

A Study on MIMO Wireless Communication Channel Performance in Correlated Channels

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A Study on MIMO Wireless Communication Channel Performance in Correlated Channels

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by

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based on research carried out

under the supervision of

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May, 2016

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May 28, 2016

Supervisor's Certificate

This is to certify that the work presented in the dissertation entitled *A Study on MIMO Wireless Communication Channel Performance in Correlated Channels* submitted by *Sourav Guha Roy*, Roll Number 214EC5202, is a record of original research carried out by him under my supervision and guidance in partial fulfillment of the requirements of the degree of *Master of Technology in Electronics and Communication Engineering*. Neither this thesis nor any part of it has been submitted earlier for any degree or diploma to any institute or university in India or abroad.

Lakshi Prosad Roy

Dedication

I humbly dedicate this work to my beloved parents and all of my teachers.

Signature

Declaration of Originality

I, *Sourav Guha Roy*, Roll Number *214EC5202* hereby declare that this dissertation entitled *A Study on MIMO Wireless Communication Channel Performance in Correlated Channels* presents my original work carried out as a postgraduate student of NIT Rourkela and, to the best of my knowledge, contains no material previously published or written by another person, nor any material presented by me for the award of any degree or diploma of NIT Rourkela or any other institution. Any contribution made to this research by others, with whom I have worked at NIT Rourkela or elsewhere, is explicitly acknowledged in the dissertation. Works of other authors cited in this dissertation have been duly acknowledged under the sections “Reference” or “Bibliography”. I have also submitted my original research records to the scrutiny committee for evaluation of my dissertation.

I am fully aware that in case of any non-compliance detected in future, the Senate of NIT Rourkela may withdraw the degree awarded to me on the basis of the present dissertation.

May 28, 2016
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Abstract

MIMO wireless communication system is gaining popularity by days due to its versatility and wide applicability. When signal travels through wireless link it gets affected due to the disturbances present in the channel i.e. different sorts of interference and noise. Plus because there may or may not be a Line of sight (LOS) path between transmitter and receiver signal copies leaving the transmitter at the same time reaches the receiver with different delays and attenuation due to multiple reflections and interfere with each other at the receiver. Therefore fading of received signal power is also observed in case of a wireless MIMO link.

In case of wireless two most important objectives can be channel estimation and signal detection. The importance of the wireless channel estimation can be attributed to faithful signal detection and transmit beam forming, power allocation etc. when Channel state information (CSI) is communicated to the transmitter via feedback loop in case of uni-directional channel or by simultaneous estimation by the transmitter itself in case of bi-directional channel.

This text introduces different aspects of signal detection and channel estimation, it also explains why channel estimation is important in context of signal detection, beam forming etc. A brief introduction to antenna arrays and beam forming procedures have been given here.

The cause of occurrence of spatial and temporal correlations have been discussed and different ways of modelling the spatial and temporal correlations involved are also briefly introduced in this text. How different link and link-end properties e.g. antenna spacing, angular spread of radiation beam, mean angle of radiation, mutual coupling present between elements of an antenna array etc. affects the channel correlations thereby affecting the overall performances of the MIMO wireless communication channel.

Modelling of antenna mutual coupling and different estimation and compensation techniques that are already in use are also discussed here.

Keywords – Wireless Communication; MIMO; Signal Detection; Channel Estimation; Spatial and Temporal Correlations; Antenna Mutual Coupling; Beam-forming.

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Chapter 1

Introduction

This chapter gives an introduction on multi-input multi-output (MIMO) wireless communication systems, the channel estimation problem for such a system and attempts to answer some fundamental questions in that context e.g. Why the MIMO wireless system is so important in communication perspective? What are problems faced in using such systems? How the problems faced may be alleviated? This chapter is organized into six sections: section 1.1 gives a basic introduction to the MIMO wireless communication systems, section 1.2 introduces the MIMO channel fading coefficients and the channel structure, section 1.3 introduces some commonly encountered terminologies in the study of MIMO wireless communication, section 1.4 presents some most important advantages and disadvantages in MIMO wireless communication, section 1.5 presents the motivation of this work, section 1.6 gives the literature review of this work, section 1.7 familiarizes the outline followed in this text and finally section 1.8 presents the challenges faced in case of correlated MIMO channel processing.

1.1 Introduction to wireless communication systems and MIMO:

When the communication medium through which signal transmission occurs is a wireless ‘ether’ medium the communication is properly termed as wireless communication. The MIMO terminology was first introduced in 1970s. While communicating through a wireless communication link a signal coming from a transmitter may/may not directly reach a receiver, a direct path is called LoS (Line of Sight) and an indirect path is called NLoS (Non-LoS). In case of NLoS the EM waves are deflected/scattered by barriers (or scatterers) on their path e.g. buildings, trees etc. while transmission through the wireless link. Finally all of the LoS/NLoS EM signals mutually interfere either constructively or destructively upon reaching the receiver with different phases and amplitudes, due to this reason the received signal power enhances/decays (i.e. fluctuates) from time-to-time and this phenomenon is called ‘fading’. In a multipath scenario every signal component is characterized by its amplitude, phase shift, delay and Direction of Arrival (DOA).^[20] A basic parameter that describes the behaviour of all this is the Power Delay Profile (PDP)

which gives the signal power as a function of the delay of the multipath components.^[20] Usually in a wireless environment the PDP has one or more peaks which indicates inherent clustering of the delays. The information obtained from the PDP can be used to calculate the delay spread,^[20] which is explained later in this text. Additive White Gaussian Noise also plays an important role in corrupting the received signal. The complexity is actually increased due to the mobility of receiver/transmitter, which aids short-term fluctuations (fading) and also long-term fluctuations (shadowing) of the received signal envelope.^[20] Block diagram of a modern digital communication system can be given as follows:

Block Diagram for a Digital Communication System

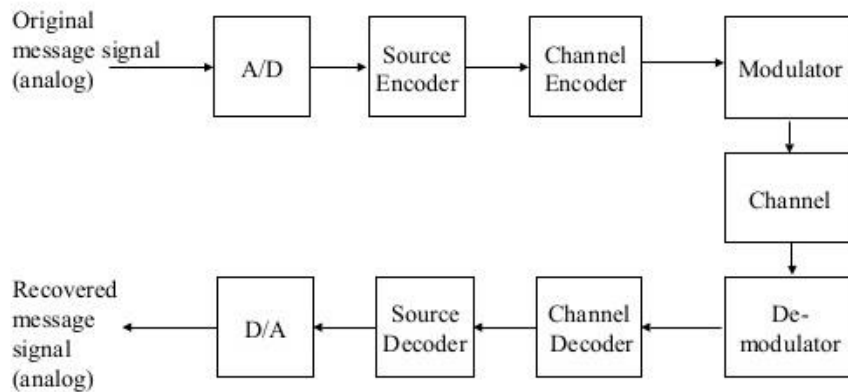


Fig. 1.1 Block Diagram of a Digital Communication System

1.2 Introduction to channel fading coefficients and MIMO channel structure:

The main disruptive effect observed in the case of MIMO wireless communication channel is fading (apart from shadowing and path-loss effects) Signal transmitted through a path in wireless medium can be represented as: $a_i \times \delta(t - \tau_i)$ for $i \in [0, L - 1]$ where 'L' is the number of individual paths and ' a_i 's are attenuations and ' τ_i 's are amount of delays for i^{th} paths therefore the channel impulse response can finally be expressed as: $h(t) = \sum_{i=0}^{L-1} a_i \times \delta(t - \tau_i)$. The basic MIMO structure can be represented by the following figure:

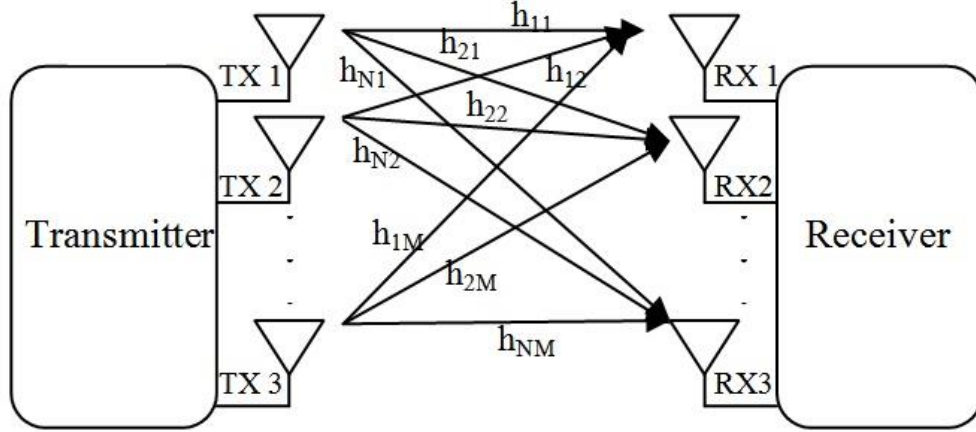


Fig. 1.2 Structure of a MIMO Wireless Communication System

On the other hand the fading coefficients can be represented in terms of fading coefficients. Now fading can be modelled in different manners, following are a few common examples:

- a) **Rayleigh fading:** In this case the channel matrix can be modelled as: $H_i = A_i e^{j\phi_i}$ or $H_i = x_i + jy_i$ where $i \in [1, MN]$ here the envelope ' A_i 's are Rayleigh distributed i.e. $pdf_{A_i}(z) = \frac{2z}{P} e^{-\frac{z^2}{P}}$ for $z \geq 0$ and '0' otherwise, for all ' i ' where ' P ' is a constant, ' ϕ_i 's are uniformly distributed i.e. $pdf(\phi_i) = \frac{1}{2\pi}$ for all ' i '. Alternatively the in-phase components ' x_i 's and quadrature-phase components ' y_i 's are i.i.d. Gaussian distributed i.e. $pdf_{x_i \text{ or } y_i}(z) = \frac{1}{\sqrt{2\pi(\sigma_x^2 \text{ or } \sigma_y^2)}} e^{-\frac{(z)^2}{2(\sigma_x^2 \text{ or } \sigma_y^2)}}$

here as mentioned ' x_i 's or ' y_i 's are assumed to be identically distributed (this is assumed quite logically when there are no direct paths between transmitter

and receiver) and the envelope: $A_i = \sqrt{x_i^2 + y_i^2}$. The final channel matrix can be

$$\text{represented as: } H = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1N} \\ h_{21} & h_{22} & \cdots & h_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ h_{M1} & h_{M2} & \cdots & h_{MN} \end{bmatrix}$$

- b) **Rician fading:** In this case the envelope of the fading coefficients ' A_i 's are Rician distributed i.e. $pdf_{A_i}(z) = \frac{2z(r+1)}{P} e^{-r-\frac{(r+1)z^2}{P}} I_0\left(2z\sqrt{\frac{r(r+1)}{P}}\right)$ for $z \geq 0$ and '0' otherwise, where $I_0(x) = \frac{1}{2\pi} \int_0^{2\pi} e^{-z \cos \phi} d\phi$ where ' ϕ 's are uniformly

distributed in $[0, 2\pi]$ gives first order Bessel function and kind zero, ‘ r ’ is the Rice factor.

- c) **Nakagami fading:** In this case the distribution of the envelopes ‘ A_i ’s is described by a central chi-squared distribution of degree ‘ m_N ’ and is given by:

$$pdf_{A_i}(z) = 2 \left(\frac{m_N}{P} \right)^{m_N} \frac{z^{2m_N-1}}{\Gamma(m_N)} e^{-\frac{m_N z^2}{P}} \text{ for } z \geq 0 \text{ and } m_N \geq 1/2 \text{ and '0' elsewhere. Here 'm}_N\text{' is also called Nakagami parameter and } \Gamma(.) \text{ is the Euler-Gamma function.}$$

While transmission through any wireless medium the baseband signal is first converted to passband using digital modulation techniques and then encoded for transmission bandwidth utilization (source-coding) and for error detection/correction and ISI removal (channel-coding). Some other techniques are also employed for improvement of quality of service (QoS) such as pre-coding e.g. water-filling type power allocation and beamforming i.e. multiplexing technique at the transmitter and maximal-ratio combining (MRC), zero-forcing (ZF) or minimum mean-squared error (MMSE) i.e. different diversity techniques at the receiver for performance (capacity or reliability) improvement. The following block diagram shows all the steps of signal transmission and reception:

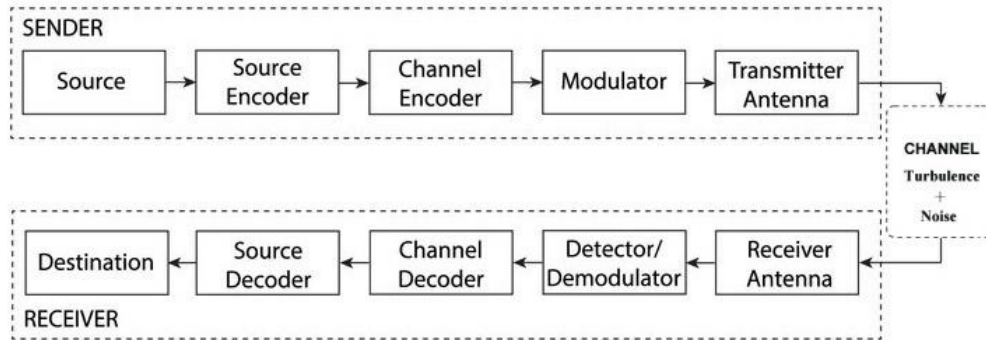


Fig. 1.3 Signal propagation in a wireless communication system

Signal detection is done at the receiver for distortion and noise reduction and proper decoding of the transmitted signal. Channel estimation is required because a good estimate of the channel is mandatory at the receiver for proper signal detection and also at the transmitter for transmit pre-coding.

1.3 Some commonly encountered terminologies in studying MIMO:

Following are a few common terminologies encountered in the study of MIMO wireless communication:

1.3.1 Channel state information:

Channel state information (CSI) is the overall information regarding the wireless channel fading characteristics. CSI can be of two types: a) full or b) average. CSI can be explicitly given by the actual fading coefficients of the channel (usually non-parametric) or by detailed description of the multi-paths and scatterers etc. present in the link (usually parametric), which is called full CSI. But instead CSI can also be given only by the description of the fading correlation(s) i.e. in this case the channel fading characteristics are known in average, which is therefore purposefully called mean/average CSI.

1.3.2 Path-loss attenuation:

It is the power density reduction of an Electro-Magnetic (EM) wave while propagation between transmitter and receiver pertaining to typical scattering scenario, other environmental effects etc. Path-loss model is defined accordingly to encompass these effects on the EM wave propagation, from approximate Friis formula (with path-loss adjustment):

$P_{rx} = \frac{P_{tx} \gamma \alpha G_{tx} G_{rx}}{r^\alpha}$ where ' P_{tx} ' and ' P_{rx} ' are respectively transmitted and received powers and ' α ' is the path-loss exponent.

1.3.3 Static vs. time-varying channel:

The channel can be of static or time-varying in nature. So for signal detection at the receiver if equalization of noise/distortion is to be done properly and therefore good channel estimate is required, the practical systems being all non-real-time in nature if the channel varies so quickly that the variation cannot be captured at the receiver it poses a difficult problem in signal detection. The same explanation also applies for the case of pre-coding which too relies on a good channel estimate. A channel may not be completely static (i.e. for any amount of time) but it may show static nature for a finite amount of time i.e. quasi-static nature.

1.3.4 Slow vs. fast fading:

The terms slow and fast fading refer to the rate at which the magnitude and phase change imposed by the channel on the signal changes. The coherence time is a measure of the minimum time required for the magnitude change or phase change of the channel to become uncorrelated from its previous value. Slow fading arises when the coherence time of the channel is large relative to the delay requirement of the application. Fast fading occurs when the coherence time of the channel is small relative to the delay requirement of the application. So for maximum Doppler frequency (often called maximum Doppler spread but both may be different) ' f_m ', channel coherence time is ' T_c ' and symbol duration is ' T_s ' then: $T_c \propto \frac{1}{f_m}$ and $T_c \gg T_s$ gives slow fading and $T_c < T_s$ gives fast fading conditions.

1.3.5 Flat vs. frequency-selective fading:

As the carrier frequency of a signal is varied the magnitude of the change in amplitude will vary. The coherence bandwidth measures the separation in frequency after which two signals will experience uncorrelated fading. In case of flat fading the coherence bandwidth of the channel is larger than the bandwidth of the signal. Therefore all frequency components of the signal will experience the same magnitude of fading. On the other hand in case of frequency-selective fading the coherence bandwidth of the channel is smaller than the bandwidth of the signal. Different frequency components of the signal therefore experience uncorrelated fading. So for channel coherence bandwidth ' B_c ', symbol duration is ' T_s ' and the root mean-square (RMS) delay-spread of the received signal is ' σ_τ ' then: $B_c \propto \frac{1}{\sigma_\tau}$ and thus $T_s \gg \sigma_\tau$ corresponds to flat-fading and $T_s \leq \sigma_\tau$ corresponds to frequency-selective fading conditions.

1.3.6 Block fading:

When the fading process is approximately constant for a number of symbol intervals it is called block fading. A channel can be doubly block-fading when it is block fading in both time and frequency domains.

1.3.7 Shadowing:

Large-scale variations in received signal level (also called large-scale fading) owing to slow-fading nature of the channel is called shadowing. This large-scale fluctuations in mean

received signal envelope occur due to the motion of receiver/transmitter node(s) which are in the proximity of objects like buildings, trees etc. which corresponds to the shadowing effect already mentioned. Shadowing can be described as a log-normal distribution:

$pdf_{envelope}(z) = \frac{1}{\mu\sigma\sqrt{2\pi}} e^{-\frac{(\log z - \mu)^2}{2\sigma^2}}$ for $z \geq 0$ and '0' elsewhere, here ' μ ' and ' σ ' are the mean and variance of $\log z$ respectively.

1.3.8 Diversity:

A diversity scheme is a method for improving the reliability of the received signal by using two or more communication links with different characteristics, such that probability of error in signal detection reduces.

1.3.9 Multiplexing:

Multiplexing is a method by which multiple digital data streams are combined into one (multiplexed) signal to be transmitted over a common medium, where the aim is to share the communication medium. In case of MIMO if multiple paths can be resolved from the common ether medium between transmitter and receiver then different signals can be effectively multiplexed for efficiency in transmission and therefore channel capacity enhancement.

1.3.10 Pre-coding and beam forming:

Pre-coding is a generalization of beamforming to support multi-stream transmission in multi-antenna wireless communications. In conventional single-stream beamforming the same signal is emitted from each of the transmit antennas with appropriate weighting such that the signal power is maximized at the receiver output, so it is a single-user (SU) technique that exploits transmit diversity. It reduces the signal corruption level at the receiver side of the communication channel. If the receiver has perfect CSI and the transmitter has average CSI, eigen-beam forming is known to achieve the MIMO channel capacity upper-bound.

1.4 Advantages and disadvantages of MIMO wireless communication systems:

1.4.1 Advantages:

There are certain advantages that MIMO wireless communication offers over SIMO/MISO or SISO wireless communication systems, such as:

1.4.1.1 Diversity gain:

In MIMO SU case multiple transmit or receive antennas are used to transmit or receive the same information stream thereby providing diversity gain and therefore more signal reliability by reducing noise and fading effects over other wireless communication systems. In case of receiver diversity, different combining techniques are used to extract the original signal transmitted from multiple copies of noisy and multi-path faded signals, interfering with each other obtained at the individual antennas of the receiver antenna array. In case of transmit diversity, space-time coding (STC) or beamforming (pre-coding) to different antenna elements of transmitter multi-element antenna system (MEA) is employed to send multiple streams of the same information in the hope that at least some of them may survive the physical path between transmission and reception in a good enough state to allow reliable reception.

1.4.1.1 Multiplexing gain:

In MIMO multi-user (MU) case multiple transmit and receive antennas are used for transmission and reception of multiple different data streams, such that the different data streams are multiplexed over available discrete paths present in the MIMO wireless communication channel. This technique boasts much superiority in comparison to several other techniques based on other wireless communication configurations. High capacity improvement is observed if all possible discrete paths between transmitter and receiver are properly utilized and water-filling type power allocation is done among them.

1.4.2 Disadvantages:

There are certain disadvantages to a wireless communication configuration like MIMO as compared to wired communication, such as:

1.4.2.1 Noise:

As MIMO corresponds to a wireless channel configuration, it observes the inevitable degrading effects of surrounding noise. Noise corrupts the transmitted signal amplitude heavily and if noise statistics are completely unknown, it is impossible to model the noise in received signals and therefore very difficult to try to mitigate its effects.

1.4.2.2 Fading:

In a multipath propagation channel the transmitted signal propagates to each receiver antenna over numerous propagation paths, where each path has an associated time delay and complex gain. In such a channel each receiver antenna receives the superposition of multiple delayed, attenuated and phase-shifted copies of the transmitted signal. Hence because of the multi-path effects e.g. reflections, refractions, diffractions, scattering etc. signal copies obtained at each receiver antenna is faded i.e. multiplied with finite channel gain and has phase offset and thus received signal power varies with time even if transmitted signal power is constant. If channel fading coefficients are static and completely known at the receiver, signal can be faithfully extracted considering noise only but if they are unknown or completely time-varying then it is very difficult to detect the original transmitted signals.

1.4.2.3 Co-channel interference:

Co-channel interference (CCI) is mainly cross-talk between different transmitters operating at the same radio frequency e.g. in case of cellular systems employing frequency re-use, assigning same frequency band to different users of adjacent clusters. If the difference in the path delays of the various propagation paths is significantly greater than the duration of a transmitted information symbol then ISI is present at the receiver. During poor weather conditions, when the radio frequencies are not properly allocated in the spectrum or due to some adverse effects present in the spectrum because of a crowded scenario, this effect is seen. If the radio spectrum is allocated properly then this problem can be mostly alleviated.

1.4.2.4 Inter-symbol interference:

Inter-symbol interference (ISI) mostly occurs in communication systems when the transmitted signals interfere with each other. This interference occurs due to overlapping of symbols and it produces signal distortion. ISI is observed for MIMO as well, owing to its

multi-path effect. While transmitted symbols interfere with each other it may be the case that the symbols overlap in the same symbol duration thus producing a distorted symbol at the receiver, which corresponds to ISI. In case of band-limited signals it can be avoided by using pulse shaping or by making the channel impulse response thinner, so that when symbols interfere with each other they don't overlap in the same symbol interval.

1.5 Motivation for this project:

Vast migration of the entire wireless communication field over the current years from the age of 2G/2.5G i.e. second generation to the age of MIMO communications i.e. third generation or 3G is because of the promise made by 3G technologies to drastically improve throughput and reliability issues i.e. overall quality of service (QoS) improvement of the communication systems. Most dominant technologies in 3G wireless communication are : multiplexing, diversity etc. techniques coupled with access methods e.g. CDMA, SDMA etc. and coding techniques like OFDM ensures higher data rate as well as more reliability in reception for wireless communication systems.

In case of unknown spatial correlation channel estimate is erroneous, which increases with higher degree of correlation although for known spatial correlation only the channel diversity gain is reduced at the link ends as well the spatial multiplexing and therefore channel capacity gain also decreases because of unavailability of discrete eigen-links. To reduce all these effects the spatial correlation needs to be estimated or needs to be previously known such that optimum beam former can be designed to improve diversity performance and power allocation strategies can be used to improve the ultimate ergodic capacity performance at the receiver by transmitting through dominant eigen-link. Channel estimation in presence of channel correlation is therefore beneficial for pre-processing applications.

For a fixed average transmit power, when $n = \min(N_T, N_R)$ grows towards ∞ if i.i.d. Rayleigh fading is assumed for the channel, the average channel capacity divided by ' r ' approaches non-zero constant determined by the average signal to noise ratio (SNR). This large capacity grows even if transmitter has no knowledge of the channel. ^[7]

If an average knowledge of the channel is available at the transmitter instead, correlated fading can be used in advantage and actually may lead to higher capacity obtainable than the uncorrelated case, only if some time/frequency is available. ^[8]

It will be later seen in this text that antenna mutual coupling (MC) also plays an important role in determining/modifying the existing spatial correlation characteristics and therefore affects signal detection and channel estimator performances, so MC needs to be estimated properly (in case of unknown MC) otherwise it needs to be accurately known for optimal performances of beam former, transmit power allocation etc. ^{[10], [16-20]}

1.6 Literature review:

Study has been going on for Multi input multi output (MIMO) wireless communication channels for a long time and many advancements have been made regarding capacity enhancement and error reduction for this type of system. Under ideal conditions the information theoretic capacity of a MIMO system grows linearly with the minimum of transmit or receive antennas is mentioned in Weichselberger et al.^[3] However various measurements show that realistic MIMO channel gives out a significantly lower capacity^[3] and this reduction of capacity is due to spatial correlation present between channel elements of the MIMO system^[3] is also mentioned in Weichselberger et al.^[3] It is mentioned in Yen-Chih Chen and Yu T. Sus paper that in comparison with single antenna systems significant capacity gains are achievable when Multi element antennas (MEAs) are used at both transmitter and receiver sides and also various spatial multiplexing techniques are used to attain high spectral efficiency for the case of rich-scattering environments.^[2] It is also mentioned that although ideal rich scattering environments de-correlate channels between different pairs of transmit and receive antennas so that maximum capacity is available, however in practice because of spatial correlation the actual capacity obtainable is often much lesser.^{[2][4]} This consideration is also important for channel estimation and receiver design.

In case of MEA, one more consideration is required, if the mean angle of separation between antenna elements of both transmitter and receiver sides are small but not 'zero' then a directional matrix is incorporated in the channel model, which is diagonal in shape and bears a typical structure^[5] is given in the Klaus I. Pedersen et al.^[5] If the directional matrix is not an identity matrix, i.e. AS is not 'zero' the overall channel matrix, because of large eigen-spread it admits a reduced-rank form. The rank-reduction is most obvious for typical urban macro-cellular environments in which an MS is surrounded by local scatterers while the BS is not obstructed by local scatterers^[2] as mentioned in Yen-Chih Chen and Yu T. Sus paper.

Finally spatial multiplexing for capacity enhancements and beamforming for MISO gain trade-off is studied in the context of the given models in Pradhan B. B. et al. [6]

In M. Biguesh and A. B. Gershmans et al. [11] a few orthogonal training signal based wireless channel estimation techniques have been considered. Here they have shown how optimal training signal for the signal detection techniques can be found out from the constraints. They have also compared the various channel estimation methods based on normalised MSE for the given values of: P/σ^2 , where 'P' is transmit power and ' σ^2 ' is the noise variance.

In Shiu et al. [7] modelling of spatial correlation among channel fading coefficients is focussed upon. For this purpose mainly Jake's 'one-ring' model is followed & extended for making it appropriate for fixed wireless communication context, where BS is at an elevated height and seldom obstructed. Here effect of correlation on capacity, when both transmitter and receiver employ MEAs have also been studied. This paper shows that for N_T transmit and N_R receive antennas being used, the system consists of $n = \min(N_T, N_R)$ subchannels (eigen-modes) so the channel capacity becomes the sum of individual capacities of the subchannels. Here the fading correlation determines the distributions of sub channel capacities. From the consideration above the upper bound and lower bound of the MEA channel capacity can be found. It is also derived and shown with simulation results that for low transmit power rank deficient channel yields higher capacity than full-rank channel due to antenna gain but for higher transmit power case due to availability of multi-stream transmission full-rank channel behaves favourably.

In Ivrlac et al. [8] it is shown that the possible availability of time-diversity (using fast-fading channel property) in case of time-selective channels have essential influence on performance with assumption that channel information is known at the transmitter side as well. Here different information theoretic measures e.g. capacity is considered and it is shown that in some cases correlated fading may offer better performance than what uncorrelated fading can offer. We know that for rank of channel $\rightarrow \infty$ capacity $C_\infty = \left(1/\ln 2\right) \times \frac{P_T}{\sigma_n^2} \approx 1.45 \times \frac{P_T}{\sigma_n^2}$, so asymptotically the capacity becomes a linear function of the transmit power. In case a MIMO channel has $N_T = N_R = r$, rank of the channel the maximum capacity condition occurs, which is called the channel matrix being 'diagonal' although in practice this situation seldom occurs. In this paper it is also shown that the rank deficiency of a MIMO channel can in fact be used to improve the channel capacity than that obtainable by even a diagonal (full-rank) channel matrix.

In Michael A. Jensen and Jon W. Wallace^[9] a detailed review of some of the factors that affects signal propagation through MIMO channel is given. It is this study we have simulated link and link-end properties such as distance between elements of an MEA, polarization properties of the individual elements of an MEA, signal correlation at the transmitter side, mutual coupling present between MEA elements etc. affects the overall performance in terms of diversity or multiplexing gains etc. of the MIMO wireless communication system. According to Michael A. Jensen and Jon W. Wallace^[9] beam forming using singular vectors of channel matrix ' H ' in case of complete CSI at both transmitter and receiver sides produces eigen-patterns that creates independent (spatially orthogonal) parallel communication channels in the multi-path environment.

In Luo et al.^[10] as well description about how different link-end channel properties affect the overall performances in terms of bit error rate (BER) of the received signal, channel capacity performances and magnitude of spatial correlation present in the channel is given and the results compared for different scenarios in case of spatially correlated Nakagami faded channel, calculation regarding spatial correlation with and without considering mutual coupling between antennas are also given in the paper.

In Ghaffar et al.^[14] the basic time correlation structure of the MIMO wireless communication channel is discussed in detail.

M. Comisso^[20] gives a detailed description how, in case of low-rank i.e. correlated MIMO channel efficient beam forming can be done for capacity enhancement.

In Kuan-Hao Chen and Jean-Fu Kiang^[16], T. Svantesson^[20], X. Liu et al.^[19] and H. T. Hui^[29] the effects of mutual coupling on estimation, capacity and also DoA estimation are discussed. In Michael A. Jensen, Jon W. Wallace^[17], S. A. Shelkunoff, H. T. Friis^[30], in J. P. Daniel^[31] and also in T. Svantesson^[25,28] structure and formulation of mutual coupling matrix (i.e. modelling) has been discussed. In M. Comisso^[20], J. Fuhl, A.F. Molisch and E. Bonek^[21], A.D. Kucar^[22] and P.H. Lehne and M. Pettersen^[23] different aspects of MC, their occurrences in MEA systems and their effects have been discussed. Several methods to accurately estimate the MC matrix and to decouple its effects are discussed in H. S. Lui^[26], Pasala^[27] and H. T. Hui^[29].

1.7 Dissertation outline:

This thesis is mainly divided into six chapters, a brief description of them are as follows:

1.7.1 Chapter 1: Introduction

This chapter introduces the basic and related concepts of wireless communication, its evolution through several generations into its current form and important notations that will be used in this text. This chapter also contains a detailed literature survey of this text. In a nutshell this chapter gives an overview of the work done in the field of wireless communication.

1.7.2 Chapter 2: Fundamentals of Signal Detection

This chapter introduces the concept of signal detection which is very important for digital communication. Because of the wireless communication link many effects such as channel fading, additive noise etc. corrupts the signal transmitted while it reaches the receiver therefore faithful detection of the transmitted signal is absolutely necessary and signal processing at the receiver end takes care of this. Comparison of detection error performance between wired and different wireless scenarios for SIMO and SISO has been shown for fixed number of information bits transmitted in all cases, where for different techniques concerning SIMO has shown considerable detection error reduction due to exploitable diversity at the receiver link-end.

1.7.3 Chapter 3: Beam forming and Capacity Enhancement

This chapter introduces the structure and radiation characteristics of multi element antenna (MEA) arrays and discusses different types of beam forming in that context. It also discusses and shows the calculations regarding ergodic capacity enhancement due to transmit beam forming.

1.7.4 Chapter 4: Fundamentals of Channel Estimation

This chapter deals with importance of channel estimation, also some basic non-statistical channel estimation techniques commonly used and their error performance comparison has been shown. The expression for optimum training signal has been found out for the case of every estimators and their minimum attainable estimation mean square error (MSE) have been calculated. The performance comparison shows that minimum mean square error (MMSE) estimator are best among non-statistical estimators considered, relaxed minimum

mean square error (RMMSE) estimator gives a notably comparable performance for high SNR values.

1.7.5 Chapter 5: Correlated MIMO Wireless Channel

This chapter discusses spatial and temporal correlations in MIMO channels and their effects in details and also how it can be modelled. This chapter also shows how different types of correlation can affect channel estimation and signal detection performances. Here a detailed description on how different link and link-end properties e.g. antenna element-spacing in an array, angular spread, mean AoA/AoD of radiation and antenna mutual coupling etc. affects the channel correlation and thus affects the channel estimation, detection error performances and also the capacity. A little elaboration on how mutual coupling between antenna elements affects the performance has been done and few mutual coupling estimation and compensation methods also discussed.

1.7.6 Chapter 6: Conclusion and Future work

This chapter concludes this text and gives a brief hint on further research scopes on the topic addressed here.

1.8 Challenges faced in case of wireless MIMO communication:

The following are some of the challenges faced in wireless MIMO communication:

- i. In practical wireless communication systems major unknown factors like timing offset, phase shift, frequency offset etc. affect the system apart from fading and additive noise etc., which if properly modelled or estimated can create trouble in faithful communication.
- ii. Without proper co-operative technology and feedback appropriate CSI cannot be communicated for uni-directional channels, which might be essential for beam forming like applications.
- iii. The assumption of homogeneous ether medium is not applicable in practice and if cell allocation is not done with proper caution then signal coverage as well as power efficiency may reduce.

- iv. Proper number of orthogonal sub-channels in the wireless link enable the MIMO system to communicate with full capability but instead due to reflectors, environmental conditions etc. rank reduction of the channel matrix is observed which in turn puts performance barrier much lower than that achievable in case of full-rank channel, resulting in a decrease in average throughput.
- v. Proper power allocation due to weak or unreliable sub-channels in the link enforces the transmitter to use complex circuitry.

Chapter 2

Fundamentals of Signal Detection

As we have seen earlier in this text that because of multi-path fading and channel noise transmitted signal gets corrupted when it reaches the receiver, therefore signal detection is an important step in wireless communication systems. From the comparison of wired vs wireless it has been shown that degree of error is more in case of wireless communication channels, the expressions of bit error rate BER in case of received signals (for known transmitted signals) for both wired and wireless communication links^[15] (SISO) are given as:

$$BER_{wired} = Q(\sqrt{SNR}) \quad (2.1)$$

$$BER_{wireless} = \frac{1}{2} \left(1 - \left(\frac{SNR}{2+SNR} \right)^{1/2} \right) \quad (2.2)$$

2.1 Benefits of using multi-antenna systems:

For satisfying the needs of high bandwidth demand (therefore high data rate) in case of different wireless communication scenarios e.g. WLAN. Traditionally user communications are separated by frequency as in Frequency Division Multiple Access (FDMA) by time as in Time Division Multiple Access (TDMA) or by code as in Code Division Multiple Access (CDMA).^[22] Recently the possibility of separating the different users by space has led to the development of a new multiplexing technique called Space Division Multiple Access (SDMA).^[23] The fundamental element of SDMA is the antenna array whose elements are dynamically controlled to produce multiple beams towards the desired directions and nulls towards the undesired ones.

“Antenna arrays can be employed to improve the link quality by combating the effects due to multipath propagation. Alternatively, adopting multiple antennas the different signal paths can be exploited to combat fading by spatial diversity techniques or can be used to increase the link capacity by allowing transmission of different data streams from different antennas. Therefore, the amount of traffic that can be sustained by a communication system for a given frequency bandwidth can be increased, leading to considerable spectral efficiency improvements. Antenna arrays can also be employed to focus the energy towards certain directions and to mitigate or adopting more sophisticated adaptive solutions to

suppress the transmission/reception towards other directions. This leads to a reduction of the transmitted/received interference and enables the spatial filtering of the incoming signals. Wireless networks in which the topology is subdivided by cells, such as the wireless mesh networks or the classical cellular systems, may obtain large benefits from the adoption of multiple antennas. In particular, the increased coverage range, which decreases the power requirements, and the possibility to track the mobile nodes using proper non-overlapping beams, reduce the number of required handovers together with the need to deploy new base stations or mesh routers. Therefore, multi-antenna technology may have a considerable impact not only in terms of performance improvement, but also in terms of cost reduction for wireless operators.”^[20]

The following gives an idea how different multi-antenna techniques can be employed for faithful signal detection and improving throughput ^[20]:

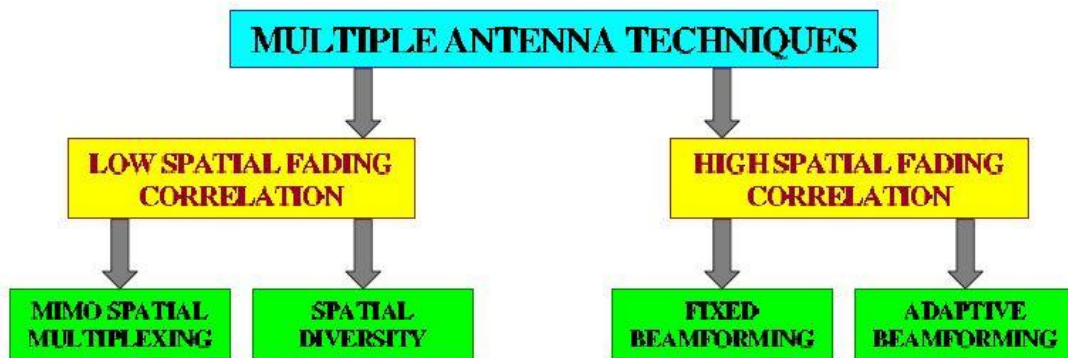


Fig. 2.4 Different multi-antenna techniques

2.2 Different types of SIMO detection methods:

We may now observe the expressions for different wireless communication detectors and also compare their performances. These types of detection methods employ receiver diversity for efficient detection of signals transmitted. Following are a few SIMO detection methods:

- a) MRC (Maximal Ratio Combination) detection
- b) EGC (Equal Gain Combination) detection

In case of MRC detection, when SIMO channel matrix ‘ h ’ is known the weight vector ‘ w ’ can be found as: $w = \frac{\bar{h}}{\|h\|}$ for multiplication with received signal vector (for one transmit antenna and ‘ r ’ receive antennas). The BER expression for BPSK in this case is:

$$\left(\frac{1-\lambda}{2}\right)^L \sum_{l=0}^{L-1} C_l^{L+l-1} \left(\frac{1+\lambda}{2}\right)^l \quad (2.3)$$

Where $\lambda = \sqrt{\frac{SNR}{2+SNR}}$. Although In case of EGC detection, knowledge of channel matrix 'h' is not required and is therefore the simplest to implement.

Now we will observe the expressions of BER for BPSK modulated signal and different MIMO wireless communication detection methods and also compare their performances.

2.3 Different types of MIMO detection methods:

Following are a few MIMO detection methods:

- a) ZF (Zero Forcing) detection
- b) MMSE (Minimum Mean Square Error) detection

In case of ZF detection, when MIMO channel matrix 'H' is known the estimate of transmitted symbols can be found out as: $\hat{x} = th((H^H H)^{-1} H^H S)$ where 'S' is the received signal vector and 'th(.)' denotes thresholding. In case of MMSE detection, when MIMO channel matrix 'H' is known the estimate of transmitted symbol can be found as:

$$\hat{x} = th(P_d(P_d H^H H + \sigma^2 I)^{-1} H^H S) \quad (2.4)$$

Where ' σ^2 ' is the noise variance and ' I ' stands for identity matrix.

2.4 Simulation of signal detection systems:

The respective error performance plots (SNR in dB vs BER plots) are given below:

The SNR vs BER plot comparison for MRC (employing receiver diversity) and the case with no diversity (single receiver) have been given for comparison here, the plots directly suggest the diversity gain for and therefore less errors for MRC receiver.

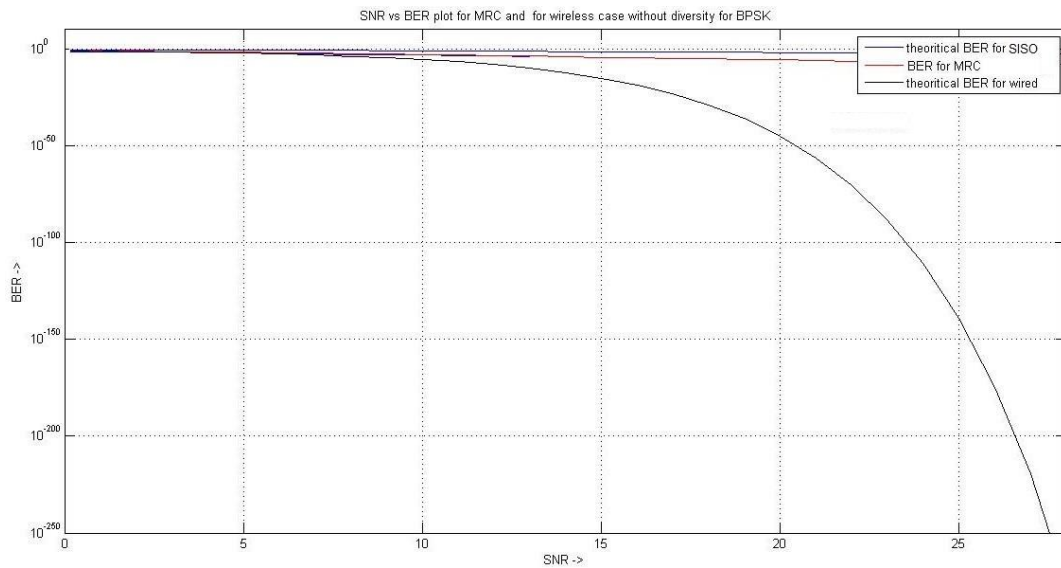


Fig. 2.5 Plot: SNR vs. BER for wired, wireless SISO and MRC

The SNR vs BER plot comparison between EGC (not requiring any information about the channel) and MRC (employing channel state information) clearly shows that MRC is advantageous than EGC in terms of detection errors.

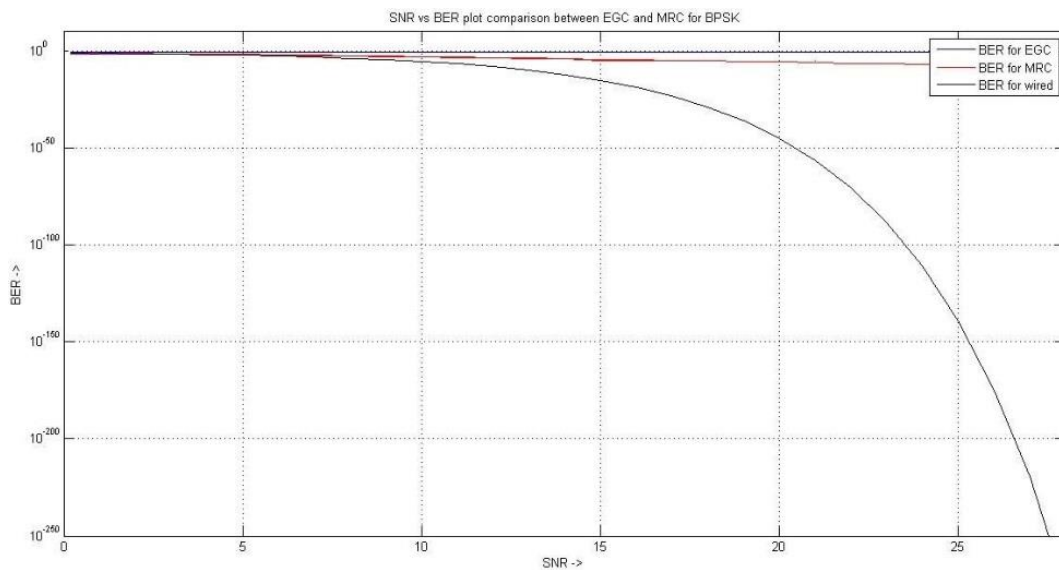


Fig. 2.6 Plot: SNR vs. BER for wired, EGC and MRC

The SNR vs BER plot comparison between ZF (employing multiplexing gain) and MRC with same nos. of receivers to show that ZF detection is indeed superior to MRC in terms of error performance.

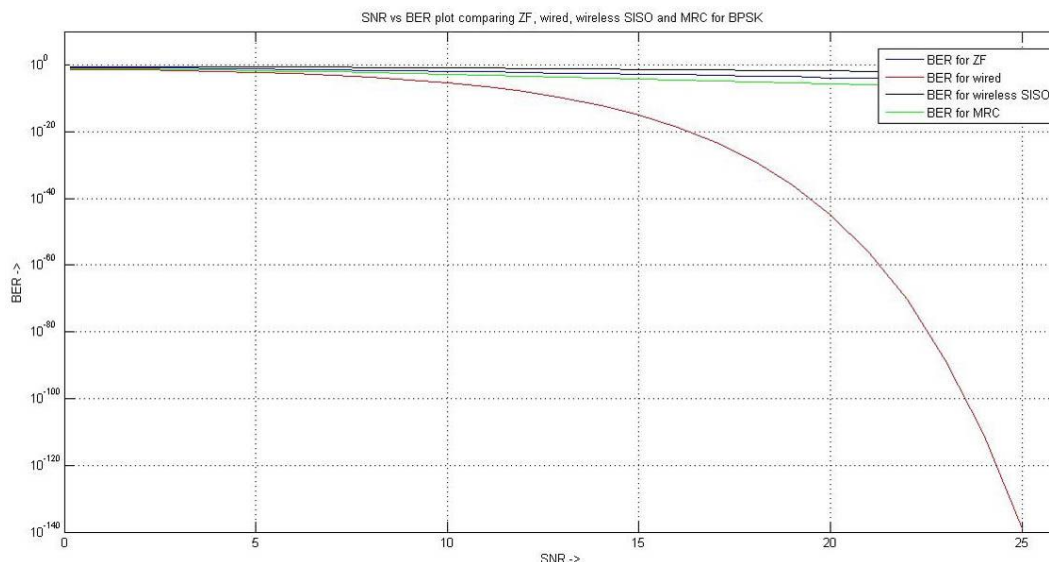


Fig. 2.7 Plot: SNR vs. BER for wired, wireless SISO, MRC and ZF

The SNR vs BER plot comparison between MMSE (also employing multiplexing gain) and ZF and also MRC with same nos. of receivers to show that MMSE detection is the better than the others in terms of error performance.

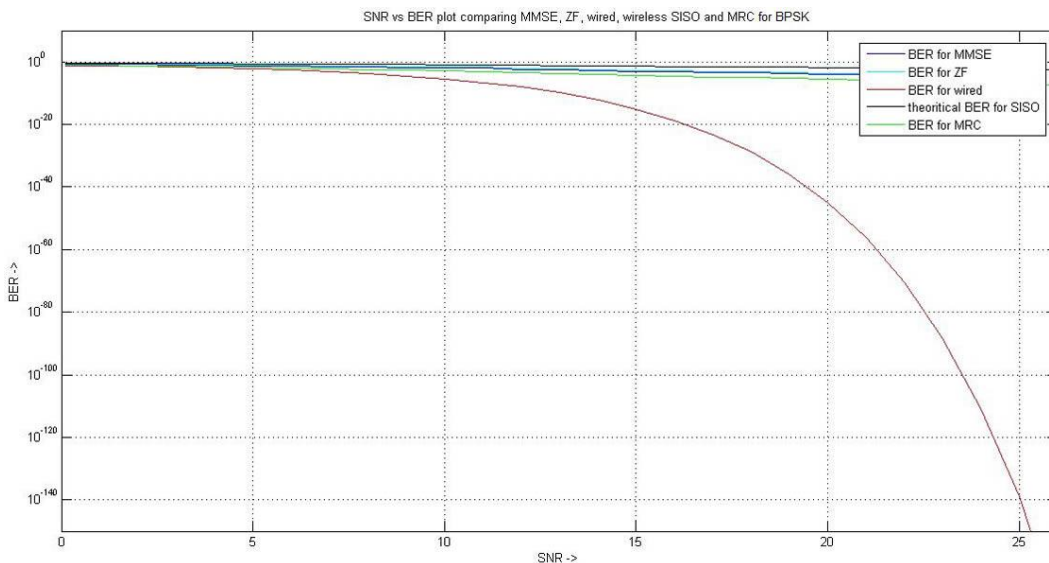


Fig. 2.8 Plot: SNR vs. BER for wired, wireless SISO, MRC, ZF and MMSE

Now we will discuss the importance of channel estimation with respect to signal detection problem. As we have seen in the above portion of this chapter for proper signal detection a complete knowledge of channel is essential to equalize the fading effects of the wireless channel, which supports our point towards channel estimation. So now in the following chapters we are going to discuss in detail the problem of channel estimation, how MIMO offers several advantages to this problem, how different spatial distribution of

channel scatterers e.g. trees, tall buildings etc. creates spatial correlation in the channel which and also other correlation properties of the channel affect the performance of channel estimation and ways to alleviate those problems.

Chapter 3

Beam forming and Capacity Enhancement

Wireless communications are based upon the transmission and reception of electromagnetic signals from antennas, which are in turn governed by Maxwell's equations. The following gives the instantaneous 'E' and 'H' fields assuming harmonic variations:

$$E(r, t) = \text{Re}\{E_s(r)e^{j\omega t}\} \quad (3.1)$$

$$H(r, t) = \text{Re}\{H_s(r)e^{j\omega t}\} \quad (3.2)$$

The power flow through an antenna can be described with the help of the quantity, Poynting vector represented as:

$$\vec{W}(r) \triangleq \frac{1}{2}E_s(r) \times H_s^*(r) \quad (3.3)$$

From this equation the radiated power can be found out as:

$$P_{rad} = \oint_{\mathcal{M}} \vec{W}(r) ds = \oint_{\mathcal{M}} \frac{1}{2} \text{Re}\{E_s(r) \times H_s^*(r)\} ds \quad (3.4)$$

The strength of EM field at any point and radiation characteristics of an antenna is determined by its physical size and operating wavelength. Based on radiation properties of an antenna the space around the antenna is divided into three different parts or regions. The different regions for antenna radiation field are depicted as below:

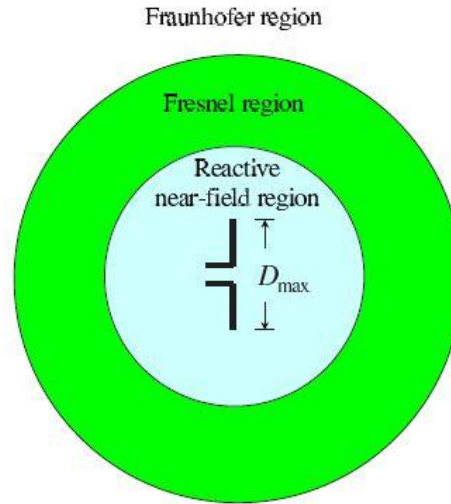


Fig. 3.9 Antenna radiation regions

The most important amongst these are the radiating near-field or Fresnel's and far-field or Fraunhofer regions which are separated approximately as: $radius < \frac{2D_{max}^2}{\lambda}$ corresponds to Fresnel's region and $radius \geq \frac{2D_{max}^2}{\lambda}$ corresponds to Fraunhofer's region where ' D_{max} ' is the maximum dimension of the radiating element i.e. antenna. Now for the far-field or Fraunhofer's region, the following can be approximately observed:

$$E_s(r) = \eta_0 H_s(r) \times \hat{r} \quad (3.5)$$

$$H_s(r) = \frac{1}{\eta_0} \hat{r} \times E_s(r) \quad (3.6)$$

Antenna radiation characteristics given as a function of space coordinates (cartesian or angular coordinates) is called antenna radiation pattern, this is represented with a function:

$$\mathcal{D}(\theta, \phi) \triangleq \frac{4\pi \mathcal{W}_u(\theta, \phi)}{P_{rad}} \quad (3.7)$$

The radiation pattern for a specific type of antenna is given below:

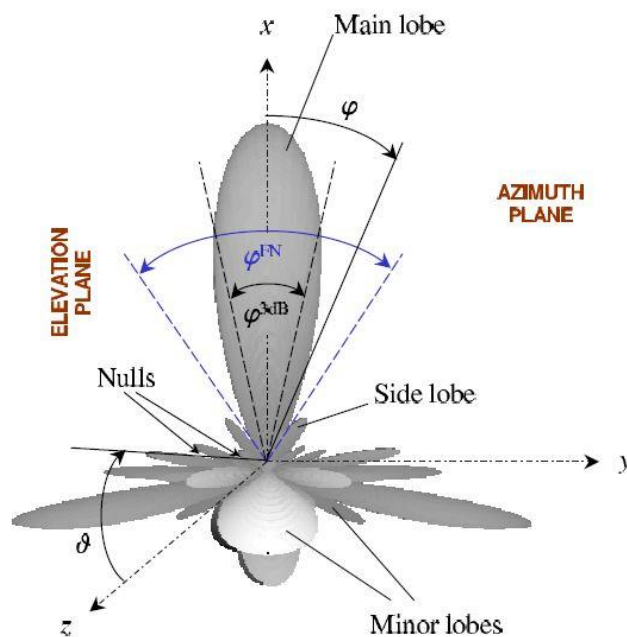


Fig. 3.10 Antenna radiation pattern

Some important definitions are given ^[20] below:

- i. Null: It is the direction in which the pattern is approximately zero.
- ii. Lobe: It is any angular region bounded by two nulls.
- iii. Main lobe: A lobe that contains the direction of maximum radiation.
- iv. Minor lobe: A lobe except the main one, where the pattern becomes maximum.
- v. Side lobe: A minor lobe that is usually adjacent to the main lobe and occupies the same hemisphere in the direction of maximum gain.
- vi. Half Power Beam Width (HPBW) or 3 dB Beam Width: The angular region containing the direction of maximum radiation that lies between the two directions in which the radiation is one-half of this maximum.
- vii. First Null Beam Width (FNBW): The entire angle spanned by the main lobe. The FNBW can be associated to the ability of an antenna to reject interference.

Now we are going to discuss about array antennas i.e. antennas containing multiple radiating elements which are also called Multi-element antennas (MEAs). Sometimes a very high directive gain is required at a certain direction but in general single antenna elements cannot provide very high directivity, so instead an array of antenna elements is used which has the capability to form a very highly directive beam (another facility in using an antenna array is its capability to steer the beam to any required direction of interest). Antenna element arrangement also plays an important role in generating the desired radiation pattern.

The total field radiated by an antenna array is determined by a vector summation, steering vector summarizes the far-field radiation pattern as:

$$a(\theta, \phi) \triangleq \left[\varepsilon_1(\theta, \phi) e^{j\frac{2\pi v_c}{\lambda} \tau_1(\theta, \phi)}, \dots, \varepsilon_N(\theta, \phi) e^{j\frac{2\pi v_c}{\lambda} \tau_N(\theta, \phi)} \right]^T \quad (3.8)$$

When the array elements are identical, the radiation pattern produced by an antenna array can be found out from the multiplication of the pattern of a single element with the array factor (AF), this principle is called the principle of pattern multiplication for finding the array response, where the array factor is represented as:

$$AF(\theta, \phi) \triangleq \sum_{k=1}^N \omega_k e^{j\frac{2\pi v_c}{\lambda} \tau_k(\theta, \phi)} \quad (3.9)$$

3.1 Different antenna arrays:

Different types of antenna arrays are encountered in practice some of which are very popular, namely:

- i. Linear arrays
- ii. Rectangular arrays
- iii. Circular arrays

3.1.1 Linear arrays:

This is the most common type of array encountered in practice. Uniform linear array (ULA) is a special case of linear arrays. The normalized AF can be given for the case of linear arrays as, for azimuth plane i.e. $\theta = \frac{\pi}{2}$:

$$AF_{norm} \left(\theta = \frac{\pi}{2}, \phi \right) = \frac{1}{N} \frac{\sin \left[\frac{N}{2} \left(\frac{2\pi \rho_l}{\lambda} \cos \phi + l \right) \right]}{\sin \left[\frac{1}{2} \left(\frac{2\pi \rho_l}{\lambda} \cos \phi + l \right) \right]} \quad (3.10)$$

Here ‘ l ’ is the progressive phase, ‘ N ’ is the number of elements. The following figure gives the structure of linear arrays:

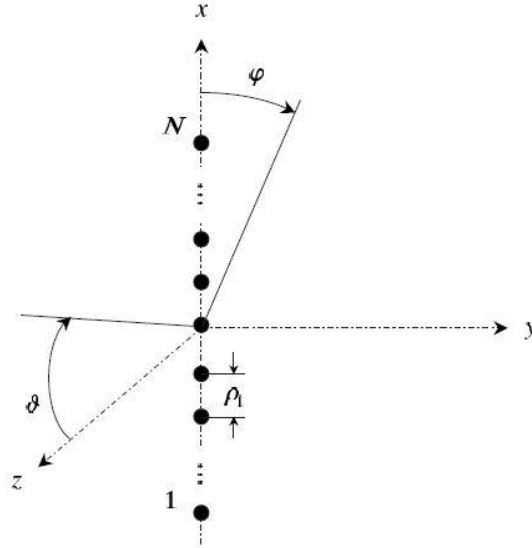


Fig. 3.11 Linear antenna array

3.1.2 Rectangular arrays:

Rectangular arrays are yet another popular type of antenna array configuration and it provides more versatility with respect to linear arrays, it can be visualized as a number of linear arrays arranged in a linear grid (or similarly array elements arranged in a rectangular grid). The construction being somewhat similar to linear arrays (except the fact that linear arrays are 1D whereas rectangular arrays are 2D) the array response matches somewhat with the linear array response but in case of 2D rectangular array more control can be achieved over steering the beam or suppress side lobes and improving the main beam directional gain. The normalized AF for the rectangular arrays is given as:

$$AF_{norm} = \frac{1}{N_x N_y} \times \frac{\sin\left[\frac{N_x}{2}\left(\frac{2\pi\rho_l^x}{\lambda}\sin\theta\cos\phi+l_x\right)\right]}{\sin\left[\frac{1}{2}\left(\frac{2\pi\rho_l^x}{\lambda}\sin\theta\cos\phi+l_x\right)\right]} \times \frac{\sin\left[\frac{N_y}{2}\left(\frac{2\pi\rho_l^y}{\lambda}\sin\theta\sin\phi+l_y\right)\right]}{\sin\left[\frac{1}{2}\left(\frac{2\pi\rho_l^y}{\lambda}\sin\theta\sin\phi+l_y\right)\right]} \quad (3.11)$$

The array construction is given in the following figure:

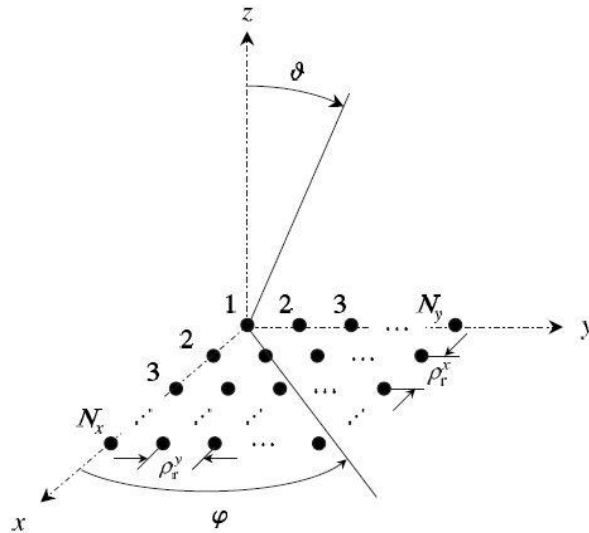


Fig. 3.12 Rectangular antenna array

3.1.3 Circular arrays:

Another commonly encountered array configuration is the circular or ring configuration, it is also a 2D configuration and provides thinner beam shaping, side lobe suppressing and beam steering capabilities that linear antenna arrays cannot provide. Uniform circular array (UCA) is a special case of circular arrays. The normalized AF in this case is as given below:

$$AF_{norm}(\theta, \phi) = \sum_{k=1}^N \omega_k e^{j\frac{2\pi}{\lambda} \rho_c \sin \theta \cos(\phi - \xi_k)} \tag{3.12}$$

The shape of the array is as given in the following figure:

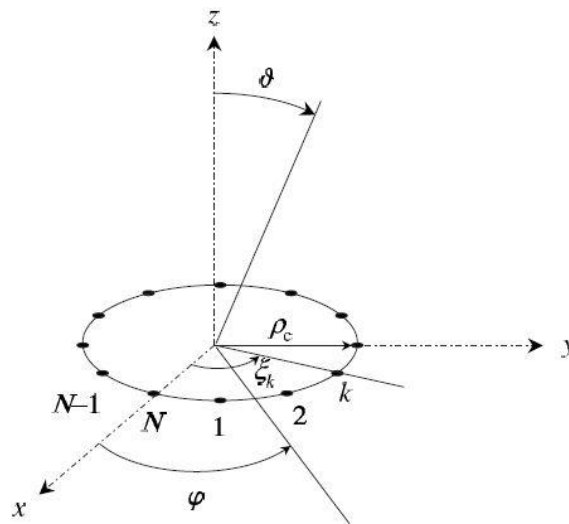


Fig. 3.13 Circular antenna array

3.2 Introduction to Beam forming:

In contrast to transmit diversity techniques the antenna elements can be used to adjust the strength of transmit/receive signals based on their direction (AoA/AoD). Focussing of the energy is achieved by choosing appropriate weights for each antenna element according to a certain criterion. Beam forming can be divided in to two principal sub-classes:

1. DoA/DoD based Beam forming
2. Eigen Beam forming

3.2.1 DoA/DoD based Beam forming:

Various signals can be characterized by their DoA/DoD (direction of arrival/departure). The DoA/DoDs can be measured using signal processing techniques and from those acquired DoA/DoDs the beam former creates a weighting factor for antenna elements to focus the signal towards a desired user while suppressing undesired interferences.

The arrival/departure time delay can be calculated as: $\tau = \left(\frac{d}{c}\right)\sin\theta$, where ‘ d ’ denotes the inter-element distance of the ULA, ‘ θ ’ is the AoA/AoD of the signal and ‘ c ’ is the velocity of the RF wave, finally the received signal vector:

$$\begin{aligned} & [y_1(t), y_2(t), \dots, y_{N_R}(t)]^T \\ &= y_1(t) \underbrace{\left[1 \exp\left(-j2\pi \frac{d\sin\theta}{\lambda}\right) \dots \exp\left(-j2\pi(N_R - 1) \frac{d\sin\theta}{\lambda}\right) \right]}_{a(\theta)} \end{aligned} \quad (3.13)$$

Where ‘ $a(\theta)$ ’ is called response vector. Now a beam forming vector: $w = [w_1, w_2, \dots, w_n]^T$ can be multiplied^[13] as:

$$w^*[a(\theta_1), a(\theta_2), \dots, a(\theta_n)]^T = [0 \ 0 \ \dots \ 1 \ \dots \ 0 \ 0]^T \quad (3.14)$$

3.2.2 Eigen Beam forming:

Instead of using array response vectors from AoA/AoDs, eigen beam forming exploits the channel impulse response of each antenna element to find array weights that satisfy a desired criterion. By using CSIT eigen beam forming utilizes eigen-decomposed channel response for focussing transmit signal to the desired user even if there are cochannel interfering signals with numerous AoA/AoDs. After transmit signal pre-coding:

$$s = V\tilde{s} \quad (3.15)$$

For $r = \text{rank}(HH^H) = \text{rank}(H)$ and $\tilde{s} \triangleq [\tilde{s}_1, \tilde{s}_2, \dots, \tilde{s}_r]^T$, 's' maps 's' to $N_T \times 1$ desired transmit signal vector. Now the received signal:

$$r = \sqrt{\rho}Hs + z = \sqrt{\rho}HV\tilde{s} + z \quad (3.16)$$

' U^H ' is pre-multiplied for detection:

$$\tilde{r} \triangleq U^H r \quad (3.17)$$

Finally:

$$\tilde{r}_i = \sqrt{\rho}\sqrt{\lambda_i}\tilde{s}_i + \tilde{z}_i \quad (3.18)$$

For $1 \leq i \leq r$ and 'U' and 'V' and ' λ_i 's can be found out from SVD of 'H': $H = UDV^H$ and $D = \text{diag}(\sqrt{\lambda_1}, \sqrt{\lambda_2}, \dots, \sqrt{\lambda_r})$. We see from the above formulation that for i^{th} eigen channel the received SNR:

$$\rho_i = \rho E\{|\tilde{s}_i|^2\}\lambda_i \quad (3.19)$$

Finally the capacity:

$$C = \sum_{i=1}^r \log_2 |1 + \rho E\{|\tilde{s}_i|^2\}\lambda_i| \quad (3.20)$$

Now the transmit power can be optimally allocated at the transmit antennas for CSIT in a water-filling manner such that ' $\sum_i P_i = P = 1$ ' for normalized case based on the Lagrange's function:

$$F = \sum_i \log_2 |1 + \rho_i \lambda_i P_i| + 1/\mu \times (P - \sum_i P_i) \quad (3.21)$$

For $\frac{dF}{dP_i} = 0$, where $P_{i_{opt}} = \left(\mu - \frac{1}{\rho\lambda_i}\right)^+$ so that^[13]:

$$C_{opt} = \sum_i \log_2 \left|1 + \rho P_{i_{opt}} \lambda_i\right| \quad (3.22)$$

3.3 Different types of Beam forming:

The name beam forming comes from the early forms of antenna arrays that were used to generate pencil beams, so as to receive signals from a specific direction and attenuate signals incoming from other directions. From this primary meaning related to propagation environments characterized by a low angular spread, beam forming has been extended to rich scattering scenarios and, at present, this term is used to denote the antenna processing techniques operating both in low and high-rank channels. In a low-rank environment, depending on the level of sophistication of the adopted processing algorithm, beam forming techniques can be subdivided in two main groups: fixed beam forming and adaptive beam forming.^[20]

Some important types of low-rank beam forming techniques are given as following:

1. Fixed Beam forming
 - a. Switched-beam Beam forming
 - b. Delay and sum Beam forming
 - c. Beam space Beam forming
2. Adaptive Beam forming

A brief description of the above mentioned techniques are given below:

3.3.1 Fixed Beam forming:

Fixed beam forming does not perform the amplitude weighting of the received signals and can be realized adopting either an analog approach (e.g. switched beam, delay and sum) or a digital approach (e.g. beam-space beam forming).^[20]

Diagram of an analog beam former:

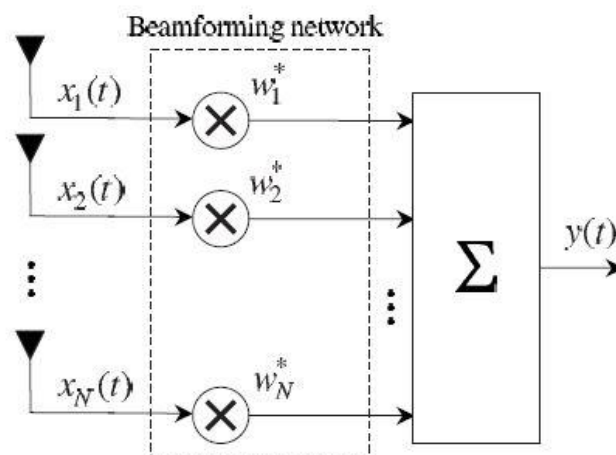


Fig. 3.14 Analog beam former

3.3.2 Adaptive Beam forming:

In case of adaptive beam forming the radiation pattern is dynamically controlled to perform the electrical beam steering to a desired direction and null steering to reject interfering signals.^[20]

3.4 Transmit Beam forming and Ergodic capacity improvement:

From the basic MIMO system structure:

$$r = \sqrt{\rho}H's + z \quad (3.23)$$

Where $\sqrt{\rho}H' = H 'r'$ is the $N_R \times 1$ received signal vector, ' H' ' is the $N_R \times N_T$ normalized channel matrix, ' s' ' is the $N_T \times 1$ transmit signal vector, ' ρ ' is the received SNR, ' σ_s^2 ' is signal covariance and ' σ^2 ' is noise power. From the capacity formula:

$$C = \log_2 \left| I_{N_R} + \frac{1}{\sigma^2} HR_{SS}H^H \right|, \text{ where } R_{SS} \triangleq E\{SS^H\} = \sigma_s^2 I_{N_T} \quad (3.24)$$

For CSIR only consideration optimal case is when equal power is distributed among transmit antennas, so that:

$$C = \log_2 \left| I_{N_R} + \frac{\rho}{N_T} H' H'^H \right| \quad (3.25)$$

Now considering the fact that multipath channels are capable of supporting transmission of multiple independent data streams (through independent eigen-modes or eigen channels) the final capacity expression can be found as:

$$C = \sum_{i=1}^r \log_2 \left| 1 + \frac{\rho}{N_T} \lambda_i \right| \quad (3.26)$$

For $\lambda_i = \lambda_1, \lambda_2, \dots, \lambda_r$ from eigen decomposition of Hermitian matrix: $HH^H = U_{N_R} D_{N_R} U_{N_R}^H$. The expression is similar to that for ' r ' number of parallel SISO channels with power gain ' λ_i ' also the effective transmitted power being ' $\frac{1}{N_T}$ ' times transmitted power. Hence we see that channel rank is a quantitative way of characterizing scattering richness of the channel. Now optimum eigen value distribution can be found out from the constrained optimization problem: $\triangleq \sum_{i=1}^r \lambda_i = \|H\|_F^2$, where ' ζ ' is the constraint constant.^[12]

Coherence distance gives a rule of thumb as to how far away antenna elements should be placed so that signals transmitted/received are statistically independent. For small coherence distance large antenna arrays give rich diversity but if coherence distance is large, more number of antennas packed in a given distance makes it impossible to exploit spatial diversity because of spatial correlation, so in that case rather than directly employing diversity techniques, beam forming is employed to extract the diversity benefits. Hence the importance of beam forming is observed.

3.5 Mutual coupling influence on MIMO channel capacity:

Here for the MIMO systems considered here, the receiver MC structure is assumed and the case of printed dipoles with $\sim 0.56\lambda$ inter-element spacing is studied for the following

discussion ^[33]. Although for multi-path scattering channels may sufficiently be able to exploit full-rank i.e. $rank = \max(n_T, n_R)$ for capacity considerations alike independent spatial paths. But in reality capacity is dependent on received SNR as well as correlations between sub channels out of channel imperfections.

Considering only receiver coupling present, the normalized pen (thin beam) formed can be expressed as:

$$E \left\{ \frac{1}{n_T n_R} \sum_{i=1}^{n_R} \sum_{j=1}^{n_T} |[CH]_{ij}|^2 \right\} = 1 \quad (3.27)$$

Where ‘ CH ’ is the matrix product, ray-tracing model is assumed with $u_t = \sin \theta_t$ and $u_r = \sin \theta_r$ where ‘ θ_t ’ and ‘ θ_r ’ are randomly chosen from specified AS and each ray is assigned arbitrary phase: $\xi \in [-\pi, \pi]$ and the rays share equal power i.e. $E = 1/\sqrt{R}$ where ‘ R ’ is the total number of rays.

From impinging wave the direct component and the coupled components from m^{th} receive array element can be expressed as:

$$v_m(u) \simeq C_{mm} E_0 \mathcal{F}^i(u) e^{jkd_m u} + \sum_{n, m \neq n} C_{mn} E_0 \mathcal{F}^i(u) e^{jkd_n u} \quad (3.28)$$

For $\mathcal{F}^i(u) e^{jkd_m u}$ giving isolated element radiation pattern, ‘ C_{mn} ’ gives coupling coefficient and ‘ u ’ is the direction cosine i.e. $u = \sin \theta$, ‘ d_m ’ is the position of the m^{th} element with respect to the phase centre.

Now the radiation pattern can be approximated as complex coupling coefficient weighted superimposed isolated element patterns of the array, in ideal case. Distortion and pattern diversity due to the coupling may initially bring forth capacity improvement but due to MC itself spatial correlation is decreased at the link-ends so that the directional nature of the channel at the link-end decreases and so does beam forming gains and capacity.

Chapter 4

Fundamentals of Channel Estimation

Here we start discussing on channel estimation because we have seen that for symbol detection CSI is essentially important therefore a good channel estimate is inevitably important for the knowledge of the receiver. For channel estimation purpose many types of techniques are employed, some of which are:

- a) Blind channel estimation
- b) Semi-blind channel estimation
- c) Training signal based channel estimation

Amongst these training signal based channel estimation is the most reliable method of channel estimation. Among different Training signal based channel estimation techniques a few are to be mentioned here, namely:

- I. LS (Least Square) based channel estimation
- II. SLS (Scaled LS) based channel estimation
- III. MMSE (Minimum Mean Square Error) based channel estimation
- IV. RMMSE (Relaxed MMSE) based channel estimation

Now an explanation about the training signal based channel estimators in brief in the following section is in order, but first the system must be introduced for which the analyses are to be done. Let us consider a block fading MIMO system with ‘ t ’ transmit and ‘ r ’ receive antennas. The $r \times 1$ complex received signal vector can be expressed as : $S = HP + V$ where $S = [s_1, s_2, \dots, s_N]$ is the $r \times N$ received signal matrix, ‘ H ’ is the $r \times t$ channel matrix, $P = [p_1, p_2, \dots, p_N]$ is the $t \times N$ training matrix and $V = [v_1, v_2, \dots, v_N]$ is the $r \times N$ noise matrix. The task of a channel estimation here is to recover the channel matrix ‘ H ’ based on the knowledge of ‘ S ’ and ‘ P ’.

4.1 LS Channel Estimator:

When received signal ‘ S ’ and training matrix ‘ P ’ are known an estimate of ‘ H ’ can be found as:

$$\hat{H}_{LS} = SP^\dagger \quad (4.1)$$

Where $P^\dagger = P^H (PP^H)^{-1}$ is the pseudo inverse of 'P' and ' $(.)^H$ ' denotes the Hermitian transpose function. A power constraint: $P = \|P\|_F^2$ where 'P' is the total power constant and ' $\|\cdot\|_F$ ' denotes Frobenius norm is also present here. For estimated channel vector ' \hat{H}_{LS} ' the MSE (Mean Square Error) expression: $J_{LS} = E\{\|H - \hat{H}_{LS}\|_F^2\}$, the expression finally becomes:

$$J_{LS} = \sigma^2 r \times \text{tr}\{(PP^H)^{-1}\} \quad (4.2)$$

Where ' $\text{tr}\{\cdot\}$ ' is the trace of a matrix and the fact: $E\{V^H V\} = \sigma^2 r I$ is used. For the following situation the LS estimator can be found when the optimization problem is solved: $\min_P(\text{tr}\{(PP^H)^{-1}\})$ for $\text{tr}\{PP^H\} = P$. Solving this problem leaves us with the equation: $PP^H = \frac{P}{t} I$ so that any orthogonal training matrix (training matrix with orthogonal rows) satisfying it will be optimal. A choice can be taken in this case as:

$$P = \sqrt{\frac{P}{Nt}} \begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & W_N & \dots & W_N^{N-1} \\ \vdots & \vdots & \dots & \vdots \\ 1 & W_N^{(t-1)} & \dots & W_N^{(t-1)(N-1)} \end{pmatrix} \quad (4.3)$$

Where all elements of the matrix have same power contribution and $W_N = e^{j2\pi/N}$. For this optimal training the LS channel estimate becomes:

$$\hat{H}_{LS} = \frac{t}{P} S P^H \quad (4.4)$$

Which is therefore: $\hat{H}_{LS} = H + \frac{t}{P} V P^H$ using condition that $S = H P + V$ the channel estimation error for optimal training^[1] is:

$$\min_P(J_{LS}) = \frac{\sigma^2 t^2 r}{P} \quad (4.5)$$

4.2 SLS Channel Estimator:

This estimator is very close in terms of operation and principle with LS channel estimator, the main focus of this type of estimator is to further reduce the channel estimation error (MSE) with a scaling component (bias) multiplied with the estimate found out using LS channel estimator. If this bias term is represented by ' γ ' then:

$$MSE = E\left\{\|H - \gamma \hat{H}_{LS}\|_F^2\right\} = (1 - \gamma)^2 \text{tr}\{R_H\} + \gamma^2 \sigma^2 r \times \text{tr}\{(PP^H)^{-1}\} \quad (4.6)$$

Which is equal to:

$$E\{\|H - \gamma \hat{H}_{LS}\|_F^2\} = (J_{LS} + \text{tr}\{R_H\}) \times \left(\gamma - \frac{\text{tr}\{R_H\}}{J_{LS} + \text{tr}\{R_H\}}\right) + \frac{J_{LS} \text{tr}\{R_H\}}{J_{LS} + \text{tr}\{R_H\}} \quad (4.7)$$

Where $R_H = E\{H^H H\}$ is the channel correlation matrix. This expression of ' J_{LS} ' is minimized by choosing:

$$\gamma_o = \frac{\text{tr}\{R_H\}}{J_{LS} + \text{tr}\{R_H\}} \quad (4.8)$$

The minimum MSE in this case is: $\frac{J_{LS} \text{tr}\{R_H\}}{J_{LS} + \text{tr}\{R_H\}} < J_{LS}$ therefore SLS estimation error is always less than LS case. Finally the SLS estimator^[1] expression is obtained as:

$$\hat{H}_{SLS} = \gamma_o \hat{H}_{LS} = \frac{\text{tr}\{R_H\}}{\sigma^2 r \times \text{tr}\{(PP^H)^{-1}\} + \text{tr}\{R_H\}} S P^\dagger \quad (4.9)$$

The optimal training matrix for SLS estimator can be found by solving : $\min_P (J_{SLS})$ for $\text{tr}\{PP^H\} = P$, now since ' J_{SLS} ' is a monotonically increasing function of ' J_{LS} ', from the expression of ' J_{SLS} ' we see that the only term bearing dependency on ' P ' is ' J_{LS} ' therefore the optimal choice of training matrix ' P ' for SLS estimator would be same as that of LS estimator. The final expression of MSE considering optimal training is:

$$\min_P (J_{SLS}) = \frac{\sigma^2 t^2 r \times \text{tr}\{R_H\}}{\sigma^2 t^2 r + P \times \text{tr}\{R_H\}} \quad (4.10)$$

Also for either ' $t = 0$ ' or ' $r = 0$ ' we observe:

$$\lim_{r/t \rightarrow \infty} \min_P (J_{SLS}) = \text{tr}\{R_H\} \quad (4.11)$$

Therefore it doesn't have a restriction on the upper limit of ' t ' or ' r ' like the case of LS estimator.

4.3 MMSE Channel Estimator:

For received signal ' S ' the MMSE estimator can be expressed as:

$$H_{MMSE} = S A_0 \quad (4.12)$$

Where $A_0 = \underset{A}{\text{argmin}} E(\|H - H_{MMSE}\|_F^2)$, the final estimation error is expressed as:

$$\begin{aligned} E = E(\|H - S A\|_F^2) &= \text{tr}(R_H) - \text{tr}(R_H P A) - \text{tr}(A^H P^H R_H) \\ &+ \text{tr}\{A^H (P^H R_H P + \sigma^2 r I) A\} \end{aligned} \quad (4.13)$$

Now forcing:

$$\begin{aligned} \frac{\partial E}{\partial A} = & \operatorname{tr}\left\{\frac{\partial}{\partial A} \operatorname{tr}(R_H) - \frac{\partial}{\partial A} \operatorname{tr}(R_H P A) - \frac{\partial}{\partial A} \operatorname{tr}(A^H P^H R_H)\right. \\ & \left. + \frac{\partial}{\partial A} \operatorname{tr}[A^H (P^H R_H P + \sigma^2 r I) A]\right\}\Big|_{A=A_0} = 0 \end{aligned} \quad (4.14)$$

Thus we observe $\operatorname{tr}(P^H R_H + P^H R_H) = \operatorname{tr}\{2(P^H R_H P + \sigma^2 r I) A_0\}$ or $A_0 = (P^H R_H P + \sigma^2 r I)^{-1} P^H R_H$ so that:

$$\hat{H}_{MMSE} = S(P^H R_H P + \sigma^2 r I)^{-1} P^H R_H \quad (4.15)$$

Now we are in a position to observe the MSE for MMSE estimator expression as:

$$J_{MMSE} = \operatorname{tr}(E\{E E^H\}) = \operatorname{tr}\left(\left\{R_H^{-1} + \frac{1}{\sigma^2 r} P P^H\right\}^{-1}\right) \quad (4.16)$$

Now from the Lagrange multiplier method J_{MMSE} can be satisfied for power constraint as:

$$\|P\|_F^2 = \operatorname{tr}(P P^H) = P \quad (4.17)$$

The problem can be stated as:

$$L(P, \mu) = \operatorname{tr}\left(\left\{R_H^{-1} + \frac{1}{\sigma^2 r} P P^H\right\}^{-1}\right) + \mu(\operatorname{tr}\{P P^H\} - P) \quad (4.18)$$

To find any solution to this problem Eigen-decomposition in the form: $R_H = Q \Lambda Q^H$ can be used where ‘ Q ’ is unitary eigenvector matrix and ‘ Λ ’ is diagonal matrix containing non-negative Eigen values. Hence the expression can be obtained:

$$J_{MMSE} = \operatorname{tr}(\{\Lambda^{-1} + \tilde{P} \tilde{P}^H\}^{-1}) \quad (4.19)$$

Where $\tilde{P} = \frac{1}{\sqrt{\sigma^2 r}} Q^H P$ so that the power constraint can be expressed as:

$$\operatorname{tr}(\tilde{P} \tilde{P}^H) = \frac{P}{\sigma^2 r} \quad (4.20)$$

For this the Lagrange multiplier expression can be given as:

$$L(\tilde{P}, \mu) = \operatorname{tr}(\{\Lambda^{-1} + \tilde{P} \tilde{P}^H\}^{-1}) + \mu(\operatorname{tr}\{\tilde{P} \tilde{P}^H\} - \frac{P}{\sigma^2 r}) \quad (4.21)$$

Now setting $\frac{\partial L(\tilde{P}, \mu)}{\partial \tilde{p}_i} = 0$ for $i = 1, \dots, t$ a Water-filling type solution can be found as:

$$\tilde{p}_i = \sqrt{\mu_0 - \lambda_i^{-1}} \quad (4.22)$$

For $\lambda_i^{-1} < \mu_0$, otherwise '0'. So that $\tilde{P} = ([\mu_0 I - \Lambda^{-1}]^+)^{1/2}$ and finally the optimal training:

$$P = \sqrt{\sigma^2 r} Q \{[(\mu_0 I - \Lambda^{-1})^+]^{1/2}, 0_{t \times N-t}\} I \quad (4.23)$$

For $N = t$ but for any arbitrary value $N \geq t$ 'I' can be replaced with a suitable matrix 'U'.^[1]

4.4 RMMSE Channel Estimator:

RMMSE channel estimator is a basic improvement over MMSE estimator, with the merits of this estimator being almost same as the MMSE estimator. The basic improvement is regarding the fact that this type of estimator uses orthogonal signal matrix as training matrix. The final expression of the estimated channel matrix using RMMSE technique is:

$$H_{RMMSE} = S [P^H P + \frac{\sigma_n^2 RT}{tr(R_H)} I]^{-1} P^H \quad (4.24)$$

While the MSE for this case is calculated^[1] as:

$$J_{RMMSE} = \frac{tr(R_H) \sigma_n^2 RT^2}{tr(R_H) P + \sigma_n^2 RT^2} \quad (4.25)$$

Finally the performance comparisons are given and the Error performances (normalized MSEs vs. SNR dB) plotted for LS and SLS estimators as follows:

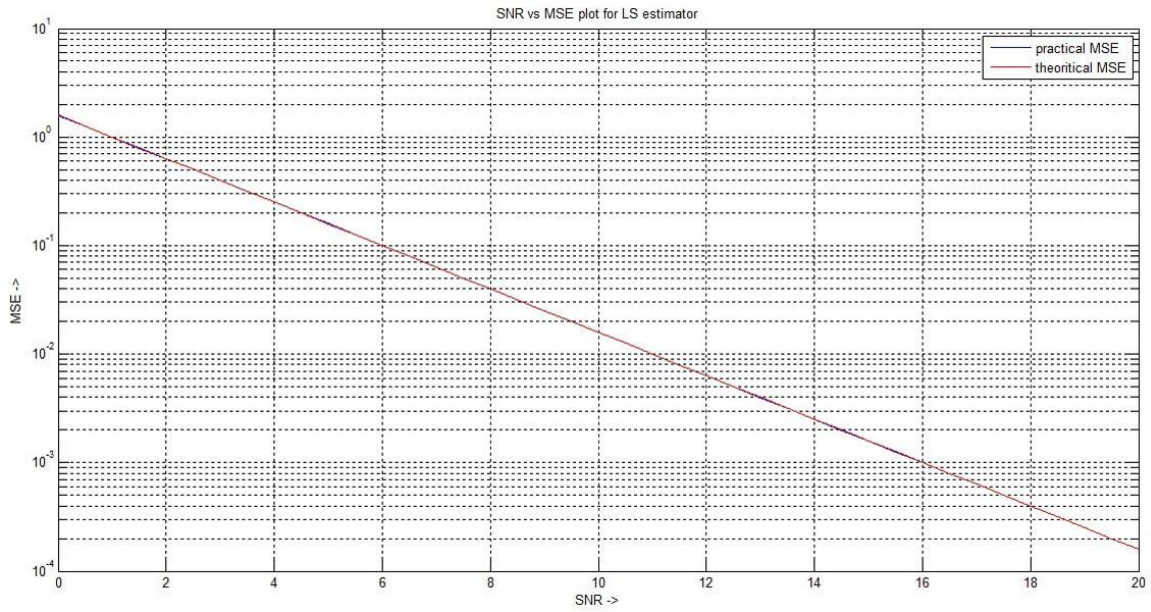


Fig. 4.15 Plot: SNR vs. MSE for LS estimator

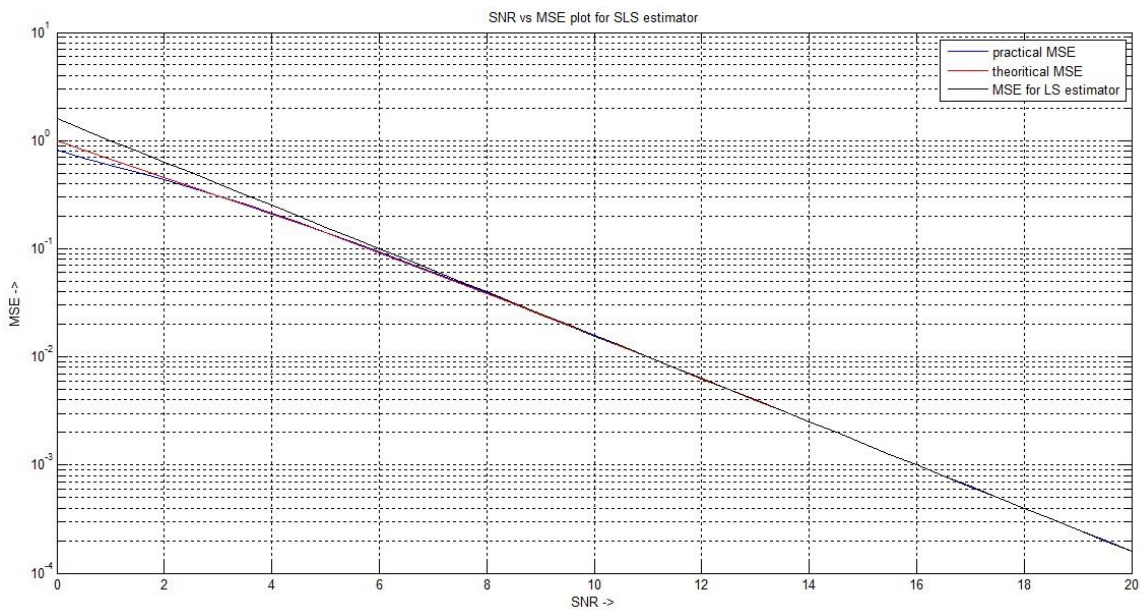


Fig. 4.16 Plot: SNR vs. MSE for SLS estimator

Here we have simulated for $r = 3$, $t = 2$, $N = 2$ and SNR dB range: $[0,20]$ we see for low SNR cases SLS estimator gives less MSE with respect to LS estimator, however for high SNR cases both give almost the same result in terms of MSE performance in estimation. Again for SLS estimator exact knowledge of channel correlation matrix is needed, we know in case when channel matrix 'H' is unknown some performance trade-off can be done by replacing $tr(R_H)$ with $tr(\widehat{R}_H)$ by using information about channel matrix from LS estimator \widehat{H}_{LS} i.e. for LS-SLS estimator. From MMSE Error performance we know

that MMSE performs better than both of LS and SLS estimators with a few demerits: no simple signal (matrix) can be used as optimal training, good estimate requires knowledge of channel correlation matrix. Pros are more than cons in case of MMSE estimator in terms of performance so MMSE estimator should be used in case of high performance estimation, some performance trade-off is always tolerable in most of the cases, so that an estimate of R_H e.g. $\hat{H}_{LS}^H \hat{H}_{LS}$ otherwise a model of R_H can also be assumed e.g. $[R_H]_{n,m} = r\varepsilon^{|n-m|}$ for $0 \leq \varepsilon < 1$ where 'n' and 'm' are arbitrary indexes can be used.

The Error performances (normalized MSEs vs SNR dB) for MMSE and RMMSE estimators can be plotted as:

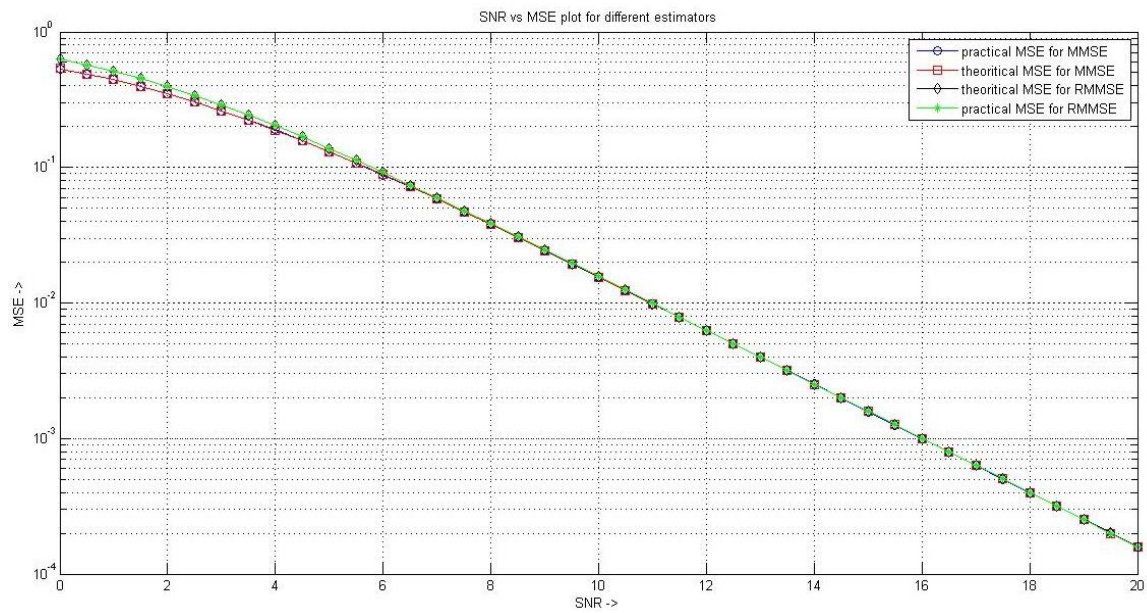


Fig. 4.17 Plot: SNR vs. MSE for MMSE and RMMSE estimators

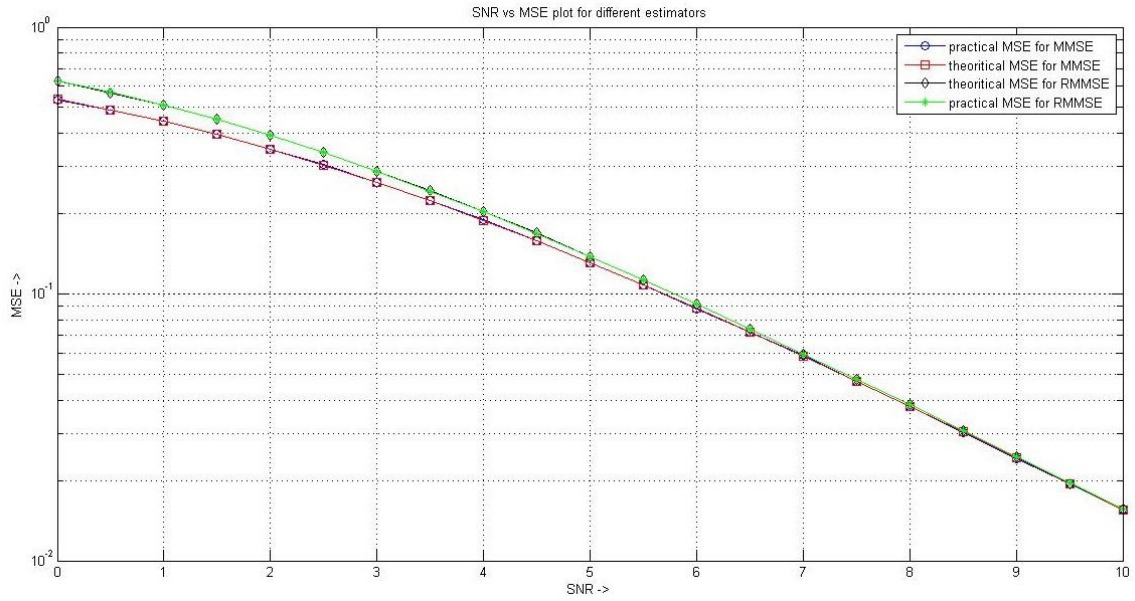


Fig. 4.18 Plot: SNR vs. MSE for different estimators

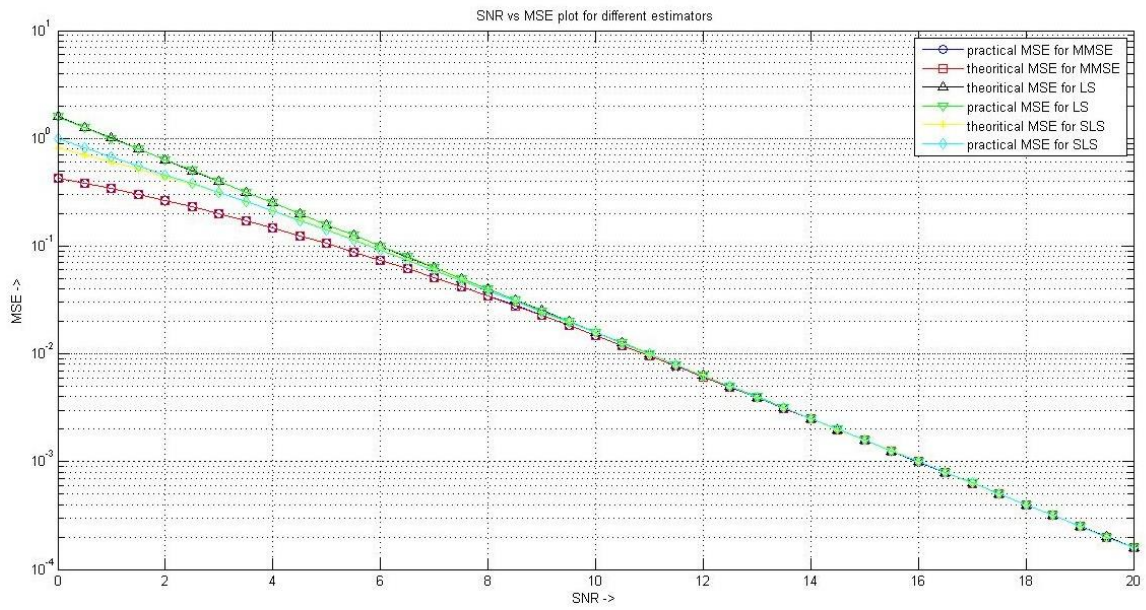


Fig. 4.19 Plot: SNR vs. MSE for LS, SLS and MMSE estimators

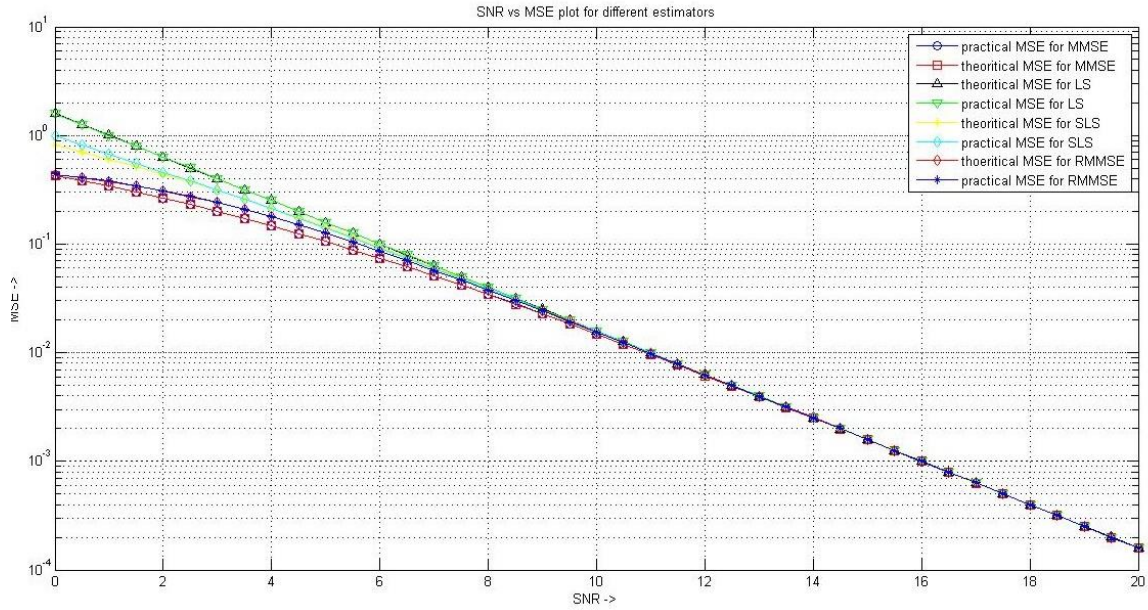


Fig. 4.20 Plot: SNR vs. MSE for LS, SLS, MMSE and RMMSE estimators

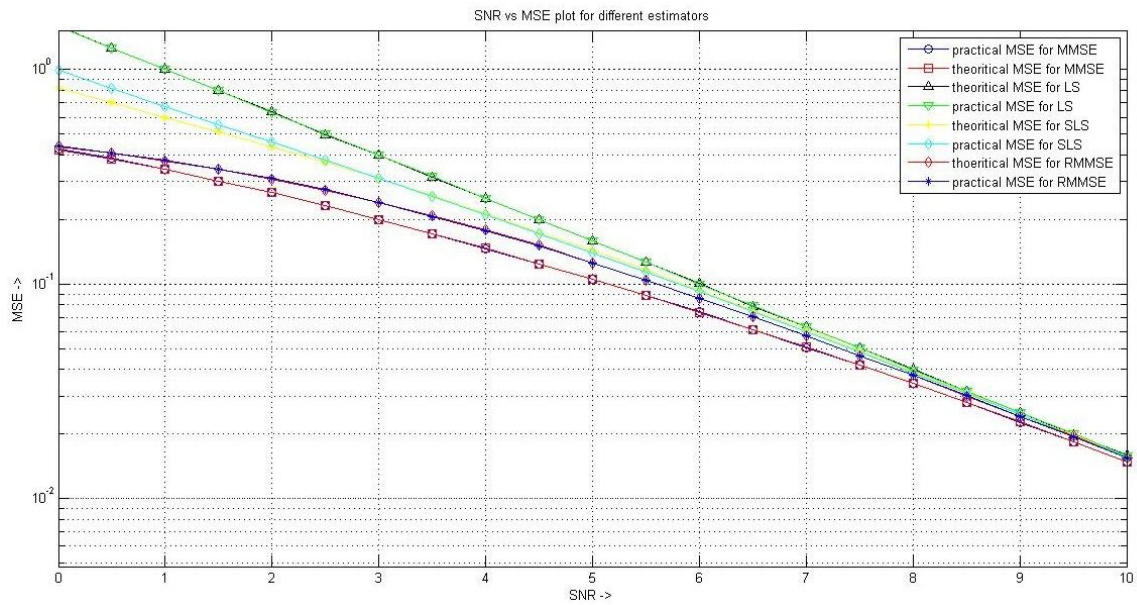


Fig. 4.21 Plot: SNR vs. MSE for different estimators

Here we have simulated for $r = 3, t = 2, N = 2$ and SNR dB range $[0, 20]$ we see for low SNR cases MMSE estimator gives less MSE with respect to RMMSE estimator but for high SNR cases both give almost the same result in terms of MSE performance in estimation, so that RMMSE gives the performance of MMSE estimator and with reduced complexity of the estimator and relaxed choice of training signal. It can be concluded that MMSE estimator gives the best error performance amongst the channel estimators that are discussed here.

Chapter 5

Correlated MIMO Wireless Channel

Channel correlation can be interpreted as dependence between time/space/frequency and average signal gain e.g. spatial correlation can be depicted as correlation of transmitted/received signal gain with a/some spatial direction(s) (or angle of arrival/departure) it can be also expressed as non-zero components in cross-diagonal elements of channel covariance matrix representing a/some specific direction(s) (similarly for temporal correlation non-zero components of cross-diagonal elements of time-autocorrelation function).

It is called correlation based on the fact that because of the existence of this phenomenon between different channel fading co-efficients (spatial correlation) or between samples of same channel fading coefficients at different time instants (temporal correlation) and because of that the received signal gains/phases are affected with respect to and because of each other.

This part of the text mainly focusses first on channel estimation for spatially correlated and next for spatio-temporally correlated flat-faded MIMO systems. For the first part, due to problem in handling with two extreme channel assumptions, namely: The Statistical model and the Physical model we may proceed with the intermediate Virtual channel representation model^[4] consideration of Sayeed Akbar M. but actually follow the improved model of Weichselberger et al.^[3] and compare the results of channel estimations and corresponding signal detections for different assumptions of ‘coupling matrices’ and that for the un-correlated case. Next we consider the model of Yen-Chih Chen et al.^[2] which encompasses the merits of all the aforementioned models and even presents those models as its special cases.

Classical models involve time and frequency domains but however when advanced antenna array techniques are adopted, the spatial characteristics of the scattering environment have an enormous impact on the system performance.^[20] With reference to the spatial domain, some parameters have been defined to take into account the antenna characteristics with respect to the spatial distribution of the different signal components.^[21] The power azimuth spectrum (PAS) ‘ $P_\phi(\varphi)$ ’ and angular spread (AS) ‘ σ_ϕ ’ are used to characterize the spatial properties of the channel in the azimuth domain.^[20] From the PAS a

low-rank channel can be defined as: when $\sigma_\phi \ll \phi^{3db}$ (and the channel is flat-fading) and a high-rank channel when $\sigma_\phi \gg \phi^{3db}$ or channel is frequency-selective. The following diagram gives an idea about this:

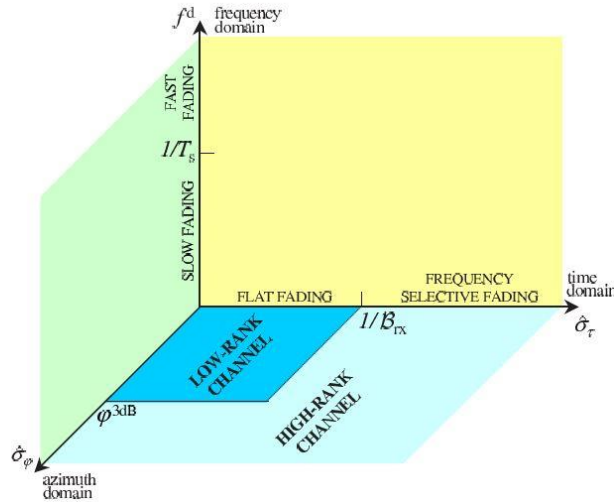


Fig. 5.22 Low-rank and high-rank channel properties

We consider the model based on assumption that the associated MIMO channel the transmit side antenna array has a small AS which occurs quite often in case of cellular downlink. [2] Therefore the need to represent the channel matrix in a reduced-rank form as well as to consider a separate matrix representing AS in the construction of channel matrix is felt. This brings forth the concept of Directional matrix, which is the main contribution of this new model and gives us a compact CSI for correlated flat-faded MIMO systems. Finally the results can be compared for channel estimation with regular MMSE and LSE with this new model and of signal detection using obtained receiver CSI from the corresponding channel estimation schemes for a correlated flat-faded MIMO system.

After the afore-mentioned discussions we next intend to show how different link and link-end properties of MIMO wireless channel (which in fact are properties of the channel itself) can affect the transmit/receive diversity, spatial/time multiplexing etc. properties of the MIMO channel which in turn affects the channel performances i.e. channel capacity, BER performance and also the error performance in channel estimation.

Here we also simulate and analyse also compare the effects of different spatial-only and spatio-temporal correlated MIMO channel examples while calculation and finding through simulations, estimation error-performances of the correlated MIMO channels at the receiver.

5.1 Introduction to Channel Correlation Modelling:

Accurate and tractable channel modelling is critical to realize the full potential of antenna arrays in wireless communication.^[2] A low-rank channel model basically consists of a local scattering model around the transmitting node and is associated with the LOS component.^[20]

The two extremes in the case of modelling:

- 1) Idealistic statistical models, representing rich scattering environments.
- 2) Realistic scattering models, represented with the angles and gains associated with different propagation paths.

Ideal rich-scattering environments de-correlate channels between different pairs of transmit and receive antennas, so that maximum number of orthogonal sub channels are available. In practice however spatial correlation exists and should be considered while designing a MIMO receiver for evaluating corresponding system behaviour.^[2] A low-rank channel can be modelled according to their construction and nature in a number of different approximate ways, few of them to mention here (without giving any descriptions):

- a) Truncated Cosine
- b) Truncated Gaussian
- c) Ring of Scatterers
- d) Disk of Scatterers
- e) Truncated Laplacian etc.

The final representation of correlated flat-faded MIMO channel, according to Yen-Chih Chen et al.^[2] is: $H = Q_A C \overline{Q_B}^T W$, where 'H' is channel matrix, 'W' is mean Directional of arrival (DoA) matrix, 'Q_A' and 'Q_B = $\overline{Q_B}^T W$ ' are unitary matrices, 'C' is the co-efficient matrix.

This kind of analytic models are often used for different purposes e.g. evaluating system capacity/performance, for designing a beam former, for channel equalization at receiver etc.

5.2 Few popular models and brief descriptions about them:

The main drawback of a full correlation matrix is its huge size therefore it's not very practical to use. So we may often resort to some analytic models for simplification of the correlation matrix. We have already mentioned that different models can be used for representing a spatially correlated MIMO channel, few of them can be named as:

- i. Kroneckers model
- ii. Virtual channel representation (VCR or Sayeed's) model
- iii. Weichselberg's model
- iv. Chen and Sus model

Following is a brief description on the models mentioned above:

5.2.1 Kroneckers model:

This model neglects the joint spatial structure of the MIMO channel and describes its spatial characteristics with the help of two separate correlation properties of the link-ends (i.e. assuming separable statistics of the link-ends).

Un-parameterised single-sided correlation matrices of link-ends 'A' and 'B' can be represented as: ' R_A ' and ' R_B ' respectively. According to Kroneckers model the overall correlation matrix ' R ' of the MIMO channel can be represented as: $R = R_A \otimes R_B$ where ' \otimes ' denotes Kronecker product. This also follows that: $R^{\frac{1}{2}} = R_B^{\frac{1}{2}} \otimes R_A^{\frac{1}{2}}$, where the square root matrix ' $R^{\frac{1}{2}}$ ' is related to ' R ' as: $R = R^{\frac{1}{2}}(R^{\frac{1}{2}})^H$. Finally the model is given as:

$$H_{kron} = R_A^{\frac{1}{2}} H_w R_B^{\frac{1}{2}} \quad (5.1)$$

Also sometimes represented as: $H_{kron} = \frac{1}{P_H} R_A^{\frac{1}{2}} G R_B^{\frac{1}{2}}$, where ' H_w ' is complex random zero mean and unit variance i.i.d. matrix (already normalized) and ' G ' is complex random zero mean i.i.d. matrix, of which the elements have to be normalized with total channel power ' P_H '.

5.2.2 Virtual channel representation (VCR or Sayeed's) model:

This representation has the distinct feature over the previously mentioned Kroneckers model that it considers the overall channel correlation as a combination of contributions from both the link-ends, not independently but as a cross-coupled form and that matrix is called the coupling matrix, ' $\tilde{\Omega}_{virt}$ '. The overall channel model is:

$$H_{virt} = A_A (\tilde{\Omega}_{virt} \odot G) A_B^T \quad (5.2)$$

Where ‘ A_A ’ and ‘ A_B ’ are DFT matrices and ‘ G ’ is the complex random i.i.d. matrix. The DFT matrices mentioned above ‘ R_A ’ and ‘ R_B ’ are pre-defined matrices and are independent of the channel being modelling.

5.2.3 Weichselbergers model:

This representation is almost identical to the previously mentioned VCR (or Sayeeds) model i.e. this model also encompasses the fact that analytical channel model should withhold the joint correlation based structure but differs a little bit in the actual construction. This model can be represented as:

$$H = U_R(\tilde{\Omega} \odot H_w)U_T^T \quad (5.3)$$

Where ‘ H_w ’ is the complex random i.i.d. matrix and ‘ $\tilde{\Omega}$ ’ is the ‘coupling matrix’. The overall structure differs from the last representation because here two unitary matrices ‘ U_R ’ and ‘ U_T ’ are used instead of the pre-defined DFT matrices ‘ A_A ’ and ‘ A_B ’ of the last representation. Now it can the two unitary matrices can be derived to be the eigen-bases of parameterized one-sided correlation matrices, for ‘ U_R ’: $R_{R,Q_T} = U_R \Lambda_{R,Q_T} U_R^H$ and for ‘ U_T ’: $R_{T,Q_R} = U_T \Lambda_{T,Q_R} U_T^H$, here ‘ Λ_{R,Q_T} ’ and ‘ Λ_{T,Q_R} ’ are real-valued diagonal matrices with non-negative entries.

Now Weichselbergers model is beneficial to use because it alleviates some restrictions of the two previously mentioned models, namely the Kroneckers model and the VCR (or Sayeeds) model:

- a. Kroneckers model doesn’t consider the joint spatial structure of the channel, which in fact the last two models do.
- b. VCR (or Sayeeds) model depends on two channel-independent DFT matrices for its construction, which might be asymptotically correct for some special cases (e.g. uniform linear arrays) but isn’t optimal for most cases.

5.2.4 Chen and Sus model:

Apart from the models already mentioned yet another model can be discussed, which basically follows the propositions of Kroneckers, Weichselbergers models but is different because of the fact that it assumes that when Multi-element antennas (MEAs) are used at both transmitter and receiver sides as antenna-arrays of the corresponding sides, a small Angular spread (AS) comes into picture. So for the construction of the correlated MIMO

channel, this model incorporates a directional matrix embedding mean DoA along with the co-efficient matrix (a linear transformation of the ‘coupling matrix’ mentioned in the previously described models). Finally the overall model is represented as:

$$H = Q_R C \overline{Q_T}^T W \quad (5.4)$$

The directional matrix, ‘ W ’ is in fact found out as: $Q_T^T = \overline{Q_T}^T W$, so this model can be also represented as : $H = Q_R C Q_T^T$.

As we can see the model bears very close resemblance with Weichselbergers and Kroneckers models. Here the unitary transformations: $U_R = Q_R P_R^H$ and $U_T = Q_T P_T^H$, are used where ‘ P_R ’ and ‘ P_T ’ are unitary matrices. So if we want to compare, the Weichselbergers model can be expressed as: $H = Q_R P_R^H (\tilde{\Omega} \odot G) P_T^* Q_T^T$, using the unitary matrix transformations mentioned above. We know unitary transformations don’t change the distribution of the elements of a random matrix, so the transformed matrix $P_R^H (\tilde{\Omega} \odot G) P_T^*$, which is represented as ‘ C ’ in the Chen and Sus model has the same nature as $(\tilde{\Omega} \odot G)$ of Weichselbergers model, similarly from eigen-decomposition of the matrices

‘ R_A ’ and ‘ R_B ’, we see: $R_A^{\frac{1}{2}} = Q_A \lambda_A$ and $R_B^{\frac{1}{2}} = Q_B \lambda_B$, from Kroneckers model we see:

$$vec(H_{kron}) = vec(Q_A \lambda_A H_w (Q_B \lambda_B)^T) = (Q_B \otimes Q_A) (\lambda_B \otimes \lambda_A) vec(H_w) \quad (5.5)$$

Where if the random matrix $(\lambda_B \otimes \lambda_A) vec(H_w)$ is represented as ‘ C ’ the Chen and Sus model can be obtained, considering ‘ A ’ and ‘ B ’ sides to be receiver and transmitter sides respectively i.e. ‘ Q_A ’ is ‘ Q_R ’ and ‘ Q_B ’ is ‘ Q_T ’.

5.3 Basic description of Spatial and Temporal Correlations:

Few important distinctive points regarding spatial and spatio-temporal correlation scenarios can be given below as:

5.3.1 Spatial correlation in MIMO channel:

Following are some important points regarding spatial correlation:

- a. For spatial-only correlation the Kronecker’s model is given as:

$$H = R_R^{1/2} H_w \left(R_T^{1/2} \right)^T \quad (5.6)$$

Where ' H_w ' is i.i.d. distributed circularly symmetric complex Gaussian with zero mean and unit variance, hence $H \sim CN(0, R_T \otimes R_R)$.

- b. According to this model spatial correlation directly depends upon the eigen value distributions of the correlation matrices ' R_T ' and ' R_R '.
- c. Each eigen vector represents a spatial direction and its corresponding eigen value describes the average signal gain along that direction of multipath propagation of the flat-fading MIMO channel.
- d. Large eigen value spread of ' R_T ' and/or ' R_R ' means some of the spatial directions are statistically stronger than others which corresponds to high spatial correlation.
- e. On the other hand if the eigen value spread is smaller, it means the signal gains are distributed along almost all spatial directions or there is low direction selectivity or low spatial correlation.
- f. Receiver side spatial correlation always degrades performance (channel capacity becomes less as less number of AoA are available for effective diversity combining at the signal receiver).
- g. Whereas in case of transmitter side spatial correlation there is chance of capacity improvement. If the channel is completely informed/uninformed at the transmitter, spatial correlation degrades performance but if only channel is averagely known (i.e. only statistical knowledge of channel ' R_T ' and ' R_R ' is present) at the transmitter correlation means less uncertainty so channel ergodic capacity increases.

5.3.2 Temporal correlation in MIMO channel:

Following are some important points regarding spatio-temporal correlation:

- a. Spatio-temporal correlation model can be expressed as the following Kronecker-like model^[14]:

$$H = R_R^{1/2} H_w \left((R_t \otimes R_T)^{1/2} \right)^T \quad (5.7)$$

- b. A fast fading channel (i.e. which has small coherence time $T_c \propto \frac{1}{f_m}$, where ' f_m ' is the maximum Doppler spread of the channel) may take the advantage of

variations in channel conditions using time diversity for increasing robustness of communication which may lead to temporary deep fade for some channel coefficients, which can be easily mitigated by successful transmission of the rest of the transmit bits at other time instances or by recovering original code by error coding at the receiver.

- c. A slow fading channel on the other hand has no time diversity because the transmitter sees only a single realization of channel (like a static channel) for the delay constraint of the channel (also a deep fade lasts forever so it can't be mitigated). Slow fading can occur because of shadowing (e.g. a hill in between a pair of transmitter and receiver).
- d. Time correlation essentially arises from slow fading nature of the channel.
- e. Time de-correlation utilizes the fast fading property of a channel and creates time-diversity thereby improving BER performance of the MIMO flat-fading channel.

5.4 Effects of Spatial Correlation on Channel Estimation and Signal Detection:

5.4.1 Methodology for this experiment:

At first it has been considered different structures of the coupling matrices according to Weichselbergers model, as given below and tried to find out the channel estimates using regular MMSE technique. For this a training matrix of size: $T \times N$, can be generated where 'T' is the number of transmitters and 'N' is the signal block-length. Next the training signal matrix is passed through correlated flat-faded MIMO channel matrices (corresponding to different structures of coupling matrices taken and using Weichselbergers model) and one un-correlated flat-faded MIMO channel case, all of size $R \times N$ and join Additive white Gaussian noise (AWGN) matrix with it. Finally we may compare the results of MMSE channel estimation Mean squared errors (MSEs) and also Bit error rates (BERs) of signal detection using the estimated channel matrices with respect to received signal Signal-to-noise ratios (SNRs) in dB scale.

Different coupling matrices considered here for performance comparisons are given along left-side and their corresponding radio environments on the right-side (black squares denote significant power, black circles are eigen-modes and grey squares are scatterers):

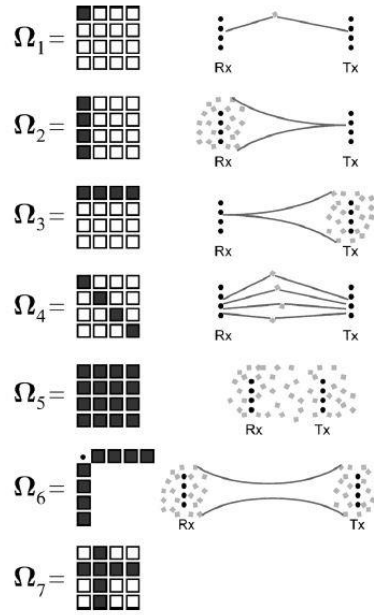


Fig. 5.23 Different Correlation structure

Finally we may assume the Chen and Sus model for correlated MIMO channel construction (Q_R and $\overline{Q_T}$ are pre-defined matrices and W has a diagonal structure with mean DoA information embedded) and use Least square estimator (LSE) for estimation of channel co-efficient matrix C i.e. \hat{C}_{LS} as well as directional matrix W i.e. \hat{W}_{LS} and therefore find the overall channel estimate as: $\hat{H} = Q_R \hat{C}_{LS} \overline{Q_T}^T \hat{W}_{LS}$, it is assumed that Q_R and $\overline{Q_T}$ pre-defined matrices are known. Here it is considered that mean AS is very small (very close to 'zero') and C is i.i.d. matrix, so according to the assumptions made channel matrix, H is full-rank. So next we simulate the MSEs with respect to SNRs in dB for \hat{H} using this technique (for correlated channel) and for un-correlated case using LSE and compare the results.

5.4.2 Results of this experiment:

The MSE performance comparison plot for channel estimation is given below (legend is arranged in order 'no correlation' represents un-correlated, ' Ω_1 ' to ' Ω_7 ')

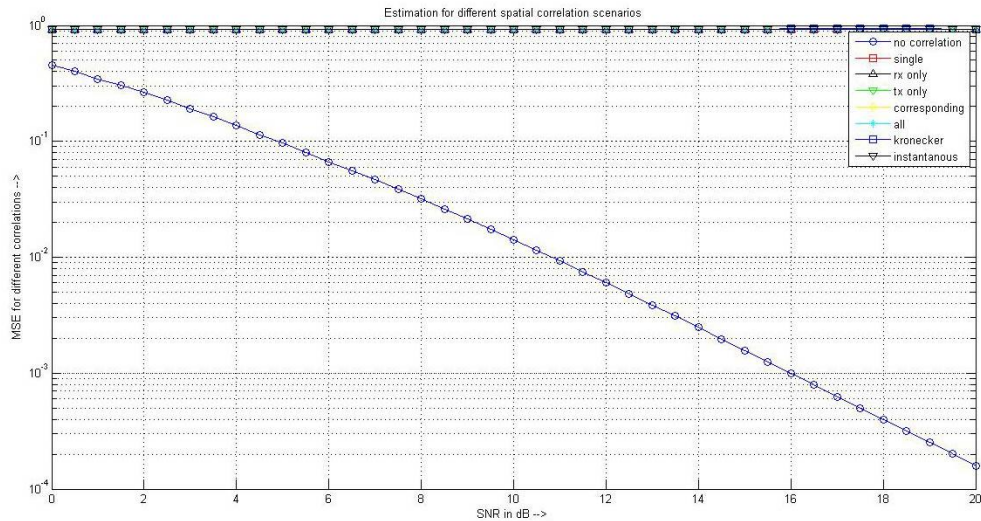


Fig. 5.24 Plot: Estimation for different Spatial Correlation scenarios

The BER performance comparison plot for signal detection is given below (legend is arranged in order ‘no correlation’ represents un-correlated, ‘ Ω_1 ’ to ‘ Ω_7 ’):

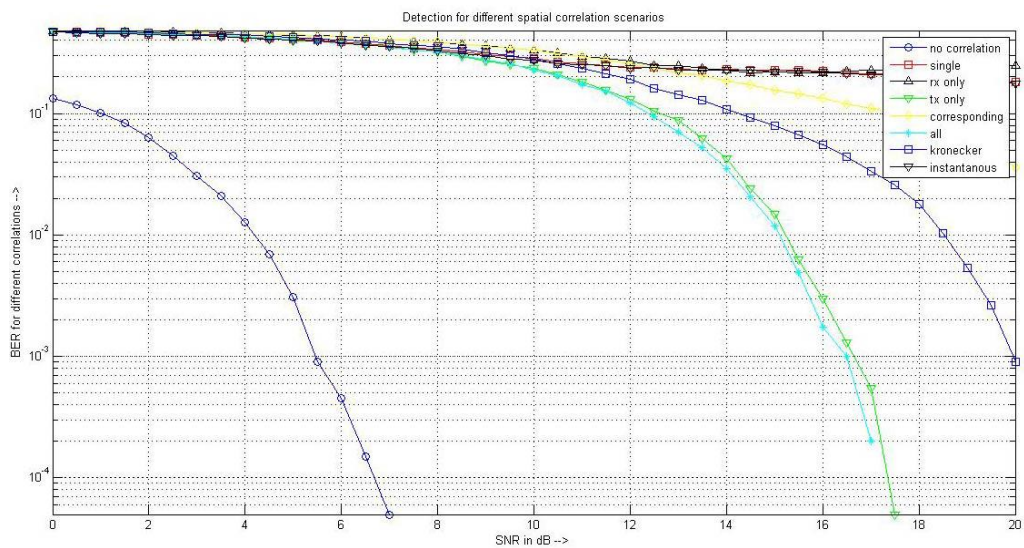


Fig. 5.25 Plot: Detection for different Spatial Correlation scenarios

Finally the discussion on the performance (BER for signal detection and MSE for channel estimation) comparison plots of correlated vs. un-correlated flat-faded MIMO channels is given below.

5.4.3 Discussions and Conclusion:

From the first plot we see that channel estimation using conventional MMSE technique (or other conventional techniques) give un-acceptable results for correlated, while we see that it gives perfectly good error-performance (as expected) for the un-correlated case.

From the second plot we see that signal detection using estimated channel information for conventional MMSE estimation (or other conventional techniques) give very poor BER performance for the correlated cases, while it gives perfectly well error-performance (as expected) for the un-correlated case.

It is also verified in the next plots that for correlated MIMO channel, the MSE performance for LS-like estimator derived from Chen and Sus model ^[2] actually converges but for much higher SNR in dB values i.e. for extracting similar MSE performances from LS like estimators for the case of correlated MIMO flat-fading channels the signals need to be transmitted with more average power. Signal detection can be done more faithfully for this case (higher received signal SNR) with better results but in the expense of more computational complexity and high average power requirements, no results have been shown for the signal detection using the given model ^[2] as our main motto in this work is to faithfully estimate the channel rather than focus on better signal detection in case of correlated flat-fading MIMO wireless communication channel.

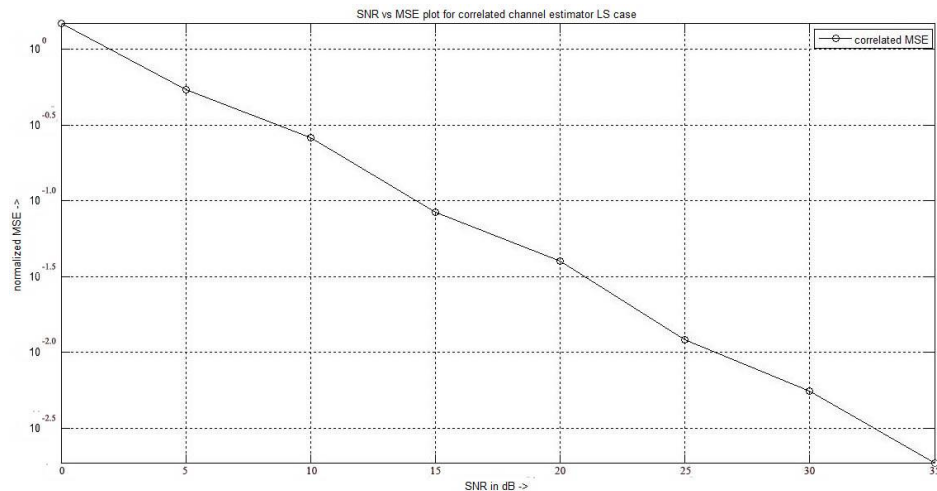


Fig. 5.26 Plot: SNR vs. MSE for LS estimator Correlated

As we can see from the above plot that MSE performance converges slowly in case of correlated MIMO, in the following figure the cases of uncorrelated Rayleigh faded MIMO channel and correlated MIMO channel both are given in the same plot for easy comparison purpose.

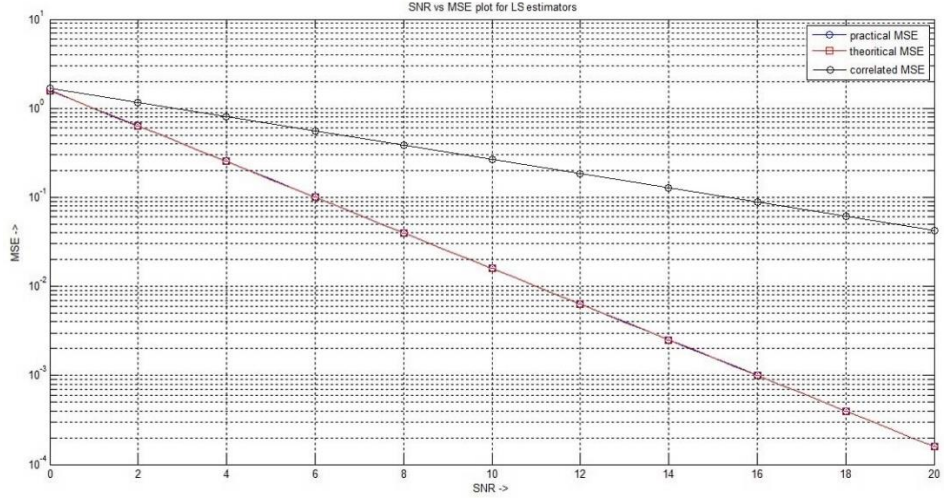


Fig. 5.27 Plot: SNR vs. MSE for LS estimators Correlated and Un-correlated

So it is seen that because of spatial correlation channel estimation as well as signal detection gives inferior performances.

5.5 Channel Estimation using Chen and Sus model:

5.5.1 Methodology for this experiment:

For considering the time-variant behaviour of a wireless MIMO channel of the system: $Y = HX + N$ and the structure of channel matrix $H = Q_R C \overline{Q_T}^T W$ is used.

Within a fixed observation windows of ‘ L ’ time units (blocks), if the snapshot of the channel is represented as X_n for n^{th} time instant or at ‘ nT_s ’: $\text{vec}(Y_{n,L}) = (X_{n,L}^T \otimes I_n) \cdot \text{vec}(H_{n,L}) + \text{vec}(N_{n,L})$. Orthogonal training matrix for any time instant can be used and the overall structure of the training matrix (in time and space) is given in Zhang et. al.^[7] two leading pilot symbol vectors can be assumed to be ‘ T ’ symbol intervals away. For an observation window of ‘ L ’ blocks: $Y_{n,L} \triangleq [Y_n, Y_{n+1}, \dots, Y_{n+L-1}]$ and $\text{vec}(Y_{n,L}) \triangleq (X_{n,L}^T \otimes I_N) \cdot \text{vec}(H_{n,L}) + \text{vec}(N_{n,L})$ where $\text{vec}(H_{n,L}) \triangleq [\text{vec}(H_n)^T, \dots, \text{vec}(H_{n+L-1})^T]^T$ and

$$\text{vec}(N_{n,L}) \triangleq [\text{vec}(N_n)^T, \dots, \text{vec}(N_{n+L-1})^T]^T \text{ and } X_{n,L}^T \stackrel{\text{def}}{=} \begin{bmatrix} X_n^T & 0 & \dots & 0 \\ 0 & X_{n+1}^T & \vdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & X_{n+L-1}^T \end{bmatrix}.$$

Now assuming eigenbases ‘ Q_T ’ and ‘ Q_R ’ remain invariant during estimation, it can be obtained as: $\Gamma_{n,L} = [v_n^T, v_{n+1}^T, \dots, v_{n+L-1}^T]^T$ where $v_n = [v_{1n}, v_{2n}, \dots, v_{(NM)n}]^T$, the stacked vector: $v(p) = [v_{pn}, v_{p(n+1)}, \dots, v_{p(n+L-1)}]^T$ represents finite duration sample of complex

random process associated with p^{th} sub channel. Now the signal matrix can be decoupled as a product in space and time domains:

$$\begin{aligned} \text{vec}(Y_{n,L}) &\approx (X_{n,L}^T \otimes I_N)[Q_{L,K_L} \otimes (W^T \bar{Q}) \otimes Q_R]c_{coef} \\ &\approx (X_{n,L}^T \otimes I_N)[Q_{L,K_L} \otimes (W^T Q_{T,K_T}) \otimes Q_{R,K_R}] \tilde{c}_{coef} \\ &\stackrel{\text{def}}{=} (((W_L X_{n,L})^T \tilde{Q}_{T,K_T}) \otimes Q_{R,K_R}) \tilde{c}_{coef} \end{aligned} \quad (5.8)$$

Where $W_L \triangleq (I_L \otimes W)$, $\tilde{Q}_{T,K_T} \triangleq Q_{L,K_L} \otimes Q_{T,K_T}$ and the steering matrix ‘ W ’ is assumed to be fixed for the period of ‘ L ’ data blocks. Now for coefficient estimation the following formulae may be used:

$$\hat{c}_{coef} = (\tilde{Z}^H \tilde{Z})^{-1} \tilde{Z}^H \text{vec}(Y_{n,L}) \stackrel{\text{def}}{=} \tilde{F}(W_{L,opt}) \quad (5.9)$$

$$\tilde{Z} \stackrel{\text{def}}{=} ((W_{L,opt} X_{n,L})^T \tilde{Q}_{T,K_T}) \otimes Q_{R,K_R} \quad (5.10)$$

For direction estimation, the formulae:

$$\text{vec}(Y_{n,L}) = \text{vec}(\tilde{G} \cdot \text{diag}(1_L \otimes w) \cdot X_{n,L}) + \tilde{N}_{n,L} \quad (5.11)$$

$$\text{vec}(\tilde{G} \cdot \text{diag}(1_L \otimes w) \cdot X_{n,L}) = ((1_{LE} \otimes \bar{G}) \odot (X_{n,L}^T \otimes 1_N))(1_L \otimes I_M)w \stackrel{\text{def}}{=} \tilde{T}w \quad (5.12)$$

Now the LS estimate can be given as: $\hat{w}_{LS} = \tilde{T}^\dagger \cdot \text{vec}(Y_{n,L})$ finally the steering vector can be found out as: $v(\theta) \stackrel{\text{def}}{=} [1, v(\theta), \dots, v^{M-1}]$ where $v(\theta) = \exp(-j2\pi \frac{d}{\lambda} \sin(\theta))$. Here in this case the AoD information is retrieved from: $\hat{\phi} = \arg \max_{-\pi \leq \theta \leq \pi} \text{Re} \{ \mathbb{P}(W_{LS})^H v(\theta) \}$

where $\mathbb{P}(\cdot)$ is the phase-extractor i.e. $\mathbb{P}(\tilde{T}^\dagger \cdot \text{vec}(Y_{n,L})) = \mathbb{P}(\overbrace{\tilde{T}^H \cdot \text{vec}(Y_{n,L})}^{\stackrel{\text{def}}{=} \tilde{w}})$. Finally it can be computed: $\hat{W}_L = I_L \otimes V(\hat{\theta})$ where $V(\hat{\theta}) = \text{diag}(v(\hat{\phi}))$ and the MSE performance can be given as: $\epsilon \stackrel{\text{def}}{=} E\{\|H - \hat{H}\|_F^2\} = E\{\|\text{vec}(H) - \text{vec}(\hat{H})\|_2^2\}$. Henceforth using these formulae an LS estimator ‘ $Q_{R,K_R} \hat{C}_{coef,opt} \tilde{Q}_{T,K_T}^T \hat{W}$ ’ is designed assuming perfect estimate of ‘ W ’.^[2]

5.5.2 Results of this experiment:

The results have been found out with the specifications:

1. 8×8 MIMO system is considered with $L = 12$.

2. Antenna spacing at both transmitter and receiver are 0.5λ .
3. Orthogonal training matrix is used for simulations.
4. AS of 15° is taken for antenna radiation elements.
5. Slow fading case with $f_d T_s = 0.3$ is considered here.
6. Spatial correlated transmitter and receiver uncorrelated assumed (urban scenario).
7. Only AoD for transmitter is estimated for directional nature of transmitted signal.

For the following MIMO wireless communication system: $Y = HX + N$. For spatial-only MIMO system we consider the following structure of flat-faded channel matrix: $H = Q_R C \bar{Q}_T^T W$ and for spatio-temporally correlated MIMO system we consider the following structure: $\tilde{H} = Q_R C (Q_t \otimes W^T \bar{Q}_T)^T$ for these experiments. Here ‘ W ’ is the directional matrix, ‘ Q_T ’ and ‘ Q_R ’ are spatial eigen-basis matrices with ‘ $Q_T = \bar{Q}_T^T W$ ’, ‘ Q_t ’ represents time-correlation and ‘ C ’ is a complex Gaussian random matrix with independent entries.

Following are the results of the experiments:

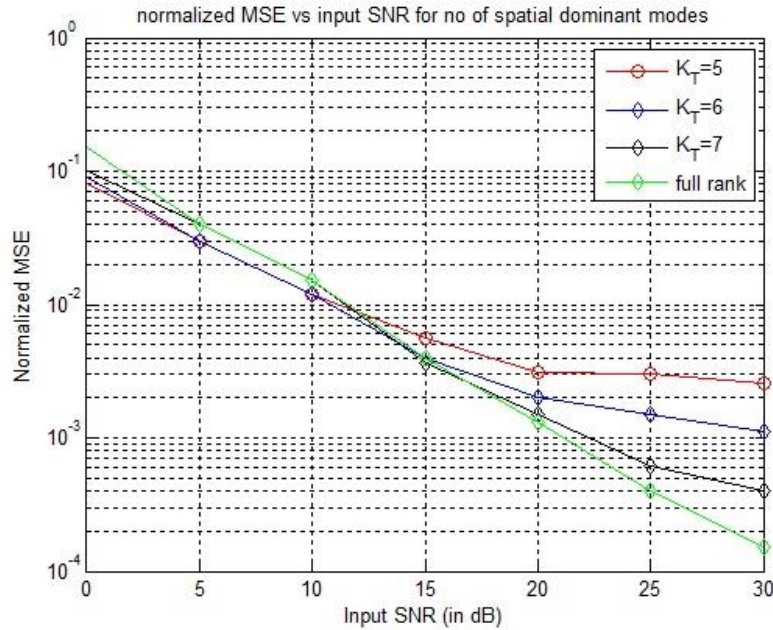


Fig. 5.28 Plot: MSE vs. SNR for a no. of dominant spatial modes

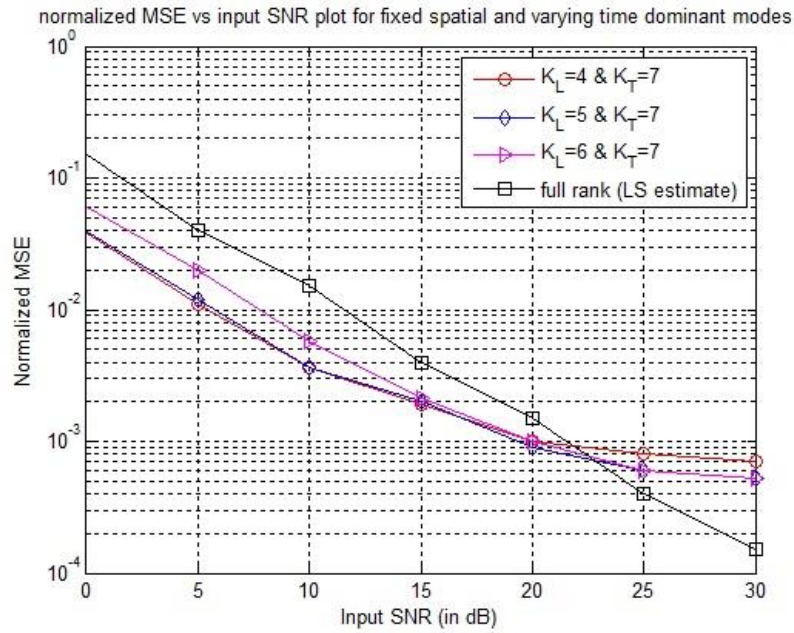


Fig. 5.29 Plot: MSE vs. SNR for fixed Spatial and variable dominant temporal modes

5.5.3 Discussions and Conclusion:

From the simulation results we see that for spatial-only correlated cases, in general when dominant rank of transmitter side ' K_T ' increases (i.e. spatial correlation decreases) the error performance eventually tends more towards conventional LS estimation performance.

For same spatial correlation if degree of freedom along time domain ' K_L ' is decreased (i.e. more temporal correlation) the performance degradation is more.

With exact direction of departure at transmitter known and assuming uncorrelated receiver a spatially and temporally correlated channel can be estimated with the technique mentioned here with fairly high accuracy (lower MSE in channel estimation) for even higher order MIMO wireless communication systems (Kronecker model is suitable for lower order systems and with more limitations in the channel's scattering environment).

One of the main contributions of the Chen and Sus model^[2] is formulating the directional matrix and estimating it simultaneously alongwith the Gaussian random matrix with independent entries. The following point highlights some points regarding the importance of the directional matrix and introduces the primary idea behind importance of DoA/DoD estimation.

5.5.4 Importance of estimation of directional matrix:

The DoA/DoD information is required because low correlation is achieved when each antenna provides a unique weighting to each individual multipath component based on the directional information in case of (AoA/AoD based) beam forming.^[9] When the mean AoA/AoD (ϕ_0) information is known from the estimation, alongwith the PAS (i.e. from the distribution one may obtain AS σ_ϕ as well) full exploitation of angular/spatial diversity is possible for a given transmitter/receiver antenna array, also we know as a rule of thumb that more diversity means more ergodic capacity. Although for a channel with known directional properties i.e. known correlation and therefore fixed AoA/AoD no more capacity improvement is possible through directional data (already known so it hardly matters for ergodic capacity improvement at the receiver). For the Chen and Sus model for the single-directional model assumed, AoD information at the transmitter side can be captured by the mean AoD information and from the principle of maximum entropy AoA at the receiver is uniformly distributed in $[0, 2\pi]$ so no mean AoA information is required at the receiver mobile station (MS) in case of down-link channel.^[2] In this case the mean AoD (in case of transmitter) information is embedded in the directional matrix ' W ' which is to be estimated from the correlated/uncorrelated random matrix ' C ' during the previous iteration of the channel estimation.

5.6 Effect of different channel properties on correlated channel:

Spatial and temporal correlations depend on the actual channel characteristics (scenario) inclusive of the link-ends properties e.g. inter-element distance between MEA elements, angular spread, mutual coupling between antennas (co and/or cross coupling), mean AoA/AoDs as well for mostly directional cases (but its effect is much less pronounced than the other mentioned properties) and also upon the exact scattering environment etc. The effects are explained as follows:

5.6.1 Effect of multi-scattering environment:

Rich scattering (i.e. rich multipath NLOS propagation) decreases selectivity of AoA/AoD for transmit/receive signal because it spreads the signal after reflection or scattering and for signal through multipath components, gain is equally spread among more and more spatial

direction due to this dispersive effect, so spatial correlation is decreased. Although reduction of spatial correlation should improve the ergodic capacity of the channel but because of decrease in received SNR from numerous reflections, capacity in turn actually decreases after a certain threshold.^{[9][10]}

5.6.2 Effect of terminal velocity of channel or scatterer's relative velocity:

A Doppler shift because of terminal velocity or a relative velocity among channel terminal(s) and any scatter(s) may contribute towards channel time-variance. Depending on rate of change of phase along different multipath the channel may have several Doppler shifts, a large Doppler spread corresponds to the phases of the received signal components changing independently over time. The maximum Doppler spread is usually considered for calculation of channel coherence time, $T_c = \sqrt{\frac{9}{16\pi f_m^2}}$. Small coherence time corresponds to less temporal correlation when channel multipath average delay ' τ_{av} ' is given.

5.6.3 Effect of close antenna spacing:

Pattern distortion which occurs from close antenna spacing creates angle diversity that can lead to reduced signal correlation but the effect becomes less pronounced because of unavoidable mutual coupling effects which are discussed next. A larger antenna spacing is needed to give better performance in case of smaller AS (angular spread). But regardless of the AS once the spacing between antennas increases beyond 2λ the BER performance of the channel starts approaching its minimum achievable BER, without further improvement. Thus lack of high AS (i.e. for more angular diversity and lower correlation) can be compensated by increasing the antenna spacing (within limit of performance improvement).^[10]

Coherence distance is a concept which gives a thumb rule regarding antenna spacing for statistical independence. For Rayleigh fading case it is given as: $D_c = \frac{9\lambda}{16\pi}$ although in general it is related to the RMS AS as: $D_c = \frac{0.2\lambda}{\theta_{rms}}$.^[13]

5.6.4 Effect of angular spread:

The angular spread or RMS angular spread can be defined as the RMS value of azimuth ' ϕ ' having certain distribution which can be calculated based on transmit/receive power profile i.e. PAS (power angular spectrum). Large σ_ϕ (AS) implies channel energy is focussed along

many spatial directions, so it is less directional, whereas small σ_ϕ corresponds to more spatial selectivity (more spatial correlation).

5.6.5 Effect of mutual coupling between antennas:

When the antenna elements come close enough to one another because of more and more packing of such elements in a MEA invariably mutual coupling plays an important role to determine the channel correlation. Capacity improvement arising from pattern distortion caused by close antenna spacing is limited by detrimental effects of mutual coupling between MEA elements. For a fixed-length array strong coupling arising from more and more packing of antenna elements ultimately leads to upper bound on maximum capacity obtainable for a system.^{[9][10]}

5.6.6 Results of varying separation or AS for a simulated MC scenario:

Following are the results of simulation of how different link and link-end properties affect the performances of spatially correlated MIMO communication channel:

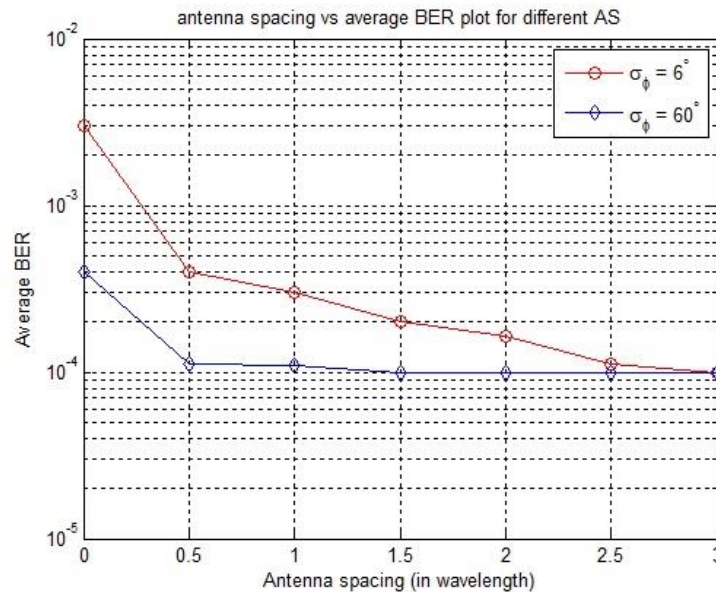


Fig. 5.30 Plot: Element separation vs. BER for fixed AS in correlated scenario

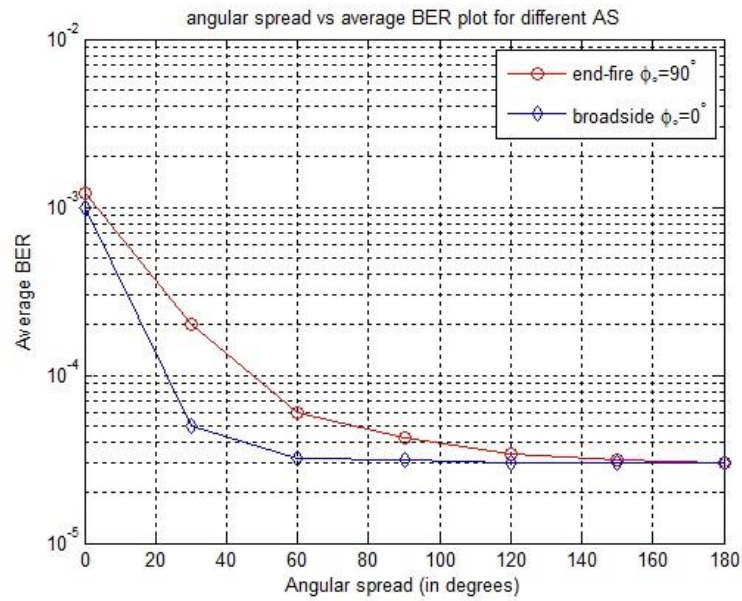


Fig. 5.31 Plot: AS vs. BER for fixed element separations in correlated scenario

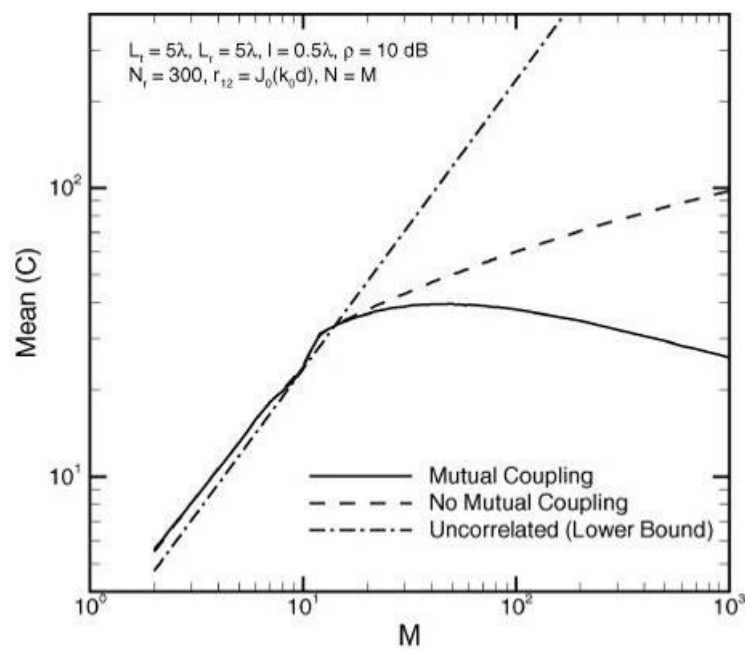


Fig. 5.32 Plot: Mean capacity vs. number of antenna elements for coupled and un-coupled

5.6.7 Discussions on the simulation results:

From the simulation result given in Fig. 4.9 we see that BER performance improves with antenna spacing with the effect being more prominent for smaller angular spread, regardless of this fact when the spacing becomes $> \sim 2\lambda - 2.5\lambda$ there is no more performance improvement i.e. the BER performance saturates.

From the simulation result given in Fig. 4.10 we see that BER performance improves with AS as well with the effect being more prominent for broadside radiating elements rather than end-fire type radiating elements.

From the simulation results given in Fig. 4.11 we see that mean capacity vs. number of transmitter/receiver antenna elements of MIMO wireless communication channel for correlated and different un-correlated scenarios, MC case corresponds to additional diversity (i.e. less correlation) hence performance improvement is observed for more closed packing of antenna elements.^[32]

5.7 Detailed description of the effects of Mutual Coupling:

Mutual Coupling arises from the fact that two antennas radiating EM waves in the vicinity of one another affects each other's radiation pattern, therefore otherwise independently radiating antennas (when both are individually radiating in free-space and isolated with respect to each other) can't radiate independently anymore and affect each other's radiation characteristic and performances. This can also be explained as following: when an antenna is placed in an isolated space, it radiates without any loss whatsoever (it's radiation impedance is assumed to be perfectly matched with that of free-space such that maximum power is transferred) but when another non-radiating conductor (e.g. another antenna with finite load) is placed in its vicinity then some of the radiated power from the first antenna is actually absorbed by the second antenna, now if both the antennas are radiating then their radiation characteristics change because of each other (e.g. their driving point as well as mutual impedances change), these effects are embedded in the mutual coupling information between antennas.

When two antennas radiate in the vicinity of each other, their driving point impedances depend on mutual impedance between each other.

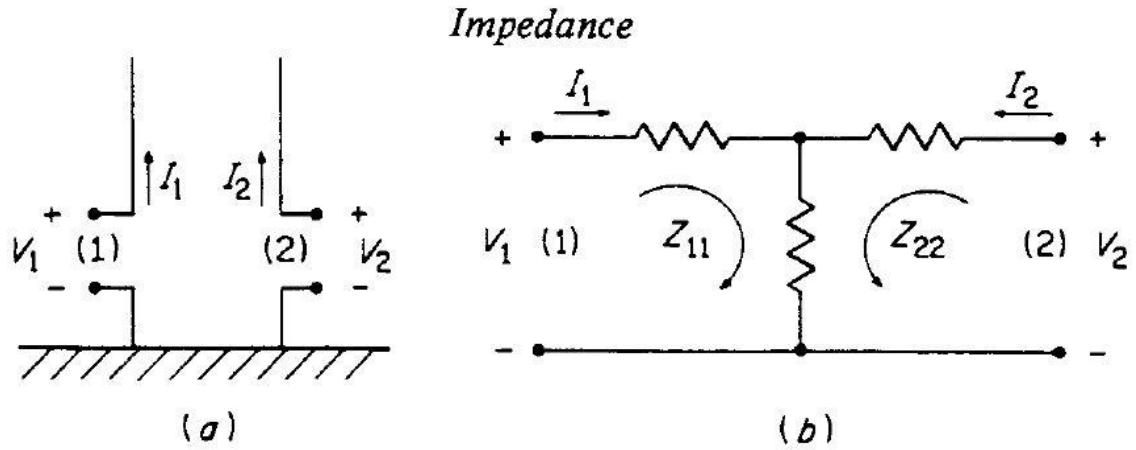


Fig. 5.33 Self and Mutual impedance calculations

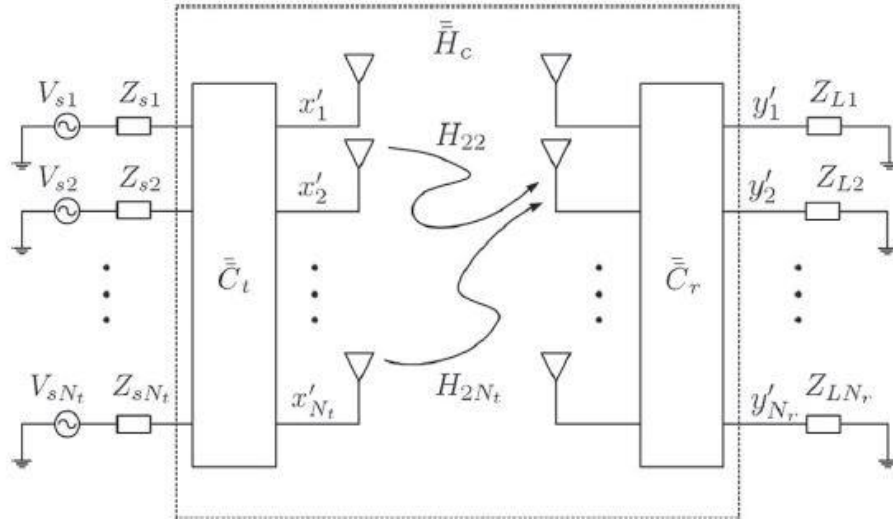


Fig. 5.13 $N_t \times N_r$ MIMO channel with MC between antennas

Considering mutual coupling in a $N_t \times N_r$ MIMO channel the transmitted signal can be expressed as: $\bar{x}' = \bar{C}_t \bar{x}$. Now the coupling matrix of transmitter array ' \bar{C}_t ' is expressed as:

$$\bar{C}_t = (\bar{Z}_s + \bar{Z}_{ant})(\bar{Z}_t + \bar{Z}_s)^{-1} \tag{5.13}$$

' \bar{Z}_t ', ' \bar{Z}_s ', ' \bar{Z}_{ant} ' are respectively: mutual impedance matrix, antenna source impedance matrix and antenna input impedance matrix. Similarly received signal matrix under mutual coupling effect can be expressed as:

$$\bar{y}' = \bar{C}_r \bar{H} \bar{x}' + \bar{n} = \bar{C}_r \bar{H} \bar{C}_t \bar{x} + \bar{n} \tag{5.14}$$

' \bar{H}_c ' is the transformed matrix considering MC effects and ' \bar{C}_r ' is the receiver coupling matrix.^[16]

From Fig. 4.9 we see the driving point impedances are: ' Z_{11} ' and ' Z_{22} ' and mutual impedances are: $Z_{12} = \left. \frac{V_1}{I_2} \right|_{I_1=0}$ i.e. for terminal one with respect to terminal two when terminal one is open circuited, similarly $Z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0}$ i.e. for terminal two with respect to terminal one when terminal two is open circuited.^[24] A few points on self and mutual impedances and also MC effect of elements of MEA systems (a detailed description of MEA systems, specially its radiation characteristics is given in CHAPTER 3A):

- i. In most cases Z_{11} is approximately equal to the self-impedance Z_{s1} , it is the input impedance with all other antennas entirely removed from the array. But when any other antenna say antenna two is radiating very close to the previous antenna say antenna one, e.g. within antenna ones resonant length, the situation is hugely different, in such case:

$$V_1 = I_1 Z_{11} + I_2 Z_{12} \quad (5.15)$$

$$V_2 = I_1 Z_{21} + I_2 Z_{22} \quad (5.16)$$

Or

$$\frac{V_1}{I_1} = Z_{11} + \frac{1}{r} Z_{12} \quad (5.17)$$

$$\frac{V_2}{I_2} = r Z_{21} + Z_{22} \quad (5.18)$$

Here $\frac{I_1}{I_2} = r$ is the complex current ratio.^[25]

- ii. Usually mutual impedance referred to loop currents is used.
- iii. Impedance presented by an antenna to a transmission line (Tx line) can be represented as a two-terminal network, in that case antenna has to be replaced with its equivalent impedance ' Z ' connected to the terminals of a transmission line.^[25]
- iv. The impedance into which a Tx line operates is called the terminal or driving point impedance.
- v. For isolated i.e. remote from ground or other conducting objects, for lossless case the terminal impedance is same as self-impedance of the antenna, self-resistance gives the radiation resistance of an antenna.^[25]

- vi. If an antenna is not isolated the assumed two-terminal network now has self as well as mutual impedances, from reciprocity (i.e. if the network is assumed to be reciprocal) then they are same for both transmitter and receiver sides.^[25]
- vii. Mutual coupling depends on:
 - a. Radiation characteristics of the antenna system
 - b. Relative separation between antenna elements
 - c. Relative orientation between antenna elements
- viii. Total contribution to the far-field radiation pattern of a particular element in the array depends on the total parasitic excitations for the element as well, which again depends on coupling from and the excitations of other nearby generators.
- ix. Different receiving mutual impedance methods are also employed apart from the conventional transmit mutual impedance explained above e.g. the Full-wave method or the Calibration method etc. for calculation of mutual impedance, especially for receiver side MC calculation and compensation.^{[26][27]}
- x. Referring to mutual impedance finding, in general current distributions need to be known and it varies with respect to direction as well (both azimuth and elevation) e.g. for receiver it varies with the direction of incoming signals.
- xi. But in case of Omni-directional antennas e.g. dipole or monopole antennas, the current distributions remain the same irrespective of the azimuth or elevation angles i.e. with respect to the AoA. So e.g. if it is assumed that the signals arrive horizontally i.e. only at $\theta = 90^\circ$, only a single elevation angle (only azimuth angle of the signals is varying) simplifies the problem of AoA estimation (i.e. estimation of mean ' ϕ 's only).^[26]

5.7.1 Effect of MC on correlation coefficients:

The transmitter and receiver side correlation matrices are given as:

$$\bar{\bar{R}}_t = \begin{pmatrix} 1 & \rho_{t,12} & \cdots & \rho_{t,1N_t} \\ \rho_{t,21} & 1 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \rho_{t,(N_t-1)N_t} \\ \rho_{t,N_t1} & \rho_{t,N_t2} & \cdots & 1 \end{pmatrix}_{N_t \times N_t} \quad (5.19)$$

$$\bar{\bar{R}}_r = \begin{pmatrix} 1 & \rho_{r,12} & \cdots & \rho_{r,1N_r} \\ \rho_{r,21} & 1 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \rho_{r,(N_r-1)N_r} \\ \rho_{r,N_r1} & \rho_{r,N_r2} & \cdots & 1 \end{pmatrix}_{N_r \times N_r} \quad (5.20)$$

Where $\rho_{t,mn}$ and $\rho_{r,mn}$ are respectively transmitter and receiver side correlation coefficients.

The covariance terms can be calculated as:

$$\begin{aligned} \rho_{m,n} = \frac{1}{\sigma_m \sigma_n} \int_0^\pi d\theta \int_{-\pi}^\pi d\phi \sin \theta & [\chi P_\theta(\theta, \phi) E_{m\theta}(\theta, \phi) E_{n\theta}(\theta, \phi) \\ & + P_\phi(\theta, \phi) E_{m\phi}(\theta, \phi) E_{n\phi}^*(\theta, \phi)] \end{aligned} \quad (5.21)$$

Here ' $E_{l\theta}(\theta, \phi)$ ' and ' $E_{l\phi}(\theta, \phi)$ ' are far-field radiation components and ' $P_\theta(\theta, \phi)$ ' and ' $P_\phi(\theta, \phi)$ ' are power angular spectra, $\chi = P_v/P_h$ is the cross-polarization ratio where:

$$P_v = \int_{-\pi}^\pi d\phi \int_0^\pi d\theta \sin \theta |E_{\theta i}(\theta, \phi)|^2 \quad (5.22)$$

$$P_h = \int_{-\pi}^\pi d\phi \int_0^\pi d\theta \sin \theta |E_{i\phi}(\theta, \phi)|^2 \quad (5.23)$$

In typical wireless environment incident waves can be attributed to several clusters over the angular domain, each of them commonly modelled by a uniform, Gaussian or truncated Laplacian PAS. [16]

The effect of mutual coupling can be captured with the coupling matrices ' \bar{C}_t ' and ' \bar{C}_r '. By applying reciprocity the received and the open-circuited voltages at the input ports of the receiver antennas are related as:

$$\begin{aligned} \hat{V}_r(\theta_i, \phi_i) &= \bar{Z}^{-1} [\bar{A} \hat{V}_{oc}(\theta_i, \phi_i) - \bar{G} E(\theta_i, \phi_i)] \\ &= \bar{B} \hat{V}_{oc}(\theta_i, \phi_i) + \bar{D}(\theta_i, \phi_i) \end{aligned} \quad (5.24)$$

Hence the following equations can be obtained:

$$Z_{mn} = \begin{cases} \frac{V_m}{Z_{Lm} I_{t,mm}(0)}, & m = n \\ 0, & m \neq n \end{cases} \quad (5.25)$$

$$G_{mn}(\theta_i) = \begin{cases} \frac{1}{I_{t,mm}(0)} \int_{-Lm/2}^{Lm/2} I_{t,mc}(t') e^{jkz' \cos \theta_i} dz', & m = n \\ \frac{1}{I_{t,mm}(0)} \int_{-Ln/2}^{Ln/2} I_{t,nm}(t') e^{jkz' \cos \theta_i} dz', & m \neq n \end{cases} \quad (5.26)$$

$$A_{mn} = \begin{cases} \frac{I_t(0)}{I_{t,mm}(0)}, & m = n \\ 0, & m \neq n \end{cases} \quad (5.27)$$

$$E_n(\theta_i, \phi_i) = E_0 e^{jk(x'_n \sin \theta_i \cos \phi_i + y'_n \sin \theta_i \sin \phi_i)} \quad (5.28)$$

$$V_{0\ cm}(\theta_i, \phi_i) = -\frac{1}{I_t(0)} E_0 e^{jk(x'_m \sin \theta_i \cos \phi_i + y'_m \sin \theta_i \sin \phi_i)} \times \int_{-Lm/2}^{Lm/2} dz' I_t(z') e^{jkz' \cos \theta_i} \quad (5.29)$$

Finally the modified correlation coefficient expression is obtained as:

$$\rho'_{mn} = \frac{1}{\sigma_m \sigma_n} \int_{-\pi}^{\pi} d\phi p(\phi) V_{rm}(\phi) V_{rn}^*(\phi) \quad (5.30)$$

With direction-dependent coupling effect considerations:

$$V_{rl}(\phi) = \sum_{p=1}^{N_r} b_{lp} V_{0\ cp}(\phi) + d_l(\phi) \quad (5.31)$$

Finally the received signal structure is given^[16] as:

$$\bar{y}' = \bar{C}_r \bar{R}'_r{}^{1/2} H_{iid} \bar{R}'_t{}^{1/2} \bar{C}_t \bar{x} + \bar{n} = \bar{H}'_c \bar{x} + \bar{n} \quad (5.32)$$

A similar construction method for MC matrix is given^[10] in other texts as well and is considered most accurate in modelling the modified spatial correlation matrix. As we can see this structure is very much complex and computationally costly, a much simpler model which encompasses the effects of MC on spatial correlation matrix is given^[18, 19] as:

$$R_{Rmu} = C_R R_R^{1/2} \quad (5.33)$$

$$R_{Tmu} = R_T^{1/2} C_T \quad (5.34)$$

5.7.3 Mutual coupling estimation and compensation:

Different MC estimation and compensation techniques are mentioned in various texts^[25-29] [34, 35]. Among them^[26] explains different techniques for MC compensation such as: Conventional-Mutual-Impedance Method (CMIM), Receiving-Mutual-Impedance Method (RMIM), Methods of Moments (MoM) based full-wave method and Calibration methods. The text also discusses the comparison between them. In^[34] a detailed description is given on the different MC compensation methods already in use and suggested a characteristic basis function (CBF) based method where CBF is modelled as the current distribution in antenna elements and electro-motive force (EMF) based calculation for finding the self and mutual impedances is given. Later in this text a detailed performance analysis with respect to actual (calculated using MoM) current distributions and DoA estimation is done.

In ^[28] a new subspace fitting based MC compensation technique is given. In this text the effects of unknown MC is compensated for by estimating the coupling matrix simultaneously with the DoA using a modified Noise sub space fitting (NSF). DoAs are obtained by numerical search with the NSF criterion. MC here is calculated by electromagnetic (EM) based methods. The usual model of MC is assumed:

$$C = (Z_A + Z_T)(Z + Z_T I)^{-1} \quad (5.35)$$

Here ‘ Z_A ’ is the antenna impedance, ‘ Z ’ is the mutual impedance matrix, ‘ Z_T ’ is the impedance for measuring equipment i.e. load connected to the antenna, proper loading condition requires ‘ Z_T ’ be the complex conjugate of ‘ Z_A ’. This model is directly valid for small dipole antennas. If the steering matrix of ULA be:

$$[A(\phi)]_{mn} = e^{-jkd(m-1)\cos\phi_n} \quad (5.36)$$

Where ‘ k ’ is wave-number, ‘ d ’ is the separation between antenna elements.

The signal model after transmission from the antenna array is:

$$x(t) = CA(\phi)s(t) + e(t) \quad (5.37)$$

Where ‘ $s(t)$ ’ is the signal transmitted, ‘ $e(t)$ ’ is added noise, ‘ C ’ is the coupling matrix and ‘ $A(\phi)$ ’ is the array steering vector.

The sub space method relies on observing eigen-decomposition of the covariance matrix ‘ R ’ (after multiplying with steering vector and noise addition) as:

$$R = E\Lambda E^H = E_s\Lambda E_s^H + \sigma^2 E_n E_n^H \quad (5.38)$$

Apart from the methods mentioned in ^[26] the text ^[35] discusses Open circuit voltage method (conventional method as described in ^[26]), S-parameter method, different element-pattern methods e.g. Isolated Element Pattern Method, Coupled Element Pattern Method etc. other methods as well. Amongst them the simplest method is obviously the Open circuit voltage method. This method depends on ‘ Z ’ parameter based MC matrix representation, already given in Equation (4.35). The open circuit voltages ‘ V_{ocj} ’ can be represented through impedance matrix:

$$\begin{pmatrix} V_{oc1} \\ V_{oc2} \\ \vdots \\ V_{ocN} \end{pmatrix} = \begin{pmatrix} 1 + \frac{Z_{11}}{Z_L} & \frac{Z_{12}}{Z_L} & \dots & \frac{Z_{1N}}{Z_L} \\ \frac{Z_{21}}{Z_L} & 1 + \frac{Z_{22}}{Z_L} & \dots & \frac{Z_{2N}}{Z_L} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{Z_{N1}}{Z_L} & \frac{Z_{N2}}{Z_L} & \dots & 1 + \frac{Z_{NN}}{Z_L} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{pmatrix} \quad (5.39)$$

Here ' Z_{ij} 's are mutual impedances and ' Z_{ii} 's are antenna self-impedances. For wire antennas, mutual impedances:

$$Z_{ij} = \frac{V_{oci}}{I_j(0)} = \frac{1}{I_i(0)I_j(0)} \int_{l=0}^L E_j(r)I_i(r) dl \quad (5.40)$$

Where ' $I_i(r)$ ' is the current distribution in i^{th} antenna at ' r ' distance, $E_j(r)$ is the surface radiation electric field of i^{th} antenna at ' r ' distance from feed point and its terminals being shorted for current distribution at j^{th} antenna. ' $I_i(r)$ ' can be obtained by driving i^{th} antenna at transmit mode while j^{th} antenna terminals are shorted and similarly ' $I_j(r)$ ' can also be found out.

The equations can be evaluated using EMF or Moments method

Chapter 6

Conclusion and Future work

6.1 Conclusion:

This text mostly focusses upon wireless MIMO channel correlations, spatial and temporal with most of the observations made for spatial-only correlated MIMO channels. It is shown here that due to spatial correlation channel estimation and signal detection performances are negatively affected, so happens for capacity also as the number of spatial multi-streams available decreases due to spatial correlation.

More error is observed in case of channel estimation i.e. higher minimum MSE achievable, in presence of spatial correlation and correspondingly signal detection error is also higher than that of un-correlated case can be attributed to the increase in the variance of the probability density function (PDF) of the received signal due to more randomness associated with the received signal because of embedded unknown spatial correlation.

Several performance comparisons show that spatial-only and spatio-temporal correlated instances of MIMO wireless communication channel performs worse than un-correlated Rayleigh fading case (this approximate assumption is made based on its popularity) in terms of channel estimation performance.

The adverse effects of unknown antenna mutual coupling for transmitter and receiver ends have been shown and ways of avoiding them have been discussed based on existing unknown mutual coupling estimation and compensation methods available in the texts mentioned here.

6.2 Future work:

We plan to explore different existing ways of estimating and de-coupling methods based on the mutual coupling modelling techniques available and find a way to statistically determine and eliminate the unknown mutual coupling for correlated MIMO scenario and use the information for beam forming like purposes.

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