

# An Economical Business-Cycle Model

Pascal Michaillat and Emmanuel Saez\*

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## ABSTRACT

In recent decades, advanced economies have experienced low and stable inflation and long periods of liquidity trap. We construct an alternative business-cycle model capturing these two features by adding two assumptions to a money-in-the-utility-function model: the labor market is subject to matching frictions, and real wealth enters the utility function. These assumptions modify the two core equations of the standard New Keynesian model. With matching frictions, we can analyze equilibria in which inflation is fixed and not determined by a forward-looking Phillips curve. With wealth in the utility, the Euler equation is modified and we can obtain steady-state equilibria with a liquidity trap, positive inflation, and labor market slack. The model is simple enough to inspect the mechanisms behind cyclical fluctuations and to study the effects of conventional and unconventional monetary and fiscal policies. As a byproduct, the model provides microfoundations for the classical IS-LM model. Finally, we show how directed search can be combined with costly price adjustments to generate a forward-looking Phillips curve and recover some insights from the New Keynesian model.

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\*Pascal Michaillat: Department of Economics and Centre for Macroeconomics, London School of Economics, Houghton Street, London, WC2A 2AE, UK; Email: [p.michaillat@lse.ac.uk](mailto:p.michaillat@lse.ac.uk). Emmanuel Saez: Department of Economics, University of California, Berkeley, CA, 94720, USA; Email: [saez@econ.berkeley.edu](mailto:saez@econ.berkeley.edu). We thank Regis Barnichon, Francesco Caselli, Giancarlo Corsetti, Wouter den Haan, Emmanuel Farhi, Etienne Lehmann, Kevin Sheedy, Carl Walsh, and numerous seminar and conference participants for helpful discussions and comments. This work was supported by the Center for Equitable Growth at the University of California Berkeley, the British Academy, the Economic and Social Research Council [grant number ES/K008641/1], the Banque de France foundation, and the Institute for New Economic Thinking.

# 1 Introduction

A first defining feature of the US macroeconomy in the past two decades is that inflation has been extremely sluggish. Inflation has not responded much to fluctuations in unemployment: although the Great Recession saw a dramatic increase in slack as unemployment rose from 4.4% to 10%, inflation remained stable, a bit lower than before but never below 1%. Nor has inflation responded much to monetary policy: most empirical studies find that monetary policy barely contributes to price movements [Christiano, Eichenbaum and Evans, 1999].<sup>1</sup> In fact, since 1994, the core inflation rate has remained in a narrow range between 1% and 2.5%, as showed in Figure 1.

A second defining feature is that a low-inflation economy is prone to enter long liquidity traps after a large negative shock. The US economy entered a liquidity trap in December 2008 when the nominal interest rate set by the Federal Reserve reached its zero lower bound. It is still in this liquidity trap in September 2014.<sup>2</sup>

To capture these two features, we propose a business-cycle model in which inflation is fixed and permanent liquidity traps can occur. Our model adds two ingredients to the money-in-the-utility-function model of Sidrauski [1967]. First, we introduce matching frictions on the labor market as in Michailat and Saez [2013], and we consider equilibria with fixed inflation. Second, we introduce wealth in the utility function as in Kurz [1968].<sup>3</sup>

The logic behind the addition of matching frictions is that in the matching framework, inflation may be fixed in equilibrium and aggregate supply and demand matter even with fixed inflation—they determine market tightness and output. This is advantageous compared to a framework with monopolistic or perfect competition because in those frameworks, assuming that inflation is fixed implies that output is determined by aggregate demand alone. Indeed, a model of monopolistic or perfect competition with fixed inflation is equivalent to a disequilibrium model in which output is

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<sup>1</sup>The behavior of inflation over the past two decades led Hall [2011] to argue that inflation is exogenous for all practical purposes. In fact, inflation has become extremely hard to forecast after 1984. Stock and Watson [2008] find that it has become exceedingly difficult to improve systematically upon simple univariate forecasting models, such as the random walk model of Atkeson and Ohanian [2001].

<sup>2</sup>These two defining features—sluggish inflation and long periods of liquidity trap—also apply to Japan and many European countries in the past two decades.

<sup>3</sup>For others models in which consumption and wealth enter the utility function, see Zou [1995] and Bakshi and Chen [1996] and Carroll [2000]. For a model in which consumption, money, and wealth enter the utility function, see Zou [1998]. These models have a different focus from ours. Kurz [1968] and Zou [1998] study long-term growth. Zou [1995] and Carroll [2000] study saving over the life cycle. Bakshi and Chen [1996] study portfolio choice and asset pricing. We study business cycles.

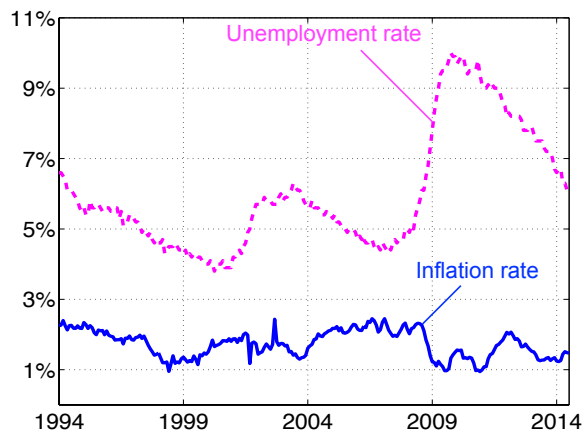


Figure 1: Inflation and Unemployment over the Past 20 Years in the US

*Notes:* The time period is January 1994–July 2014. The unemployment rate is the seasonally adjusted monthly civilian unemployment rate constructed by the Bureau of Labor Statistics from the Current Population Survey. The inflation rate is the percent change from year ago of the seasonally adjusted monthly personal consumption expenditures excluding food and energy index, constructed by the Bureau of Economic Analysis as part of the National Income and Product Accounts.

determined at the intersection of aggregate demand and a fixed price [Barro and Grossman, 1971].<sup>4</sup> But abstracting from aggregate supply does not seem completely satisfactory. Supply forces surely matter in the short run; otherwise we would not worry about the effect of cyclical changes in unemployment insurance, mismatch, labor force participation, and various subsidies and taxes.

The logic behind the assumption of wealth in the utility function is that with wealth in the utility, we can obtain permanent liquidity traps with slack on the labor market and moderate inflation. This assumption may capture a love for the social status provided by wealth, or a desire to accumulate wealth as an end in itself [Frank, 1985; Keynes, 1919; Weber, 1930]. Under this assumption, there is a unique steady-state equilibrium in liquidity traps, and this steady-state equilibrium may have positive inflation and thus a negative real interest rate. The existence of permanent liquidity traps relies on the fact that the consumption Euler equation is modified with wealth in the utility. The Euler equation defines a downward-sloping relation in a (consumption, interest rate) plane in steady state, instead of an horizontal relation without wealth in the utility. Permanent liquidity traps raise challenging questions on the role of conventional and unconventional monetary and fiscal policies, and our model is useful to address some of these questions.

Our model is simple enough to inspect the mechanisms behind cyclical fluctuations and ana-

<sup>4</sup>A number of recent papers assume fixed prices and thus work with models akin to fixprice disequilibrium models. See Mankiw and Weinzierl [2011], Caballero and Farhi [2014], and Korinek and Simsek [2014].

lyze the effects of a broad range of stabilization policies. Indeed, the steady-state equilibrium is represented with an IS and a LM curve depicted in a (consumption, interest rate) plane, and an AD and an AS curve depicted in a (consumption, labor market tightness) plane. The IS curve describes the trade-off between holding wealth and consumption. The LM curve describes the trade-off between holding money and consumption. The AD curve is obtained at the intersection of the IS and LM curves. The AS curve describes the supply of labor and the matching on the labor market. Furthermore, comparative statics completely describe the response of the equilibrium to unexpected shocks because the equilibrium jumps from one steady state to another after such shocks.

The IS-LM-AD-AS representation allows us to analyze the effects of shocks and policies. In our model, business cycles may be generated by aggregate demand and supply shocks. We find that a negative aggregate demand shock leads to lower output and lower tightness while a negative aggregate supply shock leads to lower output but higher tightness. A broad range of policies can be used to stabilize the economy. A conventional monetary policy that issues money through open market operations can stabilize the economy in normal times but not in a liquidity trap when the nominal interest rate falls to zero. In a liquidity trap, other policies can stimulate aggregate demand and stabilize the economy—for instance, a helicopter drop of money, a wealth tax, or budget-balanced government purchases. In addition, if consumers are not Ricardian in that they perceive government bonds as net wealth, government debt also stimulates the economy and government spending financed by debt is even more effective than budget-balanced spending.

Since our model is quite different from the standard New Keynesian model, the insights it offers complement those obtained in the New Keynesian model. By not imposing a forward-looking Phillips curve as in the New Keynesian model and assuming instead fixed inflation, we obtain new insights on the effects of aggregate shocks and policies in and out of liquidity traps. These insights are all the more useful than it is difficult to describe the sluggish dynamics of inflation with the forward-looking Phillips curve.<sup>5</sup> By assuming wealth in the utility and thus modifying the consumption Euler equation, we can account for permanent liquidity traps, which are typically difficult to analyze in the New Keynesian model.<sup>6</sup>

Although we have seen that the assumption that inflation is fixed is a useful and realistic ap-

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<sup>5</sup>See [Mankiw and Reis \[2002\]](#), [Rudd and Whelan \[2007\]](#), [Gordon \[2011\]](#), and [Ball and Mazumder \[2011\]](#).

<sup>6</sup>See the discussion by [Cochrane \[2013\]](#).

proximation to describe the short run in economies with a history of low and stable inflation, it would also be important to incorporate inflation dynamics in the model. To illustrate how inflation dynamics could be incorporated, we merge the price-adjustment framework of [Rotemberg \[1982\]](#) with the competitive search framework of [Moen \[1997\]](#). In this extension, sellers decrease their prices when the economy is slack and increase them when the economy is tight, which generates endogenous inflation dynamics described by a forward-looking Phillips curve similar to that of the New Keynesian model. After a negative demand shock, slack increases and inflation decreases. The lower price level stimulates aggregate demand until the economy converges back to the efficient steady state. After a negative supply shock, slack decreases and inflation increases. The higher price level depresses aggregate demand until convergence to the efficient steady state with lower output. As discussed above, empirical evidence indicates that it may be difficult to explain the behavior of inflation with such a Phillips curve. We may need to explore new avenues to understand inflation dynamics, such as pricing with rational inattention or fairness concerns.<sup>7</sup> Our model may be a useful platform for such research.

## 2 The Model

The model extends the money-in-the-utility-function model of [Sidrauski \[1967\]](#) by adding matching frictions on the labor market and wealth in the utility function. The economy consists of a measure 1 of identical households who hold money and bonds, produce labor services, and purchase labor services from other households for their own consumption.

### 2.1 Money and Bonds

Households can issue or buy riskless nominal bonds. Bonds are traded on a perfectly competitive market. At time  $t$ , households hold  $B(t)$  bonds, and the rate of return on bonds is the nominal interest rate  $i(t)$ .

A quantity  $M(t)$  of money circulates at time  $t$ . Money is issued by the government through open market operations: the government buys bonds issued by households with money. At any

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<sup>7</sup>For a survey of the literature on pricing with rational inattention, see [Sims \[2010\]](#). For macroeconomic models of pricing when consumers care about fairness, see [Rotemberg \[2005\]](#) and [Eyster, Madarasz and Michailat \[2014\]](#).

time  $t$ , the quantity of bonds issued equals the quantity of money put in circulation:  $-\dot{B}(t) = \dot{M}(t)$ . Initially,  $-B(0) = M(0)$ . Therefore, at any time  $t$ ,

$$-B(t) = M(t). \quad (1)$$

The representative household is net borrower:  $B(t) \leq 0$ . At time  $t$ , the revenue from seignorage is  $S(t) = -B(t) \cdot i(t) = i(t) \cdot M(t)$ . The government rebates this revenue lump sum to households. Without public spending or taxes, the government's budget is therefore balanced at any time.

Finally, money is the unit of account. The price level at time  $t$  is  $p(t)$ . The rate of inflation at time  $t$  is  $\pi(t) = \dot{p}(t)/p(t)$ . The quantity of real money in circulation at time  $t$  is  $m(t) = M(t)/p(t)$ .

## 2.2 Labor Market

We model the labor market as in [Michaillat and Saez \[2013\]](#). Households sell labor services on a market with matching frictions. Households would like to sell  $k$  units of services at any point in time. The capacity  $k$  of each household is exogenous. Households also consume labor services, but they cannot consume their own services, so they trade with other households. To buy labor services, households post  $v(t)$  help-wanted advertisements at time  $t$  which use up labor services as well. Hence, output of labor services is used for two purposes: consumption and recruiting which we refer to as “consumption services” and “recruiting services” below.

A matching function  $h$  with constant returns to scale gives the number of trades at time  $t$ :  $y(t) = h(k, v(t))$ . The matching function is twice differentiable, strictly increasing in both arguments, and with diminishing marginal returns in both arguments.<sup>8</sup> It also satisfies  $0 \leq h(k, v(t)) \leq k$ .<sup>8</sup> In each trade, one unit of labor service is bought at price  $p(t) > 0$ .

The labor market tightness is defined as the ratio of help-wanted advertisements to capacity:  $x(t) = v(t)/k$ . With constant returns to scale in matching, labor market tightness determines the probabilities to trade for sellers and buyers. At time  $t$ , one unit of labor service is sold at rate  $f(x(t)) = y(t)/k = h(1, x(t))$  and one help-wanted advertisement leads to trades at rate  $q(x(t)) = y(t)/v(t) = h(1/x(t), 1)$ . We denote by  $1 - \eta$  and  $-\eta$  the elasticities of  $f$  and  $q$ :  $1 - \eta \equiv x \cdot f'(x)/f(x) > 0$  and  $\eta \equiv -x \cdot q'(x)/q(x) > 0$ . We abstract from randomness at the household's

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<sup>8</sup>A matching function satisfying these properties is  $h(k, v) = (k^{-\zeta} + v^{-\zeta})^{-1/\zeta}$  with  $\zeta > 0$ .

level: at time  $t$ , a household sells  $f(x(t)) \cdot k$  units of labor services and purchases  $q(x(t)) \cdot v(t)$  units of labor services with certainty.

Households are unable to sell all their labor services since  $h(k, v) \leq k$ ,  $f(x(t)) \leq 1$ . Households are idle a fraction  $1 - f(x(t))$  of the time. The rate of idleness can be interpreted as the unemployment rate in this economy of self-employed workers. Since  $h$  is strictly increasing in its two arguments,  $f$  is strictly increasing and  $q$  is strictly decreasing in  $x$ . This means that when the labor market is slacker, it is harder for households to sell their labor services but easier for them to buy labor services from others.

Posting help-wanted advertisements is costly. The flow cost of an advertisement is  $\rho \geq 0$  units of labor services so that a total of  $\rho \cdot v(t)$  recruiting services are spent at time  $t$ . These recruiting services represent the resources devoted to matching with an appropriate worker. Recruiting services are purchased like any other labor services. As output of labor services is used for consumption, denoted  $c(t)$ , and recruiting, we have  $y(t) = c(t) + \rho \cdot v(t)$ . Only labor services for consumption enter households' utility function; labor services for recruiting do not. Thus it is consumption and not output that matters for welfare. Of course, this definition of consumption is different from that in national accounts, where  $y(t)$  would be called consumption, but defining consumption as output net of recruiting costs is the norm in the matching literature [for example, [Gertler and Trigari, 2009](#); [Ravenna and Walsh, 2011](#)].

The number of help-wanted advertisements is related to consumption by  $q(x(t)) \cdot v(t) = y(t) = c(t) + \rho \cdot v(t)$ . Therefore, the desired level of consumption determines the number of advertisements:  $v(t) = c(t) / (q(x(t)) - \rho)$ . Hence, consuming one unit of services requires to purchase  $1 + \rho \cdot v(t) / c(t) = 1 + \tau(x(t))$  units of services where  $\tau(x(t)) = \rho / (q(x(t)) - \rho)$ . The function  $\tau$  is positive and strictly increasing for all  $x \in [0, x^m)$  where  $x^m > 0$  satisfies  $\rho = q(x^m)$ . Furthermore,  $\lim_{x \rightarrow x^m} \tau(x) = +\infty$ . The elasticity of  $\tau$  is  $\eta \cdot (1 + \tau(x))$ .

We characterize the labor market tightness that maximizes consumption. In equilibrium,

$$c(t) = \frac{y(t)}{1 + \tau(x(t))} = \frac{f(x(t))}{1 + \tau(x(t))} \cdot k. \quad (2)$$

Since  $1/(1 + \tau(x)) = 1 - \rho/q(x)$  and  $q(x) = f(x)/x$ , we obtain

$$c(t) = (f(x(t)) - \rho \cdot x(t)) \cdot k. \quad (3)$$

This equation says that  $\rho \cdot x(t) \cdot k = \rho \cdot v(t)$  units of services are dissipated in matching frictions. As established by [Michaillat and Saez \[2013\]](#), the tightness that maximizes consumption given the matching frictions,  $x^* = \operatorname{argmax} \{(f(x) - \rho \cdot x) \cdot k\}$ , is uniquely defined by  $f'(x^*) = \rho$ . An equivalent definition is  $\tau(x^*) = (1 - \eta)/\eta$ . This definition will be useful when we study the Phillips curve arising from costly price adjustment in [Section 5](#). The efficient tightness,  $x^*$ , is the tightness underlying the condition of [Hosios \[1990\]](#) for efficiency in a matching model.

The labor market can be in three regimes. The labor market is slack if a marginal increase in tightness increases consumption, tight if a marginal increase in tightness decreases consumption, and efficient if a marginal increase in tightness has no effect on consumption. Equivalently, the labor market is slack if  $x(t) < x^*$ , efficient if  $x(t) = x^*$ , and tight if  $x(t) > x^*$ . If tightness is efficient on average, then business cycles are a succession of slack and tight episodes. The departure of tightness from its efficient level is the relevant measure of “output gap”.

[Figure 2](#) summarizes the relation between labor market tightness and different quantities. Capacity,  $k$ , is a vertical line, independent of tightness. Output,  $y = f(x) \cdot k$ , is increasing in tightness as it is easier to sell services when tightness is high. Consumption,  $c = f(x) \cdot k / (1 + \tau(x)) = (f(x) - \rho \cdot x) \cdot k$ , first increases and then decreases in tightness. At the efficient tightness, the consumption curve is vertical. The difference between output and consumption are recruiting services,  $\rho \cdot v = \rho \cdot k \cdot x$ . The difference between capacity and output is idle capacity,  $(1 - f(x)) \cdot k$ .

### 2.3 Intertemporal Utility Maximization

Households spend part of their labor income on labor services and save part of it as money and bonds. The law of motion of the representative household’s assets is

$$\dot{B}(t) + \dot{M}(t) = p(t) \cdot f(x(t)) \cdot k - p(t) \cdot (1 + \tau(x(t))) \cdot c(t) + i(t) \cdot B(t) + S(t).$$



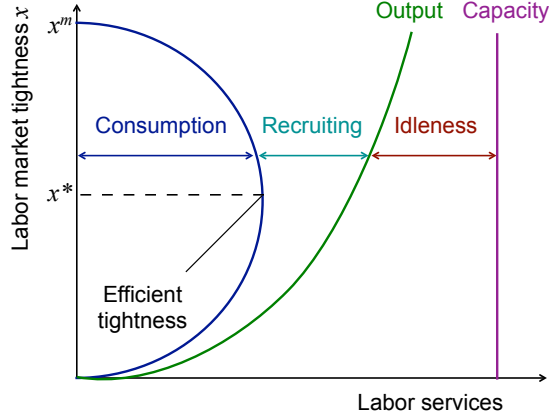


Figure 2: Capacity, Output, Idle Labor, and Consumption

Here,  $M(t)$  are money balances,  $B(t)$  are bond holdings,  $p(t)$  is the price of services,  $(1 + \tau(x(t))) \cdot c(t)$  is the quantity of services purchased,  $f(x(t)) \cdot k$  is the quantity of services sold, and  $S(t)$  is lump-sum transfer of seignorage revenue from the government. Let  $A(t) = M(t) + B(t)$  denote nominal financial wealth at time  $t$ . The law of motion can be rewritten as

$$\dot{A}(t) = p(t) \cdot f(x(t)) \cdot k - p(t) \cdot (1 + \tau(x(t))) \cdot c(t) - i(t) \cdot M(t) + i(t) \cdot A(t) + S(t).$$

Let  $a(t) = A(t)/p(t)$  denote real financial wealth at time  $t$  and  $s(t) = S(t)/p(t)$  real transfer of seignorage. Since  $\dot{a}(t)/a(t) = \dot{A}(t)/A(t) - \pi(t)$ , we have  $\dot{a}(t) = (\dot{A}(t) - \pi(t) \cdot A(t)) / p(t)$ , and the law of motion can be rewritten as

$$\dot{a}(t) = f(x(t)) \cdot k - (1 + \tau(x(t))) \cdot c(t) - i(t) \cdot m(t) + r(t) \cdot a(t) + s(t), \quad (4)$$

where  $r(t) \equiv i(t) - \pi(t)$  is the real interest rate at time  $t$ . This flow budget constraint is standard but for two differences arising from the presence of matching frictions on the labor market. First, income  $k$  is discounted by a factor  $f(x(t)) \leq 1$  as only a fraction  $f(x(t))$  of  $k$  is actually sold. Second, consumption  $c(t)$  has a price wedge  $1 + \tau(x(t)) \geq 1$  because resources are dissipated in recruiting: consuming one unit of services requires buying  $1 + \tau(x(t))$  units of services.

Households experience utility from consuming labor services and holding real money balances and real wealth. Their instantaneous utility function is  $u(c(t), m(t), a(t))$ , where  $u$  is strictly increasing in its three arguments, strictly concave, and twice differentiable. The assumptions that

real money balances and real wealth enter the utility function are critical to obtain a nondegenerate IS-LM system, and obtain permanent liquidity traps. The utility function of a household at time 0 is the discounted sum of instantaneous utilities

$$\int_0^{+\infty} e^{-\delta \cdot t} \cdot u(c(t), m(t), a(t)) dt, \quad (5)$$

where  $\delta > 0$  is the subjective discount rate. Throughout,  $[x(t)]_{t=0}^{+\infty}$  denotes the continuous-time path of variable  $x(t)$ .

**DEFINITION 1.** *The representative household's problem is to choose paths for consumption, real money balances, and real wealth  $[c(t), m(t), a(t)]_{t=0}^{+\infty}$  to maximize (5) subject to (4), taking as given initial real wealth  $a(0) = 0$  and the paths for labor market tightness, nominal interest rate, inflation, and seignorage  $[x(t), i(t), \pi(t), s(t)]_{t=0}^{+\infty}$ .*

Concretely, the model can be seen as the [Sidrauski \[1967\]](#) model with two additions. First, real wealth  $a(t)$  enters the utility function. Second, matching frictions lower labor income by a factor  $f(x(t))$  and increase the effective price of consumption by a factor  $1 + \tau(x(t))$ . Because  $x(t)$  is taken as given by households, the model can be solved exactly as the original Sidrauski model. To solve the household's problem, we set up the current-value Hamiltonian:

$$\begin{aligned} \mathcal{H}(t, c(t), m(t), a(t)) = & u(c(t), m(t), a(t)) \\ & + \lambda(t) \cdot [f(x(t)) \cdot k - (1 + \tau(x(t))) \cdot c(t) - i(t) \cdot m(t) + r(t) \cdot a(t) + s(t)] \end{aligned}$$

with control variables  $c(t)$  and  $m(t)$ , state variable  $a(t)$ , and current-value costate variable  $\lambda(t)$ . Throughout we use subscripts to denote partial derivatives. The necessary conditions for an interior solution to this maximization problem are  $\mathcal{H}_c(t, c(t), m(t), a(t)) = 0$ ,  $\mathcal{H}_m(t, c(t), m(t), a(t)) = 0$ ,  $\mathcal{H}_a(t, c(t), m(t), a(t)) = \delta \cdot \lambda(t) - \dot{\lambda}(t)$ , and the transversality condition  $\lim_{t \rightarrow +\infty} e^{-\delta \cdot t} \cdot \lambda(t) \cdot a(t) = 0$ . Given that  $u$  is concave in  $(c, m, a)$  and that  $\mathcal{H}$  is the sum of  $u$  and a linear function of  $(c, m, a)$ ,  $\mathcal{H}$  is concave in  $(c, m, a)$  and these conditions are also sufficient.

These three conditions imply that

$$u_c(c(t), m(t), a(t)) = \lambda(t) \cdot (1 + \tau(x(t))) \quad (6)$$

$$u_m(c(t), m(t), a(t)) = \lambda(t) \cdot i(t) \quad (7)$$

$$u_a(c(t), m(t), a(t)) = (\delta - r(t)) \cdot \lambda(t) - \dot{\lambda}(t). \quad (8)$$

Equations (6) and (7) imply that the marginal utilities from consumption and real money balances satisfy

$$u_m(c(t), m(t), a(t)) = \frac{i(t)}{1 + \tau(x(t))} \cdot u_c(c(t), m(t), a(t)). \quad (9)$$

In steady state, this equation yields the LM curve. It represents a demand for money. The demand for real money is declining with  $i(t)$  because  $i(t)$  is the implicit price of holding money paying zero nominal interest instead of bonds paying a nominal interest rate  $i(t)$ .

Equations (6) and (8) imply that the marginal utilities from consumption and real wealth satisfy

$$(1 + \tau(x(t))) \cdot \frac{u_a(c(t), m(t), a(t))}{u_c(c(t), m(t), a(t))} + (r(t) - \delta) = -\frac{\dot{\lambda}(t)}{\lambda(t)}, \quad (10)$$

where  $\dot{\lambda}(t)/\lambda(t)$  can be expressed as a function of  $c(t)$ ,  $m(t)$ ,  $a(t)$ ,  $x(t)$ , and their derivatives using (6). This is the consumption Euler equation. In steady state, this equation yields the IS curve. It represents a demand for saving in part from intertemporal consumption-smoothing considerations and in part from the utility provided by wealth.<sup>9</sup>

## 2.4 Equilibrium with Fixed Inflation

We now define and characterize the equilibrium with fixed inflation.

**DEFINITION 2.** *An equilibrium with fixed inflation  $\pi$  consists of paths for labor market tightness, consumption, real money balances, money supply, real wealth, nominal interest rate, and price level,  $[x(t), c(t), m(t), M(t), a(t), i(t), p(t)]_{t=0}^{+\infty}$ , such that the following conditions hold: (1)  $[c(t), m(t), a(t)]_{t=0}^{+\infty}$  solve the representative household's problem; (2) monetary policy determines*

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<sup>9</sup>If there are no matching costs ( $\rho = 0$  and hence  $\tau(x) = 0$ ) and if the utility only depends on consumption ( $u_a = u_m = 0$ ), this Euler equation reduces to the standard continuous-time consumption Euler equation:  $(r(t) - \delta) \cdot \varepsilon = \dot{c}(t)/c(t)$  where  $\varepsilon \equiv -u'(c)/(c \cdot u''(c))$  is the intertemporal elasticity of substitution.

$[M(t)]_{t=0}^{+\infty}$ ; (3) the money market clears; (4) the bond market clears; (5) actual tightness on the market for services equals the tightness taken as given by consumers for their optimization problem; and (6)  $[p(t)]_{t=0}^{+\infty}$  is a continuous function of time  $t$  satisfying the differential equation  $\dot{p}(t) = \pi \cdot p(t)$  with initial condition  $p(0) = 1$ .

Condition (1) says that consumers choose consumption, money balances, and wealth to maximize utility taking as given prices and labor market tightness. This condition in fact gives three equations but only two independent ones, the third one being redundant with the market-clearing conditions. Condition (2) says that the government determines the amount of money in circulation. Condition (3) says that consumers' money balances equal the amount of money circulated by the government. Condition (4) says that the amount of bonds outstanding for consumers equal the government's demand for bonds, which is equal to the government's supply of money. Condition (5) says that the trading probabilities taken into account by consumers for their optimization problem are realized in equilibrium (the trading probabilities depend only on tightness).<sup>10</sup>

At that point, seven variables are determined by six independent equations so the equilibrium is indeterminate. As explained in [Michaillat and Saez \[2013\]](#), the indeterminacy arises from the presence of matching frictions on the market for services. As a consequence, we need a criterion to select a unique equilibrium. This indeterminacy implies that a fixed inflation is a possible equilibrium selection mechanism. Hence, we impose for now that the price process is exogenous and grows at rate  $\pi$ . (The initial condition  $p(0) = 1$  is a normalization.) The price process never jumps and it grows at constant inflation rate  $\pi$ . If  $\pi = 0$ , the price is constant over time. The price process responds neither to equilibrium variables nor to monetary policy. This criterion may be appropriate to describe the short run because inflation responds only sluggishly to changes in macroeconomic variables, consistent with the empirical evidence for the US since the mid-1980s.

**PROPOSITION 1.** *An equilibrium with fixed inflation  $\pi$  consists of paths of labor market tightness, consumption, real money balances, money supply, real wealth, nominal interest rate, and price level,  $[x(t), c(t), m(t), M(t), a(t), i(t), p(t)]_{t=0}^{+\infty}$ , that satisfy the following seven conditions: (1) equation (9) holds; (2) equation (10) holds; (3)  $[M(t)]_{t=0}^{+\infty}$  is determined by monetary policy; (4)  $m(t) = M(t)/p(t)$ ; (5)  $a(t) = 0$ ; (6) equation (3) holds; and (7)  $\dot{p}(t) = \pi \cdot p(t)$  with  $p(0) = 1$ .*

<sup>10</sup>[Michaillat and Saez \[2013\]](#) provide more details on the equilibrium concept.

The proposition offers a simple characterization of the equilibrium. The conditions listed in the proposition follow almost immediately from those in the definition. In particular, the condition that the bond market clears yields  $a(t) = 0$ , and the condition that actual tightness equals posted tightness yields (3).

### 3 Steady-State Equilibrium

In this section we represent the steady-state equilibrium with an IS curve and a LM curve depicted in a (consumption, interest rate) plane, and an AD curve and an AS curve depicted in a (consumption, labor market tightness) plane. This graphical representation is useful to analyze the comparative static effects of shocks and policies in Section 4. We also study the equilibrium dynamics and show that the equilibrium jumps from a steady state to another after an unexpected shock. This result implies that the comparative statics of Section 4 completely describe the response of the equilibrium to unexpected shocks.

To obtain closed-form expressions for the curves, we assume that the utility function is separable in consumption, real money, and real wealth:

$$u(c, m, a) = \frac{\varepsilon}{\varepsilon - 1} \cdot \left( c^{\frac{\varepsilon-1}{\varepsilon}} - 1 \right) + \phi(m) + \omega(a). \quad (11)$$

The curvature of utility over consumption is measured by  $\varepsilon \geq 1$ . The function  $\phi$  is strictly concave and strictly increasing on  $[0, m^*]$  and constant on  $[m^*, +\infty)$ . The quantity  $m^* \in (0, \infty)$  is a bliss point in real money balances. As we shall see, a bliss point with zero marginal utility of money above is necessary to obtain liquidity traps. The function  $\omega$  is concave and strictly increasing on  $(-\infty, +\infty)$ . As wealth is zero in aggregate, the key parameter is the marginal utility of wealth at the origin,  $\omega'(0)$ . We assume that  $\omega'(0) \in (0, +\infty)$ . As we shall see, a positive marginal utility of wealth is necessary to obtain a nondegenerate IS curve. The utility functions  $\phi$  and  $\omega$  are depicted in Figure 3.

#### 3.1 Definition of the IS, LM, AD, and AS Curves

We define the IS, LM, AD, and AS curves that we use to represent the steady-state equilibrium.

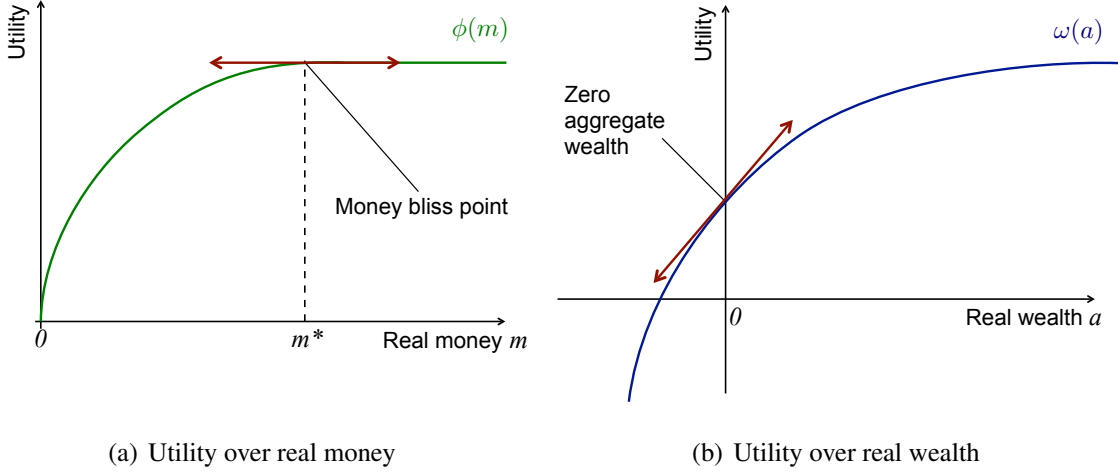


Figure 3: The Utility Functions over Money and Wealth

**DEFINITION 3.** The LM curve expresses consumption as a function of nominal interest rate, labor market tightness, and real money balances:

$$c^{LM}(i, x, m) = \left[ \frac{i}{(1 + \tau(x)) \cdot \phi'(m)} \right]^\varepsilon$$

for all  $i \in [0, +\infty)$ , all  $x \in [0, x^m]$ , and all  $m \in [0, m^*]$ . When real money balances are above the money bliss point ( $m \geq m^*$ ), the LM curve determines a unique nominal interest rate:  $i^{LM}(x, m) = 0$  for all  $x \in [0, x^m]$  and all  $m \in [m^*, +\infty)$ . In this situation, the economy is in a liquidity trap.

The LM curve is the collection of quadruples  $(c, i, x, m)$  that solves (9) so it holds at any point in time. The LM curve is defined separately for  $m$  below and above the money bliss point because when  $m$  is above the bliss point,  $\phi'(m) = 0$  so (9) imposes  $i = 0$ .

**DEFINITION 4.** The IS curve expresses consumption as a function of nominal interest rate, labor market tightness, and inflation:

$$c^{IS}(i, x, \pi) = \left[ \frac{\delta + \pi - i}{(1 + \tau(x)) \cdot \omega'(0)} \right]^\varepsilon$$

for all  $i \in [0, \delta + \pi]$ , all  $x \in [0, x^m]$ , and all  $\pi \in [-\delta, +\infty)$ . If marginal utility of wealth is zero ( $\omega'(0) = 0$ ), the IS curve determines a unique interest rate:  $i^{IS}(x, \pi) = \pi + \delta$  for all  $x \in [0, x^m]$  and all  $\pi \in [-\delta, +\infty)$ .

The IS curve is the collection of quadruples  $(c, i, x, \pi)$  that solves (10) when  $\dot{\lambda}(t) = 0$  so it holds only in steady state. The IS curve is expressed as a function of inflation and nominal interest rate but it really only depends on the real interest rate,  $r = i - \pi$ . The IS curve is defined separately when the marginal utility of wealth is positive or zero. When the marginal utility of wealth is zero, the IS curve imposes the usual condition that  $r = \delta$ .

The properties of the IS and LM curves are illustrated in Figure 4.<sup>11</sup> First, Figure 4(a) shows that the LM curve is upward sloping in a  $(c, i)$  plane. This property follows the standard logic. Demand for real money is decreasing with  $i$  as a higher  $i$  makes money less attractive relative to bonds. Demand for real money is increasing in  $c$  as a higher  $c$  reduces marginal utility of consumption, which makes real money more attractive relative to consumption. Given that real money balance is constant, an increase in  $i$  requires an increase in  $c$  to maintain equilibrium. Through the same logic, an increase in real money balances shifts the LM curve out, as illustrated in Figure 4(c).

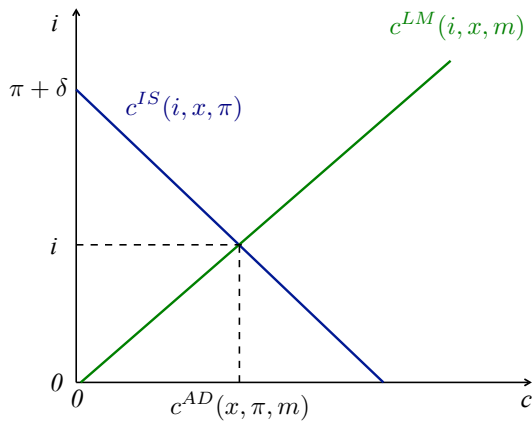
Second, Figure 4(a) shows that the IS curve is downward sloping in a  $(c, i)$  plane. The intuition is the following. For a given inflation, a higher  $i$  leads to a higher  $r$  and an increase in the marginal value of savings through bonds via the wealth effect  $r \cdot \omega'(0)$  so that holding wealth is more attractive. Since wealth is zero in equilibrium,  $c$  must decline for households to be indifferent between saving and consumption. This logic also implies that an increase in inflation, which reduces  $r$  for a given  $i$ , shifts the IS curve out, as in Figure 4(d). By the same logic, a decrease in the marginal utility of wealth shifts the IS curve out, as in Figure 4(e). An increase in the discount rate has the same effect, as showed in Figure 4(f).

Third, the IS and LM curves shift outward when labor market tightness decreases, as illustrated in Figure 4(b). However, the nominal interest rate defined by the intersection of the IS and LM curves does not depend on tightness. The IS and LM curves shift by commensurate amounts such that the equilibrium interest rate remains the same. The logic is that a lower tightness reduces the effective price of labor services,  $(1 + \tau(x)) \cdot p$ , which makes consumption of labor services more desirable relative to holding bonds or money.

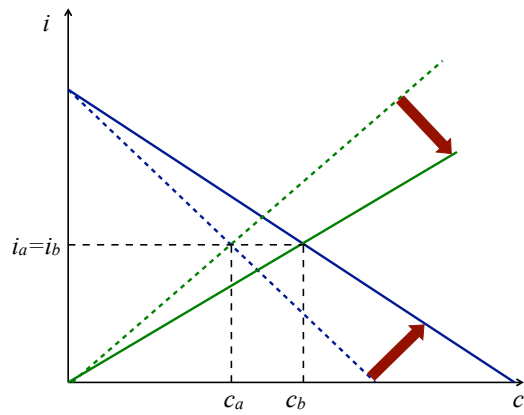
Fourth, the LM curve prevents the nominal interest rate from falling below zero because the marginal utility of money  $\phi'(m)$  is nonnegative. If the nominal interest rate were negative, money

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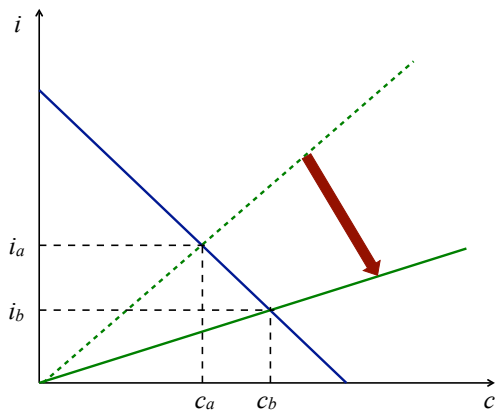
<sup>11</sup>The linear IS and LM curves in Figure 4 correspond to the case with log utility over consumption ( $\varepsilon = 1$ ).



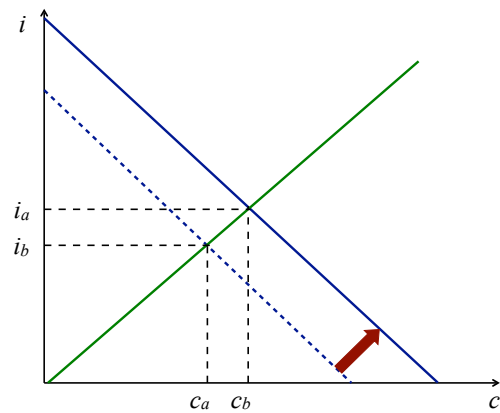
(a) Equilibrium



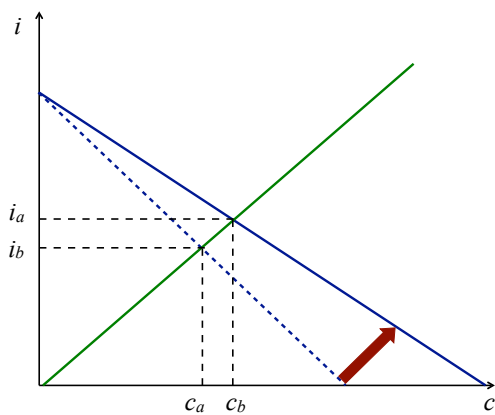
(b) Decrease in labor market tightness



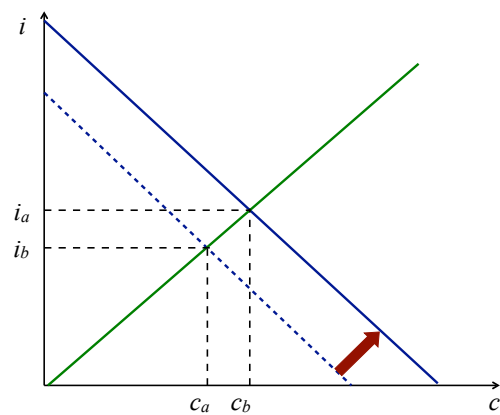
(c) Increase in real money balances



(d) Increase in inflation



(e) Decrease in marginal utility of wealth



(f) Increase in discount rate

Figure 4: IS and LM Curves in  $(c, i)$  Plane



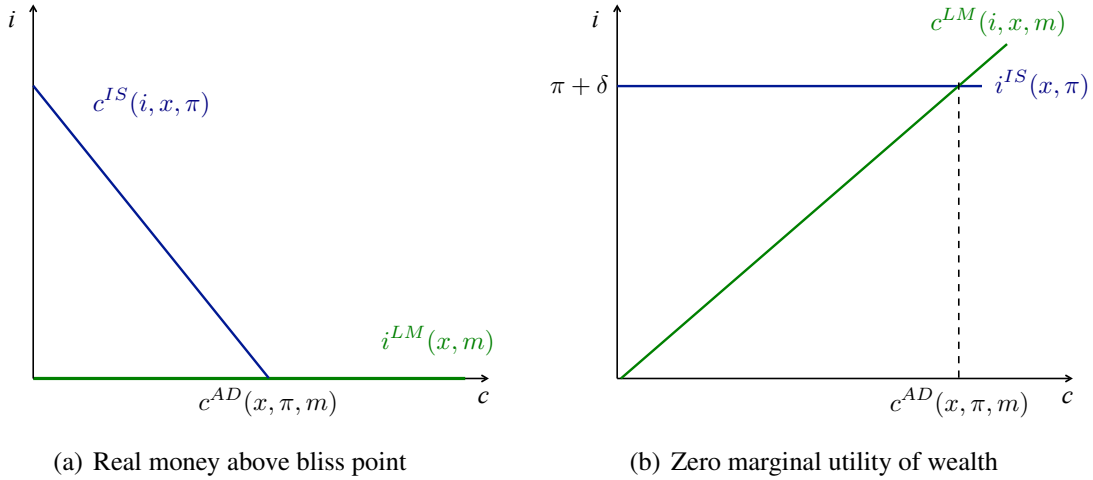


Figure 5: IS and LM Curves in Special Cases

would strictly dominate bonds. When real money is at or above the bliss point  $m^*$ , the LM curve becomes horizontal at  $i = 0$ , as illustrated in Figure 5(a). Real money balances do not affect the LM curve any more. This situation of liquidity trap has important implications to which we will come back.

Fifth, without utility of wealth, the IS curve becomes horizontal at  $i = \delta + \pi$  as depicted in Figure 5(b). The intuition is well known: steady-state consumption is constant so households hold bonds only if the return on bonds,  $r = i - \pi$ , equals the subjective discount rate,  $\delta$ . With utility of wealth,  $r < \delta$  as households also experience utility from wealth holding.

The equilibrium interest rate is given by the intersection of the IS and LM curves. The equality  $c^{IS}(i, x, \pi) = c^{LM}(i, x, m)$  implies that the equilibrium nominal interest rate is

$$i = \frac{\phi'(m)}{\phi'(m) + \omega'(0)} \cdot (\delta + \pi). \quad (12)$$

At that interest rate consumers are indifferent between money and bonds. The equilibrium real interest rate is

$$r = \frac{\phi'(m)}{\phi'(m) + \omega'(0)} \cdot \delta - \frac{\omega'(0)}{\phi'(m) + \omega'(0)} \cdot \pi.$$

Next, we construct the AD curve by plugging (12) into the LM curve:

**DEFINITION 5.** The AD curve expresses consumption as a function of labor market tightness, inflation, and real money balances defined by

$$c^{AD}(x, \pi, m) = \left[ \frac{\delta + \pi}{(1 + \tau(x)) \cdot (\phi'(m) + \omega'(0))} \right]^\varepsilon. \quad (13)$$

for all  $x \in [0, x^m]$ , all  $\pi \in [-\delta, +\infty)$ , and all  $m \in [0, \infty)$ .

The AD curve represents the consumption level obtained at the intersection of the IS and LM curves. The AD curve is downward sloping in a  $(c, x)$  plane, as illustrated on Figure 8(a). The logic for this property is displayed in Figure 4(b): by construction,  $c_a = c^{AD}(x_a, \pi, m)$  and  $c_b = c^{AD}(x_b, \pi, m)$  with  $x_a > x_b$ ; clearly,  $c_a < c_b$ . In fact, all the properties of the AD curve follow from the mechanisms illustrated in Figure 4 and discussed above. For instance, the AD curve shifts out after an increase in the discount rate, an increase in the inflation rate, or a decrease in the marginal utility of wealth, as these changes shift the IS curve out. The AD curve also shifts out after an increase in real money balances as this change shifts the LM curve out.

Last, we define the AS curve:

**DEFINITION 6.** The AS curve is a function of tightness defined for all  $x \in [0, x^m]$  by

$$c^{AS}(x) = (f(x) - \rho \cdot x) \cdot k.$$

The AS curve is the consumption level arising from the matching process on the labor market, plotted in Figure 2. The AS curve is showed in Figure 8(a). An increase in capacity shifts the AS curve out.

### 3.2 Characterization of the Steady-State Equilibrium

The steady state with fixed inflation is characterized as follows:

**PROPOSITION 2.** The steady-state equilibrium with fixed inflation  $\pi$  consists of labor market tightness, consumption, real money balances, level of money supply, growth rate of money supply, and nominal interest rate,  $(x, c, m, M(0), \dot{M}/M, i)$ , such that  $c^{LM}(i, x, m) = c^{IS}(i, x, \pi)$ ,  $c^{AD}(x, \pi, m) = c^{AS}(x)$ ,  $c = c^{AS}(x)$ ,  $M(0)$  is set by monetary policy,  $\dot{M}/M = \pi$ ; and  $m = M(0)/p(0) = M(0)$ .

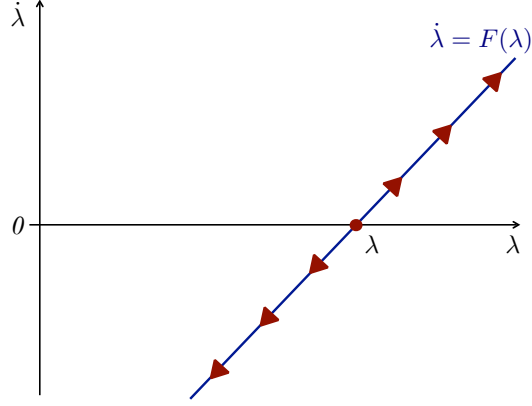


Figure 6: Phase Diagram of the Equilibrium with Fixed Inflation

The steady state consists of 6 variables determined by 6 conditions. In steady state, the price grows at a constant, exogenous inflation rate  $\pi$ . The money supply,  $M(t)$ , must also grow at rate  $\pi$  but monetary policy does not control  $\pi$ . Hence, changing the growth rate of  $M(t)$  is not within the scope of the analysis under fixed inflation. Since the price level is unaffected by monetary policy, monetary policy controls real money balances by controlling the level of money supply.

### 3.3 Dynamics Toward the Steady-State Equilibrium

Here we describe equilibrium dynamics. The dynamical system describing the equilibrium is characterized in Proposition 1. We focus here on one single endogenous variable: the costate variable  $\lambda(t)$ . All the variables can be recovered from  $\lambda(t)$ .

In equilibrium, wealth is zero so the law of motion for the costate variable from equation (8) is  $\omega'(0) = (\delta + \pi - i(t)) \cdot \lambda(t) - \dot{\lambda}(t)$ . Both money supply and price grow at a constant rate  $\pi$  so real money balances is constant:  $m(t) = M(0)/p(0) = m$ . Hence, equation (7) implies that  $i(t) \cdot \lambda(t) = \phi'(m)$  so that the law of motion of the costate variable in equilibrium is:

$$\dot{\lambda}(t) = (\delta + \pi) \cdot \lambda(t) - \omega'(0) - \phi'(m) \equiv F(\lambda(t)).$$

The steady-state value of the costate variable satisfies  $F(\lambda) = 0$  so  $\lambda = (\omega'(0) + \phi'(m))/(\delta + \pi)$ . The nature of the dynamical system is given by the sign of  $F'(\lambda)$ . Since  $F'(\lambda) = \delta + \pi > 0$ , we infer that the system is a source. We represent the phase diagram for the system in Figure 6 in the

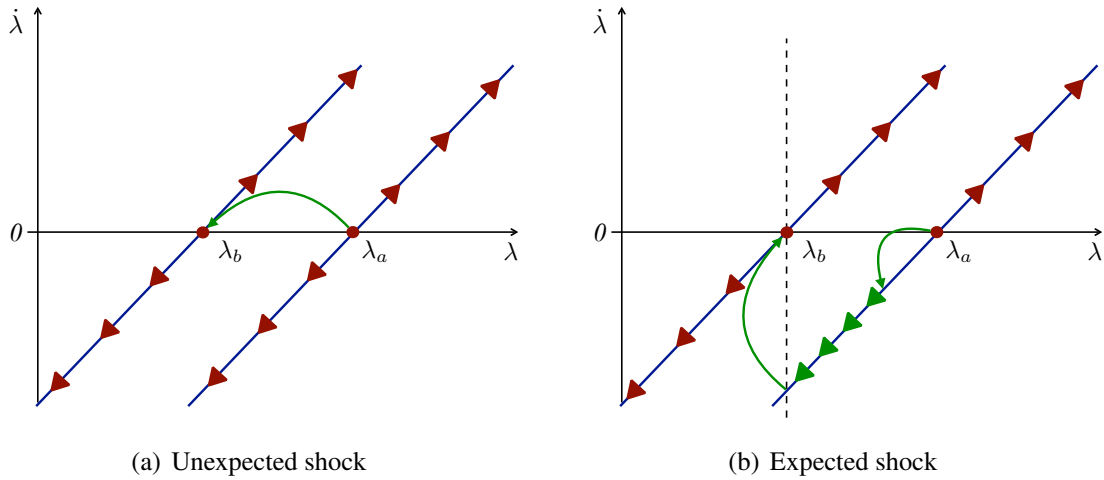


Figure 7: Response of the Equilibrium with Fixed Inflation to Unexpected and Expected Shocks

plan  $(\lambda, \dot{\lambda})$ . As there is no endogenous state variable, a source system jumps from one steady state to the other in response to an unexpected shock.

Accordingly, the transitions between steady states are immediate in response to unexpected shocks. This is illustrated in Figure 7(a) where the equilibrium jumps from  $\lambda_a$  to  $\lambda_b$  at time  $t_0$  when an unexpected shock occur. The values  $\lambda_a$  and  $\lambda_b$  are the steady-state values of  $\lambda$  for the parameters values before and after time  $t_0$ . Hence, steady-state comparative statics analysis is sufficient to capture the full dynamics following unexpected shocks.

Of course, the dynamics are a bit different in response to an expected shock. This is illustrated in Figure 7(b) where an announcement is made at time  $t_0$  that a shock changing the steady-state value of  $\lambda$  from  $\lambda_a$  to  $\lambda_b$  will occur at time  $t_1$ . The key property of the model is that absent new information,  $\lambda$  is a continuous variable of time so  $\lambda$  can only jump at time  $t_0$  but not at time  $t_1$ . Assume that  $\lambda_a > \lambda_b$ . Then  $\lambda$  jumps down at time  $t_0$ . The amplitude of the jump is such that at time  $t_1$ ,  $\lambda = \lambda_b$ . Between  $t_0$  and  $t_1$ ,  $\lambda$  falls because  $\dot{\lambda} = F(\lambda) < 0$ . We conclude that at time  $t_0$ ,  $\lambda$  jumps down part of the way toward its steady-state value, and that it keeps on falling slowly toward its new steady-state value until the expected shock occurs. The implication is that even with expected shocks, comparative statics give the correct sign of both the adjustment occurring when the announcement of the shock is made and the adjustment occurring in the long run.

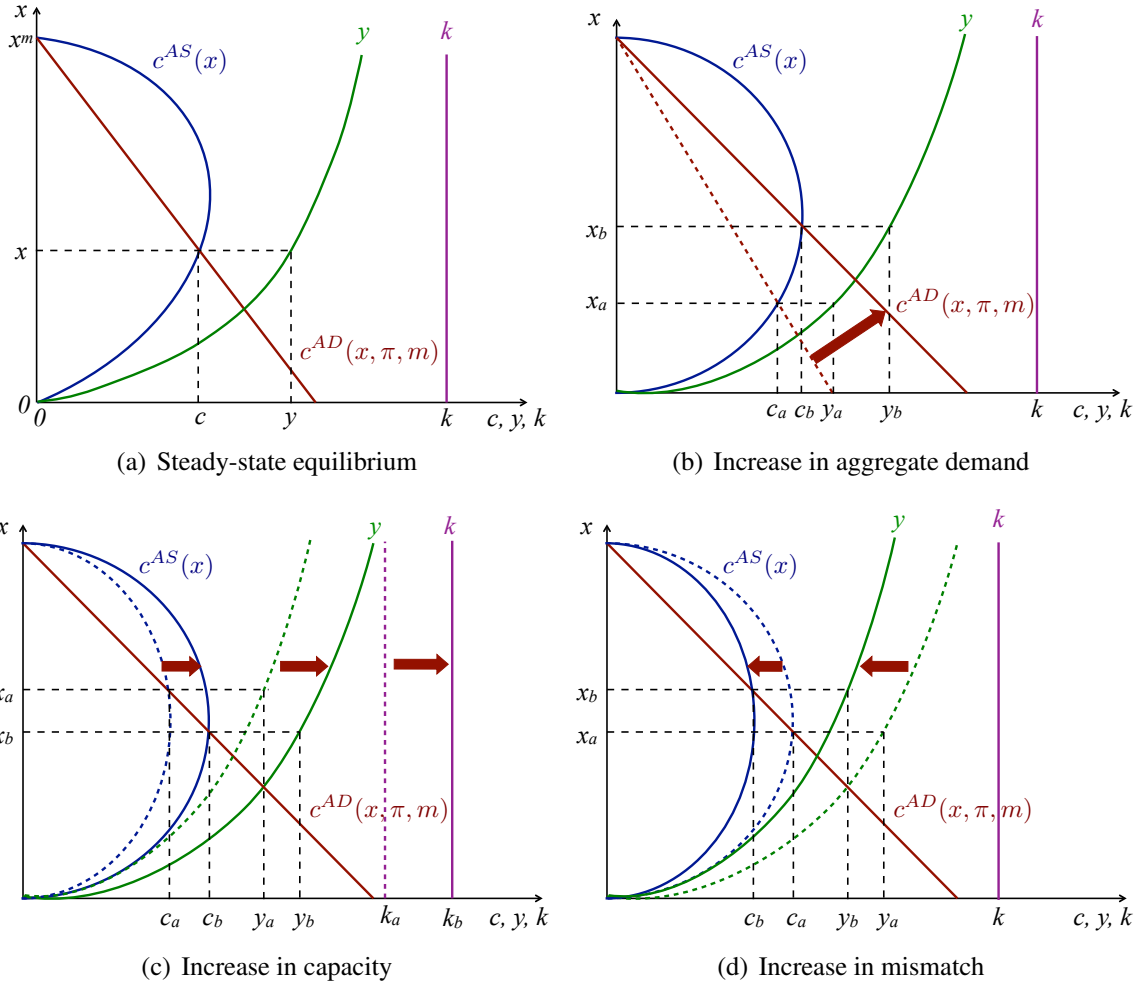


Figure 8: Steady-State Equilibrium and Aggregate Demand and Supply Shocks in a  $(c, x)$  Plane

## 4 Aggregate Shocks and Policies

In this section we use comparative statics to describe how the economy responds to aggregate demand and supply shocks, and to a number of policies. We established in the previous section that comparative statics suffice to describe both steady-state responses and transition dynamics between steady states because the equilibrium jumps from one steady state to another in response to an unexpected shock and jumps part of the way at the announcement of a future shock. Table 1 summarizes these comparative statics and Figure 8 illustrates them.

## 4.1 Aggregate Demand Shock

We first analyze aggregate demand shocks. We parameterize an increase in aggregate demand by an increase in the subjective discount rate or a decrease in the marginal utility of wealth. A positive aggregate demand shock shifts the IS curve out, as depicted in Figures 4(e) and 4(f), and it therefore raises interest rates. Note that interest rates are independent of tightness, as illustrated in Figure 4(b), so the general-equilibrium response of interest rates to the aggregate demand shock is the same as the partial-equilibrium response depicted in Figures 4(e) and 4(f).

Since the IS curve shifts out, the AD curve also shifts out, as depicted in Figure 8(b). Hence, the increase in aggregate demand leads to increases in labor market tightness and output. Since tightness is higher, the unemployment rate falls. Consumption increases if the labor market is slack and decreases if the labor market is tight. If the labor market is efficient, the aggregate demand shock has no first-order effect on consumption.

## 4.2 Aggregate Supply Shock

Next, we analyze aggregate supply shocks. We consider two types of shocks: a shock to the production capacity of households, and a labor market mismatch shock.

An increase in capacity is illustrated in Figure 8(c). This increase shifts out the AS and output curves, while the AD curve is unchanged. Hence, consumption increases, tightness decreases, and the unemployment rate increases. We can show that output increases. Interest rates do not change.

Following [Michaillat and Saez \[2013\]](#), we parameterize an increase in labor market mismatch as a decrease in matching efficacy along with a commensurate decrease in matching costs:  $h(k, v)$  becomes  $\sigma \cdot h(k, v)$  and  $\rho$  becomes  $\sigma \cdot \rho$  with  $\sigma < 1$ . Importantly, the efficient tightness and the function  $\tau$  are not affected by mismatch. Figure 8(d) illustrates an increase in mismatch. The AD curve does not change, but the AS and output curves shift inward. As a result, consumption decreases, tightness increases, and output decreases. We can show that the unemployment rate increases. Interest rates do not change.

The comparative statics are the same in a liquidity trap and away from it because the AD and AS curves retain the same properties in a trap. This property distinguishes our model from standard New Keynesian models, in which aggregate supply shocks have paradoxical effects in

Table 1: Comparative Statics: Aggregate Shocks and Policies with Fixed Inflation

Increase in:	Effect on:				
	Tightness $x$	Consumption $c$	Output $y$	Unemployment rate $1 - f(x)$	Interest rates $i, r$
Aggregate demand	+	+ / 0 / -	+	-	+
Capacity	-	+	+	+	0
Labor market mismatch	+	-	-	+	0
Money supply					
- out of liquidity trap	+	+ / 0 / -	+	-	-
- in liquidity trap	0	0	0	0	0
Helicopter money	+	+ / 0 / -	+	-	?
Wealth tax	+	+ / 0 / -	+	-	+
Government purchases	+	+ / 0 / -	+	-	0

*Notes:* An increase in aggregate demand results from an increase in the subjective discount rate or a decrease in the marginal utility of wealth. In the column on consumption, “+ / 0 / -” indicates that consumption increases when the labor market is slack, does not change when the labor market is efficient, and decreases when the labor market is tight. In the column on interest rates, “?” indicates that the response of the interest rate can be positive or negative depending on the utility functions  $\omega$  and  $\phi$ . Given that inflation is fixed, both nominal and real interest rate move in the same way. In the row on government purchase, consumption means total consumption—private plus government consumption. Private consumption always falls when government consumption increases.

liquidity traps. In these models, a negative aggregate supply shock is contractionary in normal times but expansionary in a liquidity trap [Eggertsson, 2010, 2011].<sup>12</sup>

### 4.3 Monetary Policy In and Out of a Liquidity Trap

The only lever that monetary policy chooses is the level of money supply,  $M(0)$ . A change in  $M(0)$  leads to a change in real money balances. Monetary policy cannot change the growth rate of  $M(t)$ , which must satisfy the steady-state requirement that  $\dot{M}(t)/M(t) = \pi$ . We study the comparative static effects of an increase in real money balances.

Away from a liquidity trap, an increase in real money balances shifts out the LM curve, as showed in Figure 4(c), and hence shifts out the AD curve, as showed in Figure 8(b). Higher money supply therefore leads to lower interest rates, higher tightness, lower unemployment rate,

<sup>12</sup>Whether these paradoxical effects appear in the data is debated. For instance, using a variety of empirical tests, Wieland [2013] rejects the prediction that negative aggregate supply shocks are expansionary in a liquidity trap.

and higher output. The effect on consumption depends on the state of the labor market.

As long as the nominal interest rate is positive, monetary policy can control the AD curve and thus fully accommodate shocks. Suppose that the economy starts with tightness at its efficient level, which maximizes consumption, and suppose that the government wants to use monetary policy to keep tightness at this level. A negative aggregate demand shock lowers tightness and needs to be accommodated by an increase in real money balances, and conversely for a positive aggregate demand shock. In that case, monetary policy absorbs the aggregate demand shock preventing the inefficient economic fluctuation. A positive aggregate supply shock, either an increase in capacity or a decrease in mismatch, decreases tightness and hence needs to be accommodated by an increase in real money balances, and conversely for a negative aggregate supply shock. In that case, monetary policy exacerbates the aggregate supply shock and helps the efficient economic fluctuation. Hence, in our model, monetary policy should be guided by tightness rather than output if the government aims at maximize welfare.

In a liquidity trap, monetary policy cannot accommodate shocks anymore because real money balances do not influence the LM curve and thus cannot control the AD curve. This is the situation illustrated in Figure 5(b). Monetary policy becomes ineffective. Of course, monetary policy could still be effective if it could change inflation. We know that increasing inflation stimulates the IS curve and thus the AD curve, even in a liquidity trap, as depicted in Figure 4(d). But monetary policy has no effect on inflation by assumption in our model, consistent with the empirical evidence presented in [Christiano, Eichenbaum and Evans \[1999\]](#). We consider the case with endogenous inflation in Section 5.

#### 4.4 Helicopter Drop of Money

Conventional monetary policy is not effective in a liquidity trap. We now present several unconventional policies that remain effective in this situation. What these policies have in common is that they stimulate the IS curve.

We start by analyzing a helicopter drop of money, first discussed by [Friedman \[1969\]](#). Money now comes from two sources: A quantity  $M^b(t) = -B(t)$  of money is issued by buying bonds through open market operations as before and a quantity  $M^h(t)$  of money is printed and given



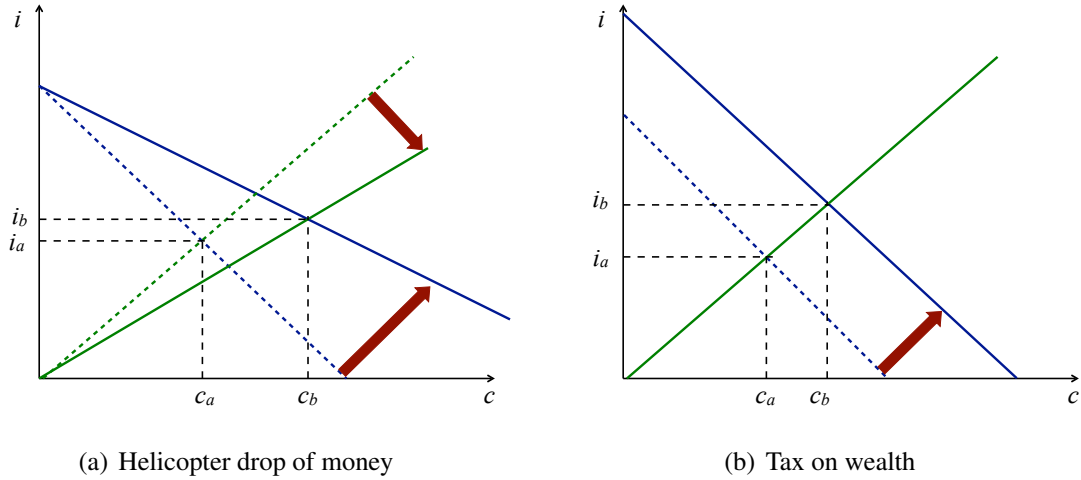


Figure 9: Unconventional Policies in the IS-LM Diagram

directly to households through a helicopter drop. Total money supply is  $M(t) = M^b(t) + M^h(t)$ . The corresponding real money balances are  $m^b(t) = M^b(t)/p(t)$  and  $m^h(t) = M^h(t)/p(t)$  and  $m(t) = M(t)/p(t)$ . Real wealth is no longer zero because helicopter money contributes to real wealth. Indeed, real wealth is  $a(t) = (B(t) + M^b(t) + M^h(t))/p(t) = m^h(t)$ .

With helicopter money, our analysis carries over by adjusting the marginal utility of wealth from  $\omega'(0)$  to  $\omega'(m^h)$ . The IS curve now depends on helicopter money:

$$c^{IS}(i, x, \pi, m^h) = \left[ \frac{\delta + \pi - i}{(1 + \tau(x)) \cdot \omega'(m^h)} \right]^\varepsilon.$$

Since the function  $\omega$  is concave, an increase in helicopter money shifts the IS curve outward in a  $(c, i)$  plane, as showed in Figure 9(a).<sup>13</sup> It also shifts the LM curve outward at it raises real money balances. The AD curve depends on both total and helicopter money:

$$c^{AD}(x, \pi, m, m^h) = \left[ \frac{\delta + \pi}{(1 + \tau(x)) \cdot (\phi'(m) + \omega'(m^h))} \right]^\varepsilon.$$

An helicopter drop of money shifts the AD curve out in a  $(c, x)$  plane, as shown in Figure 8(b).

Although open-market money cannot stimulate the AD curve in a liquidity trap, helicopter money stimulates the AD curve even in a liquidity trap. Helicopter money is effective in a liquidity

<sup>13</sup>The efficacy of a helicopter drop of money requires concave utility of wealth. With linear utility of wealth,  $\omega'(m^h)$  is constant and helicopter money does not shift the IS curve. In that case, a helicopter drop of money is ineffective.

trap because it stimulates both the LM and the IS curve, and the IS channel is immune to the liquidity trap. In contrast, open-market money only stimulates the LM curve, and the LM channel does not operate in a liquidity trap.

One drawback of a helicopter drop of money is that it is harder to reverse than open market operations. Effectively, reversing a helicopter drop of money requires to take away money from households with no compensation—taxing money held by households and destroying it.

## 4.5 Tax on Wealth

Another way to stimulate aggregate demand in a liquidity trap is to tax wealth at rate  $\tau^a(t)$ .<sup>14</sup> The wealth tax applies to the entire wealth, bond holdings plus money balances. The tax raises no revenue as the aggregate wealth is zero. But the tax changes the law of motion of the consumer's wealth and the consumption Euler equation. The law of motion becomes

$$\dot{a}(t) = f(x(t)) \cdot k - (1 + \tau(x(t))) \cdot c(t) - i(t) \cdot m(t) + (r(t) - \tau^a(t)) \cdot a(t) + s(t).$$

Therefore, the Euler equation becomes

$$(1 + \tau(x(t))) \cdot \frac{u_a(c(t), m(t), a(t))}{u_c(c(t), m(t), a(t))} + (r(t) - \tau^a(t) - \delta) = -\frac{\dot{\lambda}(t)}{\lambda(t)},$$

and the IS curve admits a new expression:

$$c^{IS}(i, x, \pi, \tau^a) = \left[ \frac{\delta + \tau^a + \pi - i}{(1 + \tau(x)) \cdot \omega'(0)} \right]^\varepsilon.$$

An increase in the wealth tax shifts the IS curve outward in a  $(c, i)$  plane, as showed in Figure 9(b).

The LM curve remains the same. The AD curve is now a function of the wealth tax:

$$c^{AD}(x, \pi, m, \tau^a) = \left[ \frac{\delta + \tau^a + \pi}{(1 + \tau(x)) \cdot (\phi'(m) + \omega'(0))} \right]^\varepsilon.$$

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<sup>14</sup>Correia et al. [2013] obtain a related result in a New Keynesian model. They show that replacing a labor income tax by a value-added tax can lift the economy of a liquidity trap. Since a value-added tax is equivalent to a tax on labor income and a one-time tax on existing wealth in a model without investment, suppressing a labor income tax and implementing a value-added tax instead amounts to implementing a one-time tax on wealth. This one-time tax on wealth is sufficient to stimulate the economy in a temporary liquidity trap. In our model with a permanent liquidity trap, a permanent wealth tax is needed.

An increase in the wealth tax shifts the AD curve outward in a  $(c, i)$  plane, as in Figure 8(b). Since the wealth tax acts on the IS curve and not the LM curve, the wealth tax is effective in a liquidity trap. The intuition for the effectiveness of the tax is the following: Taxing wealth makes wealth and future consumption more costly and hence less desirable, hereby stimulating current consumption.

## 4.6 Government Purchases With and Without Debt

The last policy that we consider is the purchase of  $g(t)$  units of services by the government. We begin by assuming that government spending is financed by a lump-sum tax  $\tau(t)$ . The government's budget constraint imposes that  $p(t) \cdot g(t) = \tau(t)$ . We assume that government purchases enters separately into households' utility function such that  $g(t)$  does not affect the consumption and saving choices of the households. Accordingly, the IS and LM curves remain the same, and  $g$  only enters in the AD curve:

$$c^{AD}(x, \pi, g) = \left[ \frac{\delta + \pi - i}{(1 + \tau(x)) \cdot \omega'(0)} \right]^\varepsilon + \frac{g}{1 + \tau(x)}. \quad (14)$$

We abuse notation and keep the labels  $c^{AD}$  and  $c^{AS}$  for the AD and AS curves, even though  $c$  is private consumption whereas the AD and AS curves measure total consumption—private plus government consumption. An increase in government purchases shifts the AD curve outward, as showed in Figure 8(b). Of course, government purchases remain effective in a liquidity trap.

An increase in government purchases leads to higher output but not always to higher total consumption. Following the usual logic, total consumption increases when the labor market is slack, decreases when the labor market is tight, and does not change when the labor market is efficient. Government consumption always crowds out private consumption. Crowding out arises because an increase in government purchases shifts the AD curve outward and raises labor market tightness; therefore, it is more expensive for consumers to purchase goods: the effective price  $(1 + \tau(x)) \cdot p$  increases. Consumers reduce consumption because of the increase in effective price. Crowding out is partial when the labor market is slack, one-for-one when the labor market is efficient, and more than one-for-one when the labor market is tight.

There is a simple interaction between fiscal and monetary policy. As long as monetary policy is able to maintain the labor market at efficiency, fiscal policy should follow public-finance con-

siderations: the economy is always efficient so there is no reason to use government spending for stabilization purposes. If monetary policy cannot maintain the economy at efficiency, fiscal policy can play a role to stabilize the economy, in addition to public-finance considerations. This would happen for instance when the economy is in a liquidity trap and monetary policy cannot stimulate aggregate demand. This result does not depend on the magnitude of the multiplier, only on its sign.

We could consider deficit spending whereby the government finances spending through an increase in government debt. In that case, if households are Ricardian in the sense that they do not count the government bonds they own as net wealth, then deficit spending is exactly equivalent to budget-balanced spending [Barro, 1974]. However, if households are not Ricardian and count government bonds as net wealth, then debt-financed government purchases have a larger impact on the economy than budget-balanced purchases. In that case, debt-financed government purchases shifts the AD curve through two channels: by the mechanical force described in (14), and by increasing aggregate wealth and thus stimulating the IS curve.

## 5 Equilibrium With a Phillips Curve

In the previous sections inflation was fixed. In this section we show how to combine directed search as in Moen [1997] with costly price adjustment as in Rotemberg [1982] to obtain a forward-looking Phillips curve. We then analyze the properties of the equilibrium with this Phillips curve. This framework could be a platform to develop new models of price dynamics, building on advances from the New Keynesian literature. It is also helpful to think about fully flexible prices.

To simplify the exposition, we specialize the utility (11) by setting  $\varepsilon = 1$  and  $\phi(m) = \ln(m)$ :

$$u(c, m, a) = \ln(c) + \ln(m) + \omega(a). \quad (15)$$

By using log utility over money, we set the money bliss point to infinity and ensure that the economy never enters a liquidity trap. Studying the properties of the equilibrium with Phillips curve in a liquidity trap is outside of the scope of this paper.<sup>15</sup> Finally, we assume that the money supply remains constant over time:  $M(t) = M$  for all  $t$ . Unlike in a New Keynesian where monetary policy

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<sup>15</sup>Such analysis raises difficult issues similar to the analysis of liquidity traps in New Keynesian models. See for instance Werning [2012] and Cochrane [2013].

follows an interest-rate rule, monetary policy is completely passive here.

## 5.1 Intertemporal Profit Maximization

We begin by solving the representative seller's problem when buyers direct their search towards the most attractive markets but adjusting prices is costly to sellers. Buyers choose the market where they buy labor services based on the price,  $p$ , and tightness,  $x$ , in that market. What matters for buyers is the effective price they pay,  $p \cdot (1 + \tau(x))$ . When a seller sets a price, she takes into account the effect of her price on the tightness she faces, which in turn determines how much labor services she sells. The solution of the seller's problem yields a Phillips curve.

As in [Moen \[1997\]](#), we assume that sellers post their price  $p(t)$  and that buyers arbitrage across sellers until they are indifferent across sellers. This means that search for labor services is not random but directed. For a given price  $p(t)$ , the tightness that a seller faces is given by

$$(1 + \tau(x(t))) \cdot p(t) = e(t) \tag{16}$$

where  $e(t)$  is the effective price in the economy. The effective price is taken as given by buyers and sellers. This condition simply says that buyers are indifferent between all sellers. Sellers can choose high prices and get few buyers or low prices and get many buyers. If a seller chooses a price  $p(t)$ , her probability to sell therefore is

$$F(p(t)) \equiv f(x(t)) = f\left(\tau^{-1}\left(\frac{e(t)}{p(t)} - 1\right)\right).$$

A useful result is that the derivative of  $F$  is  $F'(p) = -(1 - \eta) \cdot f(x) / (\eta \cdot \tau(x) \cdot p)$ . Absent any price-adjustment cost, sellers choose  $p(t)$  to maximize  $p(t)f(x(t))$  subject to (16); that is, they choose  $x(t)$  to maximize  $f(x(t)) / (1 + \tau(x(t))) = f(x(t)) - \rho \cdot x(t)$ ; thus, they set  $f'(x(t)) = \rho$  and  $x(t) = x^*$  is efficient. This is the central efficiency result of [Moen \[1997\]](#).

We add price-adjustment costs to the directed search setting. We follow the price-adjustment specification of [Rotemberg \[1982\]](#). Sellers incur a cost  $(\dot{p}(t)/p(t))^2 \cdot \kappa(t)/2$  when they change their prices, where  $\kappa(t) = \kappa \cdot p(t) \cdot y(t)$ . This cost is quadratic in the growth rate of prices,  $\dot{p}(t)/p(t)$ , and is scaled by the size of the economy  $p(t) \cdot y(t)$  as well as a cost parameter  $\kappa$ . If

$\kappa = 0$ , prices adjust at no cost.

The representative seller takes  $e(t)$ ,  $\kappa(t)$ , and  $i(t)$  as given and chooses a price level  $p(t)$ , a price growth rate  $\pi(t)$ , and a tightness  $x(t)$  to maximize the discounted sum of nominal profits

$$\int_0^{+\infty} e^{-I(t)} \cdot \left( p(t) \cdot f(x(t)) \cdot k - \frac{\kappa(t)}{2} \cdot \pi(t)^2 \right) dt, \quad (17)$$

where  $I(t) = \int_0^t i(s) ds$  is the seller's discount rate. The price level follows the law of motion

$$\dot{p}(t) = \pi(t) \cdot p(t). \quad (18)$$

To solve the seller's problem, we express  $x(t)$  as a function of  $p(t)$  using  $F(p(t)) = f(x(t))$  and set up the current-value Hamiltonian:

$$\mathcal{H}(t, \pi(t), p(t)) = p(t) \cdot F(p(t)) \cdot k - \frac{\kappa(t)}{2} \cdot \pi(t)^2 + \mu(t) \cdot (\pi(t) \cdot p(t))$$

with control variable  $\pi(t)$ , state variable  $p(t)$ , and current-value costate variable  $\mu(t)$ . The necessary conditions for an interior solution to this maximization problem are

$$\begin{aligned} \mathcal{H}_\pi(t, \pi(t), p(t)) &= 0 \\ \mathcal{H}_p(t, \pi(t), p(t)) &= i(t) \cdot \mu(t) - \dot{\mu}(t) \end{aligned}$$

together with the appropriate transversality condition. The first condition implies that

$$\frac{\kappa(t)}{p(t)} \cdot \pi(t) = \mu(t). \quad (19)$$

Recall that  $r(t) = i(t) - \pi(t)$  denotes the real interest rate. The second condition implies that

$$\dot{\mu}(t) = r(t) \cdot \mu(t) + \left( \frac{1-\eta}{\eta} \cdot \frac{1}{\tau(x(t))} - 1 \right) \cdot f(x(t)) \cdot k. \quad (20)$$

In a symmetric equilibrium,  $\kappa(t)/p(t) = \kappa \cdot y(t) = \kappa \cdot f(x(t)) \cdot k$  so the first optimality condition

simplifies to

$$\kappa \cdot f(x(t)) \cdot k \cdot \pi(t) = \mu(t). \quad (21)$$

As the elasticity of  $f(x)$  is  $1 - \eta$ , log-differentiating (21) with respect to time yields

$$\frac{\dot{\mu}(t)}{\mu(t)} = (1 - \eta) \cdot \frac{\dot{x}(t)}{x(t)} + \frac{\dot{\pi}(t)}{\pi(t)}$$

Combining this equation with (20) yields

$$\dot{\pi}(t) = \left[ r(t) - (1 - \eta) \cdot \frac{\dot{x}(t)}{x(t)} \right] \cdot \pi(t) + \frac{1}{\kappa} \cdot \left( \frac{1 - \eta}{\eta} \cdot \frac{1}{\tau(x(t))} - 1 \right). \quad (22)$$

This differential equation describes sellers' optimal pricing; it underlies the Phillips curve.

## 5.2 The Dynamical System Describing the Equilibrium

Here we derive the dynamical system describing the equilibrium. The system is composed of three equations: a consumption Euler equation, a Phillips curve, and a law of motion for the marginal utility of money.

The consumption Euler equation describes the solution to the household's problem. It is given by (10), but it is convenient to rewrite it as a differential equation in  $x$ . Using (15), (10) becomes

$$\omega'(0) \cdot f(x(t)) \cdot k + r(t) - \delta = -\frac{\dot{\lambda}(t)}{\lambda(t)}.$$

Using the utility function (15) and the matching equation (2), the first-order condition (6) becomes  $f(x(t)) \cdot k = 1/\lambda(t)$ . Log-differentiating this equation with respect to time, we obtain

$$-\frac{\dot{\lambda}(t)}{\lambda(t)} = (1 - \eta) \cdot \frac{\dot{x}(t)}{x(t)}$$

which yields the Euler equation

$$(1 - \eta) \cdot \frac{\dot{x}(t)}{x(t)} = r(t) - (\delta - \omega'(0) \cdot f(x(t)) \cdot k). \quad (23)$$

We now turn to the Phillips curve. To ease notation, we denote the tightness gap as

$$G(x(t)) = 1 - \frac{1 - \eta}{\eta} \cdot \frac{1}{\tau(x(t))}.$$

The function  $G$  increases in  $x$ , is positive if  $x > x^*$ , negative if  $x < x^*$ , and zero if  $x = x^*$ . It measures how far the labor market is from efficiency. Combining (22) with the Euler equation (23) to eliminate  $\dot{x}(t)$  yields the Phillips curve

$$\dot{\pi}(t) = (\delta - \omega'(0) \cdot f(x(t)) \cdot k) \cdot \pi(t) - \frac{1}{\kappa} \cdot G(x(t)) \quad (24)$$

The two differences with the usual forward-looking Phillips curve in New Keynesian models is that the output gap is a function  $G(x(t))$  of labor market tightness, and the effective discount rate  $\delta - \omega'(0) \cdot f(x) \cdot k$  is below the subjective discount rate  $\delta$  as wealth enters the utility function. But the properties of the Phillips curve are not affected very much by these differences.<sup>16</sup>

A last equation is required to describe the dynamics of the real interest rates,  $r(t)$ . This equation is based on the dynamics of real money balances. Let  $\psi(t) = \phi'(M/p(t)) = p(t)/M$  denote the marginal utility of real money balances. Since  $M$  is fixed and  $p(t)$  is a state variable,  $\psi(t)$  is a state variable. As  $\psi(t) = p(t)/M$ , the law of motion of  $\psi(t)$  is

$$\dot{\psi}(t) = \pi(t) \cdot \psi(t). \quad (25)$$

Using the first-order condition (7), we find that  $i(t) = \psi(t) \cdot f(x(t)) \cdot k$  and hence the real interest rate is  $r(t) = i(t) - \pi(t) = f(x(t)) \cdot k \cdot \psi(t) - \pi(t)$ . Hence we can rewrite the Euler equation as

$$\dot{x}(t) = \frac{x(t)}{1 - \eta} \cdot [f(x(t)) \cdot k \cdot (\psi(t) + \omega'(0)) - \delta - \pi(t)]. \quad (26)$$

### 5.3 Properties of the Dynamical System

The dynamical system of (25), (24), and (26) describes the behavior over time of the vector  $[\psi(t), \pi(t), x(t)]$  representing the equilibrium. It is a nonlinear system of differential equations.

<sup>16</sup>As in the New Keynesian model, inflation can be expressed as the discounted sum of future output gaps:  $\pi(t) = \int_t^\infty G(x(s)) \cdot f(x(s)) \cdot e^{R(t)-R(s)} ds / (\kappa \cdot f(x(t)))$  with  $R(t) = \int_0^t r(s) ds$ . This expression is obtained from integrating the differential equation (20) and using (21).



In this system,  $x(t)$  and  $\pi(t)$  are jump variables and  $\psi(t)$  is a state variable. Proposition 3 determines the properties of the dynamical system:

**PROPOSITION 3.** *The vector  $[\psi(t), \pi(t), x(t)]$  describing the equilibrium satisfies the dynamical system  $\{(25), (24), (26)\}$ . The dynamic system admits a unique steady state. This steady state has no inflation, efficient tightness, and an interest rate below the subjective discount rate:  $\pi = 0$ ,  $x = x^*$ , and  $i = \psi \cdot y^* = \delta - \omega'(0) \cdot y^*$ , where  $y^* = f(x^*) \cdot k$  is the efficient output level. Around the steady state, the dynamic system is a saddle, and the stable manifold is a line. Since the system has one state variable ( $\psi$ ) and two jump variables ( $x$  and  $\pi$ ), this property implies that the equilibrium is determinate. At the steady state, the stable manifold is tangent to the vector  $\mathbf{z} = [\psi/\gamma_3, 1, (y^* \cdot \psi - \gamma_3) \cdot x^* \cdot \kappa]$ , where  $\gamma_3 = (\delta/2) \cdot \left[1 - \sqrt{1 + 4/(\kappa \cdot \delta^2 \cdot (1 - \eta))}\right] < 0$ . The responses of the equilibrium to small shocks are determined by  $\mathbf{z}$  and summarized in Table 2.*

*Proof.* In steady state,  $\dot{\psi} = \dot{\pi} = \dot{x} = 0$ . Since  $\psi(t) > 0$ , (25) implies that  $\pi = 0$ . There is no inflation in steady state, which is not surprising because there is no money growth. Since  $\pi = 0$ , (24) implies that  $G(x) = 0$  and  $x = x^*$ . The labor market tightness is efficient in steady state. This means that prices always adjust in the long run to bring the economy to efficiency. The mechanism is that the price level determines real money balances and thus aggregate demand—this is the LM channel discussed in Section 4. This channel operates as long as the economy is not in a liquidity trap. Last, (26) with  $\pi = 0$  implies that  $(\psi + \omega'(0)) \cdot y^* = \delta$  with  $y^* = f(x^*) \cdot k$ . Thus  $\psi = \delta/y^* - \omega'(0)$  and  $i = \psi \cdot y = \delta - \omega'(0) \cdot y^*$ .

To study the stability properties of the system around its steady state, we need to determine the eigenvalues of the Jacobian matrix  $\mathbf{J}$  of the system evaluated at the steady state. Simple computations exploiting the fact that in steady state  $\pi = 0$ ,  $x = x^*$ ,  $G(x^*) = 0$ ,  $G'(x^*) = 1/x^*$ , and  $(\psi + \omega'(0)) \cdot f(x^*) \cdot k - \delta = 0$ , imply that

$$\mathbf{J} = \begin{bmatrix} 0 & \psi & 0 \\ 0 & \psi \cdot y^* & -\frac{1}{\kappa \cdot x^*} \\ \frac{y^* \cdot x^*}{1 - \eta} & \frac{-x^*}{1 - \eta} & \delta \end{bmatrix}.$$

The characteristic polynomial of  $\mathbf{J}$  is  $(X - \psi \cdot y^*) \cdot [-X^2 + \delta \cdot X + 1/(\kappa \cdot (1 - \eta))]$  so that  $\mathbf{J}$  admits

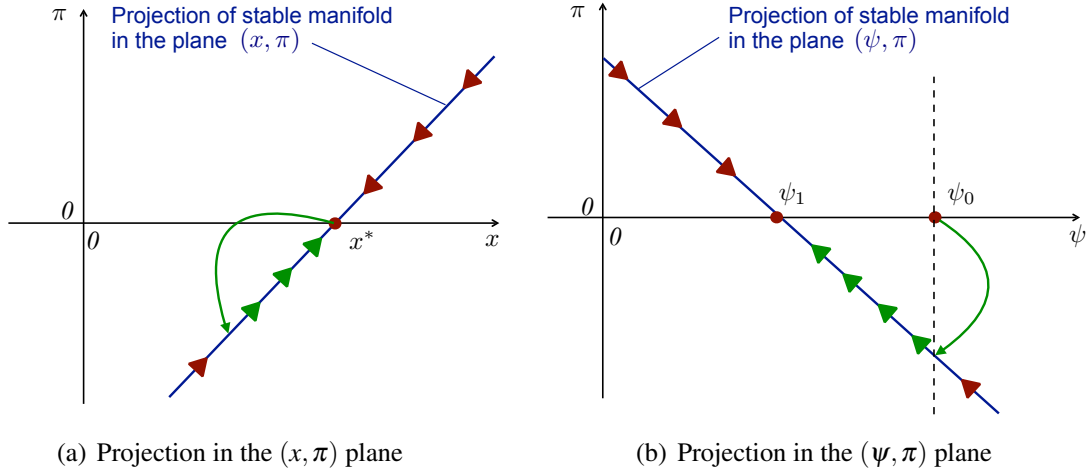


Figure 10: Response of the Equilibrium with Phillips Curve to an Unexpected Shock

three real eigenvalues:

$$\begin{aligned}\gamma_1 &= \psi \cdot y^* > 0 \\ \gamma_2 &= \frac{\delta}{2} \cdot \left[ 1 + \sqrt{1 + \frac{4}{\kappa \cdot \delta^2 \cdot (1 - \eta)}} \right] > 0 \\ \gamma_3 &= \frac{\delta}{2} \cdot \left[ 1 - \sqrt{1 + \frac{4}{\kappa \cdot \delta^2 \cdot (1 - \eta)}} \right] < 0.\end{aligned}$$

Therefore, the system is a saddle path around the steady state, and the stable manifold is a line. Since the system has one state variable ( $\psi$ ) and two jump variables ( $x$  and  $\pi$ ), this property implies that the system does not suffer from dynamic indeterminacy. Suppose that the economy is at its steady state. In response to an unexpected and permanent shock at  $t = 0$ , both  $x$  and  $\pi$  jump to the intersection of the new stable line and the plane  $\{\psi = \psi_0\}$ , where  $\psi_0$  denotes the old steady-state value of  $\psi$ . The economy remains on the plane  $\{\psi = \psi_0\}$ , orthogonal to the  $\psi$  axis, right after the shock because the state variable  $\psi$  cannot jump. This intersection is unique so the response of the system to the shock is determinate.

Finally, we compute the eigenvector  $\mathbf{z}$  associated with the negative eigenvalue,  $\gamma_3$ . The stable line is tangent to  $\mathbf{z}$  at the new steady state. Hence, this vector allows us to describe qualitatively the response of the equilibrium to aggregate demand and supply shocks, and monetary policy. The eigenvector is defined by  $\mathbf{Jz} = \gamma_3 \mathbf{z}$ . Simple calculation shows that this eigenvector is

$\mathbf{z} = [\psi/\gamma_3, 1, (y^* \cdot \psi - \gamma_3) \cdot x^* \cdot \kappa]$ . Using this vector, we obtain the responses to unexpected and permanent shocks described in Table 2. We now justify these responses.

We begin with two preliminary observations. First, irrespective of the shock,  $\pi$  and  $x$  always keep the same values in steady state, at  $\pi = 0$  and  $x = x^*$ . The steady states are therefore all aligned along a line  $\{\pi = 0, x = x^*\}$ , parallel to the  $\psi$  axis. Second, as  $\gamma_3 < 0$ , the coordinates of the eigenvector  $\mathbf{z}$  along dimensions  $\pi$  and  $x$  are both positive—the coordinates are  $1 > 0$  and  $(y^* \cdot \psi - \gamma_3) \cdot x^* \cdot \kappa > 0$ . Hence,  $x$  and  $\pi$  always move together in response to shocks—either they both increase, or they both decrease. In response to unexpected shocks, our Phillips curve therefore predicts a positive correlation between labor market tightness and inflation, or a negative correlation between unemployment and inflation, in the spirit of the traditional Phillips curve.

Next, consider a positive aggregate demand shock. For concreteness, assume that the marginal utility for wealth,  $\omega'(0)$ , decreases. In steady state, the marginal utility for money satisfies  $\psi = \delta/y^* - \omega'(0)$  so a lower  $\omega'(0)$  implies a higher  $\psi$ . Since the coordinate of the eigenvector  $\mathbf{z}$  along dimension  $\psi$  is  $\psi/\gamma_3 < 0$ ,  $\pi$  and  $x$  necessarily jump up on impact. The responses of  $y$  and  $i$  follow because  $y = f(x) \cdot k$  and  $i = \psi \cdot y$ . The mechanism is that after the shock, prices cannot adjust immediately so the economy becomes tight, unemployment is lower than efficient, and output is higher. As sellers face a tight market, they increase prices—inflation is positive. As prices rise, real money balances decrease and the marginal utility for money increases. This adjustment continues until the new steady state is reached. A monetary policy shock defined as a change in money supply has the same effects, except for the nominal interest rate.

Last, a positive aggregate supply shock has exactly the same effect on tightness and inflation as a negative aggregate demand shock. For concreteness, assume that capacity,  $k$ , increases. In steady state, the marginal utility for money satisfies  $\psi = \delta/(f(x^*) \cdot k) - \omega'(0)$  so a higher  $k$  implies a lower  $\psi$ , exactly like a negative aggregate demand shock. Hence,  $\pi$  and  $x$  necessarily jump down on impact after a positive aggregate supply shock. The response of output is more complicated because  $y = f(x) \cdot k$  and  $x$  jumps down whereas  $k$  jumps up. However, we can exploit the eigenvector  $\mathbf{z}$  to prove that the jump of  $x$  is always larger than that of  $k$ . Hence,  $y$  jumps down on impact and increases during the dynamic adjustment toward its new higher steady-state value. The response of  $i$  follows because  $i = \psi \cdot y$ . □

With directed search and costly price adjustment, prices converge slowly toward efficiency. The

Table 2: Dynamic Response of the Equilibrium with Phillips Curve to Shocks

Increase in:	Short-run / long-run effect on:				
	Tightness $x$	Inflation $\pi$	Price $p$	Output $y$	Interest rate $i$
Aggregate demand	+ / 0	+ / 0	0 / +	+ / 0	+ / +
Money supply	+ / 0	+ / 0	0 / +	+ / 0	- / 0
Aggregate supply	- / 0	- / 0	0 / -	- / +	- / -

*Notes:* The symbol “ $X/Y$ ” indicates that the response of a variable to a shock is  $X$  on impact and  $Y$  in steady state. Transition from impact to new steady state is monotonic for  $x$ ,  $\pi$ , and  $p$ . An increase in aggregate demand is an increase in the subjective discount rate or a decrease in the marginal utility of wealth. An increase in aggregate supply is an increase in capacity or a decrease in labor market mismatch.

mechanism is simple. If prices are too high, new markets are created with lower prices but higher tightness. If prices are too low, new markets are created with higher prices but lower tightness. Sellers and buyers have incentives to move to these new markets because they are more efficient so there is a larger surplus to share. Sellers are compensated for the lower price with a higher probability to sell. Buyers are compensated for the higher matching wedge by a lower price. In fact, [Lazear \[2010\]](#) finds evidence of such behavior for price and tightness in the housing market in the US.

This analysis could be used to formalize the conflict between price adjustment and inflation adjustment discussed by [Tobin \[1993\]](#). In response to a shock, the price adjustment requires an inflation change that further destabilizes the economy, possibly making the recession worse or exacerbating the overheating. This suggests that, even if price adjustments are fairly fast, the temporary inflation changes could amplify short-run fluctuations in tightness and output. For instance, after a negative aggregate demand shock inflation jumps down to allow prices to fall. This fall in prices increase real money balances and stimulates aggregate demand, which eventually brings the economy at efficiency. But a decrease in inflation has a temporary negative effect on aggregate demand and the labor market. This negative effect is illustrated in [Figure 4\(d\)](#), where we show that a decrease in inflation depresses the IS curve and thus the AD curve. The economic mechanism is that lower inflation implies higher real interest rates, which lead households to want to accumulate more wealth and hence consume less.

With costly price adjustment, assuming away liquidity traps, monetary policy can entirely accommodate all shocks. A monetary expansion, defined as a increase in money supply, can absorb a negative aggregate demand shock so that output, inflation, and tightness remain at their steady-state level at the time of the shock. Conversely, monetary tightening can absorb a positive aggregate demand shock. A monetary expansion can accommodate a positive aggregate supply shock so that the economy jumps immediately to its new steady state with zero inflation, efficient tightness, and higher output. Conversely, monetary tightening can accommodate a negative supply shock. In all cases, monetary policy should be based on tightness rather than output as efficient output varies with some shocks such as supply shocks while efficient tightness does not.<sup>17</sup>

## 5.4 The Limit with Zero Price-Adjustment Cost

On the one hand, if the price-adjustment cost is infinite then inflation is fixed at zero. This can be seen in the first-order condition (19), where  $\pi(t) = 0$  if  $\kappa(t) \rightarrow +\infty$ . This corresponds to the model studied in Sections 2–4. On the other hand, at the limit without price-adjustment cost, sellers always select a price to maintain the tightness at its efficient level as in Moen [1997]. This can be seen by combining the first-order condition (19), where  $\mu(t) = 0$  if  $\kappa(t) = 0$ , with the first-order condition (20), where  $\tau(x(t)) = (1 - \eta)/\eta$  if  $\mu(t) = 0$ ; that is,  $x(t) = x^*$  when  $\kappa(t) = 0$ .

This limit without price-adjustment cost describes an economy with flexible prices that always maintain the labor market at efficiency. Without price-adjustment cost, market forces drive prices to maintain tightness at efficiency, where consumption is maximized. If tightness were not efficient, both buyers and sellers would be better off with a price adjustment and a corresponding tightness adjustment. An implication is that aggregate demand shocks have no impact on tightness or consumption or output when prices are flexible. Aggregate supply shocks have no impact on tightness either, but they have an impact on consumption and output. Effectively, aggregate demand is irrelevant to understand the economy with flexible prices.

With a finite bliss point in the utility for money,  $m^* < +\infty$ , a large negative aggregate demand shock could bring the economy into a liquidity trap, whereby with real money balances are above

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<sup>17</sup>Alternatively, instead of choosing the level of  $M$ , the central bank could set the nominal interest rate  $i$  to follow an interest-rate rule of the form  $i = \alpha \cdot \pi$  with  $\alpha > 1$ . A negative shock increasing slack leads to negative inflation that prompts the central bank to lower  $i$ . Under this monetary policy, the economy immediately adjusts to shocks to remain at  $x = x^*$  and  $\pi = 0$ . New Keynesian models have the same property.

the bliss point but tightness is still below its efficient level. In that case, the directed search mechanism implies that sellers want to lower their price to increase the tightness they face, even though this does increase tightness in general equilibrium. The economy may fall into an instantaneous deflationary spiral with no steady-state equilibrium.<sup>18</sup> Hence, liquidity traps are worse with flexible prices than with fixed inflation, in line with the results in [Werning \[2012\]](#).

## 6 Conclusion

In this paper we provide an alternative to the standard New Keynesian model of business cycles. We use the model to re-examine the mechanisms behind cyclical fluctuations and the stabilizing effects of conventional and unconventional monetary and fiscal policies. Of course we find that monetary policy can stabilize the economy through open market operations in normal times. But in a liquidity trap, other policies are required to stimulate aggregate demand and stabilize the economy, such as a helicopter drop of money, a wealth tax, or government spending, especially if it is financed by issuing debt and consumers are not Ricardian.

The model is stylized in various aspects. As discussed above, once a realistic model of pricing has been designed, it could be easily incorporated into our framework. The model could be fruitfully extended along several other dimensions. Our theory of aggregate supply could be refined by introducing firms into the model and distinguishing between labor and product markets, as in [Michaillat and Saez \[2013\]](#). Capacity would not be exogenous but would be determined endogenously by the production decision of firms. A broad range of aggregate supply shocks, such as technology or labor-force participation shocks, could affect the economy. The concept of unemployment would be closer to what statistical agencies measure: unemployment would be the share of workers searching for a job and unable to find one instead of the share of labor services that self-employed workers cannot sell. Last, introducing firms would allow us to introduce productive capital and investment and to study the impact of investment decisions on aggregate demand.

Our theory of aggregate demand could also be refined by introducing heterogeneity across households to capture the fact that some households have higher marginal propensity to consume

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<sup>18</sup>Increasing inflation could push the economy out the trap. However, after the shock has happened and the economy is in a liquidity trap, conventional monetary policy cannot influence inflation anymore even with flexible prices. Helicopter drops or the wealth tax could still successfully pull the economy out of the liquidity trap.

than others. In that context, redistributive tax and transfer policies are likely to influence aggregate demand, and could potentially be used for stabilization. The theory could also be refined by adding a financial sector. In that model, financial crises would affect consumers' consumption and saving behavior and thus aggregate demand and unemployment.

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