Brentanian Continua

[to appear in *Brentano Studien*]

Brentano's mature theory of the continuum is developed at length in his *Philosophische Untersuchungen zu Raum, Zeit und Kontinuum* (Brentano 1976, English Brentano 1988), more specifically in the first treatise "On what is continuous" dictated in 1914, as well as in his *Kategorienlehre* (Brentano 1968, English Brentano 1981). More cursory presentations of his account are found in his 1917 essay "Vom ens rationis" (Brentano 1959: 238-281; English Brentano1995), in his *Deskriptive Psychologie* (Brentano 1982, English Brentano 1995b), in his *Untersuchungen zur Sinnespsychologie* (Brentano 1979), in *Von Sinnlichen und Noetischen Bewusstsein* (Brentano 1928, English Brentano 1981a) and in his manuscript "Vom Unendlichen" (Brentano 1963). One also finds an earlier detailed presentation of Brentano's reflections on the continuum in his *Lectures on Elementary Logics* given in 1884/1885 (Ms. Y 2, Y 3, *Die elementare Logik und die in ihr nötigen Reformen*, ed. C. Ierna, to appear).

Brentano's theory of continuity is based on his account of boundaries. The core idea of the theory is *that boundaries and coincidences thereof belong to the essence of continua*. Brentano is confident that he developed a full-fledged, boundary-based, theory of continuity[1]; and scholars often concur: whether or not they accept Brentano's take on continua they consider it a clear contender. My impression, on the contrary, is that, although it is infused with invaluable insights, several aspects of Brentano's account of continuity remain inchoate. To be clear, the theory of *boundaries* on which it relies, as well as the account of *ontological dependence* that Brentano develops alongside his theory of boundaries, constitute splendid achievements. However, the passage from the theory of boundaries to the account of continuity is rather

[1] In a letter presumed to be a recommendation for Rhees, Kastil however reports the following:

> [Brentano] indicated to me a few days before his death that his theory [of continuity] was in a process of alteration, without giving any further indication of the kind of improvements which he had in mind. (Letter quoted by Erbacher and Schirmer 2017).

sketchy. This paper pinpoints some chief problems raised by this transition, and proposes some solutions to them which, if not always faithful to the letter of Brentano's account of continua, are I believe faithful to its spirit.

§1 presents Brentano's critique of the mathematical account of the continuous. §2 introduces Brentano's positive account of continua. §3 raises three worries about Brentano's account of continuity. §4 proposes a Neo-Brentanian approach to continua that handles these worries.

Contents

1. Brentano against mathematical accounts of the continuous
    - 1.1. Against constructions
    - 1.2. A misunderstanding?
2. Brentano's account of continua
    - 2.1. Existential dependence: boundaries
    - 2.2. Essential dependence: plerosis and teleosis
    - 2.3. Reciprocal dependence: boundaries and continua
    - 2.4. Coincidence of boundaries
    - 2.5. One-sided dependence: primary and secondary continua
3. Three problems for Brentano's account
    - 3.1. Can boundaries of primary continua coincide?
    - 3.2. Do continua consist only of boundaries?
    - 3.3. What is the plerosis of boundaries within a continuous transition?
4. A Neo-Brentanian Account: varieties of continuity
    - 4.1. Primary continua: primitive continuity
    - 4.2. Abstract secondary continua: primitive similarity

1. Brentano against mathematical accounts of the continuous

1.1. Against constructions

According to the standard mathematical approaches to continuity of Dedekind, Cantor or Poincaré, continua consist of non-denumerable infinities of "point-individuals" (Dedekind 1901), which are constructed through successive insertions of numbers. Put simply, one starts with the infinitely many integers, in between which one inserts the rational numbers, in between which one inserts the irrational numbers.

An objection that Brentano raises against this approach to continuity pertains to "the origin of the concept of the continuous". True to his empiricism, Brentano maintains that the concept of continuity is, like any other concept, not possessed innately but acquired through perception. There are then only two ways to acquire a concept: either by *abstracting* marks given in perception, or by *combining* marks given in perception.

The difference between these two ways of acquiring a concept is this. Abstraction from intuition is a non-combinatorial process: to abstract something, e.g. redness, from perceptual intuitions, one has to consider various intuitions with this content, and contrast them with other intuitions lacking it, so as to make their differentiating feature(s) salient:

> As with other concepts gained not through combination of various marks but through abstraction from a unitary intuition —as for example the concept of what is coloured—we have only to bring forward different intuitions which all contain the relevant mark and then perhaps, in order still more to draw attention to the crucial point, contrast these with others where this mark is either entirely absent or at least given only in a noticeably different way. (Brentano 1988, 7)

In constructionist explanations, by contrast, one does not just extract the relevant features from the content of our perceptions, but one also *intervenes* on them, by *combining* them in various ways.

The mathematical approach to continuity is an account of the constructionist sort: it constructs the concept of continuity by successive interpolations of natural numbers. But, as Brentano notices, this mathematical approach is not the only constructionist account of continuity. Another view is that continua "can be divided *in infinitum*" (this contrasts with the view that continua consist of "infinitely many parts”). This second constructionist approach equates continua with what (following Lewis 1991) is nowadays called “gunk” or “atomless gunk”: infinitely divisible objects lacking indivisible parts, in particular point-sized parts.

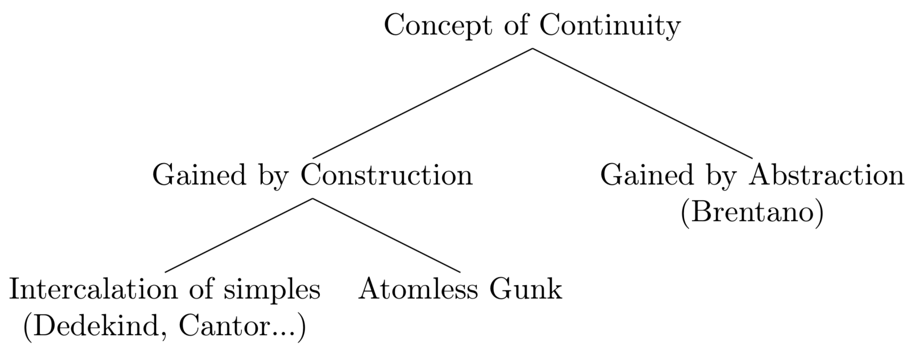

Figure 1: Ways of acquiring the concept of continuity

There are two essential differences between the gunk approach and the intercalation approach:

1. While the intercalation approach is committed to continua consisting of actual infinities of elements, the gunk approach only appeals to the modal view that continua can *possibly* be divided *in infinitum*.
2. While the mathematical approach appeals to (infinitely many) point-sized objects, the gunk approach rejects point-sized objects on the ground that they are indivisible.

Brentano raises several objections against each of these two constructionist approaches. On the whole, he considers the gunk approach more promising than the mathematical approach, as it isn’t committed to actual infinities, or to point-individuals. But he raises one objection meant to rebut them both, as well as any other possible constructionist account. Precisely because they are constructionist, such approaches render the concept of continuity too technical and inaccessible to intuition:

> we should have to ascribe to the concept of continuity an origin in operations of thought both artificial and involved. This seems unacceptable from the very start, for how could

> this concept then be found in the possession of the simple man or even of the immature child? (Brentano 1988, 4).

Given that the concept of continuity is understood pre-theoretically, construction is to be excluded and this concept can only be attained through abstraction.

A second reason why constructionist approaches are "artificial and involved" is that continuity, Brentano insists, is *presented* in all of our intuitions:

> every single one of our intuitions—both those of outer perception as also their accompaniments in inner perception, and therefore also those of memory—bring to appearance what is continuous. (Brentano 1988, 6)

> no intuition is entirely free of the concept of that which is continuous. (Brentano 1988, 9)

Since continuity is given in perception, there is no need to construct it:

> what is continuous must be given to us in individual intuition and must therefore have been abstracted therefrom. (Brentano 1988, 5)

> this concept is gained not through any intricate process of combination but rather in immediate fashion through simple abstraction from our intuition. (Brentano 1988, 6)

Before turning to Brentano's positive, anti-constructionist account of continuity, I want to rebut a common interpretation of the dispute between Brentano and the constructivists, to the effect that they are in fact not so much disagreeing as targeting different explananda.

1.2. A misunderstanding?

In their introduction to the *Philosophische Untersuchungen zu Raum, Zeit und Kontinuum,* Kölner and Chisholm argue that Dedekind, Cantor and Poincaré, on the one hand, and Brentano, on the other, do not after all hold incompatible views (Baumgartner & Simons 1994 concurs). Instead, they target different concepts of continuity. While Dedekind, Cantor and Poincaré are after the *idealized* continua central to mathematics, Brentano tries to understand the *phenomenological* continua of interest to descriptive psychology. Thus, for Kölner and Chisholm, no real disagreement is to be found here. On this reading, Brentano must have overestimated the range of his critique: its upshot is not that the mathematical approach to continuity is false, but only that it is false *as an account of phenomenal continuity*. Since phenomenal continuity was clearly not the explanandum for Dedekind, Cantor or Poincaré, they and Brentano can live together in peace.

This attempt to reconcile Brentano with his opponents by restricting the scope of their respective theories is made more plausible by the fact that, on the face of it, Dedekind's and Brentano's pre-theoretical characterisations of continuity have little in common. While Dedekind explicitly characterises continuity in terms of "completeness", "absence of gap" and "unbroken connection" (Dedekind 1901, 4-5), Brentano takes the main pre-theoretical feature of the concept of continuity to be that it is related to the concepts of "boundary" and "coincidence of boundaries":

> the concept of a boundary and the possibility of a coincidence of boundaries is essential to the concept of what is continuous. (Brentano 1988; see also 4-5, 40, 42; Brentano 1982, 112).
>
> He who does not show how we arrive at these ideas [boundaries and the possibility of their coincidence] is not, either, allowed to flatter himself with having sufficiently clarified the idea of the continuous (Brentano 1988: 6)

Kölner and Chisholm's conciliatory proposal seems to be vindicated: Brentano is after an *empirical* concept of continuity, whose core features involve *coincidences of boundaries,* while Dedekind, Cantor, Poincaré and others are attempting to shed light on an *idealized* concept of continuity, whose key feature is *gaplessness*.

Appealing as this irenic picture may be, it is, I believe, erroneous. Brentano and Dedekind, are not talking past each other, but actually disagreeing. First, Brentano not only thinks that he is in disagreement with the classical mathematical approach to the continuum, he in fact *argues* that there is genuine disagreement here. Thus, he stresses from the very beginning of his investigations into the continuum that there *must* be a single concept of continuity that we all share, which constitutes the subject-matter of such disagreements:

> The question concerning the concept of continuity cannot be framed in such a way that one would call into doubt whether we do in fact possess such a concept. For otherwise we would not be able to understand ourselves when arguing about other aspects of this concept. (Brentano 1988, 1; see also Brentano 1981b, 55)

Second, as the rest of his discussion makes clear, some of Brentano's objections target not only the application of the mathematical approach to continuity to the simple contents of perception, but also to the "number-continuum" itself (a point further documented by Ierna 2012). What Brentano sets out to uncover is what all continua —sensory *and mathematical*— have in common. Note also that Dedekind would also not welcome a restriction of his enterprise to pure mathematical continua, since he explicitly states that he intends to lay the "scientific basis for the investigation of *all* continuous domains" (Dedekind 1901, 5).

For these reasons, Kölner and Chisholm's proposal should be rejected: Brentano and the mathematicians are not after different species of continua but rather the same continuum-genus. They indeed differ in the paradigmatic examples they focus on, but they nonetheless share the same explanandum.

How then is the concept of continuity they are disagreeing about to be characterised, prior to their disagreement? If one is to disagree about continuity, one must agree on some topic-neutral identification of it. I suggest that Dedekind's pre-theoretical characterisation of continuity in terms of *gaplessness*, —which is also the standard way in which continuity is described (Bell 2005)— is in fact also the characterisation Brentano relies on: *that which is continuous has no gaps*. When Brentano insists that the coincidence of boundary is the essential feature of a continuum, he is providing an opinionated account of it, not characterising continuity in a topic-neutral way. Although Brentano is less explicit about this, he, like Dedekind subscribes to the view that gaplessness is essential to continuity and should be accounted for. In his *Lectures on Elementary Logics*, he frequently characterises continua as gapless (*lückenlos*). Later on, the assumption that gaps threaten continua surfaces in several places in his writings (Brentano 1995a, 77, 112, 114). Coincidence of boundaries, in Brentano's proposal, is precisely what *explains* gaplessness. Gaplessness therefore constitutes the explanandum common to Dedekind and Brentano.[2]

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[2] If this is right, then, Brentano's objection to the constructionist approaches on behalf of their neglecting boundaries is question-begging:

> One sees that in this entire putative construction of the concept of what is continuous the goal has been entirely missed; for that which is above all else characteristic of a continuum, namely the idea of a boundary in the strict sense (to which belongs the possibility of a coincidence of boundaries), will be sought after entirely in vain. (Brentano, 1988, 4)
>
> He who does not show how we arrive at these ideas [the peculiarity of boundaries and the possibility of their coincidence] is not, either,

Hence Brentano and Dedekind agree about *what is to be explained*, but they disagree about *how to explain it*. Where does their disagreement lie? To put it crudely, for Dedekind the difference between a continuous and a discrete object is that in the former all gaps have been *filled*, thanks to the intercalation of new "point individuals". The way to fix gaps is by *filling them up*. For Brentano, on the other hand, the difference lies in the fact that, in continuous objects, the absence of gaps is guaranteed by the coincidences of the boundaries of the subparts of this object. *Joining together* is the remedy against gaps —an idea that Brentano takes from Aristotle's account of continuous quantities (*Categories*, 4b24-5a14). Compare:

> And if we knew for certain that space was discontinuous there would be nothing to prevent us, in case we so desired, from filling up its gaps, in thought, and thus making it continuous; this filling up would consist in a creation of new point-individuals (Dedekind 1901, 6)

> it belongs to the nature of a continuum that its parts are in contact with each other as boundaries (Brentano 1988, 42)

Asked to fix a hole in a pie shell, Dedekind adds a tiny bit of pastry so as to fill the hole; Brentano's brings the sides of the hole closer so as to make them slightly overlap[3] .

2. Brentano's account of continua

What then is Brentano's positive account of the continuous? For simplicity's sake, and following Brentano's lead, we shall here focus on one species of continua: *spatial continua which are uniformly or gradually coloured*. Brentano in fact considers such continua to be not only secondary—in a sense to be introduced — but also fictitious for three reasons:

> allowed to flatter himself with having sufficiently clarified the idea of the continuous.(Brentano, 1988, 6)

This objection is a petitio principii: the necessity of appealing to boundaries to explain gaplessness is precisely what Brentano should (and elsewhere does) argue for. Continuity or gaplessness does not obviously involve the concept of boundaries — boundary-free accounts of continuity are not excluded from the start.

Brentano errs, therefore, not when he sees himself as disagreeing with the mathematicians about continuity —pace Kölner and Chisholm— but when he readily *assumes* that boundary-free accounts of continua must miss the point.

[3]One limit of this metaphor is that both Dedekind's added bit of pastry and Brentano's slight overlap, however thin, still have some thickness, which is not the case with Dedekind's points or Brentano's boundaries.

1. He thinks that time, rather than space, is the only fundamental — primary— continuum;
2. He denies that colours exist outside the mind;
3. He denies that continuous transitions between colours are possible —continuous transition between colours are "only apparent" (1981: 70) for intermediate colours such as purple are in fact chessboards made of small blue and red tiles, so that change from one simple colour to another can "only be sudden" (1981: 75 — see Massin & Hämmerli 2017 for discussion).

This notwithstanding, Brentano explicitly allows himself to appeal to uniformly or gradually coloured areas as heuristic examples to shed light on continua in general. Thus, although such cases do not strictly speaking exist, Brentano holds that it is nevertheless fruitful to:

1. Fictitiously treat space as a primary continum (1988: 27);
2. Fictitiously treat colours as real (1995: 17-20);
3. Fictitiously treat transitions between colours as continuous transitions (1988: 24; 1981: 70, 75).

One chief reason why such fictitious assumptions are fruitful is that they allow us to easily depict various kinds of continua: in the Brentanian didactic set up of colours spread over regions, continuous variation between colours represent motion across time in the real world. Since colours in space are more easily depicted on a page than changes over time, the various forms of continua and their features can be captured at a glance.

Brentano's account of boundaries and continuity has been the object of several presentations and formalizations, in particular: Rhees (in Erbacher 2017), Chisholm 1992, Baumgartner 1994, Zimmerman 1996a, Libardi 1996, Smith 1997, Bell 2000, Bell 2005, Albertazzi 2006, Poli 2012, Zelaniec 2017. The only two novelties of the following presentation are (i) it includes the main colour-figures that Brentano introduces as examples and (i) it explicitly hinges on the many kinds of ontological dependencies appealed to by Brentano (namely existential, essential, generic, individual, unilateral and reciprocal dependencies).

2.1. Existential dependence: boundaries

The central features of Brentano's approach that differentiate it from the standard mathematical approach are his use of the concept of boundary and his concomitant refusal to use point-individuals. Spatial boundaries —and correspondingly, continua— come in three varieties (1988: 10)[4]:

1. *points*, which bound lines. Lines are "one-dimensional continua", for their boundaries —points— are not themselves continuous.
2. *lines*, which bound surfaces. Surfaces are two-dimensional continua, for their boundaries —lines— are one-dimensional continua.
3. *surfaces*, which bound bodies. Bodies are three-dimensional continua, for their boundaries —surfaces— are two-dimensional continua.

Like the point-individuals of Dedekind, boundaries have no thickness (points are indivisible in all directions, lines and surfaces are indivisible along the dimension in which they lack thickness). The difference between point-individuals and boundaries, though, is that whereas point-individuals are *independent* beings –traditionally called *substances*— boundaries are *existentially dependent* entities. Boundaries cannot exist in and of themselves: they existentially depend on their insides, on what they bound:

> But just as it is certain that there are boundaries and that they must be included among things, it is also certain that a boundary is not a thing existing in itself. The boundary could not exist unless it belonged as a boundary to a continuum. (1981: 128, see also 1981: 20, 1995b: 357, 1988: 173)[5]

This constitutes, in Brentano's eyes, a further reason to reject mathematical accounts of the continuous:

> a cutting free from everything that is continuous is for [boundaries] absolutely impossible. And this allows us to grasp very clearly the topsy-turvy character of the above-mentioned attempt at construction of the concept of the continuous through interpolation of fractional numbers,

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[4]Compare Aristotle, *Categories*, 4a24-5a14.

[5]One complication to be ignored here is that, although every point is dependent on a continuum, the continuum it depends on need not be *real*. Take the present: since Brentano is a presentist and maintains that the present is boundary-dependent on the past and on the future, he has to maintain that boundaries can depend on continua that are not real, that have no being. This is also true, Brentano claims, of some spatial points, although the possibility of a spatial point depending on non-real continua is rarer (see 1995b: 357). This later claim however stands in tension with his other (and more numerous) claims to the effect that destroying the parts adjacent to a boundary modifies its plerosis (see the next sub-section).

> where every fraction is supposed to have existence without belonging to a series of fractions (1988: 10)

Mathematical accounts of continua, in other words, hypostatize boundaries. Boundaries depend on the continua that they bound, but mathematical constructions of the continuous treat them as independent bricks out of which something could be constructed.

2.2. Essential dependence: plerosis and teleosis

The dependency of boundaries on continua is even tighter, for boundaries depends on the continua they bound not only *for their existence*, but also *for their very nature:*

> the nature of the boundary is conditioned and determined by the distinctive properties of the continuum. (1981: 56, see also 60, 64n —on the distinction between existential and essential dependence at play here, see notably Correia 2008a).

What aspects of the nature of boundaries depend on what they bound?

Brentano introduces two essential features of boundaries, their *plerosis* and *teleosis*, both of which come in greater or lesser degrees, and are determined by the kind of continua that the boundaries bound.

2.2.1. Plerosis

The *plerosis* of a boundary corresponds to *the number of directions in which it bounds* (1981: 128; 1988: 11; Brentano 1995b, 157). The concept of a boundary's *plerosis* is easily grasped by contrasting *inner* and *outer* boundaries. A side of a red rectangle is an outer boundary, and only has half plerosis, for it bounds only towards the inside of the rectangle. A parallel line inside the rectangle is an inner boundary that has full plerosis, for it bounds in two opposite directions: on the left, and on the right. External boundaries are *oriented*: they so to speak "look inwards". Internal boundaries look "all around".

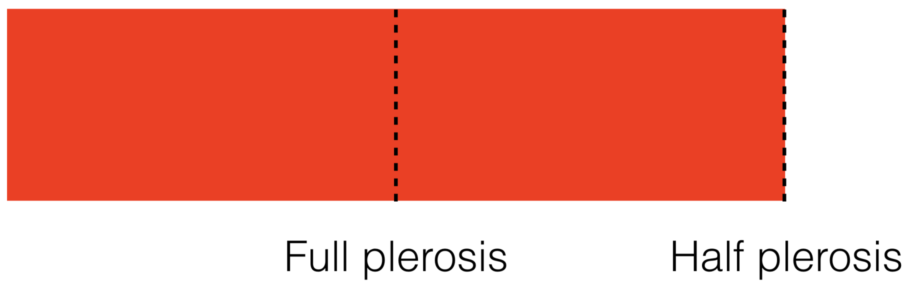

Figure 2: Boundary lines: full vs. half plerosis

What is true of lines is also true of points. Consider the centre of a uniformly red disc. This centre has a full plerosis:

> Where we have to do with the interior of a continuum, every point has full plerosis, i.e. is connected in very conceivable direction with the relevant continuum. (1988: 28)

Now suppose that one progressively removes sectors of a red disc (see Figure 3). The plerosis of the initial centre, Brentano holds, decreases as the process goes on: the point bounds in less and less directions (1981: 128).

Figure 3: Boundary points: full vs. lesser plerosis

Differences of plerosis are also found in the temporal realm. Thus, the end of a life bounds only towards the past: it has a lesser plerosis than an inner moment of a life, which is both a final and initial boundary. The same is true of the present itself, which constitutes the temporal continuum, and which bounds in two opposite directions: towards the past and towards the future. Thus, temporal boundaries can only have two degrees of plerosis: *full* plerosis —for inner boundaries of processes, and for the present itself— or *half* plerosis —which comes in two kinds: the half plerosis of initial and of final boundaries.

2.2.2. Teleosis

The *teleosis* of a boundary, by contrast, corresponds to the *velocity of change* of the continuum it belongs to, at the point where the boundary stands within the continuum. One can thus think of the teleosis of a boundary in terms of instantaneous velocity. Consider two rectangles, both continuously progressing from red on the left to blue on the right, the middle of both rectangles containing various shades of purple. Suppose that one of these rectangles is twice as long as the other (Figure 4). The velocity of colour-change will be twice as slow in the longer rectangle as in the shorter one. Brentano could have said that any vertical segment within the shorter rectangle is a boundary of a higher teleosis than any vertical segment within the longer rectangle. But he adopts the inverse convention:

> The greater the speed of the variation, the less the degree of teleosis. (1981: 129)

Thus, the vertical boundaries making up the longer rectangle are said to have a higher teleosis than the boundaries of the shorter one.

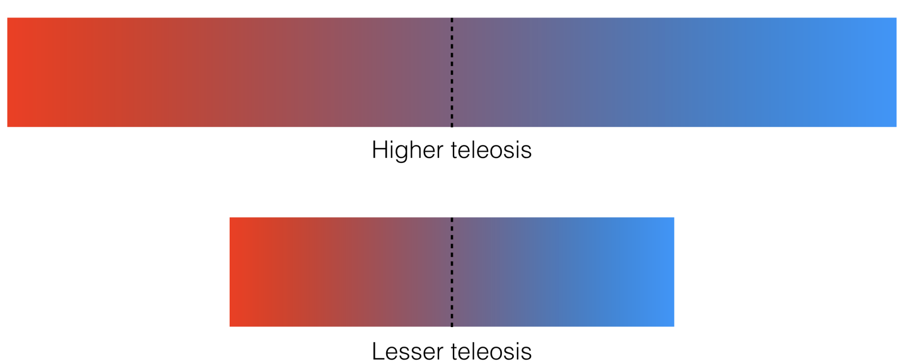

Figure 4: Higher vs. lesser teleosis

The boundaries we just been considering all have a *constant teleosis*, but it is also possible for a boundary to have a *continuously changing teleosis*. This is the case, for instance, with any spoke of a turning wheel.

But we can also have cases of continuously changing teleosis with colours in space, as Brentano points out. Consider thus the radius of a continuously changing colour circle (see Figure 5): the farther we are from the centre, the higher the teleosis of the radius —the lower its instantaneous velocity.[6]

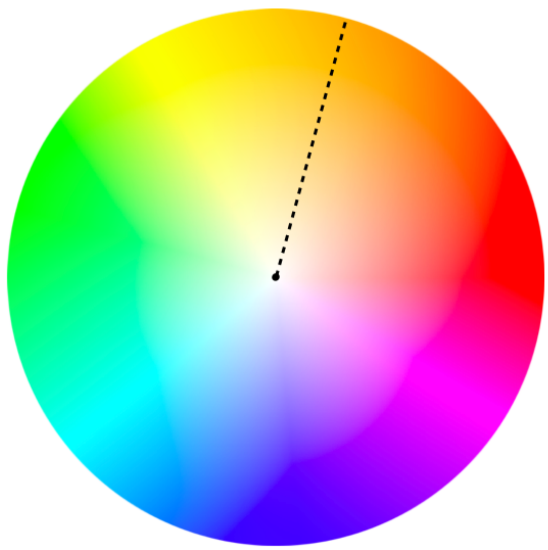

Figure 5: Boundary of continuously changing teleosis

Finally, what is the teleosis of the boundaries in the two limiting cases: *rest* (analogous to boundaries within a red rectangle); and *instantaneous motion* (analogous to the boundary between a blue and a red rectangle)? Brentano is not quite explicit about this, but the following two answers appear to follow from his proposal.

- *Rest*. The inner boundaries of a uniformly red rectangle should be said to have the *highest possible teleosis*, for the velocity of colour-change is zero (colour is at rest, so to speak).

[6] I pointed out that colour variations can be seen as standing for spatial motion in Brentano's set up. However one shouldn't be misled by the following disanalogy: on a real turning wheel, the farther we are from the center, the *fasterr* the motion; on a coloured wheel, however, the farther we are from the center, the *slower* the change between colours.

- *Instantaneous motion*. In the case of quality "jumps", by contrast, the boundaries should be said to have a plerosis, but *no or a null teleosis*. Brentano thus writes that adjacent red and blue tiles "are similar, not to teleosis, but to plerosis" (1981: 70). This can be understood in the following way: jumping immediately from one place (or quality) to another without going through intermediary places (or qualities) requires us to move with maximal speed. Since "The greater the speed of the variation, the less the degree of teleosis", the teleosis should then be zero.

2.3. Reciprocal dependence: boundaries and continua

None of what we have said so far addresses the nature of continua. On the contrary; boundaries have been introduced *thanks* to the continua on which they depend. How are we to move from the account of boundaries to an account of continua? The first step is that boundaries *belong* to the continua on which they depend, so that boundaries are *parts or constituents* of continua. Because of this, continua depend in turn on the boundaries that constitute them:

> every boundary is ... a *conditio sine qua non* of the whole continuum […] the continuum is also a *conditio sine qua non* of the boundary. (1981: 56)

> Hence it would seem to be the case both that the continuum is conditioned by the boundary and also that the boundary is conditioned by the continuum. (1981: 128)

This bi-lateral dependence between boundaries and continua is not entirely symmetrical, however. Brentano's idea is that, while continua *depend individually* on the boundaries that constitute them, boundaries *depend generically* on the continua they constitute (a similar crossing of individual and generic dependence holds between qualities and locations according to Brentano, see Massin, 2017):

> it cannot be said of any definite continuum that it is a condition of the boundary, only a universal can be designated as a condition of the boundary. In other words, what is required is not this or that particular continuum, but any continuum of the approximate kind. (1981: 56; see also 157-8, 201)

2.4. Coincidence of boundaries

As tight as the interdependencies between boundaries might be, they will not by themselves guarantee gaplessness: dependency-relations are not spatial relations. Brentano's chief way of avoiding gaps is by ensuring that the different parts of continua stand in *contact* with each other. To recall:

> it belongs to the nature of a continuum that his parts are in contact with each other as boundaries. (1988: 42)

However, the possibility of contact has long been a source of paradoxes. One chief problem is this: either the boundaries of the bodies that are in contact do not occupy the same points in space, or they do. If the boundaries occupy different points in space, and if in between two points there are always further points, the bodies are not really in contact. If, on the other hand, the boundaries occupy the same points in space, then the bodies are not impenetrable for their boundaries coincide (see Kline & Matheson 1987 and Arntzenius 2007, §6 for more detailed presentations of the paradox).

Brentano endorses a version of the later solution. Contact is for him the *coincidence* of boundaries: when two bodies are in contact some of their boundaries are at the same place at the same time (Brentano 1995b, 357). Brentano however remains strongly committed to the impenetrability of *bodies*. How is that possible? His idea is that impenetrability is compatible with the coincidence of the *surfaces* of the bodies. He also remains a fierce defender of the impenetrability of colours (see Massin & Hämmeli 2017 for discussion). But colour-impenetrability requires only that the boundaries of the *same body* can never be at once red and blue. As long as boundaries of different colours bound different entities —that is, as long as they are of incomplete plerosis and depend on distinct continua— their coincidence does not violate impenetrability:

> If a red and a blue surface are in contact with each other then a red and a blue line coincide, each with different plerosis. (1988: 41)

> the geometer's proposition that only one straight line is conceivable between two points, is strictly speaking false if one conceives the matter in terms of lines of incomplete plerosis whose pleroses, even though they coincide with one another, relate to different sides. (1988: 12)

Such coincidences of boundaries, Brentano insists, are essential to the perception of continua:

> Certainly we cannot distinguish the individual points and boundaries in the continuum that presents itself to us [...]. Yet this does not hinder us in apprehending with complete certainty that *boundaries and coincidences of boundaries are numberlessly present in the whole in question*. (Brentano, 1988, 9, my italics)

A problem may seem to arise in the case of *inner boundaries*, for, as we saw, Brentano claims that such boundaries have full plerosis:

> Where we have to do with the interior of a continuum, every point has full plerosis, i.e. is connected in every conceivable direction with the relevant continuum. (1988: 28)

The problem is this. Consider the inner boundary in Figure 2. If such a boundary indeed has full plerosis, *on what grounds are we to say that we have two coinciding boundaries here?* The only reason why there should be two boundaries, instead of just one, is if each would bound in opposite directions. But here, instead of having two lines with half plerosis, we seem to only have one boundary with full plerosis. Internal contact, then, cannot consist in boundary-coincidence but must consist instead in some boundary-sharing: the right- and left-hand sides of our red rectangle would be in contact in virtue of *having one boundary in common: the vertical line in the middle*. In other words, in the case of internal boundaries, Brentano's theory appears to face the following dilemma:

1. Either internal boundaries (in contrast with external boundaries) have full plerosis, but then internal contact does not consist in boundary-coincidence but in boundary-sharing.
2. Or internal contact (like external contact) consists in several coinciding boundaries, but then it is not the case that *all* internal boundaries have full plerosis.

Does Brentano endorse the first horn of the dilemma? I do not think so, for two reasons.

First, as Zimmerman 1996a (who attributes a boundary-sharing account to Suarez) points out, the boundary-sharing account faces a quandary when it comes to *breakage*. Suppose we split our red rectangle into two halves. Since, under the present hypothesis, there was only one boundary in the middle of the rectangle, we are left with two unattractive options. Either the boundary stays with one of the new rectangles —but which one? — and a new boundary is created on the other rectangle — *ex nihilo*? — or the inner boundary is destroyed and two boundaries are created, *ex nihilo*. Similarly, if the two rectangles are brought into contact again, one would have to say either that one boundary is destroyed and the other survives (which one?), or that the two boundaries are destroyed and a new one is created. Both options are equally unattractive. Of course, this dilemma doesn't arise if, as per the boundary-coincidence account, we have two inner boundaries to start with before the breakage. Brentano comes close to this line of thought in the following passage:

> Dedekind believes that either the number 1/2 forms the beginning of the series 1/2 to 1, so that the series 0 to 1/2 would thereby be spared a final member, i.e. an end point which would belong to it; or conversely. But this is not how things are in the case of a true continuum. Much rather is it the case that, when one divides a line, every part has a starting point, but in half plerosis. (1988, 40)

The second reason not to attribute a boundary-sharing account of inner contact to Brentano, over and above the problems it raises, is quite simply that there is no textual evidence that Brentano intends to treat inner contact in a different way to external contact. On the other hand, there is clear evidence that he is willing to endorse a boundary-coincidence account of both external and inner contact (this is also how Chisholm 1992 and Zimmerman 1996a interpret him).

So, Brentano must embrace the second horn of the dilemma: inner contact, like external contact, consists in boundary-coincidence. But how is this view to be reconciled with the claim that "Where we have to do with the interior of a continuum, *every* point has full plerosis"? I suggest that this sentence is slightly hyperbolic. Brentano should have said, more cautiously, that *at every point in the interior of a continuum, there is a boundary with full plerosis*. This more modest claim is interesting in that it does not rule out that, *at every point in the interior of a continuum, there may also be boundaries with partial plerosis*. Brentano explicitly recognises this elsewhere:

> In the case of a one-dimensional continuum these boundaries can be internal boundaries in two opposing directions and are then points of connection. *But they can also be internal boundaries in merely one direction in relation to that which they bound*, and in the other direction be external boundaries. They are then separating points, actually not one, but two of half plerosis which coincide. (1988, 108, italics mine)

One may worry that, if we admit internal boundaries of partial and full plerosis, we end up with too many coinciding boundaries: at the middle line in our red rectangle one would find not only two boundaries with half plerosis, but also a third boundary with full plerosis. The answer to this worry lies in Brentano's suggestion that coinciding boundaries may enter into part-whole relationships. *Two inner coinciding boundaries of half-plerosis form together a boundary of full-plerosis*. It is therefore not as if the inner boundary of full plerosis is a third, *additional* boundary coinciding with the two half-plerosis boundaries: rather, the third boundary is mereologically constituted by these two half-plerosis boundaries. Despite being spatially indivisible, some boundaries nevertheless have *plerotic parts* (talk of "plerotic parts" is not Brentano's, but Smith 1997's).

Likewise, at the centre of our red disc lies as many boundaries of partial plerosis as there are radii and sectors of the discs. Taken together, all these coinciding points with partial plerosis form a unique point with full plerosis:

> Euclid's supposition that a point is that which has no parts was seen already by Galileo to be in error when he drew attention to the fact that the mid-point of a circle allows the distinction of just as many parts as there are points on the circumference, since it differs in a certain sense as starting point of the different individual radii. (1988, 41)

Brentano on the other hand, implies that coinciding *external* boundaries do not form a single boundary of full plerosis. Only coinciding *inner* boundaries of partial plerosis can be mereologically summed (Figure 6).

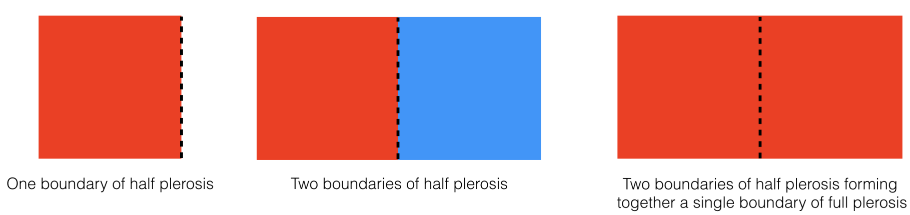

Figure 6: Mereology of Boundaries

2.5. One-sided dependence: primary and secondary continua

A colour that fills a spatial area is a *secondary continuum* while the extension it fills is a *primary continuum*. Likewise, time is a *primary continuum*, while a constant or rising tone is a *secondary continuum*.

Secondary continua therefore one-sidedly depend on primary continua: primary continua (e.g. space) “make possible” (1995a: 116) the continuity of secondary continua (e.g. single or varying colours).

> We come now to a very important respect in which to classify continuity. It is that which follows from the multiplicity of most if not all of what is continuous. Imagine, for example, a coloured surface. Its colour is something from which the geometer abstracts. For him there comes into consideration only the constantly changing manifold of spatial differences. But the colour, too, appears to be extended with the spatial surface, whether it manifests no specific colour-differences of its own—as in the case of a red colour which *fills out a surface uniformly*—or whether it varies in its colouring—perhaps in the manner of a rectangle which begins on one side with red and ends on the other side with blue, *progressing uniformly* through all colour-differences from violet to pure blue in between. In both cases we have to do with a multiple continuum, and it is the spatial continuum which appears thereby as primary, the colour-continuum as secondary. (1988: 21)

Brentano maintains that, ultimately, time is the only primary continuum, but argues that space can nevertheless be treated as a primary continuum with respect to what fills it. Teleosis, as we saw, is strictly speaking a property of boundaries, but Brentano also talks of the teleosis *of continua*, by which he means the velocity of change of the continua. In more familiar terms, the teleosis *of a boundary* corresponds to instantaneous velocity, and the teleosis *of a continuum* corresponds to velocity of change over an interval of time. Brentano investigates at length how the concept of teleosis applies to primary and secondary continua respectively. One may initially think that primary continua lack teleosis: how can that against which the velocity of change is measured —time and space—itself have a velocity of change? Brentano dismisses this worry. Since there are spatial and temporal variations, these variations must have a velocity: hence primary continua must also have teleosis. What he rejects, however, is that *the rate of the variation of primary continua can change*. In other word, the teleosis of primary continua must remain constant. Primary continua have a constant speed. They cannot accelerate:

> However, by ’velocity’ we are to understand in the end nothing other than the rate of variation, and certainly it cannot be denied that wherever variation exists, it must exist in some

determinate degree or other, whether this be constant […] or now higher, now lower. In the case of time, now, there exists a variation. What should be denied is only that, like the degree of other variations, it could come to be lowered or raise. And something quite similar hols also of the spatial as such. (1988: 23; see also 1963: 45).

Secondary continua, by contrast, may have different teleosis: motions, or colour transitions, can accelerate (see Figure 7).

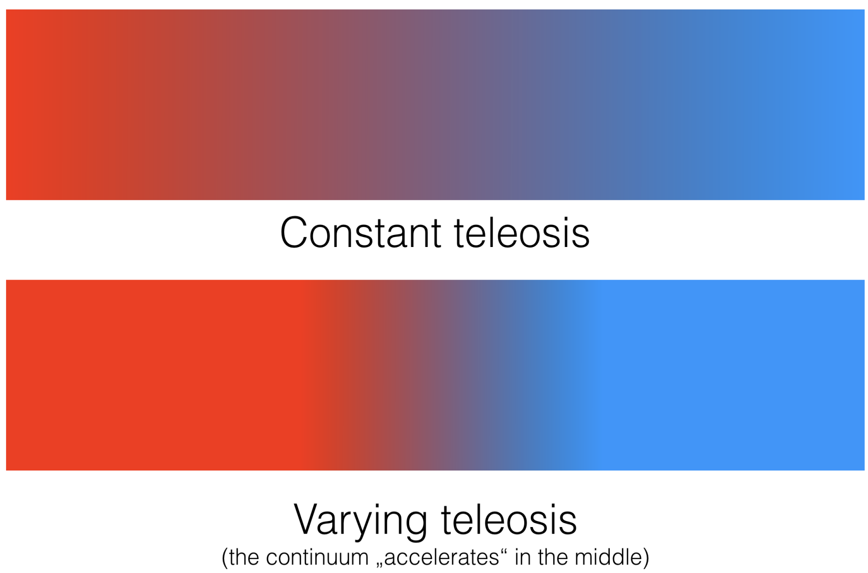

Figure 7: Constant vs. Varying teleosis

This difference between the teleosis of primary and secondary continua leads Brentano to a refined objection to constructionist accounts of the continuum. Mathematicians have to agree that the velocity of the *enumeration*[7] of fractional numbers between 0 and 1 can vary. Hence the

---

[7]It might sound surprising to target the view that the number-continuum is defined through successive acts of counting, but this was indeed Dedekind’s proposal:

> I regard the whole of arithmetic as a necessary, or at least natural, consequence of the simplest arithmetic act, that of counting, and counting it- self as nothing else than the successive creation of the infinite series of positive integers in which each individual is defined by the one immediately preceding; the simplest act is the passing from an already-formed individual to the con- secutive new one to be formed (Dedekind 1901, 2)

number-continuum must be a secondary-continuum, and therefore depends on a primary continuum, whose nature remains unexplained by mathematical accounts:

> From this it follows that the number-continuum which is supposed to have been produced would be in every case a secondary continuum which therefore included the idea of a primary continuum as previously given. And thus for this reason, too, it is apparent that what one supposed oneself able to gain by construction has already—without this being noticed—been presupposed as given. The contradictoriness of the whole enterprise thus comes to light once more in the most flagrant manner. (1988: 43).

What if, instead of appealing to enumerations, constructionist mathematicians relied on the "co-existence" of all the numbers? Then, Brentano retorts, one should be able to identify the *precise* degree of variation of the number-continuum. But any particular degree would seem arbitrary.

To recap: continua are abstracted, not constructed. Their key ingredients are boundaries. Boundaries:

1. have different plerosis —directions in which they bound;
2. have different teleosis —instantaneous velocities;
3. generically depend on the continua they bound;
4. constitute continua: hence continua depend individually on boundaries.
5. ensure the continuity of the continua they constitute by coinciding.

I shall now raise three problems for Brentano's account of continua.

3. Three problems for Brentano's account

3.1. Can boundaries of primary continua coincide?

Brentano's theory of contact as coincidence of boundaries is quite plausible in the case of things that are *in space*, such as a blue and a red book touching each other on a surface, or a blue and a red square touching each other on a line. But *can the boundaries that make up space itself coincide*? More generally, *can the boundaries constitutive of primary continua coincide?* That the boundaries of two things in space may coincide is one thing, but that the boundary of two regions of space may coincide is quite another. For boundaries to coincide, several boundaries must be at the same place. But what would it mean for a boundary that constitutes space to be at a place? How can the very boundaries making up space be located at places, since they are themselves constituent of places? We are basically saying that places are located at places.

One may retort that location is a reflexive relation (Casati & Varzi 1999, 21). Places can therefore be seen as located *at themselves*. But whatever its intrinsic merits (or problems), this proposal does not demonstrate the possibility of coincidence between the boundaries of primary continua. For even if places are located at themselves, this does not show that *two places can be located at the same place*. On the contrary: if places are (exactly) located at themselves, and if two places are exactly located at the same place, then they must be one and the same. To show this, we just need to make the additional assumption that places, if located, have only one exact location. One quick argument in favour of this assumption is that places are particulars, and particulars have only one exact location (contrary, perhaps, to universals). The argument to the effect that, if two places are located at the same place, then they are not distinct then proceeds as follows:

(1) *Coincidence*: p_1 and p_2 are exactly located at a p_3.

(2) *Reflexivity*: p_1 is exactly located at p_1 . p_2 is exactly located at p_2 .

(3) *Unicity*: Every place has exactly one exact location.

(4) p_1 is identical to p_3. (from 1, 2, 3)

(5) p_2 is identical to p_3. (from 1, 2, 3)

(6) p_1 is identical to p_2. (from 4, 5)

Thanks to reflexive location we may ,find a way of holding that the boundaries constituting space are located, but we still do not get coincidence of boundaries. Summing up: either we hold that location is irreflexive, in which case two places can never be exactly located at the same place for the reason that places simply cannot be located. Or we accept the reflexivity of location, in which case two places cannot be exactly located at the same place for the reason that they fuse into one place. Either way, two places can never be located at the same place. The relation of coincidence, therefore, must hold between things which exist in space: it cannot hold between constituents of space. If this is right, any coincidence-based account of *primary* continua is doomed to fail.

Note that Brentano himself objects to mathematical accounts of continua along quite similar lines. He repeatedly argues that the intercalation approach cannot account for primary continua because it presupposes them (1988, 3, 39, 40). Against Poincaré he presses the idea that to grasp the idea of intercalation –in particular the intercalation of transcendental numbers– we must already have the idea of some primary continua to be filled. If there is something to be filled, then there is an underlying primary continuum whose continuity is not to be explained in terms of intercalation.

Now this worry backfires against Brentano's own coincidence-account: for coincidence to take place, there must be an underlying spatial continuum in which it takes place. Whether one fixes the hole in the pie shell by filling it —à la Dedekind— or by bringing its sides together —à la Brentano— one must be working on a continuous underlying pie plate, the continuity of which remains to be explained.

3.2. Do continua consist only of boundaries?

The second problem for Brentano's account of continua pertains to the constituents of continua: do continua consist only of boundaries, or do they contain another category of entity? One strand in Brentano's thought is that continua consist *only* of boundaries, so that saying that *boundaries bound continua* amounts to saying that *boundaries bound multiplicities of boundaries*. Already in 1885, Brentano maintained:

> ultimately, every continuum is constituted by infinitely many non-continuous boundaries. (Ms. Y 3: Die elementare Logik und die in ihr nötigen Reformen II, Vienna, 1884/1885, trans. by Ierna 2012)

> These non-extended boundaries are in a certain sense the ultimate physical parts of the continuum. The continuum ultimately consists in a multiplicity of non-extensional simple boundaries. (ibid.)

(Of course, some boundaries —surfaces, lines— are continuous, but their ultimate boundaries —boundary points— aren't. Hence Brentano's claim that continua are ultimately constituted of *non-continuous* boundaries.) The idea is restated around 1890 in the Vienna lectures on descriptive psychology:

> Every continuum consists of nothing but boundary points. (Brentano 1982, 112)

Likewise, in 1915:

> Indeed we can conceive [a continuum] as a continuous multiplicity of boundaries. (Brentano 1981b, 55)

Finally, in *On what is continuous*, Brentano says that the spatial continuum "is in every one of its boundaries" (Brentano 1988, 114), and he equates boundaries with the "ultimate elements" (Brentano 1988, 176) of continua.

However, the view that continua consist merely of boundaries raises two problems:

1. Brentano insists in various places that spatial continua, which he calls continuously *many* by contrast to continuously *manifold* continua, can be partly destroyed or modified without affecting the rest of the continuum (1981: 85, 157; 1988: 32-34). But if boundaries are essentially dependent on what they bound (in virtue of their plerosis and teleosis), and if what they bound are just other boundaries, then destroying one boundary of a continuum should end up modifying all the other boundaries of that continuum, *however far apart they are*. Continua end up being holistically unified in such a way that changes in one part affect all the other parts.
2. More crucially, if coincidence is the only spatial relation between boundaries in a continuum, how can we get spatially extended continua from mere unextended boundaries? By transitivity of coincidence (Chisholm 1992, Smith 1997), all boundaries end up coinciding with each other. From unextended boundaries and coincidence alone, one never gets extension, be it temporal or spatial.

Such problems suggest that Brentano's theory requires an additional ingredient. As it happens, Brentano is not always clear about whether boundaries are the only constituents of continua. In his "Addendum to the treatise on what is continuous", he says that boundaries "in conjunction *make a contribution to the continuum*." (1988: 40, my italics), leaving it open whether other elements contribute to the continuum. In other passages, Brentano may appear to accept atomless gunk—extended substance, all of whose parts have proper parts– as he is committed to *infinite divisibility*:

> [the continuum] is made up to infinity of smaller and smaller parts, and again […] these touch each other in null-dimensional boundaries, points, which would not be conceivable if there were nothing which they would bound. (Brentano 1988, 108; see also 1981, 46, 85; 1995b, 357)

Thus, Zimmerman (1996b) argues that atomless gunk is another ingredient of Brentanian continua over and above boundaries (see also Zimmerman1996a and Chisholm 1992, 14[8]) He considers Brentano, together with Suarez, to be a leading upholder of "moderate indivisibilism", the view that extended objects are made up of atomless gunk surrounded by skins of point-sized parts constituting a surface. Let us more explicitly refer to this view as the "*bounded-gunk account*":

bounded-gunk account: (i) Bodies are made up of gunk *and* of boundaries enclosing it. (ii) Gunk and boundaries are *mutually dependent* entities. More precisely (see 2.3.): the gunk that constitutes a body *individually depends* on the body's boundaries, while the boundary of the body only *depends generically* on the gunk that constitutes it.

The introduction of extended gunk on top of unextended boundaries paves the way for a solution the two problems raised above:

1. That boundaries depend on different chunks of gunk rather than on other boundaries would put an end to the holistic regress of dependencies within a continuum —provided gunk is not itself essentially dependent on its boundaries.
2. One no longer needs to get extension purely from unextended coinciding boundaries.

As an added bonus, if we appeal to gunk we can dispense with actual infinities, for we are replacing infinities of parts with infinite divisibility. This fits with Brentano's rejection of *infinitum actu* and acceptance of *infinitum potentia* (1995b: 362).

Despite these advantages, it is doubtful whether Brentano really endorses the bounded-gunk account. His most positive remark about atomless gunk is that the idea is "not quite so absurd" as Dedekind's idea of a magnitude consisting of infinitely many parts (1988: 5). Brentano then goes on to reject the idea of constructing continua thanks to

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[8]Smith (1997) also argues that Brentanian continua are not made up just of boundaries and contain some other extended ingredients.

> Concrete continua are in contrast made up of different sorts of parts; above all, they are made up of boundaries of different numbers of dimensions, on the one hand, and of extended bodies or regions which these boundaries are the boundaries of, on the other. (Smith 1997)

Smith however remains unspecific about the nature of Brentanian bodies. They could be made of gunk, as per Zimmerman. But Smith's claim is compatible with bodies being spatial continua (as Brentano assumes in various places), themselves to be analysed in terms of multiplicities of boundaries –which would drive us back to our present problem.

atomless gunk. On top of his general objection to constructionist approaches introduced above, Brentano points out that the idea of an *extended simple* to which one ascribes continuity is conceptually consistent (1988:6). In support of this idea he mentions Democritus's extended indivisible atoms and the idea that when we divide parts in our thoughts we arrive at certain limits. None of this prevents us from ascribing continuity to these *indivisibilia*, he suggests. He concludes that "being continuous and being divisible *in infinitum* are concepts that do not coincide in their content" (1988: 6).

The bounded-gunk account raises two difficulties that may explain Brentano's reluctancy:

1. The kind of continuous phenomena that interests him the most are *continuous transitions*, such as colour gradients. But continuous variations and gunk do not make good bedfellows. Consider, as above, a rectangle whose colour varies horizontally, continuously from red to blue. Any determinate shade of purple will consist in a vertical line. Such a line is vertically indivisible, and hence has no extent. The colour shades of continuous transitions are thereby not made of extended gunk: we seem to have a continuum without extended gunk of any determinate colour (see Arntzenius and Hawthorne, 2005 for some attempts at reconciling gunk and continuous variations).
2. Accepting gunk —or any other extended element— to fill boundaries commits one to *the distinction between closed and open entities*. For if bodies are bounded gunk, then one will have to say that gunk-*minus*-boundaries —or gunk-*in-abstraction-from*-its-boundaries— is an open entity. However, such a Bolzanian distinction is highly problematic according to Brentano, who famously rejected it as "monstrous" (1988: 146-7).

   One may retort that the bounded-gunk account is in fact immune to Brentano's objection to Bolzano. Brentano argues that on Bolzano's view we end up with two kinds of bodies (open and closed), and that "contact would be possible only between a body with a surface and another without" (ibid). On the bounded-gunk account, in contrast, the open-closed distinction is not a distinction between two kinds of substances: bodies are all closed. The distinction is instead a distinction between bodies (which are all closed) and the gunk that constitutes them (which is open). Since Brentano has a compelling account of contact between bodies (namely, through coincidence of boundaries), no paradox arises in this respect: under the bounded-gunk account we have only one kind of body (closed bodies) and only one kind of contact (coincidence).

While this is true, the bounded-gunk account still faces two problems.

(a) The bounded-gunk account runs afoul of another major worry Brentano has about the open-closed distinction, namely that open entities —be they independent or dependent— *begin without having any beginning point,* which according to Brentano is absurd (1988: 41; Brentano 1981b, 128n).

(b) If the bounded-gunk account is true, one has to account for the *contact between boundaries and their insides* (Varzi 1997): any gap between the skin and the flesh would endanger continuity. On the bounded-gunk account there is indeed only one mode of contact between *bodies*, but the contact between the mutually dependent entities constitutive of bodies –gunk and boundaries– remains unexplained. Brentano nowhere mentions this sort of contact in his extensive discussion of the continuous, making it dubious that he endorses the bounded gunk account.

Wrapping up, Brentano's account of continua faces an unattractive dilemma:

- Either continua consist entirely of boundaries, but then (1) it is impossible to destroy any part of a continuum without modifying all the rest of it (due to the essential dependence of boundaries); and (2) extension becomes impossible (due to transitivity of coincidence and the view that continuity can only stem from coincidence).
- Or continua consist of atomless gunk surrounded by a bounding-skin, but then (1) continuous transitions are hard to accommodate; (2) some entities —open gunk, that is, gunk in abstraction from its boundary— begin without having any beginning point. Further, the contact between the skin of the bodies and their gunky interior remains unaccounted for.

3.3. What is the plerosis of boundaries within a continuous transition?

The third problem for Brentano's account of continua pertains to continuous transitions. What are we going to say about the plerosis of inner boundaries within a continuous colour transition, such as any vertical line within the red-to-blue rectangle above? In how many directions do such boundaries bound? Brentano does not explicitly consider this question, but there seem to be two available answers, each of which proves equally problematic.

1. Because *none* of the immediate neighbours of each inner boundary within a continuous transition have the same colour, it seems natural to ascribe a null degree of plerosis to such transitory boundaries: they bound in no direction. But this seems to contradict the nature of a boundary: in what sense is a boundary that bounds in no direction still a boundary?[9]
2. On the other hand, since the inner boundaries of a continuous transition do not mark any qualitative ruptures (by contrast with the red-blue coinciding lines discussed above), one may be tempted to ascribe to them the highest degree of plerosis. It may seem indeed that such boundaries bound in *all* conceivable directions (left and right).

   This second answer is equally problematic. If a full plerosis is ascribed to transitory colours within continuous transitions, then the intuitive contrast between such boundaries and the inner boundary of a uniformly violet rectangle is lost. The inner boundaries of a uniformly red rectangle are ascribed the maximal degree of plerosis because they all have neighbours of exactly the same colour. So, when all neighbours are qualitatively distinct, as is the case with inner boundaries within continuous transitions, they should have a lesser degree of plerosis, on pain of losing the contrast between the two cases. To illustrate, suppose we hold fixed the colour of the violet line in the middle of the blue-to-red rectangle and that we progressively colour the rest of the rectangle with the same determinate shade of violet (Figure 8). Wouldn't the

---

[9] One may reply that having a null plerosis is distinct from having no plerosis. However, whatever one's take on zero-value quantities, there are reasons to believe that the distinction between zero-value *vectorial* quantities and absences of vectorial quantities is a distinction without a difference (Massin 2016, §5.3). Indeed, it is far from clear what a null direction could mean: what would a boundary that bounds in direction 0 bound?

plerosis of this line thereby become more complete? But how is this possible if it was already bounding in all directions?

Boundaries of a same plerosis?

Figure 8: Plerosis and continuous transition

4. A Neo-Brentanian Account: varieties of continuity

These three problems, I shall argue, originate in one tacit assumption made by Brentano: that all forms of continua —primary continua such as space and secondary continua such as coloured space— call for the *same* sort of account. While Brentano distinguishes different *kinds of continua*, he assumes they must all exhibit the same sort of continuity. I propose rejecting this assumption: there are different kinds of continua, and they exhibit different kinds of continuity.

Brentano's coincidence account of continuity nicely explains the continuity of concrete secondary continua, that is, the continuity of extended things that are *in* space or *in* time: two parts of a uniform coloured shape, two shapes of different colour, two parts of a life, two lives, etc., are continuous with each other in virtue of the coincidence of their boundaries.

However, two other sorts of non-secondary continua call for explanation, but cannot be explained in terms of boundary-coincidence:

1. The continuity of primary continua –space and time themselves– cannot be explained in terms of boundary-coincidences for the reasons advanced above.
2. The continuity of any space that abstracts from spatial and temporal determinations e.g. the colour space.

I shall argue that the continuity of these two sorts of continua –primary continua and abstract secondary continua– for which boundary-coincidence is excluded, can and should be explained in a different way, namely by introducing a primitive relation of continuity.

4.1. Primary continua: primitive continuity

Coincidence of boundaries, although crucial to understanding contact and continuity between things that exist in space —or in time— is of no help when it comes to understanding the continuity of primary continua themselves. Because coincidence can only occur in secondary continua, *the continuity of primary continua is not coincidence-based*. The scope of the coincidence-account of contact and continuity should be restricted: it applies only to things that exist *in* space (e.g. colour shapes) or in time (e.g. parts of one's life).

This contradicts Brentano's repeated claim that the possibility of coincidence belongs to the nature of continua: on the present proposal, only secondary continua can be explained through boundary-coincidence. But as we shall see, abandoning the possibility of coincidence for primary continua paves the way for an arguably even more Brentanian account of primary continua.

How should we account for the continuity of primary continua, if not by boundary coincidence? Brentano does hint at an alternative account of continuity. The continuity of primary continua, Brentano seems to suggest, consists in a *primitive continuity relation* between boundaries. Such a relation is mentioned in various places:

> [what is present] cannot exist without a *relation of continuity* [Kontinualrelation] to what is earlier or later and it is thereby connected with things which are separated from it, some by a greater and some by a lesser interval. The interval may be conceived to be as small as you please but not infinitely small. (Brentano1995b, 355)
>
> One thing is easy to see, namely, that the point which belongs to a spatial continuum […] is something only in virtue of belonging to the continuum. The relational character of the continuum is essential to it. *Anyone who conceives of it must*

> *conceive of it as something in a continual relation of this sort* [ein solches kontinuales Relativ*]*. (Brentano1995b, 356 italics original; see also p. 361)
>
> we certainly can conceive a continuum as a continuous multiplicity [*Vielheit*]. Indeed we can conceive it as a continuous multiplicity of boundaries. The boundaries do not exist in and for themselves and therefore no boundary can itself be an actual thing [*ein Reales*]. But *boundaries stand in continuous relation* [kontinuierlicher Verbindung] *with other boundaries* and are real to the extent that they truly contribute to the reality of the continuum. (Brentano 1981b, 55 my italics)
>
> Actually, the Aristotelian table of relations needs to be extended. I am thinking of the relation of continuity which holds between a boundary and the continuum it bounds. (Brentano 1981b, 188)

The last quote suggests that the relation of continuity to which Brentano appeals is *primitive*. That there must be a primitive continuity-relation is a natural suggestion in view of Brentano's recurring worry that other accounts of continuity always end up presupposing continuity (as I have argued, this objection applies to his own coincidence-based account). The best and perhaps only way to avoid circularity worries of this sort is to introduce primitive continuity.

How Brentano intends to reconcile a primitivist approach to continuity with the coincidence-based account considered so far is unclear to me. Two rival account of continuity seem to co-exist in his works, although the primivist account is admittedly less salient. My proposal is to apply these two accounts of continuity to different continua. That is, one should explain secondary continua in terms of the coincidence relation, and one should take the continuity of primary continua to be primitive.

What is, however, the relation of primitive continuity that gets us what we want? *Qua* primitive, the relation cannot be defined, and is in that sense doomed to frustrate those who were yearning for a reductive account of continuity. But the *Kontinualrelation* can nevertheless be described, categorized (it is a relation) and contrasted with other relations. The continuity relation, I suggest, takes boundaries and *puts them at a distance from each other and in continuity with each other*. More specifically:

1. *Relata of continuity*. The first option is that the continuity relation takes two boundaries: *"b1* and *b2* stand in relation of continuity". The second option is that "continuity" takes a *plurality* of boundaries: "The *Bs* stand in relation of continuity with each other". The three first quotes above, together with (i) Brentano's defence of multiplicities (Brentano 1981b, 189) and (ii) his explicit admission of plural predication (Brentano 1981b, 155) suggest that he favours the plural reading over the pairwise reading. In many places Brentano also uses a third (compatible) way of predicating continuity, which takes on the one hand a single boundary, and on the other a "continuous multiplicity of boundary" —i.e. a continuum. He thus speaks in the last quote above of "the relation of continuity which holds between the boundary and the continuum it bounds" (1981:128, see also 1981: 200) or says that "each boundary point is nothing except in continuity with a host of other boundary points" (1995a, 112).
2. *The continuity relation builds in distance between boundaries*. Brentano notes that the continuity relation is distinct from the contiguity relation (1988: 104). Unlike the contiguity relation, which is explained in terms of coincidence and which relates superposed boundaries, *the continuity relation is laden with "distance" or "intervals"*(1988: 173): boundaries that stand in the continuity relation to each other are thereby at a certain distance from each other–they cannot be coincident. Primitive continuity ensures there is some non-zero distance between boundaries, which coincidence fails to do.
3. *The continuity relation grounds the infinite divisibility of continua*. Although he rejects gunk, Brentano seems willing to accept the infinite divisibility of continua ("[the continuum] is made up to infinity of smaller and smaller parts, and again", 1988, 108). I suggest that the continuity relation, because it is laden with distance, grounds infinite divisibility. That is: for any multiplicity of boundaries standing in the continuity relation, there are sub-multiplicities of boundaries standing in continuity relation.
4. *The continuity relation is distinct from the dependence relation*. To the extent that the continuity relation builds in distance between the boundaries it relates, continuity is distinct from the formal relation of dependence. Bi-lateral dependencies can relate entities without entailing anything about their distance.

If continuity and dependence are distinct, how are they related? I suggest locating the primitive continuity relation within the scope of the dependence relation, more precisely, within the dependee:

> Any boundary depends on *its standing relation of continuity with what it bounds*.[10]

With this primitive continuity relation in hand, the continuity of primary continua can then be accounted for as follows:

Primitive continuity account(PC): A *primary* continuum is a multiplicity of boundaries standing in a relation of continuity to each other, such that

1. The *continuity relation* (i) is primitive; (ii) takes multiplicities of boundaries; (iii) is distance laden and hence distinct from coincidence; (iv) grounds infinite divisibility; (v) is distinct from inseparability/dependence.
2. The *boundaries* have no thickness (like points) but (unlike points) ontologically depend on their being continuously related to the other boundaries of the continuum they bound.

Let me address three questions that the proposal may raise.

1. *Is the continuity relation gunk in disguise?* We saw that Brentano was reluctant to add any another ingredient to his continua, and, in particular, that he rejects gunk. But insofar as continuity grounds extension and infinite divisibility how different is it from gunk? I suggest that *the continuity relation is basically gunk turned into a property* –there is therefore an important truth in Zimmerman's bounded-gunk reading. The continuity-relation plays the same role in the present proposal as gunk in the bounded-gunk account: like gunk, primitive continuity brings in extension and divisibility. But unlike gunk, continuity is a relation between boundaries, not an entity or substance between them. Rather than saddling the realm of substances with extended gunk, the present proposal saddles the realm of relations with a primitive relation of continuity.[11]

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[10]Analogously, in the case of colours and extension the filling relation should be put within the dependee:

- Any colour depends *on its filling an extent*.

(See Correia 2006, 76-77 for related considerations to the effect that "x needs Fs in order to exist" often if not always means "x needs to be related in a certain way to Fs in order to exist")

[11]One may object that such an account boils down to the view that continuous multiplicities of boundaries are gunky, because they are extended and infinitely

This distinction matters for two reasons. First, because the source of continuity is in the relation between boundaries rather than in some substance or gunk in-between, PC can rescue Brentano's claim that boundaries are the only ingredients of continua. We get extension and continuity from boundaries not by adding a new ingredient such as gunk but by imposing a new structure: continuity relations between boundaries.

Second, the appeal to continuity relations rather than gunk can avoid the pitfalls of the bounded-gunk account. Recall that the bounded-gunk account entails that the gunk filling the boundaries, once abstracted from the boundaries, begins yet has no beginning point —an absurdity for Brentano. By contrast, under PC the problem does not even arise. For while it makes sense to ask "Where does gunk-minus-the-boundaries-it-fills start?", it makes no sense to ask "Where does the continuity-relation-minus-the-boundaries-it-relates start?". The continuity-relation and the boundaries it relates are not spatially adjacent —by contrast with boundaries and the gunk that fills them. For the same reason, the problem of explaining contact between boundaries and their interiors, which plagued the bounded-gunk account, does not even arise for PC.

2. *Is the continuity-relation the relation of distance?* Is there a difference between standing in a continuity relation and standing in a distance relation? I am inclined to think that, as far as boundaries of primary continua are concerned, *being at a distance from* entails *being continuous with*. One might first think that distance is not sufficient for continuity on the grounds that things at a distance from each other can be (and often are) separated by gaps –after all, discrete space is taken to be metaphysically possible (see Weyl 2009). But such a possibility hardly makes sense in the case of primary continua. First, for reasons that are now familiar, the existence of gaps in a continuum entails that the continuum in question is not primary: for there to be gaps, there must be some underlying continua, capable of being filled, on the background of which such discontinuities can arise. Second, even if gaps in primary continua were possible, it is arguable that boundaries on both sides of these gaps would not be at a distance

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divisible. However, while gunk is usually conceived of as some substance that is *primitively* extended and infinitely divisible, on the present account, by contrast, the gunky features—extension and divisibility— are grounded in more fundamental facts: boundaries standing in relations of continuity.

from each other, but simply belong to different primary continua, in the same way that a location in space is not at a distance from a point in time.

3. *Is the relation of primitive continuity, so characterised, the one Brentano appeals to?* Nearly. Brentano clearly accepts the first two features, and is sympathetic towards the third. But he appears to reject the fourth feature. He indeed tends to equate "the relation of continuity which holds between the boundary and the continuum it bounds" (1981: 200) with the crossing of dependencies which on his account also relate the boundary and the continuum (see section 2.3 above). The last quote above continues:

> Actually, the Aristotelian table of relations needs to be extended. I am thinking of the relation of continuity which holds between a boundary and the continuum it bounds. Indeed that which exists in time exists only as a boundary. But just as it is certain that there are boundaries and that they must be included among things, it is also certain that a boundary is not a thing existing in itself [*nicht etwas für sich Bestehendes*]. The boundary could not exist unless it belonged as a boundary to a continuum. Hence it would seem to be the case both that the continuum is conditioned by the boundary and also that the boundary is conditioned by the continuum. Indeed, the boundary is conditioned in its nature by the continuum; thus a point differs in kind depending upon whether it belongs to a circle or to a straight line that is a tangent of the circle. We are dealing here with a unique type of causal relation, which we may call the relation of continuity [*Kontinual-relation*] (1981: 128; see also pp. 200-201)

Likewise, in *Raum, Zeit und Continuum*:

> To the class of causal relations there belongs everything that somehow conditions the being of a thing or is conditioned by it. […] continual causation is (where we are not dealing with contiguity) a mutual causation between a boundary and what it bounds. (1988, 104)

Brentano's talk of "causal relation" may at first be found surprising. However, Brentano uses "causal" in a more encompassing sense than we do today. In Brentano's use, "causal" relations include the relation of *parthood*, the relations between *substance and accident*, or *the relation of ontological dependence between boundaries and the continua they bound* (1981: 201-2).

The upshot is that Brentano fails to clearly distinguish the *dependency relations* between boundaries and what they bound from the *continuity relations* between boundaries. This is a problem because one cannot get the relation of continuity, which incorporates distance, from a formal relation like dependence[12] . Dependencies, however tight and interwoven, will never yield continuity. The account of primary continua requires therefore at least two distinct relations between boundaries and continua: continuity and dependence.

For all its virtues, PC leaves us with one natural Brentanian worry: how are we to account for *contact* between primary continua, if not by coincidence? The question splits into two: (i) how are we to account for contact between *regions* of space? (ii) how are we to account for contact between *boundaries*?

1. *Contact between regions*. A natural answer to the first question is that regions of space are in contact iff they have boundaries in common. Contact between regions of space (or spans of time) is accounted for not by *coincidence of boundaries*, but by *sharing of boundary*. As noticed above, the chief problem for the boundary sharing account of contact –which is avoided by boundary coincidence accounts– stems from *breakage*. But in the case of primary continua this objection does not even get off the ground, for breaks in *space* do not make sense (if you think they do, then perhaps you conceive of space as a secondary continuum on the background of some more fundamental primary continuum. *That* background primary continuum in which breakage occurs cannot itself be broken).
2. *Contact between boundaries*. What is it, then, for two boundaries of space to be in contact? I suggest that this is the place to part

[12]There is arguably another instance of such a conflation between dependence and neighbouring material relations in Brentano's writings. Brentano characterises the relation between qualitative determinations —such as colours— and spatial determinations in much the same way as he characterises the relations between continua and their boundaries: while colour depends individually on location, location depends generically on colour. To characterise this mutual dependence between colours and extension, Brentano says that they are "mutually pervading parts" or that "These parts do not appear in a spatial manner side by side but are tied completely differently, in that they, one might say, penetrate one another" (1995a: 20). This suggests that he thinks of the dependency relation as building in the *filling* relation.

way with the idea that continuity always requires contact. On the proposed account, *boundaries of primary continua can —and must— stand in continuity with each other without being in contact with each other*. The reason for this is, as I argued, that two boundaries of space cannot be in contact with each other without being one and the same.

Summing up, all we have in primary continua are non-coinciding boundaries, however near, standing in continuity relations. Coincidence of boundaries is nowhere to be found among the boundaries constituting space or time. Coincidence is however ubiquitous when we turn our attention towards concrete secondary continua: things that are in space. But there is still a third kind of continua, *abstract* secondary continua, which we finally have to deal with.

4.2. Abstract secondary continua: primitive similarity

Brentano's talk of "secondary continua" tends to collapse two different sorts of secondary continua with respect to continuous transitions:

1. *The colour space,* which is an *abstract* quality space. Just like "colour is something from which the geometer abstracts" (1988: 21), space is something from which the scientist interested in the nature of colour abstracts. Although the colour space he reaches by bracketing spatial differences can only be *represented* in space (for instance, by a three-dimensional colour solid) or in space and time (such as when a shape continuously changes its colour), the colour space by itself is free of spatial and temporal determinations. When ordering colours, we try to abstract from such determinations —colours are not more or less similar depending on the space or time they occupy. Hence, although the colour space is a secondary continuum —for colours depend on extension— it is a simple and abstract continuum.
2. The *coloured space,* by contrast, is the concrete and colourful space we see. It is a double continuum formed by the encountering of two simple and abstract continua: the colour space and the (geometer's) space (Figure 9).

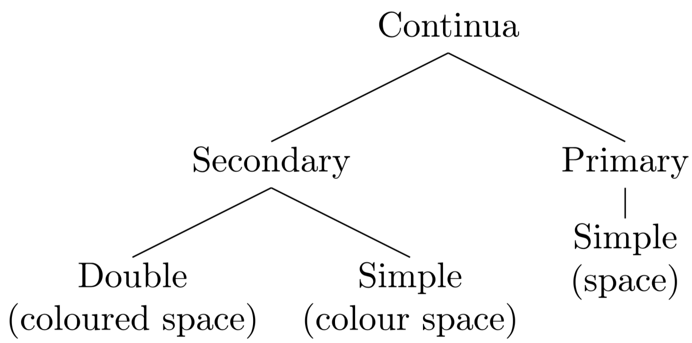

Figure 9: Kinds of continua

I have argued that the continuity of the colour*ed* space should be explained in terms of boundary coincidence, and that the continuity of space should be explained in terms of boundary sharing and a primitive relation of continuity. Our question is now how to explain the continuity of the colour space. Note first that although the colour space is a secondary continuum (because it depends on primary continua) it displays several striking similarities to the primary continua of time and space.

- First, the colour space is, like primary space, a *simple* continuum (it is one of the ingredients of the colour*ed* space which is therefore a multiple continuum).
- Second, the colour space is, like primary space for Brentano, an *abstraction*: to reach it we need to abstract from the primary continuum of space. Only double continua are concrete.
- Third, the colour space, again like primary continua, has a *constant teleosis*. There is no acceleration *within* the colour space. It is only when a portion of the colour space is projected within primary space, giving rise to the multiple colour*ed* space, that *variations* in teleosis can occur. Variations in teleosis consist in some simple secondary continua being projected heterogeneously within some primary continua. For instance, in Figure 7 above, the red-to-blue portion of the colour space is projected homogeneously into space in the first figure. In the second figure, by contrast, a greater portion of the simple red-to-blue colour continua is projected towards to middle. The reason why coloured space can have varying teleosis is not that it is a secondary

continuum, but that is a double continuum. Because the colour space is simple, its teleosis cannot vary.

Brentano insightfully pointed out a difference between continua with fixed and varying degrees of teleosis. What he should have said, however, is not that secondary continua, unlike primary continua, can have varying teleosis, but rather that *double continua*, unlike *simple* continua (primary or secondary), can have varying teleosis.

- Fourth, like the inner boundaries of space, the boundaries of the colour space have *full plerosis*. In the same way that the present bounds in two directions (contrary to a beginning, which bounds in just one direction, but pertains to multiple continua), or in the same way that a spatial point bounds in all the directions around it, any determinate shade within the colour space bounds all over it[13].
- Fifth, relatedly it is hard to see how any coincidence of boundaries could occur in the colour space: what would it be for two different determinate shades of red to occupy the same place in the colour space? Colour-shades *are* the places in the colour continua. Granting that two colours can be at the same place in the colour-space would amount to granting, absurdly, that two places in that space can be in the same place.

How then should we account for the continuity of the colour space? Given the many similarities with primary space, a natural suggestion is that, like primary space, the colour space consists in a multiplicity of boundaries; i.e., a multiplicity of determinate colour shades, standing in a relation of primitive continuity to each other. Relatedly, as with primary space, two regions of the colour space are in contact iff they share a boundary (a determinate colour); and points in the colour space can never be in contact with each other –for if they were to coincide, they would be one and the same determinate colour.

Interestingly, the primitive relation of continuity between determinate colour shades is more familiar than the primitive relation linking spatial boundaries. I suggest that it is nothing other than the

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[13] There is however one respect at least in which colour space importantly differs from space and time. Since colour space is limited or finite, it will have external boundaries. What they are will depend on how the colour space is construed — perhaps the colour spaces has corners (see Mulligan 1991), in which case not all boundaries of the colour space have full plerosis.

relation of brute *inexact similarity*[14]. One key feature of primitive continuity is that it is laden with distance. In the case of spatial boundaries, these distances are spatial distances. In the case of determinate colours, these distances must consist in various degrees of similarity –that is, in various distances within a resemblance order. That primitive continuity is, in the case of quality spaces, identical with brute inexact similarity, may help render the otherwise unconventional idea of a primitive relation of continuity more familiar.

The analogies and differences between kinds of continua and continuities according to this neo-Brentananian account are recapped in table 1.[15]

	Primary Continua	*Simple Secondary Continua*	*Double Secondary Continua*
Paradigmatic example	Space	Colour space	Coloured space
Concrete/Abstract	Abstract	Abstract	Concrete
Simple/Double	Simple	Simple	Double
Continuity	Primitive continuity relation	Primitive continuity relation	Coincidence-based
Teleosis	Necessarily constant	Necessarily constant	Possibly varying
Coincidence of boundaries	Impossible	Impossible	Possible
Contact between regions	Boundary-sharing	Boundary-sharing	Boundary-coincidence

Table 1: Varieties of continuities

[14]See Massin 2013 for a defense of such an account of determinables in terms of brute similarity.

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