# Income Smoothing as Rational Equilibrium Behavior? A Second Look

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**Abstract**

In this paper I revisit the issue of real income smoothing in the setting used by Lambert (1984). I demonstrate that the particular effect identified in his paper is actually an error: under his assumptions there is no input driven equilibrium income smoothing of the type he suggests. There are, however, several other drivers of equilibrium behavior ignored in that paper. In this paper I identify those and for the particular model structure show that when all effects are considered together there is little support for the suggestion that second-best earnings generally is being smoothed through the equilibrium behavior

# 1 INTRODUCTION

One of the earliest formal results in the Accounting Literature on (real) equilibrium earnings management is that of income smoothing provided by Lambert (1984). In a multi-period setting where the optimal first-best strategy is to implement the same expected earnings (i.e., "action") in every sub-period, the deviation in equilibrium behavior under the optimal second-best multi-period contract is not just a matter of lowering the effort level as in the similar one-period model variant, but also a matter of conditionality: in the second-best, future actions, in this class of models, generally depend on past earnings realizations. Lambert (1984) aimed to provide if not a general proof then a strong suggestion that such interdependencies would likely lead to less volatile earnings as low actions would follow high outcomes (and vice versa) and thus sub-period earnings would be mean-reverting, thereby depressing the aggregate volatility of earnings.

The fact that the result forwarded by Lambert (1984) has survived and been a key reference for over more than three decades may be attributable to the seemingly straightforward idea(s) and the intuition behind this result. Specifically, when a manager learns that "things" are on course to be better than initially expected, and thus that his total expected compensation and utility exceed his initial expectations, this manager may start to value leisure more relative to additional future compensation. Consequently, he may therefore choose to pull back a bit on future effort, causing the above-mentioned mean reversion. Because this does make some intuitive sense, the presence of negative auto-correlation in second-best earnings remains not only generally accepted as valid from a formal theoretical perspective, but also continues to be frequently cited by, in particular, empirical papers investigating issues related to managerial incentives for managing earnings.

It should be noted that Lambert (1984) is careful to point out that negative serial correlation between realized outcomes and future efforts leads to smoother earnings (in expectation) only if earnings is defined as the aggregate output of several periods (two in his case). This particular definition is not a focus of this paper, nor is it something I address directly.

Yet, while Lambert (1984) makes no attempt to extend the correlation result to alternate preference representations, he does argue that real (and perhaps also accounting) income-smoothing is a natural if not general property of the second-best to the point where the behavior should be considered empirically relevant. The results and insights provided in this paper make clear that this line of thinking is neither complete nor correct.

While my analysis (coincidentally) does expose the error(s) contained in Lambert (1984), the overall purpose here is to give a more detailed understanding of all earnings-related properties that can reasonably be predicted by a second-best agency model of the specific type explored by Lambert (1983 & 1984). In doing so, I make several points that should significantly change the status-quo thinking on this issue. As the starting point, I first establish that the proof of the Proposition in Lambert (1984) is incorrect for a number of reasons. Perhaps most significantly, Lambert (1984) implicitly over-constrains the problem in such a way that one of the key effects that potentially *does* lead to an equilibrium relation between past outcomes and future efforts, is disallowed from the set of feasible solutions and is therefore absent from his analysis.[1] This particular effect, which I refer to as the "intertemporal incentive effect" in this paper, consists of inducing outcome contingent variations in *future* (costly) workload to reduce the costly variations in future pay needed to incentivize current efforts.[2]

To establish "smoothing" as part of second-best equilibrium behavior, Lambert's (1984) proof instead relies on wealth effects argued to result from "memory" in the optimal contract. However, for this particular class of multi-period full-commitment models, the cost of providing incentives in any given sub-period is actually independent of updates to the agent's expected utility during the contracting horizon if (and only if) the agent has a power utility function where the power is one half. Therefore, as I also show, absent the above-mentioned intertemporal incentive effect, the optimal second period action for the Lambert

[1] This problem actually originates in Lambert (1983). See equation (9) on p. 445.

[2] As I show, partially rewarding (penalizing) the agent for good (bad) outcomes using reduced (increased) future workload is always optimal in this type of model.

(1984) preference specification, is actually independent of prior outcomes even if the second period's compensation is not.

Because the particular model formulation used in Lambert (1984) actually represents the case in which wealth-effect driven real earnings management does *not* take place, it also provides the cleanest setting for characterizing the real source of equilibrium demand for outcome contingent effort choice in this class of models: lowering the cost of implementing *prior* periods' actions, i.e., the intertemporal incentive effect. Specifically, the cost of having to work harder/less hard in the future represents a penalty/reward to the agent that provides current incentive just as getting a smaller/bigger bonus in the future does. As I show, splitting current incentives between variations in future compensation and variation in future (costly) work-loads is always efficient regardless of the specifics of the principal's and the agent's respective utility functions.[3]

While the intertemporal incentive effect is one-directional in the sense that, *on average*, second period effort is lower for positive first-period output-surprises than for negative output-surprises and thus, again on average, favors of the behavior suggested by Lambert (1984), the relation between current actions and past results due to the intertemporal incentive effect is generally non-monotonic unlike the wealth driven effect proposed by Lambert (1984). Moreover, the equilibrium relation between current actions and past results is determined jointly by *both* the wealth and the intertemporal incentive effect.[4] Absent the latter, wealth-effects drive the relation between past outcomes and present efforts, but there is no particular natural prediction here. For agents with utility functions for which aversion to risk, properly defined, decreases in wealth, the basic incentive is to make equilibrium effort an increasing function of past outcomes whereas the opposite is obviously the case when risk aversion is increasing in past outcomes. On top of that, this is conditional on the princi-

[3]This is true within the class of models with time additive preferences where the agent's cost of effort is denominated in utiles.

[4]It is also important to note that unlike the effect proposed by Lambert (1984), the equilibrium relation between first-period outcome and second-period effort is generally *not* monotone and thus not easily interpreted as smoothing behavior.

pal being risk neutral. With a risk-averse principal, the equilibrium implications of wealth effects, while clearly central here, become even more intractable.

Lastly, regardless of the (net) equilibrium relation between current actions and past outcomes, equilibrium actions in this type of model are in general a function of time: expected second-best effort is declining period-by-period and expected income is therefore also declining over time. This general effect of moral hazard on the time series properties of earnins is also missing from Lambert (1984) who instead suggests that if actions are not allowed to be outcome dependent, they would actually be *constant* over time. To the contrary, I show that the time-dependent decline in expected earnings is robust to the specification of the agent's preferences. More importantly, it is generally at odds with standard definitions of, motives for, or causes of income smoothing even in cases where the agent's preferences are such that the behavior suggested by Lambert (1984) actually is part of the equilibrium.

The remainder of this paper proceeds as follows. In the next section, the model and the notation used here are laid out. In section 3 the model is solved and the structure of the solution is compared with that provided by Lambert (1983 & 1984). Section 4 then identifies the unrelated and previously ignored features of the model that do make the time-series behavior of the second-best deviate from that of the first-best. Robustness of the drivers of second-best time series properties of earnings to some central model specifications is explored in section 5. Finally, concluding remarks are contained in section 6.

# 2 MODEL

For simplicity and for ease of comparison with Lambert (1984), in this paper I will concentrate on a simple two period version of the model introduced in Lambert (1983). Also in the interest of familiarity and comparability, I mainly adapt the notation of Lambert (1984). Accordingly, a risk-neutral principal, who values his end of horizon aggregate residual by the linear function $g(y) = y$, contracts with a risk- and effort-averse agent for $T = 2$ (sub-

) periods. The objective of doing so is for the agent to favorably impact the period $t \in \{1,2\}$ cash-flow probability distribution $f(x_t|a_t)$, where $x_t \in X$ is the realized (and immediately observed) cash-flow for period $t$, and $a_t \in A \subseteq R$ is the effort committed by the agent at the start of period $t$. The contract specifies the compensation paid to the agent at the end of each period $t \in \{1,2\}$ as a function of everything observed up to that point in time. Let $\overrightarrow{x}_t$ denote the vector of realized cash flows up to and including period $t$. The agent's period $t$ compensation then is denoted as $s_t(\overrightarrow{x}_t)$ .The agent is assumed to be risk-averse and have time additive preferences for consumption of the form $u\left(\{s_t\}_{t=1}^2\right) = \sum_{t=1}^2 u\left(s_t\left(\overrightarrow{x}_t\right)\right)$. Similarly, his (convexly increasing) cost of all efforts exerted at the start of each sub-period $t$ are time additive as well and thus takes the form $v\left(\{a_t\}_{t=1}^T\right) = \sum_{t=1}^T v\left(a_t\left(\overrightarrow{x}_{t-1}\right)\right)$, where $\overrightarrow{x}_0 = \emptyset$.

Denote by

$$G_t(s_t, a_t) \equiv \int \left[x_t - s_t\left(\overrightarrow{x}_t\right)\right] f\left(x_t, a_t\left(x_{t-1}\right)\right) dx_t \tag{1}$$

and

$$H_t(s_t, a_t) \equiv \int u\left(s_t\left(\overrightarrow{x}_t\right)\right) f\left(x_t, a_t\left(x_{t-1}\right)\right) dx_t - v\left(a_t\left(x_t\right)\right) \tag{2}$$

the principal's and agent's respective expected period $t$ utilities at the start of period $t$ calculated, in case of period 2, after $x_1$ has been realized.[5] Also let the principal's total expected (net) utility as of the time of contracting be denoted by

$$\begin{aligned} G(\boldsymbol{s}, \boldsymbol{a}) &\equiv G_1(s_1, a_1) + EG_2(s_2, a_2) \\ &= \int \left[x_1 - s_1\left(\overrightarrow{x}_1\right)\right] f(x_1, a_1) dx_1 \\ &\quad + \int \int \left[x_2 - s_2\left(\overrightarrow{x}_2\right)\right] f\left(x_2, a_2(x_1)\right) dx_2 f(x_1, a_1) dx_1, \end{aligned}$$

and similarly let the agent's total expected (net) utility as of the time of contracting be

[5] Note that (2) implies that the agent's utility is additively separable in utility of consumption and disutility from the cost of effort.

denoted by

$$
\begin{aligned}
H(\boldsymbol{s},\boldsymbol{a}) &\equiv H_1(s_1,a_1) + EH_2(s_2,a_2) \\
&= \int u(s_1(\overrightarrow{x}_1)) f(x_1,a_1)\,dx_1 - v(a_1) \\
&\quad + \int\int [u(s_2(\overrightarrow{x}_2)) f(x_2,a_2(x_1))\,dx_2 - v(a_2(x_1))] f(x_1,a_1)\,dx_1.
\end{aligned}
$$

As in Lambert (1984) both parties are assumed able to fully commit to the contract agreed prior to the start of period one (hereafter with a slight abuse of notation denoted period $t = 0$).[6] At the time of contracting the agent has outside opportunities worth $\theta$ utiles should he not accept the long-run contract offered by the principal. As is always assumed in this particular class of models, the principal has free access to any needed liquidity. The agent, in contrast, has no personal means of intertemporal consumption transfers here, and, thus, can neither borrow nor save privately: all income physically received (i.e., paid which is different here from what is actually earned) by the agent by the close of period $t$ therefore goes towards creating utility for that period and that period alone. For simplicity, I ignore any discounting as the implications are largely trivial here. Finally, as in Lambert (1983 & 1984), the first-order approach is assumed to be valid with the standard implications for the differentiability e.t.c. of $f(x_t|a_t)$ and $v(\cdot)$ with respect to $a_t$.

[6] After first identifying the relevant effects under the same conditions as those used in Lambert (1984), I address the implications of relaxing the agent's ability to fully commit in section 5.

# 3 BENCHMARK

Given that the first-order approach is assumed to be valid here, the principal's problem can be summarized as

$$\max_{\mathbf{s},\mathbf{a}} \quad G(\boldsymbol{s},\boldsymbol{a}) \tag{PP}$$

$$s.t. \quad H(\boldsymbol{s},\boldsymbol{a}) \geq \theta \tag{IRP}$$

$$H^{a_1}(\boldsymbol{s},\boldsymbol{a}) = 0 \tag{IC1P}$$

$$H^{a_2(x_1)}(\boldsymbol{s},\boldsymbol{a}) = 0 \quad \mathit{for\ each}\ x_1, \tag{IC2P}$$

where the super-scripts $a_1$ and $a_2(x_1)$ as usual denote the derivatives with respect to these choice variables. Let $\lambda$ be the multiplier on the agent's participation- or $IR$-constraint, $\mu_1$ be the multiplier on the first period incentive compatibility constraint, and $\mu_2(x_1)$ be the multiplier on the second period $IC$-constraint corresponding to the realized value of first period output. As is known from the initial literature detailing the solution to this class of models based on the first-order approach,[7] the optimal period 1 and 2 contracts for this case of a risk-neutral principal must satisfy the first order conditions

$$\frac{1}{u'(s_1(x_1))} = \lambda + \mu_1 \frac{f_a(x_1|a_1)}{f(x_1|a_1)}, \tag{3}$$

$$\frac{1}{u'(s_2(\overrightarrow{x}_2))} = \lambda_2(x_1) + \mu_2(x_1)\frac{f_a(x_2|a_2(x_1))}{f(x_2|a_t(x_1))}, \tag{4}$$

where

$$\lambda_2(x_1) \equiv \lambda + \mu_1 \frac{f_a(x_1|a_1)}{f(x_1|a_1)}$$

is non-decreasing in $x_1$ by the Monotone Likelihood Ratio Condition ($MLRC$ hereafter) and the fact that each $\mu_t > 0$, which in this risk-neutral principal case follows directly from Jewitt's (1988) Lemma 1.

[7] See Lambert (1983).

Similarly, the optimal effort strategy from the perspective of the principal must satisfy the first-order conditions

$$G^{a_1}(\boldsymbol{s}, \boldsymbol{a}) + \lambda H^{a_1}(\boldsymbol{s}, \boldsymbol{a}) + \mu_1 H^{a_1 a_1}(\boldsymbol{s}, \boldsymbol{a}) = 0 \tag{5}$$

and

$$G^{a_2(x_1)}(\boldsymbol{s}, \boldsymbol{a}) + \lambda H^{a_2(x_1)}(\boldsymbol{s}, \boldsymbol{a}) + \mu_1 H^{a_1 a_2(x_1)}(\boldsymbol{s}, \boldsymbol{a}) + \mu_2(x_1) H^{a_2(x_1) a_2(x_1)}(\boldsymbol{s}, \boldsymbol{a}) = 0 \tag{6}$$

The terms multiplying $\lambda$ in both (5) and (6) are here both equal to zero and the last term in both (5) and (6) is negative due the assumed validity of the first-order approach central to the formulation of the original problem. In contrast, $(A4)$ in Lambert (1984) (and eq. (9) in Lambert (1983)), which is supposed to be the same first-order condition as (6) above, in the notation used here reads

$$G^{a_2(x_1)}(s_2, a_2) + \lambda H^{a_2(x_1)}(\boldsymbol{s}, \boldsymbol{a}) + \mu_2(x_1) H^{a_2(x_1) a_2(x_1)}(s_2, a_2) = 0, \tag{A4}$$

where the term multiplying $\lambda$ is again zero due to the assumed validity of the first-order approach.[8]

Several differences are noteworthy, here. First, the term multiplied by $\mu_1$ in (6) which is absent from $(A4)$, based on the argument that the first-order approach guarantees such derivatives to be zero. $H^{a_1 a_2(x_1)}(\boldsymbol{s}, \boldsymbol{a})$ is, however, easily recognized as a cross-partial and cannot therefore safely be assumed to be zero simply based on the first derivative being zero. Indeed, to the contrary, as I will show in the next section, this cross-partial is a critical link between past performance and future actions without which there actually is *no* such link to be found in the particular setting analyzed by Lambert (1984).

A separate other perhaps more subtle difference between (6) and $(A4)$ is that the first terms in his expression corresponds to $G^{a_2(x_1)}(s_2, a_2)$ rather than to $G^{a_2(x_1)}(\boldsymbol{s}, \boldsymbol{a})$ as used in equation (6) above. While seemingly benign, as I will also show in the next section this

[8] The term is therefore not included in eq. (9) and the original version of $(A4)$ in Lambert (1983) and Lambert (1984) respectively.

discrepancy is what drives the Proposition in Lambert (1984) and accordingly is a key error in his proof. The principal solves his problem at time zero as reflected by $G^{a_2(x_1)}(\boldsymbol{s},\boldsymbol{a})$ in $(6)$. Using instead $G^{a_2(x_1)}(s_2, a_2)$ in $(A4)$ implies that he solves the second period problem at the start of the second period which he clearly does not.

Finally, it can be noted that in contrast to Lambert $(1983, 1984)$, the second period $IC$-constraints, and thus the last term in (6) here, are also written from a time zero perspective. Surely, the agent chooses the second period action to implement after $x_1$ is observed. But from a game-theoretic perspective, the agent actually chooses his strategy at the time he accepts the contract and does not deviate from plan later. While writing it the way I do is formally the correct way, in this case it is then primarily a matter of presentation that arguably only makes identifying and interpreting the multipliers on the second period $IC$-constraints more straight-forward.[9]

# 4 EQUILIBRIUM CAUSES OF SERIAL CORRELATION

The purpose of this section is to dissect the difference between the first- and second-best behavior in such a way as to isolate and identify the nature of the three unique causes of second-best serial correlation present in this model formulation: wealth-effects, intertemporal incentive effects and horizon effects. Because wealth effects are the focal point of Lambert (1984), in the next sub-section I start by establishing that for the model as specified, the particular case of a risk neutral principal and an agent with square-root preferences is actually the special case where wealth effects are not present in the model. This, in turn, helps provide the simplicity that allows me to cleanly identify the other two effects that are always present here.

[9] Formally, my approach identifies $\mu_2(x_2)$ directly, while following the Lambert (1983) approach, the identification is a two stage process. See the first paragraph of his page 446.

## 4.1 Wealth Effects

To identify the link between past outcomes and future actions, it is useful, as well as instructive, to consider a slightly different and simpler problem than the one detailed in the previous section. Specifically, let $\{\lambda_1^*, \mu_1^*, \mu_2^*(x_1), a_1^*, a_2^*(x_1)\}$ denote the values of the parameters that solve the principal's problem as captured by $(PP)$ and consider then an alternate situation where the principal does *not* face a first period moral hazard problem but where the optimal first period action as well as the structure and nature of the second period problem remain intact. Specifically, assume:

**Assumption:** Suppose *i)* $a_1$ is observable, *ii)* $f_a(x_1, a_1) = 0$ for $a_1 > a_1^*$, and *iii)* that $s_1(x_1)$ and $s_2(x_1, x_2)$ are exogenously restricted to take the form of (3) and (4) respectively with $\mu_1 = \mu_1^*$.

This alternate problem, $(AP)$, then consists of choosing $\{k, \phi_2(x_1), w(x_2), a_1, a_2(x_1)\}$ to

$$max \quad \int\left\{x_1 - s_1(x_1) + \int (x_2 - s_2(x_1, x_2)) f(x_2, a_2(x_1)) dx_2\right\} f(x_1, a_1)dx_1 \qquad \text{(AP)}$$

$$s.t. \quad \int\left\{u(s_1(x_1)) + \int u(s_2(x_1, x_2)) f(x_2, a_2(x_1)) dx_2 - v(a_2(x_1))\right\} f(x_1, a_1)dx_1$$

$$- \quad v(a_1) = \theta \qquad \text{(IRA)}$$

$$\int\int [u(s_2(x_1, x_2)) f_a(x_2, a_2(x_1)) dx_2 - v'(a_2(x_1))] f(x_1, a_1) dx_1 = 0 \quad \forall a_2(x_1) \qquad \text{(IC2A)}$$

$$\frac{1}{u'(s_1(x_1))} = k + \mu_1^* \frac{f_a(x_1, a_1^*)}{f(x_1, a_1^*)} \qquad \text{(CO1A)}$$

$$\frac{1}{u'(s_2(x_1, x_2))} = k + \mu_1^* \frac{f_a(x_1, a_1^*)}{f(x_1, a_1^*)} + \phi_2(x_1) w(x_2) \qquad \text{(CO2A)}$$

Let $\lambda$ and $\mu_2(x_1)$ represent the Lagrange multipliers on $(IRA)$ and the $(IC2A)$ constraints respectively. It is then straight-forward to verify that here $k = \lambda$ and $\phi_2(x_1) w(x_2) = \mu_2(x_1) f_a(x_2, a_2(x_1)) / f(x_2, a_2(x_1))$ so that the structure of the (constrained) optimal contracts here is the same as for (PP).

The purpose of the alternate problem represented by $(AP)$ is that it provides a means to address the following question: *if* the second period contract is irrelevant for the first period solution but the agent's second period compensation *does* depend on the first period's realized outcome (by fiat here, but none the less), what then would be the relation between second period second best action and first period realized outcome? Let $\left\{\widetilde{a}_1, \widetilde{a}_2(x_1), \widetilde{\lambda}, \widetilde{\mu}_2(x_1)\right\}$ denote the parameter values that solves the alternate problem represented by $(AP)$. The answer then is:

**Proposition 1** *For* $u(y) = 2\sqrt{y}$, $d\widetilde{a}_2(x_1)/dx_1 = 0$.

**Proof.**

Clearly, the solution to the alternate problem has $a_1 = a_1^*$. Then, $\widetilde{a}_2(x_1)$ is the solution to the Principal's second-period alternate problem:

$$\max_{a_2(x_1)} \int_{x_1}\int_{x_2} x_2 - \left(\lambda + \mu_1^* \frac{f_a(x_1, a_1^*)}{f(x_1, a_1^*)} + \mu_2(x_1)\frac{f_a(x_2, a_2(x_1))}{f(x_2, a_2(x_1))}\right)^2 f(x_2, a_2(x_1))\, dx_2 f(x_1, a_1^*) dx_1$$

where

$$\lambda = \left(\theta + v(a_1^*) + E\left[v(\widetilde{a}_2(x_1))\right]\right)/4$$

and

$$\mu_2(x_1) = v'(a_2(x_1))/2\int \frac{f_a(x_2, a_2(x_1))^2}{f(x_2, a_2(x_1))}dx_2, \quad \forall x_1$$

can be obtained directly from the $IR$-constraint and the second period incentive compatibility constraint rewritten using the properties of the agent's assumed utility function here. For simplicity define $L_1 \equiv \frac{f_a(x_1, a_1^*)}{f(x_1, a_1^*)}$ and $L_2 \equiv \frac{f_a(x_2, a_2(x_1))}{f(x_2, a_2(x_1))}$. Substituting the expressions for $\lambda$ and $\mu_2(x_1)$ back into the principal's objective function yields

$$\begin{aligned}
&\int_{x_1}\int_{x_2} x_2 - \left[\lambda(x_1)\right]^2 + 2\lambda(x_1) L_2(a_2(x_1)) + \left[L_2(a_2(x_1))\right]^2 f(x_2, a_2(x_1))\, dx_2 f(x_1, a_1^*) dx_1 \\
= \;& -\lambda^2 - (\mu_1^*)^2 \sigma_{L_1}^2 + \int_{x_1}\left(E_{x_2|x_1}\left[x_2|a_2(x_1)\right] - (\mu_2(x_1))^2 \sigma_{L_2}^2\right) f(x_1, a_1^*) dx_1 \\
= \;& -\left[\left(\theta + v(a_1^*) + E\left[v(\widetilde{a}_2(x_1))\right]\right)/4\right]^2 - (\mu_1^*)^2 \sigma_{L_1}^2 \\
& + \int_{x_1}\left(E_{x_2|x_1}\left[x_2|a_2(x_1)\right] - \left[v'(a_2(x_1))\right]^2/4\sigma_{L_2}^2\right) f(x_1, a_1^*) dx_1.
\end{aligned}$$

Then, differentiating w.r.t. $a_2(x_1)$, the first-order conditions become

$$\begin{aligned}
& E'_{x_2|x_1}\left[x_2|a_2(x_1)\right] - v'(a_2(x_1))\left[\left(\theta + v(a_1^*) + E'\left[v(\widetilde{a}_2(x_1))\right]\right)/8\right] \\
- \quad & d\left[\left[v'(a_2(x_1))\right]^2/4\sigma_{L_2}^2\right]/da_2(x_1) = 0, \qquad \forall x_1.
\end{aligned}$$

Thus, because neither the production- nor the cost-function depend directly on $x_1$, $a_2$ does not either because the $a_2$ that satisfies the first-order condition is the same regardless of the realization of $x_1$.

■

The point here is that absent first period incentive considerations, *even* if the second-period compensation paid to the agent *does* depend on the first period's outcome, the equilibrium second-period action does *not* when the principal is risk neutral and the agent has square-root preferences over consumption levels. This is significant for a number of reasons. First note that the first-order condition for the second period action choice of the alternate program $(AP)$ if following the approach of Lambert (1983, 1984) is

$$\int (x_2 - s_2(x_1, x_2)) f_a(x_2, a_2(x_1))\, dx_2$$
$$+\mu_2(x_1)\left[\int u(s_2(x_1, x_2)) f_{aa}(x_2, a_2(x_1))\, dx_2 - v''(a_2(x_1))\right] = 0 \qquad (7)$$

and thus identical to $(A4)$. Because the structure of the second period contract used here is the same as well, the implied relation between $\mu_2(x_1)$ and $a_2(x_1)$ is identical. Accordingly, all the steps of the proof offered by Lambert (1984) can be replicated here and, if done, yield the same (false) conclusion that $d\widetilde{a}_2(x_1)/dx_1 < 0$.

The key problem with relying on (7) for the purpose of that proof is that by dropping the expectation across the first period output realizations, as made explicit in the Lemma imbedded in the proof in Lambert (1984), the problem de-facto becomes one of solving a series of one period problems with interim $IR$-constraints. That is, again, *not* the problem the principal is solving in this setting. He is solving a problem at time zero with just *one* ex-ante $IR$-constraint. The technical implication of this is that the derivative of the Lagrangian w.r.t. $a_2(x_1)$ must be evaluated at time zero. The appropriate condition to use therefore is

$$G^{a_2(x_1)}(\boldsymbol{s}, \boldsymbol{a}) + \mu_2(x_1) H^{a_2(x_1)a_2(x_1)}(\boldsymbol{s}, \boldsymbol{a}) = 0$$

Using this condition instead of his $(A4)$ as the basis for the proof in Lambert (1984) yields the correct result that is the one reported as Proposition 1 above.

Before proceeding it may also be useful to point out one of the key logical inconsistencies. Lambert (1984) argues, based on his expression $(A7)$ which is the same as the expression for $\mu_2(x_1)$ in the proof of Proposition 1 above, that $\mu_2(x_1)$ only depends on $x_1$ insofar $a_2(x_1)$ does. This is, of course, also a not so subtle hint that $a_2(x_1)$ here does not depend on $x_1$ unless $\mu_2(x_1)$ does. Unlike the chicken and the egg, there actually is a defined logical sequence to the present problem. Recall that $\mu_2(x_1)$ is "fixed" at $t = 0$ as the part of the optimal contract that provides output-based variation in compensation and thus effort-incentives for the agent. The agent implements $a_2(x_1)$ subsequently as the agent's optimal response to the optimal contract. This implies conceptually that if $\mu_2(x_1)$ does not depend directly on $x_1$, neither will $a_2(x_1)$ which is exactly what is established by Proposition 1. In more technical terms, then, when taking the partial derivative of (7) with respect to $x_1$ the derivative of $\mu_2(x_1)$ with respect to $x_1$ cannot be taken to be zero as part of a proof to establish that the derivative of $a_2(x_1)$ with respect to $x_1$ is not.

The absence of wealth effects established here contrast also with, for example, Matsumura (1988) and Ramakrishnan (1988) that both attribute negative serial correlation between outcomes and future actions of the Lambert (1984) type to wealth effects stemming from compensation derived from first period effort.[10] This is based on the same misunderstanding that the cost of effort must be compensated in the state it is exerted in this two period set-up underlying the proof in Lambert (1984). If that was the case, surely higher agent wealth coming into the second period would make it more costly to compensate effort in that period. But it is not the case at all. As a quick inspection of the $IR$-constraint reveals, second period effort is compensated in expectation only and as such, there are no wealth effects in the second period other than those that affect directly the aversion to risk.[11]

The bottom line is that in the case of the square root preference representation there are

[10] Matsumura (1988) makes her claim in a setting where the agent's preferences are defined in terms of aggregate consumption and are thus not time additive. In the last period of a two period model this distinction is obviously entirely irrelevant, however.

[11] It should be noted that the proof of Proposition 3 in Ramakrishnan is mechanically correct. It is the attribution of the effect identified to changes of wealth that is incorrect. The result is due to the interim incentive effect that has been missed by the literature and that I detail in the next section.

no wealth effects, and if one considers the optimal period 2 action entirely independent of its impact on the incentives for the period 1 action, which is the purpose of the alternate program, $(AP)$, there is no demand for outcome contingent effort-variations. This is of course not true in general. As long as the principal remains risk-neutral, the nature of the wealth-effects depend directly on the functional form of $h'(\cdot)$.[12] For example, staying within the power class, it is easily verified that for $\gamma \in (1/2, 1)$, $h'(u)$ is concave while the opposite is just as easily verified to be the case for $\gamma \in (0, 1/2)$. In the former case the opposite behavior from that proposed by Lambert (1984) is the effect of responding to past realizations while the effect is as suggested in the latter case. But the direction of the wealth effect does not even have to be the same across wealth-levels: for the case where $u(y) = -e^{2y^{1/2}}$, for example, where the agent exhibits decreasing *relative* risk-aversion, $h'(u)$ is convex for relatively low values of $u$ but concave for relatively high ones.

## 4.2 Intertemporal Incentive Effects

While the second-period wealth-effects generated by the first period risk-sharing can go either way, optimal second period actions always depend on the nature of the first period incentive problem. In particular, it turns out, the more severe the first period moral hazard problem is, the more valuable it is to condition the second period action on realized first period outcome. As demonstrated above, absent a first-period moral hazard problem the optimal second-period action here is invariant to exogenously mandated wealth permutations generated by first period outcomes when the agent has a square root utility function. When the very same wealth permutation arises endogenously due to a first-period moral hazard problem, however, otherwise inefficient second-period effort variations emerge in equilibrium as a means of lowering the cost of providing first-period incentives

To see this, consider again the original problem represented by $(PP)$. First note that for the square root case, by (3) and (4) here

[12] For a nice discussion of the relation between the properties of the agent's preferences and wealth effects, see Ramakrishnan (1988), Section 2.

$$u\left(s_1\left(x_1\right)\right) = \int u\left(s_2\left(x_1, x_2\right)\right) f\left(x_2, a_2(x_1)\right) dx_2.$$

This follows because under the optimal contract, the agent's (second-period) equilibrium response is such that the expected likelihood ratio is always zero. Also note, that if we simply were to exogenously restrict attention to sharing rules that satisfy (3) and (4) and solve only for the optimal actions (along with the corresponding multiplier values) we would identify the same (second-best) solution as obtains from $(PP)$. Following this approach, $(IC1P)$ can be re-expressed simply as

$$\int \left\{2u\left(s_1\left(x_1\right)\right) - v\left(a_2\left(x_1\right)\right)\right\} f_a(x_1, a_1) dx_1 - v'(a_1) = 0.$$

The significance of this is, of course, that variations in second period actions that are dictated by first period outcomes impact the agent's first period incentives through variations in second-period costs, $v\left(a_2\left(x_1\right)\right)$, and are a direct substitute for second-period compensation-variations tied to first period outcome realizations. In particular, using this version of $(IC1P)$, the derivatives of the the Lagrangian with respect to first- and second-period effort become

$$\begin{aligned} & \int \left\{x_1 - s_1\left(x_1\right) + \int \left(x_2 - s_2\left(x_1, x_2\right)\right) f\left(x_2, a_2(x_1)\right) dx_2\right\} f_a(x_1, a_1) dx_1, \\ + \quad & \mu_1 \left[\int \left\{2u\left(s_1\left(x_1\right)\right) - v\left(a_2\left(x_1\right)\right)\right\} f_{aa}(x_1, a_1) dx_1 - v''(a_1)\right] = 0, \end{aligned} \tag{8}$$

and

$$\begin{aligned} & \int \int \left(x_2 - s_2\left(x_1, x_2\right)\right) f_a\left(x_2, a_2(x_1)\right) dx_2 f(x_1, a_1) dx_1 \\ & -\mu_1 \int v'\left(a_2\left(x_1\right)\right) f_a(x_1, a_1) dx_1 \\ & +\mu_2\left(x_1\right) \int \left[\int u\left(s_2\left(x_1, x_2\right)\right) f_{aa}\left(x_2, a_2(x_1)\right) dx_2 - v''\left(a_2\left(x_1\right)\right)\right] f(x_1, a_1) dx_1 = 0 \\ & for\ each\ x_1. \end{aligned} \tag{9}$$

The main point here is that the second period can only be viewed in isolation when there is no first period incentive problem, that is when $(IC1P)$ does not bind. If it is binding the choice of second period action as a function of first-period outcome plays a direct role in resolving the *first*-period incentive problem and $(IC1P)$ thus will not be ignored by the principal when choosing $a_2(x_1)$ as suggested by (7).

The second line of (9) above is, as discusse above, missing from equation (9) in Lambert (1983) and from $(A4)$ in Lambert (1984) based on the argument that it is the validity of the first-order approach as reflected by $(IC2P)$ makes this term equal to zero. This represents a fundamental misunderstanding of the vastly different choice problems facing the agent and the principal, however. $(IC1P)$ and $(IC2P)$ represent the agent's choice problem *after* the principal has chosen the structure of the contract. The principal's choice problem, in contrast, is to craft a deal that both attracts and appropriately incentivizes the agent. To see this clearly, consider the principal's problem of choosing an incentive compatible $a_1$. In its most general form $(IC1P)$ can here be written as:

$$H^{a_1}(\boldsymbol{s},\boldsymbol{a}) = \frac{\partial E\left[u\left(s_1\left(x_1\right)\right)\right]}{\partial a_1} + \frac{\partial E\left[u\left(s_2\left(x_1,x_2\right)\right)\right]}{\partial a_1} - v'\left(a_1\right) - \frac{\partial E\left[c\left(a_2\left(x_1\right)\right)\right]}{\partial a_1} = 0. \quad (10)$$

Because the optimal contracts always must satisfy (3) and (4), $u(\cdot) = 2\sqrt{\cdot}$ implies, as is well known, that the agent's utility from consumption under the optimal contract is additively separable in $x_1$ and $x_2$ as well. Accordingly, in the *agent's* first period choice problem,

$$\frac{\partial E\left[u\left(s_2\left(x_1,x_2\right)\right)\right]}{\partial a_1}$$

is independent of the principal's choice of which $f\left(x_2, a_2(x_1)\right)$ to implement. The agent is choosing $a_1$ *knowing* that his second period strategy, $a_2(x_1)$, will be the optimal response to the contract, which has the optimal strategy, $a_2(x_1)$ as chosen by the principal, embedded in it through the second-period likelihood ratio. The last term in the second-period contract (denominated in utiles) is therefor always zero in expectation at the time the agent chooses

$a_1$ and thus has no bearing on his expected second period utility as a function of his choice of $a_1$. But this, of course, also implies that in the *principal's* choice problem,

$$\frac{\partial\left[\frac{\partial E[u(s_2(x_1,x_2))]}{\partial a_1}\right]}{\partial a_2(x_1)} =$$
$$\frac{\partial \int\int u(s_2(x_1,x_2)) f(x_2,a_2(x_1))\,dx_2 f_a(x_1,a_1)dx_1}{\partial a_2(x_1)} = 0 \qquad \forall x_1,$$

while obviously $\partial \int c(a_2(x_1)) f_a(x_1,a_1)dx_1/\partial a_2(x_1)$ is *not.*

To crisply identify the effect of incentivizing first period action via variations in second period actions, using $(IRP)$, $(IC1P)$ and $(IC2P)$ along with (3) and (4), for this square root representation I can easily calculate

$$\lambda = \left(v(a_1) + E\left[v\left(a_2(x_1)\right)\right] + \underline{U}\right)/4, \tag{11}$$

$$\mu_1 = \frac{v'(a_1) + \int v(a_2(x_1)) f_a(x_1,a_1)dx_1}{4\int\left(\frac{f_a(x_1,a_1)}{f(x_1,a_1)}\right)^2 f(x_1,a_1)dx_1} \tag{12}$$

and

$$\mu_2(x_1) = \frac{v'(a_2(x_1))}{2\int\left(\frac{f_a(x_2,a_2(x_1))}{f(x_2,a_2(x_1))}\right)^2 f(x_2,a_2(x_1))\,dx_2}. \tag{13}$$

Again, it is immediately clear from (13) that there is no *second*-period demand for outcome-contingent variations in the second-period action here. The shadow price of the second period $IC$-constraint, $\mu_2(x_1)$, is the same for any given level of second period effort regardless of the realization of $x_1$. Accordingly, there are no wealth effects present here that change the risk-premium and thus the cost of second period incentives.

The sole reason second-period effort may depend on first-period outcome is trough the impact of $a_2(x_1)$ on $\mu_1$ via the integral in the numerator of (12). It is also immediately obvious from (12) that if $a_2^*(x_1)$ *does* depend on $x_1$, $E[a_2^*(x_1)]$ is necessarily smaller for positive than for negative values of $f_a(x_1,a_1)$, because this lowers the cost of incentivizing *first*-period effort, as represented by $\mu_1$, by making the integral in the numerator negative.

Lowering the cost of first period incentives by introducing outcome contingent variations in second period work-load comes, of course, at the expense of second period second-best efficiency, so the optimality of conditioning second period effort on first period output depends on the net of these effects. The next proposition establishes that it is always efficient to introduce some such costly variation in second period effort to lower the cost of the first period $IC$-constraint. Specifically, let $X_1^+ = \{x_1 | f_a(x_1, a_1) \geq 0\}$ and $X_1^- = \{x_1 | f_a(x_1, a_1) < 0\}$. For the model as specified we then have

**Proposition 2** *For* $u(y) = 2\sqrt{y}$, $E_{X_1^+}[a_2(x_1)] < E_{X_1^-}[a_2(x_1)]$.

**Proof.**

Start by solving for the optimal $a_1$ and $a_2$ when the latter exogenously is restricted not to depend on $x_1$. The multipliers on the IC-constraints then both take the form

$$\widetilde{\mu}_t = \frac{v'(a_t) + (2-t)\int v(a_2) f_a(x_1, a_1) dx_1}{(4/t)\int \left(\frac{f_a(x_t, a_t)}{f(x_t, a_t)}\right)^2 f(x_t, a_t)\, dx_t}, \tag{14}$$

where the integral in the numerator (of $\widetilde{\mu}_1$) is zero given that $a_2$ here is restricted to be independent of $x_2$. Consider then to add $\epsilon$ of a variation, $\delta_2(x_1) = \left\{\delta_2\left(X_1^+\right), \delta_2\left(X_1^+\right)\right\}$, to $\widetilde{\mu}_2$ that is strictly positive for $X^-$ and strictly negative for $X^+$ with $\delta_2\left(X_1^+\right) f\left(X_1^+ | a_1\right) + \delta_2\left(X_1^-\right) f\left(X_1^- | a_1\right) = 0$.

With this we have

$$\begin{aligned} v'(a_2((X_1^+))) &= \left(\widetilde{\mu}_2 + \epsilon\delta_2\left(X_1^+\right)\right) D_2 \\ v'(a_2((X_1^-))) &= \left(\widetilde{\mu}_2 + \epsilon\delta_2\left(X_1^-\right)\right) D_2, \end{aligned}$$

where $D_2$ is the denominator $(14)$.

With this

$$\left.\frac{\partial E\left[v'(a_2)\right]}{\partial \epsilon}\right|_{\epsilon=0} = \left(\delta_2\left(X_1^+\right) f\left(X_1^+ | a_1\right) + \delta_2\left(X_1^-\right) f\left(X_1^- | a_1\right)\right) D_2 = 0.$$

Further, letting the monotone relation between $v'(a_t)$ and $v(a_t)$ be represented by the function $c(\cdot)$ such that $v(a_t) = c(v'(a_t))$. Then,

$$\begin{aligned}
&\left.\frac{\partial E\left[v\left(a_2\right)\right]}{\partial \epsilon}\right|_{\epsilon=0} = \\
&f\left(X_1^{+}|a_1\right)\left.\frac{\partial c\left(\left(\widetilde{\mu}_2+\epsilon\delta_2\left(X_1^{+}\right)\right)D_2\right)}{\partial \epsilon}\right|_{\epsilon=0} + \left.\frac{\partial c\left(\left(\widetilde{\mu}_2+\epsilon\delta_2\left(X_1^{-}\right)\right)D_2\right)}{\partial \epsilon}\right|_{\epsilon=0} f\left(X_1^{-}|a_1\right) \\
= &\left(\delta_2\left(X_1^{+}\right) f\left(X_1^{+}|a_1\right) + \delta_2\left(X_1^{-}\right) f\left(X_1^{-}|a_1\right)\right) c'\left(\widetilde{\mu}_2\right) D_2 = 0.
\end{aligned}$$

Finally, let the inverse of the agent's cost function be denoted by $w(\cdot)$ such that $a_t = w(v(a_t))$, then

$$\begin{aligned}
&\left.\frac{\partial E\left[a_2\right]}{\partial \epsilon}\right|_{\epsilon=0} = \\
&f\left(X_1^{+}|a_1\right)\left.\frac{\partial w\left(c\left(\left(\widetilde{\mu}_2+\epsilon\delta_2\left(X_1^{+}\right)\right)D_2\right)\right)}{\partial \epsilon}\right|_{\epsilon=0} + \left.\frac{\partial w\left(c\left(\left(\widetilde{\mu}_2+\epsilon\delta_2\left(X_1^{-}\right)\right)D_2\right)\right)}{\partial \epsilon}\right|_{\epsilon=0} f\left(X_1^{-}|a_1\right) \\
= &\left(\delta_2\left(X_1^{+}\right) f\left(X_1^{+}|a_1\right) + \delta_2\left(X_1^{-}\right) f\left(X_1^{-}|a_1\right)\right) w'\left(c\left(\widetilde{\mu}_2\right)\right) c'\left(\widetilde{\mu}_2\right) D_2 = 0.
\end{aligned}$$

Accordingly, the effect of adding a small variation, $\delta_2(x_1)$, to the best second period contract that is restricted not to depend on $x_1$ is zero. In contrast,

$$\begin{aligned}
&\left.\frac{\partial \int v\left(a_2\right) f_a(x_1, a_1) dx_1}{\partial \epsilon}\right|_{\epsilon=0} = \\
&f_a\left(X_1^{+}|a_1\right)\left.\frac{\partial c\left(\left(\widetilde{\mu}_2+\epsilon\delta_2\left(X_1^{+}\right)\right)D_2\right)}{\partial \epsilon}\right|_{\epsilon=0} + \left.\frac{\partial c\left(\left(\widetilde{\mu}_2+\epsilon\delta_2\left(X_1^{-}\right)\right)D_2\right)}{\partial \epsilon}\right|_{\epsilon=0} f_a\left(X_1^{-}|a_1\right) \\
= &\left(\delta_2\left(X_1^{+}\right) f_a\left(X_1^{+}|a_1\right) + \delta_2\left(X_1^{-}\right) f_a\left(X_1^{-}|a_1\right)\right) c'\left(\widetilde{\mu}_2\right) D_2 < 0,
\end{aligned}$$

thus decreasing the shadow price of the first period $IC$-constraint, $\widetilde{\mu}_2$. There is therefore strict value to introduce a strictly positive amount of such a variation in the agent's contract because the reduction in the cost of providing first period incentives outweighs the cost of making second period effort outcome dependent and thus non-constant.

Finally note that for any $x_1 \in X_1^{-}$ there can be no value to setting $a_2(x_1) < a_2(\widehat{x}_1)$ if $\widehat{x}_1 \in X_1^{+}$ because doing so introduces a costly variation in second-period effort while at the same time *increasing* the cost of incentivizing first period effort.

■

The effect documented here seems quite intuitive in the additively separable preference specification: variations in future compensation and variations in future workload are substitutes when it comes to providing incentives. From a theoretical perspective, the risk premium associated with providing first period incentives using risky second period compensation can be reduced by substituting some of that compensation risk with some rewards in form of leisure. With a concave utility function over consumption and a convex cost function for effort, the optimal solution always entails splitting incentive provision between future monetary compensation and future leisure. It also fits well with rewards in terms of time off and paid vacation being tied to performance as well as with notions such as "resting on your laurels."

It is important to note that while the intertemporal incentive effect appears consistent with the Proposition in Lambert (1984), it is actually fundamentally different as it is determined by the integral in the numerator of (12) that is missing from Lambert (1983, 84). In contrast, the Proposition in Lambert (1984) is entirely driven by mistakenly over-constraining the problem with interim $IR$-constraints resulting in wealth effects that, as established in the prior section, are not part of the solution in the square root case. The significance of this is that second-period effort in Lambert (1984) is predicted to be monotone in the first period *likelihood ratio,* which is monotone in first period output by the $MLRC$.[13] The integral in the numerator of (12), in contrast, depends only on the *numerator* of the likelihood ratio which is not generally monotone in $x_1$.

To get some feel for the difference between the behavior predicted by the intertemporal incentive effect and that suggested by the analysis in Lambert (1984), consider the Gamma distribution

$$f(x,a) = \frac{1}{a}e^{-(x/a)} \times \frac{(x/a)^{k-1}}{(k-1)!}$$

of which the Exponential distribution used in the example in Lambert (1984) is the special case where (the positive integer) $k = 1$. For this distribution, the likelihood ratio is given by

$$\frac{f_a(x,a)}{f(x,a)} = \frac{x-ka}{a^2}$$

and is thus monotone (linear) in $x$ for any admissible $(k,a)$. As discussed above, however,

[13] See the second-to-last sentence of the proof in Lambert (1984).

the intertemporal incentive effect is not driven by $f_a(x,a)/f(x,a)$, but rather by $f_a(x,a)$ alone.

Figure 1 maps out $f(x,a)$ and $f_a(x,a)$ for the Gamma distribution for $k=1$ and $k=2$ to illustrate the inherent non-monotonicity of equilibrium second period effort as a function of first period outcome.

**Insert Figure 1 about here.**

The Exponential $f_a(x,a)$ is monotone over $X_1^-$ but clearly not over $X_1^+$. Moreover, the relatively small values of $f_a(x,a)$ over $X_1^+$ imply that second period effort will be less responsive to first period output over $X_1^+$ than over $X_1^-$. For $k=2$ (or greater) the Gamma distribution takes on a more "normal" shape with the mode greater than the lower bound on $x$. This increase in symmetry is mirrored in the shape of $f_a(x,a)$ which is now clearly non-monotonic over both $X_1^+$ and $X_1^-$. Note, however, that the relatively larger values here of $f_a(x,a)$ over $X_1^+$ imply that second period effort will be more responsive to first period output over $X_1^+$ than over $X_1^-$. Thus while for any $k$ the induced equilibrium behavior *on average* leads to smoother income in the Lambert (1984) sense, the induced behavior does not resemble one the principal would induce if the objective actually was to produce smoother income.

## 4.3 Horizon Effects

The third and final second-best force that shapes the time-series properties of earnings is time itself, or, remaining time to be precise. As should be obvious from (4) and the discussion throughout, the time-additive preference structure makes it optimal for the principal to spread current period's incentive risk over remaining periods. Intuitively, then, the more periods left, the closer the solution is to the first-best while the fewer, the closer it is to the standard one-period second-best. As per the argument in the previous section, certainly the last period is worse (in expectation) than the one-period second-best due to the use of otherwise inefficient outcome contingent variations in effort to provide incentives in prior periods. The effect of this is that (expected) effort decreases over time at an increasing rate. Although Lambert (1983) does show that commitment is valuable here in the sense that the more periods that are covered by a contract the better, the link to the time series properties of output is missing in Lambert (1984) as well.

To highlight the effect of the passage of time on earnings smoothness I will again use as a benchmark the case where second period action cannot depend on first period outcome but only be a function of time. Eliminating the term in the numerator of (12) that is the source of second period outcome-dependence and substituting into the objective function, the principal's constrained problem can here then be expressed as choosing $a_1$ and $a_2$ to maximize

$$E\left[x_1\right]+E\left[x_2\right]-\frac{\lambda^2}{2}-\int\frac{\mu_1^2}{2}\left(\frac{f_a(x_1,a_1)}{f(x_1,a_1)}\right)^2 f(x_1,a_1)dx_1$$
$$-\left\{\int\left(\frac{\mu_2\left(x_1\right)}{2}\right)^2\int\left(\frac{f_a(x_2,a_2(x_1))}{f(x_2,a_2(x_1))}\right)^2 f\left(x_2,a_2(x_1)\right)dx_2\right\}f(x_1,a_1)dx_1$$

or

$$E\left[x_1\right]+E\left[x_2\right]-\frac{\lambda^2}{2}-\frac{1}{2\int\left(\frac{f_a(x_1,a_1)}{f(x_1,a_1)}\right)^2 f(x_1,a_1)dx_1}$$
$$-\frac{1}{\int\left(\frac{f_a(x_2,a_2)}{f(x_2,a_2)}\right)^2 f\left(x_2,a_2\right)dx_2} \tag{15}$$

Let $\overline{a}_1^*$ and $\overline{a}_2^*$ denote the solution to (15) . Since everything is symmetric in this formulation except for the "2" in the denominator of the term representing the cost of first-period incentives, it is clear that $\overline{a}_1^* > \overline{a}_2^*$. This in turn implies that expected output is also decreasing over time here.

To establish that expected second-best effort is indeed decreasing over time, then, consider the difference between this solution and the solution to the unrestricted problem, $a_1^*$ and $a_2^*(x_1)$. Again, following the result of the prior section, second-period effort-randomization lowers the cost of first-period incentives but increases the (expected) marginal cost of second period effort. As a result, we have $a_1^* > \overline{a}_1^* > \overline{a}_2^* > E\left[a_2^*(x_1)\right]$. That this relation generalizes to other utility functions than $u(y) = 2\sqrt{y}$ is established by the final proposition.

**Proposition 3** $a_1^* > E\left[a_2^*(x_1)\right]$.

**Proof.**

The result follows almost directly from the proceeding discussion. To sketch a more formal proof, consider two different problems: *i*) the principal contracts with the agent for two periods but only facing a moral hazard problem in the first period and *ii*) the principal contracts with the agent for two periods but facing only a moral hazard problem in the second period. Because the solution to problem *i*) spreads the first period incentive related risk risk over the two remaining periods while the solution to problem *ii*) can *only* allocate risk to the second period, the marginal cost to the principal of eliciting first-period effort in problem *i*) is strictly less than that of eliciting second-period effort in problem *ii*).

Next consider the full two period problem. First note that the wealth and intertemporal incentive effects always (weakly) increase the average marginal cost of eliciting second period effort. Since the optimal contract always transfers first period risk to the second period and since that is always (weakly) inefficient from the perspective of the second period, the result follows.

■

While expected income thus is going to be declining over time, the model arguably also predicts that income volatility will be changing too. The effect that the shrinking remaining horizon has on income volatility is not guaranteed to be in one or the other direction, however. Clearly for the class of production functions identified by Jewitt (1988) for which the first-order approach is valid and of which the Gamma specification is a member, volatility is mechanically linked to expected output and is therefore also guaranteed to fall over time. For less natural specifications supportive of the first-order approach such as those identified by LiCalzi and Spaeter (2003), all that can be said is that volatility will change over time but it is conceivable that the direction of the change itself will be changing (once) over time.

For the "weighing of two distributions" specification as per Hart and Holmström (1987) the effect on volatility depends on the relative volatility of the two distributions in question, as well as their correlation. Only in very special cases, where the production-function is of the "effort-plus-noise" type and variance thus is independent of effort is the volatility guaranteed to be constant over time even as the expected income declines.[14] None of this appears generally consistent with some meaningful notion of income smoothing behavior, however.

# 5 ROBUSTNESS CONSIDERATIONS

Before concluding it seems worthwhile to provide some sense of the robustness of the 3 key drivers of equilibrium time-series behavior in the basic multi-period agency model to the specific assumptions made. A natural benchmark for this is one where neither of the 3 effects are present, namely the multiplicatively separable constant absolute risk aversion (CARA) preference representation where the agent is assumed to care only about aggregate consumption:

$$H(\boldsymbol{s}, \boldsymbol{a}) \equiv -e^{-r\left(s\left(\overrightarrow{x}_2\right)-\sum_t v(a_t)\right)}. \tag{16}$$

The lack of opportunities for intertemporal risk sharing for this specification obviously eliminate the horizon effect. CARA combined with multiplicative separability which importantly is equivalent to denominating the cost of effort in monetary units rather than in utiles as in the additively separable specification used in the proceeding analysis eliminates wealth effects as well. Finally, denominating the cost of effort in monetary units eliminates the intertemporal incentive effect: substituting variation in $v(a_2)$ for variation in $s(\overrightarrow{x}_2)$ for a given level of first period incentive is always strictly costly because of the convexity of the cost function. This does not depend on CARA but is true whenever effort cost is deducted directly from the compensation before the overall utility is assessed.

Consider then instead modifying this specification so that

$$H(\boldsymbol{s}, \boldsymbol{a}) \equiv -e^{-rs\left(\overrightarrow{x}_2\right)} - \sum_t v(a_t). \tag{17}$$

[14] Generally "effort-plus-noise" production functions are not compatible with the first-order approach. A possible specialized exception is the Laplace-Normal hybrid distribution type of Hemmer (2013) due to its likelihood ratio being bounded.

The continued absence of intertemporal consumption smoothing opportunities ensure that the horizon effect is still not present. The other two effects are present now, however. First, the risk needed in the contract to implement a particular action is now strictly increasing in wealth because of the concave transformation of $s(\overrightarrow{x}_2)$ in the agent's decision problem not present under (16). Accordingly, because of CARA the wealth effect for this specification would be consistent with Lambert (1984). This is not the case for the square root representation because its declining ARA exactly compensates for the increasing risk needed to implement a given action for higher levels of risk. Second, Proposition 2 applies here due to the concave transformation of $s(\overrightarrow{x}_2)$ as well: it is *always* optimal to substitute *some* variation in $v(a_2)$ for risk in $s(\overrightarrow{x}_2)$ used to incentivize first period effort when the utility function is additively separable as in (17).

Lastly, the role of full commitment of the agent to a long run contract for (in particular) the intertemporal incentive effect warrants some attention. From a purely technical perspective the principal can always write the contract such that the agent never wants to break it because the act of doing so is verifiable. From a practical and more descriptive perspective, however, the clauses needed to ensure this may not be enforceable based on existing law. The natural question then is whether interim $IR$-constraints imposed when the agent requires some minimum second-period expected utility at the start of the second period to remain with the agency, would diminish the demand for using future effort cost to incentivize current effort? As it turns out, the role of this approach to providing incentives instead arguably becomes more pronounced.

To eliminate confounding wealth effects I rely again on the square root specification here. Under full commitment, spreading the first period incentive risk evenly across the two periods and make the expected utility the same across the two periods is always optimal here. This is regardless of the nature of the second period incentive problem. Consider then the other extreme where neither party can credible commit and where the agent requires an expected utility at the start of the second period of $\theta/2$ to remain with the agency. In this case the two periods become independent with $a_1 = a_2$ and the same expected utility for the agent in each period. The loss of all commitment eliminates the ability to split the first period incentive risk over both periods and in doing so also eliminate the ability to provide intertemporal incentives via the second period's choice of effort.

Denote the contracts pair that solves this (no commitment) problem

$$\begin{aligned}\sqrt{s_1^\circ(x_1)} &= \lambda_1^\circ + \mu_1^\circ \frac{f_a(x_1, a_1^\circ)}{f(x_1, a_1^\circ)} \\ \sqrt{s_2^\circ(x_2)} &= \lambda_2^\circ + \mu_2^\circ \frac{f_a(x_2, a_2^\circ)}{f(x_2, a_2^\circ)},\end{aligned}$$

where $\lambda_t^\circ = (\theta/2 + v(a_t))/2$, and $\mu_1^\circ = \mu_2^\circ > 0$. Then, suppose now for a moment that the same is true also when (only) the principal can commit. Consider then modifying these period-by-period contracts as follows

$$\begin{aligned}\sqrt{s_1^\delta(x_1)} &= \lambda_1^\circ + (1-\delta)\mu_1^\circ \frac{f_a(x_1, a_1^\circ)}{f(x_1, a_1^\circ)} + \delta\mu_1^\circ \underline{L}_1^\circ \\ \sqrt{s_2^\delta(x_2)} &= \lambda_2^\circ + \delta\mu_1^\circ \frac{f_a(x_1, a_1^\circ)}{f(x_1, a_1^\circ)} + \mu_2^\circ \frac{f_a(x_2, a_2^\circ)}{f(x_2, a_2^\circ)} - \delta\mu_1^\circ \underline{L}_1^\circ\end{aligned}$$

where $\underline{L}_1^\circ$ is the lowest possible realization of $\frac{f_a(x_1, a_1^\circ)}{f(x_1, a_1^\circ)}$. This contract implements the same action pair while maintaining both the ex-ante and the second period $IR$-constraints.

The expected compensation to be paid by the principal then is

$$\begin{aligned}&\int_{x_1}\Bigg[\left(\lambda_1^\circ + (1-\delta)\mu_1^\circ \frac{f_a(x_1, a_1^\circ)}{f(x_1, a_1^\circ)} + \delta\mu_1^\circ \underline{L}_1^\circ\right)^2 \\ &+\int_{x_2}\left(\lambda_2^\circ + \delta\mu_1^\circ \frac{f_a(x_1, a_1^\circ)}{f(x_1, a_1^\circ)} + \mu_2^\circ \frac{f_a(x_2, a_2^\circ)}{f(x_2, a_2^\circ)} - \delta\mu_1^\circ \underline{L}_1^\circ\right)^2\Bigg] f(x_1, a_1^\circ)dx_1 f(x_2, a_2^\circ)dx_2.\end{aligned}$$

or after integration

$$\left(\lambda_1^\circ\right)^2 + 2\lambda_1^\circ\delta\mu_1^\circ\underline{L}_1^\circ + (1-\delta)^2\left(\mu_1^\circ\right)^2\sigma_{L_1}^2 + \left(\delta\mu_1^\circ\underline{L}_1^\circ\right)^2$$
$$+\left(\lambda_2^\circ\right)^2 - 2\lambda_2^\circ\delta\mu_2^\circ\underline{L}_2^\circ + \delta^2\left(\mu_1^\circ\right)^2\sigma_{L_1}^2 + \left(\mu_2^\circ\right)^2\sigma_{L_2}^2 + \left(\delta\mu_1^\circ\underline{L}_1^\circ\right)^2$$

The marginal effect of increasing $\delta$ on expected compensation evaluated at $\delta = 0$, however, can then be found as $-2\left(\mu_1^\circ\right)^2\sigma_{L_1}^2 < 0$. Thus it is optimal to shift some of the incentive-related risk to the second-period and thereby lower the cost of providing period one incentives also when the agent cannot commit to stay for the second period.[15] Accordingly, with the cost of effort denominated in utiles in the additively separable preference specification, Proposition 2 applies so that variations in second period work-load will continue to optimally depend on first-period output as long as the principal remains able to commit.

# 6 CONCLUSION

In this paper I explore the various ways the properties of earnings may be affected by agency problems in the early formulation of the multi-period model first proposed and analyzed by Lambert (1983 & 1984) . The enduring key insight in respect to earnings properties from his analysis is that second-best income (at least for the particular preference structure employed by Lambert (1984)) is "managed" in equilibrium in a way that results in smoother earnings, appropriately defined. I demonstrate that this result is false: under the assumptions of his model, explicit and implicit, there is no equilibrium relation between past income and future actions. Because the particular setting is actually a knife-edge case, it is clear that when such relations exist in this formulation, they are entirely due to wealth-effects in the agent's utility function. Such effects, however, can go either way: they just as plausibly lead to smoother as to less smooth income regardless of how one chooses to define "income smoothness."

[15] Lambert (1983) makes a similar point in his section 5.

I then proceed to use this benchmark case of "no relation" to identify other, generally ignored, implications for the time-series behavior of income predicted by this type of model. First, I show that when effort is not implicitly (and sub-optimally) required to be compensated in the period/state where it is exerted, output-contingent variation in future effort is optimally used to incentivize current effort. This "intertemporal incentive effect," which is separate from the wealth effect does push the solution in the direction of making future effort inversely correlated with current output. The relation is not monotonic, however, making it harder to interpret the induced equilibrium behavior as income smoothing. Second, I show expected equilibrium effort and, thus, income is going to be declining over time. This "horizon effect" is separate from the other two effects and is not output contingent. Since effort generally also impacts the volatility of income, however, this effect is hard to reconcile with standard notions of smoothing behavior as well.

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# FIGURES

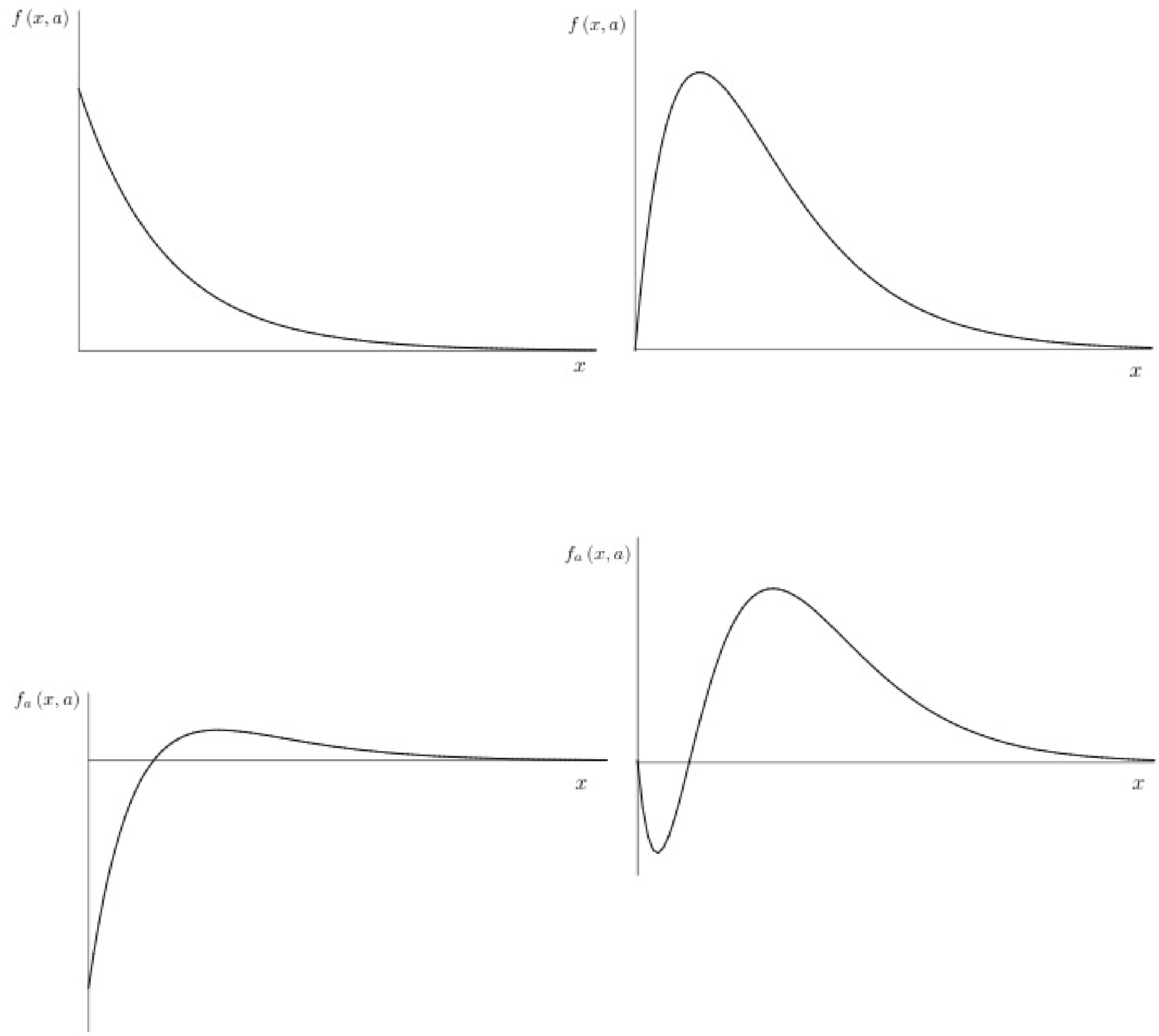


**Figure 1**. $f(x.a)$ and $f_a(x,a)$ for the Gamma Distribution with $k = 1$ (the two panels to the left) and $k = 2$ (the two panels to the right).