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# Endogenous timing decisions for product R&D investment competition with demand spillovers in a horizontally differentiated duopoly

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# Endogenous timing decisions for product R&D investment competition with demand spillovers in a horizontally differentiated duopoly

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## Abstract

By focusing on the *constructive* and *combative* spillover effects of the firms' investment in research and development (R&D), we develop a horizontally differentiated duopoly model in which R&D investment used to improve product quality influences consumer preferences and the choice of consumption goods. Applying the framework of endogenous timing decisions to the model, we examine the mutually beneficial timing of product R&D investment and demonstrate that, if there are asymmetric demand spillovers between the firms, a natural Stackelberg equilibrium persists in noncooperative product R&D investment competition in which the firm producing the product with weaker (stronger) demand spillovers moves first (second) to commit to the investment, regardless of the mode of competition. We consider the outcome of the endogenous timing decisions, based on the view of "endogenous sunk costs (i.e., The Sutton Approach)". Furthermore, we address process R&D investment competition with technology spillovers under endogenous timing.

JEL classification: L13, L15

Keywords: endogenous timing; natural Stackelberg equilibrium; product R&D investment; demand spillovers; horizontally differentiated Cournot duopoly; endogenous sunk cost

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# 1. Introduction

## 1.1 Purpose

We develop a horizontal product differentiation model that includes product research and development (R&D) investment with demand spillovers in pre-market competition. Applying the framework of endogenous timing decisions – i.e., the extended game with observable delay developed by Hamilton and Slutsky (1990) – to the developed model, the first purpose of the paper is to demonstrate how the order of product R&D investment is endogenously determined in the presence of demand spillovers. Thus, in a Stackelberg (sequential) game, the order of the moves of players is exogenously given; if we permit players to move simultaneously or sequentially, the distribution of moves can be determined. We consider under what conditions the first- or second-mover advantage persists for players. We demonstrate the counterintuitive outcome that a small firm with small demand spillovers (and a small effect on the market) commits to the investment first, whereas a large firm with large demand spillovers commits to the investment second. This order of investments is benefits both firms. Thus, the small firm gets a share of the market enhancement generated by the product R&D of the large firm. In other words, the small firm employs a free-ride strategy.<sup>1</sup>

The other purpose of this paper is to explain the economic implications of product R&D investment under endogenous timing by employing the perspective of “endogenous sunk costs”, i.e., the Sutton approach (see Sutton, 1989, 1991, 1998; Etro, 2007, 2013). As shown below, that approach posits that the size of product R&D investment under endogenous timing represents entry costs and the level of quality. Furthermore, the difference in the size of the

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<sup>1</sup> Lee et al. (1999) address whether small and medium enterprises can free-ride on the large firm’s market development efforts, taking into account their resource disadvantage.

investments between the firms depends on the difference in the degree of spillovers and product substitutability. Accordingly, the natural Stackelberg equilibrium under asymmetric spillovers implies that the small (large) firm enters to the market first (second) – incurring low (high) entry costs in the pre-market competition – and competes on quantities or on prices, providing a low (high) quality product.

## 1.2 Literature

There have been various contributions to the related literature analyzing the choice of the respective roles for firms in a market (or the timing decision for strategic variables such as price, quantity, process and product R&D investment, advertising, and other firm activities). For instance, Bulow, et al. (1985) demonstrate that a firm prefers to be a leader (follower) if the strategic relationships between the applicable firms indicates that the firms are substitutes (complements) with respect to relevant strategic variables (i.e., price and quantity), or equivalently, if the slopes of the reaction functions are negative (positive) in the relevant space. In particular, when goods are substitutes, if the firms compete on quantities (prices), a strategic substitute (complement) relationship holds between them. Thus, both firms prefer to be a leader (follower), such that the firms would engage in a simultaneous Nash game, as opposed a sequential Stackelberg game. Gal-Or (1985) and Boyer and Moreaux (1987) have shown these results in the case of horizontal product differentiation.<sup>2</sup>

Based on a horizontally differentiated duopoly model with a linear demand and asymmetric constant marginal costs, Yang et al. (2009) compare price and quantity competition under endogenous timing and demonstrate that endogenous timing in the

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<sup>2</sup> However, Albaek (1990) shows that the Stackelberg equilibrium is endogenously determined given cost uncertainty, in which the firm with the larger (smaller) cost variance will be the leader (follower).

Bertrand duopoly leads to two sequential move games, in which one firm moves first and the other moves second. Furthermore, these authors show that endogenous timing in the Cournot duopoly leads to a simultaneous move game, in which both firms move first. Recently, Tremblay et al. (2011) develop a model in which both the timing of play and the strategic choice variables (quantity and price) are endogenous. The authors show that the dynamic Cournot-Bertrand outcome can be a subgame perfect Nash equilibrium, in which the firm choosing quantity (price) moves first (second).

With respect to the literature analyzing non-price market competition such as R&D investment and advertising under endogenous timing, Amir et al. (2000) – in an analysis that is somewhat similar to the analysis in the present paper – consider the endogenous timing of process (i.e., cost-reducing) R&D investment with technology spillovers, applying D’Aspremont and Jacquemin (1988) and De Bondt and Henriques (1995).<sup>3</sup> In particular, it is assumed in these papers that a spillover effect arises because a rival firm’s R&D investment stimulates the availability of technological knowledge (i.e., incoming spillovers). These authors demonstrate the existence of a unique equilibrium in the assignment of the leader and follower roles in which the stronger firm that is better at absorbing knowledge spillovers leads and the other firm follows.

By contrast, Atallah (2005) assumes that a spillover effect arises in the form of leakage of technological information from a rival firm’s R&D investment. In this case, the outcome is the opposite of that presented by Amir et al. (2000) and others. In other words, the first mover is a firm that suffers only a small leakage of technological knowledge from its own process R&D investment. We address process R&D investment competition with technology spillover

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<sup>3</sup> De Bondt and Henriques (1995) assume asymmetric spillover between firms. See Tesoriere (2008) and Vandekerckhove and De Bondt (2008).

under endogenous timing and focus on two types of technology spillovers, i.e., incoming and leakage spillovers.

In the analysis of process R&D investment competition under endogenous timing, researchers address the technology-side spillovers among firms or intra-industry (i.e., business to business). However, this paper focuses on demand-side spillovers (i.e., business to (potential) customers) and addresses demand-enhancing investments such as product R&D investment (i.e., quality-improving) and advertising. As discussed above, by introducing the effect of product R&D investment on the demand side into the conventional utility function, we develop a horizontally differentiated duopoly model (e.g., Boyer and Moreaux, 1987; Häckner, 2000).<sup>4</sup>

At this juncture, we should note that, because product R&D investment affects consumer preferences and the demand side of the market, the investment considered in our model resembles persuasive advertising. In this case, following the terminology that Marshall (1919, pp. 304–307) uses to explain the effects of advertising, we consider both the *combative* and *constructive* effects associated with product R&D investment on the demand side. Regarding the *combative* effect, an increase in product R&D investment by a firm poaches the customers of the rival firm, thereby reducing the profits of the rival. However, an increase in product R&D investment by a firm may also attract new customers from outside markets and expand the potential size of the market. As a result, product R&D investment can also increase the customers of the rival firm and increase the profits of the rival. We refer to this result as the *constructive* effect of product R&D investment.

Similar issues have also received attention in the context of a vertically differentiated

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<sup>4</sup> In a different context, Gavazza (2011) theoretically and empirically considers the role of demand for firms' product varieties and demand spillovers in determining market conduct and market structure in the mutual fund industry. Furthermore, Cleff et al. (2009) empirically address demand-oriented innovation strategy in the European energy production sector.

duopoly model with fixed convex cost of quality. For example, Aoki and Prusa (1997) and Aoki (2003) demonstrate that firms select distinctive qualities and that a firm producing a high-quality product earns higher profits than a firm producing a low-quality product, regardless of the mode of competition. In this case, the leader (or follower) in a sequential Stackelberg game must decide to produce a high- (or low-) quality product. However, because both firms prefer to commit to the production of a high-quality product, they both choose to move first.

Lambertini (1996, 1999) considers endogenous timing with a vertically differentiated Bertrand duopoly and demonstrates that if firms endogenously decide the timing of quality choices, only simultaneous-move equilibria can arise. Jinji (2004) also examines this issue in the context of a vertically differentiated Cournot duopoly and demonstrates that the outcomes of the endogenous timing game depend on whether firms are able to choose their relative position in the quality space before deciding the timing of quality choices. In other words, if firms cannot select their relative position, similar to the result in Lambertini (1999), only simultaneous move equilibria persist. In this case, the firms have an incentive to move first because the first mover can earn higher profits than the second mover. Alternatively, if both firms can choose their relative position, only sequential-move equilibria emerge. In this case, the firm choosing the production of the low- (high-) quality product decides to be the first (second) mover. In other words, the strategic complement (substitute) relationship for the firm producing a low (high)-quality product holds in the quality space.

The remainder of the paper is structured as follows. In Section 2, we develop a horizontally differentiated Cournot duopoly model based on the assumption of a quasilinear utility function that incorporates demand spillovers associated with product R&D investments.

In Section 3, we first consider the strategic relationships between firms and demonstrate the existence of the subgame perfect Nash equilibrium (SPNE) in the noncooperative product R&D investment competition. Then, applying the framework of endogenous timing decisions to the developed model, we consider the mutually beneficial timing of product R&D investments and demonstrate the existence of a natural Stackelberg equilibrium under asymmetric demand spillovers. Furthermore, we consider the economic implication of the outcomes in the endogenous timing game, based on the view of “endogenous sunk costs”, i.e., the Sutton approach. In Section 4, we examine the endogenous timing decision in the cases of process R&D investment competition in the presence of two types of technology spillovers. Next we address the case of a horizontally differentiated Bertrand duopoly. Finally, in Section 5, we summarize the main results and discuss some remaining issues.

## 2. The model

### 2.1 Demand function and product R&D investment with demand spillovers

We assume a duopolistic Cournot competition in a market with horizontally differentiated products. The firms compete in a two-stage game. In Stage 1, each firm simultaneously chooses product R&D investment,  $x_i$ , and in Stage 2, each firm simultaneously chooses output,  $q_i$ ,  $i = 1, 2$ . We confine our attention to the SPNE in the two-stage game by solving the model using backward induction.

We focus on the spillover effects generated by product R&D investment on the demand side. In particular, we consider whether an increase in product R&D investment increases the willingness to pay of consumers for these products, thus expanding the potential size of the



market, which may also increase demand for the rival firm's products. To highlight this effect, we assume that the utility function of a representative consumer is given by

$$V = U[q_1, q_2; x_1, x_2] + q_0, \text{ and}$$

$$U = \left\{ \alpha(q_1 + q_2) - \frac{1}{2}(q_1^2 + q_2^2) - \theta q_1 q_2 \right\} + \left\{ \Omega[x_1, x_2](q_1 + q_2) + \omega_1[x_1]q_1 + \omega_2[x_2]q_2 \right\}, (1)$$

where  $q_0$  is the consumption of the outside (numeraire) good and  $p_0 = 1$ ,  $\alpha > 0$ , and  $\theta \in (0, 1)$  is a parameter representing the degree of substitutability between the products.

With respect to the second term in brackets in (1), we assume that  $\Omega = \Omega[x_1, x_2]$  is an increasing function of product R&D investment, i.e.,  $\frac{\partial \Omega}{\partial x_i} > 0$ , which is associated with aggregate market demand.<sup>5</sup> Furthermore,  $\omega_i = \omega_i[x_i]$  is also an increasing function of product R&D investment, i.e.,  $\frac{\partial \omega_i}{\partial x_i} > 0$ , which is associated with firm  $i$ 's individual demand.

For tractability, we make the following assumption.

*Assumption 1*

- (i)  $\Omega[x_1, x_2] = \varepsilon_1 x_1 + \varepsilon_2 x_2$ , where  $\varepsilon_i (\geq 0)$ ,  $i = 1, 2$  is the coefficient of the marginal effect of an increase in product R&D investment of firm  $i$  on the potential size of the market.
- (ii)  $\omega_i[x_i] = \beta_i x_i$ , where  $\beta_i (\geq 0)$ ,  $i = 1, 2$  is the coefficient of the marginal effect of an increase in product R&D investment of firm  $i$  on its own market.

The budget constraint is given by  $Y \geq p_1 q_1 + p_2 q_2 + q_0$ , where  $Y$  is the given level of

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<sup>5</sup> If  $\Omega[x_1, x_2] = 0$ , the utility function is similar to that in Häckner (2000). See also the appendix in Symeonidis (2003).

income of a representative consumer. Thus, we obtain the following expression for the optimal behavior of the representative consumer.<sup>6</sup>

$$\frac{\partial U}{\partial q_i} = A_i[x_i, x_j] - q_i - \theta q_j = p_i, \quad (2)$$

where  $A_i[x_i, x_j] \equiv \alpha + (\varepsilon_i + \beta_i)x_i + \varepsilon_j x_j$ ,  $i, j = 1, 2, i \neq j$ . In particular,  $A[x_i, x_j]$  in (2) implies that the potential size of the market depends not only on the investment of firm  $i$  but also on that of firm  $j$ . Thus,  $\varepsilon_i + \beta_i$  is the own effect of firm  $i$ 's investment on its potential market size and  $\varepsilon_j$  is the spillover effect of firm  $j$ 's investment. We obtain the following inverse demand function.

$$p_i = A_i[x_i, x_j] - q_i - \theta q_j, \quad i, j = 1, 2, i \neq j. \quad (3)$$

## 2.2 Product R&D investment cost function

To simplify the analysis, we assume that the cost incurred in product R&D investment is given by  $F_i[x_i] = \frac{f}{2}x_i^2$ ,  $f > 0$ ,  $i = 1, 2$ . We also assume that the marginal cost of production is constant, i.e.,  $(\alpha >)c_i = c > 0$ ,  $i = 1, 2$ .

## 3. Endogenous timing and product R&D investment with demand spillovers

### 3.1 Noncooperative product R&D investment competition with demand spillovers

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<sup>6</sup> Unlike Levin and Reiss (1988), who use a multiplicative marginal utility function to make empirical estimation, we employ an additive marginal utility function to simplify the analysis. By doing so, the effect of product R&D investment can be expressed as the vertical shift of the inverse demand function.

In Stage 2, firm  $i$  chooses its output to maximize profit, i.e.,  $\Pi_i = (p_i - c)q_i - \frac{f}{2}x_i^2$ . The

first-order condition for maximizing the profit of firm  $i$  is given by  $\frac{\partial \Pi_i}{\partial q_i} = p_i - q_i = 0$ .

Taking (3) into account, we derive  $A_i[x_i, x_j] - 2q_i - \theta q_j = 0$ . Thus, the Cournot–Nash equilibrium for firm  $i$  in the second stage is given by

$$q_i = \frac{2(A_i - c) - \theta(A_j - c)}{(2 - \theta)(2 + \theta)} = \frac{A + \Phi_i x_i + \Gamma_j x_j}{(2 - \theta)(2 + \theta)} \equiv q_i[x_i, x_j], \quad i, j = 1, 2, i \neq j, \quad (4)$$

where  $A \equiv (2 - \theta)(\alpha - c) > 0$ ,  $\Phi_i \equiv (2 - \theta)\varepsilon_i + 2\beta_i > 0$ , and  $\Gamma_j \equiv (2 - \theta)\varepsilon_j - \theta\beta_j$ . Given

(4), it follows that

$$\frac{\partial q_i[x_i, x_j]}{\partial x_j} \geq (<)0 \Leftrightarrow \Gamma_j \geq (<)0 \Leftrightarrow E_j \geq (<)\frac{\theta}{2}, \quad (5)$$

where  $E_j \equiv \frac{\varepsilon_j}{\varepsilon_j + \beta_j}$ ,  $j = 1, 2$ , is the strength of demand spillovers. If  $\varepsilon_j > 0$  and  $\beta_j = 0$ ,

then  $E_j = 1$ , which implies full spillovers. If  $\varepsilon_j = 0$  and  $\beta_j > 0$ , then  $E_j = 0$ , which

implies no spillovers. Thus, it follows that  $E_j \in [0, 1]$ ,  $j = 1, 2$ .<sup>7</sup>

Equation (5) shows that an increase in the product R&D investment of firm  $j$  has two effects on the output of firm  $i$ . The first is a positive effect, i.e.,  $(2 - \theta)\varepsilon_j$ , whereby an increase in the investment of firm  $j$  increases total market demand for both products, which, in turn, increases firm  $i$ 's individual demand and is known as the positive *constructive* effect. The second is a negative effect, i.e.,  $-\theta\beta_j$ , whereby an increase in the investment of firm  $j$  increases its own demand, which in turn decreases firm  $i$ 's individual demand as a result of the substitutability between the competing products, which is known as the negative

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<sup>7</sup> We ignore the case where both  $\varepsilon_j = 0$  and  $\beta_j = 0$  simultaneously hold.

*combative* effect. For that reason, if the *constructive* effect is larger (smaller) than the *combative* effect, then an increase in the rival firm's product R&D investment increases (decreases) the output of the firm.

Let us express the profit function in Stage 1 as  $\Pi_i = q_i[x_i, x_j]^2 - \frac{f}{2} x_i^2$ . Based on (4), the first-order condition is given by

$$\frac{\partial \Pi_i}{\partial x_i} = 2 \left( \frac{\Phi_i}{\Lambda} \right) q_i[x_i, x_j] - f x_i = 0, \quad i, j = 1, 2, i \neq j, \quad (6)$$

where  $\Lambda \equiv (2 - \theta)(2 + \theta) > 0$ . Using (4) and (6), we derive the reaction function of firm  $i$  as follows:

$$x_i[x_j] = \frac{2\Phi_i A}{\Lambda^2 f - 2\Phi_i^2} + \frac{2\Phi_i \Gamma_j}{\Lambda^2 f - 2\Phi_i^2} x_j, \quad i, j = 1, 2, i \neq j, \quad (7)$$

where we assume  $\Lambda^2 f > 2\Phi_i^2, i = 1, 2$ , to maintain the second-order condition.

Based on (7), and regarding the cross effect, which implies a strategic relationship between the firms, we derive the following relationship:

$$\frac{dx_i}{dx_j} = \frac{2\Phi_i \Gamma_j}{\Lambda^2 f - 2\Phi_i^2} \geq (<) 0 \Leftrightarrow \Gamma_j \geq (<) 0 \Leftrightarrow E_j \geq (<) \frac{\theta}{2}. \quad (8)$$

Furthermore, the external effect on profit is given by

$$\frac{d\Pi_i}{dx_j} = \frac{2\Gamma_j}{(2 - \theta)(2 + \theta)} q_i \geq (<) 0 \Leftrightarrow \Gamma_j \geq (<) 0 \Leftrightarrow E_j \geq (<) \frac{\theta}{2}. \quad (9)$$

To proceed with the analysis, we assume as follows.

### *Assumption 2*

*Demand spillovers for product (firm) 1 are at least as strong as those for product (firm) 2; i.e.,*

$$E_1 \geq E_2.$$

Given (8), (9), and *Assumption 2*, we derive the following lemma.

*Lemma 1*

- (i) If  $E_1 \geq E_2 > \frac{\theta}{2}$ , each firm's reaction curve slopes upward. Hence, an increase in the product R&D investment of firm 2 (1) increases the profit of firm 1 (2).
- (ii) If  $\frac{\theta}{2} > E_1 \geq E_2$ , each firm's reaction curve slopes downward. Hence, an increase in the product R&D investment of firm 2 (1) decreases the profit of firm 1 (2).
- (iii) If  $E_1 > \frac{\theta}{2} > E_2$ , the reaction curve for firm 1 slopes downward, whereas that for firm 2 slopes upward. Hence, an increase in the product R&D investment of firm 2 (1) decreases (increases) profit of firm 1 (2).

Regarding *Lemma 1 (i)* (*Lemma 1 (ii)*), if demand spillovers for both firms are stronger (weaker) than half of the level of product substitutability, an increase in the product R&D investment of the firm increases (decreases) the demand for the product of the rival firm. This result in turn increases (decreases) the rival firm's output and investment, and the rival firm's profit increases (decreases). Similarly, the firm's profit increases (decreases) with an increase in the rival firm's investment, which confirms that the strategic relationship between the firms is complementary (substitutionary). See Figure 1 (2).

<INSERT FIGURES 1 AND 2 HERE>

Regarding *Lemma 1 (iii)*, when there are asymmetric demand spillovers between the firms such that the spillover of firm 1 is larger and that of firm 2 is smaller than half of the product substitutability, an increase in firm 1's investment increases firm 2's output, whereas an increase in firm 2's investment reduces firm 1's output. In this case, firm 2 increases its output and investment, whereas firm 1 reduces its output and investment. As a result, firm 2's profit increases, whereas firm 1's profit falls. Thus, a relationship of strategic substitutability for firm 1 arises, whereas one of strategic complementarity for firm 2 arises. See Figure 3.

<INSERT FIGURE 3 HERE>

Taking (7) into account, we derive the SPNE in the noncooperative product R&D investment competition as follows.

$$x_i^N = \frac{2A\Phi_i \{\Lambda^2 f - 2\Phi_j(\Phi_j - \Gamma_j)\}}{D}, \quad i, j = 1, 2, i \neq j, \quad (10)$$

where  $D \equiv \{\Lambda^2 f - 2\Phi_1^2\} \{\Lambda^2 f - 2\Phi_2^2\} - 4\Phi_1\Gamma_1\Phi_2\Gamma_2 > 0$ . To acquire a positive equilibrium given by (10), we assume  $\Lambda^2 f > 2\Phi_j(\Phi_j - \Gamma_j) = 2\Phi_j(2 + \theta)\beta_j$ ,  $j = 1, 2$ .

For the analysis below, in comparing the equilibrium investment level of both firms, we obtain the following relationship:

$$x_1^N > (<) x_2^N \Leftrightarrow (2 - \theta)(\varepsilon_1 - \varepsilon_2) \Lambda^2 f + 2\{\Lambda^2 f + (2 + \theta)\Phi_1\Phi_2\}(\beta_1 - \beta_2) > (<) 0.$$

Thus, if the effect of the investment of firm 1 on market demand is larger than that of firm 2, the investment level of firm 1 is larger than that of firm 2. In view of *Assumption 2*, for example, if the *combative* effect of both firms is almost equal, i.e.,  $\beta_1 \cong \beta_2$ , or if the *combative* effect of firm 1 is not sufficiently lower than that of firm 2, and the *constructive* effect of firm 1 is larger than that of firm 2, i.e.,  $\varepsilon_1 > \varepsilon_2$ , then it holds that  $x_1^N > x_2^N$ .

### 3.2 Endogenous timing decision and a natural Stackelberg equilibrium

We now proceed to analyze the mutually beneficial timing of product R&D investment in the presence of demand spillovers. By applying the definition used by Albaek (1990), we extend the game-theoretic framework in Hamilton and Slutsky (1990) for the endogenous timing of an observable delay. In other words, we show the conditions necessary to sustain a natural Stackelberg equilibrium, i.e., two players are able to determine the timing of their actions and the actions themselves. If the players choose their actions at different times, the player that chooses last can observe the action chosen by the initiating player. Hence, there is a sequential-play subgame and the Stackelberg equilibrium. If the players instead choose their actions at the same time, there is a simultaneous-play subgame and the Nash equilibrium.

Consequently, based on the extended game for the endogenous timing of an observable delay, we compare the payoffs in the simultaneous-play game with those from the two sequential-play games. In this case, if one firm wants to be the first mover (superscript  $F$ ), while the other firm wants to be the second mover (superscript  $S$ ), and neither firm prefers to play a simultaneous Nash game (superscript  $N$ ), then a natural Stackelberg equilibrium results.

Without any unnecessarily complicated calculations and taking *Lemma 1* into account, the relationships between the firms' profits described below are easily derived (see Figures 1, 2, and 3).

*Lemma 2*

(i) If  $E_1 \geq E_2 > \frac{\theta}{2}$ , it follows that  $\Pi_i^F > \Pi_i^N$  and  $\Pi_i^S > \Pi_i^N$ ,  $i = 1, 2$ .

(ii) If  $\frac{\theta}{2} > E_1 \geq E_2$ , it follows that  $\Pi_i^F > \Pi_i^N > \Pi_i^S$ ,  $i = 1, 2$ .

(iii) If  $E_1 > \frac{\theta}{2} > E_2$ , it follows that  $\Pi_1^F > \Pi_1^N$  and  $\Pi_1^S > \Pi_1^N$ , and that

$$\Pi_2^F > \Pi_2^N > \Pi_2^S.$$

Under *Lemma 2 (i)*, just as in a standard Bertrand price competition model, both firms prefer being either a first mover or a second mover to playing a simultaneous Nash game. However, each firm prefers being a second mover to being a first mover because it holds that  $\Pi_i^S > \Pi_i^F$ ,  $i = 1, 2$ . Thus, there are two Stackelberg equilibria, i.e.,  $(x_1^F, x_2^S)$  and  $(x_1^S, x_2^F)$ , located in the Pareto-superior sets (see  $S_1$  and  $S_2$  in Figure 1). As proven in Lemma 1 in Yang et al. (2008), this result implies that the endogenous timing game leads to two sequential games.

Similarly, under *Lemma 2 (ii)*, as in a standard Cournot quantity competition model, both firms prefer being a first mover to being a second mover and playing a simultaneous Nash game (see Theorem V (A) in Hamilton and Slutsky, 1990; and Lemma 2 in Yang et al., 2008). Furthermore, the two Stackelberg equilibria are not located in the Pareto-superior sets (see  $S_1$  and  $S_2$  in Figure 2).

Under *Lemma 2 (iii)*, in which there are asymmetric spillovers between the firms, firm 1 prefers being either a first mover or a second mover to playing a simultaneous Nash game, whereas firm 2 prefers being a first mover to being a second mover and playing a simultaneous Nash game. In this case, because firm 1 expects that firm 2 will take the first move, firm 1 will take the second move. Taking into account the conjectural process, firm 2 will commit to the investment in advance. In other words, this commitment is credible and



preferable for firm  $I$  because it holds that  $\Pi_1^S > \Pi_1^F$ . Therefore, based on Theorem V (B) in Hamilton and Slutsky (1990), we derive the following result.<sup>8</sup>

*Proposition 1*

*The firm producing the product with weaker (stronger) demand spillovers chooses the first (second) move to commit to product R&D investment.*

For example, consider the situation in which a small firm that does not produce the recognized brand – whose product has only a small effect on market demand – decides in advance to invest on a small scale strategically. This investment makes the large firm with the recognized brand – where the product and the firm itself have a substantial effect on market demand – increase its product R&D investment. Accordingly, market demand increases, which enables both firms to raise output. As a result, both firms make higher profits. In other words, the small firm free-rides on the demand spillovers generated by the large firm's product R&D investment.

Based on the definition in Albaek (1990), we define the outcome under endogenous timing decision given in *Proposition 1* as a natural Stackelberg equilibrium, i.e.,

$$x_2^F = \frac{2A\Psi_2\{\Lambda^2 f - 2\Phi_1(\Phi_1 - \Gamma_1)\}}{D - 4\Phi_1\Gamma_1\Psi_2\Gamma_2} \quad \text{and} \quad x_1^S = \frac{2\Phi_1 A}{\Lambda^2 f - 2\Phi_1^2} + \frac{2\Phi_1\Gamma_2}{\Lambda^2 f - 2\Phi_1^2} x_2^F,$$

where  $\Psi_2 \equiv \frac{\Phi_2(\Lambda^2 f - 2\Phi_1^2) + 2\Phi_1\Gamma_1\Gamma_2}{\Lambda^2 f - 2\Phi_1^2} > 0$ . See Appendix A.

We consider the natural Stackelberg equilibrium located in the Pareto-superior sets in the case of asymmetric spillovers (see point  $S_2$  in Figure 3). Compared with the SPNE in the

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<sup>8</sup> In a rent-seeking model, Leininger (1993) demonstrates that a weaker (stronger) player moves first (second).

natural Stackelberg equilibrium, when firm  $I$  (2) produces the product with stronger (weaker) demand spillovers, its product R&D investment increases (reduces), i.e.,  $x_1^S > x_1^N$  and  $x_2^F < x_2^N$ . In this case, regarding the output of firm  $I$ , based on (4), we have the following relationship:

$$q_1^S \equiv q_1[x_1^S, x_2^F] > (\leq) q_1^N \equiv q_1[x_1^N, x_2^N] \Leftrightarrow \Phi_1(x_1^S - x_1^N) + \Gamma_2(x_2^F - x_2^N) > (\leq) 0. \quad (11)$$

Thus, it directly follows that  $q_1^S > q_1^N$ , because  $\Gamma_2 < 0$ . Compared with those in the SPNE, the product R&D investment costs increase. However, the extent of the increase in the output of firm  $I$  is sufficiently large. Thus, it follows that  $\Pi_1^S > \Pi_1^N$ .

Conversely, we derive the output of firm 2 as follows:

$$q_2^F \equiv q_2[x_1^S, x_2^F] > (\leq) q_2^N \equiv q_2[x_1^N, x_2^N] \Leftrightarrow \Phi_2(x_2^F - x_2^N) + \Gamma_1(x_1^S - x_1^N) > (\leq) 0. \quad (12)$$

Hence, we derive  $\text{sgn}\{\Phi_2(x_2^F - x_2^N) + \Gamma_1(x_1^S - x_1^N)\} = \text{sgn}\Gamma_2\{\Phi_2\Lambda^2 f - 2\Phi_1(\Phi_1\Phi_2 - \Gamma_1\Gamma_2)\}$ . It follows that  $\Phi_2\Lambda^2 f - 2\Phi_1(\Phi_1\Phi_2 - \Gamma_1\Gamma_2) > 0$ , because  $\Psi_2 > 0$  as denoted above. Thus, the output of firm 2 decreases more than that in the SPNE. However, although the output decreases because the extent of the decrease in product R&D investment costs outweighs that of the decrease in revenue, it follows that  $\Pi_2^F > \Pi_2^N$ .

Furthermore, the relationship between the product R&D investments of both firms in the natural Stackelberg equilibrium, i.e.,  $x_1^S > (\leq) x_2^F$ , is generally ambiguous because it depends on the parameters for spillovers and product substitutability, i.e.,  $\varepsilon_i$ ,  $\beta_i$ ,  $i=1,2$ , and  $\theta$ . However, for example, assuming  $\varepsilon_1 = \varepsilon > \varepsilon_2 = 0$ ,  $\beta_1 = \beta_2 = \beta > 0$ , and  $(2 - \theta)\varepsilon - \theta\beta > 0$ , we can derive  $x_1^S > (x_1^N > x_2^N) > x_2^F$  (see Appendix B). Based on (4), it follows that  $q_1^S \equiv q_1[x_1^S, x_2^F] > q_2^F \equiv q_2[x_1^S, x_2^F]$  in the Cournot duopolistic market.

Compared with SPNE, the Pareto optimality of terms of profits persists in the natural

Stackelberg equilibrium; however, the effect on consumer surplus is ambiguous. That is,

consumer surplus is given by  $CS \equiv U[q_1, q_2] - p_1 q_1 - p_2 q_2 = \frac{1}{2}(q_1^2 + q_2^2) + \theta q_1 q_2$ , where

$q_i = q_i[x_1(x_2), x_2]$ ,  $i = 1, 2$ . We derive as follows.

$$\frac{dCS}{dx_2} = (q_1 + \theta q_2) \left\{ \frac{\partial q_1}{\partial x_1} \frac{\partial x_1}{\partial x_2} + \frac{\partial q_1}{\partial x_2} \right\} + (\theta q_1 + q_2) \left\{ \frac{\partial q_2}{\partial x_1} \frac{\partial x_1}{\partial x_2} + \frac{\partial q_2}{\partial x_2} \right\}, \text{ where}$$

$$\frac{\partial q_1}{\partial x_1} \frac{\partial x_1}{\partial x_2} + \frac{\partial q_1}{\partial x_2} = \frac{\Gamma_2 \Lambda^2 f}{\Lambda(\Lambda^2 f - 2\Phi_1^2)} < 0 \quad \text{and} \quad \frac{\partial q_2}{\partial x_1} \frac{\partial x_1}{\partial x_2} + \frac{\partial q_2}{\partial x_2} = \frac{\Phi_2(\Lambda^2 f - 2\Phi_1^2) + 2\Phi_1 \Gamma_1 \Gamma_2}{\Lambda(\Lambda^2 f - 2\Phi_1^2)} > 0.$$

The first (second) term expresses the effect of a change of the investment of firm 2 on surplus regarding consumption of product 1 (2). Unless the magnitude of the effect expressed in the first term is sufficiently large, a decrease in the investment of firm 2 from the SPNE to the natural Stackelberg equilibrium reduces consumer surplus.

Furthermore, evaluating the social optimal investments at the SPNE, we derive

$$\left. \frac{\partial W}{\partial x_i} \right|_{\frac{\partial \Pi_i}{\partial x_i} = 0} = \left( \frac{q_i}{\Lambda} \right) (\Phi_i + \theta \Gamma_i) + \left( \frac{q_j}{\Lambda} \right) (\theta \Phi_i + 3\Gamma_i), \quad i, j = 1, 2, i \neq j,$$

where  $\Phi_i + \theta \Gamma_i = (2 - \theta)(1 + \theta)\varepsilon_i + (2 - \theta^2)\beta_i > 0$  and  $\theta \Phi_i + 3\Gamma_i = (2 - \theta)(3 + \theta)\varepsilon_i - \theta\beta_i$ ,

$i = 1, 2$ . If it holds that  $(2 - \theta)(3 + \theta)\varepsilon_i > \theta\beta_i, i = 1, 2$ , then we have  $\left. \frac{\partial W}{\partial x_i} \right|_{\frac{\partial \Pi_i}{\partial x_i} = 0} > 0$ . Thus, we

understand that the levels of product R&D investment in the SPNE is lower than those in the social optimality. Considering  $x_1^S > x_1^N > x_2^N > x_2^F$ , this implies either that the levels of the investments of both firms in the natural Stackelberg equilibrium are lower than the levels in the social optimality or that at least the level of firm 2, which is a first mover with a small impact on the market, is lower than that in the social optimality.

### 3.3 Implications: Product R&D investment as endogenous sunk costs<sup>9</sup>

Based on the seminal works, of John Sutton (1989, 1992, 1998), i.e., the Sutton approach denoted by Etro (2013), with respect to the outcomes demonstrated above, i.e., *Proposition 1*, we take the view that product R&D investment cost in our model can be regarded as “endogenous sunk costs”.<sup>10</sup> In this case, when a small firm commits to a first move to invest, it implies that the small firm incurs a low investment costs to enter the market in advance. This low entry costs leads to low levels of quality. Otherwise, if it chooses a second move, the small firm must incur large investment costs. However, a large firm prefers the second move to the first move, which implies that the large firm makes a substantial investment to improve the quality level and incurs large entry costs enhancing aggregate market demand. As a result, the market structure is organized such that the small (large) firm competes by providing a low (high) quality of the products and services.

Secondly, we consider how product R&D investments under endogenous timing affect potential market size (i.e.,  $A_i[x_i, x_j] \equiv \alpha + (\varepsilon_i + \beta_i)x_i + \varepsilon_j x_j$ ,  $i, j = 1, 2, i \neq j$ ), regarding both firms in point  $S_1$  and  $S_2$  in Figure 3. In this case, we derive as follows.

$$A_2[S_2] - A_2[S_1] = (\varepsilon_2 + \beta_2)(x_2^F - x_2^S) + \varepsilon_1(x_1^S - x_1^F)$$

and

$$A_1[S_2] - A_1[S_1] = (\varepsilon_1 + \beta_1)(x_1^S - x_1^F) + \varepsilon_2(x_2^F - x_2^S),$$

where  $x_1^S - x_1^F > 0$  and  $x_2^F - x_2^S < 0$ . For example, in assuming  $\varepsilon_1 = \varepsilon > \varepsilon_2 = 0$ ,  $\beta_1 = \beta_2 = \beta > 0$ ,  $(2 - \theta)\varepsilon - \theta\beta > 0$ , and  $\varepsilon > \beta$ , if a small (large) firm incurs a low (high) entry (i.e., investment) cost, the potential market size may increase more than in the opposite

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<sup>9</sup> This part is based on the suggestions of the editor and anonymous referees.

<sup>10</sup> See also Matraves (1999) and Gruber (2002).

case.<sup>11</sup> As a result, the market size is determined by the magnitude of entry costs through the endogenous timing decisions before Cournot competition in the market.

Finally, compared with the outcomes based on the vertical differentiation model, i.e., Jinji (2004), we can alternatively interpret the outcomes under endogenous timing as follows. Let us assume  $\theta = 1$ . This assumption implies that the products are homogenous and a horizontal product differentiation between the products thus vanishes. Hence, we can confirm that *Lemmas 1 and 2* and *Proposition 1* hold. In other words, our model addressing product (i.e., quality-improving) R&D investment is formally similar to the vertical differentiation model. Jinji (2004) demonstrates that the firm producing the low (high) quality product chooses the quality level first (second).<sup>12</sup> However, in our model, a small (large) firm with a small (large) effect on market demand commits to a small (large) product R&D investment first (second). As a result, the small (large) firm provides the low (high) quality product.

## 4. Discussions

### 4.1 Endogenous timing of process R&D investment competition with technology spillovers

As discussed in the Introduction, it is worth relating the demand spillovers examined to the technology spillovers. Furthermore, we demonstrate that the outcomes of endogenous timing depend on the type of technology spillovers.

We assume the following standard inverse demand function in a horizontally differentiated products market:

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<sup>11</sup> Given the example, we can derive the same result by comparing the SPNE and the natural Stackelberg equilibrium, i.e.,  $N$  and  $S_2$ .

<sup>12</sup> See Lambertini and Tedeschi (2007), and Lambertini and Tampieri (2012).

$$p_i = \alpha - q_i - \theta q_j, i, j = 1, 2, i \neq j. \quad (13)$$

First, we assume the impact of technology spillover on the marginal cost of production presented by De Bondt and Henriques (1995) and Amir, et al. (2000) as follows.

$$c_i = c - x_i - \varepsilon_i x_j, i, j = 1, 2, i \neq j, \quad (14)$$

where  $\varepsilon_i x_j$  represents technology spillovers, in which  $\varepsilon_i \in [0, 1]$  measures the extent to which the benefits of firm  $j$ 's R&D investment are available to firm  $i$ , which implies that firms can reap the rewards of another firm's technology, i.e., incoming spillovers. In this case, the Cournot–Nash equilibrium is given by

$$q_i = \frac{A + (2 - \theta\varepsilon_i)x_i + (2\varepsilon_i - \theta)x_j}{\Lambda}, i, j = 1, 2, i \neq j. \quad (15)$$

If the level of technology available to firm  $i$  from the rival firm's investment is higher (lower) than half of the product substitutability, then an increase in the investment by firm  $j$  increases (decreases) the output of firm  $i$  as follows.

$$\frac{\partial q_i}{\partial x_j} \geq (<) 0 \Leftrightarrow \varepsilon_i \geq (<) \frac{\theta}{2}, i, j = 1, 2, i \neq j. \quad (16)$$

Alternatively, we assume the impact of the technology spillover on the marginal cost of production presented by Atallah (2005) as follows.

$$c_i = c - x_i - \varepsilon_j x_j, i, j = 1, 2, i \neq j, \quad (17)$$

where  $\varepsilon_j x_j$  is the technology spillover, in which  $\varepsilon_j \in [0, 1]$  represents the leakage of technological knowledge from firm  $j$ 's R&D investment to firm  $i$ . In Stage 2, the Cournot–Nash equilibrium is given by

$$q_i = \frac{A + (2 - \theta\varepsilon_i)x_i + (2\varepsilon_j - \theta)x_j}{\Lambda}, i, j = 1, 2, i \neq j. \quad (18)$$

If the level of technological leakage from firm  $j$  is higher (lower) than half of the product

substitutability, then an increase in investment by firm  $j$  increases (decreases) the output of firm  $i$  as follows.

$$\frac{\partial q_i}{\partial x_j} \geq (<)0 \Leftrightarrow \varepsilon_j \geq (<)\frac{\theta}{2}, i, j = 1, 2, i \neq j. \quad (19)$$

Because it follows that  $E_j = \varepsilon_j, j = 1, 2$ , in assuming  $\varepsilon_i + \beta_i = 1, i = 1, 2$ , in (4), we derive (19). Thus, the effects of technological leakage from the rival firm assumed by Atallah (2005) are formally equivalent to the effects of the demand spillovers in our model.

To demonstrate that the endogenous timing of process R&D investment depends on the type of technology spillovers, let us assume  $\varepsilon_i > \frac{\theta}{2} > \varepsilon_j$ . In this case, in the models of De Bondt and Henriques (1995) and Amir et al. (2000), if firm  $i$  ( $j$ ) has a higher (lower) capability of absorbing the technology generated by firm  $j$ 's ( $i$ 's) process R&D investment, then the reaction function is upward (downward) sloping. Thus, firm  $i$  ( $j$ ) has a strategic complementary (substitutionary) relationship with firm  $j$  ( $i$ ), and we determine that the firm with a higher (lower) capability of absorbing the technology chooses to move first (second). The result is the same as Theorem 5 in Amir, et al. (2000).

Conversely, the model in Atallah (2005) predicts that the first mover is firm  $j$ , which suffers only a small leakage of technological knowledge from its own process R&D investment. Thus, we summarize the result as follows.

*Corollary 1*

*The firm with smaller (larger) technology spillovers of process R&D investment chooses to invest first (second).*

This result is contrary to the results of De Bondt and Henriques (1995) and Amir et al.

(2000), but is formally similar to *Proposition 1* in the case of demand spillovers.

#### 4.2 The Bertrand duopoly case with demand spillovers

We confirm whether the endogenous timing decision depends on the mode of competition.

Considering (2), the direct demand function of product  $i$  is given by

$$q_i = \frac{\alpha(1-\theta) + \{(1-\theta)\varepsilon_i + \beta_i\}x_i + \{(1-\theta)\varepsilon_j - \theta\beta_j\}x_j - p_i + \theta p_j}{\Sigma}, \quad i, j = 1, 2, i \neq j, \quad (20)$$

where  $\Sigma = (1-\theta)(1+\theta)$ . In this case, we obtain

$$\frac{\partial q_i}{\partial x_j} = \frac{(1-\theta)\varepsilon_j - \theta\beta_j}{\Sigma} \geq (<)0 \Leftrightarrow E_j \geq (<)\theta. \quad (21)$$

In Stage 2, firm  $i$  chooses a price to maximize its profit, i.e.,  $\Pi_i = (p_i - c)q_i - F_i[x_i]$ . The first-order condition for maximizing the profit of firm  $i$  is given by  $\frac{\partial \Pi_i}{\partial p_i} = q_i - \frac{p_i - c}{\Sigma} = 0$ .

Considering (20), we derive

$$\alpha(1-\theta) + \{\varepsilon_i(1-\theta) + \beta_i\}q_i + \{\varepsilon_j(1-\theta) - \beta_j\theta\}q_j - 2p_i + \theta p_j + c = 0.$$

Hence, the price of product  $i$  in the second stage is given by

$$p_i = \frac{A^B + \Phi_i^B x_i + \Gamma_j^B x_j}{\Lambda} + c \equiv p_i[x_i, x_j], \quad i, j = 1, 2, i \neq j. \quad (21)$$

We note parameters  $A^B \equiv (1-\theta)(2+\theta)(\alpha-c) > 0$ ,  $\Phi_i^B \equiv (1-\theta)(2+\theta)\varepsilon_i + (2-\theta^2)\beta_i > 0$ , and  $\Gamma_j^B \equiv (1-\theta)(2+\theta)\varepsilon_j - \theta\beta_j$ , where superscript  $B$  denotes Bertrand duopoly. Given (21),

the following relationship holds:

$$\frac{\partial p_i[x_i, x_j]}{\partial x_j} \geq (<)0 \Leftrightarrow \Gamma_j^B \geq (<)0 \Leftrightarrow \Theta[E_j] \geq (<)\theta, \quad (22)$$

where  $\Theta[E_j] \equiv \frac{\sqrt{E_j^{-2} + 8} - E_j^{-1}}{2} < 1$  and  $\Theta'[E_j] > 0$ ,  $j = 1, 2$ .



Equation (22) demonstrates that if the parameter representing the strength of demand spillovers for product  $j$ , i.e.,  $\Theta[E_j]$ , is larger (smaller) than a certain value of product substitutability, i.e.,  $\theta$ , then an increase in the product R&D investment of firm  $j$  increases (decreases) the price of product  $i$ . That is, firm  $i$  increases (decreases) its price because the *constructive* effect on product  $i$ 's demand that is induced by an increase in total market demand is larger (smaller) than the *combative* effect on product  $i$ 's demand resulting from the substitutability between the products.

In Stage 1, firm  $i$  chooses its profit-maximizing product R&D investment. Hence, the profit function is represented by  $\Pi_i = R_i[x_i, x_j] - \frac{f}{2}x_i^2$ , where  $R_i[x_i, x_j] = \frac{p_i[x_i, x_j]^2}{\Sigma}$ . The first-order condition is given by:

$$\frac{\partial \Pi_i}{\partial x_i} = \frac{\partial R_i}{\partial x_i} - \frac{\partial F_i}{\partial x_i} = \frac{2}{\Sigma} \left\{ \frac{\Phi_i^B}{\Lambda} \right\} p_i[x_i, x_j] - fx_i = 0, \quad i, j = 1, 2, i \neq j. \quad (23)$$

We derive the second-order condition, the cross effect, and the external effect on the profit of the rival firm, as follows.

$$\frac{\partial^2 \Pi_i}{\partial x_i^2} = \frac{\partial^2 R_i}{\partial x_i^2} - \frac{\partial^2 F_i}{\partial x_i^2} = \frac{2}{\Sigma} \left\{ \frac{\Phi_i^B}{\Lambda} \right\}^2 - f < 0, \quad (24)$$

$$\frac{\partial^2 \Pi_i}{\partial x_i \partial x_j} = \frac{2}{\Sigma} \left\{ \frac{\Phi_i^B}{\Lambda} \right\} \left\{ \frac{\Gamma_j^B}{\Lambda} \right\} \geq (<) 0 \Leftrightarrow \Gamma_j^B \geq (<) 0, \quad (25)$$

and

$$\frac{\partial \Pi_i}{\partial x_j} = \frac{2}{\Sigma} \left\{ \frac{\Gamma_j^B}{\Lambda} \right\} p_i[x_i, x_j] \geq (<) 0 \Leftrightarrow \Gamma_j^B \geq (<) 0, \quad (26)$$

where  $i, j = 1, 2, i \neq j$ .

To proceed with the analysis, it follows under *Assumption 2* that  $\Theta_1 \equiv \Theta[E_1] \geq \Theta_2 \equiv \Theta[E_2]$

and that  $\Theta_1 = \Theta_2$  if and only if  $E_1 = E_2$ . Given (25) and (26), we derive the following lemma.

*Lemma 3*

- (i) *If  $\Theta_1 \geq \Theta_2 > \theta$ , each firm's reaction curve slopes upward. Hence, an increase in the product R&D investment for product 2 (1) increases the revenue of firm 1 (2).*
- (ii) *If  $\theta > \Theta_1 \geq \Theta_2$ , each firm's reaction curve slopes downward. Hence, an increase in the product R&D investment for product 2 (1) decreases the revenue of firm 1 (2).*
- (iii) *If  $\Theta_1 > \theta > \Theta_2$ , firm 1's reaction curve slopes downward, whereas firm 2's reaction curve slopes upward. Hence, an increase in the product R&D investment for product 2 (1) decreases (increases) the revenue of firm 1 (2).*

Given *Lemmas 2* and *3*, we can obtain the same results in the case of a horizontally differentiated Bertrand duopoly as those in *Proposition 1*. However, in the vertical differentiation model, the endogenous timing of the quality decision depends on the mode of completion, e.g., Lambertini (1996, 1999) and Jinji (2004); however, the endogenous timing decision of the product R&D investment competition in a horizontally differentiated model is independent of the mode of competition. Furthermore, by the same method employed in the analysis of the Cournot duopoly case, assuming  $\varepsilon_1 = \varepsilon > \varepsilon_2 = 0$  and  $\beta_1 = \beta_2 = \beta > 0$ , we can derive  $x_1^{BS} > x_1^{BN} > x_2^{BN} > x_2^{BF}$ .

## 5. Concluding Remarks

We have considered endogenous timing decisions based on a model of product (quality-improving) R&D investment with demand spillovers in a horizontally differentiated duopoly. Furthermore, we have addressed the same problem in the case of process (cost-reducing) R&D investment with technology spillovers and in the case of a horizontally differentiated Bertrand duopoly.

We have found that if the strategic relationship of the rival firm regarding the firm is one of substitutability (complementarity), and the external effect on the rival firm's profit is negative (positive), the firm chooses to move first (second) under endogenous timing, regardless of the mode of competition. In this case, the Stackelberg equilibrium results, which is Pareto-superior for the firms but may also decrease consumer surplus.

We have considered the economic implications of the Stackelberg equilibrium under endogenous timing, considering the views of "endogenous sunk costs" and "endogenous entry". In particular, when there are asymmetric firms with various specific resources and properties, the market structure is organized such that a small firm that affects the market demand little or not at all commits to a low investment (i.e., entry) cost early, whereas a large firm that exerts substantial influence on the market commits to a high investment (i.e., entry) cost. As a result, the former (latter) competes in the market with a lower (higher) quality product. In terms of future developments, the duopoly model might be extended to an oligopoly model, and we might consider an endogenous market structure (see Etro, 2013).

Furthermore, we have shown that the endogenously decided order in the natural Stackelberg equilibrium is preferable to other equilibria for both firms, which implies that both firms non-cooperatively collude with respect to investment levels before market

competition. However, we have not addressed cooperative R&D investment, R&D agreements, or joint ventures in the presence of spillovers.<sup>13</sup> However, since the seminal paper of D'Aspremont and Jacquemin (1988), the problem has been analyzed theoretically and empirically in much of the related literature. For example, Marini and Rodano (2013) and Marini et al. (2014) recently analyzed the possibility of forming R&D agreements (cooperative ventures). Furthermore, Foros et al. (2002) consider roaming policy in the market for mobile telecommunications. In this model, firms collude at the investment stage, although they compete in the retail market, i.e., semi-collusion. In the future, we will examine how R&D agreement and cooperation affect endogenous entry and market structure.

## Appendix A: The Stackelberg equilibrium in the case of asymmetric spillovers

In the case of asymmetric spillovers, i.e.,  $\Gamma_1 > 0 > \Gamma_2 \Leftrightarrow E_1 > \frac{\theta}{2} > E_2$ , we assume that firm 2 (*I*) chooses to move first (second) in product R&D investment. Taking into account the reaction function of firm 1 given by (7), firm 2 chooses its product R&D investment to maximize the profit given by  $\Pi_2 = q_2[x_1(x_2), x_2]^2 - \frac{f}{2}x_2^2$ . The first-order condition is given

$$\text{by } \frac{d\Pi_2}{dx_2} = 2\left(\frac{\Psi_2}{\Lambda}\right)q_2[x_1(x_2), x_2] - fx_2 = 0, \quad \text{where } \Psi_2 \equiv \frac{\Phi_2(\Lambda^2 f - 2\Phi_1^2) + 2\Phi_1\Gamma_1\Gamma_2}{\Lambda^2 f - 2\Phi_1^2}. \quad \text{To}$$

sustain an interior equilibrium, we assume  $\Psi_2 > 0$ . Furthermore, the second-order condition is  $\Lambda^2 f - 2\Psi_2^2 > 0$ .

Based on the first-order condition and (4), we have

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<sup>13</sup> One of our anonymous referees comments on this point. With respect to cooperative product R&D investment (i.e., semi-collusion) in the case of the Bertrand duopoly, see Toshimitsu (2012).

$$2\Psi_2 A + 2\Psi_2 \Gamma_1 x_1 - (\Lambda^2 f - 2\Phi_2 \Psi_2) x_2 = 0. \quad (\text{A.1})$$

Substituting the reaction function of firm  $I$  given by (7) into (A.1), we derive the Stackelberg equilibrium as follows.

$$x_2^F = \frac{2A\Psi_2 \{\Lambda^2 f - 2\Phi_1(\Phi_1 - \Gamma_1)\}}{D - 4\Phi_1 \Gamma_1 \Psi_2 \Gamma_2} \quad (\text{A.2})$$

and

$$x_1^S = \frac{2\Phi_1 A}{\Lambda^2 f - 2\Phi_1^2} + \frac{2\Phi_1 \Gamma_2}{\Lambda^2 f - 2\Phi_1^2} x_2^F. \quad (\text{A.3})$$

In the case of asymmetric spillovers, bearing in mind (10) and (A.2), we obtain  $x_2^N > x_2^F$  directly. Furthermore, because the reaction function of firm  $I$  is a decreasing function of the investment of firm 2, as in (A.3), it follows that  $x_1^S = x_1(x_2^F) > x_1^N = x_1(x_2^N)$ .

## Appendix B: An example

From (A.3), we have the following relationship:

$$x_1^S > (\leq) x_2^F \Leftrightarrow 2\Phi_1 A > (\leq) \{H_1 - 2\Phi_1 \Gamma_2\} x_2^F, \quad (\text{B.1})$$

where  $H_1 \equiv \Lambda^2 f - 2\Phi_1^2 > 0$ . Furthermore, given the assumption regarding the parameters, we have  $\Phi_1 = (2 - \theta)\varepsilon + 2\beta > 0$ ,  $\Phi_2 = 2\beta > 0$ ,  $\Gamma_1 = (2 - \theta)\varepsilon - \theta\beta > 0$ , and  $\Gamma_2 = -\theta\beta < 0$ .

Substituting (A.2) into the right-hand side of (B.1), through complicated and tedious calculations, we obtain the following relationship:

$$x_1^S > (\leq) x_2^F \Leftrightarrow H_1 Z - 4\Phi_1^2 \Gamma_1 \Gamma_2 Y > (\leq) 0, \quad (\text{B.2})$$

Where  $Z \equiv \Lambda^2 f(\Phi_1 - \Phi_2) + 2\Phi_1 \Phi_2(\Phi_1 + \Gamma_2 - \Gamma_1) - 2\Phi_1 \Gamma_1 \Gamma_2 > 0$  and  $Y \equiv \Phi_2 + \Gamma_1 - \Gamma_2 > 0$ .

Because  $\Gamma_2 < 0$ , it follows that  $H_1 Z - 4\Phi_1^2 \Gamma_1 \Gamma_2 Y > 0$ . Thus, we have  $x_1^S > x_2^F$ .

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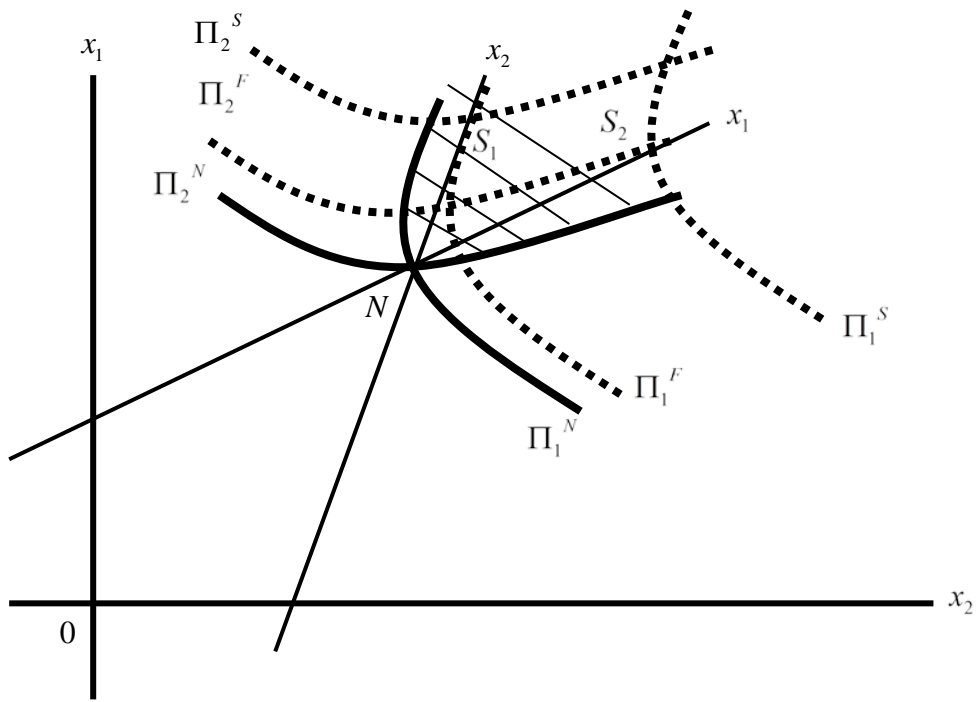


Figure 1

The case of strong demand spillovers:  $E_1 \geq E_2 > \frac{\theta}{2}$

The shaded area represents the Pareto-superior sets

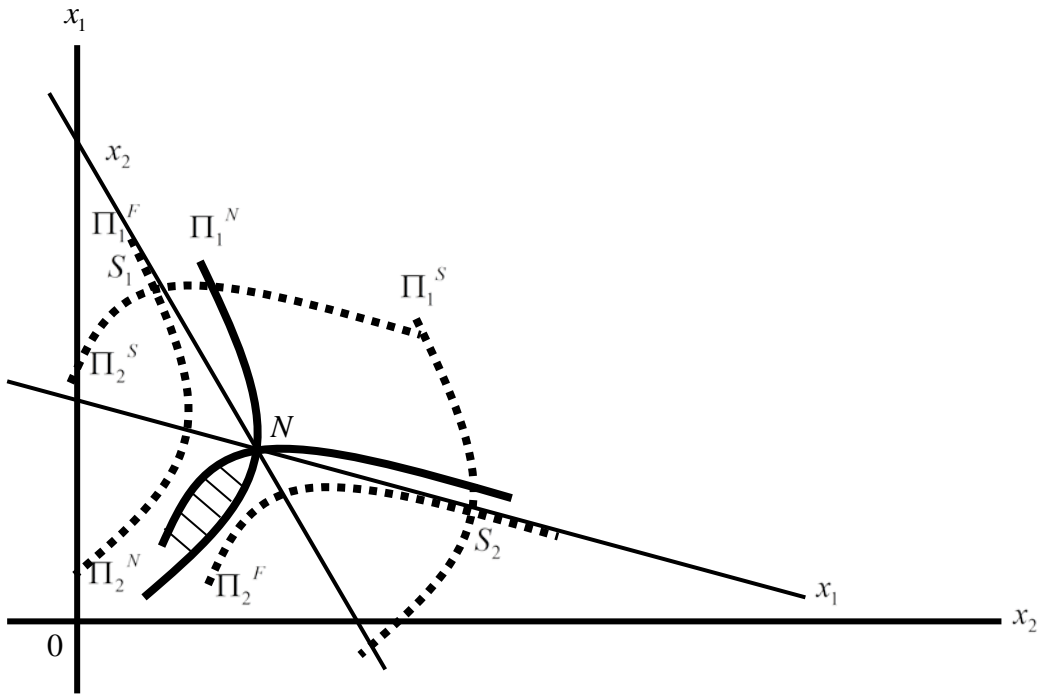


Figure 2

The case of weak demand spillovers:  $\frac{\theta}{2} > E_1 \geq E_2$

The shaded area represents the Pareto-superior sets

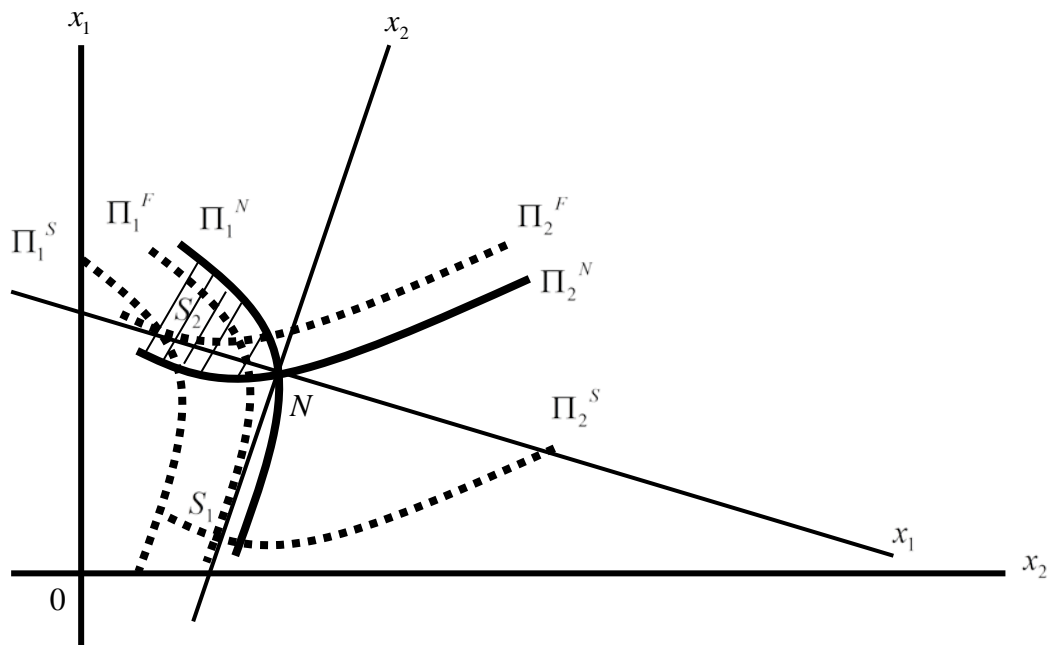


Figure 3

The case of asymmetric demand spillovers:  $E_1 > \frac{\theta}{2} > E_2$

The shaded area represents the Pareto-superior sets