Existence and Stability of Multihumped Femtosecond Solitons

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ABSTRACT
The propagation of bound soliton pairs in nonlinear photonic crystal fibers has recently been experimentally observed. The system may be modeled by a generalized nonlinear Schrödinger equation (GNLSE) which includes higher intrapulse Raman Scattering, self-steepening and higher order dispersion. Here, we find multihumped pulses as result of an accelerating similarity reduction of a GNLSE containing the intrapulse Raman scattering. Numerical simulations of the suitable GNLSE using these solutions as input showed that they are not stable, however, they may be related with the experimentally observed bound pairs since they propagate steadily for distances compared to the ones observed.
Keywords: intrapulse Raman scattering; bound solitons; photonic crystal fibers.

1. INTRODUCTION

In recent experiments, the propagation of bound pairs of short pulses was observed in highly nonlinear photonic crystal fibers (PCF) [1, 2]. The propagation of such short pulses in nonlinear fibers is affected by the standard group velocity dispersion and the Kerr effect, which are well described by the nonlinear Schrödinger equation (NLS), and by higher order effects like third order dispersion, self-steepening and intrapulse Raman scattering (IRS) [3]. The occurrence of bound solitons in the NLS is possible and predicted by the inverse scattering method as N-soliton solutions for which the velocities are all equal [4].

The NLS bound solitons propagate with periodic oscillations, with a period related to its beating frequencies, and are known to decay when they are in presence of higher order effects like the IRS [5]. Nevertheless, two-humped solutions that propagate without change of form were already associated with the model NLS plus IRS [6]. They accelerate along the fiber distance and their mean frequency linearly downshifts, in accordance to what is also observed in single hump propagation, which is known as soliton self-frequency shift [7].

Here, we obtain multihumped solutions of the NLS plus IRS equation as solutions of the ordinary differential equation (ODE) that results from an accelerating similarity reduction. Their profiles have characteristics that suffer small changes with the IRS strength. The stability of these solutions is studied by numerical simulation of the full evolution equation and their relation with the observed bound soliton pairs in PCFs is studied.

2. THE GENERALIZED NONLINEAR SCHRÖDINGER EQUATION AND THE ACCELERATED SIMILARITY VARIABLE

For spectral widths relatively small, the Raman gain spectrum may be approximately modelled by a linear function and, neglecting higher order dispersion, the dimensionless evolution equation reads [3]

$$iq_z + \tfrac{1}{2}q_{TT} + |q|^2 q = T_r\left(|q|^2\right)_T q - is\left(|q|^2 q\right)_T, \tag{1}$$

where q relates to the complex amplitude of the slowly-varying envelope A of the optical field by

$$A(z,t) = \sqrt{\frac{cA_{eff}\,|k_0''|}{\omega_0 n_2 \tau_0^2}}\, q(Z,T)$$

and the normalized distance and time are

$$Z = \frac{z|k_0''|}{\tau_0^2} \text{ and } T = \frac{\tau}{\tau_0},$$

where τ is measured in a referential that moves with the group velocity of the carrier frequency ω_0 and τ_0 is a normalization time. The normalized Raman parameter is $T_r = \tau_r/\tau_0$ and $s = 1/\omega_0\tau_0$ is associated with the self-steepening process. k_0'' is the group dispersion at the carrier frequency, A_{eff} is the effective core area of the fiber, c is the vacuum speed of light and n_2 is the Kerr coefficient. The value of τ_r can be evaluated from the Raman response function using $\tau_r = \int_0^\infty t'R(t')dt'$, where $R(t) = (1-f_R)\delta(t) + f_R\left(q_a h_a(t) + q_b h_b(t)\right)$ includes both the

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instantaneous electronic and the retarded molecular responses, as well as the isotropic and the anisotropic parts, and [8]

$$h_a(t) = \frac{\tau_1^2 + \tau_2^2}{\tau_1^2 \tau_2^2} \exp\left(-\frac{t}{\tau_2}\right) \sin\left(\frac{t}{\tau_1}\right),\ h_b(t) = \frac{2\tau_b - t}{\tau_b^2} \exp\left(-\frac{t}{\tau_b}\right).$$

By using the values [1] $\tau_1 = 12.2$ fs, $\tau_2 = 32$ fs, $\tau_b = 96$ fs, $f_R = 0.245$, $q_a = 0.79$ and $q_b = 0.21$ we easily obtain $\tau_r = 1.572$. Moreover, if we take the free-space wavelength to be $\lambda_0 = 800$ nm, we have $\frac{s}{T_r} \cong 0.27$, which implies that IRS term will be dominant in the right hand member of equation (1).

Let us first consider $s = 0$. The model (1) with $s = 0$ has been used to account for the soliton self-frequency shift since their observation in 1986. Straightforward application of an adiabatic perturbation technique to the above perturbed NLS equation yields a frequency shift given by [7]

$$\Delta\omega_0(z) = \frac{8|k_0''|\tau_r}{15} \frac{1.76^4}{t_{\mathrm{FWHM}}^4} z,$$

which is in satisfactorily agreement with the experimentally observed self-frequency shift. As the group-velocity is frequency-dependent, this frequency shift also changes the soliton velocity from its original value and produces a temporal displacement of the pulses proportional to z^2. These results anticipate that self-similar accelerating pulses might exist. Hence, inserting the *ansatz* $q(Z,T) = \exp\left[i\theta(Z,\eta)\right]W(\eta)$, where $\eta = T - \frac{a}{4}Z^2 + bZ$ and both $\theta(Z,\eta)$ and $W(\eta)$ are real, into equation (1) and requiring that $W \to 0$ as $\eta \to \infty$, we obtain the following ordinary differential equation for $W(\eta)$

$$W'' + \left(-a\eta - D + 2W^2 - 4T_r WW'\right)W = 0 \tag{2}$$

and θ given by

$$\theta(Z,\eta+) = \left(\tfrac{1}{2}aZ - b\right)\eta + \tfrac{1}{2}\left(D + b^2\right)Z - \tfrac{1}{4}baZ^2 + \tfrac{1}{24}a^2Z^3 + \theta_0. \tag{3}$$

Furthermore, the two parameters a and T_r may then be replaced by a single parameter γ through the transformation

$$W(\eta) = \frac{\gamma}{4t_r}P(\varsigma),\ \varsigma = \frac{\gamma}{4T_r}\eta,\ \gamma = a^{\frac{1}{4}}(4T_r)^{\frac{3}{4}}, \tag{4}$$

giving the following ODE

$$P'' + \left(C - \gamma\varsigma + 2P^2 - \gamma PP'\right) = 0, \tag{5}$$

where $C = D\left(\frac{4T_r}{\gamma}\right)^2$ is an arbitrary parameter.

We are interested in pulse solutions to (5) which describe steadily accelerating solutions to the full evolution equation. For such pulse solutions, the asymptotic behavior at the tails should match to the behavior of the Airy functions $A_i(z)$ and $B_i(z)$ since the linearization of (5) is equivalent to an Airy equation. Following a shooting method described elsewhere [9], we set $C = \sqrt{15/4}$ and are able to find localized solutions to (5) for γ up to 0.25. For each γ, there are one hump and several multihumped solutions, all of them located around $\varsigma = 0$.

The one-hump solutions and the upper limit of γ for their existence will be discussed in [9]. Here, we are much concerned with the multihumped profiles since we want to test them as precursors of the bound pair of pulses observed in PCFs. These multihumped profiles are easily obtained with a variant of the shooting method used for computing the single-humped ones. Figure 1 shows four examples of multihumped profiles. Whenever the solutions have a single peak, it is very close to $\sqrt{C}$. The multihumped solutions have a higher peak at the right and the following peaks to the left have decreasing amplitude. As γ increases, the peaks are closer in amplitude but further apart in ς-location. The humps are very close to a sech with a small asymmetry that is growing with γ.

It is worth noting a possible explanation for their existence. The equation (5) is a non-autonomous ODE which reduces to

$$P'' - CP + 2P^3 = 0, \tag{6}$$

in the $\gamma = 0$ case. The above equation gives us the steady travelling solutions of the NLS. They are the sech-soliton solution and several periodic solutions. The equation does not admit any solution with a finite number of

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humps since this would violate the theorem that says that there is a unique orbit through any point of the phase space [6]. However, the non-autonomous character of (5) allows this possibility. Hence for small γ, our multihumped solution may be compared to a mixture of the homoclinic orbit and some periodic orbits inside it of equation (6). Moreover, we expect they have no counterpart for $\gamma = 0$. The bound solitons of the NLS will evolve along z with periodic oscillations such that they are not steady travelling profiles and should not be solution of the above ode (6).

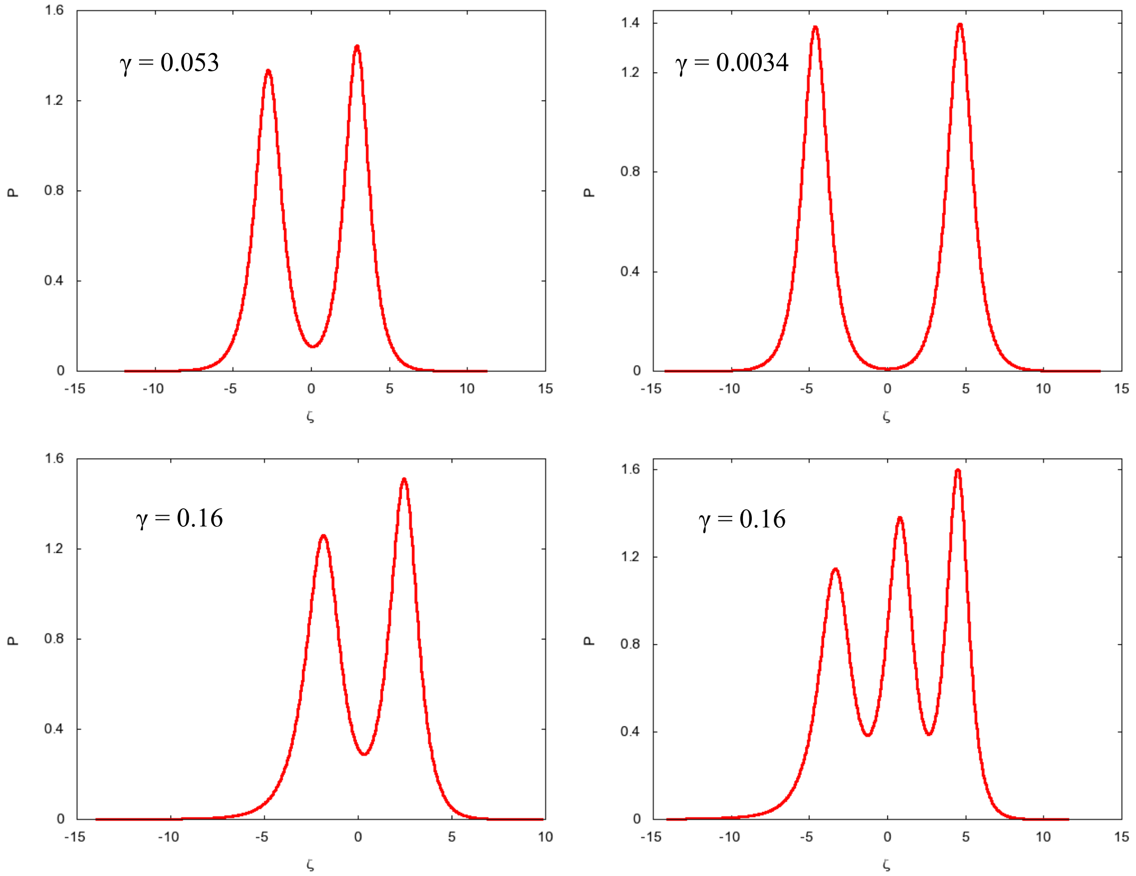

Figure 1. Two-humped and three-humped soliton profiles.

We may transfer the ODE results to the real model as follows. For a particular material, τ_r is fixed and to each pulse amplitude, or peak width, corresponds one γ. In the following text, we consider simulations of (1) with peak values of q approximately equal to 1. Since the humps are sech-type, they have $\tau_{\mathrm{FWHM}} \cong 1.76\tau_0$. Hence to simulate a particular τ_{FWHM}, we choose $\tau_0 \frac{\tau_{\mathrm{FWHM}}}{1.76}$, $T_r = \tau_r / \tau_0$ and γ using the transformation (4), namely

$$1 = q_{\max} = W_{\max} = \frac{\gamma}{4T_r} P_{\max} \cong \frac{\gamma}{4T_r} \left(\frac{15}{4}\right)^{\frac{1}{4}} .$$

3. STABILITY OF THE MULTIHUMPED PULSES

In order to study the stability of the multihumped pulses, we propagate them in the full partial differential equation (1) using a pseudospectral method.

Unfortunately, our simulations showed that these multihumped pulses are unstable since there is always a distance from where their peaks start to separate from each other. The distance up to which they travel without separation is dependent on γ which corresponds to the real width of the peaks, i.e., larger peaks stay together for longer distances. Ours simulations show also that if one pulse has a larger number of peaks, it will destabilize for smaller distances.

However, there is a chance that these solutions are analogous to the bound soliton pairs recently observed in PCFs, since they may travel without change of form for experimentally observable distances. For instance, Fig. 2a shows the steady propagation of one two-humped pulse with full width at half maximum around 50 fs until $Z = 90$. Following the procedure described above, we find $\tau_0 = 28.4$ fs, so that $Z = 90$ corresponds to $z = 3.3$ m. Moreover, in this case we have $\gamma = 0.16$. A wider pulse (around 150 fs of real full width at half

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maximum (which corresponds to $\gamma = 0.053$) travels unchanged until approximately $Z = 150$ ($z = 50$ m), as pictured in Fig. 3b.

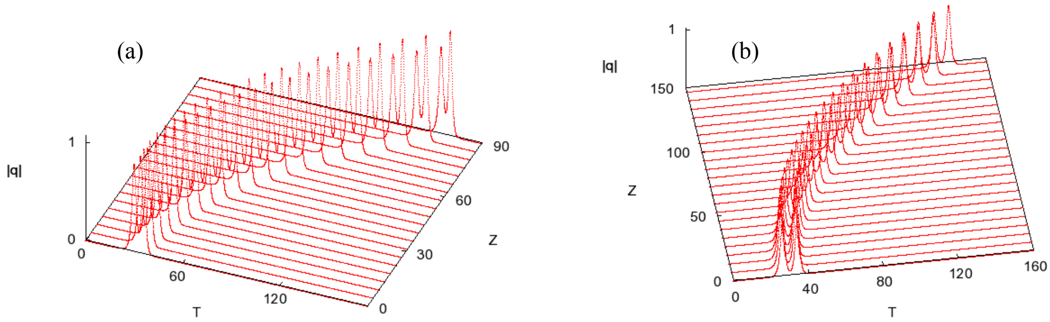

Figure 2. Stable propagation of the two-humped soliton for (a) $\gamma = 0.16$ and (b) $\gamma = 0.053$.

If we further use those solutions as input pulses in equation (1) with $s = 0.27T_r$, the individual humps separate earlier in z as is showed in Fig. 3.

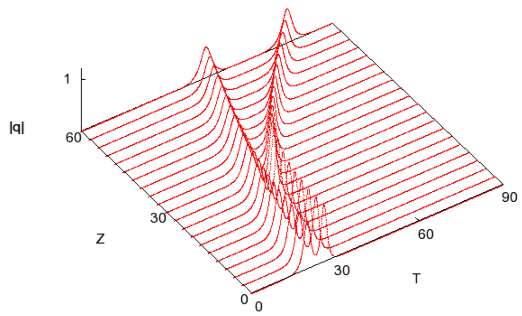

Figure 3. Propagation of the two-humped soliton for $\gamma = 0.16$ and $s \neq 0$.

4. CONCLUSIONS

We were able to obtain accelerating similar solutions to the NLS plus IRS model, both single and multihumped solutions. However, the multihumped solutions do not propagate indefinitely without peak separation. The partial differential equation simulations suggest that they may preserve their shape during several meters. To further analyse the relation of recently observed bound pairs in PCFs, we should improve the estimation of the fiber parameters and the model itself since our model is not including the Raman response of the air holes of the fiber neither higher order dispersion.

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