Faculdade de Engenharia da Universidade do Porto

The Retail Shelf Space Allocation Problem: New Optimization Methods Applied to a Supermarket Chain

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Teoria e Prática

“ *Toda a teoria deve ser feita para poder ser posta em prática, e toda a prática deve obedecer a uma teoria. Só os espíritos superficiais desligam a teoria da prática, não olhando a que a teoria não é senão uma teoria da prática, e a prática não é senão a prática de uma teoria. Quem não sabe nada dum assunto, e consegue alguma coisa nele por sorte ou acaso, chama «teórico» a quem sabe mais, e, por igual acaso, consegue menos. Quem sabe, mas não sabe aplicar - isto é, quem afinal não sabe, porque não saber aplicar é uma maneira de não saber -, tem rancor a quem aplica por instinto, isto é, sem saber que realmente sabe. Mas, em ambos os casos, para o homem são de espírito e equilibrado de inteligência, há uma separação abusiva. Na vida superior a teoria e a prática completam-se. Foram feitas uma para a outra.* ”

Fernando Pessoa,
in ´Palavras iniciais da Revista de Comércio e Contabilidade’

Abstract

While shopping, customer choices are influenced by in-store factors, in particular during unplanned purchases and product stockouts. The arrangement of products on the shelves becomes crucial in this context and a key factor to retailers' competitiveness. Recently, the shortage of shelf space and the increasing number of products available have greatly magnified the importance of how merchandise is displayed.

The Shelf Space Allocation Problem (SSAP) has long been considered by marketing professionals and the scientific community, with the first published studies tracing back to the seventies. However, academic work is far from being applied in practice: most of the optimization models have practical limitations, either because of their simplicity and lack of key-practical features or due to the large number of parameters difficult to estimate. As a result, there has been a misalignment between software applications, business practices, and research.

Motivated by the space management problems arising in the Food Retail Industry, the objective of this thesis is to tackle the SSAP and bridge the existing gap between research and practice with the development of innovative quantitative tools that support the generation of automated and optimized shelf space allocation solutions in practice. A case study in a European Food Retailer provides the right motivation to understand the current challenges faced by retailers and constitutes the perfect environment to assess the practical value of our scientific contributions.

The contributions of this thesis are aligned with two main directions. On one hand, we pushed the frontier of the shelf space literature with new mathematical models and state-of-the-art solution approaches that combine mathematical programming with heuristics. We investigated key practical features of the problem, with an emphasis in Merchandising Rules, and developed innovative approaches capable of delivering high quality solutions, suitable to business practice, in reasonable computational time. On the other hand, we developed a comprehensive decision support system for the automatic generation of shelf space solutions (*planograms*) that is nowadays being used on a daily basis in the case study company, proving the validity of our achievements. Despite the straight link with the case study, all the mathematical models and algorithms that emerged from this thesis are extensible to other food or non-food retailers sharing similar challenges.

We believe that this thesis is an important contribution both by bringing additional realism into academia and by proving the value of advanced analytics in practice. Moreover, it will ultimately contribute to the "next generation" of shelf space planning systems.

Resumo

Durante o processo de compra, as escolhas dos consumidores são influenciadas por fatores associados à disposição dos produtos nas lojas, especialmente durante as compras não planeadas e nas situações de rutura de produtos. Neste contexto, a disposição dos produtos nas prateleiras torna-se crucial e um fator-chave para a competitividade dos retalhistas. Recentemente, a escassez de espaço e o aumento do número de produtos disponíveis criou uma maior ênfase na forma como a mercadoria é exibida.

O Problema da Alocação dos Produtos nas Prateleiras (SSAP) tem sido alvo de estudo por profissionais de marketing e pela comunidade científica desde os anos setenta. No entanto, a maioria dos modelos e abordagens de otimização têm limitações que inibem a sua aplicação prática, seja por causa da sua simplicidade e falta de características-chave ou devido ao elevado número de parâmetros difíceis de estimar. Consequentemente, existe um desalinhamento entre as aplicações de software existentes, as práticas do negócio, e a investigação desenvolvida.

Motivados pelos problemas de gestão de espaço na Indústria Alimentar, o objetivo desta dissertação é abordar o SSAP com vista a um melhor alinhamento entre a prática e a teoria, através do desenvolvimento de ferramentas quantitativas inovadoras que suportem a geração de soluções de alocação de espaço na prática. Um caso de estudo num retalhista europeu constitui o ambiente ideal para compreensão dos desafios atuais e para avaliação do valor prático das contribuições científicas.

As contribuições desta dissertação estão alinhadas com duas direções principais. Por um lado, foram desenvolvidos novos modelos matemáticos e novos métodos de solução que constituem avanços científicos no SSAP. Estas abordagens integram novas características práticas do problema, com ênfase nas *Regras de Merchandising*, e oferecem soluções de alta qualidade, adequadas para a prática, e com bom desempenho computacional. Por outro lado, foi desenvolvido um sistema de apoio à decisão para a geração automática de soluções de alocação de espaço (denominados *planogramas*), que está atualmente implementado e em utilização no caso de estudo. Apesar desta estreita ligação com um retalhista, todos os modelos matemáticos e todos os métodos de solução desenvolvidos são extensíveis a outras indústrias com características semelhantes.

Acreditamos que esta tese é uma contribuição importante quer por trazer métodos inovadores e realismo adicional à investigação científica quer por provar o valor de abordagens quantitativas na prática. Além disso, acreditamos ter contribuido para a próxima geração de sistemas de planeamento e gestão de espaço no retalho.

Acknowledgments

It is with sincere gratitude and appreciation that I thank all that have assisted me during this amazing journey.

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Contents

Chapter 1

Motivation and Overview

1.1. Introduction

The retail trade sector is a key intermediary in the modern economy between thousands of product suppliers and consumers. It is the largest non-financial business economy in the European Union (EU) in terms of number of enterprises and persons employed, and the second largest in turnover, showing the importance and competitiveness of this sector. In the EU, non-specialized retailers registered a turnover of 1,065 billions of Euros and employed 6.6 million people in 2010. In Portugal, this sector represented a turnover of 11 billions of Euros and employed 68,000 people in the same year (Source: Eurostat).

In this competitive environment, retailers strive for customer satisfaction and operational efficiency, aiming to improve their stores' financial performance. To achieve such goal, retail organizations are moving towards demand driven initiatives, with the lemma "every sale counts", while trying to optimize their two most expensive resources: space and inventory.

While shopping, customer choices are highly influenced by in-store factors, in particular during frequent unplanned purchases and when the products they are searching for are not available. This is in part motivated by the low level of involvement that consumers have with in-store decisions, often made quickly and with only a minimal search. More than just displaying the merchandising, a clever product arrangement on the shelves assumes a crucial importance as a tool to increase visibility, consumer awareness and demand for the products, ultimately leading to better performance. Therefore, retailers work on getting the right goods to the right places at the right time (Chandon et al. [2009]).

The above challenges are further stressed due to the increasing number of products available. Hübner and Kuhn [2012] point out a 30% increase in the number of products in overall store assortments in nearly 10 years, between 2000 and 2009. Moreover, the short product life cycles and the increasing number of stores raise the need for constant shelf space planning. As a consequence, space management has become progressively challenging and an active field of research in retail operations management, under the name Shelf Space Allocation Problem (SSAP).

The retail sector is one of the biggest users of Information and Communications Technology (ICT), and thus a driver of innovation. Nevertheless, Hübner and Kuhn [2012] and Bai [2005] state a misalignment in shelf space management between commercial software applications and research: on the one hand software vendors focus mainly on the development of large-scale data processing technologies, with limited or no use of mathematical optimization and disregarding the space effects on consumer demand. On the other hand, state-of-the-art optimization methods have practical limitations, either because of their simplicity and lack of key features, or due to their complexity and expensive estimation requirements for parameters. A closer cooperation between retail research and practice is needed and will ultimately lead to the "next generation" of shelf space planning systems, with automated and optimized shelf space allocation solutions (Bai [2005]).

This thesis is the result of problem-driven research motivated by the space management problems arising in the food retail industry. In collaboration with a European Food Retailer, the objective is to tackle the SSAP and bridge this gap between retail research and practice with the development of quantitative tools that support the generation of automated and optimized shelf space allocation solutions in practice. The case study not only provides the motivation to understand the current challenges and flaws on the current literature approaches, but also constitutes the perfect environment to assess the practical value of our scientific contributions. Despite this straight link with the case study, all the mathematical models and algorithms emerging from this thesis are expected to be extensible to other food or non-food retailers sharing similar challenges.

This introductory chapter presents an overview of shelf space planning and defines the objectives of this thesis. The remainder of the chapter is organized as follows. In Section 1.2, shelf space planning is framed within retail operations. Section 1.3 introduces the SSAP in detail focusing on current practices, consumer demand effects and relevant decisions and constraints. The case study of the European Food Retailer that collaborated in this thesis, Sonae MC, is introduced in Section 1.4. Section 1.5 presents the research objectives and methodology, and section 1.6 contains a synopsis of the remaining chapters of this thesis.

1.2. Shelf Space Planning within Retail Operations

Getting the right goods to the right places at the right time in the most efficient way requires the coordination and cooperation of thousands of individual decisions in supply chain planning and customer management. Hübner et al. [2013] and Hübner and Kuhn [2012] present comprehensive operations planning frameworks that identify and integrate all relevant retail planning aspects. The objective is to enable practitioners and researchers to classify decisions and realize the interdependencies between them. In this section, shelf space planning is framed within both the supply chain planning and master category frameworks.

1.2.1 Demand and Supply Chain Planning

The primary objective of retail is to bridge the gap between the point of production and the point of sale, which stresses the supply chain role in this industry. Both distribution and in-store operations are costly and importantly as the first has a direct impact on the latter. Furthermore, the low value nature of grocery products leads to a higher share of distribution costs compared to manufacturing companies.

Hübner et al. [2013] present a retail oriented, consumer-backed demand and supply chain planning matrix based on the supply chain planning framework from Fleischmann et al. [2008] for manufacturing industries. This framework, depicted in Figure 1.1, follows the concept of hierarchical planning and distinguishes the planning problems horizontally along the flow of goods and vertically along the time horizon. The flow of goods is divided into four domains: Procurement, Warehousing, Distribution and Sales; and the time horizon is classified into long-, mid- and short-term. Long-term planning take strategic decisions concerning the configuration and layout of the entire network; Mid-term master planning deals with the coordination and planning of the operations and promotions for the next 6-12 months and short-term execution planning specifies the activities for the next few days or weeks. The disaggregation of data and results follows the decreasing planning horizon down the hierarchy.

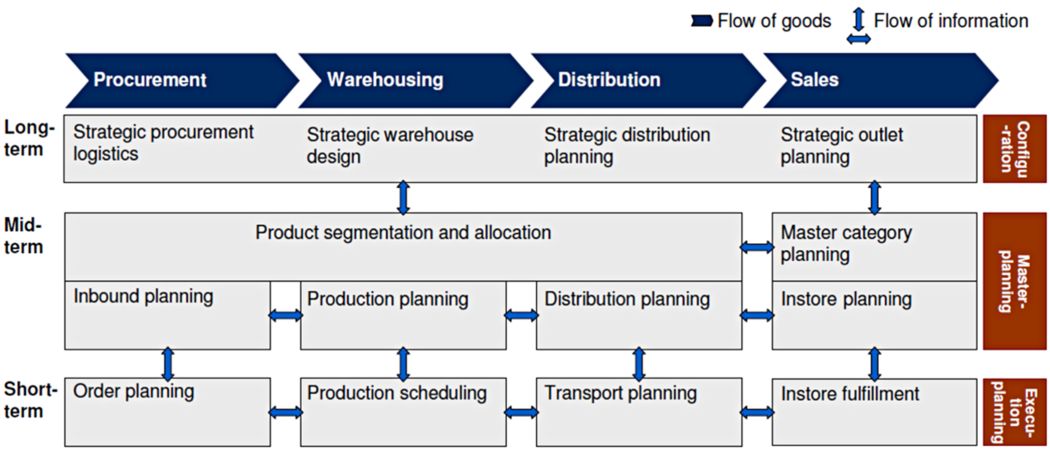

Figure 1.1 – Retail demand and supply chain planning framework (Hübner et al. [2013])

The planning modules are linked by vertical and horizontal information flows that identify the interdependencies between the activities. These activities usually belong to different organizational hierarchies and responsibilities which cause some obstacles in their cooperation. Additionally, this framework is embedded between consumer interactions on the sales side and supplier interactions on the procurement side, which affects the entire planning process and requires an integrative approach.

Shelf space planning fits within the master category planning module, which frames all the mid-term sales planning tasks of category management. It also has major interactions with other planning activities. The layout and infrastructure of stores are long-term decisions from strategic outlet planning that highly constrain the space available for shelf place planning. Nowadays, retailers have to balance the conflict of an increasing number of products to display versus the limited amount of store space available (Bai [2005]).

Despite being a sales-oriented decision, shelf space planning impacts and is impacted by supply chain decisions. Backstage is limited and scarce and shelf space should hold enough inventory until restocking to ensure product availability and avoid the occurrence of stockouts. Therefore, distribution decisions such as shipping frequencies, lead times and order sizes have to be properly coordinated with shelf space allocation.

At last, all these decisions affect in-store planning, with a high impact on shelf-replenishment operations and personnel planning.

1.2.2 Master Category Planning

Master category planning covers the sales planning tasks of category management, divided into four major hierarchical activities: category sales planning, assortment planning, shelf space planning and in-store logistics planning, as proposed by Hübner and Kuhn [2012] (Figure 1.2).

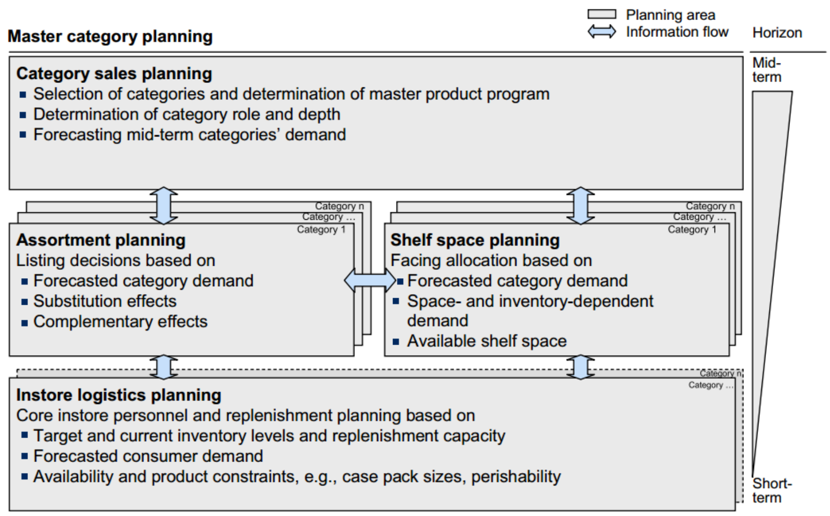

Figure 1.2 – Interdependencies between retail problems (Hübner and Kuhn [2012])

Category sales planning start by identifying the set of categories to have in each store type, their role and depth, price position, space share and mid-term demand forecasting. Moreover, it sets the guidelines for subordinated planning problems, to ensure that the categories' role is always present. From that moment on, categories are planned individually in a shorter planning horizon. Assortment planning involves deciding the products to carry in the stores. Optimizing assortment planning requires the consideration of the consumer demand for the products, including both substitution and complementary demand (substitution demand from non-existing products and complementary demand from related products) (Kök et al. [2009]). On the other hand, shelf space planning assigns and locates the space to the individual products of the assortment, under capacity and restocking constraints. Both assortment and shelf space planning activities are usually accomplished for clusters of stores with similar demand and space patterns. At the end of master category planning, in-store planning includes store personnel planning and store logistics planning. Similarly to the rest of the supply chain, the above decisions differ in their planning horizon, decision owners and IT areas. Ultimately, they also diverge in terms of research domains.

The interdependency between the activities is evident. Large assortments drive lower inventory levels of individual products, which reduce their visibility on the shelves, increase the risk of stockouts and impose frequent replenishment operations, leading to high restocking costs. However, Hübner and Kuhn [2012] alert that those problems are not yet sufficiently integrated. On the one hand, assortment decisions disregard space elasticity effects and to a large extent shelf space constraints too. On the other hand, shelf space allocation assume lost sales and no consumer substitution for non-available products. Furthermore, the shelf inventory is not carefully handled to obtain synergies in replenishment activities.

1.3. The Shelf Space Allocation Problem

According to a survey to US retailers (Keltz and Sterneckert [2009]), the main drivers for space planning initiatives rely on the improvement of overall profitability (and overall sales), reduction of the stock levels, improvement of product availability and in delivering a differentiated consumer shopping experience. However, the same survey concludes that the benefits realized are not meeting the expectations. This chapter presents an overview of the shelf space allocation problem and its current challenges.

1.3.1 Current Practices

As mentioned above, shelf space planning follows assortment planning and is done separately for each category, in a mid-term planning horizon. It is often called micro-space planning because of its precedence by store space planning (known as macro-space planning, a long-term decision). The increasing number of stores turns impractical individual plans and often lead to store clustering based on demand and space patterns. However, current trends towards customer centricity state that "One plan does not fit all" and defend store-specific space planning.

Retailers use planograms to plan the products placement on the shelves. A planogram is an illustration of a category specific part of a store, showing exactly where each product should physically be displayed and how many faces that product should hold. An example of a planogram and its corresponding implementation in a store is present in Figure 1.3.

Figure 1.3 – Example of a planogram and its implementation in a supermarket

Products are usually placed on shelves following merchandising rules which specify associations of products in families that are placed together on the shelves (such as color, brand, type and flavor). These rules try to reproduce the way customers search for the products while shopping and have also in mind the identity of the retailer and its strategy for each category. The complexity of the merchandising rules vary from retailer to retailer but can include more than one level of family types, family sequences, special display locations, among other requirements. In some situations, those rules are defined together with category captains, which are key suppliers with deeper knowledge about each category (for more information about category captains, please consult Kurtulus and Toktay [2009]), or using techniques such as market basket analysis. In Figure 1.4 one can see a planogram where products are organized and highlighted by brand. Note that products are placed in rectangular shapes, one shape for each brand.

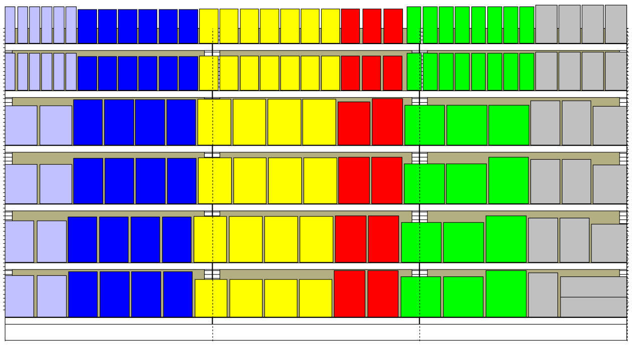

Figure 1.4 – Planogram with products organized and highlighted by brand

Generating planograms is a highly time consuming activity - the industry standard for manually creating a single planogram is three hours (JDA [2009]). Therefore, adequate Information Technology (IT) systems are essential. Despite this, 30% of the retailers did not use any kind of IT support in 2009, and only 36% had up-to-date technology, as seen in Figure 1.5. This reality is changing as retailers are realizing the benefits of space planning.

Current commercial IT solutions are similar in their purpose and scope and focus on simplicity, allowing for realistic views of the shelves, the ability to quickly handle products and providing different data and powerful analysis reporting. Those systems already incorporate tools for the automatic generation of planograms based on simple heuristics such as proportional-to-market share or proportional-to-profit share and require a significant tunning effort for additional requirements. Among the space planning solutions currently on the market the top three vendors are: Spaceman suite (AC Nielsen) and Space planning (JDA), with over 2000 users each, and Apollo professional (MEMRB/IRI), with over 800 users (Hübner and Kuhn [2012]).

Nevertheless, today's commercial IT solutions for space planning have been essentially used for visual and handling purposes, and the planograms are still generated with significant human interaction. As a matter of fact, many authors argue that no "real" optimization takes places due to the limited or non existing use of mathematical optimization and consumer demand effects (Irion et al. [2011], Hansen et al. [2010], Hübner and Kuhn [2012], Drèze et al. [1994], Desmet and Renaudin [1998]). As a result, automatically generated planograms are most likely to receive significant manual adjustments by the end users.

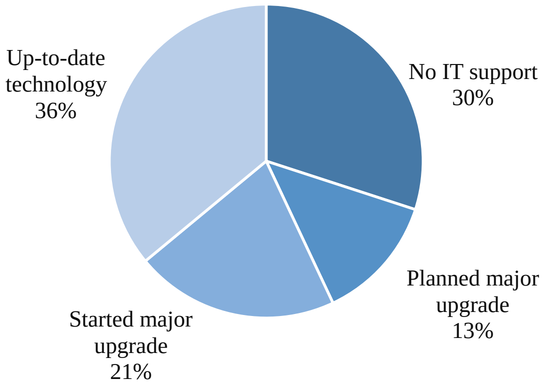

Figure 1.5 – Status of retailers IT usage for shelf space planning (Hübner and Kuhn [2012])

1.3.2 Consumer Demand Effects

Most shoppers enter the store with only a general idea of what to purchase, becoming susceptible to in-store marketing. Additionally, reduced assortments and stockouts force consumers to search for substitution products, highlighting the role of space management. The low level of involvement that customers have with in-store decisions, often made quickly and with only a minimal search, reinforces the importance of in-store marketing.

Experimental studies have consistently proven the positive effect of space on the demand of the products. These studies point to three main elasticities: *space elasticity* measures the increasing responsiveness of sales as more space is allocated to a product, experiencing declined marginal returns at some point - see Fgure 1.6 (Curhan [1972], Chandon et al. [2009]); *location elasticity* highlights key display locations that bring a better exposure, such as the eye- or hand-level (Drèze et al. [1994]); lastly, *cross elasticity* measures the interdependency between adjacent products and is assumed to be positive for complementary products and negative for substitute products (Corstjens and Doyle [1981]). Additionally, the way products are arranged on the shelves can also have an important role on gaining the consumers' attention. Thus, carefully organizing them in families can increase interest, while disorganized or excessive complexity (i.e. variations in the basic visual content) damages the buying experience (Pieters et al. [2010]). We call to this effect *design complexity*.

1.3.3 Problem Definition

Shelf Space Allocation is the scientific name for the problem of distributing the scarce shelf space of a retail store among a set of products of a category. The definition of the problem may vary depending on the retail segment, company's strategy, relation with vendors, store layout, among others. This section will define the problem as generic as possible, considering food retail environments.

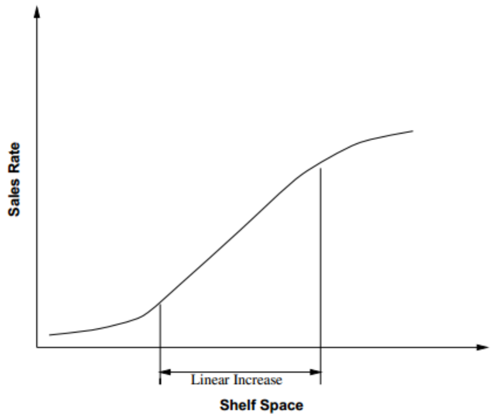

Figure 1.6 – Sales rate in function of shelf space for a single product (Abbott and Palekar [2008])

To start with, there is the need to review some concepts related to shelf space allocation, some of them already mentioned before. A *Stock-Keeping Unit (SKU)* is a unique identifier for each distinct product that can be purchased. Each SKU belongs to a *supplier*, is part of a *brand*, and contains a set of other attributes, such as colour, size and packaging, that distinguishes it from all other products. We call to these attributes *family types*. A *family* is a set of products sharing the same value for a given attribute. The highest organizational structure of products are *categories*. Usually in a planogram all SKUs belong to the same category.

A retailer usually displays a limited part of the inventory of a SKU on the shelves, leaving the rest in the *backroom*. The visible stock of each product can be characterized by the number of *facings wide, high and deep*, as depicted in Figure 1.7. The number of facings wide is most commonly known as the facings of the product. The way each product is placed on the shelves defines its *orientation*: front, back, top or side. The *days-supply* value of a product measures the number of demand days covered by its shelf stock until the need for replenishment.

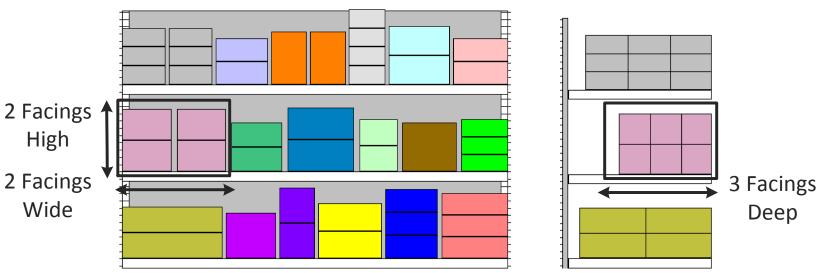

Figure 1.7 – Facings wide, high and deep of a product (front view on the left and side view on the right)

Fixtures are located in *segments* that are placed end-to-end (i.e. horizontally stacked against each other) to form the *aisles* of the stores. Each segment has its own shelf placement. Shelves can be aligned with the shelves of the other segments, forming continuous long shelves from the beginning to the end of the aisle or they can be placed differently, forming misalignments that need to be taken into consideration while placing the products. Whenever a planogram has misalignments between the shelves we refer to it as *irregular planogram*, as opposed to *regular planogram*. Figure 1.8 presents an irregular planogram with 3 segments.

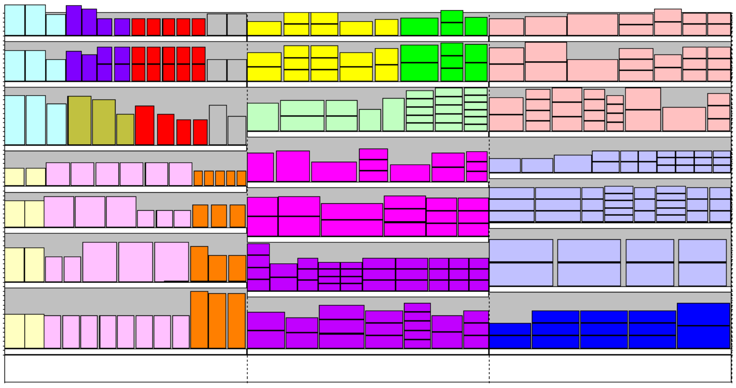

Figure 1.8 – Example of an irregular planogram

Shelf space planning may involve other fixtures than shelves, such as chests, pallets and pegboards. Chests are enclosed spaces for storing non-organized products and pegboards are bars with steel rods sticking out to hold peggable products like shewing gums. Nevertheless, these fixtures are out of the scope of this thesis.

Objectives

The aim of the SSAP is usually to obtain the maximum profit or sales out of the available space, considering consumer demand in function of the space allocated to the products. Section 1.3.2 presented a brief overview of the most commonly considered consumer demand effects: space-, location- and cross-elasticities.

Some authors also include a cost reduction approach, with a higher emphasis in inventory management. However, Bai [2005] points out that if the products' demand is dependent on the space, a cost-minimization objective may not be appropriate as it may reduce the number of product facings which is against the intent of the problem.

Decisions

The most common SSAP decisions are the number of facings (wide) for the products and their placement on the shelves. The problem is usually seen in a 2D fashion because the items placed behind each facing cannot be seen directly and hence do not have an impact on consumer demand (inventory requirements can be considered by determining the number of items behind each facing).

The fixtures location is frequently given as input to the model because retailers are not likely to change the layout of the shelves during products reallocation. However, it is also possible to consider the shelf location as a decision. Several approaches also integrate other non-space decisions that have an impact on consumer demand, including assortment, replenishment, promotions and advertising.

Constraints

There are several potential constraints for the SSAP, ranging from hard to soft, depending on the need to entirely satisfy these requirements. A list of the most commonly considered constrains is presented below, organized from the hardest to the softest.

- Integrality Constraints - the space allocated to a product on each shelf should be an integral number of times the size of that product;
- Physical Constraints - The total shelf space available cannot be exceeded. Physical constraints can be one or two dimensional, whether they consider height constraints. Nevertheless, these are often ignored when the vertical location of the shelves can be readjusted at the end.
- Control Constraints - Many retailers set lower and upper bounds to the number of facings to ensure that a minimum and a maximum exposure is given to the products. Other bounds can result from special contracts with important suppliers, with the power to influence the location and shelf space of their products. These contracts usually set minimum space shares for brands. Retailers may also try to maintain a minimum and maximum number of days-supply for each product, in order to control stockouts, inventory costs and replenishment costs.
- Family Constraints - Merchandising rules identify families of products that should be placed together on the shelves, preferably in rectangular shapes. These rules may also specify predefined family sequences and shapes orientation.

Most part of the complexity in the SSAP comes from the inclusion of the space effects on the demand function, which are hard to estimate and non-linear by nature. The literature presents a great variety of models which incorporate different estimates of (some of) those effects. These models also differ in the level of detail of the decisions, ranging from facings calculation to almost complete planogram descriptions. As a consequence, there is no definitive shelf space allocation model. Moreover, models that can be adapted to reality are particularly difficult to find either because of their simplicity and lack of key practical features, or due to their complexity and expensive estimation requirements for parameters. One important practical limitation is that most models disregard that product allocation must follow merchandising rules which specify associations of products on the shelves.

Finding the best products allocation from the set of all possible arrangements of products is clearly a combinatorial problem. From a simplistic point of view, it consists in placing a set of small items (products) into a set of large objects (fixtures). Therefore, it can be related to the literature of Cutting and Packing Problems. More specifically, the SSAP can be considered a extension of a placement type of problem, according to the Wäscher's improved typology of Cutting and Packing Problems Wäscher et al. [2007]. In this paper, Placement Problems are described in the following way:

> " (...) a weakly heterogeneous assortment of small items has to be assigned to a given limited set of large objects. The value or the total size (as an auxiliary objective) of the accommodated small items has to be maximized (...) "

The simplest form of the Placement Problem, with one large object, is already NP-hard. The SSAP further extends Placement Problems by integrating the effect of space variables on the products' demand. Several practical constraints are additionally added to the problem.

1.4. Case Study Presentation

This thesis had the collaboration of Sonae MC with whom we carried out a project aiming at developing a tool for the automatic generation of planograms, where we integrated and validated the main outcomes of this thesis.

Sonae MC operates a food retail business in Portugal and is one of the biggest Portuguese companies (ranked the 4^{th} in 2014, with 3.33 billion annual sales). Its brand *Continente* is the country food retail market leader, with a network of 478 stores (and additionally 162 stores under franchising) covering the entire country, with three major formats: convenience stores (*Continente Bom dia*), supermarkets (*Continente Modelo*), and hypermarkets (*Continente*).

Sonae MC has a centralized operations management activity, responsible for planning all the operations for the stores nationwide. The space planning department is engaged with managing the space available at the stores, an activity that comprises two main levels: a macro-space planning level that defines, on a long-term basis, the layout of the stores; and a micro- (or shelf-) space planning level that defines, for each category, the products' placement on the shelves. Shelf space plans are updated with an average rate of 2 to 3 times a year for more than 300 categories. This activity fully occupies 23 space managers that generate an average of 60,000 planograms each year.

Similarly to other retailers, shelf space is managed on a cluster based approach. Space managers start by creating generic shelf space plans that fit the average sales of each cluster of stores. Once validated with category managers to check its consistency with the category strategy, these generic plans are then replicated for all the stores of the clusters, by adjusting the product facings to each store, while keeping the same allocation rules. Figure 1.9 summarizes this shelf space planning process.

Sonae MC uses a space planning software from one of the top three vendors, the JDA Software Group, Inc. Although automatic tools for planogram generation are available in JDA software, these tools do not accommodate all the intrinsic complexity of

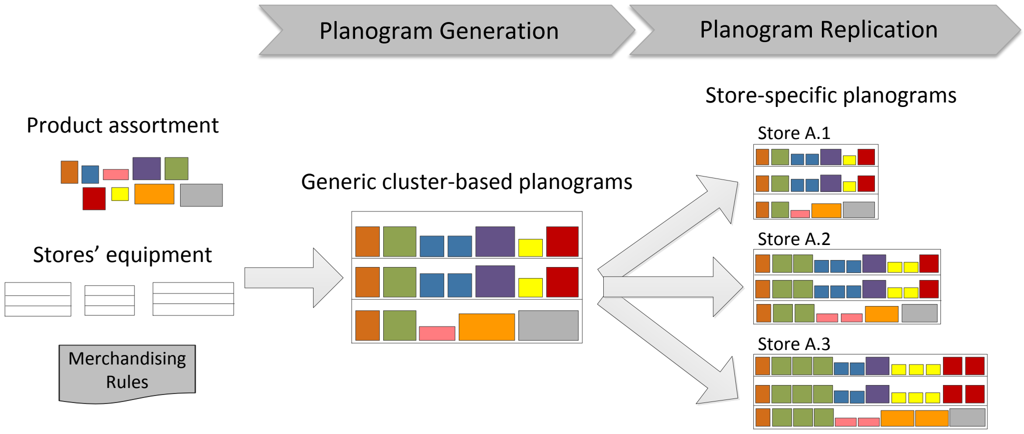

Figure 1.9 – The micro-space planning process has a major interaction with the commercial department and comprises two main processes: Generation and Replication

the company's merchandising rules. Therefore, space managers manually developed the planograms by dragging and dropping the products onto the shelves.

1.5. Research Objectives and Methodology

As aforementioned, this thesis is motivated by the space management problems arising in food retail. From his deep analysis of retail category management problems, Hübner and Kuhn [2012] identified four directions of future research in this field: (1) Alignment of software applications and science; (2) Alignment of assortment and shelf space management; (3) Alignment with other planning objectives and (4) Alignment within shelf space competition (for instance, study of the category captainship effects). The overview presented in this initial chapter motivated the need for future research in these areas.

The research objectives of this thesis are related with the first direction for future research: alignment of software applications and science. We aim to develop quantitative tools to support the generation of automated and optimized shelf space allocation solutions in practice, thus contributing to the "next generation" of shelf space planning systems. To achieve such objective, a close collaboration is held with a Food Retailer (Sonae MC), which gives the necessary proximity to the practice. Meanwhile, we also aim to make important contributions to the current state-of-the-art of shelf space management with the development of new formulations and solution approaches to the SSAP and related problems.

Throughout this thesis we will follow the typical OR methodology that starts by the problem definition, then mathematical modeling, solution methods and at the end testing and validation.

Problem Definition – In spite of the existing commercial applications for the process automation, planograms are still in most of the cases manually generated. We aim to study the retailers' space planning process and identify what should be done for a better fit of those applications and how quantitative methods can help in that task.

Mathematical Modeling – The SSAP can be seen as an optimization problem and given the type of decisions involved we use Mixed Integer Programming (MIP) to model this problem. MIP is one of the most widely used techniques to solve hard optimization problems due to its flexibility in capturing problems' details. Although some models exist in the literature devoted to SSAP, these models are either too simplistic and lacking key practical features, or too complex and with parameters hard to estimate. We aim to study in depth those models, identify their flaws for practical implementation and develop new formulations that can be used in practice.

Solution Methods – To solve MIP problems both *branch-and-bound* and *branch-and-cut* algorithms are normally applied using modern commercial solvers. In spite of the remarkable improvements in the quality of these general purpose solvers, and the recent developments in hardware, the SSAP is a complex problem and it is not expected that a straightforward implementation of a MIP model can solve realistic size instances. Under these circumstances, alternative solution methods are required. Metaheuristics (heuristic search) algorithms are often used being tailored to solve large-scale combinatorial optimization problems which require the efficient exploration of large scale neighborhoods. Recently, a trendy and successful research line is the creation of algorithms combining metaheuristics with exact methods, the so-called *matheuristics*. We will base our solution strategy in MIP-based heuristics, which are a class of matheuristics relying on the heuristic solution of the mathematical formulation. We aim to design and develop new matheuristics for the problem that ensure scalability and efficiency. We believe that this solution technique is a good choice for this problem due to the high and constantly evolving number of practical features which are not easily grasped with a traditional metaheuristic. Furthermore, in general, these algorithms are flexible enough to cope with different model extensions and new features.

Testing and Validation – As our ultimate objective is to bridge the gap between theory and practice, the work would not be complete without a practical validation. We intend to implement the methods on the case study and evaluate the results.

1.6. Thesis Synopsis

Figure 1.10 presents an overview of the main chapters of this thesis, that are organized around the typical shelf space planning process shown in section 1.4. We start by giving an overview of the shelf space literature and then we approach two different space-related problems: First we tackle the traditional SSAP using a more practical oriented perspective, with a higher emphasis on merchandising rules. Secondly, we look at the problem of generating store-specific planograms from generic planograms, which we called Shelf Space Replication Problem. To the best of our knowledge, this is the first time that the replication problem is introduced in the literature and we believe that it is a major step towards the use of operations research (OR) in the practice of shelf space management. The outcomes from the two problems were implemented and tested on the case-study, giving rise to a Decision

Support System (DSS) that is now used on a daily basis by the company's space managers. This DSS and the key implementation issues that supported its success are described in the last chapter.

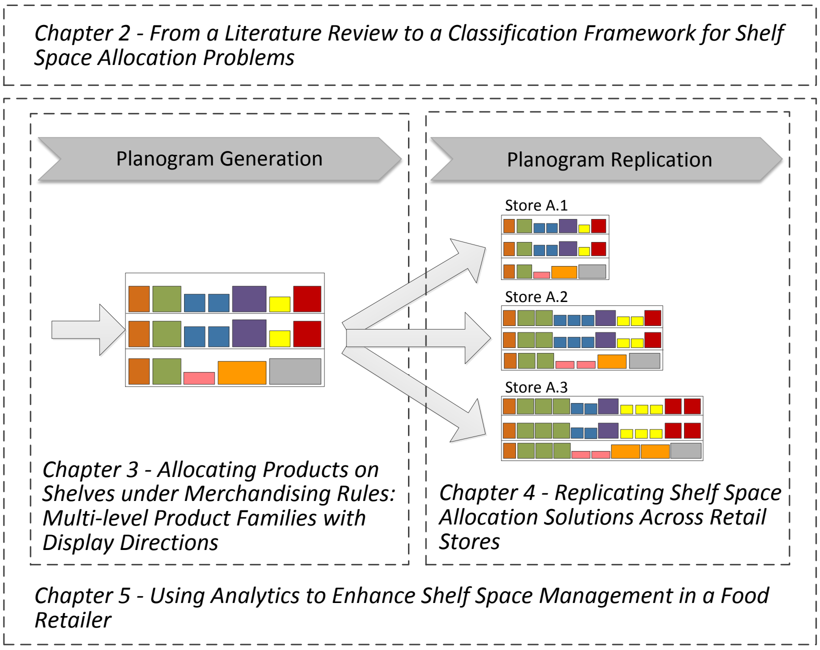

Figure 1.10 – Chapters of the thesis

This thesis is paper-oriented, which means that the main chapters consist of a collection of papers. Despite the many advantages of using this type of approach, it has the drawback of resulting in multiple descriptions of the same definitions.

In this section, we overview the main aspects covered in each chapter and the most substantial contributions associated with each one.

Chapter 2 provides a description and a state-of-the-art literature review of the SSAP focusing on mathematical modeling approaches. Based on this review, a classification framework is proposed with the intent to systematize the research into a set of sub-problems. Future research lines point to the most promising open questions in this field.

Chapter 3 presents a novel mixed integer programming formulation for the SSAP considering two innovative features emerging from merchandising rules: hierarchical product families and display directions. The formulation uses single commodity flow constraints to model product sequencing and explores the hierarchy in product families to reduce the combinatorial nature of the problem. Based on the formulation, a mathematical programming-based heuristic is also presented which uses the product families to decompose the problem into a sequence of sub-problems.

Chapter 4 describes the novel problem of transforming generic cluster-based shelf space plans into store-specific plans, a process that is called Replication in this paper. Two mathematical programming formulations are presented to address the Shelf Space

Replication Problem, with different levels of practical details. The formulations use a novel inventory-related objective function that balances the products' inventory level in order to trigger joint shelf replenishments. Based on the formulations, a mathematical programming-based heuristic is also introduced in order to ensure the process scalability.

Chapter 5 introduces GAP, the DSS that is nowadays used on a daily basis by the space management team of Sonae MC. We developed a modular Operations Research (OR)-approach that systematically applies tailor-made mathematical programming models combined with heuristics to efficiently create planograms. On top of its algorithmic advances, one of the most relevant features of GAP is its flexibility to incorporate different types of merchandising rules, allowing the company to test different strategies for the products allocation. Nevertheless, it goes beyond the straightforward implementation of merchandising rules and it trades-off customization with optimization. GAP enhances shelf space management in three axis: process automation, space optimization and image standardization.

Finally, Chapter 6 summarizes the work and suggests directions for future research.

We included at the end two Appendices. Appendix A summarizes the nomenclature used to describe the three mathematical models present in chapters 3 to 5, and Appendix B presents additional planograms generated by the DSS and the corresponding merchandising rules.

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Chapter 2

From a Literature Review to a Classification Framework for Shelf Space Allocation Problems

Teresa Bianchi-Aguiar* · **Maria Antónia Carravilla*** · **José F. Oliveira***

Abstract The Shelf Space Allocation Problem has long been addressed by marketing professionals and researchers, with the first studies tracing back to the 1970s. Nevertheless, this field presents a wide range of different approaches, that deal with different decisions and space elasticity effects in more comprehensive or simplistic formulations. As a result, there is not a unique model and, consequently, no benchmark sets are available. This paper provides a description and a state-of-the-art literature review of this problem focusing on mathematical modeling approaches. Based on this review, a classification framework is proposed with the intent to systematize the research into a set of sub-problems. Future research lines point to the most promising open questions in this field.

Keywords Retail operations · Shelf space allocation · Framework

2.1. Introduction

The Shelf Space Allocation Problem (hereafter referred as SSAP) consists of distributing the scarce shelf space of a retail store among the different products to be displayed. While shopping, customer choices are highly influenced by in-store factors that positively impact the visibility, awareness and demand for the products. This fact highlights the role of shelf space planning in retail operations with the ultimate objective of maximizing the profit obtained from the limited available retail space. Moreover, the shorter product life cycles, the rising number of products available for selling and the high number of stores have turn this activity increasingly challenging and an active field of research.

Most part of the complexity in the SSAP comes from the consideration of the space effects on the customer's demand function, which are difficult to estimate and non-linear

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by nature. The literature presents a great variety of models which incorporate different estimates of some of those effects. Models also differ in the level of detail of the decisions, ranging from rough space estimations to almost complete shelf space descriptions. Moreover, shelf space problems may also differ from company to company, depending on strategies, managerial style, categories of products, retailer-supplier relationships, among others. Additionally, the SSAP is usually addressed together with other related retail problems that integrate other sales' elasticities, further increasing the complexity of the demand functions. As a consequence, there is no unique shelf space allocation model and no instance benchmarks set are available.

In this paper, a literature review of the shelf space allocation models is made. The SSAP has long ago been addressed by marketing professionals and researchers, with the first studies tracing back to the 1970s. This stream of research has been growing ever since without any work systematizing published research. The only exception comes from Hübner and Kuhn [2012] in 2011 that presented an overview of the state-of-the-art research and software applications in retail category management, which includes shelf space management. Nevertheless, the vast scope of the paper did not allow a deep analysis of shelf space allocation problems.

Figure 2.1 shows the increasing number of publications between 1970 and 2015, with a total of 43 papers. We highlight the decade of 2000, when the number of published works sharply increased to the double, and also the projection for the current decade that anticipates a sustained growth. The European Journal of Operational Research has been the major journal for the presentation of new developments in this field (with 9 published articles). Other journals such as the Journal of Retailing (6 articles) and the Journal of Operational Research Society (5 articles) have also presented significant contributions. These publications focused both on experimentally measuring the effects of shelf space allocation on the products' demand and on building decision models and optimization algorithms.

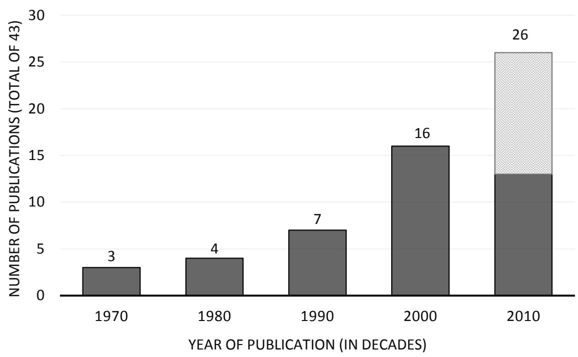


Figure 2.1 – The evolution of the publication activity on Shelf Space Allocation (the last decade is projected based on the data available for the period 2010-2015)

Despite the recent progresses that this research area has been experiencing, Bai [2005] and Hübner and Kuhn [2012] state a misalignment in shelf space management between existing commercial software applications and research agenda. Software vendors focus mainly on the development of applications with large-scale data processing technologies, with limited or no use of mathematical optimization and disregarding the space effects on consumer demand. As a consequence, these applications require significant human interaction and are essentially used for visual and handling purposes. State-of-the-art optimization methods, on the other hand, have practical limitations, either because of their simplicity and lack of key features, or due to their complexity and expensive parameter estimation requirements. Therefore, many challenges still exist and the SSAP is still an open problem.

With this literature review we intent to stimulate further research and boost more practical approaches to the problem. For that purpose, the remainder of the review is organized as follows. In Section 2.2 we present the problem and identify the basic features that models must capture to support decision making. Section 2.3 reviews the existing literature of the SSAP focusing on mathematical models with an emphasis in 3 main building blocks: decisions, demand and cost functions, and problem constraints. We also analyze the integration of the problem with other interdependent decisions and identify the main areas of application. Section 2.4 introduces a classification framework that systematizes the different types of approaches. Based on this framework, section 2.5 draws the final conclusions and disagnoses existing gaps in order to identify promising future research lines.

2.2. Shelf Space Allocation Problem

Space management comprises two hierarchical levels. A store (macro) level, deciding the space for product categories, and a product category (micro) level, which allocates individual products within each category. The SSAP is usually connected with the micro level and considers the allocation of a category of products onto the shelves to which it has been previously assigned.

The traditional space planning tool is a *planogram*, representing an illustration of a specific part of a store, showing exactly where each product should physically be displayed and how much space that product should have. Figure 2.2 presents an example of a planogram where we can see the number of different decisions that must be taken to create a full planogram. The shelf stock of each product can be characterized by the number of facings wide, high and deep. The number of facings wide is most commonly known as the product's facings (as often the remaining decisions are not tackled) and is the key space decision. The location of each product is defined by the shelf allocated to the product and its placement within the shelf. Other decisions include the products' *orientation* that specify the way products are displayed on shelves: front, back, top or sideway. Product *A* in Figure 2.2 has 2 facings wide, 3 facings high and 4 facings deep. It is located in the first shelf of the planogram in the first position (0 cm measured from the lower-left corner).

The aim of the SSAP is to maximize the outcome obtained from the available retail space. The problem focuses on demand and in the center of the problem there is the objective of maximizing the profit obtained with consumer demand, which in turn depends on the space allocated to the products. This problem has also a cost side and besides the

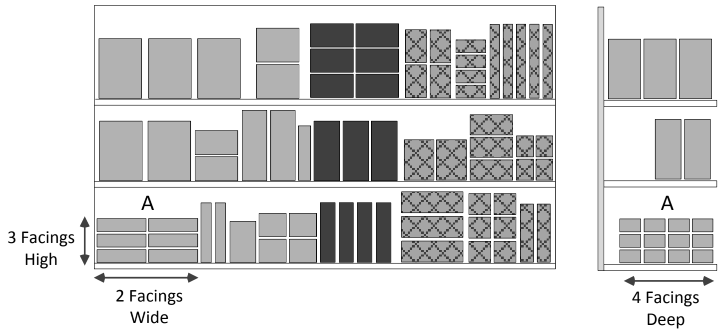


Figure 2.2 – Example of a planogram (front view on the left and side view on the right)

product costs, it sometimes considers operation costs originated by replenishment, holding and ordering activities.

There are several potential constraints for the SSAP, ranging from hard to soft, depending on the need to entirely satisfy these requirements. Besides the *integrality* and *capacity* constraints, typical of placement problems, *Control* constraints for lower and upper bounds may also be set for the products. Lower bounds are defined when the retailer wants to maintain a minimum number of facings of all products or because of new products which need a chance to make an impact. Upper bounds are set for products at a later stage of the life cycle, or for sales' champions, when the retailer wants to leave space for refreshing the product assortment of the stores. *Availability* constraints also limit products' sales by a production or availability limit.

On top of the previous requirements, many companies also define merchandising rules that identify families of products that should be placed together on the shelves, preferably in rectangular shapes. For example, the products of the planogram in Figure 2.2 are grouped into 3 rectangular-shaped families. These are called *product grouping* constraints. Merchandising rules may also specify other company-specific requirements such as family sequences, shapes' orientation (either columns or lines), special locations for special products, among others. Merchandising rules try to reproduce the way customers search for the products while shopping and are obtained with the help of category captains (key suppliers with deeper knowledge about each category - Kurtulus and Toktay [2009]) and techniques such as market basket analysis.

Shelf space allocation has a close interaction with other related retail problems, such as assortment planning, inventory management and shelf replenishment operations. When planning stores' assortment, retailers have to carefully consider the effect of carrying large assortments due to the space limitations. Increasing the assortment reduces the visibility of the products on the shelves and drive lower inventory levels, which leverages the risk of stockouts and imposes frequent replenishment operations. On the other hand, not carrying some products in the assortment may generate lost-demand from loyal costumers. Therefore, a careful alignment is needed between space, assortment and inventory levels. Moreover, retailers only display a limited amount of the inventory on the shelves, storing the remaining products in the *backroom*. Hence, the alignment between shelf replenishment operations and shelf space allocation is also vital to avoid stockouts and design efficient shelf space plans.

Nonetheless, most part of the complexity in the SSAP comes from the inclusion of the space effects on the demand function. We will now review the key space effects that were studied in the literature and that are usually present in the demand functions.

2.2.1 Space Effects on Consumer Demand

Marketing studies have proven the positive influence of shelf space in stimulating consumer demand and identified three main types of product elasticities that should be incorporated into the demand functions to represent the consumers' behavior: space elasticity, location elasticity and cross elasticity.

Space Elasticity was originally defined by Curhan [1972] as "the ratio of relative change in unit sales to relative change in shelf space". Experiments have concluded that products' demand increases as more space is allocated to them. However, the increasing rate slows down until a steady point, resembling an "S" shape. An average increasing rate of 20% was reported by Curhan [1972] and 9% by Corstjens and Doyle [1981]. These values are only an indication, as the space elasticity strongly differs with the products category and shelves features.

Location Elasticity measures the impact of the vertical and horizontal location on the demand of the products. Studies show a higher impact of products located on the top- and middle- shelf positions (at eye and hand level) and at the beginning of the aisles, with the vertical effects dominating the horizontal ones (Chandon et al. [2009]). Drèze et al. [1994] reported an average of 39% and 15% sales' increase from the worst to the best vertical and horizontal position, respectively.

Cross Elasticity was introduced by Corstjens and Doyle [1981] to evaluate the interdependency between two different products. Ranging between [-1,1] cross elasticities are considered to be positive for complementary products and negative for substitution products. Drèze et al. [1994] experienced a boost of sales of above 5% in complementary merchandising. However, most retailers reveal the difficulty to attain a real estimation of such values, due to the complicated merchandizing relationships between products and the quantity of data required.

Additionally, the way products are arranged on the shelves can also have an important role on gaining the consumers' attention, which we call here the *Design Complexity* effect. Pieters et al. [2010] show that carefully organizing a display in families increases the viewers' attention but its excessive complexity (i.e. variations in the basic visual content) can indeed decrease their interest. As a result, products are organized in families in rectangular shapes (Geismar et al. [2014], Russell and Urban [2010]). The need to follow structured shapes is sometimes further stressed by assuming a direction for the shapes, either vertical or horizontal (forming straight columns or straight lines). To the best of our knowledge, the impact of this latter effect on demand has never been studied or included in any model.

Due to the high testing costs, experiments have not been sufficiently extensive, with most of them dating from the 1960s and 1970s. In addition, some results are contradictory, with few conclusions that can be generalized. As an example, while Chandon et al. [2009] reveal that the variation in the number of facings is the most significant in-store factor, Drèze et al. [1994] state that location has a larger impact, as long as a minimum inventory is maintained to avoid stockouts.

2.3. Shelf Space Allocation Review

As aforementioned, the demand estimation is at the center of the problem and products are to be organized having in mind the effect of space in their demand. Nevertheless, (1) the types of decisions, (2) the space effects considered, (3) the way the demand and cost functions are estimated and (4) the problem constraints vary considerably from one approach to the other, which creates a high level of inconsistency in this field. This section reviews the literature of SSAP focusing on the existing mathematical formulations. The review mainly tackles the four topics just mentioned (1)-(4) plus an additional topic that analysis the instances used in the publications.

Hereafter consider the following notation: N products, indexed by $i, j \in \mathcal{N}$ are to be placed on K shelves, indexed by $k \in \mathcal{M}$. Products and shelves features are described below, the remaining parameters are introduced as required.

a_i	width of product i,
l_i (u_i)	lower bound (upper bound) on the number of facings of product i,
p_i	unitary profit of product i,
c_i	unitary cost of product i,
W	total capacity of the planogram,
w_k	width of shelf k,
h_k	vertical location of shelf k.

2.3.1 Space Decisions

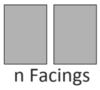
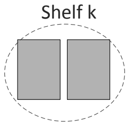
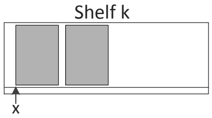
Space allocation models generally consider 3 different decisions: the space occupied by each product, often measured by the number of facings (*Space*), the allocation of products to the shelves (*Allocation*) and the location of the products on each shelf (*Location*). The following 3 variables are associated with these decisions:

W_i	the space allocated to product $i \in \mathcal{N}$, either measured in facings or linear space,
Y_{ik}	=1 weather product $i \in \mathcal{N}$ in allocated to shelf $k \in \mathcal{M}$ or not,
X_{ik}	the continuous horizontal location of product $i \in \mathcal{N}$ in shelf $k \in \mathcal{M}$ (measured from the most left point of the shelf).

Table 2.1 shows the publications that consider each of the 3 decisions mentioned above. Note that the existing mathematical models focus specially on determining the space for the products. The reason behind this emphasis lies in the fact that most authors consider

space elasticity as the most important effect, with a higher impact on demand than the remaining elasticities. The table also shows that only 4 publications consider location decisions. Nevertheless, if location decisions are disregarded, solutions do not translate into a complete description of a planogram.

Table 2.1 – Shelf space decisions

Decisions		References
Space		Anderson and Amato [1974], Hansen and Heinsbroek [1979], Anderson [1979], Corstjens and Doyle [1981], Corstjens and Doyle [1983], Zufryden [1986], Bultez et al. [1989],Preston and Mercer [1990], Borin et al. [1994], Drèze et al. [1994], Brown and Lee [1996], Urban [1998], Yang and Chen [1999], Yang [2001], Urban [2002], Lim et al. [2004], Bai [2005], Hwang et al. [2005], Reyes and Frazier [2005], Maiti and Maiti [2006], Hariga et al. [2007], Reyes and Frazier [2007], Bai and Kendall [2008], van Nierop et al. [2008], Abbott and Palekar [2008], Hwang et al. [2009], Ranaseshan et al. [2009], Raut et al. [2009], Gajjar and Adil [2010], Hansen et al. [2010], Murray et al. [2010], Russell and Urban [2010], Irion et al. [2011], Lotfi et al. [2011], Gajjar and Adil [2011a], Gajjar and Adil [2011a], Lotfi and Torabi [2011], Hübner and Kuhn [2011], Irion et al. [2012], Geismar et al. [2014]
Allocation		Drèze et al. [1994], Yang and Chen [1999], Yang [2001], Lim et al. [2004], Bai [2005], Hwang et al. [2005], Hariga et al. [2007], van Nierop et al. [2008], Hwang et al. [2009], Raut et al. [2009], Gajjar and Adil [2010], Hansen et al. [2010], Murray et al. [2010], Russell and Urban [2010], Gajjar and Adil [2011a], Gajjar and Adil [2011a], Lotfi and Torabi [2011], Geismar et al. [2014]
Location		van Nierop et al. [2008], Hwang et al. [2009], Hansen et al. [2010], Raut et al. [2009], Russell and Urban [2010]

The variables defined before construct the problem in a 2D fashion. Such an approach is based on the fact that items placed behind each facing cannot be seen directly and hence do not have an impact on consumer demand. As a result, it is not also common to see other type of space-related decisions such as the number of facings high and deep, and products' orientation. Moreover, products tend to have a preferred orientation, specified by the suppliers. Nonetheless, as these quantities impact inventory related decisions recent works (see Ranaseshan et al. [2009] and Murray et al. [2010]) are starting to consider 3D space-related decisions.

Due to the large number of products within categories, some authors also argue that it is not practical to optimize shelf space plans having in mind all products. As a result, the decisions are sometimes aggregated and tackled at the brand level (or subcategory level).

All the above decisions are product-related but planograms also have shelves whose location needs to be determined. The shelf-related decisions are usually given as input to the models because retailers are not likely to change the layout of the shelves during products reallocation. However, some authors consider the shelf height as a decision: Hwang et al. [2009] and Coskun [2012] are two examples. To the best of our knowledge, the determination of the number of shelves to place on planograms has never been tackled.

Several approaches also integrate other non-space decisions that have an impact on consumer demand, with an emphasis on assortment and inventory management. Other variables such as promotions, advertising and pricing also appear in some formulations but these decisions are most of the times considered as fixed (i.e. decided beforehand) and only affect the demand function. The investigation of these related problems is considered beyond the scope of this paper.

2.3.2 Demand and Cost Estimation

The objective functions have usually three main components: the profit obtained with the sales of the products, which depend on the demand d_i, subtracted by purchasing costs c_i and other estimated costs o_i, as identified in equation 2.1. The complexity of the shelf space formulation relies on the demand and cost estimations which will be analyzed next.

$$P = \sum_{i \in \mathcal{N}} (p_i \cdot d_i - c_i \cdot d_i - o_i) \tag{2.1}$$

Demand Estimation

The literature has a great variety of demand functions which incorporate different elasticities' estimates, as well as different ways of aggregating these effects. Moreover, many proposed functions tend to focus on particular effects while disregarding the others. Table 2.2 presents the characteristics that each publication has considered for the demand estimates. We distinguish between the space related elasticities (space-, cross-, vertical location- and horizontal location-), other demand effects that were also taken into consideration, and two different types of aggregation: additive and multiplicative. The consideration of the space elasticity is common to all approaches and this effect is frequently aggregated with other elasticities using a multiplicative form. Cross and vertical location elasticities are also frequently considered but only five approaches aggregate both effects. The horizontal dimension is often disregarded.

Despite the existence of many demand estimates, this field presents some key demand-functions that are almost consensual and used across multiple publications. Next, we review the most important ones.

One of the first and most important demand functions in shelf-space allocation literature was introduced by Corstjens and Doyle [1981], which influenced most future research. They formulated the problem in a non-linear multiplicative form and included space- and cross- elasticities. The demand of product i was formulated as:

$$d_i = \alpha_i \cdot W_i^{\beta_i} \cdot \prod_{j \in \mathcal{N}: j \neq i} W_j^{\delta_{ij}} \tag{2.2}$$

where W_i is defined in terms of (linear) space allocated to product i, α_i is a scaling constant identified as the base demand for the product (demand with one facing), β_i is the space elasticity expressed as a power function and δ_{ij} is the cross-elasticity between products i and j. Note that δ_{ij} can be positive or negative depending upon whether i and j are complementary or substitute of each other, and that δ_{ij}, is not necessarily equal to δ_{ji}.

Table 2.2 – Consumer demand effects in demand estimation

Reference	SE	CE	LE		Other	EA	
			V	H		A	M
Anderson and Amato [1974]	L				out-of-assortment substitution	•	
Hansen and Heinsbroek [1979]	P						
Anderson [1979]	P						
Corstjens and Doyle [1981]	P	•					•
Corstjens and Doyle [1983]	P	•					•
Zufryden [1986]	P						
Bultez et al. [1989]	P	•					•
Preston and Mercer [1990]	P						
Borin et al. [1994]	P	•			Stockout and out-of-assortment substitution		•
Drèze et al. [1994]	P		•			•	
Brown and Lee [1996]	P	•					•
Urban [1998]	P	•					•
Yang and Chen [1999]	P	•	•				•
Yang [2001]	L		•				
Lim et al. [2004]	L		•			•	
Bai [2005]	P						
Hwang et al. [2005]	P	•	•		Inventory level		•
Reyes and Frazier [2005]	L	•				•	
Maiti and Maiti [2006]	P	•			Inventory level and price elasticity		•
Hariga et al. [2007]	P	•	•		Inventory level		•
Reyes and Frazier [2007]	P				Price elasticity		•
Bai and Kendall [2008]	P				Inventory level and decay (freshness)		•
van Nierop et al. [2008]	P		•	•			•
Abbott and Palekar [2008]	L	•				•	
Hwang et al. [2009]	P	•	•	•			•
Ranaseshan et al. [2009]	P	•					•
Raut et al. [2009]	L	•	•			•	
Gajjar and Adil [2010]	P						
Hansen et al. [2010]	L	•	•	•		•	
Murray et al. [2010]	P		•		Own- and cross-price elasticity		•
Russell and Urban [2010]	P		•	•		•	
Gajjar and Adil [2011a]	L		•				
Gajjar and Adil [2011b]	P						
Irion et al. [2011]	P	•					•
Lotfi and Torabi [2011]	P		•				•
Lotfi et al. [2011]	P				Own- and cross-price elasticity		•
Hübner and Kuhn [2011]	P	•			Out of assortment substitution	•	
Irion et al. [2012]	P	•					•
Geismar et al. [2014]	L		•				

SE - Space Elasticity (L - Linear and P - Polynomial), CE - Cross Elasticity, LE - Location Elasticity (V - Vertical and H - Horizontal), EA - Effects Aggregation (A - Additive and M - Multiplicative)

This demand formulation uses polynomial terms to model the decreasing demand rate as the number of facings increases. These polynomial forms were widely used by other authors who extended this formulation in many different ways. Urban [1998] stated that the unit of measure of the products, W_i, could also be in terms of facings, as long as all parameters reflect the appropriate measure, and many following formulations used facings instead. Yang and Chen [1999] additionally integrated the location effect of a product, by using variables W_{ik} instead, and considering different parameter values depending on the shelf k. They also included additional marketing variables. Hwang et al. [2005] assumed an average location effect in case the same product is displayed on different shelves at the same time. Corstjens and Doyle [1983] and Raut et al. [2009] included the time dimension and assumed that past demand influences current period demand, and Gajjar and Adil [2010] and Irion et al. [2012] presented piecewise linearization approaches for this formulation. Nevertheless, Bai [2005] argues that the polynomial form is intrinsic linear as it can be easily transformed to a linear function by a logarithmic transformation and the parameters can then be estimated by a simple linear regression. To the best of our knowledge there is not any paper with such approach though.

At last, Borin et al. [1994] and Urban [1998] extended the original function by considering the demand coming from the consumers which are willing to purchase a replacing product if their preferred product is not included in the assortment or is temporarily stock-out. For that porpuse, consider products $l \in \mathcal{N}^-$ which are not present on the shelves. The demand function becomes:

$$d_i = \alpha_i \cdot W_i^{\beta_i} \prod_{j \in \mathcal{N}: j \neq i} W_j^{\delta_{ij}} \left[1 + \sum_{l \in \mathcal{N}^-} (1 - \Theta_l) \cdot f(\alpha_l, \delta_{li}) \right] \tag{2.3}$$

where Θ_l is the resistance to compromise and f is a function that represents the distribution of demand amongst the displayed products.

Due to the highly non-linear nature of the space elasticities, all the models that include these effects are complex and generally hard to solve. Yang and Chen [1999] proposed a simplified and yet practical alternative model in the form of a linear multi-knapsack problem that started a new trend in SSAP. The authors state that in practice it is difficult to obtain an estimation for the sales volume elasticity and make the assumption that the profit of any product is linear with regard to a range of facings, if it is kept in a controlled range defined by proper upper and lower bounds. Those bounds are to be determined by the management according to the policy of the store. They propose a formulation that maximizes the benefit of including items (additional facings) in a set of knapsacks (each shelf is a knapsack) while not exceeding the knapsack capacity. The resulting objective function is the following:

$$P = \sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{M}} p_{ik} \cdot W_{ik} \tag{2.4}$$

where p_{ik} is the per-facing profit of product i on shelf k. By associating the shelf space allocation problem to a knapsack problem, the authors also proved that even a simplified version of the problem is NP-hard. Nevertheless, its simplicity was criticized by several authors because it contradicted previous experiments revealing an S-shaped curve for the effects of space elasticity. Other authors, such as Lim et al. [2004], further defended the linearity assumption stating that retailers prefer to operate on the linear portion of the S-shaped curve of marginal returns. This simplified version has been used by several authors to develop efficient heuristics and matheuristics for the problem.

The discrete nature of the aforementioned models disregards the exact location of the products and assigns the same effect regardless of the product's location on a particular shelf. Hansen et al. [2010] and Russell and Urban [2010] considered location decisions and proposed (quasi) horizontal effects on the demand function that we will review now.

Hansen et al. [2010] extended the simplified version of Yang and Chen [1999] and presented a formulation where the variables were discretized to consider the horizontal location of the products on the shelves. The authors divided the shelf in multiple horizontal segments and defined the binary decision variables W_{jkhf} as the decision of allocating product i on shelf k starting at horizontal segment h for face-length f. The resulting objective function is as follows:

$$P = \sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{M}} \sum_{h=1}^{T_k} \sum_{f=1}^{u_i} p_{ikhf} \cdot W_{ikhf} + \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} \frac{V_{ij}}{2} \cdot e_{ij} \tag{2.5}$$

where p_{ikhf} is the profit of product i associated with W_{jkhf}. A non-linear profit function is considered, with a decreasing demand rate as the number of facings increase. However, the non-linearity is absorbed by parameter p_{ikhf}, as it depends on the number of facings assigned to each product. The second part of the objective function is a linearization of the cross-elasticity effect. The authors defined parameters e_{ij} as the unitary incremental profit or loss due to cross-elasticity effects between products i and j, which is multiplied by $min(b_i, b_j)$, where b_i and b_j are the total lengths of products i and j on the shelf. The function is linearized by introducing variables $V_{ij} = min(b_i, b_j)$.

Russell and Urban [2010] introduced the first formulation with continuous horizontal locations for the products. They based their demand function on a previous work by Drèze et al. [1994] who noted that sales tend to be quadratic in the horizontal dimension and cubic in the vertical one. As for space elasticity, the authors chose to use a quadratic formulation, not only for consistency and tractability, but also because it reflects diminishing returns. The expected demand for each product i is then expressed by:

$$\begin{aligned} d_i = {} & \beta_{0i} + \beta_{1i} \cdot X_i + \beta_{2i} \cdot X_i^2 + \\ & \sum_k \left[\beta_{3i} \cdot (h_k \cdot Y_{ik}) + \beta_{4i} \cdot (h_k \cdot Y_{ik})^2 + \beta_{5i} \cdot (h_k \cdot Y_{ik})^3 + \beta_{6i} \cdot W_{ik} + \beta_{7i} \cdot W_{ik}^2 \right] \end{aligned} \tag{2.6}$$

where $\beta_{\bullet i}$ are appropriate coefficients for the specific implementation. Since Y_{ik} is a binary variable, the demand formulation is expressed as a quadratic function.

Cost Estimation

The literature on shelf space allocation has focused less on the cost side of the problem and many authors only subtract to the selling price of each unit sold its corresponding purchasing costs, which result in its gross margin. Corstjens and Doyle [1981] considers a second component on the objective function that estimates the operating costs (which they called store expense). This component takes into consideration the concept of economies of scale and considers marginal costs:

$$o_i = \gamma_i \cdot d_i^{\tau_i} = \gamma_i \left[x_i^{\beta_i} \cdot \prod_{j \in \mathcal{N}: j \neq i} x_j^{\gamma_{ij}} \right]^{\tau_i} \tag{2.7}$$

γ_i is the cost scale parameter of product i and τ_i is the operating cost elasticity associated with increased sales. This element has been used by other authors to represent operating costs.

Reyes and Frazier [2005] present an interesting alternative approach to model operating costs. They consider three additional factors besides the purchasing costs, namely, ordering costs, holding costs and stockout costs: ordering costs consider a constant cost rate that is multiplied by the number of orders per day; holding costs are given as a percentage of the purchase cost of an item; and stockout costs are assumed near zero (which occur when the space is well planned). The cost formulation is as follows:

$$o_i = O \cdot \frac{d_i}{s_i} + H \cdot c_i \cdot x_i \tag{2.8}$$

where O is the ordering cost rate, (d_i/x_i) represent the number of orders per day, H is the daily ordering cost rate and c is the product purchasing cost.

2.3.3 Problem Constraints

The type of constraints present in the shelf space formulations have also evolve over the years. Table 2.3 indicates the publications that consider each constraint type identified on Section 2.2, namely: integrality, capacity, control of lower and upper bounds, availability and product grouping constraints.

Integrality, capacity and control constraints are the most common features of the problem, both in simplistic and linear approaches as well as in comprehensive and polynomial models. Availability constraints were introduced by Corstjens and Doyle [1981] but Yang and Chen [1999] concluded that retailers can prevent stockout occurrences by building effective logistics systems. Since that moment on, this type of constraint was rarely considered.

One important practical limitation from the current literature is that it neglects merchandising rules during products' allocation. To overcome this, Lim et al. [2004] introduced an additional element on the objective function and attributed additional benefits if two products with affinity were placed on the same shelf. Russell and Urban [2010] explicitly considered the products as part of a family, which can be based on a variety of

Table 2.3 – Shelf space constraints

Constraints	References
Integrality $W_i \in \mathbb{N}, \forall i \in \mathcal{N}$ $(W_{ik} \in \mathbb{N}, \forall i \in \mathcal{N}, k \in \mathcal{M})$ $Y_{ik} \in \{0,1\}, \forall i \in \mathcal{N}, k \in \mathcal{M}$	Anderson and Amato [1974], Hansen and Heinsbroek [1979], Zufryden [1986], Preston and Mercer [1990], Borin et al. [1994], Drèze et al. [1994], Yang and Chen [1999], Yang [2001], Lim et al. [2004], Bai [2005], Hwang et al. [2005], Reyes and Frazier [2005], Maiti and Maiti [2006], Hariga et al. [2007], Reyes and Frazier [2007], Bai and Kendall [2008], van Nierop et al. [2008], Ranaseshan et al. [2009], Raut et al. [2009], Gajjar and Adil [2010], Hansen et al. [2010], Murray et al. [2010], Russell and Urban [2010], Gajjar and Adil [2011a], Gajjar and Adil [2011b], Lotfi and Torabi [2011], Lotfi et al. [2011], Hübner and Kuhn [2011], Irion et al. [2012], Geismar et al. [2014]
Capacity $\sum_{i \in \mathcal{N}} a_i W_i \leq W$ $(\sum_{i \in \mathcal{N}} a_i W_{ik} \leq w_k, \forall k \in \mathcal{M})$	Anderson and Amato [1974], Hansen and Heinsbroek [1979], Anderson [1979], Corstjens and Doyle [1981], Corstjens and Doyle [1983], Zufryden [1986], Bultez et al. [1989], Preston and Mercer [1990], Borin et al. [1994], Drèze et al. [1994], Brown and Lee [1996], Urban [1998], Yang and Chen [1999], Yang [2001], Lim et al. [2004], Bai [2005], Hwang et al. [2005], Reyes and Frazier [2005], Maiti and Maiti [2006], Hariga et al. [2007], Reyes and Frazier [2007], Bai and Kendall [2008], van Nierop et al. [2008], Abbott and Palekar [2008], Hwang et al. [2009], Ranaseshan et al. [2009], Raut et al. [2009], Gajjar and Adil [2010], Hansen et al. [2010], Murray et al. [2010], Russell and Urban [2010], Gajjar and Adil [2011a], Gajjar and Adil [2011b], Irion et al. [2011], Lotfi and Torabi [2011], Lotfi et al. [2011], Hübner and Kuhn [2011], Coskun [2012], Irion et al. [2012], Geismar et al. [2014]
Control $l_i \leq x_i, \forall i \in \mathcal{N}$ $(l_i \leq \sum_{k \in \mathcal{M}} x_{ik}, \forall i \in \mathcal{N})$ $u_i \geq x_i, \forall i \in \mathcal{N}$ $(u_i \geq \sum_{k \in \mathcal{M}} x_{ik}, \forall i \in \mathcal{N})$	Hansen and Heinsbroek [1979], Corstjens and Doyle [1981], Zufryden [1986], Preston and Mercer [1990], Borin et al. [1994], Reyes and Frazier [2005], Urban [1998], Yang and Chen [1999], Yang [2001], Lim et al. [2004], Bai [2005], Hwang et al. [2005], Maiti and Maiti [2006], Hariga et al. [2007], Reyes and Frazier [2007], Bai and Kendall [2008], van Nierop et al. [2008], Hwang et al. [2009], Ranaseshan et al. [2009], Raut et al. [2009], Gajjar and Adil [2010], Hansen et al. [2010], Murray et al. [2010], Russell and Urban [2010], Gajjar and Adil [2011a], Gajjar and Adil [2011b], Irion et al. [2011], Lotfi and Torabi [2011], Lotfi et al. [2011], Hübner and Kuhn [2011], Irion et al. [2012], Geismar et al. [2014]
Availability $d_i \leq$ supply limit, $\forall i \in \mathcal{N}$	Corstjens and Doyle [1981], Zufryden [1986], Urban [1998], Yang and Chen [1999], Ranaseshan et al. [2009]
Family Grouping	Lim et al. [2004], Russell and Urban [2010]

characteristics, such as brand, flavor, price set, among others. Products of these families should be kept together and, for aesthetic reasons, in uniform and rectangular shapes. To the best of our knowledge, this was the first publication that explicitly considered product families as a constraint.

2.3.4 Instances

Similarly to the variety of SSAP models, there is not a unique set of benchmark instances, and authors have been mainly generating and using their own data sets, with few comparisons between approaches. Moreover, few instances are published online and made available to the research community.

Table 2.4 in Appendix 2.A presents an overview of the widely different data sets studied in this review, we identify the following information: instance type (randomly generated or real-world), solution method (heuristic or exact method), instance size and motivation. Note that there are many real-world instances coming from a wide range of categories, such as quality candies, bottled juices, canned dog food and distilled-spirits.

The size of the instances can be measured by the number of products considered. We observe that real-world instances have usually small problem sizes as they often aggregate products into brands or subcategories. Indeed, a great share of the instances approach the problem from an aggregated perspective or consider a very small set of products, leading to instances with less than 10 products / brands. Other instances present a more detailed perspective by approaching the problem at the product level, with more than 100 products. Nevertheless, we noticed that these bigger instances are tackled in less-constrained and simplistic problem descriptions. For instance, Drèze et al. [1994] allocates 115 products but considers a discretized version of its polynomial demand function and assigns products to shelves only limited to capacity constraints.

Table 2.4 also shows that many instances were solved using heuristics as the authors commonly argue that shelf space models are difficult to solve using exact methods.

2.4. Classification Framework for the Shelf Space Allocation Problem

Based on the literature reviewed in the previous section, we propose a classification framework for the shelf space allocation problem, with the intent to systematize the different types of formulations. As it became clear through the paper, the demand function estimation is at the center of the problem, and different approaches aggregate different types of decisions and elasticities. These are the two main dimensions used for classifying the problems, using the framework present in Figure 2.3.

Note that both the decisions and space effects tend to evolve as there are precedences between them. Allocation decisions are always considered on top of facings calculation. In the same way, no problem considers location elasticities without considering space elasticities. This is the basis of the framework. The more decisions and elasticities are considered, the more on the lower right side the problem is. The only exception are the cross-

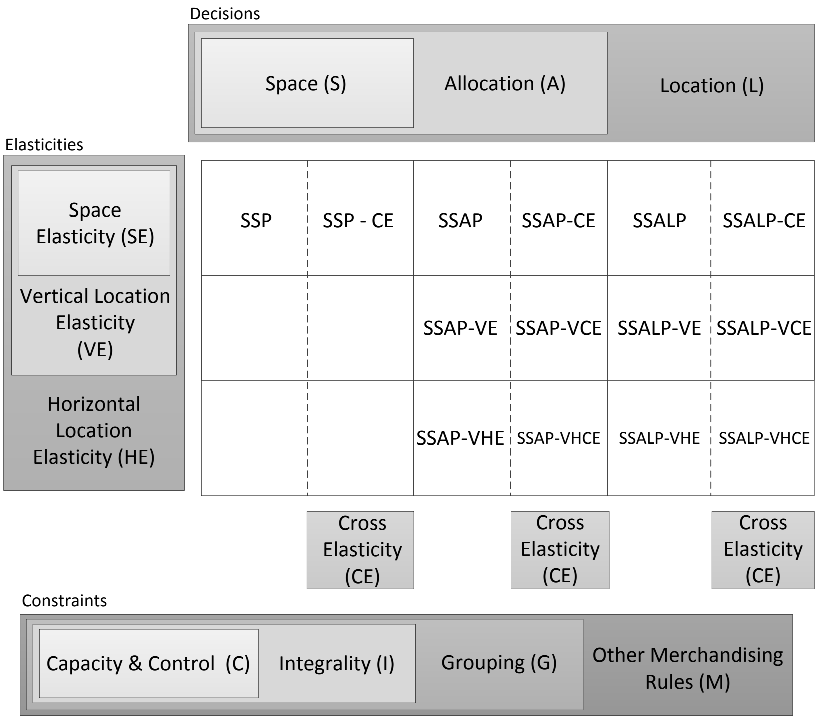


Figure 2.3 – Classification Framework for the Shelf Space Allocation Problem

elasticities, that are used without any known precedence. Because of that, this last effect was included in an additional dimension.

The names of the problems were generated using the acronyms associated with both the decisions and the elasticities. The first part of the names are connected with the decisions and the second part with the elasticities. As an example, the problem that considers all decisions and elasticities is called *SSALP-VHCE*, Shelf Space Allocation and Location Problem with vertical-, horizontal- and cross- elasticity effects.

In the lower part of the framework we distinguish between the different types of problem constraints, that also evolve under a precedence scheme. These constraints can also be included in the name of the problems by considering an additional (third) part. As an example, if the problem *SSALP-VHE* is considered with product grouping constraints, we call to this problem *SSAP-VHE-G*.

Figure 2.4 presents the distribution of the publications across the framework. The gray scale indicates the amount of research addressing the problem. The darker areas reveal more publications tackling that problem. The figure shows that the basic problem SSP, with and without cross-elasticity effects, are the most tackled problems in the literature. It also reveals that the right side of the framework has received less attention from the research community. This fact corroborates the initial findings concerning the lack of approaches considering location decisions and gives a clear indication of the research gaps in this field, as discussed in the following final section.

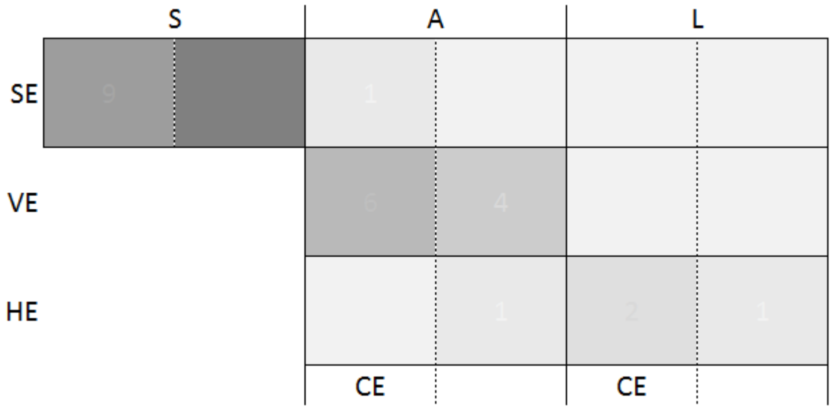


Figure 2.4 – Distribution of the publications across the framework

2.5. Conclusion and Directions for Future Research

In this paper we reviewed the literature of the Shelf Space Allocation Problem and presented a classification framework that systematized the widely different approaches to the problem. We also distributed the existing literature across the different problems proposed in the framework in order to identify the main research gaps. This study revealed the following suggestions for future research, that we divided into five main directions:

Exploration of the gaps identified by the framework – Section 2.2 revealed the importance of merchandising rules and strengthen the need to consider product families as this is a key requirement from most retailers in practice. However, most of the current literature on shelf space allocation disregards location decisions. As a result, existing formulations are not capable of being extended to included these constraints and additional work is required to develop new formulations and solution approaches to the *SSALP*.

Development of efficient solution methods – The shelf space allocation literature has mainly focused on developing formulations and exact mathematical approaches and few heuristics and matheuristics have been proposed. The only exception is the linear formulation from Yang and Chen [1999] (whose profit function is present in equation (2.4)), that has been widely used to develop many solution approaches mainly to the *SSAP* problems. Moreover, most computational experiments have used instances with a very small number of products and larger problems are usually randomly generated. Hence, we find crucial to gather a set of benchmark instances that can be used to compare solution approaches covering realistics features and sizes.

Consideration of space-related extensions – There are many space-related extensions that are worth studying. We highlight three of them in this review.

Firstly, shelf space models could be extended to identify the family types that should be used to group products, seeking to increase display attractiveness. Until now, family groups have been used as constraints and given as inputs to the models. To the best of our knowledge, the quantification of the impact of these groups on the consumer demand is still to be studied. Figure 2.5 presents the corresponding framework extension, in terms of types of elasticities, with the inclusion of an additional effect which we called *Design Complexity*. Note that more experimental studies need to be conducted to correctly evaluate the impact of the design complexity on the demand.

Figure 2.5 – Framework extension to consider Design Complexity

Secondly, most literature on shelf space allocation considers that the location of the shelves is defined beforehand. The reasons behind this assumption mostly come from the observation that retailers are not likely to change the shelf positions during operations due to the high cost that such endeavor would represent. However, shelves' placement is an important decision to consider during stores' opening and refurbishment as well as during significant assortment modifications. Therefore, this problem extension is not only a valuable pratical contribution but also a scientific contribution given the challenge that it holds. This extension could easily be included in the framework by adding a fourth decision with the name *Shelf Design*, as seen in Figure 2.6. Note that these two extensions perfectly fit the framework because they also have precedences with the remaining decisions and elasticities.

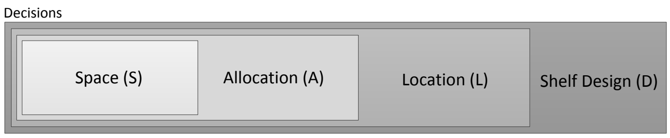


Figure 2.6 – Framework extension to consider Shelf Design

Thirdly, as stores usually have other types of fixtures, their study and integration are also potential research topics. Pegboards are a particularly engaging fixture type for future studies because of their inherit complexity and the fact that they have been hardly studied in the literature. Nevertheless, in our opinion, this research line would not fit in this stream, but it would rather open a new one.

Alignment with other retail problems – The interdependency of this problem with other retail planning activities was also highlighted through this review. Many opportunities lie in this research direction. The solutions generated using any shelf space model strongly depend on the category space and product assortment previously defined, considered as inputs to these models. Both the literature and practice of retail shelf space would benefit from an integration or a sensitivity analysis on the impact of these upstream decisions in shelf space plans. Some authors have already tackled assortment and shelf space jointly but usually disregard the space-elasticity effects (typical in shelf space literature) or the substitution effects (typical in assortment literature). Moreover, the problem would benefit from deeper analysis on inventory-related concerns with a focus on replenishment synergies. Both problems are highly interdependent as shelf replenishment operations have expensive handling costs and are limited to the shelf merchandisers available to immediately fill the shelves after stockout. The inclusion of the concept of target service level would be an interesting approach.

Alignment of software applications with science – Besides the need for efficient algorithms able to cope with high number of products, the expensive estimation requirements for parameters is also a major barrier for a better alignment between science and software applications. Nevertheless, there are other ways of improving the use of shelf space theory in practice. One example is to study the creation of shelf space solutions taking into account the current planogram implemented in the stores in order to trade-off potential profit and the costs of changes, namely, handling costs.

Appendix 2.A Problem Instances Table

This section presents the details of the data sets discussed in this review, in section 2.3.4. We identify the following information: instance type (randomly generated or real-world), solution method (heuristic or exact method), instance size and motivation.

Table 2.4 – Problem Instances

	Data			Instances	
Reference	**G**	**R**	**Method**	**Size**	**Motivation**
Anderson and Amato [1974]	•		H	4 brands	Illustrative example
Hansen and Heinsbroek [1979]		•	H	6443 products	Based on a LOEB-IGA study
Corstjens and Doyle [1981]		•	E	5 product categories	Quality candy, ice cream and greeting cards with 140 stores
Corstjens and Doyle [1983]	•		E	4 product groups	Illustrative example
Zufryden [1986]	•		E	up to 40 products	
Bultez et al. [1989]		•	H	20 products	Canned dog food (Belgium retailer)
Drèze et al. [1994]		•	E	average size 115 products (27 min. and 235 max.)	Analgesics, Bottled Juices, Canned Seafood, Canned Soup, Oral Care, Refrigerated Juices (US supermarket - 60 stores)
Borin et al. [1994], Borin and Farris [1995]	•	•	H	6 products (generated), 18 products (field study)	Ketchup (local supermarket)
Brown and Lee [1996]		•	E	one instance with 2 product categories	Juice (US grocery stores)
Urban [1998]	•		H	up to 54 products	Based on Borin et al. [1994]
Yang [2001]	•		H	up to 10 products, 4 shelves	
Urban [2002]	•		H	6 products	Based on Borin et al. [1994]
Lim et al. [2004]	•		H	up to 100 products and 30 shelves	
Bai [2005]	•		H	up to 100 products and 40 shelves	
Hwang et al. [2005]	•		H	4 products and 6 shelves	
Reyes and Frazier [2005]	•		E	6 products	
Maiti and Maiti [2006]	•		H	up to 5 products	
Hariga et al. [2007]	•		E	4 products and 4 display areas	
Reyes and Frazier [2007]	•		E	4 products	Random but based on real world data collected from a US grocery store
Bai and Kendall [2008]	•		H	up to 64 products	Small instances based on Borin et al. [1994] and generated large instances.
van Nierop et al. [2008]		•	H	81 products and 5 shelves	Canned Soup from Drèze et al. [1994].
Hwang et al. [2009]	•		E/H	4 products	
Ranaseshan et al. [2009]		•	H	6, 10 and 14 products	Generated from a category of size 300 of Baked Beans and Noodles. Data collected from a medium size national grocery retailer.
Raut et al. [2009]		•	E/H	6 screens / 6 products, 5 weeks	Exhibitors movie allocation problem in a multiplex.
Gajjar and Adil [2010], Gajjar and Adil [2011a], Gajjar and Adil [2011b]	•		H	up to 200 products and 50 shelves	
Hansen et al. [2010]	•	•	E H H	up to 10 products and 2 shelves up to 100 products and 10 shelves (generated) 67 products and 7 shelves (case study)	Health and beauty
Murray et al. [2010]	•		E	up to 100 products (3 orientations) and 10 shelves	
Russell and Urban [2010]		•	E H	10 products, 5 families, 4 shelves 103 products, 36 families, 25 shelves	Distilled-spirits
Irion et al. [2011]		•	E	9 categories	Home improvement-product retailer
Lotfi and Torabi [2011]	•		E H	up to 20 products up to 80 products	
Lotfi et al. [2011]	•		E	4 products	
Hübner and Kuhn [2012]	•		E	up to 250 products	
Irion et al. [2012]	•	•	E	up to 50 products	Home improvement-product retailer
Geismar et al. [2014]	•		H	200 and 500 products	Motivated by a case study in a blockbuster store

Data: G - Generated and R - Real, Method: H - Heuristic and E - Exact

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Chapter 3

Allocating Products on Shelves under Merchandising Rules: Multi-level Product Families with Display Directions

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Abstract Retailers' individual products are categorized as part of product families. Merchandising rules specify how the products should be arranged on the shelves using product families, creating more structured displays capable of increasing the viewers' attention. This paper presents a novel mixed integer programming formulation for the Shelf Space Allocation Problem considering two innovative features emerging from merchandising rules: hierarchical product families and display directions. The formulation uses single commodity flow constraints to model product sequencing and explores the product families' hierarchy to reduce the combinatorial nature of the problem. Based on the formulation, a mathematical programming-based heuristic was also developed that uses product families to decompose the problem into a sequence of sub-problems. To improve performance, its original design was adapted following two directions: recovery from infeasible solutions and reduction of solution times. A new set of real case benchmark instances is also provided, which was used to assess the formulation and the matheuristic. This approach will allow retailers to efficiently create planograms capable of following merchandising rules and optimizing shelf space revenue.

Keywords Retail · Shelf space allocation · Single commodity flow formulation · MIP-based heuristics

3.1. Introduction

While shopping, customer choices are highly influenced by in-store factors, in particular during frequent unplanned purchases and when the products they are searching for are not

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available. In this context, more than just displaying the merchandise, a clever product arrangement on the shelves can boost demand and ultimately the stores' financial performance. With the increasing number of products available for the same scarce space, shelf space planning has become more and more challenging and an active field of research in retail operations management, under the name Shelf Space Allocation Problem (SSAP).

The SSAP consists of distributing the scarce shelf space of a retail store among the different products to be displayed. Marketing studies have proven that space allocation has a positive impact on the visibility, consumer awareness and demand for the products (Drèze et al. [1994], Chandon et al. [2009], Curhan [1972], Desmet and Renaudin [1998]). As a result, for the past 40 years several models have tried to address the various objectives associated with product-to-shelf allocation, ranging from comprehensive to simplistic forms.

In practice, the traditional space planning tool is a planogram, which is a blueprint of the shelves where retailers develop their merchandising plan, showing exactly the location where each product should physically be displayed and the number of facings that the product should hold. Planograms are usually created separately for each category, whose space is determined beforehand on a macro or upstream level. There are space planning software systems which can assist retailers in this activity. These systems provide realistic views of the shelves and allow retailers to quickly handle products through the planograms. Moreover, they have powerful analysis reports and automatic tools for product-to-shelf allocation.

However, Hübner and Kuhn [2012] and Bai [2005] identified a misalignment in shelf space planning between commercial software applications and research: on one hand, software vendors focus mainly on the development of large-scale data processing technologies, with limited or no use of mathematical optimization and disregarding consumer demand effects. On the other hand, state-of-the-art optimization methods have practical limitations, either because of their simplicity and lack of key features, or due to their complexity and expensive estimation requirements for parameters.

Retailers' individual products are categorized as part of product families. One important practical limitation from the current literature is that it disregards that product allocation must follow merchandising rules which specify associations of products on the shelves. Merchandising rules try to reproduce the way customers search for the products while shopping and are obtained with the help of category captains (key suppliers with deeper knowledge about each category - Kurtulus and Toktay [2009]) and techniques such as market basket analysis. Those rules vary from retailer to retailer, and can include more than one level of product association.

Another key practical feature of the problem is the way families are arranged on the shelves. Pieters et al. [2010] show that carefully organizing a display increases the viewers' attention but its excessive complexity (i.e. variations in the basic visual content) can indeed decrease their interest. These concepts are applied in the Shelf Space Allocation Problem by imposing that both the products and the families are arranged in rectangular shapes (Geismar et al. [2014], Russell and Urban [2010]). The need to follow structured shapes is sometimes further stressed by assuming a direction for the shapes, either vertical or horizontal (forming columns or lines). To the best of our knowledge, the display direction is for the first time tackled in this paper.

This paper presents a novel and realistic mathematical model for the SSAP with multi-level product families. The model uses a linear profit function, as suggested by Yang and Chen [1999], and considers space and location decisions, similarly to Russell and Urban [2010]. Considering product families requires the definition of the exact location of the products on the shelves (not common in SSA literature) and thus products need to be sequenced. This additional requirement turns the models much more complex as sequencing decisions are known to pose hard analytical challenges mainly due to subtour elimination constraints. Following the research done in other combinatorial problems such as the asymmetric traveling salesman problem (Öncan et al. [2009]), we improved the modeling of product location using commodity-based constraints which are known to yield very tight models. This formulation is embedded in a matheuristic aiming at delivering quasi optimal solutions in short computational times. The matheuristic solves a sequence of sub-problems, exploring the hierarchy present in the product families. Using instances taken from a European grocery retailer, we demonstrate the applicability of the formulation, and report the improvements obtained with both the formulation and matheuristic over the existing literature.

The contributions of this paper are as follows. A novel mathematical model has been developed for the Shelf Space Allocation Problem with location decisions based on the commodity flow formulation. This model additionally explores the existence of product families to reduce the combinatorial nature of the problem and introduces a new practical constraint imposed to product families: the display direction. On the algorithmic front, an innovative matheuristic is presented that was tailor-made to the formulation as it is based on the existence of multi-level product families. To improve the matheuristic performance, its original design was adapted following two directions: recovery from infeasible solutions through backtracking (improving feasibility) and reduction of solution times by adjusting the model's detail (improving efficiency). Finally, a new set of real case benchmark instances is provided for the shelf space allocation problem with location decisions, allowing for future research in this area.

The remainder of this paper is structured as follows. Section 3.2 begins with a literature review on the Shelf Space Allocation Problem defining the basis of this research. The problem is formally defined in section 3.3 with a focus on the definition of real world features. Section 3.4 is dedicated to describing the novel realistic mathematical formulation with single commodity flow constraints, and section 3.5 describes the solution approach that was tailor-made for the model. The computational results are presented and analyzed in section 3.6. The final section 3.7 pinpoints the conclusions and potential topics for future research.

3.2. Literature overview

The shelf space allocation problem has long been addressed by marketing professionals and researchers, with the first studies tracing back to the 1970s.

Marketing studies have proven the positive influence of shelf space on stimulating consumer demand and identified three main demand effects: space elasticity, location and cross-elasticity. Implicit in most experiments is the assumption of diminishing marginal returns (in the form of an S-shaped curve), which results in non-linear and complex profit functions. Previous literature has focused broadly on finding different ways of addressing the various effects associated with product-to-shelf allocation, and several models with different goals have been developed. As a result, there is no definitive shelf space allocation model(s) and no benchmarks are available (Lim et al. [2004]).

The basic Shelf Space Allocation Problem consists of maximizing profit by deciding how much space is required for each product, under shelf capacity, control (lower and upper bounds) and availability constraints. In this basic SSAP, the amount of shelf space is considered the only significant factor and the location is disregarded. In very early works, several authors proposed different formulations for this problem. Among them, Corstjens and Doyle [1981] and Zufryden [1986] proposed comprehensive models with multiplicative polynomial forces that were important landmarks used as a reference by many researchers. More recently, the problem has been extended to incorporate other issues, such as assortment decisions, wholesale prices and inventory control (Borin et al. [1994], Urban [1998], Hwang et al. [2005], Hariga et al. [2007], Murray et al. [2010], Hübner and Kuhn [2011]).

In a large-scale experimental study, Drèze et al. [1994] concluded that the location of a product is more important to stimulate sales than the number of facings allocated to it, as long as a minimum threshold is maintained to avoid stockouts. In accordance, Yang and Chen [1999] formulated a model that divides the shelf space into a distinct set of shelves, and defined the demand of an item according to the shelf where the item is displayed. However, the discrete nature of the model provides the same effect no matter the exact place where a product is located on a particular shelf. Russell and Urban [2010], Hansen et al. [2010] and Geismar et al. [2014] proposed (quasi) continuous horizontal effects. Despite this, none of the models were able to solve to optimality instances with more than 10 products.

Several efforts have been recently made by researchers to develop more tractable models. Yang and Chen [1999] proposed a simplified and yet practical alternative model in the form of a linear multi-knapsack problem that started a new stream of Shelf Space allocation (called LSSAP in Gajjar and Adil [2011]), and proved that even a simplified version of the problem is NP-hard. The authors state that in practice it is difficult to obtain an estimation for the sales volume elasticity and make the assumption that the profit of any product is linear with regard to a range of facings, for which an upper and lower bound should be controlled. Those bounds are to be determined by the management according to the policy of the store. Lim et al. [2004] further defended the linearity assumption stating that retailers prefer to operate on the linear portion of the S-shaped curve of marginal returns. This simplified version of the problem has been used by several authors to develop efficient heuristics and matheuristics for the problem (Yang [2001], Lim et al. [2004], Hansen et al. [2010], Gajjar and Adil [2011], Castelli and Vanneschi [2014]).

Another key practical feature which has been neglected in the current state-of-the-art is the existence of product family arrangements on the shelves. The relationship between products has long been addressed by some authors, by assigning additional (less) profit if two complementary (substitute) products are assigned to the same shelf. However, Russell and Urban [2010] and Geismar et al. [2014] are the only authors who allocate the space in such a manner that keeps product families together, and in uniform and rectangular shapes, to improve the aesthetics of the planograms.

3.3. Problem Description

There are two decision levels in shelf space allocation: a store (macro) level, deciding the space for product categories, and a product category (micro) level, which allocates individual products within each category. This paper describes and tackles the problem faced by a retailer when defining the micro-space. The objective is, for a given category, to determine the optimal allocation of products, as well as their shelf location. The problem is defined as follows.

Consider a specific category of a store, whose space has been previously determined. There are K shelves available (indexed by $k \in \mathcal{K}$), where the retailers wish to display N products (indexed by $i, j \in \mathcal{N}$). Without loss of generality, the shelves are numbered from bottom to top. The length and height of shelf k are respectively w_k and h_k and each facing (i.e. visible unit containing other units stacked behind for inventory purposes) of product i is a_i long and b_i high. One product might be placed in more than one shelf, as long as it is vertically aligned.

The profit per facing of product i is p_i. Similarly to Yang and Chen [1999] and Lim et al. [2004], we assume a linear profit function with the number of facings by considering that retailers operate on the linear portion of the S-shaped curve of marginal returns. In accordance, a lower and upper bound of l_i and u_i are defined for the products. The problem also considers of vertical location effects by using the effectiveness of each shelf k, γ_k, to generate revenue. This parameter is multiplied by the profit to obtain the revenue potential from displaying each unit of product i on each shelf k. Cross-elasticities and the horizontal effects are disregarded as cross-elasticities have parameters which are difficult to estimate and the horizontal effects are less important (according to Drèze et al. [1994]), and can be tackled in a downstream problem by rearranging the products and product families.

Under the given operating conditions, the decisions to be made for each product are: the number of facings to be displayed on each shelf and its horizontal location within the shelf.

Up to this point we have described the standard shelf space allocation problem as commonly tackled in the literature (except for the location decisions that are usually disregarded). However, there are still key practical features associated with product grouping that need to be addressed. These are introduced next.

3.3.1 Product Grouping

Retailers specify merchandising rules that impact how products are placed on shelves. Those merchandising rules identify a set of product families whose products should be grouped together. Moreover, if a product family spans more than one shelf, the products should maintain a uniform and rectangular shape, with a small deviation v allowed between shelves. For that purpose, consider M product families (indexed by $u \in \mathcal{M}$), each one containing one or more products, defined by the set $\mathcal{N}_u$. Due to its uniform shape, product families are also called *blocks*. Both terms are used interchangeably in the remainder of the paper.

Merchandising rules often comprehend multi-level hierarchical structures of product families, trying to further capture the consumer buying behavior. As an example, retailers might want to organize yogurts firstly by type, differentiating classic from drinkable yogurts, secondly by brand, and thirdly by flavor. Two distinct product families may also have different downstream families or a different number of levels (for instance, drinkable yogurts further divided by package size and classic yogurts by brand and flavor). Figure 3.1 presents an example of a planogram with two levels of product families. This multi-level structure can be better captured by a network of product families and represented using a directed acyclic graph, as seen in figure 3.2. The graph starts with a dummy node connecting all the product families from the first level. Each product family is then connected to its downstream families, which form the second level of the graph, and so forth until the product level is reached. Consider the sets $\mathcal{M}_u$ that define the downstream product families belonging to each upstream (or parent) family u.

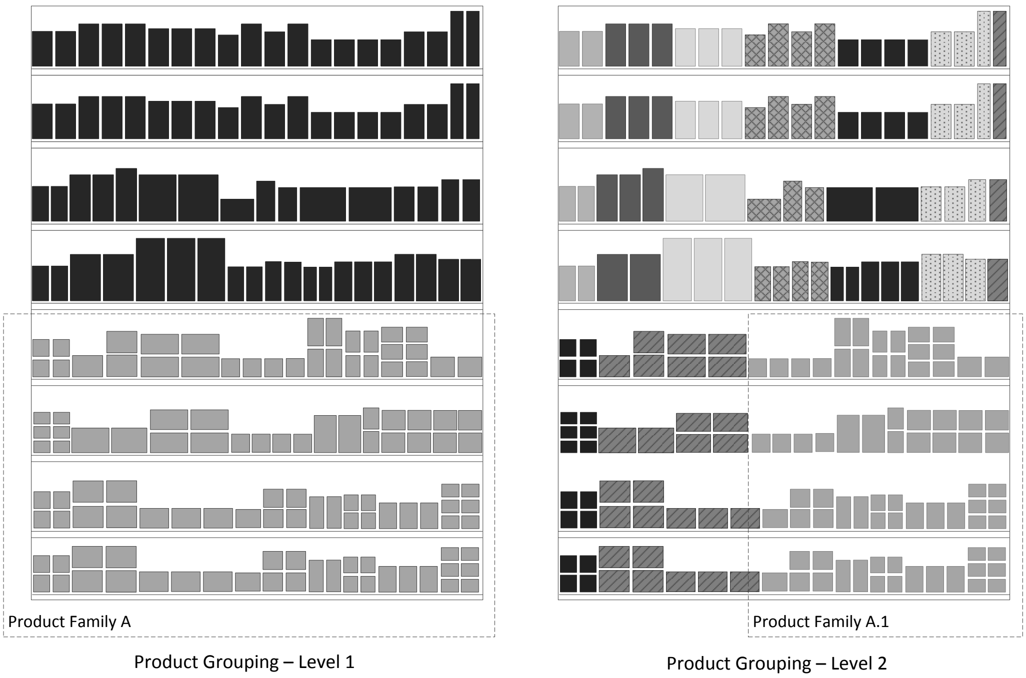

Figure 3.1 – Example of a Planogram with two levels of product grouping

Retailers may also specify whether product families should have a horizontal or vertical shape when placed on the shelves. A horizontal family is predominantly located on one shelf (or more, as far as it occupies the whole length of its parent), whereas a vertical family occupies as much as possible all the shelves from the parent family. In Figure 3.1, the

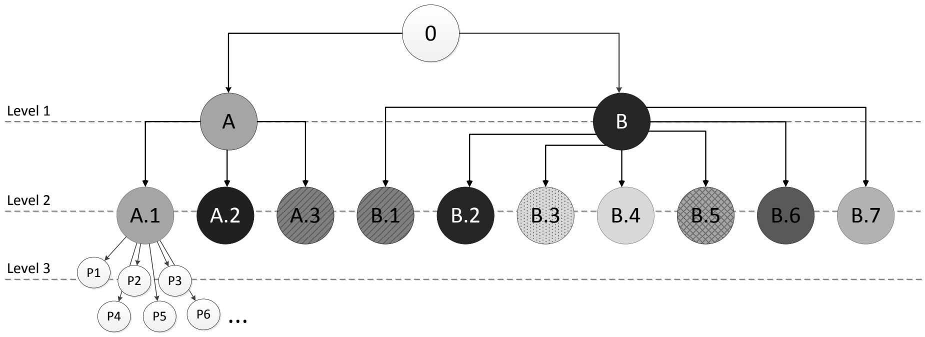

Figure 3.2 – Product grouping diagram from Figure 3.1

families from the first level are placed horizontally, whereas the families from the second level assume a vertical orientation. The same direction is defined for all families from the same parent but, the direction among different parents may differ within each level. For this purpose, consider the sets $\mathcal{S}^H$ and $\mathcal{S}^V$, containing the families that should have their downstream blocks with a horizontal and vertical shape, respectively.

3.4. Model Formulation

This section presents the formulation proposed for the Shelf Space Allocation Problem with location decisions, as described in the previous chapter. The starting point was the work by Russell and Urban [2010]. To the best of our knowledge, this is the first paper that presents a model that treats product location as a continuous variable, aiming for the complete description of the planogram. They additionally considered practical requirements related to product families, once again aligned with the motivation for this paper. To improve the applicability of the formulation, we follow Yang and Chen [1999] and other subsequent researchers, and consider a linear profit function. For clarification purposes, the section is divided into three subsections. Firstly we define the objective function and allocation constraints. At this point, the formulation corresponds to the basic shelf space allocation problem. The next two subsections present the necessary constraints to define the exact location of the products, divided into sequencing constraints and family grouping constraints.

The necessary decision variables are presented along the text; however, two sets of decision variables are sufficient to define the solution of the problem:

W_{ik} the integer number of facings of product $i \in \mathcal{N}$ on shelf $k \in \mathcal{K}$,

X_i^s the continuous horizontal location of product $i \in \mathcal{N}$, measured from the lower-left corner of the planogram to the lower-left corner of the first facing of the product.

3.4.1 Objective Function and Allocation Constraints

The objective function and the allocation constraints are similar to Yang and Chen [1999], and define the number of facings allocated to each product on each shelf:

$$\text{Maximize} \quad Z = \sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{K}} p_i \cdot \gamma_k \cdot W_{ik} \tag{3.1}$$

$$\text{subject to:} \quad \sum_{k \in \mathcal{K}} W_{ik} \leq u_i, \quad \forall i \in \mathcal{N} \tag{3.2}$$

$$\sum_{k \in \mathcal{K}} W_{ik} \geq l_i, \quad \forall i \in \mathcal{N} \tag{3.3}$$

$$\sum_{i \in \mathcal{N}} a_i \cdot W_{ik} \leq w_k, \quad \forall k \in \mathcal{K} \tag{3.4}$$

$$W_{ik} = 0, \quad \forall i \in N, k \in \mathcal{K} : b_i \leq h_k \tag{3.5}$$

$$W_{ik} \in \mathbb{N}_0, \quad \forall i \in \mathcal{N}, k \in \mathcal{K} \tag{3.6}$$

As previously mentioned, the objective function (3.1) is a linear profit function where the profit associated with each product is linear with regard to a shelf and a range of facings, controlled by an upper and lower bound imposed by constraints (3.2) and (3.3). The product $p_i\gamma_k$ is analogous to the parameter p_{ik} used in Yang and Chen [1999], but it additionally specifies how the value was obtained (Geismar et al. [2014]). Constraints (3.4) ensure that shelf capacities are not exceeded and (3.5) prevent products from being placed on shelves where they do not fit. Lastly, constraints (3.6) guarantee that the decision variables W are non-negative integers.

3.4.2 Sequencing Constraints

Considering product families requires the definition of the exact location of the products on the shelves and thus products need to be sequenced. The traditional sequencing T_{ij} variables defining whether product i precedes (=1) or proceeds (=0) product j would be responsible for the exponential increase of the model's size. To overcome this fact, we explore the existence of a multi-level hierarchy of product families. For each shelf, we start by defining a sequence with the blocks from the first level. Afterwards, for each first level block, we define a sequence with the corresponding blocks from the following level and so forth, until the products are reached. At the end of the levels, by placing all the product sequences in the correct order, the overall sequence is then obtained. This approach also guarantees that the products belonging to the same block are consecutively placed on each shelf, which is another requirement from the problem.

To capture this idea, consider a network where the nodes are associated with the products and the arcs represent the precedence in the allocation of the products. The network has two additional nodes: one source node, which is connected to the first product, and a sink node, connected to the last. In this approach, instead of using one product network on each shelf for the sequencing decisions, we consider a set of networks, one for each parent block, whose nodes are the corresponding blocks from the downstream level. This is illustrated in Figure 3.3. To model these networks, consider that single products might

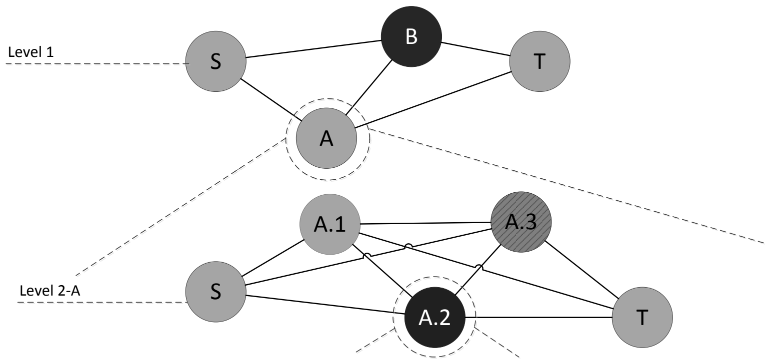

Figure 3.3 – Network for Level 1 and 2-A, based on the diagram from Figure 3.2

also be seen as blocks, containing one product each. Each block $u \in \mathcal{M}$ is then associated with the set $\mathcal{V}_u$ containing the blocks from the immediate downstream level, either product families ($m,n \in \mathcal{M}_u$) or products ($m,n \in \mathcal{N}$). In Figure 3.2, while the set from block A (V_A) contains the blocks A.1, A.2 and A.3 ($\in \mathcal{M}$), the set from block A.1 ($V_{A.1}$) contains the products P1-P6 ($\in \mathcal{N}$). Additionally, consider a dummy node 0 associated with the source and the sink node of the networks and a new set $\mathcal{V} = \mathcal{M} \cup \mathcal{N}$ that aggregates all the blocks. Accordingly, the following additional decision variables are introduced into the model:

T_{mnk} = 1 if block m is displayed immediately after block n on shelf $k \in \mathcal{K}$, $u \in \mathcal{M}, m,n \in \mathcal{V}_u \cup \{0\}$,

Y_{mk} = 1 if block $m \in \mathcal{V}$ is located on shelf $k \in \mathcal{K}$.

For each network associated with the block $u \in \mathcal{M}$, the following constraints determine the sequence of its downstream blocks $m,n \in \mathcal{V}_u$ on each shelf k:

$$\sum_{m \in \mathcal{V}_u} T_{0mk} = Y_{uk}, \qquad \forall\, u \in \mathcal{M}, k \in \mathcal{K} \tag{3.7}$$

$$\sum_{m \in \mathcal{V}_u} T_{m0k} = Y_{uk}, \qquad \forall\, u \in \mathcal{M}, k \in \mathcal{K} \tag{3.8}$$

$$\sum_{n \in \mathcal{V}_u \cup \{0\}} T_{nmk} = Y_{mk}, \qquad \forall\, u \in \mathcal{M}, m \in \mathcal{V}_u, k \in \mathcal{K} \tag{3.9}$$

$$\sum_{n \in \mathcal{V}_u \cup \{0\}} T_{mnk} = Y_{mk}, \qquad \forall\, u \in \mathcal{M}, m \in \mathcal{V}_u, k \in \mathcal{K} \tag{3.10}$$

$$Y_{mk} \le Y_{uk}, \qquad \forall\, u \in \mathcal{M}, m \in \mathcal{V}_u, k \in \mathcal{K} \tag{3.11}$$

$$W_{ik} \ge Y_{ik}, \qquad \forall\, i \in \mathcal{N}, k \in \mathcal{K} \tag{3.12}$$

$$Y_{mk} \in \{0,1\}, \qquad \forall\, m \in \mathcal{V}, k \in \mathcal{K} \tag{3.13}$$

$$T_{mnk} \in \{0,1\}, \qquad \forall\, u \in \mathcal{M}, m,n \in \mathcal{V}_u \cup \{0\}, k \in \mathcal{K} \tag{3.14}$$

For each block u, constraints (3.7)-(3.10) ensure that if a block is present on a shelf, a tour exists among its downstream blocks, starting and ending in the source and sink node.

Constraints (3.7) and (3.8) force the tour to pass once through the source and sink node, while constraints (3.9) and (3.10) ensure that, if a downstream block is on the shelf, it should be immediately preceded and proceeded by exactly one node, either a block or the source/sink. Constraints (3.11) state that one block is only present on a shelf if the parent block is also present and finally constraints (3.12) make the connection with the allocation part of the problem. Constraints (3.13) and (3.14) guarantee that the variables are binary.

Inspired by the good results in the Asymmetric Traveling Salesman Problem (Öncan et al. [2009]), we used single commodity flow constraints to guarantee that sequences are connected. The disconnected subtours are eliminated with additional decision variables representing a commodity flow through each network, which has to satisfy conservation constraints. The commodity is associated with the length of the corresponding upstream block on the shelf. Every time the commodity goes from one downstream block to another, the length assigned to the first block is added to the commodity flow. At the end, when the commodity enters the sink node, its value should be equal to the total length occupied by the upstream block. In the first block, the total length is equal to the width of the shelf, meaning that the entire space of the planogram is occupied. For this purpose, two sets of decision variables are added to the model:

F_{mnk} the continuous flow from block m to block n on shelf $k \in \mathcal{K}$, $u \in \mathcal{M}$, $m,n \in \mathcal{V}_u \cup \{0\}$,

L_{ik} shelf length assigned to product $i \in \mathcal{N}$ on shelf $k \in \mathcal{K}$.

The single commodity flow constraints that enforce the existence of a path from the source to the sink node of each network associated with block $u \in \mathcal{M}$ are the following:

$$\sum_{m \in \mathcal{V}_u} F_{0mk} = 0, \qquad \forall u \in \mathcal{M}, k \in \mathcal{K} \tag{3.15}$$

$$\sum_{m \in \mathcal{V}_u} F_{m0k} = \sum_{i \in \mathcal{N}_u} L_{ik}, \qquad \forall u \in \mathcal{M}, k \in \mathcal{K} \tag{3.16}$$

$$\sum_{\substack{n \in \mathcal{V}_u \cup \{0\}: \\ m \neq n}} F_{nmk} + \sum_{i \in \mathcal{N}_m} L_{ik} = \sum_{\substack{n \in \mathcal{V}_u \cup \{0\}: \\ m \neq n}} F_{mnk}, \qquad \forall u \in \mathcal{M}, m \in \mathcal{V}_u, k \in \mathcal{K} \tag{3.17}$$

$$F_{mnk} \leq w_k \cdot T_{mnk}, \qquad \forall u \in \mathcal{M}, m,n \in \mathcal{V}_u \cup \{0\} : m \neq n, k \in \mathcal{K} \tag{3.18}$$

$$\sum_{i \in \mathcal{N}} L_{ik} = w_k, \qquad \forall k \in \mathcal{K} \tag{3.19}$$

$$a_i \cdot W_{ik} \leq L_{ik}, \qquad \forall i \in \mathcal{N}, k \in \mathcal{K} \tag{3.20}$$

$$L_{ik} \geq 0, \qquad \forall i \in \mathcal{N}, k \in \mathcal{K} \tag{3.21}$$

$$F_{mnk} \geq 0, \qquad \forall u \in \mathcal{M}, m,n \in \mathcal{V}_u \cup \{0\}, k \in \mathcal{K} \tag{3.22}$$

Constraints (3.15) force the commodity flow to leave the source of each network with no length, and constraints (3.16) ensure that in the end the total flow amount must be equal to the total length of the upstream block on the shelf (equal to zero if the block is not on the shelf). The flow balance constraints are expressed by (3.17), which ensure that the flow that enters each node plus its block's length is equal to the flow that leaves the node. Constraints (3.18) guarantee that the flow only traverses active arcs, and in (3.19) the total width of each shelf should be occupied by the products. Constraints (3.20) ensure that enough space is reserved on each shelf for the facings of the products. Finally, the nonnegativity of the product's length and commodity flows are ensured by (3.21)) and (3.22)).

3.4.3 Product Grouping Constraints

For aesthetic reasons, both the families and the products should have rectangular shapes on the shelves, which means that each block should be placed on contiguous shelves and aligned, with a small deviation v allowed between shelves. This leads to a new set of decision variables and an extension of variable X_i^s, from product to block range:

- X_m^s the horizontal location of the block $m \in \mathcal{V}$ (left coordinate),
- X_m^e the horizontal location of the block $m \in \mathcal{V}$ (right coordinate),
- FL_{mk} = 1 if $k \in \mathcal{K}$ is the first shelf of block $m \in \mathcal{V}$,
- LL_{mk} = 1 if $k \in \mathcal{K}$ is the last shelf of block $m \in \mathcal{V}$.

The sequencing constraints (3.9) - (3.14) already impose the connectivity of the blocks within each shelf. The following constraints guarantee the rectangular shape and also define the horizontal location of the blocks:

$$X_m^s \geq X_u^s + \sum_{\substack{n \in \mathcal{V}_u \cup \{0\}: \\ n \neq m}} F_{nmk}, \quad \forall u \in \mathcal{M}, m \in \mathcal{V}_u, k \in \mathcal{K} \tag{3.23}$$

$$X_m^s \leq X_u^s + \sum_{\substack{n \in \mathcal{V}_u \cup \{0\}: \\ n \neq m}} F_{nmk} + W \cdot (1 - Y_{mk}), \quad \forall u \in \mathcal{M}, m \in \mathcal{V}_u, k \in \mathcal{K} \tag{3.24}$$

$$X_m^e - X_m^s \geq \sum_{i \in \mathcal{N}_m} L_{ik}, \quad \forall m \in \mathcal{V}, k \in \mathcal{K} \tag{3.25}$$

$$X_m^e - X_m^s - v \leq \sum_{i \in \mathcal{N}_m} L_{ik} + W \cdot (1 - Y_{mk}), \quad \forall m \in \mathcal{V}, k \in \mathcal{K} \tag{3.26}$$

$$\sum_{k \in \mathcal{K}} FL_{mk} = 1, \quad \forall m \in \mathcal{V} \tag{3.27}$$

$$\sum_{k \in \mathcal{K}} LL_{mk} = 1, \quad \forall m \in \mathcal{V} \tag{3.28}$$

$$FL_{m,k+1} + Y_{mk} = Y_{m,k+1} + LL_{mk}, \quad \forall m \in \mathcal{V}, k \in \mathcal{K} : k \neq K \tag{3.29}$$

$$FL_{m0} = Y_{m0}, \quad \forall m \in \mathcal{V} \tag{3.30}$$

$$LL_{mK} = Y_{mK}, \quad \forall m \in \mathcal{V} \tag{3.31}$$

$$X_m^s, X_m^e \geq 0, \quad \forall m \in \mathcal{V} \tag{3.32}$$

$$FL_{mk}, LL_{mk} \in \{0, 1\}, \quad \forall m \in \mathcal{V}, k \in \mathcal{K} \tag{3.33}$$

Constraint sets (3.23) and (3.24) establish the horizontal location of each block (left coordinate) according to the location of its parent block and the flow coming from the preceding block (that equals the length of the blocks since the beginning of the parent block). Constraints (3.25) and (3.26) define the right coordinate for each block and keep its location within v units from one shelf to the others. Constraint sets (3.27) and (3.28) establish the top and bottom shelf where each block is located, while constraints (3.29)-(3.31) keep the block on adjacent shelves.

Finally, if the blocks have predefined orientations, either horizontal or vertical, the following constraints guarantee the desired shapes:

$$Y_{mk} = Y_{uk}, \quad \forall u \in \mathcal{S}^V, m \in \mathcal{M}_u, k \tag{3.34}$$

$$\sum_{n \in \mathcal{V}_u : n \neq m} Y_{nk} \leq M \cdot (2 - Y_{mk} - Y_{m,k+1}), \quad \forall u \in \mathcal{S}^H, m \in \mathcal{M}_u, k \in \mathcal{K} : k \neq K \tag{3.35}$$

Constraints (3.34) ensure that the vertical blocks are present on the same shelves as their parent block, while constraint set (3.35) only allows a horizontal block to occupy more than one shelf if the first one is fully occupied by the block. Note that these two last requirements can be defined as soft constraints by introducing two new sets of variables that can relax the constraints, although there is a penalty on the objective function value.

For the sake of simplicity, this model will hereafter be referred to as *BAP* – Block Allocation Problem.

3.5. Solution Approach

When the instance size increases, the shelf space allocation problem with location decisions becomes intractable, which limits the straightforward use of standard mathematical programming approaches, in particular when dealing with real world instances. This fact motivated the development of an approximate method. We chose a mathematical programming based approach because the high number of constraints associated with family grouping would make it difficult to develop a constructive heuristic capable of generating high quality feasible solutions within reasonable time limits.

Our solution approach decomposes the original problem into smaller sub-problems that can be more easily solved using exact methods. Following the idea that most of the computational burden comes from the integer variables, we used an approach based on the relax-and-fix (R&F) framework (Pochet and Wolsey [2006]). This framework decomposes the integer variables of large-scale MIP problems into subsets, and then sequentially solves relaxed MIP sub-problems containing each subset. As the number of integer variables in each sub-problem is significantly smaller than the original problem, the solution times to solve each one to optimality is lower.

Consider the set $\mathcal{G}$ composed of the integer variables Y, T associated with the blocks, and W with the products. At each iteration l, the integer variables are grouped into three subsets: $\mathcal{G}_l^F$ - variables whose values have been fixed in previous iterations to the values Y', T' and W', $\mathcal{G}_l^I$ - variables required to be integer in the current iteration, and finally $\mathcal{G}_l^R$ - the relaxed variables. The sub-problem to be solved, labeled $subBAP^l$, corresponds to the original SSA model where equations (3.6), (3.13) and (3.14) are replaced by:

$$Y = Y', T = T', W = W' \qquad \forall (Y, T, W) \in \mathcal{G}_l^F \tag{3.36}$$

$$(Y, T) \in \{0, 1\}, W \in \mathbb{N}_0, \qquad \forall (Y, T, W) \in \mathcal{G}_l^I \tag{3.37}$$

$$(Y,T,W) \geq 0, \qquad \forall (Y,T,W) \in \mathcal{G}_l^R \tag{3.38}$$

As the matheuristic progresses, the three subsets are updated as follows: part or all the integer variables are fixed (moved from $\mathcal{G}^I$ to $\mathcal{G}^F$) and part or all the remaining relaxed variables are turned into integer variables (from $\mathcal{G}^R$ to $\mathcal{G}^I$). The way subsets evolve defines both the quality of the solution and the computational burden of the *R&F* heuristic. The heuristic finishes when a feasible integer solution is found for the entire problem or when a sub-problem is infeasible.

In the SSAP, the family blocks and their hierarchical structure define a natural partition of the problem. The relation between the blocks within each upper block is indeed the most computational demanding feature of the problem due to the T variables (for sequencing purposes). In accordance, a block partition is used to define the subsets. The heuristic starts by solving sub-problems corresponding to the blocks from the first level and progressively moves down until it reaches the blocks in the last level. For that reason, this matheuristic will hereafter be called *H-BAP* – hierarchical resolution of *BAP*. For clarification purposes, figure 3.4 depicts three successive iterations of the heuristic (the first iteration corresponds to the initial one).

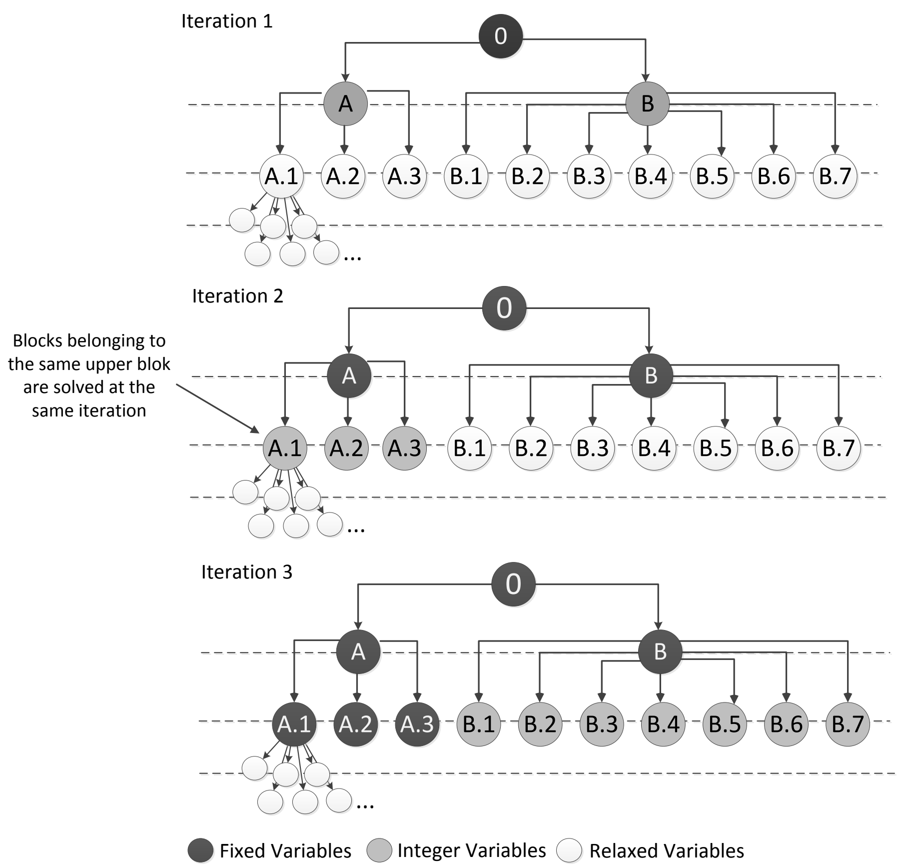

Figure 3.4 – Successive iterations of the MIP based heuristic

The blocks in dark gray are those in which the value of the integer variables are fixed to the solution obtained in previous iterations (equations 3.36). The blocks in light gray are restricted to assume integer values (equations 3.37), and finally, the blocks in white are relaxed to fractional values (equations 3.38). From one iteration to the next, the subsets are updated so that at each time the variables in the integer set correspond to all blocks from the same upstream block. This evolution scheme was chosen to take into consideration the shape's orientation, as it impacts all the blocks from the same parent.

The pseudocode for the heuristic is presented below (Algorithm 1), where the function *getNextIntegerSet* returns the integer variables for each iteration. The algorithm is a straightforward implementation of what has been previously described. Note that line 11 guarantees that the subset $\mathcal{G}^F$ (fixed variables) is updated correctly if the integer variables of two successive iterations overlap. The code is sufficiently generic to allow other evolution schemes for the subsets (function *getNextIntegerSet*).

Algorithm 1: Pseudocode for the MIP based heuristic (*H-BAP*)

1 **begin**
2     $l \leftarrow 1$
3     $\mathcal{G}^I_l$:= Solve *getNextIntegerSet*
4     $\mathcal{G}^F_l := \emptyset$
5     $\mathcal{G}^R_l := \mathcal{G} \backslash \{\mathcal{G}^I_l\}$
6   **while** $\mathcal{G}^R_l \notin \emptyset$ **do**
7   Status := Solve $subBAP_l$
8   **if** *Status = Feasible* **then**
9   Y' := Y, T' := T, W' := W
10             $\mathcal{G}^I_{l+1}$:= Solve *getNextIntegerSet*
11             $\mathcal{G}^F_{l+1} := \mathcal{G}^F_l \cup (\mathcal{G}^I_l \backslash \{\overline{\mathcal{G}^I_l \cap \mathcal{G}^I_{l+1}}\})$
12             $\mathcal{G}^R_{l+1} := \mathcal{G}^R_l \backslash \{\mathcal{G}^I_{l+1}\}$
13   **else**
14   Return Infeasible
15   **end**
16         $l \leftarrow l+1$
17   **end**
18   Status :=Solve $subBAP_l$
19   Return Status
20 **end**

3.5.1 Improving Feasibility

Finding a feasible solution while using a *R&F* heuristic is not always guaranteed. Even though a top-down approach for the SSAP explores the problem structure, it risks creating top level assignments that constitute infeasible product allocations. As it goes further down, the heuristic might not reserve enough space to guarantee the minimum facings for all the products, especially when forcing integrality on the W variables.

Therefore, to minimize the chances of infeasibility, we have created a new set of constraints that take into consideration product-related features at an earlier stage. For that purpose, consider a new parameter w_m^{max} with the width of the largest product from each block m ($w_m^{max} = max\{a_i | i \in \mathcal{N}_m\}$). As the family (and product) blocks have to form rectangular shapes, and all the products have to be allocated, the minimum width of a block is w_m^{max}. In accordance, we introduce the new set of constraints (3.39):

$$\sum_{i \in \mathcal{N}_m} L_{ik} \geq w_m^{max} \qquad \forall m \in \mathcal{M}, k \in \mathcal{K} \tag{3.39}$$

Additionally, the heuristic was also changed to include a backtracking scheme. Whenever a sub-problem is infeasible, the heuristic shifts backward instead of forward, and solves a larger sub-problem by unfixing previous parts of the solution while maintaining the current integer variables. The heuristic starts by unfixing all the integer variables from the previous level, and while the problem remains infeasible it moves further backwards until reaching a maximum number of backward moves, or the set $\mathcal{G}^F$ is empty. Notice that the backtracking scheme unfixes the variables by level instead of blocks. This gives the necessary freedom to change the blocks arrangement. This backtracking only requires changes in the way the sets are updated and the above formulation does not suffer any change.

Algorithm 2 presents the backtracking scheme that should replace line 14 in Algorithm 1 for handling infeasible sub-problems. The new subset $\mathcal{G}_l^{\overline{F}}$ contains the variables that should be unfixed in the next iteration. This subset is updated using the function *getUnFixSet*. Once again, the code allows other backtracking schemes (by changing the function *getUnFixSet*). We will hereafter call to this extension *Improving Feasiblity* (*H-BAP-IF*).

Algorithm 2: Pseudocode for improving feasibility – replaces line 14 in Algorithm 1

if $\mathcal{G}_l^F \notin \emptyset$ **then**
  $\mathcal{G}_{l+1}^{\overline{F}}$:= Solve *getUnFixSet*
  $\mathcal{G}_{l+1}^{I} := \mathcal{G}_l^I \cup \mathcal{G}_{l+1}^{\overline{F}}$
  $\mathcal{G}_{l+1}^{F} := \mathcal{G}_l^F \backslash \{\mathcal{G}_{l+1}^{\overline{F}}\}$
else
  Return Infeasible
end

3.5.2 Improving Efficiency

As previously mentioned, most of the problem's complexity comes from the family groups which impose hard constraints on the positioning of the products. One possible approach to improve the problem's efficiency would be to reduce the level of detail by not considering the products until a later stage (by removing the single product blocks from the model's formulation, it is possible to substantially reduce its size). The products could be handled afterwards in a downstream problem by using a knapsack model (multiple examples are provided in the literature) or a heuristic method. Such approach is only possible if all the family groups are handled in this upstream model.

However, there are two drawbacks when the products are removed from the formulation: firstly, the final space assigned to each block may not be enough to guarantee the minimum number of facings for the corresponding products, and secondly, the blocks may be assigned to shelves where the products do not fit because of their height.

To avoid potential downstream infeasibilities, we chose to disregard the products' sequencing and exact positioning, and yet consider the products' linear shelf space allocation to shelves. This change requires the use of the length (L) instead of the number of facings (W) both in the objective function and to ensure the products' lower and upper bounds. Constraints (3.2)-(3.6), (3.12)) and (3.20) are replaced by:

$$\text{Maximize} \quad \sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{K}} Z = p_i \cdot \gamma_k \cdot L_{ik} \tag{3.40}$$

$$\text{subject to:} \quad \sum_{k \in \mathcal{K}} L_{ik} \leq u_i \cdot w_i, \quad \forall i \in \mathcal{N} \tag{3.41}$$

$$\sum_{k \in \mathcal{K}} L_{ik} \geq l_i \cdot w_i, \qquad \forall i \in \mathcal{N} \tag{3.42}$$

$$L_{ik} = 0, \qquad \forall i \in N, k \in \mathcal{K} : b_i \leq h_k \tag{3.43}$$

$$L_{ik} \geq Y_{ik} \cdot w_i, \qquad \forall i \in \mathcal{N}, k \in \mathcal{K} \tag{3.44}$$

Additionally, the sequencing and family block variables and constraints do not apply to single product blocks, except Y variables. For that purpose, consider a new set M^U containing all the blocks from M except the lowest ones ($M^U \in \{0, A, B\}$ in the diagram from Figure 3.2). In constraints (3.7)-(3.10),(3.14),(3.15)-(3.18),(3.22),(3.23) and (3.24) the set M is replaced by M^U, and in constraints (3.25)-(3.33) the set V is replaced by M. We will hereafter call to this extension *Improving efficiency* (*H-BAP-IE*).

3.6. Experimental Analysis and Computational Results

This section presents the results of the computational study to validate and assess the performance of both the formulation and the solution approach. For this purpose, we provide a set of benchmark instances for the shelf space allocation problem that capture the different features of real world problems as described in Section 3.3. All the computational experiments were conducted on Intel @2.40GHz processing units limited to 4.0Gb of Random Access Memory using the Linux operating system. The IBM ILOG CPLEX 12.4 was used both as the mixed integer and linear programming solver.

3.6.1 Problem Instances

A total of 54 problem instances were obtained from a European Grocery Retailer. The company defines very complex block diagrams that try to reproduce the way customers search for the products while shopping. Blocks can be defined by different criteria and the most common ones are brand, type, package size and flavor. To ensure that different realities are covered, the instances belong to 22 different categories, ranging from low to high sales products, light to heavy block diagrams, vertically to horizontally shaped blocks, among other features. Table 3.1 presents the key information about the instances:

number of products (N), number of family blocks (M), number of shelves (K), number of hierarchical levels (L) and number of vertical (MV) and horizontal blocks (MH). The instances are grouped by category and organized by increasing number of products (for example, instances *AZ*_1, *AZ*_2 and *AZ*_3 belong to category *AZ*, whose average number of products is higher than category *FL*). As it is possible to see, the instances vary in size and are significantly bigger than the ones reported in the literature, with up to 240 products, 9 shelves and 5 hierarchy levels.

One critical information for planogram design is the lower and upper bounds on the number of product facings. The lower bounds were set to one in all instances and the upper bounds were determined by setting a days supply baseline for the planogram (i.e. number of days that the shelf inventory should last), based on the revenue potential and the space available. Other information was taken into consideration, such as the maximum number of facings determined by the management, product shelf life, supplier contracts, among others. The exact calculus behind the upper bounds is beyond the scope of this paper. Another key parameter is shelf effectiveness. We used beta functions to model the way the management considers the shelves' attractiveness to the consumers, always privileging eye-level shelves.

The instances are available online in Bianchi-Aguiar et al. [2014]. It was not possible to test the instances found in the literature either because they were not available, or because they did not consider family groups. As aforementioned, family groups are of major importance in practice and are a key feature of our problem definition and the basis of our formulation.

Table 3.1 – Problem Instances

Name	N	M	K	L	MV	MH	Name	N	M	K	L	MV	MH	Name	N	M	K	L	MV	MH
FL_1	16	16	6	4	5	10	**CR_2**	32	13	7	3	0	12	**SM_3**	49	10	6	3	7	2
AZ_3	10	11	5	3	8	2	**CR_1**	82	44	5	4	4	39	**SM_1**	171	47	6	4	36	10
AZ_2	25	12	5	3	9	2	**PT_1**	38	22	6	5	0	21	**CA_2**	77	34	6	5	2	31
AZ_1	32	27	5	3	0	26	**AI_1**	37	22	7	4	5	16	**AG_2**	19	19	7	5	0	18
LS_1	26	5	8	2	0	4	**AI_3**	41	27	9	5	4	22	**AG_1**	39	6	7	2	5	0
VG_3	7	4	7	2	0	3	**AI_4**	47	24	9	4	7	16	**AG_4**	85	32	5	4	10	21
VG_2	19	18	6	4	2	15	**AI_2**	47	17	7	3	3	13	**AG_3**	113	24	8	4	17	6
VG_4	28	15	6	3	2	12	**VN_1**	45	32	6	3	0	31	**AB_1**	28	14	8	3	0	13
VG_5	42	17	6	4	10	6	**SP_1**	49	15	7	4	6	8	**AB_2**	160	42	8	4	0	41
VG_1	60	29	8	4	0	28	**OM_3**	22	16	5	4	2	13	**VV_2**	84	3	5	2	2	0
CP_2	8	11	6	3	0	10	**OM_1**	54	41	5	4	6	34	**VV_1**	121	9	5	2	8	0
CP_1	24	18	5	3	5	12	**OM_2**	78	17	6	3	0	16	**LO_2**	15	4	5	2	0	3
CP_4	47	29	8	3	28	0	**LC_1**	46	21	6	4	4	16	**LO_1**	206	128	7	3	122	5
CP_3	51	27	6	5	21	5	**LC_2**	59	25	6	3	4	20	**BH_1**	108	25	7	3	0	24
CR_6	16	8	5	2	0	7	**SM_6**	19	7	6	3	4	2	**BH_2**	131	26	5	4	17	8
CR_5	19	10	5	3	0	9	**SM_2**	31	8	6	3	5	2	**CH_1**	190	99	6	4	15	83
CR_4	22	14	5	3	0	13	**SM_4**	34	11	6	3	8	2	**BC_1**	239	121	6	5	10	110
CR_3	25	11	5	3	0	10	**SM_5**	38	10	6	3	7	2	**DE_1**	240	45	8	4	40	4

N - Number of Products, M - Number of Family Blocks, K - Number of Shelves, L - Number of Hierarchical Levels, MV - Number of Vertical Blocks, MH - Number of Horizontal Blocks

3.6.2 Model Validation and Performance Evaluation

The model analysis will firstly focus on the validation of the practical constraints by analyzing one of the instances in three different scenarios: (1) the first scenario does not consider family blocks. For that purpose, $\mathcal{M}$ was replaced in all instances by a single block containing all the products: $\mathcal{M} = \{1\}$, $\mathcal{N}_1 = \mathcal{N}$; (2) the second scenario considers the family blocks without the shapes' direction, i.e. $S^H = \{\}$ and $S^V = \{\}$; (3) and the third scenario considers the blocks to their full extent. Secondly, the performance of the formulation will be assessed. Although there is no comparable formulation in the literature, we have adapted Russell and Urban [2010] formulation (*RU*), which is, to the best of our knowledge, the only formulation present in the literature that also considers continuous location decisions, and have compared the results achieved by the two formulations. Appendix 3.A presents all the details about this adapted formulation.

Figure 3.5 presents the planogram obtained for one of the smallest instances, *AZ_3*, in each of the three scenarios. The results were obtained using the IBM ILOG CPLEX Optimization Studio to run the monolithic model *BAP* until optimality. *AZ_3* contains 10 products organized into two levels of product families, the first one being horizontal and the second one vertical. Each planogram is firstly highlighted by the families of level 1, secondly by the families of level 2, and lastly by the products. By analyzing *Scenario 1*, it is possible to see that the products with the highest profits were on the highest shelves and reaching the maximum number of facings, while the lowest profit products were on the lowest shelves and with the minimum number of facings. This allocation is in accordance with the objective function and resulted in the highest objective function value among the three scenarios (71751.4, 71749.4 and 70535.7 respectively). However, the resulting planogram does not follow any implementation logic, which may make the search for the products in the stores difficult, specially when the size of the planogram increases. In *Scenario 2*, which already organizes the products by product families, the blocks with the highest average profit are also pushed to the top, although they might include products with low profit. The objective function is lower than the one of the first scenario, but the families bring a better understanding of the planogram. Finally, by including the shapes' direction in *Scenario 3*, we obtain the lowest objective function value but with a clear identification of the allocation rules. Moreover, this new feature brings more realism, providing shelf space layouts similar to what is seen in practice. This analysis demonstrates that, even though the objective function decreases with the new features, benefits are obtained by organizing products into families. These benefits are hard to grasp in the model as they are linked to the customers' response to the complexity of the planograms and to the way costumers search for the products while shopping. However, merchandising rules have been and are carefully studied by marketeers and the gains obtained by taking these rules into consideration are supposed to overcompensate the decreasing values in the objective function.

Table 3.2 summarizes the results obtained for all the instances in the three scenarios and the two formulations (*BAP* and *RU*). The detailed information is presented in Table 3.4 from Appendix 3.B. For each instance, we provide information about the linear relaxation (Z^{lr}), best integer solution found (Z), total execution time in seconds (T) and the deviation

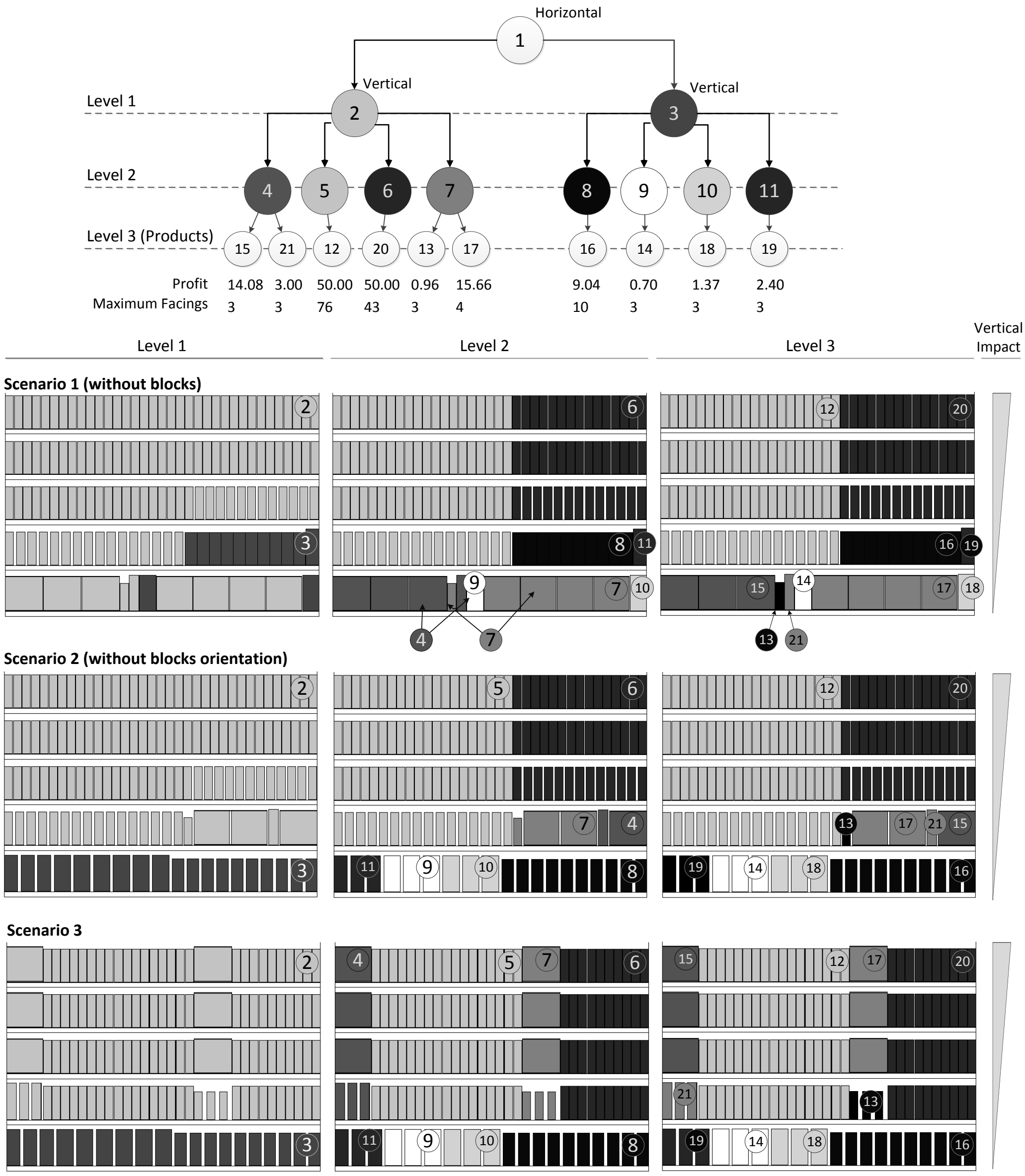

Figure 3.5 – Solution Analysis of instance *AZ_3* in the three scenarios

of the best integer solution found from the best upper bound available at the stopping criteria (*GAP*). The models were implemented using the IBM ILOG CPLEX Optimization Studio and a time limit of 3600 seconds was used as stopping criteria. As the vertical shapes are common and can easily lead to infeasible solutions if the capacity is tight for the products' lower bound, the vertical shape constraints were considered soft constraints (in both formulations) by introducing a new decision variable R_{mk} that equals 1 if the constraint associated with block m and shelf k is violated. In that case, the following penalty P is subtracted from the objective function:

$$P = \sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{K}} p_i \cdot \gamma_k \cdot u_i \tag{3.45}$$

Table 3.2 – Summary of results for the two formulations *BAP* and *RU* in the three scenarios

	Scenario 1		**Scenario 2**		**Scenario 3**	
	BAP	*RU*	*BAP*	*RU*	*BAP*	*RU*
# Feasible solutions	25	38	39	26	44(4*)	30 (0*)
# Optimal solutions	5	8	9	8	22	17
# Better solutions	0	32	22	3	22	4
# GAP < 1%	10	27	19	13	30	23
# Average GAP (%)	3.9	2.9	2.5	2.3	2.2**	2.5**
Average time for optimal solutions*** (sec)	576.8	60.6	699.9	782.7	129.8	189.3

* # instances violating at least one vertical shape's direction (Z < 0); ** Instances with Z < 0 were not considered; *** Considering only instances in which both models obtained optimal solutions.

In the third scenario, it was possible to find feasible solutions within the time limit for 44 instances (4 of those instances violated at least one vertical shape's direction). Out of these instances, 22 were solved until proving optimality and 8 ended within at most 1% from the optimal solution. Interestingly, as the new constraints are introduced into the model, the execution times decrease 6.1% from the first to the second scenario and 35.4% to the third. In accordance, the number of feasible (and optimal) solutions increases by 14 (and 5) from the first to the second scenario, and by 4 (and 15) to the third. Nevertheless, one hour is still a prohibitive amount of time to obtain solutions in practice and 10 instances (15 in *Scenario 2*) still need to be solved. This motivated the development of a mathematical programming based heuristic whose computational study is presented in the next section.

When comparing our formulation to the adapted version of Russell and Urban [2010] (Appendix 3.A), we were able to outperform the latter in the last two scenarios, both in the number of feasible and optimal solutions. In *Scenario 2* we obtained more 13 feasible and 1 optimal solutions and in *Scenario 3* we obtained more 14 feasible and 5 optimal solutions. Additionally, it also took less time in the instances where both approaches obtained provably optimal solutions: 9% less in the second scenario and 31% less in the third. However, when looking at the first scenario, Russell and Urban [2010] were able to find a further 13 feasible and 3 optimal solutions and took less 89% of the time to solve the instances where both approaches obtained provably optimal solutions. As our formulation was developed to take advantage of the blocks' hierarchy, it was not expected to perform as well as Russel's in this scenario.

3.6.3 Solution Approach Computational Results

We tested the *H-BAP* matheuristic and its two extensions, *Improving Feasibility (H-BAP-IF)* and *Improving Efficiency (H-BAP-IE)*, in all instances using both *Scenario 2* and *Scenario 3*. *Scenario 1* was not tested as the matheuristic is based on the natural partition of the products in family blocks which were not considered in this scenario.

The matheuristic was implemented in C++ and compiled using a gcc compiler. The formulation was embedded in the code and the resulting sub-problems were solved with the CPLEX library for C++. For each sub-problem $subBAP_l$, a time limit was defined according to the level and number of blocks in the integer set G_l^I. The overall matheuristic time was limited to 3600 seconds, and whenever a sub-problem stopped before its time limit, the remaining time was used in the subsequent sub-problems. The downstream level of the *Improving Efficiency* extension was solved using the adapted Russel and Urban's formulation (at this level, with no family blocks). All sequencing variables T_{ij} were previously fixed by decreasing order of sales. The reason why a common knapsack formulation was not used is because it was necessary to align each product across its assigned shelves. The vertical shapes' constraints were also considered soft constraints.

Table 3.3 summarizes the results obtained with the matheuristic and compares them with the plain resolution of the same formulation (*BAP*) by CPLEX. The detailed information is presented in Table 3.5 of Appendix 3.B. For each instance, we provide information about the best integer solution found (Z), the total execution time in seconds (T) and the deviation of the best integer solution from the best upper bound available (taken from the previous section) (GAP).

The performance profiles from Figures 3.6 (for *Scenario 2*) and 3.7(for *Scenario 3*) further explore the comparison between the four approaches by presenting cumulative distribution functions for two performance metrics: execution times and solution quality (for additional information about performance profiles, please see Dolan and Moré [2002]). Each approach is represented by a curve where each point (τ,ρ) means that the execution times of that approach took at most τ times more than the execution times of the fastest approach (or the solution is within a factor of τ from the best integer solution found), in $100\rho\%$ of the instances. For example, in *Scenario 2*, the approach *H-BAP* has a better objective function value on 30% of the instances ($\tau = 0, \rho = 0.30$), is within 10% of the best solution in 76% of the cases ($\tau = 1.1, \rho = 0.76$) and solves a total of 78% of the instances ($\tau \rightarrow \infty, \rho = 0.78$). To build the graphs, we used all the 54 instances and whenever we had an instance with no integer solution found, infeasible or violating the shapes' direction we assumed that it was within an infinite distance from the best solution.

With regard to the *H-BAP* matheuristic (without extensions), and comparatively to the *BAP* formulation, it was possible to improve the results in two directions: the number of feasible solutions was increased (by 3 and 4 in the second and third scenario, respectively) and the running times were reduced (by 73% and 72%). The results were obtained without compromising the quality of the solution whose GAP increased on average 1.7% in *Sce-*

Table 3.3 – Summary of results for the *H-BAP* matheuristic and its extensions using *Scenario 2* and *Scenario 3*

	Scenario 2				**Scenario 3**			
	BAP	*H-BAP*	*H-BAP-IF*	*H-BAP-IE*	*BAP*	*H-BAP*	*H-BAP-IF*	*H-BAP-IE*
# Feasible solutions	39	42	49	52	44(4*)	48(6*)	54(7*)	53(4*)
# Optimal solutions	9	2	2	1	22	11	13	5
# Better solutions	–	7	8	3	–	11	12	6
Average GAP (%)**	2.5	4.2	4.8	8.2	2.2	3.0	2.8	4.4
Average time (sec)**	3154.9	750.3	866.4	184.8	2249.9	559.6	517.6	105.5
Avg.deviation to *BAP* (%) **	–	-2.2	-2.4	-6.5	–	-0.2	0.1	-1.5

*#instances violating at least one vertical shape's direction (Z<0); **Instances with Z<0 were not considered.

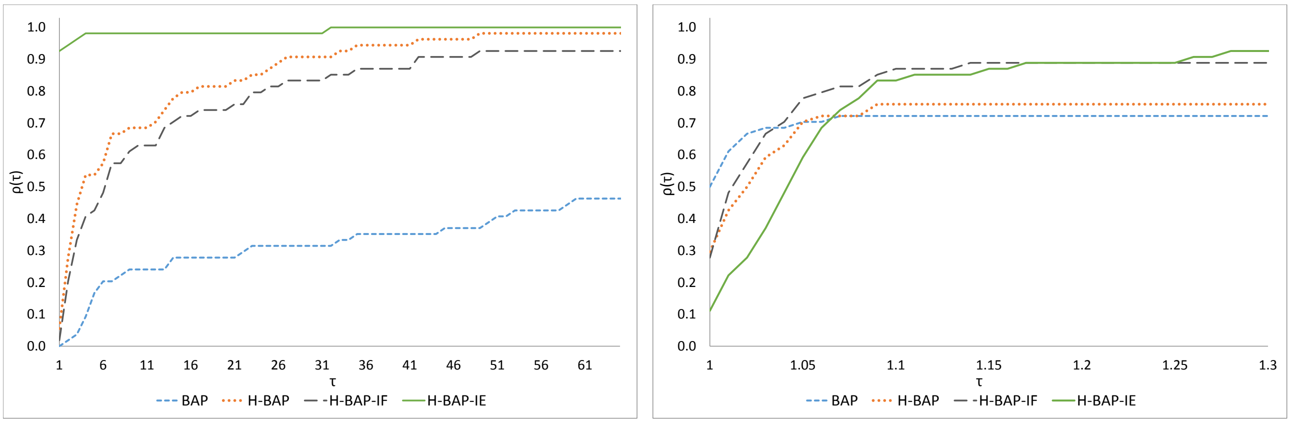

Figure 3.6 – Performance profile of *Scenario 2* using computational times (on the left side) and solution quality (on the right side) as performance measures

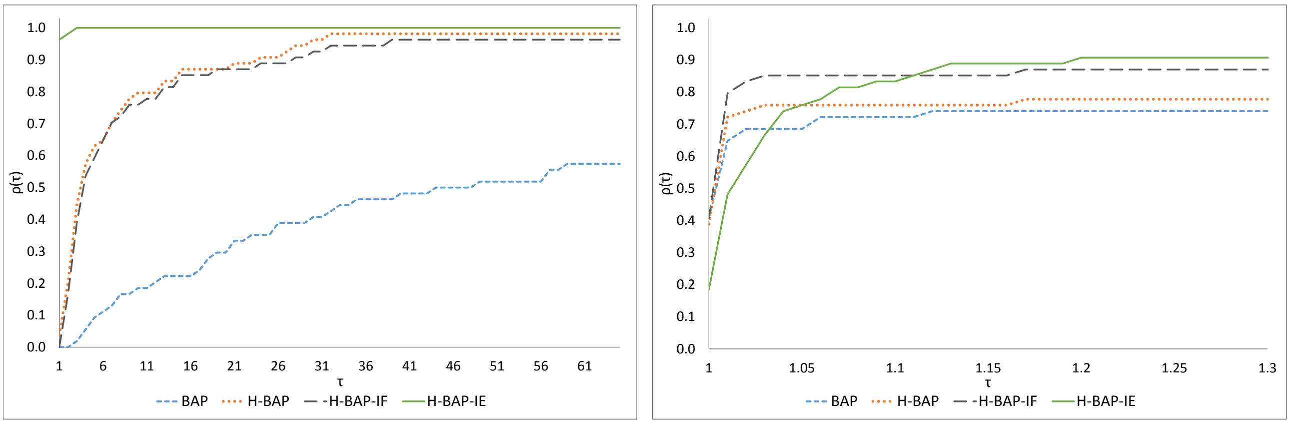

Figure 3.7 – Performance profile of *Scenario 3* using computational times (on the left side) and solution quality (on the right side) as performance measures

nario 2 and 0.8% in *Scenario 3*, with a distance of 2.2% and 0.2% to the *BAP* solutions. The performance profiles further prove these findings. 70% of the instances are solved within at most 1% from the best solution in the third scenario, and 5% in the second. Moreover, in *Scenario 3*, *H-BAP* has a performance similar to *BAP*, with the advantage of generating feasible solutions for a higher number of instances.

The *Improving Feasibility (H-BAP-IF)* extension made it possible to obtain feasible solutions for the instances that were infeasible in the *H-BAP* matheuristic. As a result, the number of solutions increased by 7 in the second scenario, and all instances were solved in third scenario. Both the execution times and the quality of the solution remained similar for the instances that were already feasible before, as confirmed in the performance profiles, that present similar patterns. Moreover, *H-BAP-IF* dominates both *H-BAP* and *BAP* in *Scenario 3*: the solution quality performance profile for this approach lies above the other two for all performance ratios. The performance profiles also show an execution time overhead caused by the backtracking scheme.

The *Improving Efficiency (H-BAP-IE)* extension explores a trade-off between the execution times and the quality of the solution. An additional reduction of 79% (*Scenario 2*) and 80% (*Scenario 3*) was obtained in the execution times by sacrificing 3.2% and 1.6% of the average GAP. This trade-off is also visible in the performance profiles, as the curves from *H-BAP-IE* start with the lowest percentage of intances close to the best solution. In 70% of the intances the solution is within 7% (*Scenario 2*) and 4% (*Scenario 3*) from the best solution, still the highest figures among the different heuristics. On the other hand, the execution times are substantially better, dominating all other approaches.

3.7. Conclusions

This paper presents a novel and realistic mixed integer programming formulation for the Shelf Space Allocation Problem with location decisions and a new practical constraint imposed to product families: the shapes' direction. The novelty in the formulation comes from introducing single commodity flow constraints to model product sequencing and exploring product families to reduce the combinatorial nature of the problem. Based on the formulation, a mathematical programming-based heuristic was also developed that explores the hierarchy present in the product families to decompose the problem into a sequence of sub-problems. To improve the matheuristic's performance, its original design was adapted following two directions: recovery from infeasible solutions (improving feasibility) and reduction of solution times (improving efficiency). We also provide a set of real case benchmark instances for the shelf space allocation problem with location decisions which was used to assess the formulation and matheuristic and will allow future research in this area.

We proved the validity of the real-world features by analyzing a concrete example and by showing its impact on the solution. Three different scenarios were used for this purpose: in the first scenario no product families were considered; in the second scenario product families did not have shapes' direction; and lastly, in the third scenario, product families were fully considered. The scenario analysis showed that, even though the objective function decreases with the new features, benefits are obtained with a more clever product arrangement on the shelves. However, these benefits are hard to grasp in the model as they are linked to the customers' response to the complexity of the planograms and to the way customers search for the products while shopping. The example also made clear that it is important to define good lower and upper bounds for the number of product facings, by ensuring that the lower bounds define satisfactory quantities for the products (in accordance with replenishment policies to avoid stockouts) and the upper bounds take into consideration the products' shelf life and holding costs. This importance is stressed due to the use of a linear profit function in the novel formulation. Moreover, the example showed (in line with the computational experiments) that introducing the new features made the problem easier to solve.

Extensive computational tests were performed on both the formulation and the matheuristic. The formulation was able to find feasible solutions within a time limit of one hour for 44 out of 54 instances (50% of them were solved to optimality). However, as the instance size increases using an exact approach such as branch-and-cut on the formulation fails to generate feasible solutions. The matheuristic improved these results in two directions: it was able to solve a higher number of instances and decreased running times by over 70%. The results were obtained with a limited impact on the quality of the solutions. The matheuristic's *Improving Efficiency* extension further explored a trade-off between the execution times and the solution quality. For the scenario with shape's direction, execution times were on average below 120 seconds and within 1.5% of the best known solution.

The proposed formulation (*BAP*) was also compared to an adapted version of the formulation present in Russell and Urban [2010]. In the presence of product families the new formulation outperformed the state-of-the-art both in terms of feasible (and optimal) solutions and execution times. This proves that the model was successful in taking advantage of the product families and in considering commodity based constraints for subtour elimination.

We identify the following interesting topics of future research. The solutions generated using any SSA model strongly depend on the assortment and merchandising rules defined previously. Both the literature and practice of retail shelf space would benefit from an integration of those decisions and a sensitivity analysis on the impact of the upstream decisions in shelf space plans. From an algorithmic perspective, an improving heuristic on top of the existing solution approach could further increase the quality of the solutions with limited impact on the execution times.

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Appendix 3.A Adapted Formulation from Russel and Urban

This section presents an adapted version of the formulation by Russell and Urban [2010], according to the Shelf Space Allocation Problem as described in section 3.3. The notation presented in that section is used again.

The key difference of this adapted version is the possibility of placing the same product on more than one shelf, and its alignment across shelves whenever that happens. To take advantage of the existing alignment constraints, we considered the products as single product family blocks, which are interchangeably represented by index i or m. Additionally, products may occupy a length greater than its number of facings, giving more freedom to the block alignment process (the alignment process becomes difficult when the widths of the products are not compatible). In accordance, consider the following decision variables:

- W_{ik} the integer number of facings from product $i \in \mathcal{N}$ on shelf $k \in \mathcal{K}$,
- L_{ik} shelf length assigned to product $i \in \mathcal{N}$ on shelf $k \in \mathcal{K}$,
- Y_{mk} = 1 if block $m \in \mathcal{V}$ is located on shelf $k \in \mathcal{K}$,
- $X^s_m(X^e_m)$ the horizontal location of the block $m \in \mathcal{V}$ - left (right) coordinate,
- T_{ij} = 1 if product j is located to the left of product i, $i, j \in \mathcal{N}$,
- $FL_m(LL_m)$ first (last) shelf of block $m \in \mathcal{V}$.

The formulation is presented next, divided again into three parts: allocation, sequencing and family grouping constraints. The objective function and allocation constraints are the following:

$$\text{Maximize} \quad \sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{K}} p_i \cdot \gamma_k \cdot W_{ik} \tag{3.46}$$

$$\text{Subject to:} \quad \sum_{k \in \mathcal{K}} W_{ik} \leq u_i, \quad \forall\, i \in \mathcal{N} \tag{3.47}$$

$$\sum_{k \in \mathcal{K}} W_{ik} \geq l_i, \quad \forall\, i \in \mathcal{N} \tag{3.48}$$

$$a_i \cdot W_{ik} \leq L_{ik}, \quad \forall\, i \in \mathcal{N}, k \in \mathcal{K} \tag{3.49}$$

$$\sum_{i \in \mathcal{N}} L_{ik} \leq w_k, \quad \forall\, k \in \mathcal{K} \tag{3.50}$$

$$W_{ik} = 0, \quad \forall\, i \in N, k \in \mathcal{K} : b_i \leq h_k \tag{3.51}$$

$$W_{ik} \in \mathbb{N}_0, \quad \forall\, i \in \mathcal{N}, k \in \mathcal{K} \tag{3.52}$$

$$L_{ik} \leq 0, \quad \forall\, i \in \mathcal{N}, k \in \mathcal{K} \tag{3.53}$$

Both the objective function and allocation constraints are similar to the proposed formulation. The objective function maximizes the profit of the planogram in accordance with the number of facings that each product has on each shelf. The maximum and minimum number of facings are maintained with constraints (3.47) and (3.48). Constraints (3.49) ensure that the length allocated to each product on each shelf is enough for the corresponding number of facings, and constraints (3.50) ensure that the total length allocated to each shelf does not exceed its capacity. Constraints (3.52) ensure that the products do not exceed the height of the shelves.

To guarantee that there is no physical overlap between the products, the formulation includes:

$$L_{ik} \leq w_k \cdot Y_{ik}, \quad \forall i \in \mathcal{N} \tag{3.54}$$
$$X^s_{ik} \geq 0, \quad \forall i \in \mathcal{N}, k \in \mathcal{K} \tag{3.55}$$
$$X^s_{ik} \leq w_k - a_i \cdot W_{ik}, \quad \forall i \in \mathcal{N}, k \in \mathcal{K} \tag{3.56}$$
$$X^s_{ik} \leq X^s_{jk} + L_{jk} - w_k \cdot T_{ij} - w_k \cdot (2 - Y_{ik} - Y_{jk}), \quad \forall i, j \in N : i \neq j, k \in \mathcal{K} \tag{3.57}$$
$$T_{ij} + T_{ji} = 1, \quad \forall i, j \in N : i < j, k \in \mathcal{K} \tag{3.58}$$
$$T_{ij} \in \{0, 1\}, \quad \forall i, j \in \mathcal{N} \tag{3.59}$$

Constraints (3.54) relate variables L and Y: a product can only have length within the shelves it was assigned to. The limits of the shelves are imposed by constraints (3.55) and (3.56). Constraints (3.57) define the product sequencing by ensuring that the left coordinate of product i is to the right of the right coordinate of product j, unless product i is to the left of product j, or both products are on different shelves. Constraints (3.58) guarantees that a product is either to the left or to the right of another product. The sequencing part of the formulation is one of the key differences for the formulation proposed in this paper, which takes advantage of the family blocks to reduce the combinatorial nature of T_{ij}.

Finally, product families are kept in rectangular and continuous blocks through the following constraints:

$$Y_{mk} \leq \sum_{i \in \mathcal{N}_m} Y_{ik}, \quad \forall m \in \mathcal{V}, k \in \mathcal{K} \tag{3.60}$$
$$Y_{mk} \geq Y_{ik}, \quad \forall m \in \mathcal{V}, i \in \mathcal{N}_m, k \in \mathcal{K} \tag{3.61}$$
$$X^s_{mk} \leq X^s_{ik} + w_k \cdot (1 - Y_{ik}), \quad \forall m \in \mathcal{V}, i \in \mathcal{N}_m, k \in \mathcal{K} \tag{3.62}$$
$$X^e_{mk} \geq X^e_{ik} + L_{ij} - w_k \cdot (1 - Y_{ik}), \quad \forall m \in \mathcal{V}, i \in \mathcal{N}_m, k \in \mathcal{K} \tag{3.63}$$
$$X^e_{mk} - X^s_{mk} = \sum_{i \in \mathcal{N}_m} L_{ik}, \quad \forall m \in \mathcal{V}, k \in \mathcal{K} \tag{3.64}$$
$$FL_m \leq K - (K - k) \cdot Y_{mk}, \quad \forall m \in \mathcal{V}, k \in \mathcal{K} \tag{3.65}$$
$$LL_m \geq k \cdot Y_{mk}, \quad \forall m \in \mathcal{V}, k \in \mathcal{K} \tag{3.66}$$
$$LL_m - FL_m = \sum_{k \in \mathcal{K}} Y_{mk} - 1, \quad \forall m \in \mathcal{V} \tag{3.67}$$
$$LL_m \geq FL_m, \quad \forall m \in \mathcal{V} \tag{3.68}$$
$$X^s_{m,k+1} - X^s_{mk} \leq v + w_k \cdot (2 - Y_{mk} - Y_{m,k+1}), \quad \forall m \in \mathcal{V}, k \in \mathcal{K} : k < K \tag{3.69}$$
$$X^s_{mk} - X^s_{m,k+1} \leq v + w_k \cdot (2 - Y_{mk} - Y_{m,k+1}), \quad \forall m \in \mathcal{V}, k \in \mathcal{K} : k < K \tag{3.70}$$
$$X^e_{m,k+1} - X^e_{mk} \leq v + w_k \cdot (2 - Y_{mk} - Y_{m,k+1}), \quad \forall m \in \mathcal{V}, k \in \mathcal{K} : k < K \tag{3.71}$$
$$X^e_{mk} - X^e_{m,k+1} \leq v + w_k \cdot (2 - Y_{mk} - Y_{m,k+1}), \quad \forall m \in \mathcal{V}, k \in \mathcal{K} : k < K \tag{3.72}$$

$$X^s_{mk}, X^e_{mk} \leq 0, \quad \forall m \in \mathcal{V}, k \in \mathcal{K} \tag{3.73}$$

$$Y_{mk} \in \{0,1\}, \quad \forall m \in \mathcal{V}, k \in \mathcal{K} \tag{3.74}$$

$$FL_m, LL_m \in \mathbb{N}_0, \quad \forall m \in \mathcal{V} \tag{3.75}$$

Constraints (3.60) and (3.61) associate the families' placement on the shelves with the products' placement, while constraints (3.62)-(3.64) define the family blocks' left and right coordinates in accordance with the products' coordinates. The blocks' vertical adjacency is ensured by constraints (3.65)-(3.68), and the horizontal adjacency by (3.69)-(3.72). Constraints (3.34) and (3.35) from section 3.4.3, pertaining to the novel display direction feature, were additionally added to the formulation.

Appendix 3.B Result Tables

This section presents the detailed results obtained for all the instances present in section 3.6.1. Table 3.4 contains the results using both the *BAP* and *RU* formulations in the three scenarios. For each instance, we provide information about the linear relaxation (Z^{lr}), best integer solution found (Z), total execution time in seconds (T) and the deviation of the best integer solution found from the best upper bound available at the stopping criteria (*GAP*). Note that Z is defined in equation (3.1) and additionally includes a penalty P defined in (3.45) whenever a shape's vertical direction is violated. This penalty explains the negative values in some instances. The results are further explained in section 3.6.2.

Table 3.5 contains the results obtained using the *H-BAP* matheuristic and its extensions in the last two scenarios. For each instance, we provide information about the best integer solution found (Z), the total execution time in seconds (T) and the deviation of the best integer solution from the best upper bound available (taken from the previous section) (*GAP*). The results are explained in section 3.6.

Table 3.4 – Model Validation and Performance Results

Instance Name	Scenario 1								Scenario 2								Scenario 3							
	Z^{lr}		Z		$T(s)$		$GAP(\%)$		Z^{lr}		Z		$T(s)$		$GAP(\%)$		Z^{lr}		Z		$T(s)$		$GAP(\%)$	
	BAP	*RU*	*BAP*	*RU*	*BAP*	*RU*	*BAP*	*RU*	*BAP*	*RU*	*BAP*	*RU*	*BAP*	*RU*	*BAP*	*RU*	*BAP*	*RU*	*BAP*	*RU*	*BAP*	*RU*	*BAP*	*RU*
FL_1	10183.5	10183.5	10089.1	10089.1	2290	259	0.0	0.0	**10178.4**	10181.9	9927.5	9927.5	31	79	0.0	0.0	**10112.4**	10181.9	9761.0	9761.0	18	30	0.0	0.0
AZ_3	71751.4	71751.4	71399.8	71399.8	13	5	0.0	0.0	**71749.4**	71751.2	70875.1	70875.1	1	7	0.0	0.0	**70535.7**	71750.8	64203.2	64203.2	2	3	0.0	0.0
AZ_2	52398.2	52398.2	52268.0	**52340.0**	*Tl*	*Tl*	0.2	0.0	**52376.8**	52390.1	51695.4	51695.4	1090	1431	0.0	0.0	**52107.9**	52375.9	**51107.5**	51040.4	*Tl*	*Tl*	0.2	0.7
AZ_1	4936.2	4936.2	4885.2	**4919.4**	*Tl*	*Tl*	0.8	0.1	**4934.3**	4935.4	4819.5	**4829.7**	*Tl*	*Tl*	0.6	0.5	**4934.3**	4935.4	4746.4	4746.4	255	96	0.0	0.0
LS_1	7393.2	7393.2	6969.1	**7302.3**	*Tl*	*Tl*	5.7	1.2	7393.2	**7393.0**	**7166.7**	7147.1	*Tl*	*Tl*	2.1	2.5	7393.2	**7392.8**	7124.2	7124.2	*Tl*	*Tl*	0.3	0.2
VG_3	18904.2	18904.2	18718.2	18718.2	16	7	0.0	0.0	**18902.1**	18903.1	18062.3	18062.3	21	20	0.0	0.0	**18901.9**	18903.0	17431.5	17431.5	11	8	0.0	0.0
VG_2	141171.9	141171.9	134769.0	**140226.8**	*Tl*	58	4.0	0.0	**141150.8**	141169.8	138756.0	138756.0	*Tl*	*Tl*	0.2	0.3	**140699.9**	141169.5	135736.8	135736.8	29	58	0.0	0.0
VG_4	18013.3	18013.3	17826.3	**17853.0**	*Tl*	*Tl*	0.4	0.2	18013.3	**18012.8**	**17231.2**	17091.9	*Tl*	*Tl*	0.4	2.4	**18006.0**	18012.2	16040.5	16040.5	34	282	0.0	0.0
VG_5	70919.1	70919.1	*	**70729.2**	*Tl*	*Tl*		0.2	70919.0	**70912.7**	**69202.0**	66490.9	*Tl*	*Tl*	1.7	5.7	**69752.5**	70893.6	**68151.0**	*	*Tl*	*Tl*	2.1	
VG_1	11325.8	11325.8	*	*	*Tl*	*Tl*			**11323.1**	11325.2	*	*	*Tl*	*Tl*			**11323.1**	11325.2	**10421.8**	*	1119	*Tl*	0.0	
CP_2	782.8	782.8	760.1	760.1	8	3	0.0	0.0	782.8	782.8	690.2	690.2	12	9	0.0	0.0	782.8	782.8	690.2	690.2	1	2	0.0	0.0
CP_1	472.5	472.5	471.8	471.8	*Tl*	1329	0.0	0.0	472.5	472.5	463.3	**465.2**	*Tl*	*Tl*	1.8	1.1	**472.3**	472.5	453.6	453.6	152	63	0.0	0.0
CP_4	3278.5	3278.5	*	*	*Tl*	*Tl*			3278.5	3278.5	*	*	*Tl*	*Tl*			**3167.3**	3278.3	*	*	*Tl*	*Tl*		
CP_3	10380.4	10380.4	*	**10369.0**	*Tl*	*Tl*		0.1	10380.4	**10380.0**	**10054.0**	*	*Tl*	*Tl*	2.4		**10082.7**	10379.7	**-355477.4**	*	*Tl*	*Tl*	100.0	
CR_6	718.4	718.4	619.6	**625.3**	*Tl*	*Tl*	11.2	6.0	718.4	718.4	617.9	617.9	3405	3594	0.0	0.0	718.4	718.4	608.3	608.3	7	9	0.0	0.0
CR_5	1162.7	1162.7	1003.1	**1009.9**	*Tl*	*Tl*	11.0	10.4	1162.6	**1162.5**	900.7	900.7	*Tl*	*Tl*	4.5	6.0	1162.6	**1162.5**	632.1	632.1	7	13	0.0	0.0
CR_4	571.6	571.6	561.1	**562.6**	*Tl*	*Tl*	1.3	0.7	571.6	**571.5**	536.2	536.2	*Tl*	*Tl*	1.0	0.0	571.6	**571.5**	442.6	442.6	43	59	0.0	0.0
CR_3	641.3	641.3	536.1	**545.8**	*Tl*	*Tl*	12.7	11.4	**641.0**	641.2	516.0	516.0	*Tl*	*Tl*	13.2	13.3	**641.0**	641.2	460.6	460.6	18	60	0.0	0.0
CR_2	1017.3	1017.3	996.0	**1006.4**	*Tl*	*Tl*	2.0	0.8	1017.3	1017.3	**975.4**	852.0	*Tl*	*Tl*	2.9	15.1	1017.3	1017.3	872.2	872.2	1034	1616	0.0	0.0
CR_1	3801.5	3801.5	*	*	*Tl*	*Tl*			3801.5	**3800.7**	**3395.2**	*	*Tl*	*Tl*	8.9		**3762.7**	3798.5	**3124.6**	*	*Tl*	*Tl*	8.1	
PT_1	90096.2	90096.2	60866.3	**66477.9**	*Tl*	*Tl*	8.5	0.0	**90074.2**	90091.9	62304.6	62304.6	93	*Tl*	0.0	0.0	**90074.2**	90091.9	62034.9	62034.9	19	*Tl*	0.0	0.0
AI_1	4039.6	4039.6	3792.6	**4007.8**	*Tl*	*Tl*	6.1	0.7	4039.4	**4039.3**	**3882.5**	3854.2	*Tl*	*Tl*	1.3	2.1	**3938.1**	4039.0	**3748.6**	3736.3	*Tl*	*Tl*	4.7	4.8
AI_3	7100.5	7100.5	*	*	*Tl*	*Tl*			**7100.3**	7100.4	**6437.2**	*	*Tl*	*Tl*	9.1		**7080.8**	7100.4	**6474.0**	*	*Tl*	*Tl*	8.4	
AI_4	20423.2	20423.2	*	**20328.3**	*Tl*	*Tl*		0.3	**20422.6**	20423.0	**19758.3**	*	*Tl*	*Tl*	2.1		**20267.6**	20422.4	**19831.4**	19528.2	*Tl*	*Tl*	0.8	3.0
AI_2	3234.7	3234.7	2939.6	**3110.1**	*Tl*	*Tl*	9.1	3.8	**3234.3**	3234.6	**2790.6**	*	*Tl*	*Tl*	12.8		**3195.4**	3234.4	**2470.2**	2120.6	*Tl*	*Tl*	22.2	33.3
VN_1	18270.0	18270.0	*	**18025.3**	*Tl*	*Tl*		0.4	18269.9	**18269.8**	**17571.4**	*	*Tl*	*Tl*	0.5		18269.9	**18269.8**	15605.5	15605.5	216	*Tl*	0.0	0.2
SP_1	9761.4	9761.4	*	*	*Tl*	*Tl*			9761.4	**9760.6**	**9591.0**	*	*Tl*	*Tl*	0.7		**9719.7**	9759.0	**9332.0**	*	*Tl*	*Tl*	1.9	
OM_3	527.7	527.7	502.1	**503.9**	*Tl*	1719	0.4	0.0	527.7	527.7	469.8	469.8	*Tl*	*Tl*	0.1	2.0	**519.4**	527.7	358.3	358.3	81	117	0.0	0.0
OM_1	983.3	983.3	*	**975.3**	*Tl*	*Tl*		0.1	983.2	**983.0**	**955.8**	*	*Tl*	*Tl*	1.8		**938.3**	982.6	-24368.6	**891.1**	*Tl*	*Tl*	100.0	3.1
OM_2	299.5	299.5	*	*	*Tl*	*Tl*			299.5	299.5	*	*	*Tl*	*Tl*			299.5	299.5	**259.1**	*	716	*Tl*	0.0	
LC_1	21061.9	21061.9	*	**21017.5**	*Tl*	*Tl*		0.1	21061.6	**21059.8**	**20349.3**	20259.2	*Tl*	*Tl*	0.5	1.4	21061.5	**21059.8**	**20281.9**	20007.1	*Tl*	*Tl*	0.5	2.5
LC_2	8443.5	8443.5	*	**8062.3**	*Tl*	*Tl*		4.5	8443.4	**8443.0**	**7938.8**	*	*Tl*	*Tl*	5.6		**8410.7**	8442.2	**7626.1**	6253.1	*Tl*	*Tl*	9.1	25.6
SM_6	17652.6	17652.6	17150.5	**17296.3**	*Tl*	*Tl*	1.3	0.4	**17651.6**	17652.1	16939.1	16939.1	3373	*Tl*	0.0	0.0	**17646.6**	17651.2	16784.5	16784.5	2019	*Tl*	0.0	1.6
SM_2	28455.5	28455.5	27131.5	**28282.4**	*Tl*	*Tl*	4.2	0.1	**28446.4**	28454.1	28066.7	**28067.5**	*Tl*	*Tl*	0.5	0.4	**28394.2**	28453.9	**27804.1**	27665.3	*Tl*	*Tl*	0.3	1.0
SM_4	22976.6	22976.6	*	**22845.4**	*Tl*	*Tl*		0.2	**22971.3**	22975.2	**22481.3**	22369.4	*Tl*	*Tl*	0.8	1.6	**22926.9**	22972.8	**22303.1**	22302.8	*Tl*	*Tl*	0.4	0.3
SM_5	10765.4	10765.4	9655.4	**10693.8**	*Tl*	*Tl*	10.2	0.4	**10764.7**	10765.1	**10455.5**	*	*Tl*	*Tl*	0.9		**10734.4**	10764.8	**10246.5**	*	*Tl*	*Tl*	0.4	
SM_3	58355.6	58355.6	*	**57268.3**	*Tl*	*Tl*		1.2	**58345.3**	58355.2	**53897.1**	*	*Tl*	*Tl*	5.9		**57901.9**	58351.9	**53382.4**	*	*Tl*	*Tl*	6.9	
SM_1	136291.7	**136291.6**	*	*	*Tl*	*Tl*			136291.7	**136278.5**	*	*	*Tl*	*Tl*			**135908.5**	136266.4	*	*	*Tl*	*Tl*		
CA_2	290.6	290.6	*	*	*Tl*	*Tl*			290.6	290.6	*	*	*Tl*	*Tl*			**290.4**	290.6	**243.2**	*	*Tl*	*Tl*	7.7	
AG_2	2540.4	2540.4	2472.0	**2504.0**	*Tl*	*Tl*	2.6	1.2	**2539.9**	2540.3	**2381.1**	2372.2	*Tl*	*Tl*	2.7	3.7	**2539.9**	2540.3	2154.0	2154.0	54	138	0.0	0.0
AG_1	2743.8	2743.8	*	**2725.7**	*Tl*	*Tl*		0.6	2743.8	2743.8	**2559.9**	*	*Tl*	*Tl*	5.9		**2730.3**	2743.7	**-142247.9**	*	*Tl*	*Tl*	100.0	
AG_4	5909.1	5909.1	*	**5812.7**	*Tl*	*Tl*		1.6	5909.1	**5908.7**	**5603.0**	*	*Tl*	*Tl*	3.9		**5762.3**	5908.5	**-148264.8**	*	*Tl*	*Tl*	100.0	
AG_3	33623.9	33623.9	*	*	*Tl*	*Tl*			33623.3	**33622.4**	*	*	*Tl*	*Tl*			**33598.4**	33622.4	*	*	*Tl*	*Tl*		

Instance Name	Scenario 1								Scenario 2								Scenario 3							
	Z^{lr}		Z		$T(s)$		$GAP(\%)$		Z^{lr}		Z		$T(s)$		$GAP(\%)$		Z^{lr}		Z		$T(s)$		$GAP(\%)$	
	BAP	*RU*	*BAP*	*RU*	*BAP*	*RU*	*BAP*	*RU*	*BAP*	*RU*	*BAP*	*RU*	*BAP*	*RU*	*BAP*	*RU*	*BAP*	*RU*	*BAP*	*RU*	*BAP*	*RU*	*BAP*	*RU*
AB_1	538.7	538.7	510.9	**531.2**	^{Tl}	^{Tl}	5.0	1.1	**538.5**	538.7	**523.3**	522.7	^{Tl}	^{Tl}	1.7	1.9	**538.5**	538.7	500.7	500.7	377	627	0.0	0.0
AB_2	7213.3	7213.3	*	*	^{Tl}	^{Tl}			7213.3	**7213.2**	*	*	^{Tl}	^{Tl}			7213.3	**7213.2**	**6222.8**	*	^{Tl}	^{Tl}	13.1	
VV_2	3266.8	3266.8	*	**3258.4**	^{Tl}	^{Tl}		0.1	3266.8	**3266.4**	*	*	^{Tl}	^{Tl}			**3252.9**	3266.3	*	*	^{Tl}	^{Tl}		
VV_1	61951.0	61951.0	*	**22599.5**	^{Tl}	^{Tl}		63.4	61951.0	**61943.7**	*	*	^{Tl}	^{Tl}			**61016.7**	61942.7	*	*	^{Tl}	^{Tl}		
LO_2	69.1	69.1	68.3	68.3	557	29	0.0	0.0	69.1	69.1	65.8	65.8	339	339	0.0	0.0	69.1	69.1	51.3	51.3	84	37	0.0	0.0
LO_1	4343.3	4343.3	*	*	^{Tl}	^{Tl}			4343.3	**4343.1**	*	*	^{Tl}	^{Tl}			4343.3	**4342.7**	*	*	^{Tl}	^{Tl}		
BH_1	42878.0	42878.0	*	*	^{Tl}	^{Tl}			42876.6	**42875.7**	*	*	^{Tl}	^{Tl}			42876.6	**42875.3**	**40820.1**	*	^{Tl}	^{Tl}	0.0	
BH_2	77697.2	77697.2	*	*	^{Tl}	^{Tl}			77689.4	**77673.4**	*	*	^{Tl}	^{Tl}			**74591.3**	77661.5	*	*	^{Tl}	^{Tl}		
CH_1	1367.9	1367.9	*	*	^{Tl}	^{Tl}			1367.9	1367.9	*	*	^{Tl}	^{Tl}			**1361.2**	1367.8	*	*	^{Tl}	^{Tl}		
BC_1	66069.7	66069.7	*	*	^{Tl}	^{Tl}			66069.7	**66065.4**	*	*	^{Tl}	^{Tl}			**65517.9**	66057.6	*	*	^{Tl}	^{Tl}		
DE_1	6720.9	**6720.8**	*	*	^{Tl}	^{Tl}			6720.9	**6720.5**	*	*	^{Tl}	^{Tl}			**6719.7**	6720.4	*	*	^{Tl}	^{Tl}		

* No feasible solution was found.

^{Tl} Time limit of 3600 s was reached.

Table 3.5 – Solution Approach Results

Instance Name	H-BAP						H-BAP-IF						H-BAP-IE					
	Scenario 2			Scenario 3			Scenario 2			Scenario 3			Scenario 2			Scenario 3		
	Z	GAP(%)	*T(s)*	*Z*	GAP(%)	*T(s)*	*Z*	GAP(%)	*T(s)*	*Z*	GAP(%)	*T(s)*	*Z*	GAP(%)	*T(s)*	*Z*	GAP(%)	*T(s)*
FL_1	9708.7	2.2	2.1	9761.0	0.0	1.2	9708.7	2.2	2.1	9761.0	0.0	1.2	9645.0	2.9	1.0	9718.7	0.4	0.9
AZ_3	70875.2	0.0	0.6	64203.2	0.0	0.6	70875.2	0.0	0.6	64203.2	0.0	0.6	70287.5	0.8	0.4	61907.8	3.6	0.3
AZ_2	51416.1	0.6	19.3	50758.2	0.9	37.9	51416.1	0.6	19.2	50758.2	0.9	40.0	50968.1	1.4	3.0	50504.1	1.4	7.0
AZ_1	*		1309.2	*		4.1	4733.2	2.4	2406.9	4743.1	0.1	6.2	4587.4	5.4	860.3	4723.9	0.5	1.7
LS_1	*		2359.7	7124.2	0.3	36.0	7115.8	2.8	2975.8	7124.2	0.3	36.0	7169.9	2.1	72.1	7118.7	0.4	2.9
VG_3	17649.1	2.3	1.3	17409.1	0.1	1.1	17649.1	2.3	1.5	17409.1	0.1	1.0	17000.0	5.9	0.4	16878.1	3.2	0.3
VG_2	133028.0	4.3	1.5	135737.0	0.0	1.7	133028.0	4.3	1.8	135737.0	0.0	1.8	131314.0	5.5	0.9	135265.0	0.4	0.9
VG_4	16940.4	2.1	7.4	16040.6	0.0	4.7	16940.4	2.1	7.3	16040.6	0.0	4.9	16485.2	4.7	3.1	16000.1	0.3	1.5
VG_5	68972.8	2.1	90.7	67996.4	2.3	321.1	68972.8	2.1	90.9	67996.4	2.3	324.8	68567.4	2.6	10.9	66622.1	4.3	22.7
VG_1	10870.6	3.1	579.8	10411.8	0.1	25.8	10870.6	3.1	607.3	10411.8	0.1	27.5	10041.4	10.5	49.0	10399.4	0.2	9.1
CP_2	690.2	0.0	0.3	690.2	0.0	0.2	690.2	0.0	0.3	690.2	0.0	0.3	690.2	0.0	0.4	690.2	0.0	0.2
CP_1	463.7	1.7	1177.7	453.6	0.0	42.6	461.4	2.2	1178.4	453.6	0.0	41.5	447.9	5.0	761.0	439.7	3.1	11.9
CP_4	3203.8	2.2	1159.1	-47366100.0	100.0	1158.5	3203.8	2.2	1159.4	-47366100.0	100.0	1158.3	3045.5	7.1	761.3	2638.0	15.0	302.2
CP_3	10204.6	1.0	102.2	-18659500.0	100.0	1176.4	10204.6	1.0	101.3	-18659500.0	100.0	1175.9	10045.8	2.5	34.3	-705417.0	100.0	842.4
CR_6	617.2	0.1	16.2	*		5.6	617.2	0.1	14.3	608.3	0.0	31.7	483.6	21.7	7.4	541.2	11.0	0.2
CR_5	*		3.9	632.1	0.0	0.7	843.5	10.6	1794.5	632.1	0.0	0.6	510.5	45.9	1.1	632.1	0.0	0.3
CR_4	493.5	8.9	2.7	442.6	0.0	1.4	493.5	8.9	2.5	442.6	0.0	1.4	492.2	9.1	1.1	442.6	0.0	0.8
CR_3	*		10.6	460.6	0.0	0.8	469.5	21.1	1795.3	460.6	0.0	0.8	410.5	31.0	2.8	460.6	0.0	0.3
CR_2	*		25.3	872.2	0.0	9.9	959.9	4.4	1805.5	872.2	0.0	9.3	918.6	8.6	74.2	872.2	0.0	2.2
CR_1	3255.0	12.7	127.7	3165.9	6.9	27.1	3255.0	12.7	117.7	3165.9	6.9	25.1	3181.1	14.6	37.3	3022.6	11.1	8.1
PT_1	*		8.6	*		1.7	54840.4	12.0	33.9	62035.0	0.0	2.3	54323.4	12.8	4.2	56182.9	9.4	1.0
AI_1	3762.4	4.4	134.0	3760.7	4.4	27.3	3762.4	4.4	139.8	3760.7	4.4	27.0	3702.1	5.9	20.5	3686.5	6.3	11.6
AI_3	6750.9	4.7	1176.7	6805.2	3.7	246.6	6750.9	4.7	1177.2	6805.2	3.7	225.1	6422.3	9.3	24.4	6465.4	8.5	39.1
AI_4	19444.4	3.7	203.8	19805.5	0.9	35.6	19444.4	3.7	209.7	19805.5	0.9	33.9	19122.0	5.3	7.8	19235.3	3.7	5.5
AI_2	*		19.8	2616.8	17.5	26.0	2980.7	6.9	449.2	2616.8	17.5	24.8	2852.1	10.9	624.3	2598.6	18.1	9.1
VN_1	16800.0	4.9	25.4	*		35.4	16800.0	4.9	25.6	15596.8	0.1	467.8	16703.8	5.4	15.2	15481.3	0.8	12.2
SP_1	9545.1	1.1	95.5	9321.3	2.1	1812.3	9545.1	1.1	95.0	9321.3	2.1	1811.1	9376.2	2.9	16.4	9216.7	3.2	4.5
OM_3	355.6	24.4	2.9	307.4	14.2	1.4	355.6	24.4	3.1	307.4	14.2	1.5	335.6	28.6	1.1	300.7	16.1	0.6
OM_1	962.0	1.1	65.5	889.8	3.6	214.1	962.0	1.1	67.1	889.8	3.6	214.6	918.3	5.6	25.8	795.7	13.8	35.5
OM_2	*		3600.3	259.1	0.0	28.1	*		3600.3	259.1	0.0	26.7	267.5	10.0	103.1	257.8	0.5	3.8
LC_1	19686.7	3.8	248.7	20276.0	0.5	7.6	19686.7	3.8	246.5	20276.0	0.5	7.6	19475.6	4.8	11.2	20184.8	1.0	1.7
LC_2	8018.9	4.7	533.9	7696.7	8.3	58.5	8018.9	4.7	533.9	7696.7	8.3	58.9	7226.3	14.1	25.7	7519.8	10.4	22.7
SM_6	16469.9	2.8	30.7	16469.9	1.9	30.2	16469.9	2.8	30.9	16469.9	1.9	30.2	16431.2	3.0	2.4	16431.2	2.1	2.1
SM_2	27393.9	2.9	45.8	27672.1	0.7	2020.6	27393.9	2.9	46.1	27672.1	0.7	2021.0	27115.2	3.9	7.2	27763.7	0.4	63.7
SM_4	22018.0	2.9	14.3	22156.8	1.0	23.1	22018.0	2.9	13.9	22156.8	1.0	23.1	21772.4	4.0	2.1	22098.1	1.3	1.8
SM_5	10394.6	1.5	35.6	10002.6	2.8	39.4	10394.6	1.5	35.8	10002.6	2.8	39.3	9704.5	8.0	2.1	10130.1	1.6	1.5
SM_3	54833.9	4.3	1818.2	54013.8	5.8	1686.9	54833.8	4.3	2279.9	54013.7	5.8	1502.0	54637.6	4.6	71.4	52799.9	7.9	82.4
SM_1	132377.0	2.7	3463.2	131061.0	2.8	3283.8	132541.0	2.6	2410.0	131207.0	2.7	2833.8	130469.0	4.1	257.5	129632.0	3.9	466.9
CA_2	*		621.7	243.5	7.6	21.7	*		3600.4	243.5	7.6	20.7	*		2400.4	239.2	9.2	4.8
AG_2	2282.6	6.7	13.1	2132.8	1.0	1.9	2282.6	6.7	12.4	2132.8	1.0	1.9	2043.0	16.5	7.6	2132.8	1.0	0.7
AG_1	2550.4	6.3	2452.3	2558.7	5.0	2398.6	2575.7	5.3	2443.1	2558.8	4.9	2397.4	2399.4	11.8	165.8	2351.8	12.6	1071.1
AG_4	5720.9	1.9	837.3	5505.3	4.2	621.3	5720.9	1.9	838.5	5505.3	4.2	626.2	5559.1	4.7	61.2	5184.0	9.8	210.1
AG_3	32647.5	2.7	3272.7	32094.6	4.3	3285.5	32645.5	2.7	3348.7	32087.0	4.3	3271.9	31195.1	7.0	260.0	31793.4	5.2	140.0
AB_1	513.7	3.5	44.4	500.4	0.1	4.2	513.7	3.5	44.3	500.4	0.1	4.1	507.1	4.7	6.3	500.4	0.1	1.7
AB_2	*		1165.2	*		867.6	*		3601.7	6944.2	3.0	980.5	6987.3	2.8	745.5	6953.4	2.9	92.2

Instance Name	H-BAP						H-BAP-IF						H-BAP-IE					
	Scenario 2			Scenario 3			Scenario 2			Scenario 3			Scenario 2			Scenario 3		
	Z	GAP(%)	*T(s)*	*Z*	GAP(%)	*T(s)*	*Z*	GAP(%)	*T(s)*	*Z*	GAP(%)	*T(s)*	*Z*	GAP(%)	*T(s)*	*Z*	GAP(%)	*T(s).*
VV_2	3244.2	0.3	2529.1	3204.8	1.4	3600.8	3240.2	0.5	2529.5	3204.9	1.4	3600.8	3243.1	0.4	60.4	3230.3	0.6	120.3
VV_1	61597.9	0.1	1330.3	-69175900.0	100.0	1788.8	61597.9	0.1	1520.7	-69175900.0	100.0	1789.1	61596.8	0.1	19.1	60209.9	1.2	1972.9
LO_2	60.4	8.2	1495.2	51.3	0.0	32.7	60.4	8.2	1471.7	51.3	0.0	32.9	60.4	8.2	60.2	51.3	0.0	4.0
LO_1	*		2871.4	-11239700.0	100.0	1574.3	*		3602.3	-11239700.0	100.0	1561.0	*		2400.9	-481437.0	100.0	906.0
BH_1	41637.1	1.9	1852.4	40807.2	0.1	73.8	41592.6	2.0	1856.3	40807.2	0.1	75.4	40791.4	3.9	1156.9	38246.0	6.4	198.0
BH_2	76362.3	1.0	2263.3	-218294000.0	100.0	653.6	76362.3	1.0	2272.3	-218294000.0	100.0	653.6	75697.2	1.8	438.8	73174.9	1.8	212.6
CH_1	1135.0	16.9	3601.6	-393847.0	100.0	1224.4	1147.9	16.0	3601.4	-393847.0	100.0	1202.6	1120.4	18.0	1079.6	-53232.5	100.0	743.9
BC_1	55736.7	15.5	636.7	51320.0	21.4	3407.1	55736.7	15.5	635.9	50416.3	22.8	3410.0	58771.4	10.8	460.2	*		385.3
DE_1	*		2964.4	*		1904.8	*		3601.7	-27167400.0	100.0	2725.2	6316.2	5.7	1182.6	-2249420.0	100.0	495.0

* No feasible solution was found.

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Chapter 4

Replicating Shelf Space Allocation Solutions Across Retail Stores

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Abstract Consumer-centric merchandising policies require store-specific shelf-space planning to better account for local consumer demand. Nevertheless, the high number of stores and the time spend developing merchandising plans force retailers to cluster the stores, and develop generic shelf space plans. In this paper we introduce the novel problem of transforming generic cluster-based shelf space plans into store-specific plans, a process that is called Replication in this paper. Two mathematical programming formulations are presented to address the Shelf Space Replication Problem, with different levels of practical details. The formulations use a novel inventory-related objective function that balances the products' inventory level in order to trigger joint shelf replenishments. Based on the formulations, a mathematical programming-based heuristic was also developed. Both approaches were tested using real data from a European Food Retailer that motivated this project, proving their suitability for practical use.

Keywords Retail operations · Shelf space allocation · Replicating solutions · Store-specific planograms · MIP-based heuristic

4.1. Introduction

In today's increasing competitive environment, retailers strive for customer satisfaction and operational efficiency, aiming to improve the stores' financial performance. To achieve such goal, retail organizations are moving towards demand driven initiatives, with the lemma "every sale counts", while trying to optimize their two most expensive resources: space and inventory.

Shelf space planning is a mid-term operational planning activity that defines the allocation of the products on the shelves for a period of 6-12 months. Two major goals are associated with this activity: maximize selling space effectiveness and tighter inventory control. As a matter of fact, marketing studies have long proved the positive influence of

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shelf space on stimulating consumer demand, highlighting the amount of space allocated to the products (space elasticity) and specific locations, such as eye- and hand- level (Curhan [1972], Drèze et al. [1994], Chandon et al. [2009]). On the inventory side, a loose inventory control may increase the risk of stockouts and impose high restocking costs due to the need for frequent replenishment.

Shelf space planning is performed using *planograms*, blueprints of the shelves where retailers develop their merchandising plans. Planograms are usually created separately for each category, whose space is determined before, at an upstream level. Each planogram shows, for a specific category, the exact display location for each product and the number of facings (i.e. visible items). The items behind the facings determine the product shelf inventory.

To perform this activity, retailers are often assisted by space planning software systems. These systems are large-scale data processing technologies that have powerful analysis reports and automatic tools for product-to-shelf allocation. Nevertheless, many studies argue that no "real" optimization takes places due to the limited or nonexistent use of mathematical optimization, and because consumer demand effects are disregarded (Irion et al. [2011], Hansen et al. [2010], Hübner and Kuhn [2012], Drèze et al. [1994], Desmet and Renaudin [1998]). Moreover, the existing rules are simplistic and the automatically generated planograms are not suitable or very likely to receive significant manual adjustments. Thus, generating planograms is still a time consuming activity - the industry standard for creating a single planogram is around three hours (JDA [2009]).

The increasing number of stores turns store-specific planograms impractical, and retailers usually cluster the stores based on demand and space patterns, generating a unique planogram for each cluster. This results in average space plans that are distributed to all the stores of the same cluster. However, current trends state that "One plan does not fit all" because these cluster-based planograms do not balance regional differences, and do not accurately represent consumer demands at a localized level.

In this project we worked with a European Grocery Retailer that generates an average of 60,000 planograms a year for over 400 stores. The company has complex merchandising rules for product allocation, which makes space planning a time consuming process, as it is manually executed by a team of space managers. Similarly to common practice, stores are grouped within clusters. A generic planogram is firstly created for each cluster and later it is manually adjusted, following dispatching rules, in order to fit the space and sales of each store. The goal of this project was to develop an approach that from the generic planogram automatically generates these store-specific planograms, while optimizing shelf inventory. From now on, we will call this process *Planogram Replication* and the generic planogram *Role Planogram*. An example of a replication process is shown in Figure 4.1. This paper presents the MIP model and the solution approach that have been successfully used by the company during the replication process. Its use made it possible to reduce stock levels, replenishment costs, and to reallocate the existing space team to manage categories more efficiently, by focusing less on the role planogram replication, and more on category analysis and market trends.

Replicating a generic planogram to a specific store mainly requires deciding how much space to allocate to each product in the new space. This is a challenging and relevant

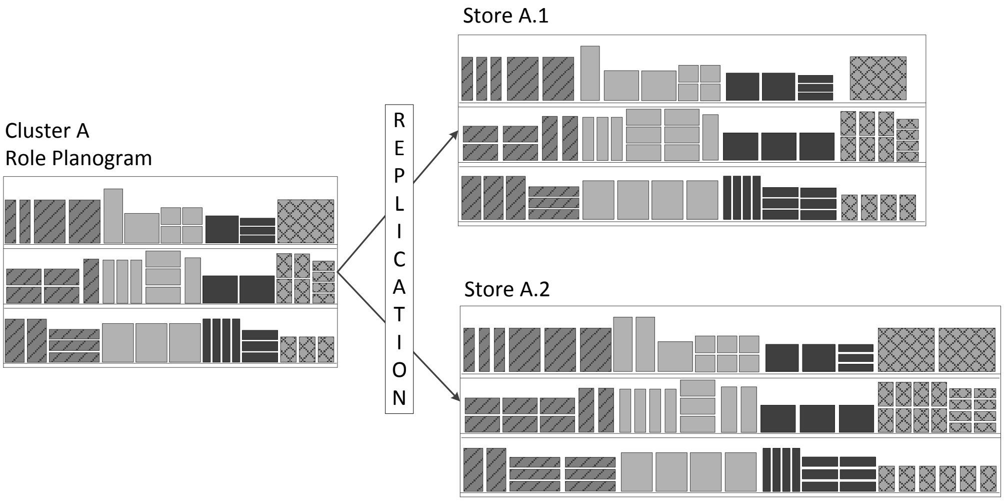

Figure 4.1 – Example of a replication process, from role planogram A to stores A.1 and A.2

problem for several reasons:

- To begin with, products are categorized as part of product families and, based on these families, merchandising rules often specify associations of products on the shelves, which should be kept in continuous rectangular shapes. Those rules depend on the retailers' strategy for each category, and therefore it is difficult to automatically generate planograms that accommodate all the specific details of each category, without any human interaction. By previously constructing a role planogram, a single moment of validation and manual adjustments is necessary for each cluster. Moreover, the replication process guarantees the standardization across the stores of the cluster.
- Merchandising rules frequently define multiple layers of product families which create many product alignments that need to be taken into consideration during the replication process. For example, a planogram can be organized firstly by brand, and secondly by format. Each brand, and each format within each brand, has a continuous rectangular shape. While replicating this planogram, we have to maintain these shapes. This fact makes it necessary to determine the exact location of products on the shelves, and poses many constraints to product facings. By hand, this alignment requires a trial and error facing allocation.
- Some planograms are not continuous from the beginning until the end, but instead the shelves may have misalignments and interruptions (see Figure 4.3). This fact poses additional constraints to product placement. Additionally, the way shelves are placed on the role planogram is not necessarily the way how shelves are placed in all stores. Consequently, replicating a planogram is not always a straightforward task and requires some initial analysis.

- Retailers have contracts with suppliers that commonly specify lower and upper bounds for the amount of shelf space allocated to each brand (space share). This is an additional requirement to take into consideration when determining product facings. Bearing in mind that some planograms have over 200 products, adjusting product facings to the stores' space and demand variations is not an easy nor time efficient task, and it can indeed benefit from specialized optimization methods.

The replication problem naturally falls into the Shelf Space Allocation Problems, which already present a strong body of literature. These problems focus mainly on the demand side, with complex polynomial profit functions that take into consideration the impact that space variables have on consumer demand. The basic shelf space allocation problem consists of maximizing profit by deciding how much space is required for each product, considering shelf capacity and lower and upper bound constraints for the products (Corstjens and Doyle [1981], Zufryden [1986]). More recently, other features have been introduced, such as the effects of product location on demand by Drèze et al. [1994], the use of a simplified linear objective function by Yang and Chen [1999], and the organization of products in families by Russell and Urban [2010]. Only the latter considers the exact location of the products on the shelves. This problem is also often integrated with other retail problems, such as assortment (Borin et al. [1994], Hübner and Kuhn [2011]) and pricing (Murray et al. [2010]). Despite the recent advances towards more practical models, Bai [2005] and Hübner and Kuhn [2012] still identify a misalignment.

The literature on shelf space allocation has focused less on the cost side of the problem, and most models do not explicitly consider inventory related decisions. Baker and Urban [1988] presented the first model that considered the demand dependent on the instantaneous inventory level of an item, based on the economic order quantity (EOQ) model. Several authors propose extensions to this model, implicitly assuming that all the inventory is displayed and has a direct impact on demand. Urban [1998] proposes the first attempt to include shelf space allocation in the inventory decision-making process, in a comprehensive model that integrates assortment, allocation and replenishment decisions. Other research by Hwang et al. [2005] and Hariga et al. [2007] additionally consider the effect of location on demand, and Abbott and Palekar [2008] includes replenishment costs but requires an initial space assignment as input. Nevertheless, these models have practical limitations: they consider continuous shelf replenishment operations from the backroom and determine individual product replenishment policies while, in practice, replenishment activities are not continuous but constrained to the shelf merchandisers available, and products usually have joint delivery cycles from warehouses (Hübner and Kuhn [2012]). Moreover, these inventory-related comprehensive models are only solved to optimality for a small number of products and to the best of our knowledge, there is not any approach in this inventory stream that tackles the existence of product families.

Our contributions are as follows. To the best of our knowledge, we are the first to introduce the replication problem in the shelf space literature. Therefore, we are establishing a rich novel problem within shelf space allocation that considers many practical requirements, including product families, minimum and maximum space shares and irregular shelf placements. Secondly, we present two mathematical programming formulations to this problem, with different levels of detail. The formulations use a novel inventory related objective function that is also a contribution from this paper. We do not determine order quantities, but instead we balance the products' inventory level in order to trigger joint shelf replenishments. Finally, we present a MIP based approach to ensure fast solutions in practice.

The remainder of the paper has the following structure. Section 4.2 formally describes the replication process and introduces the notation used in the remainder of the paper. Section 4.3 starts by describing the novel inventory related objective function, and then proposes two formulations for the new Shelf Space Replication Problem. Based on the formulations, section 4.4 describes a MIP based heuristic for the problem. This section also describes the practitioners' heuristic that inspired this work. Section 4.5 describes a computational study and section 4.6 draws conclusions.

4.2. Problem Description

The Replication Process consists of generating store-specific planograms for all stores of a cluster for which a role planogram has been previously generated. We assume that the role planogram conforms to all the necessary merchandising rules, and takes into consideration the space effects on consumer demand. Our objective is to create new planograms by adjusting the role planogram to fit the space and sales of all the stores, while keeping the rules present in the role planogram. Hence, the replication process focuses more on inventory control and less on maximizing space effectiveness.

When analyzing shelf inventory, a common metric used in practice is the *days-supply*, which measures the number of demand days covered by the shelf stock. This operational metric is also used to estimate the maximum number of days between two consecutive shelf replenishment operations. Additionally, if the retailer wants to reduce the backroom inventory, it can also be used to specify ordering policies. By assuming that D_i is the daily demand of product i, and S_i is the total shelf inventory, the resulting days-supply value, R_i, is obtained by equation (4.1).

$$R_i = \frac{S_i}{D_i} \tag{4.1}$$

Bearing in mind the expensive handling costs, the stores' limited resources that impose constraints to shelf replenishment, and the fact that products normally have joint delivery cycles from central warehouses, we consider the objective of balancing days-supply values across the products of each new planogram. By having all products covered for a similar number of days, we decrease the number of shelf replenishment operations, control the stock level for long-tale products, prevent stockouts for fast moving products, and we give the possibility of reducing backroom inventory (determined at a later stage, when selecting the inventory policy). Moreover, we meet common practices and evaluation metrics.

Because the space has a direct impact on consumer demand, we consider that the daily demand D_i is dependent on two factors: the visible inventory, i.e. the number of facings, and the shelves where the product is located. Cross-elasticities and horizontal effects are disregarded as cross-elasticities have parameters which are difficult to estimate in practice, and the horizontal effects have a lower impact on demand (according to Drèze et al. [1994]).

Despite the operational gains that balancing days-supply brings, merchandising rules are essential to give a meaningfully organization to the display, having a direct impact on the planograms' profitability. This is what the role planogram is used for. Therefore, the replication process is constrained to keep the rules in the role planogram, in particular:

- products should keep the same relative position as in the role planogram. For that purpose, products from the same shelf level should be placed at the same shelf level and with the same sequence.

- product families should keep their continuous rectangular shapes. The shapes' continuity within each shelf is already ensured by keeping the same sequence. The rectangular shape is obtained by vertically aligning the first and the last product of the shape on each shelf. Note that retailers may have exceptions to the rectangular shapes and one family can have more than two product alignments (for example, 'L" shapes have 3 alignments). Moreover, planograms usually have multi-levels of product families, resulting in a high number of alignments. Each product may also belong to more than one alignment.

Figure 4.2 presents an example of a role planogram, with 17 products and 3 product families. The new planograms have to keep products $A.1-A.4$, $B.1$ and $C.1-C.2$ on the first shelf level, products $A.5-A.7$ and $B.2$ on the second shelf, and products $A.8-A.9$, $B.3$ and $C.3-C.5$ on the third. For keeping the rectangular shape of the first family, products $A.1$, $A.5$ and $A.8$ have to be aligned on the left, and products $A.4$, $A.7$ and $A.9$ on the right. The same reasoning applies to the second and third families.

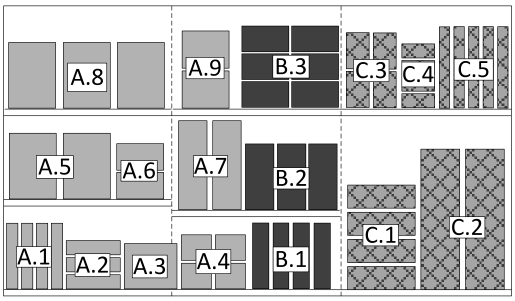

Figure 4.2 – Example of a role planogram.

Besides the rules present in the role planogram, there are also minimum and maximum display quantities to ensure, as well as minimum family space share requirements (due to supplier contracts) that must be taken into consideration.

Planograms are physically made of segments that are stacked together to form an aisle. Each segment has its own shelves that can be aligned with the shelves of the other segments, forming continuous long shelves from the beginning to the end of the planogram (hereafter called *levels*); or they can be placed differently, forming irregular levels that need to be taken into consideration while placing the products. Whenever a planogram has misalignments between the shelves we refer to it as *irregular planogram*, as opposed to *regular planogram*. Figure 4.2 presents an irregular planogram with 3 segments where two different irregular situations arise: shelves that belong to the same level but are misaligned (consequently, no products can be placed in between), and shelves that do not exist. Note that if the two planograms are not compatible in terms of the number of levels, replication is not possible.

4.2.1 Problem Definition and Notation

Assuming the aforementioned description, the problem is formally described as follows. Consider a specific category of a store whose space contains O segments (indexed by $o \in \mathcal{O}$) and K levels (indexed by $k \in \mathcal{K}$). Without loss of generality, the segments are numbered from left to right, and the levels are ordered from bottom to top. On the intersection of segment o with level k, there is the shelf (o, k). Shelves from irregular planograms need to be standardized in order to find the number of levels from these planograms. This involves two steps: if two misalignment shelves are close apart, they are placed at the same level and all levels have all shelves, even if in practice some shelves do not exist. Figure 4.3 presents the standardized version of the planogram from Figure 4.2 along with the parameters associated with the shelves, which are the following:

W	total width of the planogram,
w_{ok} (h_{ok})	width (height) of shelf (o, k),
e_{ok}	(=1) if shelf (o, k) exists in the planogram. (=0 otherwise),
c_{ok}	(=1) if shelf (o, k) is aligned with the following shelf $(o+1, k)$ (=0 otherwise),
n_{ok}	(=1) segment of the next existing shelf of level k after shelf (o, k) (if there is no next shelf, $n_{ok} = -1$).

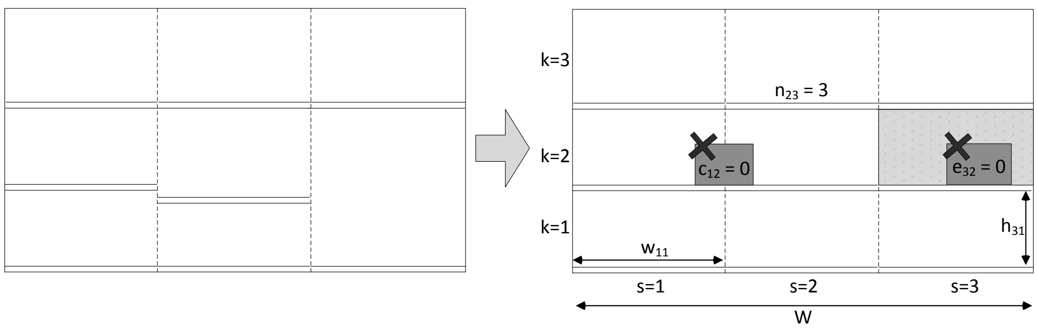

Figure 4.3 – Standardization of the planogram from Figure 4.2.

The retailer wishes to display N products (indexed by $i, j \in \mathcal{N}$) and the role planogram identifies the rules for their placement: the sequence of products on each shelf and the product families, defined by their left and right alignments. Consider M^L left-alignments and M^R right-alignments (indexed by $m \in \mathcal{M}^R \cup \mathcal{M}^L$). Each alignment has two or more products and no more than one product on each shelf. A small deviation v is allowed between the products of each alignment. If one product is placed in more than one shelf, it should also be vertically aligned, with the same number of facings in all shelves. There are also Q minimum space share requirements to consider (indexed by $u \in \mathcal{Q}$. The parameters and sets associated with the products are the following:

a_i (b_i)	width (height) of product i,
l_i (u_i)	lower bound (upper bound) on the number of facings of product i,
s_i	total stock of product i for each facing,
K_i	number of shelves where product i is,
$\mathcal{N}_k$	set of products of level k, ordered by order of appearance,
N_m^L (N_m^R)	set of products of each left (right) alignment m,
q_u	minimum percentage of space to allocate to the products belonging to the space share requirement u,
$\mathcal{N}_u^Q$	set of products of each minimum space share requirement u.

Under the given operating conditions, the decisions to be made for each product are: the number of facings to be displayed on each shelf and its horizontal location. Both decisions are determined while ensuring that the display area is fully occupied (*full shelf merchandising* policy). The objective is balancing days-supply, assuming inventory-dependent and shelf-dependent demand.

This new problem will hereafter be called *SSRP* – Shelf Space Replication Problem.

4.3. Model Development

This section presents the formulation proposed for the SSRP, as described in the previous section. For clarification purposes, we present two versions of the formulation: a simplified version that only considers one segment (Single-Segment Shelf Space Replication Model), and a more complex version with multiple segments (Multi-Segment Shelf Space Replication Model).

The necessary decision variables are presented along the section; however, two sets of decision variables define the overall problem solution (common to both formulations):

W_i	the integer number of facings of product $i \in \mathcal{N}$ on each of the shelves where the product is located,
X_i	the continuous horizontal location of product $i \in \mathcal{N}$, measured from the lower-left corner of the planogram to the lower-left corner of the first facing of the product.

Note that both the W and the X variables do not distinguish between shelf levels. This fact is because products are required to have a rectangular shape, which imposes the same number of facings in all shelves where the product is, and to be vertically aligned. Before presenting the two formulations, we will first define the objective function for balancing days-supply.

4.3.1 Objective Function

The objective of the SSRP is balancing days-supply values across the planogram, using equation (4.1) provided in the previous section. We consider a goal programming-based approach, where the idea is to minimize the deviation of R_i, the days-supply value of each product i, from a common days-supply goal R'. R' is determined to be the average of R_i values, and the objective function is similar to the variance (or the squared standard deviation). This objective function is provided in equation (4.2).

$$\text{minimize} \quad P = \sum_{i \in \mathcal{N}} (R_i - R')^2 \tag{4.2}$$
$$\text{where :} \quad R = \frac{\sum_{j \in \mathcal{N}} R_j}{N}$$

The days-supply of each product i is defined in terms of the product demand, D_i, which in turn depends on the space allocated to it. A diminishing return polynomial function (resembling a "s-shaped curve") has been widely used by several researchers in the literature to model space dependent demand. Similarly to Yang and Chen [1999] and Lim et al. [2004], we consider that retailers prefer to operate on the linear portion of the S-shaped curve, by defining minimum and maximum display quantities that place the demand on this linear part. In accordance, the demand can be determined by the linear equation (4.3), where D_i^0 is the base demand (for the minimum display quantity), γ_i and α_k are scale parameters that reflect the variation in demand with respect to the number of facings of product i and to the shelf k where the product is placed (respectively) and η_i is an additional parameter aggregates the shelf impact in case the product is placed on more than one shelf.

$$D_i = \eta_i \cdot (D_i^0 + \gamma_i \cdot K_i \cdot W_i) \tag{4.3}$$
$$\text{where :} \quad \eta_i = \frac{\sum_{k \in \mathcal{K}: i \in \mathcal{N}_k} \alpha_k}{K_i}$$

This objective function does not incorporate the time effect on demand and consequently, it does not consider a decreasing demand effect as the shelf inventory is being purchased. We consider instead that the minimum stock between shelf replenishments is enough to accommodate at least one item on each product facing, ensuring that the product is always fully visible. The days-supply values (R_i) are at last determined by equation (4.4).

$$R_i = \frac{s_i \cdot K_i \cdot W_i}{\eta_i \cdot (D_i^0 + \gamma_i \cdot K_i \cdot W_i)} \tag{4.4}$$

4.3.1.1 Alternative Objective Function

Figure 4.4 shows the days-supply values of two instances with 10 products, where each product is associated with two values: the days-supply value when the product has a minimum display quantity (in dark gray) and the days-supply value when it has the optimal display quantity (in black). The optimal days-supply values were obtained using objective function (4.2) and considering a single capacity constraint that ensured that the entire space was fully stocked. Both instances are similar, with the exception of one product that has a significantly higher minimum days-supply value in the second instance. The figure shows that, while in the first instance days-supply values are fairly close to each other, in the second instance they were influenced by the low-sales product (that increased the average), resulting in more spread values. Note that this situation occurs very often in shelf space allocation because of long-tale products. Moreover, the family alignments also constraint the products' facings and may lead to extreme differences in terms of days-supply values.

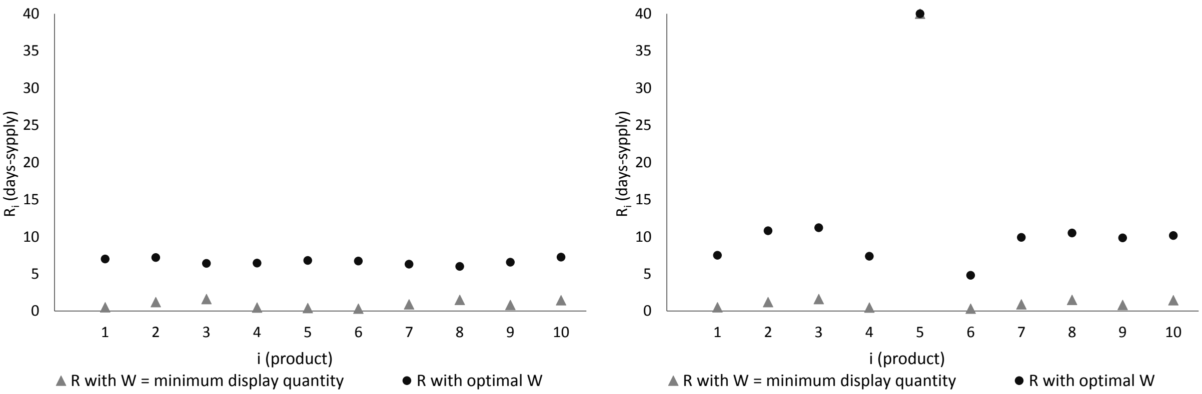

Figure 4.4 – Days-supply values of two instances with the optimal values obtained using objective function (4.2)

To overcome this fact, we propose an alternative objective function. Consider an iterative approach to the problem, in particular, an improvement heuristic which uses the minimum display quantities (minimum days-supply values) as the initial solution, and iteratively increases the product facings one at a time, choosing at each iteration the product with the lowest minimum days-supply value. The heuristic stops when the capacity is met or when there are no more products that can increase their facings due to the remaining constraints. This heuristic can be interpreted as the problem of maximizing the minimum days-supply values among the products. However, if one product is forced to have a low days-supply value, the iterative approach continues to maximize the minimum of the remaining products (as opposed to the problem of maximizing the minimum value, that does not optimize the remaining values). At the end, all products have close days-supply values, except the ones with high minimum days-supply values (or the ones that have to ensure alignments, when applicable). This is confirmed in Figure 4.5, where the same instances were solved with this improvement heuristic.

The alternative objective function tries to mimic this behavior. Consider the following binary decision variables, which correspond to a partition of variables W_i according to their range of facings (u_i):

W_{ip} (=1) if product $i \in \mathcal{N}$ has the p^{th} facing on the planogram, $p = 1, \ldots, u_i$.

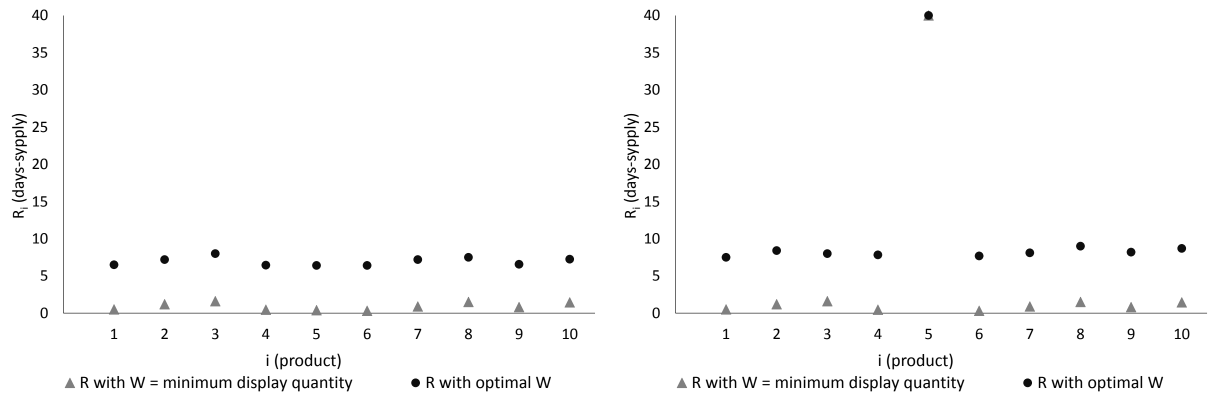

Figure 4.5 – Days-supply values of the same two instances of Figure 4.4 with the optimal values obtained using the improvement heuristic

The parameters R_{ip} are associated with W_{ip} and consist of the days-supply value of each product i prior to introducing the p^{th} facing (note that R_{ip} is a parameter because it does not depend any more on the number of facings). We mimic the iterative procedure by setting variables W_{ip} to one by descending order of the replenishment frequency $F_{ip} = 1/R_{ip}$ (for p=1, $F_{ip} > max\{F_{it}|t = 2, \ldots, u_i\}$), using the linear objective function (4.5).

$$\text{maximize} \quad \sum_{i \in \mathcal{N}} \sum_{p=1}^{u_i} F_{ip} \cdot W_{ip} \tag{4.5}$$

$$\text{where :} \quad W_i = \sum_{p=1}^{u_i} W_{ip}, \qquad \forall\, i \in \mathcal{N} \tag{4.6}$$

One drawback from this objective function is the fact that the model may not set the products by decreasing order of F_{ip} if there are severe differences in the products' width (if product i is twice the width of product j, then it is more advantageous to give two facings of product j instead of one to product i, as long as $F_{jp} + F_{j,p+1} > F_{ip}$). This is not a common case when replicating planograms where the products belong to the same category. Nevertheless, when that happens, it can be compensated with an extra coefficient on the objective function, which increases the F_{ip} value of the largest products.

4.3.2 Single-Segment Shelf Space Replication Model

In this subsection we formulate a simplified version of the replication problem where the segments are not considered, with the assumption that the planogram has only one segment. Besides W_i, W_{ip} and X_i, consider the following decision variables:

- L_i shelf length assigned to product $i \in \mathcal{N}$ on each of the shelves where the product is located,
- X_m^L the horizontal location of left alignment $m \in \mathcal{M}^L$,

X_m^R the horizontal location of right alignment $m \in \mathcal{M}^R$.

The Single-Segment Replication Model can be represented by the objective function present in equation (4.5) and the following remaining linear constraints:

$$K_i \cdot W_i \leq u_i, \quad \forall i \in \mathcal{N} \tag{4.7}$$
$$K_i \cdot W_i \geq l_i, \quad \forall i \in \mathcal{N} \tag{4.8}$$
$$L_i - a_i W_i \geq 0, \quad \forall i \in \mathcal{N} \tag{4.9}$$
$$\sum_{i \in \mathcal{N}_k} L_i = W, \quad \forall k \in \mathcal{K} \tag{4.10}$$
$$X_i = \sum_{j \in \mathcal{N}_k : j < i} L_j, \quad \forall k \in \mathcal{K}, i \in \mathcal{N}_k \tag{4.11}$$
$$X_m^L - v \leq X_i, \quad \forall m \in \mathcal{M}^L, i \in \mathcal{N}_m^L \tag{4.12}$$
$$X_m^L \geq X_i, \quad \forall m \in \mathcal{M}^L, i \in \mathcal{N}_m^L \tag{4.13}$$
$$X_m^R + v \geq X_i + L_i, \quad \forall m \in \mathcal{M}^R, i \in \mathcal{N}_k^R \tag{4.14}$$
$$X_m^R \leq X_i + L_i, \quad \forall m \in \mathcal{M}^R, i \in \mathcal{N}_k^R \tag{4.15}$$
$$\sum_{i \in \mathcal{N}_u} (a_i \cdot K_i \cdot W_i) \geq q_u, \quad \forall u \in \mathcal{Q} \tag{4.16}$$
$$W_i \in \mathbb{N}_0, \quad \forall i \in \mathcal{N} \tag{4.17}$$
$$L_i, X_i \geq 0, \quad \forall i \in \mathcal{N} \tag{4.18}$$
$$X_m^L \geq 0, \quad \forall m \in \mathcal{M}^L \tag{4.19}$$
$$X_m^R \geq 0, \quad \forall m \in \mathcal{M}^R \tag{4.20}$$

Constraints (4.7) and (4.8) impose the lower and upper bounds of the number of facings. Note that W_i only specifies the number of facings on each of the shelves where the product is placed, and it has to be multiplied by the number of shelves. Constraints (4.9) ensure that the shelves have enough space reserved for product facings. (4.10) are capacity constraints that additionally guarantee the *Full Shelf Merchandising* policy. As the shelves are fully occupied by the products, constraints (4.11) identify the horizontal location of the products as the sum of the lengths from the preceding products. These constraints also guarantee that the products have the desired sequence. The left and right alignments (with the tolerance v) are preserved by constraints (4.12)-(4.13) and (4.14)-(4.15), respectively. Constraints (4.16) introduce the space share requirements and the remaining constraints ensure the integrality and non-negativity properties of the variables.

4.3.3 Multi-Segment Shelf Space Replication Model

In the presence of more than one segment, the single-segment formulation may not be suitable as it does not take into account the non-existing shelves and the misalignments throughout the levels. Therefore, we have developed a multi-segment extension, where the decisions regarding the product shelf space and product location consider the existence of segments. Accordingly, consider the following additional variables:

L_{io} shelf length assigned to product $i \in \mathcal{N}$ in segment $o \in \mathcal{O}$, on each of the levels where the product is located,

Y_{io} (=1) if product $i \in \mathcal{N}$ is located in segment $o \in \mathcal{O}$,

X_{io} the continuous horizontal location of product $i \in \mathcal{N}$ in segment $o \in \mathcal{O}$, measured from the lower-left corner of the planogram.

In this extension, constraints (4.10) and (4.11) are replaced by:

$$\sum_{i \in \mathcal{N}_k} L_{io} = w_{ok} \cdot e_{ok}, \quad \forall\, o \in \mathcal{O}, k \in \mathcal{K} \tag{4.21}$$

$$L_{io} \leq Y_{io} \cdot w_{ok} \cdot e_{ok}, \quad \forall\, o \in \mathcal{O}, k \in \mathcal{K}, i \in \mathcal{N}_k \tag{4.22}$$

$$Y_{in} + Y_{i+1,o} \leq 1, \quad \forall\, o \in \mathcal{O}, n \in \mathcal{O} : n > o, k \in \mathcal{K}, i \in \mathcal{N}_k^- \tag{4.23}$$

$$X_{io} = \sum_{n \in \mathcal{O}: n < o} w_{nk} + \sum_{j \in \mathcal{N}_k : j < i} L_{io}, \quad \forall\, o \in \mathcal{O}, k \in \mathcal{K}, i \in \mathcal{N}_k \tag{4.24}$$

$$Y_{io} + Y_{i,next_{ok}} \leq 1 + c_{ok}, \quad \forall\, o \in \mathcal{O} : next_{ok} > 0, k \in \mathcal{K}, i \in \mathcal{N}_k \tag{4.25}$$

$$b_i \cdot Y_{io} \leq h_{ok}, \quad \forall\, o \in \mathcal{O}, k \in \mathcal{K}, i \in \mathcal{N}_k \tag{4.26}$$

$$X_i \leq X_{io} + W \cdot (1 - Y_{io}), \qquad \forall i \in \mathcal{N}, \forall o \in \mathcal{O} \tag{4.27}$$

$$X_i \geq X_{io} - \sum_{n \in \mathcal{O}: n < o} L_{in} - W \cdot (1 - Y_{io}), \qquad \forall i \in \mathcal{N}, \forall o \in \mathcal{O} \tag{4.28}$$

$$L_i = \sum_{o \in \mathcal{O}} L_{io}, \qquad \forall i \in \mathcal{N} \tag{4.29}$$

$$L_{io}, X_{io} \geq 0, \quad \forall\, i \in \mathcal{N}_k, o \in \mathcal{O} \tag{4.30}$$

$$Y_{io} \in \{0, 1\}, \quad \forall\, i \in \mathcal{N}_k, o \in \mathcal{O} \tag{4.31}$$

Note that the sets $\mathcal{N}_k^-$ have all the products from each level k except the last. Constraints (4.21) are similar to the previous constraints (4.10) and ensure that the full width of the existing shelves is occupied by the products. Constraints (4.22) relate variables L_{io} and Y_{io} by stating that the length L_{io} is equal to zero in case product i is not assigned to segment o. These constraints also guarantee that a product is not assigned to a shelf that does not exist in the store. The product sequence of each level is ensured by constraints (4.23) and (4.24). The first set of constraints ensures the sequence between segments and the second set ensures the sequence inside each segment. The latter also specifies the horizontal location of each product inside each segment. Note that this variable only has a meaningful value if the product is part of the segment. Constraints (4.25) do not allow a product to be on both shelves k and n_{ko} in two cases: if the shelves are not consecutive, or if the shelves are misaligned. Constraints (4.26) prevent products from being placed on shelves where they do not fit (because of their height), and finally, the values of L_i and X_i are specified with regard to L_{io} and X_{io}, respectively.

The consideration of segments increases the complexity of the formulation, as we will analyze later, during the computational results. Its motivation is the necessity to take into account the non-existing shelves and misalignments thought the levels, that impose additional constraints to the product's placements. However, potential benefit may be achieved by aggregating the segments that have similar shelf placements. In the more extreme case, when all segments are equal, we obtain the Single-Segment SSRP formulation. Therefore, both the Single and Multi-Segment formulations have practical relevance.

4.4. MIP-based Heuristic

The SSRP is a practical and relevant operational problem in retail and it is crucial to develop methods that can allow its use in practice. Because the SSRP is a mid-term operational planning activity, retailers do not need real time solutions obtained in less than a few seconds. Nevertheless, the high number of planograms that Space Managers have to handle every year is not compatible with high generation times and the European Food Retailer that motivated this work defined "the time to have a cup of coffee" as the maximum period of time space managers would be willing to wait for the solutions.

This section starts by describing the methodology currently used by the Space Managers at the European Food Retailer. Then, we propose a MIP-based Heuristic for the problem, based on the formulation presented in section 4.3.

4.4.1 Methodology Currently Used

Replicating planograms is a manual and time consuming activity, mainly because of the different product family alignments that need to be taken into consideration when placing the products. The role planogram is, by definition, smaller than the remaining planograms, and the ultimate decision that Space Managers have to make is on the number of facings that each product will have. Nevertheless, aligning products is most of the time a trial and error activity, and managers try to give one extra facing to a product, take one facing from of a product or spread out the facings, so that all alignments are strictly fulfilled. The company has a space planning software that assists in this task by providing realistic views of the shelves to where the products are drag and dropped to create the planograms. This software also has powerful analysis tools.

In general terms, the process is as follows. Space Managers start by copying the role planogram to the new space, which leaves an empty space to fill. Product facings are then iteratively increased based on the product's days-supply values, in an approach similar to the iterative procedure described in section 4.3.1. The process continuously alternates between shelves for preserving the alignments. The days-supply values are automatically computed by the space planning software and are only based on past sales. Therefore, the company does not explicitly take into account the impact that the space has on demand. Moreover, the high number of alignments strongly complicates the process, making it difficult to balance days-supply values.

4.4.2 MIP-based Heuristic

Preliminary experiments showed us that by solving the problem using a commercial MIP solver (ILOG CPLEX in our implementation) we were able to generate optimal solutions within the expected period, but only for small sized instances. Therefore, our goal when developing the MIP-based heuristic was to make sure that the approach was scalable, especially in the multi-segment case.

Table 4.1 presents the number of decision variables and constraints of both formulations (single and multi-segment) with regard to the problem parameters. Two parameters arise as the most important: the number of segments and the number of products. However, we can also relate the number of segments to the number of products (or vice versa) because the more space is available, the more products the assortment has.

Table 4.1 – Size of the formulations as a function of the problem parameters

	Single-segment SSRP	**Multi-segment SSRP**
# Decision Variables	$\sum_i u_i$	$\sum_i u_i$
# Constraints	$3N + 2\sum_m N_m^R + 2\sum_m N_m^L + Q$ $+ \sum_k N_k + K$	$4N + 2\sum_m N_m^R + 2\sum_m N_m^L + Q$ $+O(K + 2N + \sum_k N_k + \frac{1}{2}O\sum_k N_k)$

Figure 4.6 depicts the general idea of the proposed MIP-based heuristic which has three main steps. The first step originates an initial solution for the problem with only the minimum display quantities for the products. The second step iteratively adds the remaining product facings to the planogram until no more space is available. At last, the third step tries to improve the solution by allowing the removal and insertion of new facings. As it is possible to see, this MIP-based heuristic is inspired in the methodology currently used by space managers at Sonae MC.

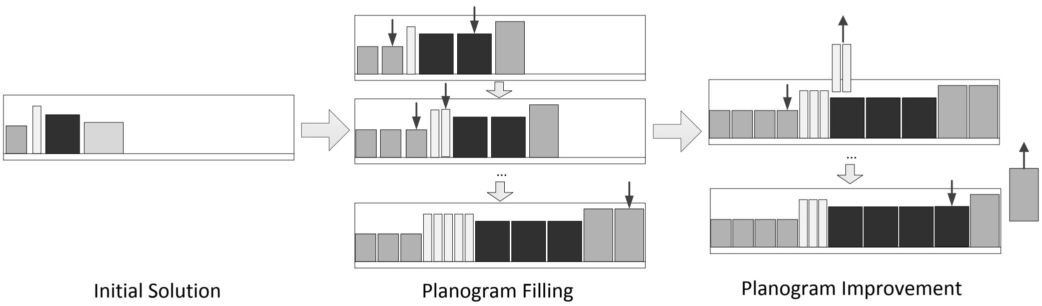

Figure 4.6 – Steps of the MIP-based Heuristic (for one shelf)

Technically, this approach is an integration of two well-known MIP-based improvement heuristics: *fix-and-optimize* (Pochet and Wolsey [2006]) and *local branching* (Fischetti and Lodi [2003]). In each iteration, we solve the SSRP formulation with the W_{ip} variables partially constrained in one of two different ways: a subset of the variables are fixed to the values obtained in the incumbent solution (*fix-and-optimize*), or a limited number of changes can be made to the values obtained in the incumbent solution (*local branching*). Figure 4.7 depicts the evolution of W_{ip} variables throughout the iterations. The variables are divided into K subsets $\mathcal{G}_k$, depending on the products on each shelf k: $W_{ip} \in \mathcal{G}_k : i \in \mathcal{N}_i$. Each set has the variables sorted by increasing days-supply values (R_{ip}), so that the products with low days-supply values will be considered first. Within each subset, the first variables to be analyzed correspond to the minimum display quantities for the products (i.e. the first l_i variables of each product i). At each iteration, the variables in gray are the ones optimized by the formulation. The remaining ones are already fixed to zero (white variables) or to one (dark gray variables).

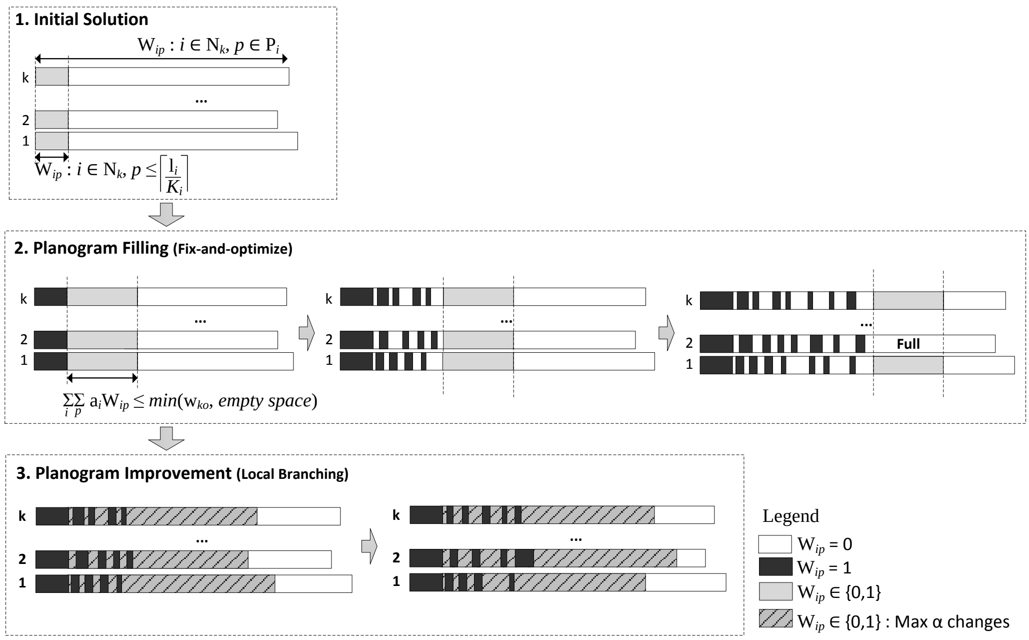

Figure 4.7 – State of W_{ip} variables throughout the iterations

Step 1 - Initial Solution – In the first step, the formulation is solved with the number of product facings limited to the minimum display quantities (first part of the $\mathcal{G}_k$ subsets), resulting in an initial, feasible, but still empty planogram. As the number of facings of each product i should be the same across all the shelves where the product is, the number of facings is indeed limited to the first multiple of K_i, greater than or equal to l_i.

Step 2 - Planogram Filling – In the second step, the variables considered at each iteration are limited to a portion of each subset $\mathcal{G}_k$, so that the space occupied by adding those facings does not exceed a maximum length, which we defined as the minimum value between the empty space and the length of one segment. This step ends when the space available is insufficient for any extra facing, or at least one W_{ip} variable was set to zero in all products. This iterative procedure ensures the scalability of the method, and is not expected to strongly deteriorate the solution as the variables are introduced in the formulation according to their expected order of usage.

Step 3 - Planogram Improvement – In the third step, a subset $\mathcal{S}$ of the W_{ip} variables is optimized but limited to a maximum of α changes to the values obtained in the incumbent solution (W'_{ip}). This improvement phase is done by adding constraint (4.32), which counts the number of changes on the variables and limits them to α. As the formulation would not set W_{ip+1} to one without setting W_{ip} first, the subset $\mathcal{S}$ contains, at each time, the β variables with lowest days-supply values (R_{ip}) previously set to one and the β variables with the highest R_{ip} values previously set to zero.

$$\sum_{W_{ip}\in\mathcal{S}:W'_{ip}=1}(1-W_{ip})+\sum_{W_{ip}\in\mathcal{S}:W'_{ip}=0}W_{ip}\leq\alpha \tag{4.32}$$

Note that this MIP-based heuristic is valid for both formulations. The pseudocode is present in Appendix 4.A.

4.5. Experimental Results

This section presents a computational study to assess this novel SSRP problem. In particular, we will focus on the suitability of the formulation and the MIP-based heuristic to solve the practical problems for which they were designed. For that purpose, we tested and validated the approaches using real data provided by the company, but some of the parameters were masked to protect the company's confidentiality. The test data include 20 role planograms ($A-T$) that were replicated in three stores ($1-3$). Only the first corresponds to a real store and the remaining ones were obtained by increasing one segment in store 2 and two segments in store 3 (each segment with approximately 130 cm). Table 4.2 presents the key information of each instance, namely the number of products (N), the number of segments of store 1 (O), the number of segments with different shelf placements (O'), and lastly, the number of left and right alignments (M^L and M^R, respectively). Note that the instances vary from 26 to 256 products and from 1 to 13 segments, with the expected positive correlation between the number of segments and the number of products. In general, role planograms have a high number of family groups which result in up to 47 alignments. The instances are available online in Bianchi-Aguiar et al. [2015].

Table 4.2 – Problem Instances

Regular Planograms						Irregular Planograms					
Instance	N	O	O'	M^L	M^R	Instance	N	O	O'	M^L	M^R
$A_{\{1,2,3\}}$	26	1	1	8	6	$M_{\{1,2,3\}}$	122	3	2	17	18
$B_{\{1,2,3\}}$	45	2	1	6	5	$N_{\{1,2,3\}}$	114	5	2	17	17
$C_{\{1,2,3\}}$	16	3	1	3	5	$O_{\{1,2,3\}}$	239	12	2	30	47
$D_{\{1,2,3\}}$	38	3	1	7	7	$P_{\{1,2,3\}}$	239	13	2	21	27
$E_{\{1,2,3\}}$	49	3	1	3	4	$Q_{\{1,2,3\}}$	154	5	3	44	43
$F_{\{1,2,3\}}$	190	3	1	20	12	$R_{\{1,2,3\}}$	172	8	3	27	33
$G_{\{1,2,3\}}$	240	3	1	32	35	$S_{\{1,2,3\}}$	256	13	4	36	33
$H_{\{1,2,3\}}$	188	5	1	12	11	$T_{\{1,2,3\}}$	156	9	5	24	39
$I_{\{1,2,3\}}$	205	6	1	12	13						
$J_{\{1,2,3\}}$	107	8	1	11	10						
$K_{\{1,2,3\}}$	67	9	1	10	12						
$L_{\{1,2,3\}}$	171	12	1	23	23						

The parameters R_{ip} are provided in the instances. These values were obtained using equation (4.4) and considering the company sales forecast for the products as the base demand (D_i^0). The forecasts already incorporated the impact of the products' usual location and space and, due to the lack of additional information, the scale parameters ηi and γi were set to 1 and 0, respectively. The product base demand for stores 2 and 3 (indexed by l) was generated according to formula (4.33), creating normally distributed values with mean D_{i1}^0 (base demand from store 1) and standard deviation σ_i, which we considered was 25% of the forecast. For that purpose, δ_{il} is a normally distributed random number with zero expectation and one unit of variance.

$$D_{il}^0 = D_{i1}^0 + \sigma_i \cdot \delta_{il} \tag{4.33}$$

All computational experiments were conducted on Intel @2.40GHz processing units limited to 4.0Gb of Random Access Memory, using the Linux operating system. The IBM ILOG CPLEX 12.4 was used as the mixed integer solver. The formulations and the MIP-based approach were implemented in C++, compiled with the gcc compiler and solved using CPLEX with the Concert Technology library. The overall execution time was limited to 300 seconds, and each iteration of the heuristic was also limited to 90 seconds during planogram filling (step 1 and 2), and 10 seconds during the improvement phase (step 3).

Table 4.3 summarizes the results obtained for the instances considered, grouped into regular and irregular planograms, and into stores 1, 2 and 3. For each group we report the performance of the approaches in terms of the average optimality gap - deviation of the best integer solution found from the best upper bound available (*GAP*), average execution time in seconds (T) and number of instances with no solutions found within the time limit (IS). The instances were solved using three approaches: the multi-segment SSRP formulation (*M-SSRP*), the multi-(single-) SSRP formulation with the aggregation of the segments that have similar shelf placements (*J-SSRP*), and finally the MIP-based heuristic with the *J-SSRP* (*J-SSRP-H*). The optimality gap of the MIP-based heuristic is measured with respect to the upper bound obtained when solving the *J-SSRP* formulation. The detailed information is presented in Table 4.4 of Appendix 4.B.

A straightforward use of the multi-segment formulation is not suitable for the practical requirements, as the solver was not able to find one feasible solution within the time limit for 28 out of 60 instances. Nevertheless, in those instances where the solver did find at least one feasible solution, the optimality gaps were on average below 0.14% (with a maximum of 1.45%). By joining the segments with similar shelf placements (in a pre-processing step), the results improved significantly, both in terms of execution times (which were reduced by 40%) and quality of the solution (whose optimality gap decreased on average 0.11%). Note that this is obtained without compromising the use of an exact approach to the problem. The MIP-based heuristic further reduced the execution times by 88%; however, it already presents a trade-off to the solution quality, as the GAP increased from 0.03% to 0.19% (comparatively to *J-SSRP*).

When comparing the regular and irregular planograms, the latter had, on average, higher execution times. In the *M-SSRP* approach, this increase is not so significant (18%) and is most probably caused by the fact that irregular planograms have on average more

Table 4.3 – Summary of results (full results on Table 4.4 in Appendix 4.B)

		M-SSRP			J-SSRP			J-SSRP-H		
		GAP	T (sec)	#IS*	GAP	T (sec)	#IS*	GAP	T (sec)	#IS*
Regular Planograms	Store 1	0.05%	178.8	6	0.03%	90.8	0	0.19%	8.8	0
	Store 2	0.03%	200.9	6	0.02%	92.1	0	0.21%	7.0	0
	Store 3	0.28%	208.5	7	0.02%	101.5	0	0.24%	5.8	0
	Total	*0.12%*	*196.4*	*19*	*0.02%*	*95.0*	*0*	*0.21%*	*7.2*	*0*
Irregular Planograms	Store 1	0.21%	197.0	2	0.05%	175.9	0	0.14%	27.2	0
	Store 2	0.15%	261.8	3	0.04%	149.4	0	0.11%	29.0	0
	Store 3	0.08%	263.8	4	0.03%	210.9	0	0.17%	26.3	0
	Total	*0.16%*	*239.9*	*9*	*0.04%*	*177.3*	*0*	*0.14%*	*27.5*	*0*
Total		**0.14%**	**213.1**	**28**	**0.03%**	**126.5**	**0**	**0.19%**	**15.0**	**0**

* IS – Number of instances with no solution found within the time limit.

segments (and more products as well). However, in the *J-SSRP* and *J-SSRP-H* approaches, this increase is higher (46% and 74%, respectively) because regular planograms use the Single-Segment formulation. When looking at the increasing number of segments for the same role planogram (from store 1 to store 3), although the execution times tend to increase in stores 2 and 3, a significant increase was not perceived.

The plot presented in Figure 4.8 shows that the execution times of the *J-SSRP-H* approach increase linearly with the number of products times the square of the number of segments (the instances T were considered outliers and removed from the plot due to their fast results). This independent variable ($N \cdot O^2$) is actually derived from the highest term in the number of constraints (Table 4.1), and corroborates the initial findings about the impact of the number of segments and products. Therefore, other practical instances, even if bigger, are expected to increase the execution times linearly.

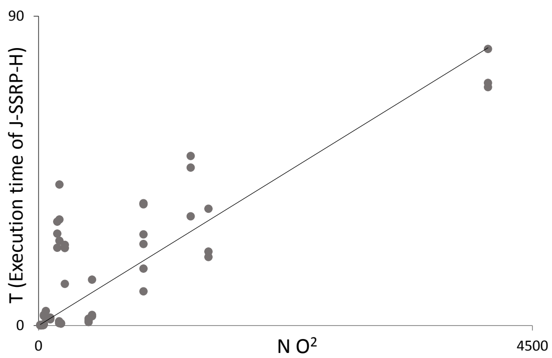

Figure 4.8 – Plot of the execution times of *J-SSRP-H* in function of the number of products (N) and the number of segments (O)

Figure 4.9 presents part of the solution from instance T_1. This instance has 5 rectangular product families, highlighted in the figure, with a small "L" shape at the end. A total of 41 products are placed in the planogram, with the overall space and the days-supply values also identified in the figure (the facings of each product are not visible). Days-supply values are balanced as much as possible throughout the planogram, as all family alignments have to be strictly followed.

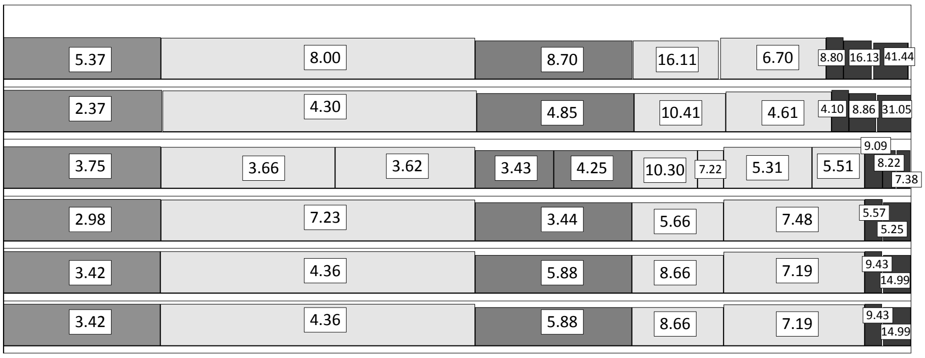

Figure 4.9 – Part of the solution from instance T_1

We compared the solutions (only store 1 instances,using the *J-SSRP-H* approach) to the hand-made planograms generated by the Space Managers of the European Food Retailer. Figure 4.10 shows for each instance the variation in the resulting days-supply values, both in terms of average and standard deviation. The majority of the solutions suffered a reduction in the two dimensions, with an average reduction of 16.6% and 13.1%. The size of the circles in the figure indicate the density of each planogram in terms of products and alignments, measured as $N \cdot (M^L + M^R)/O$. As expected, it shows that planograms with higher density have lower reductions in term of the two days-supply dimensions. Note that these findings are only estimates, as the space manager from the case study may have taken other requirements into consideration which were not considered in this study.

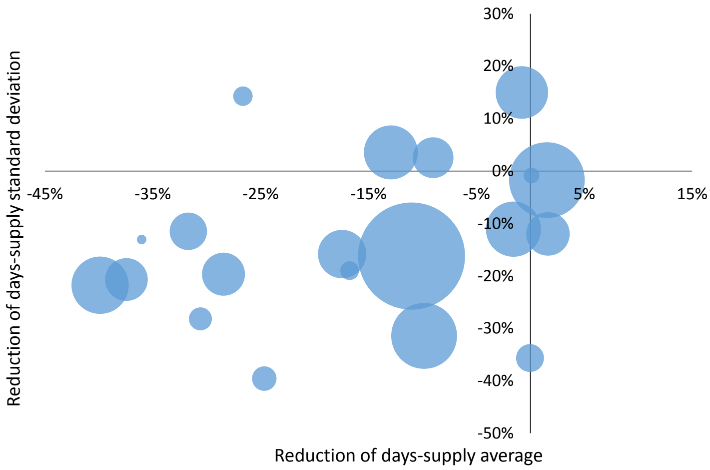

Figure 4.10 – Reduction of days-supply average and standard deviation, compared to hand-made planograms

4.6. Conclusions

In this paper we have presented the retail problem of transforming generic cluster-based shelf space solutions (role planograms) into store-specific planograms. This problem is very challenging because of merchandising rules, present in the role planogram, which must be considered in the new planograms. This is a practical problem faced by most retailers that are forced to cluster their stores in order to manage categories efficiently. To the best of our knowledge, we are the first to introduce this problem, which we call Shelf Space Replication Problem (SSRP), in the shelf space literature.

We present two mathematical programming formulations to solve the SSRP: the single- and multi- segment, with the latter having misaligned shelves along the planogram. The formulations use a novel inventory related objective function that is also a key-contribution from this paper. Instead of determining individual order quantities for the products, the formulation balances the products' inventory level in order to trigger joint shelf replenishments. This objective is in line with common practices and evaluation metrics. To ensure fast solutions in practice, we also present a MIP-based heuristic based on the formulation, that uses two state-of-the-art techniques: *fix-and-optimize* and *local-branching*.

To test both formulations and the MIP-based heuristic, we used real data from the European Food Retailer that motivated this project. The results were encouraging. By performing a pre-processing step, we were able to generate a solution for all the instances with an average optimality gap of 0.03% in an average of 126.5 seconds. The MIP-based heuristic explored a trade-off between execution times and solution quality, by reducing 88% of the time with an optimality gap of 0.19%.

The fact that the replication process is based on a role planogram makes the problem suitable for most retail companies. By creating awareness on the problem, we hope to stimulate further research and encourage the use of optimization in the practice of shelf space management. More specifically, this paper opens the following opportunities for future research. Other replication methods can be studied, with more evasive changes to the role planogram (for instance, making it possible to change the product shelf). The problem could also benefit from deeper analysis of the new objective of balancing days-supply, and from further integrating shelf-replenishment and shelf-inventory policies. Finally, on the algorithmic side, further work could be done to generate on-demand solutions, for instance, by using heuristics.

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Appendix 4.A Pseudocode of the MIP-based heuristic

The MIP-based heuristic has two parts, the first corresponding to steps 1 and 2 (*Initial Solution* and *Planogram Filling*) and the second part to step 3 (*Planogram Improvement*), whose pseudocode is presented in Algorithms 3 and 4, respectively. Both parts successively run the formulation from section 4.3 (*SSRP*) with some W variables fixed or additionally constrained based on a set of subsets that evolve throughout the iterations. The first part of the algorithm divides each set $\mathcal{G}_k$ into three subsets: variables not yet considered and fixed to zero ($\mathcal{V}_k^0$); variables that will be considered in the next iteration and can take either the value 0 or 1 ($\mathcal{V}_k^C$); and variables that are fixed to the values obtained in incumbent solutions ($\mathcal{V}_k^F$). The second part of the algorithm considers three different subsets: variables fixed to one in the incumbent solution ($\mathcal{S}^1$); variables fixed to zero in the incumbent solution ($\mathcal{S}^0$); and variables that will be considered in the next iteration whose values can change until a maximum of α changes ($\mathcal{S}$).

Algorithm 3: Pseudocode of the MIP-based heuristic (Steps 1 and 2)

begin
    $l \leftarrow 1$
    $\mathcal{G}_k \leftarrow W_{ip} : i \in \mathcal{N}_k, p \in P_i$ (variables from products on shelf k, sorted by decreasing value of F_{ip})
    $\mathcal{V}_k^F \leftarrow \emptyset$ (empty sets)
    $\mathcal{V}_k^C \leftarrow W_{ip} \in \mathcal{G}_k : p \leq \left\lceil \frac{l_i}{K_i} \right\rceil$ (variables corresponding to the minimum display quantities)
    $\mathcal{V}_k^0 \leftarrow \mathcal{G}_k \backslash \{\mathcal{V}_k^C\}$ (remaining variables)
    while $\mathcal{V}_l^C \neq\notin \emptyset$ *or there is at least one i with no variables fixed to zero* **do**
        Solve $SSRP_l : W = W', \forall k, W \in \mathcal{V}_k^F; W = 0, \forall k, W \in \mathcal{V}_k^0$
        W' := W (save incumbent solution)
        $\mathcal{V}_k^F := \mathcal{V}_k^F \cup \mathcal{V}_k^I$
        $\mathcal{V}_k^C :=$ next variables from $\mathcal{G}_k$ until $\sum_i \sum_p a_i W_{ip} \leq min(w_{ko},$empty space)
        $\mathcal{V}_k^0 := \mathcal{V}_k^0 \backslash \{\mathcal{V}_k^C\}$
        $l \leftarrow l + 1$
    end
end

Appendix 4.B Results

This section presents the detailed results obtained for all the instances presented in section 4.5, using approaches *M-SSRP*, *J-SSRP* and *J-SSRP-H*. For each instance, we provide information about the best integer solution found (Z), the total execution time in seconds (T) and the deviation of the best integer solution found from the best upper bound available at the stopping criteria - optimality gap (*GAP*). In the MIP-based heuristic (*J-SSRP-H*), the optimality gap is obtained with regard to the upper bound from the *J-SSRP* approach.

Algorithm 4: Pseudocode of the MIP-based heuristic (Step 3)

begin
  $\mathcal{S}^0 \leftarrow W \in \mathcal{G}_k : W' = 0$ (variables fixed to 0, sorted by decreasing value of F_{ip})
  $\mathcal{S}^1 \leftarrow W \in \mathcal{G}_k : W' = 1$ (variables fixed to 1, sorted by decreasing value of F_{ip})
  while *W' different than previous solution or maximum number of iterations* **do**
    $\alpha \leftarrow$ update maximum number of variables that can be changed
    $\beta \leftarrow$ update number of variables to consider
    $\mathcal{S}^C \leftarrow$ last β variables from set $\mathcal{S}^1$ and first β variables from set $\mathcal{S}^0$
    Solve $SSRP_l : W = W', \forall k, W \in \mathcal{S}^1_k \cup \mathcal{S}^0_k$;
    $\sum W \in \mathcal{S} : W' = 1(1 - W_{ip}) + \sum_{W \in \mathcal{S}: W'=0} W_{ip} \leq \alpha$
    W' := W (save incumbent solution)
    $l \leftarrow l + 1$
  end
end

Table 4.4 – Results

Name	*M-SSRP*			*J-SSRP*			*J-SSRP-H*			Name	*M-SSRP*			*J-SSRP*			*J-SSRP-H*		
	Z	GAP	*T(s)*	*Z*	GAP	*T(s)*	*Z*	GAP	*T(s)*		*Z*	GAP	*T(s)*	*Z*	GAP	*T(s)*	*Z*	GAP	*T(s)*
A_1	54564	0.00%	0	54564	0.00%	0	54564	0.00%	0.0	K_1	*	*	^{Tl}	419175	0.01%	93.2	418025	0.28%	4.2
A_2	79822	0.00%	0	79822	0.00%	0	79822	0.00%	0.1	K_2	*	*	^{Tl}	528489	0.01%	4.9	526827	0.32%	2.2
A_3	76839	0.01%	1	76839	0.00%	0	76839	0.00%	0.1	K_3	*	*	^{Tl}	433809	0.01%	31.4	432422	0.33%	2.3
B_1	140909	0.00%	0	140901	0.01%	0	140690	0.16%	0.1	L_1	*	*	^{Tl}	581002	0.04%	^{Tl}	578295	0.51%	30.2
B_2	272817	0.01%	7	272816	0.01%	0	272111	0.27%	0.4	L_2	*	*	^{Tl}	727802	0.03%	^{Tl}	725282	0.38%	26.7
B_3	315621	0.00%	6	315626	0.00%	0	315006	0.20%	0.4	L_3	*	*	^{Tl}	649167	0.03%	^{Tl}	647023	0.36%	22.6
C_1	18809	0.01%	0	18809	0.00%	0	18765	0.23%	0.1	M_1	25085	0.01%	6.7	25085	0.01%	5.8	25082.4	0.02%	2.7
C_2	16250	0.01%	1	16250	0.00%	0	16231	0.12%	0.2	M_2	22557	0.08%	^{Tl}	22557	0.01%	88.3	22553	0.03%	13.4
C_3	22863	0.01%	2	22863	0.01%	0	22762	0.45%	0.2	M_3	18577	1.45%	^{Tl}	18795	0.03%	^{Tl}	18777	0.13%	3.1
D_1	419687	0.01%	2	419687	0.01%	0	419567	0.04%	0.1	N_1	63997	0.01%	1.4	63997	0.01%	0.2	63996.8	0.01%	1.0
D_2	499451	0.01%	3	499451	0.01%	0	499254	0.05%	0.1	N_2	61646	0.01%	19.9	61646	0.01%	0.7	61626.7	0.04%	2.0
D_3	449704	0.01%	2	449704	0.01%	0	449446	0.07%	0.2	N_3	80153	0.01%	46.7	80153	0.01%	0.7	80129.7	0.04%	1.5
E_1	19620	0.01%	42	19620	0.01%	14	19598	0.12%	2.8	O_1	*	*	^{Tl}	1103960	0.02%	^{Tl}	1100970	0.29%	26.5
E_2	22726	0.03%	^{Tl}	22726	0.01%	89	22708	0.09%	3.1	O_2	*	*	^{Tl}	1353880	0.01%	103.6	1352040	0.15%	16.6
E_3	21147	0.22%	^{Tl}	21148	0.01%	27	21126	0.12%	0.4	O_3	*	*	^{Tl}	1193150	0.01%	151.6	1190840	0.20%	9.9
F_1	23649	0.24%	^{Tl}	23654	0.09%	^{Tl}	23653	0.09%	41.0	P_1	*	*	^{Tl}	239139	0.09%	^{Tl}	238591	0.32%	23.7
F_2	*	*	^{Tl}	30281	0.08%	^{Tl}	30204	0.33%	24.7	P_2	*	*	^{Tl}	216715	0.08%	^{Tl}	216224	0.30%	35.6
F_3	*	*	^{Tl}	24704	0.04%	^{Tl}	24667	0.20%	30.9	P_3	*	*	^{Tl}	327311	0.04%	^{Tl}	326921	0.16%	35.3
G_1	*	*	^{Tl}	159537	0.12%	^{Tl}	159248	0.30%	23.5	Q_1	56529	0.01%	67.8	56536	0.01%	123.1	56526	0.03%	46.0
G_2	*	*	^{Tl}	168879	0.09%	^{Tl}	168442	0.35%	22.6	Q_2	65902	0.01%	274.0	65908	0.01%	50.1	65902.9	0.02%	49.3
G_3	*	*	^{Tl}	202660	0.06%	^{Tl}	201922	0.43%	12.1	Q_3	55583	0.05%	^{Tl}	55591	0.01%	123.5	55554.8	0.08%	31.8
H_1	*	*	^{Tl}	52992	0.01%	6	52924	0.14%	0.9	R_1	314363	0.41%	^{Tl}	314481	0.12%	^{Tl}	314292	0.18%	34.0
H_2	54817	0.11%	^{Tl}	54852	0.01%	7	54702	0.28%	1.2	R_2	446907	0.45%	^{Tl}	447152	0.05%	^{Tl}	446776	0.14%	19.9
H_3	*	*	^{Tl}	74515	0.01%	6	74389	0.18%	0.7	R_3	*	*	^{Tl}	382637	0.05%	^{Tl}	381840	0.26%	21.5
I_1	*	*	^{Tl}	1435310	0.01%	0	1433430	0.14%	0.6	S_1	139501	0.79%	^{Tl}	139860	0.11%	^{Tl}	139682	0.24%	69.4
I_2	*	*	^{Tl}	1453780	0.01%	14	1452370	0.11%	0.7	S_2	*	*	^{Tl}	158051	0.10%	^{Tl}	157861	0.22%	80.5
I_3	*	*	^{Tl}	1282900	0.01%	3	1280760	0.18%	0.6	S_3	*	*	^{Tl}	189182	0.09%	^{Tl}	188719	0.33%	70.6
J_1	*	*	^{Tl}	450872	0.01%	76	449773	0.25%	1.8	T_1	647848	0.03%	^{Tl}	647851	0.01%	77.9	647651	0.04%	14.4
J_2	*	*	^{Tl}	520909	0.01%	89	519793	0.22%	2.3	T_2	669144	0.22%	^{Tl}	669354	0.01%	52.6	669244	0.03%	14.6
J_3	*	*	^{Tl}	383631	0.01%	51	381684	0.52%	2.3	T_3	715809	0.19%	^{Tl}	715958	0.01%	^{Tl}	715422	0.09%	13.3

* No feasible solution was found.

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Chapter 5

Using Analytics to Enhance Shelf Space Management in a Food Retailer

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Abstract This paper describes a collaboration project with the Portuguese leading food retailer which addresses shelf space planning, for the allocation of products on shelves. Prior to this project, the shelf space planning process was very time consuming, with an empirical use of space elasticities, lacking formal performance evaluation criteria, and heavily dependent on the space managers' experience. Our challenge was to bring analytical methods into the practice in order to enhance shelf space management in three axes: process automation, space optimization, and image standardization, without disrupting (but somehow questioning) the company's policies. This led to the creation of GAP, a decision support system that is today used on a daily basis by the space management team of the company. We developed a modular Operations Research (OR)-approach that systematically applies tailor-made mathematical programming models that were combined with heuristics to improve its efficiency. On top of the algorithmic advances, one of the most relevant features of GAP is its flexibility to incorporate different types of merchandising rules, allowing the company to test several strategies for the product allocation. Nevertheless, it goes beyond the straightforward implementation of merchandising rules and it trades-off customization with optimization.

Keywords Retail operations · Shelf space allocation · MIP-based heuristic

5.1. Introduction

Sonae MC is one of the biggest Portuguese companies (ranked the 4^{th} in 2014, with 3.33 billion annual sales) that operates a food retail business in Portugal. It is one of the core

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businesses of the Sonae Group, which also acts in other areas such as specialized retail (selling sports goods, fashion and electronics), shopping centers and telecommunications.

Its brand *Continente* is the country's food retail market leader, and has been considered one of the most trusted brands in Portugal over the last 13 years. The company is a benchmark in the Portuguese market, after having launched the country's first hypermarket in 1985. Today it has a network of 478 stores (and additionally 162 stores under franchising) covering the entire country, with three major formats: *Continente Bom dia*, convenience stores with average sales areas of 800 m^2 (8,611 square feet); *Continente Modelo*, supermarkets located in medium sized population centers with an average of 2,000 m^2 (21,528 square feet); and, finally, *Continente*, hypermarkets located in prime locations and offering an extensive and varied range of products and services with average sales areas of 9,000 m^2 (96,875 square feet). In total, Sonae MC has a sales area of 595,000 m^2 (6,404,527 square feet) and its strategy is to grow its convenience channel and to look for international growth opportunities.

Sonae MC is aware of the impact of in-store planning on customer satisfaction, sales effectiveness and operations efficiency. In particular, it believes that a clever product organization on the shelves leads to higher visibility, consumer awareness and demand for the products, as well as reduced inventory holding and handling costs. However, the short product life cycles, the increasing number of products available and the progressively higher number of stores has lead to a continuous need for shelf space planning which turned the process more and more challenging for the company.

Innovation is a priority at Sonae MC, which is constantly seeking for opportunities to improve their products, services and processes. This paper describes the development, implementation and impact of an OR-based approach to better plan the allocation of products on the shelves. It is the result of a collaborative work between the Information Systems and Innovation Department (ISI) and the Space Planning Department (SP) of Sonae MC, and a group of researchers from the Industrial Engineering and Management Department of the Faculty of Engineering of the University of Porto (FEUP).

Prior to this work, the shelf space planning process was very time consuming, with an empirical use of space elasticities, lacking formal performance evaluation criteria and heavily dependent on the space managers' experience. The challenge consisted of incorporating analytical methods into the practice in order to automate the process, improve the return on space, and reduce stockouts and inventory costs, without disrupting (but somehow questioning) the company's policies. Based on these objectives, three axes were defined for the project: Process Automation, Space Optimization, and Image Standardization.

The resulting Decision Support System (DSS) is called GAP and is today used by space managers on a daily basis to automatically generate shelf space plans. GAP is developed on top of a modular architecture and systematically applies tailor-made mathematical programming models, combined with heuristics, to derive the best allocation of products on the shelves. The key benefit of the approach is its flexibility to incorporate different types of merchandising (placement) rules, including hierarchies of product families, family precedences, display shapes, and special locations. This customization level allows space managers to control the entire process and to test different strategies for allocating the products. Moreover, GAP goes beyond the straightforward implementation of merchandising rules, hence combining customization with optimization.

The remainder of this paper is structured as follows. We start by describing how shelf space is managed, firstly in Sonae MC (section 5.2) and secondly in a more generic perspective, both from a practical and theoretic point of view (section 5.3). GAP is presented next, in section 5.4, with a general discussion of its analytical approach and a description of the decision support system. We also offer some details about the project development that were critical for its success. The impact of the project is carefully analyzed in Section 5.5. We end with some brief concluding remarks emphasizing the fact that we are presenting a generic approach that is suitable for other retail companies. Note that this is a practice oriented paper and many details were omitted for the sake of simplicity. Additional papers will be referred throughout the text for more technical details.

5.2. Shelf Space Management at Sonae MC

The primary objective of retail is to bridge the gap between the point of production and the point of sales, which stresses the role of logistics and operations in this industry. Sonae MC has a centralized operations management activity, responsible for planning all the operations for the stores nationwide. The space planning department, as its name implies, is engaged with managing the space available at the stores, an activity that comprises two main levels: a macro-space planning level that defines, on a long-term basis, the layout of the stores (divided by categories); and a micro- (or shelf-) space planning level that defines, for each category, the products' placement on the shelves. Shelf space planning is a mid-term activity that updates shelf space plans with an average rate of 2 to 3 times a year for more than 300 categories. This activity fully occupies 23 space managers.

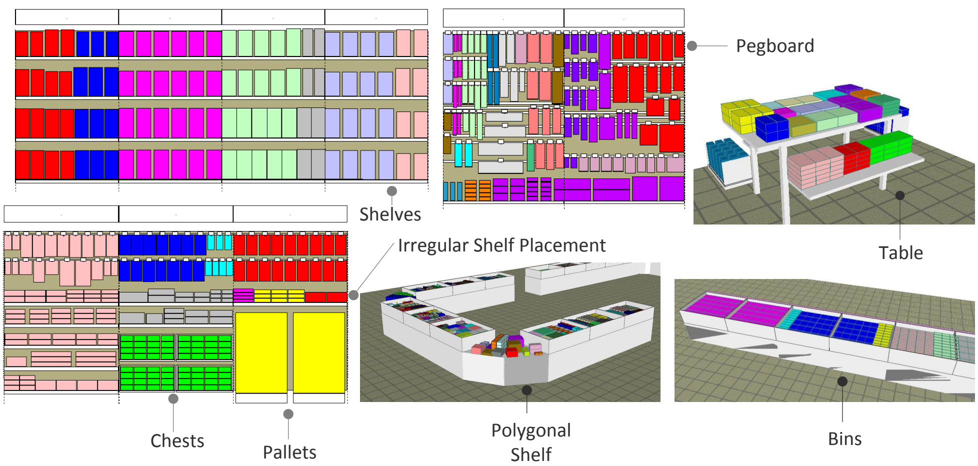

Figure 5.1 – Planograms with different types of fixtures. Some include more than one fixture type or present irregular shelf placements, resulting in irregular planograms.

The traditional micro-space planning tool is a *planogram*, which is a virtual representation of the shelves, showing exactly where each product should be physically displayed and the inventory that it should hold. One planogram comprehends plenty of information that has to be carefully planned: the location of the products, the number of facings (visible items), number of items stacked behind and above each facing, packaging style, orientations (front, side, back, top), among others. Besides the most commonly used shelves, stores have also other fixture types such as chests, pallets and pegboards (bars with steel rods sticking out to hold peggable products like pens and pencils). Moreover, planograms are physically made of segments that are stacked together to form an aisle. Each segment has its own shelves, which can be placed vertically aligned with the shelves of the other segments, or be placed differently, forming irregular planograms. Some examples are present in Figure 5.1.

At Sonae MC, planograms follow a complex structure of merchandising rules that try to reflect the consumer buying behavior and the strategy of the company (and of the suppliers) for the categories. To do so, the company has a superior customer insight due to its successful loyalty card, which covers 3 out of 4 Portuguese households and is associated with 90% of the sales. Morever, the company maintains key partnerships with suppliers that have a deep knowledge about their categories, assuming the role of category captains. Space managers are also committed to developing planograms with a compelling visual look, and put a great effort on it. Nevertheless, the attractiveness of the planogram is a subjective field and planograms depend on the space manager in charge. Figure 5.2 presents an example of a merchandising manual for a category, where we can see that products are usually grouped by families which are placed in rectangular shapes. Each planogram has a hierarchy of families that typically range from 2 to 5 criteria. For each criterion, the merchandising manual specifies the family type, the display orientation (either vertical or horizontal), the family precedences and, in some cases, additional information about preferred locations. Due to the strategic character of merchandising rules, this figure does not represent a real situation.

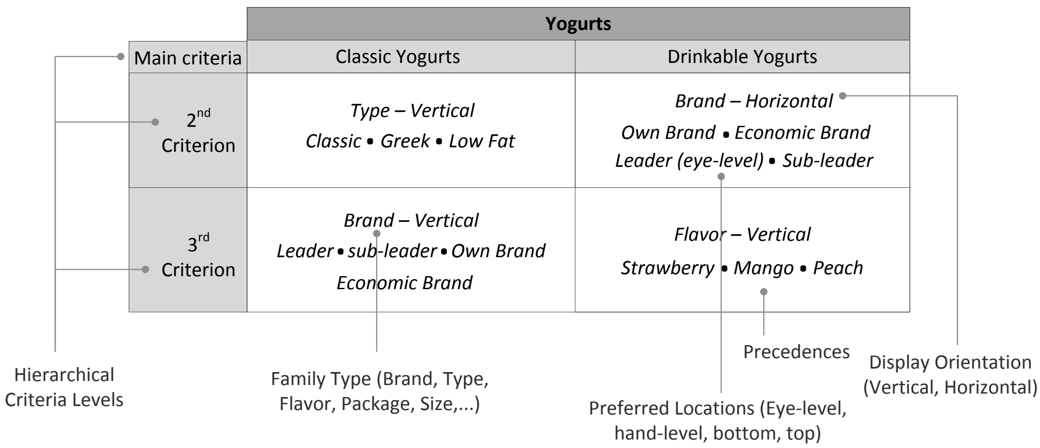

Figure 5.2 – Merchandising rules for implementing a given category: an example with yogurts. Each manual specifies from 2 to 5 hierarchical criteria levels with different types of information.

The process of updating shelf space plans has a major interaction with the commercial department, that is responsible for managing categories. The process for a given category is as follows. The category manager (from the commercial department) triggers the process after specifying the product portfolio (assortment) for the stores, as well as the key merchandising rules for their implementation. Product portfolios are not store-specific but are instead specified for clusters of stores with similar sales and space patterns in order to manage complexity and effort. Space managers start by generating a template planogram (known as role planogram) for each cluster, where they carefully check how merchandising rules fit the space. In a collaborative work between the space and category managers, the planogram is then discussed and merchandising rules are tuned. Once validated by the category manager, it is then replicated for the remaining stores, by adjusting the product facings to the space of each store, while maintaining the same allocation rules. Figure 5.3 summarizes this shelf space planning process, where the two key processes are highlighted: The Generation Process and the Replication Process. Note that the company has many categories to update and, at the beginning of each year, the category space planning processes are scheduled for the entire year.

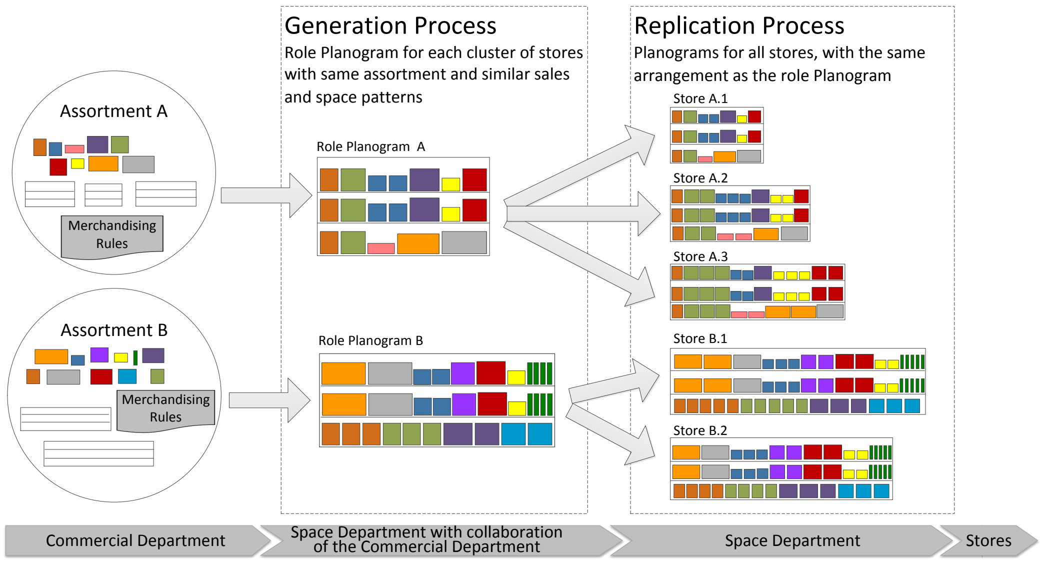

Figure 5.3 – The micro-space planning process has a major interaction with the commercial department and comprises two main processes: Generation and Replication.

During the shelf space updating processes, space managers generate an average of 60,000 planograms each year. For this purpose, Sonae MC uses a space planning software from one of the world top three vendors, the JDA Software Group, Inc. This software gathers the necessary capabilities for creating and maintaining the planograms, including a space database with all the key information about the products and store equipments, and a visualization tool that provides realistic views of the shelves, the ability to easily handle products and powerful reporting. Although automatic tools for planogram generation are available in JDA software, they do not accommodate all the inherent complexity of the Merchandising Rules. Therefore, space managers manually developed their planograms by dragging and dropping the products onto the shelves, in a time consuming activity that lasts on average 3 hours.

One of the most difficult challenges that we faced in the beginning of the project was the lack of formal criteria for evaluating planograms. Space managers were creating and evaluating planograms based on their intuition and personal judgment, as opposed to analytical methods. Nevertheless, in most situations, they were empirically considering space elasticities and balancing the product days-supply values. When analyzing shelf inventory, *days-supply* is a common operational metric, measuring the number of demand days covered by the shelf stock. For balancing days-supply values, space managers were using a software highlighting tool that colored the products according to predefined days-supply intervals and they sought to fit all the products within one interval. Moreover, some categories had alternative objectives such as meeting the brand market-shares.

In 2011, the stores went through a successful lean process that, among other things, changed their shelf replenishment policy from just-in-time (shelves were replenished frequently in small quantities during the day) to a single shelf replenishment operation each day, before the morning opening. This change of policy, and the fact that products normally have joint delivery cycles from the central distribution centers, explain the reasoning behind balancing days-supply values across the products. By having all products covered for a similar number of days, the number of shelf replenishment operations are reduced, the stock level for long-tale products is better controlled, stockouts for fast moving products are prevented, and it also possible to reduce the backroom inventory.

Sonae MC believed that analytics could help to improve shelf space management going beyond a straightforward planogram automation tool and this is when this projected started.

5.3. Theory and Practice of Shelf Space Management

Most shoppers are susceptible to in-store marketing, mainly because of the low level of involvement that consumers have with in-store decisions. Additionally, reduced assortments and stockouts force the search for substitute products, highlighting the role of space management. Experimental studies have been addressing the effect of space variables on the demand of the products. These studies point to three main elasticities: *space elasticity* measures the increasing responsiveness of demand as more space is allocated to a product, experiencing declined marginal returns at some point (Curhan [1972], Chandon et al. [2009]); *location elasticity* highlights key display locations that bring a better exposure, such as the eye- or hand-level (Drèze et al. [1994]); lastly, *cross elasticity* measures the interdependency between adjacent products and is assumed to be positive for complementary products and negative for substitute products (Corstjens and Doyle [1981]). Additionally, the way products are arranged on the shelves can also have an important role on gaining the consumers' attention. Thus, carefully organizing them in families can increase interest, while disorganized or excessive complexity (i.e. variations in the basic visual content) damages the buying experience (Pieters et al. [2010]).

According to a survey to US retailers (Keltz and Sterneckert [2009]), the main drivers for space planning initiatives rely on two main axes: maximizing selling space effectiveness, powered by the aforementioned effects, and tighter inventory control. However, the same survey concludes that the benefits are not meeting the expectations. As a result, space planning investments are required because "conventional assortment analytics and space tools do not deliver the optimization capabilities needed for success". Software vendors mainly tackle the development of large-scale data processing technologies capable of addressing the complexity of shelf space in practice, but with limited or no use of mathematical optimization, and a complete disregard for consumer demand effects. Therefore, automatically generated planograms are still a mirage for most retailers and they often opt for generic planograms that fit clusters of stores.

Shelf space management is an active field of research in retail operations management, under the name Shelf Space Allocation Problem (SSAP). Despite the practical relevance of the problem, there has been somehow a misalignment of the scientific knowledge with the practice as most state-of-the-art mathematical models have strong limitations (Hübner and Kuhn [2012], Bai [2005]).

The literature presents a great variety of models, mostly differing in their demand functions, which incorporate different estimates of (some of) the consumer demand effects, ranging from complex multiplicative polynomial forms to simplistic linear profit functions. Nevertheless, most of these models have the common goal of maximizing demand by deciding the product facings on each shelf, without considering their location within the shelves. The most relevant approaches to this work are from Corstjens and Doyle [1981] who were the first to present space elasticity in a polynomial form, Gajjar and Adil [2010] who propose a piecewise linearization to the space elastic demand function, and Yang and Chen [1999], who use an alternative model in the form of a linear multi-knapsack problem.

Perhaps the most important practical limitation from the aforementioned literature is that it neglects merchandising rules; more specifically, it disregards the existence of product families that specify associations of products on the shelves. Russell and Urban [2010] and Geismar et al. [2014] are the only authors who define the exact location of the products on the shelves and allocate the space in such a manner that keeps product families together, in uniform and rectangular shapes. Despite this, none of the models were able to solve to optimality instances with more than 10 products.

Another key-point is that the shelf space allocation literature has focused less on the cost side of the problem and most models do not explicitly consider inventory related decisions. Two authors stand out in a more inventory-related stream: Baker and Urban [1988] presented the first model that considered the demand in function of the instantaneous inventory level of an item, based on the economic order quantity (EOQ) model, and Urban [1998] proposed the first attempt to include shelf space allocation in the inventory decision-making process. Nevertheless, these models are comprehensive and are only solved to optimality for a reduced number of products. The models also include practical limitations: they consider continuous shelf replenishment operations from the backroom and determine individual product replenishment policies. Our approach can also relate to this stream as we give a special emphasis to inventory.

Finally, the literature regarding Category Captains is also interesting to this work, and a more general work is found in the book chapter from Kurtulus and Toktay [2009]. Category Captains are key suppliers that help retailers manage their categories, for instance by consulting on the definition of merchandising rules. The use of external consultants with a deep knowledge about the categories and the large number of factors behind merchandising rules reinforce the consideration of the rules as inputs to the space planning process.

5.4. GAP Overview

This section presents an overview of GAP. More precisely, we describe the new OR-based process that was developed to help space managers at Sonae MC perform their daily basis activities more efficiently and effectively. GAP stands for Automatic Generation of Planograms and its main functionality is precisely the generation of planograms bearing in mind the intrinsic complexity of the company space management process.

We soon realized that one single planogram generation process was not enough to accommodate the space management process in Sonae MC, as directly generating store specific planograms would break the validation point in the middle of the process, and would result in a higher validation effort for both the category and space managers. Moreover, the tremendous amount of guidelines would make unlikely the acceptance of fully automatically generated planograms and would result in an excess of manual adjustments. Therefore, following the company's current practice, we divided GAP into two major processes: *GAP Generation* and *GAP Replication*. GAP Generation is responsible for generating planograms from scratch, following a set of guidelines, and fits the Generation Process from Figure 5.3, for the construction of role Planograms. GAP Replication, on the other hand, adjusts a given planogram to different spaces, by adapting the product facings to the performance and space of the new stores, while keeping all the allocation guidelines. GAP Replication fits the Replication Process, for the generation of store specific planograms based on the role planogram. Both processes present relevant analytical advances that will be detailed later in this paper.

On top of the analytical advances, one of the most relevant features of GAP is the possibility that is given to users to control the entire planogram generation process. GAP Generation can incorporate different types of merchandising rules that can change on the fly, allowing space managers to test different strategies or to shape the planograms according to the current practice (see Figure 5.2 for the main types of rules that can be included). More importantly, the users choose the level of customization, giving more or less freedom to the program to decide the products' allocation on the shelves. Moreover, GAP Replication replicates planograms with any allocation rules, including non-standard rules such as family non-rectangular shapes, using either handmade or automatically generated role planograms. Finally, GAP can be tuned to meet different performance evaluation criteria, such as the equilibrium of days-supply values across the products (the most typical planogram evaluation criteria), the correspondence with sales shares or the maximization of expected demand based on space elasticity effects. All these features provide the necessary flexibility that is crucial for an activity that is so highly dependent on the market, and is so interconnected with the companies' strategy.

GAP consists of two main building blocks: (1) *GAP optimizer* is the system's heart and contains all the analytical methods from GAP Generation and GAP Replication for generating planograms; and (2) *GAP User Interface* provides all the tools for handling data and for managing the insertion of merchandising rules. The remainder of the section provides details on the analytical approach behind GAP Optimizer, the description of the overall decision support system, and an analysis of the key-factors that led to a successful deployment in Sonae MC. Although GAP considers and integrates many types of fixtures, all of them depicted in Figure 5.1, we will focus on the most common type: shelves.

5.4.1 Analytical Approach

Figure 5.4 presents the architecture of the GAP Optimizer. Both the GAP Generation and the GAP Replication processes were developed in a modular fashion, and systematically apply innovative tailor-made optimization models that were solved using mathematical programming-based heuristics (also known as matheuritics) to ensure fast solutions. As depicted in the figure, some of the modules are common to the two processes.

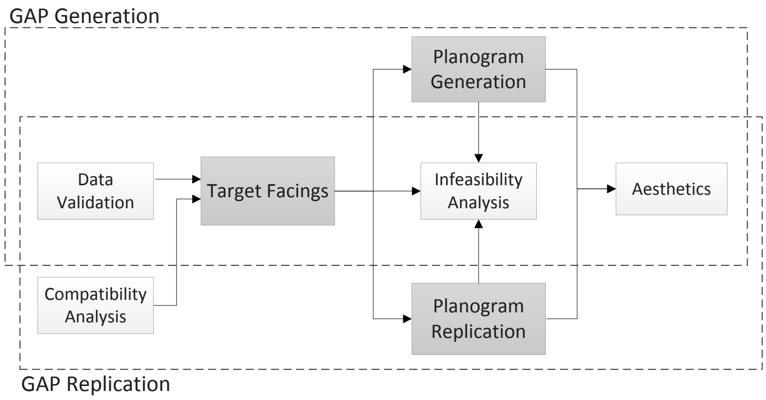

Figure 5.4 – GAP Optimizer has a modular architecture and some modules are common to both GAP Generation and GAP Replication. The key modules are highlighted in gray.

Both the GAP Generation and GAP Replication processes start with a thorough data validation and compatibility analysis to guarantee that all the data are present and in accordance with the requirements. If so, the processes proceed to the calculation of the target number of facings for the products, with regard to one of the possible performance evaluation criteria. These values are then used as goals while generating the planograms in the next modules, planogram generation or replication. In case any of the three dark gray modules are unable to generate valid solutions, the processes end with what we called infeasibility analysis, in order to infer about the causes. Finally, as the visual attractiveness is very important, the last step is designed to improve the planogram aesthetics.

We will now describe all the modules, with a high-level overview of the modeling and solution approaches included the GAP Optimizer. Additional details are provided in the appendix and in two technical-oriented companion papers: Bianchi-Aguiar et al. [2015b,a].

Data Validation and Compatibility Analysis

Sometimes data has some minor inconsistencies that are not compatible with an automated process. While space planning was a manual process, these inconsistencies were either not visible to the human eye, and therefore disregarded, or were solved, case by case, by the space manager. Examples of common data inconsistencies are overlapping or out-of-border equipment and products without dimensions. Other inconsistency problems may be generated when specifying merchandising rules or when configuring GAP. The *Data Validation* module is responsible for tracing these inconsistencies and return the necessary warnings to the user. Based on the severity of the inconsistencies, the processes may or may not proceed.

Other problems arise when the role planogram is not compatible with the store-specific equipment and the replication process cannot be executed. Examples of planogram incompatibilities are: different number of available shelf levels, different types of equipment and products outside the assortment of the role planogram. This type of problems are traced by the *Incompatibility Analysis* module, only executed in GAP Replication.

Target Facings

The *Target Facings* module decides the number of facings that each product should have in order to maximize planogram performance, without considering any merchandising rules or other allocation constraints but only the shelf-space capacity. The reasons for estimating the target facings beforehand are twofold: it allows us to consider alternative performance evaluation criteria, and it reduces the complexity of the subsequent allocation problems. For the sake of brevity, this paper will focus on the most frequent planogram evaluation objective, which aims at balancing days-supply values across the products while considering space elastic demand.

At the center of the target facings calculation is the *space-to-sales* curve depicted in Figure 5.5 which predicts the demand of a product as a function of the allocated shelf space. This curve embeds the experimental findings regarding the space elasticity effect: the more space is allocated to a product, the more consumer awareness the product has, leading to increasing demand. Nevertheless, the marginal returns decrease as the shelf space reaches a saturation point, resembling an "S" shape. We included a control parameter in the curve that specifies the maximum demand variation that can be explained by the shelf space allocated to products and consider demand forecasts (given as inputs) as the maximum value. Having captured the space elastic demand, we address the objective of balancing days-supply values by defining a set of days-supply intervals, and by limiting all the products to a unique interval. The possible days-supply intervals are calculated in a preprocessing phase using a user-defined interval length. These intervals also depicted in the figure using dashed lines.

The target facings optimization model is formulated as a mixed-integer program (MIP) and embeds piecewise-linear approximations of each product's *space-to-sales* curve, obtained using the days-supply intervals. The model determines the target facings for each product that maximizes the planogram expected demand, subject to the shelf-space capac-

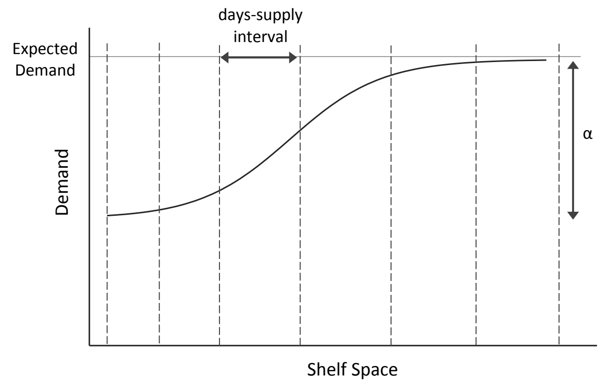

Figure 5.5 – Demand of a product in function of its shelf space, limited to a maximum variation α. The vertical lines represent the days-supply intervals.

ity, minimum and maximum number of facings and limited to the selection of a single days-supply interval. Additional details about this formulation are presented in Appendix 5.A.

GAP Generation

The *GAP Generation* module decides the products' placement on the shelves, subject to the user-defined merchandising rules. For each product, it determines the shelf (or shelves) where the product is to be allocated, its horizontal location within the shelves, and the number of facings to be displayed. Therefore, we may say that the outcome is a fully defined planogram. This module aims to meet the target facings specified upstream (Target Facings module), while considering the location elasticity effects when choosing the products' placement. For that purpose, we have defined a set of *shelf attractiveness* curves that model the attractiveness of the shelves depending on their vertical locations (Figure 5.6). The shelf attractiveness may vary with the category and fixture type, which explains the different alternative shapes.

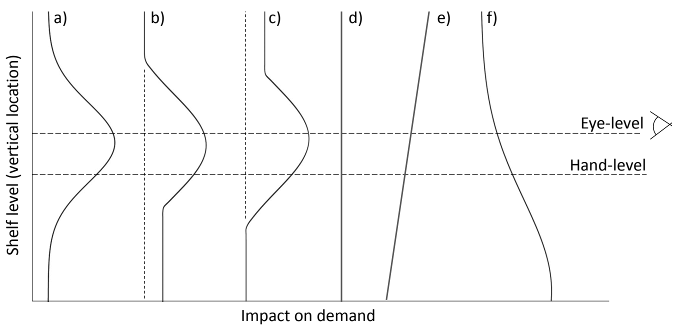

Figure 5.6 – *Shelf attractiveness* curves that model the attractiveness of the shelves depending on their vertical locations. We have defined 6 different curves (a-f).

Merchandising rules present an hierarchical structure of product families that are a key-feature of every shelf space plan. The products of each family must be placed together, in adjacent positions, and if a family spans more than one shelf, products have to maintain a continuous, uniform and rectangular shape that can either be vertical (like columns, occupying the full height of the planogram) or horizontal (like lines, occupying the full length of the planogram). The hierarchical structure creates complex relations between the products that highly constrain the solutions. We capture these relations using a diagram tree, such as the one presented in Figure 5.7. The diagram tree starts with an initial node connecting the families from the first criteria, which define the first level of the tree. Each family is then connected to its downstream families, leading to a multi-level tree of product families.

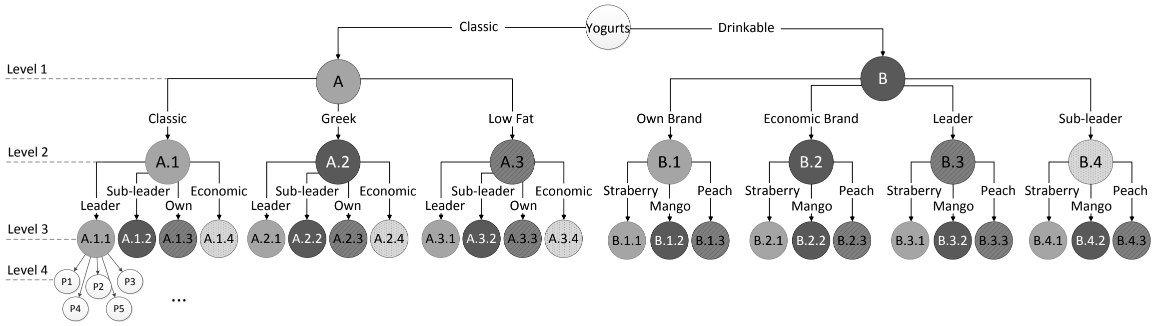

Figure 5.7 – Diagram tree representing all the product family relations for yogurts, present in the merchandising manual from Figure 5.2

We formulated this problem using an innovative network flow MIP model. An intuitive network approach would associate one node to each product to find the sequence of products on each shelf. However, such approach would lead to an extremely complex and intractable model as the traditional sequencing variables T_{ij}, stating whether each product i precedes or succeeds product j, would increase exponentially with the number of products. To overcome this, we explore the existence of a hierarchy in the product families and consider a set of multi-level family dependent networks. For each shelf, we start by defining a network with the families from the first level of the diagram tree. Afterwards, for each first level family, we define a network with the corresponding downstream families and repeat the process until the last level is reached, with a set of disjoint product networks (see Figure 5.8 for a partial definition of the network resulting from the tree diagram in Figure 5.7). The model decides on the network sequences and then the overall product sequence is obtained by conveniently joining the product-level sequences. This approach also guarantees that the products belonging to the same family are consecutively placed on each shelf, which is another requirement of the problem. Additional constraints ensure the coordination of each family between shelves for the rectangular shapes, as well as the display orientations. The precedence and special location rules, if any, correspond to variables that are fixed during a preprocessing step.

One of the most relevant features of the formulation is that it accommodates all the levels of flexibility the user may want to use when generating planograms. In a less flexible

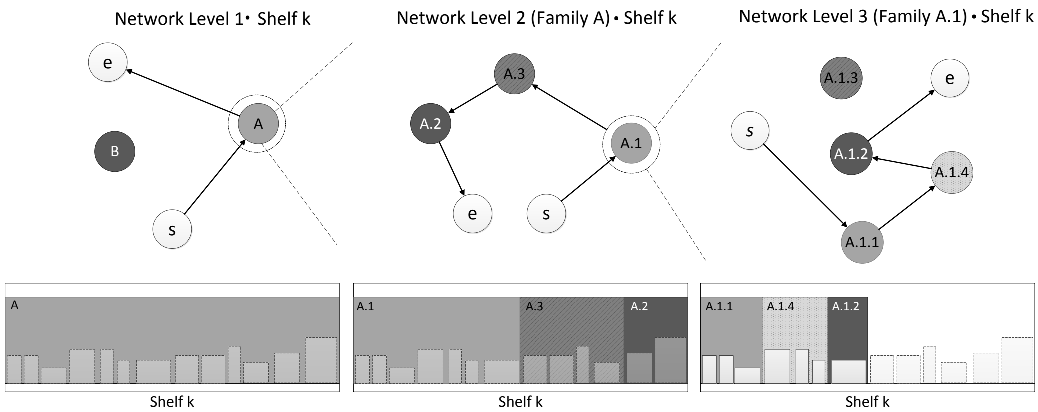

Figure 5.8 – Partial representation of the multi-level family networks resulting from the diagram tree in Figure 5.7.

scenario, when the user wants to control all the planogram details, the formulation will be more constrained with many variables fixed. However, even in these cases optimization still takes place but with a more limited scope. For instance, if the user chooses to set high-level family precedences, the remaining families are still sequenced by the model. In a more extreme case, if all sequences are defined, the formulation still decides on the optimal shelf space for each product.

When the problem size increases, it becomes intractable, even when resorting to the multi-level family networks. This fact limits the straightforward use of a commercial solver on the standard mathematical programming model. This fact motivated the development of an approximate method. We chose a mathematical programming based approach because it would be difficult to develop a highly customized heuristic considering all family-related merchandising rules, that would still be capable of generating high quality feasible solutions within reasonable time limits. Moreover, a MIP-based approach makes it possible to introduce new features in the problem with none or limited effort. The formulation is then embedded in a matheuristic that successively solves a sequence of sub-problems exploring the hierarchy present in the product families. The matheuristic starts by allocating families from the first level and progressively moves down until reaching the product level. Technically, this approach is based on the relax-and-fix (R&F) framework: we consider the entire formulation in all iterations but families already considered in previous subproblems have their variables fixed; families not yet considered have their variables relaxed to continuous values; and families from the current subproblem have integer variables. The approach additionally includes a backtracking scheme. Whenever a sub-problem is infeasible, the heuristic shifts backward instead of forward, and solves a larger sub-problem by unfixing previous parts of the solution.

Both the formulation and the matheuristic are formally defined in Bianchi-Aguiar et al. [2015b].

GAP Replication

Given a fully defined planogram (role planogram), the *GAP Replication* module reproduces a similar product placement for a new store without the need to provide merchandising rules or other type of reasoning behind the planogram construction. The new space is usually larger (in width, as most of the times planograms have the same height) but should have a similar shelf layout to ensure the compatibility between the two planograms. Since the new store has a different demand pattern, the objective is to meet new target number of facings, specified upstream and suitable for this store.

We formulated the replication problem as a MIP model. Although we mainly aim to adjust the product facings, the model necessarily determines the products' location, in order to guarantee that the new planogram fully complies with the role planogram. In particular, the following product placement information is considered:

- products are required to keep the same relative position as in the role planogram. In the case of shelves, this means that products maintain the same shelf level and they are placed following the same sequence;
- product families are required to keep their uniform rectangular shapes. The family continuity within each shelf is already ensured by keeping the same sequence. The rectangular shape is obtained by vertically aligning the first and the last products of the shape, which we call the left and right alignments. This brings flexibility to consider shapes from the role planogram that may not be necessarily rectangular.

In other words, one may say that the role planogram suffers a controlled "expansion" in order to keep all alignments. Note that during this process, we do not consider location effects on demand, as the relative product placement constrains such decision.

Although solving the formulation in a commercial solver is able to generate solutions within time limits that are acceptable in practice, we developed a second MIP-based heuristic to ensure the scalability of the approach, especially for the planograms with irregular shelf placements (mis-alignments and interruptions in each shelf level), whose additional constraints greatly impacted the performance of the formulation.

Generically speaking, this matheuristic has three main steps. The first step generates an initial solution for the problem with the minimum display quantities for the products. The second step iteratively adds the remaining product facings to the planogram until no more space is available (or no more facings can be added). The last step tries to improve the solution by allowing the removal and insertion of new facings. Technically, this approach is an integration of two well-known MIP-based improvement heuristics: *fix-and-optimize* and *local branching*. Thus, in each iteration, we solve the model with some variables partially constrained in one of two different ways: a subset of the variables are fixed to the values obtained in the incumbent solution (*fix-and-optimize*) or there is a limited number of changes allowed to the values obtained in the incumbent solution (*local branching*). One of the interesting aspects of this matheuristic is that it mimics the process followed by the space managers when manually replicating planograms.

Both the formulation and the matheuritic are formally defined in Bianchi-Aguiar et al. [2015a].

Aesthetics

The formulations for planogram generation and replication focus on ensuring that the shapes are rectangular and disregard other aesthetic details, resulting in planograms with some display issues, such as such as large and irregular gaps between the products. The *Aesthetics* module is responsible for improving the attractiveness of the planogram and it considers two key factors for obtaining attractive displays: the way products are spaced throughout the planogram and whether the planogram is fully merchandised (i.e. full of facings). For that purpose, the generation or replication formulation (depending on whether it is a GAP Generation or Replication process) is re-executed again with all the decisions fixed to the incumbent solution, with the exception of the horizontal location of the products. The objective function is changed, firstly to minimize the empty space, and secondly, when no more products fit the planogram, to minimize the maximum spacing between two consecutive products. This latter objective distributes the empty space throughout the products.

Infeasibility Analysis

Highly customized and detailed merchandising rules lead to significantly constrained generation and replication formulations which can compromise the existence of a feasible solution for the problems. Moreover, the target facings formulation can also be infeasible, which result in too many possible causes for the process ending without a valid solution. To overcome these data related issues, we have developed an *Infeasibility Analysis* module that searches for the possible infeasibility causes in a structured and logical procedure. It performs multiple runs of the infeasible formulation and, at each one, a problem feature or requirement is removed from the formulation. The process stops after identifying a source for the infeasibility (i.e. whenever the formulation is able to find a valid solution for the relaxed problem).

5.4.2 Decision Support System

GAP Optimizer requires the integration of different types of information obtained from multiple sources. If this information is not handled carefully, it may jeopardize the successful use of the application. For that purpose, another important building block is the GAP User Interface that manages all the data handling process, executes the Replication and Generation processes with real time status messages, and presents the generated planograms at the end, together with a full execution report. Among other things, this report provides all the warnings and errors that occurred during the process and when applicable, the infeasibility causes. In other words, this interface is present throughout the entire process, and it works as the liaison with the users. Figure 5.9 depicts the most relevant flows of information as well as snapshots from two interface forms: the project manager for handling the data and the generation manual for managing rules and configurations.

The generation manual is one of the key-parts of the overall system. Inspired by handmade merchandising manuals (c.f. Figure 5.2), this form presents a familiar interface (to space managers) for the configuration of merchandising rules in a very intuitive way. Space

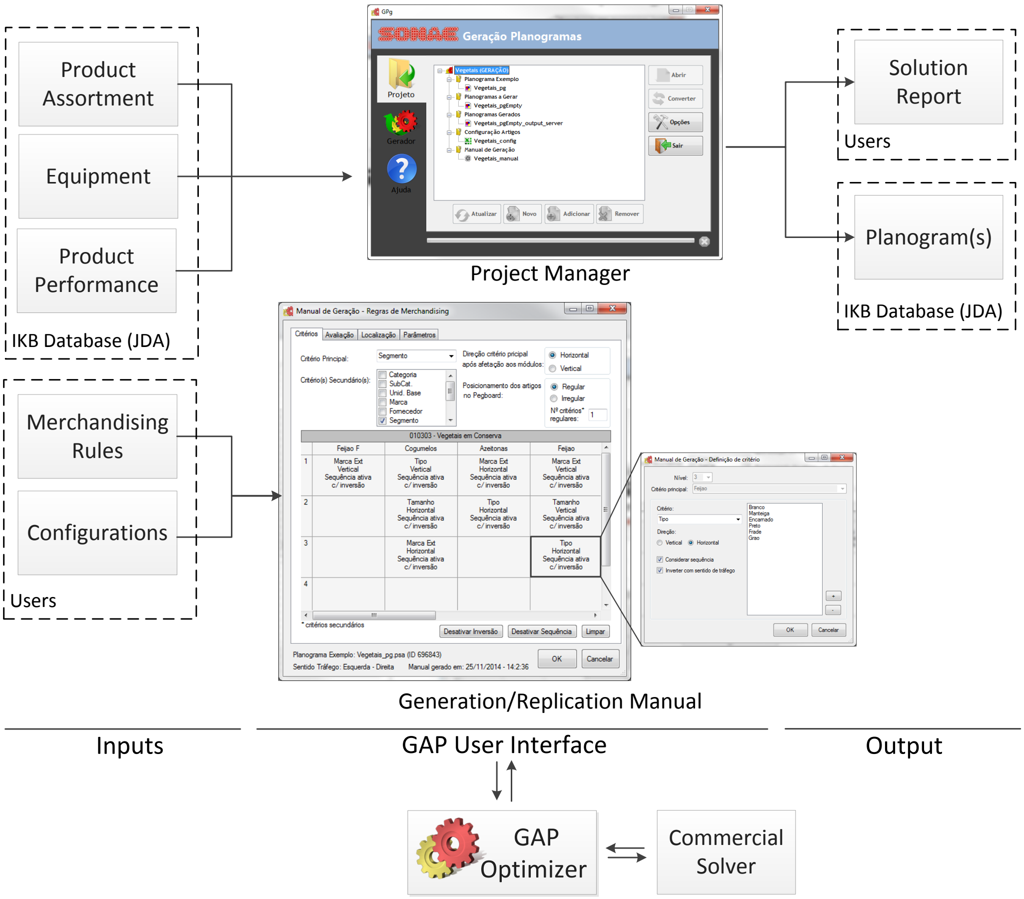

Figure 5.9 – Inputs and Outputs of GAP

managers have high flexibility to define these rules, and in each run they can choose the level of customization that they want to have in the generated planograms. For the advanced users, several other configurations are available, from alternative location-elasticity curves and planogram evaluation criteria to control parameters and tolerances. Each generation manual can be saved, consulted and reused in multiple processes. Most importantly, it can evolve as the space managers evaluate planogram solutions and realize possible changes to the planograms.

With regard to the IT implementation, the GAP Optimizer is a C++ program with all the models embedded in the code. The company acquired a commercial solver and the formulations are executed using a C++ library from the solver. The GAP User Interface is developed using Windows Forms, and all the communications between the two building blocks use XML files. Both the GAP Optimizer and the GAP User Interface are executed on a dedicated server and all Space Managers have access to the interface using a remote desktop connection in a terminal-server architecture. At the moment, GAP does not have a direct connection with the space database and the information is manually exported and imported to the interface. Given the success of the project, Sonae MC is now studying more efficient infrastructures, both for communicating with the space planning database and for accessing the server from the space managers terminals.

5.4.3 Project Development

The project kick-off was on March 2012 and it lasted until July 2014. The two processes, GAP Generation and GAP Replication, were developed sequentially and each one involved three main phases: *Requirements Definition*, *Prototype & Proof-of-concept* and *Testing & Validation*. From the organizational standpoint, it included a team from FEUP, responsible for the complete development of GAP, and two teams belonging to Sonae MC: a team from the Space Department, responsible for validating requirements and testing GAP, and a team from ISI (the Information Systems and Innovation Department), responsible for integrating GAP with the Information Systems of the company.

We strongly believe that there were some key factors in GAP's design and project management which had a crucial contribution to the project's success. To start with, the decision of dividing GAP in two processes played a vital role both from the space and commercial department perspectives, as it did not disrupt current practices. Starting the implementation with GAP *Replication* has also proved to be a wise decision mainly for two reasons. Firstly, because the replication process was faster to implement and provided more consensual solutions, contributing to an earlier engagement of the space managers with GAP. Secondly, it allowed us to obtain a deeper knowledge about the complex structure of merchandising rules, which was vital for the GAP *Generation*.

Another key aspect was the close collaboration with the 3 space managers that were part of the project team, whose role was essential all the way from the requirements gathering and problem definition to the testing and validation phases. Weekly meetings between FEUP and the space managers were important milestones to validate new developments. This continuous process conferred great flexibility to GAP and led to the development of an application tailored to the Sonae MC reality. Additionally, the same 3 space managers tested and validated GAP using different categories of products, which was also significant for building (and communicating) their internal confidence in the application.

Nevertheless, there were some challenges in this collaboration between academic researchers and industry practitioners who have different objectives, incentives and time horizons. In particular, at the beginning of the project we found a high resistance from the space managers to adapt to the new process. This was gradually overcome as we attempted to keep them updated on the evolution of the project, which improved their commitment to GAP and allowed them to understand the potential of the application.

Training sessions were also performed before the roll-out of each of the two processes and, given the systems' complexity, they were crucial to engage space managers. These sessions included the analysis of solutions with unexpected characteristics obtained while using GAP and the development of a check-list for systematically looking for alternative solutions in these situations. Moreover, the execution report also played a vital role in dealing with the disappointment of space managers when GAP produces a solution that is not expected and, more importantly, when it is not possible to generate one.

Lastly, and perhaps one of the most important key factors for the success of the GAP implementation was the enormous commitment of the space planning and innovation directors and their sponsorship during the whole project.

5.5. Impact

Today, GAP is used on a daily-basis by the entire micro-space team at Sonae MC, both for the generation and replication of planograms. This section describes how it enhanced shelf space management in each of the three axes that were identified for the project: Process Automation, Space Optimization and Image Standardization.

5.5.1 Automation: from planogram construction to planogram evaluation

Perhaps the first and most straightforward impact of the project was on the space management process, as it led to better processing times as well as a positive change of paradigm.

During the first months after the roll-out, all space managers were encouraged to use GAP in their daily tasks, and to register the number of planograms that GAP could generate, as well as the process execution times (including the duration of data handling, creation of the generation/replication manuals and handmade adjustments to the final solution). The execution times were later compared with the legacy process and the results were encouraging: based on a preliminary analysis, space managers were able to automatically generate 80% of the planograms and the category space management processes took on average 48% less time (46% less in the generation processes and 50% less in the replication processes). Moreover, space managers also highlighted that these time reductions can be more significant in the future, after becoming more agile using the new software and by, totally or partially, reusing the generation manuals that they carefully developed for the first processes.

Additionally, GAP shifted the space managers' focus from planogram construction to planogram evaluation, allowing them to concentrate in additional activities, such as market trend studies and experiments with alternative merchandising rules. Therefore, this change of paradigm brought increasing responsibilities to space managers, and gave to them an analytical tool to support their decisions during the meetings with the commercial department.

5.5.2 Optimization: targeting optimality in all customization levels

In what concerns optimization, it is necessary to measure independently the impact of GAP Generation and GAP Replication.

One of the greatest advantages of GAP Generation is its flexibility to generate either highly customized solutions or more demand driven (and at the same time innovative) solutions, based on what is specified in the generation manual. Figure 5.10 depicts this flexibility by showing three planograms: the first was handmade by a space manager and the remaining two were generated with GAP, firstly using a fully defined generation manual (high customization) and then using the same manual but removing family precedences and display directions (low customization). While the first generated planogram is almost similar to the handmade planogram, the second presents an alternative reasoning behind its creation that intended to put the most popular families in the premium vertical and horizontal locations.

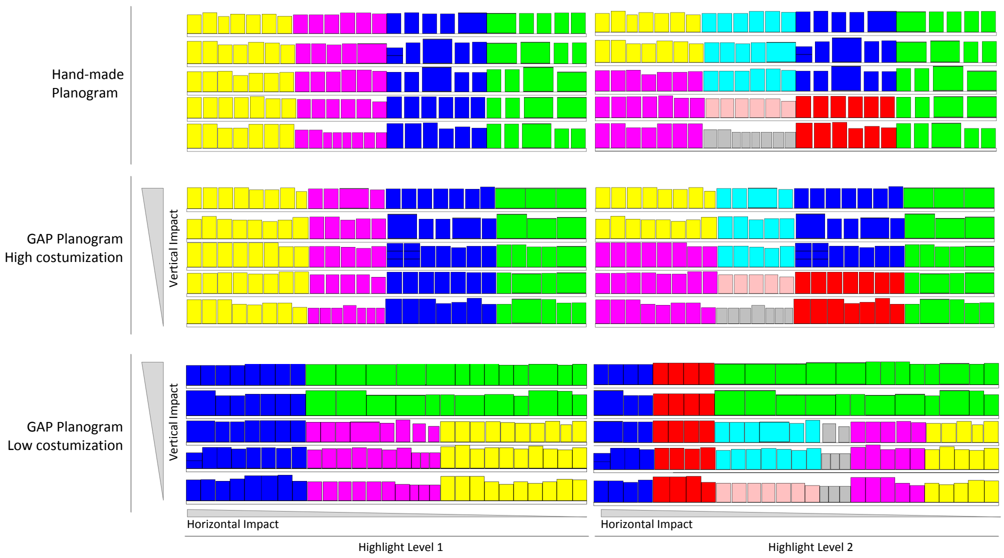

Figure 5.10 – Example of two planograms generated with GAP using two different levels of customization, and comparison with a handmade planogram.

Regardless of the customization level, GAP Generation always uses the available degrees of freedom to optimize the number of product facings and the products' location. To assess the impact of GAP Generation on the planograms' performance, we have analyzed 25 different generation processes (one of which was the example described above). These examples were carefully selected during the proof-of-concept phase in order to guarantee that all specificities of the categories were covered. The impact was evaluated by measuring four performance metrics: potential sales increase (estimated using the location elasticity curves with a maximum impact of 20%); days-supply balance measured in terms of the average and the standard deviation reductions; and planogram filling rate (defined by the ratio of the linear space utilized and the overall available space). Table 5.1 summarizes the results when generating these planograms using high and low customization levels. The percentage values are relative to the handmade version. GAP Generation is able to improve the manual planogram performance in all four metrics, both in the low and high costumization versions. As expected, reducing the number of rules imposed to the planogram yields additional gains with an average increase of potential sales from 0.4% to 1.4%. We observed that the gains were more relevant in the metrics regarding to the day-supply values (whose average and standard deviation were reduced in 38% and 61% respecitvely), which is consistent with our primary objective. Note that days-supply values have a major impact in replenishment operations, holding costs and product availability. The planograms' filling rate is also increased in both versions by 3% compared to the handmade planograms.

Table 5.1 – Summary of the planograms' performance in 25 Generation and Replication processes.

	GAP Generation		GAP Replication
	Low Customization	High Customization	
Potential sales increase *	1.4%	0.7%	–
Average days-supply reduction*	37.6%	35.3%	34.3%
Standard deviation days-supply reduction*	60.8%	51.4%	56.3%
Space Occupation**	96.5%	96.7%	97.3%
Execution Time (hh:mm:ss)	00:07:12	00:01:50	00:02:10

* with respect to the handmade planogram; ** 94% in handmade planogram

GAP Replication, by definition, has less flexibility to optimize, only deciding on the number of facings that products have in the new planograms, subject to many allocation rules that were extracted from the role planogram. Nevertheless, a smarter product facing allocation can optimize the day-supply values. We used the same 25 examples to assess GAP Replication performance by replicating the handmade planograms to the same store. The results are also present in Table 5.1 and prove that we are still able to significantly improve days-supply balancing.

This project also caused a deep cultural change in the company. The success of the project motivated the use of OR-approaches (and more generically speaking, of analytical approaches) in other operations planning activities. In particular, it already triggered many other projects with the same OR group from the University of Porto, both in space related problems, such as backroom optimization, and in other related areas, such as marketing, store operations and logistics.

5.5.3 Standardization: knowledge management for a global process

Space Managers are divided into groups responsible for subsets of categories. The categories' know-how is kept inside each group, supported by manuals that report the implementation details (using a template similar to Figure 5.2). Nevertheless, these manuals are frequently limited to the upper criteria levels, giving only a general idea of the reasoning behind the planograms. Consequently, space managers keep most of the category know-how, which is partially lost when organizational changes occur. GAP also had a major impact on standardizing information and managing knowledge. Firstly, the use of electronic generation manuals (and the possibility of reusing them) centralized the categories' space planning know-how and made it possible to systematize the tacit knowledge available into information that can be shared among peers. Secondly, it reduced the subjectivity of the process, which is nowadays less dependent of the managers' experience.

5.6. Concluding Remarks

In the highly competitive retail environment of today, retailers can benefit from analytic tools for better decision making and many successful examples are reported in the literature. Shelf space planning is one area that is still to be explored mainly because of its complexity and high dependency on merchandising rules. We believe that this work is an important contribution in this direction both from a theoretical and practical point of view. On the scientific front, we provide innovative mathematical models and efficient algorithms to the shelf space allocation problem and bring more realism to the scientific approaches to this problem. From the application perspective, we give insights on how to tailor analytical approaches to the practice of shelf space management, namely by introducing the replication problem and by allowing users to control the level of customization from solutions, while still applying optimization in every step of the process. We also provide project management details that were critical during GAP implementation in Sonae MC, the major Portuguese retail company.

Although this paper describes a real application of shelf space planning, the approach does not intrinsically depend on any company specific policies, as it is based on rules that are defined in run-time. Therefore, it is sufficiently generic to be suitable to other retail companies working in the grocery or similar markets. Its modular nature also enables its adaptation and integration with other realities and IT systems.

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Appendix 5.A Target Facings Model

In this appendix, we provide a mathematical formulation of the Target Facings Model. Consider a specific category of a store with overall capacity C. The retailer wants to allocate N products, indexed by $i \in \mathcal{N}$, with length a_i. Each product is associated with a *space-to-sales* curve presented in Figure 5.5 that is linearized with piecewise lines, obtained using the facings associated with the days-supply intervals (see Figure 5.11). There are T days-supply intervals, indexed by $n \in \mathcal{T}$. For each product i, the minimum and maximum facings of each interval n are ds_i^n and ds_i^{n+1}.

This *space-to-sales* curve represented in the figure is widely used in the literature and is associated with a polynomial function depending on the space-elasticity parameter as firstly introduced by Corstjens and Doyle [1981]. Other authors already proposed piecewise linear approximations such as Gajjar and Adil [2010]. However, we are the first to consider the problem with days-supply intervals.

The objective is to maximize the planogram's expected demand by determining the

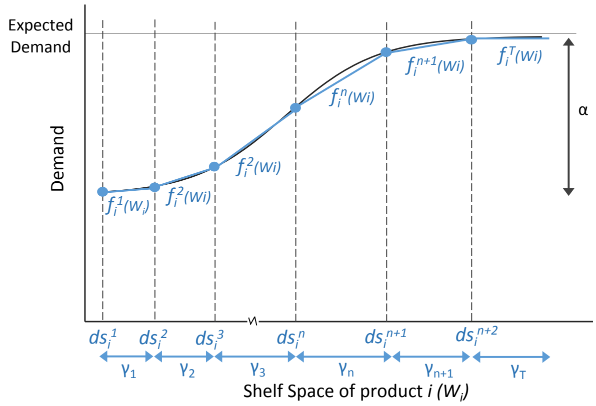

Figure 5.11 – Piecewise linearization of the space elasticity curve considering the days-supply intervals.

number of facings for each product i. The decisions to be made are: γ_n, which specifies whether the days-supply interval n is selected for all products; and W_i^n, which indicates the number of facings of product i if the days-supply interval is γ_n. The formulation is as follows:

$$\text{Maximize} \quad \sum_{i \in \mathcal{N}} \sum_{n \in \mathcal{T}} f_i^n(W_i^n) \tag{5.1}$$

$$\text{Subject to:} \quad \sum_{i \in \mathcal{N}} \sum_{n \in \mathcal{T}} W_i^n \cdot a_i \leq C \quad \text{(shelf-space capacity)} \tag{5.2}$$

$$l_i \leq \sum_{n \in \mathcal{T}} W_i^n \leq u_i, \forall\, i \in \mathcal{N} \quad \text{(minimum and maximum facings)} \tag{5.3}$$

$$ds_i^n \cdot \gamma_n \leq W_i^n \leq ds_i^n \cdot \gamma_n, \forall\, i \in \mathcal{N}, n \in \mathcal{T} \quad \text{(number of facings)} \tag{5.4}$$

$$\sum_{n \in \mathcal{DS}} \gamma_n = 1 \quad \text{(single day-supply interval)} \tag{5.5}$$

$$W_i^n \in \{0,1\}, \forall\, i \in \mathcal{N}, n \in \mathcal{T}; \gamma_n \in \{0,1\}, \forall\, n \in \mathcal{T} \quad \text{(integrality)} \tag{5.6}$$

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Chapter 6

Conclusion

This thesis is the result of problem-driven research motivated by the space management problems arising in the food retail industry. Instigated by the current challenges faced by Sonae MC, a Food Retailer operating in the Portuguese market, we approached two space-related problems. Firstly, we tackled the traditional Shelf Space Allocation Problem (SSAP) extending it in a practical oriented perspective, with a special emphasis on merchandising rules. Secondly, we looked into the problem of generating store-specific planograms from generic cluster-based planograms, introducing a innovative problem which we called Shelf Space Replication Problem (SSRP). This is a practice-oriented problem that was designed to help retailers which are forced to cluster their stores in order to efficiently manage categories. The case study of Sonae MC provided the motivation to understand these current challenges and was also the perfect environment to assess the practical value of our work.

The contributions of this thesis are aligned in two main directions. On one hand, we pushed the frontier of the shelf space literature with new formulations and solution approaches. On the other hand, we developed a Decision Support System (DSS) for the automatic generation of planograms that is nowadays being used on a daily basis by the shelf space management team at Sonae MC. In what concerns the shelf space literature, we started by reviewing the different SSAP models and developed a classification framework for this problem (Chapter 2). Subsequently, we proposed a commodity flow based formulation to the SSAP with two additional practical features: hierarchical product families and display directions. Later, we introduced an innovative application of Operations Research (OR) within the shelf space literature for replicating a shelf space plan across different stores, the aforementioned SSRP. Connected to this problem, we also proposed a promising objective function for shelf space problems that targets inventory leveling in order to obtain replenishment synergies. On the algorithmic front, we focused on MIP-based heuristics and developed two approaches, one for each of the two problems tackled. Chapters 3 and 4 describe the advances in the SSAP and SSRP, respectively. Regarding the DSS, we developed a modular system that successively applies OR-based approaches for the automatic generation of planograms. One of the most relevant features of the DSS is the flexibility to incorporate different merchandising rules, allowing the users to test different strategies for the products allocation. Chapter 5 details on how the analytical developments were tailored to fit the practice of shelf space management and the key-factors that led to a successful implementation.

In the highly competitive retail environment of today, there is no doubt that retailers can benefit from analytic tools to better managing space and increase their profitability. We believe that this thesis is an important contribution in this direction both by bringing additional realism into academia and by proving the value of advanced analytics in practice. Moreover, it fulfills its ultimate objective with the creation of one of the first applications of the "next generation" of shelf space planning systems.

In the remaining conclusions, we will make a brief description of the key-contributions of each chapter and highlight future research lines.

6.1. Contributions

The key-contributions of each chapter are the following.

Chapter 2 presented a description and a state-of-the-art literature review of the SSAP focusing on mathematical modeling approaches. This review emphasized the existence of a wide range of different approaches dealing with different decisions and space elasticity effects, which created a high level of inconsistency in the field. Based on this review, a classification framework was proposed with the intent to systematize the research into a set of sub-problems. The distribution of the existing publications across the sub-problems gave a clear indication of the research gaps in this field and we highlight the need to consider merchandising rules. Future research lines pointed to the most promising open questions in this field and suggested possible extensions to the framework. This chapter originated the following research paper:

- T. Bianchi-Aguiar, Maria Antónia Carravilla and José F. Oliveira. From a literature review to a classification framework for shelf space application problems. *Working paper*, 2015.

Chapter 3 presented a novel and realistic mixed integer programming formulation for the SSAP that considers location decisions and two novel features: hierarchical product families and display directions. The novelty in the formulation comes from introducing single commodity flow constraints to model product sequencing and from exploring the hierarchy in the product families to reduce the combinatorial nature of the problem. Based on the formulation, a MIP-based heuristic was also developed that uses product families to decompose the problem into a sequence of sub-problems that are solved using a *relax-and-fix* approach. To improve the matheuristic's performance, its original design was adapted following two directions: recovery from infeasible solutions (improving feasibility) and reduction of solution times (improving efficiency). We also provided a set of real case benchmark instances for the shelf space allocation problem with location decisions which was used to assess the formulation and the matheuristic and will hopefully allow future research in this area.

We proved the validity of the real-world features by analyzing a concrete example where we tested different scenarios: we removed the display shapes and then the product families to check the changes on the solution. This example showed that, even though the objective function decreases with the inclusion of these features, benefits are obtained with a more clever product arrangement on the shelves. However, these benefits are hard to grasp in the model as they are linked to the customers' response to the complexity of the planograms and to the way customers search for the products while shopping.

The new formulation (*BAP*) was compared to a state-of-the-art formulation and proved the benefit of the commodity flow based constraints to obtain better solutions and better execution times by exploring the product families hierarchy. Extensive computational tests were also performed to compare the formulation with the matheuristic. The matheuristic improved the solutions in two directions: it was able to solve a higher number of instances and it decreased the running times by over 70%. Nevertheless, this was obtained with a limited decrease on the solution quality. The *Improving Efficiency* extension further explored the trade-off of seeking lower execution times.

Chapter 3 resulted in the following research paper:

- T. Bianchi-Aguiar, Elsa Silva, Luis Guimarães, Maria Antónia Carravilla and José F. Oliveira. Allocating Products on Shelves under Merchandising Rules: Multi-level Product Families with Display Directions. *Submitted for publication*, 2015.

In Chapter 4 we presented the SSRP, the retail problem of transforming generic cluster-based shelf space solutions (role-planograms) into store-specific planograms. To the best of our knowledge, we are the first to introduce this problem in the shelf space literature. As the replication process is done by systematically extracting all the features out of the role planogram, we believe that it is suitable for most retail companies and may improve the use of analytics in practice.

We presented two mathematical programming formulations to solve the SSRP: the single- and multi- segment, with the latter having misaligned shelves along the planogram. The formulations use a novel inventory related objective function that is also a key-contribution of this chapter. Instead of determining individual order quantities for the products, the formulation balances the products' inventory level in order to trigger joint shelf replenishments. This objective is in line with common practices and evaluation metrics. To ensure the process scalability, we also present a MIP-based heuristic based on the formulation, that combines two well-known mathematical programming based heuristics: *fix-and-optimize* and *local-branching*. Moreover, we provided a set of real case benchmark instances to promote future research in this novel problem.

We tested the formulations and the MIP-based heuristic using the benchmark instances and the results were encouraging. We were able to generate a solution for all the instances with an average optimality gap of 0.03% in approximately two minutes on average. The MIP-based heuristic used 88% of the time and obtained an optimality gap of 0.19%. We also compared the solutions to hand-made planograms generated by the space managers of Sonae MC to evaluate inventory leveling. The majority of the solutions suffered a reduction of 16.6% in terms of average days-supply values and 13.1% in terms of days-supply standard deviation.

Chapter 4 resulted in the following research paper:

- T. Bianchi-Aguiar, Maria Antónia Carravilla and José F. Oliveira. Replicating Shelf Space Allocation Solutions Across Retail Stores. *Submitted for publication*, 2015.

Chapter 5 introduced GAP, the DSS that is today used on a daily basis by the space management team of Sonae MC. GAP is built on a modular basis and two of its modules integrate the approaches that were developed on the previous chapters (GAP Generation and GAP Replication). This chapter also introduces a novel formulation that was developed to estimate the number of facings for the products before their placement on the shelves. At the center of this formulation is a piecewise representation of a space-to-sales curve that embeds the experimental findings regarding the space elasticity effect, while still considering the objective of inventory leveling.

GAP enhanced shelf space management in three axes: process automation, by bringing better processing times (54% of reduction) and a positive change of paradigm from planogram construction to planogram evaluation; space optimization, with a potential sales increase of 1.4%, higher space occupation rates (on average 97%) and days-supply values with 61% less variability (standard deviation); and image standardization, with less subjectivity and a centralization of the categories' know-how.

Although this chapter describes a real application of shelf space planning, the approach does not explicitly integrate any company specific policies as it works by rules that are defined on the fly. Therefore, it is sufficiently generic to be suitable to other retail companies working in the grocery or similar markets. Its modular nature also enables the adaptation and integration with other realities and IT systems.

Besides the DSS, this chapter also resulted in the following paper:

- T. Bianchi-Aguiar, Elsa Silva, Luis Guimarães, Maria Antónia Carravilla and José F. Oliveira. Using Analytics to Enhance Shelf Space Management in a Food Retailer. *Working Paper*, 2015.

6.2. Future Work

By creating awareness to this problem, we hope to stimulate further research and encourage the use of optimization in the practice of shelf space management. We identified a set of future research topics connected to each of the chapters of this thesis, that we will briefly describe now.

Chapter 1 framed shelf space planning within retail operations and emphasized the interdependency between shelf space and other planning activities such as assortment, inventory and replenishment. Many opportunities lie in the integration of these activities. Nevertheless, as they usually belong to different organizational hierarchies and responsibilities, there may be some obstacles in their cooperation in practice. Therefore, besides integration, we also suggest the use of sensitivity analysis to measure the impact of upstream decisions in shelf space plans.

Chapter 2 suggested many relevant extensions to the current shelf space formulations that are worth tackling. In particular, it refers that most literature on shelf space allocation, including this thesis, considers that the location of the shelves are given as inputs to the models and suggests the integration of shelf-decisions in the problems. Additionally, as stores usually have other types of fixtures, their consideration and integration are also potential research topics. Pegboards are a particularly engaging fixture type for future studies because of their inherit complexity and the fact that they have been hardly studied in the literature.

Chapter 3 gave emphasis to the use of merchandising rules in shelf space models. Nevertheless, all rules were given as inputs to the formulation. Shelf space models could be extended to identify the family types that should be used to group products, seeking to increase display attractiveness.

Chapter 4 introduced the replication problem. Other replication methods can be worthwhile studying, especially with more evasive changes to the role planogram. Two possible extensions that could benefit the current practice are: (1) allowing changes to the products' shelf while still considering their relative positions; (2) minor variations on the product assortments by introducing for instance regional products. From an algorithmic perspective, further work could be done to generate real-time solutions by using more traditional heuristics.

Last but not the least, Chapter 5 presented a successful implementation of analytics in the practice of shelf space management. Additional work can be done to further enhance the DSS and for a better alignment between the theory and practice of shelf space management. We suggest the creation of shelf space solutions taking into account the current planogram implemented in the stores in order to trade-off potential profit and the costs of changes, namely, handling costs.

Appendix A

Notation

A.1. Shelf Space Allocation Problem

Indices

k	shelves
i, j	products
u, m	product families

Parameters

K	number of shelves
M	number of product families
N	number of products to display
w_k	width of shelf k
h_k	height of shelf k
a_i	width of product i
b_i	height of product i
p_i	profit of product i
l_i	minimum number of facings of product i
u_i	maximum number of facings of product i
γ_k	effectiveness of shelf k to generate revenue
v	maximum deviation of product families between shelves
w_m^{max}	width of the largest product from each block m

Sets

$\mathcal{K}$	set of shelves
$\mathcal{N}$	set of products
$\mathcal{M}$	set of family products (also known as blocks)
$\mathcal{N}_u$	set of products belonging to each family u
$\mathcal{M}_u$	set of downstream families belonging to each family u

$\mathcal{S}^H$ set of families that should have their downstream blocks with horizontal shape
$\mathcal{S}^V$ set of families that should have their downstream blocks with vertical shape
$\mathcal{V}_u$ set of blocks from the immediate downstream level, either product families ($m,n \in \mathcal{M}_u$) or products ($m,n \in \mathcal{N}$)

Decision Variables

W_{ik} the integer number of facings of product $i \in \mathcal{N}$ on shelf $k \in \mathcal{K}$
X_i^s the continuous horizontal location of product $i \in \mathcal{N}$, measured from the lower-left corner of the planogram to the lower-left corner of the first facing of the product
T_{mnk} = 1 if block m is displayed immediately after block n on shelf $k \in \mathcal{K}$, $u \in \mathcal{M}, m,n \in \mathcal{V}_u \cup \{0\}$
Y_{mk} = 1 if block $m \in \mathcal{V}$ is located on shelf $k \in \mathcal{K}$
F_{mnk} the continuous flow from block m to block n on shelf $k \in \mathcal{K}$, $u \in \mathcal{M}$, $m,n \in \mathcal{V}_u \cup \{0\}$
L_{ik} shelf length assigned to product $i \in \mathcal{N}$ on shelf $k \in \mathcal{K}$
X_m^s the horizontal location of the block $m \in \mathcal{V}$ (left coordinate)
X_m^e the horizontal location of the block $m \in \mathcal{V}$ (right coordinate)
FL_{mk} = 1 if $k \in \mathcal{K}$ is the first shelf of block $m \in \mathcal{V}$
LL_{mk} = 1 if $k \in \mathcal{K}$ is the last shelf of block $m \in \mathcal{V}$

A.2. Shelf Space Replication Problem

Indices

k levels
o segments
i, j products
m family alignments (left and right)
p facings
u minimum space share requirements

Parameters

K number of level
O number of segments
W total width of the planogram
N number of products
N_k number of products of level k
M^L number of left (right) alignments
M^R number of right alignments
Q number of minimum space share requirements
w_{ok} width of shelf (o,k)
h_{ok} height of shelf (o,k)
e_{ok} (=1) if shelf (o,k) exists in the planogram. (=0 otherwise)

c_{ok} (=1) if shelf (o,k) is aligned with the following shelf $(o+1,k)$ (=0 otherwise)
n_{ok} (=1) segment of the next existing shelf of level k after shelf (o,k) (if there is no next shelf, $n_{ok} = -1$)
a_i width of product i
b_i height of product i
l_i lower bound (upper bound) on the number of facings of product i
u_i upper bound on the number of facings of product i
s_i total stock of product i for each facing
K_i number of shelves where product i is
q_u minimum percentage of space to allocate to the products belonging to the space share requirement u
v maximum deviation between
D_i daily demand of product i
γ_i scale parameter that reflects the variation in demand with respect to the number of facings of product i
α_k scale parameter that reflects the variation in demand with respect to the shelf k where the product is placed
R_{ip} the days-supply value of product i prior to introducing the p^{th} facing
F_{ip} the replenishment frequency of product i prior to introducing the p^{th} facing ($F_{ip} = 1/R_{ip}$)

Sets

$\mathcal{K}$ set of levels
$\mathcal{O}$ set of segments
$\mathcal{N}$ set of products
$\mathcal{N}_k$ set of products of level k, ordered by order of appearance
$\mathcal{N}_k^-$ set of products of level k, ordered by order of appearance, except the last product
$\mathcal{M}^R$ set of left alignments
$\mathcal{M}^R$ set of right alignments
N_m^L set of products of each left alignment m
N_m^R set of products of each right alignment m
$\mathcal{Q}$ set of minimum space share requirements
$\mathcal{N}_u^Q$ set of products of each minimum space share requirement u

Decision Variables

R_i days-supply value of product i
W_i the integer number of facings of product $i \in \mathcal{N}$ on each of the shelves where the product is located
X_i the continuous horizontal location of product $i \in \mathcal{N}$, measured from the lower-left corner of the planogram to the lower-left corner of the first facing of the product,
W_{ip} (=1) if product $i \in \mathcal{N}$ has the p^{th} facing on the planogram, $p = 1,\ldots,u_i$

L_i	shelf length assigned to product $i \in \mathcal{N}$ on each of the shelves where the product is located
X_m^L	the horizontal location of left alignment $m \in \mathcal{M}^L$
X_m^R	the horizontal location of right alignment $m \in \mathcal{M}^R$
L_{io}	shelf length assigned to product $i \in \mathcal{N}$ in segment $o \in \mathcal{O}$, on each of the levels where the product is located,
Y_{io}	(=1) if product $i \in \mathcal{N}$ is located in segment $o \in \mathcal{O}$
X_{io}	the continuous horizontal location of product $i \in \mathcal{N}$ in segment $o \in \mathcal{O}$, measured from the lower-left corner of the planogrammytablesmalllinespace

A.3. Target Facings Problem

Indices

i	products
n	days-supply intervals

Parameters

N	number of products
T	number of days-supply intervals
w_i	width of product i
C	planogram overall linear capacity

Sets

$\mathcal{N}$	set of products
$\mathcal{T}$	set of days-supply intervals

Decision Variables

γ_n	(=1) if the days-supply interval n is selected to all products
W_i^n	the number of facings of product i if the days-supply interval is γ_n

Appendix B

Planogram Solutions

B.1. Example 1

	Category A
Main criteria	Subcategory A.1
1st Criterion	*Family-type 1 – Horizontal*
2nd Criterion	*Family-type 2 – Vertical*
3rd Criterion	*Family-type 3 – Horizontal*

Figure B.1 – Merchandising Rules from Example 1

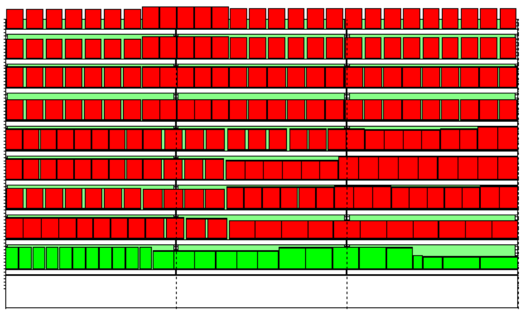

Figure B.2 – Highlight family-type 1 from Example 1

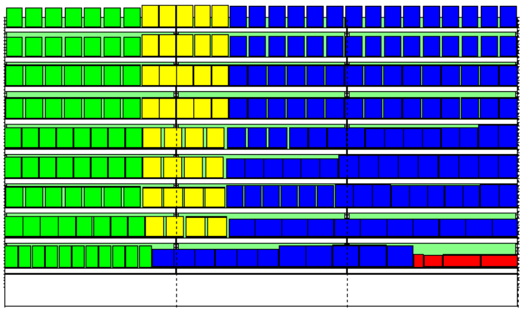

Figure B.3 – Highlight family-type 2 from Example 1

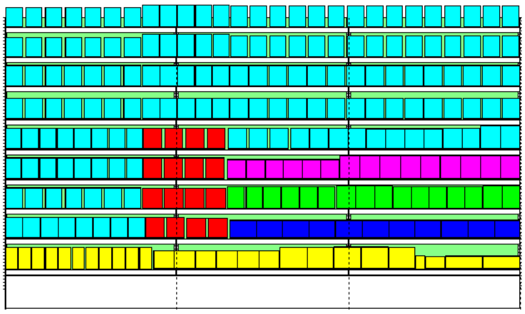

Figure B.4 – Highlight family-type 3 from Example 1

B.2. Example 2

	Category B		
Main Criteria	Subcategory B.1	Subcategory B.2	Subcategory B.3
1st Criterion	*Family-type 3 – Vertical*	*Family-type 2 – Horizontal*	*Family-type 1 – Vertical*
2nd Criterion		*Family-type 3 – Vertical*	*Family-type 2 – Horizontal*
3rd Criterion			*Family-type 3 – Vertical*

Figure B.5 – Merchandising Rules from Example 2

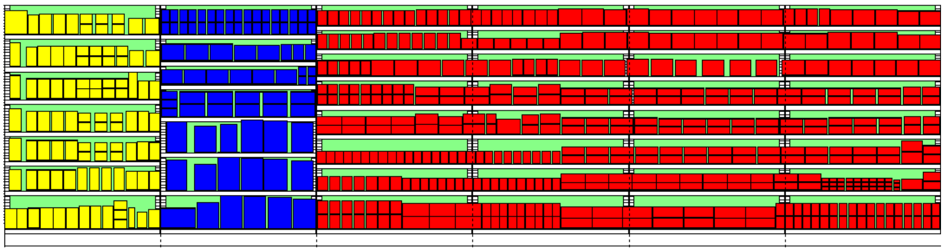

Figure B.6 – Highlight subcategory from Example 2

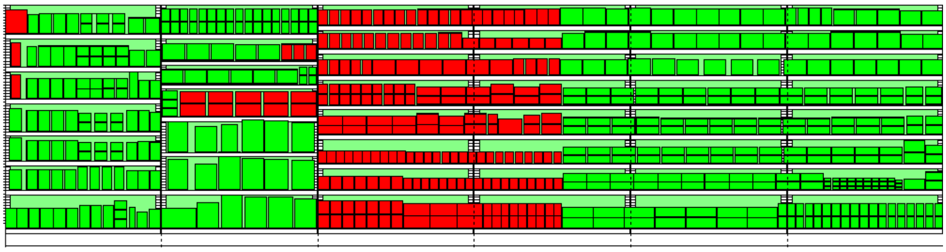

Figure B.7 – Highlight family-type 1 from Example 2

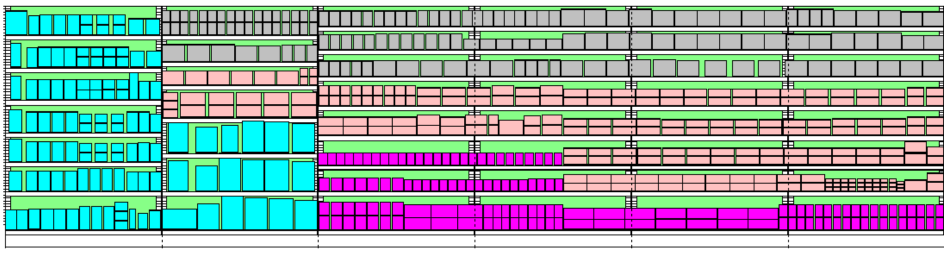

Figure B.8 – Highlight family-type 2 from Example 2

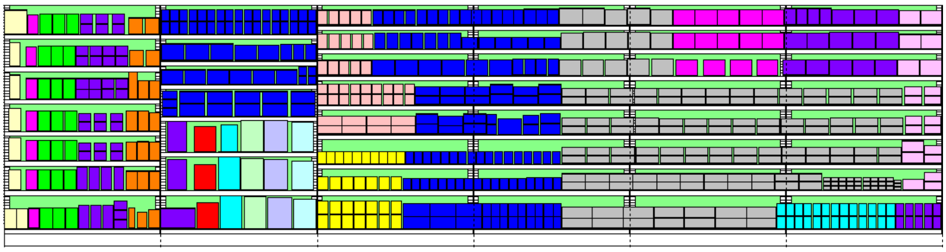

Figure B.9 – Highlight family-type 3 from Example 2

B.3. Example 3

	Category C						
Main Criteria	Subcat. C.1	Subcat. C.2	Subcat. C.3	Subcat. C.4	Subcat. C.5	Subcat. C.6	Subcat. C.7
1st Criterion	*Family-type 1 Vertical*	*Family-type 1 Horizontal*	*Family-type 1 Vertical*	*Family-type 1 Vertical*	*Family-type 1 Vertical*	*Family-type 1 Horizontal*	*Family-type 1 Vertical*
2nd Criterion	*Family-type 2 Horizontal*		*Family-type 2 Horizontal*	*Family-type 2 Horizontal*	*Family-type 2 Horizontal*		*Family-type 2 Horizontal*

Figure B.10 – Merchandising Rules from Example 3

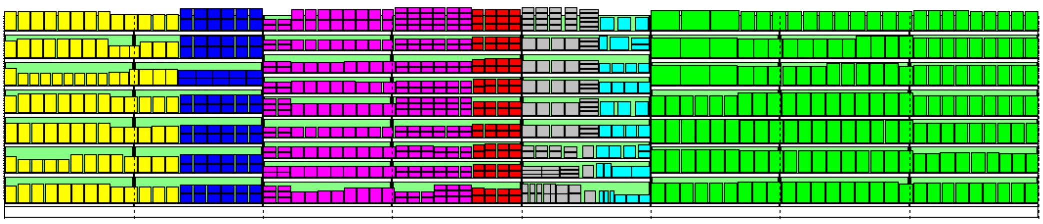

Figure B.11 – Highlight subcategory from Example 3

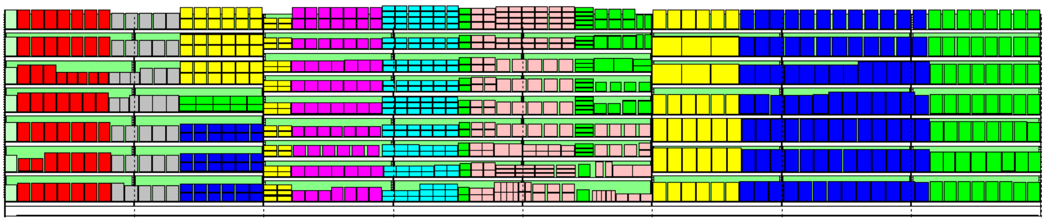

Figure B.12 – Highlight family-type 1 from Example 3

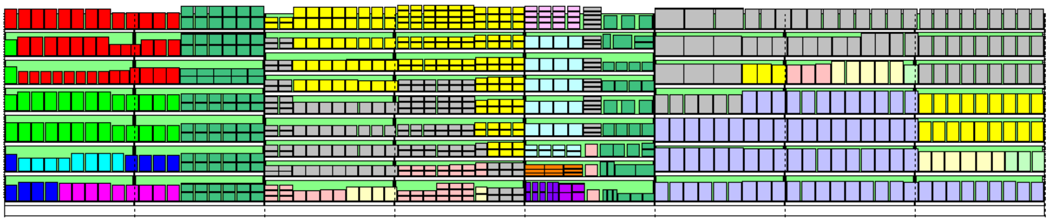

Figure B.13 – Highlight family-type 2 from Example 3

B.4. Example 4

	Category D		
Main Criteria	Subcategory D.1	Subcategory D.2	Subcategory D.3
1st Criterion	*Family-type 1 – Horizontal*	*Family-type 1 – Horizontal*	*Family-type 1 – Horizontal*
2nd Criterion	*Family-type 2 – Horizontal*	*Family-type 2 – Vertical*	*Family-type 2 – Vertical*

Figure B.14 – Merchandising Rules from Example 4

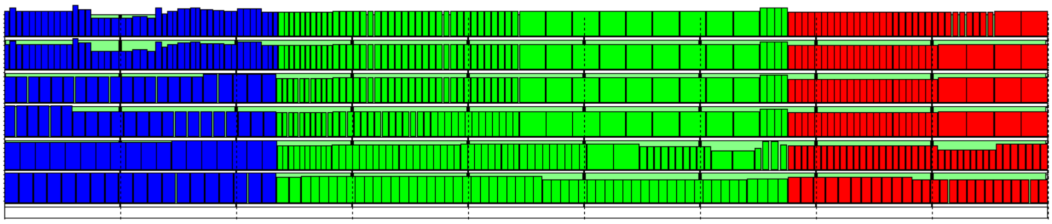

Figure B.15 – Highlight subcategory from Example 4

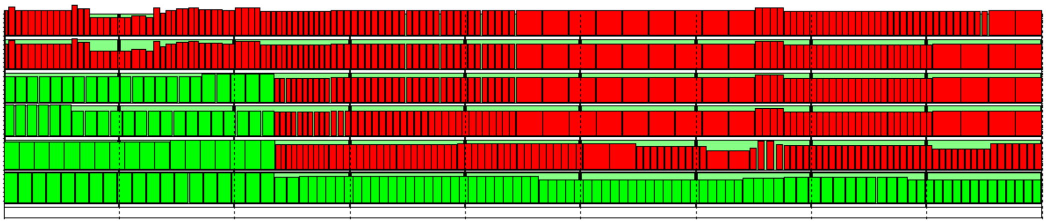

Figure B.16 – Highlight family-type 1 from Example 4

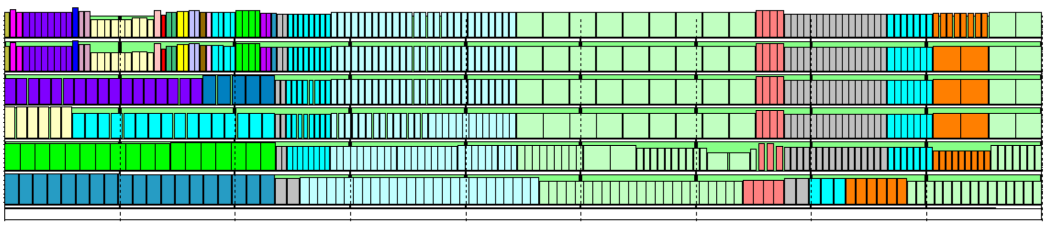

Figure B.17 – Highlight family-type 2 from Example 4