.5838 22.609 20.9492 23.7412 24.2824
24.2140 23.8666 25.2803 24.2063 23.2978 22.8670 22.0536 23.9668 24.5141
24.0802 23.4107 23.6176 22.9126 25.0120 25.1899 26.1407 26.3089 23.3550
26.1163 25.1925 25.2020 24.9177 23.4260 23.7433 25.8431 25.3830 24.0776
24.8372 24.9110 23.1333 22.3675 25.3314 24.610 23.2107 22.7774 23.3402
23.9829 23.7380 23.0144 24.6234 22.5940 23.6057 25.5378 23.7493 24.1868
25.2953 23.4501 25.2205 24.2712 23.5385 23.8798 25.9435 23.2227 24.1938
26.7975 24.9259 25.7857 25.9398 24.0773 23.5003 24.2702 24.0289 24.3270
24.1538 24.6134 25.5821 23.1902 23.9782 23.1891 24.3518 23.1271 24.5449

rho -0.6 g Var = 1

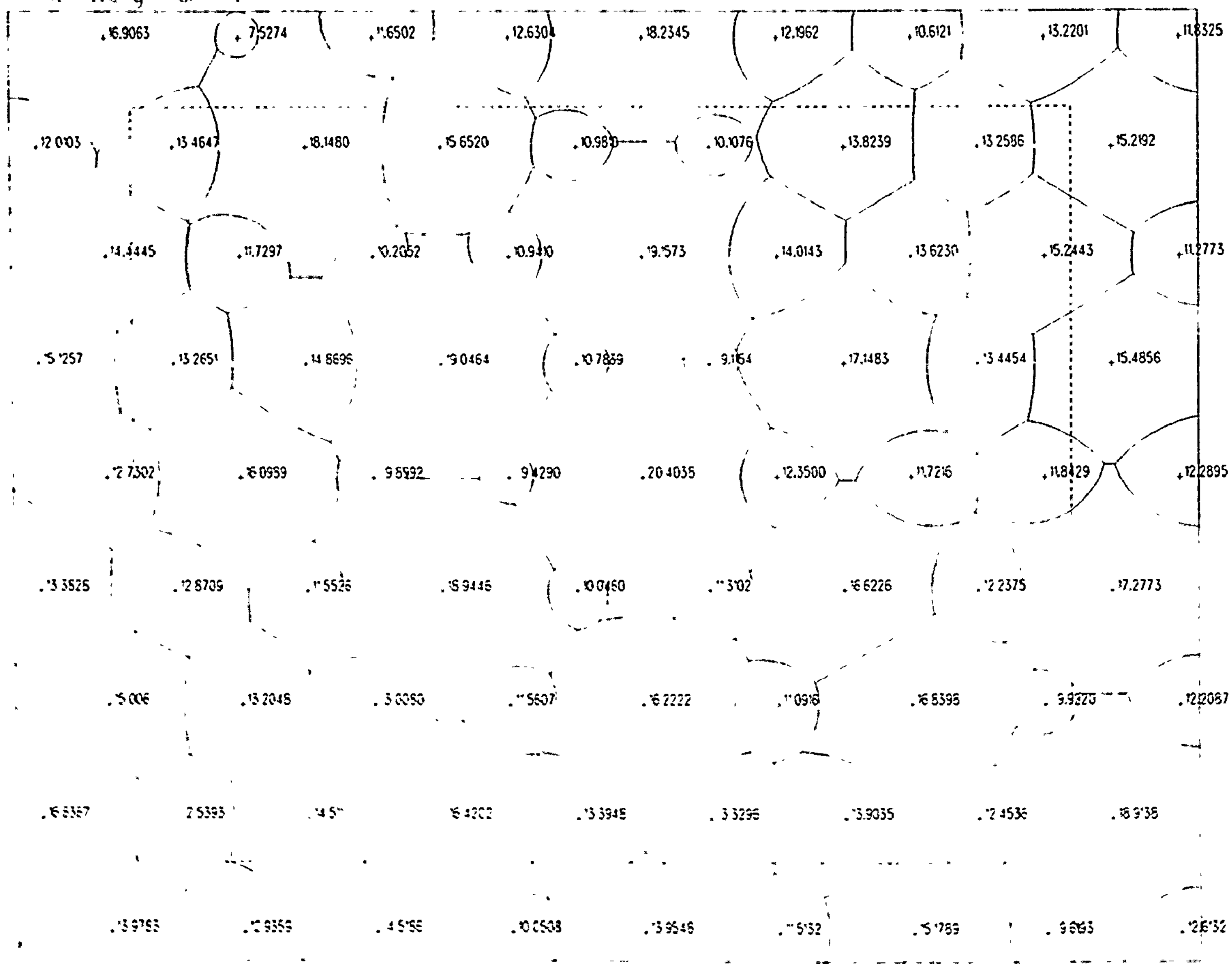


17.0460
13.3402
16.0115
16.1246
19.0250
15.2410
13.7295
16.0742
16.2487
16.0995
16.1463
17.9715
16.4679
15.0991
14.7552
14.4982
16.1296
16.5319
15.9247
15.1003
15.3862
14.9152
17.6995
17.1818
17.6559
18.0282
15.4164
17.9430
16.8145
17.2420
17.6525
15.4306
15.0250
17.6169
17.0214
16.3333
16.2505
17.0407
15.0146
14.3339
18.1858
16.3618
14.8776
14.9251
15.8279
15.9450
15.6063
15.0165
17.3834
14.6559
15.4756
17.6667
15.7883
16.7010
17.1300
17.0037
16.0747
16.0595
15.6248
18.0528
14.9178
16.2190
16.3641
16.2075
15.7879
16.2793
15.9853
17.0575
15.9659
14.6577
16.2208
15.2663
14.7614

rho -1.2 g Var = 1

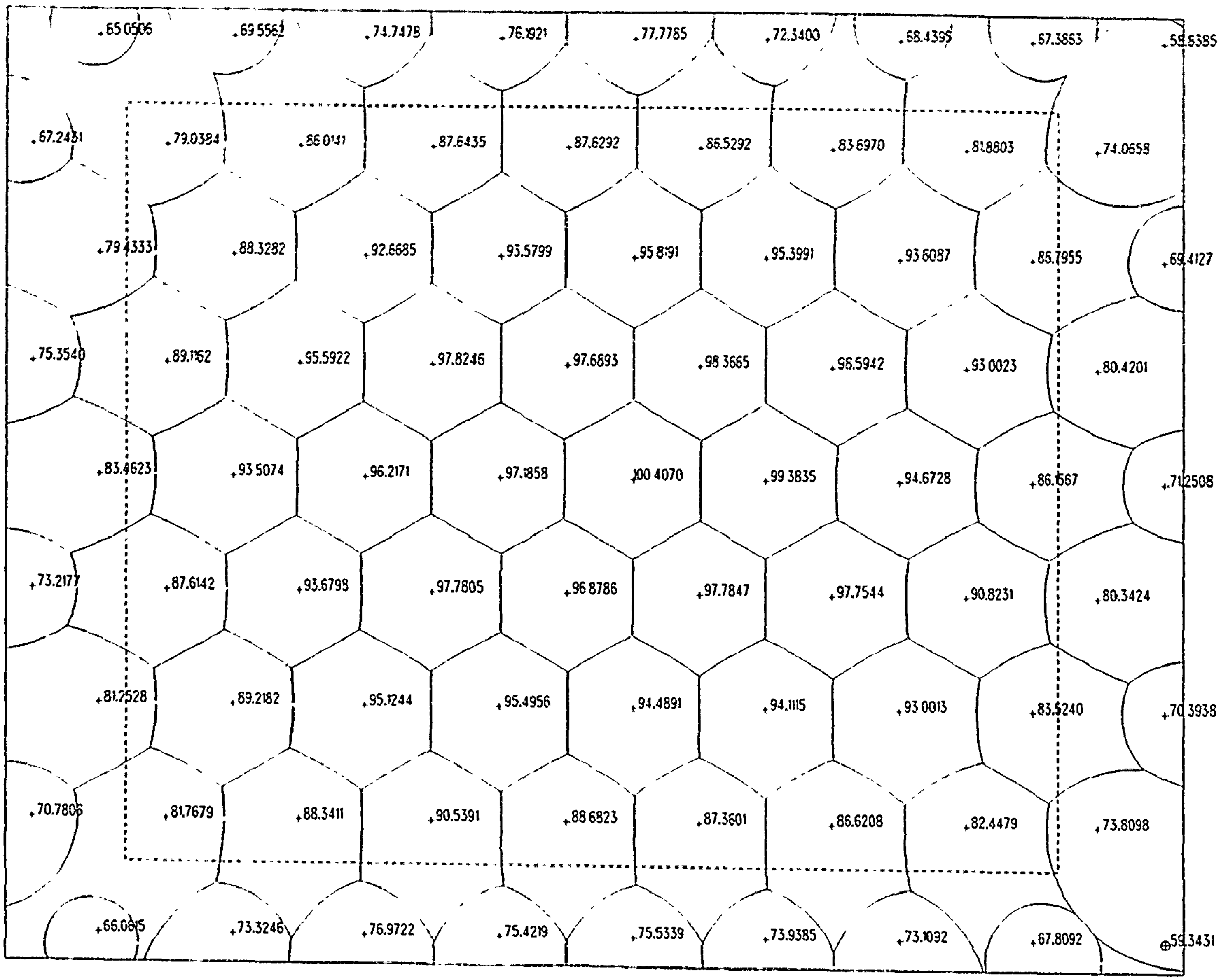

15.8535
10.5336
13.8181
14.0505
17.6519
13.5149
11.9930
14.2606
13.8253
13.8932
14.729
17.1245
15.1709
13.1189
12.6418
13.2265
14.5092
15.1147
14.3798
13.0427
13.1671
12.9600
16.9199
15.5077
15.5383
16.2279
13.2560
15.9912
14.998
15.6319
16.8402
13.4267
12.4465
16.2228
15.1316
14.9956
14.1516
15.6355
12.8714
12.1654
17.5932
14.4603
12.9477
13.2354
13.9449
14.0287
14.2724
13.1730
16.6047
12.6677
13.4381
16.2827
14.0068
15.6966
15.5227
13.4437
15.1101
13.9555
14.9405
13.5204
16.6522
12.6784
14.0845
16.9986
14.4514
15.5077
16.0963
14.6608
14.2657
14.6190
14.1770
16.4968
14.2110
14.1649
15.6907
12.2610
14.4587
13.2404
15.1768
12.3256
14.4386

ıho 1.8 g ''ar = 1



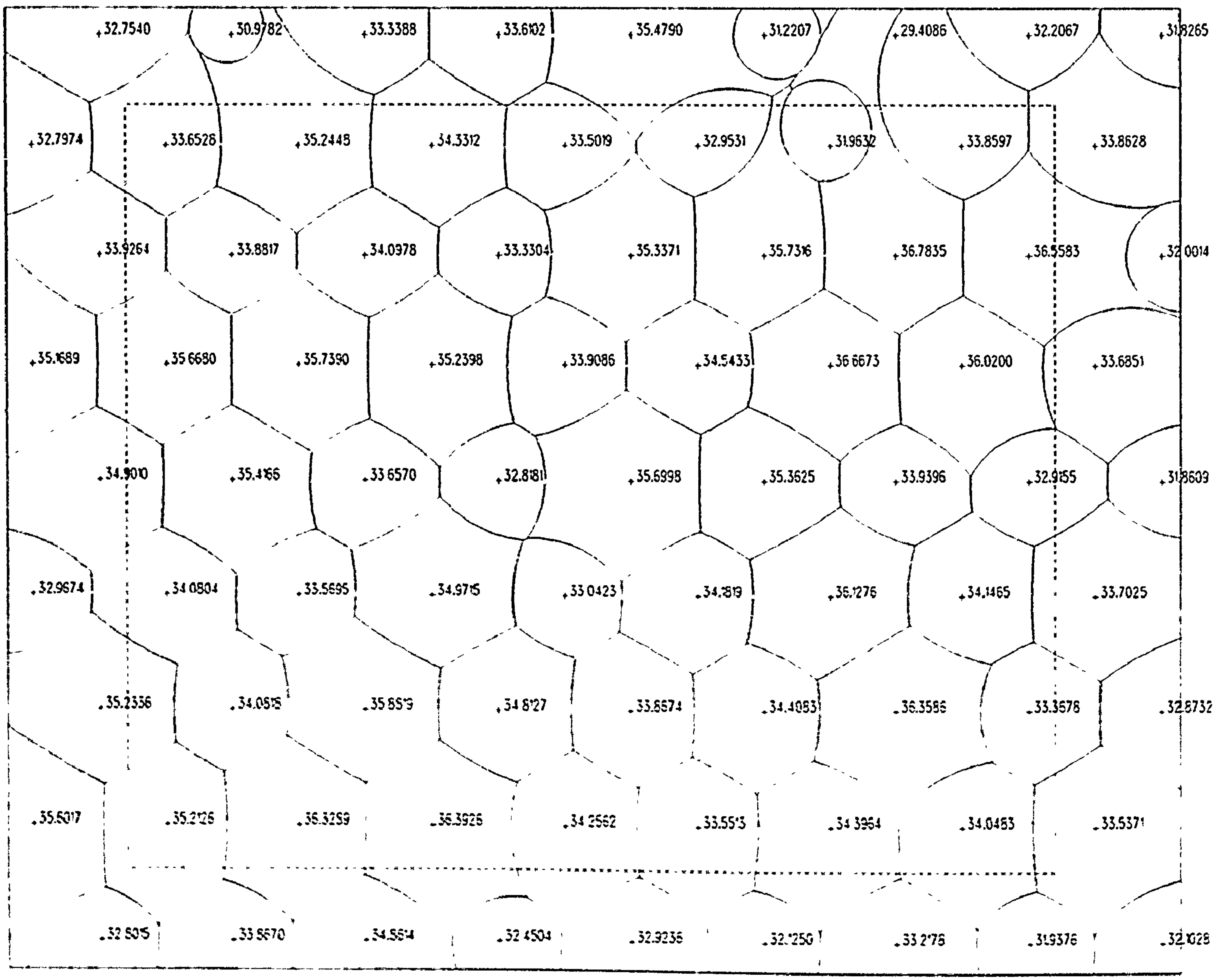
16.9063
7.5274
12.6304
18.2345
12.1962
10.6121
13.2201
11.8325
12.0103
13.4647
18.1480
15.6520
10.9830
10.1076
13.8239
13.2586
15.2192
14.4445
11.7297
10.2052
10.9410
14.0143
13.6230
15.2443
11.2773
13.2651
14.8696
10.7859
9.1164
17.1483
13.4454
15.4856
16.0959
9.5592
9.4290
20.4035
12.3500
11.7216
11.8429
12.2895
12.8709
10.0460
17.2773
13.2045
12.2087

rho 0.9 m Var = 1

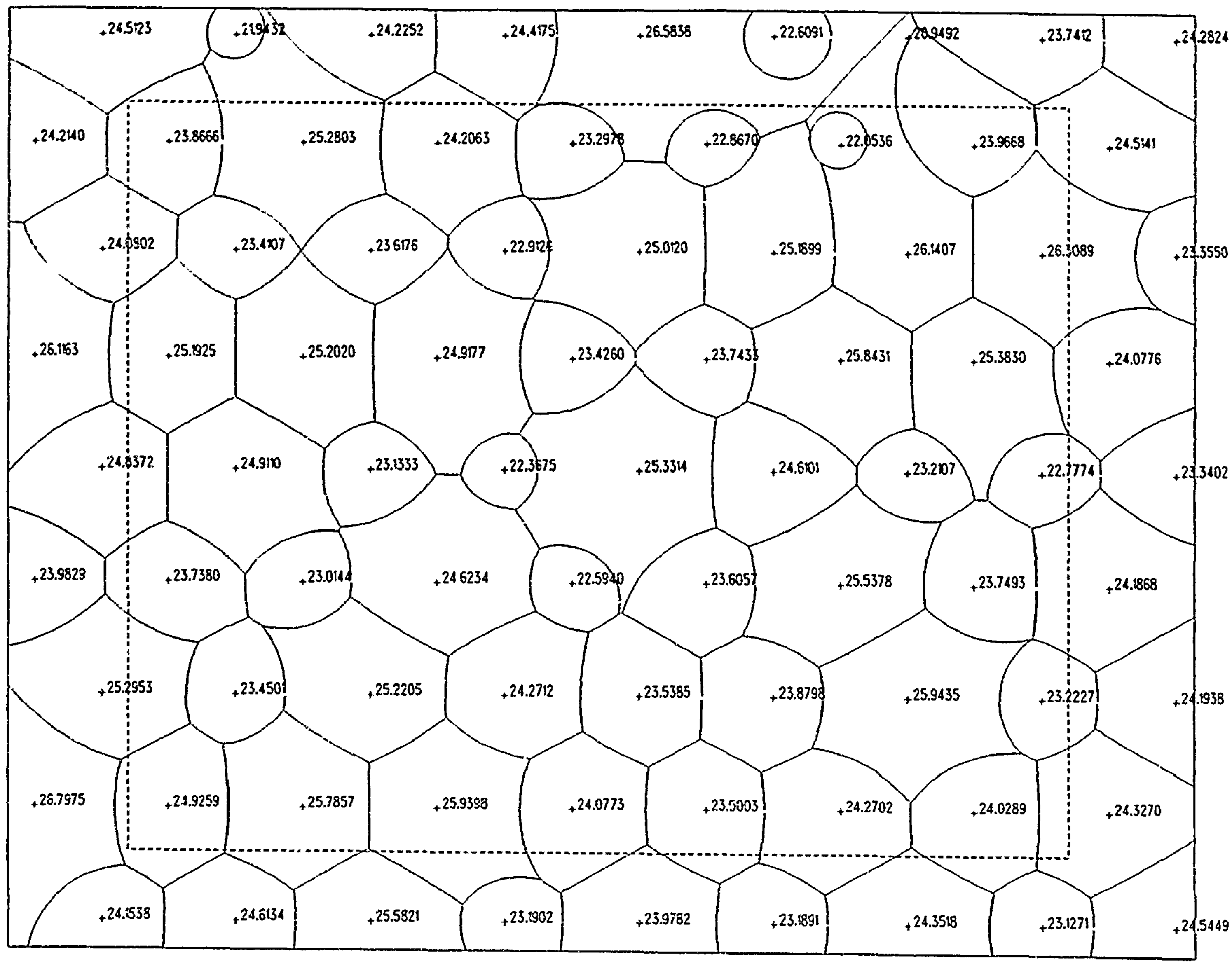


65.0506
69.5562
74.7478
76.1921
77.7785
72.3400
68.4395
67.3853
59.6385
67.2431
79.0384
86.0141
87.6435
87.6292
86.5292
83.6970
81.8803
74.0658
79.4333
88.3282
92.6685
93.5799
95.8191
95.3991
93.6087
86.7955
69.4127
75.3540
83.1152
95.5922
97.8246
97.6893
98.3665
96.5942
93.0023
80.4201
83.4623
93.5074
96.2171
97.1858
100.4070
99.3835
94.6728
86.1567
71.2508
73.2177
87.6142
93.6798
97.7805
96.8786
97.7847
97.7544
90.8231
80.3424
81.7528
89.2182
95.1244
95.4956
94.4891
94.1115
93.0013
83.5240
70.3938
70.7806
81.7679
88.3411
90.5391
88.6823
87.3601
86.6208
82.4479
73.8098
66.0615
73.3246
76.9722
75.4219
75.5339
73.9385
73.1092
67.8092
59.3431

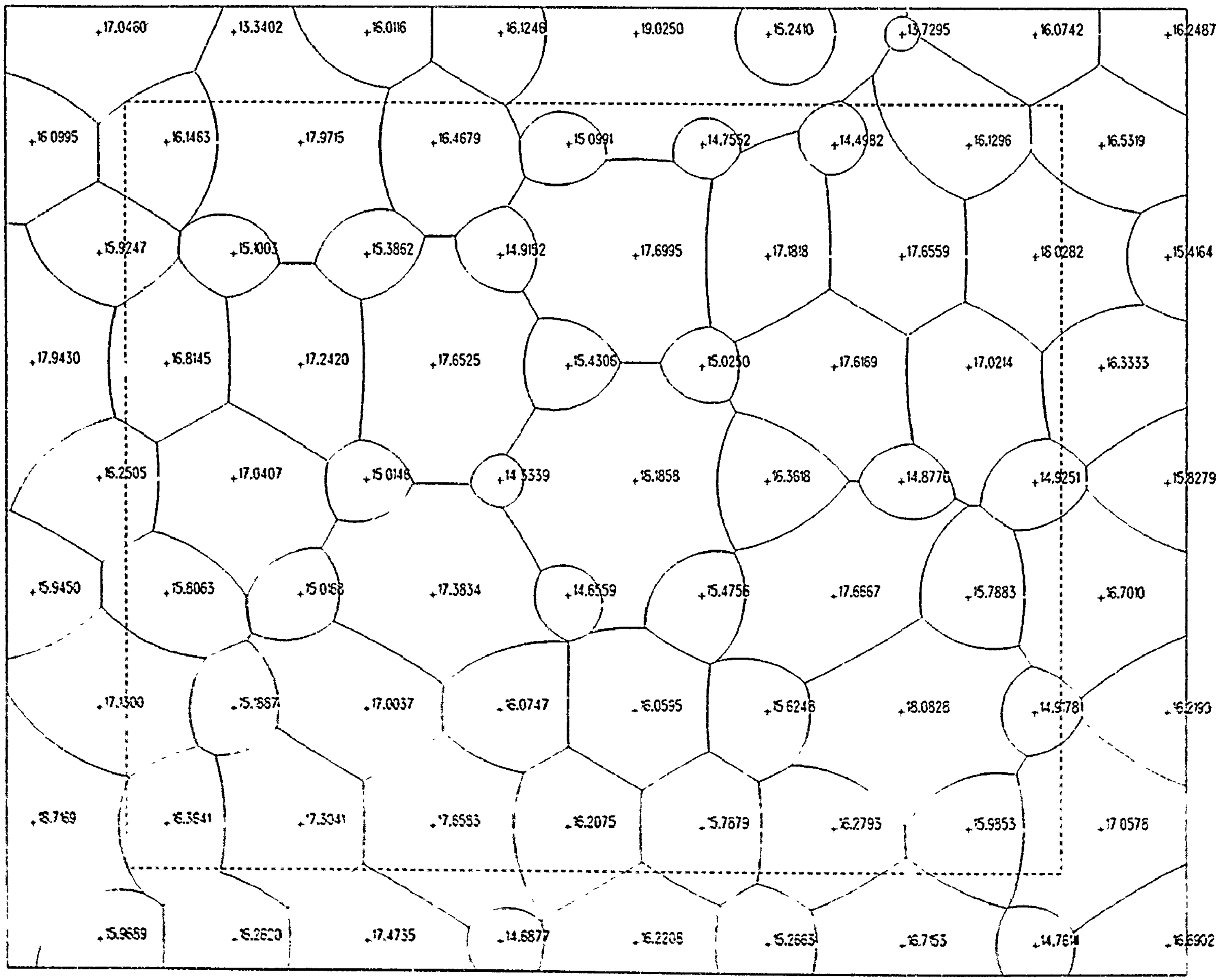
rho 0.6 m Var = 1


32.7540 30.9782 33.3388 33.6102 35.4790 31.2207 29.4086 32.2067 31.8265
32.7974 33.6528 35.2448 34.3312 33.5019 32.9531 31.9632 33.8597 33.8628
33.9264 33.8817 34.0978 33.3304 35.3371 35.7316 36.7835 36.5583 32.0014
35.1689 35.6680 35.7390 35.2398 33.9086 34.5433 36.6673 36.0200 33.6851
34.9010 35.4166 33.6570 32.8181 35.6998 35.3625 33.9396 32.9155 31.8609
32.9674 34.0804 33.5695 34.9715 33.0423 34.7819 36.1276 34.1465 33.7025
35.2336 34.0515 35.8559 34.8127 33.8674 34.4083 36.3586 33.3676 32.8732
35.5017 35.2126 36.3259 36.3926 34.2562 33.5513 34.3954 34.0483 33.5371
32.5015 33.5670 34.5514 32.4504 32.9236 32.1250 33.2176 31.9376 32.1028

rho 0.3 m Var = 1


24.5123
21.9432
24.2252
24.4175
26.5838
22.6091
20.9492
23.7412
24.2824
24.2140
23.8666
25.2803
24.2063
23.2978
22.8670
22.0536
23.9668
24.5141
24.0902
23.4107
23.6176
22.9124
25.0120
25.1699
26.1407
26.3089
23.3550
26.1163
25.1925
25.2020
24.9177
23.4260
23.7433
25.8431
25.3830
24.0776
24.8372
24.9110
23.1333
22.3675
25.3314
24.6101
23.2107
22.7774
23.3402
23.9829
23.7380
23.0144
24.6234
22.5940
23.6057
25.5378
23.7493
24.1868
25.2953
23.4501
25.2205
24.2712
23.5385
23.8798
25.9435
23.2227
24.1938
26.7975
24.9259
25.7857
25.9398
24.0773
23.5003
24.2702
24.0289
24.3270
24.1538
24.6134
25.5821
23.1902
23.9782
23.1891
24.3518
23.1271
24.5449

rho -0.6 m Var = 1


17.0460
13.3402
16.0116
16.1248
19.0250
15.2410
13.7295
16.0742
16.2487
16.0995
16.1463
17.9715
16.4679
15.0991
14.7552
14.4982
16.1296
16.5319
15.9247
15.1003
15.3862
14.9152
17.6995
17.1818
17.6559
18.0282
15.4164
17.9430
16.8145
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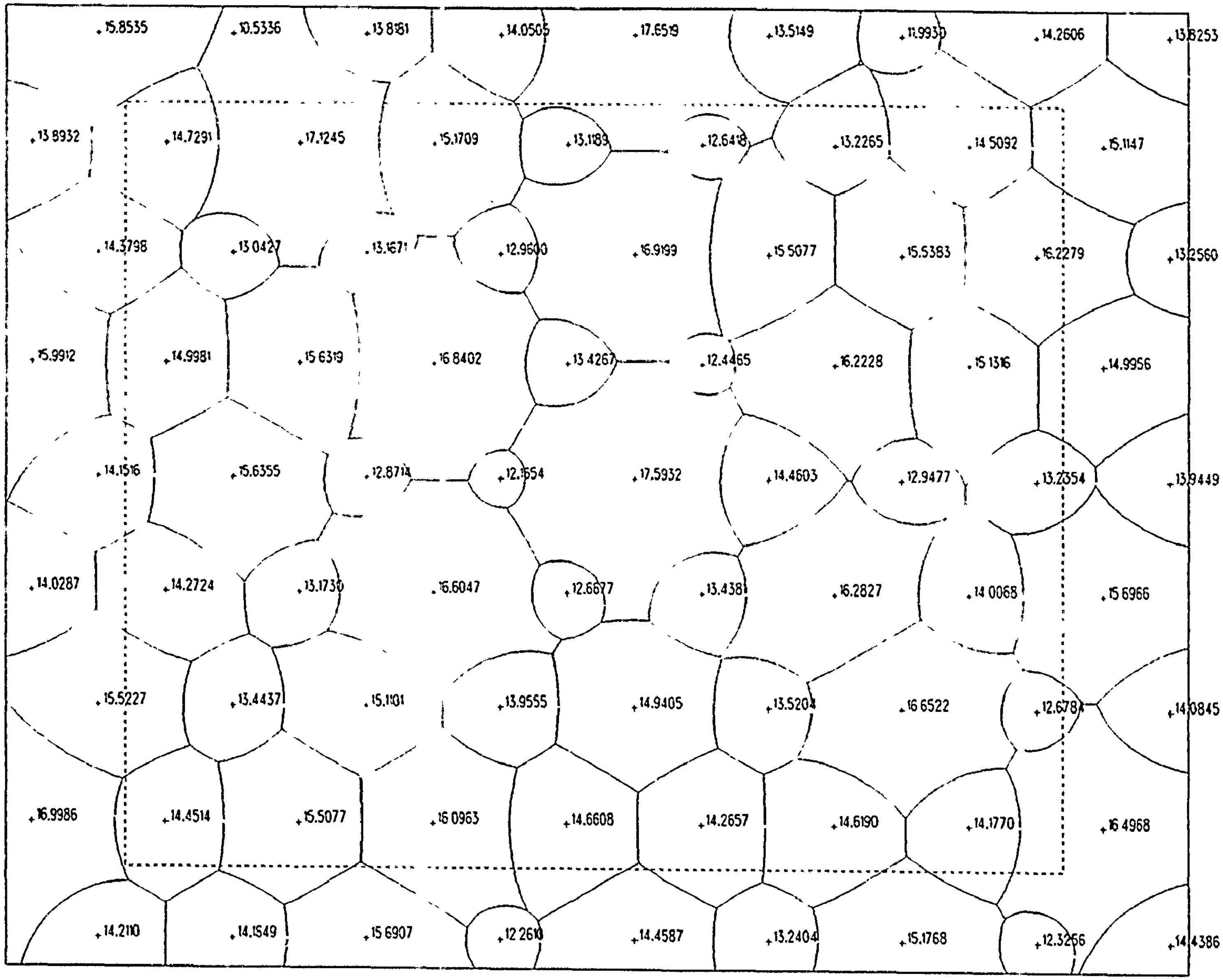
rho 1.2 m Var = 1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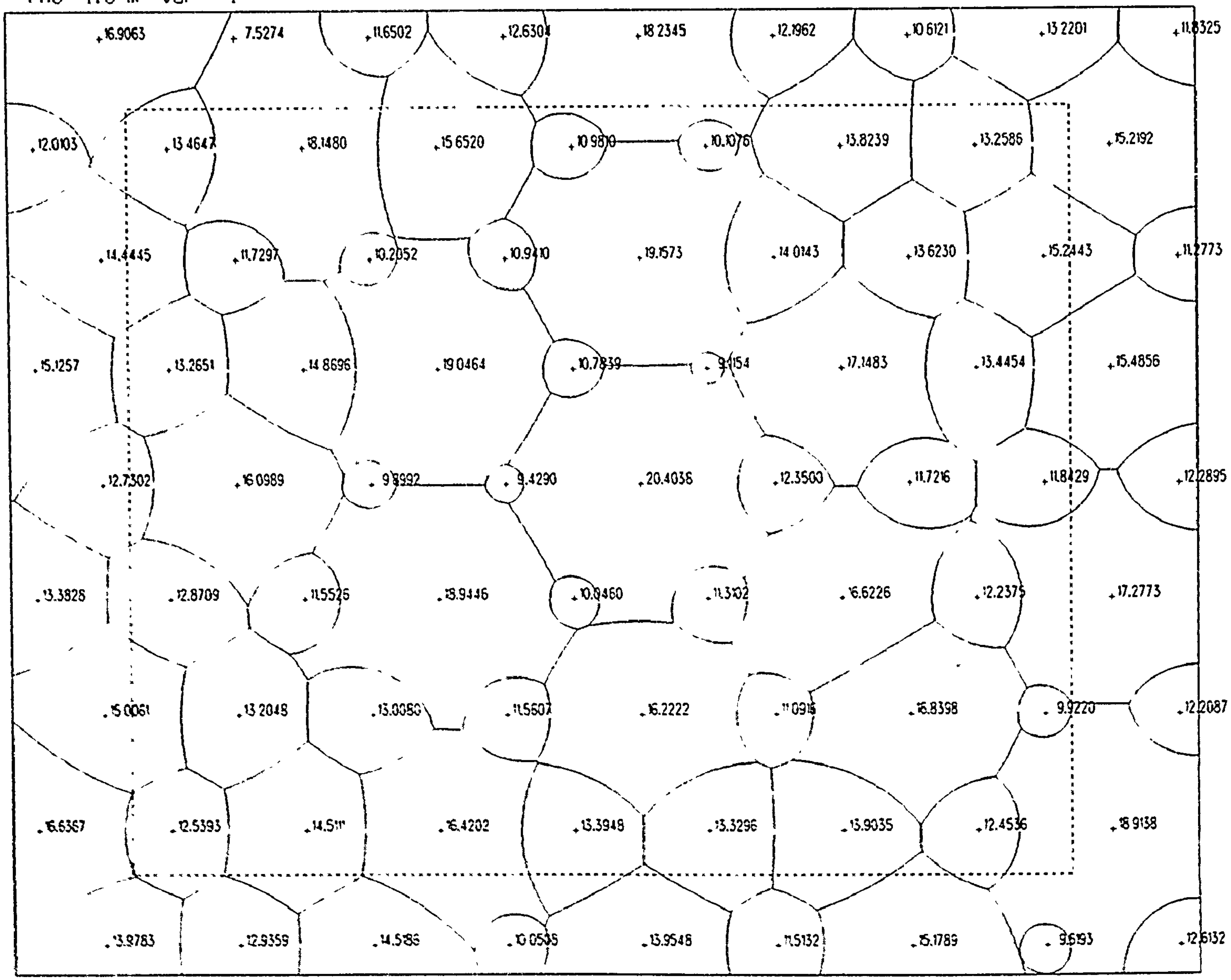
rho -1.8 m Var = 1



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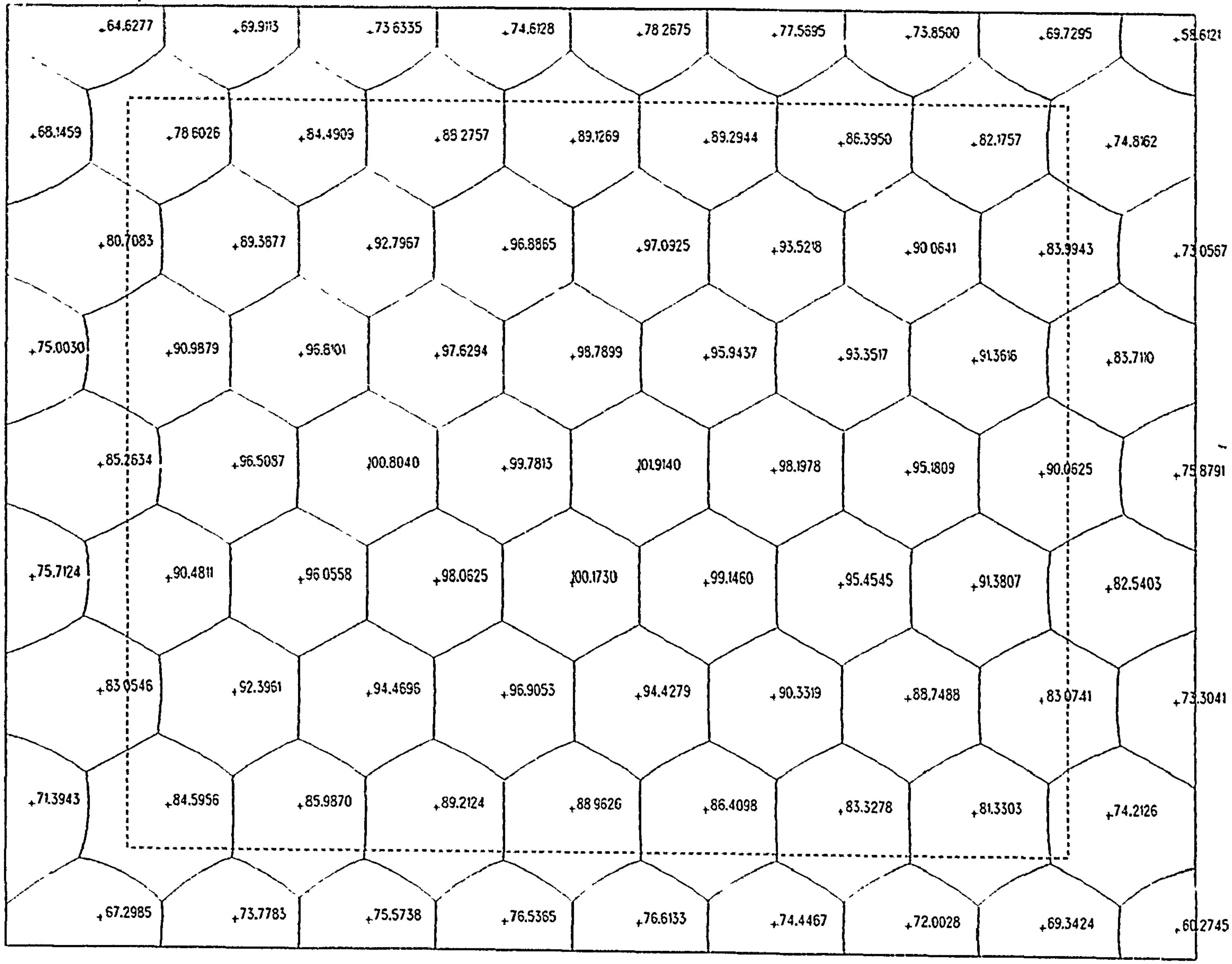
rho 0.99 Var = 2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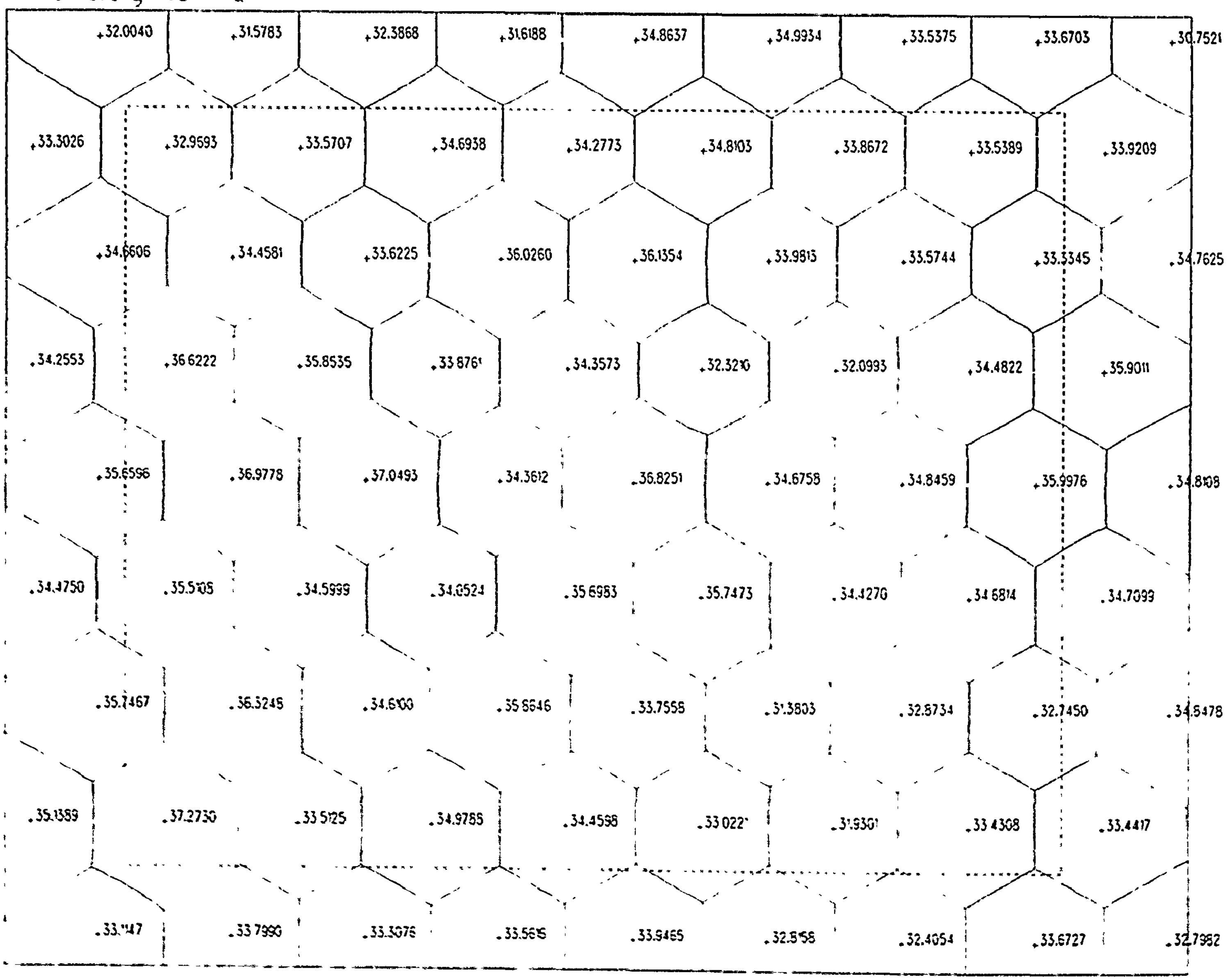
rho 0.6 g Var = 2



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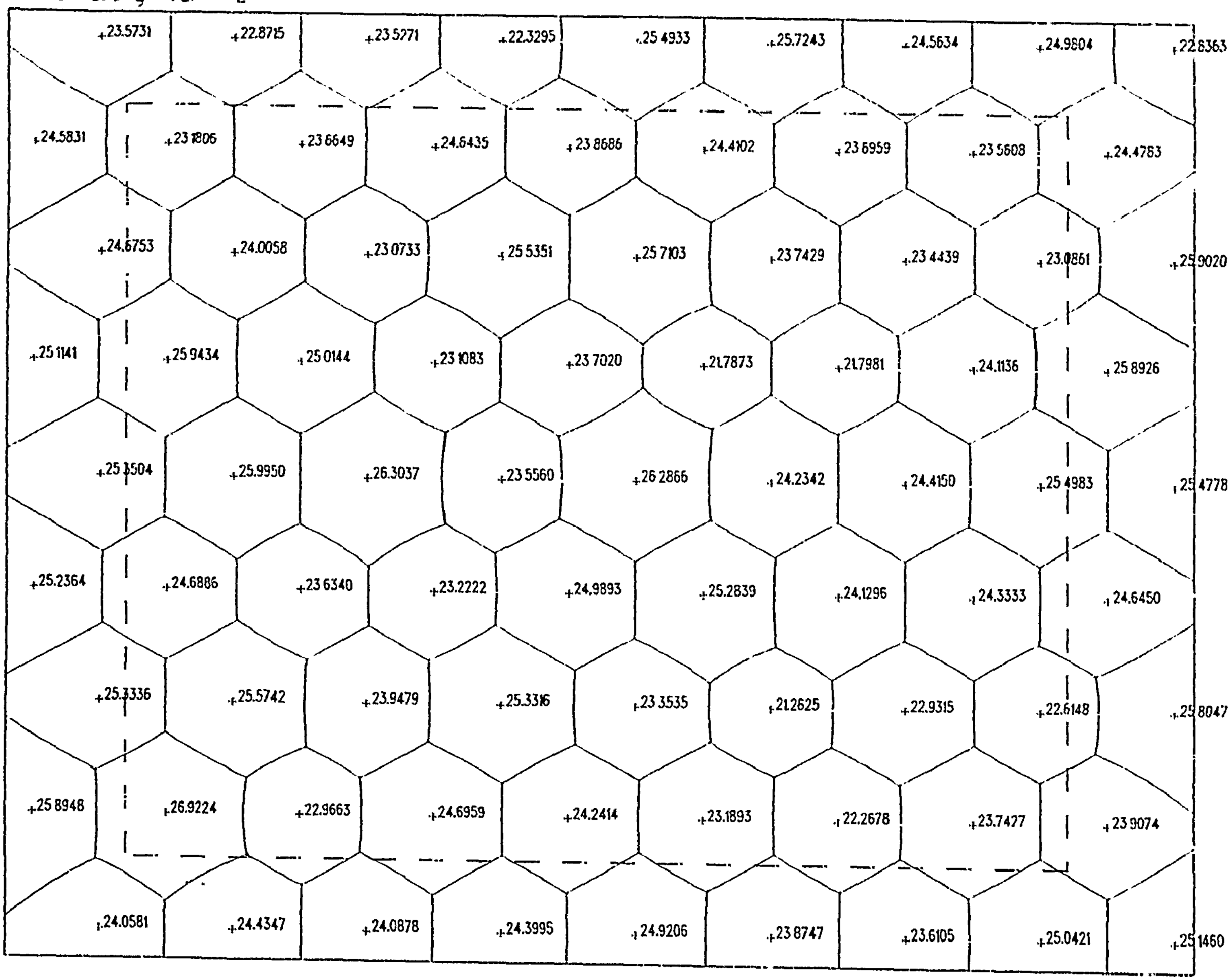
rho 0.3 g Var = 2



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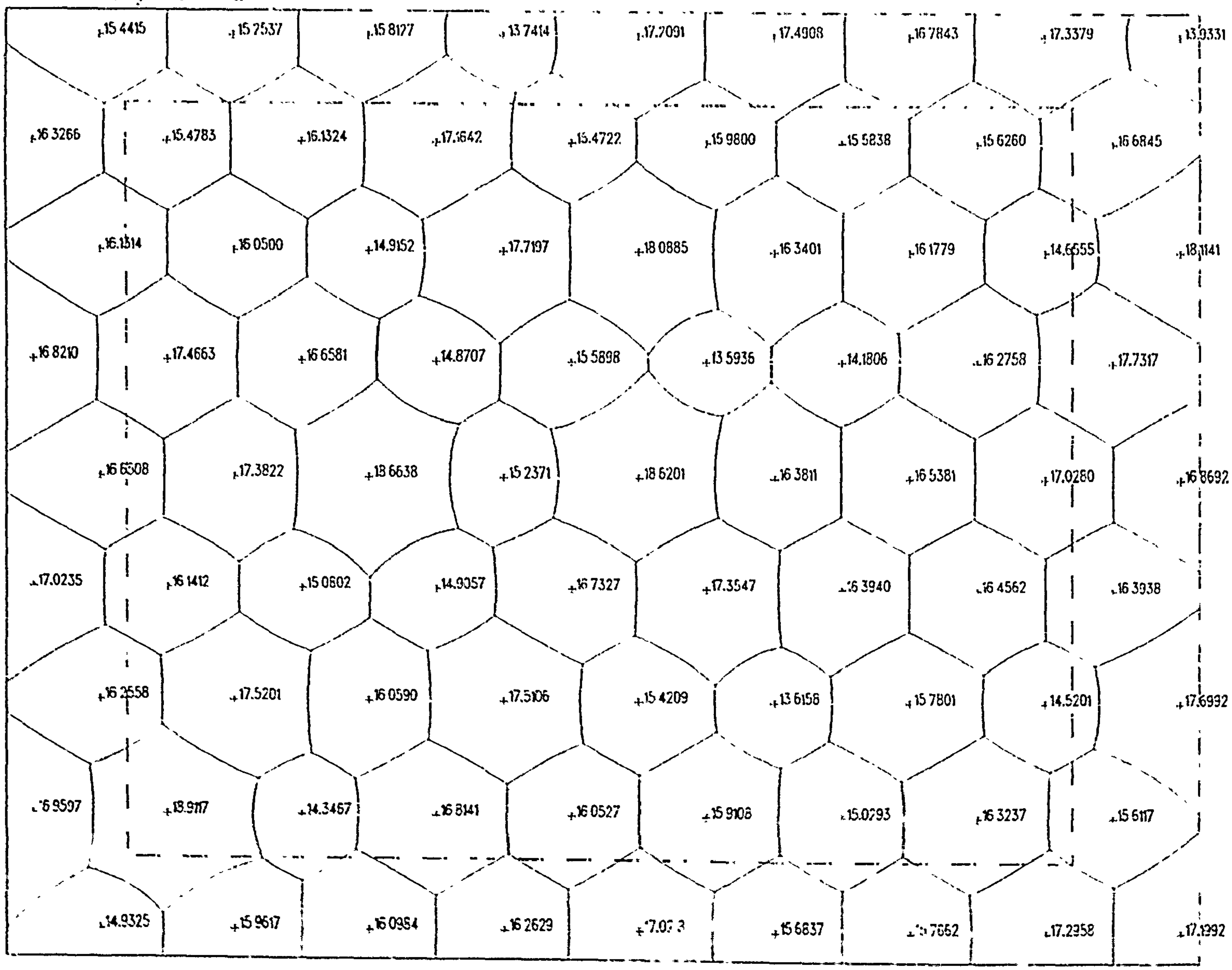
rho -0.6 q Var = 2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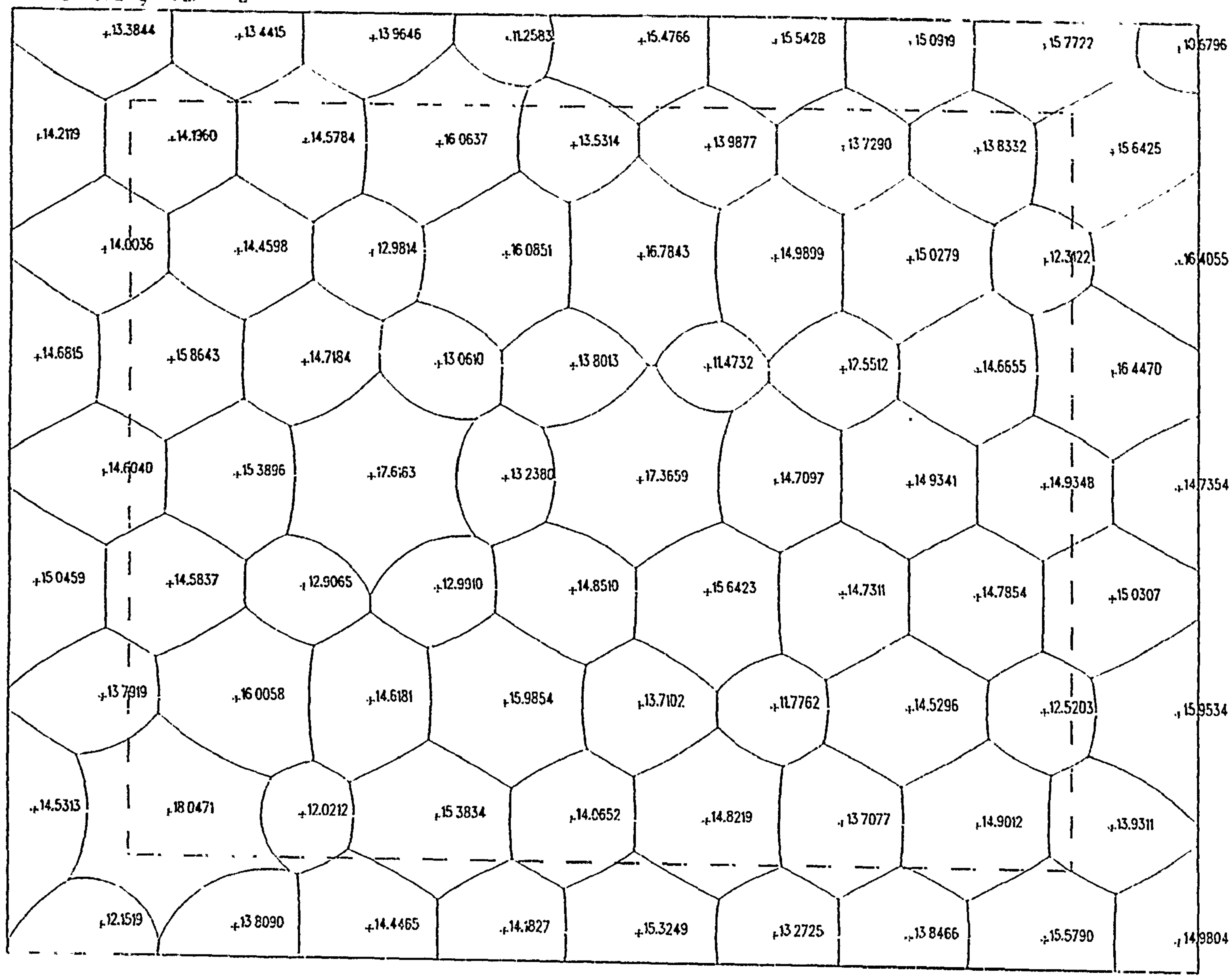
rho -1.2 g Var = 2



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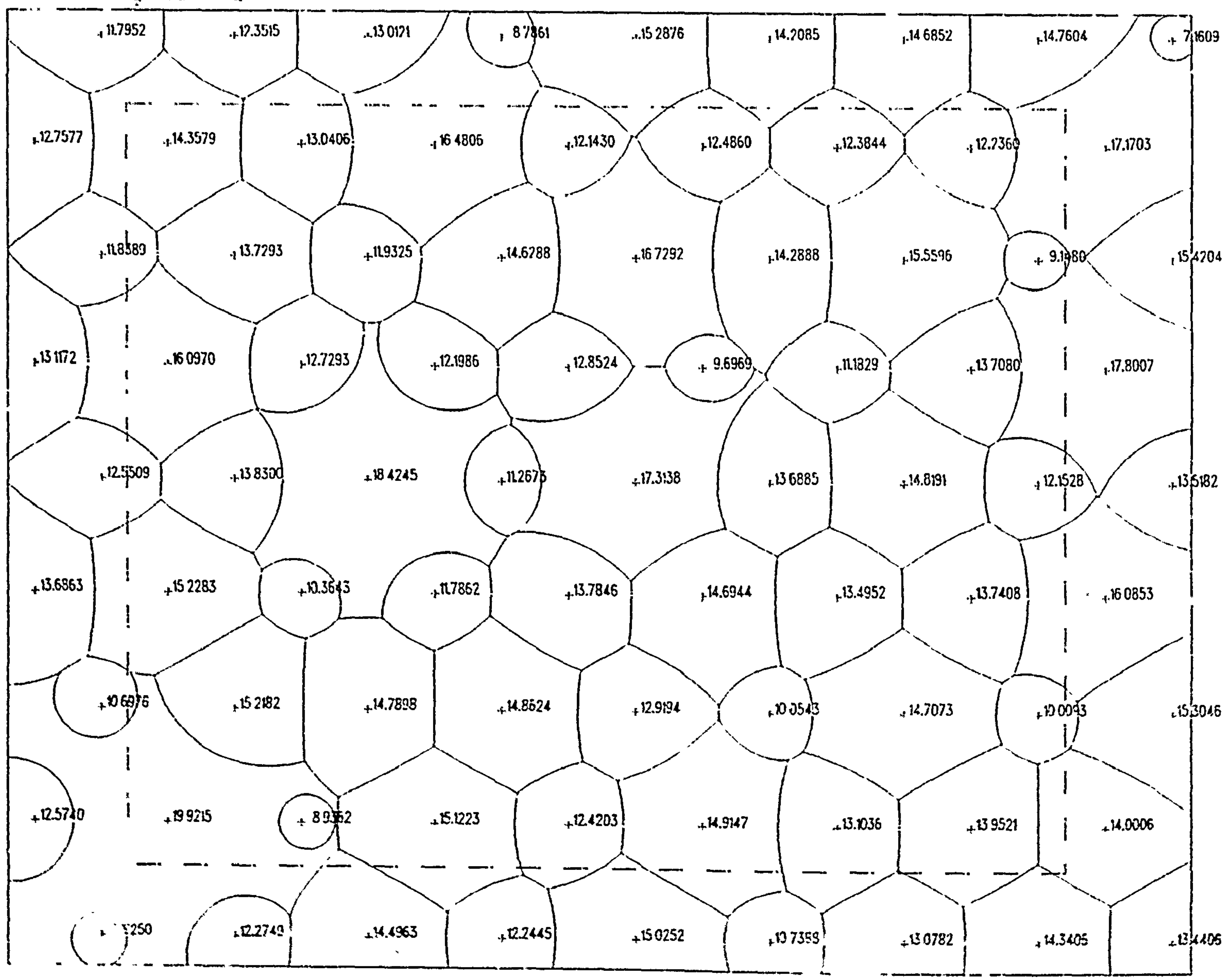
rho -1.8 q Var = 2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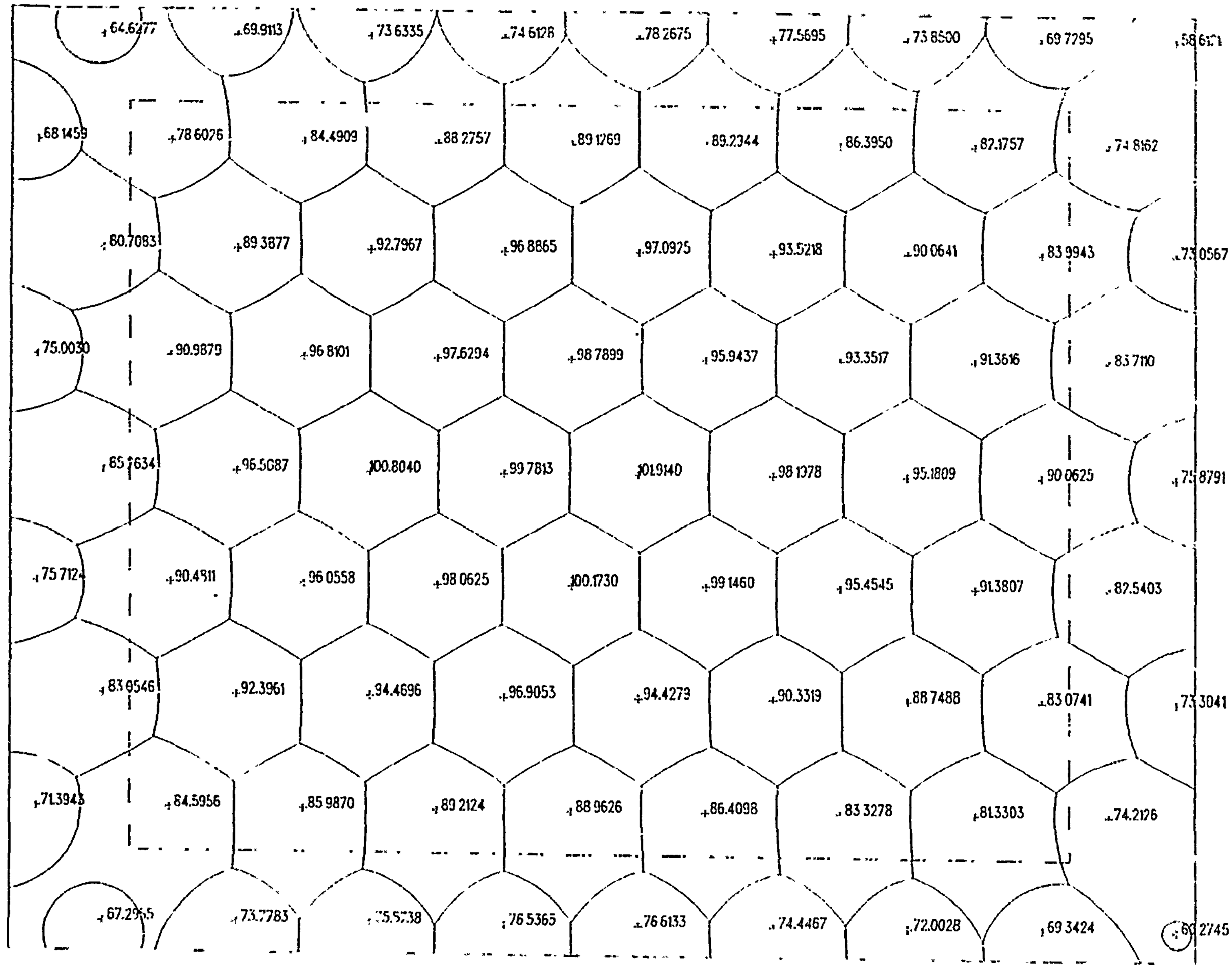
rho 0.9 m Var = 2



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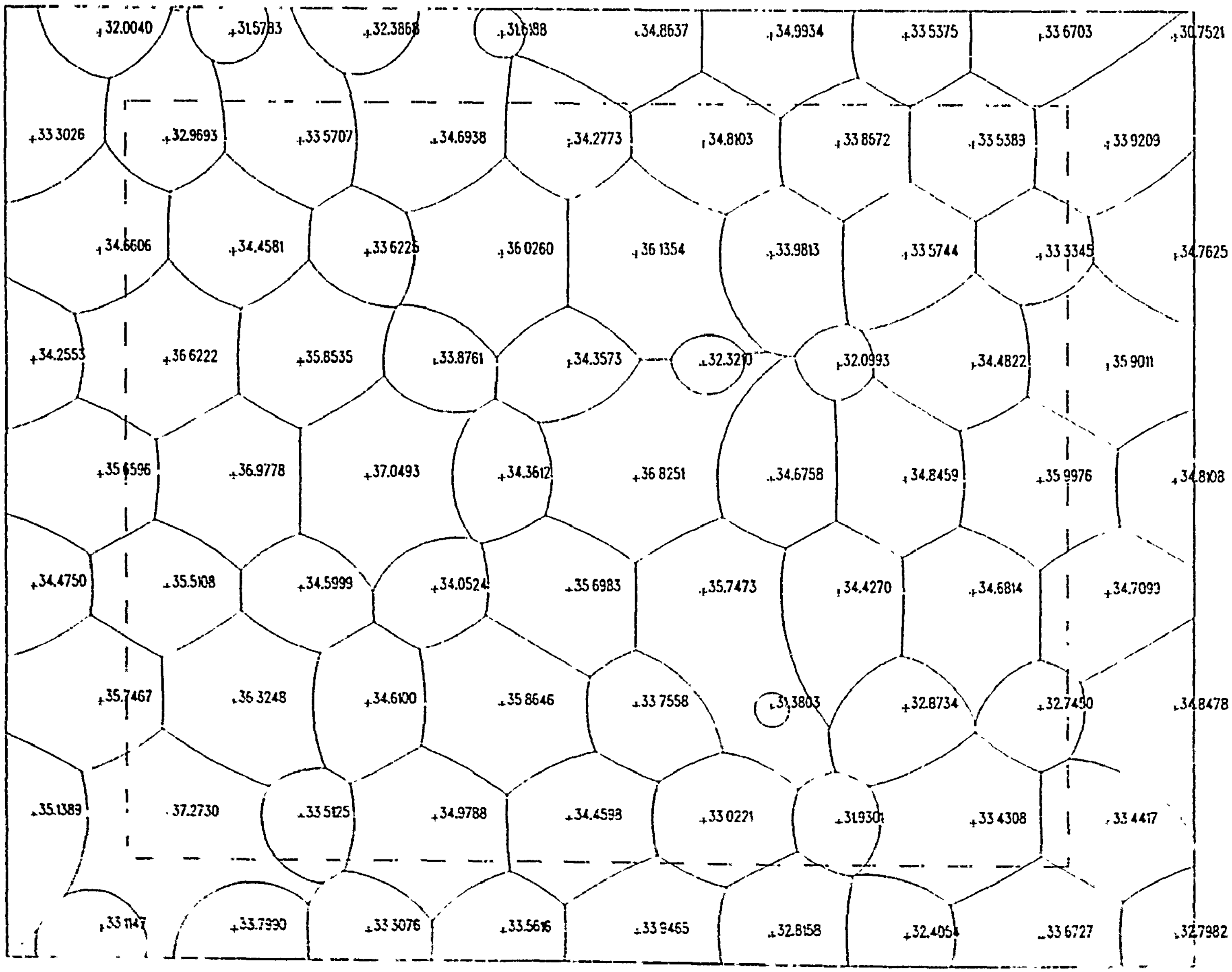
rho 0.6 m Var = 2



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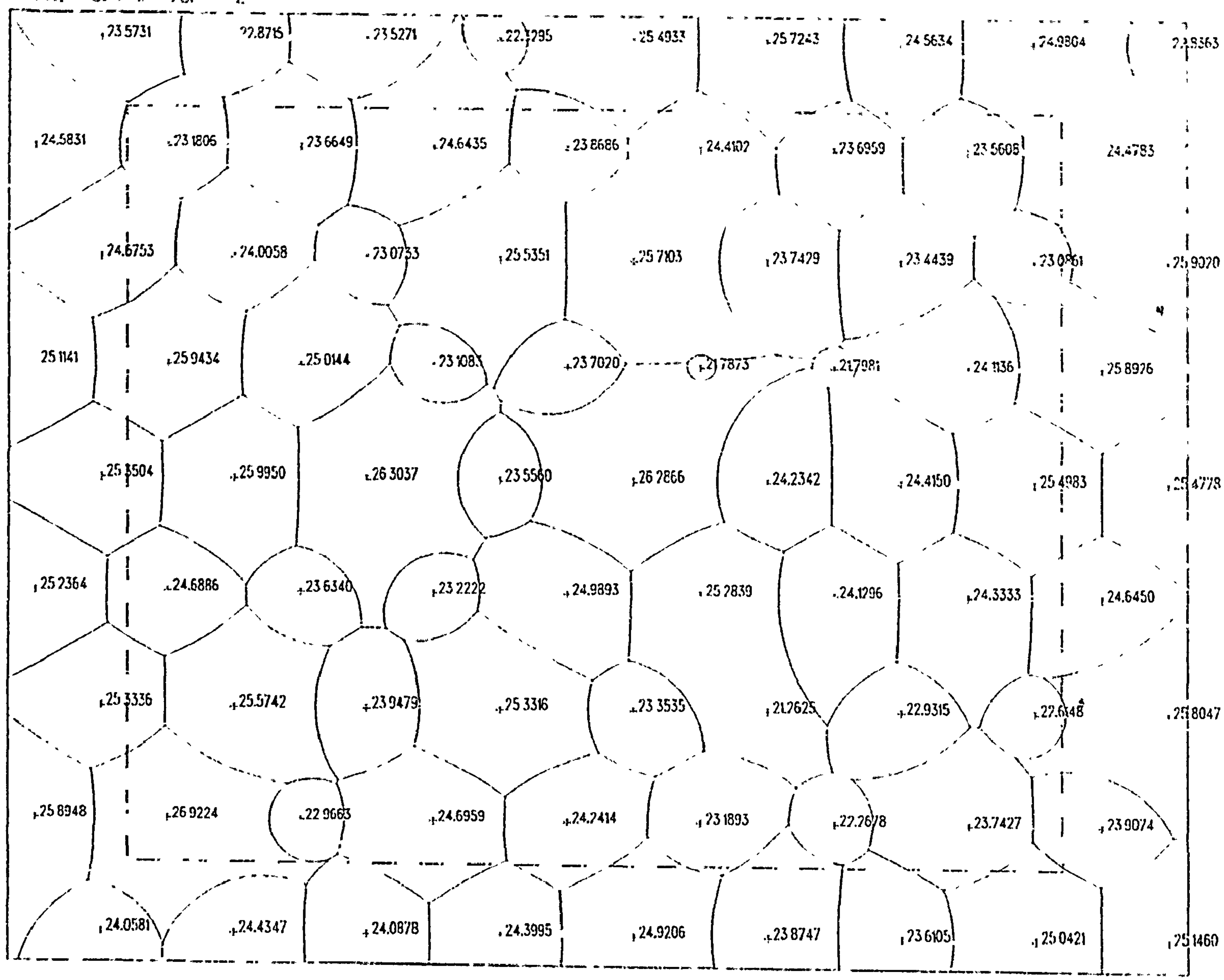
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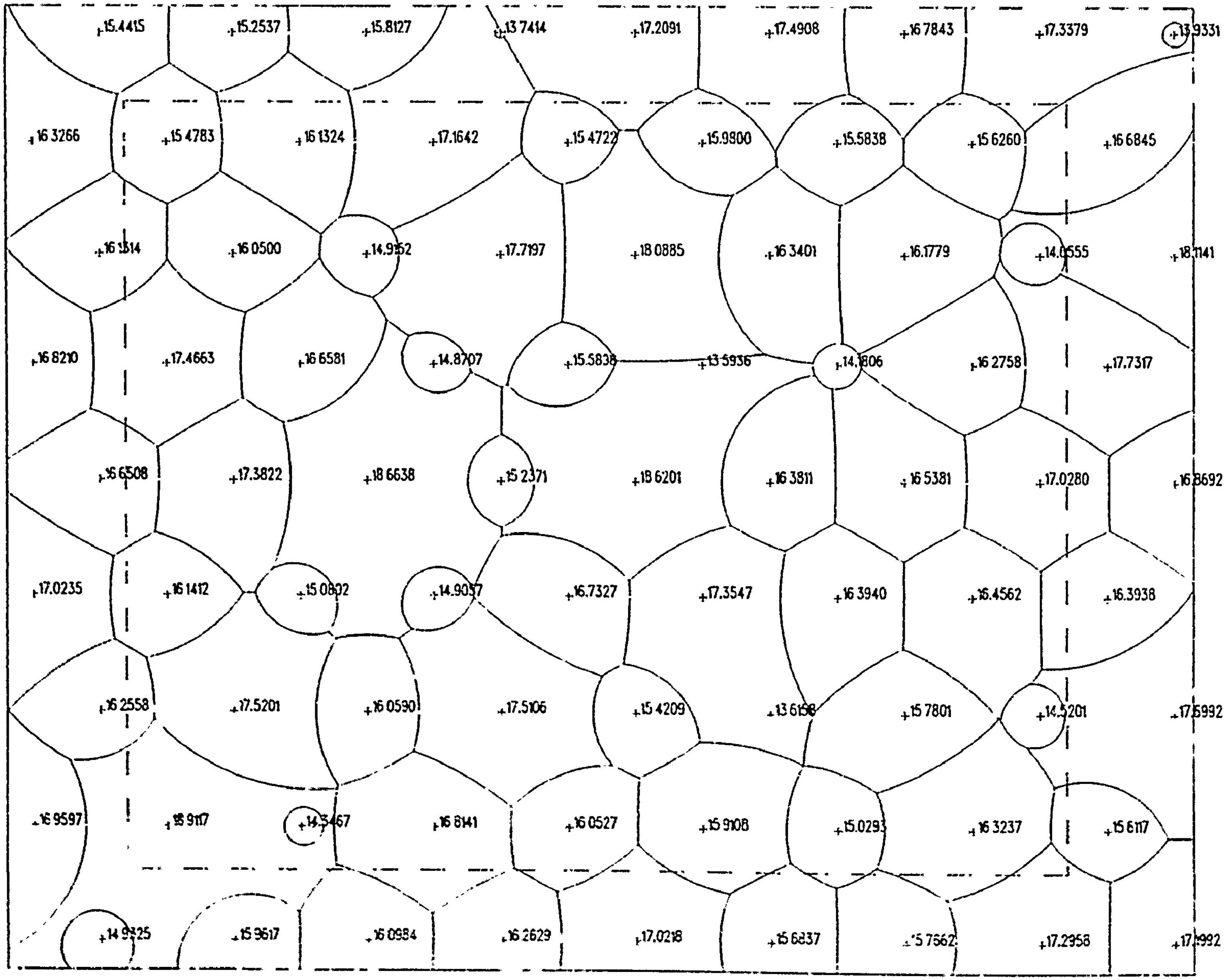
rho -0.6 m Var = 2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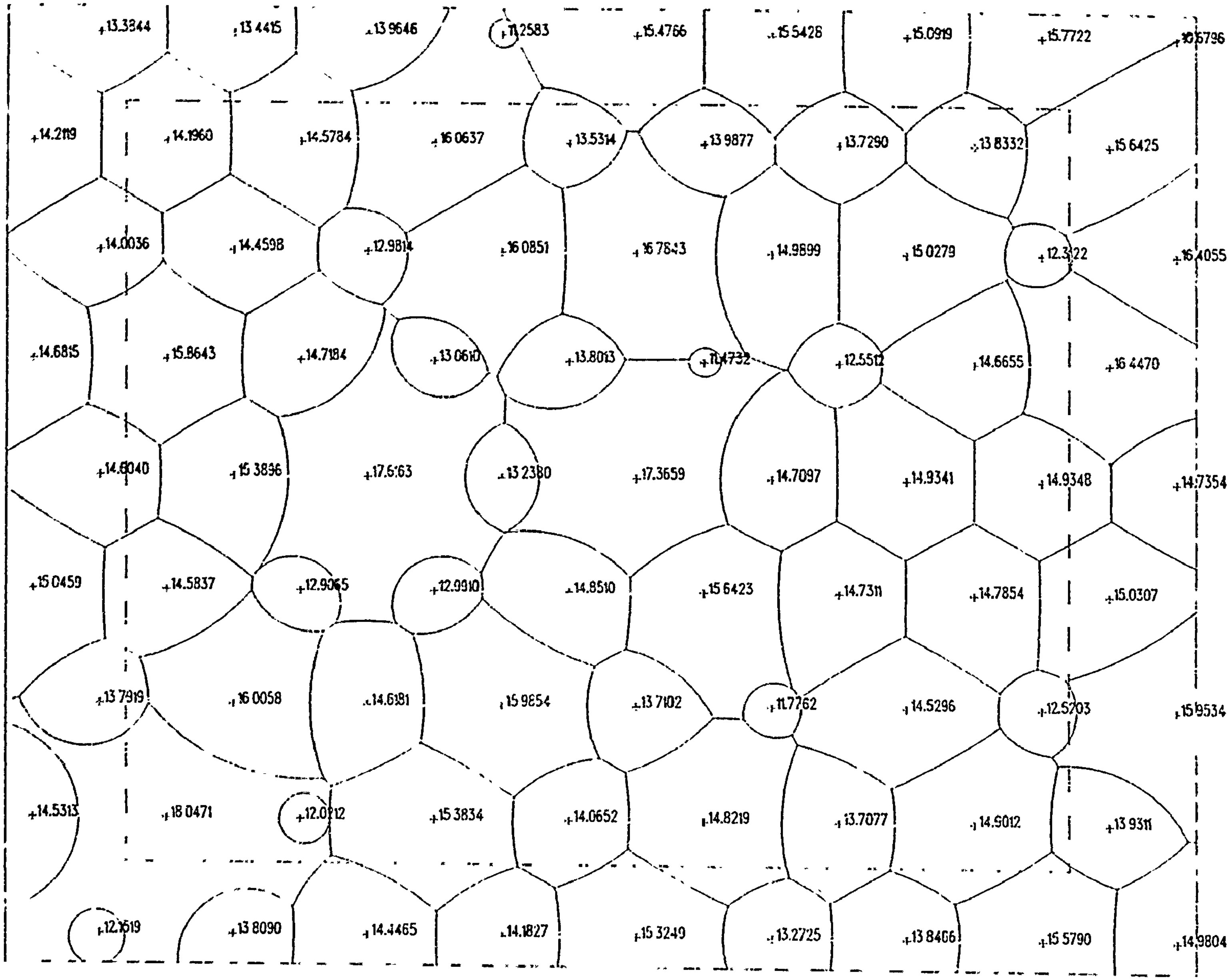
rho -1.2 m Var = 2



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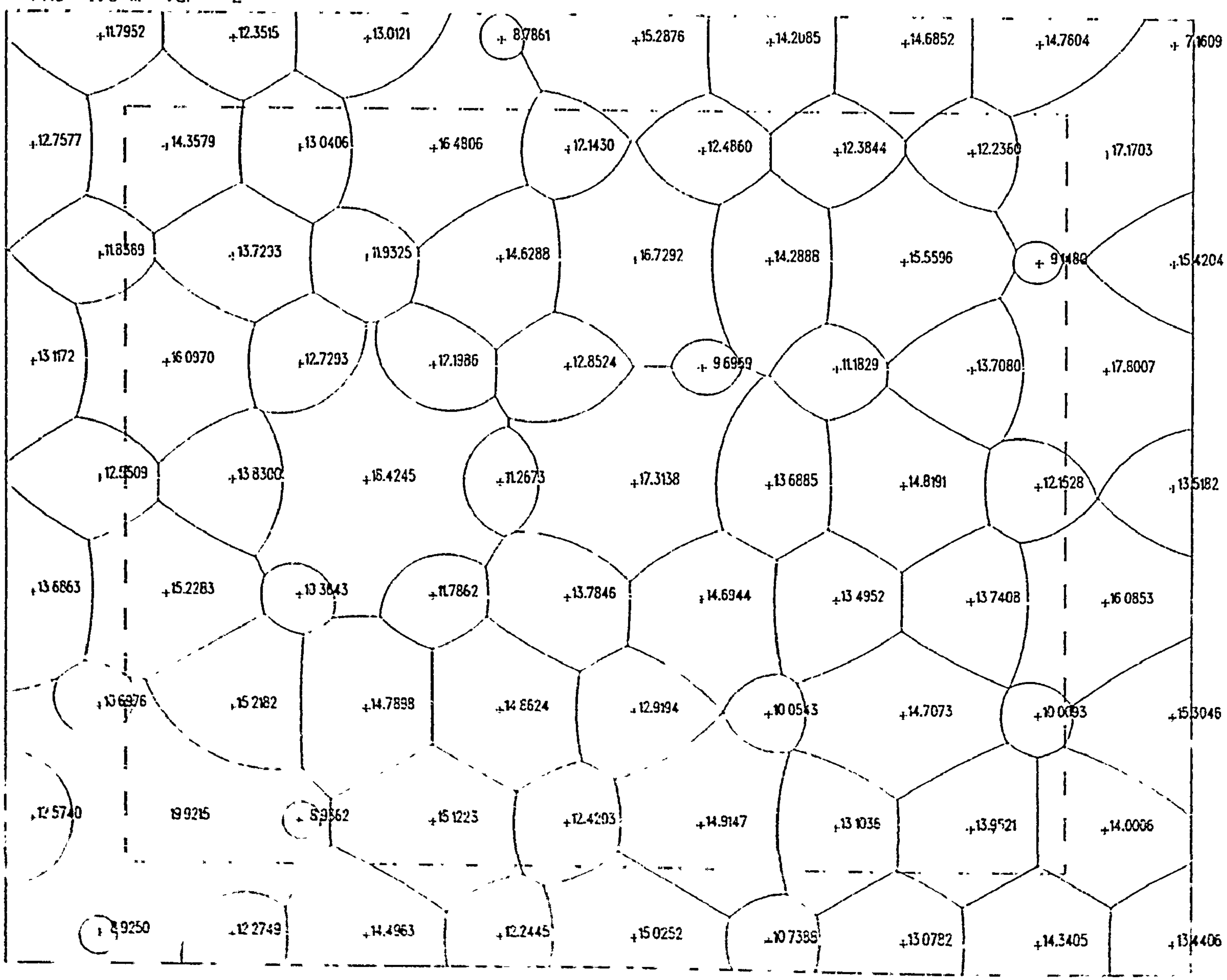

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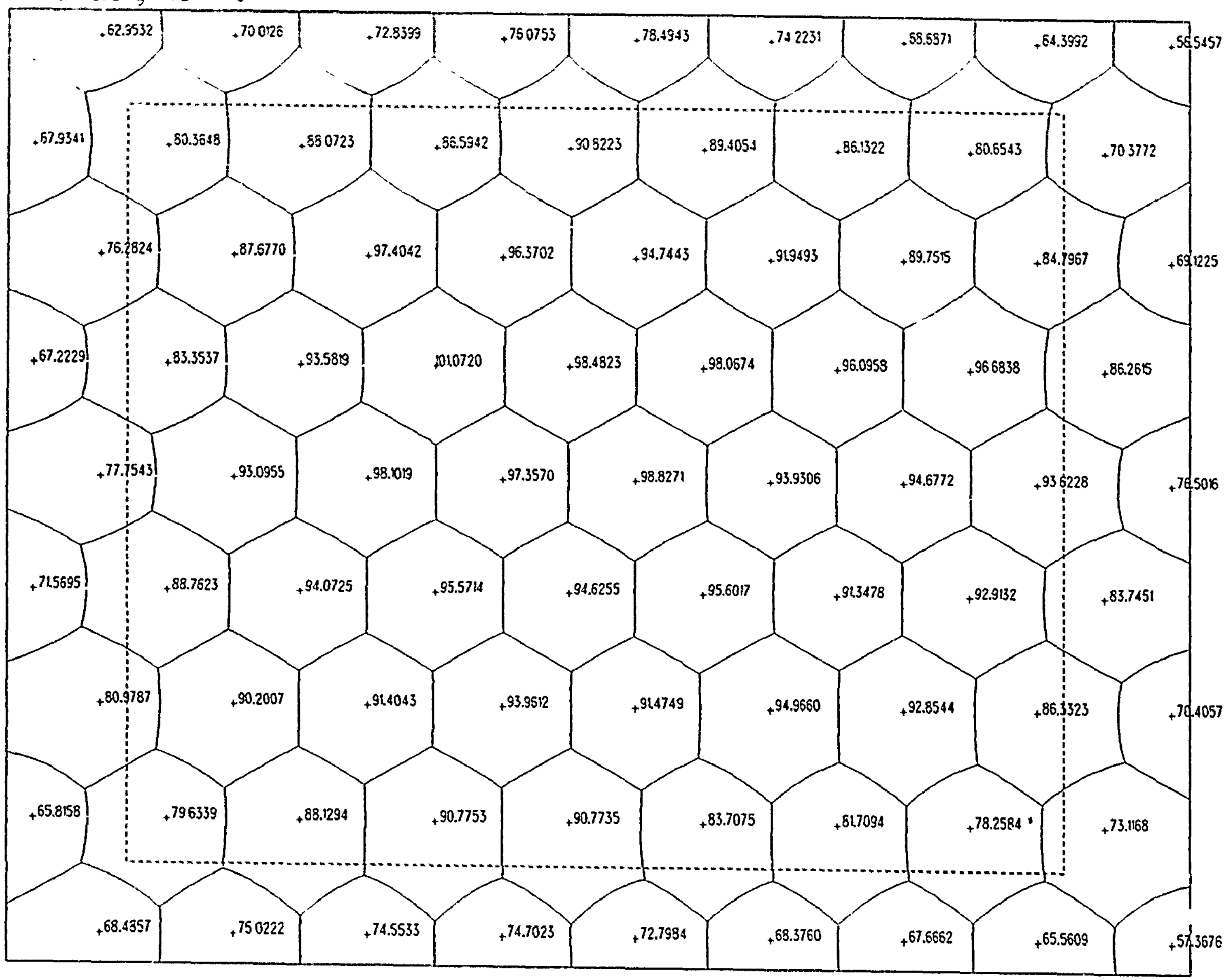
rho 0.9 g Va = 3



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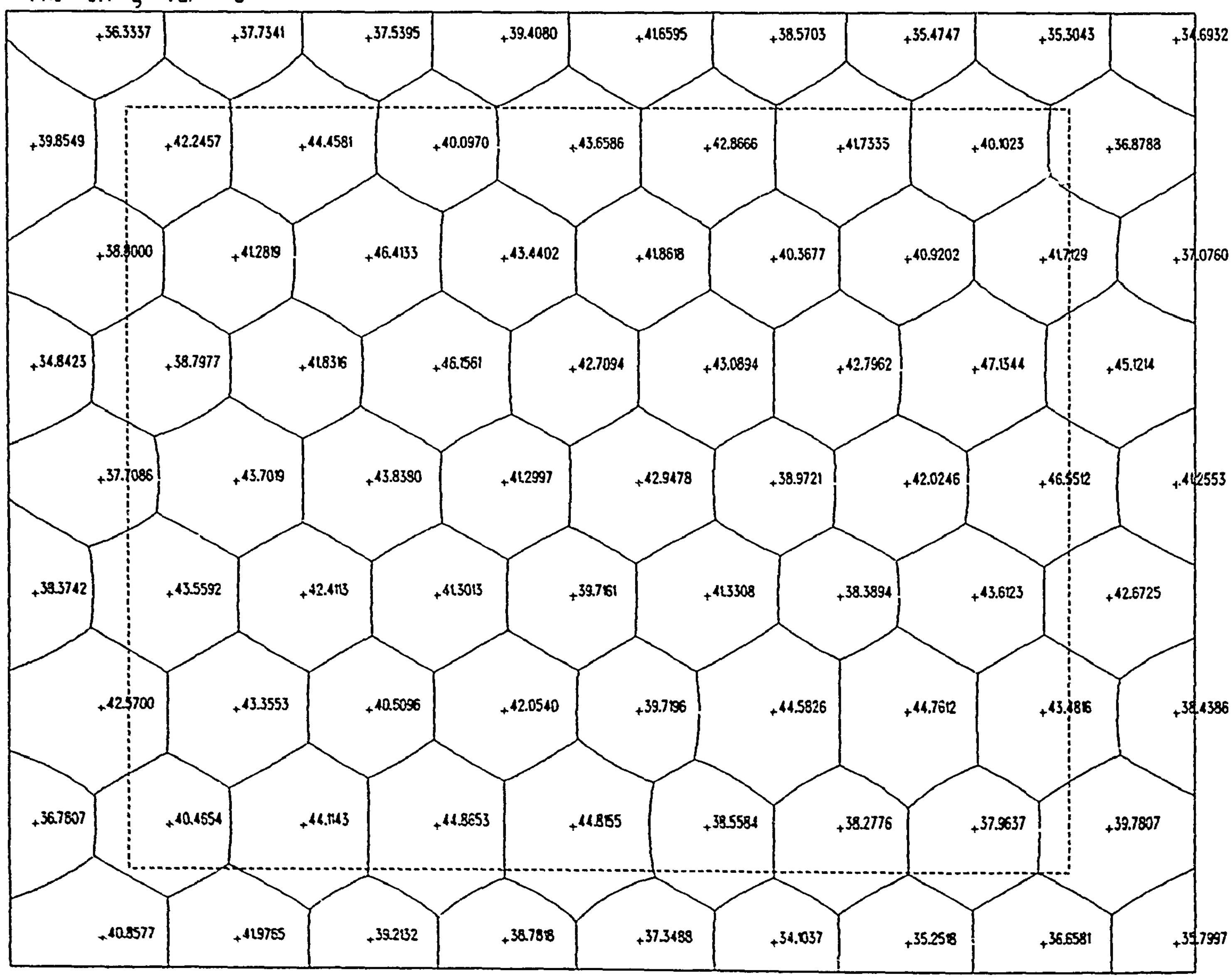
rho 0.7 g Var = 3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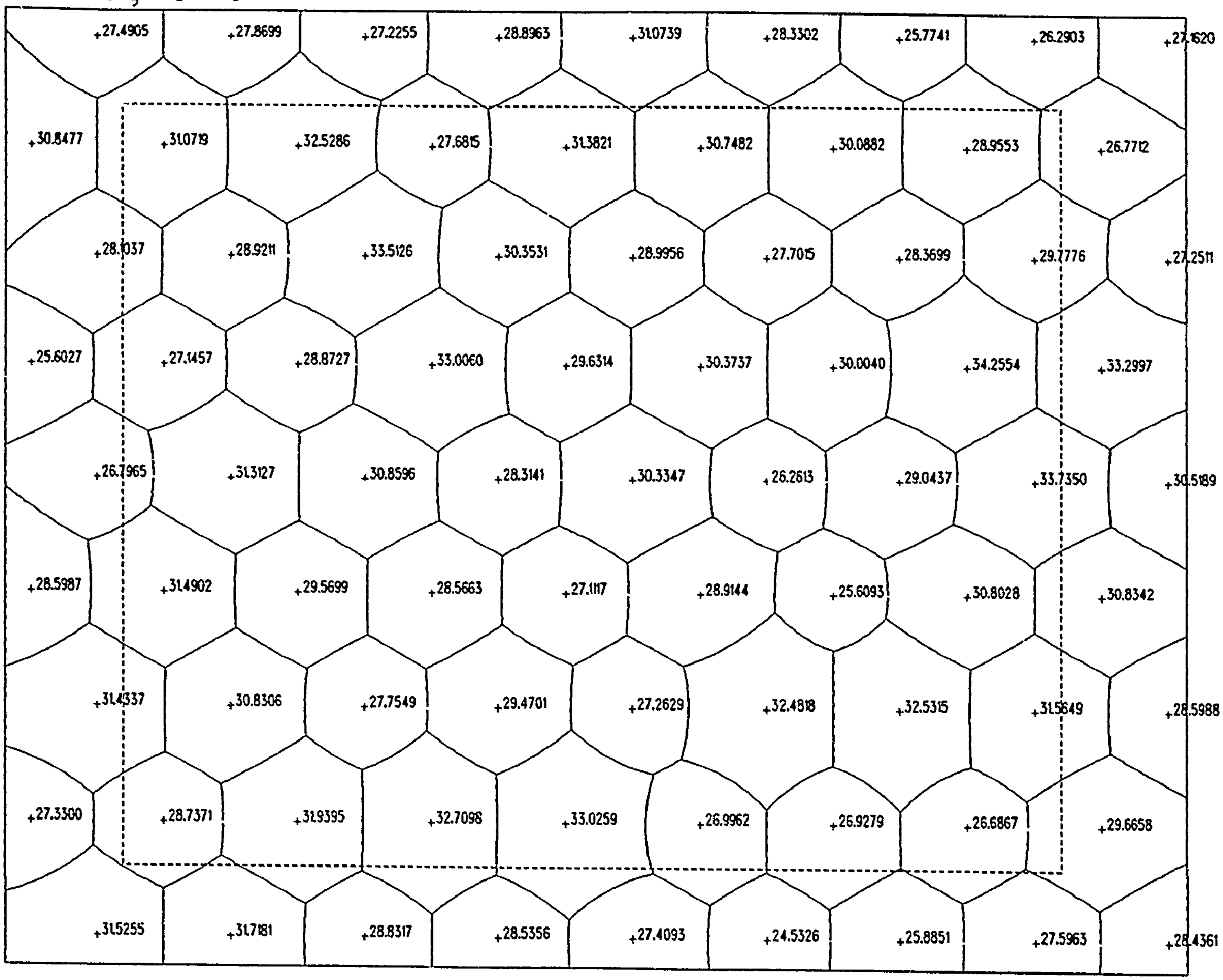
rho 0.5 g Var = 3



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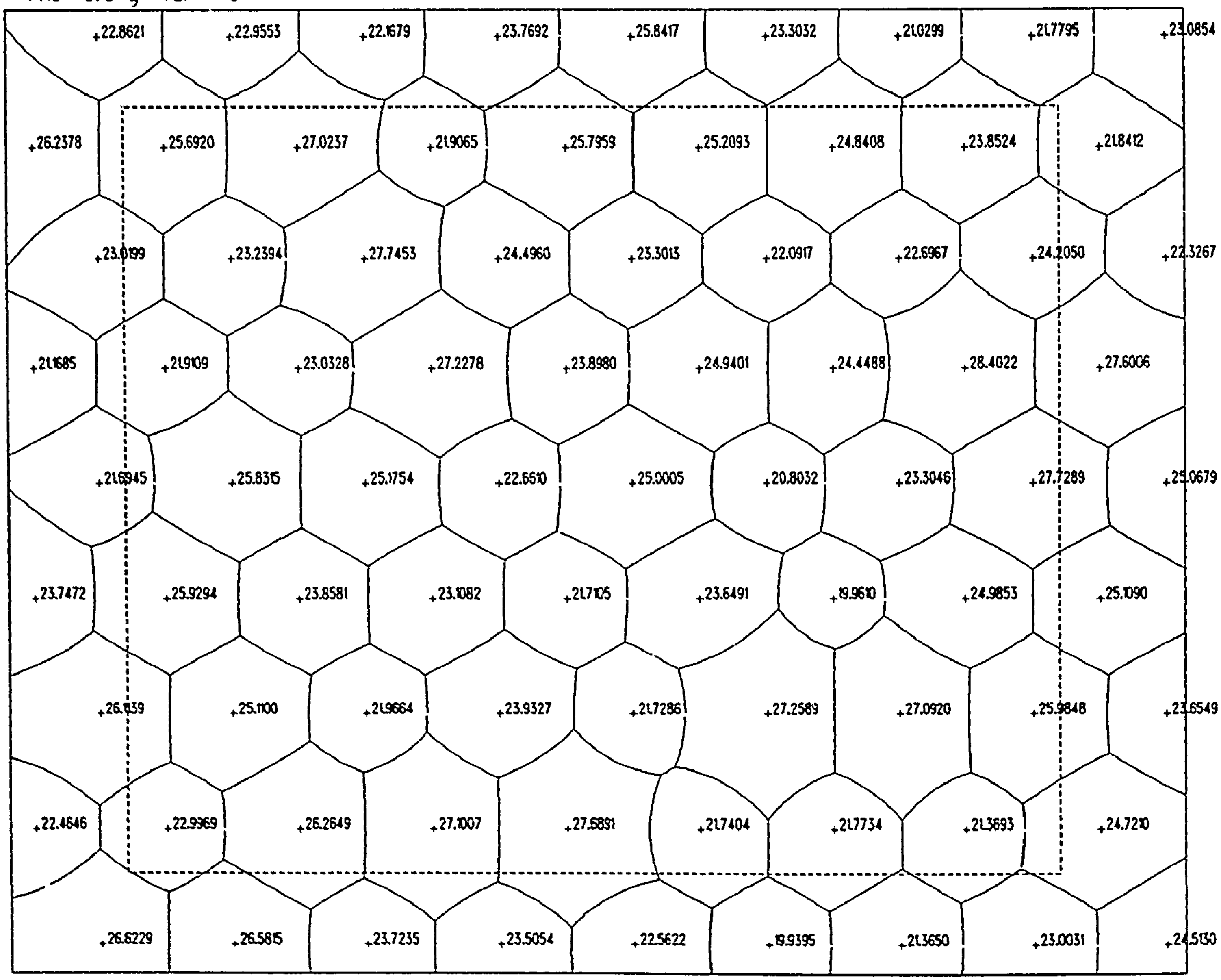
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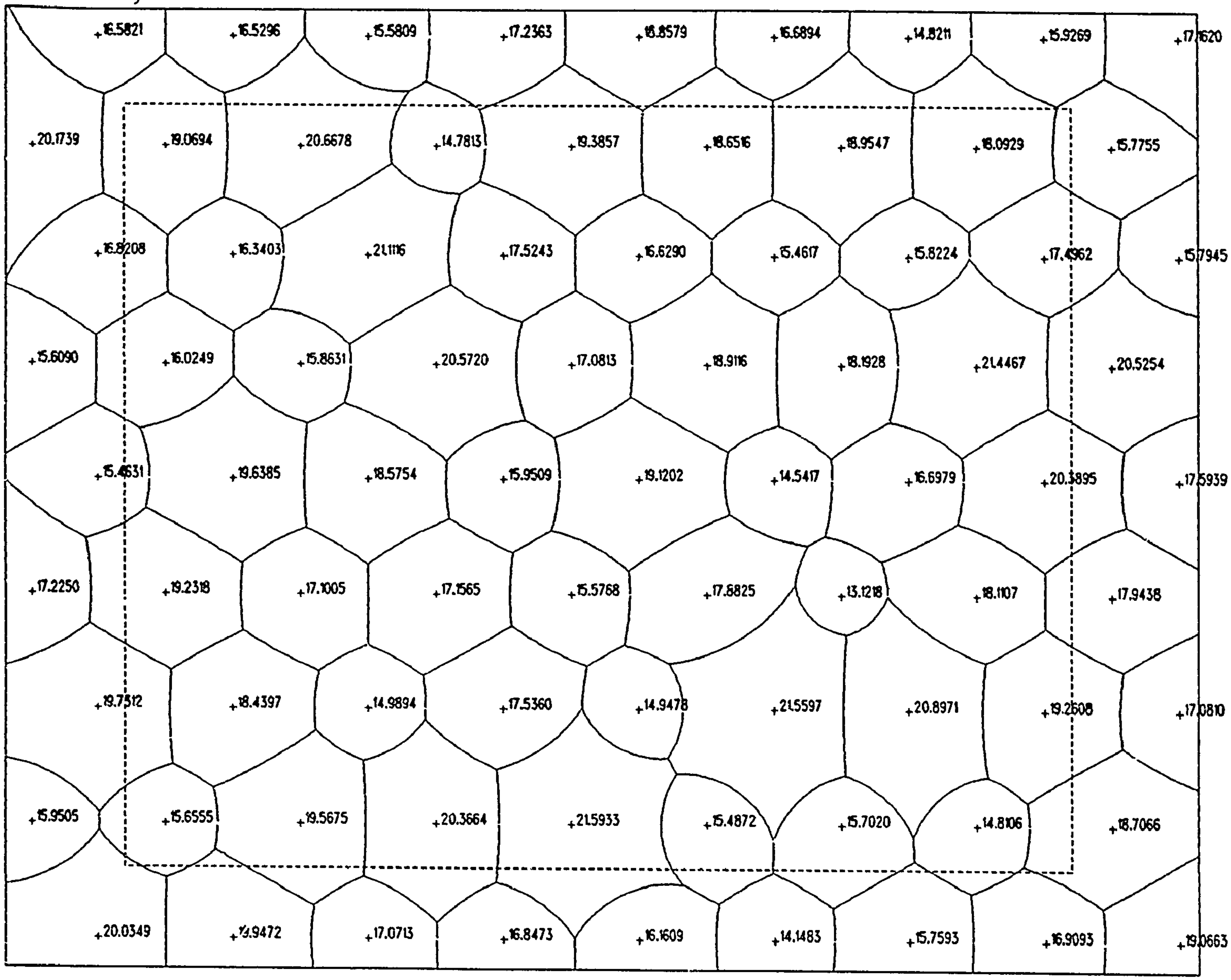

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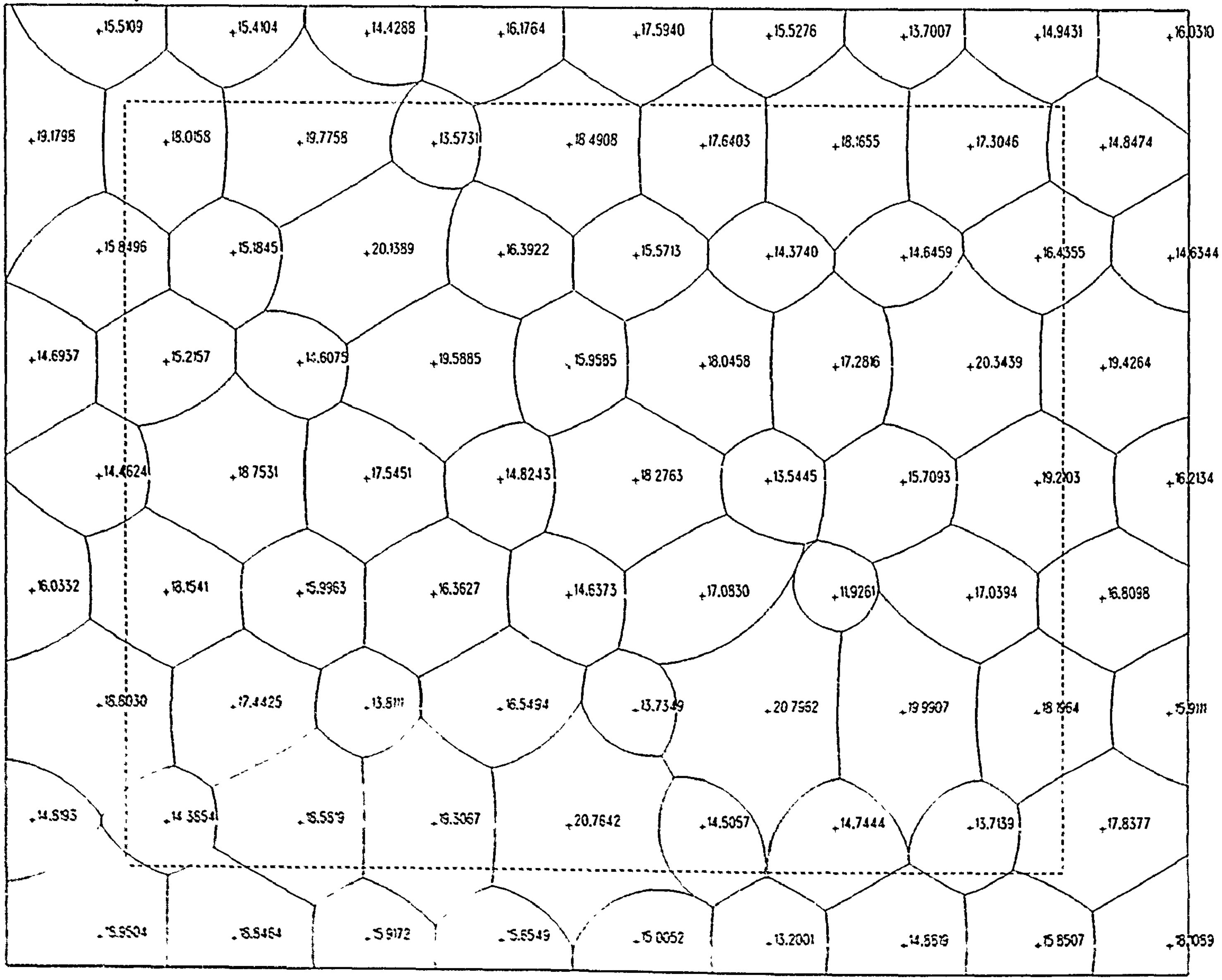

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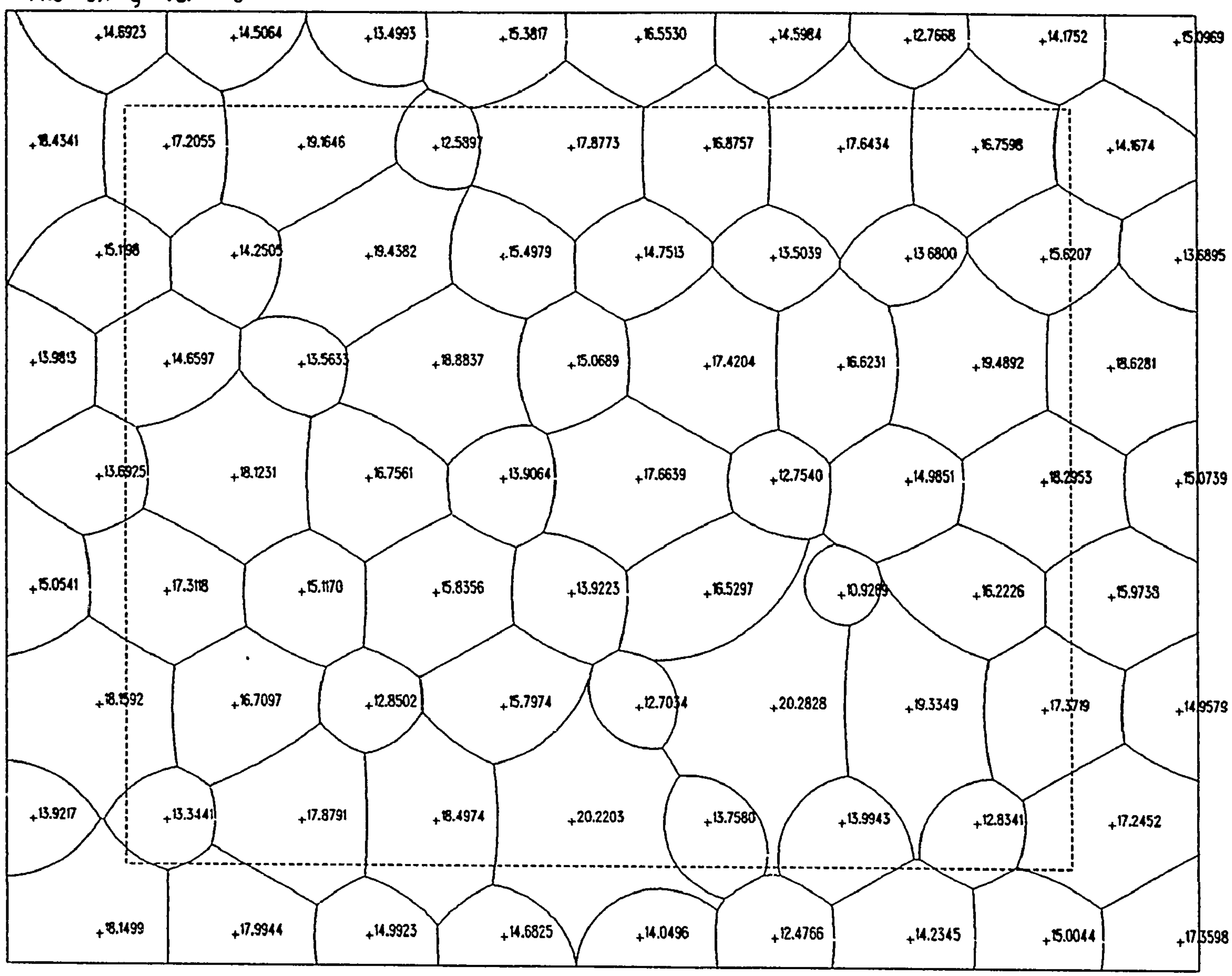
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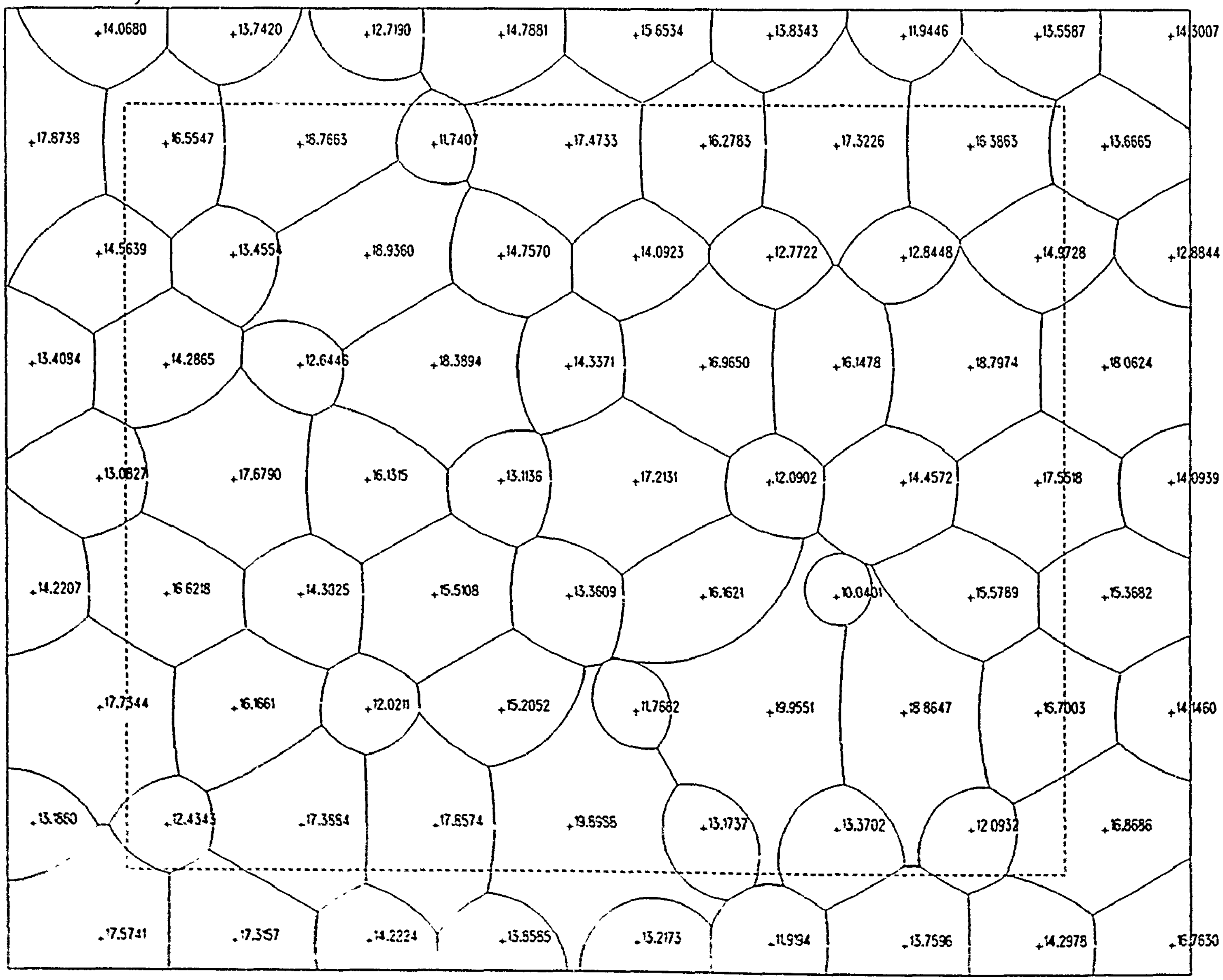
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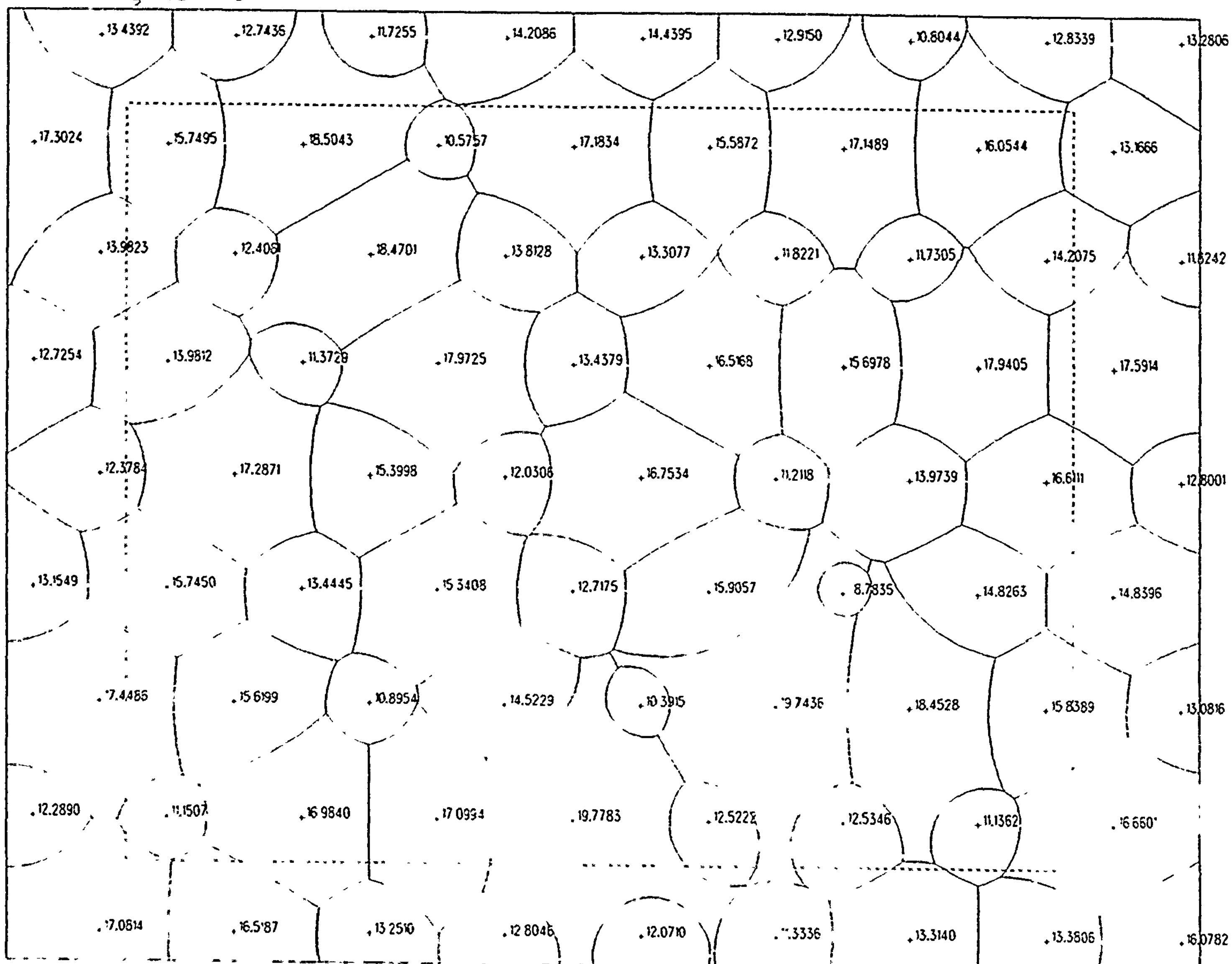

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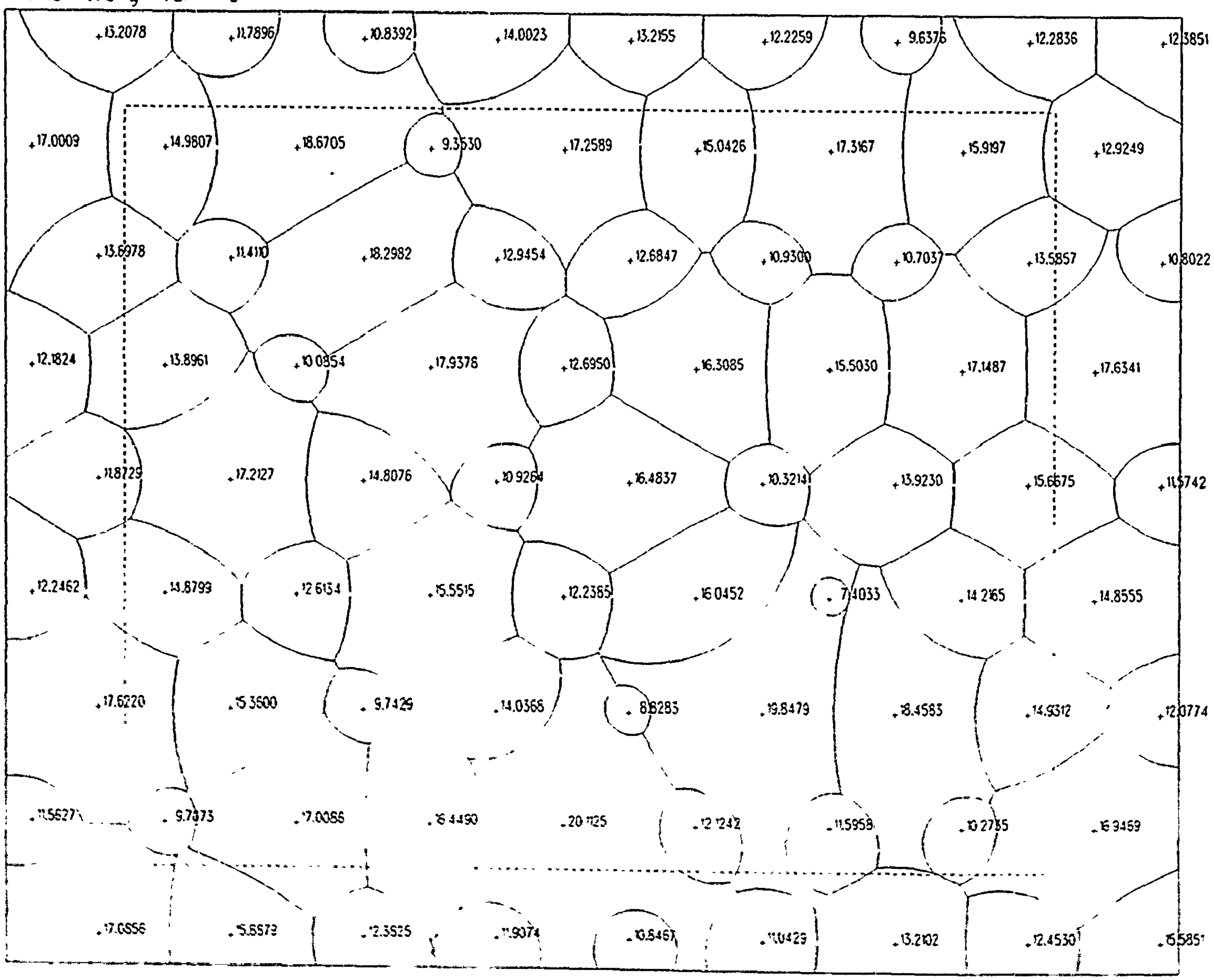

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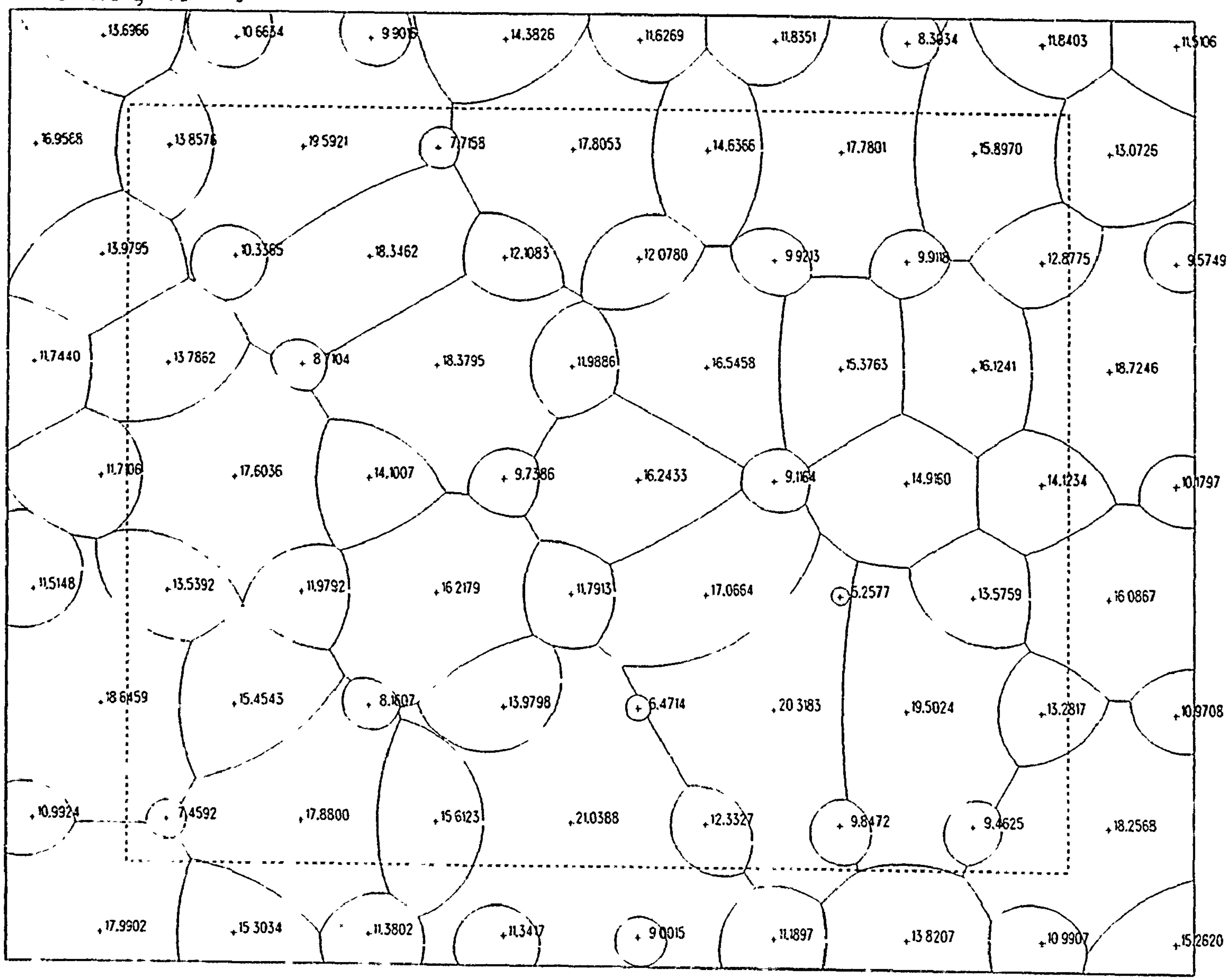
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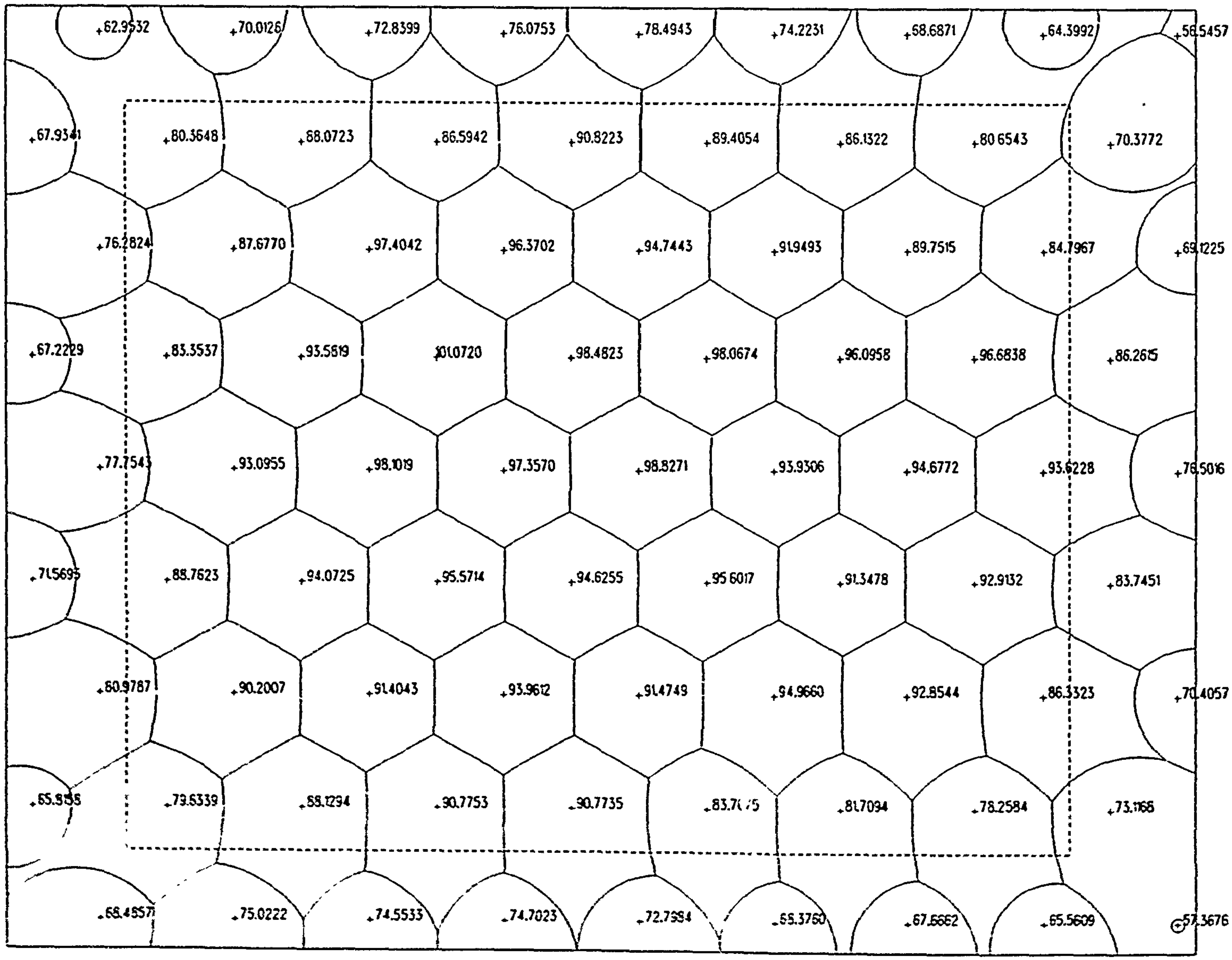
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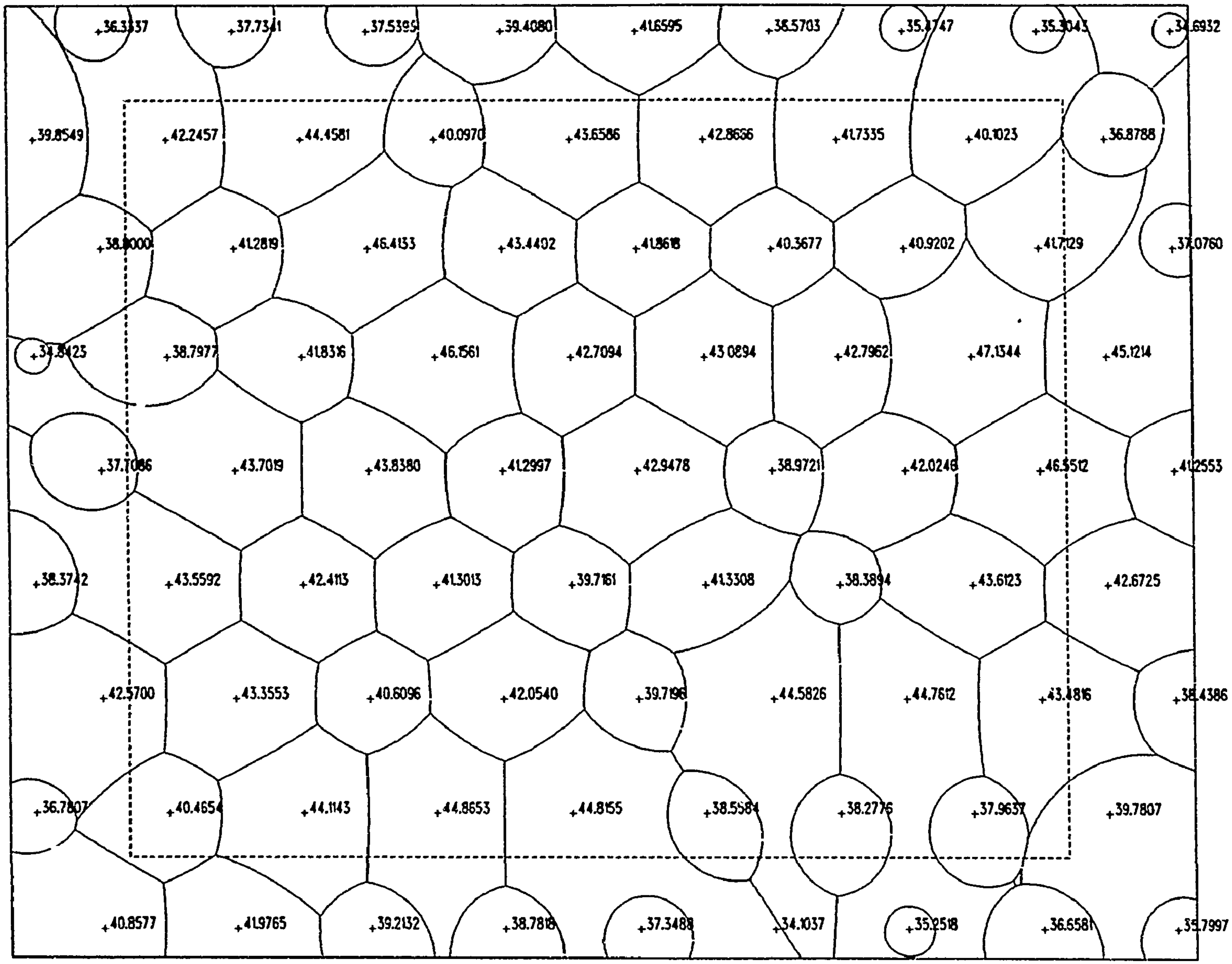
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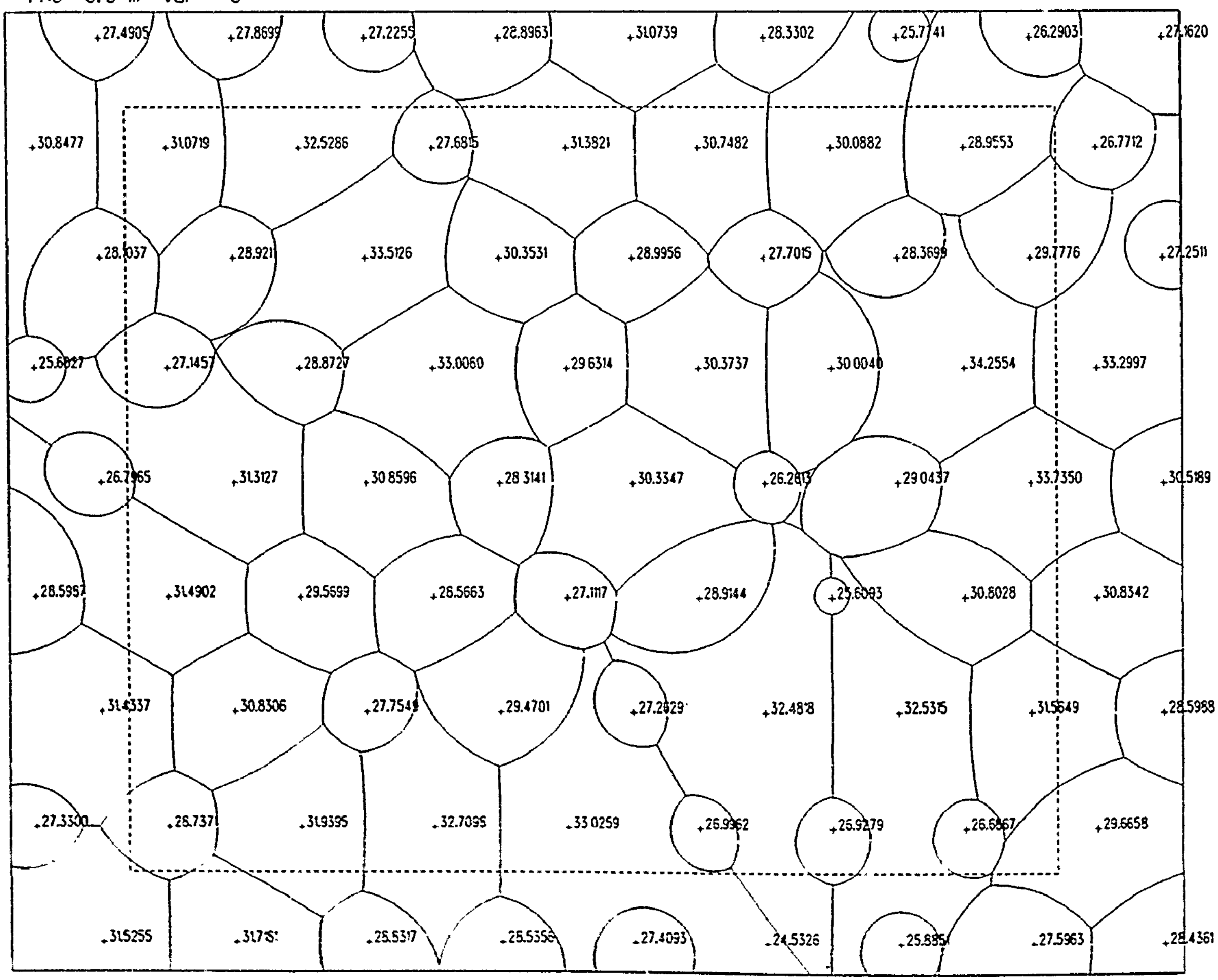

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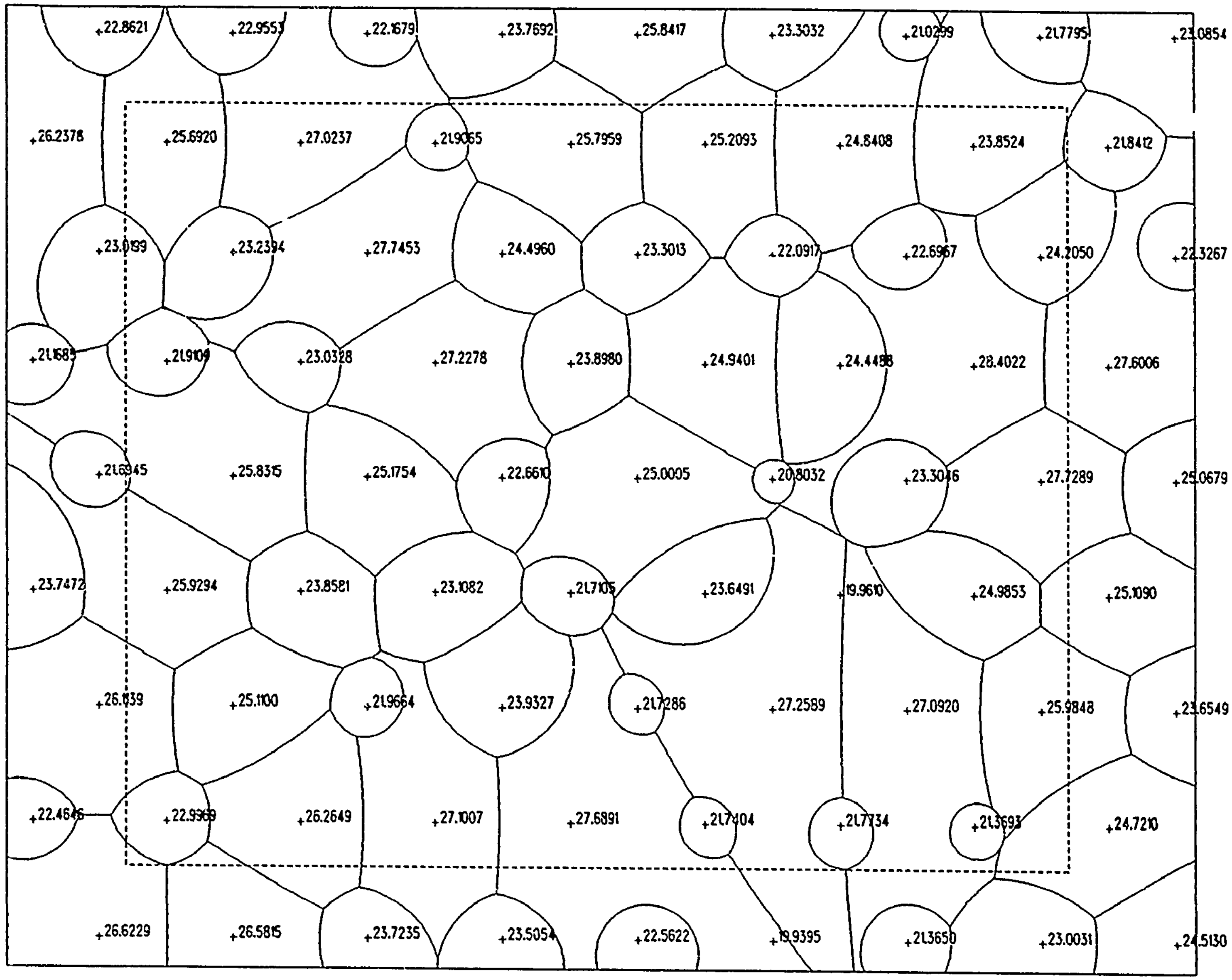
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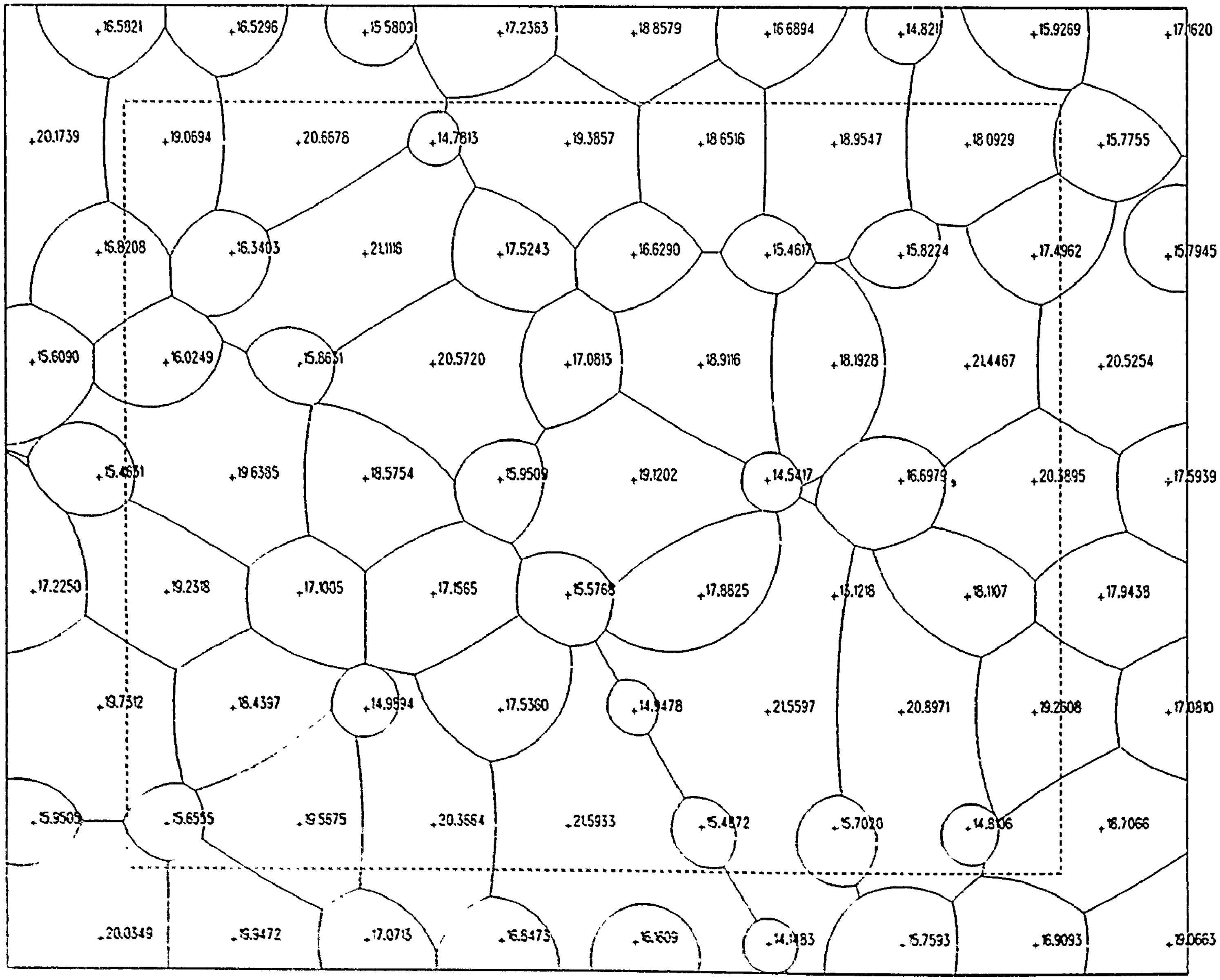
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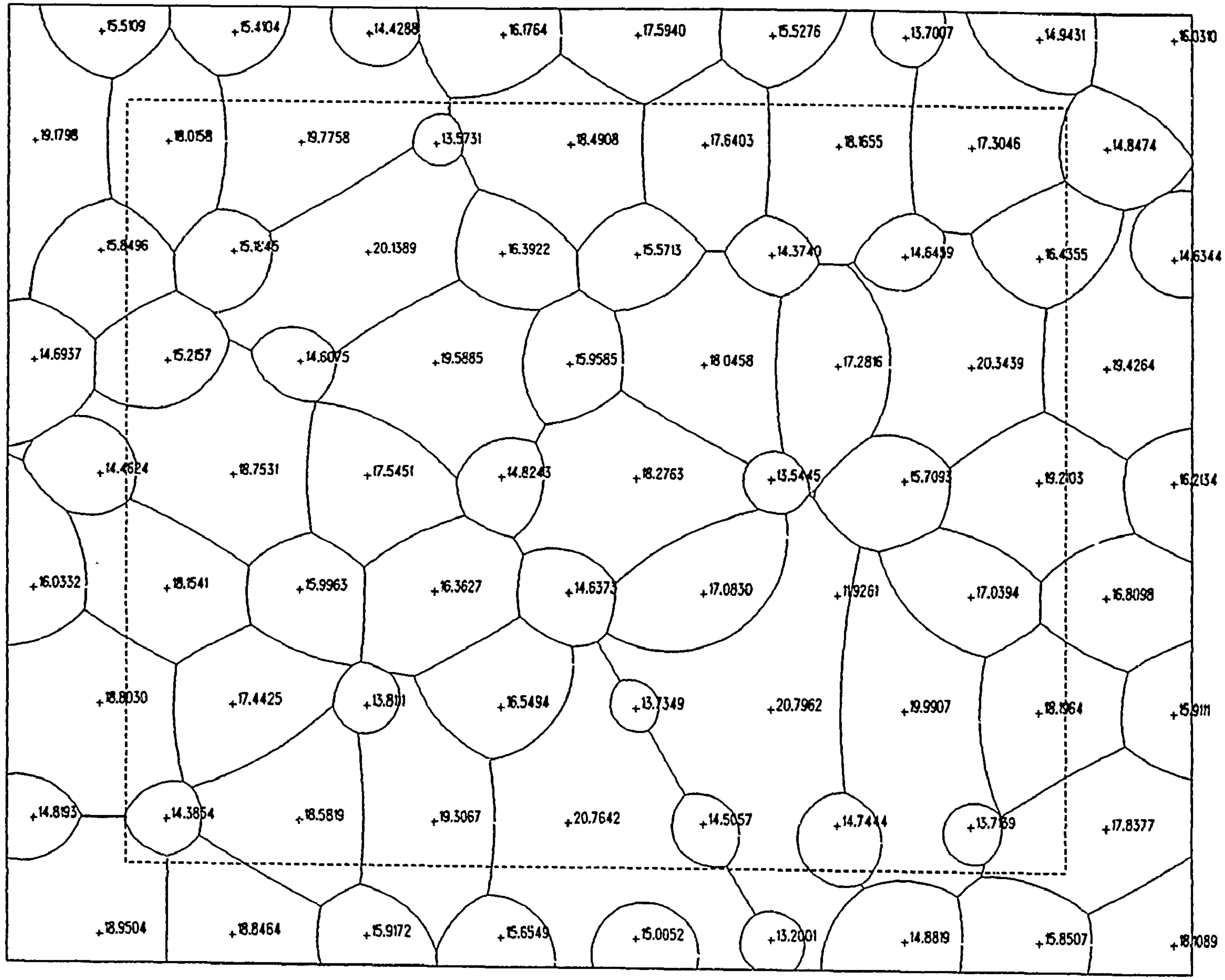
rho -0.5 m Var = 3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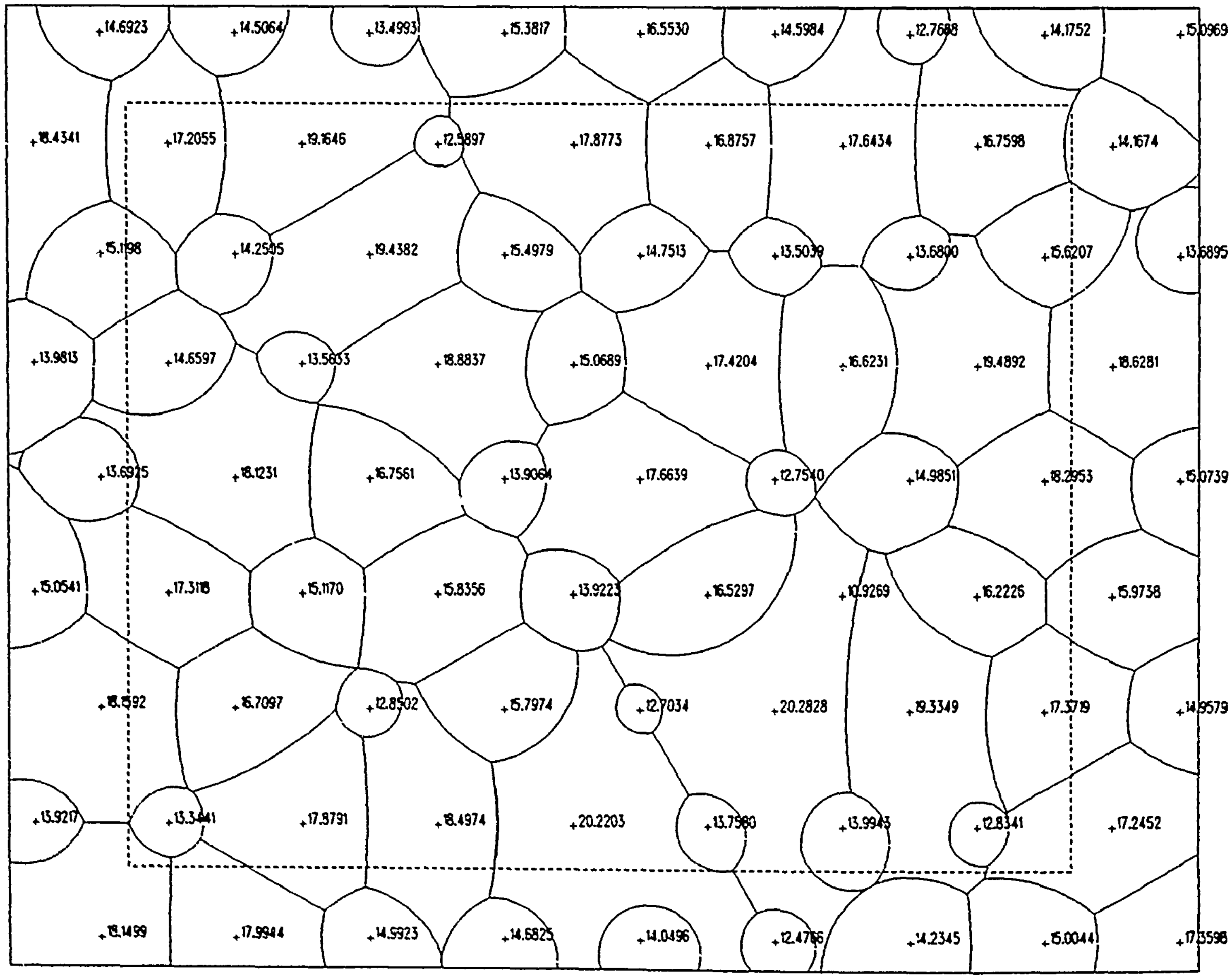

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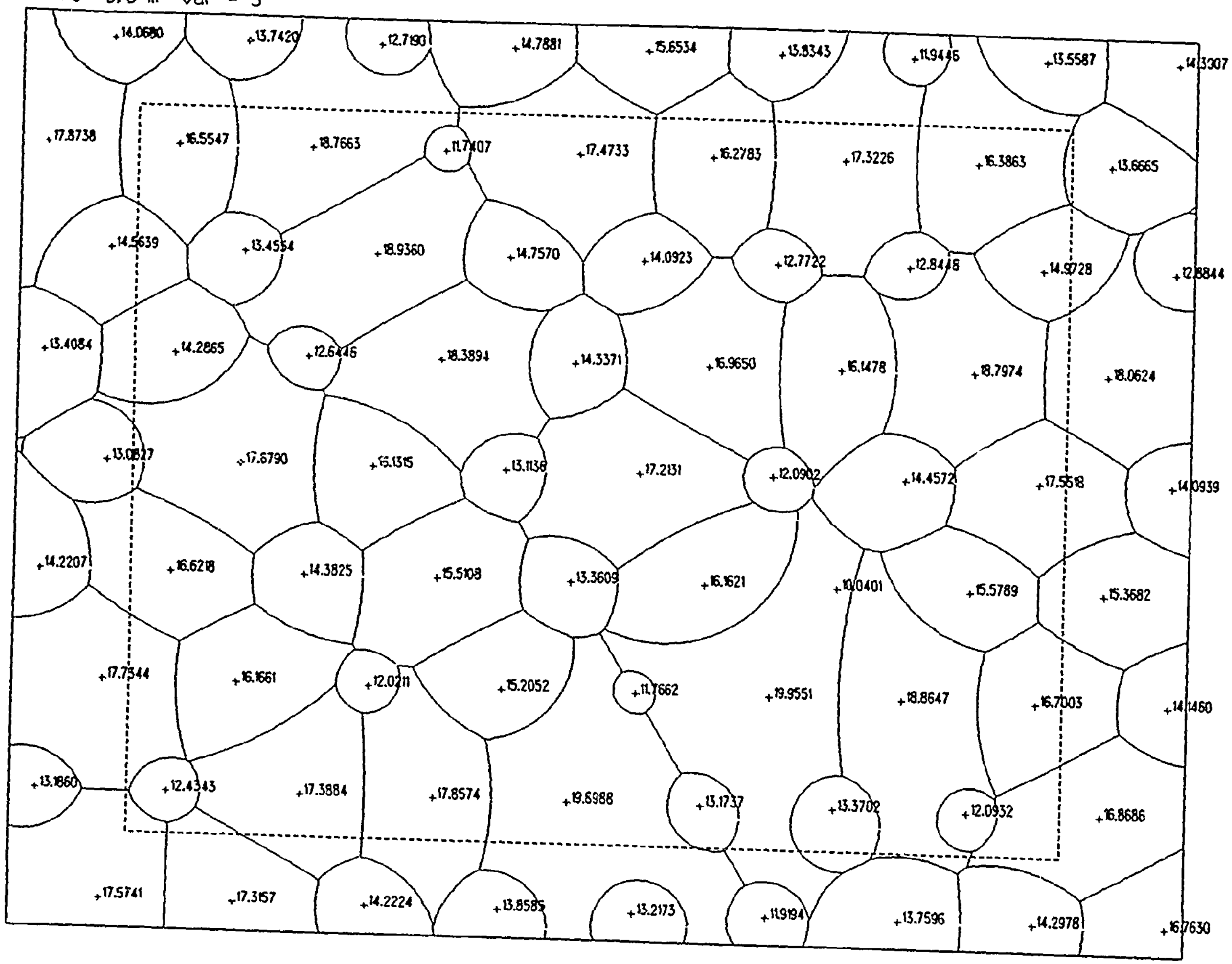
rho -0.9 m Var = 3



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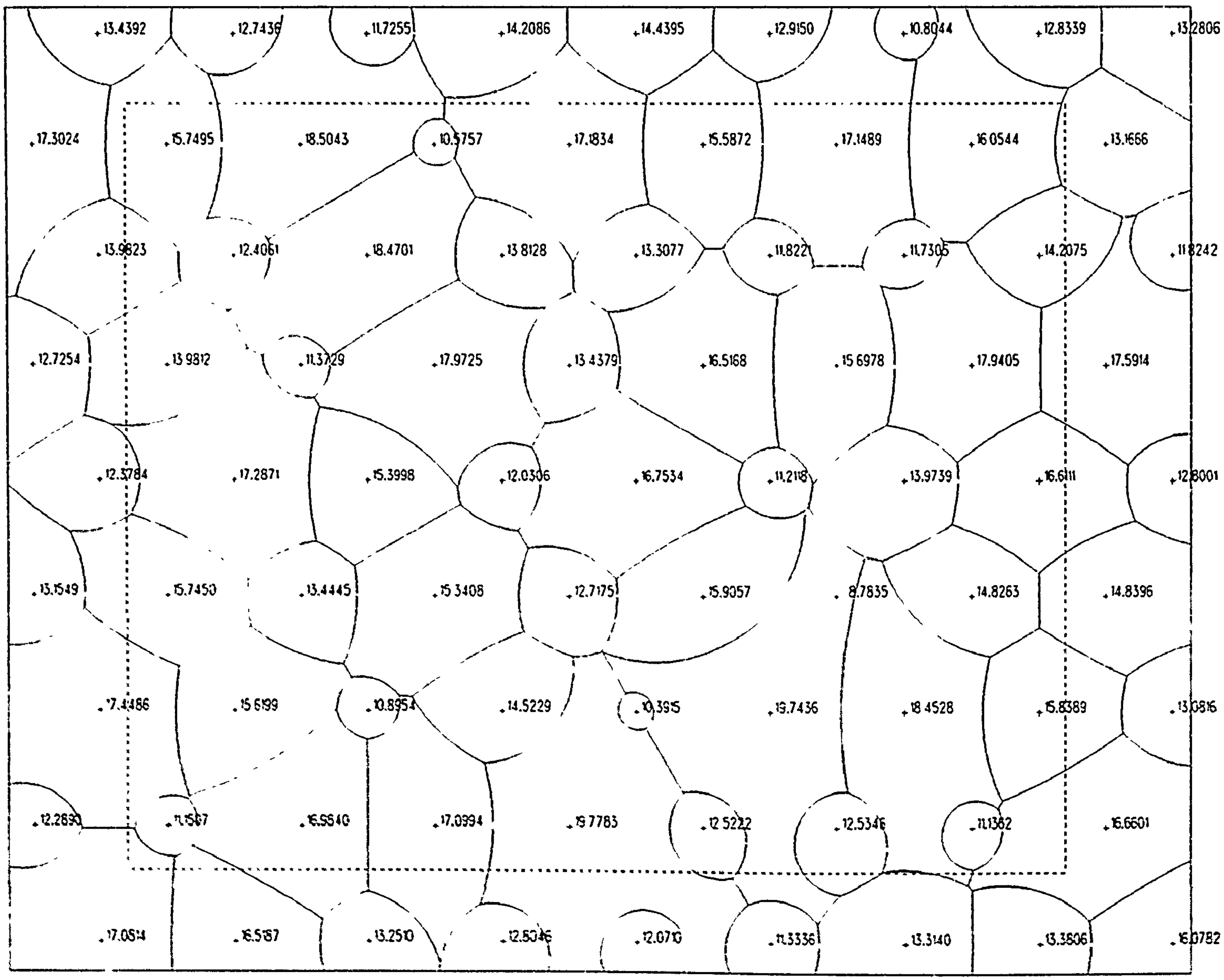
rho -1.2 m Var = 3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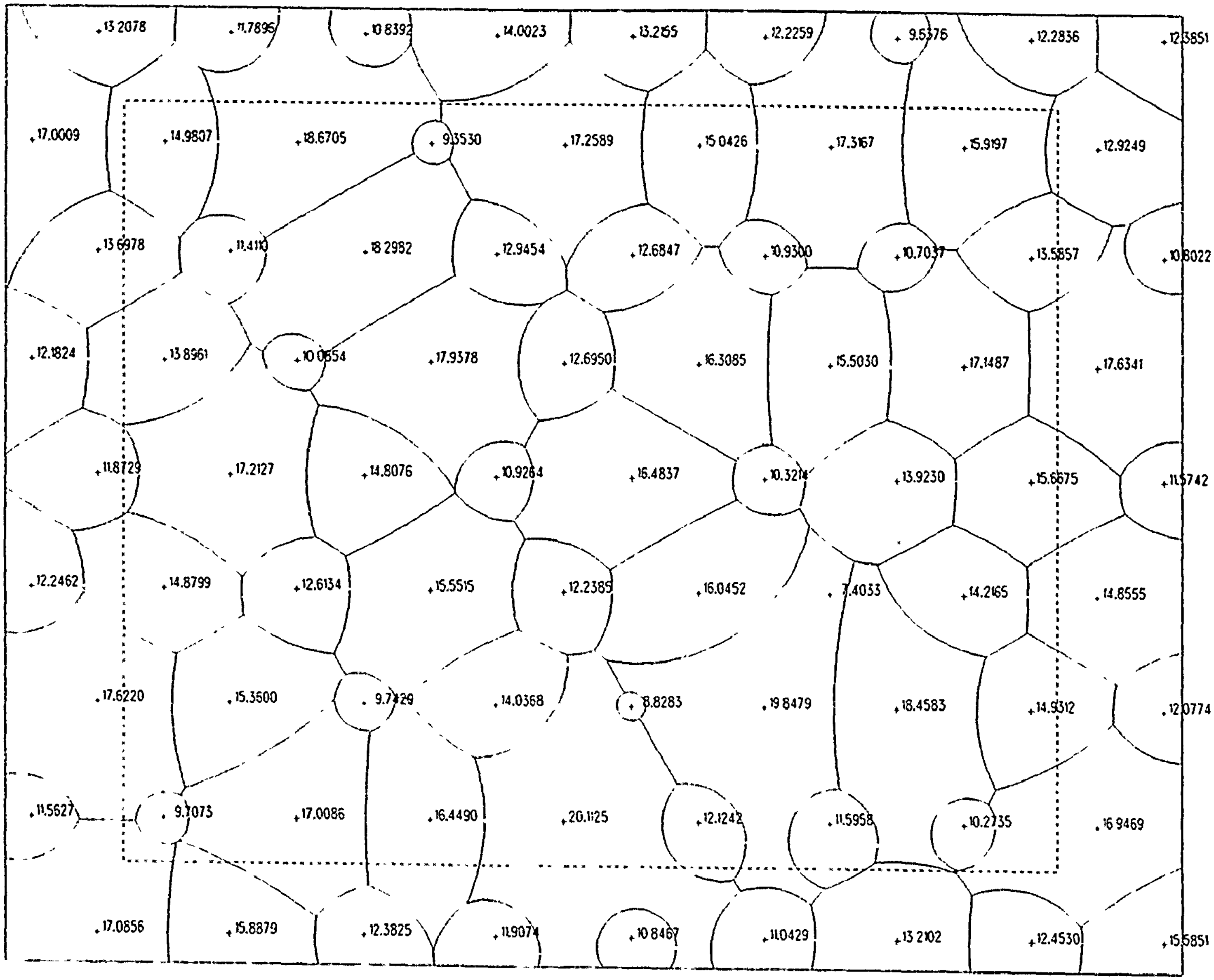
rho -1.5 m Var = 3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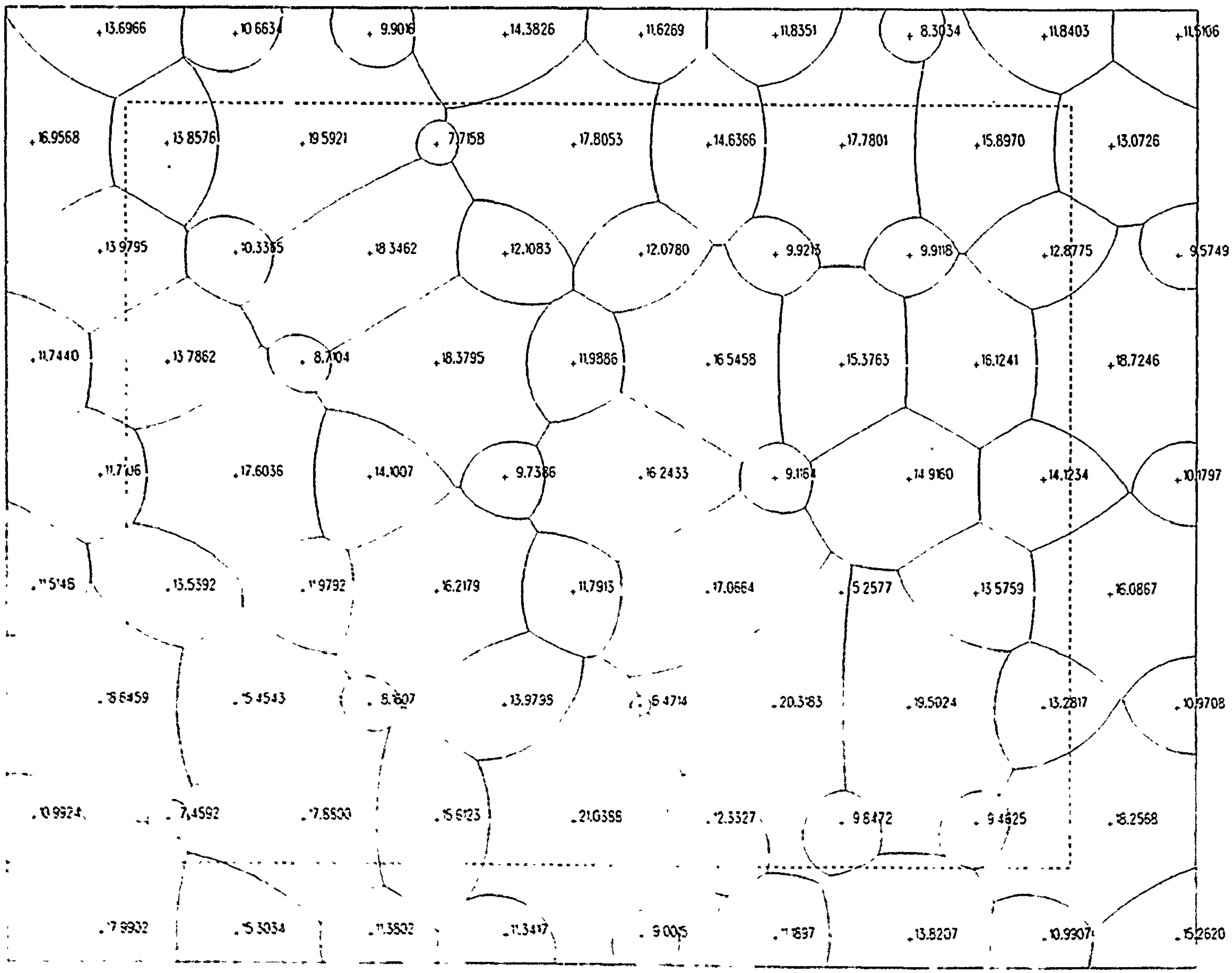
rho -1.8 m Var = 3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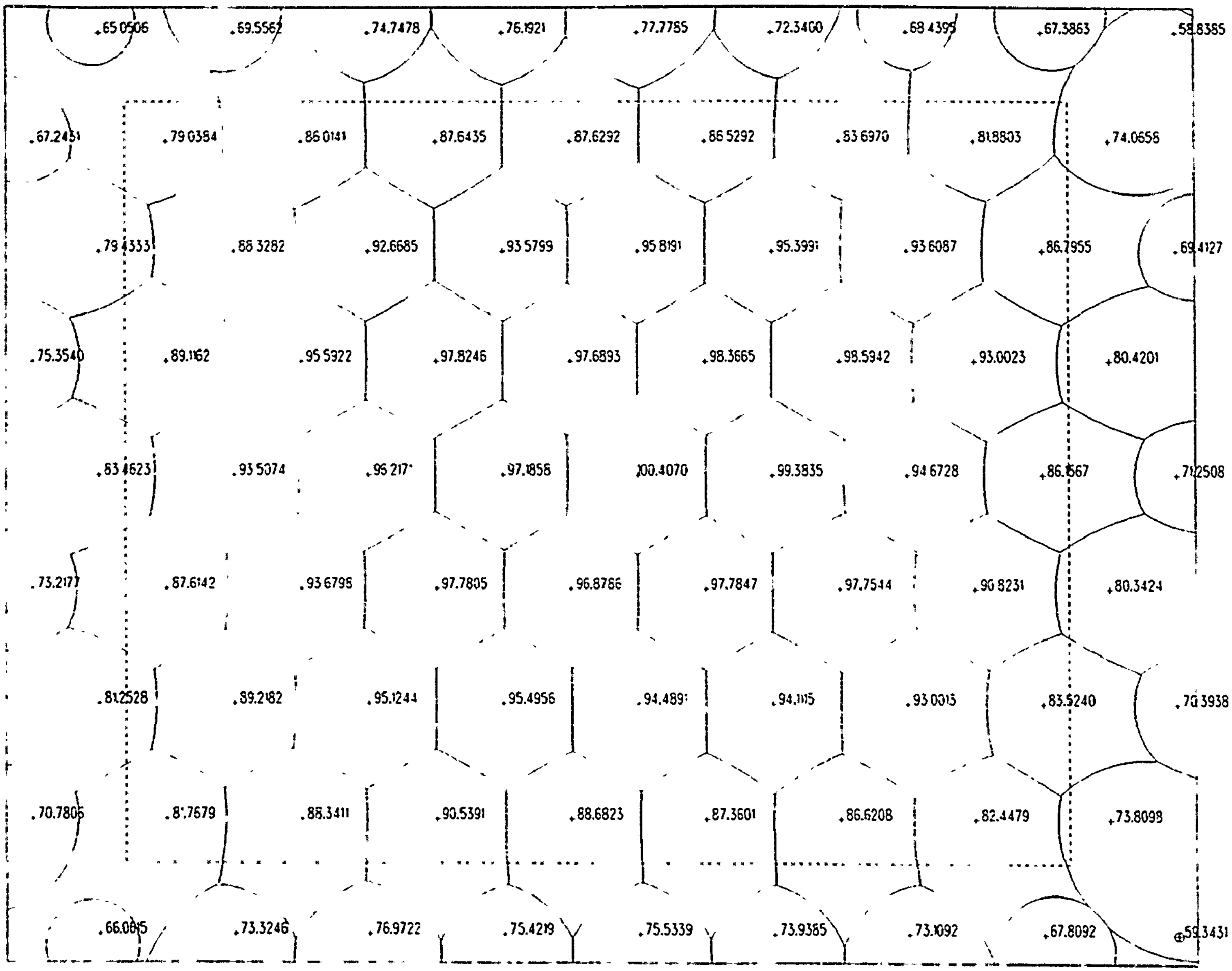
rho 0.9 m Var : 4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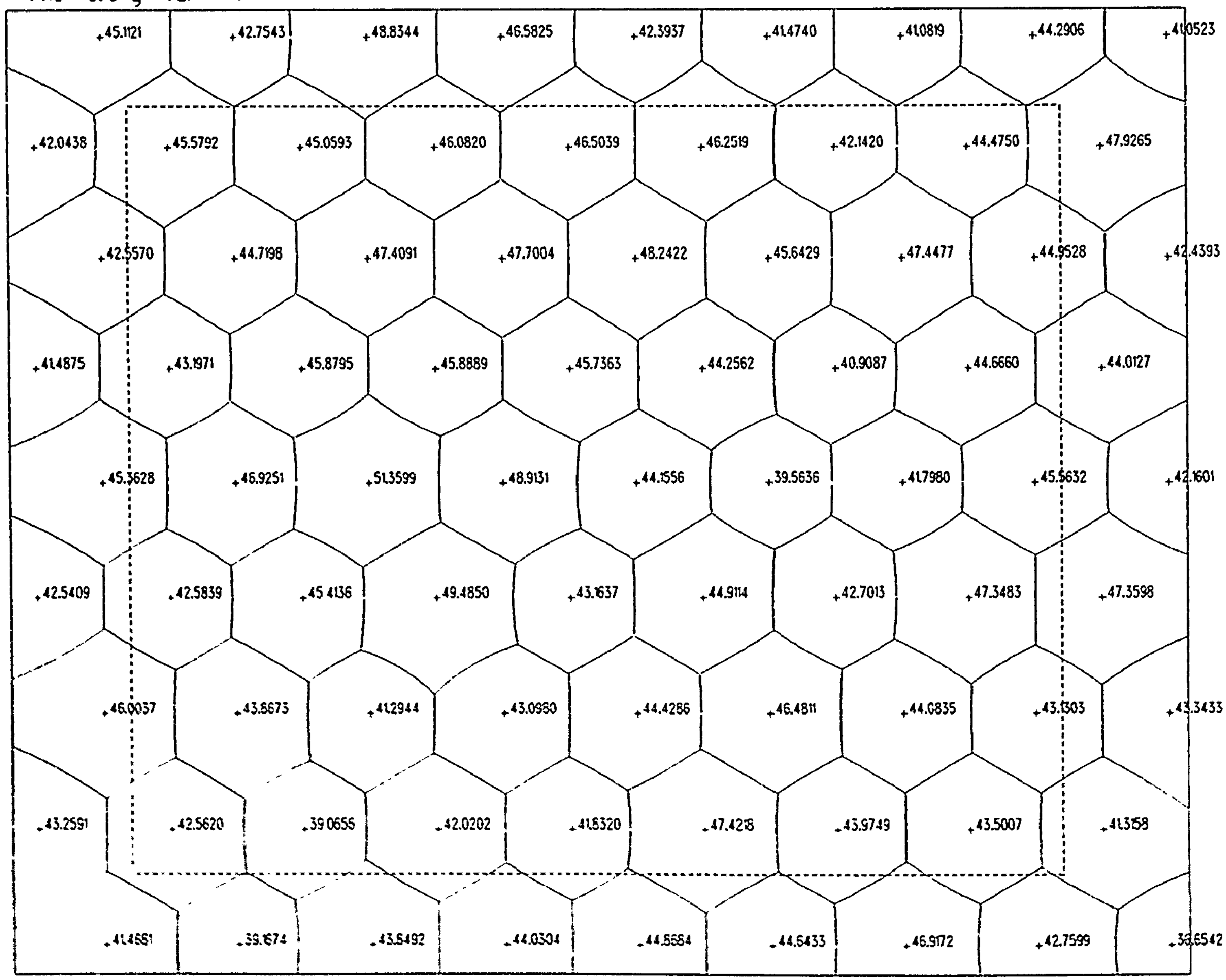
rho 0.6 g Var = 4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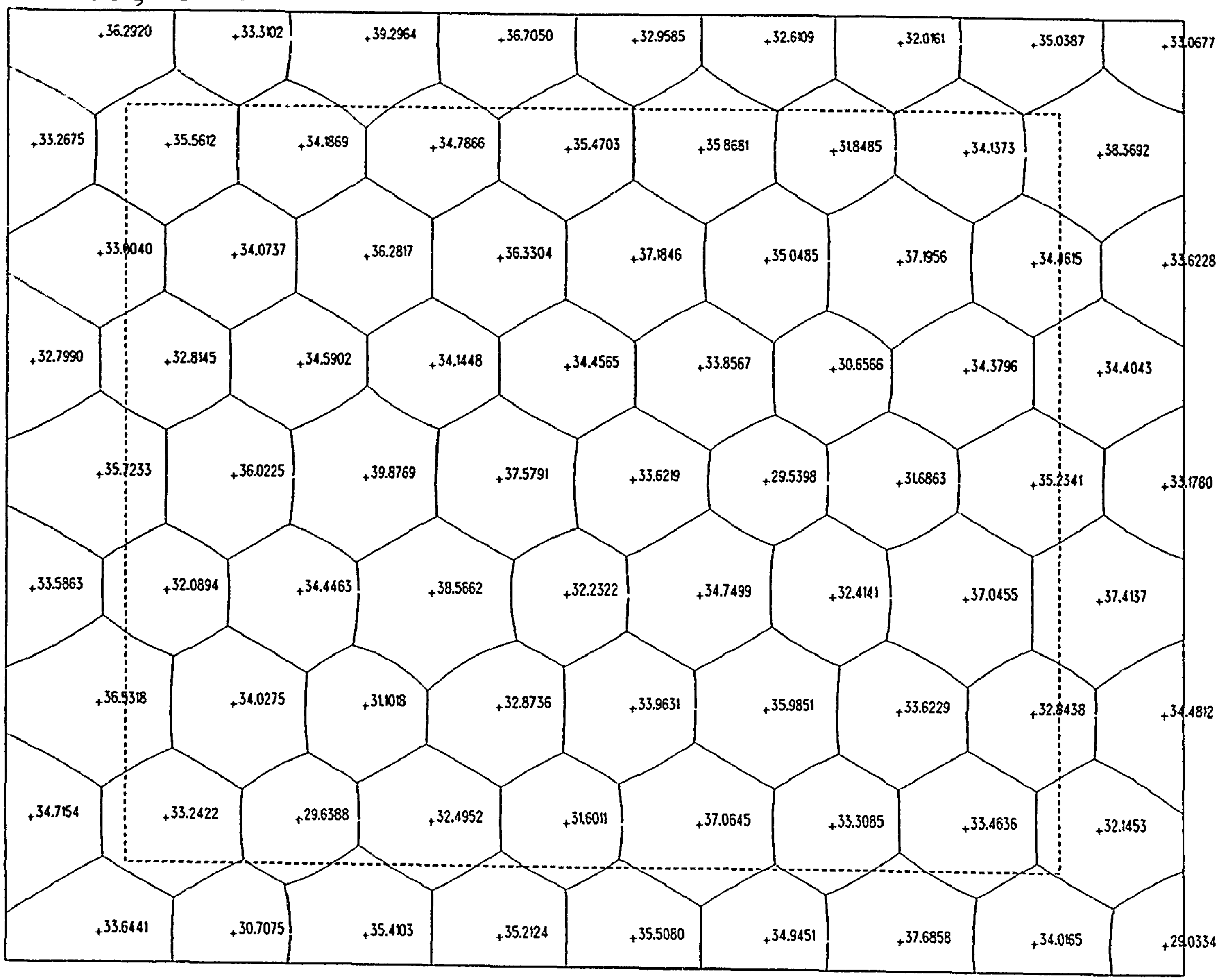
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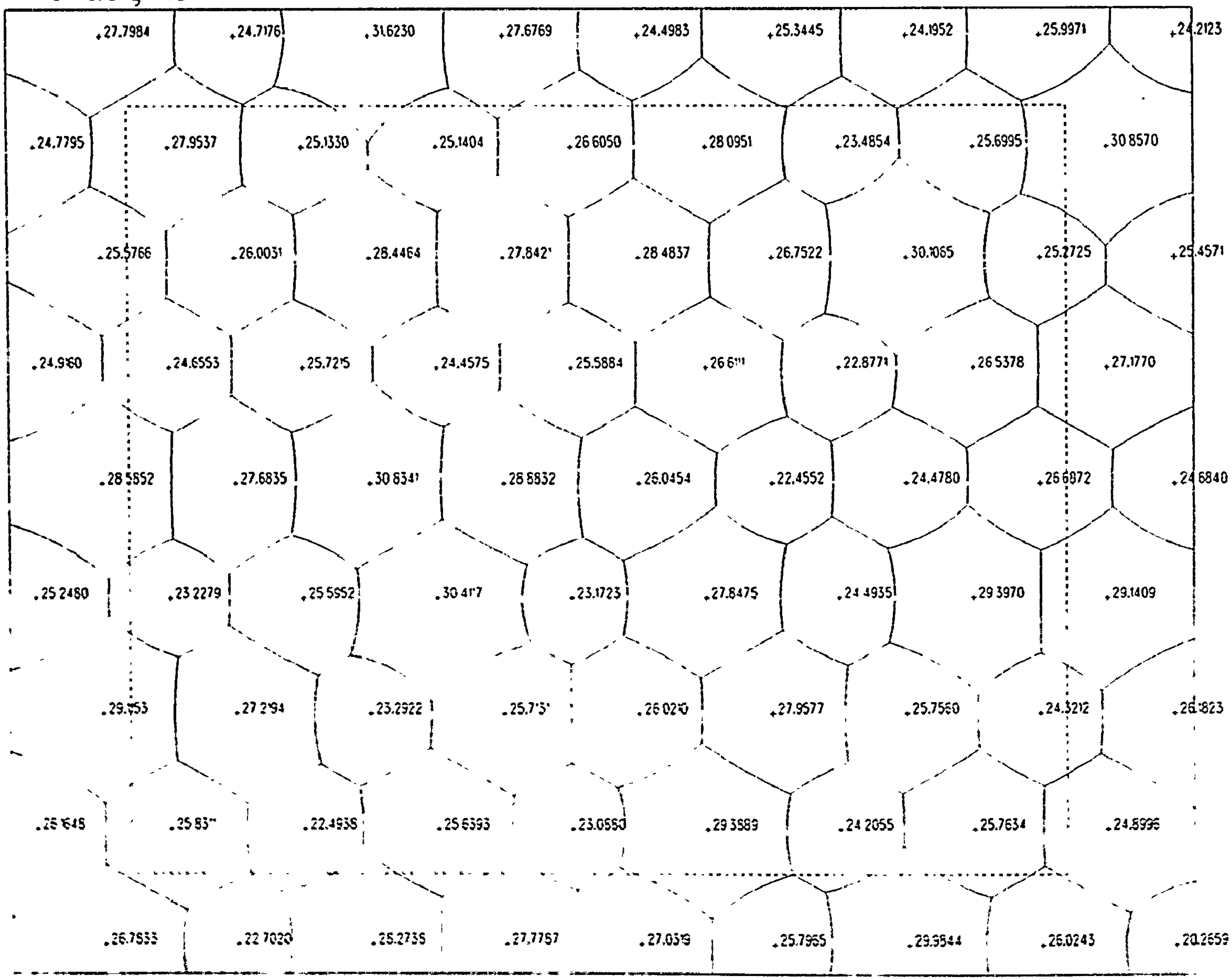
rho -0.6 g Var = 4



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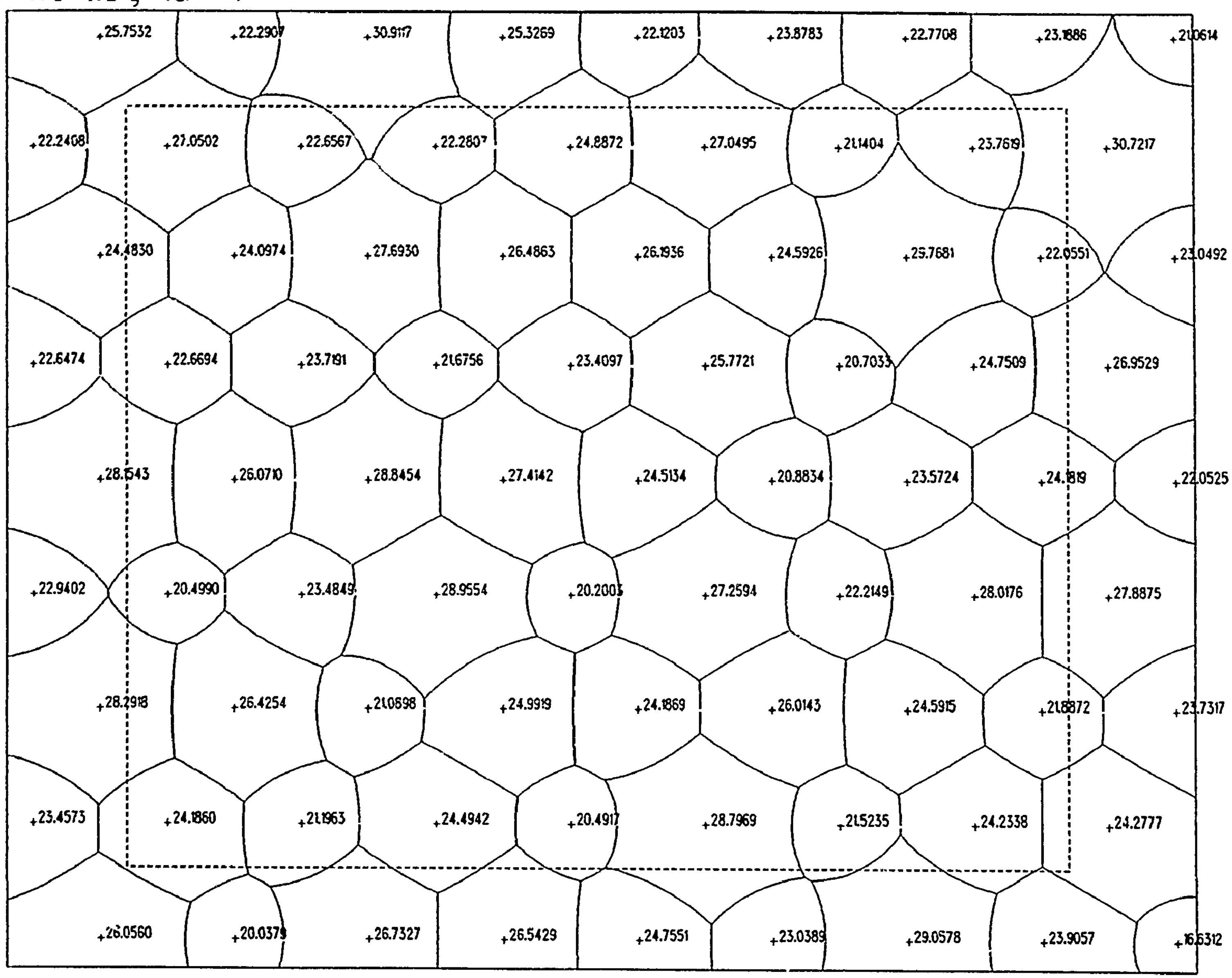
rho -1.2 g Var = 4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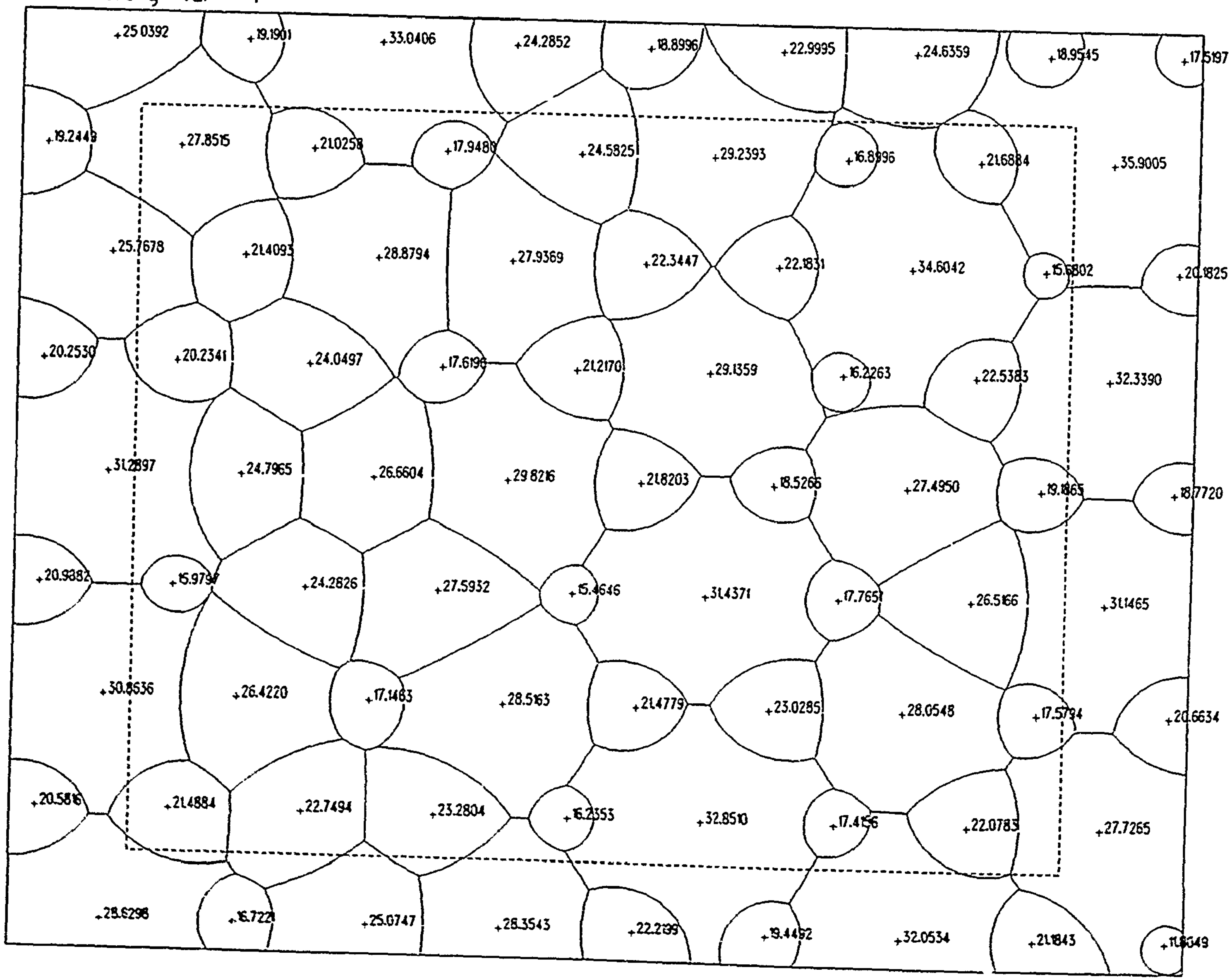
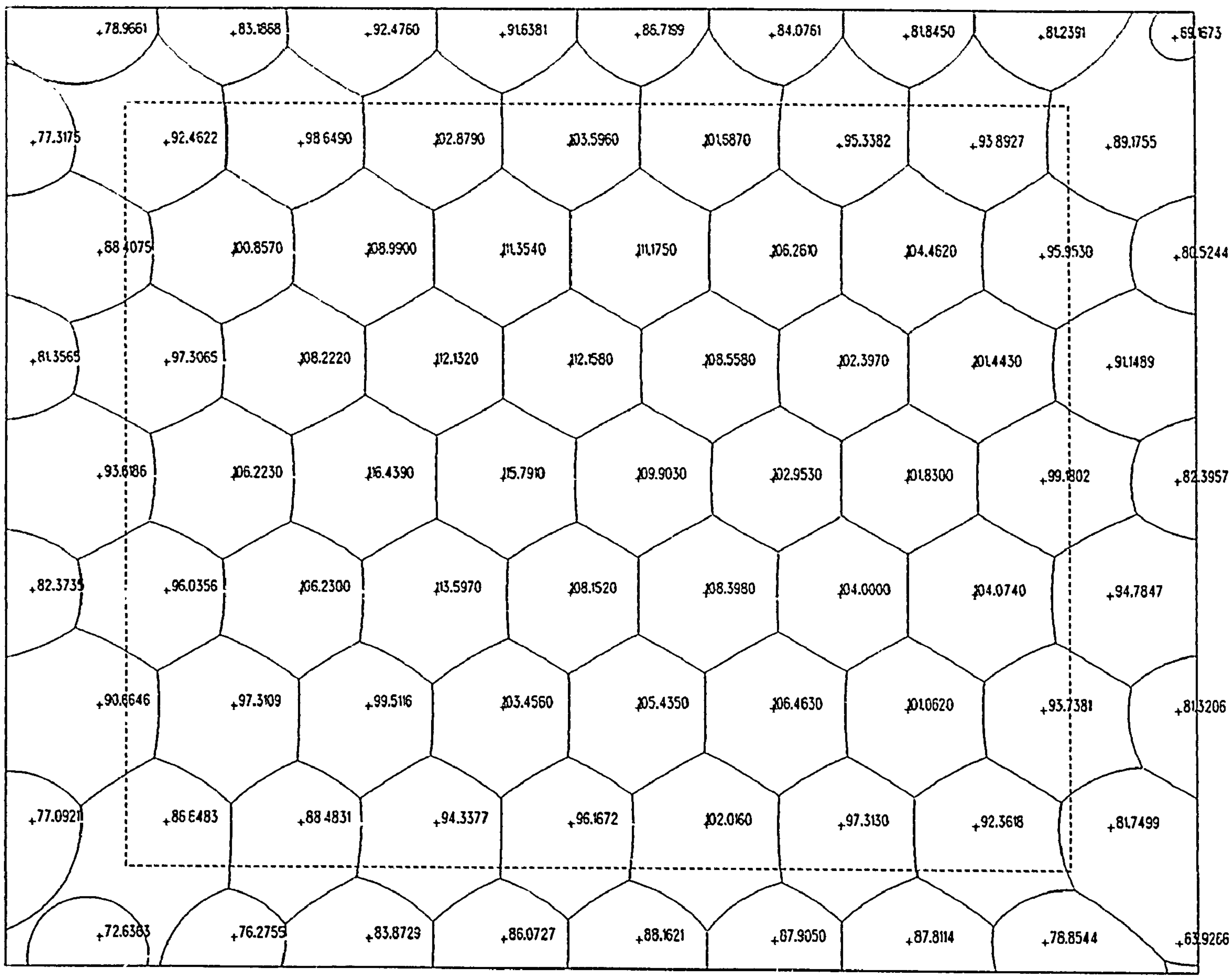
rho -1.8 g Var = 4



rho 0.9 m Var = 4



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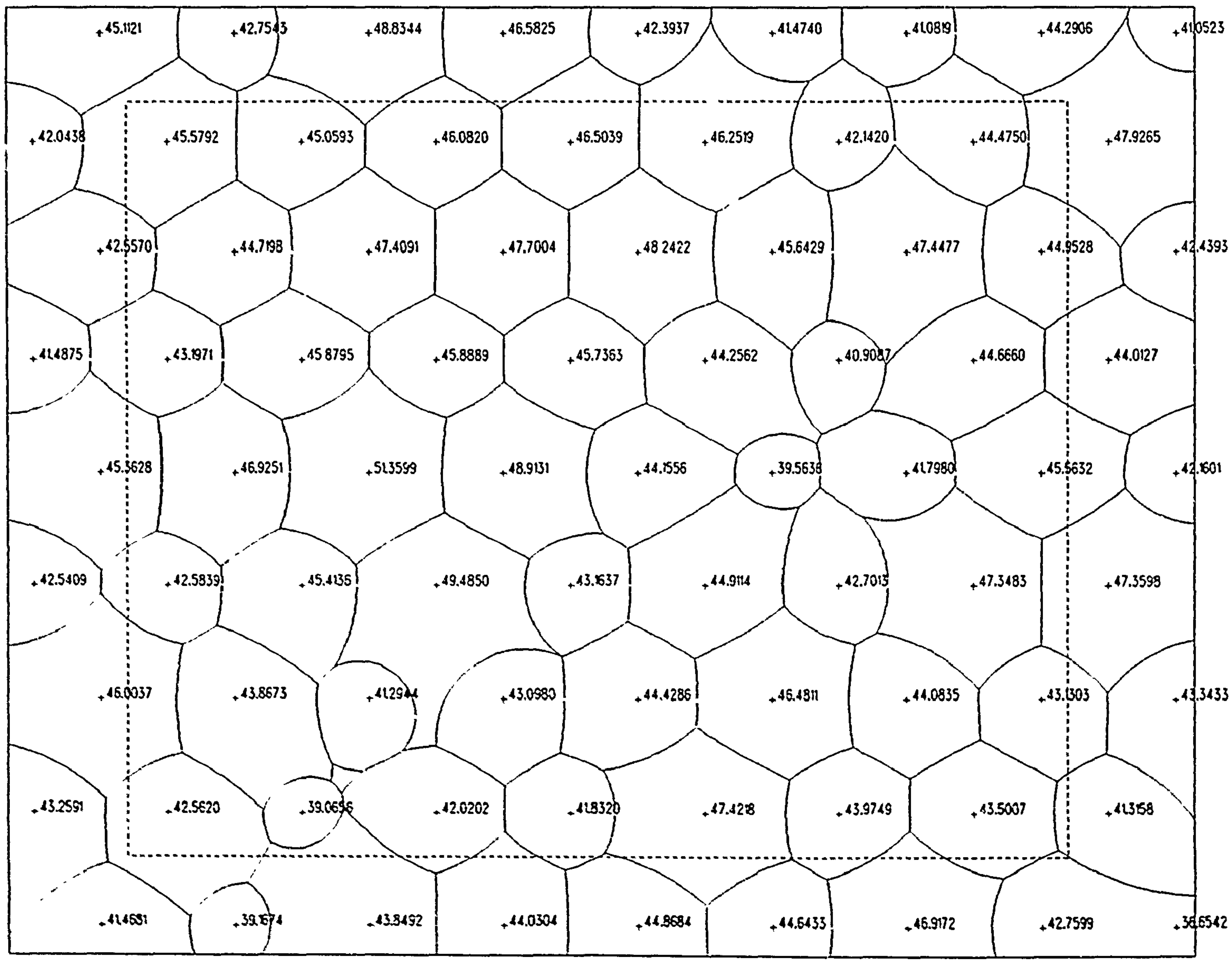
rho 0.6 m Var = 4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42.5409 42.5839 45.4136 49.4850 43.1637 44.9114 42.7013 47.3483 47.3598
46.0037 43.8673 41.2944 43.0980 44.4286 46.4811 44.0835 43.1303 43.3433
43.2591 42.5620 39.0696 42.0202 41.8320 47.4218 43.9749 43.5007 41.3158
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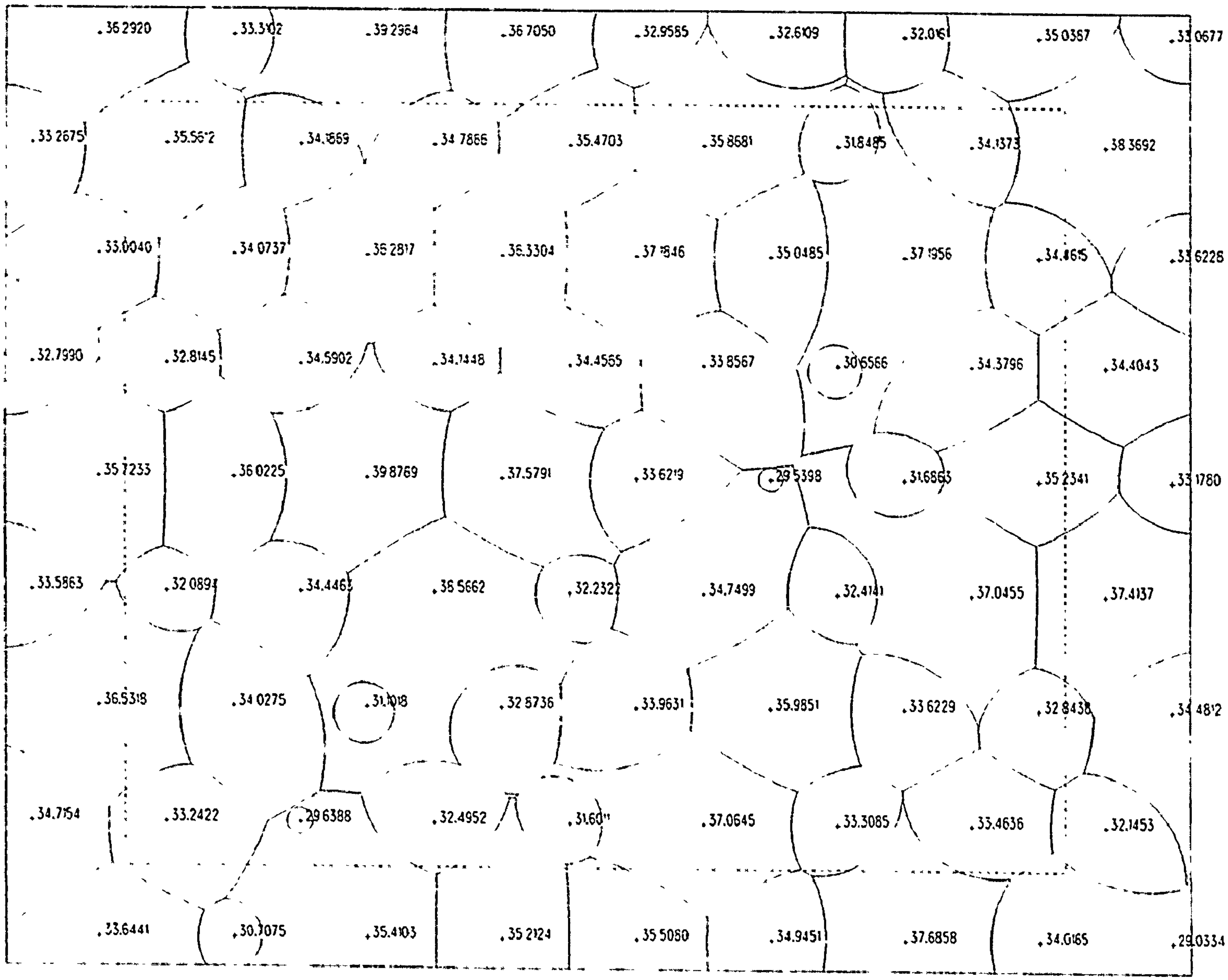

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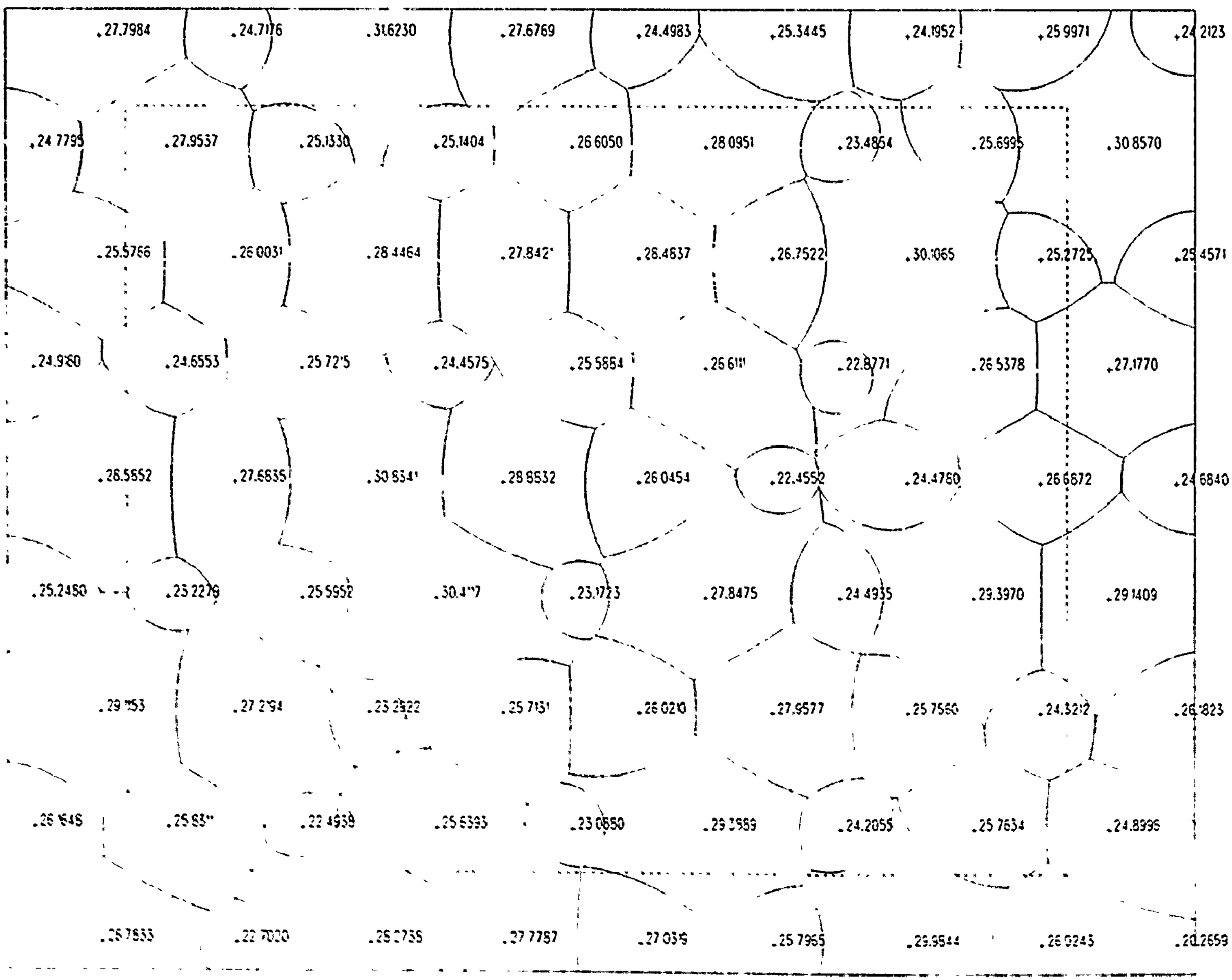
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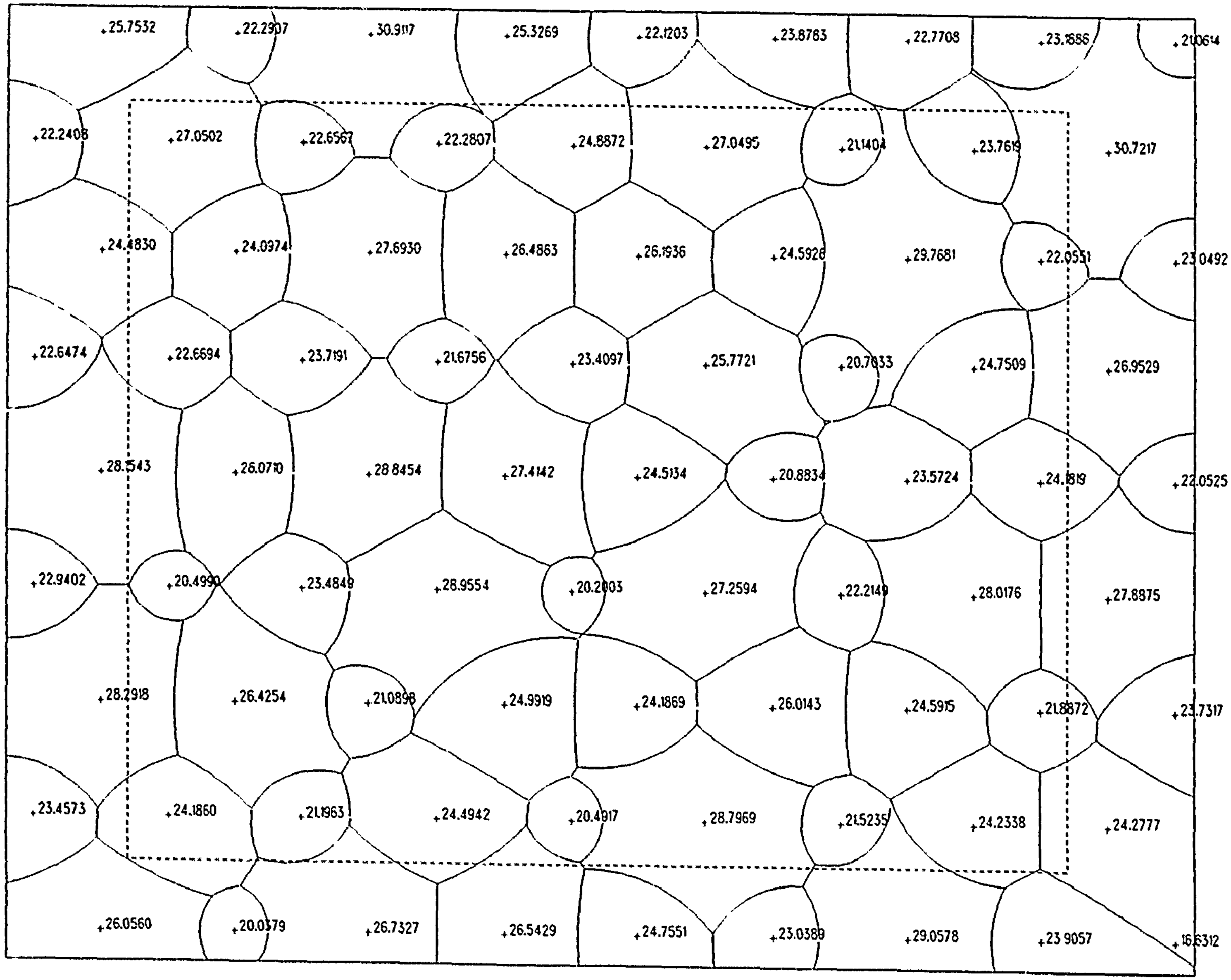
rho -1.2 m Var = 4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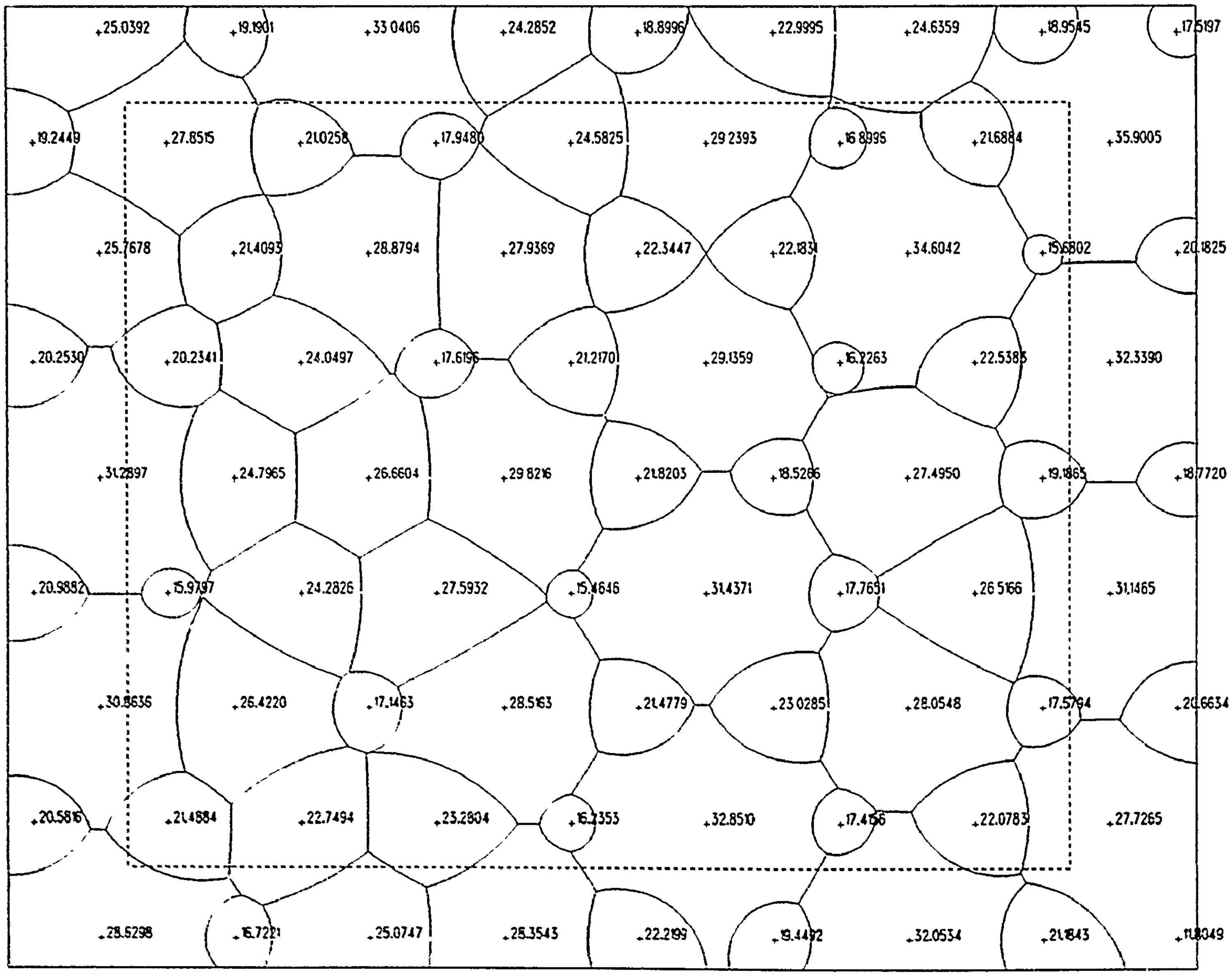

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