Monetary Policy with Ambiguity Averse Agents*

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Abstract

We study a prototypical new-Keynesian model in which agents are averse to ambiguity, and where the ambiguity regards the monetary policy rule. We show that ambiguity has important effects even in steady state, as uncertainty about the policymaker's response function affects the rest of the model via the consumption-saving decision. A reduction in ambiguity - e.g. due to credible monetary policy actions and communications - results in a fall in inflation and the policy rate, and an increase in welfare. Moreover while, absent ambiguity, the policymaker's actual responsiveness to inflation does not matter as long as the Taylor principle is satisfied, in the face of ambiguity the exact degree to which the central bank responds to inflation regains importance. Indeed, a high degree of responsiveness to inflation mitigates the welfare costs of ambiguity. We also present various results regarding the optimal choice of an inflation target, both when ambiguity is given and when assuming the policymaker can affect ambiguity with increased transparency and communications.

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1 Introduction

Communication and transparency have become more important for central banks in recent decades. Since the demise of Lehman Brothers and the subsequent Great

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Recession, central banks have adopted a range of unconventional monetary policy measures: great emphasis has also been put on enhanced communication, aimed at clarifying the policymakers' reaction function[1]. As with all policy actions, it is crucial to have models that can help us understand how improved central bank communications affect the economy. Addressing this question however is possible only in a model that relaxes the commonly held assumption that all the laws of motion of the variables in the economy are perfectly known by the agents.

In this paper we study a prototypical new-Keynesian model in which agents are averse to ambiguity, or Knightian uncertainty, and where the ambiguity regards the monetary policy rule. Ambiguity describes a situation in which there is uncertainty about the probability distribution over contingent states of the world. In our model, ambiguity does not affect a purely exogenous process like total factor productivity, as in recent work by Ilut and Schneider (2014), but rather the perceived response function of the central bank. Effectively, we model the ambiguity as a further component of the monetary policy rule that can take values in a range the agents entertain as possible, but to which they cannot assign probabilities, and we investigate how this affects the design of monetary policy[2].

When agents are averse to ambiguity, they will make their decisions *as if* the worst-case scenario were to materialise. Therefore the first step of our analysis must be to characterise the worst-case steady state. We first analyse a model in which a subsidy to factor inputs is aimed at offsetting the distortions arising from monopolistic competition. In this model, and in absence of ambiguity, the central bank can implement a monetary policy that delivers the first-best allocation. We show that the agents' welfare is locally concave with respect to ambiguity so that only the two extreme values in the range of uncertainty considered by agents are candidate minima. We also show that, in the first-best model, the worst-case steady state is, for any level of ambiguity, the one in which agents act *as if* the interest rate were lower than that implied by rational expectations. This results in a "too high" demand for consumption on the part of the agents, which generates an increase in the marginal costs firms face, in inflation and consequently in the interest rate set by the central bank. In the absence of indexation, as inflation deviates from its first-best value, price dispersion (Yun, 2005) arises. Price dispersion is a key driver of the loss of welfare that characterises the steady state under ambiguity.

Ambiguity, therefore, has important effects even in steady state. First, in a world with lower ambiguity - *i.e.* where the range of models agents consider possible but to which they cannot assign probabilities is smaller - inflation and the policy rate will be lower, and welfare will be higher, than a world with higher ambiguity about the monetary policy rule. Indeed, clearer and more transparent central bank communications since the 1990s plausibly reduced ambiguity about monetary policy and could, in principle, be a concurrent explanation for the Great Moderation. Figures 1 and 2 report, for the US and the UK respectively, inflation and the policy

[1]Yellen (March 2013) of the Federal Reserve, Bean (August 2013) of the Bank of England and Draghi (April 2014) of the European Central Bank have all described the guidance they had given as "clarifying the reaction function."

[2]Unlike Benigno and Paciello (2014), who study how the presence of doubts and ambiguity *about the economic environment* influences the characterisation of optimal monetary policy, we focus on the uncertainty about the policymaker's reaction function.

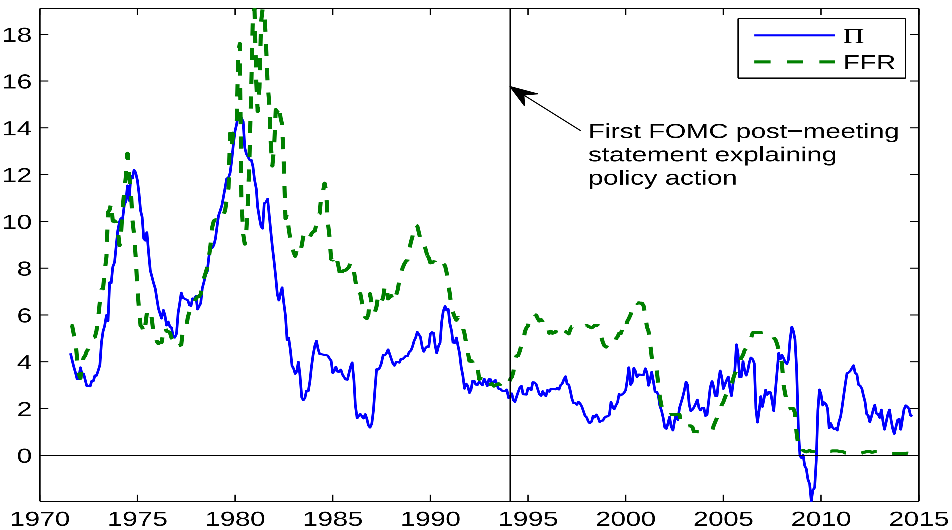

Figure 1: **US**: Annual inflation and the Fed Funds rate from the end of the Bretton Woods system (1971) to September 2014. The shaded areas indicate NBER recessions, while the black vertical lines indicate the dates of the main changes in the communication of the Federal Reserve Board

rate from the the end of the Bretton Woods system in 1971 [3] to 2014. In both charts the black vertical line indicates the first change in the communication of the central bank aimed at reducing the uncertainty about the behaviour of the policymakers: namely, the first post-meeting FOMC statement in the case of the Federal Reserve and the introduction of an inflation target in the case of the Bank of England. Both charts are suggestive of some break in the behaviour of inflation and the interest rate around the time of the increase in transparency. This observation is supported for example by work done by Levin and Piger (2004), who investigate the time series properties of inflation in industrialised economies and find strong evidence of a structural break at the beginning of the 1990s. This evidence suggests that indeed periods of lower transparency - and thus higher ambiguity about monetary policy - are associated with higher inflation and higher interest rates.

We also look at a version of our model in which there is no subsidy offsetting the distortions arising from monopolistic competition, and where consequently no monetary policy can achieve the first-best allocation. Also in this case the agents' welfare is locally concave with respect to ambiguity, and only the two extreme values in the range of uncertainty considered by agents are candidate minima. However the worst-case scenario can be the low-inflation or the high-inflation one, depending on the level of ambiguity. When the ambiguity is very low, the worst-case scenario will be the low-inflation one; it will be the high-inflation one otherwise.

We then study how the design of monetary policy is affected by the presence of ambiguity. First, we find that, in the presence of ambiguity, the degree of responsiveness to inflation of the central bank regains a role in determining welfare.

[3] Or the earliest available data for the UK

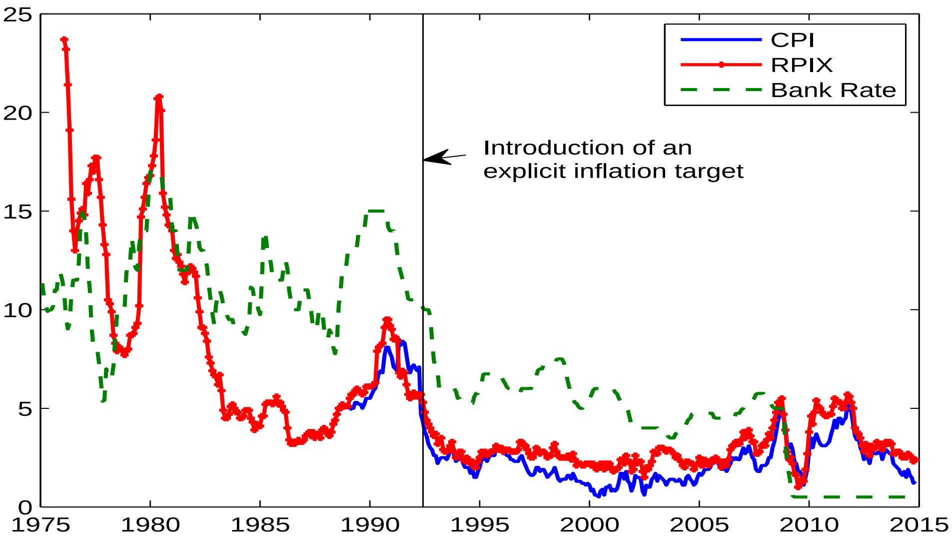

Figure 2: **UK**: Two measures of inflation and Bank Rate from 1975 to September 2014. The black vertical lines indicate the dates of the main changes in the communication of the Bank of England

As shown for example in Schmitt-Grohe and Uribe (2007) and Ascari and Ropele (2007), in models like the one we consider but with no ambiguity the policymaker's degree of responsiveness to inflation plays virtually no role in determining welfare, provided that the Taylor Principle is satisfied[4]. In our framework, instead, there is an interplay between the amount of ambiguity, the degree of responsiveness to inflation and welfare. In particular, in a world with low ambiguity about monetary policy the costs associated with a lower responsiveness to inflation are smaller than in a world with high ambiguity. We also study if and how the policymaker should set an inflation target in order to maximise welfare for a given level of ambiguity. In a model in which a subsidy to factor inputs offsets the distortions arising from monopolistic competition, steady-state inflation is always higher than in the first-best case. We show that, for a given level of ambiguity, it is welfare enhancing for the central bank to implement an inflation target. The target reduces the relative price distortion associated with the price dispersion introduced by ambiguity and this naturally enhances welfare[5]. This is true also for a model in which the distortion arising from monopolistic competition cannot be offset, though with a different value of the optimal target.

We then analyse the case in which the policymaker can affect ambiguity about monetary policy rather than simply taking it as given. This seems like a natural thought experiment given the massive increases in transparency about policymaking that happened in the last decades, as well as the more and more explicit commu-

[4]In models as simple as the one we use for our analysis, the degree of responsiveness to inflation would be entirely irrelevant, if the Taylor principle is satisfied, because a TFP shock would not generate any inflation irrespective of the degree of responsiveness.

[5]This is in line with Yun's (2005) result for a model with no ambiguity but with price dispersion.

nications of the last years. Since ambiguity is welfare reducing, the best strategy would be for the policymakers to completely dispel ambiguity - though it seems rather unrealistic that they might succeed in removing ambiguity completely, as there might be lingering credibility issues, or simply the inability to pin down the exact dynamics of committee decision-making. In the first best-model, this would be enough to bring the model back to the first-best steady state. For the second-best model, Ascari and Ropele (2007) show that, even when ambiguity is zero, there is an incentive for the policymaker to introduce an inflation target. However the target could be counter-productive from a welfare perspective, if the uncertainty is not dispelled. Our qualitative results suggest that the introduction of the inflation target is beneficial only if ambiguity about monetary policy is very close to zero, while it becomes damaging above a certain threshold of uncertainty. This is in line with evidence on the minor effectiveness of inflation targeting in emerging market economies, due to the lack of credible institutions (for a recent review of the evidence see Brito and Bystedt, 2010).

We conclude our analysis studying the transitional dynamics from a high-ambiguity steady state to a low-ambiguity steady state. Consumption increases, while the interest rate and inflation fall. Price dispersion subsides, as a result of the fact that optimally set prices are closer to the prevailing average price, which follows from the lower level of inflation. Therefore welfare increases and the output gap closes. A lower level of ambiguity, makes agents better off, so in the new steady state they want to consume more goods and enjoy more leisure as well. Price stickiness, however, causes the price dispersion to fall falls quite slowly, which hinders productivity. As a result hours actually jump up on impact before starting to slowly converge to the new, lower, steady state. A combination of high hours and high consumption, together with sticky prices, drive real wages temporarily above their new steady state level.

The paper is organised as follows. Section 2 provides a description of the model. In Section 3 we characterise the worst case with respect to which the agents make their decisions and present simple comparative statics results that cast light on the workings of the distortion determined by the presence of ambiguity, while section 4 investigate the design of monetary policy under ambiguity. In section 5 we summarise the results about transitional dynamics from a steady state with high ambiguity to one with low ambiguity and we report our conclusions in section 6.

2 The Model

We modify a textbook New-Keynesian model (Galí, 2008) by assuming that the agents suffer ambiguity about the expected future policy rate. Absent ambiguity, the first-best allocation is attained thanks to a sufficiently strong response of the Central Bank to inflation, *i.e.* for $\phi > 1$ and to a Government subsidy that corrects the distortion introduced by monopolistic competition.

2.1 Households

Let $s_t \in S$ be the vector of exogenous states. We use $s^t = (s_1, ..., \ s_t)$ to denote the history of the states up to date t. A consumption plan $\overrightarrow{C}$ says, for every history s^t, how many units of the final good $C_t(s^t)$ a household consumes and for how many hours $N_t(s^t)$ a household works. The consumer's felicity function is:

$$u(\overrightarrow{C}_t) = \log(C_t) - \frac{N_t^{1+\sigma}}{1+\sigma}$$

Utility conditional on history s^t equals felicity from the current consumption and labour mix plus discounted expected continuation utility, *i.e.* the households' utility is defined recursively as

$$U_t(\overrightarrow{C}; s^t) = u(\overrightarrow{C}_t) + \beta \min_{p \in \mathcal{P}_t(s^t)} \mathbb{E}^p U_{t+1}(\overrightarrow{C}; s_t,\ s_{t+1}) \tag{1}$$

where $\mathcal{P}_t(s^t)$ is a set of conditional probabilities about next period's state $s_{t+1} \in S$. The recursive formulation ensures that preferences are dynamically consistent. The multiple priors functional form (1) allows modeling agents that have a set of multiple beliefs and also captures a strict preference for knowing probabilities (or an aversion to not knowing the probabilities of outcomes), as discussed in Ilut and Schneider (2014)[6]. A non-degenerate belief set $\mathcal{P}_t(s^t)$ means that agents are not confident in probability assessments, while the standard rational expectations model can be obtained as a special case of this framework in which the belief set contains only one belief.

As discussed in more detail below, we parametrise the belief set with an interval $[-\overline{\mu}_t,\ \overline{\mu}_t]$ of means centered around zero, so we can think of a loss of confidence as an increase in the width of that interval. That is, a wider interval at history s^t describes an agent who is less confident, perhaps because he has only poor information about what will happen at $t+1$. The preferences above then take the form:

$$U_t(\overrightarrow{C}; s^t) = u(\overrightarrow{C}_t) + \beta \min_{\mu \in [-\overline{\mu}_t,\ \overline{\mu}_t]} \mathbb{E}^\mu U_{t+1}(\overrightarrow{C}; s_t,\ s_{t+1}) \tag{2}$$

The households' budget constraint is:

$$P_t C_t + B_t = R_{t-1} B_{t-1} + W_t N_t + T_t \tag{3}$$

where T_t includes government transfers as well as a profits, W_t is the hourly wage, P_t is the price of the final good and B_t are bonds with a one-period nominal return R_t. There is no heterogeneity across households, because they all earn the same wage in the competitive labor market, they own a diversified portfolio of firms, they consume the same Dixit-Stiglitz consumption bundle and face the same ambiguity. The only peculiarity of households in this setup is their perceived uncertainty about the return to their savings R_t. As we describe in more detail in the Subsection 2.3, R_t is formally set by the Central Bank after the consumption decision is made, while the agents make their decisions based on their perceived interest rate $\tilde{R}_t$, which is a function of the ambiguity μ. The Central Bank sets R_t based on current inflation and the current level of the natural rate, so absent ambiguity, the private sector would know its exact value and it would correspond to the usual risk-free rate. In this context, however, agents do not fully trust the Central Bank's response function and so they will consider a range of interest rates indexed by μ.

[6]More details and axiomatic foundations for such preferences are in Epstein and Schneider (2003).

The household's intertemporal and intratemporal Euler equation simply read:

$$\frac{1}{C_t} = \mathbb{E}_t^\mu \left[\frac{\beta R_t}{C_{t+1}\Pi_{t+1}} \right] \tag{4}$$

$$N_t^\sigma C_t = \frac{W_t}{P_t} \tag{5}$$

While they both look absolutely standard the expectation for the intertemporal Euler equation reflects agents' ambiguous beliefs.
In particular, we restrict ambiguity to the policy rate as follows:

$$\mathbb{E}_t^\mu \left[\frac{\beta R_t}{C_{t+1}\Pi_{t+1}} \right] \equiv \mathbb{E}_t \left[\frac{\beta \tilde{R}_t}{C_{t+1}\Pi_{t+1}} \right]$$

Hence the intertemporal Euler equation becomes:

$$\frac{1}{C_t} = \mathbb{E}_t \left[\frac{\beta \tilde{R}_t}{C_{t+1}\Pi_{t+1}} \right] \tag{6}$$

where $\tilde{R}_t \equiv R_t e^{\mu_t}$ and $\mathbb{E}_t$ is the rational-expectations operator.

2.2 Firms

The final good Y_t is produced by final good producers who operate in a perfectly competitive environment using a continuum of intermediate goods $Y_t(i)$ and the standard CES production function

$$Y_t = \left[\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{7}$$

Taking prices as given, the final good producers choose intermediate good quantities $Y_t(i)$ to maximize profits, resulting in the usual Dixit-Stiglitz demand function for the intermediate goods

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} Y_t \tag{8}$$

and in the aggregate price index

$$P_t = \left[\int_0^1 P_t(i)^{1-\varepsilon} di \right]^{\frac{1}{1-\varepsilon}}.$$

Intermediate goods are produced by a continuum of monopolistically competitive firms with the following linear technology:

$$Y_t(i) = A_t N_t(i), \tag{9}$$

where A_t is a stationary technology process [7]. Prices are sticky in the sense of Calvo (1983): only a random fraction of firms $(1-\theta)$ can re-optimise their price at any

[7] We have also considered a version of the model which included capital in the production function and an investment decision and our qualitative results follow through.

given period, while the others must keep the nominal price unchanged. Whenever a firm can re-optimise, it sets its price maximising the expected presented discounted value of future profits

$$\max_{P_t^*} E_t \left[\sum_{s=0}^{\infty} \theta^s Q_{t+s} \left(\left(\frac{P_t^*}{P_{t+s}} \right)^{1-\epsilon} Y_{t+s} - \Psi \left(\left(\frac{P_t^*}{P_{t+s}} \right)^{-\epsilon} Y_{t+s} \right) \right) \right] \tag{10}$$

where Q_{t+s} is the stochastic discount factor, Y_{t+s} denotes aggregate output in period $t+s$ and $\Psi(\cdot)$ is the net cost function. Given the simple linear production function in one input the (real) cost function simply takes the form $\Psi\left(Y_t(i)\right) = (1-\tau)\frac{W_t}{P_t}\frac{Y_t(i)}{A_t}$, where τ is the production subsidy.

The firm's price-setting decision in characterised by the following first-order condition:

$$\frac{P_t^*(i)}{P_t} = \frac{\mathbb{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t+s} \left(\frac{P_{t+j}}{P_t}\right)^{\epsilon} \frac{\epsilon}{\epsilon-1} MC_{t+j}}{\mathbb{E}_t \sum_{j=0}^{\infty} \theta^j Q_{t+s} \left(\frac{P_{t+j}}{P_t}\right)^{\epsilon-1}},$$

which ultimately pins down inflation, together with the following equation derived from the law of motion for the price index:

$$\frac{P_t^*(i)}{P_t} = \left(\frac{1-\theta \Pi_t^{\epsilon-1}}{1-\theta} \right)^{\frac{1}{1-\epsilon}}. \tag{11}$$

This setup results in a purely forward looking log-linear Phillips Curve, since firms do not index to past inflation[8]. We will consider both the case in which the production subsidy successfully offsets the distortions arising from monopolistic competition and the case in which there is no subsidy, *i.e.* $\tau = 0$. In the former case, absent ambiguity, the Central Bank's Taylor rule attains the first-best allocation, while in the latter case the first-best cannot be attained. The presence of the subsidy has implications for the determination of the worst case scenario, which will be discussed in detail in Section 3.1.

2.3 The Government and the Central Bank

The Government runs a balanced budget and, if $\tau \neq 0$ finances the production subsidy with a lump-sum tax. Out of notational convenience, we include the firms' profits and the deadweight loss resulting from price dispersion Δ_t, which is defined in the next section, in the lump-sum transfer:

$$\begin{aligned} T_t &= P_t \left(-\tau \frac{W_t}{P_t} N_t + Y_t \left(1 - (1-\tau) \frac{W_t \Delta_t}{P_t A_t} \right) \right) \\ &= P_t Y_t \left(1 - \frac{W_t \Delta_t}{P_t A_t} \right). \end{aligned}$$

The first expression explicitly shows that we include in T_t the financing of the subsidy, the second refers to the economy-wide profits, which include the price-dispersion term Δ_t, formally defined in the following Subsection

[8] As also mentioned above, we assume zero inflation in the first-best steady state, but it would simple to re-write the whole problem under the assumption of positive first-best steady state inflation; all results would carry through provided that the firms indexed to that level of inflation.

The Central Bank follows a very simple Taylor rule:

$$R_t = R_t^n (\Pi_t)^{\phi}, \tag{12}$$

here R_t is the gross nominal interest rate paid on bonds maturing at time $t+1$ and $R_t^n = \mathbb{E}_t \frac{A_{t+1}}{\beta A_t}$ is the gross natural interest rate[9]. For simplicity of exposition we have assumed an inflation target of zero, however all our results would through with any inflation target, provided that the firms indexed at that same value of the target.

The Central Bank formally sets rates after the private sector makes their economic decisions, but it does so based on variables such as the current natural rate and current inflation, which are known to the private sector as well. There are no monetary policy shocks, nor the Central Bank is trying to take advantage of this timing as in the inflation bias literature. Therefore if the private sector were to fully trust the Central Bank, *i.e.* under the standard modeling assumption $\overline{\mu}_t = 0$ then clearly:

$$\tilde{R}_t \equiv R_t e^{\mu_t} = R_t = R_t^n (\Pi_t)^{\phi}$$

which is the nominal rate that implements first-best allocations (together with the subsidy). And, clearly, there is no uncertainty in the standard sense of the word around the expected value, which is then a risk-free rate.

In the context of our analysis, however, agents suffer ambiguity about the policymaker's response function ($\mu_t \neq 0$) and dislike such ambiguity, so they will base their decision on the interest-rate level that, within the range they entertain possible, would hurt their welfare the most if it was to prevail. In this case, even in the presence of the production subsidy, we cannot achieve the first-best allocation, despite the Central Bank following a Taylor rule (12) that would normally implement it, because the private sector will use a somewhat different interest rate for their consumption-saving decision. In this stylized setup, we thus capture a situation in which, despite the policymakers actions, the first-best allocation fails to be attained because of a lack of confidence and/or understanding on the part of the private sector, which sets the stage for studying the benefits resulting from making the private sector more aware and confident about the implementation of monetary policy.

2.4 Market clearing conditions

Market clearing in the goods markets requires that

$$Y_t(i) = C_t(i)$$

for all firms $i \in [0, 1]$ and all t. Given aggregate output Y_t is defined as in equation (7), then it follows that

$$Y_t = C_t.$$

[9]While there is an expectation in the definition of the natural rate, under rational expectations the expectations of the Central Bank will coincide with those of the private sector, hence the natural rate will be known by both sides and there will be no uncertainty about it.

Market clearing on the labour market implies that

$$
\begin{aligned}
N_t &= \int_0^1 N_t(i)di. \\
&= \int_0^1 \frac{Y_t(i)}{A_t}di \\
&= \frac{Y_t}{A_t}\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\varepsilon} di
\end{aligned}
$$

where we obtain the second equality substituting in the production function (9) and then use the demand function (8) to obtain the last equality. Let us define $\Delta_t \equiv \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} di$ as the variable that measures the relative price dispersion across intermediate firms. Δ_t represents the inefficiency loss due to relative price dispersion under the Calvo mechanism: the higher Δ_t, the more labor is needed to produce a given level of output.

3 Steady state analysis

Following Ilut and Schneider (2014), we study our model economy log-linearized around the worst-case steady state, because ambiguity-averse agents will make their decisions *as if* that were the steady state. Therefore, we must first identify the worst-case scenario and characterise it. We derive the steady state of the agents' first-order conditions as a function of a generic constant level of μ and we rank the different steady states (indexed by the level of distortion induced by the ambiguity) to characterize the worst-case steady state.

3.1 General Properties of the Steady States

3.1.1 Inflation and the Policy Rate

All steady states, albeit distorted, are characterised by a constant consumption stream. As a result, the intertemporal Euler equation pins down the perceived *real* interest rate, *i.e.* the rate that determines the intertemporal substitution of consumption. Combining this with our simple Taylor rule then delivers the steady state level of inflation consistent with the distortion and the constant consumption stream, as the following result states.

Result 3.1. *In a steady state with no real growth, inflation depends on the ambiguity distortion parameter as follows:*

$$\Pi(\mu,\cdot) = e^{-\frac{\mu}{\phi-1}}, \tag{13}$$

while the policy rate is:

$$R(\mu,\cdot) = \frac{1}{\beta}e^{-\frac{\phi\mu}{\phi-1}}. \tag{14}$$

Hence, $\phi > 1$ implies that for any $\mu > 0$:

$$\Pi(\mu,\cdot) < \Pi(0,\cdot) = 1 \qquad R(\mu,\cdot) < R(0,\cdot) = \frac{1}{\beta}, \tag{15}$$

and the opposite for $\mu < 0$.

Proof. Proof in Appendix A. □

Result 3.1 clearly shows that inflation is a decreasing function of μ as long as $\phi > 1$. The mapping from μ to $\Pi(\mu, \cdot)$ implies that the steady state of the model and it's associated welfare, can be equivalently characterised in terms of inflation or in terms of μ.

To build some intuition on the steady state formula for inflation and the interest rate, let us consider the case in which household decisions are based on a level of the interest rate that is systematically lower than the true policy rate ($\mu < 0$)[10]. Other things equal, this will induce too a high a demand pressure, causing an increase in inflation. In the end, higher inflation will be matched by higher nominal interest rate so that constant consumption in steady state is attained. The result of this is that the policy rate will end up being higher than in the first-best steady state[11] ($\frac{1}{\beta}$).

3.1.2 Pricing

In our model firms index their prices based on the first-best inflation, which is zero in this case. Because of ambiguity, steady-state inflation will not be zero and therefore there will be price dispersion in steady state:

$$\Delta(\mu, \cdot) \quad = \quad \frac{(1-\theta)\left(\frac{1-\theta\Pi(\mu,\cdot)^{\epsilon-1}}{1-\theta}\right)^{\frac{\epsilon}{\epsilon-1}}}{1-\theta\Pi(\mu,\cdot)^{\epsilon}} \tag{16}$$

Δ is minimised for $\Pi_{ss} = 1$ - or, equivalently, $\mu = 0$ - and is larger than unity for any other value of μ. As in Yun (2005), the presence of price dispersion alters the behaviour of the monetary policymaker in sticky price models with staggered price-setting. Price dispersion reduces labour productivity and ultimately welfare.

3.1.3 Hours, Consumption and Welfare

In a steady state with no real growth[12], steady-state hours are the following function of μ:

$$N(\mu, \cdot) = \left(\frac{\left(1-\theta\Pi(\mu,\cdot)^{\epsilon-1}\right)\left(1-\beta\theta\Pi(\mu,\cdot)^{\epsilon}\right)}{\left(1-\beta\theta\Pi(\mu,\cdot)^{\epsilon-1}\right)\left(1-\theta\Pi(\mu,\cdot)^{\epsilon}\right)}\right)^{\frac{1}{1+\psi}}, \tag{17}$$

while consumption is:

$$C(\mu, \cdot) = \frac{A}{\Delta(\mu, \cdot)} N(\mu, \cdot) \tag{18}$$

[10]In our analysis we do not consider the zero lower bound, because in case it was binding that could not be be a steady state. If one wanted to explicitly account for that, a restriction on range of μ would simply take the following form: $\mu < -\frac{\phi-1}{\phi}\log(\beta)$.

[11]Here we assume that, absent distortions, inflation would be zero in steady state, but, as discussed previously, all results follow through with non-zero steady state inflation, as long as the firms index to that value of inflation.

[12]We expect the results to follow through for a model with a stochastic trend in technology, as long as the variables are appropriately detrended.

Hence the steady state welfare function takes a very simple form:

$$\mathbb{V}(\mu, \cdot) \quad = \quad \frac{1}{1-\beta}\left(\log\left(C(\mu, \cdot)\right) - \frac{N(\mu, \cdot)^{1+\psi}}{1+\psi}\right). \tag{19}$$

3.2 Calibration

In many instances, we will try to provide general results that do not depend on the specific parameter values. However, to illustrate our points we will make frequent use of charts, so we discuss the model calibration in this Subsection, while also noting that the model is intended for a more qualitative analysis rather than sharp quantitative predictions.

We calibrate $\beta = .995$ to deliver a 2 percent annual steady-state interest rate. $\psi = 1$ implies a unitary Frisch elasticity of the labor supply, which is within the commonly accepted range. We assign value .83 to the fraction of firms θ that cannot re-optimise in each period: this corresponds to an average price duration of about one and half years. While this is towards the high end of the accepted range, since this is the only friction in the model at the moment, a high value helps us get a richer dynamics in Section 5. We set the elasticity of goods' demand ϵ to 11, which implies a ten percent markup, and set the inflation response coefficient in the Taylor rule to $\phi = 2$. We choose this value for the baseline calibration, because, in log-linear terms, it makes inflation vary one for one with the distortion while being within commonly accepted ranges. Finally, for what concerns the parameter governing ambiguity, μ, we will consider values up to .01. Translating this into annual interest rates, that implies uncertainty around the policy rate of the order of ± 4 percent, larger than one could reasonably consider. We consider this value primarily to illustrate the robustness of our local analytical results.

3.3 Properties of the Welfare Function and Worst-Case Steady State

So far we have considered the optimal behaviour of consumers and firms for a given μ, *i.e.* a given misperception of the expected policy rate. To pin down the worst-case scenario we need to consider how the agents' welfare is affected by different values of μ and find the μ that minimises the their welfare.

Ilut and Schneider (2014) assume that agents are ambiguous about the *exogenous* TFP process. It follows quite naturally that the worst-case steady state is one in which agents under-estimate TFP growth. In our case, however, two differences arise:

I. Given that the interest rate is set optimally, any under- or overprediction will generate a welfare loss. Hence, it is not a priori clear what the worst case might be.

II. Because the policy rate responds to the endogenously determined inflation rate, distortions in expectations have a *feedback effect* via their impact on the steady state level of inflation.

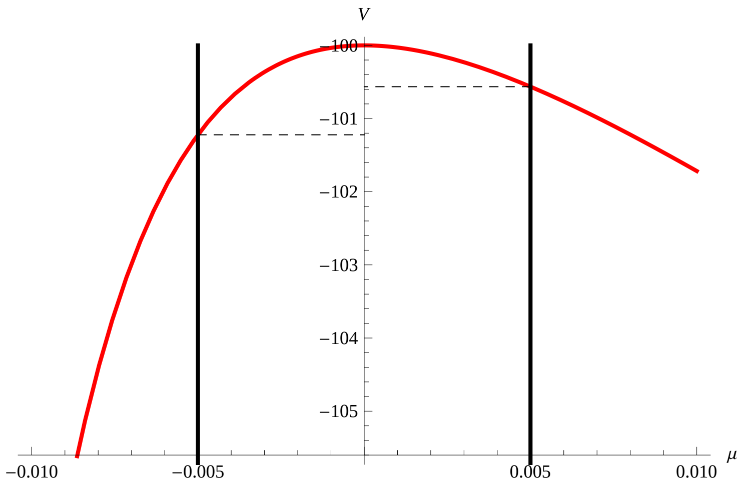

Figure 3: Steady-state welfare as a function of μ in the model with the production subsidy.

We now proceed by first studying the model with the production subsidy and then the case in which the there is no subsidy, *i.e.* $\tau = 0$.

3.3.1 The first-best model

In the our simple model with the production subsidy, monetary policy implements the first-best allocation, so it is clear that any $\mu \neq 0$ will generate a welfare loss. However, it is not a priori clear if a negative μ is worse than a positive one of the same magnitude.

The following result rules out the presence of interior minima for sufficiently small ambiguity ranges, given the weakest restrictions on parameter values implied by economic theory.

Result 3.2. *For* $\beta \in [0,1)$, $\epsilon \in (1,\infty)$, $\theta \in [0,1)$, $\phi \in (1,\infty)$, $\psi \in [0,\infty)$, $\mathbb{V}(\mu,\cdot)$ *is continuously differentiable around* $\mu = 0$ *and:*

$$\frac{\partial \mathbb{V}(0,\cdot)}{\partial \mu} = 0 \quad \textit{and} \quad \frac{\partial^2 \mathbb{V}(0,\cdot)}{\partial \mu^2} < 0$$

As a consequence, for small enough $\overline{\mu}$*, there are no minima in* $\mu \in (-\overline{\mu}, \overline{\mu})$*.*

Proof. Proof in Appendix A. □

Result 3.2 illustrates that the welfare function is locally concave around the first-best (see Figure 3, drawn under our baseline calibration described above). Realistic calibrations show that the range of μ for which the value function is concave is in practice much larger than any plausible range for the ambiguity.

While this is hardly surprising, given that our economy attains the first best in the absence of ambiguity, it is worth stressing that Result 3.2 depends on the production subsidy that undoes the price markup in steady state. Absent the subsidy $\frac{\partial \mathbb{V}(0,\cdot)}{\partial \mu}|_{\tau=0} < 0$, a situation we address in the next Subsection.

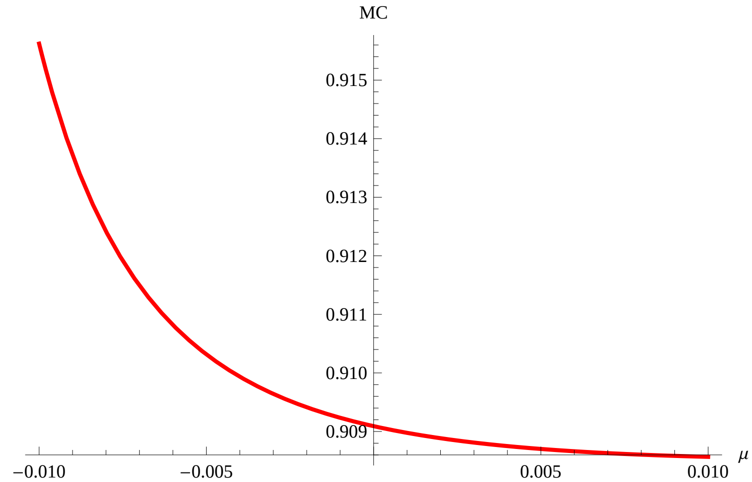

Figure 4: Steady state marginal cost as a function of μ.

Result 3.2 rules out interior minima, but it remains to be seen which of the two extremes is worse from a welfare perspective. Graphically and numerically it is immediate to see from Figure 3 that the welfare function is concave but not symmetric with respect to μ. More generally, it is possible to establish this sufficient condition for the characterisation of the worst-case scenario.

Result 3.3. *For β sufficiently close but below 1 and all the other parameters in the intervals defined in Result 3.2, $\mu = -\overline{\mu}$ minimizes $\mathbb{V}(\mu, \cdot)$ over $[-\overline{\mu}, \overline{\mu}]$, for any sufficiently small $\overline{\mu} > 0$.*

Proof. Proof in Appendix A. □

In practice, this sufficient condition is not at all restrictive because any sensible calibration would set β to a value that is well within the range for which our result holds.

The intuition for the asymmetry of the welfare function is the following. While the effect of μ on inflation is symmetric (in logs) around zero, the impact of inflation on welfare is not. In particular, positive steady-state inflation - associated with negative levels of μ as shown in Result 3.1 - leads to a bigger welfare loss than a corresponding level of negative inflation. This results from the fact that positive inflation tends to lower the relative price of firms who do not get a chance to re-optimise. These firms will face a very high demand, which in turn will push up their labour demand, and ultimately their marginal costs, due to the convexity of the marginal cost function with respect to μ, as Figure 4 shows. On the other hand, negative inflation will reduce the demand for firms which do not re-optimise and this will reduce their demand for labour and their marginal costs[13]. In the limit, as the relative price goes to zero, firms will incur huge marginal costs while as their relative price goes to infinity their demand goes to zero.

[13]As this is the first best model, when there is no ambiguity, *i.e.* where the MC function crosses the y-axis, the marginal cost is equal to the inverse of the mark-up.

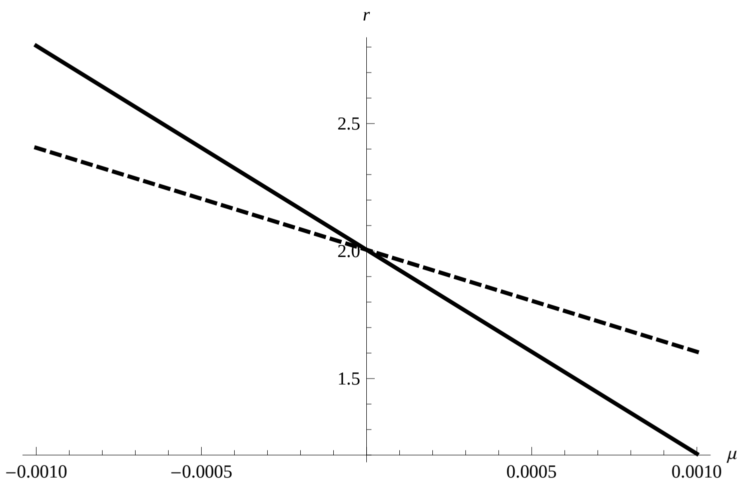

Figure 5: Steady State policy rate R_t (solid line) and interest rate expected by ambiguity-averse agents $\tilde{R}_t$ (dashed line) as a function of μ.

Having characterised the worst-case steady state, we can now use Result 3.1 to directly infer that, in an ambiguity-ridden economy, inflation will be higher than in the first-best allocation and so will be the policy rate, as evident from Figure 5. Figure 5 illustrates the wedge between the actual and the perceived policy rates as a function of μ. When agents base their decisions on a perceived interest rate $\tilde{R}_t$ that is lower than the actual policy rate, the under-estimation of the policy rate tends to push up consumption and generate inflationary pressures, which, in turn, lead to an increase in the policy rate (the afore mentioned *feedback effect*).

The combined effects of higher inflation, higher policy rates and negative μ make the perceived real rate of interest equal to $\frac{1}{\beta}$, which is necessary to deliver a constant consumption stream, *i.e.* a steady state. The level of this constant consumption stream (and ultimately welfare) depends, in turn, on the price dispersion generated by the level of inflation that characterises the steady state.

The following Result summarises Results 3.1 and 3.3 and establishes more formally the effects of ambiguity on the worst-case steady state levels of inflation and the policy rate. In particular, we study the effect of a permanent unforseen reduction in the range over which agents are ambiguous.

Result 3.4. *For β sufficiently close but below one and all the other parameters in the intervals defined in Result 3.2, for any small enough $\overline{\mu} > 0$:*

$$\mathbb{V}_w(\overline{\mu}') > \mathbb{V}_w(\overline{\mu}) \quad \Pi_w(\overline{\mu}') < \Pi_w(\overline{\mu}) \quad R_w(\overline{\mu}') < R_w(\overline{\mu}) \quad \forall\ 0 \leq \overline{\mu}' < \overline{\mu} \tag{20}$$

where the w subscript refers to welfare-minimizing steady state value of each variable over the interval $[-\overline{\mu}, \overline{\mu}]$.

Proof. Proof in Appendix A. □

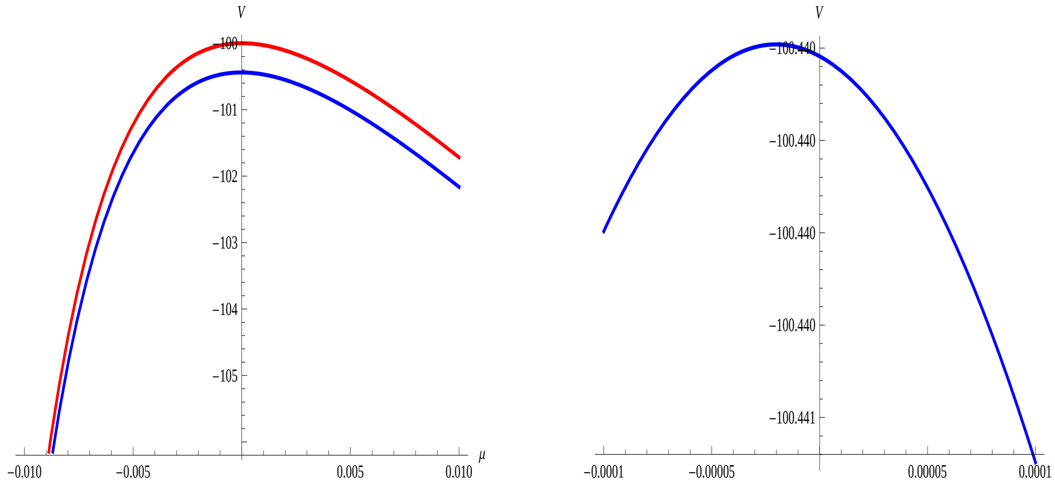

Figure 6: On the left pane, the first-best welfare function (red) and that in the absence of the subsidy (blue). On the right pane a detail of the welfare function without subsidy around zero.

3.3.2 The second-best model

So far, we have focused on an economy that, in the absence of ambiguity, will attain the first-best allocation. Removing the subsidy implies that, even if there is no ambiguity, the first-best allocation is not achievable, because distortions arising from monopolistic competition are not offset. This experiment casts light on what can happen when a richer set of frictions are added to the model.

As shown in the left panel of Figure 6, the welfare function of our calibrated model[14] shifts down and to the left, when we remove the subsidy ($\tau = 0$). The right hand panel of Figure 6 zooms into the welfare function for the model with no subsidy to show that welfare actually does not have a maximum in zero, but slightly shifted too the left. We can interpret the downward shift as a general efficiency loss, while the movement to the left - *i.e.* the fact that the welfare function's maximum is achieved for a negative value of μ) and a positive value of inflation - follows from the fact that a moderate positive inflation reduces the monopoly power of firms and enhances welfare, as also highlighted Ascari and Ropele (2009) among others, in models with rational expectations. Both movements are detrimental, the first for completely obvious reasons, the second because clearly the set of worst-case steady states over symmetric intervals around zero is bounded by $\mathbb{V}(0) < \mathbb{V}(\mu_{SB})$, where μ_{SB} refers to the level of μ that would achieve the second best allocation. In other words, even absent ambiguity the second-best cannot be achieved.

Notice that, when there is no subsidy, the first derivative of the welfare function evaluated at zero ambiguity is negative,$\mathbb{V}'_{NT}(0) < 0$. This means that for low levels of ambiguity the worst-case steady state will correspond to $\mu = \overline{\mu}$, *i.e.* worst-case inflation will be inefficiently low. For higher levels of $\overline{\mu}$, however, welfare drops faster for higher levels of inflation than it does for levels of inflation below the first-best

[14]These results seem absolutely robust to the specifics of the calibration.

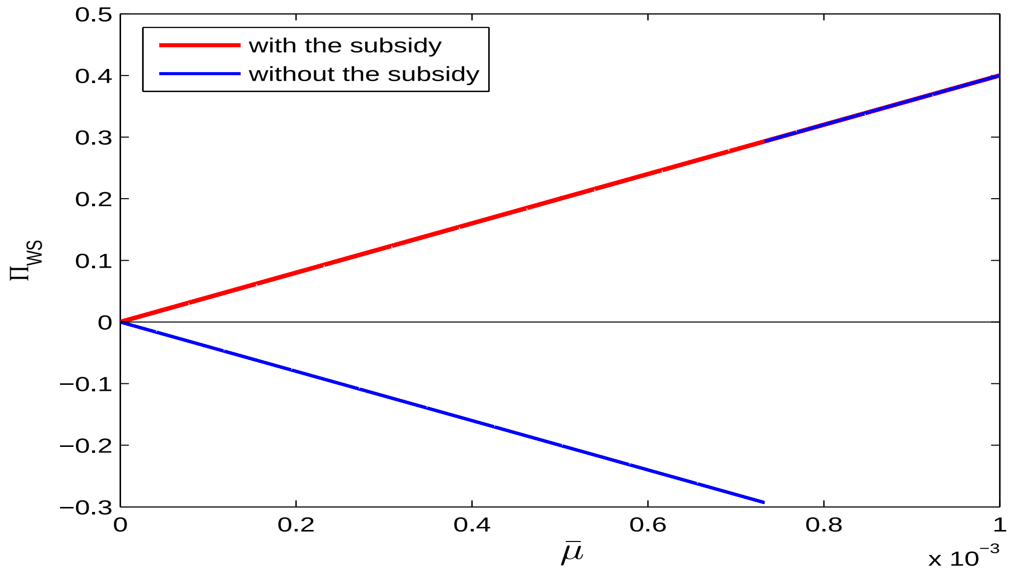

Figure 7: Annualized worst-case steady state inflation as a function of $\overline{\mu}$ in a model with production subsidy (red line) and a model without subsidy (blue).

benchmark, just as in the case with the subsidy: in that case the worst case reverts to being the one in which $\mu = -\overline{\mu}$ and inflation is "too high". In other words, in the absence of the subsidy, the worst-case steady state inflation can be both too high or too low, depending on the prevailing degree of ambiguity. Figure (7) graphically illustrates the point by showing the worst-case steady-state level of inflation as a function of $\overline{\mu}$ both in the presence and the absence of the subsidy.

At this stage, our model is too stylised to make sharp quantitative conclusions, however we can make two general points. First, it is entirely possible that discontinuities in the worst-case steady states for the model variables arise as $\overline{\mu}$ varies. Second, the effects of policy interventions might interact with ambiguity in a non-trivial way. Building on this, in the next Section we study the design of monetary policy in the presence of ambiguity.

4 Monetary Policy Design

In this section we analyse the design of monetary policy under ambiguity. We first study the case in which the policymaker takes the level of ambiguity about policy as given, while in the second subsection we consider the case in which the Central Bank can reduce ambiguity through transparency and better communication of its intentions and strategy.

4.1 For a given level of ambiguity $\overline{\mu}$

The presence of ambiguity in the model affects the characteristics of policy quite substantially with respect to the case in which there is no ambiguity. Crucially, the inflation response coefficient regains an important role in determining welfare, as we show in what follows. Moreover, if a policymaker is aware of the ambiguity, he/she can also choose an inflation target to minimise its effect. We will present the results below for the model in which the production subsidy is present, and will point out if and when the results are different when the subsidy is absent.

4.1.1 The role of the inflation response coefficient ϕ

In the model with the production subsidy and no ambiguity, any inflation response coefficient ϕ larger than one would deliver the first-best allocation (see Galí, 2008), both in steady state and, absent unwarranted monetary policy shocks, even period by period[15]. As a result, from a welfare perspective any value of $\phi > 1$ would be equivalent. Once ambiguity enters the picture, however, things change and the responsiveness of the Central Bank to inflation interacts with ambiguity in an economically interesting way.

First, it is possible to view a reduction in ambiguity and the responsiveness to inflation as substitutes in terms of welfare, which can be formalized as follows.

Result 4.1. *While parameter values are in the intervals defined in Result 3.2 and* $\overline{\mu}$ *is a small positive number, given any pair* $(\mu, \phi) \in [-\overline{\mu}, 0) \times (1, \infty)$*, for any* $\mu' \in [-\overline{\mu},\ 0)$ *there exists* $\phi' \in (1,\ \infty)$ *such that:*

$$\mathbb{V}(\mu, \phi') = \mathbb{V}(\mu', \phi) \tag{21}$$

And $\phi' \geq \phi$ *iff* $\mu' \geq \mu$.
A corresponding equivalence holds for $\mu \in (0,\ \overline{\mu}]$.

Proof. Proof in Appendix A. □

This result illustrates how the presence of ambiguity brings the inflation responsiveness coefficient to centre stage. The intuition behind this relation between responsiveness to inflation and ambiguity is the following. What matters for agents is the steady state level of inflation: if ambiguity is taken as given, the only way of getting close to its first-best value is for the central bank to respond much more strongly to deviations of inflation from its first-best level. A higher value of ϕ works as an insurance that the response to inflation will be aggressive, which acts against the effect of ambiguity about policy. At the same time, as Schmitt-Grohe and Uribe (2007) suggest, it is practically not very sensible to consider very high values for ϕ, for instance because of the possibility that a modest cost-push shock would cause the policy rate to hit the Zero Lower Bound. So a very high value for ϕ is ultimately not a solution.

Figure (8) illustrates Result 4.1 graphically for our preferred calibration. Basically, a higher responsiveness of the Central Bank to inflation flattens the welfare function with respect to μ, thus reducing the loss induced by ambiguity. As a result,

[15] Limited to the fact in this economy the only other shock is a technology shock.

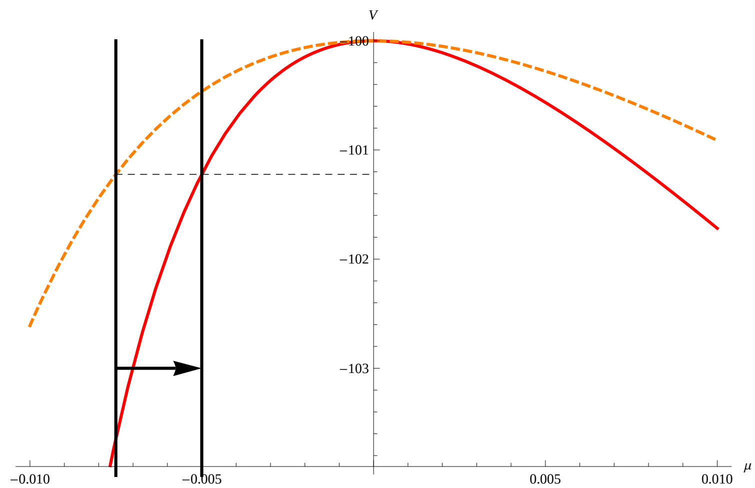

Figure 8: Welfare function for $\phi = 2$ (solid red line) and $\phi = 2.5$ (orange dashed line).

it is possible to find different combinations of ambiguity and inflation responsiveness which are welfare equivalent. We can also show that the impact of different values of ϕ is smaller for lower values of ambiguity, as the gap between the welfare functions of two economies, identical except for the value of ϕ, vanishes as μ approaches zero.

4.1.2 A different Taylor rule

In this section we investigate what a Central Bank can do if it takes the level of ambiguity about policy as given - and cannot or will not set ϕ to infinity. The first-best allocation is not attainable under these circumstances, but the policymaker can still intervene to improve welfare.

To begin our discussion, it is important to note that the first-best welfare can be achieved if and only if:

i. $\mu_{FB} \in [-\overline{\mu},\ \overline{\mu}]$

ii. $\mathbb{V}(\mu, \cdot) = \mathbb{V}(\mu_{FB}, \cdot) \quad \forall\ \mu \in [-\overline{\mu},\ \overline{\mu}]$.

Let us express the first derivative of the welfare function as:

$$\frac{\partial \mathbb{V}(\mu, \cdot)}{\partial \mu} = \frac{\partial \mathbb{V}(\mu, \cdot)}{\partial \Pi(\mu, \cdot)} \frac{\partial \Pi(\mu, \cdot)}{\partial \mu}. \tag{22}$$

We can think of $\frac{\partial \mathbb{V}(\mu,\cdot)}{\partial \Pi(\mu,\cdot)}$ as depending on the structure of the economy - for example, the degree of indexation, the existence of subsidies, the degree of price stickiness - and therefore being beyond the reach of monetary policy action. The policymaker can control the second component of the right-hand side of equation (22), $\frac{\partial \Pi(\mu,\cdot)}{\partial \mu}$, by either shrinking the range over which μ can vary or by increasing ϕ. That is because the choice of μ pertains to private sector, the Central Bank can only induce the private sector to make the best choice.

Note that our baseline case meets both conditions when $\overline{\mu} = 0$, *i.e.* there is no ambiguity. The conditions are met even for positive $\overline{\mu}$ in the limiting case in which

$\phi \to \infty$, as it makes $\frac{\partial \Pi(\mu,\cdot)}{\partial \mu} \to 0$[16]. So achieving the first-best allocation would only be possible if the welfare function was made completely flat over $[-\overline{\mu}, \overline{\mu}]$. Any finite value of ϕ would prevent the Central Bank from implementing it. This said, there is something policymakers can do to improve of the welfare level $\mathbb{V}(-\overline{\mu}, \cdot)$ which would prevail in this economy.

In very general terms, we consider an extra term in our Taylor rule that could possibly depend on some endogenous variables and can capture the idea that, if policymakers are faced with households that constantly underestimate the interest rate they set, they might naturally consider responding responding to the agents' distorted beliefs by increasing the interest rate they set above the level implied by the inflation level.

$$R_t = R_t^n \Pi_t^{\phi} e^{\delta(\cdot)} \tag{23}$$

It would be tempting to say that policymakers should use δ to increase the policy rate by the same amount by which the agents underestimate the policy rate. The intuition is not completely unwarranted, as Result 4.2 shows, but misses the fact that such policy action would change the worst-case scenario and would not be the most successful strategy.

Before getting to the determination of the optimal value for $\delta(\cdot)$, following the same logic as for Result 3.1, it is easy to show that this modification will only change the steady state level of inflation, which becomes:

$$\Pi(\mu, \delta(\cdot), \cdot) = e^{-\frac{\mu+\delta(\cdot)}{\phi-1}} \tag{24}$$

The rest of the model equations are unaffected and the changes in welfare come from the fact at a certain level of μ corresponds a different inflation level. In particular, what determines inflation, and ultimately welfare, is the sum of μ and δ, hence it is very easy to characterise the welfare function for this economy (using a subscript δ):

$$\mathbb{V}_{\delta}(\mu, \cdot) = \mathbb{V}(\mu + \delta(\cdot), \cdot) \quad \forall\ \mu \in [-\overline{\mu},\ \overline{\mu}]. \tag{25}$$

Graphically, δ shifts the welfare function horizontally.

The optimal level of $\delta(\cdot)$ as a function of the level of ambiguity $\overline{\mu}$ can then be easily computed numerically based on the following general characterization.

Result 4.2. *Assuming that $\mathbb{V}(\mu, \cdot)$ takes only real values over some interval $(-\overline{a}, \overline{a})$, is continuous, strictly concave and attains a finite maximum at $\mu = \mu_0 \in (-\overline{a}, \overline{a})$; if ϕ is fixed and $\overline{\mu} > 0$, then the optimal level of δ is pinned down by the following condition.*

$$\delta^*(\overline{\mu}): \qquad \mathbb{V}\left(-\overline{\mu} + \delta^*(\overline{\mu}), \cdot\right) = \mathbb{V}\left(\overline{\mu} + \delta^*(\overline{\mu}), \cdot\right) \tag{26}$$

Moreover, in the context of the economy described in Section 2, it is the case that:

$$0 < \delta^*(\overline{\mu}) < \overline{\mu}. \tag{27}$$

[16]Another situation in which the conditions are met even for positive levels of ambiguity is indexation, as it makes $\frac{\partial \mathbb{V}(\mu,\cdot)}{\partial \Pi(\mu,\cdot)} = 0$.

Proof. Proof in Appendix A. □

Intuitively, this result illustrates how the $\delta(\cdot)$ should be used to shift the welfare function sideways, to the point in which the worst case would move to the other extreme of the interval. In our economy, this implies charging a higher interest rate than warranted by inflation and the natural rate, supporting the intuitive claim made above. At the same time though, if one was to mechanically increase rates by the amount agents underestimate them (which would correspond to setting $\delta = \overline{\mu}$), that would induce the worst-case scenario to be one in which agents overestimate rates, thus making the correction miss its point. That is why the optimal level of δ is strictly positive and yet smaller than the amount by which agents underestimate the policy rate in the original model.

4.2 When the policymaker can affect $\overline{\mu}$

We now consider a situation in which the monetary policy maker is able to reduce the ambiguity about monetary policy by increasing transparency and improving communications. If it were possible, the policymaker should reduce the ambiguity to zero. In that case the agents' belief set would collapse to only one, rational, belief.

In the case of the model with the production subsidy, completely removing ambiguity would allow the attainment of the first-best allocation. In the model without the production subsidy, when there is no ambiguity the Central Bank should implement an inflation target, aiming to achieve the second-best steady state allocation - *i.e.* the highest level of steady-state welfare achievable in the model economy without the subsidy. It is easy to show that the following modification of the Taylor rule:

$$
\begin{aligned}
R_t &= R_t^n \left(\frac{\Pi_t}{\Pi_*}\right)^{\phi} \\
\Pi_* &\equiv e^{-\frac{\mu_{SB}}{\phi}} \qquad \mu_{SB} : \mathbb{V}'(\mu_{SB}) = 0
\end{aligned}
$$

will ensure that $\mathbb{V}_*{}'(0) = 0$, *i.e.* that the welfare function of the economy without the subsidy but with the inflation target[17] will achieve its maximum at $\mu = 0$.

As Figure 9 shows, the inflation target causes a shift to the right of the welfare function. By removing ambiguity and setting the inflation target to the level defined above, the Central Bank would achieve the best possible steady state in the absence of the production subsidy.

This well known policy-design strategy is proven to work, when ambiguity is completely removed. If high level of ambiguity persist after the implementation of the target, it might be possible that the inflation target defined above might be counterproductive[18].

[17] A star subscript will denote the welfare function when there is an inflation target but no subsidy.

[18] This is not to say that *any* inflation target would be counterproductive, but simply that the optimal inflation target should reflect the degree of ambiguity in the economy, i.e. the inflation target that would be optimal in the absence of uncertainty might not be so when ambiguity is accounted for (see Result 4.2 for an illustration of this point).

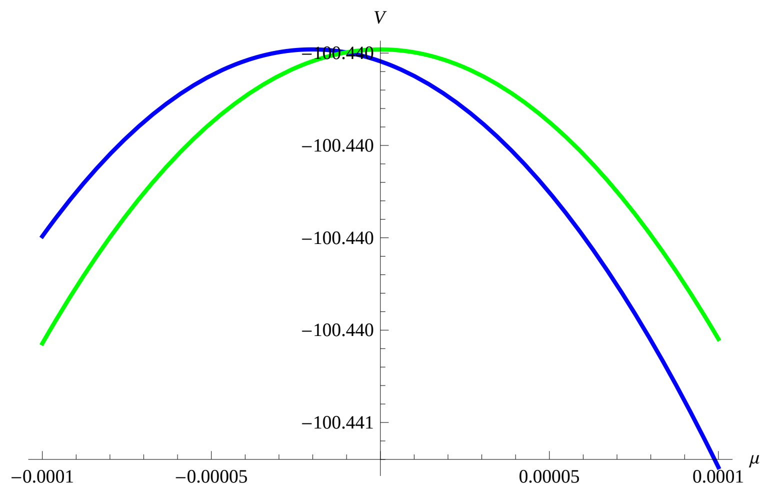

Figure 9: Welfare function for the second-best model with no inflation target in blue and with the optimal inflation target in green.

When the optimal inflation target is implemented, the worst-case will always correspond to high inflation, yet inflation will be higher than in the first-best, as Figure (11) shows. On the one hand, we know that in this class of models, higher inflation will drive up price dispersion reducing welfare. At the same time, though, in the absence of a production subsidy, it is well known that a moderate level of positive inflation reduces the adverse welfare effects of monopolistic competition, i.e. effectively reduces the markup. The target defined above exploits the benefit of moderate positive inflation to the fullest.

When ambiguity is too high, however, a target designed to work in an ambiguity-free economy might end up being counterproductive as Figure (10) shows. The basic intuitive reason is that the premise under which it is computed is that inflation would be zero in the absence of the target, while we know that inflation will not be zero under ambiguity. This does not mean that an inflation target could not be devised for an ambiguity-ridden economy, as the analysis in Section 4.1.2 suggests. Rather, it suggests that, ideally a Central Bank will want to reduce ambiguity as much as possible and if that proves not feasible that should be accounted for when setting an optimal target.

5 Transitional Dynamics - a primer

Given a steady state we can log-linearize around it in the usual way. As explained in Ascari and Ropele (2007), having price dispersion in steady state essentially results in an extra term in the Phillips Curve. Appendix B presents the log-linear approximation around a generic steady state indexed by μ, simply setting $\mu = -\overline{\mu}$ one obtains the log-linear approximation to the worst-case steady state.

For a moment, let us consider the case in which ambiguity is constant and the only shock hitting the economy is a standard TFP shock. The fact that the Central Bank tracks the natural rate of interest makes the dynamic solution extremely simple

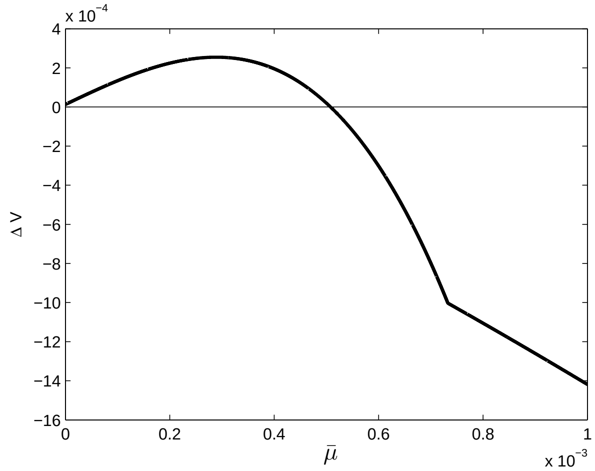

Figure 10: Difference in the worst-case welfare between the case in which an inflation target is implemented and that in which is not for different values $\overline{\mu}$. In formulas: $\min_{\mu\in[-\overline{\mu},\overline{\mu}]} \mathbb{V}_*(\mu,\cdot) - \min_{\mu\in[-\overline{\mu},\overline{\mu}]} \mathbb{V}_{NT}(\mu,\cdot)$

in that only consumption/output would respond one for one to a TFP shock, while all other variables are constant. This is true for any given level of ambiguity. An immediate consequence is that while welfare would increase with a positive TFP shock, the output gap would be constant at at its steady state level which depends on the degree of inefficiently high inflation in the economy. In other words the response of the central bank would be optimal in a dynamic sense but would not get the economy closer to first-best. Moreover, it is essential to note that the determinacy region of the solution described above would shrink with the degree of ambiguity. As Ascari and Ropele (2009), Kiley (2007) and Coibion and Gorodnichenko (2011) have documented, non-zero trend inflation has important effects on the determinacy region. In particular, other things equal, it tends to drive up the level of inflation responsiveness ϕ that guarantees determinacy to a value larger than unity. This has interesting implications in our setting, because trend inflation in turn depends on ϕ. In other words, a low level of ϕ generates, for a given level of ambiguity, a higher level of trend inflation, which in turn requires a higher ϕ to keep the model economy in the determinacy region. So, once more, ambiguity makes the Central Bank's responsiveness to inflation all the more crucial.

If we then consider the transmission of other shocks, a key parameter to understand the economy's response is the coefficient on marginal costs in the Phillips Curve, sometimes referred to as *the slope of the Phillips Curve*. The presence of ambiguity changes the Phillips Curve in our model in two ways: first, there will be an extra term relative to the standard textbook case due to price dispersion, and, second, the *slope* of the curve falls with higher levels of worst-case steady state inflation. Hence, the Central Bank's attitude towards inflation ϕ will, in the context of our model, also impact the transmission of shocks via the responsiveness of inflation to the marginal cost or the output gap.

Up to this point we have restricted our analysis to a constant level of ambiguity.

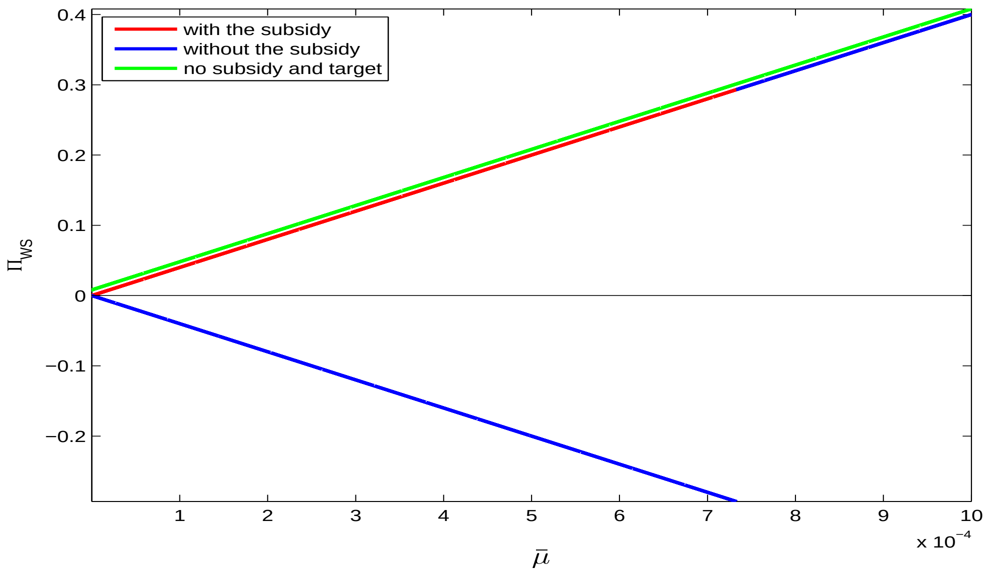

Figure 11: Annualized worst-case steady state inflation as a function of $\overline{\mu}$. Model with production subsidy in red, without neither subsidy nor inflation target in blue and without subsidy but with inflation target in green.

We now consider a transition between two levels of ambiguity, *i.e.* a situation in which $\overline{\mu}$ is permanently and unexpectedly reduced from .00125 to 0. This amounts to reducing the ambiguity around the annualized policy rate from ± 50 to ± 0 basis points. Studying a permanent unexpected shift is desirable in at least a couple of ways. First, it dispenses from making an *ad hoc* assumption on a known process for the evolution for ambiguity. Second, it captures the spirit of a policy announcement that changes the agents' level of confidence permanently. Otherwise we would end up capturing the fact that agents make their consumption decision fully aware that the economy would eventually drift back to the initial level of ambiguity.

To study the transitional dynamics, we log-linearize the economy around $\mu = -.00125$ and initialize at the old steady state values (in percent deviations from the new). Figure 12 presents the responses for some of the variables of the model. It shows how consumption, which equals output, increases while the interest rate and inflation fall. Along the way welfare increases and the output gap closes. On the pricing side, price dispersion subsides, as a result of the fact that optimally set prices are closer to the prevailing average price (which follows from the lower level of inflation). The dynamics of hours is a bit more striking. A lower level of ambiguity, makes agents better off, so in the new steady state they want to consume more goods, and enjoy more leisure. Price stickiness, however, generates the dynamics depicted in Figure 12. Demand for consumption jumps up very quickly, while Δ_t, as a result of the Calvo friction, falls quite slowly. This hinders labor productivity and, as a result, hours actually jump up on impact before starting to slowly converge to the new steady state. A combination of high hours and high consumption, together with sticky prices, drive real wages temporarily above their new steady state level.

Notice that in the new equilibrium ambiguity is zero, so the policy rate r and the rate on the basis of which the agents take their decision $\tilde{r}$ coincide.

Clearly the transition is quite fast and the changes numerically small, since the model has a minimum amount of frictions and the change in ambiguity is relatively small too. However at this point we are not trying to provide a quantitative assessment of the effects of this permanent unforeseen reduction in ambiguity but rather to build our intuition.

6 Conclusions

We study a prototypical new-Keynesian model in which agents are averse to ambiguity, or Knightian uncertainty, and where the ambiguity regards the monetary policy rule. Ambiguity is welfare reducing, but in our framework the policymaker can dampen its impact, if he or she can credibly implement policy actions and communications that reduce such ambiguity. Effectively, we model the ambiguity as a further component of the monetary policy rule that can take values in a range the agents entertain as possible, but to which they cannot assign probabilities, and we investigate the role for policy actions that can credibly reduce this range.

Already when doing comparative statics with this model, interesting results emerge. First, a reduction in the parameter governing steady state ambiguity - i.e. a tightening of the range of models agents consider possible but to which they cannot assign probabilities - results in a fall in inflation and the policy rate, and an increase in welfare. Indeed, clearer and more transparent Central Bank communications since the 1980s plausibly reduced ambiguity about monetary policy and could, in principle, be a concurrent explanation for the Great Moderation. Second, the policymaker's responsiveness to inflation regains centre stage. In particular, if the Central Bank is very responsive to inflation, it will dampen the negative effects of welfare brought about by ambiguity. Similarly, the welfare benefits of a reduction in ambiguity also depend of the degree of responsiveness to inflation of the Central Bank: with lower ambiguity, the costs associated with a lower responsiveness to inflation are also reduced. Third, the introduction of an inflation target can be beneficial, but the optimal value of the target crucially depends on the existing amount of ambiguity. So, if the Central Bank misjudges the degree of ambiguity about his behaviour, or if it does not have enough credibility, then it could be that an inflation target is welfare-reducing. Our qualitative results suggest that the introduction of the inflation target is beneficial only if there is a limited amount of uncertainty about the implemented policy action, while it becomes damaging above a certain threshold of uncertainty. This is in line with evidence on the minor effectiveness of inflation targeting in emerging market economies, due to the lack of credible institutions.

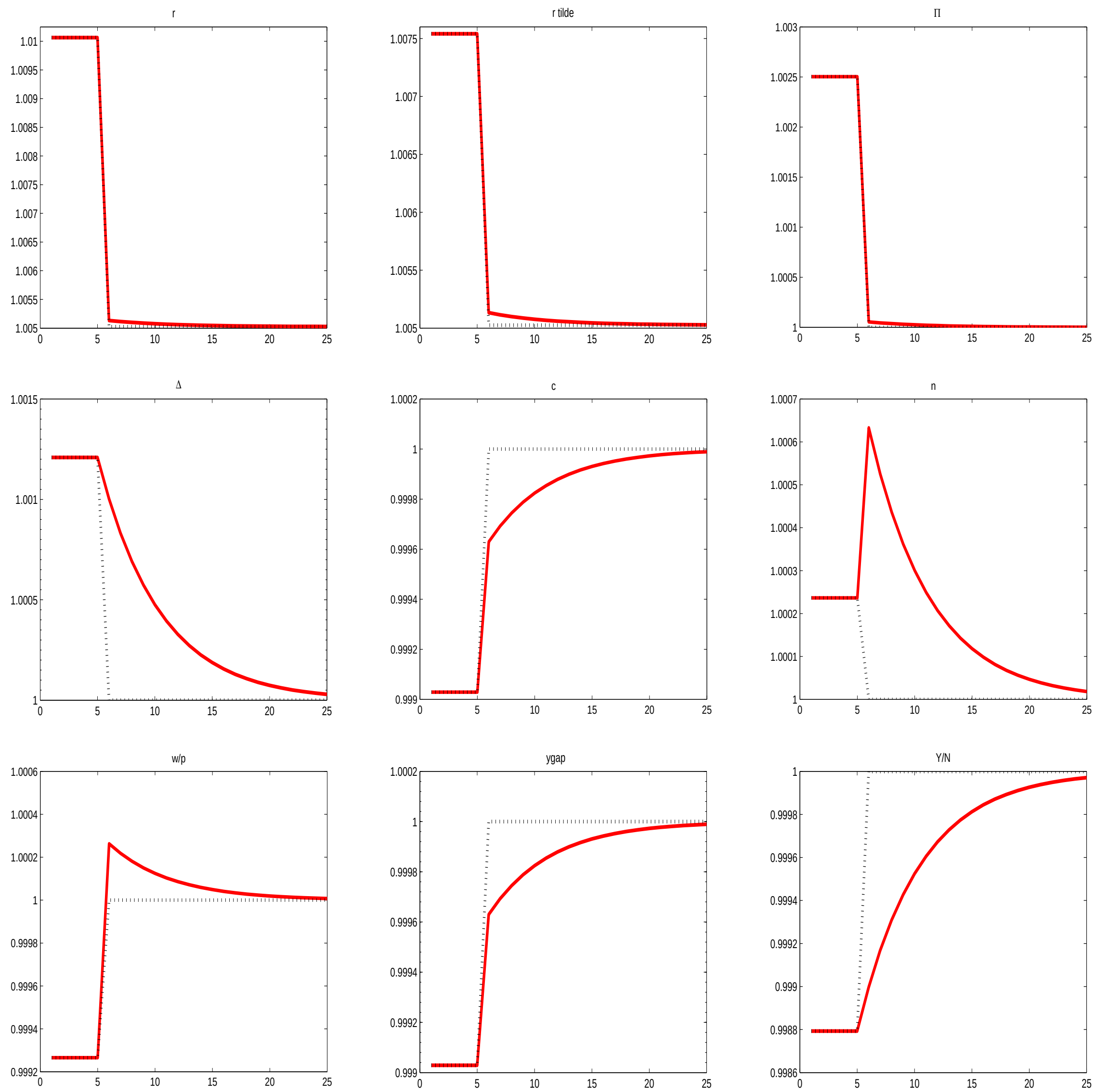

Figure 12: Responses of the policy rate, the interest rate perceived by the agents and inflation (top row), output/consumption, price dispersion, consumption and hours (middle row), real wages, the output gap and labor productivity (bottom row) to a permanent and unexpected reduction in ambiguity, in the model with production subsidy.

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A Proofs

Proof of Result 3.1

In steady state, equation 6 becomes:

$$1 = \frac{\beta \tilde{R}(\mu, \cdot)}{\Pi(\mu, \cdot)} \tag{28}$$

From the Taylor rule we get:

$$\tilde{R}(\mu, \cdot) = R^n(\mu, \cdot)\,\Pi(\mu, \cdot)^{\phi} e^{\mu} = \frac{1}{\beta}\Pi(\mu, \cdot)^{\phi} e^{\mu} \tag{29}$$

Combining the two, delivers the first part of the result.
The second follows immediately by plugging the resulting expression for inflation into the Taylor rule.
The inequalities result by noting that $\phi > 1$. □

Proof of Result 3.2

$\mathbb{V}(\mu, \cdot)$, as defined in equation 19, is continuously differentiable around zero. Direct computation, or noting that the first-best allocation is attained in our model when $\mu = 0$, shows that $\frac{\partial \mathbb{V}(\mu, \cdot)}{\partial \mu} = 0$.
Direct computation also delivers:

$$\left.\frac{\partial^2 \mathbb{V}(\mu, \cdot)}{\partial \mu^2}\right|_{\mu=0} = -\frac{\theta\left((\beta-1)^2\theta + \epsilon(\beta\theta-1)^2(1+\psi)\right)}{(1-\beta)(\theta-1)^2(\beta\theta-1)^2(\phi-1)^2(1+\psi)} \tag{30}$$

All the terms are positive given the minimal theoretical restrictions we impose, hence the second derivative is strictly negative and there are no interior minima in a neighbourhood of zero. □

Proof of Result 3.3

Direct computation shows that the third derivative evaluated at $\mu = 0$ can be expressed as:

$$\left.\frac{\partial^3 \mathbb{V}(\mu, \cdot)}{\partial \mu^3}\right|_{\mu=0} = \frac{\epsilon(2\epsilon-1)\theta(1+\theta)}{(1-\beta)(1-\theta)^3(\phi-1)^3} + \mathcal{R}(\beta) \tag{31}$$

Where, given our parameter restrictions, the first term on the RHS is positve and $\mathcal{R}(\beta)$ is a term in β such that $\lim_{\beta \to 1^-} \mathcal{R}(\beta) = 0$.
Hence, $\lim_{\beta \to 1^-} \left.\frac{\partial^3 \mathbb{V}(\mu, \cdot)}{\partial \mu^3}\right|_{\mu=0} = +\infty$.
Moreover, $\partial \left(\left.\frac{\partial^3 \mathbb{V}(\mu, \cdot)}{\partial \mu^3}\right|_{\mu=0} \right) / \partial \beta$ exists, which ensures continuity of the third derivative in β. Hence the third derivative is positive for any β sufficiently close to but below unity.
A third-order Taylor expansion around zero can be used to show that:

$$\mathbb{V}(\mu_0, \cdot) - \mathbb{V}(-\mu_0, \cdot) = \left.\frac{\partial^3 \mathbb{V}(\mu, \cdot)}{\partial \mu^3}\right|_{\mu=0} \frac{2\mu_0^3}{6} + o(\mu_0^4), \tag{32}$$

which is positive for a generic, positive but small value μ_0 thus showing that, the steady state value function attains a lower value at $-\mu_0$ than it does at at μ_0. This, combined with the absence of internal minima (Result 3.2), delivers our result. $\square$

Proof of Result 3.4

The first inequality follows immediately, as a weak inequality, by considering that $\mathbb{V}_w(\overline{\mu}')$ is the minimum value of welfare on a smaller set than$\mathbb{V}_w(\overline{\mu})$.

The strict inequality follows from the characterization of the worst case in Results 3.2 and 3.3; in particular from the fact that $\mathbb{V}_w(\overline{\mu}) = \mathbb{V}(-\overline{\mu}, \cdot)$ and that $\left.\frac{\partial \mathbb{V}(\mu,\cdot)}{\partial \mu}\right|_{\mu<0} > 0$ in the vicinity of $\mu = 0$.

For what concerns inflation, given the formula in Result 3.1, $\phi > 1$ and given that the worst case corresponds to $\mu = -\overline{\mu}$, it is immediate to verify that $\frac{\overline{\mu}}{\phi-1} > \frac{\overline{\mu}'}{\phi-1}$. $\phi > 1$ also ensures that the Taylor rule is increasing in inflation more than one for one, which delivers the last inequality. $\square$

Proof of Result 4.1

Inspection reveals that μ and ϕ only enter steady-state welfare through the steady-state inflation term $\Pi(\mu, \cdot) = e^{\frac{\mu}{1-\phi}}$. It follows immediately that, for a given μ', $\phi' = 1 + \frac{(\phi-1)\mu}{\mu'}$ implies that (μ, ϕ') is welfare equivalent to (μ', ϕ). $(\mu, \mu') \in [-\overline{\mu}, 0) \times [-\overline{\mu}, 0)$ ensures that $\mu' \cdot \mu > 0$ and so $\phi' \in (1, \infty)$ for any $\phi > 1$. The inequalities follow immediately from the definition of ϕ' given above and the fact that both μ and μ' have the same sign.

A similar argument would go through for $(\mu, \mu') \in (0, \overline{\mu}] \times (0, \overline{\mu}]$. $\square$

Proof of Result 4.2

First we define μ_0 to be the value that maximizes $\mathbb{V}(\mu, \cdot)$. Strict concavity ensures it is unique.

A number of different cases then arise:

1. $\mu_0 \in (-\overline{\mu}, \overline{\mu})$: then $\mathbb{V}'(-\overline{\mu}, \cdot) > 0 > \mathbb{V}'(\overline{\mu}, \cdot)$[19]

   a. $\mathbb{V}(-\overline{\mu}, \cdot) < \mathbb{V}(\overline{\mu}, \cdot)$. Together with strict concavity this implies that $\mu_{ws} = -\overline{\mu}$. Then there exists a small enough $\delta > 0$ such that

   $$\mathbb{V}(-\overline{\mu}, \cdot) < \mathbb{V}(-\overline{\mu} + \delta, \cdot) < \mathbb{V}(\overline{\mu} + \delta, \cdot) < \mathbb{V}(\overline{\mu}, \cdot)$$

   . So now the worst case $\mu'_{ws} = -\overline{\mu} + \delta$ generates a higher level of welfare. The worst-case welfare can be improved until the second inequality above holds with equality. Continuity ensures such a level of δ^* exists. Any value of $\delta > \delta^*$ will, however, make welfare in the worst case decrease, and the second inequality above would reverse the sign.

[19]With an abuse of notation we use derivatives here but we do not need differentiability. We just need the function to be strictly increasing and strictly decreasing for values of μ respectively smaller and larger than μ_0, which is ensured by strict concavity.

b. $\mathbb{V}(-\overline{\mu},\cdot) > \mathbb{V}(\overline{\mu},\cdot)$. Together with strict concavity, this implies that $\mu_{ws} = \overline{\mu}$. Then there exists a small enough $\delta < 0$ delivering the same as above

c. $\mathbb{V}(-\overline{\mu},\cdot) = \mathbb{V}(\overline{\mu},\cdot)$. There is no room for improvement. Any $\delta \neq 0$ would lower the worst-case welfare.

2. $\mu_0 \geq \overline{\mu}$. Strict concavity implies that $\mathbb{V}(-\overline{\mu},\cdot) < \mathbb{V}(\overline{\mu},\cdot)$. Hence $\mu_{ws} = -\overline{\mu}$. For all $0 \leq \delta \leq \mu_0 - \overline{\mu}$

$$\mathbb{V}(-\overline{\mu},\cdot) < \mathbb{V}(-\overline{\mu}+\delta,\cdot) < \mathbb{V}(\overline{\mu}+\delta,\cdot) \leq \mathbb{V}(\mu_0,\cdot)$$

For δ just above $\mu_0 - \overline{\mu}$ we fall in case 1a above.

3. $\mu_0 \leq -\overline{\mu}$. Strict concavity implies that $\mathbb{V}(-\overline{\mu},\cdot) > \mathbb{V}(\overline{\mu},\cdot)$. Hence $\mu_{ws} = \overline{\mu}$. For all $\mu_0 - \overline{\mu} \leq \delta \leq 0$

$$\mathbb{V}(\mu_0,\cdot) \geq \mathbb{V}(-\overline{\mu}+\delta,\cdot) > \mathbb{V}(\overline{\mu}+\delta,\cdot) > \mathbb{V}(\overline{\mu},\cdot)$$

For δ just above $\mu_0 - \overline{\mu}$ we fall in case 1b above.

To prove the second part, note that in our baseline economy the welfare function is continuous and strictly concave in the vicinity of zero and attains a maximum at $\mu_0 = 0$. Since $0 \in (-\overline{\mu}, \overline{\mu})$ for all strictly positive $\overline{\mu}$, then we are in case 1a. This proves that $\delta^*(\overline{\mu}) > 0$. Suppose now that $\delta^*(\overline{\mu}) \geq \overline{\mu}$. That would push the argmax of the welfare function outside (or on the boundary) of $[-\overline{\mu}, \overline{\mu}]$, which can never be optimal given strict concavity (similar arguments to cases 2 and 3 above). □

B Log-Linearized Equations

The following equation describe the dynamics of the variables of interest around the worst-case steady state:

$$
\begin{aligned}
c_t &= \mathbb{E}_t c_{t+1} - (\tilde{r}_t - \mathbb{E}_t \pi_{t+1}) && (33)\\
w_t &= c_t + \psi n_t && (34)\\
\pi_t &= \kappa_0(\mu,\cdot)\, mc_t + \kappa_1(\mu,\cdot)\,\mathbb{E}_t \widehat{F2}_{t+1} + \kappa_2(\mu,\cdot)\,\mathbb{E}_t \pi_{t+1} && (35)\\
r_t &= r_t^n + \phi \pi_t && (36)\\
\tilde{r}_t &= r_t + \mu_t && (37)\\
mc_t &= w_t - a_t && (38)\\
y_t &= a_t - \tilde{\Delta}_t + n_t && (39)\\
c_t &= y_t && (40)\\
\hat{\Delta}_t &= \Pi(\mu,\cdot)^{\epsilon}\theta \hat{\Delta}_{t-1} + \epsilon \left(\Pi(\mu,\cdot)^{\epsilon}\theta - (1 - \Pi(\mu,\cdot)^{\epsilon}\theta) \frac{\theta}{\left(\frac{1}{\Pi(\mu,\cdot)}\right)^{\epsilon-1} - \theta} \right) \pi_t && (41)\\
\widehat{F2}_t &= (\epsilon-1)\beta\theta\Pi(\mu,\cdot)^{\epsilon-1}\mathbb{E}_t\pi_{t+1} + \beta\theta\Pi(\mu,\cdot)^{\epsilon-1}\mathbb{E}_t\widehat{F2}_{t+1} && (42)
\end{aligned}
$$

Where $\kappa_0(\mu,\cdot) \equiv \frac{\left(\left(\frac{1}{\Pi(\mu,\cdot)}\right)^{\epsilon-1} - \theta\right)(1-\beta\theta\Pi(\mu,\cdot)^{\epsilon})}{\theta}$, $\kappa_1(\mu,\cdot) \equiv \beta\left(\left(\frac{1}{\Pi(\mu,\cdot)}\right)^{\epsilon-1} - \theta\right)(\Pi(\mu,\cdot) - 1)\,\Pi(\mu,\cdot)^{\epsilon-1}$ and $\kappa_2(\mu,\cdot) \equiv$, $\kappa_2(\mu,\cdot) \equiv \beta\Pi(\mu,\cdot)^{\epsilon-1}\left(\theta(\epsilon-1)(\Pi(\mu,\cdot)-1) + (\epsilon(1-\Pi(\mu,\cdot)-1))\left(\frac{1}{\Pi(\mu,\cdot)}\right)^{\epsilon-1}\right)$.