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On Emissions Trading and Market Structure: Cap-and-Trade versus Intensity Standards*

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Abstract

This paper examines the interdependence between imperfect competition and emissions trading. We particularly analyze the long run equilibrium in a two-sector ("clean" and "dirty") model with Cournot competition among firms who face a fixed cost of production. The clean sector is defined as the sector with the highest long run cost margin on emissions. We compare the welfare implications of a cap-and-trade scheme with an emissions trading scheme based on relative intensity standards. It is shown that a firm's long run equilibrium output in the clean or dirty sector does not depend on the emissions trading format, but only depends on the fixed cost of producing in the respective sector. Intensity standards can result in clean firms selling allowances to dirty firms, or dirty firms selling to clean firms. The former outcome yields higher welfare. It is demonstrated that cap-and-trade outperforms the intensity-based trading scheme in terms of long run welfare with free entry and exit. With intensity standards the size of the clean sector is too large.

Keywords: cap-and-trade, emissions trading, imperfect competition, industrial change, intensity standards, pollution control

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1 Introduction

Governmental authorities increasingly embark upon emissions trading schemes to efficiently curtail environmental pollution. This paper analyzes and compares the two main market configurations for organizing trade in emission allowances: cap-and-trade versus intensity standards. Although tradable pollution markets have been studied extensively in recent years, a comparison of these pollution market designs in an imperfectly competitive, multi-sector model is still lacking. The aim of this paper is to fill this gap by focusing on the connection between emissions trading and market structure. Since polluting industries are often concentrated in nature, we allow firms to strategically interact in an imperfectly competitive output market and assess the sectoral implications of emissions trading in the long run equilibrium. Analyzing the interaction between emissions trading and output market effects is an important dimension in policy assessments, since it is often not optimal to completely eliminate the pollution-intensive sector, even though reducing pollution is the underlying policy goal.

Cap-and-trade and intensity-based emissions trading represent schemes that are based on an absolute cap on emissions and on relative emission intensities, respectively. Whereas under cap-and-trade a control authority fixes the total supply of emissions, in the case of intensity-based trading a source-specific level of emissions abatement is set, implying a fixed average emissions intensity (e.g., Tietenberg 1999). Prime examples of cap-and-trade schemes in the U.S. are the acid rain programme and the RECLAIM programme to reduce sulfur dioxide and nitrogen oxide emissions from stationary sources in the Los Angeles basin. The European counterpart of large scale cap-and-trade currently occurs within the European Union Emissions Trading System (EU ETS) for carbon dioxide emissions (e.g., Ellerman and Buchner 2007). In the 1980s the U.S. established intensity-based trading arrangements between refineries as part of the lead phasedown (e.g., Hahn and Hester 1989; Kerr and Newell 2003). Another more recent intensity-based scheme is California's Low Carbon Fuel Standard (Holland et al. 2009). In Canada an intensity-based trading system was launched in

1996 under the Pilot Emission Reduction Target. This type of scheme is currently also one of the main design features of Canada's climate policy (Environment Canada 2007). Also for developing countries intensity targets have been suggested (Philibert and Pershing 2000), which has entered the post-Kyoto emissions trading design debate (e.g., Michaelowa et al. 2005; Jotzo and Pezzey 2007).

Our paper illustrates that entry and exit in the output market is a prime factor in the interplay between sectoral choice, production and emissions trading. We show that a firm's equilibrium level of output in the long run does not depend on the specific design of the pollution market. Under either a no-policy regime, cap-and-trade or intensity standards, equilibrium output in the clean or dirty sector only depends on the fixed cost of producing in that sector. Due to the existence of fixed cost, in our model we illustrate that in the long run equilibrium a firm's 'price-cost margin' is therefore positive, even with free entry and exit. For a given level of aggregate emissions in the long run, and given the zero-profit output level per firm, we find that a cap-and-trade scheme generates higher welfare than emissions trading via intensity standards. Relative to the first-best outcome, the size of the clean (dirty) sector is too large (small) under the trading regime based on intensity standards.

These results complement and extend the finding by Helfand (1991), Fischer (2001) and Holland et al. (2009) that intensity standards are generally inefficient, and the more recent studies by Boom and Dijkstra (2009) and Holland (2012) showing that in the absence of market power intensity standards cannot attain the first-best outcome whereas an absolute emissions trading scheme can.[1] Boom and Dijkstra (2009) find that the welfare comparison between the two schemes under imperfect competition is ambiguous in both the short run and the long run. By contrast, we find in our specific setting that cap-and-trade yields higher welfare in the long run.

Our welfare result may seem surprising in light of the literature. Boom and Dijkstra (2009) and Holland (2012) show that cap-and-trade maximizes welfare under perfect competition (in the short run and the long run), and emission trading based on an intensity standard does not. Boom and Dijkstra (2009) find that the welfare comparison

[1] Note that Holland (2012) does not consider an absolute emissions trading scheme explicitly as we do but makes use of an emission tax instead, which is equivalent to an absolute cap on emissions.

is ambiguous under imperfect competition. On the one hand, if competition is 'close to' perfect, one would expect the perfect-competition result of higher welfare under cap-and-trade. On the other hand, output is higher under emissions trading via intensity standards, which counteracts the output-reducing tendency of imperfect competition. The difference in results stems from our assumption that the emission-to-output ratio in each sector is fixed. In Boom and Dijkstra (2009) this ratio is variable, so that with intensity standards the industry can expand its output while still implementing the pollution target by reducing its emission intensity. In the present paper, emissions trading on the basis of intensity standards leads to an output expansion in the clean sector and a (drastic) output reduction in the dirty sector. This is contrary to the optimal policy prescription, which is for both sectors to contract according to their relative emission intensities. The latter is exactly what cap-and-trade achieves.

Allowing for the presence of market power in the output sector, our study also adds to the literature that examines the interdependence between market structure and environmental policy. Seminal contributions in this domain are Buchanan (1969) and Barnett (1980), which show that the optimal emission tax for a monopoly falls short of the marginal damage from pollution.[2] Other studies that compare emissions trading on the basis of absolute and relative targets have ignored the multi-sectoral implications under imperfect competition. Dewees (2001) makes a welfare comparison between the two emissions trading schemes in a single perfectly competitive industry, whereas Boom and Dijkstra (2009) make the comparison for a perfectly as well as an imperfectly competitive sector. Fischer (2003) analyzes emissions trading between two perfectly competitive sectors, one of them regulated by a cap-and-trade scheme and the other by a scheme based on intensity standards. Boom and Dijkstra (2009) analyze the same scenario for two perfectly competitive and two imperfectly competitive sectors.

The paper proceeds as follows. In the next section we introduce the benchmark model. Section 3 develops and analyzes the emissions trading regimes, followed by a welfare comparison in Section 4. Conclusions are given in Section 5.

[2] See Millimet et al. (2009) for a survey on the interaction between environmental policy and market structure.

2 Basic model

Consider an imperfectly competitive market consisting of $n \geq 2$ firms that choose output to maximize profit. Firms can choose to produce output in either one of two sectors $i = c, d$, with $i = c$ referring to the clean sector and $i = d$ to the dirty sector. We shall define 'clean' and 'dirty' at the end of this Section. For simplicity, we treat the number of firms in each sector, n_i, as a continuous variable.[3]

Firm-level emissions $e_i > 0$ are assumed to vary proportionally with firm-level output $q_i > 0$ for both goods:

$$e_i = \epsilon_i q_i \qquad i = c, d \tag{1}$$

with $\epsilon_i > 0$. Aggregate output produced by firms in the clean and dirty sector is simply $Q_i = n_i q_i$, with the two sectors facing the following inverse demand functions:

$$p_i = \alpha_i - Q_i, \qquad i = c, d \tag{2}$$

where p_i is the price of good i.[4] A higher α_i (relative to α_{-i}) implies an absolute advantage in demand (at equal output levels) enjoyed by the firm in sector i. Put differently, $\alpha_i - \alpha_{-i}$ reflects a price premium for sector i. Production in sector i incurs fixed cost $F_i > 0$ and constant marginal cost $c_i > 0$. Following Dixit (1979), the cost margin for a firm in sector i is $\alpha_i - c_i > 0$, and a firm in sector i has a margin advantage if $\alpha_i - c_i > \alpha_{-i} - c_{-i}$. Further, let us define a firm's *full marginal cost,* k_i, as its marginal cost of production, c_i, plus its cost of emissions from the extra output. Without environmental policy, $k_i = c_i$. We shall see that with emissions trading based on an absolute cap and a relative intensity standard, k_i is given by (16) and (25) respectively. Both with and without environmental policy, k_i is a constant to the individual firm.

We can now solve for the profit (π_i)-maximizing output level of a firm with full marginal cost k_i and fixed cost F_i. From (2):

$$\max_{q_i} \pi_i = (\alpha_i - \widehat{Q}_i - q_i - k_i)q_i - F_i \qquad i = c, d \tag{3}$$

[3] This is a standard assumption in the literature; Boom and Dijkstra (2009) is an exception.

[4] Note that the slope of both inverse demand functions is normalized to -1. In appendix A.1 it is demonstrated that this normalization procedure has no impact on the subsequent analysis.

with $\widehat{Q}_i$ the aggregate output of all other firms in sector i. The first-order condition is:

$$\alpha_i - \widehat{Q}_i - 2q_i - k_i = 0 \qquad (4)$$

By symmetry, $\widehat{Q}_i = (n_i - 1)q_i$ so that the equilibrium quantities are:

$$q_i = \frac{\alpha_i - k_i}{n_i + 1} \qquad i = c, d \qquad (5)$$

Substituting (5) and $\widehat{Q}_i = (n_i - 1)q_i$ back into (3), profits can be written as:

$$\pi_i = \left(\alpha_i - k_i - \frac{n_i(\alpha_i - k_i)}{n_i + 1}\right) q_i - F_i = q_i^2 - F_i \qquad i = c, d. \qquad (6)$$

In the long run firms exit from a sector when they incur losses, whereas profits attract new firms, until profit is driven to zero. Setting $\pi_i = 0$ in (6), we find:

Proposition 1 *Absent environmental policy, or with emissions trading either in the form of an absolute cap or on the basis of relative intensity standards, the long run equilibrium output per firm in sector i is:*

$$q_i = f_i \equiv \sqrt{F_i} \qquad i = c, d. \qquad (7)$$

Let us now complete the solution for the unconstrained benchmark, i.e., the long run equilibrium without environmental policy. Substituting $k_i = c_i$ and (7) into (5) yields:[5]

$$\bar{n}_i = \frac{\alpha_i - c_i - f_i}{f_i} \qquad i = c, d \qquad (8)$$

An interior equilibrium exists (e.g., $\bar{n}_i > 0$) if and only if:

$$\gamma_i \equiv \alpha_i - c_i - f_i > 0 \qquad i = c, d \qquad (9)$$

where γ_i can be seen as the long run cost margin on production. In the long run, each unit of output should not only cover its marginal production cost, but also contribute its share f_i to cover the fixed cost, F_i. Equations (7) to (9) then imply:

$$\bar{n}_i \bar{q}_i = \bar{n}_i f_i = \gamma_i \qquad i = c, d. \qquad (10)$$

[5] Overbars represent the value of a variable in the unconstrained benchmark.

The total amount of emissions generated by the firms in the clean and dirty sector in the unconstrained benchmark is:

$$\bar{E} = \epsilon_c \bar{n}_c \bar{q}_c + \epsilon_d \bar{n}_c \bar{q}_c = \epsilon_c \gamma_c + \epsilon_d \gamma_d. \tag{11}$$

where the second equality follows from (10).

We shall define the clean sector as the sector with the greater long run cost margin on emissions γ_i/ϵ_i:[6]

$$\frac{\gamma_c}{\epsilon_c} > \frac{\gamma_d}{\epsilon_d}, \tag{12}$$

with γ_i given by (9) and ϵ_i by (1). This definition implies that when total emissions are below the unconstrained level (11) and the number of clean and dirty firms is equal, clean production contributes more to welfare than dirty production. Hence, it is optimal to have more clean firms than dirty firms.[7]

3 Emissions trading

This section applies the model developed in the previous section to emissions trading on the basis of an absolute cap on emissions in the form of a cap-and-trade scheme (section 3.1) and emissions trading on the basis of a relative intensity standard (section 3.2). We denote these two emissions trading schemes by A and R, respectively.

We shall analyze and compare the trading schemes for a given level L of total emissions:

$$L = \epsilon_c n_c q_c + \epsilon_d n_d q_d = \epsilon_c n_c f_c + \epsilon_d n_d f_d. \tag{13}$$

The second equality follows from Proposition 1. Throughout the analysis we assume that total emissions L exceed a threshold $L_{\min}$:

$$L > L_{\min} \equiv \gamma_c \epsilon_c = \bar{E} - \gamma_d \epsilon_d, \tag{14}$$

where the second equality follows from (11). Condition (14) is necessary and sufficient for interior equilibria (with $n_c, n_d > 0$) to exist with emissions trading under the two design configurations.

[6] Since γ_i is measured in money per unit of output i and ϵ_i is measured in emissions per unit of output i, the long run cost margin on emissions, γ_i/ϵ_i, is measured in money per unit of emissions.

[7] This follows formally from equation (A5) given in Appendix A.3.

In order to ensure that $p_i > 0$ $(i = c, d)$ we shall assume:

$$\alpha_c - \frac{\gamma_c + \sqrt{\gamma_c^2 + \gamma_d^2}}{2} > 0. \tag{15}$$

Note that while we consider imperfect competition in the output market, it is assumed that firms act as price takers in the tradeable pollution market. Although this may seem restrictive, it is a credible assumption and not in conflict with the imperfectly competitive nature of the output market. For instance, the EU ETS for carbon emissions allows trade between firms from different industries such as electric power plants, glass manufacturers, steel producers, the cement industry as well as firms from the paper industry. The pollution market can therefore be competitive while competition in the respective output markets is imperfect.

3.1 Cap-and-trade

The regulator auctions allowances, each valid for one unit of emissions. The allowances in total sum up to the absolute cap, L, and the allowance price, v, is established on the pollution market. The profit-maximization problem of firm i, taking the allowance price as given, can then be written as (3) with:

$$k_i^A = c_i + v\epsilon_i. \tag{16}$$

Equation (16) shows that the full marginal production costs under cap-and-trade, k_i^A, equal marginal production costs, c_i, plus the cost of buying the allowances for the ϵ_i emissions from the extra output. We see that a cap-and-trade policy increases the marginal cost of both the dirty and the clean firm.

The cap is non-binding if it is greater or equal to the unconstrained level of emissions given by equation (11), i.e., if $L \geq \bar{E}$. A non-binding cap on pollution will result in an allowance price $v = 0$; a cap $L < \bar{E}$ is binding, implying that the allowance price $v > 0$. This ensures that the demand for allowances is equal to its supply shown in (13). Using (5), (7), (13) and (16), we can now solve for n_c^A, n_d^A and v for a given level of total emissions L. The long run equilibrium allowance price under a cap-and-trade regime is:

$$v = \frac{\bar{E} - L}{\epsilon_c^2 + \epsilon_d^2}, \tag{17}$$

where $\bar{E}$ is the unconstrained emission level given by (11).

The long run equilibrium number of firms is:

$$n_i^A = \frac{\epsilon_{-i}(\epsilon_{-i}\gamma_i - \epsilon_i\gamma_{-i}) + \epsilon_i L}{f_i\left(\epsilon_c^2 + \epsilon_d^2\right)} \qquad i = c, d. \tag{18}$$

Since n_i^A is increasing in $L < \bar{E}$, we have:

$$n_i^A < \bar{n}_i \qquad i = c, d. \tag{19}$$

By (12), n_c^A in (18) is always positive. However, $n_d^A > 0$ if and only if:

$$L > L_{\min}^A \equiv \frac{\epsilon_c\left(\epsilon_d\gamma_c - \epsilon_c\gamma_d\right)}{\epsilon_d} = \bar{E} - \frac{\gamma_d\left(\epsilon_c^2 + \epsilon_d^2\right)}{\epsilon_d}. \tag{20}$$

The second equality follows from (11). Comparing $L_{\min}^A$ in (20) to threshold $L_{\min}$ in (14), one directly obtains $L_{\min} - L_{\min}^A = \frac{\gamma_d\epsilon_c^2}{\epsilon_d} > 0$. Condition (14) is therefore a sufficient condition for (20) to hold, meaning that the cap is sufficiently lax such that both the clean and dirty sector coexist.

Let us illustrate our findings with a specific numerical example where:

$$\epsilon_c = 1,\ \epsilon_d = 2,\ \alpha_c = \alpha_d = \alpha = 300,\ c_c = c_d = c = 92,\ f_c = f_d = f = 8, \tag{21}$$

so that $\gamma_c = \gamma_d = \gamma = 200$ by (9) and $q_c = q_d = f = 8$ by (7). To simplify the graphical exposition, we assume that the two sectors are identical except for their emissions-to-output ratios. Figure 1 shows the inverse demand curve for sector $i = c, d$ as $p_i(Q_i)$ and the long run average production costs as $c + f$. The unconstrained benchmark is at point B with $Q_i = 200,\ p_i = 100,\ n_i = 25$ and $\bar{E} = 600$.

Figure 1 illustrates the long run cap-and-trade equilibrium for our numerical example (21).[8] We take as our starting point a certain value of Q_d as the equilibrium value of total dirty good production for a certain exogenous emission cap L (with the value of L yet to be inferred). We wish to know what would be the equilibrium value of Q_c that goes with this value of Q_d. Once we have established the equilibrium combination of Q_d and Q_c, we can infer the associated exogenous level of L from (13) and (21):

$$L = \epsilon_c Q_c + \epsilon_d Q_d = Q_c + 2Q_d \tag{22}$$

[8]In Figures 1 and 2, parameters and variables are shown in italics, while points (ordered pairs) are shown in roman type.

Returning to the question of what is the equilibrium value of Q_c for a given equilibrium value of Q_d, this follows from (18) and (21) with $Q_i = n_i f_i,\ i = c, d$:

$$Q_c = \frac{1}{\epsilon_d}(Q_d\epsilon_c + \gamma_c\epsilon_d - \gamma_d\epsilon_c) = \frac{Q_d}{2} + 100 \tag{23}$$

When $Q_d = 120$, for instance, $Q_c = 160$. In order to illustrate this relation between the equilibrium values of Q_c and Q_d in Figure 1, it is useful to define $p_c^A(Q_d)$ as the long run equilibrium price of the clean good, given that Q_d is the long run equilibrium quantity of the dirty good with emission trading based on cap-and-trade where the exogenous level of total emissions is given by (22). From (2), (21) and (23), the expression for $p_c^A(Q_d)$ in general and for our numerical example is, respectively:

$$p_c^A(Q_d) = \alpha_c - \frac{1}{\epsilon_d}(Q_d\epsilon_c + \gamma_c\epsilon_d - \gamma_d\epsilon_c) = 200 - \frac{Q_d}{2}. \tag{24}$$

Figure 1 shows the $p_c^A(Q_d)$ curve for our numerical example. We see that when $Q_d = 120$, then $p_c = 140$ so that $Q_c = 160$. Applying (22), we see that the combination $Q_d = 120$, $Q_c = 160$ is the equilibrium outcome for the exogenous emission cap of $L = 400$. The complete characterization of the cap-and-trade equilibrium is then: When $L = 400$, then $Q_d = 120,\ p_d = 180,\ n_d = 15,\ Q_c = 160,\ p_c = 140,\ n_c = 20$, and by (17) $v = 40$. For any given Q_d, the $p_c^A(Q_d)$ curve given by (24) is halfway between $p_i(Q_i)$ and $c + f$. This is because by (3) with $\pi = 0$ and (16), the vertical distance between p_d and $c + f$ equals $v\epsilon_d$, while the vertical distance between p_c and $c + f$ equals $v\epsilon_c$. The ratio between the two distances is thus ϵ_d/ϵ_c, which equals 2 in our numerical example (21).

3.2 Intensity standards

In contrast to a cap-and-trade system, consider now the case where the government sets a pollution intensity standard δ_i for sector $i = c, d$. Under such an intensity-based trading system, if a firm wants to emit more *per unit* than the standard allows, it can buy allowances from firms that emit less *per unit* than the standard allows. The result is that, on average, the economy as a whole complies with the emission standard but the individual firm has the flexibility to deviate from it. With our specification

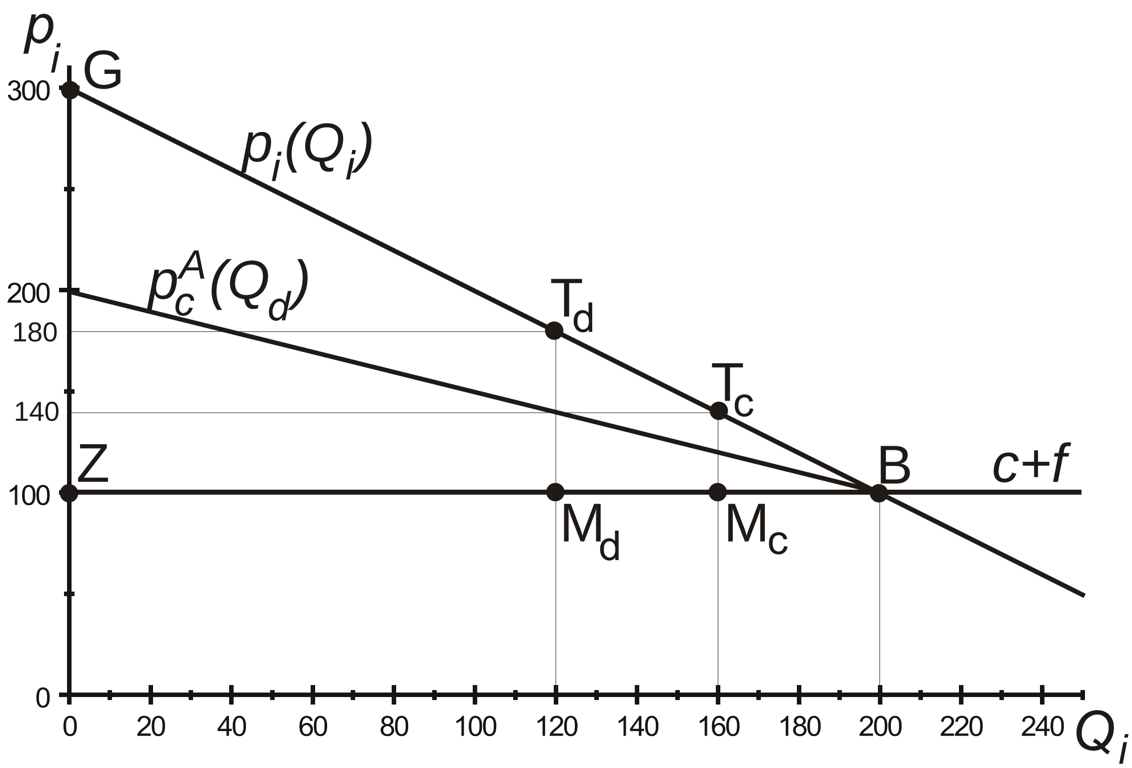

Figure 1: Long run equilibrium with cap-and-trade and welfare optimum

of the demand function as shown in (2) we have implicitly defined one unit of good i such that when the price p_i increases by one unit, demand Q_i decreases by one unit. However, there is no reason why the regulator should adopt this definition of a unit and set $\delta_i = \delta$. We therefore allow δ_c to differ from δ_d.[9]

The profit-maximization problem of firm i, taking the allowance price under intensity standards, w, as given, can then be written as (3) with:

$$k_i^R = c_i + w(\epsilon_i - \delta_i). \tag{25}$$

The full marginal production costs under intensity standards, k_i^R, are equal to marginal production costs, c_i, plus the cost of buying the allowances for the extra emissions exceeding the standard. Each extra unit of output comes with ϵ_i extra emissions as well as with permission for δ_i extra emissions. If $\epsilon_i > \delta_i$, firm i has to buy allowances from other firms; if $\epsilon_i < \delta_i$, the firm can sell allowances.

[9] Note that the definition of a unit of production does not affect our definition of the clean and dirty sector as given by (12), because the latter definition is in terms of the long run cost margin *on emissions* (see also footnote 6).

Substituting (25) and (7) into (5), we find:

$$f_i = \frac{\alpha_i - c_i + w(\delta_i - \epsilon_i)}{n_i + 1} \qquad i = c, d. \tag{26}$$

Using (7), the pollution market clears via the constraint:

$$n_d^R f_d(\epsilon_d - \delta_d) = n_c^R f_c(\delta_c - \epsilon_c). \tag{27}$$

This constraint reveals the key difference in the functioning of the two different allowance market configurations. Whereas the supply of allowances, L, is fixed under a cap-and-trade regime, the supply of allowances under intensity standards — reflected by the RHS of (27) — varies with aggregate clean output $Q_c^R = n_c^R f_c$.

We now have four conditions for the long run equilibrium: (13) for the total level L of emissions, (26) for each sector i, and (27). However, we have five variables: δ_c, δ_d, n_c, n_d and w. This implies that the solution is not uniquely determined in the long run equilibrium. In order to reduce the number of variables to four, let us define:

$$h_i \equiv w(\delta_i - \epsilon_i) \tag{28}$$

as the revenue per unit of output that a firm in sector i receives from selling allowances. In equilibrium, if h_c is positive, h_d must be negative and vice versa, as is clear from (27). Substituting (28) into (26) and (27) respectively yields:

$$n_i f_i = \gamma_i + h_i \qquad i = c, d, \tag{29}$$

$$n_c f_c h_c = -n_d f_d h_d. \tag{30}$$

We now have a system of four equations: (29) for each sector $i = c, d$, (30) and (13). This system can then be solved for the four unknown variables: n_c, n_d, h_c and h_d. Thus, while the allowance price w as well as the sector-specific standards δ_c and δ_d are not uniquely determined in equilibrium, the amount that each firm receives from selling (or spends on buying) allowances per unit of output *is* determined.

We shall see that this system of four equations has two solutions, which can be compared on welfare. Since both solutions have the same level of emissions (L) in the long run equilibrium, they feature the same level of environmental damage. This implies that we can abstract from the environmental damage component in the welfare function explicitly.

Under both emissions trading schemes, output per firm is given by (7). From (2), (7) and (13), welfare for a given level L of total emissions, with $q_i = f_i$ in both sectors is given by:

$$W = \sum_{i=c,d} \left[\alpha_i n_i f_i - \frac{1}{2}(n_i f_i)^2 - c_i n_i f_i - n_i F_i \right] - \lambda \left(\sum_{i=c,d} \epsilon_i n_i f_i - L \right). \qquad (31)$$

The first two terms between square brackets on the RHS denote the consumption utility from the good (the area below the inverse demand curve); the third term denotes aggregate variable cost, and the fourth term aggregate fixed cost. The second term on the RHS of (31) is the emissions constraint (13) with $q_i = f_i$ $(i = c, d)$. The only difference between the two solutions consists of the number n_i of firms in either sector, since output per firm is fixed.

We can now state:

Proposition 2 *The long run equilibrium with emissions trading based on a pollution intensity standard is given by equations (13), (29) for each sector $i = c, d$, and (30). This system can be solved for n_c, n_d, h_c and h_d, with n_i denoting the number of firms in sector i and h_i the revenue per unit of output that a firm in sector i receives from selling allowances.*

1. *There are two solutions: r and ρ. Solution r features $n_c^r > \bar{n}_c$, $n_d^r < \bar{n}_d$ (with the number $\bar{n}_i$ of firms in sector i in the unconstrained equilibrium given by (8)), and clean firms selling allowances to dirty firms. Solution ρ features $n_c^\rho < \bar{n}_c$, $n_d^\rho > \bar{n}_d$, and dirty firms selling allowances to clean firms.*

2. *Solution r leads to higher welfare than solution ρ.*

3. *Solution r exists if and only if both inequalities (14) and (15) hold.*

Proof. See Appendix A.2. ■

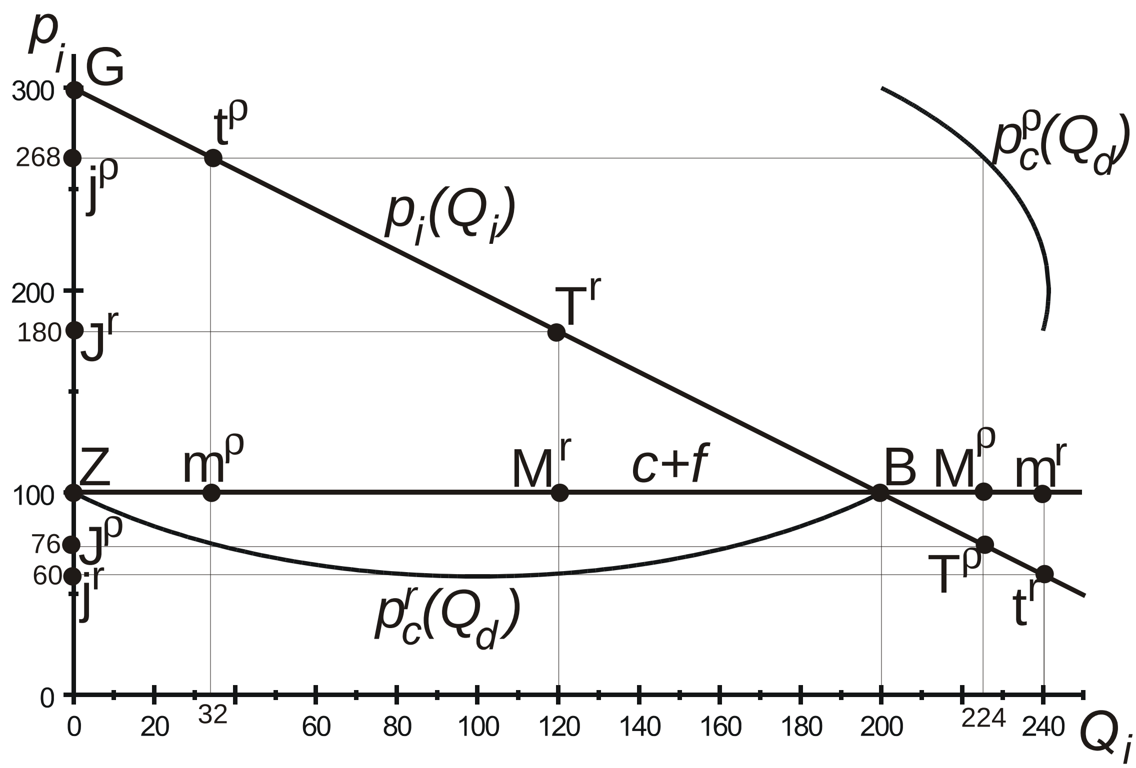

Figure 2: Long run equilibrium with intensity standards

Figure 2 illustrates Proposition 2 for our numerical example (21). Analogous to $p_c^A(Q_d)$ in Figure 1, $p_c^r(Q_d)$ gives the equilibrium clean output price for a given equilibrium level of Q_d in solution r and $p_d^\rho(Q_d)$ does the same for solution ρ, with the exogenous level L of emissions given by (22).[10]

In solution r, clean firms sell allowances to dirty firms. For instance, when $Q_d = 120$, Figure 2 shows that $p_c^r = 60$ so that $Q_c^r = 240$, implying that this is solution r for the exogenous emission level $L = 480$. Figure 2 illustrates equation (30) for solution r: The amount $Q_c h_c$ that clean firms receive from selling allowances equals the amount $-Q_d h_d$ that dirty firms pay for allowances. In Figure 2, $Q_c^r h_c^r$ is given by the area $Zm^r r^r j^r = 240 \times 40 = 9600$, while $-Q_d^r h_d^r$ is given by $ZJ^r T^r M^r = 120 \times 80 = 9600$. The full solution r for $L = 480$ is thus: $Q_d^r = 120,\ p_d^r = 180,\ n_d^r = 15,\ h_d^r = -80$ and $Q_c^r = 240,\ p_c^r = 60,\ n_c^r = 30,\ h_c^r = 40$.

In solution ρ, dirty firms sell allowances to clean firms. For instance, when $Q_d =$

[10]The expressions for $p_c^r(Q_d)$ and $p_c^\rho(Q_d)$ are derived in Appendix A.4.

224, Figure 2 shows that $p_c^\rho = 268$ so that $Q_c^\rho = 32$ which implies that this is solution ρ for the exogenous emission level $L = 480$. Figure 2 illustrates equation (30) for solution ρ: The amount $Q_d h_d$ that dirty firms receive from selling allowances equals the amount $-Q_c h_c$ that clean firms pay for allowances. In Figure 2, $Q_d^\rho h_d^\rho$ is given by the area $ZM^\rho T^\rho J^\rho = 224 \times 24 = 5376$, while $-Q_c^\rho h_c^\rho$ is given by $Zj^\rho t^\rho m^\rho = 32 \times 168 = 5376$. The full solution ρ for $L = 480$ is thus: $Q_d^\rho = 224,\ p_d^\rho = 76,\ n_d^\rho = 28,\ h_d^\rho = 24$ and $Q_c^\rho = 32,\ p_c^\rho = 268,\ n_c^\rho = 4,\ h_c^\rho = 168$.

Next, let us compare welfare in both solutions r and ρ for the numerical example (21) with $L = 480$. In Figure 2, welfare in sector i with output Q_i is given by the area between the demand curve $p_i(Q_i)$ and the long run average cost curve $c+f$. In solution r, welfare in the clean and dirty sector together is, respectively:

$$\begin{aligned} W^r &= (ZGB - Bm^r t^r) + (ZGB - BM^r T^r) \\ &= \frac{(200^2 - 40^2) + (200^2 - 80^2)}{2} = 36000. \end{aligned}$$

This exceeds welfare in solution ρ, which can be calculated in the same way as:

$$\begin{aligned} W^\rho &= (ZGB - Bm^\rho t^\rho) + (ZGB - BM^\rho T^\rho) \\ &= \frac{(200^2 - 168^2) + (200^2 - 24^2)}{2} = 25600. \end{aligned}$$

The intuition is as follows. Emissions trading via intensity standards inevitably leads to the subsidization and expansion of one sector (relative to the unconstrained benchmark), and the taxation and contraction of the other sector. It is better for the clean sector to expand, because this leads to a relatively small increase in emissions so that the dirty sector does not have to contract a lot in order to reach the desired emission level. By contrast, expansion of the dirty sector leads to a large emission increase, so that the clean sector has to contract significantly.

Since solution r yields higher welfare than solution ρ, we shall assume from now on that the regulator will implement solution r where clean firms sell allowances to dirty firms. Thus, $n_i^R = n_i^r$ and $h_i^R = h_i^r$, with n_i^r and h_i^r $(i = c, d)$ given by equations (A1a) through (A1d) in Appendix A.1. Solution r can be implemented with a range of sector-specific intensity standard combinations (δ_c, δ_d). It follows from (28) that:

$$\frac{h_c^r}{-h_d^r} = \frac{\delta_c - \epsilon_c}{\epsilon_d - \delta_d}.$$

In the above example with $\epsilon_c = 1$ and $\epsilon_\delta = 2$, where $L = 480$ implies $h_c^r = 40$ and $h_d^r = -80$, this becomes:

$$\frac{1}{2} = \frac{\delta_c - 1}{2 - \delta_d}.$$

Thus we have $\delta_c \in (1, 2]$ and $\delta_d \in [0, 2)$. Note that the range of solutions includes the uniform standard $\delta_c = \delta_d = \frac{4}{3}$. By (28), the allowance price w decreases as δ_c and δ_d move further away from ϵ_c and ϵ_d respectively, ultimately dropping to $w = 40$ for $(\delta_c, \delta_d) = (2, 0)$.

4 Welfare comparison

In this section we compare welfare under the two emission trading policies. Since we are comparing cap-and-trade and intensity-based emissions trading for a given equal level of emissions, we can abstract from the environmental damage component in the welfare function explicitly. Welfare for a given level L of total emissions, with long run output per firm $q_i = f_i$ in both sectors, is given by (31) as explained in subsection 3.2.[11] We find that:

Proposition 3 *Emissions trading via an absolute cap-and-trade scheme maximizes welfare for a given level of total emissions under the constraint that $q_i = f_i$ $(i = c, d)$. Emissions trading via intensity standards results in too many clean firms and too few dirty firms.*

Figure 1 illustrates the optimality of cap-and-trade given that $q_i = f_i$. We know from section 3.1 that the long run cap-and-trade equilibrium for our numerical example (21) with $L = 400$ is $Q_d = 120$ and $Q_c = 160$. In Figure 1, welfare in sector i with output Q_i is given by the area between the demand curve $p_i(Q_i)$ and the long run

[11] Without the zero-profit result that $q_i = f_i$ (or any other constraints on q_i), the welfare optimum would feature n_c and n_d arbitrarily small (ignoring the integer constraint) or $n_c = n_d = 1$ (taking the integer constraint into account) in order to minimize aggregate fixed costs.

average cost curve $c + f$. The sum of welfare in the dirty and clean sector respectively is then:

$$\begin{aligned} W^A &= (ZGB - M_d T_d B) + (ZGB - M_c T_c B) \\ &= \frac{(200^2 - 80^2) + (200^2 - 40^2)}{2} = 36000. \end{aligned}$$

How would welfare change if we slightly decreased production of the dirty good and increased production of the clean good, so that total emissions remain at 400. Since $\epsilon_d = 2\epsilon_c$, we can increase clean output by twice the dirty output reduction. A marginal reduction in dirty output reduces welfare in Figure 1 by $M_d T_d = 80$. A double marginal increase in the production of the clean good raises welfare by $2M_c T_c = 80$. Total welfare thus remains unchanged, which means that the long run cap-and-trade equilibrium must be the welfare optimum.

Emissions trading on the basis of intensity standards cannot implement the welfare optimum. As we know from subsection 3.2, the clean sector is subsidized and the dirty sector is taxed under such a system. Thus clean output is higher than in the unconstrained benchmark, and dirty output is lower. It is easily seen with the aid of Figure 1 that the optimal response to emission reduction is output reduction in both sectors.[12] This leads to higher welfare than output reduction in one sector only, which in turn is better than output reduction in one sector and output expansion in the other sector. Under a regime of intensity standards the dirty sector is inefficiently small to compensate for the growth in the clean sector.

5 Conclusions

The design of markets for tradeable emission allowances can generally take two forms: organizing trade on the basis of an *absolute* cap or on the basis of *relative* pollution intensity standards. The design has implications for the functioning of these markets, particularly in relation to their interaction with output markets and the impact on entry and exit. This paper analyzes these interactions and assesses the corresponding long run welfare performance of these emissions trading schemes in a two-sector ('clean'

[12] See Holland et al. (2009) for a similar assessment of Low Carbon Fuel Standards.

and 'dirty') model with imperfectly competitive output markets, where we define the clean sector as the sector with the highest 'value' per unit of emissions.

With intensity standards we allow the maximum allowed level of emissions per unit of output to differ between the two sectors. This means that the standard does not depend on the definition of a unit of output. We find that intensity standards could result in clean firms selling allowances to dirty firms, or in dirty firms selling to clean firms. Since the former outcome always yields higher welfare, we assumed that the regulator will set the standards so as to implement this outcome.

With free entry and exit driving profits to zero, output per firm in either of the two sectors does not depend on how emissions trading is organized but only depends on the fixed cost of producing in a sector. This is because a firm faces constant full marginal cost, comprising both the production and pollution cost. It is shown that an absolute cap-and-trade regime always generates the first-best outcome in the long run, given the zero-profit output level per firm with free entry and exit in the output market. Emissions trading on the basis of relative intensity standards leads to too many clean firms in the long run equilibrium, i.e., the size of the clean sector is too large compared to the size of the clean sector under cap-and-trade.

A Appendix

A.1 Normalization of the Slope of a Demand Function

The slope of the demand function for a good, when using conventional units for measuring the good as well as for money, is usually different from -1. In this appendix we show how to normalize the slopes of the demand functions for two goods (gasoline and coal) to -1 by changing the unit of measurement of the respective goods. We leave the money measurement intact, so that consumer surplus from the two goods can still be added together after normalization.

Suppose the inverse demand function for gasoline (gas) is:

$$P_{gas} = A - BY_{gas}$$

with the quantity of gasoline Y_{gas} measured in gallons and its price P_{gas} in dollars per gallon. Thus the units on A and B are "\$/gallon" and "\$/(gallon)^2" respectively. Total revenue is $P_{gas}Y_{gas}$ and units for total revenue are dollars. Now to normalize the demand function, we first divide both sides by $b \equiv \sqrt{B}$ "1/b-gallons" per gallon (or equivalently, we multiply by $1/b$ gallons per "1/b-gallon"). The demand function then becomes:

$$p_{gas} = \alpha_{gas} - bY_{gas}$$

with $p_{gas} \equiv P_{gas}/b$ and $\alpha_{gas} \equiv A/b$. Now p_{gas} and α_{gas} are measured in "\$ per 1/b-gallon" and $b = B/b$ in "\$/(gallon $\times$ 1/b-gallon)". Finally, we introduce the quantity measure Q_{gas} which is expressed in "1/b-gallon" so that $Q_{gas} = bY_{gas}$. This turns the demand function into:

$$p_{gas} = \alpha_{gas} - \beta_{gas}Q_{gas},$$

with $\beta_{gas} = \$1/(1/b\text{-gallon})^2$. *The slope of the demand function is now* -1.

As a specific example, let us set $A = \$500$/gallon and $B = \$100/(\text{gallon})^2$ in $P_{gas} = A - BY_{gas}$. This means the vertical intercept is \$500 per gallon and the horizontal intercept is 5 gallons. Writing the unit of measurement below each parameter and variable, we have:

$$\underset{\frac{\$}{gallon}}{P_{gas}} = \underset{\frac{\$}{gallon}}{500} - (\underset{\frac{\$}{(gallon)^2}}{100} \times \underset{gallon}{Y_{gas}})$$

Multiplying the left-hand side and the right-hand side by 0.1 gallon/decigallon yields:

$$(\underset{\frac{\$}{gallon}}{P_{gas}} \times \underset{\frac{gallon}{decigallon}}{0.1}) = (\underset{\frac{\$}{gallon}}{500} \times \underset{\frac{gallon}{decigallon}}{0.1}) - (\underset{\frac{\$}{(gallon)^2}}{100} \times \underset{\frac{gallon}{decigallon}}{0.1} \times \underset{gallon}{Y_{gas}})$$

Simplifying and noting that $Y_{gas} = 0.1Q_{gas}$ yields:

$$\underset{\frac{\$}{decigallon}}{p_{gas}} = \underset{\frac{\$}{decigallon}}{50} - (\underset{\frac{\$}{(gallon)^2}}{100} \times \underset{\frac{gallon}{decigallon}}{0.1} \times \underset{\frac{gallon}{decigallon}}{0.1} \times \underset{decigallon}{Q_{gas}})$$

Simplifying this gives:

$$\underset{\frac{\$}{decigallon}}{p_{gas}} = \underset{\frac{\$}{decigallon}}{50} - (\underset{\frac{\$}{(decigallon)^2}}{1} \times \underset{decigallon}{Q_{gas}})$$

After normalization, the vertical intercept is \$50 per decigallon and the horizontal intercept is 50 decigallons.

In the same way, let the inverse demand function for coal be $P_{coal} = C - DY_{coal}$ with the quantity of coal measured in tons and its price in dollars per ton. We normalize this demand function to $p_{coal} = \alpha_{coal} - Q_{coal}$ with $p_{coal} \equiv P_{coal}/d$, $\alpha_{coal} \equiv C/d$ and $Q_{coal} \equiv dY_k$ where $d \equiv \sqrt{D}$. Now the quantity of coal is measured in "$1/d$-tons" and its price in dollars per $1/d$-ton. As a specific example, let us set $C = \$100$ per ton and $D = \$4/(\text{ton})^2$. This means the vertical intercept is \$100 per ton and the horizontal intercept is 25 tons. We normalize this demand function by expressing the quantity of coal Q_{coal} in "half tons," with its price p_{coal} expressed in dollar per half ton. After normalization the demand curve is $p_{coal} = 50 - Q_{coal}$. The vertical intercept is \$50 per half ton and the horizontal intercept is 50 half tons.

A.2 Proof of Proposition 2

Proposition 2.1 There are two solutions to equations (29) for each sector $i = c, d$, (13) and (30) which we shall denote by r and ρ. Solution r is:

$$h_c^r = \frac{-\epsilon_d\left(\gamma_c\epsilon_d - \gamma_d\epsilon_c\right) - 2\epsilon_c(\bar{E} - L) + \epsilon_d\sqrt{\left(\gamma_c\epsilon_d - \gamma_d\epsilon_c\right)^2 + 4L\left(\bar{E} - L\right)}}{2\left(\epsilon_c^2 + \epsilon_d^2\right)} \quad \text{(A1a)}$$

$$h_d^r = \frac{\epsilon_c\left(\gamma_c\epsilon_d - \gamma_d\epsilon_c\right) - 2\epsilon_d(\bar{E} - L) - \epsilon_c\sqrt{\left(\gamma_c\epsilon_d - \gamma_d\epsilon_c\right)^2 + 4L\left(\bar{E} - L\right)}}{2\left(\epsilon_c^2 + \epsilon_d^2\right)} \quad \text{(A1b)}$$

$$n_c^r = \frac{2L\epsilon_c + \epsilon_d\left(\gamma_c\epsilon_d - \gamma_d\epsilon_c + \sqrt{\left(\gamma_c\epsilon_d - \gamma_d\epsilon_c\right)^2 + 4L\left(\bar{E} - L\right)}\right)}{2f_c\left(\epsilon_c^2 + \epsilon_d^2\right)} \quad \text{(A1c)}$$

$$n_d^r = \frac{2L\epsilon_d + \epsilon_c\left(\gamma_d\epsilon_c - \gamma_c\epsilon_d - \sqrt{\left(\gamma_c\epsilon_d - \gamma_d\epsilon_c\right)^2 + 4L\left(\bar{E} - L\right)}\right)}{2f_d\left(\epsilon_c^2 + \epsilon_d^2\right)}, \quad \text{(A1d)}$$

with $\bar{E} > L$ being the unconstrained emissions given by (11) and γ_i by (9). We see that h_d^r in (A1b) is negative, i.e., dirty firms are buying allowances so that $n_d^r < \bar{n}_d$ by (10) and (29). Market clearing with $n_c^r, n_d^r > 0$ then requires by (30) that $h_c^r > 0$: clean firms are selling allowances, and $n_c^r > \bar{n}_c$ by (10) and (29).

Solution ρ is:

$$h_c^\rho = \frac{-\epsilon_d\left(\gamma_c\epsilon_d-\gamma_d\epsilon_c\right)-2\epsilon_c\left(\bar{E}-L\right)-\epsilon_d\sqrt{\left(\gamma_c\epsilon_d-\gamma_d\epsilon_c\right)^2+4L(\bar{E}-L)}}{2\left(\epsilon_c^2+\epsilon_d^2\right)} \quad \text{(A2a)}$$

$$h_d^\rho = \frac{\epsilon_c\left(\gamma_c\epsilon_d-\gamma_d\epsilon_c\right)-2\epsilon_d\left(\bar{E}-L\right)+\epsilon_c\sqrt{\left(\gamma_c\epsilon_d-\gamma_d\epsilon_c\right)^2+4L(\bar{E}-L)}}{2\left(\epsilon_c^2+\epsilon_d^2\right)} \quad \text{(A2b)}$$

$$n_c^\rho = \frac{2L\epsilon_c+\epsilon_d\left(\gamma_c\epsilon_d-\gamma_d\epsilon_c-\sqrt{\left(\gamma_c\epsilon_d-\gamma_d\epsilon_c\right)^2+4L\left(\bar{E}-L\right)}\right)}{2f_c\left(\epsilon_c^2+\epsilon_d^2\right)} \quad \text{(A2c)}$$

$$n_d^\rho = \frac{2L\epsilon_d+\epsilon_c\left(\gamma_d\epsilon_c-\gamma_c\epsilon_d+\sqrt{\left(\gamma_c\epsilon_d-\gamma_d\epsilon_c\right)^2+4L\left(\bar{E}-L\right)}\right)}{2f_d\left(\epsilon_c^2+\epsilon_d^2\right)}. \quad \text{(A2d)}$$

We see that h_c^ρ in (A2a) is negative, i.e., clean firms are buying allowances so that $n_c^\rho < \bar{n}_c$ by (10) and (29). Market clearing with $n_c, n_d > 0$ then requires by (30) that $h_d^\rho > 0$: dirty firms are selling allowances so that $n_d^\rho > \bar{n}_d$ by (10) and (29). ■

Proposition 2.2 Substituting (29) and (30) into (31) gives welfare W^R under intensity standards:

$$W^R = \frac{\sum_i \left(n_i f_i\right)^2}{2}. \quad \text{(A3)}$$

Substituting (A1c) and (A1d) into (A3) yields:

$$W^r = \frac{\left(\gamma_c\epsilon_d-\gamma_d\epsilon_c\right)^2+2L\left(2L-\bar{E}\right)+\left(\gamma_c\epsilon_d-\gamma_d\epsilon_c\right)\sqrt{\left(\gamma_c\epsilon_d-\gamma_d\epsilon_c\right)^2+4L\left(\bar{E}-L\right)}}{2\left(\epsilon_c^2+\epsilon_d^2\right)}.$$

Substituting (A2c) and (A2d) into (A3) yields:

$$W^\rho = \frac{\left(\gamma_c\epsilon_d-\gamma_d\epsilon_c\right)^2+2L\left(2L-\bar{E}\right)-\left(\gamma_c\epsilon_d-\gamma_d\epsilon_c\right)\sqrt{\left(\gamma_c\epsilon_d-\gamma_d\epsilon_c\right)^2+4L\left(\bar{E}-L\right)}}{2\left(\epsilon_c^2+\epsilon_d^2\right)}.$$

By definition (12), $W^r > W^\rho$. ■

Proposition 2.3 Since $n_d^r < \bar{n}_d$ and $n_c^r > \bar{n}_c$, we have to ensure that $n_d^r > 0$ and $P_c(Q_c^r) > 0$. From (A1d), $n_d^r > 0$ if and only if (14) holds. Maximizing n_c^r in (A1c) with respect to L yields:

$$L = \frac{\bar{E}+\epsilon_c\sqrt{\gamma_c^2+\gamma_d^2}}{2}. \quad \text{(A4)}$$

Substituting (A4) into (A1c) yields, using Proposition 1:

$$Q_c^{\max} = n_c^{\max} f_c = \frac{\gamma_c+\sqrt{\gamma_c^2+\gamma_d^2}}{2}.$$

Then $P_c(Q_c^{\max}) > 0$ if and only if (15) holds. ■

A.3 Proof of Proposition 3

Maximizing welfare (31) with respect to n_i $(i = c, d)$ yields:

$$(\alpha_i - c_i - f_i)f_i - n_i f_i^2 - \lambda \epsilon_i f_i = 0. \tag{A5}$$

This is the same condition as the first-order condition under the cap-and-trade regime, substituting (7) and (16) into (4). The shadow price λ of emissions in (A5) therefore equals the allowance price v in (17), and n_i in (A5) equals n_i^A in (18). This means that a cap-and-trade scheme implements the welfare optimum for a given level of total emissions with $q_i = f_i$. Combining (19) and Proposition 2.1, we find $n_c^A < \bar{n}_c < n_c^r = n_c^R$. Combining $n_c^A < n_c^R$ with (13) yields $n_d^A > n_d^R$. ■

A.4 Derivation of $p_c^r(Q_d)$ and $p_c^\rho(Q_d)$ curves

Solving (29) and (30) for Q_c yields two solutions:

$$Q_c = Q_c^+(Q_d) \equiv \frac{\gamma_c + \sqrt{\gamma_c^2 + 4Q_d(\gamma_d - Q_d)}}{2}, \tag{A6}$$

$$Q_c = Q_c^-(Q_d) \equiv \frac{\gamma_c - \sqrt{\gamma_c^2 + 4Q_d(\gamma_d - Q_d)}}{2}. \tag{A7}$$

The highest possible value of Q_d in (A6) and (A7) is where the term under the square root is zero:[13]

$$Q_d^{\max} = \frac{\gamma_d + \sqrt{\gamma_c^2 + \gamma_d^2}}{2} > \gamma_d. \tag{A8}$$

The Q_c^+ solution (A6) includes the unconstrained benchmark, since $Q_c^+(\gamma_d) = \gamma_c$. Substituting (A6) into (13), we find that total emissions are:

$$L^+(Q_d) = \epsilon_d + \epsilon_c \left(\frac{\gamma_c}{2} + \sqrt{\gamma_c^2 + 4Q_d(\gamma_d - Q_d)} \right). \tag{A9}$$

The first and second derivatives are:

$$L^{+\prime}(Q_d) = \epsilon_d - \frac{\epsilon_c\,(2Q_d - \gamma_d)}{\sqrt{\gamma_c^2 + 4Q_d(\gamma_d - Q_d)}}, \qquad L^{+\prime\prime}(Q_d) = \frac{-2\epsilon_c\,(\gamma_c^2 + \gamma_d^2)}{(\gamma_c^2 + 4Q_d(\gamma_d - Q_d))^{\frac{3}{2}}} < 0. \tag{A10}$$

[13] The other solution $Q_d^{\min} = \frac{\gamma_d - \sqrt{\gamma_c^2 + \gamma_d^2}}{2} < 0$ is irrelevant.

From (A8) and (A10) we find:

$$L^{+\prime}(\gamma_d) = \frac{\epsilon_d \gamma_c - \epsilon_c \gamma_d}{\gamma_c} > 0, \qquad \lim_{Q_d \to Q_d^{\max}} L^{+\prime}(Q_d) = -\infty. \tag{A11}$$

The inequality follows from (12). Equation (A11) together with $L^{+\prime\prime}(Q_d) < 0$ from (A10) implies that $L^+(Q_d)$ has a unique stationary point, which is a maximum, between γ_d and $Q_d^{\max}$. Thus $L^{+\prime}(Q_d) > 0$ for $Q_d \in [0, \gamma_d]$. From (A6) it follows that $Q_c^+(Q_d) \geq \gamma_c = \bar{n}_c f_c$ for $Q_d < \gamma_d = \bar{n}_d f_d$. Thus, $Q_c^+(Q_d)$ implements solution r for $Q_d < \gamma_d$. Substituting (A6) into (2), the expression for $p_c^r(Q_d)$ is then:

$$p_c^r(Q_d) = \alpha_c - \frac{\gamma_c + \sqrt{\gamma_c^2 + 4Q_d(\gamma_d - Q_d)}}{2} \text{ for } Q_d \in [0, \gamma_d].$$

In our numerical example (21), this becomes:

$$p_c^r(Q_d) = 200 - \sqrt{10\,000 + Q_d(200 - Q_d)} \text{ for } Q_d \in [0, 200].$$

By (A11), $L^+(Q_d) > L^+(\gamma_d) = \bar{E}$ for Q_d just above γ_d. The other solution to $L^+(Q_d) = \bar{E}$ is:

$$Q_d = \tilde{Q}_d \equiv \frac{\epsilon_d \left(\epsilon_d \gamma_d + \epsilon_c \gamma_c\right)}{\epsilon_c^2 + \epsilon_d^2}. \tag{A12}$$

Since $L^+(Q_d)$ has a unique stationary point, which is a maximum, between γ_d and $Q_d^{\max}$, $L^{+\prime}(Q_d) < 0$ for $Q_d \in [\tilde{Q}_d, Q_d^{\max}]$ and $L^+(Q_d) < \bar{E}$ for $Q_d \in (\tilde{Q}_d, Q_d^{\max}]$. With $Q_d \in (\tilde{Q}_d, Q_d^{\max}]$, Q_d exceeds γ_d and $L^+(Q_d) < \bar{E}$, so that Q_c must be below γ_c, which means this is part of solution ρ.

The other part of solution ρ is found on $Q_c^-(Q_d)$ in (A7) with $Q_c^-(Q_d^{\max}) = \gamma_c/2$ by (A8), and $Q_d = 0$ and $Q_d = \gamma_d$ the only solutions to $Q_c^-(Q_d) = 0$. Thus $Q_d \in [\gamma_d, Q_d^{\max}]$. Substituting (A7) into (13), total emissions are:

$$L^-(Q_d) = \epsilon_d + \epsilon_c \left(\frac{\gamma_c}{2} - \sqrt{\gamma_c^2 + 4Q_d(\gamma_d - Q_d)} \right)$$

with

$$L^{-\prime}(Q_d) = \frac{2\epsilon_c(2Q_d - \gamma_d) + \epsilon_d \sqrt{\gamma_c^2 - 4Q_d^2 + 4\gamma_d Q_d}}{\sqrt{\gamma_c^2 - 4Q_d^2 + 4\gamma_d Q_d}} > 0.$$

The inequality follows from $Q_d \geq \gamma_d$. The correspondence $p_c^\rho(Q_d)$ is therefore given by:

$$p_c^\rho(Q_d) = \begin{cases} \alpha_c - \frac{\gamma_c + \sqrt{\gamma_c^2 + 4Q_d(\gamma_d - Q_d)}}{2} & \text{for } Q_d \in [\tilde{Q}_d, Q_d^{\max}] \\ \alpha_c - \frac{\gamma_c - \sqrt{\gamma_c^2 + 4Q_d(\gamma_d - Q_d)}}{2} & \text{for } Q_d \in [\frac{\gamma_c}{2}, Q_d^{\max}] \end{cases}$$

In our numerical example (21), this becomes:

$$p_c^\rho(Q_d) = \begin{cases} 200 - \sqrt{10\,000 + 200Q_d - Q_d^2} & \text{for } Q_d \in [240; 241.42] \\ 200 + \sqrt{10\,000 + 200Q_d - Q_d^2} & \text{for } Q_d \in [200; 241.42] \end{cases}$$

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