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OPTIMAL SIGNAL TIMING SIGNALIZED INTERSECTION BY GLOBAL OPTIMIZATION (OPT-*I*)

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Abstract. *Accurate measurements of signal control parameters at signalized intersections are very important for designing and operating traffic control systems. In this work, a queuing system has been modelled as a result from the characterization of the vehicles behaviour approaching and passing of a signalized intersection. This intersection is part of a network of urban traffic and two problems are formulated, distinguished by the type of control used: pre-timed, semi-actuated or fully-actuated. Subsequently global optimization and complementarity can be used to determine the parameters of the control signal. This formulation includes the green times and cycle lengths that minimize the total waiting time of vehicles at the intersection. At intersections regulated by actuated control, this methodology allows us to estimate the effective green time in actuated streams as well as the length of each cycle. It should be noted that the duration of departure of vehicles when signal is green or yellow is constant. The vehicles arrivals are random, following a Poisson probability distribution.*

*The models in question were formulated as linear programs with linear complementarity constraints (*LPLCC*). In the present study, the Sequential Linear Complementarity Algorithm (*SLCP*) was also analysed to calculate the global minimum for the* LPLCC*. Furthermore, several scenarios of traffic intersections are created to demonstrate the method's efficiency, particularly to verify the accuracy of the solutions of the problems.*

1 INTRODUCTION

Solving the problem of severe traffic congestion has become a top priority in many cities. Since 1950, traffic engineers and researchers have used Webster's formulation in order to determine the optimal cycle length and green split allocation at isolated intersections, regulated by traffic signals with a pre-timed control. However, this formulation has been proven to be ineffective under saturated conditions, and is thus inappropriate for the particular case of actuated control. Despite the acknowledgement of these limitations, an analytical solution capable of determining the optimal signal timing in the case of actuated control has yet to be found. Thus traffic engineers continue to rely on computer simulation to generate signal timing plans [11].

It is common knowledge that vehicles only leave the intersection during the green period and that the signal phases in actuated control are not of a fixed length. Nevertheless, this does not foster the application of the queuing theory which is restricted to traffic conditions of a simpler nature. Thus, in this work, a methodology for determining and carrying out timing decisions is presented. Timing plan parameters, including both the cycle's length and the green times, have been optimized based on minimizing the total waiting time at the intersection in question. As such, a signalized intersection regulated by a pre-timed, semi-actuated or fully-actuated control with two phases has been analysed and adopted. The main performance measures of the signalized intersections under consideration in the selection of a phasing plan are the queue length in addition to the delay drivers are subject to.

The formulation proposed by Webster to determine the optimal cycle length generates an unreasonably long cycle as the critical intersection flow ratio nears saturated conditions. Additionally, it is inappropriate whenever the intersection's critical flow ratio is subject to saturated conditions. The critical intersection flow ratio is the sum of the flow ratio for critical movements which are characterized by the fact that the ratio of the arrival flow to the saturation flow is the highest in the intersection. The optimal cycle length formulation proposed by [12] can be expressed as follows:

$$C_o = \frac{5 + 1.5L}{1 - \sum_i y_{c_i}}, \quad 25 \leq C_o \leq 120 \tag{1}$$

where L is the total lost time (s) and $\sum_i y_{c_i}$ represents the intersection critical flow ratio (veh/s).

Lan [3, 4] provides us with a novel formulation for determining the optimal cycle length in pre-timed control under saturation conditions, in which a nonlinear regression analysis of the functional relationship is established between the optimal cycle lengths and the traffic flow parameters, including the intersection critical flow rates, the total lost time and the duration of the analysis period. This formulation can be expressed as:

$$C_o = \frac{a + bL + c\ln(T)}{1 + d\exp(e\sum_i y_{c_i})}, \quad 25 \leq C_o \leq 120 \tag{2}$$

where T (in hours) represents the duration of the analysis period and a, b, c, d and e are the model parameters.

Both of the above mentioned methods begin by stating the cycle length and then move on to calculating the optimal green split allocation. It should be noted that in the case of the latter, the timing plan parameters, including both the cycle length and the green times, have been optimized based on the delay minimization criterion. Furthermore, it should be noted that the available literature offers very few methods for estimating delays, dealing instead exclusively with semi-actuated signals.

Traffic signals operating with semi-actuated control have been widely used on secondary streets, since they provide flexible controls adjustable to traffic volumes, thereby reducing the vehicle's delay. The actuated control is thus a strategy of response to important variations in the traffic conditions. The main problem to be found when using semi-actuated control is the difficulty in selecting an optimal combination of the maximum, minimum and unit extension of green time.

On the other hand, the queuing theory is an alternative tool which is widely used to compute performance measures for traffic signals, in the case of customers (vehicles) arriving at a service point (intersection regulated by traffic control signals). Akelik and Lin [1, 5] have proposed analytical methods for estimating green times and cycle lengths for actuated signals.

According to the available literature, semi-actuated signal operations are currently being used separately or as part of a system of coordinated traffic signals. The study of the influence of the characteristics of the arrivals and departures of traffic in the optimal performance of signalized intersections, which have this type of control, commenced shortly after the concept of actuated control was first used [1]. The need to optimize the parameters of the controller and the position of the detector, as well as to investigate the relationship between these factors has resulted in a number of studies [6, 9].

Schutter [10] have developed an algorithm to solve an Extended Linear Complementarity Problem, having subsequently applied it in order to determine the optimal timing plan in traffic control. However, the algorithm in question seems incapable of solving problems of this nature for a reasonable number of switching instants (it only appears to be effective in the case of a maximum of seven instants).

Therefore, a queuing system resulting from a signalized intersection control in an urban traffic network is considered by this paper. In addition, a model that describes the evolution of the queue lengths as a function of time is introduced. The input data for the model in question are the arrival and departure rates of the vehicles to be found at the intersection. The departure rate of vehicles during green and yellow times have been found to be constant. As for the arrivals of vehicles a Poisson probability distribution has been considered.

In the case of traffic control by vehicles, the green time associated with an actuated stream, is directly influenced by the time intervals between the consecutive detections of vehicles, and is limited by a maximum and minimum value. Therefore the estimation of arrival headways is fundamental to determining actuated signal timings.

The model in question has been formulated as the following Linear Program with Linear Complementarity Constraints (LPLCC)

$$\begin{array}{lll} (\text{LPLCC}) & \textit{Minimize} & c^T z + d^T y \\ & \textit{subject to} & Ew = q + Mz + Ny \\ & & z \geq 0, \;\; w \geq 0 \\ & & y \in K_y \\ & & z^T w = 0 \end{array} \tag{3}$$

where $q \in \mathbb{R}^p$, c, $z \in \mathbb{R}^n$, d, $y \in \mathbb{R}^m$, M, $E \in \mathbb{R}^{p\times n}$, $N \in \mathbb{R}^{p\times m}$ and

$$K_y = \{y \in \mathbb{R}^m : \; Cy = b, \; y \geq 0\}$$

with $C \in \mathbb{R}^{l\times m}$ and $b \in \mathbb{R}^l$.

The manner in which Global Optimization and Complementarity can be used to attain the optimal control parameters for an isolated signalized intersection is hereby determined. Subsequently, a Sequential Linear Complementarity (SLCP) algorithm is used to calculate a global minimum for a linear LPLCC. This algorithm determines a sequence of stationary points of the LPLCC with strictly decreasing values. The final stationary point of this sequence has been proven to represent the global minimum of the LPLCC. In general, the results present in the available literature demonstrate that the SLCP algorithm is rather efficient in determining a global optimum. However, the situation is significantly more complex when it comes to establishing the achievement of this global minimum. However, since each stationary point in the sequence corresponds to a feasible solution for a given problem, the engineer is thus able to have access to a number of possible solutions (equal to the number of iterations of the algorithm SLCP) over a reasonable period of time.

Under the scope of this study, we have investigated the nature of the performance of this algorithm when subjected to a number of traffic problems. The numerical results of various experiments that have been carried out reveal that it is possible, even in the specific instance of a long period of time instants, to efficiently determine the optimal control parameters for an isolated intersection. While, in the case of pre-timed control, the SLCP algorithm always finds the optimal solution in the first stationary point of sequence, when it comes to semi-actuated or fully-actuated controls, the SLCP determines various stationary points until the optimal solution is obtained. In all cases, the computational time required by the SLCP is very reduced and the attained solution corresponds to the minimum total delay of the intersection even when subjected to saturation conditions.

The remainder of this chapter is organized as follows. In Section 2, the types of traffic signal control are presented. The model is then introduced in Section 3, whilst Section 4 addresses the formulations of the underlying problems. A report of the computational experiments and some conclusions are presented in the last section of this paper.

2 TRAFFIC CONTROL

As far as the regulation of traffic signals is concerned, three types are considered by this paper: pre-timed, semi-actuated and fully-actuated signal operation.

In the case of pre-timed traffic control, each signal phase or traffic movement is serviced in a programmed sequence that is repeated throughout the day. Main street traffic receives a fixed amount of green time followed by the yellow and red clearance intervals. The same interval timing is then repeated for the minor or side street. The amount of time it takes to service all conflicting traffic movements is referred to as the cycle length.

The signal timings and cycle lengths may vary according to the time of day in order to reflect changes in traffic volumes and patterns. During peak traffic periods for example, cycle lengths may range up to 90 seconds to accommodate heavier volumes. During off-peak periods of the day, cycle lengths are more reduced as traffic volumes are much lighter, and therefore, not as much green time is required to effectively service all the movements. In the case of pre-timed signals, the pedestrian signal indications are automatically displayed in conjunction with the green signal for vehicles. Pre-timed signals can provide fairly efficient operation during peak traffic periods, assuming the signal timing settings reflect current conditions. However, during off-peak times, particularly at night, traffic on the main streets is often stopped for no particular reason due to little or no traffic, or pedestrians on the cross streets. In the case of pre-timed signals the only means of avoiding this unnecessary delay has been to program the signals to a flashing operation mode during the night period. Night flash operation was once common practice in many cities and municipalities. However, with advances in signal technology and detection devices, it has rarely been used over the last few years.

Actuated signal control differs from pre-timed control in that it requires actuation by a vehicle or pedestrian in order for certain phases or traffic movements to be serviced. Actuation is achieved due to vehicle detection devices and pedestrian push buttons. The most common method of detecting vehicles is to install inductive loop wires in the pavement located at or near the stop line. Video detection is also used at select locations. Actuated signals are of two types: namely, semi-actuated and fully-actuated.

Semi-actuated control is a traffic management resource deployed essentially at intersections where a main street intersects a secondary street. As such, a detector is installed along the secondary street. The main street is always allocated the green signal for at least a fixed minimum green time of $7 - 10$ seconds during a signal cycle. If the detector is activated during this interval the main street remains on green mode until the minimum green time is reached. The green signal is then passed on to the secondary street until the traffic has been cleared or until the green time reaches a fixed maximum (whichever occurs first) upon which point, the green signal is transferred back to the main street. If no vehicle is detected along the minor approach, the period of green in the main street is prolonged until a vehicle is detected by the sensor located in the secondary street. This procedure is illustrated in Figure 1.

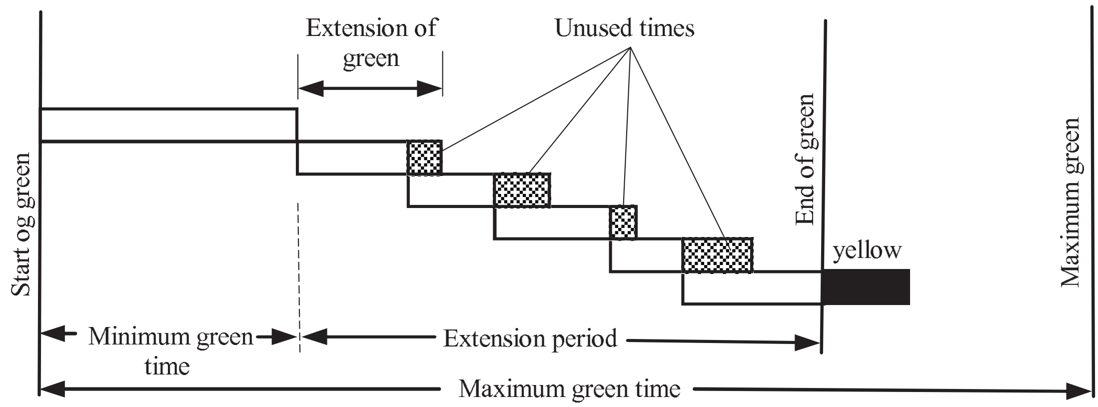

Figure 1: Extension sequence in vehicle-actuated control.

An actuated signal is assumed to be extremely efficient in the management of the available green time. The efficient use of actuated control requires careful selection of the phasing plan, timing design and detector configuration. In the case of semi-actuated control [6] the regulation of traffic signals is permanently adapted to the traffic demands, in real time in the intersection, in order to guarantee the highest possible level of efficiency.

In the case of fully-actuated control, detector loops and pedestrian push buttons are installed at all approaches. All signal phases, including left turn arrows have preset minimum and maximum greens and will be serviced on demand only. Fully-actuated signals are most efficient in isolated locations where coordination with adjacent signals is not a concern, and where the intersecting streets have similar traffic volumes. Actuated signal control provides greater efficiency in comparison to pre-timed signals by servicing cross street traffic and pedestrians only when required. The primary disadvantage of pre-timed signals is avoided as main street traffic is not interrupted unnecessarily. This is particularly beneficial during off-peak hours. The result is fewer stops and delays to the traffic on the main streets, whilst simultaneously guaranteeing safe pedestrian crossings upon demand, which ultimately leads to a decrease in fuel consumption and pollution.

3 MODEL DESCRIPTION

In this study, three models are presented. The first one is associated with a signalized intersection regulated by pre-timed control in which the randomness of the vehicle arrivals has been taken into account. The formulation of this problem is presented in [7], however the vehicle arrivals were considered deterministic. The second and third are associated with a signalized intersection regulated by semi-actuated [8] and fully-actuated control, respectively.

In this section, an intersection with four traffic streams, S_1, S_2, S_3 and S_4 has been considered, without loss of generality. Each of the traffic streams is controlled by a traffic signal, T_1, T_2, T_3 and T_4, respectively (see Figure 2).

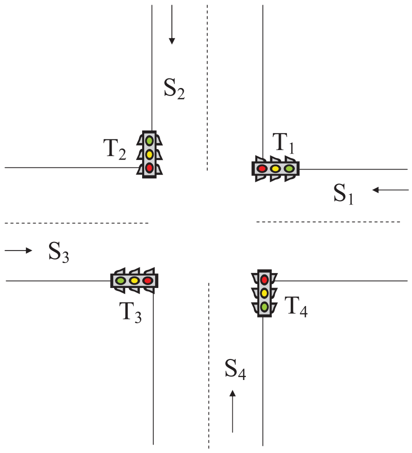

Figure 2: Sketch of a signalized intersection with four traffic streams.

The intersection presented in Figure 2 is controlled by two phases (A and B). During Phase A, the traffic signals T_1 and T_3 have a green light and the same occurs in Phase B for T_2 and T_4. In both phases, the cycle has three states: green, yellow and red.

The arrival rate of vehicles in traffic stream S_i at the particular time instant t is $\lambda_i(t)$ for $i = 1, 2, 3, 4$. When the traffic signal T_i is green, the departure rate in traffic stream S_i at the time instant t is $\mu_i(t)$ and in the case of the traffic signal being yellow, the departure rate in traffic stream S_i at time t is $\kappa_i(t)$ for $i = 1, 2, 3, 4$. The design of the timing plan is illustrated in Figure 3. Let $t_0, t_1, t_2, \ldots$ represent the time instants when a change in the traffic signals occurs.

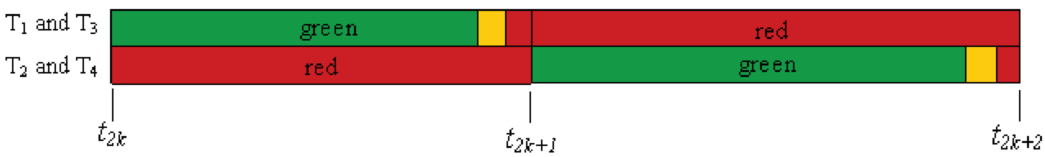

Figure 3: Diagram of signal timing.

It has been assumed that the duration of the yellow time and the clearance time are fixed and have been set equal to the d_Y and d_C values, respectively.

The time instants when the traffic signals T_1 and T_3 initiate a green period and T_2 and T_4 begin a red period are $t_0, t_2, t_4, \ldots$. The time instants when the traffic signals T_1 and T_3 initiate a red period and T_2 and T_4 begin a green period are $t_1, t_3, t_5, \ldots$.

Consequently, one is lead to $t_{2k+1} - t_{2k} = y_G + d_Y + d_C$ and $t_{2k+2} - t_{2k+1} = y_R + d_Y + d_C$, $k \in \mathbb{N}_0$. Therefore, y_G represents the green time and y_R represents the red time in traffic signals T_1 and T_3. A cycle length is equal to $y_G + y_R + 2(d_Y + d_C)$. Clearly, one should then have $y_R, y_G \geq d_Y \geq 0$. Furthermore, $\lambda_i(t), \mu_i(t), \kappa_i(t) \geq 0, \forall i, t$ and $t_k < t_{k+1}, \forall k$.

The queue length in the traffic stream S_i at instant time t, $L_i(t)$, is thus clearly equal to or greater than zero for all i and t.

Whenever the traffic signal T_i is red, arrivals at traffic stream S_i are verified, which are characterized by the arrival rate function $\lambda_i(t)$. It should also be noted that there are no departures in this case. Alternatively, when the traffic signal T_i is green or yellow, both arrivals and departures occur at traffic stream S_i. In these cases, the net queue growth rate at the time instant t is $\lambda_i(t) - \mu_i(t)$ or $\lambda_i(t) - \kappa_i(t)$, respectively. Accordingly, for streams S_1 and S_3, the evolution of the queue length is obtained by

$$\frac{dL_i(t)}{dt} = \begin{cases} \lambda_i(t) - \mu_i(t), & t \in [t_{2k}, t_{2k+1} - d_Y - d_C] \\ \lambda_i(t) - \kappa_i(t), & t \in [t_{2k+1} - d_Y - d_C, t_{2k+1} - d_C] \\ \lambda_i(t), & t \in [t_{2k+1} - d_C, t_{2k+2}] \end{cases} \tag{4}$$

for $i = 1, 3$ and $k \in \mathbb{N}_0$.

Similarly, for traffic signals T_2 and T_4, the evolution of the queue lengths in traffic streams S_2 and S_4 are obtained by

$$\frac{dL_i(t)}{dt} = \begin{cases} \lambda_i(t), & t \in [t_{2k}, t_{2k+1}] \cup [t_{2k+2} - d_C, t_{2k+2}] \\ \lambda_i(t) - \mu_i(t), & t \in [t_{2k+1}, t_{2k+2} - d_Y - d_C] \\ \lambda_i(t) - \kappa_i(t), & t \in [t_{2k+2} - d_Y - d_C, t_{2k+2} - d_C] \end{cases} \tag{5}$$

for $i = 2, 4$ and $k \in \mathbb{N}_0$.

The relation of the queue length between the time instants t_{2k} and t_{2k+2} may be represented by the following equations:

$$\begin{aligned} L_i(t_{2k+2}) &= L_i(t_{2k+1}) + \int_{t_{2k+1}}^{t_{2k+2}} \lambda_i(t)dt \\ L_i(t_{2k+1}) &= L_i(t_{2k}) + \int_{t_{2k}}^{t_{2k+1}-d_Y-d_C} (\lambda_i(t) - \mu_i(t))dt + \\ &+ \int_{t_{2k+1}-d_Y-d_C}^{t_{2k+1}-d_C} (\lambda_i(t) - \kappa_i(t))dt + \int_{t_{2k+1}-d_C}^{t_{2k+1}} \lambda_i(t)dt \end{aligned}$$

for $i = 1, 3$ and $k \in \mathbb{N}_0$.

The equations describing the relationship of the queue length at traffic streams S_2 and S_4 are obtained in a similar manner.

Let us assume that, for each $i \in \{1, 2, 3, 4\}$, $\bar{\mu}_i$ and $\bar{\kappa}_i$ represent the average departure rates when the traffic signal is green or yellow, respectively. Let one further assume that $\mu_i(t) = \bar{\mu}_i$ and $\kappa_i(t) = \bar{\kappa}_i$ for all time instants t in addition to the fact that queue is not empty. In the latter case the departure rates are equal to zero. Therefore departures are considered to be deterministic.

As for the arrivals distribution, a model that represents a random arrival process has been considered. The Poisson distribution has been found to be the random process which better suits the particular situation in question. It is a discrete distribution and is commonly referred to as a counting distribution, representing the count distribution of random events. The assumption of the arrival of random vehicles also implies a random distribution of the time intervals between the arrivals of the successive vehicles.

The Poisson distribution can describe the probability of observing n arrivals in a period from 0 to t with the aid of the following expression:

$$p_n(t) = \frac{(\bar{\lambda}t)^n}{n!} e^{-\bar{\lambda}t}$$

where $\bar{\lambda}$ is the average arrival rate in vehicles per unit of time and t is the duration of the time interval over which vehicles are counted. This equation provides information as to how probability is distributed over a time interval in terms of the total number of vehicles. In a sequence of n arrivals one can observe vehicles passing with a random headway distance.

Due to the fact that, in the case of these approaches, the arrival and the departure rates are nonnegative values and $\bar{\kappa}_i \leq \bar{\mu}_i$, the queue lengths will never be negative values if equations (4)-(3) are considered. Therefore, it is clear that this non-negativity condition must be included in the equations describing the evolution of queue lengths. Thus,

$$\begin{aligned}
L_i(t_{2k+2}) &= \max\{L_i(t_{2k+1}) + \lambda_i(t_{2k+2})\,(y_R + d_Y + d_C)\,, 0\} \\
L_i(t_{2k+1} - d_Y - d_C) &= \max\{L_i(t_{2k}) + (\lambda_i(t_{2k+1}) - \bar{\mu}_i)y_G, 0\} \\
L_i(t_{2k+1} - d_C) &= \max\{L_i(t_{2k}) + (\lambda_i(t_{2k+1}) - \bar{\mu}_i)y_G + \\
&\quad +(\lambda_i(t_{2k+1}) - \bar{\kappa}_i)d_Y, (\lambda_i(t_{2k+1}) - \bar{\kappa}_i)d_Y, 0\} \\
L_i(t_{2k+1}) &= \max\{L_i(t_{2k}) + (\lambda_i(t_{2k+1}) - \bar{\mu}_i)y_G + (\lambda_i(t_{2k+1}) - \bar{\kappa}_i)d_Y \\
&\quad +\lambda_i(t_{2k+1})d_C, (\lambda_i(t_{2k+1}) - \bar{\kappa}_i)d_Y \\
&\quad +\lambda_i(t_{2k+1})d_C, \lambda_i(t_{2k+1})d_C\}
\end{aligned}$$

for $i = 1, 3$ and $k \in \mathbb{N}_0$.

It should be noted that according to a deterministic approach $\lambda_i(t_k) = \bar{\lambda}_i$ for all $k \in \mathbb{N}_0$ and for traffic streams S_2 and S_4 the equations can be obtained in a similar manner.

4 PROBLEMS FORMULATION

If the following vectors are considered

$$\begin{aligned}
x_k &= [L_1(t_k), L_2(t_k), L_3(t_k), L_4(t_k)]^T \\
b_{1_{k+1}} &= [\lambda_1(t_{2k+1}) - \bar{\mu}_1, \lambda_2(t_{2k+1}), \lambda_3(t_{2k+1}) - \bar{\mu}_3, \lambda_4(t_{2k+1})]^T \\
b_{2_{k+1}} &= [\lambda_1(t_{2k+2}), \lambda_2(t_{2k+2}) - \bar{\mu}_2, \lambda_3(t_{2k+2}), \lambda_4(t_{2k+2}) - \bar{\mu}_4]^T \\
b_{3_{k+1}} &= [(\lambda_1(t_{2k+1}) - \bar{\kappa}_1)d_Y + \lambda_1(t_{2k+1})d_C, \lambda_2(t_{2k+1})(d_Y + d_C), \\
&\quad (\lambda_3(t_{2k+1}) - \bar{\kappa}_3)d_Y + \lambda_3(t_{2k+1})d_C, \lambda_4(t_{2k+1})(d_Y + d_C)]^T \\
b_{4_{k+1}} &= [\lambda_1(t_{2k+2})(d_Y + d_C), (\lambda_2(t_{2k+2}) - \bar{\kappa}_2)d_Y + \lambda_2(t_{2k+2})d_C, \\
&\quad \lambda_3(t_{2k+2})(d_Y + d_C), (\lambda_4(t_{2k+2}) - \bar{\kappa}_4)d_Y + \lambda_4(t_{2k+2})d_C]^T \\
b_{5_{k+1}} &= [\max\{(\lambda_1(t_{2k+1}) - \bar{\kappa}_1)d_Y + \lambda_1(t_{2k+1})d_C, \lambda_1(t_{2k+1})d_C\}\,, 0, \\
&\quad \max\{(\lambda_3(t_{2k+1}) - \bar{\kappa}_3)d_Y + \lambda_3(t_{2k+1})d_C, \lambda_3(t_{2k+1})d_C\}\,, 0]^T \\
b_{6_{k+1}} &= [0, \max\{(\lambda_2(t_{2k+2}) - \bar{\kappa}_2)d_Y + \lambda_2(t_{2k+2})d_C, \lambda_2(t_{2k+2})d_C\}\,, 0, \\
&\quad \max\{(\lambda_4(t_{2k+2}) - \bar{\kappa}_4)d_Y + \lambda_4(t_{2k+2})d_C, \lambda_4(t_{2k+2})d_C\}]^T
\end{aligned}$$

then

$$x_{2k+1} = \max\left\{x_{2k} + b_{1_{k+1}}y_G + b_{3_{k+1}}, b_{5_{k+1}}\right\}$$

$$x_{2k+2} = \max\left\{x_{2k+1} + b_{2_{k+1}}y_R + b_{4_{k+1}}, b_{6_{k+1}}\right\}$$

for $k \in \mathbb{N}_0$.

In this study, non-saturated intersections have been taken in consideration, which implies that the queue lengths may disappear when the traffic signal is green.

Let us assume that the arrival and departure rates have been previously determined. The model then seeks an optimal cycle length and optimal green split allocation for each phase. The objective function represents the total average waiting time experienced by the vehicles in all queues:

$$J = \frac{1}{t_N - t_0} \sum_{i=1}^{4} \int_{t_0}^{t_N} \frac{1}{\lambda_i(t)} L_i(t) dt \tag{6}$$

where N is the number of time instants and $t_N - t_0$ is the time interval considered.

One of the advantages of using criteria based on time averaged values is that the objective function has a finite value even if N or t_N tend to infinity, provided that the queue lengths remain finite.

Some additional conditions, such as the minimum and maximum durations for the red and green times or the maximum queue lengths, have been also added to the model since short cycles imply more stops, and long cycles cause longer delays. As such, they are unsuitable for the variations in the daily flow of traffic. This leads one to the following mathematical programming program:

$$\begin{aligned}
(P1) \quad & Min \quad J & \\
& s.t. \quad g_{min_A} \leq y_G \leq g_{max_A} & (7)\\
& \qquad\; g_{min_B} \leq y_R \leq g_{max_B} & (8)\\
& \qquad\; 0 \leq x_k \leq x_{max} & (9)\\
& \qquad\; x_{2k+1} = \max\left\{x_{2k} + b_{1_{k+1}} y_G + b_{3_{k+1}}, b_{5_{k+1}}\right\} & (10)\\
& \qquad\; x_{2k+2} = \max\left\{x_{2k+1} + b_{2_{k+1}} y_R + b_{4_{k+1}}, b_{6_{k+1}}\right\} & (11)
\end{aligned}$$

where $k \in \mathbb{N}_0$, g_{min} and g_{max} are the minimum green and maximum green time, respectively, in phase A(B) and x_{max} is the maximum queue length in each traffic stream.

However, for each index k, the nonlinear constraints (10) and (11) can respectively be rewritten as

$$\left\{\begin{array}{l}
x_{2k+1} \geq x_{2k} + b_{1_{k+1}} y_G + b_{3_{k+1}} \\
x_{2k+1} \geq b_{5_{k+1}} \\
(x_{2k+1} - x_{2k} - b_{1_{k+1}} y_G - b_{3_{k+1}})^T (x_{2k+1} - b_{5_{k+1}}) = 0
\end{array}\right. \tag{12}$$

and

$$\left\{\begin{array}{l}
x_{2k+2} \geq x_{2k+1} + b_{2_{k+1}} y_R + b_{4_{k+1}} \\
x_{2k+2} \geq b_{6_{k+1}} \\
(x_{2k+2} - x_{2k+1} - b_{2_{k+1}} y_R - b_{4_{k+1}})^T (x_{2k+2} - b_{6_{k+1}}) = 0
\end{array}\right. \tag{13}$$

and thus the objective function (6) can be replaced by

$$J = \frac{1}{N} \sum_{i=1}^{4} \frac{1}{\bar{\lambda}_i} \left(\sum_{k=1}^{N-1} (x_k)_i + \frac{(x_N)_i}{2} \right). \tag{14}$$

Note that (12) and (13) can be stated as

$$z_{2k+1} = x_{2k+1} - b_{5_{k+1}} \geq 0 \tag{15}$$
$$z_{2k+2} = x_{2k+2} - b_{6_{k+1}} \geq 0 \tag{16}$$
$$w_{2k+1} = x_{2k+1} - x_{2k} - b_{1_{k+1}} y_G - b_{3_{k+1}} \geq 0 \tag{17}$$
$$w_{2k+2} = x_{2k+2} - x_{2k+1} - b_{2_{k+1}} y_R - b_{4_{k+1}} \geq 0 \tag{18}$$
$$w \geq 0, z \geq 0 \tag{19}$$
$$w^T z = 0 \tag{20}$$

Due to the fact that, in this work, two kinds of traffic signals regulation have been considered, namely pre-timed and semi-actuated control, some different characteristics are expected to occur in the formulation of each problem.

4.1 Pre-timed control

In the case of pre-timed control, in particular, the equations (15)-(20) can be rewritten as

$$w_{2k+1} = z_{2k+1} - z_{2k} - b_{1_{k+1}} y_G - b_{3_{k+1}} + b_{5_{k+1}} - b_{6_{k+1}} \tag{21}$$
$$w_{2k+2} = z_{2k+2} - z_{2k+1} - b_{2_{k+1}} y_R - b_{4_{k+1}} - b_{5_{k+1}} + b_{6_{k+1}} \tag{22}$$
$$w \geq 0, z \geq 0 \tag{23}$$
$$w^T z = 0 \tag{24}$$

Thus, by taking into account the constraints (7)-(9) and (21)-(23), the problem (P1) can be reduced to the following Linear Program with Linear Complementarity Constraints (LPLCC):

$$\begin{array}{ll} Minimize & c^T z \\ \text{s. t.} & w = q + Mz + Ny \\ & l \leq y \leq u \\ & 0 \leq z \leq z_{max}, w \geq 0. \\ & w^T z = 0. \end{array} \tag{25}$$

where $M \in \mathbb{R}^{4N \times 4N}$, $N \in \mathbb{R}^{4N \times 2}$, $q \in \mathbb{R}^{4N}$, $c \in \mathbb{R}^{4N}$, $l, u \in \mathbb{R}^2$, $z_{max_{2k+1}} = x_{max} - b_{5_{k+1}}$ and $z_{max_{2k+2}} = x_{max} - b_{6_{k+1}}$ for $k \in \mathbb{N}_0$.

4.2 Semi-actuated and fully-actuated control

In a signalized intersection regulated by actuated control for the semi-actuated streams it is well known that:

- the green time can assume different values for each cycle;
- the traffic signal remains green until:
  - traffic is cleared;
  - green time reaches a fixed maximum value.

Since in the fully-actuated control all streams at the intersection are actuated, the formulation of the underlying model is an extension of the case of semi-actuated control which is detailed in the following. Without loss of generality, for the semi-actuated control, let us assumed that

S_2 and S_4 are the actuated streams at the intersection. Therefore, to represent the characteristics of the semi-actuated control model, the following expressions should be incorporated into the problem

$$\begin{aligned} a_{k+1} &= max\left\{x_{2k+1}(2) + b_{2_{k+1}}(2)y_{R_{k+1}}, 0\right\} \\ c_{k+1} &= max\left\{x_{2k+1}(4) + b_{2k+1}(4)y_{R_{k+1}}, 0\right\} \end{aligned} \tag{26}$$

where a_{k+1} and c_{k+1} are the queue lengths of streams S_2 and S_4 at the time instant when traffic signals T_2 and T_4 change from green to yellow. Due to the fact that, in some of cases, the traffic in one of these two streams has not been cleared at this time instant, the green time must forcibly reach a fixed maximum, the following constraint must consequently be considered in the formulation:

$$(a_{k+1} + c_{k+1})^T (g_{max_B} - y_{R_{k+1}}) = 0 \tag{27}$$

The equations (26) and (27) can be rewritten in the following manner:

$$\begin{cases} d_{k+1} = a_{k+1} - x_{2k+1}(2) - b_{2_{k+1}}(2)y_{R_{k+1}} \\ e_{k+1} = c_{k+1} - x_{2k+1}(4) - b_{2_{k+1}}(4)y_{R_{k+1}} \\ t_{k+1} = a_{k+1} + c_{k+1} \\ yy_{R_{k+1}} = g_{max_B} - y_{R_{k+1}} \\ t \geq 0, a \geq 0, c \geq 0, d \geq 0, e \geq 0, yy_{R_{k+1}} \geq 0 \\ a^T d = 0, c^T e = 0, t^T yy_{R_{k+1}} = 0 \end{cases} \tag{28}$$

Taking into account the sets of constraints (15)-(20) and (28) as well as the objective function, (14) the semi-actuated control problem can be formulated as the following LPLCC:

$$\begin{aligned} Min \quad & J \\ & z_{2k+1} = x_{2k+1} - b_{5_{k+1}} \geq 0 \\ & z_{2k+2} = x_{2k+2} - b_{6_{k+1}} \geq 0 \\ & w_{2k+1} = x_{2k+1} - x_{2k} - b_{1_{k+1}} y_G - b_{3_{k+1}} \geq 0 \\ & w_{2k+2} = x_{2k+2} - x_{2k+1} - b_{2_{k+1}} y_R - b_{4_{k+1}} \geq 0 \\ & d_{k+1} = a_{k+1} - x_{2k+1}(2) - b_2(2)y_{R_{k+1}} \\ & e_{k+1} = c_{k+1} - x_{2k+1}(4) - b_2(4)y_{R_{k+1}} \\ & t_{k+1} = a_{k+1} + c_{k+1} \\ & yy_{R_{k+1}} = g_{max_B} - y_{R_{k+1}} \\ & w \geq 0, z \geq 0, t \geq 0, a \geq 0, c \geq 0, d \geq 0, e \geq 0 \\ & 0 \leq x \leq x_{max}, yy_{R_{k+1}} \geq 0 \\ & g_{min_B} \leq y_{R_{k+1}} \leq g_{max_B}, g_{min_A} \leq y_G \leq g_{max_A} \\ & w^T z = 0, a^T d = 0, c^T e = 0, t^T yy_{R_{k+1}} = 0 \end{aligned}$$

This problem can be easily stated as

$$\begin{array}{ll} Minimize & d'^T y' \\ \text{s. t.} & Ew' = q + Mz' + Ny' \\ & l \leq y' \leq u \\ & z' \geq 0, w' \geq 0. \\ & w'^T z' = 0. \end{array} \tag{29}$$

where $E \in \mathbb{R}^{10N\times 10N}$, $M \in \mathbb{R}^{10N\times 10N}$, $N \in \mathbb{R}^{10N\times(4N+1)}$, $q \in \mathbb{R}^{10N}$, $d' \in \mathbb{R}^{4N+1}$ and $l, u \in \mathbb{R}^{4N+1}$.

These problems can be solved with the aid of the Sequential Linear Complementarity algorithm [2] which reaches a global minimum of the LPLCC by computing a set of stationary points with strictly decreasing objective function values.

5 COMPUTATIONAL EXPERIMENTS

In this section, some of the more relevant computational experience relating to the proposed traffic model, which exploits the LPLCC formulation and uses the algorithm SLCP, which has been carried out under the scope of this study, is reported. All the computations have been performed on a Intel(R) Core (TM) 2 Duo CPU 2.4 GHz machine with 2 GB RAM.

Table 1 presents parameters that distinguish each test problem (PROB), namely, the four arrival rates, $\bar{\lambda}_i$ (veh/h), $i = 1, 2, 3, 4$. In each of the test problems, a signalized intersection has been considered with the following specifications:

$\bar{\mu}_i$ = 1800 veh/h $\quad\quad$ $\bar{\kappa}_i$ = 720 veh/h $\quad$ for $i = 1, 2, 3, 4$

d_Y = 3 s $\quad\quad$ d_C = 2 s

PROB	λ_1	λ_2	λ_3	λ_4
P1	600	370	400	240
P2	400	400	700	320
P3	150	550	700	400
P4	700	350	300	600
P5	750	650	250	500
P6	600	540	400	450
P7	400	480	420	700
P8	650	300	550	325
P9	450	300	600	750
P10	450	300	750	300
P11	250	300	400	450
P12	650	230	200	550
P13	500	375	325	400
P14	250	350	650	150
P15	650	540	540	600

Table 1: Parameters for the test problems

Note that the problems should be solved by a sufficiently large number of intervals, in order to eliminate the instability resulting from the initial conditions. In this study, N=121 has been considered, which corresponds to a period of time over 60 cycles.

5.1 Pre-timed control

In the computational experiments relating to pre-timed control, the green time is limited by g_{min}=10 s and g_{max}=60 s in both phases.

Table 2 displays an optimal cycle length for each test problem and the corresponding green splits for phase A (traffic signals T_1 and T_3) obtained by SLCP algorithm. The resulting solutions lead to a minimum total delay at the intersection in question. The numerical results attained indicate that the optimal cycle length, in each test problem, reflects the correspondence with the traffic flow on each stream.

To confirm the applicability of this methodology, the same test traffic problems have been used to estimate the optimal cycle length and the green time split for phase A by the Webster and Lan formulations (equations (1) and (2), respectively). In Table 2, the solutions attained by the various approaches mentioned above are presented by:

GR: Green time split (s)

RD: Red time split (s)

CY: Cycle length (s)

For all test problems relating to pre-timed control the cycle length can be obtained by CY=RD+GR+2(d_Y+d_C).

	SLCP			WEBSTER			LAN		
PROB	GR	RD	CY	GR	RD	CY	GR	RD	CY
P1	15.8	10.0	35.8	21.1	12.4	43.5	17.1	10.0	37.1
P2	16.7	11.6	38.3	25.3	14.7	50.0	21.9	12.7	44.6
P3	18.2	15.7	43.9	29.3	23.2	62.5	25.7	20.2	55.9
P4	19.1	16.2	45.3	32.4	29.0	71.4	27.4	24.5	61.9
P5	24.0	23.1	57.0	43.6	37.3	90.9	33.0	28.2	71.2
P6	18.6	15.8	44.4	24.2	21.4	55.6	21.3	18.8	50.1
P7	12.7	16.9	39.6	16.5	26.1	52.6	14.4	22.9	47.3
P8	18.1	10.9	39.0	22.3	11.2	43.5	18.1	9.0	37.1
P9	18.6	21.6	50.2	32.8	40.5	83.3	26.0	32.1	68.1
P10	18.6	10.7	39.3	27.2	10.4	47.6	23.2	8.8	42.0
P11	10.0	10.4	30.4	13.1	15.4	38.5	9.4	11.0	30.4
P12	17.0	14.0	41.0	26.6	22.2	58.8	23.5	19.5	53.0
P13	12.7	10.5	33.2	16.8	13.2	40.0	12.6	9.9	32.5
P14	14.2	10.0	34.2	22.8	12.7	45.5	19.0	10.5	39.5
P15	22.3	20.6	52.9	29.2	27.5	66.7	25.1	23.8	58.9

Table 2: Comparison of solutions obtained by SLCP, Webster and Lan.

Figure 4 shows a comparison of the estimated average uniform delay of the intersection resulting of the solutions presented in Table 2. In general, the optimal cycles attained by SLCP for the proposed problems lead to minor delays.

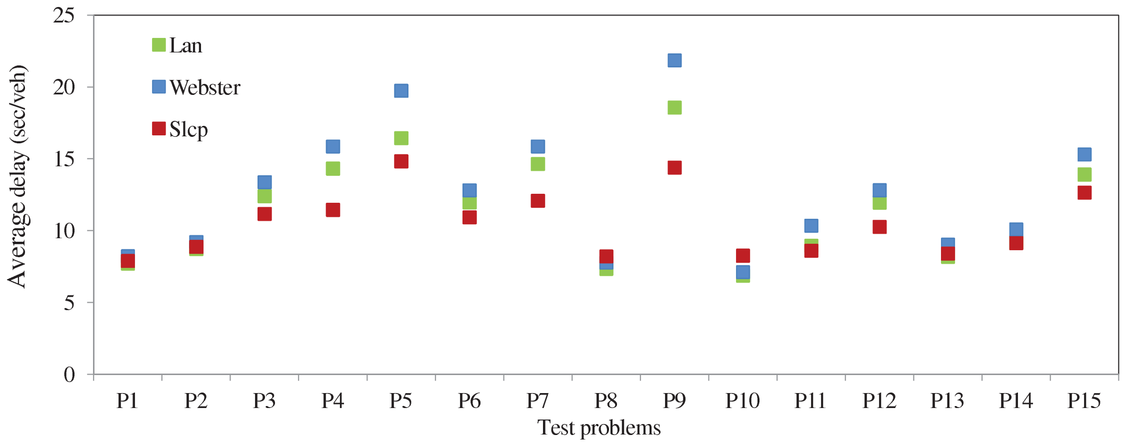

Figure 4: Comparison of the average delay of the intersection resulting of the solutions obtained by SLCP, Webster and Lan.

5.2 Semi-actuated control

In this particular experience, an isolated intersection regulated by semi-actued control with three traffic streams (S_1, S_2 and S_3) has been considered, since this type of control is the most used for T-intersections. In this case, the actuated stream is S_2. The lower bound for the green time is g_{min}=7 s, however the upper bounds can assume two values:

- if the stream is actuated g_{max_B}=40 s;
- alternatively, g_{max_A}=60 s.

For each test problem, Table 3 displays the corresponding solution attained by SLCP for the actuated stream (phase B): average cycle lengths, average green times and red splits. The numerical results indicate that the signal timings, in each test problem, reflect the correspondence with the traffic flow on each stream. Thus, this methodology is useful to estimate signal timings of semi-actuated control.

PROB	Red time	Average green time	Average cycle length
P1	26.5	13.2	49.7
P2	27.3	14.4	51.7
P3	25.1	18.2	53.4
P4	22.9	11.3	44.1
P5	35.9	26.6	72.5
P6	29.3	19.5	58.8
P7	19.7	14.4	44.1
P8	21.7	9.2	40.8
P9	18.2	8.7	36.9
P10	25.1	9.7	44.8
P11	13.9	8.1	32.0
P12	19.7	7.7	37.5
P13	22.1	12.4	44.4
P14	20.0	10.6	40.6
P15	31.6	20.5	62.0

Table 3: Experimental results attained to semi-actuated intersection.

5.3 Fully-actuated control

For the fully-actuated control, a signalized intersection with four actuated traffic streams (S_1, S_2, S_3 and S_4) is considered in this computational experience. In all streams, the lower bound for the green time is g_{min}=7 s while the upper bound is g_{max}=40 s.

For each test problem, Table 4 displays the corresponding solution attained by SLCP for the phase A: average cycle lengths, average green times and average red times. This methodology is useful to estimate signal timings for every cycle of fully-actuated control.

PROB	Average green time	Average red time	Average cycle length
P1	17.0	12.7	39.7
P2	18.5	12.8	41.2
P3	17.8	15.2	43.0
P4	21.6	19.8	51.4
P5	25.7	25.2	60.9
P6	20.1	18.3	48.4
P7	15.8	20.6	46.5
P8	19.7	11.5	41.1
P9	22.1	22.9	55.0
P10	18.4	10.2	38.9
P11	8.9	9.7	28.5
P12	18.1	17.3	45.4
P13	13.2	12.5	35.7
P14	14.5	9.2	33.6
P15	24.9	22.5	57.4

Table 4: Experimental results attained to fully-actuated intersection.

6 CONCLUSIONS

- In this paper, the solution of a LPLCC associated with traffic control problems has been duly investigated. The problems formulation describes the evolution of the queue lengths at signalized intersections regulated by pre-timed or actuated control. Consequently, a Sequential Linear Complementarity algorithm has been considered throughout this study to find the global minimums.

- The numerical results of several test problems reveal that it is possible to accurately determine the optimal cycle length and the effective green time for an signalized intersection regulated by pre-timed control. This methodology is also useful since it allows us to estimate the average cycle length and average green time for actuated signal operations. In this manner, the solutions obtained can be useful for estimating the delay of vehicles using any existing model in the literature.

- The SLCP algorithm always determines the optimal solution and requires reduced computational time. The solutions attained by the present study correspond to the minimum total delay of the intersections even when under saturation conditions.

- The future work of under the scope of this study will be to focus on the applications of these methodologies to real traffic intersections with the aim of demonstrating and highlighting the utility of these procedures.

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