Reliability Assessment of Reinforced Concrete Beam with Embedded PVC Pipes Below the Neutral Axis

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***Abstract*— This research presents the reliability of reinforced concrete beam with embedded PVC pipes below the neutral axis. First Order Reliability Method (FORM) and the theory of Statistics are reviewed and adopted for designing the beam to a pre-determined safety level using a FORTRAN subroutine created and linked with the reliability software (FORM5). Experimental investigation of beams without PVC pipe (RCBM) and beams with one (RCPVC1), two (RCPVC2) and three (RCPVC3) PVC pipes were carried out using ASTMC 293. Results indicate satisfactory performance of RCPVC1 and RCBM with similar ultimate failure load. Other beams have reduced failure load. Reliability analysis using FORM5 revealed area of reinforcement 38.9% higher than deterministic design for RCBM and RCPVC1 beam with safety index, β=3.3-4.4 meeting the probabilistic code's requirement and same area of reinforcement for RCPVC2 and RCPVC3 using deterministic design. It is concluded that the method is suitable for application.**

***Keywords*— Beam, PVC Pipes, Reliability, FORM and Safety Index**

I. INTRODUCTION

Beams can be described as members that are mainly subjected to flexure and it is essential to focus on the analysis of bending moment, shear, and deflection. When the bending moment acts on the beam, bending strain is produced. The resisting moment is developed by internal stresses. Under positive moment, compressive strains are produced in the top of beam and tensile strains in the bottom.

Beams are capable of withstanding load primarily by resisting bending. Concrete beams are widely in use as roof supports not only in industrial and residential buildings but also in bridges. It is an efficient, economical, and widely used structural system. The bending force included into the material of the beam as a result of the external loads, own weight, span and external reactions to these loads is called a *bending moment.*

Beams are major structural elements in structures, other than slabs and columns. Standardized and optimized beams can significantly enhance safety and durability of structures. This requires special techniques to achieve standardized and optimized beams which can satisfy all the important design standards.

In addition, when the span of the building is increasing, deflection of beam and slab are more important. Therefore, the beam and slab thickness is on the increase. Increasing beam thickness makes the beam and slab heavier, and it leads to increased column and base size. Thus, it makes buildings consume more materials such as concrete and steel [1], [2].

In other to avoid these disadvantages caused by increasing self-weight of beams, reliability assessment of a system consisting of PVC pipes cast into the concrete to create a grid of void inside the beam is suggested with a major contribution to the objective of sustainable buildings. This beam system could optimize the sizes of vertical members like walls, columns and base by lightening the weight of beams.

Reinforced concrete is one of the most important building materials in the world and it is widely used in many types of engineering structures in different departments. The economy, the efficiency, the strength and the stiffness of reinforced concrete make it an attractive material for a wide range of structural applications [3]. To use concrete for construction it must satisfy the following conditions:

i). The structure must be strong and safe. The proper application of the fundamental principles of analysis, the laws of equilibrium and the consideration of the mechanical properties of the component materials should result in a sufficient margin of safety against collapse under accidental overloads.

ii). The structure must be stiff and appear unblemished. Care must be taken to control deflections under service loads and to limit the crack width to an acceptable level.

iii). The structure must be economical. Materials must be used efficiently, since the difference in unit cost between concrete and steel is relatively large.

Reinforced concrete structures are commonly designed to satisfy criteria of serviceability and safety. To ensure the serviceable requirement it is necessary to predict the cracking and the deflections of reinforced concrete structures under service loads. In order to assess the margin of safety of reinforced concrete structures against failure an accurate estimation of the ultimate load is essential and the prediction of the load-deformation behavior of the structure throughout the range of elastic and inelastic response is desirable.

Ultimate strength design of reinforced concrete building frames, slabs and beams is based on semi-empirical methods using results from extensive laboratory testing programs.

Deterministic design criteria for reinforced concrete beams attempt to guard against unforeseeable event by imposing factors of safety in the design equations. These safety factors are empirical values that are subject to certain uncertainties and as such may not result in safe and economic design.

The design domains are subjectively derived quantitative evidence of the uncertainly inherent in designs. Changes to either the tools or domains require a change to the design margin. Unfortunately, with less reliance on engineering Judgments, the traditional criteria often provide an undetermined level of safety and performance that experience has shown is not always adequate, even for traditional floor structural configurations. This inadequacy will be on the sensitive with the use of new design approaches beyond the traditional design domain, where implied assumptions in the criteria no longer apply, and with the increasing demand of multiple, competing design and performance objectives as focused for future hollow floor constructions.

The experienced engineering designer, while aiming for increased accuracy and perfection in his work, has always been conscious of the limitation in human knowledge, the unreliability of data about materials, and the approximation inherent in modeling and mathematical methods .Aware too that he will hear the blame for poor performance on "failures", he tend to be cautious and prefers to err on the side of safety. Unfortunately, demands for lower costs, higher performances, and enhanced efficiency, lower weight- to power ratio, more sophisticated Computer-aided analytical techniques as well as the demand for higher reliability assurance have increased the pressure on engineering designers to increase the safety margin of his design.

The old-fashioned "factor of safety" which in reality does not represent the actual operating conditions and true performance capabilities, was use to provide a sensible reserve against such unknowns and was usually based on years of accumulated in-service experience. Today engineers are sentient that reliability is associated with risk taking, cost levels, life pattern and life expectancy.

Reliability is a diverse field of study that covers all aspect of life, from our daily household to the most sophisticated structure of the modern world. The importance of reliability has been in the increase in the field of structural engineering. Every user of a structure looks for the most reliable one in order to avoid unexpected failures, which might involve large amount of money for repairs or total collapse.

Reliability has been defined in different ways by a number of internationally respected bodies. The most prominent among these bodies are the United Kingdom ministry of Defense [5] which defines reliability as "the ability of an item to perform, or to be capable of performing, a required function without Failure under Stated Condition for a stated period of time on unit of operation. Reliability is usually specified in terms of probability of failure.

II. REVIEW OF LITERATURE

Reliability and deterministic approaches to design differ in principle. Deterministic design is based on total discounting of the occurrence of failure. Partial factors of safety are used to cater for these uncertainties. On the other hand, reliability design is concerned with the probability that the structure will realize the functions assign to it. It is a measure of the ability of the structure to perform, or to be capable of performing, a required function without Failure under Stated Condition for a stated period of time on unit of operation. Reliability is usually specified in terms of probability of failure [4].

A common measure of reliability of structural members is through safety index (β). This is expressed in terms of resistance (R) and load effect (S) of the structure. R and S are random variables. The purpose of reliability analysis of any system or component is to ensure that R is greater than or equal to S. In practice, R and S are usually functions of different variables. In order to evaluate the effect of the, variables on the performance of the structural system, a limit state equation is required. This limit state equation is called performance function and expressed in the form:

$$g(x_i) = g(x_1, x_2, x_3 \ldots\ldots, x_n) = R - S \ldots\ldots\ldots\ldots\ldots \quad (1)$$

Where, n= 1,2,3,4,………………

The limit state is expressed as:

$$g(x_i) = 0 \ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

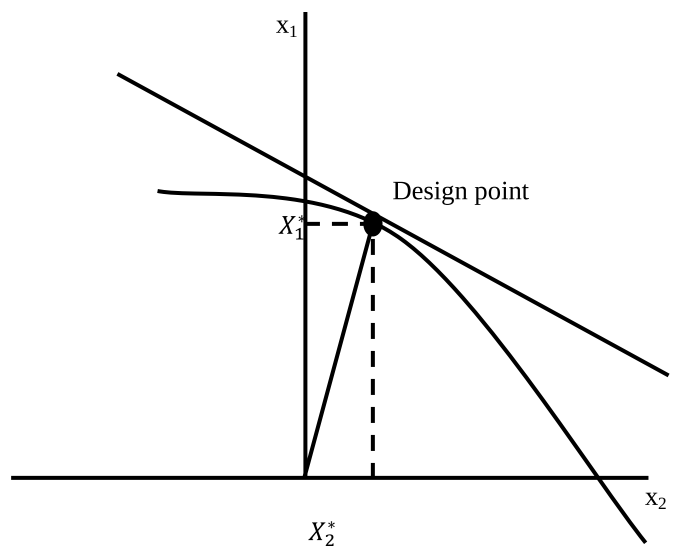


Fig. 1: Hasofer – lind reliability index

For uncorrelated reduced variates,

$$x' = \frac{x_i - \mu_{xi}}{\sigma_{xi}} \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

Where $i = 1,2, \ldots\ldots\ldots\ldots\ldots .. n$

The limit state in terms of reduced variates is given by:

$$g(\sigma_{xi}x'_i + \mu_{xi}, \sigma_{x2}x'_2 + \mu_{x2} \ldots\ldots\ldots\ldots. \sigma_{xn}x'_n + \mu_{xn}) \ldots. \quad (4)$$

Where, μ and σ are the means and standard deviations of the design variables. The distance D, from a point $x'_i = (x'_1, x'_2, x'_3, \ldots, x'_n)$ on the failure boundary $(g) = x'_i$ to the origin of x'_i space is given by:

$$D = \sqrt{x'^2_1 + x'^2_2 + \cdots \ldots + x'^2_n} \ldots \quad (5)$$

Equation 4 and 5 can be solved by transforming it into vector gradient:

$$G'x' = -\frac{G'GD}{\sqrt{(G'G)}} = -(G'G)^{\frac{1}{2}}D \ldots \quad (6)$$

$$D = -\frac{G'x'}{\sqrt{(G'G)}} \ldots \quad (7)$$

The minimum distance from the origin describing the variable space to the line representing the failure surface equals β and equation (7) becomes:

$$\beta = -\frac{G^{*\prime}x^{*\prime}}{\sqrt{(G'G)}} \ldots \quad (8)$$

Where $G^{*\prime}$ is the gradient vector at the most probable failure point $(x^*_1, x^*_2, x^*_{3,}\ldots, x^*_n)$ and the value of safety index, β is the measure of the safety of any given design under uncertainties in the decision variables. Therefore equation (8) can be represented in scalar form as:

$$\beta = \frac{\sum_i x^{*i}\left(\frac{\partial g}{\partial x'_i}\right)}{\sqrt{\sum_i\left(\frac{\partial g}{\partial x'_i}\right)}} \ldots \quad (9)$$

evaluated at design point

Equation (9) can be truncated at first order linear term and simplified to:

$$\beta = \frac{\mu_g}{\sigma_g} \ldots \quad (10)$$

III. MATERIALS

The materials used in this research include:

1) Coarse Aggregate

Coarse Aggregate consists of large chunks of materials in a concrete mix. Generally, coarse gravel or crushed rocks such as limestone or granite are commonly used. For the purpose of this research, aggregate size of 20 mm was used.

2) Fine Aggregate

Sand is the product of natural or artificial disintegration of rocks and minerals. Sand is an important constituent of concrete and is extremely abundant as a surface deposit along the course of rivers, on the shores of lakes, seas and in arid regions. Sharp sand used in the concrete mixture was obtained from Ado Ekiti, Ekiti state.

3) Cement

Cement is a binding material which binds the coarse and fine aggregate together and also solidifies and hardens the concrete through a chemical process called hydration. The cement used in this research is the Lime Portland Cement (Dangote product) of grade 42.5 which is the most common type available.

Fig. 2: Coarse aggregates

Fig. 3: Fine aggregates

4) Water

Water is mixed with the dry powder and aggregates which produces a semi-liquid that can shape typically by pouring it into a mould. The strength and workability of concrete depends greatly on the amount of water used in mixing. The purpose of using water is to cause the hydration of cement. Water to be used for the production of concrete must be free of suspended particles, inorganic salts, acids and alkalis, oil contamination and algae. ABUAD water was used for this research.

5) PVC Pipes

PVC (polyvinyl chloride) is a common, strong but lightweight plastic used in construction. It is made softer and more flexible by the addition of plasticizers. The rigid form of PVC is used in construction for pipe and in profile applications such as doors and windows. The PVC pipe of 50 mm diameter was used in this research work.

6) Steel Bars

Concrete can be formulated with high compressive strength, but always has lower tensile strength. For this reason it is usually reinforced with materials that are strong in tension, often steel. Reinforcing bar or rebar is used for the concrete reinforcement. Steel rebar were used as a tensioning devise to reinforce concrete. The various sizes of reinforcement bar used include $Y5.5, Y10$.

Fig. 4: Reinforcement cage with shear links

Fig. 5: 50mm diameter PVC pipes

IV. METHODS

The proposed study was conducted in three phases namely; experimental approach, deterministic and Reliability analysis.

A) Experimental Approach

The experimental procedure required the production of 56 beams of size 1000 x150 x 150 mm. Eight number (8 Nos.) of reinforced concrete beams with one PVC pipe (0.00196m^3) and specimen label RCPVC1, eight number (8 Nos.) of reinforced concrete beams with two PVC pipes (0.00393m^3) and specimen label RCPVC2, eight number (8 Nos.) of reinforced concrete beams with three PVC pipes (0.00442m^3) and specimen label RCPVC3 and eight number (8 Nos.) of reinforced concrete beams and specimen label RCBM respectively. Fig. 6 – Fig. 8 shows the casting of the beams and curing process.

Fig. 6: Beam formwork with and without PVC pipes

Fig. 7: Casting of beams with and without PVC pipes

Fig. 8: Curing of beams in a curing tank

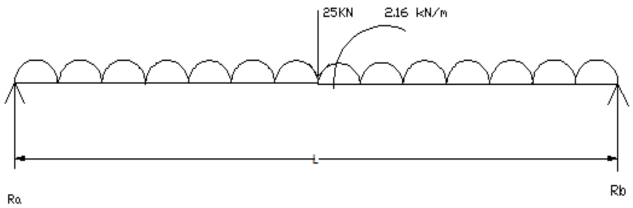
The experimental set up using ASTM C 293 centre point loading system is shown in Fig 9. Load at first crack, strain and ultimate load at failure were measured as shown in Fig. 10.

Fig. 9: Experimental setup

Fig. 10: Failure of beam under applied maximum load

B) Deterministic Design

Deterministic designs were carried following the procedure [8] on beams with and without PVC pipes using grade C25 concrete with the following procedure below:



Loading

Beam self weight= $= 0.15 \times 0.15 \times 24 = 0.54\ kN/m$

Finishes = 1.0 kN/m

Total = 1.54 kN/m

Design Load $= 1.4G_k = 1.4 \times 1.54 = 2.16\ kN/m$

Moment, $M = \frac{PL}{4} + \frac{wL^2}{8} = \frac{25\times0.9}{4} + \frac{2.16\times0.9^2}{8} = 5.84\ kNm$

$R_a = R_b = \frac{P}{2} + \frac{wl}{2} = 13.47\ kN$

$k = \frac{M}{bd^2 f_{cu}} = 0.1046$

$d = 150 - 20 - 5 - 3 = 122$

$I_a = 0.5 + \sqrt{0.25 - \frac{0.1046}{0.9}} = 0.865$

$z = I_a d = 105.5\ mm$

$A_s = \frac{5.84 \times 10^6}{0.95 \times 410 \times 105.5} = 142\ mm^2$

Provide

$A_s = 226\ mm^2$

$A_{smin} = 0.15\frac{bd}{100} = 0.15 \times 150 \times \frac{150}{100} = 33.75\ mm^2$

C) Reliability Analysis

Reliability analysis was carried out using FORM5 reliability software with subroutine programs RCPVC1, RCPVC2, RCPVC3 and RCBEAM for limit state in bending; NDEFLECT for limit state deflection and RCPVC1S, RCPVC2S, RCPVC3S, RCBEAMS for limit state in shear. All subroutines are written in FORTRAN to generate the safety indices (β) as shown in Fig. 11 – Fig. 13.

The reliability analysis was carried out by determining the means, standard deviations, coefficient of variations and the distributions of the basic variables. The probabilistic model code [9] specified that:

(a) Material properties such as concrete strength characteristic strength etc. are treated as log-normal distribution

(b) Geometric properties are modeled as normal or log-normal distribution

(c) Load considered in this research is permanent loading and are such treated as normal-distribution. The imposed load will be treated as lognormal

Limit State

The limit states considered in this research are:

- Bending,
- Shear, and
- Deflection on beams

1) Bending

For a beam in bending, the nominal resistance is given by:

$$R_n = A_s\ f_y\left(d - 0.59\frac{A_s\ f_y}{f_c^{'} b}\right) = A_s\ f_y d - 0.59\frac{(A_s\ f_y)^2}{f_c^{'} b}$$

Source: [7]

The beam is examined for the limit state exceeding the beam capacity in bending. The performance function or limit state would be:

$$g\left(A_s,\ f_y, f_c, M, b, d\right) = A_s\ f_y d - 0.59\frac{(A_s\ f_y)^2}{f_c^{'} b} - M$$

Where,

M is the moment (load effect) due to the applied load.

A_s = area of reinforcement

f_y = characteristics yield strength of steel

f_s = concrete strength.

2) Shear

For a beam in shear, the code [8] specifies:

$$v = \frac{V}{b_V\ d} \quad (1)$$

Where,

v = shear stress

V = Shear force

b_v = width

d = Effective depth of beam

The limit state is given by:

$$g\left(v_C, b_V, d, f_{yV}, A_{SV}, S_V, V\right) = v_C b_V d + 0.95 f_{yV}\frac{A_{SV}}{S_V} d - V \quad (2)$$

3) Deflection

The code specifies:

Limiting deflection $\alpha\ \frac{Allowable\ span}{effective\ depth} = 20 \times m.f_{TS} \times m.f_{CS}$

Actual deflection $\alpha\ \frac{Actual\ span}{effective\ depth}$

$$m.f_{TS} = 0.55 + \frac{\left(477 - \frac{2}{3} f_y \frac{As}{A_{sp}}\right)}{120\left(0.9 + \frac{M}{b_v d^2}\right)}$$

$$M = \frac{wl^2}{8} + \frac{Pl}{4}$$

Therefore, the limit state in deflection is given by:

$$g(d, f_y, A_s, A_{sp}, P, w, b_v) = d - \frac{l}{20 \times \left[0.55 + \frac{\left(477 - \frac{0.67 f_y A_s}{A_{sp}}\right)}{\left(108 + \frac{30Pl + 15wl^2}{b_v d^2}\right)}\right]} \quad \text{.. .....(13)}$$

The stochastic models for the basic variables in the different limit state will be calculated from equation 10-13.

$$COV = \frac{S(X)}{E(X)} \text{ ....(14)}$$

$$E(X) = N \times \lambda \text{ ....(15)}$$

$$S(X) = COV. \times E(X) \text{ ....(16)}$$

Where,

COV = Coefficient of variation of the basic variables,

S(X) = Standard deviation of the basic variables,

E(X) = Mean of the basic variables,

$\lambda =$ Bias factor of the basic variables

N = Nominal values of the basic variables obtained from the deterministic analysis of the beam. The coefficient of variation and the bias factor are computed accordingly.

Reliability Analysis Using Manual Method:

The Reliability analysis using equations 1-10 was carried manually as shown below and results compared with that generated from the Reliability software (FORM5).

The procedures for the manual analysis using FORM are itemized below:

Data

$\mu_A = 201\ mm^2$, $\sigma_{As} = \mu_{As}.CoV_{As} = 201 \times 0.02 = 4.02$

$\mu_{fy} = 410 N/mm^2$, $\sigma_{fy} = \mu_{fy}.CoV_{fy} = 410 \times 0.098 = 40.18$

$\mu_M = 6820000\ N\ mm$, $\sigma_M = \mu_M.CoV_M = 6540000 \times 0.12 = 818899 Nmm$

$\mu_z = 111\ mm$, $\sigma_z = \mu_z.CoV_z = 111 \times 0.1 = 11.1\ mm$

1st iteration

The limit state in bending is given by:

$g = (A_s, f_y, z, M) = 0.95 A_s A f_y\ z - M$, evaluated at mean values

At g=0,

$$X_z = \frac{\mu_M}{0.95 \mu_{As} \mu_{fy}} = 87.11\ mm$$

(a) Determining the reduce variates in the form

$$Z = \frac{X - \mu}{\sigma}$$

$$Z_{As} = \frac{X_{As} - \mu_{As}}{\sigma_{As}} = 0$$

$$Z_{fy} = \frac{X_{fy} - \mu_{fy}}{\sigma_{fy}} = 0$$

$$Z_M = \frac{X_M - \mu_M}{\sigma_M} = 0$$

$$Z_z = \frac{X_z - \mu_z}{\sigma_z} = -2.152$$

(a) Determining the vector $\{G\}$

$$G_1 = -\frac{\partial g}{\partial A_s}\sigma_{As} = -0.95 X_{fy} X_z \sigma_{As} = -0.95 \times 410 \times 87.11 \times 4.02 = -136400$$

$$G_2 = -\frac{\partial g}{\partial f_y}\sigma_{fy} = -668360$$

$$G_3 = -\frac{\partial g}{\partial M}\sigma_M = \quad 818496$$

$$G_4 = -\frac{\partial g}{\partial z}\sigma_z = - \ 869013$$

Reliability index, β= $\frac{\{G^T\}\{z\}}{\sqrt{\{G^T\}\{G\}}} = 1.36$

<u>Iteration 2</u>

$$\alpha_1 = \frac{G_1}{\sqrt{\{G^T\}\{G\}}} = -0.0992$$

$$\alpha_2 = \frac{G_2}{\sqrt{\{G^T\}\{G\}}} = -0.4861$$

$$\alpha_3 = \frac{G_3}{\sqrt{\{G^T\}\{G\}}} = 0.5953$$

$$\alpha_4 = \frac{G_1}{\sqrt{\{G^T\}\{G\}}} = -0.6320$$

(b) Determining new design point in reduce variate

$Z_{11} = \alpha\beta = -0.1349$
$Z_{12} = -0.6612$
$Z_{13} = \ \ 0.8097$
$Z_{14} = -0.8597$

(c) Determining design point in original coordinates

$X_{11} = \mu_{As} + Z_{11}\sigma_A = 200.1$
$X_{12} = 383.43$
$X_{13} = 6820000$

$$X_{14} = \frac{X_{13}}{0.95 X_{11} X_{12}} = 102.48\ mm$$

(a) Reliability index, β= $\frac{\{G^T\}\{z\}}{\sqrt{\{G^T\}\{G\}}} = 1.33$

The procedures are continued until β- value converges.

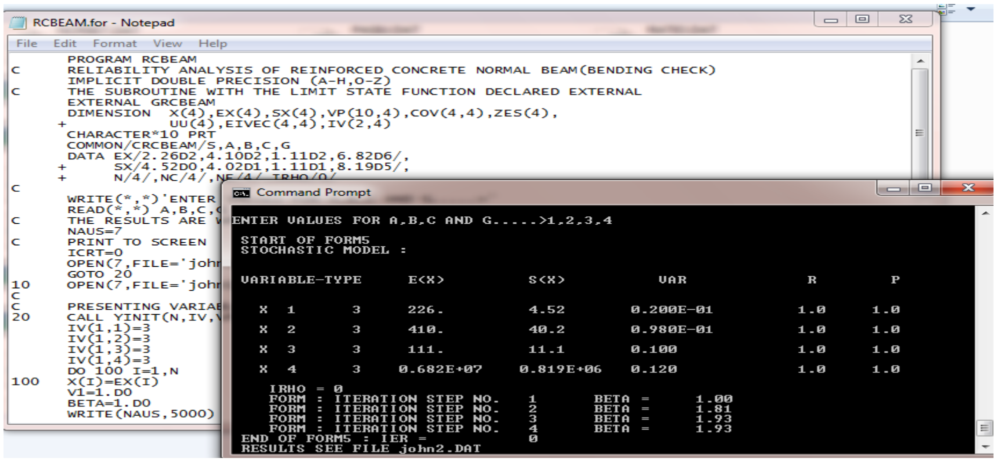


Fig. 11: Reliability Analysis using FORM5 for limit state in bending

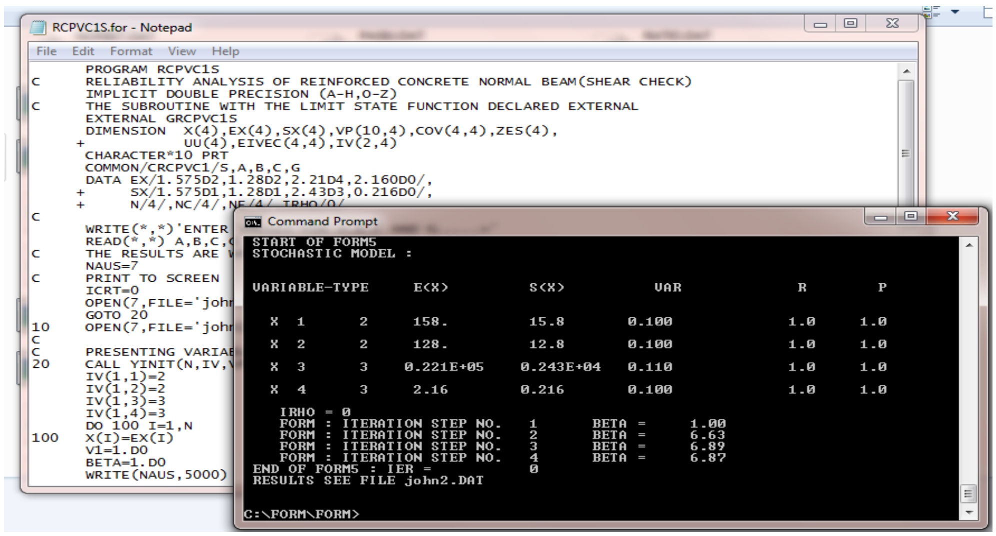


Fig. 12: Reliability Analysis using FORM5 for limit state in shear

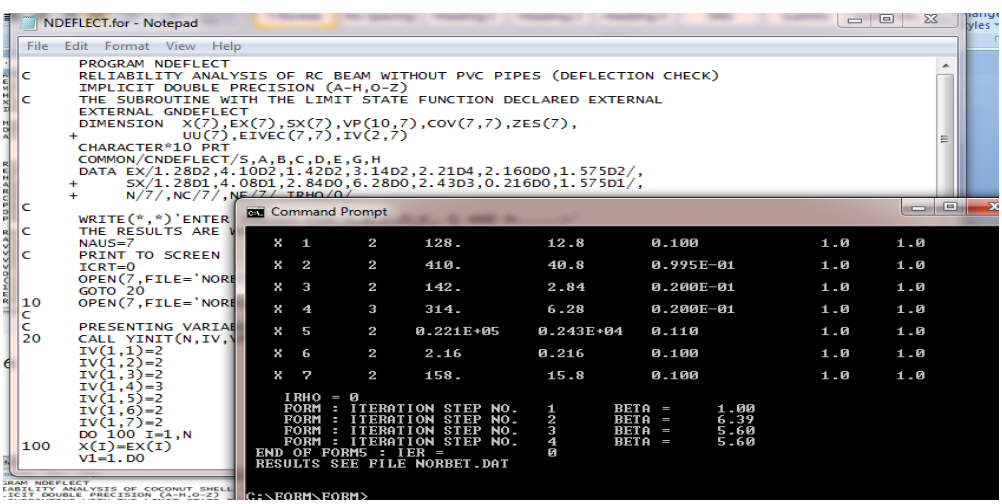


Fig. 13: Reliability Analysis using FORM5 for limit state in deflection

V. RESULTS AND DISCUSSION

Table I shows the failure load, stress, strain and deflection of each specimen.

The results for the reliability analysis using the reliability software (FORM5) and manual method is presented in Table II for limit state in bending, shear and deflection.

TABLE I: EXPERIMENTAL FAILURE LOAD AND STRESSES

S/No	Beam Specimen	Faillure load (kN)	Stress (Mpa)	Strain x 10^{-2}	Deflection (mm)
1	RCBM	26	59.9	1.56	14
2	RCPVC1	25	62.3	1.10	9
3	RCPVC2	15	38.3	1.06	7
4	RCPVC3	15	38.3	0.67	6

TABLE II: RELIABILITY ANALYSIS USING FORM AND MANUAL COMPUTATION

	Reliability Analysis (FORM)			**Reliability Analysis (Manual)**				
SAMPLE	β			β			m^3	%
	Bending	Shear	Deflection	Bending	Shear	Deflection	Volume	reduction
RCBEAM	*1.93,5.04	6.87	*5.40,5.66	*1.98,5.05	6.53	*5.43,5.68	0.02025	-
RCPVC1	*2.16,5.27	6.99	*5.43,5.69	*2.19,5.25	6.62	*5.48,5.71	0.01829	9.67
RCPVC2	*4.81,7.92	8.11	*5.78,6.06	*4.87,7.18	7.53	*5.76,6.09	0.01632	20.00
RCPVC3	*4.81,7.16	8.18	*5.81,6.08	*4.87,7.17	7.57	*5.83,6.12	0.01583	21.80

* Safety index (β) obtained using A_s=226mm^2, - Safety index(β) obtained using A_s=402mm^2

TABLE III: COMPARISON BETWEEN RELIABILITY AND DETERMINISTIC DESIGN

	Reliability Analysis(FORM)				**Deterministic Design**			
SAMPLE	β							
	Bending	Shear	Deflection	Area (mm^2)	Bending	Shear	Deflection	Area (mm^2)
RCBEAM	3.83	6.87	5.60	314(4Y1001)	ok	ok	ok	226(2Y1201)
RCPVC1	3.31	6.98	5.61	314(4Y1001)	ok	ok	ok	226(2Y1201)
RCPVC2	4.81	8.11	5.85	226(2Y1201)	ok	ok	ok	226(2Y1201)
RCPVC3	4.81	8.17	5.88	226(2Y1201)	ok	ok	ok	226(2Y1201)

TABLE IV: VARIATION OF SAFETY INDEX WITH LENGTH OF BEAM SPECIMEN

	Samples	**RCBM**	**RCPVC1**	**RCPVC2**	**RCPVC3**
S/No	Length		β		
1	500	7.64	6.93	6.22	5.99
2	750	6.27	5.48	4.71	4.46
3	1000	4.68	3.94	3.2	2.96
4	1250	3.39	2.66	1.96	1.66
5	1500	2.31	1.62	0.92	0.7
6	1750	1.39	0.71	0.036	-0.18
7	2000	0.59	-0.86	-0.73	-0.95
8	2500	-0.76	-1.41	-1.54	-2.2

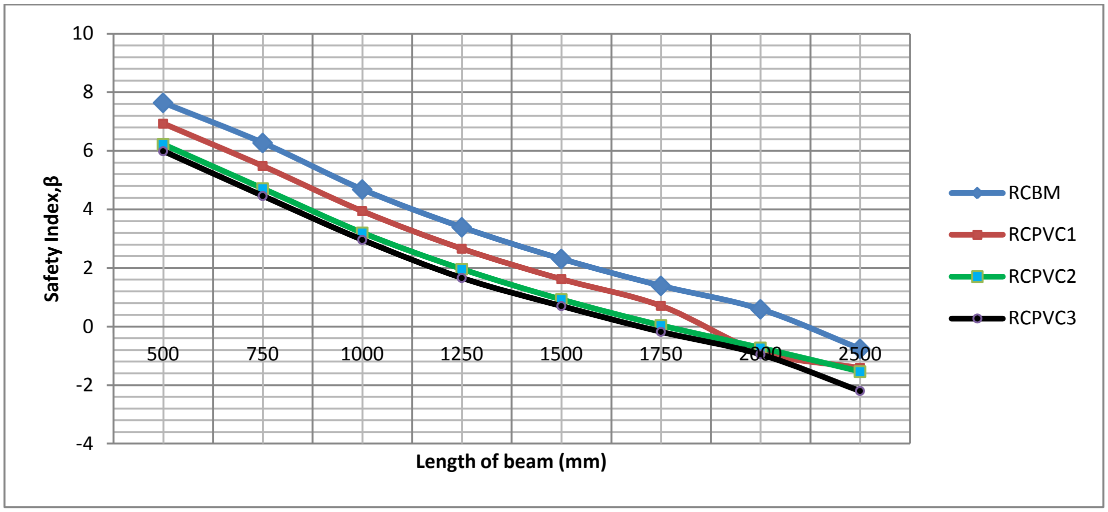


Fig. 14: Graph of Reliability index against length of beams

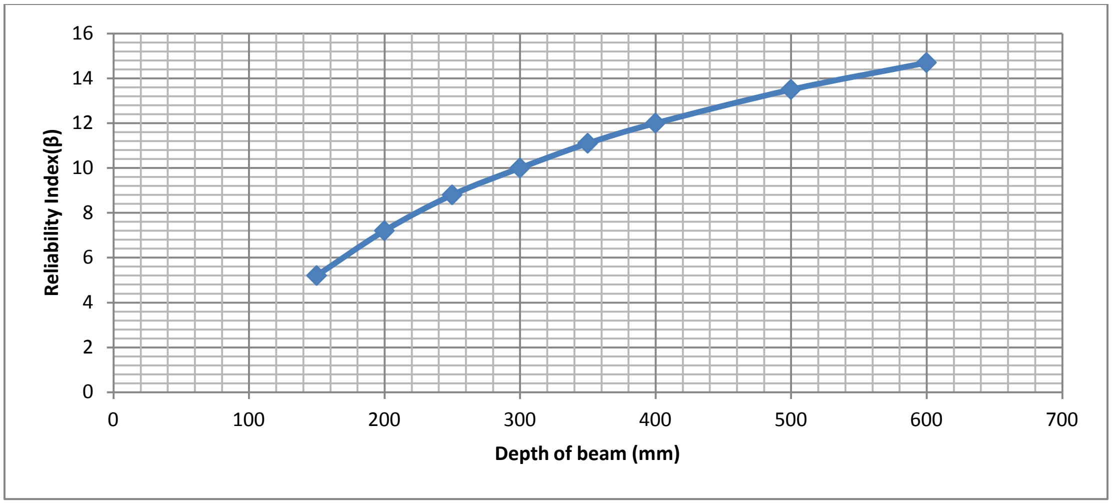


Fig. 16: Variation of reliability index with depth of beam

Table V: TENTATIVE TARGET RELIABILITY INDICES (AND ASSOCIATED TARGET FAILURE RATES)

1	2	3	4
Relative cost of safety measure	Minor consequences of failure	Moderate consequences of failure	Large consequences of failure
Large (A)	β=3.1 (p_F≈10^{-3})	β=3.3 (p_F ≈ 5 10^{-4})	β=3.7 (p_F ≈ 10^{-4})
Normal (B)	β=3.7 (p_F≈10^{-4})	β=4.2 (p_F ≈ 10^{-5})	β=4.4 (p_F ≈ 5 10^{-6})
Small (C)	β=4.2 (p_F≈10^{-5})	β=4.4 (p_F ≈ 5 10^{-6})	β=4.7 (p_F ≈ 10^{-6})

Source: [9]

Where,
RCBEAM= Beam without embedded PVC pipe
RCPVC1= Beam with one embedded PVC pipes below neutral axis.
RCPVC2= Beam with two embedded PVC pipes below neutral axis.
RCPVC3= Beam with three embedded PVC pipes below neutral axis.

A) Discussion of Results

a) Reinforced concrete beams with one embedded PVC pipes (RCPVC1) indicates maximum deflection at the centre of magnitude $\delta = 9$mm and maximum stress of 62.3 MPa. The strain reading ranges between 0.0044-0.010 and maximum failure load of 25kN. The failure pattern observed was mainly bending with visible cracks at support and at loading point.

b) The failure pattern of RCPVC2 and RCPVC3 beams was similar in nature to that of RCPVC1 but with lesser cracks at support and at loading point. The beams failed in both shear and bending. They both have maximum stress of 38.32MPa and maximum ultimate load of 15kN. Maximum deflection at the centre are; $\delta = 7$ mm and 6mm respectively. Strain ranges between 0.0033-0.0078.

c) For reinforced concrete beams without PVC pipes (RCBM), results indicate maximum deflection of $\delta = 14$ mm and maximum stress of 64.71 MPa. Strain ranges between 0.0067-0.0156 and maximum failure load of 26kN. Failure was mainly in bending with visible crack pattern at supports and at loading area.

d) Deterministic design yields an area of reinforcement of 226mm^2 (2Y1201) for all beams and seemed to be safe in bending, shear and deflection in accordance with design based on [8]. But, reliability analysis using the first order reliability method (FORM) and [9] revealed an area of reinforcement of 314mm^2 (4Y1001) for RCBM and RCPVC1 and 226mm^2 (2Y1201) for RCPVC2 and RCPVC3. Each given a safety index (β=3-4) within the specified code [9] and depending on the consequences of failure. Reliability analysis was also performed using the manual computation (Hasofer Lind) and results compared with analysis using reliability software (FORM5). Table V shows the comparison.

e) Fig. 14 shows the variation of safety index with the length of beam. As the length of beam increases, the safety index decreases. Fig. 15 also shows the variation of depth of beam with safety index.

VI. CONCLUSION

a) Experimental investigation revealed that RCBM and RCPVC1 can withstand approximately the same stress. Therefore, RCPVC1can replace RCBM due to its economy, strength and the PVC pipe can serve the purpose of conduit pipes for electrical wire installation.

b) Deterministic design following the [8] yields an area of reinforcement within the specified limits but may not give the required safety (since optimum design point is not known). Reliability analysis on the other hand, gives a specified safety index (pre-determined safety index) according to [9] which [8] cannot give.

c) Reliability analysis using the first order reliability method (FORM) revealed that RCBM and RCPVC1 are capable of resisting applied load effects on a structure at the same time enhancing structural safety of the beams. RCPVC1 beams will yield an economical design compared to RCBM beams in terms of volume reduction at the same time satisfying the condition of safety (β= 3-4) recommended by [9] when considering moderate consequences of failure.

d) Reliability analysis using FORM is very essential in all engineering designs to predetermine safety of structural elements even before they are constructed. It is therefore recommended in design processes to ascertain safety level of structural elements whether they satisfy code's requirements or not.

ACKNOWLEDGMENT

The authors would like to express their gratitude to ABUAD and UNILORIN for their support in carrying out this research.

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