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On the indeterminacy of the meter

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Abstract In the International System of Units (SI), 'meter' is defined in terms of seconds and the speed of light, and 'second' is defined in terms of properties of cesium 133 atoms. I show that one consequence of these definitions is that: if there is a minimal length (e.g., Planck length), then the chances that 'meter' is completely determinate are only 1 in 21,413,747. Moreover, we have good reason to believe that there is a minimal length. Thus, it is highly probable that 'meter' is indeterminate. If the meter is indeterminate, then any unit in the SI system that is defined in terms of the meter is indeterminate as well. This problem affects most of the familiar derived units in SI. As such, it is highly likely that indeterminacy pervades the SI system. The indeterminacy of the meter is compared and contrasted with emerging literature on indeterminacy in measurement locutions (as in Eran Tal's recent argument that measurement units are vague in certain ways). Moreover, the indeterminacy of the meter has ramifications for the metaphysics of measurement (e.g., problems for widespread assumptions about the nature of conventionality, as in Theodore Sider's *Writing the Book of the World*) and the semantics of measurement locutions (e.g., undermining the received view that measurement phrases are absolutely precise as in Christopher Kennedy's and Louise McNally's semantics for gradable adjectives). Finally, it is shown how to redefine 'meter' and 'second' to completely avoid the indeterminacy.

Keywords Meter · Planck length · Measurement · Indeterminacy · Metaphysics · Measure phrases

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0 Introduction

The International System of Units (SI) is the global standard for measurement. As of 2017, it has been adopted by every nation on Earth except Myanmar, Liberia, and the United States of America as its official system of measurements, and even in these exceptions, it is unofficially adopted by scientists, engineers, and just about anyone else who is interested in precise measurement. Moreover, it is not as if these rogue nations *reject* the SI system—they just do not take the units of the SI system to be their official units of measurement. (For example, the United States of America uses miles.) The fundamental SI unit of length is the meter. Of course, one can define derivative units in terms of meters (e.g., nanometer, kilometer, inch, and megaparsec); for example, since 1959, an inch has been defined as exactly 0.0254 m. Thus, even the rogue nations accept what I will call a *standard* unit of length (i.e., any unit of length definable in terms of the meter).

It is tempting to think that the meter is perfectly determinate. Indeed, this seems to be a common view amongst those who even pause to consider the issue.[1] However, there is a good—although not definitive—reason to think that the meter is indeterminate. It follows that any standard unit of length is indeterminate as well. To be more precise, it is *very likely* that the meter is indeterminate; thus, probably, any standard unit of length is indeterminate as well.

The indeterminacy of the meter has significant consequences for several philosophical disputes. For example, in Theodore Sider's recent influential book on metaphysical disputes, he introduces the notion of structure, which is supposed to be a generalization of naturalness. Sider makes several claims about the features of structure, but two of them are that the linguistic expressions that are indispensible for formulating our best physical theories are structural, and what is structural is completely determinate.[2] Another example is the received view on the semantics of measure phrases. The semantics defended by Christopher Kennedy and Louise McNally for gradable adjectives entails that phrases like 'one meter long' are perfectly determinate.[3] If 'meter' is indeterminate, then some central views in metaphysics and semantics are unacceptable. Before presenting the argument, we need to get clear on the SI system of units and discuss what it is for something to be a minimal length.

1 The meter

Over the last sixty years, the official definition of 'meter' has changed several times. Those philosophers influenced by Saul Kripke but unfamiliar with the actual history of how 'meter' has been defined might think that 'meter' is defined by as the length

1 Examples include Kennedy and McNally (2005b) and Sider (2011) For opposition, see Tal (2011).

2 Sider (2011: pp. 19–20, 2011: p. 137).

3 Kennedy and McNally (2005b) and Kennedy (2007).

of a particular physical rod dubbed *the standard meter*.[4] However, when Kripke used this example in his 1970 lectures that were published as *Naming and Necessity*, it was already a decade out of date. From 1960 to 1983 'meter' was defined as 1,650,763.73 wavelengths of the orange-red emission line in the electromagnetic spectrum of the krypton-86 atom in a vacuum. Since 1983, the official definition of 'meter' has been:

(Meter) One meter = the distance light travels in 1/299,792,458 of a second in a vacuum.

That is, (Meter) defines 'meter' in the SI system of units, which enjoys almost universal acceptance (and even where it is not officially recognized, one can define the recognized units of length in terms of meters). According to (Meter), 'meter' is defined, in part, in terms of seconds. So the next question to ask is: what is a second?

The history of the official definition of 'second' is almost as convoluted as that of 'meter', but the current definition of 'second' is:

(Second) One second = the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of a cesium 133 atom at rest at a temperature of 0 K.

This definition is considerably more complex than (Meter). Cesium atoms, like all atoms, have electrons and a nucleus composed of neutrons and protons; the electrons generate a magnetic field, with which the nucleus interacts. This field generates what are called hyperfine structure effects. One of these effects is a transition associated with splitting the energy levels of the electrons. When this transition occurs while the cesium 133 atom is in its ground state at 0 K, the cesium atom emits electromagnetic radiation. This radiation has a certain wavelength and a certain period, which is the time it takes to oscillate once. A second is defined as the length of time of 9,192,631,770 of these periods. In what follows, I use 'cesium period' as an abbreviation for 'period of the radiation corresponding to the transition between the two hyperfine levels of the ground state of a cesium 133 atom at rest at a temperature of 0 K'. It is these official definitions of 'meter' and 'second' that are in play throughout what follows until the final section where I suggest two new definitions.

To return to our topic, when we put (Meter) and (Second) together, we derive the following identity:

(Meter*) One meter = the distance light travels in the duration of 9,192,631,770/299,792,458 cesium periods.

How many cesium periods is that? Between 30 and 31. In fact, it is 30 and 198,858,030/299,792,458 cesium periods (roughly, 30.6633189884 cesium periods); or, to use the reduced fraction, it is 30 and 14,204,145/21,413,747 cesium periods.

To be clear: (Meter*) is not a definition of 'meter'. It is a logical consequence of the definitions of 'meter' and of 'second'. We can already see a potential problem. For 'meter' to be completely determinate, it had better be the case that every single cesium 133 atom at 0 K transitions in the way in question every single time in exactly

[4] See Kripke (1972/1980: pp. 54–63, 75, 96, 107). Kripke borrows the example from Wittgenstein, who used it in Wittgenstein (1953: section 50). Since Kripke's work, the example has often been used in debates about whether there are a priori contingent truths; that topic is not our focus here.

the same amount of time, no matter how precise we are about an "amount of time". This assumption built into the official definition of 'meter' might be implausible. Nevertheless, in the remainder of the paper, I grant it for the sake of bringing out a different (and what I take to be more fundamental) problem.

2 The minimal length

The *Planck length* is defined as the square root of the product of the gravitational constant and the reduced Planck's constant over the speed of light cubed. We can measure all three of these fundamental physical constants, and, so, we can calculate the value of the Planck length, which turns out to be about 1.616199×10^{-35} m. Without the benefit of scientific notation, Planck length is about 0.00000000000000000000000000000000001616199 m. (There is a gigantic margin of error, which is about $.000097 \times 10^{-35}$ m due to the margin of error in measuring the gravitational constant.)[5]

Related to the Planck length is the *Planck time*, which is the time it takes light in a vacuum to travel one Planck length.[6] It turns out to be about 5.391×10^{-44} s, or 0.00000000000000000000000000000000000000000005391 s. One can derive the Planck time from the same three fundamental constants (the square root of the product of the gravitational constant and the reduced Planck's constant over the speed of light to the fifth).[7]

When discussing Planck length or Planck time, one has to be extremely careful to get the physics right. What, then, is their significance? The short answer is: we don't know. Planck length is an interesting length for one because it is defined entirely in terms of physical constants rather than arbitrary features of aspects of the universe

[5] In the interest of clarity, I avoid sarcasm and irony—the margin of error is very large compared to the numerical value of the constant in question.

[6] Strictly speaking, the term 'Planck time' is a misnomer. Space is associated with length (or distance—recall that these terms are synonyms), and length is a quantitative relation among points in space. One can define various units for measuring length, like the meter, which is the topic of the present investigation. This three-part distinction between space, length (a quantitative relation among points in space), and meter (a unit for measuring length) is essential to respect in order to avoid confusion. Notice that the most famous choice for a minimal length is the Planck length. The term 'length' is in the name, and this is exactly right. However, when we come to the term 'Planck time' we see immediately that there is a problem. For time, we have the same three-part distinction between time, duration (a quantitative relation among points in time), and second (a unit for measuring duration). The Planck length is a particular length—that is it is a particular value of the relation among points in space. Hence, the phrase 'Planck length' is appropriate. On the other hand, the temporal analog of the Planck length is called the Planck *time*, but it should be clear now that this expression is confused. It should be 'Planck duration', not 'Planck time'. The phrase 'Planck time' would be most intuitively interpreted as referring to either a particular dimension (e.g., 'real time' or 'imaginary time') or perhaps a particular temporal point, just like 'tea time'. Unfortunately, it seems like the term 'Planck time' is firmly entrenched, but is important for the reader to keep in mind that it refers to a certain duration, not a certain instant or moment in time.

[7] A value that is about midway between the Planck time and the current age of the universe (on a logarithmic scale) is on the order of 100 femtoseconds (a femtosecond is 10^{-15} s), which is about the duration of one pulse of our most sophisticated lasers today. The shortest time interval that we are capable of measuring right now is only about 10,000 times shorter at 12 attoseconds (an attosecond is 10^{-18} s); see Koke et al. (2010). There are 26 orders of magnitude between an attosecond and the Planck time.

(e.g., arbitrary numbers of cycles of light or the distance light travels in some arbitrary period). Given the significance of the gravitational constant (i.e., the strength of the gravitational force), the reduced Planck constant (i.e., the smallest possible change in angular momentum), the speed of light (i.e., the invariant velocity), and the fact that the Planck length is the natural unit of length that is defined in terms of these constants, it has a *natural* significance. In other words, if one takes the gravitational constant, the reduced Planck constant, and the speed of light as units of force, angular momentum (action), and velocity, respectively, to define a system of units (often called *natural* units) then the Planck length is the unit of length and the Planck time is the unit of duration in this system.[8] In a system of this sort, many natural laws have simpler formulations. Beyond that, there is nothing significant about the Planck length that one can glean from its definition alone.

The Planck length is often said to be a *minimal* length and the Planck time a minimal duration. However it is rarely made clear what is meant by these claims. Neither of the two major traditions in contemporary physics, Quantum Mechanics (and its development into Quantum Field Theory and the Standard Model of Particle Physics) and Relativity (and its development from Special to General and into the Standard Model of Cosmology), imply anything in particular about the Planck length, nor does either one by itself imply that there is a minimal length. Instead, the definition of the Planck length borrows from each theory. Quantum Mechanics (QM) and General Relativity (GR) are notorious for being incompatible, but the incompatibility is rather subtle and complex. It is not as if one of them states that p and the other states that not p, which would make them just flat inconsistent. Instead, one can use parts of one and parts of the other to get meaningful results (as many of the arguments below will illustrate). However, if one uses the wrong parts, the one gets nonsensical results.[9]

The Planck length has a certain significance that derives from QM and GR together—it is significant because one can argue from certain aspects of QM and GR that there is a minimal length (in a sense to be clarified below) and that it is near the Planck length (by ‘near’ I mean within a couple of orders of magnitude). Moreover, the Planck length is the scale below around which one needs a successor to both QM and GR to figure out what happens (i.e., the nonsensical answers one sometimes gets from combining QM and GR occur for phenomena at the Planck length and smaller). There are many theories that are designed to incorporate some aspects of QM and some aspects of GR without causing these problems. It is common to call these theories of *quantum gravity* (although that can be misleading because ‘quantum gravity’ sometimes refers to specific ones of these theories). They include loop quantum gravity, superstring theory, noncommutative geometry, causal sets, and doubly

[8] Note that this Planck scale is not the only one that could be called natural. The Compton scale (defined in terms of electron mass and electron charge) is as well; see Sidharth (2006) for discussion. The Planck length attracts much more attention than the Planck time, but see He (2009) and Wetterich (2012) on Planck time.

[9] QM and *special* relativity are compatible, and Quantum Field Theory (QFT) is usually thought of as their unification. See Callender and Huggett (2001: pp. 3–13) and Rovelli (2004) for discussion.

special relativity (I discuss some of these below).[10] Many of these successor theories imply that there is a minimal length (often in very strong ways) and many also imply that it is near the Planck length. So, the significance of the Planck length is subtle and depends on whether one is considering just its definition, some aspects of QM and GR, or quantum gravity.[11] As such it can be misleading to say that a theory of quantum gravity entails that the Planck length is a quantum of spacetime—it is better to say that the theory entails that there is a minimal length and that it is near the Planck length. For this reason and the fact that a more general presentation will be beneficial later, I focus on whether there is a minimal length, regardless of whether it happens to be identical to the Planck length. I designate the minimal length ℓ_M.

There are several things one might mean by saying that ℓ_M is the minimal length or distance.[12] One might mean that:

(1) spacetime is quantized or granular or discrete (one sees all these terms in the literature), which entails that spatial quantities come in chunks and can only have values that are multiples of ℓ_M.
(2) 'distance' has no meaning for values less than ℓ_M.[13]
(3) there is no such thing as a distance shorter than ℓ_M.
(4) it is impossible to measure a distance less than ℓ_M.
(5) any scale to measure length must have a minimal value of ℓ_M (i.e., have a unit that is an integral multiple of ℓ_M).[14]

Some discussion of these minimal length theses is in order.

(1) *Spacetime is quantized* This is a claim exclusively about the physical world—it says nothing at all about how we think about the physical world or how we represent the physical world or about the language we use to talk about the physical world. It also says nothing at all about our knowledge of the physical world. In essence, it says that spacetime has a particular structure, namely, that there is no such thing as a distance that is not an integral multiple of the minimal distance. We might formulate this point in terms of existence—the only lengths or distances that exist are those that are integral multiples of the minimal length. There exist no other distances.

We already know (or at least we have a colossal amount of evidence to support) that certain things are quantized, like the energy of an atom and the frequency of light. The claim about spacetime being quantized is meant to be similar to these familiar claims. The claim that energy is quantized comes down to the claim that it is impossible for there to be certain energy states; as something changes energy, the increase or decrease

[10] Baez (2000) on loop quantum gravity, Ng (2011) on superstring theory, Majid (2000) on noncommutative geometry, Sorkin (2005) on causal sets, and Burton (2009) on doubly special relativity.

[11] Other aspects of the Planck length's significance are: (i) a minimum wavelength for photons—see Amelino-Camelia (2003) and Pesci (2011), (ii) as the scale of spacetime foam and related phenomena like the holographic principle, the cosmological constant, and dark energy—see Nieto et al. (2007) and Ng (2011), and (iii) as an invariant length in doubly special relativity—see Rovelli and Speziale (2002), Amelino-Camelia (2003) and Burton (2009).

[12] See Hossenfelder (2013: pp. 5–15) for a history of the idea.

[13] I use 'length' and 'distance' interchangeably in what follows to refer to a scalar quantity, which should be distinguished from displacement, a vector quantity.

[14] x is an *integral multiple* of y iff there is an integer n such that $yn = x$.

in energy occurs in "packets" or quanta. It is impossible for there to be an energy state in between the units of the quanta. Likewise, the claim that spacetime is quantized comes down to the claim that it is impossible for there to be certain distances or durations; as the spatial distance between two things changes or the temporal interval between two things changes, the increase or decrease in distance or duration occurs in "packets" or quanta. It is impossible for there to be a distance or duration in between the units of the quanta.

(2) *'Distance' has no meaning for values less than the minimal length* This is a claim about the meaning of a particular English word, 'distance'. As such, it is not a claim about the physical world, but rather a claim about our language. It might seem like an odd way to capture the claim that there is a minimal length, but I recommend thinking of it in the following way. Intuitively, it might seem that any positive number might be a real distance, but it turns out that there are certain values (any number less than the minimal length in the given units) the cannot be real distances. In other words, beyond the minimal length, there is nothing to refer to when using the word 'distance'. Therefore, I think this sort of statement is just a roundabout way of saying that there are no distances below a certain minimal value.

One difference between claims (1) and (2) is that the latter is entirely about spatial intervals (distances) and not about temporal intervals (durations). However, special and general relativity entail that spatial and temporal intervals cannot be neatly separated. Therefore, I assume throughout that what goes for spatial intervals goes for temporal intervals as well.

(3) *There is no such thing as a distance shorter than the minimal length* This is a claim about the physical world, not a claim about our language or thought or knowledge. It is essentially the same claim as the one made by (2), when (2) is properly interpreted as being about the world rather than about the meaning of 'distance'. Both say that distances less than the minimal length do not exist. That is, there simply are no spatial intervals less than the minimal length. Again, what goes for distance goes for duration as well; so, given our background information, claim (3) should be interpreted as entailing that there is no such thing as a duration less than the minimal duration.

(4) *It is impossible to measure a distance less than the minimal length* This is a claim about what it is possible to measure, and so it should be interpreted as a claim about what in principle can be done in human practice. In particular, it should be interpreted as a limit to what anyone can measure in practice. As such, it is a practical claim about what humans (or maybe any rational entity) can do. In this presentation, I am assuming that 'measure' is a success term; if someone measures a length of certain sort, then there is a length of that sort. If the reader does not share this assumption, then such a reader should replace all occurrences of 'measure' with 'successfully measure'. It should be clear that if there is no such thing as a distance less than the minimal length, then it is impossible to measure a distance less than the minimal length. However, the converse does not hold—it might be that there are certain lengths despite our (in principle) inability to measure them. If (4) turned out to be true, then it would be analogous to an epistemic reading of the uncertainty principle in quantum mechanics (for example, in hidden variable theories), which states that the certain properties cannot be measured despite there being a fact of the matter about their values. There are, of course, myriad

controversies and difficulties with interpreting quantum mechanics, so this analogy should be taken as a heuristic, which is intended as an aid to understanding.

(5) *Any scale to measure length must have a minimal value of the minimal length* This claim is not about the world nor is it about how we talk or think, nor is it about what we can know about the world. Instead, it is a claim about any legitimate measurement scale for length. It has been said already, but it is worth emphasizing that the International System of Units (SI) is far and away the most common and popular in the world, and it employs the meter as its basic unit of length. If it turns out that the meter is not an integral multiple of the minimal length, which, as we will see, is very likely, then any measurement system that includes the meter would be unacceptable. If there is a minimal length, and the meter turns out to not be an integral multiple of minimal lengths, then why would that be a problem? The problem is that a measurement system like this would not accurately represent reality. There would be no such thing as the distance defined as a meter. There would be distances a bit shorter and there would be distances a bit longer, but there would be no such thing as a distance that is exactly a meter. Consider an analogy with some quantity we already think is quantized, like energy. Any system of measurement that includes a unit for measuring energy that is not an integral multiple of the minimal energy is obviously problematic. The same goes for measuring distance in the event that there is a minimal length.

We need to establish the logical relationships between these five interpretations. Based on the discussion above, it should be clear that (1) entails all the rest. Moreover, when properly interpreted, (1), (2), and (3) are equivalent. That is, spacetime is quantized iff there is nothing smaller than the minimal distance for ‘distance’ to refer to (and nothing shorter than the minimal duration for ‘duration’ to refer to) iff there is no such thing as a distance smaller than the minimal distance (and no such thing as a duration shorter than the minimal duration).

It should be obvious as well that (1) entails (5). One might think that (2) and (3) do not entail (5); for example, one think that there could be no distance values less than 5 but anything greater is fair game. But then if entity A is 5 distant from entity B and entity C is 6 distant from B in the same direction, then it looks like A is 1 distant from C, which violates our assumption. Thus, it looks like (2) and (3) entail (5) (given some basic assumptions about the acceptability of a measurement scale). It should be equally clear that (5) does not entail (1), (2) or (3). After all, it might be that considerations pertaining to the nature of measurement scales mandate (5) but these do not entail anything about whether there exist distances less than the minimal length (or durations less than the minimal duration).

(4) does not entail (5); i.e., (4) is compatible with the scale of measurements greater than ℓ_{M} being continuous (i.e., it is not the case any value of a metric—distance function—is an integral multiple of ℓ_{M}). Why doesn’t the same problem as above occur with (4) as with (2) and (3)? The reason is that there is a distinction between measurement and calculation. Say one measures the distance between event A and event B as $1.5\ell_{\mathrm{M}}$ and between event A and event C in the same direction as $2\ell_{\mathrm{M}}$. Neither measurement violates (4). We then *calculate* the distance between event B and event C as $.5\ell_{\mathrm{M}}$. The calculation does not violate (4) either.

Overall, (1), (2), and (3)—properly interpreted—are equivalent. They entail (4) and (5). Our focus throughout what follows will be (5) and it will turn out that we will not be concerned with (4) after this section.

Why believe that there is a minimal length? There are plenty of arguments for minimal length theses, but they are rarely if ever distinguished by which minimal length thesis is the conclusion of the argument. Let us review some of them.[15]

A. *Light and distance* When measuring a distance by measuring the time it takes light to traverse it, the accuracy of the measurement increases as the wavelength of the light decreases. However, as the wavelength of the light decreases, its energy increases. As the energy increases, it deforms spacetime more. An analysis of these relationships reveals that once one decreases the wavelength of the light past a certain point, the spacetime deformation decreases the accuracy of the measurement, so there is a limit to how accurate such a measurement can be. It turns out to be around a Planck length. This argument supports interpretation (4).
B. *Light and volume* If we try to measure some properties of a region of space then we need to use light with high enough energy so that the region does not change while we measure it. Once one increases the energy enough, the light deforms the spacetime of the region so much that it no longer constitutes an accurate measurement. This limit occurs at about the Planck length. This argument supports interpretation (4).
C. *Density* If we begin with some mass in a certain regular volume and begin increasing its density by decreasing the volume, then, according to GR, we eventually reach a point at which the process stops because we create a black hole; the radius of the volume at which this occurs is proportional to the mass (i.e., smaller masses result in a smaller radius) According to QM, the same system eventually reaches a point at which the process stops because the uncertainty of the energy in the system reaches a maximum; the radius of the volume at which this occurs is inversely proportional to the mass (i.e., larger masses result in a smaller radius). Using these two results, we can solve for the smallest radius possible, which turns out to be about the Planck length. Because this argument is not about measurement, it seems to support an interpretation stronger than (4), like (2) or (3).
D. *Uncertainty and position* The uncertainty principle from QM entails that there is an inverse relationship between the precision with which we may measure a particle's momentum and its position. If we alter this principle to include the effect of gravity, then we arrive at the generalized uncertainty principle, which entails that any particle has a minimum position uncertainty of around a Planck length, no matter how uncertain its momentum is. If position measurements have a minimum uncertainty of a Planck length, then it is impossible to measure any distance less than a Planck length, so this argument supports interpretation (4). However, if the uncertainty in question is taken to be with respect to position itself rather than our measurement of position, then this argument supports a stronger interpretation like (2) or (3).

[15] For more information on these and other arguments, see the surveys by Garay (1995), Calmet (2007a, b), Adler (2010), Ng (2011), Hossenfelder (2013) and the references therein.

E. *Energy density and gravity* A gravitational field has a certain energy and the density of that energy is related to the strength of the field. Because of the energy-time uncertainty in QM, any region has some fluctuations in gravity that limit our measurements of gravitational energy in that region. These fluctuations in energy correspond to distortions in the spacetime of the region. It turns out that uncertainty in the energy density of the field corresponds to an uncertainty in the spatial specifications of the region. The specifications of the region are defined only to about the Planck length. This argument supports interpretation (4) for sure, and if the uncertainty in question pertains to energy itself rather than our measurements of it, then the argument seems to support stronger interpretations as well (e.g., (2) and (3)).
F. *Weakness of gravity* Of the four acknowledged fundamental forces (i.e., gravity, electromagnetism, the weak nuclear force, and the strong nuclear force), gravity is the weakest, and it is dramatically weaker than the others. The standard model of particle physics incorporates theories of electromagnetism, the weak force, and the strong force. However, at around the order of the Planck length, gravitational effects of particles described by the standard model are no longer negligible. That is, one needs to account for gravitational effects when considering processes that occur around the scale of the Planck length. That is exactly what we cannot do in a straightforward way because of problems integrating GR and QM. Thus, it is natural to think that we need some new physical theory to describe processes that occur around the Planck length. This argument does not directly support any of the interpretations because of its heuristic character, but if we think of distance as implicitly defined by our best physical theories (e.g., QM and GR), then the concept of distance no longer makes sense at scales less than the Planck length. This consideration supports interpretation (2).
G. *Superstring theory* An attempt to unify QM and GR is superstring theory, which posits very small strings whose features explain the central claims of QM and GR. These strings are around the size of the Planck length. It is unclear which interpretation of minimal length superstring theory supports, but it is reasonable to think that it is (2) or (1).
H. *Loop quantum gravity* An alternative attempt to unify QM and GR is loop quantum gravity, which describes spacetime geometry in a novel way so that area and volume are quantized. Loop quantum gravity supports interpretation (1) of minimal length.

Arguments (A)-(F) appeal to aspects of QM and GR, while arguments (G) and (H) depend on theories that are designed to incorporate both QM and GR while avoiding the problems we have applying them together. It is these successor theories that provide us with the arguments for the strongest versions of the minimal length thesis.

In what follows, it is the claim that any meaningful distance must be an integral multiple of the minimal length that plays a central role. This claim is supported by theses (1), (2), (3), and (5), and we have strong arguments for these theses. Thus, we have good reason to believe that there is a minimal length in a strong sense even if we ignore thesis (4) and the arguments for it.

On the other hand, there is some evidence *against* the Planck length being a minimal length. This evidence usually falls into one of two types: (i) theoretical calculations of

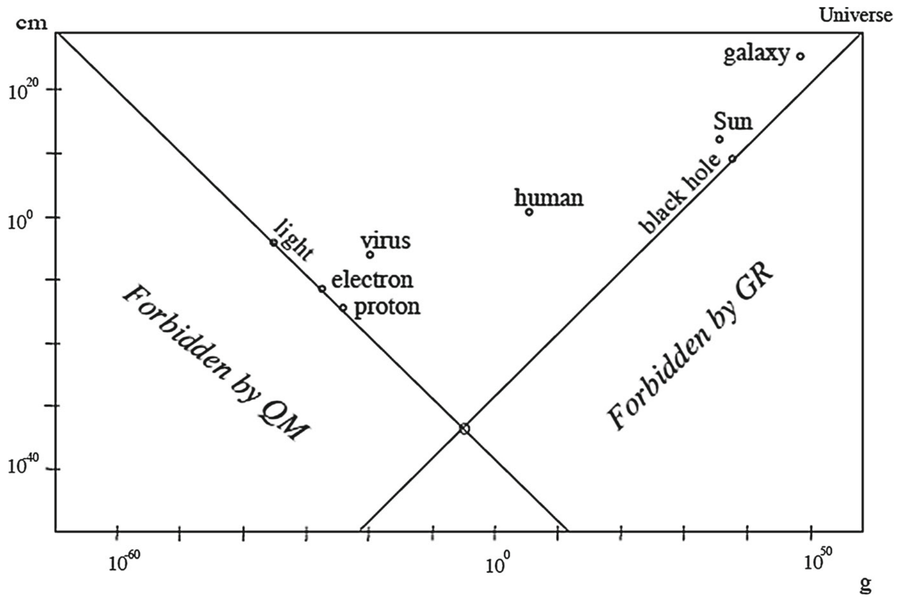

Fig. 1 Possible length/mass combinations

meaningful distances less than the Planck length and (ii) astrophysical measurements of the smoothness of spacetime.[16] For example, in the famous paper by Bekenstein, he calculates that if one adds one photon to a solar mass black hole, then its radius would increase by about 10^{-71} m. The astrophysical measurements typically use theoretical considerations to predict that a minimal length of spacetime would cause certain features (e.g., polarization) in light that travels through spacetime, and over long enough intervals, we would be able to detect these features. So far, we have not detected any.

I do not think that the calculations constitute evidence against there being a minimal length because these calculations are usually based on GR, which is consistent with there being no minimal length. So it is not a surprise to find out that one can use GR to calculate something being smaller than the Planck length. Figure 1 shows which combinations of masses and distances are ruled out by QM and GR.[17] In it, the vertical axis is length in centimeters and the horizontal axis is mass in grams. One arrives at a minimal length only by considering both QM and GR (and the right aspects of them at that).

The astrophysical measurements on the other hand do provide evidence against some values of a minimal length, but they are far from conclusive. For example, they often rely on Lorentz invariance considerations (i.e., a minimal length would be

[16] On the former, see Bekenstein (1973) and Reifler and Morris (2003); on the latter, see Bernadotte and Klinkhamer (2007), Klinkhamer (2007), Stecker (2011), Christiansen et al. (2011) and Laurent et al. (2011). See Cunliff (2012) for criticism of some theoretical arguments for a minimal length.

[17] Figure from Majid (2000: p. 15).

invariant across reference frames), but this is a controversial assumption.[18] Moreover, at best, they tell against certain values of a minimal length, not against a minimal length per se.

To sum up, we do not know whether there is a minimal length in any of the above senses. We do not know whether a minimal length would be the Planck length or some other length. We do have many good reasons to think that there is a minimal length. Think of what follows as an investigation into the consequences for the SI system if there is a minimal length in the sense of (5), which is entailed by (1), (2), and (3).

3 The indeterminacy of the meter

So far we have seen: (i) how 'meter' and 'second' are defined, (ii) five interpretations of the claim that there is a minimal length and the entailments between them, and (iii) the arguments from physics that there is a minimal length and which interpretation each argument favors. Before turning to the main argument that 'meter' is indeterminate, it will be helpful to discuss exactly what is meant by 'indeterminacy'.[19]

It is standard to distinguish between two or three kinds of indeterminacy: semantic, metaphysical, and epistemic. The following is a quote from Matti Eklund that sets out the distinction:

> The 'indeterminacy' we speak of can in principle be held to be semantic (or more, generally: representational), metaphysical, or epistemic. To say that it is semantic is to say that the indeterminacy in question is a matter of how we represent the world; of the relations between our representations and the world. To say that it is metaphysical is to say that the world in itself, as opposed merely to how we represent it, is indeterminate. To say that it is epistemic is to say that we are dealing with a case where our ignorance is in a certain way principled.[20]

Some theorists focus only on the distinction between semantic and metaphysical indeterminacy, thinking of epistemic indeterminacy as a misnomer. For example, David Taylor and Alexis Burgess write:

> In broad strokes, semantic indeterminacy, is supposed to be indeterminacy—in some more generic sense—stemming specifically from the semantic or otherwise representational features of expressions/concepts used to articulate any given instance of the phenomenon. Metaphysical indeterminacy, by contrast, is supposed to consist in portions or aspects of reality itself being somehow unsettled, quite independently of whether and how we think or talk about them.[21]

I follow Taylor and Burgess in excluding so called epistemic indeterminacy as well. Doing so allows us to use a single way of making sense of indeterminacy in general.

[18] See Rovelli and Speziale (2002) and Burton (2009).

[19] The reader should be careful to distinguish indeterminacy from uncertainty. Uncertainty is related to the measurement of various physical quantities; see BIPM (2008).

[20] Eklund (2011: p. 150).

[21] Taylor and Burgess (2015: p. 298).

In the literature, the most common way of glossing indeterminacy in general is to say that when something is indeterminate, there is "no fact of the matter" about it. Then one distinguishes between semantic and metaphysical indeterminacy on the basis of *why* there is no fact of the matter. In the case of semantic indeterminacy, there is no fact of the matter because of the meanings or other semantic features of our words or concepts—the way we represent the world. In the case of metaphysical indeterminacy, there is no fact of the matter because of the way the world is, independently of the way we represent it.

The idea that there is no fact of the matter is common between semantic and metaphysical indeterminacy, and the two varieties are distinguished on the basis of the *source* of the indeterminacy. For example, Elizabeth Barnes and J. Robert G. Williams characterize metaphysical indeterminacy in the following way: "It is metaphysically indeterminate whether p iff (1) it is [indeterminate] whether p, and (2) the source of this [indeterminacy] is the non-representational world." (Barnes and Williams 2011: p. 108).[22] Likewise, semantic indeterminacy is characterized as indeterminacy whose source is the semantic features of one or more of our words.

Taylor and Burgess provide the following example to help illustrate the distinction between metaphysical and semantic indeterminacy.

For some large, carefully chosen integer n, we might be inclined to endorse a claim like:

(0) It is indeterminate whether Kilimanjaro contains more than n molecules.

> We can call facts of this form "cases" or "instances" of indeterminacy, whether they turn out to be semantic or metaphysical. To call (0) a case of [semantic indeterminacy] is basically to say that the indeterminacy at issue "owes" to certain semantic properties of one or more of the words in its complement: 'Kilimanjaro contains more than n molecules'. The pertinent semantic fact might be that 'Kilimanjaro' fails to pick out a unique, mountain-shaped composite of molecules. On the other hand, to call (0) an instance of [metaphysical indeterminacy] is effectively to say that Kilimanjaro itself—that natural volcanic peak in Tanzania—is somehow indeterminate with respect to the number of molecules it contains.[23]

All these quotes make clear that the distinction between metaphysical and semantic indeterminacy comes down to the source of the indeterminacy—is it due to the semantic features of our words or is it due to the world itself? If we make use of the 'no fact of the matter' locution, we can rephrase the question as—is there no fact of the matter as to what such and such linguistic expression refers to or is there no fact of the matter about the world itself? As we will see, it makes the most sense to think of the indeterminacy associated with 'meter' to be *semantic* indeterminacy—indeterminacy that has its source in the semantic features of 'meter' and 'second'. Indeed, one major reasons for thinking that this indeterminacy is semantic is that its source is the inter-

[22] Barnes and Williams (2011: p. 108). Barnes and Williams distinguish between indefiniteness and indeterminacy, but this distinction does not matter for our purposes (they are interested in the relationship between vagueness and indeterminateness).

[23] Taylor and Burgess (2015: pp. 298–299).

action between the official definition of 'meter' and the official definition of 'second', provided we assume that there is a minimal length in the sense of (5) described above. And the second major reason is that if we change the official definitions of 'meter' and 'second' in the way suggested in the final section of this paper, the indeterminacy disappears. Let us turn to the arguments.

We are assuming that there is a minimal length and that any measurement of distance has a value that is a multiple of ℓ_{M}. It will be helpful to have an actual value for ℓ_{M}. We might as well take it to be the Planck length, with the understanding that we will return to the issue of alternative minimal lengths below.

The natural question to ask at this point is, how many Planck lengths are in a meter? The trouble with answering this question is the uncertainty in the value of the Planck length. However, we can establish upper and lower bounds to an answer with our estimate together with the margin of error. It turns out that there are between 61,873,940,138,076,647,333,329,084,656,110,518.736883111402…and 61,873,197,440,404,819,006,357,285,417,409,273.938446192004…Planck lengths in a meter. But, of course, there cannot be a fraction of a Planck length since it is the minimal length. So we should round the first number down and round the second number up. That still leaves us with a huge margin: a difference of 742,697,671,828,326,971,799,238,701,244 Planck lengths. Presumably, there is an exact number in this range that constitutes the exact number of Planck lengths in a meter. If so, then the meter is completely determinate.

Not so fast. Recall that (Meter*), which is entailed by the definitions of 'meter' and 'second' together, is an identity relating the meter to cesium periods—a meter is exactly the distance traveled by light in a vacuum in 30 and 14,204,145/21,413,747 cesium periods. There must be an exact integer number of Planck lengths in the distance traveled by light in a vacuum in 30 and 14,204,145/21,413,747 cesium periods. One Planck length is the distance traveled by light in a vacuum in one Planck time; thus, for there to be an exact integer number of Planck lengths in a meter, that fraction, 14,204,145/21,413,747, must cut the number of Planck times in a cesium period exactly.

To see the problem, let us simplify the math a bit. Imagine that one cesium period is 3 Planck times and that a meter is the distance light travels in 2 and 1/2 cesium periods. 2 cesium periods would take 6 Planck times, but that extra 1/2 cesium period would take 1.5 Planck times. However, there is no such thing as taking 1.5 Planck times. A process can take 1 Planck time or it can take 2 Planck times, but it is impossible to take any value in between. Thus, if the meter were the distance light travels in 2 and 1/2 cesium periods and a cesium period takes 3 Planck times, then the number of Planck times it takes light to travel one meter would be indeterminate; the number of Planck lengths in a meter would likewise be indeterminate. In order for a meter to be completely determinate—to be an exact number of Planck lengths— the fractional remainder of the number of cesium periods required for light to travel one meter must cut the number of Planck times in a cesium period exactly. How likely is that?

Consider again the problem with the math simplified. If that fractional remainder were 1/2, then there would have to be an even number of Planck times in a cesium period. If that fractional remainder were 1/10, then the number of Planck times in a cesium period would have to be exactly divisible by 10. If that fractional remainder were 33/100, then the number of Planck times in a cesium period would have to be exactly divisible by 100. Let n/m be the fractional remainder. For which values of t, the number of Planck times in a cesium period, does n/m pick out an integer number of Planck times? (n/m)t = x for x a positive integer. For that to happen t must be divisible by m without remainder. That is, t must be a multiple of m.

Given that the fractional remainder is exactly 14,204,145/21,413,747, the number of Planck times in a cesium period must be exactly divisible by 21,413,747. Given our current estimate for Planck length and our margin of error, there are between 61,873,940,138,076,647,333,329,084,656,110,517 and 61,873,197,440,404,819,006,357,285,417,409,273 Planck lengths in a meter. Call the set of integers between these two the *margin set*. The least natural number in the margin set that is exactly divisible by 21,413,747 is 61,873,940,138,076,647,333,329,084,667,011,124. If there are exactly that many Planck times in a cesium period, then 'meter' is completely determinate. How many natural numbers in the margin set are exactly divisible by 21,413,747? 34,683,218,767,286,592,664,013. So, there are that many elements in the margin set for which 'meter' to is completely determinate. That is, only for 1 out of every 21,413,747 members of the margin set does 'meter' end up being determinate.

Notice that the particular value for the Planck length plays no role in the above calculations. To see why, let the margin set be {x : a < x < b, where a, b, x are positive integers}. For t, the number of minimal lengths in the distance light travels in a cesium period, to be a positive integer, t must be a multiple of m, the denominator of the fractional remainder, which in our case is 21,413,747 (regardless of what the minimal length turns out to be). How many multiples of 21,413,747 turn up in the margin set? As long as the difference between b and a is much greater than 21,413,747, it will be 1/21,413,747 of the margin set.

If we assume that all the options are equally likely, then that is about a 1 in 21 million chance of there being the right number of Planck times in a cesium period for the meter to be completely determinate.[24] Those are not good odds. Therefore, probably, the meter is indeterminate (if there is a minimal length). If it is, then it follows that any standard unit of length is indeterminate as well.[25] All these conclusions follow as long as there is some minimal length, regardless of how small it turns out to be: if the chance that 'meter' is completely determinate is nonzero, then it is about 1 in 21 million.

Furthermore, if the meter is indeterminate, then the problem ripples through the SI system because any derived unit based on the meter will be indeterminate too. Before going through them, we need to deal with the distinction between units and quantities. The meter is a *unit* for measuring the *quantity* of length. 'meter' is probably indeterminate, but 'length' is not (so long as there is some exactly determinate length). Many SI units have special names (for example, the coulomb is the SI unit for measuring

24 In making this claim, I am assuming that; the important point is not the exact probability but rather the fact that 'meter' is very likely indeterminate.

25 Note that it need not be the case that 'second' is indeterminate. If there is an exact number of Planck times in a cesium period, regardless of whether it is divisible by 21,413,747, then 'second' is completely determinate.

Table 1 Indeterminate SI units

Unit	Formula	Quantity
Meter	–	Length
The SI unit for area	meters^2	Area
The SI unit for volume	meters^3	Volume
The SI unit for velocity	meters/second	Velocity
The SI unit for acceleration	meters/second^2	Acceleration
Newton	kilograms * meters/second^2	Force
Joule	newtons * meters	Energy
Gray	joules/kilogram	Absorbed radiation dose
Pascal	newtons/meter^2	Pressure
Watt	joules/second	Power
Volt	watts/ampere	Potential
Farad	coulombs/volt	Capacitance
Ohm	volts/ampere	Resistance
Siemen	ohms^{-1}	Conductance
Weber	volts/second	Magnetic flux
Henry	webers/ampere	Inductance
Tesla	webers/meter^2	Magnetic flux density
Lux	lumens/meter^2	Illuminance
Ampere	–	Electric current
Candela	–	Luminous intensity
Coulomb	amperes * seconds	Electric charge
Lumen	candelas * steradians	Luminous flux

electric charge, and electric charge is electric current * time).[26] The problem is, lots of units in the SI system do not have special names (e.g., the quantity, area, is length squared, but it does not have a special name). I use the convention ‘the SI unit for Q’ where ‘Q’ is replaced by a quantity term for naming those units of the SI system that do not have special names.

The SI system has seven base units and dozens of derived units. ‘Meter’ is a base unit, along with ‘ampere’ (the SI unit for current), ‘second’ (the SI unit for time), ‘kelvin’ (the SI unit for temperature), ‘mole’ (the SI unit for quantity of substance), ‘candela’ (the SI unit for luminous intensity), and ‘kilogram’ (the SI unit for mass). If ‘meter’ is indeterminate, then all the SI units on Table 1 are indeterminate as well. These include derived units that are explicitly defined in terms of meters in addition to two base units whose definitions appeal to meters—e.g., an ampere is defined as that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 meter apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newtons per meter of

[26] ‘*’ is just a multiplication sign.

length. The other example is the candela, which is defined as the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540×10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian. The definition of 'ampere' explicitly mentions the meter, so if 'meter' is indeterminate, then 'ampere' is too. The definition of 'candela' explicitly mentions the watt, which is defined in terms of meters. Thus, if 'meter' is indeterminate, then 'candela' is too. These indeterminacies in base units affect some other derived units that do not show up on the list—'coulomb' and 'lumen' would be indeterminate as well.

See Fig. 2 for a diagram of the SI system; the impact of the indeterminacy of the meter is the shaded area.[27] Little is unaffected.

4 Discussion

A quick summary of the argument so far is in order. From the definitions of 'meter' and 'second', it follows that a meter is the distance light travels in the duration of 30 and 14,204,145/21,413,747 cesium periods. If there is a minimal length and a minimal duration (these two assumptions stand or fall together because space and time are just two aspects of a single underlying spacetime), then the number of minimal durations in a cesium period must be an integral multiple of 21,413,747. Otherwise, there would not be an exact number of minimal lengths in a meter. One way of thinking about this problem is that, together, the definitions impose what we might call a certain *requirement on reality*—that the number of minimal lengths in a meter is an integral multiple of 21,413,747 and that the number of minimal durations in a cesium period is an integral multiple of 21,413,747.[28] If the number of minimal lengths in a meter (and the number of minimal durations in a cesium period) might have any value at all, and there is an equal likelihood for any of the available values, then there is only a 1 in 21,413,747 chance that the value is just right. If the value is not just right, then 'meter' is indeterminate in the sense that there is no exact number of minimal lengths in a meter.

It is helpful to consider how the value might turn out to be *not* just right. Let us simplify the math. If a meter is the distance light travels in 1/100 of a second and a second is the duration of 130 cesium periods, then a meter would be the distance light travels in 1 and 3/10 cesium periods. Assume that we eventually arrive at a value for the minimal duration that makes it exactly 1/100 of a cesium period. In that case, a cesium period would be 100 minimal durations. One second would be the duration of 13,000 minimal durations (130 * 100), and one meter would be the distance light travels in 130 minimal durations (1/100 of 13,000). This is an example of everything turning out just right.

Instead, imagine that we eventually arrive at a value for the minimal duration that makes it exactly 1/99 of a cesium period. In that case, a cesium period would be 99

[27] Figure 2 is based on a diagram from the National Institute of Standards and Technology (NIST) Physical Measurement Laboratory (http://physics.nist.gov/cuu/Units/SIdiagram2.html).

[28] I borrow this terminology from Williams (2012).

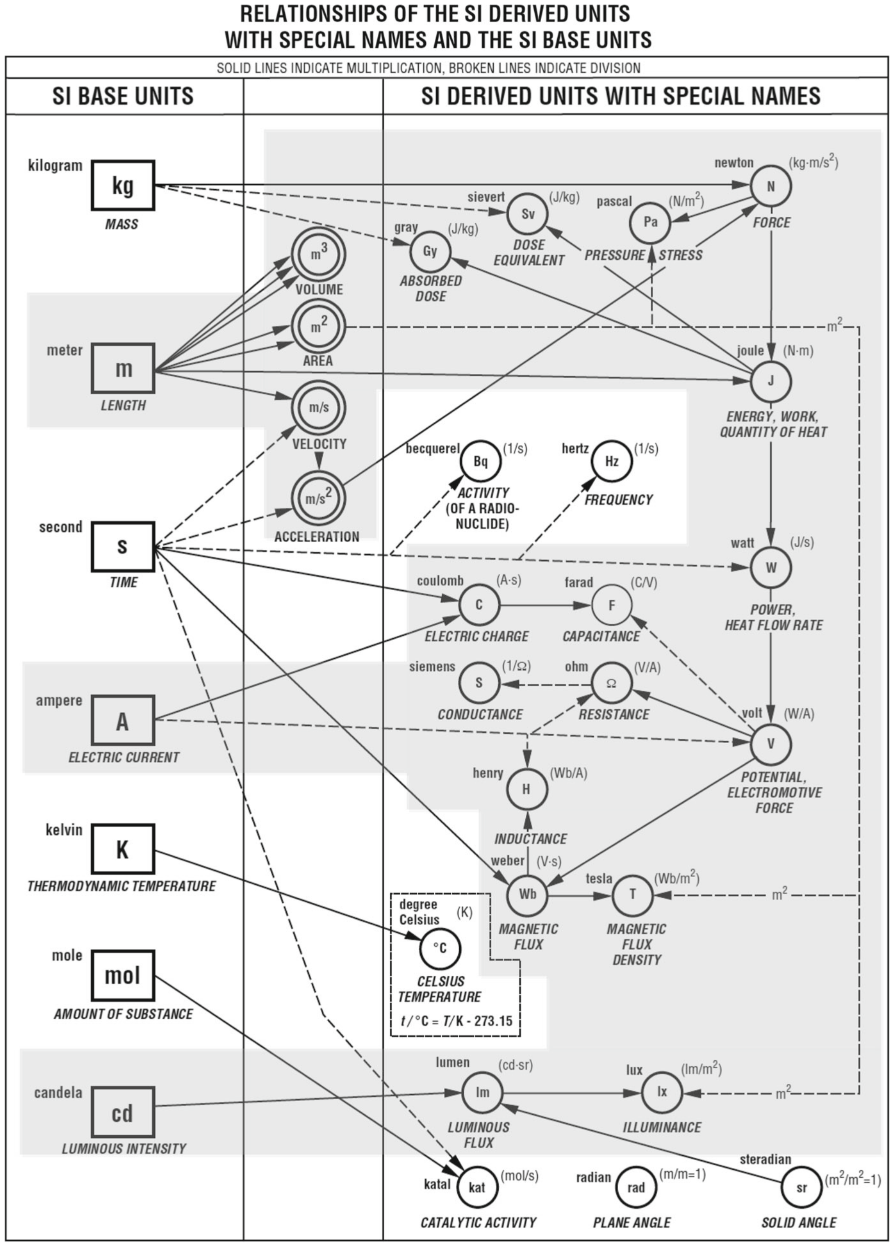

Fig. 2 Indeterminacy in the SI System

minimal durations. One second would be the duration of 12,870 minimal durations (130 × 99), and one meter would be the distance light travels in 128.7 minimal durations (1/99 of 12,870). That is, it would be 128.7 minimal lengths. But that is impossible! By hypothesis, nothing can be 128.7 minimal durations, and nothing can be 128.7 minimal lengths.

Return for a moment to the question of what kind of indeterminacy is involved if the meter turns out to be indeterminate—that is, if there turns out to not the right number of minimal durations in a cesium period. I claim that this is a case of *semantic* indeterminacy rather than metaphysical indeterminacy because it makes more sense to say that the source of indeterminacy is in the definitions of 'meter' and 'second'. Together these two definitions impose a requirement on reality—that the number of minimal times in a cesium period must be evenly divisible by 21,413,747. If it turns out by sheer luck that this requirement is met, then the definitions are in fine order and 'meter' would be determinate. However, it is much more likely that the requirement fails, and in this case, the problem would obviously be due to the interaction between these two definitions. That is, this is clearly a semantic indeterminacy. This is one of the two major reasons for thinking that the indeterminacy in question is semantic. The second reason is that we can redefine 'meter' and 'second' in a way that is compatible with present usage and completely avoids the indeterminacy problem that is the topic of this paper. I show exactly how to do that in the final section. If that is right, then there is no reason to think that the source of the indeterminacy is in the world rather than in the way we represent the world. That is, it is not a metaphysical indeterminacy, but instead a semantic indeterminacy because changing the definitions of 'meter' and 'second' would not eliminate a metaphysical indeterminacy.

Let us turn to some objections. One might think that if there are, say, 1 and 1/10 Planck times in each cesium period, and a second is 5 and 1/5 cesium periods, then there would be an exact number of Planck times in a second even though the number of Planck times in a cesium period does not exactly divide the denominator of the fractional remainder. Thus, the objection goes, the chances of a completely determinate meter might be much better.

My reply is that, above, I assumed that there is an exact natural number of Planck times in a cesium period. If that assumption turns out to be false, then 'cesium period' is indeterminate, which makes 'second' indeterminate, which makes 'meter' indeterminate. Thus, the assumptions of the objection would support my conclusion without having to calculate the probability at all. It would be assured that the meter is indeterminate.

Another objection is that even though no positive integral multiple of Planck lengths satisfies 'the distance light travels in the duration of 9,192,631,770/299,792,458 cesium periods' and so no positive integral multiple of Planck lengths makes (Meter) true, nevertheless, there is an exact positive integral multiple of Planck lengths in a meter. This sort of view might be supported by what has come to be known as *reference magnetism*. The key to understanding reference magnetism is the idea of carving nature at its joints. This metaphor goes back to Plato, but is probably more familiar to contemporary philosophers in the form of David Lewis' theory of natural properties.[29] Natural properties are those that are the most basic and fundamental constituents of reality. For example, the property of being green is thought to be more natural than

[29] See Plato (1997) and Lewis (1983). Do not confuse this use of 'natural' with the one in the above discussion of planck units. More on this topic below.

the property of being grue. (Something is grue iff it is green and it is examined before a certain time t, or it is blue and not examined before t.) Although controversial, they are helpful in dealing with many of the most intractable problems in metaphysics. I am going to assume that the reader is familiar with Lewisian naturalness.[30]

Reference magnetism is the claim that natural properties need not satisfy a definition in order to be the referent of the term defined. Even if the definition is somehow faulty (in that no thing satisfies it), the term defined ends up with a determinate semantic value. If this view is correct, then (Meter) might succeed in giving ‘meter’ a referent even though nothing satisfies the definiens because of the peculiar mismatch between (Meter), (Second), and the fact that there is a minimal length.

First, I should say that reference magnetism is a controversial minority view amongst philosophers of language with very little to support it beyond some armchair metaphysical speculations. It is supposed to help solve some very difficult problems about how our words come to have determinate content, so most of the support for it is of the “hey, it would be great if this were true because it would solve so many of our problems!” There is little to no empirical support for the idea but many problems with it. For example J. Robert. G. Williams argues convincingly that if reference magnetism is true, then we have no reason to think that our words refer to things in the world instead of to numbers.[31] Moreover, John Hawthorne and Cian Dorr recently catalogued the most important features of naturalness (which is the notion on which reference magnetism rests), and pointed out dozens of potential inconsistencies between these features, and reference magnetism is the feature most involved in these inconsistencies.[32]

Second, it is not even clear that reference magnetism would be an effective response here. Imagine we get a better estimate for the natural constants in terms of which Planck length is defined (i.e., the speed of light, the gravitational constant, and the reduced Planck constant), and this estimate narrows the estimate for Planck length to the point that P− and P+ are the new lower and upper bounds, respectively. Assume as well that no integer between P− and P+ is exactly divisible by 21,413,747. None of the values between P− and P+ will determine a positive integral multiple of Planck lengths in a meter that satisfies (Meter). So, which one of these equally unsatisfactory numbers correctly describes how many Planck lengths are in a meter? The numbers themselves are all equally natural, so that consideration does not help. It is not clear how the referent of ‘meter’ would get determined in this case even if we assume reference magnetism is true. Thus, not only is the view unmotivated and internally inconsistent, but it isn’t even efficacious in this case.

Consider a related objection: the argument given above that ‘meter’ is probably indeterminate illicitly presupposes a tight connection between semantic determinacy and naturalness. Imagine, for example, that the average American family has 1.5 children, and we define a unit for this quantity—call it *avekid*. There is nothing wrong with the definition of ‘avekid’; it is perfectly determinate and we could use it without

30 See Dorr and Hawthorne (2013) for discussion.

31 Williams (2007). For commentary, see Hawthorne (2007), Bays (2007) and Sider (2011: ch. 3).

32 Dorr and Hawthorne (2013). See also Schwartz (2014).

any problem despite the fact that it is unnatural because children naturally come in single units. The same lesson applies to ‘meter’; namely, if there is no exact integral multiple of minimal lengths in a meter, ‘meter’ is still determinate despite the fact that it won’t be natural.

My reply is that the analogy is not apt. In section two I considered five ways of interpreting ‘there is a minimal length’. When we consider the analogs of these claims for children it is obvious that not a single one is remotely plausible. For example, there is such a thing as half a child, it is possible to measure half a child, there is no reason to think that children come only in discrete indivisible chunks. Thus, the objection misses the most important point about a minimal length—that it is impossible for there to be smaller lengths. The problem is not that there is nothing *natural* for ‘meter’ to represent (that is, if no integral multiple of minimal lengths make up a meter)—it is that there is *nothing at all* for it to represent. The argument I have given does not rely on the concept of naturalness at all.

5 Measurement and philosophy of science

One might agree with my conclusion but object to its significance by noting that the SI system is shot through with indeterminacy; thus, the point about the meter being indeterminate is not worrisome at all. In fact, ‘kilogram’ is still defined as the weight of a particular platinum and iridium cylinder called the *International Prototype of the Kilogram* (IPK). How precise is that? Probably not very precise at all. How many atoms are in the cylinder? There probably is not a stable answer because the platinum and iridium atoms that compose the IPK are interacting with the surrounding air all the time. In fact, we have good evidence that the IPK is losing weight (on the order of 10^{-8} kg/year, which is gigantic).[33] So ‘kilogram’ is wildly indeterminate given the standards of precision at play in the above considerations about ‘meter’. Notice that this is a different sense of ‘indeterminate’ that I have been using. I argued that there is probably nothing for ‘meter’ to represent—no distance at all that is an integral multiple of minimal lengths. The problem with ‘kilogram’ is that what it is supposed to represent is changing and also that ‘cylinder’ is vague. For example even at a time slice, it will be difficult to determine the boundary of IPK exactly (say, down to the Planck length) due to quantum indeterminacy—and even though these differences in mass would be extraordinarily small, the inability to say for sure whether they are or are not part of IPK makes ‘kilogram’ vague. (The same can be said for ‘meter’ when it was defined as the length of the meter bar, which is a primary reason the definition was changed.) In the parlance of our times, vagueness is a kind of indeterminacy—whether it is epistemic, semantic, or metaphysical depends on the particular theory of vagueness.[34] Thus, even if we ignore the changes in IPK, ‘kilogram’ is still indeterminate.

My reply to this “who cares?” objection is that I agree that the SI system has a dramatic indeterminacy because of the conventional definition of ‘kilogram’. However, the Bureau International des Poids et Mesures (BIPM), which is the international

33 Jabbour and Yaniv (2001).

34 See Eklund (2011) for example.

organization whose responsibility is maintaining and promoting the SI system, is well aware of this problem and is probably going to adopt the following new definition of ‘kilogram’:[35]

> (Kilogram) the unit of mass whose magnitude is set by fixing the numerical value of the Planck constant to be equal to exactly 6.62606X $\times 10^{-34}$ m^2 kg s^{-1}.

Notice that this change in the SI system would eliminate definitions in terms of physical objects (like the IPK) and define all base units in terms of fundamental constants. Any indeterminacy in the SI system because of the IPK definition of ‘kilogram’ would disappear as well. In fact, removing this indeterminacy in the SI system is the primary motivation for the change.[36]

The lesson is that there is a push to make the SI system as determinate as possible given our best physical theories, our measurement technology, and our interests. Pointing out that the kilogram is indeterminate (and its indeterminacy is a problem for our current measurements) spurred the scientists and engineers dedicated to improving the SI system to suggest changes that would eliminate that indeterminacy. Of course, they could not do that if no one noticed that source of indeterminacy. The problem discussed above for ‘meter’ is a hitherto unnoticed source of probable indeterminacy in the SI system. It does not pose a practical problem for us given the state of our measurement technology yet, but might very well pose theoretical problems within physics (for example, calculations about the very early universe—within a few Planck times after the big bang).

Instead of focusing on the kilogram in particular, one might motivate the same kind of objection from general considerations about measurement locutions. That is, in effect, what Eran Tal and Paul Teller have argued in recent work.[37] Here are two quotes from Tal’s paper on ‘second’:

> [A] question arises as to how the reference of ‘second’ is fixed. The traditional philosophical approach would be to propose some ‘semantic machinery’ through which the definition succeeds in picking out a definite duration, for example, a possible world semantics of counterfactuals. However, this sort of approach is hard pressed to explain how metrologists are able to experimentally access the extension of ‘second’ given the fact that it is physically impossible to instantiate the conditions specified by the definition. By contrast, the approach adopted in this article takes the definition to fix a reference only indirectly and approximately by virtue of its role in guiding the construction of atomic clocks. … The activities of constructing and modeling cesium clocks are therefore taken to fulfill a semantic function, that is, that of approximately fixing the reference of ‘second’ rather than simply measuring an already linguistically fixed time interval.[38]

[35] The ‘X’ in the definition stands for any additional significant digits that might be added by the time the new definition is adopted. Discussion of when the new definition is set to take effect is ongoing.

[36] However, the change leaves the indeterminacy of the meter unaffected.

[37] Tal (2011) and Teller (forthcoming). In what follows I focus on Tal’s paper.

[38] Tal (2011: p. 1087)

> [U]nit definitions do not completely fix the reference of unit terms unless fixing is understood in a manner that is utterly divorced from practice.[39]

Tal claims that because the definition of the second involves some kind of idealization (e.g., one cannot get a Cesium 133 atom to 0 K), ‘second’ cannot be determinate. Moreover, the actions of metrologists in calibrating the official atomic clocks, which is the frontier for measuring a temporal duration, impact the semantic value of ‘second’ by making it more precise as the uncertainty in these measurements decreases. To accomplish this calibration, metrologists engage in a process of de-idealization, which involves estimating ways in which various concrete features of atomic clocks diverge from the ideal definition of ‘second’ (e.g., estimating the impact of curved spacetime on the operation of the clock).

In response, I want to emphasize taht these points made by Tal pretty clearly confuse the semantic features of ‘second’ with epistemological issues associated with how we calibrate our instruments. It is perfectly coherent—and much more intuitive and plausible—to say that the definition of ‘second’ fixes its extension and that metrologists use all the techniques Tal describes to arrive at an estimate of the duration defined by ‘second’ in the SI system. Given the radical nature of Tal’s position, one would expect way more argumentation for the main conclusion. But the two places Tal alludes to a problem with the received view (quoted above) can be addressed without much trouble at all. How do metrologists experimentally access the extension of ‘second’? They don’t, but metrologists estimate it using the method of de-idealization Tal describes. Is the definition that supposedly fixes the reference of ‘second’ utterly divorced from practice? No. Indeed, Tal explains exactly how it is related to practice. Again, the primary connection to practice is in methods of de-idealization as Tal describes them. However, instead of contributing to the semantic value of ‘second’, this process helps us estimate the duration of a second in a way that can be readily applied to all sorts of problems associated with estimating very short temporal durations. If these answers to the Tal’s questions from the perspective of the received view are inadequate in some way, then Tal has yet to explain why. In sum, my response to this objection from Tal’s work is that it is almost completely unmotivated.

6 Measurement and metaphysics

All the standard units of length mentioned so far are conventional in the sense that they are arbitrary. That is, there is nothing about the universe as far as we can tell that would adjudicate in favor of any one of them or any multiple of any one of them. For example, let a pimeter be exactly pi meters. There is no feature of the universe that would render pimeters over meters more reasonable or perspicuous as our basic unit of length. It will pay to be a bit more careful about this notion of conventionality.

One way of making sense of conventionality that I find particularly helpful for the topic at hand is suggested by Theodore Sider’s theory of substantivity. According to Sider, a sentence s is *nonsubstantive* iff for some expression e occurring in s, the

[39] Tal (2011: p. 1094).

semantic candidates for e are such that for one of e's semantic candidates s is true and for another of e's candidates, s is not true, and all of e's semantic candidates are equally natural (in the sense of naturalness discussed above).[40] Of semantic candidates, Sider writes, "if a linguistic community, roughly in our circumstances, could have use E to mean m without seeming "semantically alien"—could have used E to reach "the same semantic goal" as we use E to reach, albeit perhaps by a different route—then m is a candidate for E."[41] Semantic candidates are meanings that an expression could have had without changing the proper use of the expression too much. It will be helpful to have a definition of 'substantive': a sentence s is *substantive* iff for every expression e in s, one of e's semantic candidates is more natural than the others or s has the same truth value on each of e's semantic candidates.

Sider uses this account of substantivity to define a notion of conventionality. A linguistic expression is *conventional* iff it has multiple candidate meanings that are equally natural and each would serve the semantic goal of the expression equally well. Sider writes:

> To illustrate, consider the word 'inch'. The purpose of 'inch' is to be a convenient measure for smallish things, the kinds of things we can hold in our hands. But there is a range of very similar lengths that would each have served this purpose. We chose one of these to mean by 'inch', but that choice was arbitrary; any of the others would have served our purposes equally well.[42]

He continues:

> All length-words achieve a general semantic goal of allowing speech of absolute and relative sizes, but 'inch' has a more specific goal: to be a convenient measure of smallish things. This goal could have been achieved by many lengths within a certain range. But if 'inch' had meant mile, it would not have achieved exactly this goal, since measuring smallish lengths in miles would be inconvenient. And if 'inch' had meant something other than a length—for instance it has meant happiness—then it would not have achieved anything like its actual semantic goal. All words for units of measure are conventional in this way.[43]

By the same reasoning, 'meter' and all the other standard length expressions are conventional as well. However, as we will see, Sider is wrong to think that *all* measure expressions are conventional.

Because one can derive the Planck length as the natural unit of length by considering fundamental constants of the universe alone, it is free from the kind of arbitrariness one finds in standard units of length like the meter. Thus, the property of having the Planck length is a natural property in Lewis's sense. Using Sider's theory of substantivity and

40 Above, I criticized the idea that natural properties are reference magnets, so it might seem odd to appeal to Sider's theory here. However, there is no inconsistency in saying that the concept of naturalness can play an important role in an account of substantivity and conventionality even though there are no reference magnets. See Williams (2010) for a theory of fundamentality that is similar in spirit.

41 Sider (2011: p. 50).

42 Sider (2011: p. 54).

43 Sider (2011: p. 55).

his theory of conventionality, we can see that 'Planck length' is not conventional. That is, it has among its semantic candidates one that is most natural, namely, its actual semantic value—the Planck length. As such, it is a counterexample to Sider's claim that all measure expressions are conventional. Moreover, a sentence like 'A is n Planck lengths long' (where 'A' is the name of some object and 'n' is a positive integer) is substantive. It is substantive because the measure expression has a semantic candidate that is much more natural than any of its others, namely the Planck length, and we have no reason to think that it would be nonsubstantive because of one of its other constituents. All the same considerations apply to the Planck time as well.

The next consideration is whether the naturalness of the Planck length would render the meter natural as well if there turned out to be an exact number of Planck lengths in a meter. If 1 meter is n Planck lengths, where n is a positive integer, and 'A is n Planck lengths long' is substantive, then why wouldn't 'A is 1 meter long' be substantive? After all, '1 meter long' has a semantic candidate that is more natural than the others: *n Planck lengths long*. Still, there is good reason to think that even in this case, 'meter' is still conventional because it has several equally natural semantic candidates that would fulfill its semantic role. Let n be the number of Planck lengths in a meter. Then the semantic candidates *n Planck lengths* and *n-1 Planck lengths* are equally natural and each would have fulfilled the semantic role for 'meter' equally well. Thus, even if 'meter' turns out to be completely determinate, it is still conventional. One cannot say the same for 'Planck length'.

7 Measurement and language

We often speak and think as if the world is a certain way. That is, attempts to understand or explain the semantic and pragmatic features of our linguistic activity and the contents of our mental states often require making assumptions about the world. For example, one might say that 'dog' designates the property of being a dog or that 'Barack Obama' refers to Barack Obama. That works fine as long as there is a property of being a dog and Barack Obama exists. However, when we discover something about the world that conflicts with the assumptions made when we explain our thought or talk, then we have a potential problem. For example, if we say that 'phlogiston' designates the property of being phlogiston, then we do not really have a satisfying explanation because we have good reason to think that there is no such property. Likewise, we don't want to say that 'A is simultaneous with B' is true iff A is simultaneous with B; because of special and general relativity, simultaneity is relative to a reference frame—there is no absolute simultaneity. Any time we have a mismatch between a scientific description of the world and what linguists and cognitive scientists tell us about what our thought and talk presupposes about the world, we need some way of reconciling the two. This topic has come to be known as *natural language metaphysics*.[44]

In our case, it turns out that most, if not all, of the proposed semantics for measure phrases (e.g., '6 m long') presuppose that these phrases are completely determinate. For example, Kennedy and McNally claim that gradable adjectives (e.g., 'long') have

44 The term comes from Bach (1986); see Pelletier (2011) for discussion.

semantic values that map individuals to degrees, where a degree is a point or interval on a scale, which is an ordered set of degrees. Measure phrases pick out a degree in the scale relevant to the gradable adjective in question (e.g., '6 m long' picks out a real number on the meter scale. The assumption is that any measure phrase like '6 m long' is completely determinate—there is no possibility for indeterminacy of any kind.[45]

One might say that if a semantic theory presupposes that 'meter' is completely determinate and 'meter' turns out to be indeterminate, then the semantic theory is false—end of story. However, this dramatic stance is implausible and unnecessary. Analytic metaphysicians have recently given this topic considerable attention and arrived at a new way of thinking about the relation between discourse that aims at the fundamental nature of reality and discourse that does not. This new work is largely motivated by discontent over the available options for dealing with this kind of situation. For example, one might be a mereological nihilist (i.e., hold that only simples exist and no composite objects—things with parts—exist). Presumably, a table has parts. Thus, a mereological nihilist wants to deny that tables exist. However, that same person might find discourse about tables to be perfectly legitimate as long as that discourse is not aiming to specify the fundamental nature of reality. How do we reconcile the claims that aim at the way reality is fundamentally with claims that do not? The typical responses to this conundrum are: (i) error theory (the description of fundamental reality is true and nonfundamental discourse that contradicts it is false), (ii) fictionalism (the description of fundamental realty is true and nonfundamental discourse that contradicts it should be treated as if it is a useful fiction), and (iii) nonfactualism (the description of fundamental reality is true and nonfundamental discourse that contradicts it should be treated as if it does not aim at representing the world at all—perhaps statements of nonfundamental discourse aim to express attitudes of the speaker rather than state facts about the world). The reasons to be dissatisfied with these responses are the primary reasons for seeking an alternative.[46]

Kit Fine's proposal is a good example of a metaphysical approach to the relation between fundamental and nonfundamental discourse. Fine introduces an operator, 'in reality' that allows one to say things like 'there is a table' and 'in reality there are no tables' without contradicting oneself. He also motivates the metaphysical relation of grounding, which is intended to make sense of the 'in virtue of' locution. Proposition p grounds another proposition q iff q holds in virtue of p and the truth of q consists in nothing more than the truth of p. Together, the notion of reality and the notion of grounding allow Fine to specify what it is for a proposition to be factual: a proposition is factual iff it is real or it is grounded in the real.[47]

Does Fine's approach help us reconcile the claim that 'meter' is indeterminate (assuming it turns out to be) with the claim that measure phrases involving 'meter' are completely determinate? We could say that 'meter' is completely determinate, but in reality 'meter' is indeterminate. That seems strange especially if 'in reality'

[45] See Kennedy and McNally (2005a); see also Krifka (1989), Schwartzchild (2005) Kennedy (2007), Sassoon (2010) and van Rooij (2011).

[46] I am not going to cover the debate about these responses; see Fine (2001) for discussion.

[47] An alternative to Fine's theory is Sider's account of structure and metaphysical semantics; see Sider (2011: pp. 105–124).

is factive, then we're back with a contradiction. Instead, we should say that 'meter' is completely determinate, but it is not the case that in reality 'meter' is completely determinate. Then, there is no contradiction. However, we'd need to also say something about whether 'meter' is factual, which would require us to say that the proposition that 'meter' is completely determinate is grounded in some other propositions that are real.[48]

8 Replacing the definitions of 'meter' and 'second'

Even if we get very lucky and there turns out to be exactly the right number of minimal durations in a cesium period, making 'meter' perfectly determinate, one might think that these definitions are defective for imposing such an arbitrary and unlikely requirement on reality. In this final section, I consider one way of fixing the problem with the definitions of 'meter' and 'second'.

One suggestion for fixing the indeterminacy problem: defining 'meter' directly in terms of Planck lengths instead of in terms of seconds and the speed of light. There are about 10^{30} options for doing this, and it is hard to say what the costs and benefits might be for choosing one over the others, so we might just try one that is close to the middle of the margin set. Then the meter would be defined as exactly that many Planck lengths. Because Planck lengths are highly natural—defined in terms of the gravitational constant, the speed of light, and Planck's constant—the meter would then inherit this desirable feature. Moreover, as long as 'Planck length' is determinate, 'meter' would turn out to be determinate as well.

However, this suggestion immediately runs into a serious objection: defining the meter directly in terms of Planck lengths severs the conceptual tie between meters and seconds. As it is right now, meters are defined directly in terms of seconds, and because of this direct connection between the two, we know the speed of light in meters per second exactly—it is 299,792,458 m/s. If we change the definition of 'meter' so that it is an exact number of Planck lengths, then the speed of light in m/s might no longer an exact whole number, and it would have to be determined experimentally. This result would entail some uncertainty about its value. Of course for all intents and purposes *right now*, treating it as exactly 299,792,458 m/s would be fine even if we redefine 'meter' directly in terms of Planck lengths. Still, this problem would be a serious cost to the proposal for redefining the meter.

There are two obvious replies to this objection and each one involves redefining 'second'. The first would be to define a second as the duration it takes light to travel 1/299,792,458 of a meter. Then seconds would be defined in terms of meters rather than the way it is currently, where meters are defined in terms of seconds. This proposal would reestableish the exact value for the speed of light in meters per second.

[48] I am not sure how that story about grounding would work. Still, I think this sort of view is worth considering, especially given the fact that there are multiple ways of implementing it, from Schaffer's entity grounding view to Sider's concept of structure and metaphysical semantics, to Robert Williams' take on a regimented representation of reality instead of a representation of a regimented reality. See Schaffer (2009), Williams (2010) and Sider (2011).

The second option would be to define the second directly in terms of Planck times. If the number of Planck lengths defining a meter and the number of Plank times defining a second were chosen to coordinate properly, then the speed of light in meters per second would again be an exact value. Here are the candidates—these choices are very close to the middle of the current range of uncertainty for the value of the Planck length and Planck time in our current units:

(Proposed Meter) 1 m = 61,873,568,789,240,733,169,843,185,036,759,895 Planck lengths.

(Proposed Second) 1 s = 18,549,229,272,558,563,350,709,419,916,719,069,277, 871,910 Planck times.

It is easy to see that, because a Planck length is the distance traveled by light in a vacuum in one Planck time, these proposed definitions would make the speed of light in meters per second have exactly the same value it has right now: 299,792,458 m/s.

There are plenty of other issues associated with these proposed revisions that cannot be discussed here—e.g., how they would interact with the proposed changes to the SI system discussed above in section five—but I want to emphasize that these new definitions of 'meter' and 'second' would eliminate the indeterminacy of the meter entirely. As such, it makes the most sense to think of the indeterminacy of the meter, which has been the topic of this paper, as a *semantic* indeterminacy, not a metaphysical indeterminacy. A metaphysical indeterminacy would not disappear by changing the definitions of 'meter' and 'second'.

9 Conclusion

We have good reason to think that there is a minimal length (whether or not it is the Planck length), and if there is a minimal length, then there is a hitherto unnoticed problem with the International System of units. The definition of 'meter' depends on the definition of 'second' in such a way that it is extremely unlikely that there is an exact number of minimal lengths in a meter (even if there is an exact number of minimal times in a second). If there is no positive integer value for the number of minimal lengths in a meter, then 'meter' is indeterminate. And if 'meter' is indeterminate, then all the units in the SI system that depend on meters are indeterminate as well. That is most of the system.

The existence of a minimal length and the indeterminacy of the meter have implications for the conventionality of measurement and the semantics of measure phrases. On a plausible account of conventionality, not all measurement terms are conventional—in particular, 'Planck length' is not conventional even if it turns out that there is no minimal length or the minimal length has some other value. Moreover, even if 'meter' turns out to be determinate, it is still conventional. In addition, the received view of measure phrases is that they are perfectly determinate. Thus, if 'meter' is indeterminate, then the received view of measure phrases is false. Nevertheless, there are some strategies available for reconciling the claims of linguists about measure phrases and the facts about the indeterminacy of 'meter'.

Let me conclude with a puzzle raised by these considerations. I began Section Two by presenting our best estimate of the Planck length in meters (i.e., 1.616199 x 10^{-35}

meters). However, if 'meter' is indeterminate because there is a minimal length and that minimal length is the Planck length, then in what sense do we have any estimate at all of the Planck length? What is the status of the claim that the Planck length is about 1.616199 x 10^{-35} meters? Wouldn't the indeterminacy of 'meter' undermine the legitimacy of this measurement? Moreover, we do not currently know whether 'meter' is indeterminate because we do not currently know whether there is an exact number of minimal lengths in a meter. With increases in technology, we could conceivably investigate this matter by getting a better estimate of the Planck length. But why should we think that getting a more precise measurement of the Planck length, in meters, would help clear up whether 'meter' is indeterminate? I think there are answers to these questions, but they are far from easy and require some reflection on the nature of scientific and conceptual change that will have to wait for future work. Fortunately, we have some time before the potential indeterminacy in 'meter' becomes technologically significant.

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