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Topologic structure of cellular networks a spatial autocorrelation approach

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TOPOLOGIC STRUCTURE OF CELLULAR NETWORKS: A SPATIAL AUTOCORRELATION APPROACH

BY

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CHAPTER ONE

INTRODUCTION

A traditional aim of geographic research has been to describe and explain spatial patterns of objects and events (Harvey, 1969). Pattern can be defined as a characteristic of spatial arrangement and implies some sort of detectable organization or spatial relationship between elements of the observed phenomena. Since it is quite unlikely that geographic distributions are a result of equally probable events and thus conceptualized as purely random, it is therefore expected that most map patterns will reflect some sort of system or order (Dacey, 1964).

Geographic phenomena distributed in space can be conceptualized in three basic geometric forms; points, lines and areas. These geometric forms are representative of numerous objects and by conceptualizing the elements of spatial patterns in one of these ways it is possible to translate information about the pattern into mathematical language. Typical examples of patterns of points are the distribution of cities in Canada, adopters of an innovation in a study area or supermarkets in an urban area. Examples of patterns of lines include railway networks of particular countries and information channels such as telephone links. Patterns of areas include such things as maps of soil types, climatic types and surface geology. However, many patterns of areas can be represented in two dimensions as a set of contiguous, non-overlapping, space exhaustive polygons. Such patterns are usually referred to as cellular networks by geographers (Haggett, 1967). Examples of such occurances in physical geography are drainage basins (Woldenburg, 1970), cooling cracks in basalt (Beard, 1959; Smalley, 1966) and territories of some birds and mammals (Nice, 1964; Hamilton, 1971) to name a few. In human geography (man - made patterns) there are such examples as central place hinterlands (Berry, 1967; Woldenburg and Berry, 1967), politico-administrative units (Haggett, 1967), market areas and bus service centres (Getis and Boots, 1978)

This research concerns itself with patterns of areas which can be considered cellular networks. However, there are many aspects of cellular networks which can be studied. These include for example, the features of individual units in the patterns such as size, shape, relative locations and so on. Further, such studies may concentrate on the networks at one point in time or be dynamic and examine changes in these features.

Geometry is an appropriate mathematical technique which may be used to discuss aspects of spatial relationships and morphology and to generalize about spatial patterns. One branch of geometry that is particularly useful in this regard is topology (Harvey, 1969, p. 218). In the present research on cellular networks it is the topologic property of contact number which is studied. Contact number is simply a count of the number of contacts between one cell (area) in a network and adjacent cells. For example in Figure 1, the contact number of cell H is 6. It is a topologic measure since the extent and nature of each contact is not considered.

More particularly, the study is concerned with the spatial arrangement or pattern of contact numbers within cellular networks. The arrangement of contact numbers is studied because it can provide information about the spatial structure of the cellular networks.

There are two fundamental reasons why it is important for geographers to study spatial structure. Firstly, the relationship between process and form (spatial structure) is generally accepted as a fundamental concern of geographers (Harvey, 1969, p. 129). It is this concern which prompts the researcher to examine map patterns in an effort to reveal signs of spatial process. The researcher acknowledges that to identify and describe a pattern within a network is not necessarily to know the process at work in that network. Properties of spatial structure may facilitate the analysis concerning the processes influencing the pattern. When this information is then set in a wider context it can contribute to the understanding of process by providing a 'stepping stone'

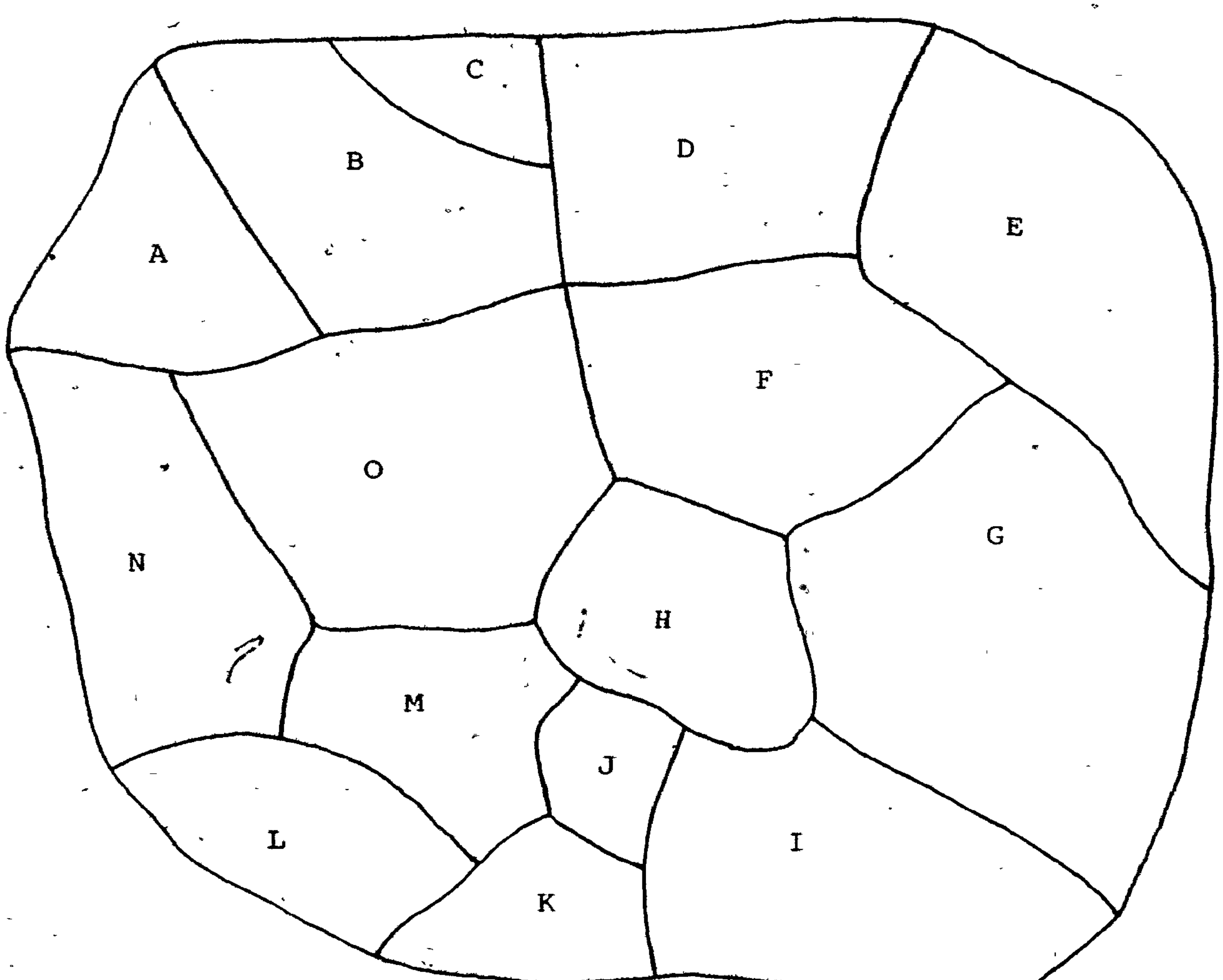

Figure 1: A typical cellular network

in the explanation (Smith, 1975, p. 348). Secondly, quite often in geographic research many processes (for example migration, diffusion and demographic change) are examined by studying their rates of occurance over space. Frequently, the collection units for such data are in the form of cellular networks, for example census tracts, police precincts, trade areas, counties, provinces. In such instances, there is the possibility that the behavior of the data may be a function of two kinds of forces: a) those inherent in the processes themselves; and b) those imposed by the nature of the collection units. It is necessary, then, to know the latent spatial structure (convolution of spatial interdependence and spatial arrangement) of the areal units in the cellular network in order to isolate the processes. In the present research, this is done by examining the variable contact number.

Having obtained information (numerical) about the latent spatial structure of the cellular network, researchers may then relate this to the information about specific processes operating over the network. In turn, evaluation of these relationships permits a conclusion to be drawn regarding whether or not the processes and their rates are independent of spatial structural components or are an artifact of the spatial structure of the areal units (Griffith and Jones, 1980, p.190). If the latter is the case, then the numerical information about the cellular network structure must be incorporated into any models devised for evaluating spatial process.

Discussion of Related Studies

The first studies of cellular networks were mainly analytical in nature and focused on the derivation of summary statistics for various properties, for example, area, length of perimeter, edge length, shape of individual cells (Evans, 1945; Meirjering, 1953; Dacey, 1963; Corte and Higashi, 1964). The most widely examined variable is contact number. Haggett (1965) was one of the first to examine contact number. At first he suggested that examination of the mean contact number value of a network might reveal something about the nature of the process responsible for the network evolution (Haggett, 1965, p.51). As a result of his work other researchers began to undertake similar studies. (Rasheed, 1973; Smalley, 1966). However, it was subsequently established (eg. Woldenburg, 1972 and Matschinski, 1969) that many of the properties studied, including contact number, were insensitive to differences in the nature of the processes generating the cellular networks and consequently were of little analytical value. Thus studies were developed primarily by means of Monte Carlo simulation techniques to examine the complete distribution of random variables describing properties of the individual cells. Crain (1972) for example derived the expected contact number proportions for a Poisson generated pattern by a simulation approach and Boots (1977) examined the distribution of contact number values under three different network generations; Poisson process, Compound negative binomial process and Smalley process, to determine whether different generating processes produce cellular networks whose contact number properties are distinctive.

There were also attempts to determine the nature of the relationships between individual properties either in pairs (Crain, 1978) or collectively (Boots, 1975). However, regardless of the approach used there was a limitation. The studies were aspatial. Neither moment nor distribution measures reveal anything about the arrangement of individual contact numbers within the network.

Initially, there were only a few exceptions to this approach in geography which provided some vague and implicit considerations of geometric structure (Cox and Agnew, 1974, and Boots, 1975). Some studies unrelated to geography however have concerned themselves with the arrangement characteristics and exclusively with contact number. Aboav (1970) examined a sample of 3,000 cells produced by taking sections through polycrystalline magnesium oxide. For this sample Aboav found that the value of $\bar{m}_n$, the average contact number for cells of a given contact number n, is described by

$$\bar{m}_n = 5 + 8 / n \qquad (3 \leq n \leq 8) \qquad (1)$$

which indicates that on average, cells with larger contact number values are likely to be surrounded by those with smaller contact numbers and vice versa. Aboav interprets this result as an indication that the arrangement of grains in the polycrystal is significantly non random.

In response to Aboav's empirical findings, Weaire(1974) argues that Aboav's results are most likely to occur under the basic laws of topology and are thus indicative of a random situation. He suggests that in networks which consist exclusively of trivalent vertices (ie. three and only three edges are incident at each vertex in the network), all edges are straight lines and any cell and its immediate neighbours average six cells, then

$$\bar{m}_n = 5 + 6 / n \qquad (3 \leq n \leq \infty) \tag{2}$$

Since Aboav's material and most other empirical situations do not satisfy the third of his conditions, Weaire suggests a more general form of (2), which is

$$\bar{m}_n = 5 + (6 + \mu_2) / n \qquad (3 \leq n \leq \infty) \tag{3}$$

where $\mu_2 = \sum (n - 6)^2 f_n$

and f_n is the proportion of cells with n sides.

Despite the discrepencies between these three equations on the whole they indicate that on average cells with larger contact number values are likely to be surrounded by cells of smaller contact number values and vice versa.

Boots (1979; 1980) explored the arrangement of contact number values in 'geographical' cellular networks, specifically Indian administrative divisions and English civil parishes, using the Aboav and Weaire equations and concluded that the arrangement of the divisions within the overall map patterns are not significantly different from those likely to arise under chance conditions.

In further work with soap films, Aboav (1980) finds the expression

$$\bar{m}_n = 6 - a + (6a + \mu_2) / n \qquad (4)$$

where a is a constant equal to 1.2, and μ_2 is the same as in equation (3). However, Boots (1982) is unable to find support for Aboav (1980) in studies of theoretic models of random networks.

Outline of the Study

Chapter One has introduced the topic of the research and the research objective and also reviewed some related studies. Chapter Two will deal with a description of the methodology used in the analysis of the topologic structure of cellular networks. Chapter Three introduces the data used in the analysis. Chapter Four will present and analyze the results obtained from the application of spatial autocorrelation statistics to the networks and also deal with problems associated with applying this procedure to the data. Chapter Five presents an evaluation of the spatial autocorrelation statistics and their behavior in the context of the research. Chapter Six is a discussion of the 'boundary problem' as it affects the statistics and the analysis of the cellular networks. Chapter Seven summarizes the study's findings and discusses possible future research in this area.

CHAPTER TWO

METHODOLOGY

The approach adopted here to examine spatial structure of cellular networks is to test the topologic measure contact number for spatial autocorrelation. Spatial autocorrelation is a measure of dependency within the map pattern and ascertains the extent to which the location of a variate value at some location within the pattern is related to values at other locations. If there is a dependency over space, the data is said to be spatially autocorrelated. Basically, if high values of the variate at one locality are associated with high values at neighbouring localities, the spatial autocorrelation is positive and if high and low values alternate the spatial autocorrelation is negative. Thus it may be argued that spatial autocorrelation is an explicit measure of spatial structure (Gatrell, 1977a; 1977b). Here the variable (variate value) examined is contact number value of an individual cell (areal unit) and so it can be suggested that what is being considered is the topologic structure of the network. The specific research question then becomes, 'What is the relationship between the contact number value of a cell and that of neighbouring cells?' However, the intent of the research is not only to explore the topologic structure but also to illustrate an approach to the analysis. Spatial autocorrelation studies represent a direct contribution to the search for order in spatial phenomena and test the notion of spatial independence, an assumption underlying the use of various statistical procedures (Silk, 1979, p. 114).

Before attempting to discuss the research question and how one might measure this relationship using spatial autocorrelation techniques it is necessary to consider the ways in which cellular networks may be represented. Besides its original form (see Figure 2) the network may be represented by its dual, an adjacency graph, G. In this transformation the cells of the original cellular network become the vertices of the adjacency graph (location for such vertices may exist already as nodes in individual cells or may have to be arbitrarily assigned, but their location is of no consequence since the research is concerned with the topologic structure) and the boundaries between cells become the edges of G (see Figure 2). In this way contact numbers in the cellular network become equated with the vertex degrees in G. In this form the network is a connected planar graph. G, in turn can then be represented as a binary matrix A (see Figure 3) in which the rows and columns represent individual elements. The row and column sums of G are equal to the individual contact numbers of the original network. The entries in the body of the matrix represent the relationship between the individual elements ij. If an edge (join) exists between any pair of vertices i and j, δij and δji equals 1. For every pair of localities that are not connected $\delta ij = \delta ji = 0$. The resulting

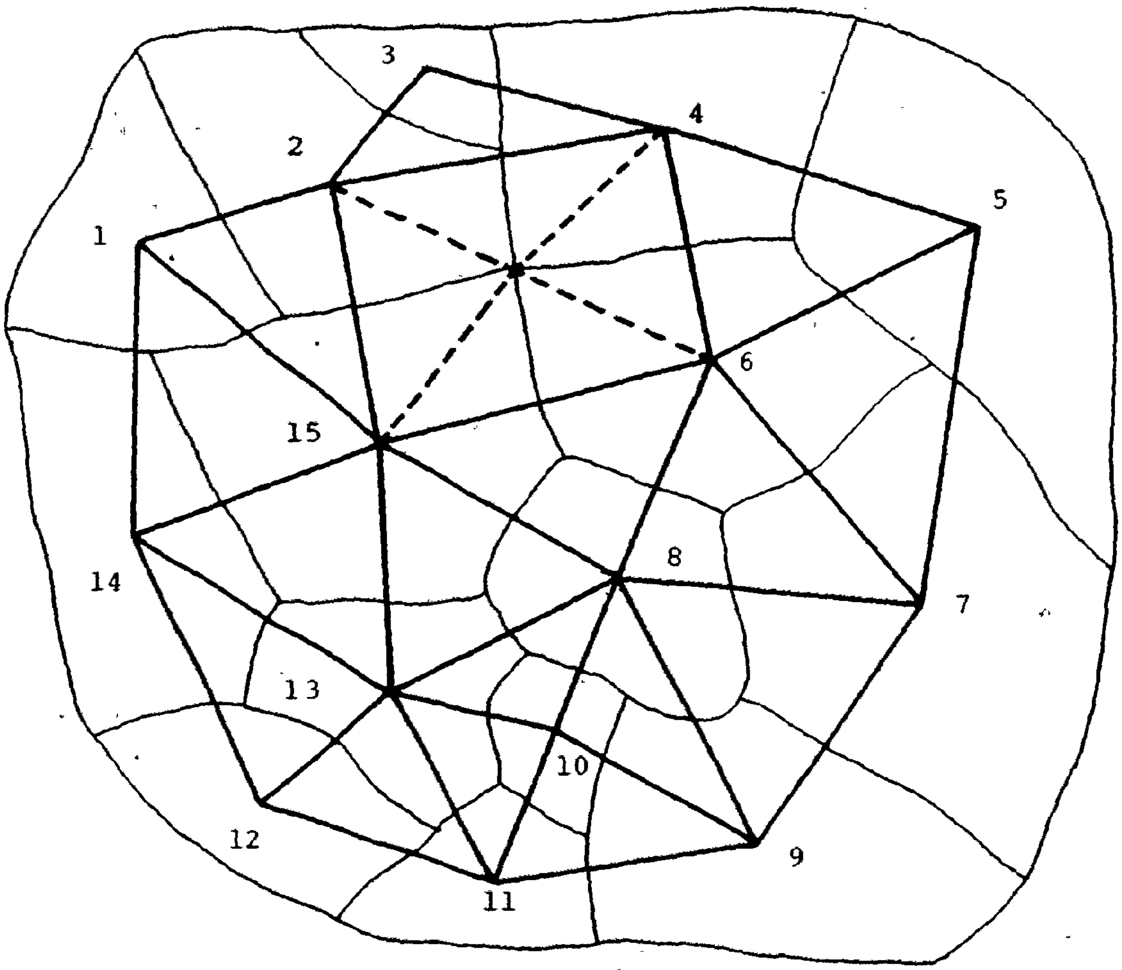

Figure 2: Representation of a cellular network as a graph

———	Cellular network
———	Graph representation
———	Rook's case
- - - -	Queen's case

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	Σ
1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	1	3
2	1	0	1	1	0	0	0	0	0	0	0	0	0	0	1	4
3	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	2
4	0	1	1	0	1	1	0	0	0	0	0	0	0	0	0	4
5	0	0	0	1	0	1	1	0	0	0	0	0	0	0	0	3
6	0	0	0	1	1	0	1	1	0	0	0	0	0	0	1	4
7	0	0	0	0	1	1	0	1	1	0	0	0	0	0	0	4
8	0	0	0	0	0	1	1	0	1	1	0	0	1	0	1	6
9	0	0	0	0	0	0	1	1	0	1	1	0	0	0	0	4
10	0	0	0	0	0	0	0	1	1	0	1	0	1	0	0	4
11	0	0	0	0	0	0	0	0	1	1	0	1	1	0	0	4
12	0	0	0	0	0	0	0	0	0	0	1	0	1	1	0	3
13	0	0	0	0	0	0	0	1	0	0	1	1	0	1	1	5
14	1	0	0	0	0	0	0	0	0	0	0	1	1	0	1	4
15	1	1	0	0	0	1	0	1	0	0	0	0	1	1	0	6
																60

Figure 3: Matrix representation of Figure 2.

adjacency matrix is symmetrical in that $\delta_{ij} = \delta_{ji}$ and in this research the elements of the matrix are restricted to these binary weights. The criterion for connecting any pair of vertices with an edge will vary with the problem and purpose of the investigation. There is more than one way to obtain join counts. There is the rook's case (cells have a common edge), the bishop's case (cells have a common vertex), and the queen's case (cells have either a common vertex or a common edge) (see Figure 2). In this study the edges of the graph are based on first order connectedness between the points and the form of the connection is the rook's case. In this way the resulting graphs are planar. For analysis, this research will deal with cellular networks in the form of either G or A.

The study will now turn to considering ways to measure the extent of spatial autocorrelation in the vertex degrees (contact number). Potentially, several approaches are possible. One would be to fit trend surfaces to the data as suggested by Haggett et al. (1977), with the x, y, values representing the location of the vertices and the z values their degrees or the values of the contact number. Such a method would identify positive spatial autocorrelation but would probably be unable to distinguish between negative and no spatial autocorrelation because in the case of negative spatial autocorrelation, the technique has to take into account abrupt changes in characteristics from one locality to another.

Instead of showing a distinct departure of one locality from another, the distiction would tend to be 'smoothed out' and would not appear defined. In addition the surfaces produced would be at least partially dependent on the location of the vertices which may be, on occasion, arbitrarily assigned.

A second technique to measure spatial autocorrelation involves examining join counts. Here one would record the frequency with which a vertex of a given degree k is linked to vertices of the same or other degrees n. Using the frequency distribution of vertex degrees for the entire graph, the expected frequency of k-k and k-n links could be determined under the assumptions of random linkages. The observed and expected frequencies could then be compared for goodness of fit using a chi-squared or similar test. However, there are various problems associated with this technique. The problem in applying such a procedure stems mainly from the nature of the empirical networks. Typically in such networks the number of vertices is less than 100 while the range of degree values may be relatively large (say from 1 to 14). Thus most of the expected frequencies will be small and retaining sufficient degrees of freedom for the goodness of fit test will be difficult, if not impossible.

A third approach, and the one adopted in this study, is to utilize spatial autocorrelation coefficients. Most of those now used have their origins in the two coefficients proposed by Moran (1950) and Geary (1954). These are basically join count statistics. Dacey (1968) used the Geary and Moran correlation coefficients in his study of measures of contiguity for two and k coloured maps. Cliff and Ord modified the statistics and proposed a generalization thus providing for a variable weighting of the various geographic regions rather than a binary classification of pairs of regions as joined or not joined. The use of a generalized weighting matrix as opposed to a binary connection matrix allows the investigator to choose a set of weights which are appropriate for the particular analysis (Cliff and Ord, 1973). The modified statistics of Cliff and Ord have been used by various researchers, for example, Jones, 1978; Lebert, 1969; Sokal and Oden, 1978; Jumars et al, 1977; Haggett et al, 1977; to mention a few. This study uses the Moran statistic I and the Geary coefficient c in their original form as the weights under consideration are binary.

Moran's statistic has the following form

$$I = \frac{n}{2A} \frac{\sum_{i}\sum_{j,\, i \neq j} \delta ij (z_i zj)}{\sum_{i=1}^{n} zi^2} \qquad (5)$$

while Geary's coefficient c, is given by

$$c = \frac{(n-1)\sum_{i}\sum_{j} \delta ij(x_i - x_j)^2}{4A \sum_{i=1}^{n} z_i^2} \quad (6)$$

with $i \neq j$ in the double sum.

where : n is the number of observations (cells)

x_i, x_j are the variate values of the ith and jth observations respectively

$\bar{x} = \sum_i x_i/n$

$z_i = x_i - \bar{x}$

$z_j = x_j - \bar{x}$

$\delta ij = 1$ if i and j are neighbours, 0 otherwise

$A = \frac{1}{2}\sum_{i=1}^{n} L_i$ where L_i is the number of neighbours of the ith observation

Although the results computed by both statistics are similar they are not identical. Both are analagous to the Pearson correlation coefficient in that the numerator term is a measure of covariance among the $\{x_i\}$ observations and the denomentor is a measure of variance of the observations (Sokal and Oden, 1978, p. 207). The Geary coefficient c is a squared term and is the ratio of the sum of squared differences between contiguous regions and the sum of the squared differences of all observations from the mean (Taylor, 1977, p.121). Thus as c will depend on the absolute differences between neighbouring variates, it will be more significant when testing patterns of negative spatial autocorrelation (stresses unlike joins). The Moran statistic on the other hand is based upon the cross product of deviations of the x_i (variate values) from the mean (Cliff and Ord, 1973, p.8). Therefore I will be more strongly affected by marked join departures of two neighbouring values from their mean and will be more significant when testing patterns for positive spatial autocorrelation (stresses like joins).

The Geary statistic ranges from 0 for maximal positive autocorrelation to a positive value for high negative autocorrelation. Its expected value in the absence of spatial autocorrelation is 1. The Moran coefficient I varies from a maximum value of -1 for negative spatial autocorrelation and produces a positive score to a maximum of +1 for positive spatial autocorrelation, the expected value approaching 0 in the absence of spatial autocorrelation (Sokal and Oden, 1978).

In applying the spatial autocorrelation coefficients to cellular networks, one must specify the conditions under which the expected values would be realized (Cliff and Ord, 1973). In this research the specification takes the form of the null hypothesis. The null hypothesis tested is that there is no spatial autocorrelation in the contact numbers. That is to say, that the contact numbers within the network are randomly distributed within the network. Conversely the hypothesis would state that the contact numbers are autocorrelated and exhibit some sort of spatial dependency. This hypothesis may be tested under the assumptions of either normality or randomization. Under the assumption of normality one assumes that the $\{x_i\}$ are the result of n independent drawings from a normal population or populations (Cliff and Ord, 1973). The randomization assumption is used when the underlying distribution cannot be considered normal. It involves considering the observed value of I or c relative to the set of all possible values which I or c could take if the $\{x_i\}$ were repeatedly randomly permutated around the set of observations. There are n! such permutations (Cliff and Ord, 1973, p. 8). In effect we are asking if the observed pattern of $\{x_i\}$ values as judged by I or c is in any sense unusual in the set of all possible patterns that the $\{x_i\}$ could have formed (Haggett et al, 1977). Since we have no prior knowledge of the underlying distribution of contact number except for special model cases (Crain, 1972; 1978; Boots, 1977) which are known to be normal, this research adopts the randomization assumption,

Under the assumption of randomization, expectations of I and c are as follows

$$E(I) = -(n-1)^{-1} \tag{7}$$

$$E(c) = 1 \tag{8}$$

$$E(I^2) = \{n\ 4A(n^2-3n+3)-8(A+D)n+12A^2 \quad (9)$$
$$-b_2\ 4A(n^2-n)-16(A+D)n+24A^2 \} /$$
$$4A^2(n-1)(n-2)(n-3)$$

$$\mathrm{Var}(c) = \frac{1}{n(n-2)(n-3)2A^2} \{2A^2 - (n-1)^2 b_2 + (n^2-3)$$
$$+2A(n-1) - (n-1)b_2 + n^2 - 3n + 3 \quad (10)$$
$$+(D+A)(n-1) \ (n^2-n+2)b_2 - (n^2+3n-6) \}$$

where all the symbols are the same as in the previous equations 5 and 6 p.20 and

$$D = \tfrac{1}{2}L \sum_i (L_i - 1)$$

b_2 is the sample kurtosis coefficient $m_4/m_2{}^2$, m_j being the jth sample moment of the $\{x_i\}$ about the sample mean.
(Cliff and Ord, 1973, p. 8-9)

Since many of the empirical networks are finite and typically small $(n<100)$ it is necessary to exhaustively sample the population of values in the graph. Such a system generates the problem of what to do with the boundary cells (vertices). Boundary cells are those polygons or areal units of the cellular network lying on the circumference of the study area consisting of n areal units. They delimit the cellular network as a finite size. Several possibilities are available. The first is to retain them in the analysis, but if this is done a ring of boundary vertices with low degrees (which in the trivalent vertex case will average 4) is imposed on the inside vertices (average 6). The effect of such a boundary is not consistent and depends on the nature of the correlation (see Chapter 5). A second possibility is to disregard the boundary vertices, however, in doing so some of the remaining vertices will lose some of their edges as elements of the matrix so that $\sum_j \delta_{ij}$ will no longer equal the degree of such vertices. Thus, the procedure results in a new ring of 'outer vertices' which are distinctive in having some of their original edges truncated. The bias introduced by such a sampling procedure would seem greatest when there is positive spatial autocorrelation present among the interior interfaces, the reason for this will become apparent in Chapter 6. An alternative to both the above procedures is to sample at random from the population of edges in the graph. However, although the probability of an edge being selected in this way is constant, the probability of a cell being selected is not since larger cells (in a topologic sense) would tend to be over-represented. Obviously in all instances the effect of the boundary cells decreases as the size of n increases. In this research the second procedure is adopted, that of discounting boundary cells and sampling exhaustively from amongst the remainder for all the empirical patterns. If boundary cells are included, A in expressions (5), (6), (9) and (10) is equal to $\frac{1}{2}\sum_i x_i$.

CHAPTER THREE

THE DATA

In order to study the spatial arrangement of contact numbers in cellular networks, three kinds of data are examined. Firstly, real world data, which is comprised of 45 empirical networks representative of cellular networks in geography and consisting of politico-administrative patterns eg. counties of England and Wales; economic map patterns eg. central place hinterlands in S.W. Ontario; social map patterns eg. family land holdings in Ghana; biological map patterns eg. red grouse territories; and physical map patterns eg. a 2-D section of crystal grain. The real world data is collected from maps contained in various sources which are listed in the Appendix. In the case of politico-administrative patterns such as standard census units, these maps can be considered highly reliable sources. In the case of the other types of patterns, there is a possibility that the maps may not truly represent the phenomena, particularly the actual form of the boundaries between individual cells may not be exact. However, in the present research such a limitation is not important since it is the pattern of linkages which is being examined.

Secondly, theoretic data is examined. This is comprised of 39 networks generated through computer simulation using two simple stochastic models, each of which produces networks through a different generating process. Thus, the theoretic networks are mathematical constructs in which the elements (nodes and cells) remain unidentified in terms of real world phenomena but the processes generating them are known. By knowing the processes responsible for generating the theoretic patterns, we are essentially using them as normative models to help us understand the less familiar empirical networks. Comparison can be made between the geometric structure of the theoretic networks (generated by known processes) and empirical networks (thought to be generated by processes similar to those incorporated in the models).

The first model is a very simple one and widely used. It generates a network of cells such as that in Figure 4 and is commonly known as the random Voronoi polygon (RVP). It involves a simple two step process. First, a set of labelled points or nuclei ($a_1, a_2, \ldots a_n$) are located in the plane E^2 at random (a homogeneous Poisson point process), with a density of λ points per unit area. Then, with each point a_i, is associated the set of all locations in E^2 that are closer to a_i than to any other point a_j ($j \neq i$). The result is to produce a set of Voronoi polygons (cells) $A1, A2, \ldots A_n$. Such polygons are also known elsewhere under the names of Dirichlet and Wigner-Seitz polygons and geographers normally refer to them as Theissen polygons. The resulting polygons form a contiguous, space exhaustive tesselation that is unique for any given distribution of points. It is clear that such

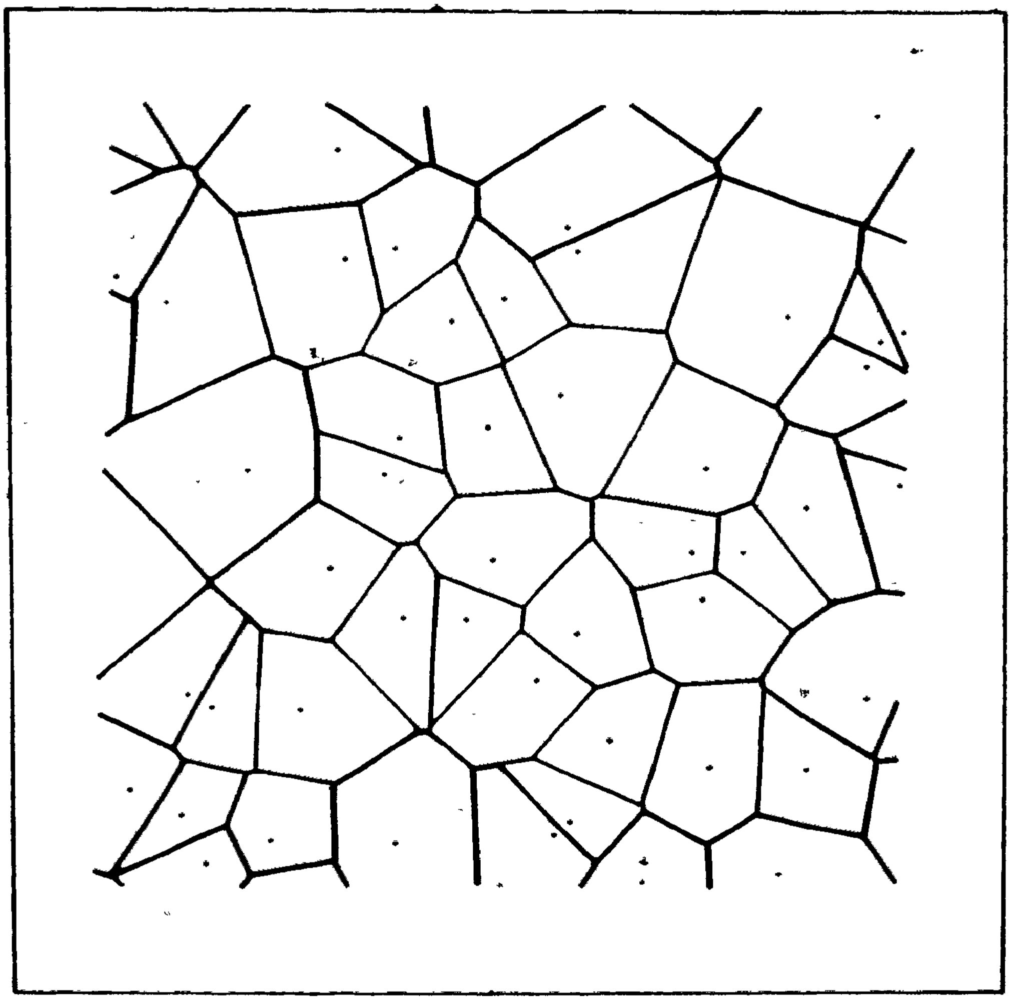

Figure 4: Portion of network generated by the RVP model

Source: Boots, B.N. The arrangement of cells in "Random" Networks, Metallography 15: p.54, 1982.

a tesselation constitutes a random division of E^2 (Boots, 1982, p. 54-55).

The second model used is identical to the first with one exception: it is assumed that the nuclei are generated by an inhomogeneous Poisson point process. The portion of the network shown in Figure 5 is generated using this model. More specifically, λ is assumed to vary spatially according to a gamma distribution. Statistically, this is achieved by compounding the Poisson with a gamma distribution, thus producing a negative binomial model. The effect of this change is to produce a pattern in which the individual nuclei are more clustered than in realizations of the RVP model. Once the nuclei are located, cells are produced as in the RVP model. This model may be labelled the compound negative binomial (CNB) model.

The third type of patterns are termed 'abstract' and consist of highly simple but rigid and regular patterns (eg. a network of triangles and squares) and some other patterns devised by hand which are tested in an attempt to learn more about the nature of the statistics by knowing the statistical outcome beforehand and then manipulating the pattern arrangement.

The analysis of the networks is organized into two phases. i) abstraction of the data into graph theoretic terms and

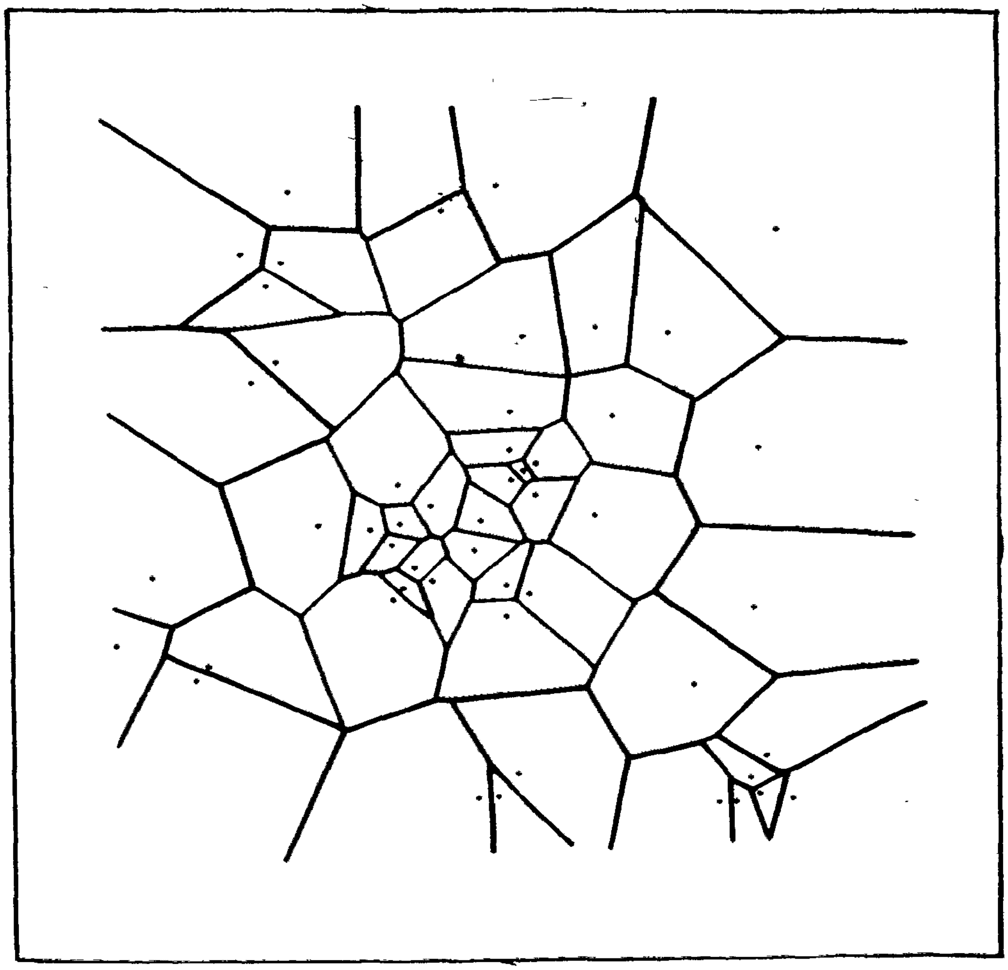

Figure 5: Portion of network generated by CNB model

Source: Boots, B.N. The arrangement of cells in "Random Networks, Metallography 15: p.56, 1982

ii) analysis of the resultant networks by means of spatial autocorrelation coefficients.

CHAPTER FOUR

RESULTS

The analysis of cellular networks using the autocorrelation coefficients c of Geary and I of Moran has produced some interesting results. The autocorrelation coefficients computed under the case of randomization are found in Table 1 (empirical networks), Table 2 (theoretical networks) and Table 3 (abstract and hand made patterns). An appropriate test of significance is provided by evaluating the coefficients as standard normal deviates, the equations for which are as follows

$$z(c) = \frac{E(c) - c}{\sigma(c)} \qquad (11)$$

$$z(I) = \frac{I - E(I)}{\sigma(I)} \qquad (12)$$

(Cliff and Ord, 1973, p. 12)

For a definition of I and c see p.19 and p.20 respectively and for E(I) and E(c) see p.22. The associated standard normal deviates are also found in Tables 1 and 2.

The Geary coefficient is structured so that under the null hypothesis (no spatial autocorrelation) the expected value of c is 1. Values of c less than 1 indicate positive spatial autocorrelation and values greater than 1 indicate negative spatial autocorrelation. Positive values of the standard normal correspond to positive spatial autocorrelation while negative values correspond to negative spatial autocorrelation. The statistic I ranges from -1 to +1 with the expected value approaching zero. A negative standard normal deviate indicates negative spatial autocorrelation while a positive value is indicative of positive spatial autocorrelation. The H_O is examined at various levels, $\alpha = 0.1$ and .05. Since the H_1 (research hypothesis) does not specify direction, a two tailed test is appropriate. When $\alpha = 0.1$ we reject the H_O if a z value is greater the ± 1.65 and when $\alpha = .05$ we reject the H_O if a z value is greater than ± 1.96.

Of the 45 empirical networks examined the number of networks exhibiting significant negative spatial autocorrelation was 11 or 24%. There were no patterns exhibiting significant positive spatial autocorrelation. The most dominant were patterns showing an absence of either negative or positive spatial autocorrelation. This set comprised 76%. The results can be found in Table 1.

An examination of the results of analysis of the 39 theoretic networks reveals that 10 or 25.6% exhibited significant negative spatial autocorrelation, no patterns showed significant positive spatial autocorrelation and 29% or 74.3% show an absence of either negative or positive spatial autocorrelation

Table 1 Spatial autocorrelation coefficients for contact number values for selected networks

Network type	Network	Number of cells	I	Standard normal deviate	c	Standard normal deviate
Politico-Administrative	Regions of Tanzania	6	-.435	-.824	.978	.0644
	Planning Regions Bundelkhand, India	9	-.109	.088	1.218	-.909
	Drobin County, Poland	10	-.190	-.392	1.125	-.541
	Merseyside Traffic Zones	10	-.119	-.048	1.018	-.064
	Provinces of China	11	-.122	-.128	.986	.062
	Administrative Districts of Ghana	18	-.114	-.391	1.186	-.908
	Boroughs of Greater London	19	-.211	-1.136	1.136	-.850
	Administrative Districts of Romania	20	-.090	-.277	1.168	-.965
	Administrative Districts of Kenya	24	.036	.612	.821	1.136
	Provinces of Spain	23	-.036	.072	.731	1.431

	Quartz-filled Joints	27	-.252	-2.019**	1.571	-2.791**
Economic	Shopping Centre Hinterlands in N.W. England	7	-.034	.752	.904	.322
	Urban Hinterlands Lower Saxony	15	-.309	-1.434	1.245	-1.176
	Central Place Hinterlands, Skane Sweden	15	-.044	.158	.943	.287
	Central Place Hinterlands, S.W. Ontario	26	-.171	-1.056	1.085	-.520
	Urban Hinterlands Netherlands	24	.088	.707	1.049	-.242
	Urban Spheres of Influence, U.S.A.	28	-.036	.004	.812	1.299
	Urban Spheres of Influence, S.W. England	30	-.206	-1.257	1.356	-2.074**
	Bus Service Areas England and Wales	24	-.153	-.812	1.286	-2.232**
	Urban Hinterlands Ireland	23	-.115	-.460	1.094	-.477
	Urban Hinterlands Wales	41	.0136	.370	1.186	-1.110

* significant at $\alpha = .10$

** Significant at $\alpha = .05$

	Family Land Holdings Akwapim, Ghana	21	-.241	-2.096**	1.386	-2.516**
	Neighbourhood Areas Portsmouth, England	22	-.263	-1.529	1.299	-1.656*
	African Tribal Territories	27	-.241	-2.096	1.387	-2.516**
Biological	Red Grouse (1) Territories	4	-.888	-1.666*	1.583	-1.649
	Red Grouse (2) Territories	5	-.375	-.816	1.250	-.816
	Fur Seal Territories	7	-.171	-.021	.829	.739
	Song Sparrow (1) Territories	11	-.237	-.681	.807	.699
	Song Sparrow (2) Territories	21	-.212	-1.328	1.302	-1.549
	Red Grouse (3) Territories	22	-.265	-1.643	1.426	-2.381**
	Ordivician Corals, North America	26	-.257	-1.889*	1.270	-2.005**
Physical	Crystal Grain (2-D Section)	22	-.125	-.644	1.140	-.804
	Basalt (Devil's Post Pile)	14	-.102	-.174	.842	.713

	Counties of England and Wales	23	-.263	-1.618	1.242	-1.380
	Counties of Eire	10	.058	.776	.799	.847
	Counties of S.E. Texas	25	-.130	-.730	1.074	-.579
	Counties of California	25	-.246	-1.573	1.112	-.723
	Counties of West Virginia	26	-.209	-1.396	1.199	-1.389
	Administrative Districts of Nigeria	30	-.125	-.827	1.390	-2.525 **
	Municipos of Puerto Rico	32	-.094	-.522	.938	.382
	Hundreds of Wales	34	-.241	-.836	1.138	-1.163
	Counties of Alabama	40	-.241	-2.252**	1.138	-1.163
	Counties of South Carolina	22	-.229	-1.378	1.178	-1.083
	Administrative Zones of Reading Subregion, England	12	-.158	-.402	1.016	-.081
Social	Residential Blocks Benin, Nigeria	15	-.357	-1.941**	1.406	-1.300

From these results it can therefore be concluded that in those instances where spatial autocorrelation was detected it way always negative. Aboav (1970) and Weaire (1974) suggest that a preponderance of negative spatial autocorrelation is to be expected since trivalent vertices dominate in all the networks examined and this is typically the commonest type of significant spatial autocorrelation found. However, the most frequent result is an absence of significant spatial autocorrelation. This can perhaps be explained in three ways. Of the networks examined, most of them have fewer than 35 cells so it can be said that the sample size is not sufficiently large for significant spatial autocorrelation to appear. The smaller sample size allows for a greater influence of the boundary vertices on the statistical results (see Chapter 6). The absence of expected negative spatial autocorrelation could be the result of adopting the randomization assumption where the observed value of I or c are observed relative to the set of n! permutations. Some of the permutations will inevitably be non planar and thus not representative of cellular networks. Since the nature of the planarity is important, the set of all possible I and c that could be used will be less than n! It is impossible to determine the effect of this situation and we can only anticipate that it should make it more difficult to identify significant spatial autocorrelation. This may be borne out in comparative analysis of the statistics (see Chapter 5). Finally the actual results could be an indication of the impact of non-geometric forces

Table 2 Spatial autocorrelation coefficients for contact number values for theoretic networks

Network Generation	Network	Number of cells	I	Standard normal deviate	c	Standard normal deviate
Poisson	1	42	-.077	-.573	1.237	-1.889*
	2	40	-.084	-.600	1.016	-.147
	3	38	-.127	-1.251	1.220	-1.999*
	4	33	-.068	-.348	1.100	--.809
	5	32	-.066	-.312	1.215	-1.346
	6	25	-.019	-.521	1.250	-1.185
	7	31	.069	.888	.877	.801
	8	36	-.088	-.588	1.220	-1.705*
	9	33	-.010	.186	.931	.515
	10	46	-.127	-1.758*	1.220	-1.752*
	11	41	-.075	-.528	1.047	-.401
	12	31	-.133	-.953	1.119	-.941
	13	32	-.154	-1.243	1.030	-.191
	14	32	-.329	-2.722**	1.284	-2.307**
	15	39	.025	.548	.863	1.173

	16	34	-.067	-.362	1.003	-.030
	17	36	-.289	-2.716**	1.366	-2.363**
	18	32	-.098	-.617	1.211	-1.490
	19	31	-.142	-1.010	1.114	-.851
	20	38	-.183	-1.616	1.219	-1.799*
	21	32	-.134	-.988	1.137	-1.048
	22	37	-.079	-.525	1.085	-.625
	23	28	.084	1.154	.905	.654
	24	29	-.201	-1.492	1.143	-1.034
	25	49	-.042	-.254	1.048	-.482
	26	38	-.268	-2.535**	1.618	-4.171**
	27	35	-.187	-1.441	1.122	-.971
	28	36	-.276	-2.612**	1.625	-4.026**
	29	40	.116	1.388	.882	.969
Compound Negative Binomial	1	28	-.138	-1.075	1.265	-1.884*
	2	29	-.013	.193	1.328	-2.004**
	3	30	.060	.813	1.180	-1.630
	4	32	-.072	-.361	1.170	1.387
	5	35	.065	.813	1.180	1.191

6	31	-.078	-.286	.979	.138
7	40	.166	1.613	.904	.678
8	43	-.018	.099	1.109	-.802
9	37	.011	.384	.986	.090
10	35	.049	.768	1.118	-.654

Table 3 Spatial autocoreelation coefficients for contact number values for 'abstract' networks

Network	Number of cells	I	Standard normal deviate	c	Standard normal deviate
Foureig combination of 4 and 8 sided cells	6	.000	1.224	.833	1.224
Foureig1 as above but with addition of a boundary ring	33	.434	4.142**	.728	2.330**
Foureig2 1 and 2 plus another boundary ring	66	.414	5.512**	.705	3.766**
Abstract Combination of 4 and 8 sided cells	5	-.375	-2.000**	1.250	-2.000**
Abstract1 1st ring of boundary cells added	25	-.374	-2.865**	1.250	-2.107**
Abstract2 2nd ring of boundary cells added	58	-.371	-4.540**	1.287	-3.696**

Squaretri1 Combination of squares and triangles	30	-.384	-2.259**	1.450	-2.872**
Square tri2 Combination of squares and triangles	36	-.174	-.944	.984	.099
Square Network of all squares	12	.000	.384	.000	4.957**
OctSquare Network of Octagons and Squares	21	-.333	-2.115**	1.338	-2.935**
Tri Network of all triangles	21	.000	.249	.000	5.553**
TruncSquare Network of truncated squares	32	-.302	-2.401**	1.526	-4.299**
Dodtri Network of dodecagons and triangles	36	-.040	-.113	2.213	-4.894**
Hextess Network of hexagonal tesselations	37	-.501	-4.953**	1.913	-6.099**
Hexagon Network of all hexagons	9	.000	.629	.000	9.165**

	Pentagon Network of all pentagons	24	.000	.297	.000	8.502**
Hypothetical						
	Sevfiv1	42	-.223	-2.180**	1.256	-2.839**
	Sevfiv2	42	-.176	-1.678*	1.268	-2.315**
	Sevfiv5	42	-.307	-3.101**	1.331	-3.671**
	Sevfiv6	42	-.222	-2.178**	1.245	-2.728**
	Sevfiv8	42	-.195	-1.804*	1.256	-2.703**
	Altern	29	.768	6.183**	.118	6.428**
	Altern1	29	.109	1.625	.658	2.423**
	Altern2	29	.492	3.800**	.364	4.240**
	Clumps	41	.646	6.145**	.414	4.899**

* significant at $\alpha = .10$

** significant at $\alpha = .05$

in the formation of the networks such as social or man-made forces.

The results of experimentation with abstract cellular networks (that is cellular networks composed of all similar cells ie. triangles, pentagons, hexagons, etc. or a combination ie squares and triangles) are found in Table 3 and reveal that 8 or 54% exhibit significant negative spatial autocorrelation, 33% exhibit significant positive spatial autocorrelation and 13% do not exhibit either. Significant positive spatial autocorrelation is found in those patterns of cells with equal contact number values, except in the case of Foureig, Foureig1 and Foureig2 which will be discussed further in Chapter 6. Significant negative spatial autocorrelation is found in those networks where the pattern of cells is an alternation of two different contact number values. These networks provide an ideal sample set for exemplifying this type of spatial autocorrelation.

It is interesting to note that the two patterns of triangles and squares (Squaretri1 and Squaretri2) give different results (see Figure 6 and 7). In both patterns all vertices are non trivalent. In Squaretri1, visual inspection and the results in Table 3 establish that the contact number arrangement is definitely that of negative spatial autocorrelation. Using the rook's criteria for connecting cells, it is noted that there are no connections between any pair of four sided cells and the three sided cells are only connected to three sided cells on one side. Because of the absence of 'like' joins

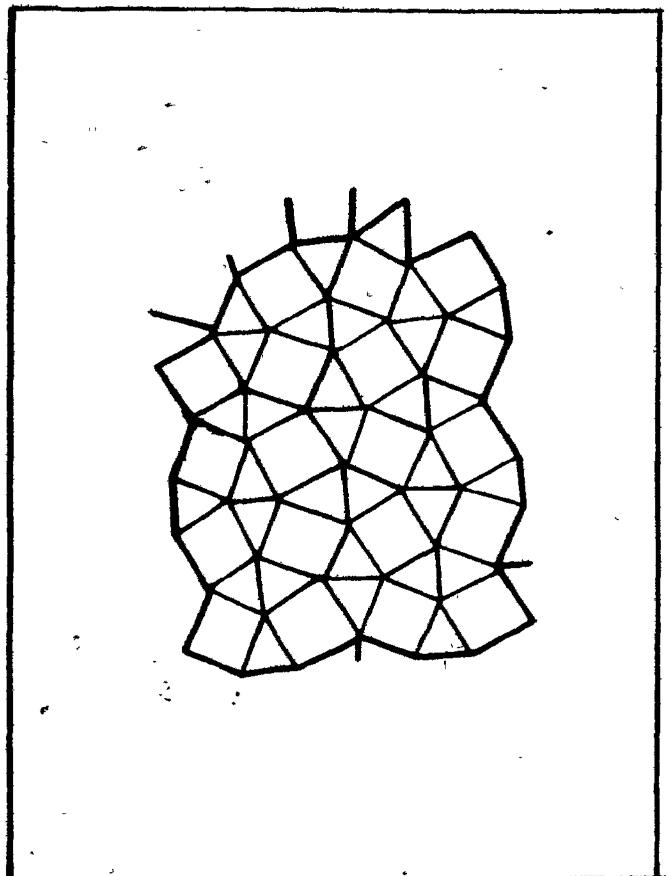

Figure 6: Squaretril

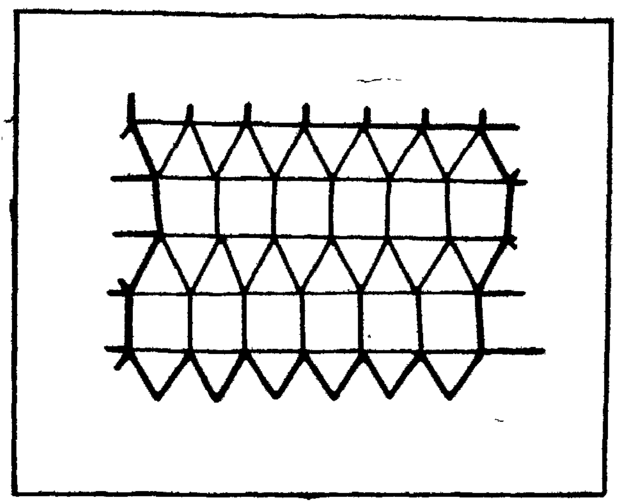

Figure 7: Squaretri2

one would expect negative spatial autocorrelation to ensue. In the case of Squaretri2, on the other hand, there is no evidence of spatial autocorrelation in the results but there are connections between pairs of three sided and four sided cells. The presence of a lot of 'like' joins would lead one to believe that positive spatial autocorrelation should be exhibited. The effect of the boundary cells appear to be so dominant as to resist the pattern's approach to positive spatial autocorrelation. A further discussion of the boundary effects will follow in Chapter 6.

The results of the inclusion of the boundary cells are found in Table 4. Twenty eight randomly selected empirical networks had a boundary ring added and the statistics were then recomputed. Of these patterns, two or 7.14% exhibited significant negative spatial autocorrelation, 16 or 57.14% exhibited positive spatial autocorrelation and 35.7% exhibited no significant spatial autocorrelation. Of the twenty eight patterns two of the statistical interpretations changed from showing significant negative spatial autocorrelation to showing positive spatial autocorrelation, 13 or 46.43% showed no spatial autocorrelation but with the addition of the boundary cells, the patterns then exhibited significant positive spatial autocorrelation. Conversely, two or 7.1% went from showing no spatial autocorrelation to showing significant negative spatial autocorrelation and three of 10% which were significantly negatively spatially autocorrelated

Table 4 Spatial autocorrelation coefficients for contact number values for selected networks with the addition of a ring of 'boundary cells'

Network	Number of cells	I	Standard Normal Deviate	c	Standard Normal Deviate
Counties of Alabama	67	.222	3.230**	.720	3.436**
Counties of California	58	.126	1.744*	.854	1.398
Counties of S.E. Texas	48	.162	2.081**	.822	1.817*
Administrative Districts of Romania	39	.214	2.527**	.830	1.572
Planning regions, Bundelkhand, India	23	.261	2.547**	.790	1.244
Counties of West Virginia	48	.223	2.784**	.746	2.482**
Provinces of Spain	50	.146	1.886*	.839	1.482
Municipos of Puerto Rico	50	.175	2.604**	.816	2.242**
Drobin County Poland	32	.284	2.898**	.722	2.240**

Boroughs of Greater London	33	.154	1.799*	.820	1.675*
Administrative Districts of Ghana	40	.159	1.916*	.847	1.365
Administrative Districts of Nigeria	50	.133	1.841*	.961	.356
Counties of South Carolina	47	.298	3.552**	.765	2.093**
Administrative Districts of Kenya	39	.136	1.679*	1.007	-.065
Regions of Tanzania	20	.144	1.478	.787	1.246
Counties of Eire	25	.129	1.388	1.275	-.181
Administrative Zones of Reading	23	-.045	-.003	1.100	-.717
Mersyside Traffic Zones	25	.129	1.467	.843	1.048
Urban Spheres of Influence, U.S.A.	72	.315	4.264**	.812	1.299
Bus Service Areas England Wales	55	.147	1.924*	1.286	-1.772*
Urban Hinterlands Ireland	57	-.007	.210	1.523	-3.564**

Shopping Centre Hinterlands N.W.England	23	.024	.652	1.948	-3.062**
Urban Hinterlands Lower Saxony	35	.038	.658	1.135	-1.008
Central Place Hinterlands, Skane Sweden	37	.008	.362	1.123	-.853
Red Grouse Territories (3)	44	.070	1.057	1.156	-1.225
Song Sparrow Territories (2)	44	.112	1.514	1.073	-.575
Residential Blocks Benin, Nigeria	28	-.009	.258	1.253	-1.648
Neighbourhood Areas Portsmouth, England	40	-.008	-.655	1.201	-1.462

* significant at = .10

** significant at = .05

became void of any significant spatial autocorrelation.

If significant positive spatial autocorrelation had been detected in patterns (other than those where the boundary cells had been added), a visual comparison of the two types of patterns may have revealed some interesting findings. Looking at those patterns which did exhibit negative spatial autocorrelation amongst the contact number values, it is apparent that in most cases large cells are surrounded by smaller cells and trivalent vertices dominate. These observations coincide with the finding of Aboav (1970) and Weaire (1974). It is also interesting to note that of all the cellular networks examined before the boundary cells were added, percentage wise those categorized as 'social' and 'biological' contain a dominance of patterns exhibiting significant negative spatial autocorrelation.

With regard to the statistics themselves, the results of Tables 1 and 2 add credibility to the fact that the Geary statistic measures unlike joins while the Moran statistic stresses like joins. Of the significant cases of negative spatial autocorrelation (empirical, bounded and theoretic networks) 18 or 75% of the time the Geary statistic is more significant than the Moran statistic. Of these results, 50% of the time the Moran statistic shows significant negative spatial autocorrelation as well. In the case of positive spatial autocorrelation it is found that 93% of the time, the Moran statistic is more significant than the Geary statistic. Although the Geary statistic is consistent and 53% of the time shows significant positive spatial autocorrelation as well. It is interesting that positive spatial autocorrelation only shows up when the 'boundary' cells are added. In some situations the statistics appear to be consistent with each other, however, the Geary statistic appears to regard positive spatial autocorrelation more than the Moran statistic regards negative spatial autocorrelation. More specifically, Jones has determined the relation between the two autocorrelation statistics so that one can be read in terms of the other. Expansion and multiplication of each equation so that the two statistical values are the same, may shed light on the relationship between the two.

The purpose of this Chapter was to introduce the results stemming from applying the spatial autocorrelation coefficients to sets of 'geographical' cellular networks. This leads on to an evalualtion of the results (Chapter 5) and then a discussion of the boundary problem (Chapter 6).

CHAPTER FIVE

ANALYSIS

Evaluation of the Networks

In light of the results in Chapter 4, examination of the networks leads to some speculation as to the reason for a prevalence of negative spatial autocorrelation in those cases where spatial autocorrelation exists. Firstly, trivalent vertices dominate in the networks. Secondly, it is interesting to note that generally those patterns exhibiting significant negative spatial autocorrelation have cells whose contact number value is eight or greater. These cells are adjacent to cells with a contact number value of at least 4 degrees less. For example in the pattern of family land holding in Ghana, there are interfaces of 9-3 and 9-4 sided cells; in the pattern of Ordivician corals there is an interface of 8-3; red grouse territories, a 9-4; and quartz filled joints a 10-4 interface. (see Figures 8,9,10,11). These interfaces are generally central to the pattern (network) which is important as there will be no interference from the boundary cells. So, as in the findings of Aboav and Weaire, large cells are surrounded by smaller cells.

In the case of patterns of only two different contact degrees as illustrated in some of the 'abstract' patterns,

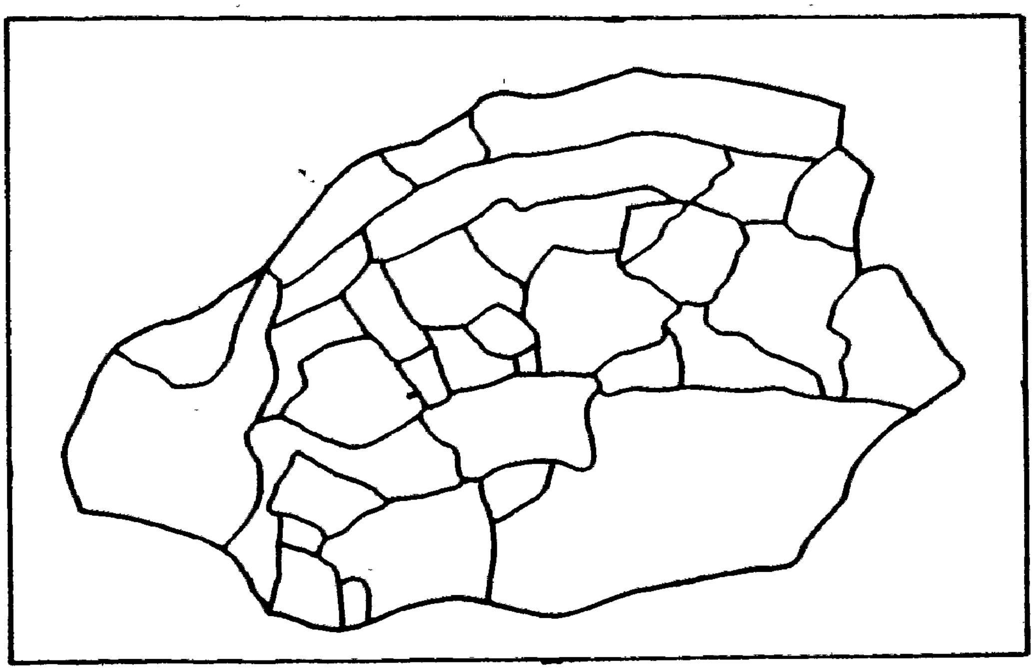

Figure 8: Family land holdings, Akwapia, Ghana

Source: Cliff, A.D and J.K Ord Spatial Autocorrelation Pion, London, 1973.

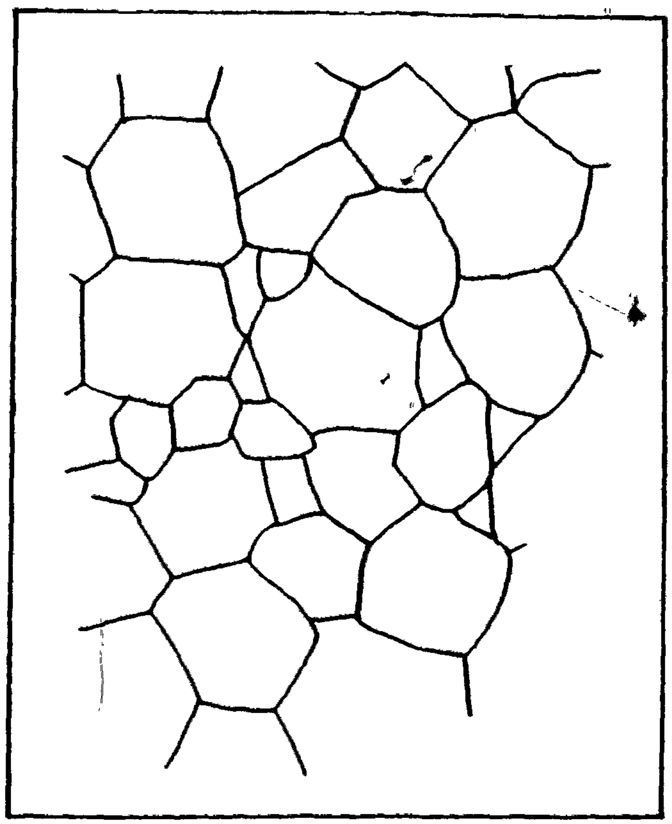

Figure 9: Ordivician corals, North America

Source: Stevens, P.S. Patterns in Nature, Atlantic-
Little, Brown and Company, Boston, 1977.

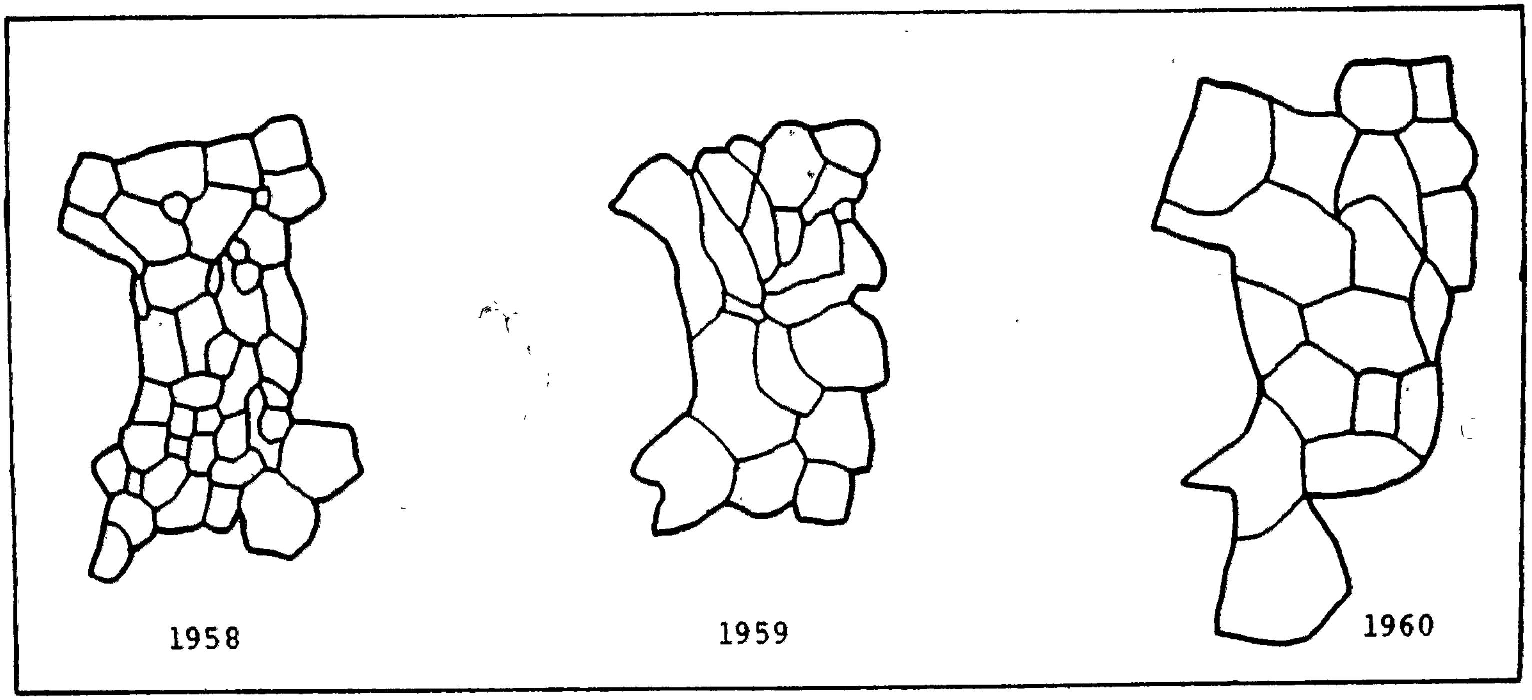

Figure 10: Red grouse territories.

Source: Watson, A. and R. Moss Behavior and Environment: The use of space by animals and men, Plenium Press, New York 1977.

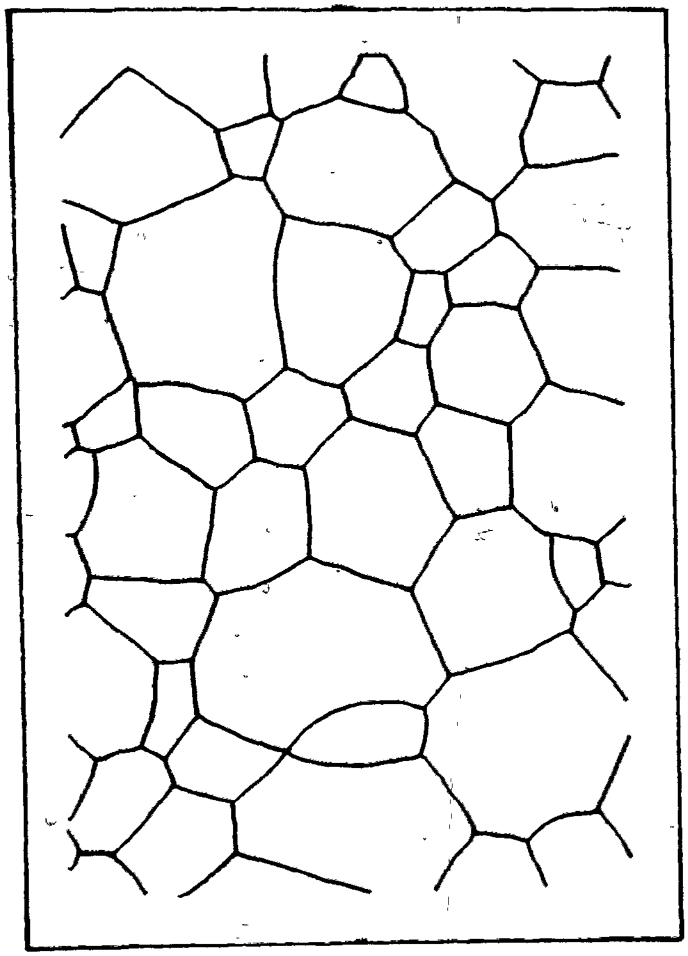

Figure 11: Crystal grain (2-D section)

Source: Stevens, P.S. Patterns in Nature, Atlantic-Little, Brown and Company, Boston, 1977.

spatial autocorrelation is almost always negative. In these cases negative spatial autocorrelation is distinguished even when the contact number value is very close as in the case of four and five sided cells. However, when the networks contain cells of all different contact number values it appears that the statistics will distinguish more between cells of extreme values ie. 8's and 3's than 5's and 6's. So in networks of mixed contact number values, those which have values which are close in degree, are less perceptable to the statistics than those with a larger difference between them. It appears then that what is important in determining the degree of negative spatial autocorrelation is the relative difference between contact number values of adjacent cells and this appears to be most significant when the differences are greater than 4 in patterns of contact number values ranging from 2 to 10.

Visually comparing the networks which exhibit no spatial autocorrelation and significant negative spatial autocorrelation, it is difficult to assess the difference. Both appear to have a dominance of trivalent vertices so this is not necessarily indicative of negative spatial autocorrelation. It is known that a pattern of complete alternation of contact number values will result in strong negative spatial autocorrelation while pattern of cells of all the same contact number values will show very strong positive spatial autocorrelation, no spatial autocorrelation falls somewhere between some alternation and some clustering of like cells. It appears that those patterns exhibiting negative spatial autocorrelation tend to have few or no trivalent vertices where all three cells converging at a common vertex all have the same contact number value (that is clustering of like contact numbers), while networks of no spatial autocorrelation have trivalent vertices of this type. This tends to disrupt the pattern's approach to negative spatial autocorrelation.

The association of contact number value is not a simple one. There appear to be several other factors which may contribute to the negative spatial autocorrelation of cellular networks. One of these may be the frequency distribution of particular contact number associations ie. the number of 5 and 6 joins and 7 and 5 joins. It may also be suggested that the ratio of dislike to like joins is another contributing factor. Generally in the patterns examined there were more dislike joins than like joins and the association varies between networks. According to the size of n, there may be a critical number of like to dislike joins which will determine the sign and degree of spatial autocorrelation but this ratio will also be dependent on other factors such as trivalent vertices and clumping.

In order to examine more carefully network arrangement, hypothetical patterns are devised, the statistics computed, and the results examined. Five patterns of seven and five sided cells are experimented with (Sevfivl,2,5,6,8, Table3). Each pattern contains the same number of cells (42), the same mean contact number value ($\bar{x}$ = 5.8092) and the same distribution of five sided nad seven sided cells (25-5's and 17-7's). All the vertices are trivalent, only the arrangement of the cells vary within the network. For figures 12, 13, 14, 15, 16) the statistics show that negative spatial autocorrelation is the case in every network. In this example the boundary cells are not included in the derivation of the statistical results.

Ranking these five hypothetical patterns according to the degree of negative spatial autocorrelation, it is found that the first is Sevfiv6 showing the most significance, then Sevfiv5,Sevfivl, Sevfiv8 and last is Sevfiv2 showing the least significance. In ranking these patterns according to the number of trivalent vertices where cells of equal contact number meet, we find that the order is the same with Sevfiv6 having the least and Sevfiv2 having the most. Correlating these two facts, one can conclude that as observed in the empirical networks, the increased occurance of these 'trivalent vertices' tends to decrease negative spatial autocorrelation. Clumping of three or more similar cells, even though isolated within the network tends to affect the spatial autocorrelation more than pairs or rows of similar cells. This is further supported by analysis of the pattern 'Clumps' (Table 3, Figure 17). In

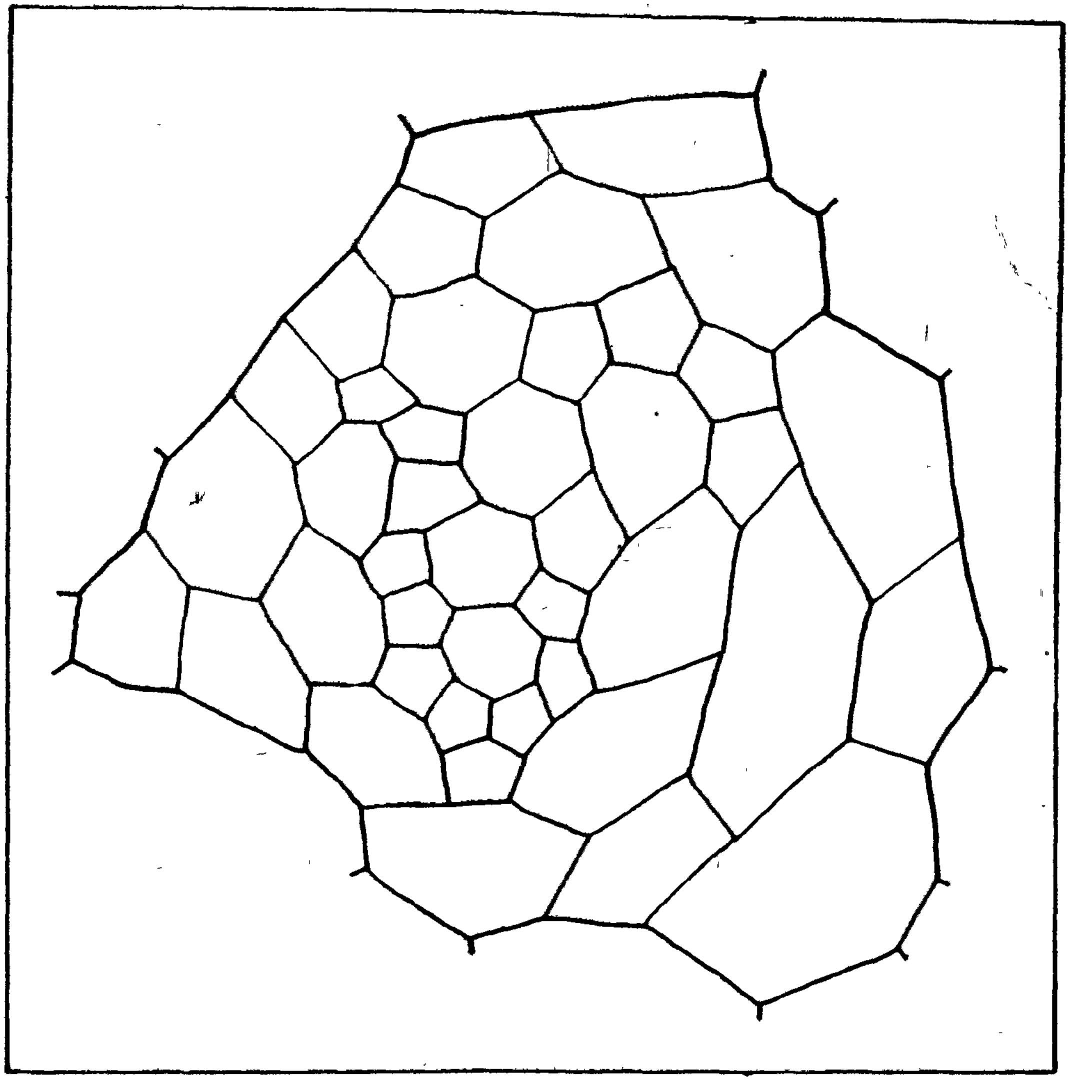

Figure 12: SevFivl

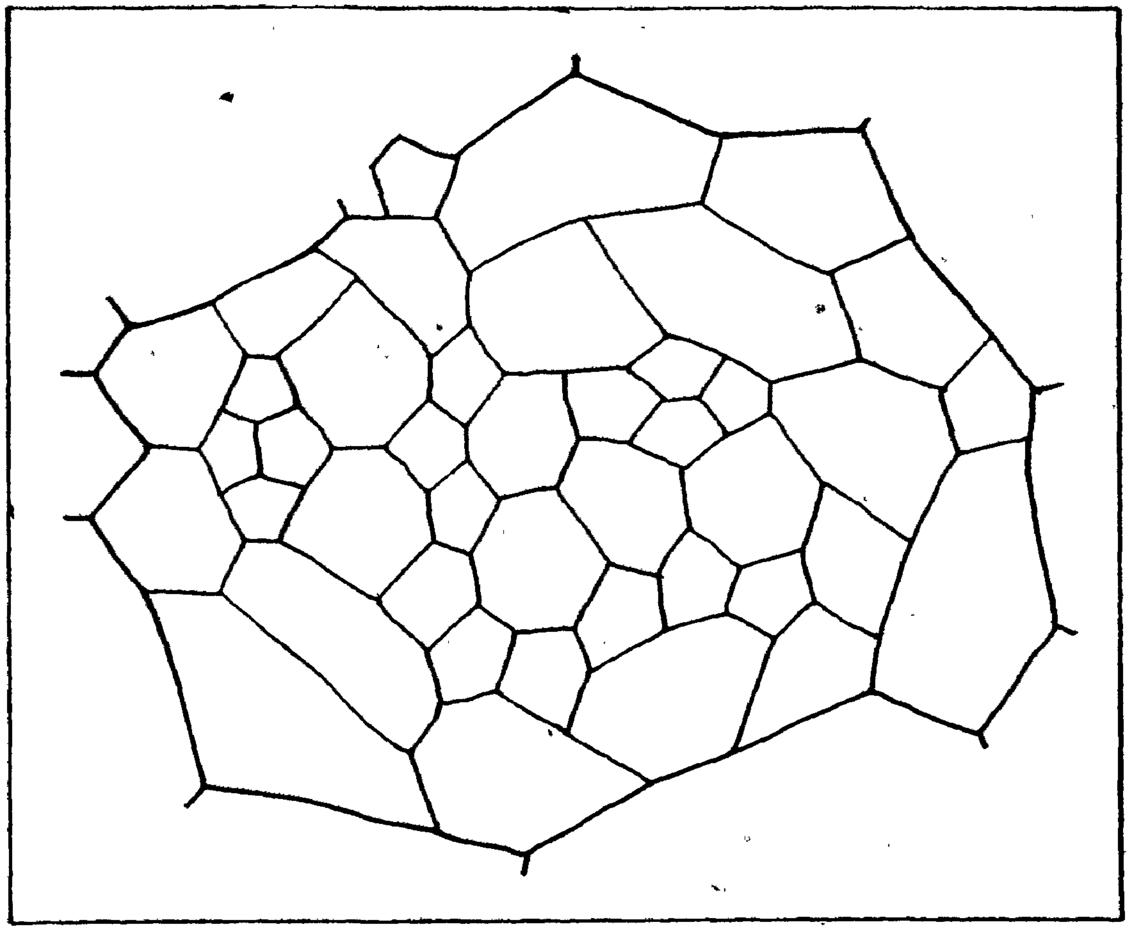

Figure 13: Sevfiv 2

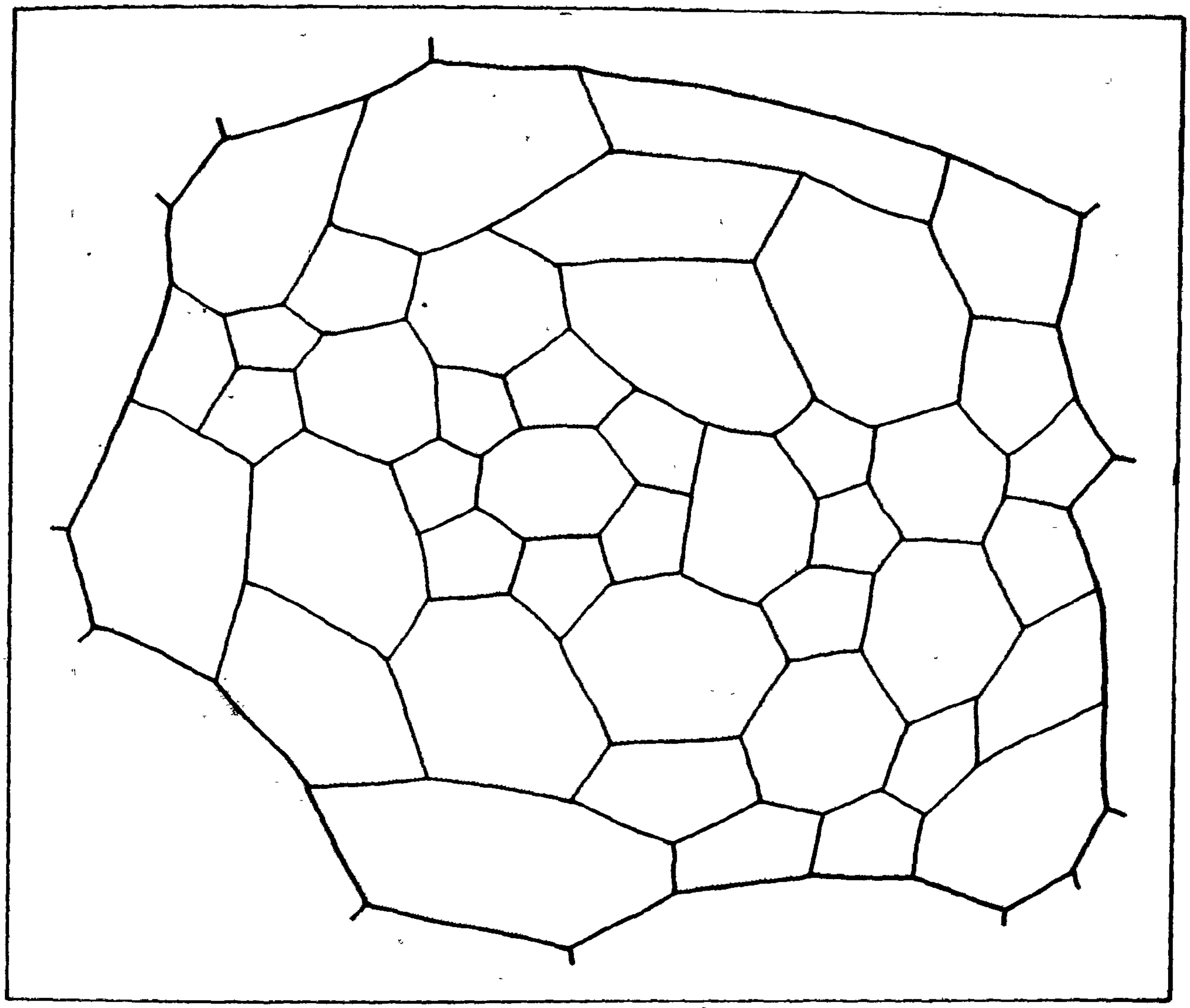

Figure 14: Sevfiv5

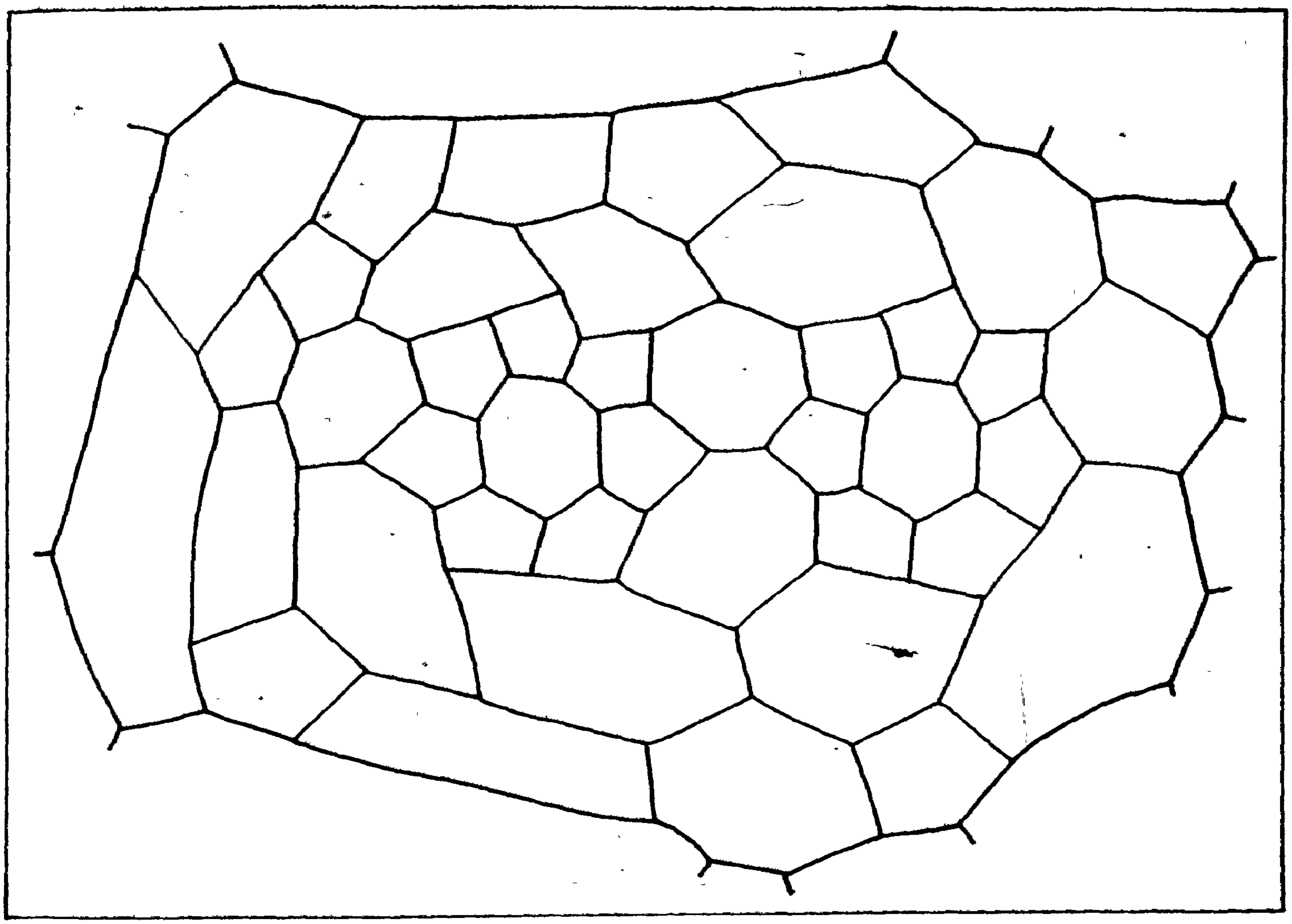

Figure 15: Sevfiv6

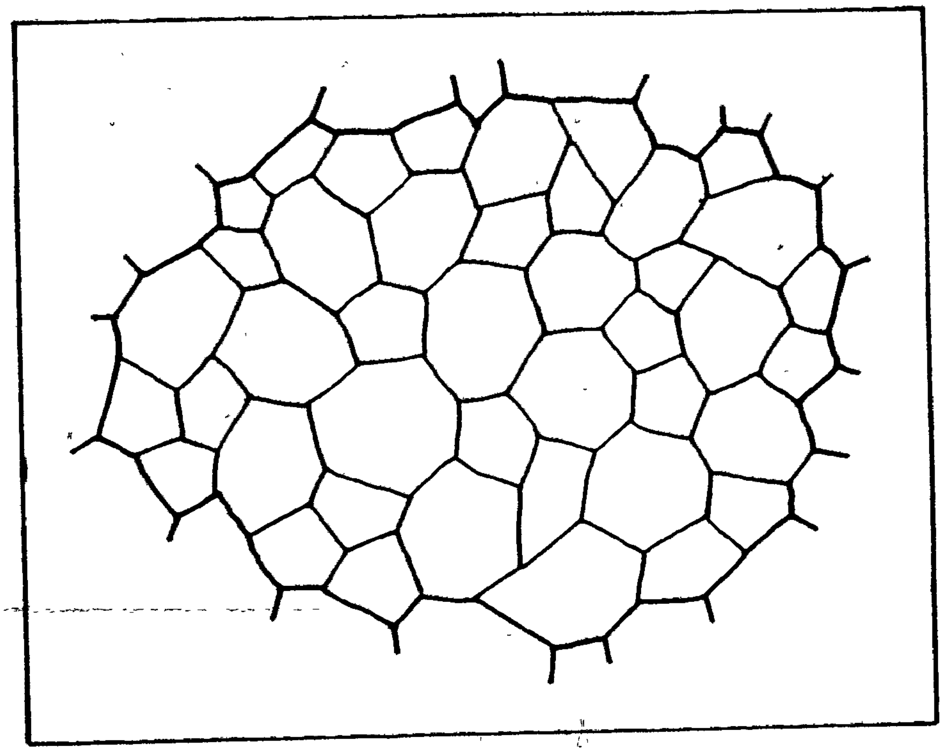

Figure 16: Sevfiv8

this instance cells of like value are clumped together and surrounded by cells of dissimilar values. Even though the unlike joins out number the like joins, the pattern still exhibits significant positive spatial autocorrelation.

Experimentation was undertaken with cellular networks where rows of cells of similar contact number values are situated adjacent to rows of cells all of similar contact number but different from the rows on either side (Figures 18, 19, 20). The number of cells and mean contact number value remain invariate as rows are rearranged. In the first pattern (Figure 18) Altern, the pattern has a row of cells with a contact number value of three next to fours which are next to fives and so on. The significance level of the positive spatial autocorrelation is very high even though there are more unlike joins than like joins. The contact number values are so close (only one degree apart) that the statistics do not appear to make a strong distinction between them. However, when the pattern is rearranged as in Altern1, (Figure 19), no row of cells is adjacent to a row of cells less than two degrees apart. There is also a loss of joins between the four sided cells and so there is a decrease in the degree of positive spatial autocorrelation because the difference between the contact number values is greater than in Altern. In Altern2 (Figure 20) degrees of three and four are adjacent which serves to strenghten the overall positiveness of the spatial autocorrelation in the

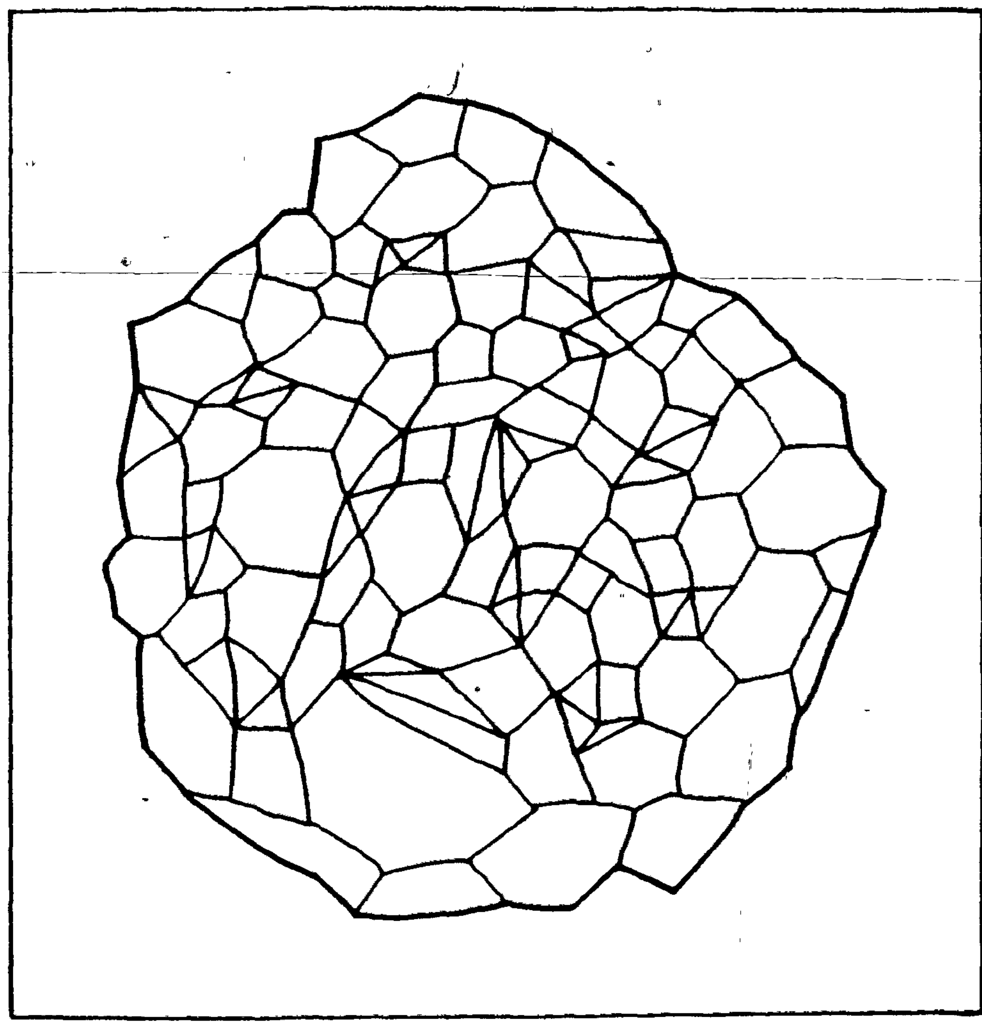

Figure 17: Clumps

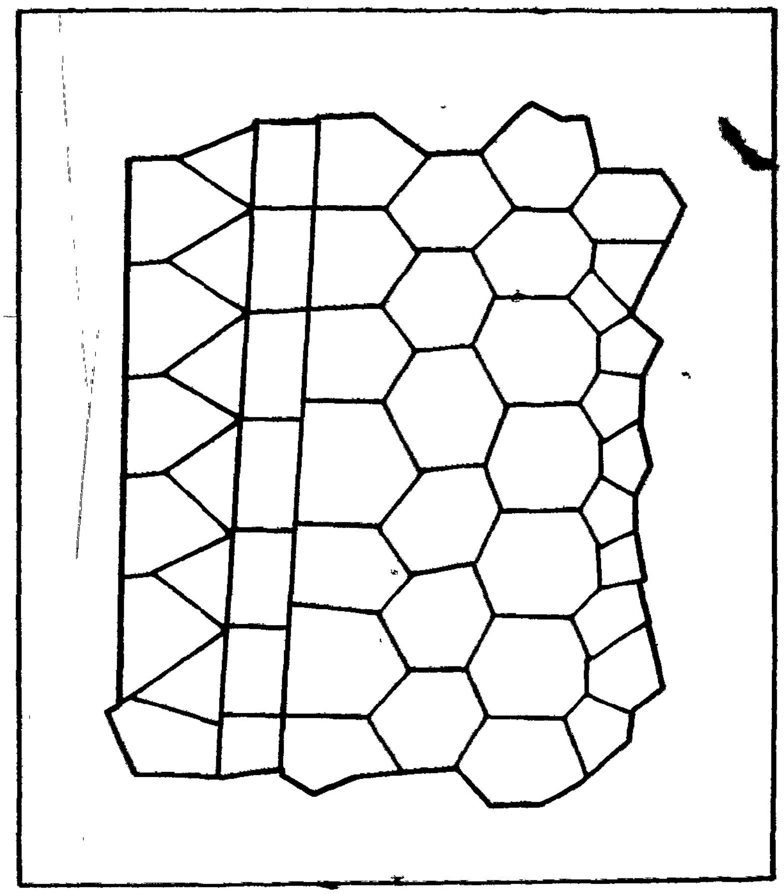

Figure 18: Altern

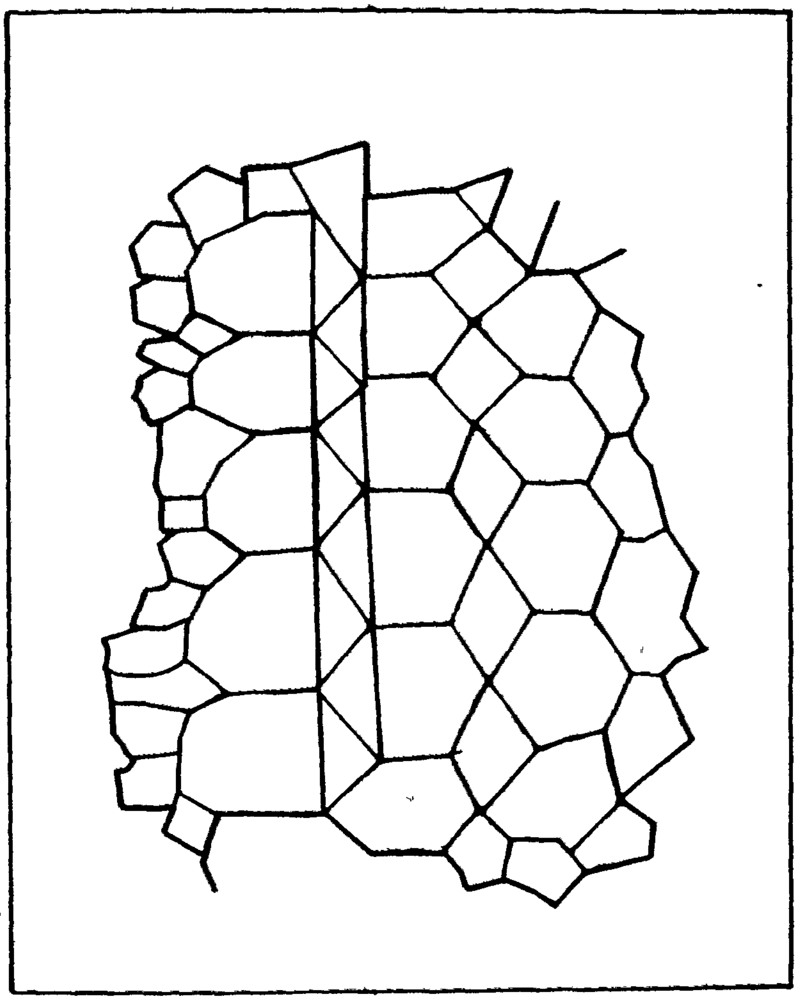

Figure 19: Alternl

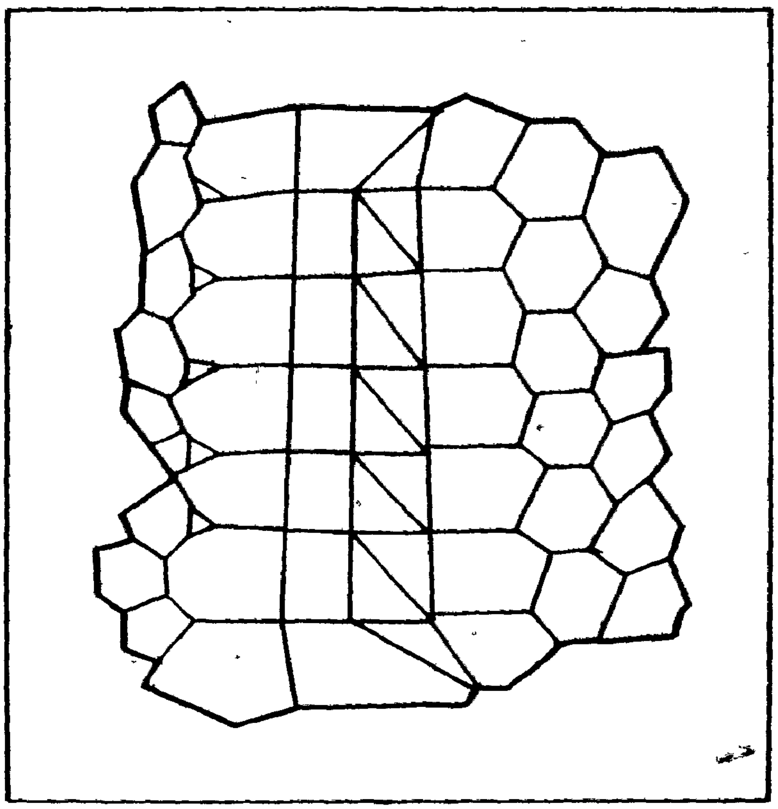

Figure 20: Altern2

network. (Also prevalent in these hypothetical patterns are non-trivalent vertices which may contribute to the positive spatial autocorrelation).

Evaluation of the Statistics

The results of Chapter 3 support the notion that the Geary statistic is the most appropriate when measuring negative spatial autocorrelation (unlike joins) and the Moran statistic for measuring positive spatial autocorrelation (like joins). Generally the statistics are consistent with each other; that is if negative spatial autocorrelation is prevalent it will show up in both coefficients. The patterns generated by the compond negative binomial process did not reveal any patterns which exhibited positive spatial autocorrelation, instead we find that there are only two cases of significant negative spatial autocorrelation and the signs of the z scores are not generally consistent with each other.

In order to compare better the results of the statistics much can be gained by plotting them on a graph (Figure 21). We can see from the graph that on the whole the statistics are consistent with each other. Even when no spatial autocorrelation is evident, the Moran statistic leans towards positive spatial autocorrelation and the Geary towards negative spatial autocorrelation. There are of course anomolies where the Moran statistic leans towards negative and the Geary towards positive. This can be explained by the fact that these patterns tend to have a value of n less than 20. In yet other cases the Geary statistic shows significant negative spatial autocorrelation while the Moran although exhibiting no significant spatial autocorrelation, leans towards positive. All of these results are for patterns which have had the boundary cells included in the computation and so it can be concluded that it is the affect of the addition of the boundary cells which leads to the inconsistency of the statistics with each other.

The fact that no positive spatial autocorrelation is found in the empirical and theoretic networks except when the boundary cells are added seems to indicate that perhaps negative spatial autocorrelation is the most prevalent case in the real world. The close correspondance between the empirical results and the theoretic results would seem to suggest that the arrangement of contact number in empirical networks is not significantly different from the geometric structure of networks generated by RVP and CNB models which may well represent an appropriate description of the generation of particular cellular networks.

One unique occurance is exhibited in the pattern of bus service areas in England and Wales (Boundary included). The Moran statistic shows significant positive spatial auto-

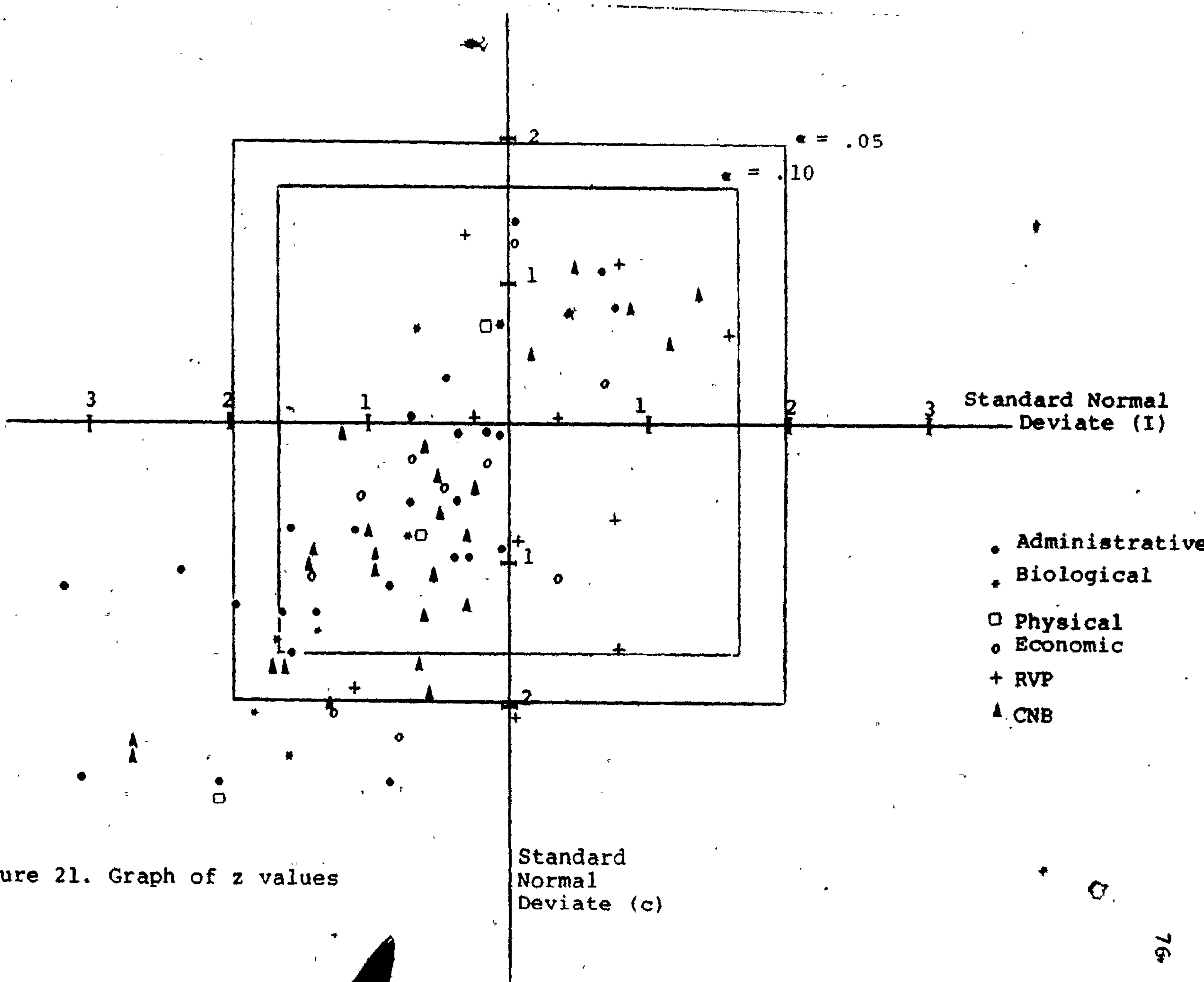

Figure 21. Graph of z values

correlation while the Geary statistic shows significant negative spatial autocorrelation. This represents an instance where the two statistics are clearly measuring different aspects of the pattern and the boundary does have a very strong influence on the pattern.

The abstraction of the cellular network into graph theoretic terms and the generation of the adjacency matrix is important in the outcome of the statistics used in this study. In this study the form of the connection used is the rook's case where any two vertices i and j in G are linked by an edge if the two cells they represent have a common border. Other methods include the bishop's count in which only cells with a common vertex are considered connected and the queen's count where either a vertex or border are common (see Figure 22). The rook's count is used because it ensures that G will always be planar whereas the bishop's or queen's count will lead to a G which in non-planar (see Figure 23). When the rook's count is used there is a loss of connections in the case of a 4 or 5 edged vertex. There is no diagonal association between the cells. Some of these connections might actually be realized without disturbing the planarity of the graph. With a 4 edged vertex there are two more connections realized with the queen's count than with the rook's count but in order to realize this the graph can no longer lie in a plane. However, one of these connections can be realized and still maintain the planarity

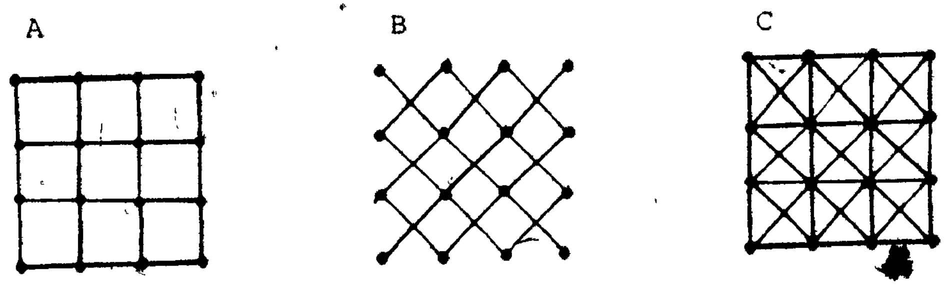

Figure 22. Simple connections in a regular grid.

The 16 points or localities in this regular grid are connected by rook's moves in panel A, bishop's moves in panel B, and queen's moves in panel C.

(Sokal et al. 1978)

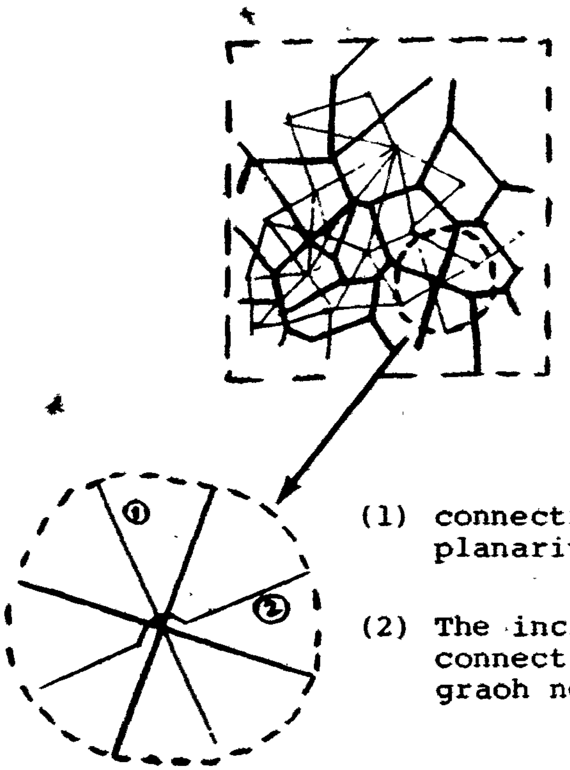

Figure 23 . Illustration of loss of planarity in bishop's or queen's count

of the graph (see Figure 23). When the queen's count is used there is an over representation of connections so that each locality will have a greater number of neighbours of similar or dissimilar contact number values. With the rook's count however, there is an under representation so the connection criteria becomes a problem in the analysis of cellular networks using Geary and Moran correlation coefficients because these statistics are very sensitive to the number of connections or join counts in the matrix.

CHAPTER SIX

BOUNDARY PROBLEM

This chapter discusses the boundary problem introduced in Chapter 2. It was noted that the delineation of the sample set will affect the overall spatial autocorrelation results due to the fact that the ring of cells on the boundary of the cellular network will necessarily be lower in degree value than those inside. The lower nature of these 'boundary cells' is due to the fact that they are only connected between themselves and the inner cells while lacking cells exterior to them. (see Figure 24). The boundary has always posed a problem to geographic research. It will always inevitably bias the results but since it is a problem that is not easily overcome it seems acceptable for the results to be used to draw theoretic conclusions.

Due to the small size of the empirical patterns in this study ($n < 100$) when the boundary cells are added to the 28 randomly selected networks, over 50% of these show that the number of boundary cells exceeds the number of internal cells so the boundary undoubtedly has a marked importance. In most cases, the affect or error factor caused by the boundary becomes less as n increases.

By comparing the results of the 28 bounded empirical networks with the results obtained before the boundary cells

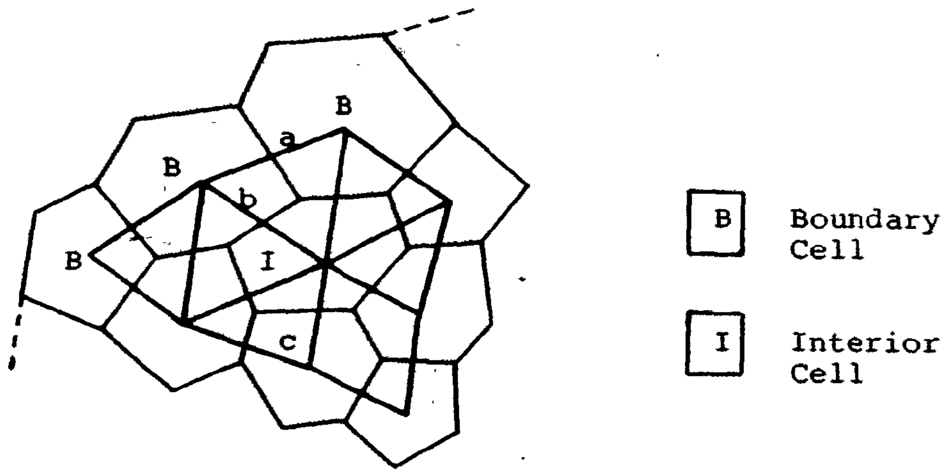

Figure 24: Illustration of loss of Boundary Connections

Contact number of I = 4 if boundary not included

Contact number of I = 6 if boundary included

If boundary connection is left there are three types of connections between cells

Boundary to boundary (B-B) (eg link a)

Boundary to interior (B-I) (eg link b)

Interior to interior (I-I) (eg link c)

Only the (I-I) links are realized if the boundary connections are not included so the # of 1st order neighbours and join counts are not necessarily the same.

were added, evidence can be obtained to examine the effects the addition of these lower value boundary cells will presumably have on the internal cells.

In a network which consists of all trivalent vertices the average contact number $\bar{m}$ is between 5 and 6. The actual value depends in the number of cells n (as $n \rightarrow \infty$, $\bar{m} \rightarrow 6$). In the discussion below, H will represent a cell where contact number is 6 or greater and L where the contact number is less than 6. We know that the cells of the boundary ring are almost exclusively low values (<6) due to the truncation of some of the edges. Joins between the boundary cells and internal cells will then be of the types L/L and H/L. With this in mind we may now consider the possible outcomes of the addition of the boundary cells to patterns exhibiting positive spatial autocorrelation, negative spatial autocorrelation and no spatial autocorrelation.

Positive Spatial Autocorrelation

There are three typical patterns resulting in positive spatial autocorrelation (see Figure 25). For pattern 25a, there are two types of boundary/interior interfaces (Figure 26a and 26b).

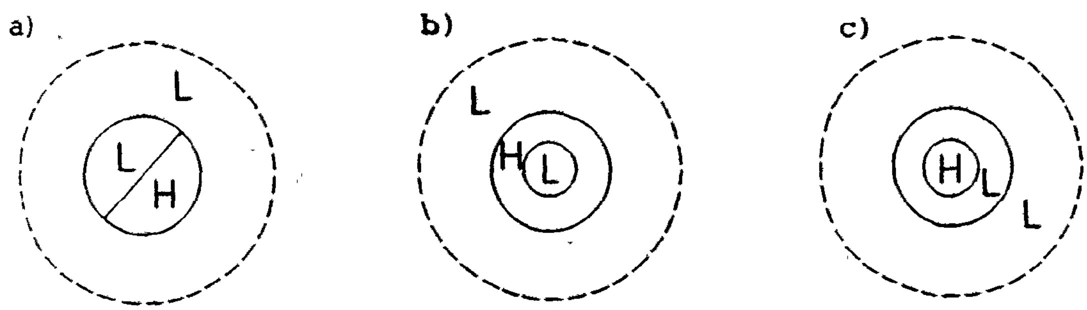

Figure 25: Positive Spatial Autocorrelation

a) b)

Figure 26: Boundary/interior interfaces for Figure 25a.

In pattern 25a, a typical interface consists of 4 L/L contacts which is equivalent to 8 L/L joins. In pattern 25b there are 2 L/L contacts and 2 H/L contacts or 4 L/L and 4 H/L joins. Thus on average a typical boundary cell adds 6 L/L and 2H/L joins to the calculations. Since like joins dominate, the effect will be to reinforce the positive spatial autocorrelation coefficient.

In pattern 25b, the situation is exclusively that of the addition of 4 L/L and 4 H/L joins. Since the joins balance there should be little affect on the spatial autocorrelation coefficient.

In pattern 25c, the situation is exclusively that described in 25a, and so a typical boundary cell adds 8 L/L joins. Since only like joins are added the value of the spatial autocorrelation coefficient should be increased.

Since there are no empirical or theoretic networks in this study which exhibit significant positive spatial autocorrelation, these theories cannot be verified with real world examples. However, one network that comes close to exhibiting significant positive spatial autocorrelation is provinces of Spain (see Figure 27). With the addition of the ring of boundary cells, the degree of positive spatial autocorrelation does in fact increase especially on the Moran statistic.

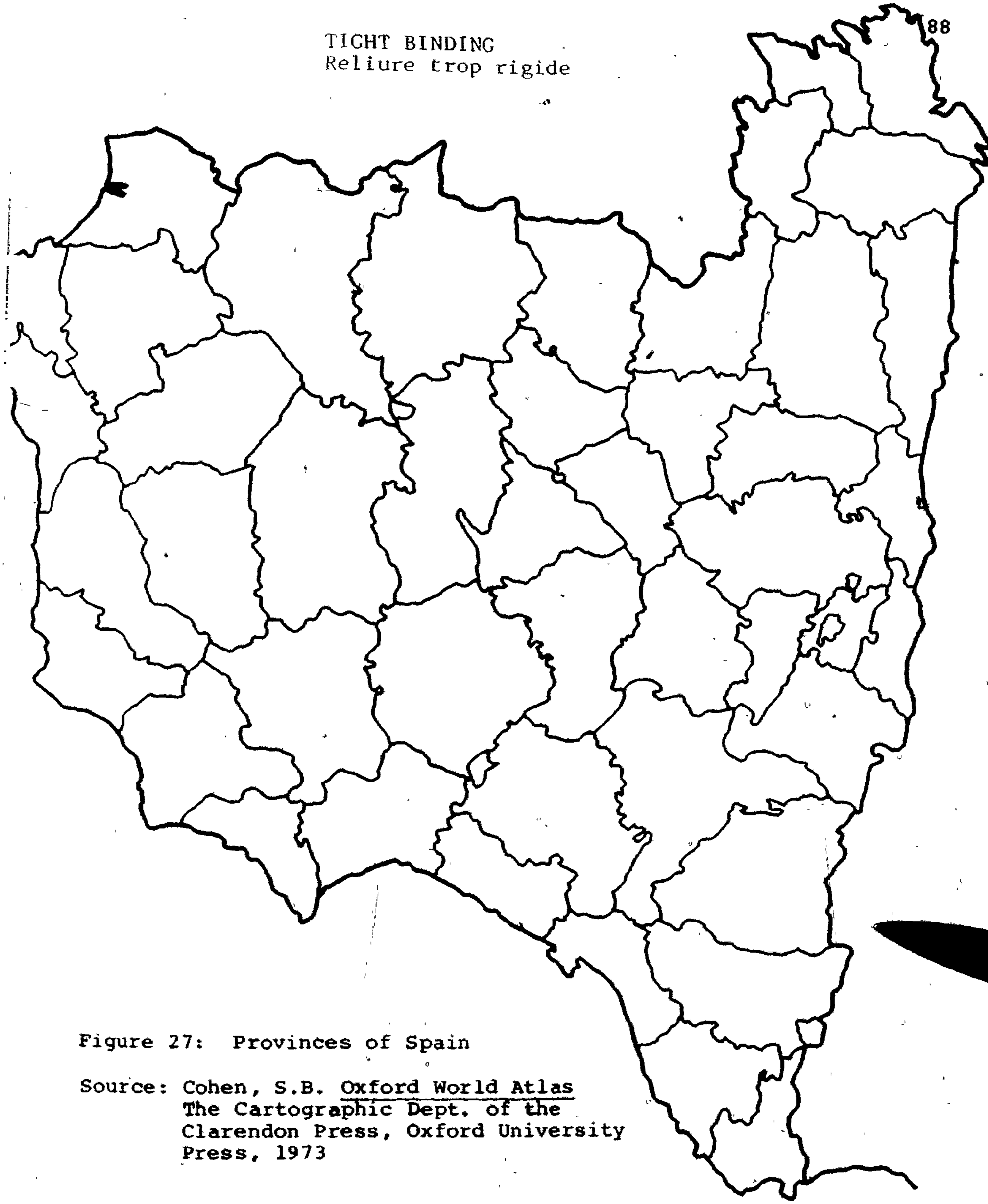

Figure 27: Provinces of Spain

Source: Cohen, S.B. Oxford World Atlas
The Cartographic Dept. of the
Clarendon Press, Oxford University
Press, 1973

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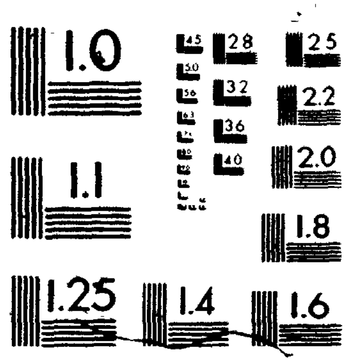

This is in accordance with the pattern of 25a. The network of administrative districts in Kenya (see Figure 28) also shows an increase in positive spatial autocorrelation. In this pattern some of the boundary cells have the same contact values as the internal cells so like connections are added between the boundary cells and internal cells as well as between the boundary cells and other boundary cells as in 25a. This serves to increase the number of like joins and thus strenghten the positive spatial autocorrelation.

Negative Spatial Autocorrelation

In the case of negative spatial autocorrelation, there is only one 'typical pattern which is illustrated in Figure 29. The situation here is the same for the one described in pattern 25a, with a typical boundary cell adding 6 L/L and 2 H/L joins. Since like joins dominate the spatial autocorrelation should be weakened.

No Spatial Autocorrelation

Here in the interior of the pattern, H and L values are distributed randomly as illustrated in Figure 30. The situation is approximately equal to that described by the pattern in Figure 25a, although there may be more H/L than L/L

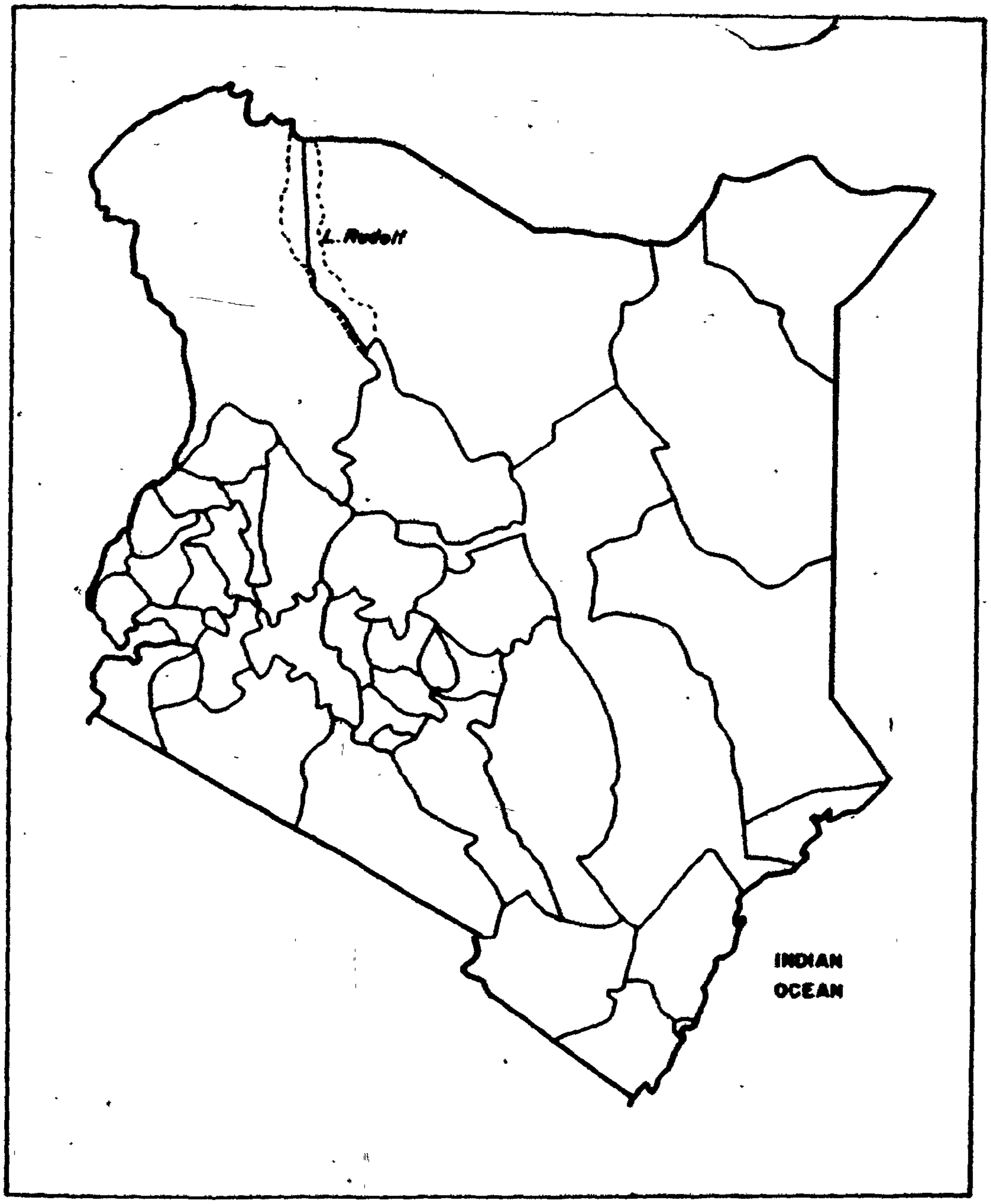

Figure 28: Administrative districts in Kenya

Source: Davis, J.T. Development of the small farm sector in Kenya, The Canadian Geographer, Vol. xxll, 1977.

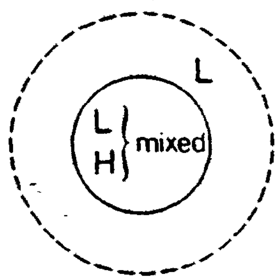

Figure 29; Negative Spatial Autocorrelation

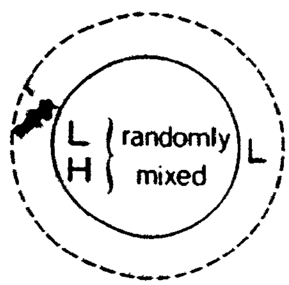

Figure 30: No Spatial Autocorrelation

joins added or vice versa depending on the particular circumstances. Thus, there may be a weak tendency to move the coefficient in either direction (Boots and Morgan, 1979).

Of the 28 empirical networks for which the statistics were computed with the addition of the boundary cells, only 17 showed consistent significant results. We can categorize the patterns and then fit them to the expected results. The majority of the patterns go from showing no spatial autocorrelation to exhibiting positive spatial autocorrelation. (76.47%) while 12% go from no spatial autocorrelation to significant negative spatial autocorrelation and 12% go from significant negative spatial autocorrelation to significant positive spatial autocorrelation.

The situation illustrated in Figure 29 is clearly found in 76% of the networks. In this case negative spatial autocorrelation is weakened to such an extent that the statistics become positive meaning that there are more L/L interfaces after the boundary cells are added.

It is interesting to note that there are two cases (counties of Eire and shopping centre hinterlands of N.W. England) where the results of the statistical analysis before the addition of the boundary was no spatial autocorrelation while the statistics show significant negative spatial autocorrelation after the addition of the boundary cells. In these two patterns we find that there are boundary cells with contact values of greater than 6 (10,12) so that a H/L interface exists but it is in the opposite direction from the normal case of 26b. Instead of H/L with the connection between high interior and low exterior cells, it is opposite. This serves to increase the dislike joins so that the patterns exhibit negative spatial autocorrelation.

Of the 13 cases where the statistics change from showing no spatial autocorrelation to showing significant positive spatial autocorrelation, the majority of the statistics do tend towards negative spatial autocorrelation before the addition of the boundary and this is weakened by the addition of the low value boundary cells. There is an increase in L/L interfaces and so positive spatial autocorrelation ensues.

In some cases there is no change at all in the statistics with the addition of the boundary cells. There is no spatial autocorrelation before and none is evident after. For example in the networks of the central place hinterlands of Skane, Sweden (Figure 31, Table 1) and zones of Reading region (Figure 32, Table 1). These results are consistent with the expectations of Figure 30.

The addition of the boundary cells in the computation

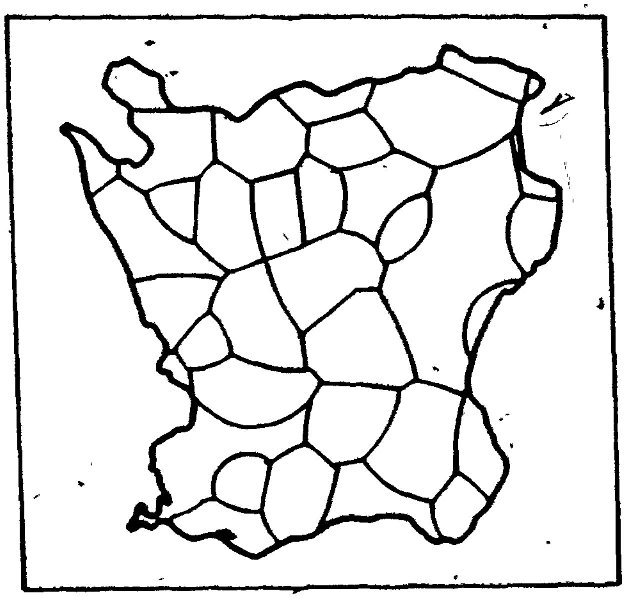

Figure 31: Central place hinterlands Skane Sweden.

Source: Morrill, R.L. The Spatial Organization of Society, Duxbury Press, Mass. p. 78.

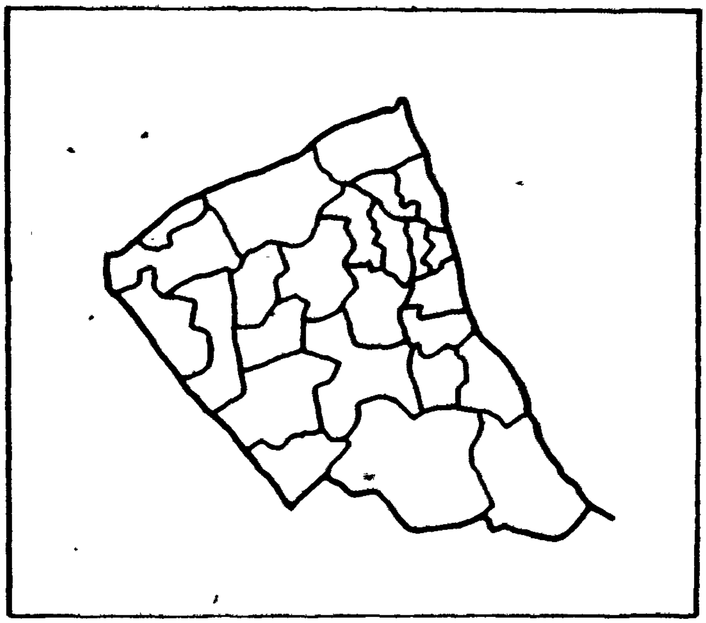

Figure 32: Merseyside Traffic Zones

Source: Thomas, R. W. An interpretation of the journey to work on Merseyside using entropy maximising methods, Environment and Planning A, 1977, Vol. 9, p. 820.

of the spatial autocorrelation statistics has a very strong influence on the outcomes. This is evident in the fact that in cases where very strong significant positive spatial autocorrelation is the case after the addition of the boundary there was no spatial autocorrelation before and patterns with a tendency towards negative spatial autocorrelation (not quite significant at the .10 level) are found to become strongly positively spatially autocorrelated with the inclusion of the boundary cells. This strong reversal in sign shows that the inclusion of the boundary cells in the statistical analysis has a very distorting affect.

Experimentation with 'abstract' patterns is carried out to attempt to try and discover more about the inclusion of boundary cells in the calculation of spatial autocorrelation coefficients. In a pattern of 4 sided and 8 sided cells (see Figure 33), addition of the boundary cells to the original six cells (labelled 1-6, Figure 33) serves to illustrate that positive spatial autocorrelation increases. When another ring of boundary cells is added to the original cells and the first ring of boundary cells, again the degree of positive spatial autocorrelation increases. It must be noted that the first set of boundary cells added to the original sample set comprises 82% of the total cells of the cellular network while the boundary cells only contribute 50% of the total cells in the second addition. The spatial autocorrelation of the internal cells

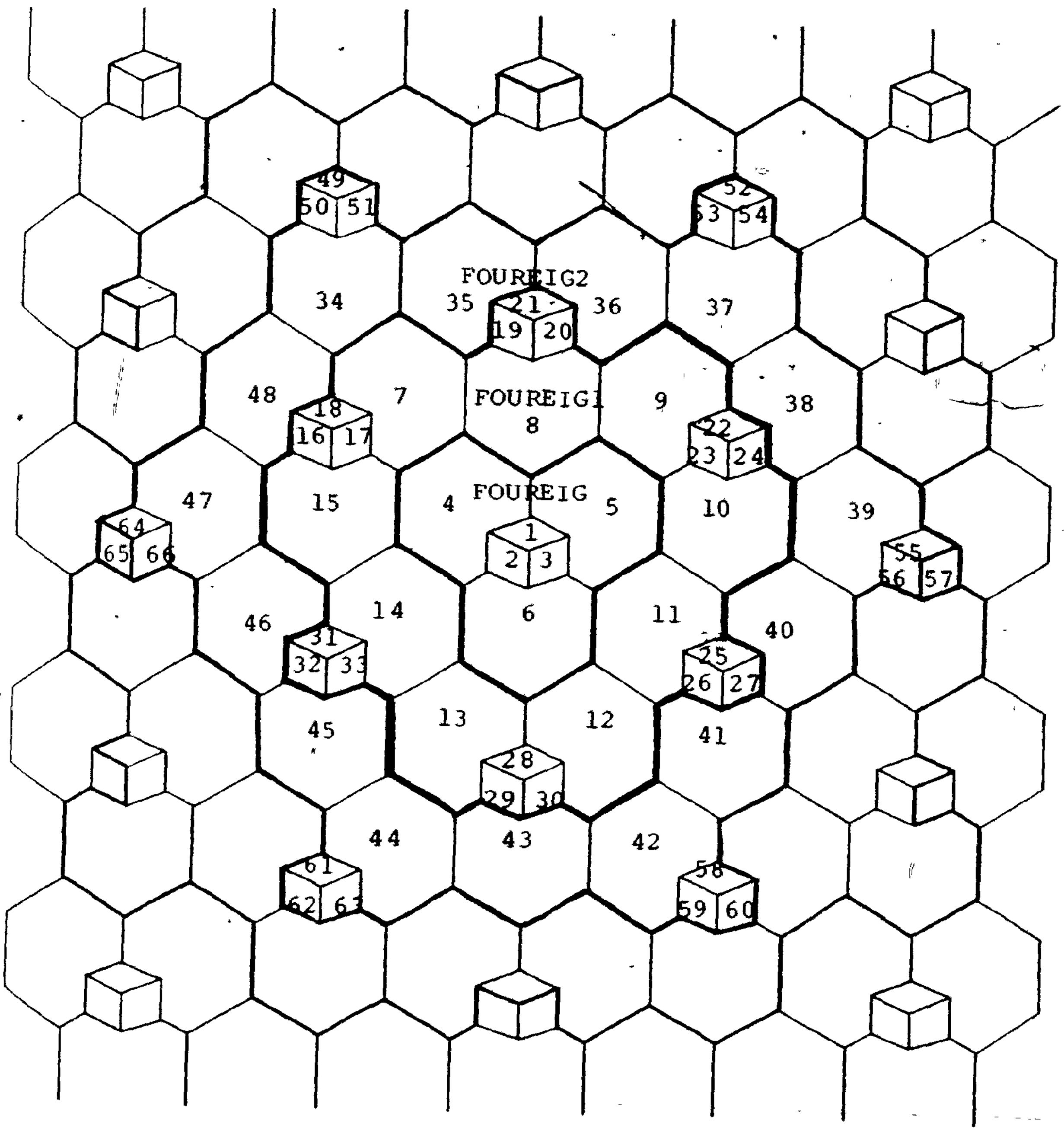

Figure 33: Foureig

is disturbed less by the inclusion of the boundary cells as the sample size n increases and the degree of positive spatial autocorrelation is not as high with the second addition of boundary cells. The correlation coefficients for 'Foureig' are found in Table 3.

In the second example a pattern of 4 and 8 sided cells is also looked at (see Figure 34, Abstract). The spatial autocorrelation in this network is negative and becomes increasingly so with the addition of each 'boundary ring'. In the pattern 'Abstract' there are no like joins between four sided cells as there are in 'Foureig'. With the addition of the first boundary ring in both networks, there are more H/H interfaces added but this is more pronounced in 'Foureig' and so one would assume that there would be an increase in the degree of positive spatial autocorrelation in this network which is in fact the case. The ratio of dislike to like joins is much less in 'Foureig' after the first addition of the boundary than in 'Abstract', being 18:39 and 20:32 respectively. Dislike joins increase in 'Abstract' with the addition of the second boundary ring, 30:64 and so as indicated by this ratio negative spatial autocorrelation increased. Also in 'Abstract' all the trivalent vertices are a junction of one interface between 8 sided cells (8/8) and two interfaces between 8 sided and 4 sided cells (8/4), whereas in 'Foureig' the trivalent vertices consist of predominately 8/8, 8/8, 8/8, and 4/4, 4/4,

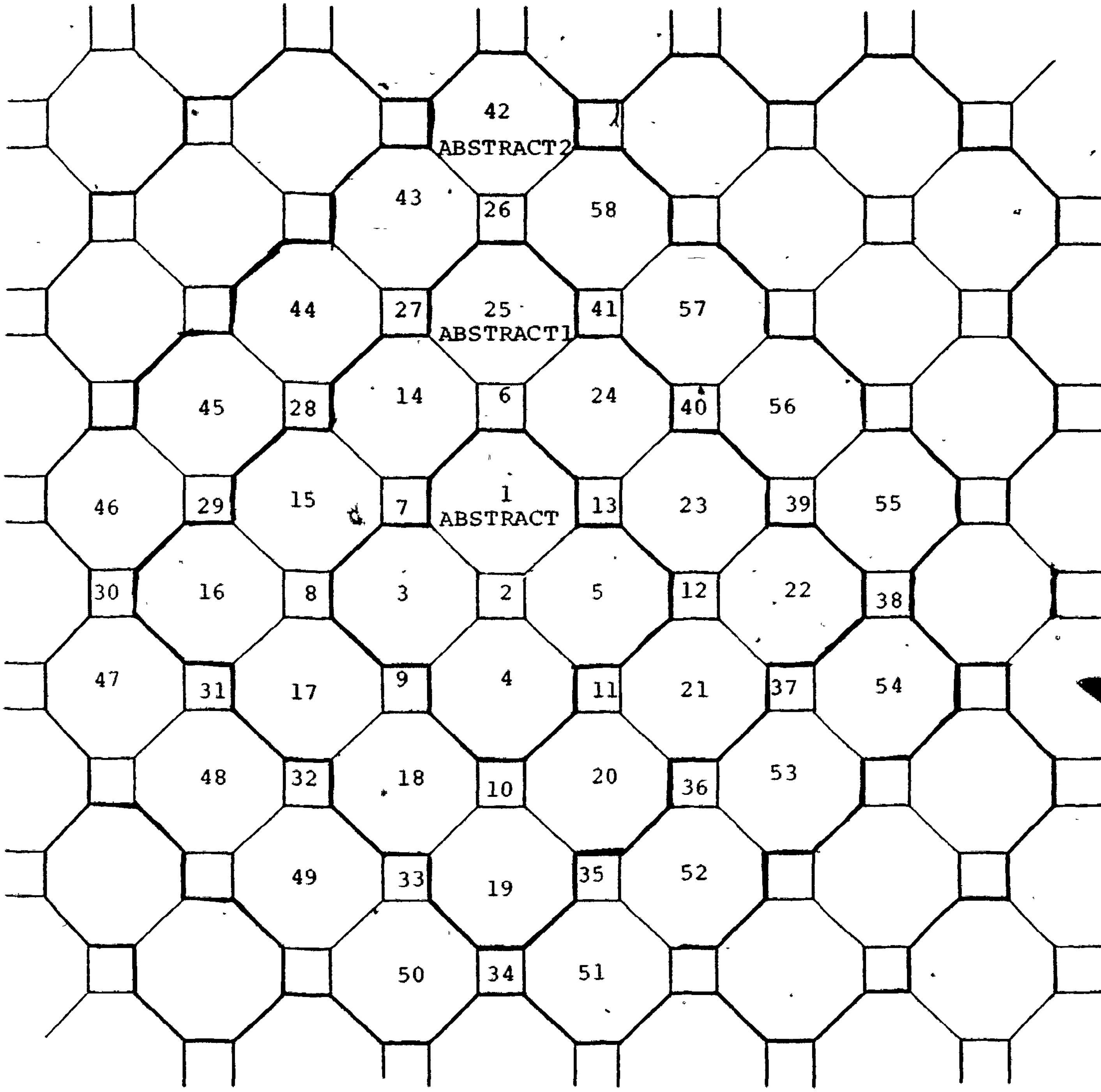

Figure 34: Abstract

4/4, and just a few 8/8, 4/8, 4/8.

It appears then that the percentage of boundary cells to the total number of cells is important to consider when evaluating the effect of the addition of the boundary cells; the type of trivalent vertices that are added to the internal pattern with the addition of the boundary and the ratio of dislike to like joins that are added. All serve to help explain the significant impact the boundary cells have on the spatial autocorrelation of the network.

CHAPTER SEVEN

CONCLUSIONS

The primary objective of this thesis was to obtain some insight into the spatial structure of cellular networks. In doing so it has been determined that there is a relationship between the type of network and the structure. The study shows that there are two predominant structures of cellular networks. Those which, by the arrangement of the cellular units exhibit negative spatial autocorrelation and those that exhibit no spatial autocorrelation. Spatial structure is related to process, therefore we might anticipate that the structure of a cellular network has different impacts on the processes studied, the data for which is collected over these units. Since there is evidence of some special ordering of the cellular units in relation to one another in cellular networks, it is important to take this into consideration when testing the map patterns for evidence of particular process.

A pattern exemplifying negative spatial autocorrelation will have cells of dissimilar contact number values, x_i adjacent to each other. Aboav(1970) and Weaire(1974) suggested that cells of smaller contact number values tend to be surrounded by those of larger contact number values and vice versa. A structure such as this will exhibit negative spatial autocorrelation. Aboav states that an arrangement of this nature is a result of a process which minimizes the instability of the pattern. The

The arrangement is such that unstable cells (those of low contact value) are located next to stable cells (those of large contact value). The average cell size (number of sides analagous to number of contacts) at this stable state is nearly always 5.85 (Aboav, 1970). D'arcy Thompson (1942) has stated that structures of all or predominately six sided cells (eg. honeycombs, cooling cracks in basalt, some corals, crystal structures) are very stable forms. It seems likely then that the introduction of a cell of size greater than six will always mean introducing cells of size less than six to compensate for the larger cells and obtain a desired mean contact number value of close to six for maximum stability of the structure. Aboav also states that the growth of the process behind the development of a pattern should then stop once the $\bar{m}$ is very close to six. In nature, networks evolve to their fittest form and tend towards a configuration with the least expenditure of energy, the tightest fit and the least motion (Stevens, 1974). To increase the stability of natural structures, interfaces between areas are minimized and it is not very often that greater than triple joints of trivalent vertices occur. 120° joints are the most favoured in nature (Stevens, 1974).

The interpretation of the majority of cases exhibiting no spatial autocorrelation may then perhaps be explained by the fact that the observed patterns are not stable (most of the patterns are man-made and not in a natural state) or when examined were in a generative state, in the process of conforming to a stable state.

The I or c indices enable the examiner to communicate aspects of dispersion patterns which would otherwise be difficukt to convey, however, while finding the extreme I and c values negates the null hypothesis it does not necessarily guarantee that a specified alternative is the best possible description of the observed autocorrelation. It has been noted that there are certain factors that have a very influential affect on the autocorrelation coeffivients other than the pattern of the cellular contacts, namely the boundary cells, the connection criteria and the type of trivalent vertices.

The ring of boundary cells has an average cell size of four contacts and biases the results of the investigation. The examiner must determine whether the bias produced by the inclusion of the boundary cells is greater than just using the sample of the internal cells. This will partly depend on the size of n as the boundary effect diminishes as n increases. Apart from keeping the ratio of boundary cells to internal cells low, the investigator may also introduce the idea of weighting the boundary cells so that the bias is minimized.

If a pattern is infinite in size, delimiting the pattern (large size may be constraining in analysing these patterns) and just using a sample ~~size~~ may result in the analysis of a network which is not truly representative of the variation exhibited in the larger network because the boundary is perhaps located at a critical point in the overall network. Large cells which serve to stabilize the effect of the smaller cells may have been left out when the boundary was arbitrarily chosen. The network under examination may not necessarily furnish the basis for inferences about the entire structure.

In this study $\delta ij = 1$ for connection between adjacent cells and 0 for lack of connection however, weights need not necessarily be binary. Further investigation could be done in altering the weights according to connectedness for example joins between internal cells and boundary cells. Instead of just considering 1st order connectedness, different connections could be considered and weighted accordingly, (2nd, 3rd, etc. order). The method of connecting cells, eg. rook's, bishop's and queen's count could be experimented with.

If the results discussed in Chapter 4 are inherent in the character of the networks and their generating processes, inquiring about what conditions are necessary for specific results is appropriate. Since this research is concerned with topologic network structure, investigation may be concentrated in a search for other related properties such as the incidence and type of trivalent vertices, cell shape, and areal size. A more thorough examination of the contact number properties eg variance, the incidence of connections between cells of specific contact size (for large patterns) and skewness amy also prove instructive as well as a further investigation into chains and clumping of like cells(Aboav, 1970).

The testing for spatial autocorrelation as operationalized in this study tells us whether or not spatial autocorrelation exists but not how likely that result is in a particular context. The best immediate solution to this would be an enumeration approach where a sufficiently large number of planar graphs would be generated using the same constraints (contact numbers) as the original network. Such a response has already been adopted in graph theory applications in architecture (eg. Combes, 1976; Korf, 1977). One way to do this would be to use Tinkler's graph enumeration algorithm (Tinkler, 1978) and then to test each of the graphs for planarity using an algorithm such as that developed by Hopcroft and Tarjan (1974). Spatial autocorrelation coefficients would then be computed for each of the graphs so that the resulting distribution of such values could be used to evaluate the particular value obtained for the empirical graph. This might help to determine whether a particular cellular network is in fact common (and perhaps considered stable) or perhaps a rare arrangement (unstable)

The researcher realizes that there are problems with the statistics in that their behavior in certain circumstances and in relation to one another is not completely understood and thus further investigation is needed to expand the relevance to research in geography, Nevertheless, this thesis has presented an approach to cellular network analysis which, by using contact number, concentrates on an evaluation of topologic structure. The nature of the results suggests that there is a latent spatial structure of the areal units in cellular networks which should be considered when determining any process operating over the network. The author believes that this research has provided both an approach to describing the morphology and structure of cellular networks and findings which will be useful in the development of more sophisticated models for isolating and describing processes operating over cellular networks.

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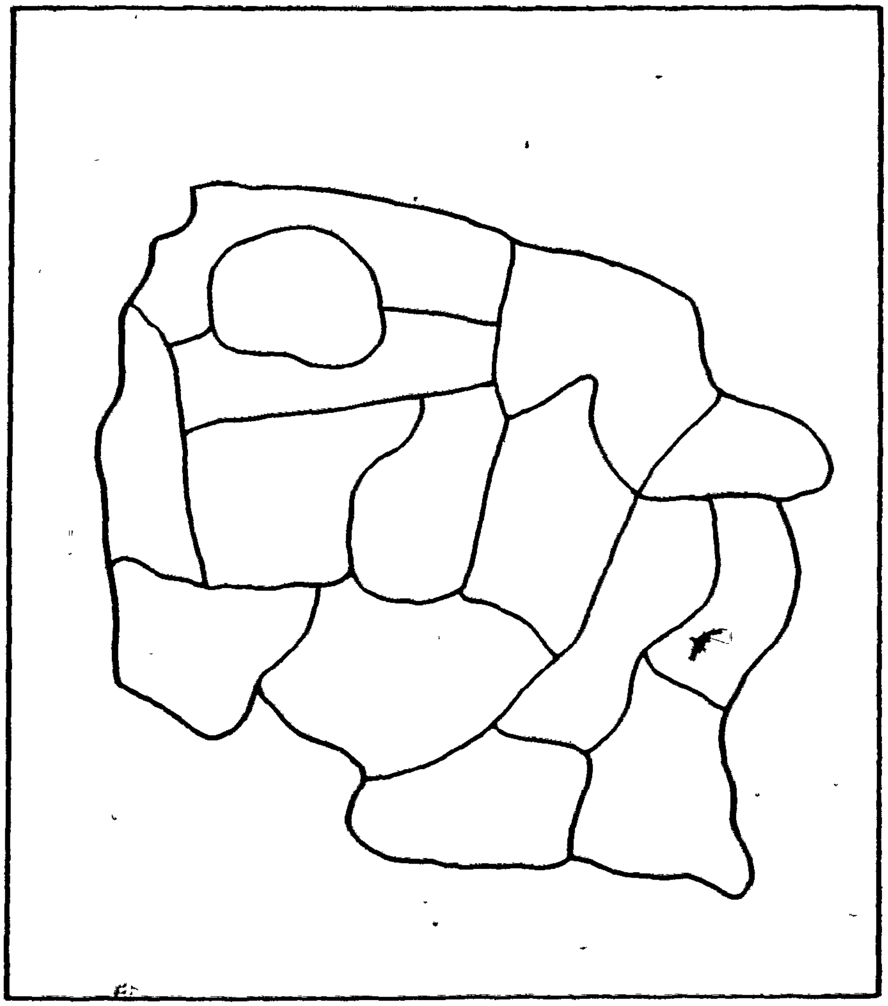
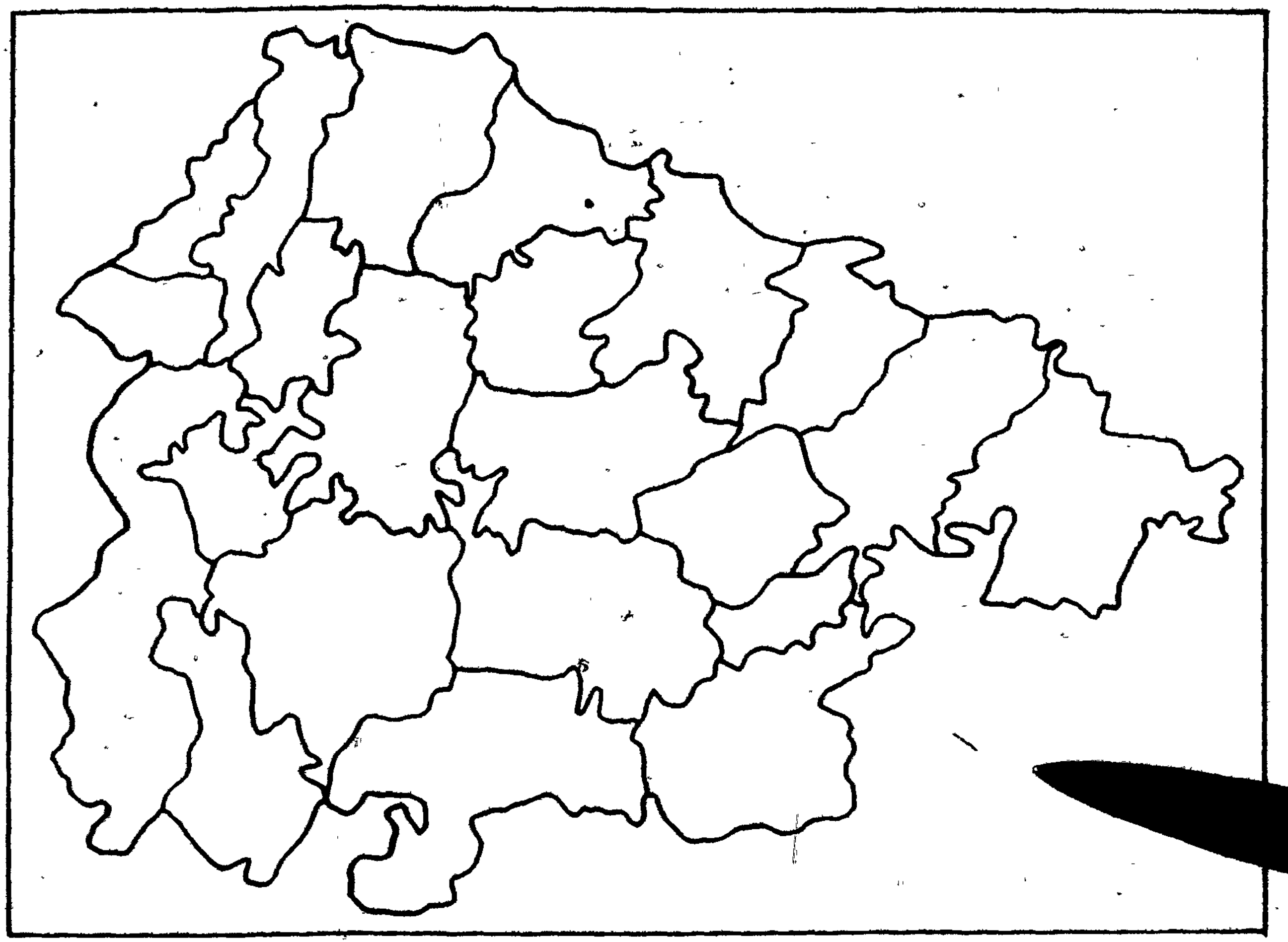
APPENDIX

POLITICO-ADMISTRATIVE CELLULAR NETWORKS



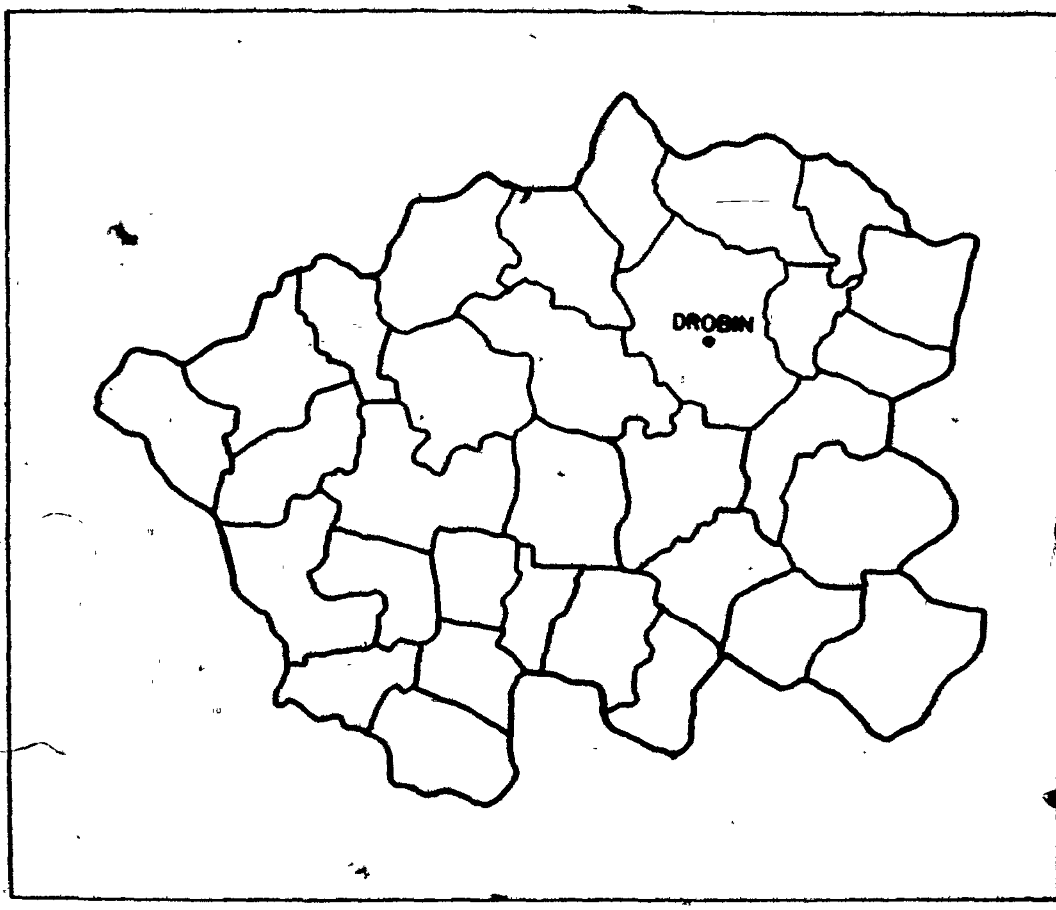
Regions of Tanzania

Source: Cohen, S.B. <u>Oxford World Atlas</u>, The Cartographic Dept. of the Clarendon Press, Oxford University Press, 1973



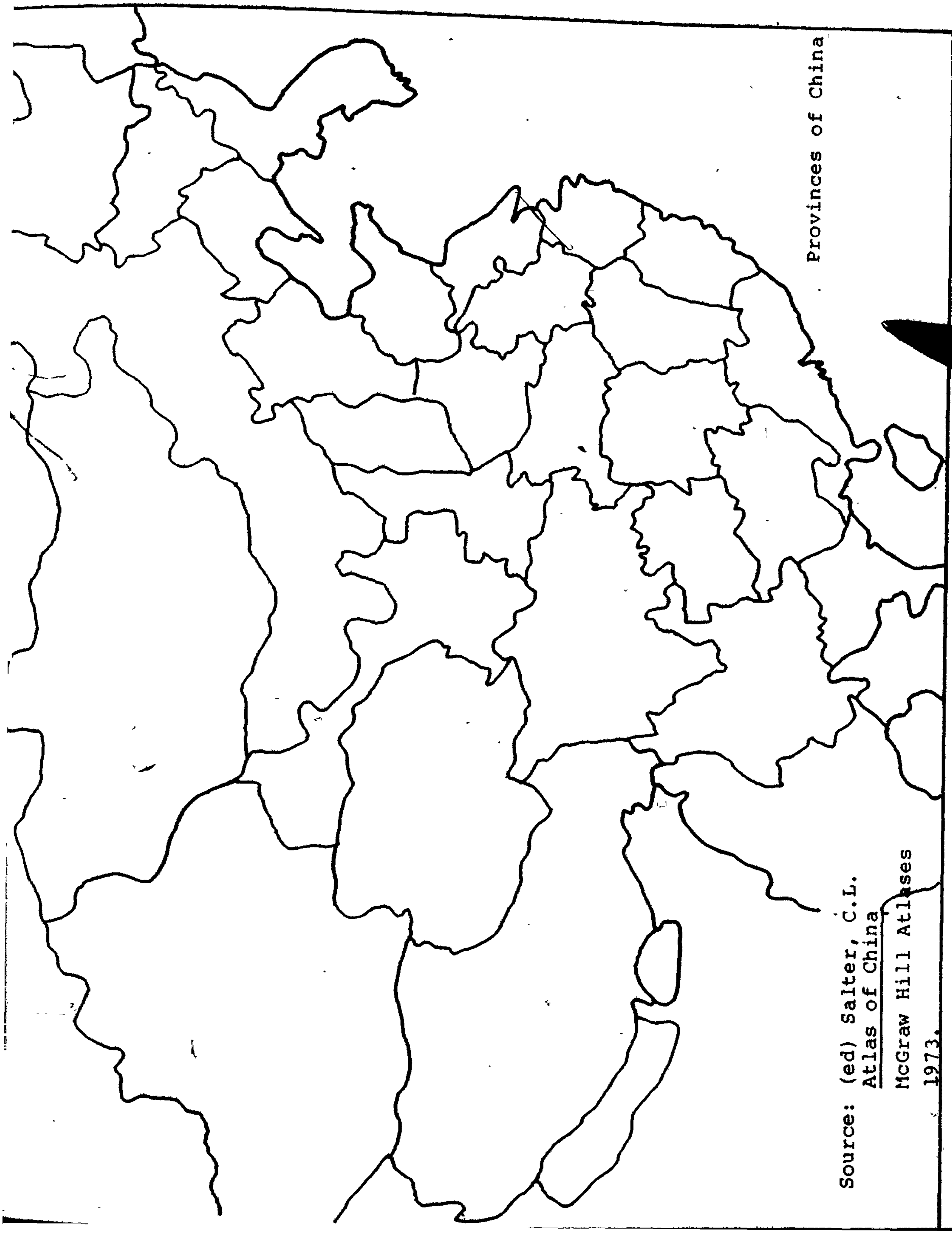
Planning Regions, Bundelkhand, India

Source: Singh, Harendra Pal, Delimitation of planning regions in Bundelkhand <u>The Geographer</u>, Vol. 13, n0.1, January 1976.



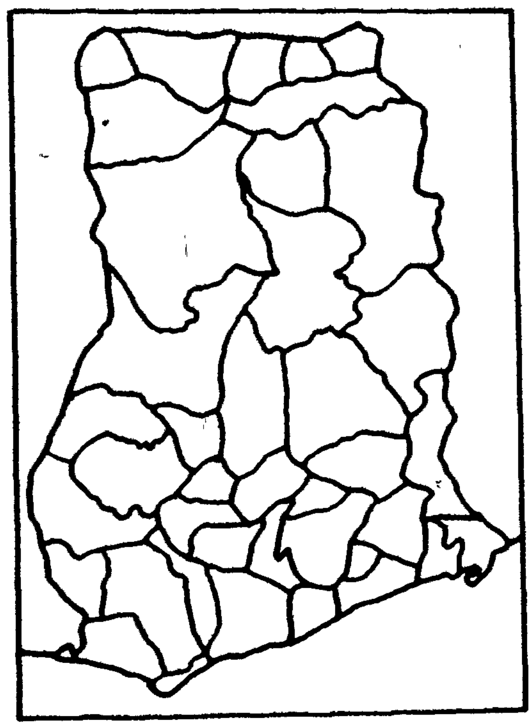
Drobin County, Poland

Source: Kulikowski, R., Optimization of rural -urban development and migration, Environment and Planning A, 1978, Vol. 10, p. 585.



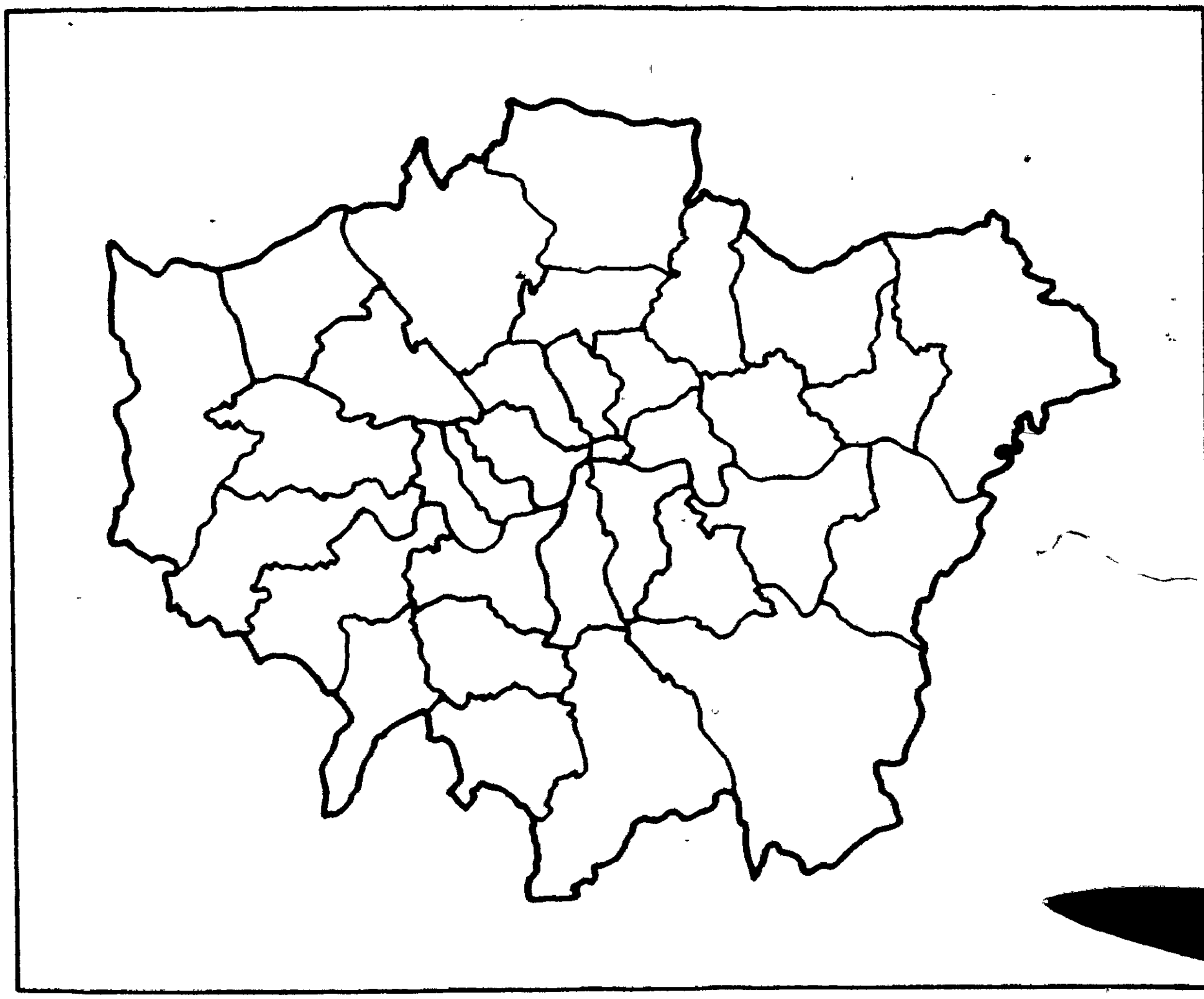
Provinces of China

Source: (ed) Salter, C.L. Atlas of China McGraw Hill Atlases 1973.



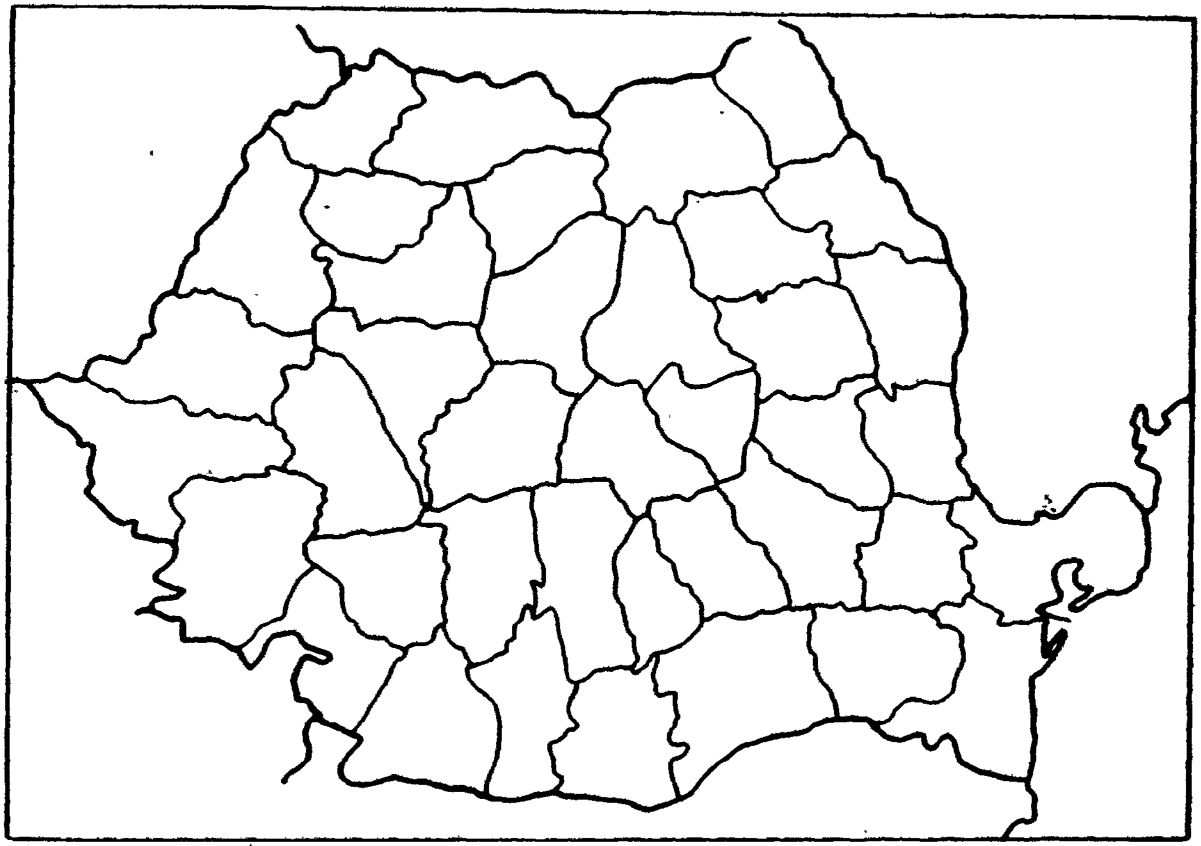
Administrative Districts of
Ghana

Source: Cliff, A.D. and J.K. Ord. Spatial Autocorrelation, 1973, Pion, London p. 115.



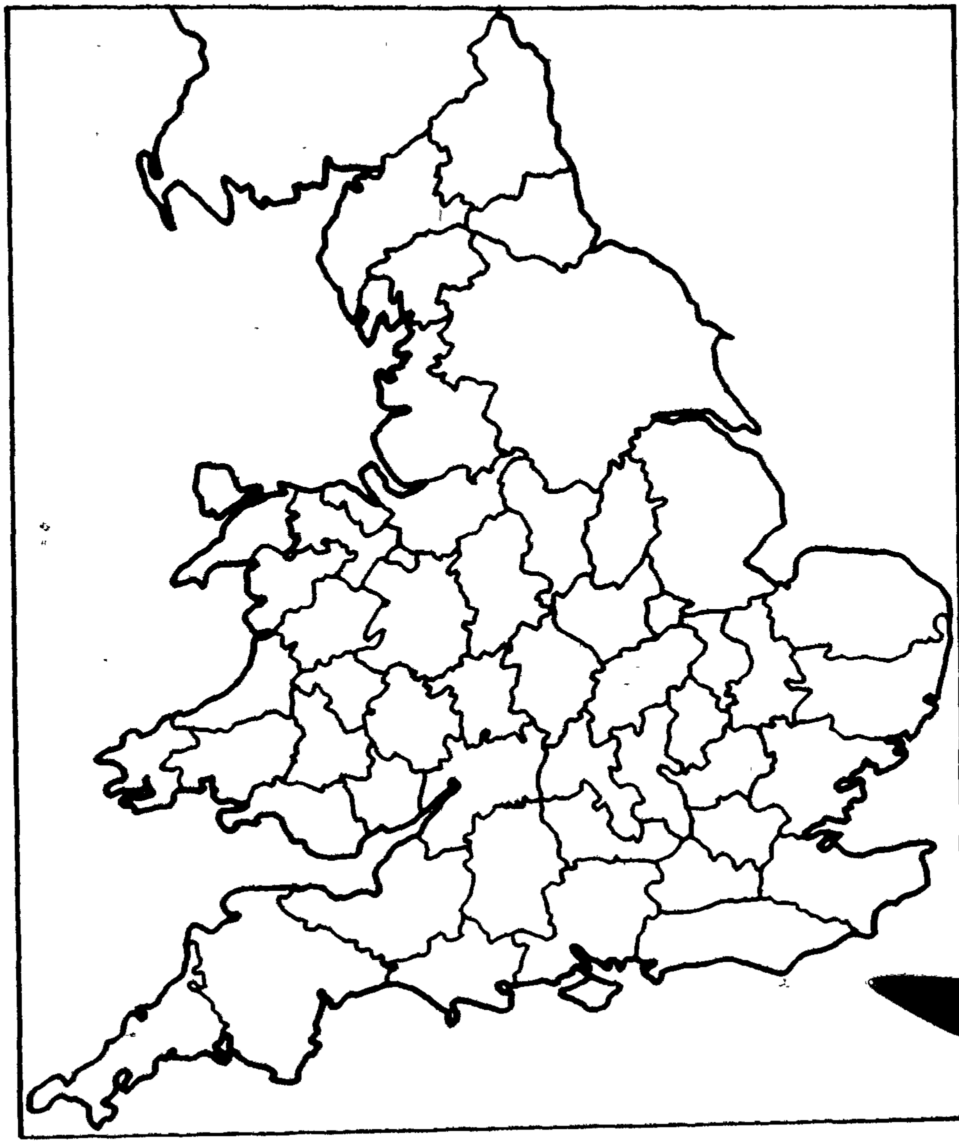
Boroughs of Greater London

Source: King, R., Italian migration to Great Britain, Geography Vol. 62, July 1977, p. 185.



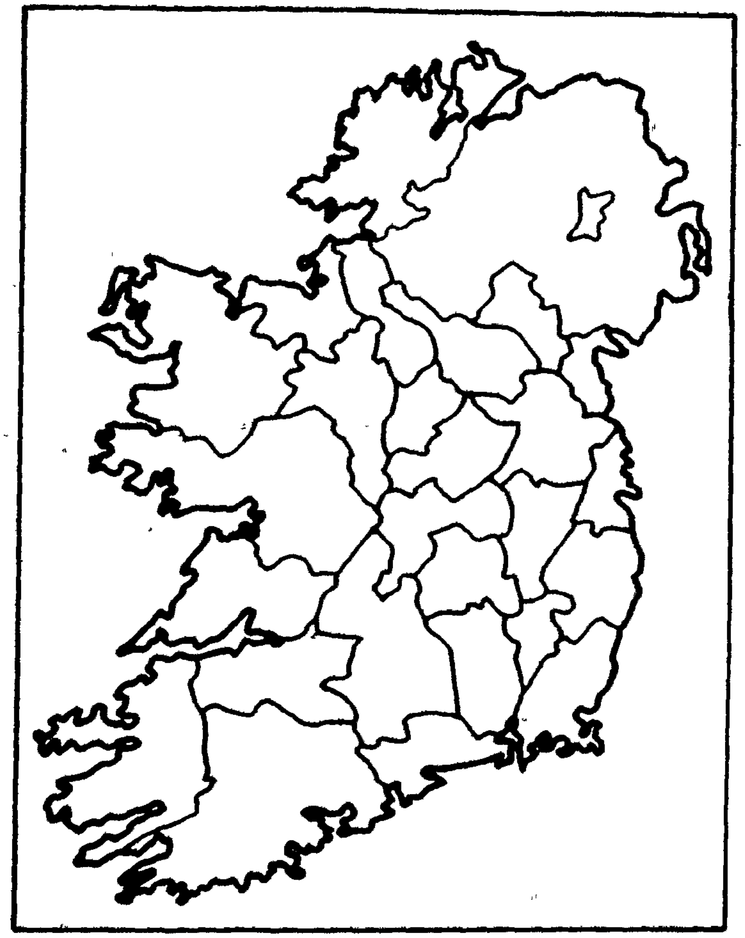
Administrative Districts
of Romania

Source: Posea G and I. Velcea The socialist republic of Romania, <u>Geoforum</u>, Vol. 6, 1975, p. 18.



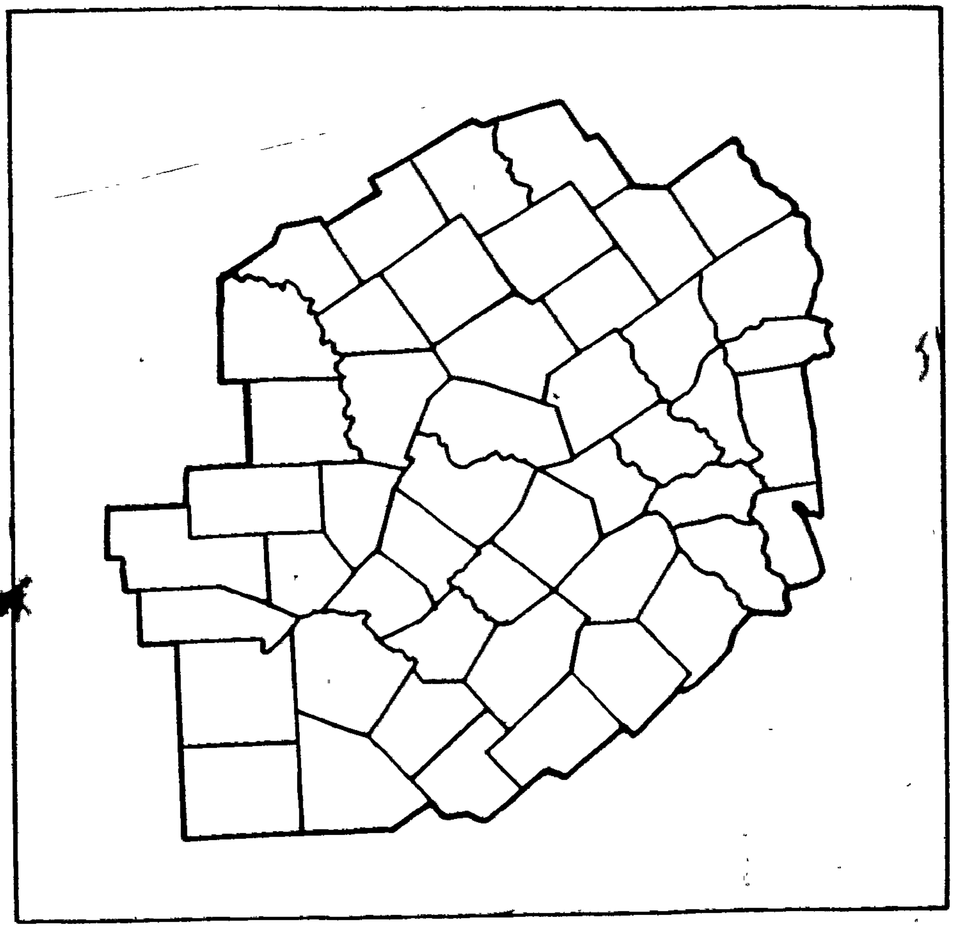
Counties of England and Wales

Source: King R., Italian migration to Great Britian
Geography, Vol. 62, July, 1977, p. 184.



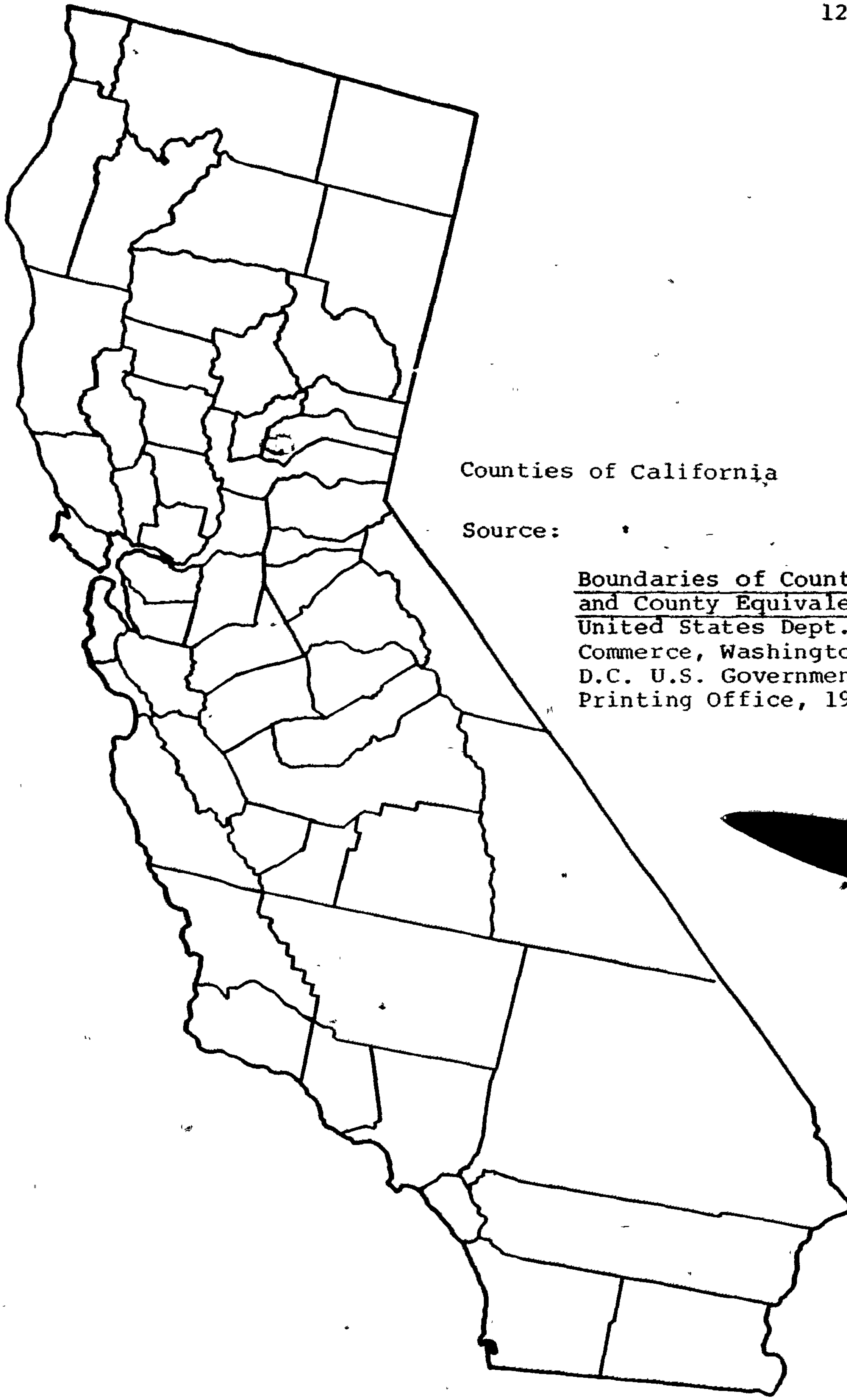
Counties of Eire

Source: Geary, R.C. The contiguity rato and statistical mapping, The Incorporated Statistician, 1954, 5, p. 119.



Counties of S.E. Texas

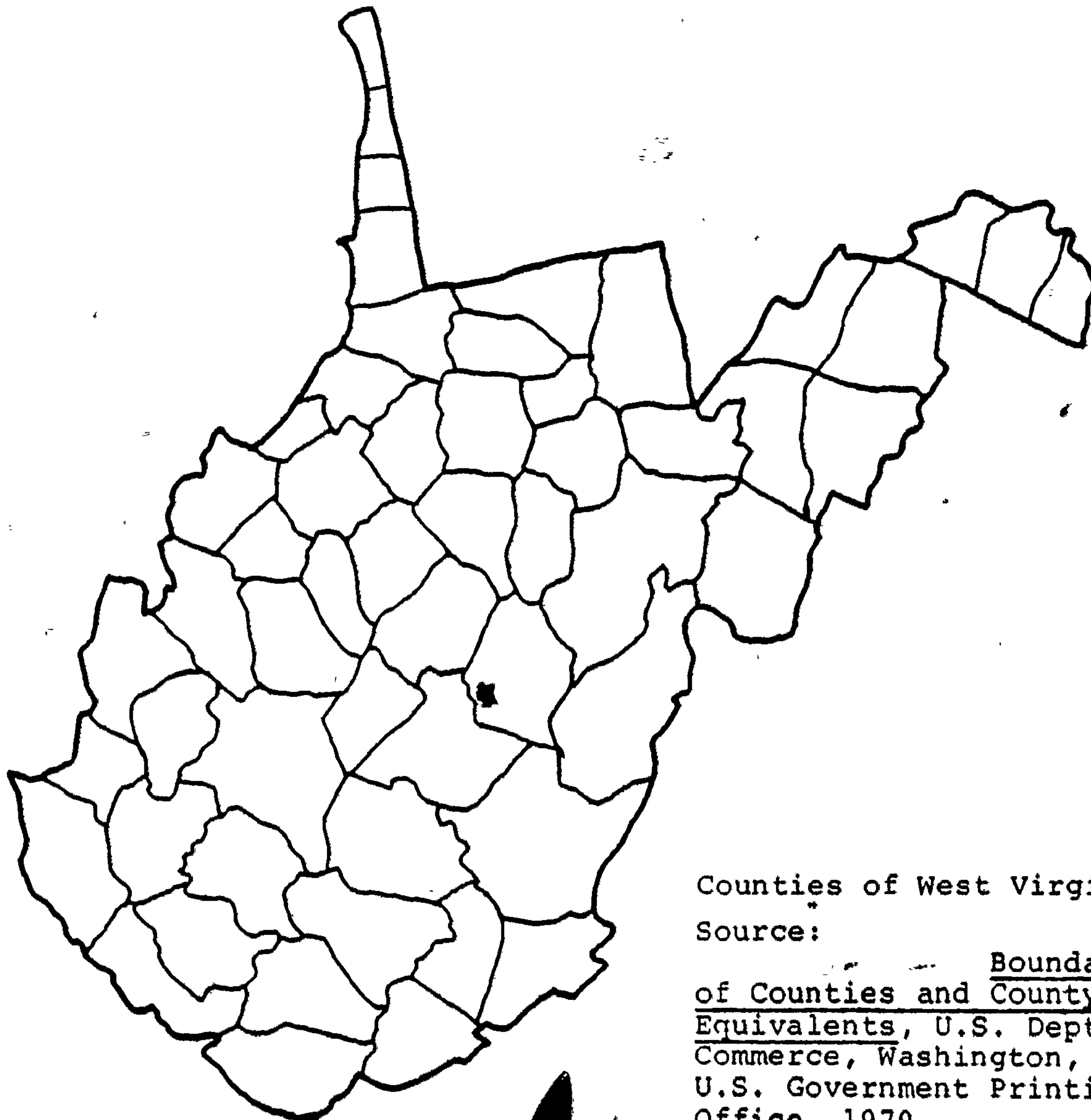
Source: Achabal, D.D. The development of a spatial delivery system of emergency medical services, Geographical Analysis, Jan. 1978 Vol. 10, No. 1.



Counties of California

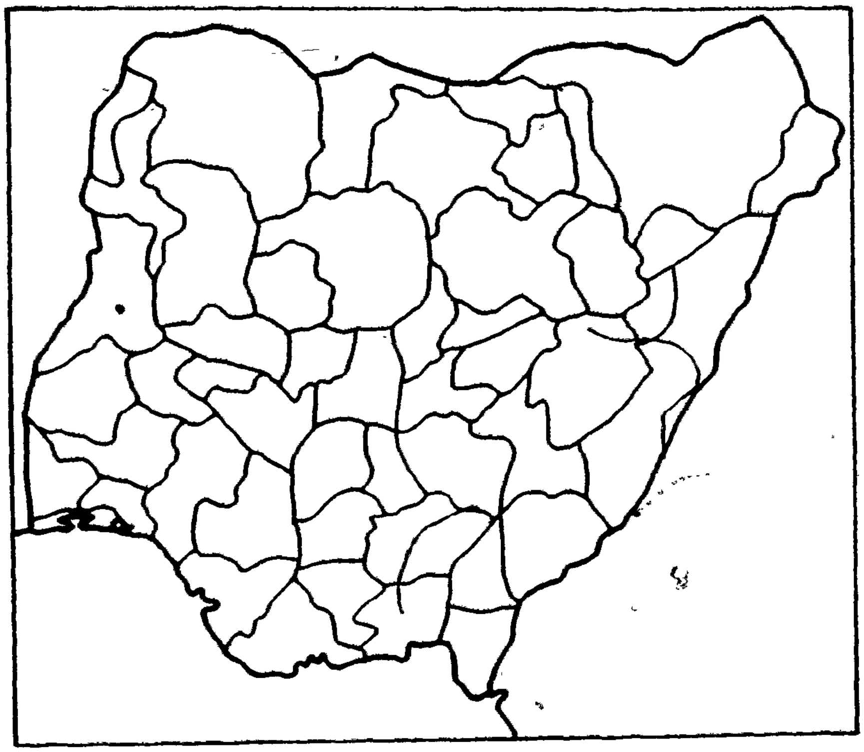
Source:

Boundaries of Counties and County Equivalents United States Dept. of Commerce, Washington, D.C. U.S. Government Printing Office, 1970.



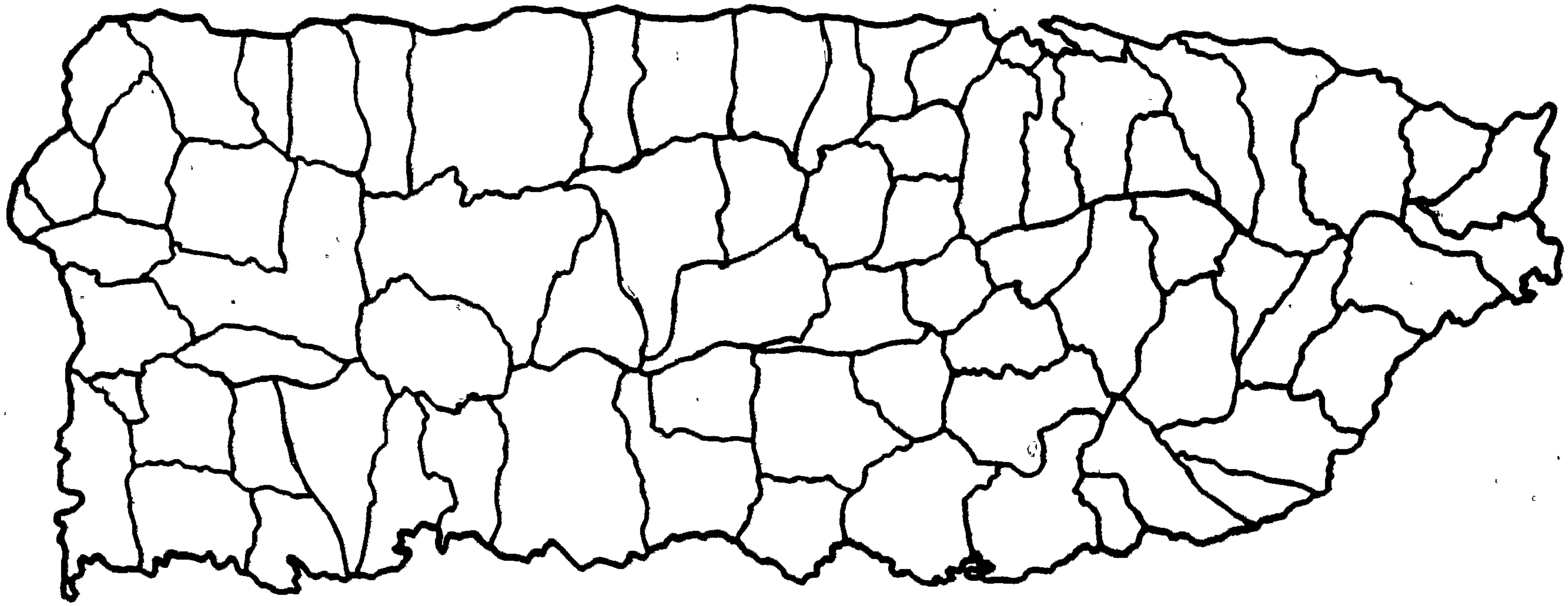
Counties of West Virginia

Source: <u>Boundaries of Counties and County Equivalents</u>, U.S. Dept. of Commerce, Washington, D.C U.S. Government Printing Office, 1970.



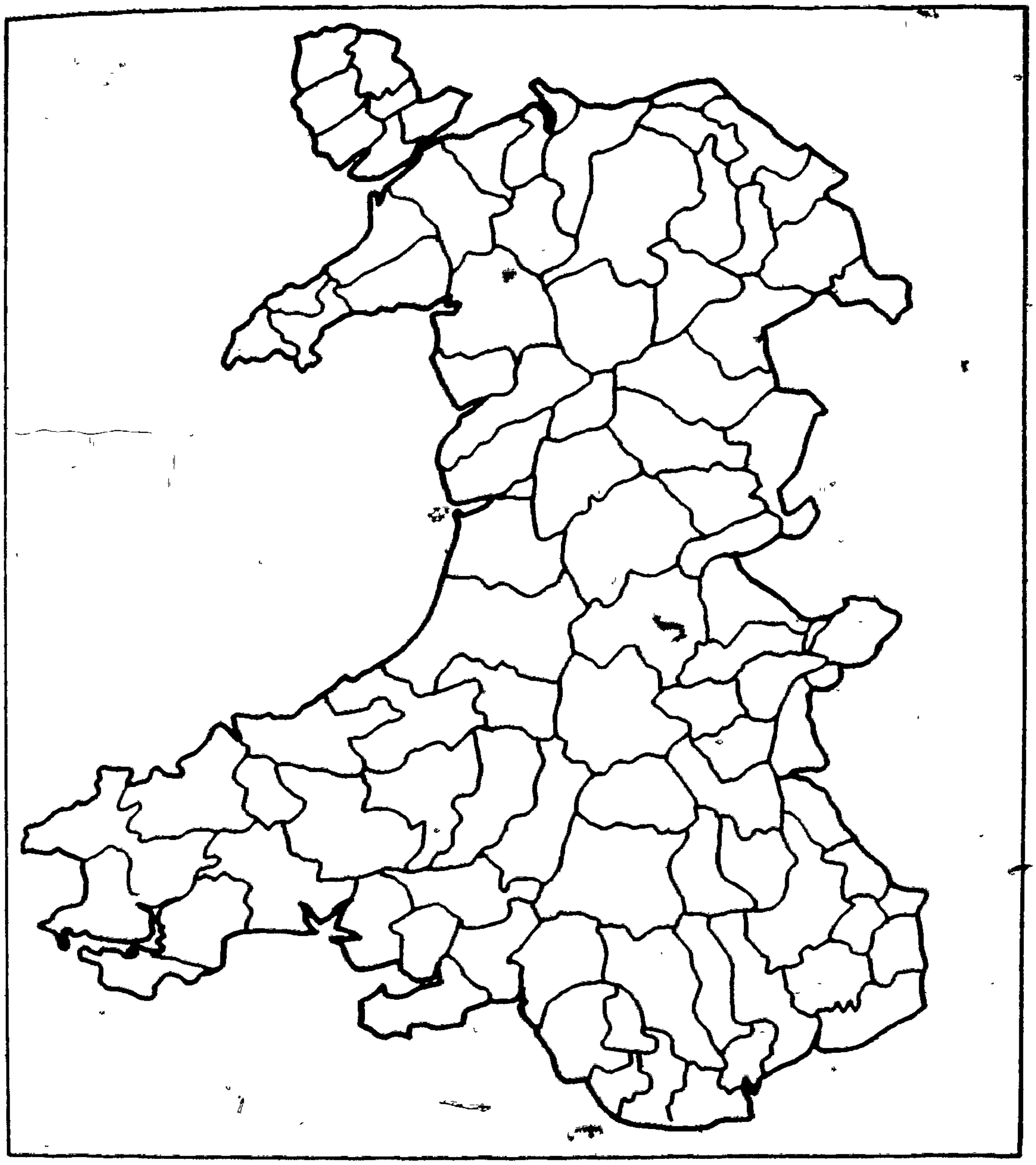
Administrative Districts
Nigeria

Source: Cliff, A.D. and J.K. Ord, <u>Spatial Autocorrelation</u>, 1973, Pion, London p. 116.



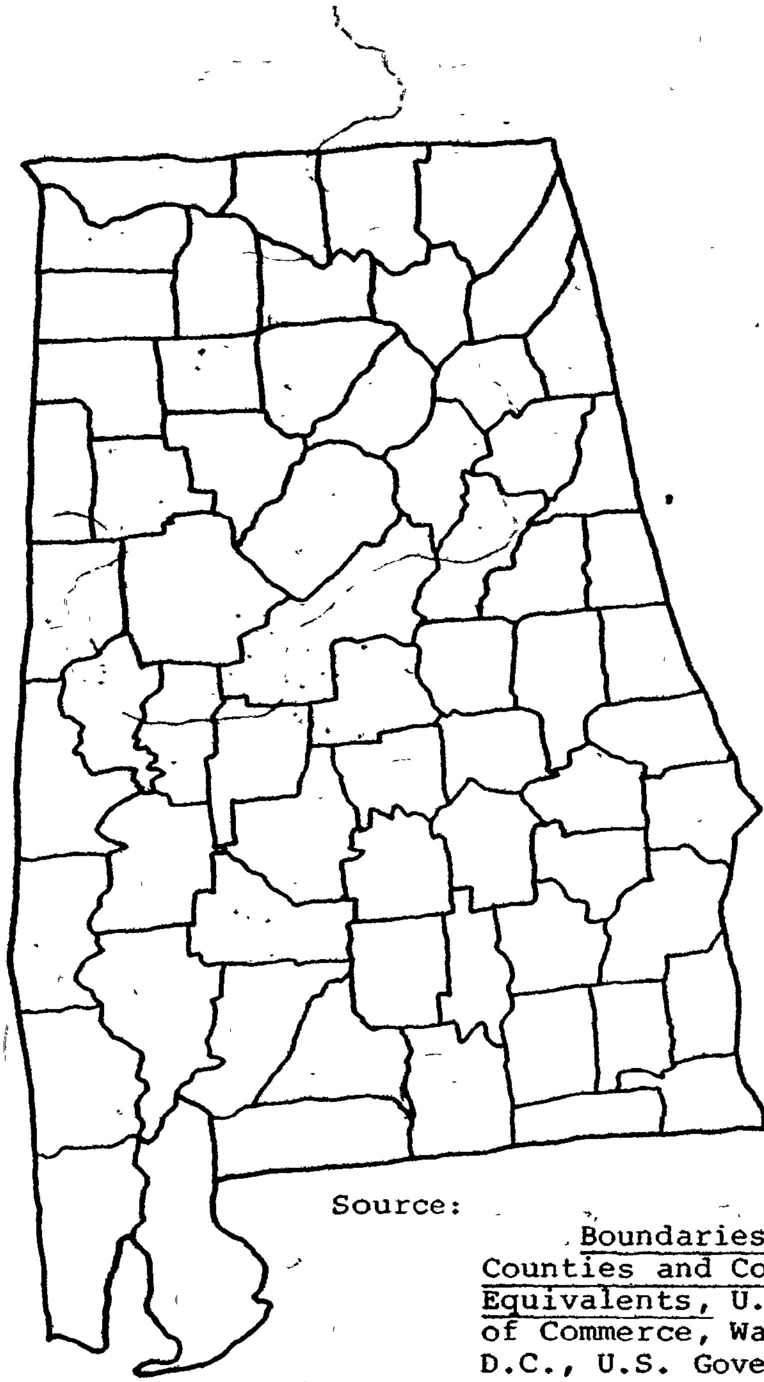
Municipos of Puerto Rico.

Source: Griffith, D. The Impact of Configuration and Spatial Autocorrelation on the Specification and Interpretation of Geographical Models, PhD Thesis, Dept. Of Geography, University of Toronto, 1972



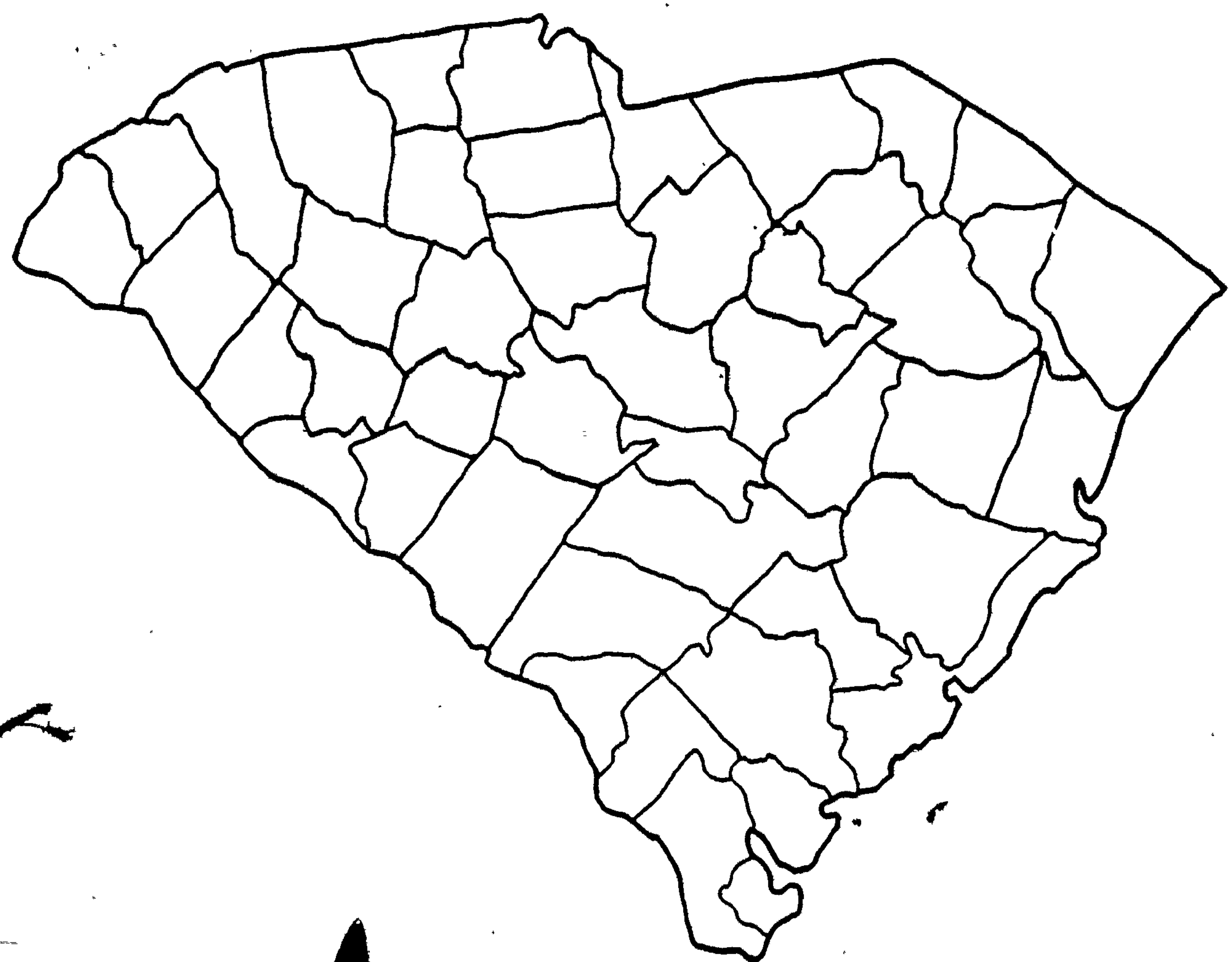
Hundreds of Wales

Source: Rees, W. A Historical Atlas of Wales, Faber and Faber, London, 1972, plate 57.



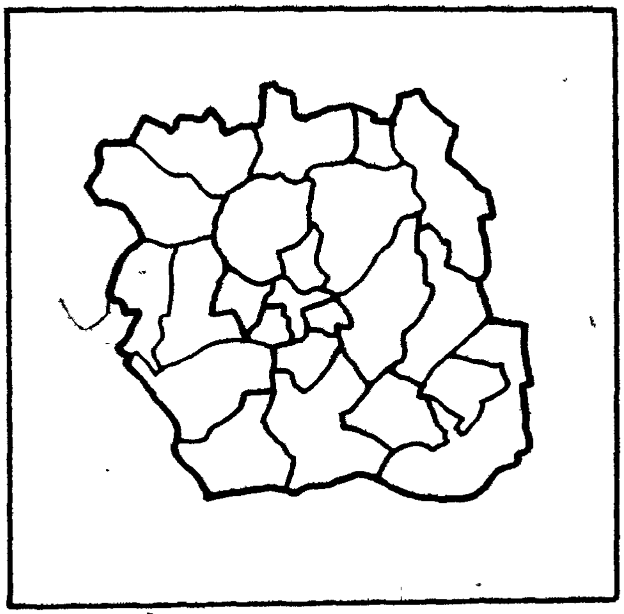
Source: Boundaries of Counties and County Equivalents, U.S. Dept. of Commerce, Washington, D.C., U.S. Government Printing office, 1970

Counties of Alabama



Counties of South Carolina

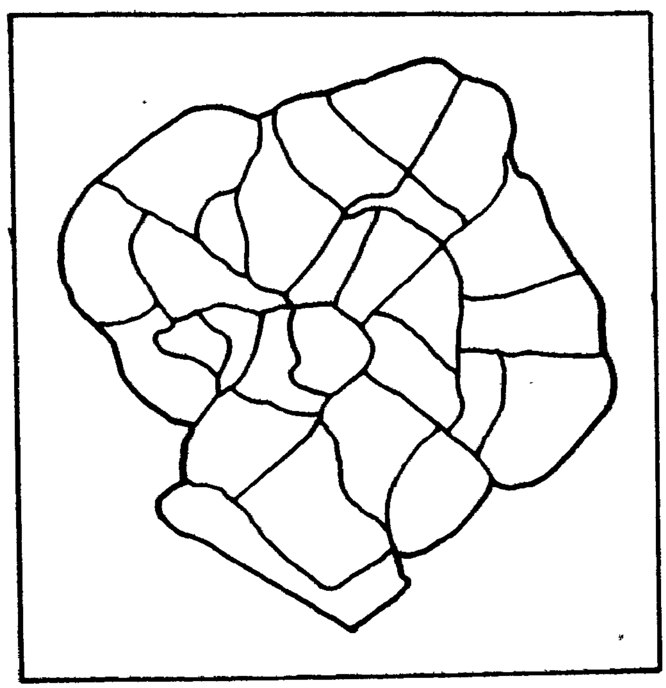
Source: *Boundaries of Counties and County Equivalents*, U.S. Dept. of Commerce, Washington, D.C. U.S. Government Printing Office, 1970.



Administrative Zones of Reading
Subregion, England.

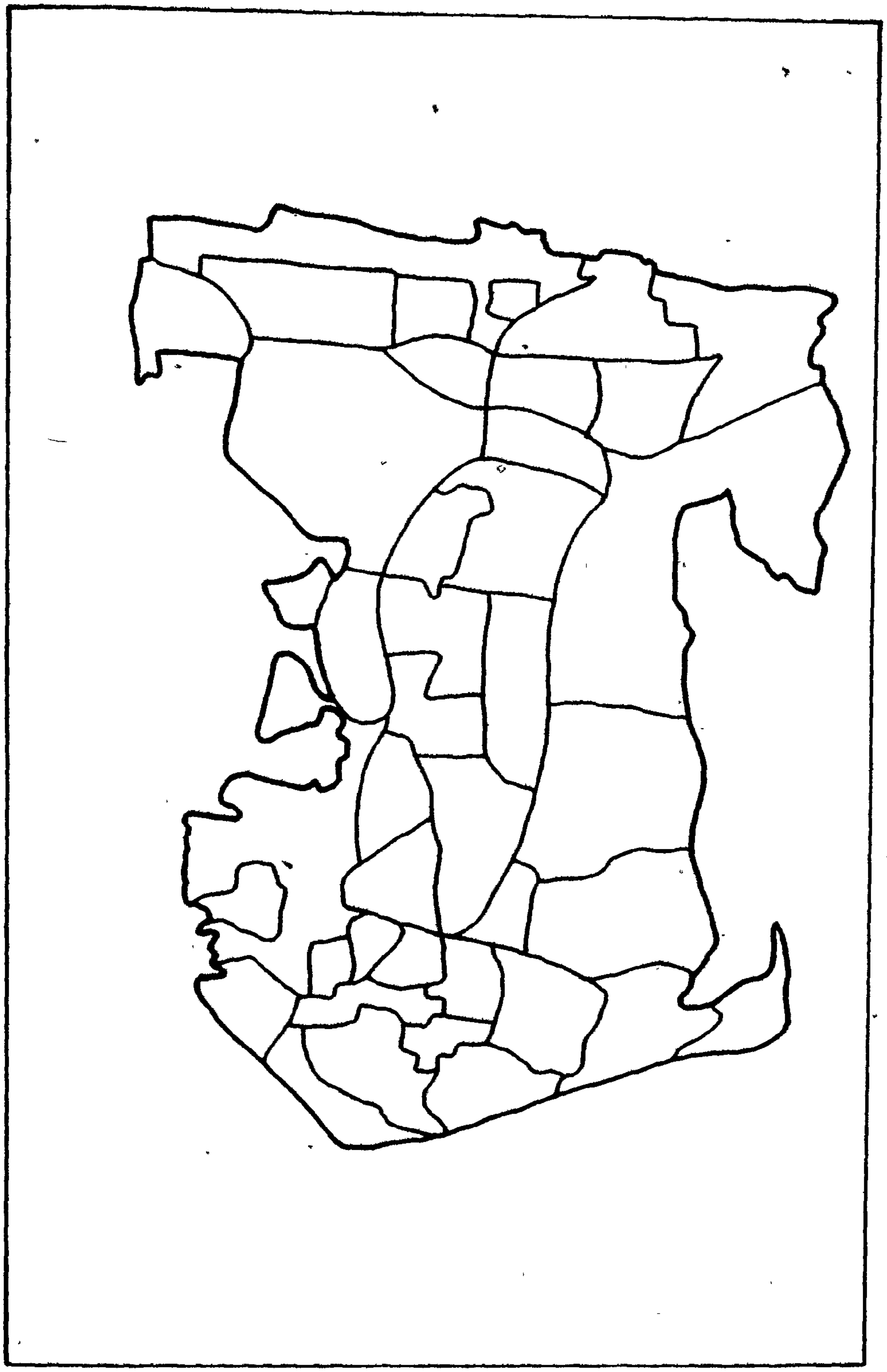
Source: Batty, M., Reilly's challenge, new laws of retail gravitation which define systems of cental places, Environment and Planning A, 1978, Vol. 2, No. 10. p. 213

SOCIAL CELLULAR NETWORKS



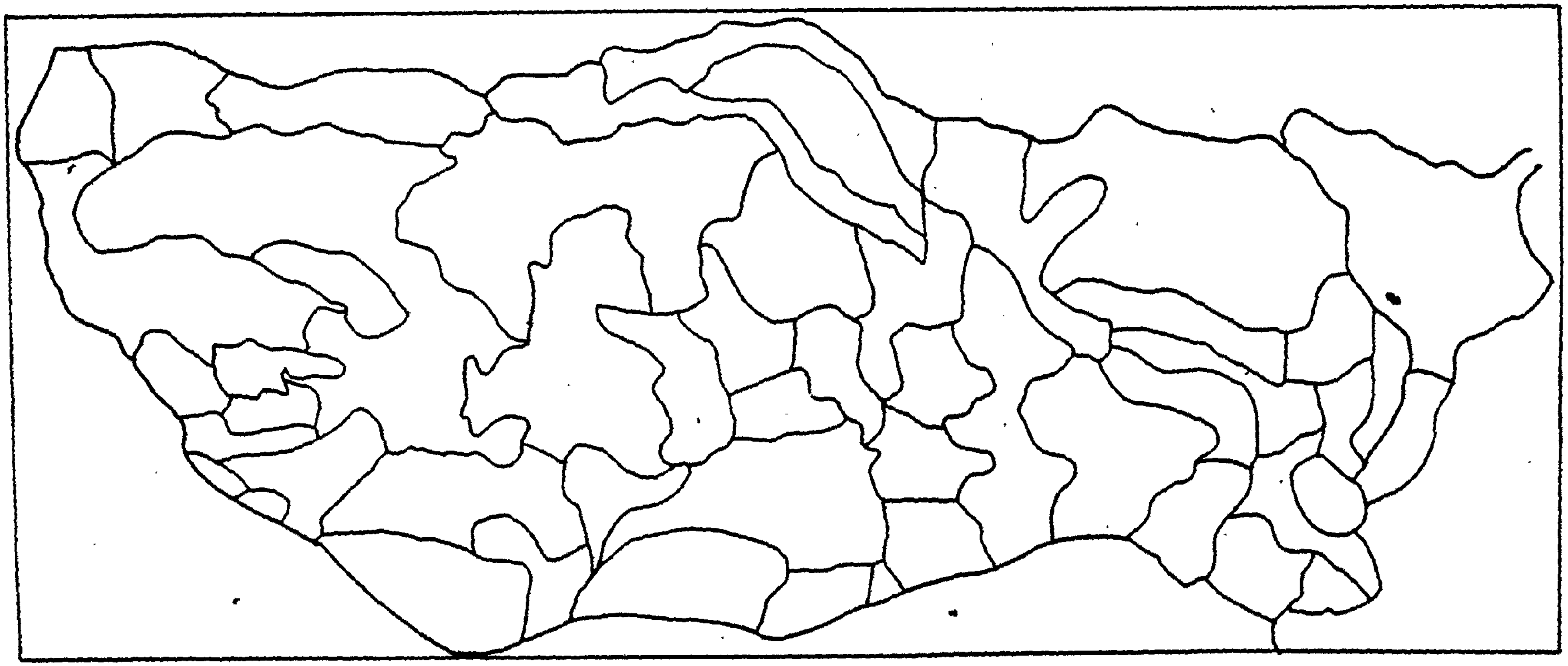
Residential Blocks, Benin, Nigeria.

Source: Onokerhoraye, A.G. The spatial pattern of residential districts in Benin, Nigeria, Urban Studies Oct. 1977, Vol. 14, No.3.



Neighbourhood Areas, Portsmouth, England

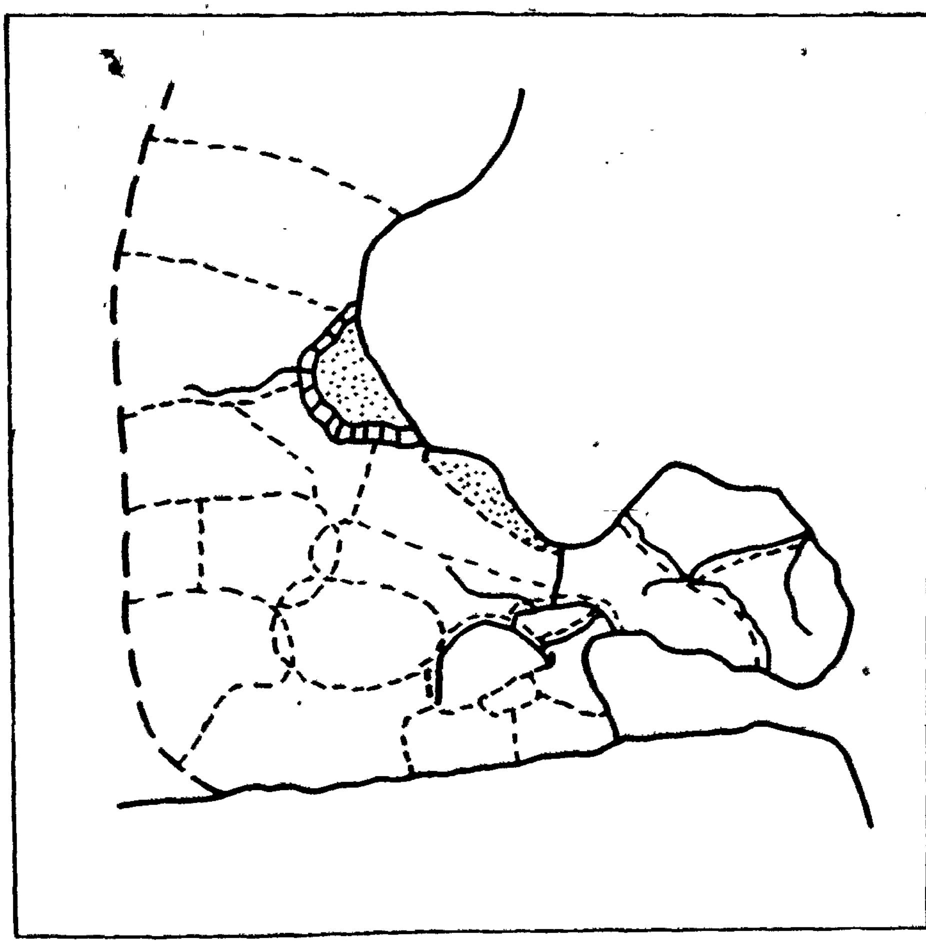
Source: Hall, D.R. Defining and evaluating areas for urban neighbourhood councils, Geoforum, Vol. 8, 1977, 277-310.



African Tribal Territories

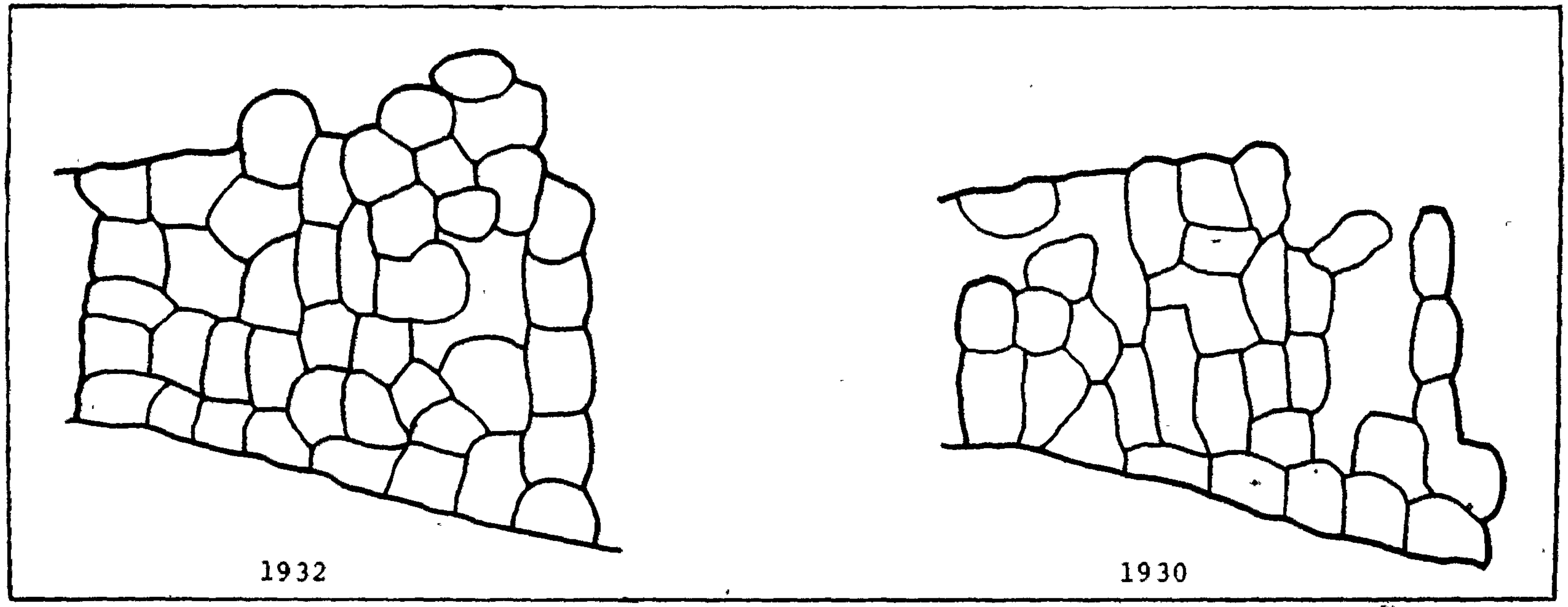
Source: 'The Times' Map of the tribes, peoples and nations of modern Africa, by Roy Lewis and Yvonne Fox, John Bartholomew & Son Ltd., Edinburgh , 1978
ISBN 0-7320-0088-3

BIOLOGICAL CELLULAR NETWORKS



Fur Seal Territories

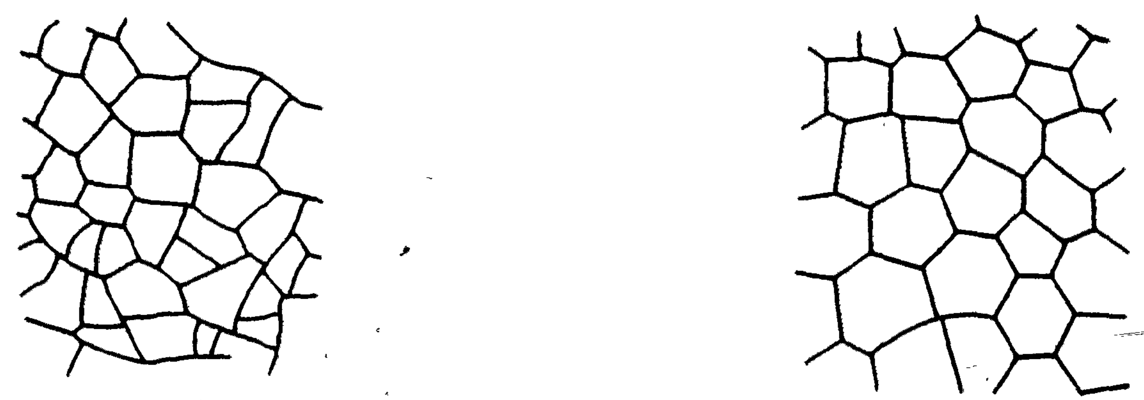
Source: Bartholomew, G.A. and P.G. Hoel, Journal of Mammology, Nov. 1953, p. 429.



Song Sparrow Territories

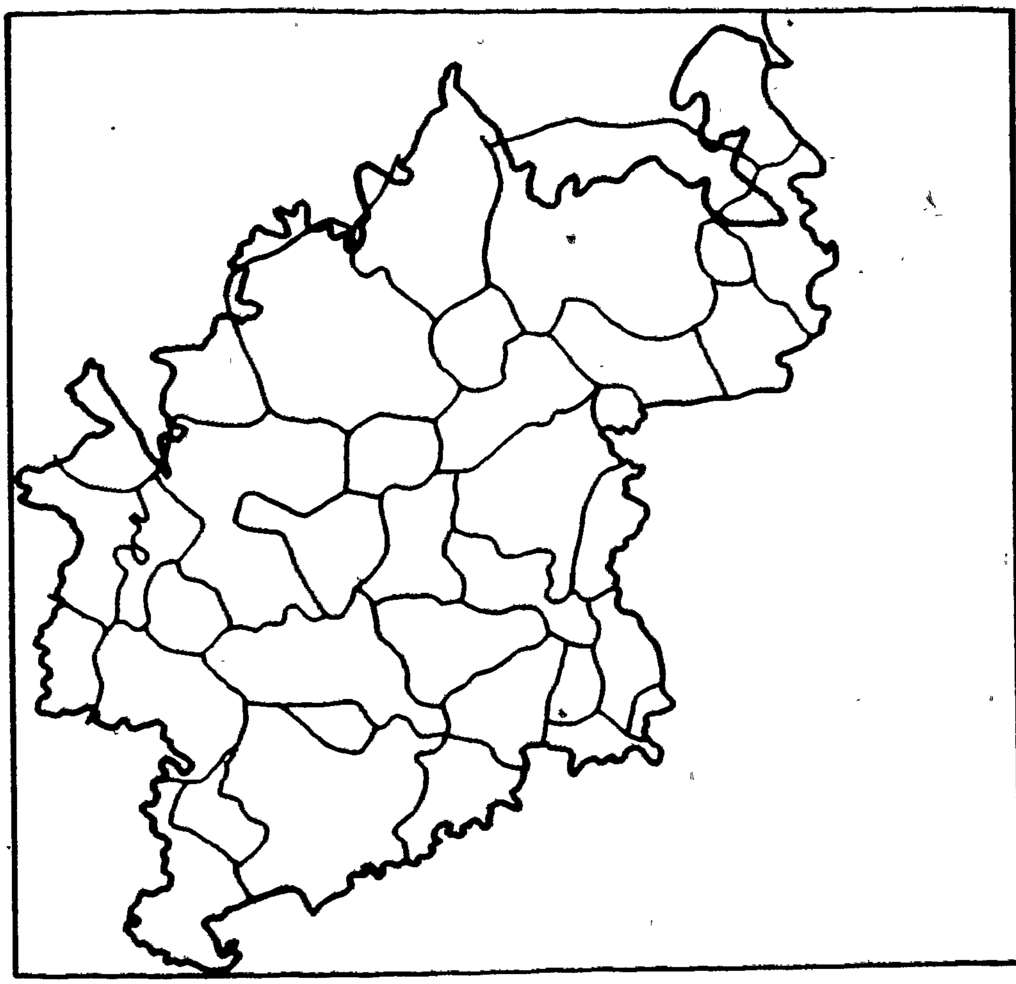
Source: Nice, M.M. Territories from year to year, Life History of the song sparrow, Vol:1: A Population of The Song Sparrow, New York.

PHYSICAL CELLULAR NETWORKS



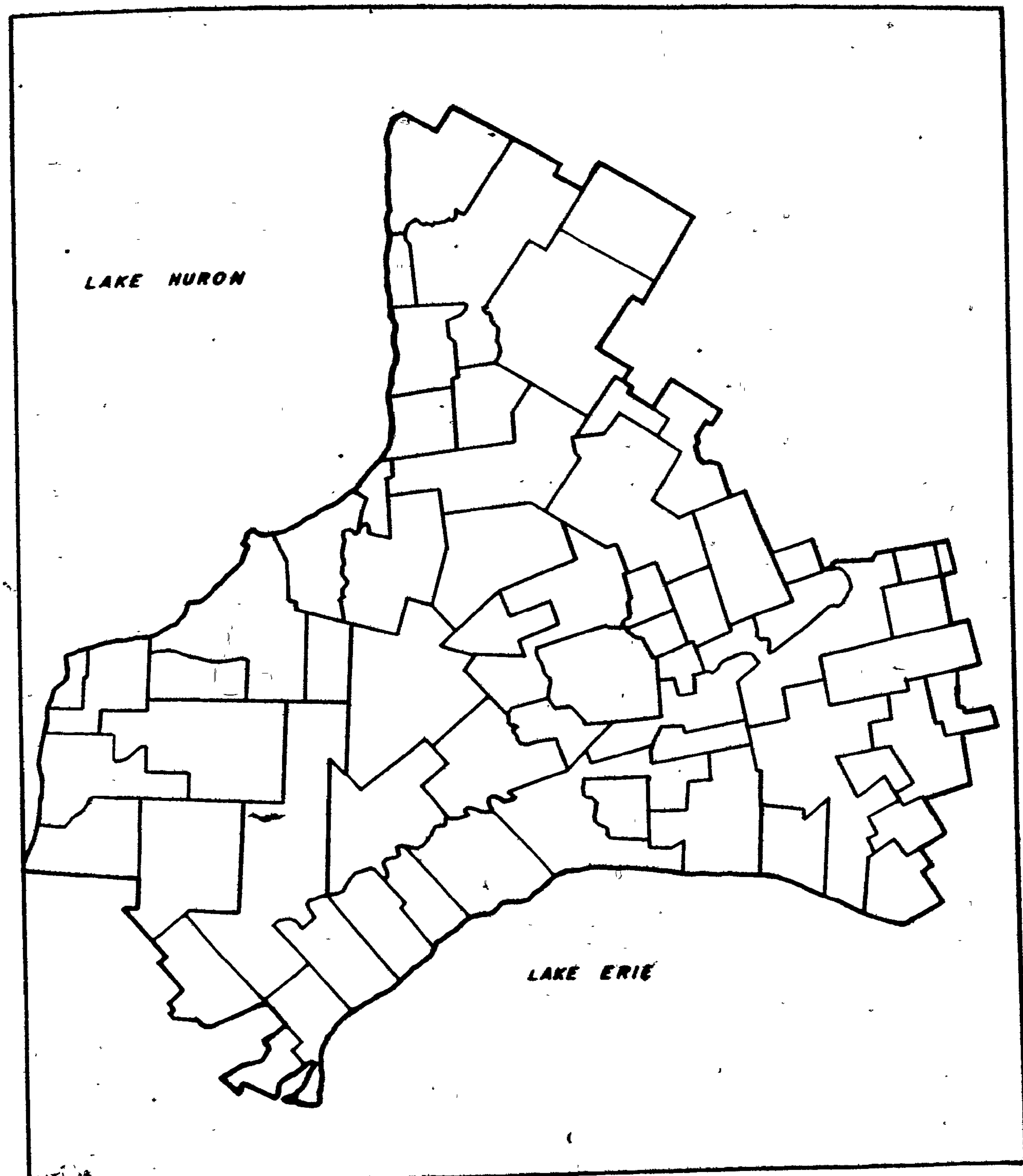
Source: Gray, N.H. J.B. Anderson, J.D. Devine, and J.M. Kwasnik, Topologic properties of random crack networks, International Association for Mathematical Geology Journal, 1976, 8, p. 624

ECONOMIC CELLULAR NETWORKS



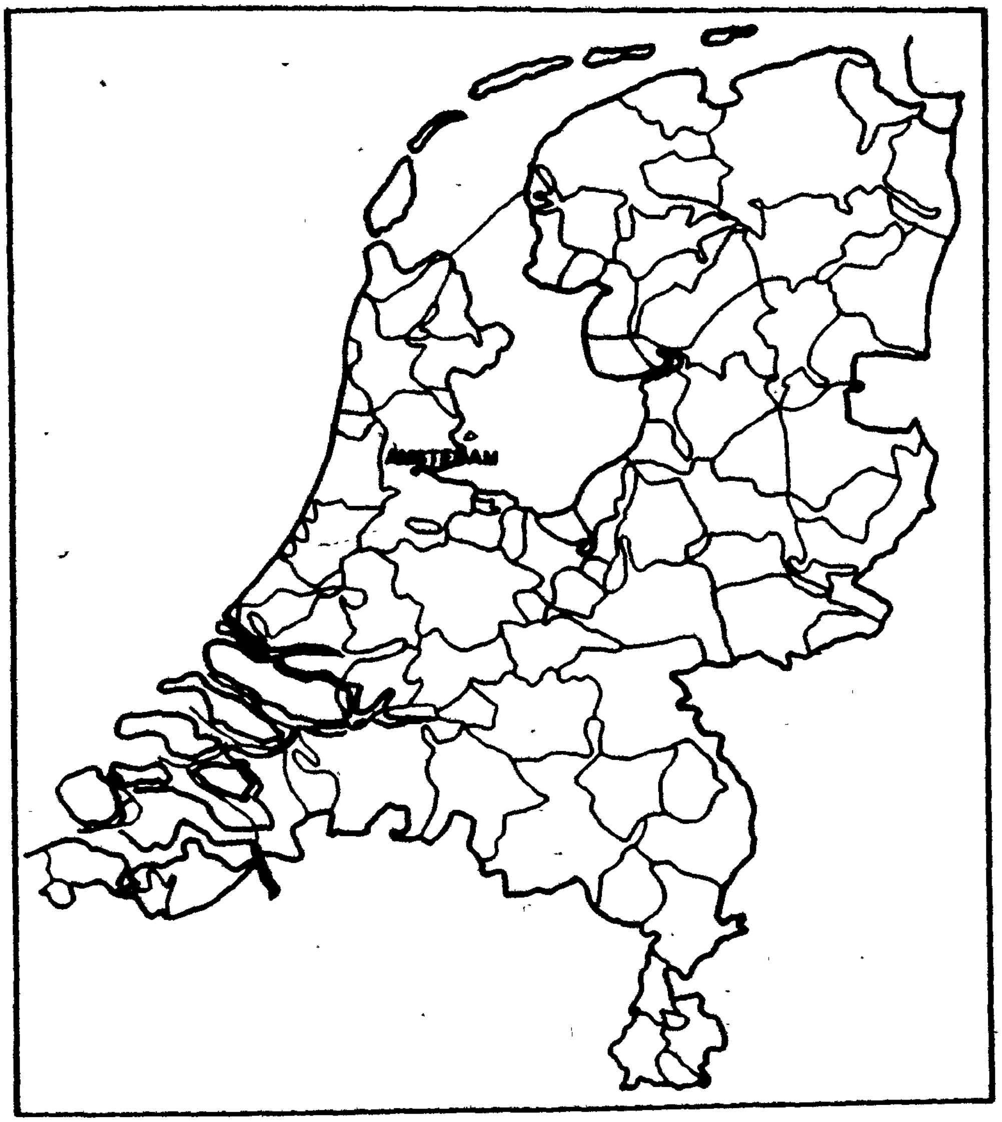
Urban Hinterlands, Lower Saxony

Source: Green, F.H.W. Community of interest areas in Western Europe - Some geographical aspects of local passenger traffic, Economic Geography October, 1953, Vol. 29, No. 4, p. 286.



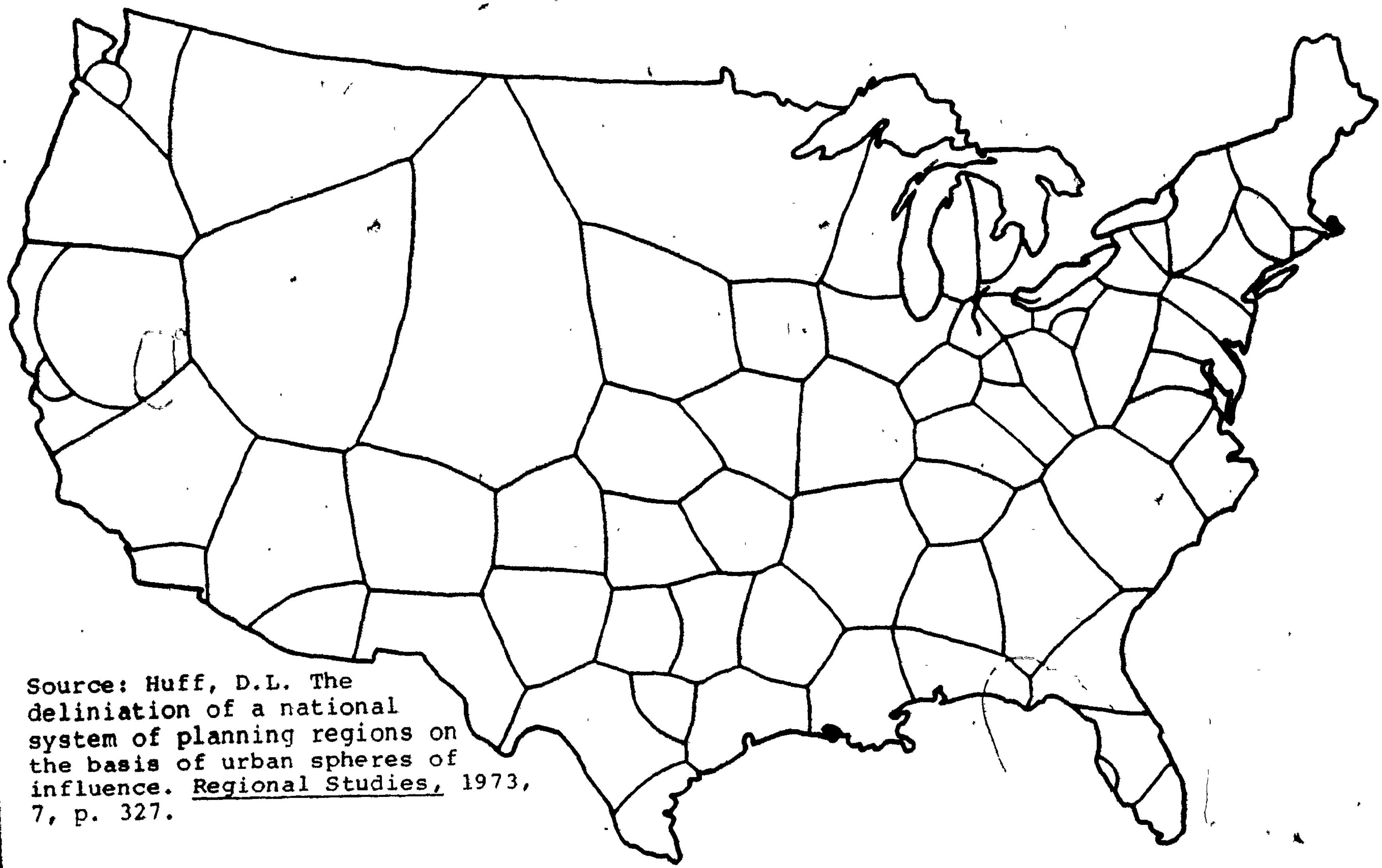
Central Place Hinterlands, S.W. Ontario

Source: Muncaster, R.W. A model for mixed urban place heirarchies: An application to the London, Ontario, urban place system. Graduate School of Geography, Clark Univ. Worcester, Mass. 1972.



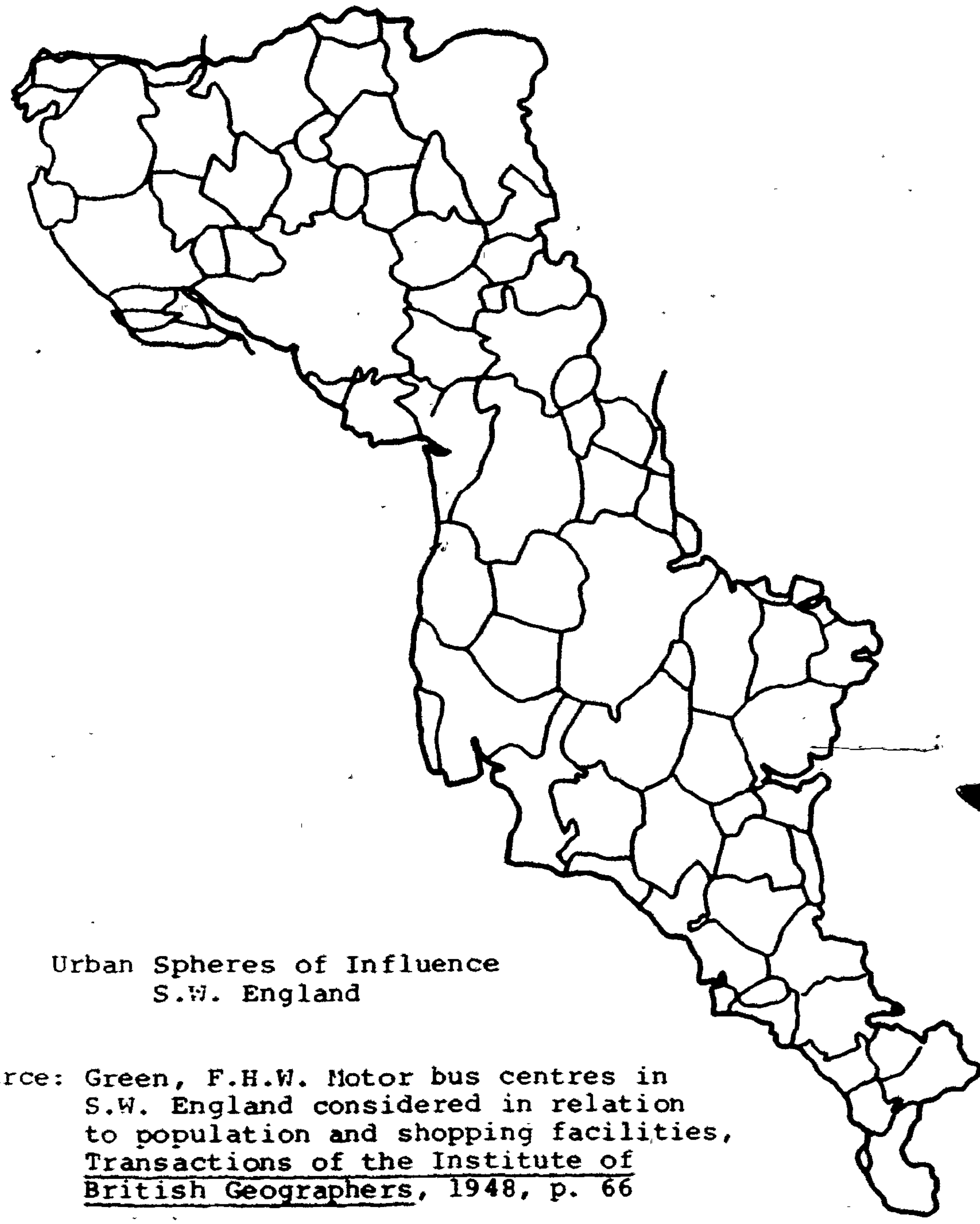
Urban Hinterlands, Netherlands

Source: Green, F.H.W. Community interest areas in Western Europe - Some geographical aspects of local passenger traffic, <u>Economic Geography</u> October, 1953, Vol. 29, No. 4, p.293.



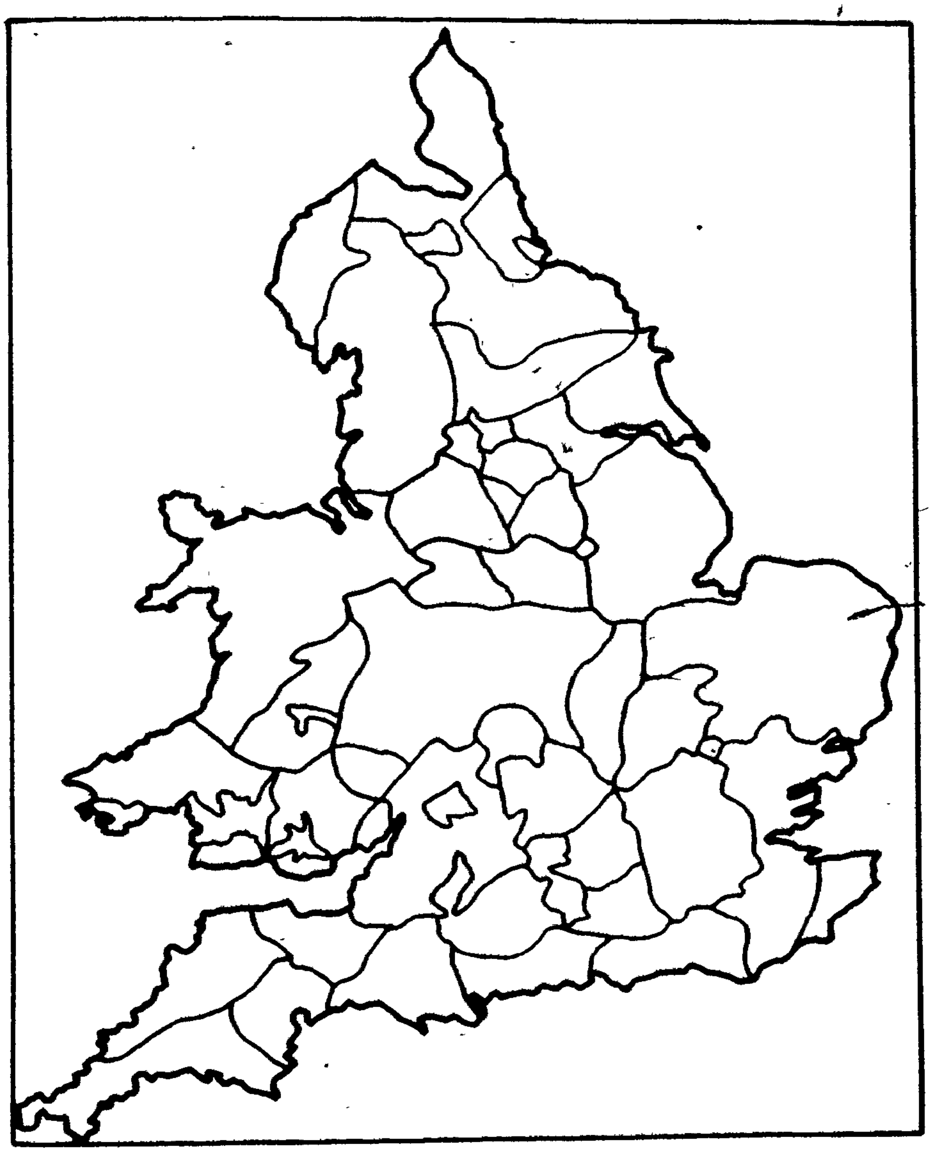
Source: Huff, D.L. The deliniation of a national system of planning regions on the basis of urban spheres of influence. Regional Studies, 1973, 7, p. 327.

Urban Spheres of Influence, U.S.A.



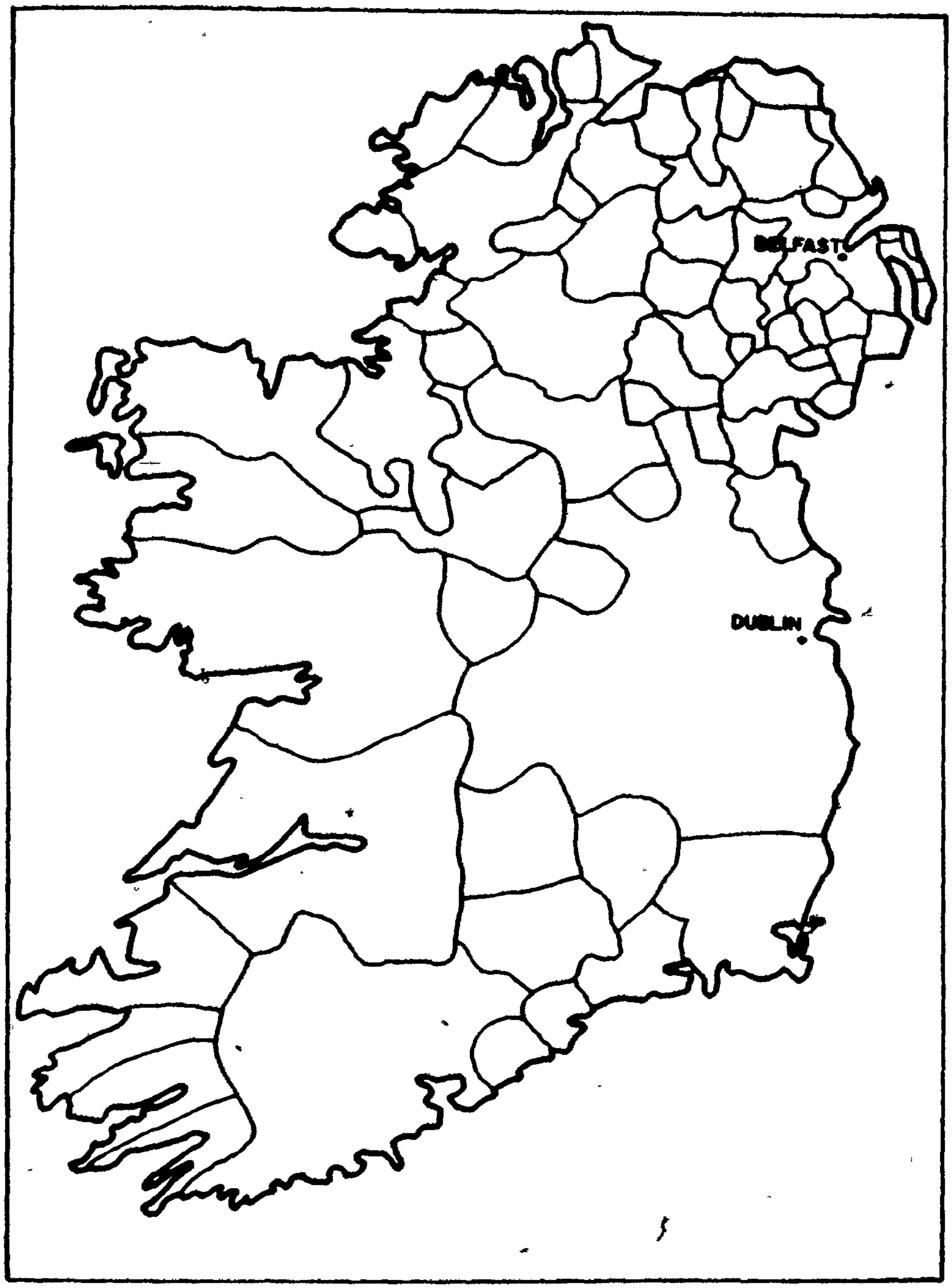
Urban Spheres of Influence
S.W. England

Source: Green, F.H.W. Motor bus centres in S.W. England considered in relation to population and shopping facilities, Transactions of the Institute of British Geographers, 1948, p. 66



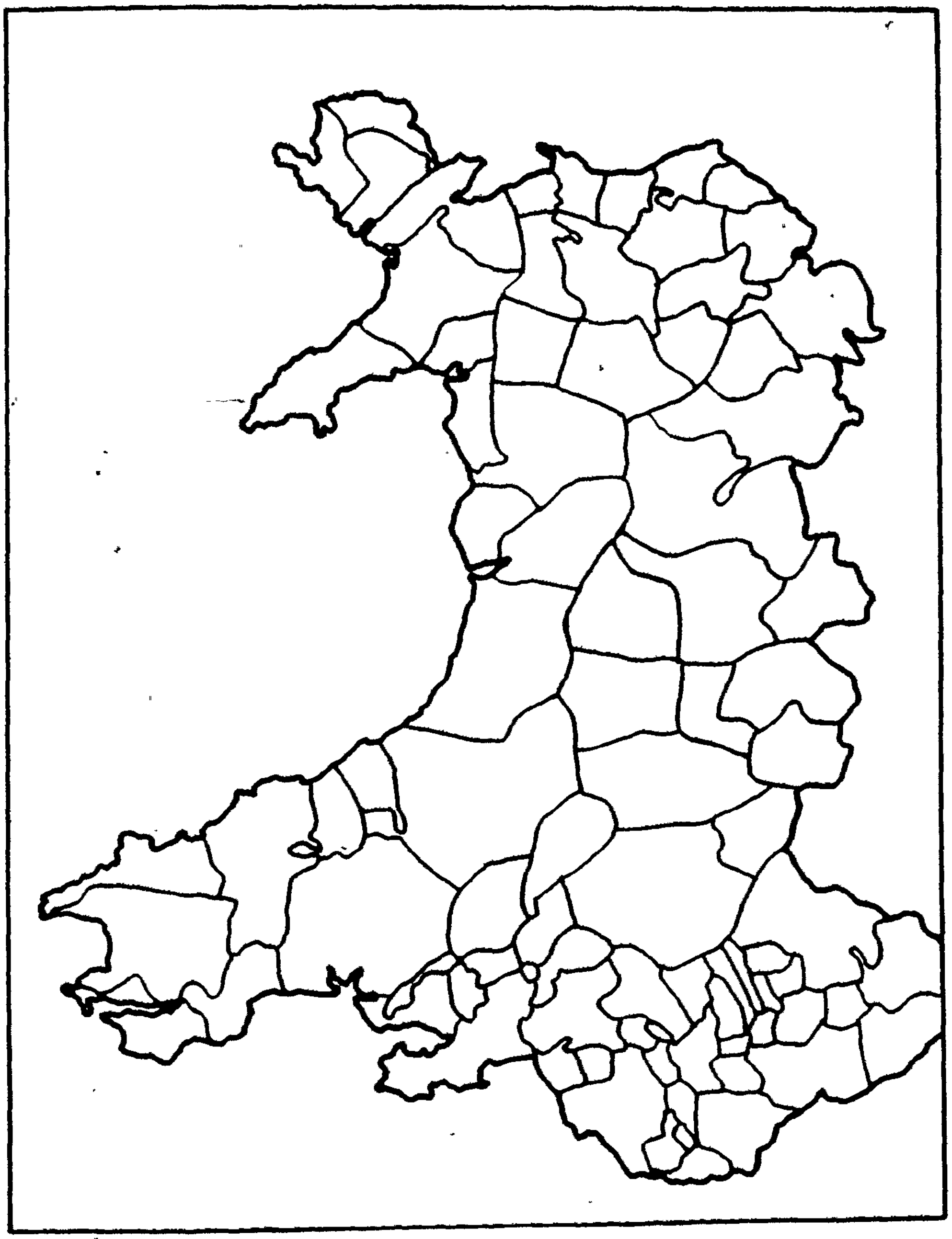
Bus Service Areas, England and Wales

Source: Green, F.H.W. Bus sevices in the British Isles, Geographical Review, 41, 1951, Vol. 41, p. 648.



Urban Hinterlands, Ireland

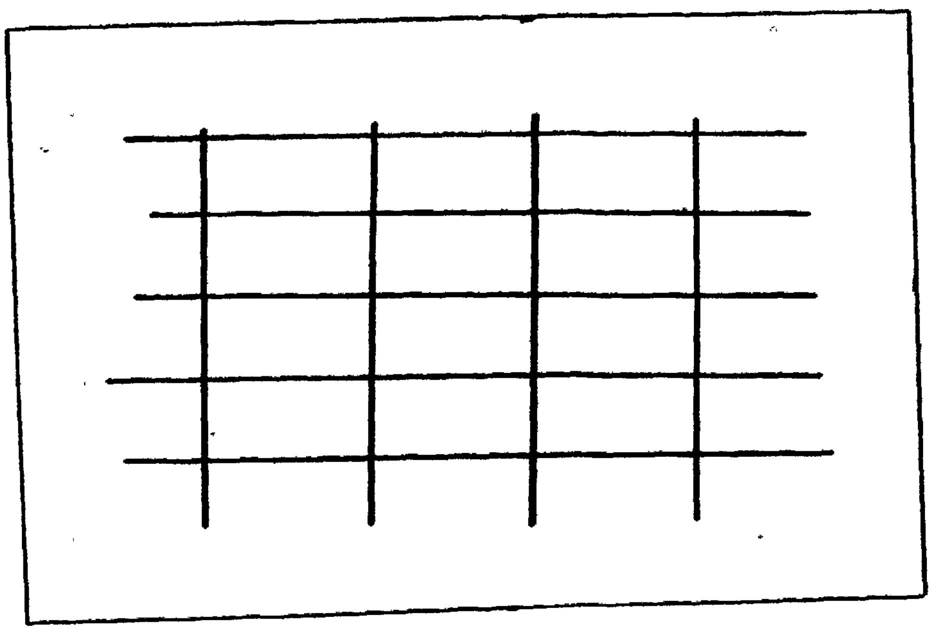
Source: Green, F.H.W. Community of interest areas in Western Europe - Some geographical aspects of local passenger traffic, Economic Geography October, 1953, Vol. 29, No. 4



Urban Hinterlands, Wales

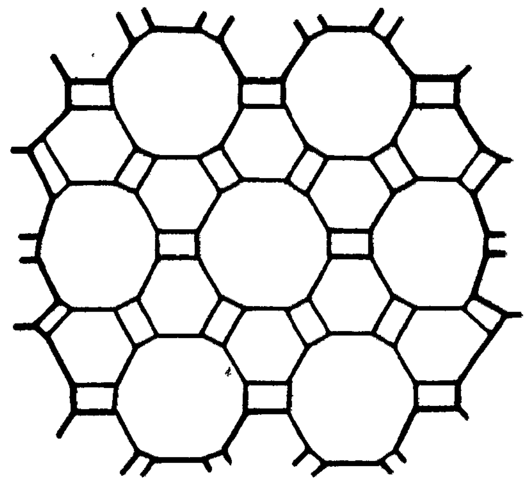
Source: Green, F.H.W. Community of Interest areas in Western Europe - Some geographical aspects of local passenger traffic, Economic Geography October, 1953, Vol. 29, No. 4, p. 286

ABSTRACT CELLULAR NETWORKS

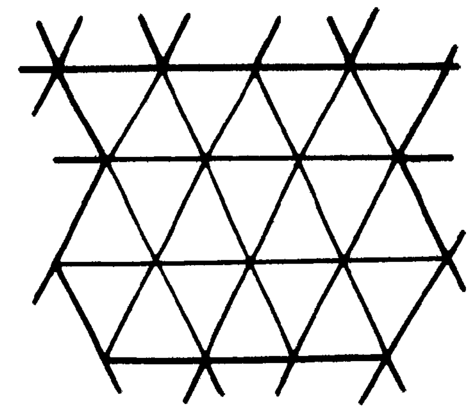


Square

Source: Loeb, A.C. Space Structures, Harmony, and Counterpoint
Reading, Mass. 1976

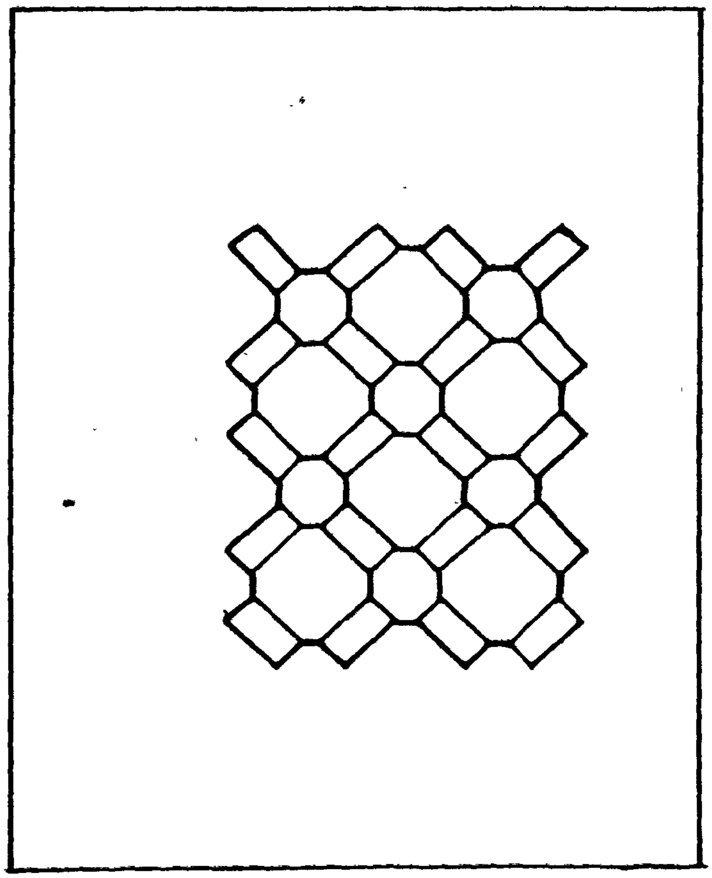


Hextess



Tri

Source: Loeb, A.C. Space Structures, Harmony, and Counterpoint
Reading, Mass. 1976



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Source: Loeb, A.C. Space Structures,
Harmony and Counterpoint
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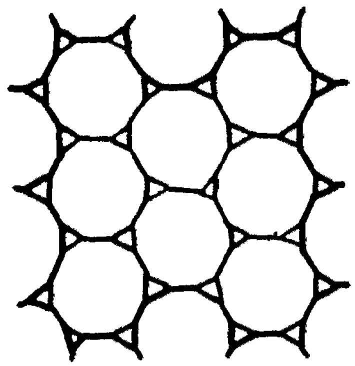

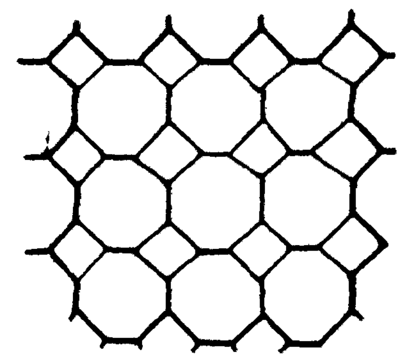

Dodtri — Octsquare

Source: Loeb, A.C. Space Structures, Harmony and Counterpoint
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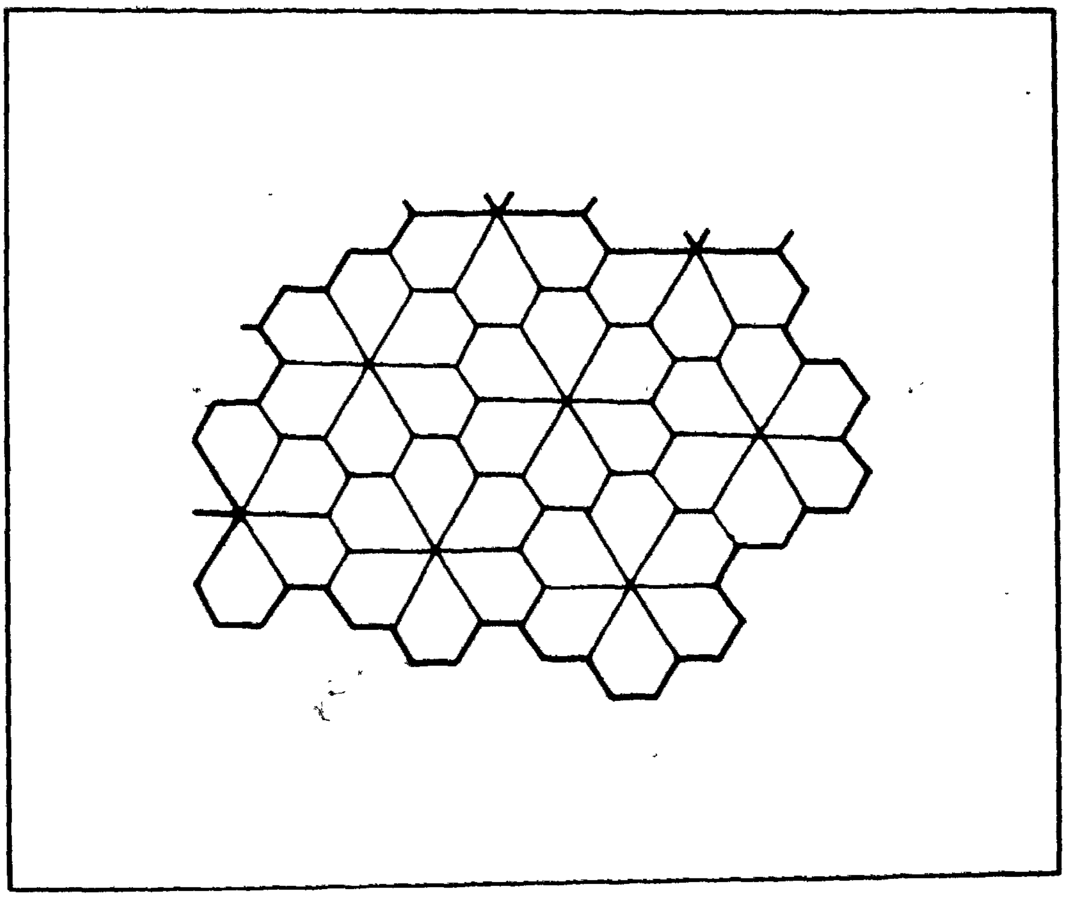

Pentagon

Source: Loeb, A.C. Space Structures, Harmony and Counterpoint, Reading, Mass. 1976