

U. PORTO

**FEUP FACULDADE DE ENGENHARIA
UNIVERSIDADE DO PORTO**

**PROBABILISTIC METHODOLOGIES FOR THE
SAFETY ASSESSMENT OF SHORT SPAN RAILWAY
BRIDGES FOR HIGH-SPEED TRAFFIC**

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2015

A dissertation presented to the Faculty of Engineering of the University of Porto
for the degree of Doctor in Civil Engineering.

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*“Light thinks it travels faster than anything but it is wrong.
No matter how fast light travels, it finds the darkness has
always got there first, and is waiting for it.”*

Terry Pratchett

“Yehi 'Or! (Let there be light!)”

The Book of Genesis 1:3

Aos meus Pais e à minha Avó

Abstract

Transportation systems are vital in modern societies as they represent one of the most important means to achieve both social and economic development on a more than ever global world. Presently a fast, safe, efficient and, if possible, environmentally friendly mobility of people and goods is paramount. It is in this context that the high-speed railway systems arises as one of the most interesting alternatives to other means of transportation, explaining its rapid expansion across the globe in the last decades.

Bearing in mind the importance of a safe and reliable high-speed railway network it was decided that the focus of this thesis should be directed to the development of a probabilistic methodology that enables the accurate safety assessment of short span railway bridges. This would complement most of the works done on this field to this date (that have adopted a deterministic approach) and would help understanding the effects of the intrinsic variability of parameters related to the train, track and bridge parameters on the dynamic behaviour of the system.

A review of the state of the art regarding the dynamics of railway bridges is presented, with particular attention given to the resonance phenomenon due to its significance to small and medium span bridges. Excessive accelerations have caused several problems to short span railway bridges within the European high-speed railway network and it is important to understand the advantages, drawbacks and limitations of the different methods used to assess the dynamic response of railway bridges. Additionally, it is important to review the limit states prescribed by the current European standards and understand their origin to evaluate the adequacy of these proposals.

Different probabilistic approaches were analysed and, taking into account the problem being studied, it became evident that the use of simulation techniques would be the most adequate approach. Two different simulation methods, namely the Monte Carlo and the Latin Hypercube, are combined with two different procedures to enhance the efficiency of the assessment. One of the methods is a tail modelling approach based on the extreme value theory

that uses adequate functions to model the tail of the obtained distribution. The other one is an Enhanced Simulation procedure which uses an approximation procedure based on the estimates of the failure probabilities at moderate levels for the prediction of the far tail failure probabilities by extrapolation.

To test the efficiency and accuracy of the different methodologies that are proposed a case study bridge was selected based on carefully selected range of potential solutions and taking into consideration the existing bridge portfolio on the current high-speed railway network. A ballasted filler beam bridge composed by six simply supported spans of 12 m each was selected and its behaviour was analysed for the crossing of a TGV-Double train. Several modelling particularities, including the complexity of the model used for both the train and the track, the consideration of train-bridge interaction and the existence of track irregularities are discussed in detail to understand their impact on the accurate assessment of the dynamic response of short span railway bridges. Additionally, a sensitivity analysis was carried out to determine the influence of each of the basic random variables on the dynamic behaviour of the train-bridge system.

The proposed probabilistic methodologies are used to assess the safety of the case study bridge for two different criteria: the running safety of trains due to loss of contact between the wheel and the rail and the track instability due to excessive deck vibrations. The use of these criteria provide examples of limit state functions with varying degrees of complexity that test and validate their efficiency, robustness and reliability. Globally, the Enhanced Simulation procedure proved to be significantly more efficient than the tail modelling approach. It was also observed that if the response is not monotonic the use of Latin Hypercube simulation may affect the efficiency of safety assessment. Furthermore, it was demonstrated that if the computational costs can be reduced through a refined simulation method then the use of the Enhanced Simulation approach will results in even further benefits. The obtained results are extremely promising and indicate the feasibility of the application of this type of methodology more frequently due to the reasonable computational costs that are required.

Resumo

Os sistemas de transporte são vitais para as sociedades modernas uma vez que são um dos mais importantes meios de desenvolvimento económico e social num Mundo cada vez mais globalizado. Atualmente, a deslocação rápida, segura, eficiente e, se possível sustentável, de pessoas e bens assume extrema importância. Por estes motivos o transporte ferroviário de alta velocidade emergiu como uma alternativa muito apelativa aos meios de transporte mais tradicionais, explicando-se assim o seu rápido desenvolvimento nas últimas décadas.

Tendo em consideração a relevância de uma rede ferroviária de alta velocidade segura e eficiente, a presente dissertação dedica a sua atenção ao desenvolvimento de uma metodologia probabilística que permita a avaliação da segurança de pontes ferroviárias de pequeno vão. Esta opção permite complementar o trabalho anteriormente desenvolvido nesta área (de carácter predominantemente determinístico) ajudando ainda a compreender os efeitos da variabilidade intrínseca aos parâmetros que condicionam a resposta do sistema ponte-via-comboio.

Foi realizada uma revisão do estado da arte relativa ao comportamento dinâmico de pontes ferroviárias, dando-se particular atenção aos fenómenos ressonantes que são especialmente importantes para pontes de pequeno e médio vão. Diversos problemas devido a acelerações excessivas neste tipo de estruturas inseridas na rede ferroviária de alta velocidade Europeia foram reportados no passado. Deste modo é imperativo compreender-se as vantagens, desvantagens e limitações das diferentes metodologias utilizadas para aferir a resposta dinâmica das pontes. Adicionalmente, é importante rever os estados limites definidos nas normas Europeias e compreender a sua génese de modo a avaliar se são adequados.

Foram analisadas diferentes metodologias probabilísticas para abordar o problema a estudar tendo-se concluído que a utilização de métodos de simulação seria a solução mais adequada. Deste modo, dois métodos de simulação, o método de Monte Carlo e o método do Hipercubo Latino, foram utilizados. Estes métodos foram combinados com dois procedimentos distintos de modo a melhorar a eficiência da avaliação da segurança. A primeira técnica baseia-se na teoria dos valores extremos e consiste na modelação das caudas das distribuições através de

funções apropriadas. A outra metodologia é uma técnica de simulação melhorada que utiliza um ajuste para níveis de probabilidade mais moderados estimando a probabilidade de falha para os valores mais extremos por extrapolação.

Atendendo às soluções tipicamente adotadas na rede ferroviária de alta velocidade Europeia para pontes de pequeno vão procedeu-se à seleção de um caso de estudo adequado de modo a permitir testar a eficiência e a precisão das metodologias propostas. Uma ponte mista do tipo “filler beam” com via balastrada e composta por seis tramos simplesmente apoiados de 12 m de vão foi escolhida. O comportamento dinâmico da ponte selecionada foi avaliada para a passagem de um comboio TGV duplo. De modo a melhorar a compreensão do impacto de alguns aspetos relacionados com a modelação numérica na avaliação rigorosa da resposta dinâmica, foram analisados e discutidos em detalhe aspetos específicos como o tipo de modelo a utilizar tanto para o comboio como para a via, a influência da interação entre comboio e ponte e ainda a importância de tomar em consideração a existência de irregularidades da via-férrea. Para além disso, foi ainda levada a cabo uma análise de sensibilidade de modo a avaliar a influência apresentada por cada uma das variáveis aleatórias no comportamento dinâmico do sistema comboio-ponte.

As metodologias probabilísticas propostas são aplicadas na avaliação da segurança da ponte adotada como caso de estudo tendo a análise sido efetuada para dois critérios de segurança distintos: um relacionado com a segurança de circulação dos comboios devido à perda de contacto entre roda e carril e outro relacionado com a estabilidade da via associada a vibrações excessivas do tabuleiro da ponte. A utilização de dois estados limites distintos permite a avaliação de funções com graus de complexidade distintos, permitindo assim testar e validar a eficiência, robustez e precisão das metodologias propostas. Foi possível verificar que, no geral, a técnica de simulação melhorada é mais eficiente que a metodologia baseada na modelação das caudas das distribuições. Conclui-se ainda que caso a função estado limite não seja monotónica, a utilização do método do Hipercubo Latino pode afetar a eficiência da avaliação da segurança. Finalmente, foi ainda demonstrado que caso a utilização de uma técnica de simulação mais refinada torne possível obter ganhos de eficiência, a utilização da metodologia de simulação melhorada introduz ainda um maior benefício. Pode então dizer-se que os resultados obtidos são extremamente promissores e apontam para a viabilidade da utilização deste tipo de análise de uma forma mais recorrente tendo em consideração os custos computacionais bastante razoáveis que lhe estão associados.

Acknowledgements/ Agradecimentos

The completion of this thesis would not have been possible without the contribution of several people. Therefore, I would like to express my sincere gratitude to those who have helped and contributed to the achievement of this thesis. In particular, I would like to thank:

- Ao meu orientador, Prof. Abel Henriques, por me acompanhar há já vários anos e me ter apoiado sempre de forma incondicional e por todo o encorajamento que me transmitiu. O desafio que me lançou para embarcar num projeto de investigação numa área na qual não me sentia muito confortável apenas foi superado através da sua orientação, por todo o conhecimento que transmitiu e pela sua total disponibilidade para me auxiliar nos momentos menos bons.
- Ao meu co-orientador, Prof. Rui Calçada, gostaria de agradecer por todo o conhecimento que me transmitiu na área da dinâmica de pontes e de toda a componente ferroviária. Gostaria ainda de agradecer o seu apoio, encorajamento e otimismo transmitidos em todas as fases deste trabalho.
- Devo ainda acrescentar que tive de facto uma equipa de orientadores, cujo contributo foi fundamental para o sucesso do meu trabalho. As discussões científicas mantidas permitiram-me crescer imenso como Engenheiro e estou certo que a relação de amizade mantida se irá manter pelo futuro.
- To Prof. Anders Rönnquist, for approaching me during a conference and for the discussion on alternatives to the methods I was using at the time. His scientific knowledge and expertise were important to improve the quality of my work.
- Ao João Barbosa e ao Sérgio Neves por toda a ajuda, paciência e dedicação que tiveram comigo para me ensinar e ajudar na utilização do FEMIX. A sua ajuda foi crucial para melhorar a eficiência das análises e para o sucesso do meu trabalho.
- Aos colegas e Engenheiros do NCREP, Alexandre Costa, Bruno Quelhas, Filipe Neves, Tiago Ilharco e Valter Lopes, pela sua amizade e por terem sido os meus

primeiros “colegas de trabalho”. Tento levar comigo a atitude relaxada, divertida e bem-disposta que me transmitiram e que considero fundamental para um bom ambiente de trabalho.

- A todos os meus colegas do grupo de investigação, Alejandro de Miguel, Andreia Meixedo, Carlos Albuquerque, Cristiana Bonifácio, Cristina Ribeiro, Diogo Ribeiro, Joana Delgado, João Barbosa, João Xico, Joel Malveiro, Nuno Santos, Nuno Ribeiro Pedro Jorge e Sérgio Neves, e aos pela sua amizade e apoio e por todos os excelentes momentos que me proporcionaram ao longo deste trabalho.
- Aos restantes colegas da Faculdade, Ádám Zsarnóczy, Ana Sofia, André Monteiro, Athanasios Tappas, Despoina Skoulidou, Diogo Coutinho, Diogo Figueira, Jiang Yadong, João Milheiro, José Pedro Silva, Luis Macedo, Luis Martins, Miriam López, Nuno Pereira e Rui Barros.
- To all my colleagues at Network Rail for making me feel welcome and helping me to adapt to a new country and culture. I would also like to thank Ben, Myles, Richard Crew, Richard Newell and Steve for taking the time to review the grammar in this document.
- Aos meus amigos da “Confraria”, Ana, Artur, Filomena, Joana, João Neves, Marisa Simão por me terem recebido tão bem e me terem mostrado tantas partes fantásticas de Londres. Agradeço ainda o facto de, apesar da minha menor disponibilidade, me incluírem sempre nas atividades.
- A todos os meus amigos a quem não pude dar a atenção merecida nos últimos tempos, mas cuja amizade, apoio e carinho que demonstram por mim foi também essencial para a realização deste trabalho.
- À “Mãe” Florinda, à Maria, à Leste, à Ana e à Fátima por todo o carinho que têm por mim, por se preocuparem sempre comigo e por me tratarem como se fosse da família.
- Ao meu irmão, Ricardo, pelo apoio e carinho e pelas discussões sobre Engenharia em que agora, por vezes, já me ensina umas coisitas.
- Aos meus Pais, por todo o apoio incondicional, carinho e incentivo ao longo não só deste trabalho mas de toda a minha vida. Peço-lhes desculpa pelos momentos de mau

humor e menor atenção e agradeço toda a paciência que têm comigo. São, sem dúvida, os grandes responsáveis por todos os meus sucessos.

- Por último, mas não menos importante, gostava de agradecer todo o carinho e apoio da Joana, por toda a paciência que tem comigo e por todo o ânimo que sempre me transmite nos momentos em que mais preciso.

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Chapter 1

Introduction

1.1 Scope of the thesis

Transportation systems play a key role in modern societies. Social and economic development depend on the fast and efficient mobility of both people and goods. In this context, the high-speed railway system emerges as a reliable and appealing alternative to road-based means of transportation. The popularity and success of the high-speed railway network relies on two main aspects: firstly it is a safe and low energy means of transportation and secondly because of its high on-schedule rate. For this reason the high-speed railway network has developed and expanded rapidly across the world in the last decades. The first high-speed railway line was the Tōkaidō Shinkansen in Japan which connected Tokyo to Osaka and started operating in 1964, with trains running at 210 km/h. In Europe the first high-speed railway line started operating nearly 20 years later in 1981 in France, connecting Paris to Lyon, with a maximum speed of 260 km/h. Since then the European high-speed railway network has been expanding continuously and the operation speed has also increased. Nowadays it is already well established in several countries such as France, Germany, Italy, Spain and the United Kingdom and is planned to keep expanding in the near future, as is shown in Figure 1.1 and Figure 1.2.



Figure 1.1 – European High-speed railway network [adapted from Institut d'aménagement et d'urbanisme - Île-de-France (2012)].

Driven by the socio-economic potential of a European high-speed railway network the European directive 69/48/EC was published in July 1996 and was the initial step to establish the basis of the operational standards within the network. This work finally culminated with the publication of the Technical Specifications for Interoperability [TSI (2002)]. This document aimed to harmonise the European standards regarding the several components of the network, the rolling stock, the infrastructure and operation, and defines the technical basis for operational safety within the European high-speed railway network.

Amongst the several components of the railway network, railway bridges have been identified as some of the most sensitive elements. Despite this fact, due to the several constraints involved in the planning of high-speed railway networks, which can either be political, territorial or technical, it can be observed that most networks have a significant percentage of bridges and viaducts. In China and Japan, for example, the percentage of bridges and viaducts in several of the lines exceeds 75% of the total length [Ishibashi (2004); Dai *et al* (2010)]. The European reality is slightly different and the percentage of bridges or viaducts tends to be lower. However, if we analyse the case of HS2, which is the biggest planned high-speed development in Europe and previewed to begin in 2017 in the UK, in a line with a length of approximately 200 km there will be around 350 bridges, which is still quite significant.

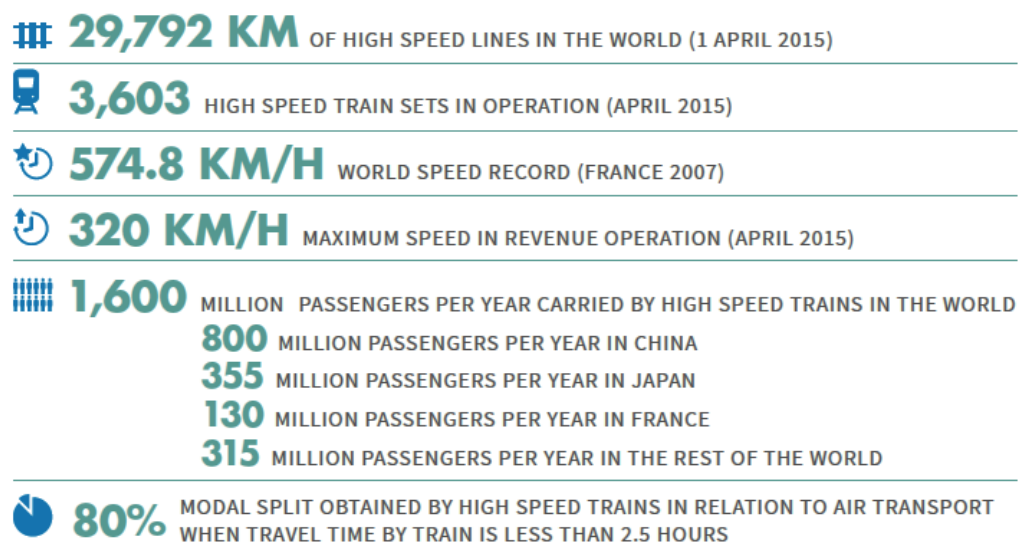


Figure 1.2 –High-speed railway network in numbers [<http://www.uic.org/highspeed>].

Therefore, the dynamic behaviour of railway bridges has been the research subject for many engineers since the introduction of railway as a means of transportation due to the importance of the dynamic effects caused by trains crossing over them. This is one of the most relevant aspects to take into account for a safe design and operation. However, the generalisation of high-speed railway lines over the last decades, the continuous demand for higher operational speed and the introduction of high-speed freight traffic has introduced new engineering challenges. Within the high-speed railway network the dynamic amplifications caused by trains moving over the bridges have proven to be the governing factor for bridge design. Recent research has shown that excessive vibrations have a higher tendency to occur for speeds above 200 km/h as a consequence of the resonance phenomena [ERRI (1999)]. This can lead to several problems, namely the instability of the ballast layer, the loss of contact between the wheel and the rail, the increase of fatigue-related damage or even affect the comfort of the passengers. The current European standards reflect the concern for excessive vibrations and all the consequences that arise from inadequate/inefficient design by imposing the application of a dynamic analysis in almost every case where the maximum line speed exceeds 200 km/h [EN1991-2 (2003)].

Short span railway bridges have been particularly identified as most critical as they are more prone to experiencing resonant effects. This has become evident following the reports of problems due to excessive vibrations in several bridges in the first European high-speed railway

line in France [Hoorpah (2005); Zacher & Baeßler (2009)], which required immediate repairs to the bridges shortly after construction to ensure they had adequate dynamic behaviour. Besides being very sensitive to resonance the dynamic response of short span railway bridges is also strongly affected by the non-structural environment, namely the track and train properties and even by the structural boundary conditions. All these aspects are particularly difficult to adequately quantify during the design stage and this was identified as one of the reasons why the initial design proved ineffective.

It became evident that a better understanding of the resonance mechanisms was required in order to try to establish which parameters govern the dynamic response of these structures. The works of the European Railway Research Institute ERRI (1999), Majka & Hartnett (2008) and more recently of Doménech *et al* (2014) have tried to provide a response to this problem. Despite being extremely useful and providing some excellent insight into this issue none of the works has simultaneously considered the variability of the parameters related to the bridge, the track and the trains nor have they taken into consideration the existence of track irregularities in the analysis.

It is also clear that the majority of the research studies on the assessment of the dynamic behaviour of railway bridges are deterministic in nature. The analysis is often limited to a specific scenario even when the existence of track irregularities is accounted for. This conventional approach uses fixed values to define the parameters, it may use a parametric analysis to assess the dynamic response and evaluates the results and safety based on deterministic limits defined in the standards which are the result of semi-probabilistic methods. In the few works that took the variability of parameters into account it is noticed that generally the variability is defined for only one of the components of the system and usually limited to a specific train speed.

Taking advantage of the increasing computational capacity, the use of reliability based methodologies to assess the safety of railway bridges can be a reality in the present. This type of approach enables a more realistic assessment of the dynamic behaviour of the train-bridge system as it reflects the real variability of the parameters that affect the dynamic response. Furthermore, the output of this approach is a probability of a defined limit being reached (usually understood as a probability of failure) which allows the adjustment of the risk level according to the limit state being analysed and the problem that is being studied. This versatility is also reflected in the type of problems that can be addressed which are not limited to the design of new structures but

can also be applied to the assessment of existing structures when the lines are upgraded for higher speeds or heavier traffic.

In the absence of sufficient studies and due to the existing knowledge gap in this topic the present thesis intends to propose an efficient, robust and accurate probabilistic methodology to assess the safety of short span railway bridges.

1.2 Motivation and objectives

The motivation for the research work presented in this thesis is the paucity of studies carried out to assess the safety of high-speed railway bridges using probabilistic approaches that enable the variability of the parameters that govern the dynamic behaviour of the train-bridge system to be taken into account. As previously highlighted, most of the work developed in this field is deterministic in nature and the analysis is often limited to a specific scenario even when the existence of track irregularities is accounted for. As a consequence, there has been some debate about the adequacy of some of the limits defined in the current European standards, particularly with respect to the bridge deck acceleration limits.

Due to the importance of accurately assessing the response of the train-bridge system, both for a cost efficient and safe design and operation, the development of adequate and efficient probabilistic methodologies is of the utmost importance and is extremely useful for Engineers both at the design stage and for the assessment of existing structures. Therefore, the main objective of this thesis consists of developing a probabilistic methodology that enables an accurate and efficient assessment of the safety of short span high-speed railway bridges whilst accounting for the variability of parameters of the bridge, the track and the train as well as for the existence of track irregularities.

The use of such an approach enables a more realistic analysis of the behaviour of the train-bridge system which is translated into an increase in the accuracy of the assessment. Furthermore, this type of approach also enables the definition of the safety/risk level by an adequate selection of the probability threshold which can be adjusted according to the problem being studied. For this reason, and despite the fact that in this thesis the design stage view is adopted, this type of methodology can bring benefits to the analysis of either new or existing structures.

Additionally, the research presented in this thesis focus its attention on the short span high-speed railway bridges due to the fact that these structures are particularly sensitive to resonant effects as well as to the influence of the non-structural elements (such as the track properties and the train-bridge interaction effects) on the response of the train-bridge system. It is also an objective of this study to identify the parameters that govern the dynamic behaviour of this type of bridges and how their intrinsic variability affects the safety of the train-bridge system.

In order to achieve these objectives a case study bridge that is representative of the population of short span bridges in the European high-speed railway network is selected and different limit states are analysed using the proposed methodologies in order to check the accuracy, efficiency and robustness of the methodologies as well as analysing the feasibility of the application of these methods in the future.

1.3 Layout of the thesis

Taking into account the motivation and objectives of this thesis that have just been presented in the previous section, the contents of this thesis have been divided into seven chapters. A schematic representation of the contents of the thesis can be seen in Figure 1.3.

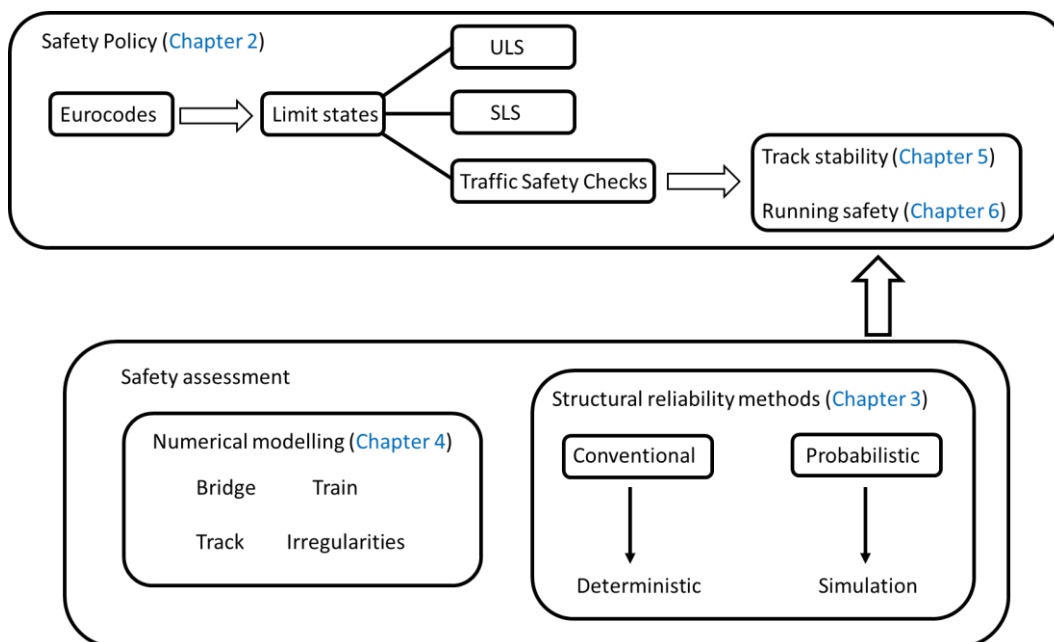


Figure 1.3 – Schematic representation of the contents of the thesis.

The current chapter is the introduction and aims to present the scope and the reasons for the research work that has been developed as well as the objectives that were aimed to be achieved by this work.

Chapter 2 is dedicated to the review of the state of the art regarding the dynamic behaviour of railway bridges, with special attention given to the resonance phenomenon which is particularly important for small to medium span bridges. A review of the limit states defined in the European standards to ensure structural and traffic safety in the high-speed railway network is also presented along with a review of some of the most up to date research studies and recommendations about the safety topic. Due to its importance for an accurate assessment of the dynamic response of the train bridge-system a review of the most common numerical methods is presented and the main advantages and drawbacks of each methodology are discussed.

As described in the previous section the main objective of the current thesis is the development of an efficient and accurate probabilistic methodology to assess the safety of railway bridges while accounting for the variability of the parameters that govern the dynamic behaviour of the train-bridge system. Therefore, it is key to review the basic concepts of structural reliability and this is presented in Chapter 3 along with some of the most usual methods used to address these problems in Civil Engineering. In this chapter the methodologies that are proposed to assess the safety of the train-bridge system are presented, the selection of the methodologies is justified and the main advantages and potential drawbacks are analysed.

In Chapter 4 the case study bridge is presented along with the numerical models developed for both the bridge and the train. A literature review of the usual numerical models is presented and the options chosen in the modelling are explained. Furthermore, some particular aspects of the numerical models like the track length before the bridge, the generation of the track irregularities profiles and the track model are discussed in greater detail due to the importance that the adequate modelling of these elements has to the accurate assessment of the dynamic response of the train-bridge system. Finally, a discussion on the influence that each of the selected random variables has on the safety of the train-bridge system, based on the results obtained by a sensitivity analysis, is presented and the variables that govern the dynamic response of the train-bridge system are identified.

In Chapter 5 the proposed probabilistic methodologies are applied to the analysis of the track stability assessment. A preliminary assessment is carried out using a simpler approach (moving loads) and accounting only for the variability of bridge related parameters. The aim of this

preliminary analysis was to test the applicability of simulation methods to assess the safety of the train-bridge system and to identify the parameters that govern the dynamic behaviour. Afterwards, a safety assessment that accounts for the train-bridge interaction effects, including the variability of parameters related to the bridge, the track as well as for the existence of track irregularities is carried out and the obtained results are discussed and the efficiency and accuracy of the different methods is analysed.

Chapter 6 is dedicated to the assessment of train running safety which is analysed by the evaluation of the wheel unloading rates. The two most efficient methodologies identified in Chapter 5 are applied and the obtained results and the efficiency of the different methodologies are again discussed, highlighting the advantages and drawbacks of each of methods.

Finally, Chapter 7 presents the main conclusions and contributions from the work carried out in this thesis and discusses some lines of research that could be worthy of pursuing in the future in order to complement the work developed in this dissertation.

Dynamic behaviour of railway bridges

2.1 Introduction

The introduction of high-speed railway has brought new challenges to Engineering, particularly to what concerns assessing and designing bridges, which have been identified as being some of the most sensitive elements of the network. The generalisation of high-speed railway lines over the last decades, the continuous demand for higher operational speed and the introduction of high-speed freight traffic represent some of these challenges.

High-speed trains can travel at speeds greater than 300 km/h which introduce loads of higher intensity due to the significant increase of the dynamic component of the loading. The dynamic amplifications originated by trains moving over a bridge at such high speeds have proved to be the governing factor for bridge design in most cases. Short span railway bridges (spans up to 15 m), in particular, have been reported as problematic since the introduction of the first high-speed lines in France due to excessive deck vibrations related to resonance effects [Hoorpah (2005); Zacher & Baeßler (2009)]. Research carried out by the European Rail Research Institute (ERRI) [ERRI (1999)] also indicates that excessive vibrations have a higher tendency to occur for speeds above 200 km/h, as a consequence of the resonance phenomena. This can lead to several problems, namely the instability of the ballast layer, the loss of contact between the wheel and the rail, the increase of fatigue-related damage or can even affect the comfort of the passengers.

Despite the small number of accidents that have occurred in the past in high-speed railway lines, the safety subject requires continuous research in order to guarantee a safe and reliable operation. The European Standards and the Technical Specifications for Interoperability (TSI) define several criteria for the rolling stock, the infrastructure and the operation in order to guarantee a safe design and adequate performance of railway bridges throughout their design life.

In this Chapter the general aspects of the dynamic behaviour of railway bridges are analysed and discussed. Special attention is dedicated to the identification of the main aspects that influence the dynamic response of railway bridges and consideration is given to a particular response case that is critical for the safety of railway bridges: resonance. In addition to this, the numerical evaluation of the dynamic response is also addressed. The most typical approaches are presented, their advantages and drawbacks are discussed and the influence of each method on the accuracy of the obtained results is analysed. Lastly, an overview of the several limit states defined in the European standards, which aim to guarantee both the structural and operational safety of railway bridges in high-speed railway lines, is presented.

2.2 Dynamic effects

2.2.1 Overview

Generally, the most relevant loads acting on railway bridges have a dynamic nature. This dynamic nature results from the crossing of vehicles over the bridge at a certain speed, transmitting their loads and inertial actions to the structure. At very low speeds the load effects still vary with time but are equivalent to those of a static loading. However, as the train speed increases the load effects also rise due to dynamic amplification and, consequently, the bridge response (stresses, displacements, etc.) also tends to increase. It is well known that the response of a structure subjected to dynamic loading can be significantly higher than the response obtained in the case of static loading. The works of Timoshenko & Young (1974), Frýba (1996), *Yang et al* (1997) and *Xia et al* (2003) are some of the most comprehensive studies on this topic. The increment of the dynamic loading compared to static effects corresponds to the dynamic amplification factor and depends on the following aspects [ERRI (1999)]:

- The response of the bridge to the loads travelling at a certain speed and the inertial response of the bridge;
- The effects of regularly spaced groups of loads crossing the bridge at a certain speed, which can lead to resonance phenomena;
- The effects of loads passing over track defects, the existence of track irregularities or wheel defects that lead to an increase of the loading due to impulsive effects.

Therefore, it can be said that the dynamic response of the bridge results from a complex system composed by three different parts: the bridge, the track and the train. Regarding the bridge some of the most important properties are the span length, the mass, the natural frequencies (itself a function of the span, the stiffness, the mass and support conditions) and the damping [Frýba (1996)]. With respect to the track one of the most important properties are the track irregularities, in particular their profiles (shape and amplitude) and the existence of regularly spaced defects or isolated defects. Finally, the train properties with more influence on the dynamic behaviour of the system are the train speed, the spacing of regular groups of axles, the variation in magnitude of the axle loads and the length of the train (which influences the number of regularly repeated loads) [ERRI (1999)]. These conclusions were backed up by results obtained by Majka & Hartnett (2008) who analysed the effects of several parameters on the dynamic amplification of railway bridges. A parametric analysis was performed in order to identify the key variables influencing the dynamic response of railway bridges. The authors concluded that the speed of the train, train-to-bridge frequency, mass and span ratios, as well as bridge damping were the most significant variables.

2.2.1.1 Influence of the mass and stiffness of the bridge

Other authors [Rigueiro (2007); Figueiredo (2007)] have also carried out parametric studies to complement some of the findings reported by the ERRI and to identify how different parameters affect the dynamic response of railway bridges. These studies analysed, amongst other aspects, the influence of the mass of the structure and its stiffness on the dynamic response. It was concluded that the dynamic amplification is inversely proportional to the mass of the bridge. These results were expected as the increase of the mass leads to the increase of the inertial effects that directly oppose to the effects of the dynamic loading. However, the study showed that despite the acceleration levels are reduced for an increase of the bridge mass, the

displacement level is unaffected by this parameter (in terms of maximum value). As for the stiffness of the bridge, it affects mostly the displacements of the structure and being the relation also inversely proportional. It was also noted that the stiffness of the bridge only influences the level of displacement and does not affect the maximum acceleration value.

2.2.1.2 Influence of the structural damping

Damping is another key structural parameter for the dynamic response of the train-track-bridge system. Structural damping occurs due to energy dissipation caused by the loss of energy during oscillation cycles. The damping mechanisms are extremely complex and may be due to a combination of several different aspects such as the energy dissipation due to the bending of materials, friction at the bearings or energy dissipation along structural boundaries, energy dissipation in the ballast layer and the opening and closing of cracks in materials (particularly in concrete structures). Due to the complexity of the damping phenomenon its exact quantification is extremely difficult to calculate and usually structural damping values are determined using the results obtained during experimental tests. When this is not possible, a lower bound value is selected based on the bridge material. An ERRI (1999) study concluded that the structural response at resonance is extremely dependant on the structural damping value. Since the dynamic response is inversely proportional to the structural damping it is important to adopt a lower bound value for the damping in order to ensure conservative results. This study also carried out several experimental tests on different bridges and bridge types in order to try to identify trends and the existence of correlation between damping and other structural parameters. However, the results were inconclusive due the limited amount of data gathered for each bridge type. Nevertheless, these results provide a useful insight on structural damping to be considered for railway bridges and allowed the definition of a lower bound to be used for different bridge types according to the span length, which has been reflected in EN1991-2 (2003).

2.2.1.3 Influence of the numerical modelling

When assessing the dynamic response of the train-track-bridge system another important aspect is the adequate modelling of each subsystem. Due to the advances in the processing capacity of computers the models used to model the bridge, the track and the train have been continuously increasing in complexity, enabling a more detailed simulation of the phenomena.

Despite the several modelling possibilities the model should be selected carefully in order to guarantee the best possible balance between the accuracy of the results and the computational costs. Dieleman & Fournol (2003) refer that in the particular case of short span bridges a significant difference between numerical and experimental results is often observed. The authors identified the following aspects as the main reasons for the observed differences: i) inaccurate definition of the span length; ii) inaccurate definition of the boundary conditions; iii) neglecting the track-bridge composite effect; iv) the use of moving loads to model the train; and v) neglecting the train-bridge interaction effects. Amongst the several aspects these authors focused mostly on the influence of the bearings stiffness on the dynamic response and were able to conclude that the use of elastic supports generally leads to satisfactory results. However, they point out that the bearings stiffness is a very difficult parameter to be accurately measure and represents a very sensitive modelling parameter.

With respect to the modelling of the track several different proposals can be found in the literature. However, there are some aspects to distinguish between these models. Primarily they can be split according to the number of dimensions that are considered, dividing them into 2D [Calçada (1995); Zhai *et al* (2004)], 3D [Chellini & Salvatore (2007); Zabel & Brehm (2009)] and even 2.5D models, for some specific problems, such as the analysis of ground vibrations, [Costa (2011)]. This choice is mostly related with the type of analysis that is intended to be carried out. Another aspect concerns the track elements that are included in the numerical model, with some including all the track elements such as the rail, rail pads, sleepers, ballast and the existence of track irregularities, as well as the track-bridge composite effect [Calçada (1995); Zhai *et al* (2004)], whereas other authors choose fewer elements [Lou (2005)]. The latter models often neglect this effect but a parametric analysis carried out for a simply supported bridge with a small span shows the impact of the track-bridge composite effect on the dynamic response. The analysis considered both the higher and lower limits proposed in the European Standard EN1991-2 (2003), which represent a loaded and an unloaded track, respectively, as well as a scenario where the composite effect is neglected. The results are presented in Figure 2.1 and clearly indicate that the variation from loaded to unloaded track has a small impact on the dynamic response. However, when the track-bridge composite effect is neglected significant changes can be observed and the dynamic amplification shows an important increase.

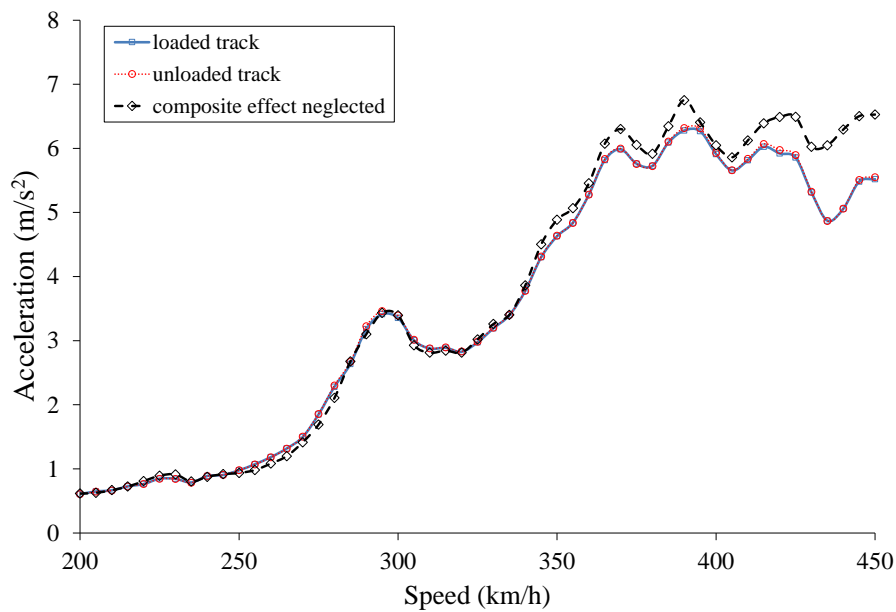
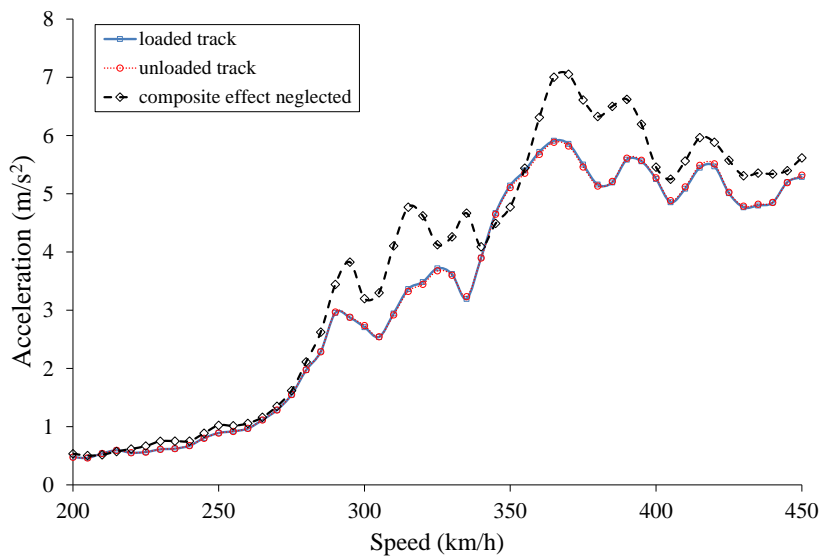
a) Maximum deck acceleration at $\frac{1}{2}$ span.b) Maximum deck acceleration at $\frac{3}{4}$ span.

Figure 2.1 – Example of the influence of the track-bridge composite effect on the dynamic response.

Finally, regarding the train model a different range of options can also be found in the literature. There are generally two main types of formulation used to model trains: one is the multibody dynamics formulation [Pombo (2004); Kwark *et al* (2004)], very common in Mechanical Engineering, and the other is the Finite Elements Method, mostly used in Civil Engineering [Calçada (1995); Yang *et al* (2004)]. The numerical models can either be two dimensional [Zhai *et al* (2001); Goicolea *et al* (2004); Doménech & Museros (2011)] or three

dimensional [Kwark *et al* (2004); Xia & Zhang (2005); Zhang *et al* (2008); Lee & Kim (2010)]. The 2D models are the most commonly found in the literature and this can be explained by the fact that these are simpler, require less information about the train dynamic properties (which is often difficult to find) resulting in a model with fewer degrees of freedom and, consequently, with higher computational efficiency. However, the main drawback of these models is that they limit the analysis to a single direction (generally, the vertical direction). The 3D models are more versatile as they allow the analysis of all directions. These models tend to be more realistic and for some particular cases, namely when assessing passenger riding comfort, are necessary to obtain accurate results. The consequence of using more complex models is the increase of the computational time as the number of degrees of freedom that are taken into account is considerably larger.

Given the importance of numerical modelling on the accuracy and efficiency of the dynamic analysis this topic is discussed in greater detail on Chapter 4. Several types of models that can be found in the literature are presented and the models used in this work are introduced and discussed. It is therefore important that the selection of the numerical model is carefully made and the degree of complexity to employ should always be related with the results that are to be analysed. This is to ensure that the computational costs of the analysis are feasible and adequate to the aim of the study.

Besides the modelling of each subsystem their interaction is accounted for in the analysis is another aspect of great importance. Generically, this can be divided into two distinct groups: analysis that do or do not account for the train-bridge interaction. Due to the relevance of this topic for the present dissertation this is discussed in more depth on Section 2.3 where the different alternatives are presented and their advantages and disadvantages are analysed.

2.2.2 Resonance

A particular case of dynamic amplification, which is particularly relevant due to the consequences of its occurrence, is resonance. Bridge resonance occurs when the excitation frequency, λ , matches or is a multiple of a natural frequency, n_j , of the bridge. The resonant speed, v_{res} , can then be estimated by:

$$v_{res} = n_j \cdot \lambda_i = n_j \cdot \frac{d}{i}, i = 1, 2, 3, \dots \quad (2.1)$$

where d represents the spacing between regular groups of axles.

Recent research has shown that excessive vibrations have a higher tendency to occur for speeds above 200 km/h as a consequence of the resonance phenomena [ERRI (1999)]. Therefore, this is a crucial aspect to take into consideration when designing bridges for high-speed railway lines.

The existence of resonance can significantly increase the dynamic response, which can lead to several problems. Some of these problems include the instability of the ballast layer, loss of contact between the wheel and the rail, increase of fatigue-related damage, discomfort of the passengers and, in more extreme cases, it can even lead to the collapse of the bridge.

In the first high-speed railway line in Europe, which connected Paris to Lyon, several problems were reported due to the existence of resonance, resulting in significant maintenance and repair costs [ERRI (1999); Hoorpah (2005); Zacher & Baeßler (2009)]. Museros (2002) summarised the main problems that were detected due to excessive vibrations. The problems reported included the flying ballast phenomenon, which reduced the support to the rails, the enhancement of the ballast layer deterioration rate that affected the ballast bonding and promoted the appearance of voids beneath some sleepers (originating hanging sleepers). Moreover, the cracking of concrete elements was also observed, leading to a global stiffness reduction of the bridge which changed the critical train speeds over the structure. In order to provide a better understanding of this dynamic phenomenon and to prevent the serious consequences associated to its occurrence several researches have been dedicated to this topic in recent years. The aim of these studies is mainly to ensure the adequate performance of the train-track-bridge system and to develop methodologies that enable guaranteeing the safe design and operation of railway bridges and also the minimisation of operational/maintenance costs.

The European Rail Research Institute (ERRI) carried out extensive studies that aimed to expand the knowledge on the dynamic behaviour of bridges in high-speed lines and to serve as a guideline to verify the safety demands and guarantee an adequate performance in service [ERRI (1999)]. The scope of the study was quite broad and dedicated some attention to the resonance phenomena. The report points out that for simply supported structures the design is generally

governed by resonant effects that originate excessive deck accelerations, affecting the stability of the ballast layer and, consequently, creating a running safety risk.

Yang *et al* (1997) investigated the dynamic behaviour of simply supported beams subjected to the passage of high-speed trains. This study identified the parameters that govern the dynamic behaviour of the structure and proposed an optimal design criterion for high-speed railway bridges based on both the condition of resonance and suppression.

Xia *et al* (2006) evaluated the resonance effects on a high-speed railway bridge using theoretical formulations, numerical simulation and experimental tests. The authors divide the resonant response into three types according to different resonance mechanisms: the first is due to the periodical effects of the loading, the second is due to the rate of the loading and the third is a result of the periodically loading of the swing caused by track irregularities and wheel hunting movements.

The work of Rigueiro (2007) was focused on the study of the dynamic behaviour of short to medium span railway bridges. The study analysed both the resonance and the suppression phenomena, highlighting the governing parameters. Furthermore, the influence of the track model in the dynamic response of the train-track-bridge system was also studied. The author observed that accounting for the track in the numerical model enabled the dissipation of the higher frequencies. It was also possible to conclude that neglecting the track in the model leads to an overestimation of the maximum bridge deck accelerations, particularly for resonant speeds.

Rauert *et al* (2010) dedicated his attention to the influence of the continuous ballast layer over bridges with two structurally independent decks since generally this aspect is neglected in most research works. Through a monitoring campaign the authors observed that there is a considerable load transfer from the loaded to the unloaded deck (see Figure 2.2). This study concluded that the continuous ballast layer leads to a considerable increase of the overall stiffness of the structure, leading to a significantly lower dynamic response of the bridge, particularly for resonant speeds. This is explained by the fact that the resonant effects are generally associated with the structure's first natural frequencies, which are the most affected by the additional stiffness provided by the continuous ballast. For research purposes the ballast interaction was simulated through spring elements that coupled the two separated bridge decks, requiring a complex 3D model. However, the authors recognised that in terms of common design practice a simplified approach would be of great advantage. For this reason, the introduction of a

component of additional bending stiffness which reproduces the effects of the continuous ballast over the independent decks was suggested.

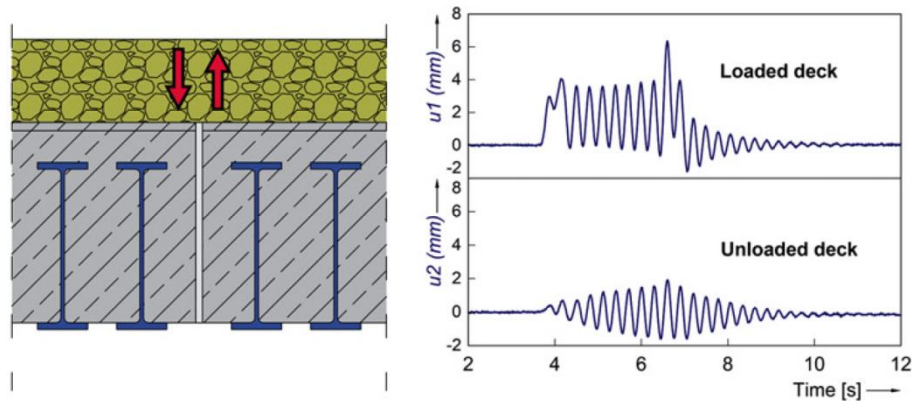


Figure 2.2 – Example for load transfer between loaded and unloaded bridge decks [Rauert *et al* (2010)].

Martínez-Rodrigo *et al* (2010) applied passive control techniques to mitigate the excessive vibrations due to resonance on short simply supported railway bridges. Two real bridges in Spain, one inserted in the high-speed railway network and the other inserted in the conventional railway network, were investigated in order to reduce their dynamic responses under the circulation of high-speed traffic. This work had two main objectives: to verify if the use of passive control techniques, through the application of fluid viscous dampers (FVDs), enabled the correction of the dynamic behaviour of the bridge, reducing the level of vertical acceleration to admissible values according to the European standards and to evaluate the technical feasibility of the proposed solution to real structures. The authors concluded that the introduction of the FVDs allowed an efficient control of the deck vibrations (see Figure 2.3) and that its application in a real bridge did not prove problematic from a technical point of view.

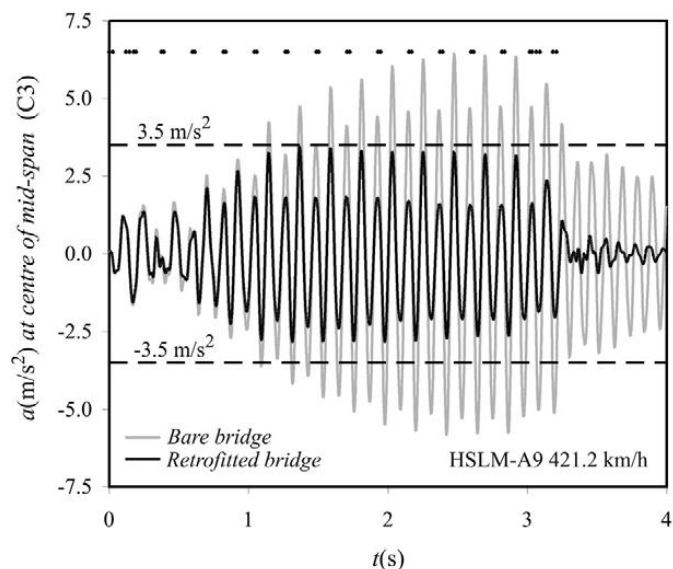


Figure 2.3 – Influence of the fluid viscous dampers in the dynamic response [Martínez-Rodrigo *et al* (2010)].

Shin *et al* (2010) on the other hand focused their attention in the vehicles, namely in Korean Train eXpress (KTX) train. In order to prevent resonance due to the periodical effects of the loading the authors suggest the modification of the high-speed train configuration by inserting size-adjusted vehicle(s) into the existing train arrangement (see Figure 2.4). However, from a practical point of view this approach does not seem feasible due to the large variability of the dynamic properties of bridges that are inserted in the high-speed railway networks. Nevertheless, the work presents some very interesting observations regarding the potential of exploring resonance suppression.

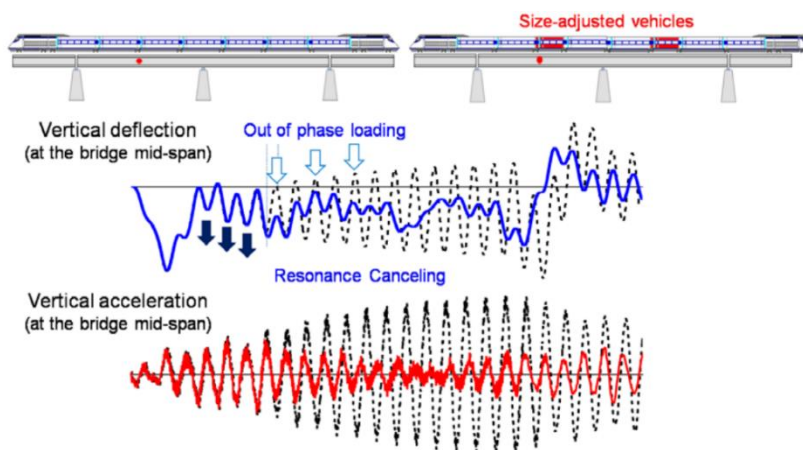


Figure 2.4 – Vibration reduction scheme based on insertion of size-adjusted vehicles [Shin *et al* (2010)].

Romero *et al* (2011) studied the problem of confined structures, which represent a quite common case in the railway network for hydraulic passages or box culverts. The typical configuration of these structures is characterised by very short spans which are very prone to experience resonance phenomena. The main objective of this work was to understand the influence of soil-structure interaction in the dynamic response of the railway bridge. The results showed that this aspect is particularly important for the resonant speeds, as disregarding it may lead to a significant overestimation of the dynamic response (see Figure 2.5).

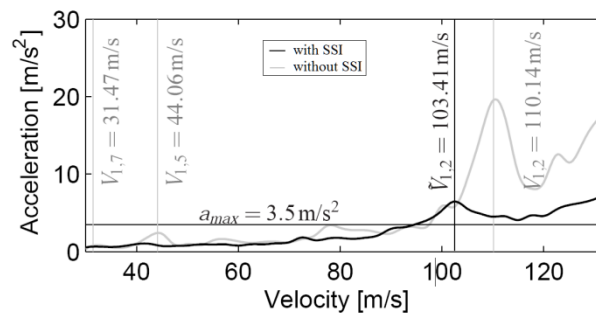


Figure 2.5 – Influence of the soil-structure interaction in the dynamic behaviour of confined structures [Romero *et al* (2011)].

2.3 Numerical evaluation of the dynamic response

The methodologies to assess the dynamic behaviour of railway bridges have been evolving throughout the years. The *Union International de Chemin de Fer* (UIC) has been responsible for recommendations and codes of practice for the design and assessment of railway bridges in Europe. An extensive historical review of the UIC recommendations can be found in the works of James (2003) and Museros (2002). In the work of Museros (2002) an overview of the most important publications in the design and evaluation of the dynamic behaviour of railway bridges, highlighting their main contributions, is also included.

Presently, the most recent recommendations of UIC have been incorporated in EN1991-2 (2003), which is the current guideline for the design of railway bridges. In order to numerically assess the dynamic response of a railway bridge EN1991-2 (2003) allows two different types of approach: quasi-static analysis or dynamic analysis. The selection of the most suitable method can be done by consulting the flowchart that is illustrated in Figure 2.6.

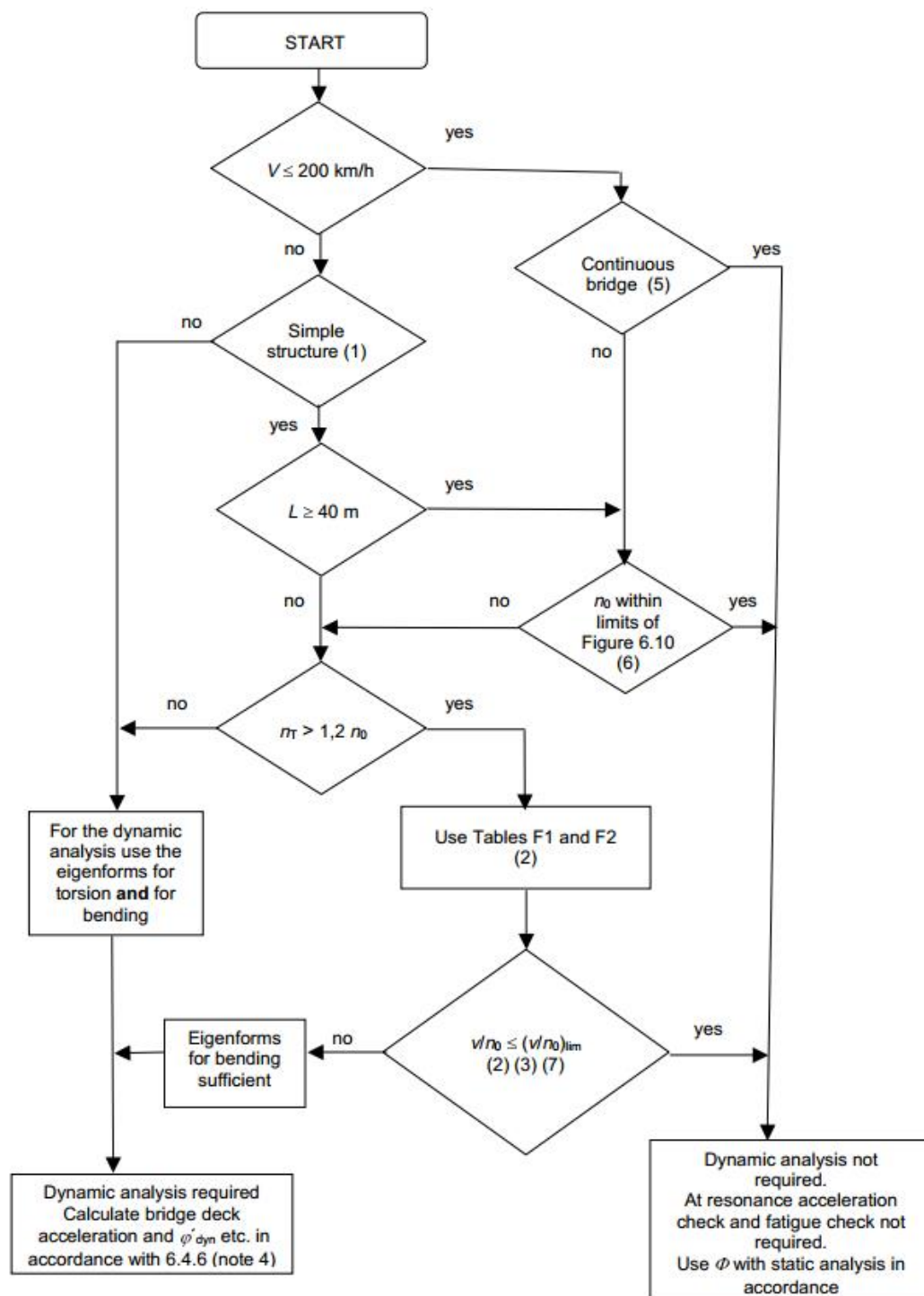


Figure 2.6 – Flowchart for determining the need of performing a dynamic analysis [EN1991-2 (2003)].

It can be observed that for some very specific cases EN1991-2 (2003) allows using a quasi-static analysis, which results from the response obtained by a static analysis multiplied by a dynamic factor, Φ , that is introduced in order to account for dynamic nature of the loading. This

method is addressed in Section 2.3.1. For all the remaining cases EN1991-2 (2003) proposes the use of dynamic analysis, as described in Section 2.3.2.

2.3.1 Quasi-static calculation method

If the requirements established by EN1991-2 (2003), shown in Figure 2.6, are verified a static analysis can be performed. The static response of the bridge must be evaluated under Load Models 71, SW/0 and SW/2. Furthermore, these results must be multiplied by a dynamic factor, Φ , which enhances the static effects of the load models. This dynamic factor is not to be perceived as a dynamic amplification factor since its application is limited to load models. The value this coefficient takes was defined according to a study that involved the analysis of several real bridges and intends to cover the dynamic envelope of all trains operating in the European high-speed railway network.

The value of the dynamic factor depends on the quality of track maintenance and also on the “determinant” length, L_ϕ , which represents the length of the influence line for deflection of the element being analysed. Initially the application of this factor was limited to simply supported bridges, however, the introduction of L_ϕ enabled its application to any type of bridge.

For standard maintenance tracks the dynamic factor, Φ_3 , can be determined by:

$$\Phi_3 = \frac{2.16}{\sqrt{L_\phi - 0.72}} + 0.73 \quad (2.2)$$

Additionally, the value of Φ_3 is limited to the interval $1 \leq \Phi_3 \leq 2$.

In the case of carefully maintained tracks the dynamic factor, Φ_2 , is given by:

$$\Phi_2 = \frac{1.44}{\sqrt{L_\phi - 0.20}} + 0.82 \quad (2.3)$$

where Φ_2 is limited to the interval $1 \leq \Phi_2 \leq 1.67$. The values to be adopted for L_ϕ can be consulted in EN1991-2 (2003).

This simplified approach is particularly useful for preliminary design stages. As long as the analysis is limited to structures that present properties that fall within the range of application of a general case scenario it enables simplifying some of the calculations while ensuring an adequate design of the structure. Due to these simplifications the approach is necessarily more conservative. Furthermore, it should be noted that the dynamic factor does not account for resonance effects. For such cases a dynamic analysis must be carried out in order to accurately determine the bridge response.

2.3.2 Dynamic analysis

Another methodology involves carrying out a dynamic analysis. In this approach two types of trains must be considered: real trains, which currently operate in the European railway networks, and the High Speed Load Models (HSLM) trains, which were defined taking into account the interoperability criteria defined in TSI (2002). Several train speeds should be analysed up to a maximum of 1.2 times the maximum allowable speed on the line. Furthermore, the selected speed step must enable a clear identification of the dynamic behaviour of the bridge, particularly of the resonant peaks.

EN1991-2 (2003) suggests the determination of the dynamic amplification, φ , obtained in the dynamic analysis. This dynamic amplification also includes the effects of track irregularities and vehicle imperfections and can be written as follows:

$$1 + \varphi = 1 + \varphi' + \lambda \cdot \varphi'' \quad (2.4)$$

φ' – factor related with the dynamic amplification due to the train crossing the bridge at speed, assuming a perfect track;

φ'' – corrective factor to account for the existence of track irregularities and vehicle imperfections;

λ – factor that accounts for the quality of the track maintenance.

Regarding the dynamic analysis several methodologies, presenting different degrees of sophistication, including analytical [Timoshenko & Young (1955); Frýba (1996); Yang *et al*

(1997)], simplified [ERRI (1999)] or numerical [Frýba (2001); Museros & Alarcon (2005); Johansson *et al* (2013)], are suggested in the literature. The procedures can be formulated either in time domain or in the frequency domain. The selection of the formulation is generally dependent on the problem that is being studied. The frequency domain formulation is usually applied in problems where the soil properties are a key aspect, as it tends to be more efficient from a computational point of view. However, this approach presents some limitations, namely the fact that it may impose some restrictions when dealing with non-periodic effects and nonlinear structural models [Popp *et al* (1999)]. The use of the time domain formulation is more common and tends to be selected in the analysis of the dynamic behaviour of railway bridges. According to Neves *et al* (2012) there are several nonlinearities in the vehicle–structure system that should be considered, such as the nonlinear contact or the state-dependent rail pads and ballast properties. In these cases the use of time domain methods is more appropriate. In the present dissertation the time domain formulation was adopted. A brief review of some of these methods is presented in the following sections.

2.3.2.1 Analytical methodologies

The analytical methodologies enable a complete understanding of the basic principles of the dynamic behaviour of railway bridges. Typically, this problem tends to be particularly complex limiting the application of such an approach to very simple cases. A classic example is the analysis of the dynamic response of a simply supported beam subjected to the passage of a single moving load [Timoshenko & Young (1955); Frýba (1996)] or a set of moving loads [Yang *et al* (1997)]. Yau *et al* (2001) formulated the solution of the dynamic response of a simply supported beam with elastic bearings subjected to a moving load. There is also an analytical formulation for determining the solution of the problem related to the response of a simply supported beam subjected to a moving sprung mass [Yang *et al* (2004)]. Recently, Johansson *et al* (2013) proposed a closed-form solution for the analysis of the dynamic response of multi-span beam bridges under moving loads. The authors point out that the solution assumes the Bernoulli beam theory and that its accuracy depends on how well the structure can be represented by this theory. As it can be seen the application of analytical methods is very limited. Nevertheless, these methodologies were very useful to understand the parameters that govern the dynamic behaviour of railway bridges and were used to set out the basis for simplified methodologies that will be present in the following section.

2.3.2.2 Simplified methodologies

Based on the analytical solutions for the problem of a series of loads moving over a simply supported beam some other methods have been proposed. In this section two different simplified methods are presented: the Decomposition of Excitation at Resonance (DER) method, proposed by ERRI Committee D 214 [ERRI (1999)], and the Virtual Influence Line method, which was proposed by SNCF as alternative to the DER approach. The application of such methods is limited to isostatic bridges that present a behaviour similar to that of a simply supported beam and also that the dynamic response can be adequately represented by simply taking into account the contribution of the first vertical bending mode.

a) Decomposition of Excitation at Resonance (DER) method

The DER method is particularly suitable for analysing the response of simple supported spans at resonance. It decomposes the dynamic response of the bridge in Fourier series and assumes that the incremental contribution to the acceleration is provided by the term corresponding to resonance. Therefore, in the analysis only the term from the Fourier series closest to resonance is used.

The acceleration response of a beam, \ddot{y} , is given by:

$$\ddot{y} = C_t \cdot A\left(\frac{L}{\lambda}\right) \cdot G(\lambda) \quad (2.5)$$

The response can be divided into three terms, where the first two terms depend on the characteristics of the bridge, whereas the final term depends on the properties of the train. In particular, C_t is a constant term that can be computed by:

$$C_t = \frac{4}{m \cdot \pi} \quad (2.6)$$

where m is the mass per unit length of the bridge deck. The term $A(L/\lambda)$ represents a influence line:

$$A\left(\frac{L}{\lambda}\right) = \frac{\left| \cos\left(\frac{\pi \cdot L}{\lambda}\right) \right|}{\left| \left(\frac{2 \cdot L}{\lambda}\right)^2 - 1 \right|} \quad (2.7)$$

where L is the span length of the bridge and λ is the wavelength of the excitation which is a function of the train speed, v , and the natural frequency of the bridge, n_0 .

Finally, the term $G(\lambda)$ reflects the excitation due to the train and the response of the deck to resonance and is designated by train spectrum, which is the so called dynamic signature of the train:

$$G(\lambda) = \max_{i=1}^{N-1} \frac{1}{\xi x_i} \left[\sqrt{\left(\sum_{k=0}^i P_k \cos\left(\frac{2\pi x_k}{\lambda}\right) \right)^2 + \left(\sum_{k=0}^i P_k \sin\left(\frac{2\pi x_k}{\lambda}\right) \right)^2} + \left(1 - e^{-2\pi \xi \frac{x_i}{\lambda}} \right) \right] \quad (2.8)$$

where P_k and x_k represent the load k and the corresponding position, respectively, relative to the first load that crosses the bridge (first axle of the first locomotive) and ξ represents the damping rate of the bridge. The maximum response is not necessarily obtained when the whole train has crossed the bridge. The train spectrum must correspond to the envelope of the response, thus explaining the use of the maximum function in Eq. (2.8).

b) Virtual Influence Line method

The Virtual Influence Line method divides the response of the beam into two different stages: the first stage corresponds to the forced vibration of the beam, which represents the period of time where the train loads are on the bridge, whereas the second stage corresponds to the free vibration of the beam, representing the period of time after the loads have crossed the bridge.

The main assumptions of the method rely on the fact that the length of the train is much longer than the span of the bridge and also in the fact that the maximum dynamic response is obtained in the moment where the last train load leaves the structure. Therefore, the dynamic response of the beam with respect to the accelerations is given by:

$$\ddot{y} = C_{accel} \cdot A(r) \cdot G(\lambda) \quad (2.9)$$

where C_{accel} is a constant term that depends on the mass per unit length of the bridge deck:

$$C_{accel} = \frac{1}{m} \quad (2.10)$$

$A(r)$ is designated by dynamic response factor, which only depends on parameters related to the bridge and the train speed:

$$A(r) = \frac{r}{1-r^2} \cdot \sqrt{e^{-\frac{2\xi\pi}{r}} + 1 + 2 \cdot \cos\left(\frac{\pi}{r}\right) \cdot e^{-\frac{\xi\pi}{r}}} \quad (2.11)$$

where r is given by:

$$r = \frac{v}{2 \cdot L \cdot n_0} \quad (2.12)$$

The term $G(\lambda)$ depends solely on parameters related to the train and translates the accumulation of the effects of several train loads:

$$G(\lambda) = \max_{i=1}^{N-1} \sqrt{\left(\sum_{k=0}^i P_k \cos\left(\frac{2\pi x_k}{\lambda}\right) e^{-2\pi \xi \frac{x_i}{\lambda}} \right)^2 + \left(\sum_{k=0}^i P_k \sin\left(\frac{2\pi x_k}{\lambda}\right) e^{-2\pi \xi \frac{x_i}{\lambda}} \right)^2} \quad (2.13)$$

Like in the DER method the maximum function must be used since the maximum response is not necessarily obtained when the whole train has crossed the bridge.

As a final remark it should be noted that both methodologies presented in this section have a very narrow practical application range and their use is limited to preliminary design stages of simply supported structures in order to estimate the maximum dynamic response and identify the critical train speeds.

2.3.2.3 Moving loads method

One of the most commonly used methodologies when analysing the dynamic behaviour of a bridge subjected to the loading of a vehicle is the moving loads method. This numerical method is relatively simple but enables the analysis of any type of structure, regardless of its complexity and represents the simplest way to represent a moving vehicle. Some examples of the application of such a method in the analysis of the dynamic response of railway bridges can be found in the works of Klasztorny & Langer (1990), Yang *et al* (1997), and Yau *et al* (2001).

This method is based on the time integration of the dynamic equations for the structure when subjected to a series of moving loads, representing each axle of a given train. As was previously stated the method is fairly simple, which translates into both reduced computational times and effort. However, this approach assumes some simplifications, namely by neglecting the interaction effects between the train and the structure.

The equations of motion represent the force equilibrium for every degree of freedom of the system and can be expressed as:

$$\mathbf{F}_i(t) + \mathbf{F}_a(t) + \mathbf{F}_e(t) = \mathbf{F}(t) \quad (2.14)$$

This equation indicates that for any given time step t the external forces, $\mathbf{F}(t)$, and the internal forces, that can be divided into inertial forces, $\mathbf{F}_i(t)$, damping forces, $\mathbf{F}_a(t)$, and elastic forces, $\mathbf{F}_e(t)$, are in equilibrium.

Scrutinising Eq. (2.14) and taking into account that the inertial forces can be obtained by multiplying the mass matrix, \mathbf{M} , the viscous damping matrix, \mathbf{C} , and the stiffness matrix, \mathbf{K} , by the acceleration vector, \mathbf{u} , the velocity vector, $\dot{\mathbf{u}}$, and the displacement vector, \mathbf{u} , respectively, it is noticed that it can be re-written in the following form:

$$\mathbf{M} \cdot \ddot{\mathbf{u}} + \mathbf{C} \cdot \dot{\mathbf{u}} + \mathbf{K} \cdot \mathbf{u} = \mathbf{F}(t) \quad (2.15)$$

Due to its simplicity it has been reported that this method may not be sufficiently accurate for cases where the coupled behaviour of the train and the bridge significantly affect the dynamic response, which corresponds to cases where the mass of the vehicle is not negligible compared to that of the bridge. Short to medium span bridges are within the sort of structures where the effects of the mass of the vehicle are not small compared to that of the bridge and, in these cases, the moving loads method tends to overestimate the maximum accelerations of the bridge deck.

Museros *et al* (2002) compared the dynamic behaviour of 25 simply supported bridges using both the moving loads method and the train-bridge interaction method. This study concluded that the interaction effects lead to a significant reduction (around 25%) of the maximum dynamic response.

Similar conclusions were drawn in the research work of Goicolea *et al* (2002). In this work the two methods were also compared and scenarios with different bridge spans and damping were analysed. It was noticed that the differences were higher for structures with structural damping lower than 2%.

For this reason EN1991-2 (2003) suggests the use of an additional damping, $\Delta\zeta$, which is a function of the span length, to reproduce the effects of the vehicle-bridge interaction and to overcome the referred limitations. Nonetheless, the additional damping method should be used with caution as Doménech *et al* (2014) concluded that in certain cases this approach overestimates the interaction benefits, which leads to a non-conservative prediction of the bridge dynamic response.

2.3.2.4 Train-bridge interaction method

To perform a more realistic and accurate method to assess the dynamic behaviour the train-bridge interaction effects need to be taken into consideration. The train-bridge interaction method accounts not only to the dynamic properties of the bridge but also takes into account the dynamic properties of both the train and the track. In addition, and contrary to what is observed in the moving loads method, the load of each wheel varies in time due to the variation of the wheel-rail contact forces. This is a more advanced assessment method which requires specific software to be used. For this reason, this approach requires higher computational capacity and is more time consuming when compared to the moving loads approach.

Due to the significance of the train-bridge interaction effects for an accurate assessment of the dynamic behaviour of railway bridges, particularly those with short to medium spans, the methodology is becoming more usual. The principles of the train-bridge interaction method are similar to those presented in the case of the moving loads method. Several approaches have been proposed to solve the train-bridge interaction problem. Regardless of the approach that is employed it is necessary to establish the equations of motion of the two systems, vehicle and structure, which can be achieved using either coupled or uncoupled sets of equations. A review of the advantages and disadvantages of each approach can be found in the works of Yang & Fonder (1996) and Lei & Noda (2002).

As the name indicates, in the coupled approach the equations of motion of the vehicle and the structure are coupled into a single system of equations. Examples of application of this methodology can be found in Yang *et al* (1999), Yang *et al* (2004) and Neves *et al* (2012). Yang & Wu (2001) point out that this approach may demand a considerable computational effort since the position of each contact point changes over time, thus generally making the system matrix time-dependent and requiring it to be updated and factorised at each time step.

The uncoupled approach formulates the interaction problem using two distinct sets of equations (one for each subsystem) which are solved separately. The uncoupled equations of motion are complemented with an additional equation in order to ensure the compatibility of the two subsystems. However, the compatibility of the two systems, train and bridge, must be verified at each instant in order to guarantee contact. This means that both subsystems are coupled by the compatibility of displacements and equilibrium of forces at the contact points. On the one hand, the train loading results in the deformation of the bridge. Simultaneously these displacements translate into actions on the train similar to an imposed settlement. The reactions

on each of the train's axles (which represent the contact points) represent the dynamic interaction forces between the two systems. Examples of application of this approach can be found in [Cruz (1994); Calçada (1995); Yang & Fonder (1996)]. To solve these two sets of equations iterative procedures are often used [Cruz (1994); Calçada (1995); Yang & Fonder (1996)]. Antolín (2013) points out that this approach has the advantage of reducing the size of the matrix used in the dynamic calculations. The equations of motion can be written as follows:

$$\begin{bmatrix} M_s & 0 \\ 0 & M_v \end{bmatrix} \cdot \begin{bmatrix} \ddot{u}_s(t) \\ \ddot{u}_v(t) \end{bmatrix} + \begin{bmatrix} C_s & 0 \\ 0 & C_v \end{bmatrix} \cdot \begin{bmatrix} \dot{u}_s(t) \\ \dot{u}_v(t) \end{bmatrix} + \begin{bmatrix} K_s & 0 \\ 0 & K_v \end{bmatrix} \cdot \begin{bmatrix} u_s(t) \\ u_v(t) \end{bmatrix} = \begin{bmatrix} F_s(t) \\ F_v(t) \end{bmatrix} \quad (2.16)$$

where the subscripts s and v represent the structure and the vehicle, respectively.

However, Yang & Wu (2001) indicate that this procedure may exhibit a slow rate of convergence, particularly when a large number of contact points is considered. In order to overcome this drawback Neves *et al* (2012) developed a procedure in which the equations of motion are solved directly using an optimised block factorisation algorithm. Ribeiro (2012) proposed another development to the iterative procedure used by Cruz (1994) and Calçada (1995) by solving the equations of motion combining both the Mode Superposition method to solve the bridge subsystem and the Newmark method to solve the train subsystem. The use of the Mode Superposition method allows a more efficient analysis from a computational point of view, thus reducing the computational timings. However, Ribeiro (2012) points out that the existence of dampers on the trains does not enable the equations of motion of the train to be uncoupled, requiring therefore the use of the Newmark Method. In this work the approach introduced by Neves *et al* (2012) was used.

Furthermore, it should be noted that most of the work done on this field, including the present dissertation, limits the analysis to the vertical interaction phenomenon. However, lateral interaction effects are extremely important in cases where lateral forces (such as wind or seismic actions) act on the trains. For this reason, this has been the subject of research work in recent years and several methodologies account for full interaction, i.e. account for both vertical and lateral interaction, have been proposed [Zhai *et al* (2009); Nguyen *et al* (2009); Antolín (2013); Montenegro (2015)].

2.3.2.5 Methods to solve the equations of motion

Besides the procedure selected to formulate the train-bridge interaction problem there are also several techniques to solve the system of equations of motion when using the Finite Element Method. The most common techniques are the Mode Superposition method and the direct integration methods [Clough & Penzien (1993); Chopra (1995)].

The Mode Superposition method consists on uncoupling the equations of motion by transforming the generalised coordinates into modal coordinates. Due to the orthogonality properties of the mode shapes the equations of motion from the set of N coupled differential equations can be transformed into a system of N independent linear equations. Therefore, it is possible to assess the contribution of each of the mode shapes to the global structural response.

$$\ddot{\mathbf{Y}}_i(t) + 2 \cdot \zeta_i \cdot \omega_i \cdot \dot{\mathbf{Y}}_i(t) + \omega_i^2 \cdot \mathbf{Y}_i(t) = \frac{\mathbf{P}_i(t)}{\mathbf{M}_i}, \quad i = 1, 2, \dots, N \quad (2.17)$$

where $\ddot{\mathbf{Y}}_i$, $\dot{\mathbf{Y}}_i$ and \mathbf{Y}_i represent the generalised acceleration, speed and displacement, respectively, ζ_i is viscous damping ratio, ω_i is the angular frequency, \mathbf{P}_i is the generalised load and \mathbf{M}_i is the generalised mass for mode i .

To obtain the global response of the structure one simply needs to solve the N uncoupled modal equations and superposing the effects of each of the modal contributions, hence explaining the name of the methodology.

$$u(t) = \phi_1 \cdot Y_1(t) + \phi_2 \cdot Y_2(t) + \dots + \phi_n \cdot Y_n(t) \quad (2.18)$$

where ϕ_n is the natural mode of order n .

One of the features of this method is the fact that it enables the selection of the mode shapes that will be accounted for in the analysis of the dynamic response. This is particularly useful because it simplifies solving the system of equations of motion, making the method particularly efficient. Another advantage of the method is the possibility to define the damping ratio for each mode, which is more convenient and generally more reasonable since the modal damping ratios

can normally be determined experimentally or estimated with adequate precision in many cases. One of the drawbacks of the method is the fact that it requires performing a modal analysis prior to the dynamic analysis in order to identify the structure mode shapes and the corresponding natural frequencies. Owing to the fact that superposition is applied this method is not suitable for the analysis of nonlinear problems.

Another typical option to solve the equations of motion is based on direct integration methods, which adopt a step-by-step approach and makes use of integration techniques. Unlike the previous method, no transformation of the integration space is required. Among the several methods that use such an approach are the Newmark method, the Wilson- θ method and Hilber-Hughes-Taylor (HHT) method [Cruz (1994); Calçada (1995); Neves (2008)]. In the present dissertation only the Newmark method is addressed.

The Newmark method adopts a step-by-step approach and, as previously stated, makes use of integration techniques to solve the dynamic equilibrium problem. In this method the contribution of all natural frequencies is taken into account. Two parameters define the variation of acceleration over a time step and determine the stability and accuracy of the method: γ and β . The factor γ provides a linearly varying weighting for the contribution of the initial and the final accelerations on the change of velocity, whereas the factor β provides a weighting between the initial and the final accelerations on the change of displacement.

The Newmark method becomes unconditionally stable for:

$$\gamma \geq \frac{1}{2} \tag{2.19}$$

It is also possible to observe that the maximum efficiency in terms of numerical dissipation is registered for:

$$\beta = \frac{(\gamma + 0.5)^2}{4} \tag{2.20}$$

If $\gamma = 1/2$ and $\beta = 1/4$ are adopted, the Newmark method is usually referred as the constant average acceleration method and satisfactory results are obtained from all points of view, including that of accuracy.

Contrarily to the Mode Superposition method when using the Newmark method it is not possible to define a modal damping ratio. For this reason it is necessary to derive appropriate proportional damping matrices. Typically Rayleigh damping is used and is expressed as being proportional to both the mass and the stiffness matrices [Clough & Penzien (1993)]:

$$\mathbf{C} = a_1 \cdot \mathbf{M} + a_2 \cdot \mathbf{K} \tag{2.21}$$

The two Rayleigh damping factors, a_1 and a_2 , are obtained from the following equation:

$$\begin{Bmatrix} a_1 \\ a_2 \end{Bmatrix} = 2 \cdot \frac{\omega_m \cdot \omega_n}{\omega_n^2 - \omega_m^2} \cdot \begin{bmatrix} \omega_n & -\omega_m \\ -\frac{1}{\omega_n} & -\frac{1}{\omega_m} \end{bmatrix} \cdot \begin{Bmatrix} \xi_m \\ \xi_n \end{Bmatrix} \tag{2.22}$$

where ω_m and ω_n are the angular frequencies of the modes of order m and n , respectively, and ξ_m and ξ_n are the corresponding damping ratios.

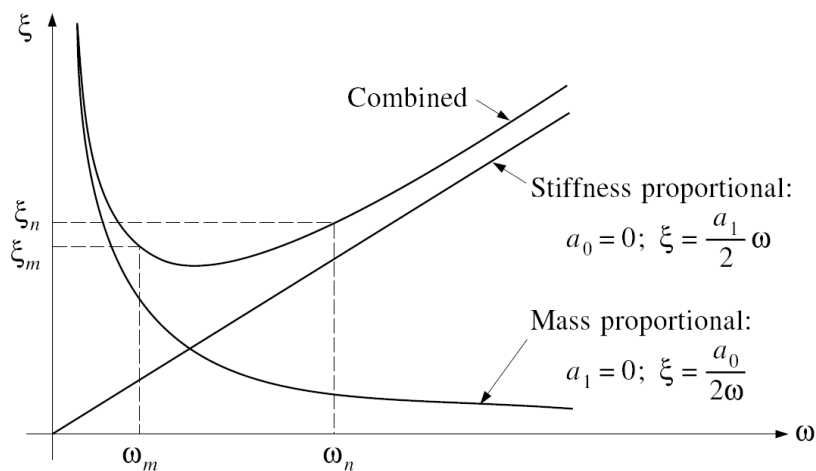


Figure 2.7 – Rayleigh damping [Clough & Penzien (1993)].

2.3.2.6 Time step selection

A particularly important aspect to take into consideration when performing a dynamic analysis is the selection of the time step, Δt , to adopt in the analysis. The selection of an adequate time step is essential to accurately assess the dynamic response and to ensure that the most important modes of vibration are considered.

There are several criteria in the literature regarding the selection of the most adequate time steps to adopt in dynamic analysis. One criterion is adopting a time step between $T_n/10$ and $T_n/20$, where T_n is the period of the highest mode that is to be considered in the dynamic analysis.

ERRI (1999) suggests that the selected time step should be the minimum of the following two values:

$$\Delta t = \frac{1}{8 \cdot f_{\max}} \quad (2.23)$$

where f_{\max} is the frequency of the highest mode of vibration considered in the analysis, and

$$\Delta t = \frac{L}{4 \cdot n \cdot v_{\max}} \quad (2.24)$$

where L is the span of the bridge, n is the number of modes considered in the analysis and v_{\max} is the maximum train speed. The first criterion guarantees that the highest mode of vibration is represented by a minimum of eight points. This criterion is similar to those that had been referred previously, although it is slightly less conservative. The second criterion is defined in order to ensure that the excitation is accurately accounted in the dynamic analysis. It intends to guarantee that a given load moving over the bridge at a speed, v_{\max} , is discretised into $4n$ intervals. Meixedo (2012) points out that the time step selection should not only be a function of the bridge natural frequencies but should also take into consideration the train frequencies as well as the frequencies related with the excitation promoted by track irregularities. Generally, the train frequencies are less limitative than those of the bridge. However, depending on the train speed to be analysed and on the irregularities wavelength range that is used in the analysis the latter criterion might

sometimes require a smaller time step than that predicted by the bridge natural frequencies to adequately model the excitation and obtain an accurate response.

It is important to mention that when using the Newmark method and $\gamma = 1/2$ and $\beta = 1/4$ is adopted the method does not introduce any dissipation to the higher frequencies. When proceeding to the integration of the equations of motion the contribution from very high frequencies related to higher modes can be observed. However, these effects are generally spurious due to the fact that these frequencies are not being adequately taken into account in the numerical integration [Rigueiro (2007)]. Since in this method no upper limit to the frequency of the modes considered is established the time step acts as a cut-off, solving this problem.

2.4 Limit states

The European standards define several limit states that must be verified in order to guarantee the structural and traffic safety in the high-speed railway network. In order to perform according to the safety demands of the European standards a structure must verify Ultimate Limit States (ULS) as well as Serviceability Limit States (SLS), related to passenger comfort. Besides the traditional verifications, the European standard EN1990-A2 (2005) also define some specific verifications for bridges inserted in the high-speed railway network, which are labelled as Traffic Safety Checks and apply limits regarding deformations and vibrations of the bridge deck. In the following sections a review of the verifications required by the European standards to assess the safety of railway bridges is presented.

2.4.1 Structural safety

Regarding structural safety, the design of bridges requires performing a dynamic analysis for the most unfavourable value of the two following scenarios:

$$\left(1 + \varphi' + \frac{\varphi''}{2}\right) \times \begin{pmatrix} HSLM \\ or \\ RT \end{pmatrix} \quad (2.25)$$

or

$$\Phi \times (LM71 + SW/0) \quad (2.26)$$

where RT represents the real trains operating in the European high-speed railway network, HSLM represent the High Speed Load Models, LM 71 represents the static effects of vertical loading due to normal rail traffic and SW/0 which represents the static effects of vertical loading due to normal rail traffic on continuous beams.

For the cases where no dynamic analysis is required the dynamic amplification factor, φ' , is given by:

$$\varphi' = \begin{cases} \frac{K}{1 - K - K^4}, & K < 0,76 \\ 1,325, & K \geq 0,76 \end{cases} \quad (2.27)$$

with

$$K = \frac{v}{2 \cdot L_{\Phi} \cdot n_0} \quad (2.28)$$

The term φ'' that has been previously introduced and represents an amplification factor to account for the existence of track irregularities and vehicle imperfections can be determined by:

$$\varphi'' = \frac{\alpha}{100} \cdot \left(56 \cdot e^{-\left(\frac{L_{\Phi}}{10}\right)^2} + 50 \cdot \left(\frac{L_{\Phi} \cdot n_0}{80} - 1 \right) \cdot e^{-\left(\frac{L_{\Phi}}{20}\right)^2} \right) \quad (2.29)$$

where the parameter α is given by:

$$\alpha = \begin{cases} \frac{v}{22}, & v \leq 22 \text{ m/s} \\ 1, & v > 22 \text{ m/s} \end{cases} \quad (2.30)$$

For the cases of carefully maintained track EN1991-2 (2003) allows a 50% reduction of the parameter φ'' .

2.4.2 Traffic Safety Checks

These verifications correspond to Serviceability Limit States from a structural point of view that must be verified in order to guarantee an adequate performance of the railway bridge. Despite corresponding to SLS from a structural perspective they represent an Ultimate Limit State for the running safety of the trains.

The verifications regarding the running safety of trains over bridges include the assessment of the vertical acceleration of the deck, deck twist, vertical deformation of the deck, transverse deformation and vibration of deck and the longitudinal displacement of the deck. Furthermore, EN1990-A2 (2005) also includes a Serviceability Limit State that aims to guarantee passenger comfort. Since the focus of this dissertation is the vertical behaviour of the bridge only the verifications related to this behaviour will be discussed in the following sections.

2.4.2.1 Vertical acceleration of the deck

Limiting the vertical acceleration of the bridge deck has two main objectives: the first is to avoid track instability due to the loss of interlock between ballast grains in ballasted tracks and the second is to prevent the loss of contact between the wheel and the rail due to excessive reduction of the wheel-rail contact forces. The ballast instability leads to the loss of the lateral resistance of the track which affects the running safety of the trains. The loss of contact between the wheel and the rail can originate the derailment of the train.

For bridges that require performing a dynamic analysis the maximum deck acceleration must be assessed for both real trains and HSLM considering frequencies up to 30 Hz or 1.5 times the value of the frequency of the fundamental mode of vibration and should include a minimum of three vibration modes.

EN1990-A2 (2005) limits the maximum peak values of the bridge deck acceleration to 3.5 m/s^2 for ballasted track and to 5 m/s^2 for ballastless tracks. More recently, Zacher & Baeßler (2009) carried out laboratory tests in order to obtain a better understanding of the behaviour of the ballast layer and to assess what deck acceleration level leads to its instability. The test rig consists of a steel box that was filled with ballast to a thickness of 35 cm. A concrete bloc representing the sleeper was embedded in the ballast. The box was guided with rails on both sides and was supported with a servo-hydraulic shaker which is able to excite the box to vertical vibrations up to 2g and to frequencies up to 60 Hz. A second servo-hydraulic shaker can also load the sleeper. A schematic representation of the test rig and the built steel box are illustrated in Figure 2.8.

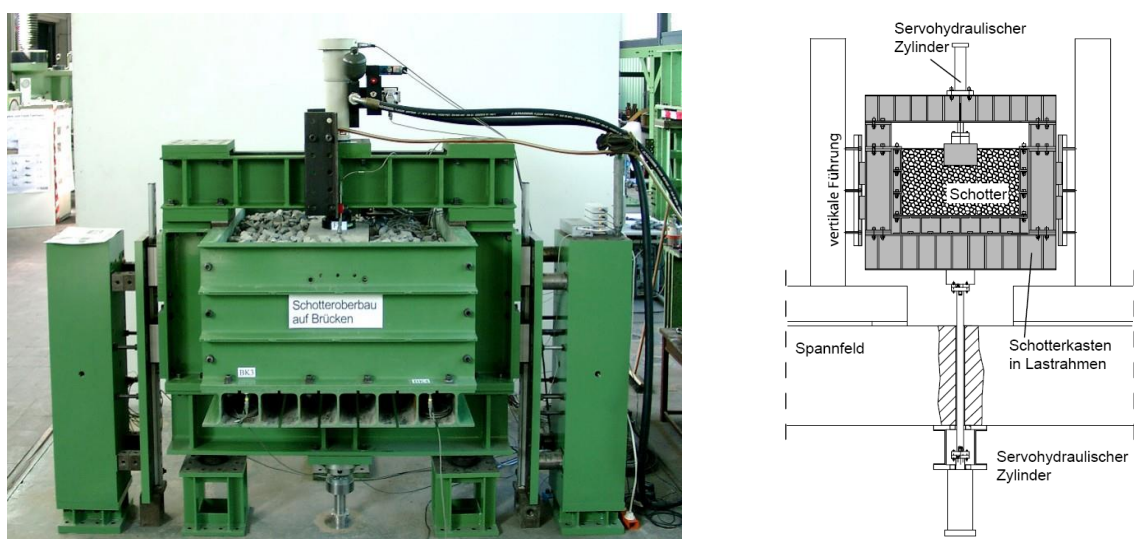


Figure 2.8 – Test rig used to test the ballast dynamic behaviour [Zacher & Baeßler (2009)].

Zacher & Baeßler (2009) point out that the results of the test are strongly dependent on the initial state of the ballast configuration. Therefore, prior to the test the ballast was pre-loaded in

order to reproduce its configuration on a real track and to ensure that the initial state is approximately the same for all test series. The obtained results are shown in Figure 2.9.

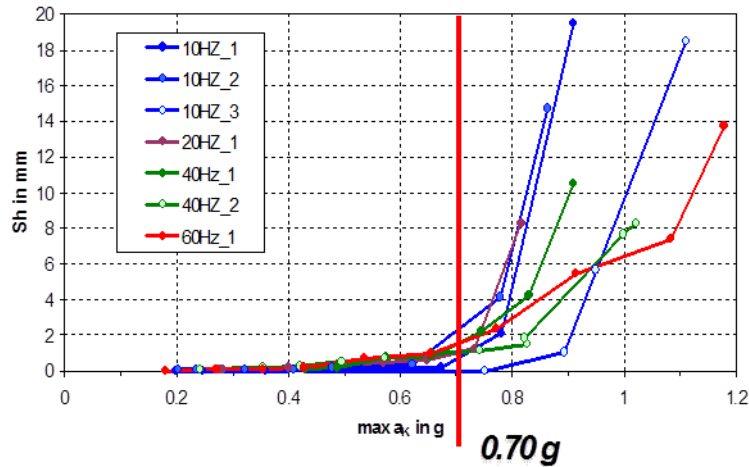


Figure 2.9 – Lateral displacement of the sleeper [Zacher & Baeßler (2009)].

The results show that the lateral resistance of the sleeper, which is an indicator of the interlock of the ballast grains, is significantly affected when the acceleration level reaches 7 m/s^2 . Furthermore, the results indicate that the acceleration limit suggested in the European standards to ballasted tracks results from the application of a safety factor of 2, which seems too conservative. Since this criterion often turns out to be the most restrictive aspect of the dynamic response of railway bridges, particularly those with short to medium spans, Zacher & Baeßler (2009) propose a reduction of the safety factor from 2 to 1.3, which would translate into limiting the deck acceleration on ballasted tracks to 5.5 m/s^2 and to 7.5 m/s^2 in ballastless tracks.

Norris (2005) also analysed the influence of the deck vibration level on the stability of the ballast layer and his research was based on more than 100 experimental tests carried out on railway bridges across the United Kingdom. It was possible to conclude that even if in some small areas the acceleration levels are very high the ballast layer remains stable as adjacent ballast provides confinement and prevents local instability (see Figure 2.10). Based on these results Norris suggests that the deck acceleration limit be changed to 5 m/s^2 for most of ballasted bridges and this value be even increased to 6 m/s^2 for bridges with higher structural damping levels.

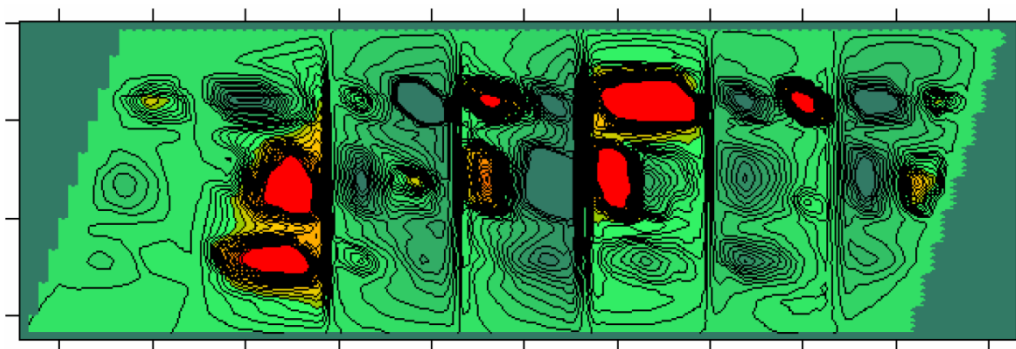


Figure 2.10 – Confinement of the localised areas of ballast of high acceleration levels [Norris (2005)].

In addition, the highest frequency that should be included in the analysis was also a topic of the research. As previously mentioned, the European standards define that the maximum frequency to be considered should be the maximum between the frequency of the third mode shape or 30 Hz. However, Zacher & Baeßler (2009) observed that the maximum value of the transfer function occurs at a frequency of 60 Hz, as depicted in Figure 2.11.

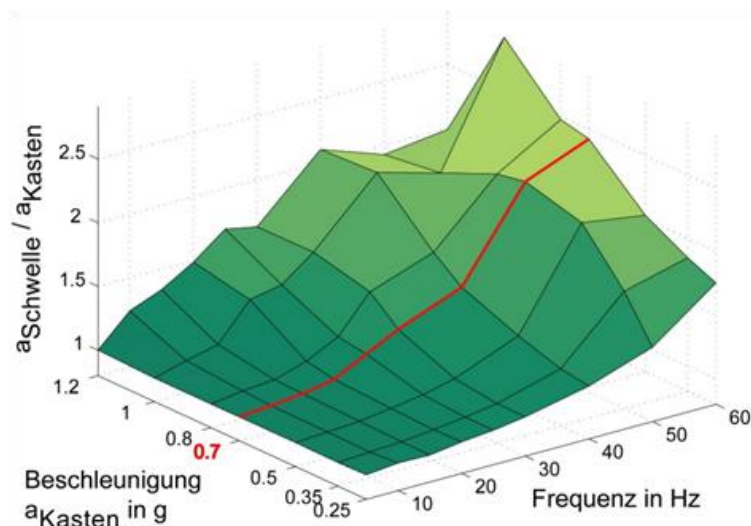


Figure 2.11 – Transfer function between ballast and sleeper [Zacher & Baeßler (2009)].

This topic has been further investigated in Baeßler *et al* (2012) and it was concluded that higher frequencies cannot be regarded as less critical than the lower frequencies. Nevertheless, the higher frequencies correspond to smaller wavelengths, which are generally associated with a more localised phenomenon, restricting this problem to a small area. For this reason it can be

said that, normally, performing the analysis using the first three modes of vibration related with vertical bending of the bridge (which typically are below 60 Hz) is sufficient to accurately analyse the dynamic behaviour of railway bridges. Even so, in order to prevent ballast instability phenomena due to effects related with modes with higher frequencies a modification of the current European standards would be advisable. Instead of the 30 Hz limit suggested in the current version, this value should be extended to frequencies up to 60 Hz.

2.4.2.2 Vertical deformation of the deck

The excessive deformation of the bridge deck can lead to unacceptable changes in both the vertical and lateral geometry of the track and to excessive stresses in the rails. For this reason the European Standards limit the maximum deflection in order to prevent excessive bending of the track, which can accelerate its deterioration process, and which also indirectly controls the stress level on the rails, preventing their instability and failure. EN1990-A2 (2005) defines that the maximum total vertical deflection measured along any track due to rail traffic actions must not exceed $L/600$, where L represents the span of the bridge.

This standard also limits the relative longitudinal displacements between two consecutive decks or between the end of a deck and the adjacent abutment in order to reduce transition zones degradation and to prevent discontinuities in zones near rail expansion devices or expansion joints. For this reason the vertical displacement, δ_v , of the upper surface of a deck relative to the adjacent element (another deck or abutment) is limited to:

- 3 mm for sites where the maximum speed is limited to 160 km/h;
- 2 mm for sites where the maximum speed exceeds 160 km/h.

2.4.3 Serviceability Limit State – Passenger riding comfort

Another important aspect that needs to be taken into considering during the design of railway bridges is passenger riding comfort. Although this aspect exceeds the scope of the investigation that is presented in the current dissertation it is a topic that needed to be referred.

The railway service should not only be attractive from the point of view of travelling time but should also provide a comfortable journey to its clients. For this reason it is crucial to limit the

vibration levels that the passengers experience during their journey in order to prevent unpleasant and discomfort sensations. EN1990-A2 (2005) defines the passenger comfort with respect to the acceleration level, b_v , in the interior of the train carriages. The three different comfort levels that are recommended in EN1990-A2 (2005) are presented in Table 2.1.

Table 2.1 – Recommended levels of comfort [EN1990-A2 (2005)].

Level of comfort	Acceleration (m/s^2)
Very good	1.0
Good	1.3
Acceptable	2.0

In order to assess the vertical acceleration level in the vehicle EN1990-A2 (2005) allows two different approaches. The first one is a simplified approach that relates the acceleration level with the vertical deflection of the bridge as a function of the train speed, the configuration of the bridge, the span length and the number of spans. This resulted in a chart that indicates the maximum allowable vertical deflection, δ , in a bridge in order to ensure a very good level of comfort as represented in Figure 2.12.

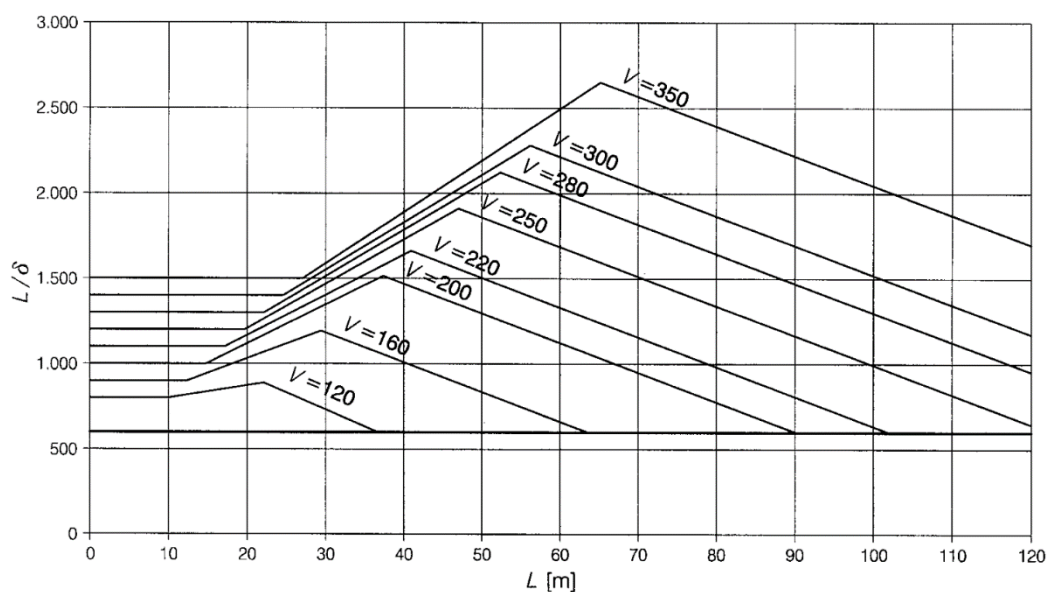


Figure 2.12 – Maximum acceptable vertical deflection to ensure a very good level of comfort [EN1990-A2 (2005)].

It should be noted that these vertical deflections should be determined with LM 71 multiplied by the dynamic factor, Φ . Furthermore, the limits shown in Figure 2.12 are given for a succession of simply supported beam with three or more spans. For different bridge configurations a corrective factor should be used. Additionally, since the limits of the ratio L/δ indicated in Figure 2.12 correspond to a scenario of very good level of comfort, these values must be divided by the corresponding b_v for the analysis of other levels of comfort.

As an alternative the vertical acceleration of the train carriages can be determined by a dynamic train-bridge interaction analysis. This approach is more complex since it requires modelling the train but it is also more accurate. The selected train models can vary in complexity and some works even propose including the passenger-seat interface [Wei & Griffin (1998); Carlbom (2000)]. A comprehensive review and discussion on this approach can be found in Carlbom (2000). More recently Ribeiro (2012) also assessed the passenger riding comfort in a real bridge on the Portuguese railway network for the passage of the Alfa-Pendular train using a dynamic train-bridge interaction analysis.

2.4.4 Running safety

Besides the criteria that have been previously presented and that indirectly assess the running safety of trains, there are also a few criteria that directly assess this issue, namely by analysing the possibility of train derailment. In these criteria the assessment of the running safety of the trains is generally based on the wheel rail contact forces. A detailed review of the several derailment mechanisms and the criteria that are typically used can be found in Antolín (2013) and Montenegro (2015). In the current dissertation a summary of the main criteria is presented in this section.

2.4.4.1 Wheel flange climbing

One of the most commonly used criteria to assess the train running safety is related with the wheel climbing the rail flange. This tends to occur for the combination of high wheel lateral forces with low vertical contact force on the flanging wheel. Nadal's criterion [Nadal (1896)] is possibly the most common criterion to assess derailment due to wheel flange climbing. This criterion determines derailment according to the ratio of lateral, Y , to vertical, Q , contact force in

each wheel and depends on the dynamic friction coefficient, μ , and the contact angle, γ . The Nadal factor, η_N , can be expressed as:

$$\eta_N = \frac{Y}{Q} = \frac{\tan \gamma - \mu}{1 + \mu \tan \gamma} \quad (2.31)$$

Nadal's criterion is of static nature and neglects the longitudinal creep forces. Furthermore, it was demonstrated to be conservative, particularly for small or negative values of angle of attack, since it does not account for the effects of friction in the non-flanging wheel. Therefore, a less conservative criterion was proposed by Weinstock (1984). This criterion presents a more realistic approach and is less sensitive to the variation of the friction coefficient. Weinstock's criterion evaluates the flanging wheel using Nadal's criterion but accounts for the non-flanging wheel by considering a Y/Q ratio equal to the friction coefficient. Therefore, the Weinstock factor, η_w , can be expressed as:

$$\eta_w = \sum \frac{Y}{Q} = \frac{\tan \gamma_A - \mu_A}{1 + \mu_A \tan \gamma_A} + \mu_B \quad (2.32)$$

where the subscript A represents the flanging wheel, whereas the subscript B represents the non-flanging wheel.

According to the TSI (2002), the Y/Q ratio should not exceed 0.8 for either of the wheel flange climbing criteria.

2.4.4.2 Track panel shift

Another derailment mechanism is the track panel shift, which corresponds to the lateral displacement of the track panel. This phenomenon is due to excessive lateral forces acting on the track. As the lateral displacement of the track panel builds up in some of the track elements, such as the rails and the sleepers, the loss of guidance of the wheels might be observed resulting in one of the wheels falling between the rails and the outer side of the track. The track panel shift

has become increasingly important due to generalised use of continuously welded rail and also to the higher operational train speed. Prud'homme (1967) proposed a criterion that limits the maximum lateral force applied on the track by a wheelset in order to prevent track panel shift:

$$\sum_{wheelset} Y \leq 10 + 2 \cdot \frac{Q_{stat}}{3} \quad (2.33)$$

where Q_{stat} represents the static load per wheel. This criterion was adopted by TSI (2002). Recently some variations have been proposed to the original criterion in order to account for different track maintenance levels, ballast compaction level and sleeper type [Iwnicki (2006)].

2.4.4.3 Wheel unloading

The derailment by wheel unloading might occur when one or more wheels lose contact with the rails as a result of excessive vibrations. These excessive vibrations can be due to aspects such as track irregularities, crosswind or earthquakes.

There are several methods to determine wheel unloading. A common method is the vector intercept, which is based on a geometrical analysis of the acting point of the overall resultant vertical forces [Andersson *et al* (2004)]. In its most usual form the wheel unloading factor, η_U , measures the reduction of the wheel load compared to its static value and is calculated by:

$$\eta_U = \frac{Q_{sta} - Q_{dyn}}{Q_{sta}} = 1 - \frac{Q_{dyn}}{Q_{sta}} \quad (2.34)$$

where Q_{dyn} is the dynamic wheel load. When there is no unloading the parameter η_U takes the value of 0. On the contrary if a limit situation where the wheel loses contact with the rail is observed the dynamic wheel load is null and the parameter η_U is equal to 1.

Carrarini (2006) refers that analysing a single wheel can be excessively conservative, as this criterion is not valid when complete unloading is reached, regardless of the severity of the wheel lift. A less conservative approach uses a similar principle but consists on analysing a complete

bogie/wheelset. It should be noted that both approaches are allowed by the current European standard as indicated in EN14067-6 (2009), which limits the wheel unloading to 90% of the value of the static load, corresponding to a value of η_U equal to 0.9.

A criterion that is strongly associated with wheel unloading is the train overturning criterion since the degree of unloading of the critical wheels is usually also used as a criterion for the assessment of the risk of overturning. The overturn factor, η_O , can be expressed as:

$$\eta_O = \frac{\sum_{wheelset} |Q_1 - Q_2|}{\sum_{wheelset} |Q_1 + Q_2|} \quad (2.35)$$

where Q_1 and Q_2 are the vertical loads of each wheel of the same wheelset. Similarly to the wheel unloading TSI (2002) limit the overturn factor to 0.9.

Structural reliability assessment

3.1 Introduction

The safety of structures and their adequate performance while in service are the two key issues to take into account during the design stage. However, the complete safety of a structure is a concept that is not real as there is always some risk of failure. This risk is due to the uncertainties that characterise the parameters that influence the structural behaviour, which makes the structural reliability problem non-deterministic in nature. In order to classify the structural safety EN1990 (2002) defines that structures should be designed and maintained in order to display an adequate performance throughout their service life, with an appropriate balance between economy and safety level. For this reason structural reliability theory is based on mathematical statistics where the uncertainties are modelled by stochastic variables.

Until the 19th century the design of Civil Engineering structures was carried out in an empirical way, mostly based on the experience of masons and builders. However, it was clear that uncertainty was a significant part of the structural reliability problem. The structural behaviour depends on several parameters that display, in most cases, a certain degree of uncertainty thus making it intrinsically a probabilistic problem.

With the research carried out in this field a scientifically based method was developed for the assessment of structural safety: the admissible stress method. The fundamental idea of the method was to ensure that the maximum stress in the critical zones was lower than the material

capacity divided by a safety factor. The selection of the safety factor was somewhat empirical. The enhancements in Structural Engineering, namely a better understanding of the behaviour of materials and a more adequate evaluation of the loadings resulted in a reduction and a diversification of the safety factors. However, the method proved to have some imperfections, particularly by providing different safety levels for different structural elements and by making it difficult to assess the overall safety of the structure.

For this reason the introduction of a probabilistically based approach to the safety problem was a natural development. This approach offers a more realistic model of the real phenomena. However, it requires statistical information for the description of the parameters involved to be used properly. Significant research has been dedicated to the topic of probabilistic structural reliability and it is possible to say that nowadays this is a field that is adequately documented and consolidated within the academic community [Jacinto (2011)]. Despite this this approach is not yet a common practice in Structural Engineering. This can be explained by the adoption of semi-probabilistic approaches by the design codes. In recent years the adoption of probabilistic approaches to assess structural safety has been increasing, indicating that these methods can now become more common and due to the advances in computational capacity their application to real structures is now becoming more feasible.

In the particular field of railway bridges the use of probabilistic methods for safety assessment is very limited. From the literature review it can be seen that most of the work done in this research field was performed through deterministic analysis. This reveals that not much attention has been given to the variability of the parameters that are known to influence the dynamic response of the bridge or to the identification of the parameters that have a significant effect in the structural response. One of the few exceptions is the work of Cho *et al* (2010) which accounted for the variability of some parameters of both the bridge and the train and performed a reliability analysis of a box-girder railway bridge using an improved Response Surface Method. A prestressed concrete box girder bridge was used as a case study and the reliability analysis showed that bridge-related uncertainties have greater influence in the reliability indexes than train-related uncertainties. Nonetheless, the analysis was limited to a single train speed not enabling to conclude how the reliability of the train-bridge system is affected by this parameter.

Even when the existence of track irregularities is accounted for, the analysis is often limited to a specific scenario. Au *et al* (2002) studied the behaviour of a cable stayed railway bridge for different track irregularity profiles and different track quality scenarios. It was found that the

impact factor is not proportional to the magnitude of the roughness. However, it was noticed that this impact factor tends to increase for lower track quality. Lu *et al* (2009) proposed an extension of the pseudo excitation method for the analysis of the behaviour of vehicle-bridge coupled systems. Several examples are shown proving the efficiency of the proposed method against the Monte Carlo method. However, for both research works, the variability is limited to the track irregularity profiles.

Johansson *et al* (2014) recently proposed a methodology for the preliminary assessment of existing railway bridges for high-speed traffic. The methodology divides the bridges into groups with similar characteristics. Afterwards, prediction bounds for the properties of the bridges within a group are determined through statistical analyses for each group of bridges and a probability distribution is generated for the entire bridge network. The dynamic response is assessed using a closed-form solution previously developed by the same authors [Johansson *et al* (2013)]. To take into account the uncertainties associated with the prediction bounds the Monte Carlo method is used to assess the dynamic behaviour of the bridges and based on the simulation results the probability of failure is determined.

In this chapter the basis of structural reliability is presented. An overview of the fundamental concepts and the different approaches used to address this problem is provided along with the description of different methods used to assess structural safety. Furthermore, some enhancements of the most usual methods are presented in order to increase the efficiency of the safety assessment. The chapter ends with the presentation of the safety assessment framework that includes the basis of the methodology used to assess the safety of the train-bridge system.

3.2 Fundamentals

In this section the fundamentals of structural reliability analysis are summarised. An overview of the basic concepts is presented along with the theoretical background to structural reliability.

3.2.1 Basic concepts

Since the safety and reliability of structures assumes such an important role in Engineering, structural reliability is a field that has been extensively studied in order to create tools for the

economic design of structures as well as accurate assessment of the safety level. A fundamental aspect to understand is the concept of reliability. According to EN1990 (2002) the term ‘reliability’ should be considered as the ability of a structure to fulfil the specified requirements for which it has been designed during its service life. This code states that generally the ability to comply with the specified requirements is evaluated in terms of a probability, thus making reliability a probabilistic concept. Therefore the reliability is a quantity that can be calculated or estimated.

For this reason, in order to be realistic, structural reliability methods have a probabilistic nature, to account for the several sources of uncertainty that characterise these problems. Both in Thoft-Christensen & Baker (1982) and in Melchers (1999) the sources of uncertainty are divided into four main groups: physical uncertainty, statistical uncertainty, modelling uncertainty and uncertainties due to human factors. Therefore, an important part of the method is the identification of the parameters that influence the structural behaviour, known as the basic variables, as well as the definition of an appropriate probability distribution family and the estimation of the parameters of this distribution. Usually the selection of the distribution family is based on subjective information whereas the parameters of the distribution are estimated on the basis of available data or experience [Faber (2012)].

While accounting for the several sources of uncertainty is important, the definition of boundaries that measure the performance level that a structure must display during its service life is another key aspect. Nowak & Collins (2000) define the limit state as the boundary between desired and undesired structural performance. Therefore, the limit state functions or performance functions can be perceived as the basic requirements for adequate structural behaviour. Generally, two types of limit states are defined: serviceability limit states and ultimate limit states. Ultimate limit states are associated with severe damage to the structure, that reduces the structural capacity and raises concerns regarding structural and/or people safety [EN1990 (2002)]. In bridges the most common causes for ultimate limit states are bending, shear and loss of stability [Wisniewski (2007)]. Serviceability limit states are associated with less severe damage to the structure and generally concern the performance of the structure under normal use, the comfort of people, the structural appearance and its durability [EN1990 (2002)]. In railway bridges the most common serviceability problems are related with excessive deformations or vibrations and cracks. It is worth mentioning that EN1990 (2002) also distinguishes between reversible and irreversible serviceability limit states. Wisniewski (2007) points out that some

authors separate the Fatigue Limit state from the ULS and recently some authors also propose the use of another limit state with respect to structural durability.

If a structure does not meet the requirements established in the limit states it is considered to have failed. Since structural reliability relies on a probabilistic approach the structural response can be assessed statistically and the probability of violating a limit state, which represents the probability of failure, can be estimated. Assuming that the structural response is determined by the parameter x , the probability of failure can be expressed as:

$$p_f = \int_{\Omega_F} f_x(x) dx \quad (3.1)$$

where Ω_F represents the failure region, which corresponds to the region where a given limit condition is not satisfied. The severity of the consequences that result from failing to meet a specific limit state is used to define the target reliability. Limit states that have greater consequences when exceeded must have a very small probability of occurrence and, consequently, higher reliability. Contrary to the reliability, safety is a qualitative concept [Schneider (2006)]. It is not possible to quantify the safety of a structure as it can only be classified as safe or unsafe. Therefore, it is necessary to assess the structural reliability and a structure is considered to be unsafe when its reliability is lower than a target reliability, which represents the minimum acceptable structural performance.

3.2.2 Semi-probabilistic approach

One of the most common methods to evaluate structural safety and reliability is based on a semi-probabilistic approach. It constitutes the basis of most of the current standards and guidelines for the design of structures. In this approach the actions, E , and the resistance, R , are represented by their design values, E_d and R_d , respectively. In order to ensure safety the following condition must be met:

$$E_d \leq R_d \quad (3.2)$$

The semi-probabilistic approach reflects the uncertainty in a simplified manner, through the introduction of partial safety factors. These safety factors are applied to the characteristic values of the actions, E_k , and resistance, R_k , in order to enhance the loading and reduce the resistance. Typically the characteristic value for the actions corresponds to the 95% quantile whereas in the case of the resistance the 5% quantile is used. However, other quantiles can be used provided that safety factors are adequately adapted. Therefore, Eq. (3.2) can be written as:

$$\gamma_f \cdot E_k \leq \frac{R_k}{\gamma_M} \quad (3.3)$$

where γ_f and γ_M are partial safety factors with values greater than one.

This method is based on more complex probabilistic approaches. However, the main purpose of this approach is to define a simple method to assess structural safety while ensuring the same safety level as more complex methodologies. It is within the designated Level I methodologies, which is a topic that is addressed in more detail in Section 3.3.

3.2.3 Probabilistic approach

Another possibility for the assessment of structural reliability is adopting a probabilistic approach. In this method both the structural response and the loading are characterised by random variables, offering a more realistic representation of the structural reliability problem. The probability of failure is evaluated by structural reliability techniques. The basic variables are defined according to real or theoretical distributions and comprise the mechanical properties of the materials, the geometric imperfections and the loading on the structure among other significant properties.

The use of probabilistic approaches in structural reliability problems has become more widespread in recent decades mainly due to the evolution of computers and their increasing capacities. The application of these methodologies in Civil Engineering is not limited to structural design assessment but has been also frequently applied to the structural assessment of existing structures. Since most European standards do not differentiate the structural assessment of existing structures from the design of new ones this approach is very useful to provide an

adequate evaluation of the remaining service life of this type of structures, enabling the selection of adequate reliability levels.

In order to assess structural safety the probability of occurrence of a given limit state is evaluated taking into consideration the uncertainties in the basic variables. The estimated probability is then compared to reference limit values, defined by EN1990 (2002) or JCSS (2001) in order to understand if the structure complies with the required target reliability and is able to perform adequately.

Some methods to assess structural reliability adopting this approach are presented in Section 3.3.

3.2.4 The Fundamental Reliability Case

The basic reliability problem is a two dimensional case where both the resistance, R , and the effect of actions, E , are described by a single stochastic variable. These parameters can be described by their probability distribution functions, f_R and f_E , respectively. The probability of failure can then be defined by:

$$p_f = P(R \leq E) \quad (3.4)$$

This represents the probability of a given limit state not being satisfied. The limit state function, M , is often addressed as the difference between actions and resistance:

$$M = g(R, E) = R - E \quad (3.5)$$

Considering the probability distribution function for each parameter along with the combined probability distribution function, f_{RE} , and assuming that Ω_F is the failure region, the probability of failure can be written as:

$$p_f = P(M \leq 0) = \iint_{\Omega_F} f_{RE}(r, e) dr de \quad (3.6)$$

If R and E are independent, which corresponds to the most common assumption, then:

$$f_{RE} = f_R(r) \cdot f_E(e) \quad (3.7)$$

and Eq. (3.6) can be written as:

$$p_f = P(M \leq 0) = \int_{-\infty}^{+\infty} \int_{-\infty}^{e>r} f_R(r) \cdot f_E(e) dr de \quad (3.8)$$

Since the cumulative distribution function, $F_X(x)$, of a generic variable X is given by:

$$F_X(x) = P(X \leq x) = \int_{-\infty}^x f_x(y) dy \quad (3.9)$$

Therefore, the probability of failure can be expressed as:

$$p_f = \int_{-\infty}^{+\infty} F_R(x) \cdot f_E(x) dx \quad (3.10)$$

The integral in Eq. (3.10) is often referred to as the convolution integral [Melchers (1999)]. Figure 3.1 shows a schematic representation of the convolution integral.

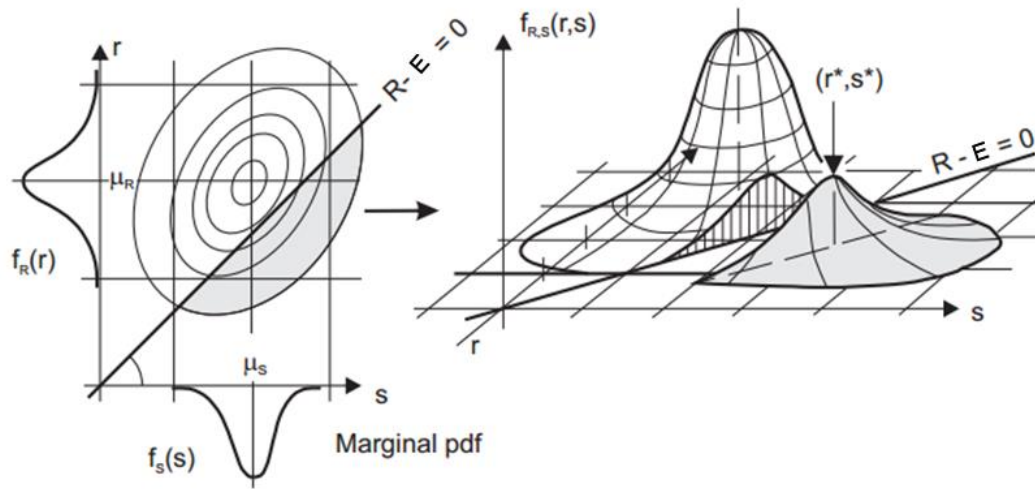


Figure 3.1 – Graphical representation of the convolution integral [adapted from Wisniewski (2007)].

The analytical resolution of this integral is only possible for a few particular cases, namely when the resistance and the actions follow a Gaussian distribution. When this is the case, assuming the variables have a mean μ_R and μ_E and a variance σ_R^2 and σ_E^2 and taking into account that the safety margin, M , represents the difference between the resistance and the effect of actions, taking advantage of the additive properties of independent Gaussian random variables the probability of failure can be expressed as:

$$P_f = P(R - E \leq 0) = P(M \leq 0) = \Phi\left(\frac{0 - \mu_Z}{\sigma_Z}\right) \quad (3.11)$$

where

$$\mu_Z = \mu_R - \mu_E \quad (3.12)$$

and

$$\sigma_Z^2 = \sigma_R^2 + \sigma_E^2 \quad (3.13)$$

Due to the relation between the mean and variance of both variables E and R with the mean and variance of the safety margin, M , the probability of failure can be re-written in the following form:

$$p_f = \phi \left[\frac{-(\mu_R - \mu_E)}{(\sigma_R^2 + \sigma_E^2)^{1/2}} \right] = \phi(-\beta) \quad (3.14)$$

The parameter β represents the reliability index and is also commonly named the Cornell reliability index (Cornell, 1969):

$$\beta = \frac{\mu_M}{\sigma_M} \quad (3.15)$$

This index can be interpreted as the number of standard deviations by which the mean value of the safety margins exceeds zero (which represents the safety limit). There is an equivalent geometrical interpretation of the reliability index which is interesting and is the basis of the FORM and SORM methods that are discussed in Section 3.3.2, in which the index is interpreted as the distance from the mean value of the safety margin to the most likely failure point (see Figure 3.2).

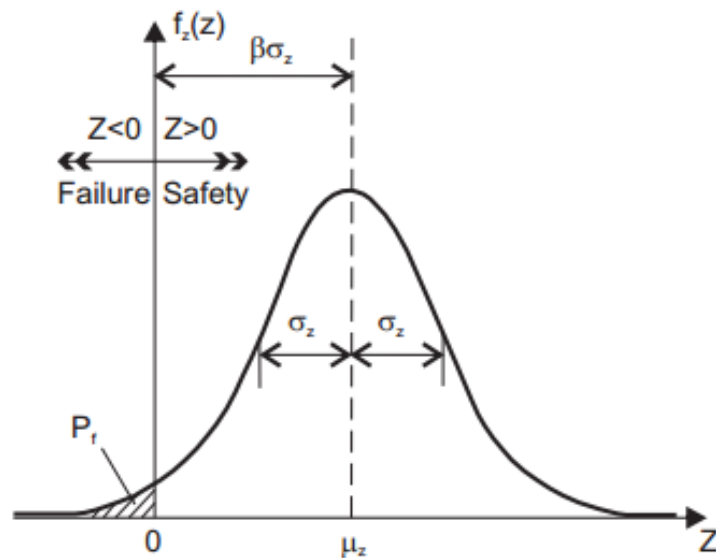


Figure 3.2 – Distribution of the safety margin [Wisniewski (2007)].

3.3 Methods for the evaluation of the probability of failure

The overview of the structural reliability theory provides a basis for a better understanding of the problem. It is also important to present the different methodologies that can be used to evaluate the probability of failure in Structural Engineering. The different levels that are presented correspond to increasingly probabilistic approaches. According to Henriques (1998) it is also possible to include Level 0 methodologies, which correspond to a purely deterministic approach. However, this has stopped being a common practice in Engineering and it is not within the scope of the present dissertation and, for this reason, only a reference is made to such methodologies.

3.3.1 Level I Methods

Usually the Level I methodologies are associated with semi-probabilistic approaches. The aforementioned method of partial safety factors is an example of such a method. The loading and the resistance are represented by their characteristic values and in order to guarantee the structural safety the characteristic value for the effect of actions must be lower than the characteristic value of the resistance, as expressed in Eq. (3.2).

This approach is the main one that is currently incorporated in the European standards. Despite the simplifications the aim of these methods is to provide a reliability level similar to that obtained by a probabilistic approach. For this reason, the adopted safety factors should be carefully selected in order to reflect the target reliability level obtained by more advanced methodologies. The calibration of the safety factors can result from either an empirical basis, resulting from previous experience, or by comparison with purely probabilistic methods [Jacinto (2011)]. In the case of EN1990 (2002) the safety factors are mostly obtained through an historical record of successfully designed and constructed structures.

The drawback of this type of method is the fact that the Designer is not responsible for defining the reliability level but instead uses an unknown reliability target. For this reason trust is required when adopting the safety factors that are recommended.

3.3.2 Level II Methods

Level II methods include the First Order Reliability Method (FORM) and the Second Order Reliability Method (SORM) which are based on a geometrical interpretation of the reliability index. When the basic random variables, X_i , are all independent and follow a Gaussian distribution and the limit state function (or performance function), $g(X)$, is linear then the reliability index, β , is easy to determine, regardless of the number of variables, by these methods, due to the additive properties of the Gaussian distribution, as previously shown. In this case the reliability index represents the minimum distance between the limit state to the origin of the standardised Gaussian distribution space.

However, this is not the most common scenario for reliability problems. This fact has two main consequences: the first one is that both the mean and variance cannot be obtained by the additive properties of Gaussian distribution and the second is the fact that the safety margin, M , can also be non-Gaussian. In this case the results obtained by this approach are an approximation. Usually, the two statistical moments are obtained by adjusting an approximate function to the most representative point of the studied problem that is known as the design point [Henriques (1998)]. Jacinto (2011) points out that the degree of linearity is only important in the vicinity of this design point as this is what defines the quality of the estimated probability of failure. This means that despite the limit state function being highly non-linear as long as this is not observed near the design point the approximation can actually prove to be quite good.

The approximation function can be defined by expressing the performance function, $g(X)$, in a Taylor series:

$$Z = g(X^*) + \nabla g|_{X^*} (X - X^*) + \frac{1}{2} (X - X^*)^T \nabla^2 g|_{X^*} (X - X^*) + \dots \quad (3.16)$$

Where X^* represent the design point and $\nabla^k g$ corresponds to the partial derivatives in terms of k . Therefore, it can be said that the essence of the FORM or SORM methods is to find the point that leads to the minimum distance between the limit state function and the origin of the normalised space [Melchers (1999)].

As the name indicates the FORM only accounts for the first order terms of the Taylor series of the function $g(X)$ relative to the design point X^* :

$$Z \cong g(X^*) + \frac{\partial g}{\partial X} \bigg|_{X^*} (X - X^*) \quad (3.17)$$

The limit state function is therefore approximated by a hyper surface tangent to the design point X^* as illustrated in Figure 3.3.

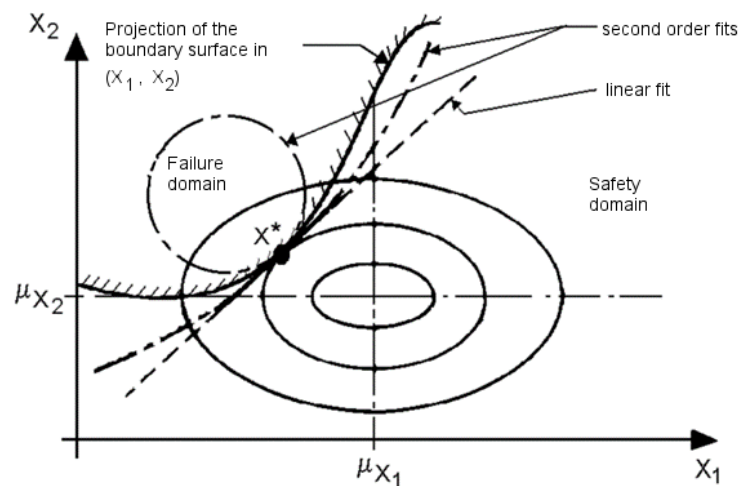


Figure 3.3 – Approximation to the performance function.

Generally second order surfaces such as paraboloids or spheres are used for the approximation in order to reduce the errors that result from using an approximation to the limit state function.

Cornell (1969) introduced the reliability index concept by calculating the mean and the variance using first order approximation functions.

$$\beta = \frac{a_0 + \sum_{i=1}^n a_i \cdot X_i}{\sqrt{\sum_{i=1}^n a_i^2 \cdot \sigma_{X_i}^2}} \quad (3.18)$$

However, this formulation has an important drawback which is the dependency of the reliability index, β , on the design point. In order to overcome this problem Hasofer & Lind (1974) proposed a methodology that performs a transformation of the original space into the normalised space (zero mean and unit variance). The variables X_i are substituted by the reduced variables Y_i . The limit state function is also reformulated in the new coordinate system. The subsequent procedure is similar to the procedure described for the fundamental reliability case, the design point Y^* is determined and afterwards the reliability index, β , can be calculated. If the limit state function is non-linear the determination of the design point is carried out using an iterative procedure. An interesting concept that is associated to this methodology is related to the physical interpretation of the direction cosines. The obtained value represents a sensitivity measure of the performance function at the design point for each of the basic variables, X_i .

In the case of correlated and/or nonnormal basic variables the problem is more complex and some specific methods need to be applied. The work of Carrarini (2006) discusses some of them, including the Rosenblatt transformation, Rackwitz-Fiessler formula and the Orthogonal transformation.

One drawback of the FORM and SORM approach is that in its formulation the probability distributions of basic variables, X_i , are not considered but only the first and second moments.

3.3.3 Level III Methods

The Level III reliability methods try to determine the probability of failure by calculating the integral that defines it. Considering performance function $M = g(X)$, where X is the vector of the basic random variables, the probability of failure is represented by the following integral:

$$p_f = P(g(X) \leq 0) = \int_{\Omega_f} f_x(x) dx \quad (3.19)$$

This integral can be solved analytically (only possible for a very limited number of cases) or numerically. Jacinto (2011) notes that the numerical solution of the integral can be difficult for problems with a significant number of basic random variables (generally higher than 5) or when the failure regions displays a complex geometry. Furthermore, it is not always possible to explicitly obtain the failure region function. This is typically the case when a problem is solved by the Finite Element Method, where these functions can only be defined point by point. The main consequence is that the failure domain is unknown and, consequently, the probability of failure cannot be calculated from an integral such as expressed in Eq. (3.19) as the domain is not completely known. In these cases one usual option to assess structural safety is by response surface methods.

Seeing that the limit state function is not explicit the response surface method provides an analytical performance function, $\tilde{g}(X)$, generally of the polynomial type, that defines the limit state that is to be assessed from a few properly selected deterministic analyses. In order to obtain an accurate assessment of the structural reliability the approximation function must adequately represent the limit state function in the vicinity of the design point. The response surface method can be summarised in the following steps [Henriques (1998); Wisniewski (2007)]:

- Define a set of values for the basic variables in order to obtain $\tilde{g}(X)$ and assess the quality of the approximation;
- Assess the structural response for each set of values that were defined using the Finite Element Method;
- Determine the function coefficients using regression techniques based on the FEM analysis results;

- Based on the analytical performance function, $\tilde{g}(X)$, determine structural reliability through FORM/SORM methods or using simulation techniques.

A careful choice of the polynomial degree of the analytical performance function must be carried out. The degree of $\tilde{g}(X)$ must be smaller or equal to the degree of $g(X)$ in order to obtain a system of well-conditioned linear equations during the determination of the coefficients [Henriques (1998)]. However, Deng (2006) points out that this method becomes computationally impractical for problems involving a large number of nonlinear random variables, particularly when dependent random variables are involved.

An enhancement to the traditional response surface methods is Artificial Neural Networks (ANN). When studying complex limit state functions this method proves to be particularly useful as obtaining an adequate fit, using the response surface method, might be very difficult. ANN are numerical algorithms that attempt to replicate the behaviour of the biological neural network in a computational model. One of the most interesting features of this methodology is the possibility of *learning*. This means that given a task to solve, such as determining the limit state function, ANN is capable of defining a model of the limit state surface from training examples and finding meaningful solutions without the need to specify the relationship between variables [Deng (2006)]. The main advantage of this methodology is that obtaining the limit state surface represents a small fraction of the computational time required by the numerical model, thus making it possible to analyse complex structures that display a non-linear behaviour [Chojaczyk *et al* (2012)]. Usually, and similarly to what is done with the traditional response surface methods, after determining the limit state surface using ANN the method is combined with Monte Carlo simulation or FORM/SORM methods in order to evaluate structural reliability.

An alternative methodology is based on simulation techniques. This approach does not require an explicit limit state function, thus making it suitable to be used when FEM is applied in the assessment of the structural response. Since the determination of the integral presented in Eq. (3.19) is extremely difficult for most structural reliability problems, the use of simulation techniques allows one to obtain unbiased estimates of the value of the integral while adequately accounting for the non-regular structural behaviour. This method has emerged as an interesting alternative due to its simplicity and also because it is almost unaffected by the complexity of the studied problem and the number of basic random variables involved [Rubinstein (1981)].

The Monte Carlo method is the basis of most simulation methods and is generally selected for the analysis of complex systems [Rubinstein, 1981; Shinozuka (1972)]. In this method values are generated for the random variables, $X^{(i)} = (X_1^{(i)}, X_2^{(i)}, \dots, X_n^{(i)})$, according to their distribution. Next the structural response, $Y_i^{(i)}$, is assessed for each trial and the global structural response is analysed as a sample with distribution of Y . Monte Carlo simulation is based on the random sampling concept and aims to artificially simulate a large number of experiments.

The probability of failure, p_f , or the structural reliability, β , can then be assessed using two distinct approaches. The most usual is a simple process that consists on counting the number of trials where the safety limit is exceeded over the total number of simulations:

$$p_f \approx \frac{z_0}{N} \quad (3.20)$$

where z_0 corresponds to the number (or realizations) where structural safety is not verified and N represents the total number of trials. The total number of trials depends on desired accuracy for p_f and also on the order of magnitude of the target p_f . The necessary number of trials is directly proportional to the desired accuracy and inversely proportional to the value of the target p_f .

The second approach consists of statistical analysis of the results of all the realisations using the limit state functions. This enables the determination of the probability density function of the performance function, $g(X)$, as well as the mean value, μ_M , and the standard deviation, σ_M . Assuming that $g(X)$ is normally distributed the reliability index, β , can be determined using Eq. (3.15).

The use of Monte Carlo simulation for structural reliability problems can be divided into two different classes: pure simulation methods and semi-analytical methods. The first class corresponds to the original formulation whereas the second class corresponds to cases where simulation techniques are combined with other methods that enable a more efficient assessment of the structural reliability [Henriques (1998)].

In its most usual form the method is also known as Crude Monte Carlo (CMC), due to its somewhat brute force nature, and can be adequately described by the following steps [Haldar & Mahadevan (2000)]:

- Definition of the structural reliability problem with all its basic variables;

- Definition of an appropriate probability distribution family and the estimation of the parameters of this distribution for each of the basic variables;
- Numerical sampling of the basic variables based on their distribution;
- Assessment of the structural response for each trial;
- Obtain all the necessary statistical information from the N trials;
- Evaluation of the quality of the estimation based on the accuracy and efficiency of the structural reliability assessment.

The methodology is simple and its application depends essentially on the necessary number of trials, which define the computational costs. One way to estimate the necessary number of trials for an accurate assessment of the structural reliability is by analysing the variance or the coefficient of variation (CV) of p_f . The variance or CV can be estimated assuming that each realisation is a Bernoulli trial. This way the number of failures in N Bernoulli trials has a Bernoulli distribution. The variance of p_f can be expressed as:

$$\sigma_{p_f}^2 \approx \frac{(1 - p_f) \cdot p_f}{N} \quad (3.21)$$

Consequently, the CV can be written as:

$$CV_{p_f} \approx \frac{\sqrt{(1 - p_f) \cdot p_f}}{p_f} \quad (3.22)$$

The statistical accuracy in the estimate of p_f is inversely proportional to the value of CV.

Brodig *et al* (1964) suggested the following formula to make an initial estimate of the necessary number of trials, N , based on the confidence level, c :

$$N > \frac{-\ln(1-c)}{p_f} \quad (3.23)$$

Bjerager (1990) on the other hand suggested that the number of trials only needed to be related with the value of p_f and a value in the range of $1/p_f$ to $10/p_f$ should be selected. [Melchers 1999] points out that despite being useful these ‘rules’ do not provide information about the accuracy of the method. One useful tool to assess accuracy is to analyse the evolution of both the estimated p_f and its estimated variance for increasing sample sizes.

The main criticism that is appointed to this method is the typically large number of simulations required for accurately assessing the reliability of Civil Engineering structures. Due to the rather small probabilities that are typically used in structural safety engineering problems [EN1990 (2002)], the computational costs of this method can, in some cases, be prohibitive. As an alternative to the Monte Carlo approach, the use of variance reduction techniques enables the refinement of the sampling process and increases the efficiency of the simulation. Among them the importance sampling techniques, the directional simulation method and the stratified sampling method are worth mentioning. The works of Rubinstein (1981) and Melchers (1999) provide a good overview of the various strategies for variance reduction when using simulation methods to solve structural reliability problems.

Schuëler (2009) carried out a study in order to compare the efficiency of the standard Monte Carlo against methods where variance reduction techniques are employed. Using a building as a case study it was possible to observe that the use of importance sampling and directional simulation results in significant efficiency gains.

Hurtado (2007) proposed a methodology that combines pattern recognition techniques with importance sampling. This work made it possible to conclude that the use of pattern recognition techniques achieves a significant improvement in the efficiency in the selection of the sample to use in the structural reliability analysis, resulting in an important reduction in the required number of samples.

Grooteman focused his studies in directional simulation methods. In Grooteman (2011) an adaptive directional importance sampling was presented whereas in Grooteman (2008) a radial-based importance sampling method is discussed. The author recognises that importance sampling can prove more efficient but in some cases it requires information about the limit state which can

prove hard to define. For this reason he suggests that directional simulation is a good option to assess structural reliability problems due to the efficiency displayed by the method.

Another variance reduction technique is based on stratified sampling methods. These methods divide the entire sample space, S , of X into m strata of equal marginal probability, Ω_i . The probability of failure associated with each strata, Ω_i , is defined by:

$$p_{f_i} = \int_{\Omega_i} f_X(X) \cdot h(X) dX \quad (3.24)$$

Each interval is characterised by the following probability:

$$p_i = \int_{\Omega_i} h(X) dX \quad (3.25)$$

$$\sum_{i=1}^m p_i = 1 \quad (3.26)$$

Thus, the probability of failure is determined by:

$$p_f = \int_{\Omega_i} f_X(X) \cdot h(X) dX = \sum_{i=1}^m \int_{\Omega_i} f_X(X) \cdot h(X) dX = \sum_{i=1}^m p_{f_i} \quad (3.27)$$

Using discrete Monte Carlo simulation an estimate of p_f can be obtained by:

$$\tilde{p}_f = \sum_{i=1}^m \frac{P_i}{n_i} \sum_{k_i=1}^{n_i} f_X^{(i)}(\hat{X}_i^{(k)}) \quad (3.28)$$

with a variance of:

$$\sigma_{\bar{p}_f} = \sum_{i=1}^m \frac{P_i}{n_i} \cdot \sigma^2 [f_X^{(i)}(x)] = \sum_{i=1}^m \frac{P_i \cdot \sigma_i^2}{n_i} \quad (3.29)$$

where:

$$\sigma_i^2 = \sigma^2 [f_X^{(i)}(x)] = \frac{1}{P_i} \int_{\Omega_i} f_X^2(X) \cdot h(X) dX - \frac{P_{f_i}^2}{P_i^2} \quad (3.30)$$

n_i being the number of samples within the sub-space Ω_i .

Eq. (3.30) demonstrates that if an adequate stratification is selected a significant reduction of the variance is obtained, thus significantly increasing efficiency.

One of the most common methods that uses stratified sampling techniques is the Latin Hypercube sampling method [Mckay *et al* (1979); Florian (1992)]. In this method the range of each variable X_i is divided into n strata of equal marginal probability, Ω_i , ensuring that each variable X_i has all portions of its distribution represented by input values, sampling once from each stratum. Each interval is represented in the sample by the representative parameter which can be taken randomly within the interval or may be taken as the centroid of the interval [Florian (1992)]. Stein (1987) shows that this method is superior to standard Monte Carlo simulation with respect to both efficiency and precision of estimators provided that the response is a monotonic function (a function that is either entirely increasing or entirely decreasing) of the basic variables. The sampling process allows several possible configurations of the sample space. Therefore, the proper selection of samples representing the stratified sampled space is decisive for the efficiency of the method. The works of Morris & Mitchell (1995), Vořechovský & Novák (2003), Stocki (2005) and Beachkofski & Grandhi (2002) provide examples of different approaches regarding the selection of the optimal configuration of the sample space.

3.3.4 Level IV Methods

The Level IV methods combine structural reliability with risk concepts. There are several definitions for risk, but the most general in the context of structural reliability is to understand it as the product of probability of occurrence of a given event, which in this case can correspond to the probability of violating a given limit state, by the consequences of the occurrence of such event, C_F , namely the damages that are caused, the loss of lives, etc. [Melchers (1999)]. This perspective can be similar to a cost-benefit analysis and, therefore, the risk can be interpreted as the cost of the occurrence of a given event. The whole-life total cost of the structure, C_T , corresponds to the sum of this cost with the initial costs (both of project and construction), C_I , and the maintenance costs expected during the service life, C_M .

Taken this into consideration, the structural reliability assessment can be interpreted as an optimisation of problem with regard to finding an optimal probability of failure in order to maximise the following:

$$\max(B - C_T) = \max(B - C_I - C_M - p_f \cdot C_F) \quad (3.31)$$

where B represents the benefits and, as previously explained, the last term represents the risk.

Another formulation that can be employed is estimating the risk and ensuring that it is lower than the maximum admissible risk for structure. This type of approach is generally limited to structures that are extremely important, which are characterised by severe consequences when failure occurs.

The use of methodologies that take risk into consideration has been more frequent in structural reliability problems in recent years. However, since this method exceeds the scope of the present dissertation this serves only as a brief reference and overview, acknowledging the existence of this approach and the potential that it offers when addressing this type of problem.

3.4 Enhanced methodologies for estimating the probability of failure

One of the main objectives of the present dissertation is to analyse and implement an efficient methodology for the safety assessment of railway bridges while account for the variability of the parameters that govern their dynamic behaviour. For this reason the most adequate methodologies to achieve this goal are within the Level III methods. Due to the complexity of the problem that is being studied, in particular considering the fact that the obtained response might not be unimodal, the use of response surface methods is less attractive than the use of simulation techniques.

With the aim of enhancing efficiency the simulation method is combined with other techniques in order to reduce the necessary number of simulations. In the following sections the several methods used in this dissertation are presented and discussed.

3.4.1 Tail modelling

One common way of reducing the necessary number of simulations and increasing the efficiency is combining simulation methods with tail modelling techniques. This approach does not take into account the central behaviour. Instead, it focuses on the upper or lower tail behaviour, which fits for the purpose of structural reliability analysis. Since structural reliability problems are determined by the tail of the obtained statistical distributions, the computational cost can be significantly reduced if an extrapolation of the Cumulative Distribution Function (CDF) is made using such an approach [Ramu *et al* (2010)]. The classical tail modelling is based on the extreme value theory and consists of approximating the tail portion of the CDF above a certain threshold, u , by the Generalised Pareto Distribution (GPD) [Castillo (1988)]. The approximation function, $F_{\xi,\psi}(z)$, can be written as [Ramu *et al* (2010)]:

$$F_{\xi,\psi}(z) = \begin{cases} 1 - \left(1 + \frac{\xi}{\psi} \cdot z\right)^{-\frac{1}{\xi}} & \text{if } \xi \neq 0 \\ 1 - \exp\left(-\frac{z}{\xi}\right) & \text{if } \xi = 0 \end{cases} \quad (3.32)$$

where z is the exceedance, ξ and ψ are the shape and scale parameters respectively.

Examples of application of this procedure can be found in Caers & Maes (1998), Ramu *et al* (2010) and Acar (2011). However, the method should be applied with care as the tail needs to be modelled accurately, since small variations in the tail of the distribution can result in variation of the safety level by an order of magnitude.

Besides the GPD other functions can also be used to model the tail of the distributions and typically include normal, lognormal, Weibull or exponential functions. Several of the most traditional functions were tested along with some less usual ones, namely sigmoid functions, which for the particular case of work carried in this dissertation provided the best fit to the obtained distribution tails. The sigmoid functions that were used to model the tails are expressed by:

$$P = y_0 + \frac{d}{\left[1 + e^{-\left(\frac{a-x_0}{b}\right)} \right]^c} \quad (3.33)$$

An important aspect of the tail modelling approach relies on the threshold selection. Despite the importance of this aspect there is not a straightforward globally accepted method to select the threshold [Ramu (2007)]. Caers & Maes (1998) highlight the importance of the threshold selection by pointing out that it has important repercussions on the estimated value of the shape factor, extreme quantiles and other parameters that are important when addressing structural reliability problems. On the one hand, if the threshold is too close to the central data it will bias the estimation towards the central values thus affecting the quality of the fit to the tail. On the other hand, if the threshold is too high there is the risk of using a reduced number of points which can result in high estimation variance. For this reason, Ramu (2007) stated that the selection of the threshold is a trade-off between bias and variance. Several suggestions regarding the selection of the threshold can be found in the literature. Smith (1989) took a fixed threshold and a fixed number of points above the threshold. Boos (1984) suggested that the ratio between the number of points above the threshold, N_{ex} , over the total number of data points, N , should be 0.02 for $50 \leq N < 500$ and 0.1 for $500 \leq N < 1,000$. Hasofer (1996) proposed taking $N_{ex} = 1.5 \cdot \sqrt{N}$. Caers & Maes (1998) suggested using a finite sample mean square error as a criterion for the threshold

selection. A parametric study was carried out in order to identify the most suitable threshold for the problem that is studied in the current dissertation. This will be discussed in Section 5.3.2.2.

One possible drawback of this approach is the fact that the method relies significantly on the most extreme values, which are the ones that present the largest uncertainty. For this reason the estimated probability of failure may require, in some cases, a larger number of simulations until it stabilises.

3.4.2 Enhanced simulation method

Naess *et al* (2009) proposed an enhanced simulation method which is able to overcome some limitations of high computational cost due to large samples needed for a robust estimation as in the previously presented method. It exploits the regularity of the tail probabilities to set up an approximation procedure based on the estimates of the failure probabilities at more moderate levels for the prediction of the far tail failure probabilities. The safety margin, M , represents the difference between the capacity and the demand to define the probability of failure as $p_f = \text{Prob}(M \leq 0)$ and is extended to a parameterised class of safety margins in the following way:

$$M(\lambda) = M - \mu_M \cdot (1 - \lambda) \quad (3.34)$$

where μ_M is the mean value of the safety margin M and λ is the scaling parameter that assumes values in the interval $0 \leq \lambda \leq 1$, putting the emphasis on the more reliable data points. Thus, the original system is obtained for $\lambda = 1$ while $\lambda = 0$ represents a system highly disposed to failure.

For a sample of size N an empirically estimated probability of failure is given by:

$$\hat{p}_f(\lambda) = \frac{N_f(\lambda)}{N} \quad (3.35)$$

where $N_f(\lambda)$ represents the number of realisations where there was failure for the given λ level. Since the probabilities of failure are small, the coefficient of variation of this estimator can be approximated by:

$$C_v(\hat{p}_f(\lambda)) = \sqrt{\frac{1-p_f(\lambda)}{p_f(\lambda)N}} \approx \frac{1}{\sqrt{p_f(\lambda) \cdot N}} \quad (3.36)$$

The 95% confidence interval for the value $p_f(\lambda)$ can be reasonably estimated by [Wald & Wolfowitz (1939)]:

$$[P_{LB}, P_{UB}] = \hat{p}_f(\lambda) \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_f(\lambda)(1-\hat{p}_f(\lambda))}{N}} \quad (3.37)$$

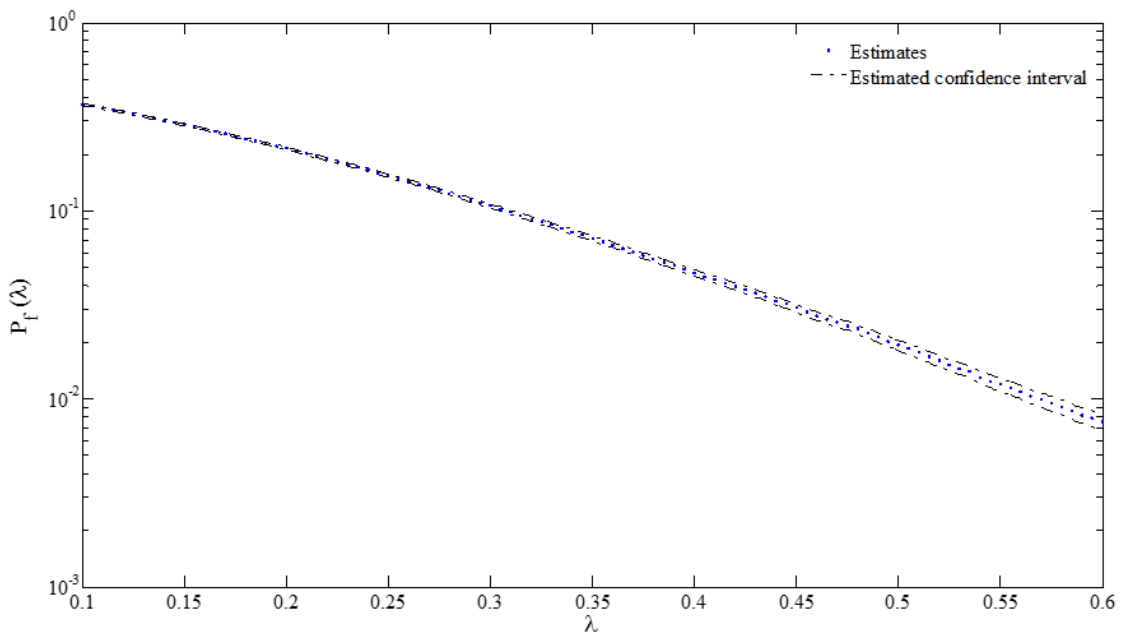


Figure 3.4 – Example of the empirically estimated probability of failure and corresponding confidence intervals.

Introducing an approximation function fitted to the estimates allows estimating the target probability of failure by extrapolation. As proposed by Naess *et al* (2009), it is assumed that the probability of failure in the tail is dominated by a function that can be written as a function of λ :

$$p_f(\lambda) \underset{\lambda \rightarrow 1}{\approx} q(\lambda) \cdot \exp\{-a \cdot (\lambda - b)^c\} \quad (3.38)$$

where $q(\lambda)$ is a function that varies slowly compared with the exponential function $\exp\{-a(b - \lambda)^c\}$. Thus, for practical applications it can be assumed that $q(\lambda) = q$ and this can then be applied in the following form for a suitable value of λ_0 [Naess *et al* (2009)]:

$$p_f(\lambda) \approx q \cdot \exp\{-a \cdot (\lambda - b)^c\} \quad (3.39)$$

Therefore, an important part of the method is the identification of this suitable λ_0 so that Eq. (3.38) represents a good approximation of $p_f(\lambda)$ for $\lambda \in [\lambda_0, 1]$, and at least such that $\lambda_0 > b$. The optimum values for the four parameters q , a , b and c can be obtained through a least square optimisation method using the Levenberg-Marquardt algorithm to the failure probabilities obtained by the Monte Carlo simulation. This consists of optimising the fit on the log level by minimising the following mean square error function with respect to all the four arguments:

$$F(q, a, b, c) = \sum_{j=1}^M w_j \cdot [\log \hat{p}_f(\lambda_j) - \log q + a \cdot (\lambda_j - b)^c]^2 \quad (3.40)$$

where w_j represents a weight factor that puts more emphasis on the more reliable data points and is calculated by:

$$w_j = [\log C^+(\lambda_j) - \log C^-(\lambda_j)]^{-\theta} \quad (3.41)$$

Similarly to Naess *et al* (2009), in the current dissertation $\theta = 2$ was adopted for the optimised fitting. By scrutinising Eq. (3.40), Naess *et al* (2009) noticed that the mean square error function could be simplified. If b and c are fixed, the optimisation problem is reduced to a standard weighted linear regression problem. This way the optimum values of a and $\log q$ are found using a closed form linear weighted regression formula in terms of w_j , $y_j = \log \hat{p}_f(\lambda_j)$ and $x_j = (\lambda_j - b)^c$. Hence, the optimum values of a and $\log q$ are given by:

$$a^*(b, c) = - \frac{\sum_{j=1}^M w_j \cdot (x_j - \bar{x}) \cdot (y_j - \bar{y})}{\sum_{j=1}^M w_j \cdot (x_j - \bar{x})^2} \quad (3.42)$$

and

$$\log q^*(b, c) = \bar{y} + a^*(b, c) \cdot \bar{x} \quad (3.43)$$

The Levenberg-Marquardt algorithm can then be used on the function $\tilde{F}(b, c) = F(q^*(b, c), a^*(b, c), b, c)$ to find the optimum values b^* and c^* , and the corresponding a^* and q^* can be determined by Eqs. (3.42) and (3.43). For the estimation of the confidence intervals the empirical confidence band is re-adjusted to the optimal curve through Eq. (3.37). Then, the optimised confidence intervals are determined using a similar optimisation procedure based on the re-adjusted confidence band. An example is illustrated in Figure 3.5.

The fact that the confidence intervals can also be obtained by extrapolation, reducing significantly the influence of the sample size, is an important feature of this method. This enables it to be used to determine the accuracy of the estimated probability of failure. The fitted curves, extrapolated to the level of interest, will determine an optimised confidence interval of the estimated target failure probability. This procedure seems to give confidence intervals that are consistent with the results obtained by a nonparametric bootstrapping method [Karpa & Naess (2013)].

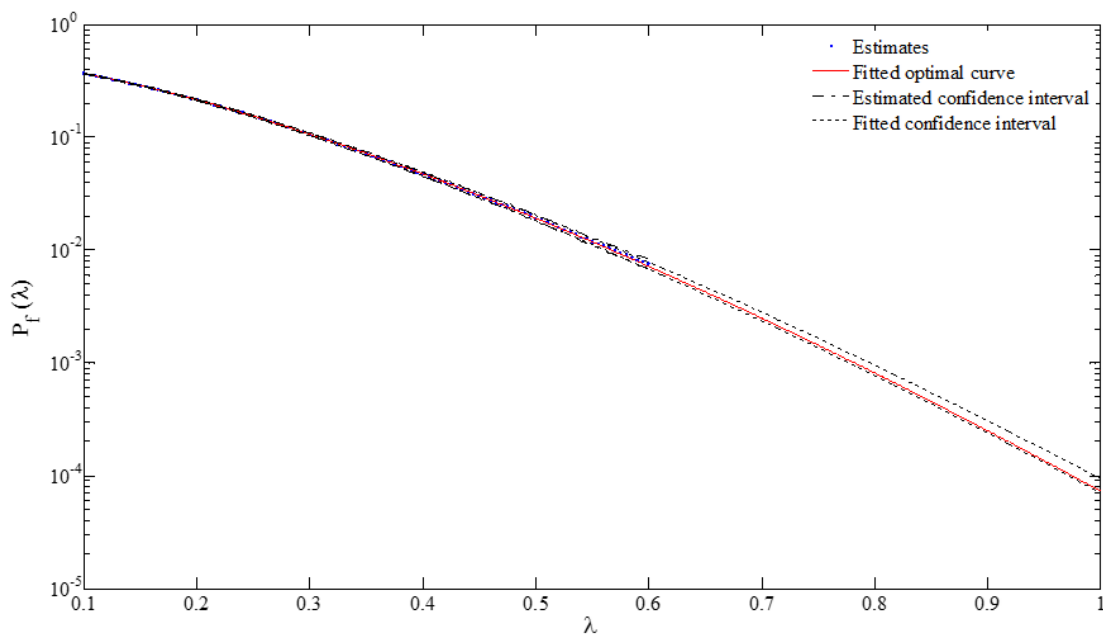


Figure 3.5 – Example of the empirically estimated probability of failure and corresponding confidence intervals.

3.5 Safety assessment framework

One of the objectives of this work was creating a simple, efficient and automatic procedure, which requires as little intervention from the user as possible, that allows the identification of the critical train speeds over a bridge and the assessment of the safety of the train-bridge system. A schematic representation of the proposed probabilistic methodology can be observed in the flowchart presented in Figure 3.6.

After defining the basic random variables, which should include all the parameters which variability affects the dynamic behaviour of the train-bridge system, one has to generate the values that each variable will take for each simulation. To do so a random number generator can be used. This type of tool is available in a wide range of computer software. Afterwards the data files can be created. In this particular case study, two different data files need to be created: one for the structure, which includes the bridge and the track, and another for the train. An automatic data file generation procedure was developed for efficiency reasons.

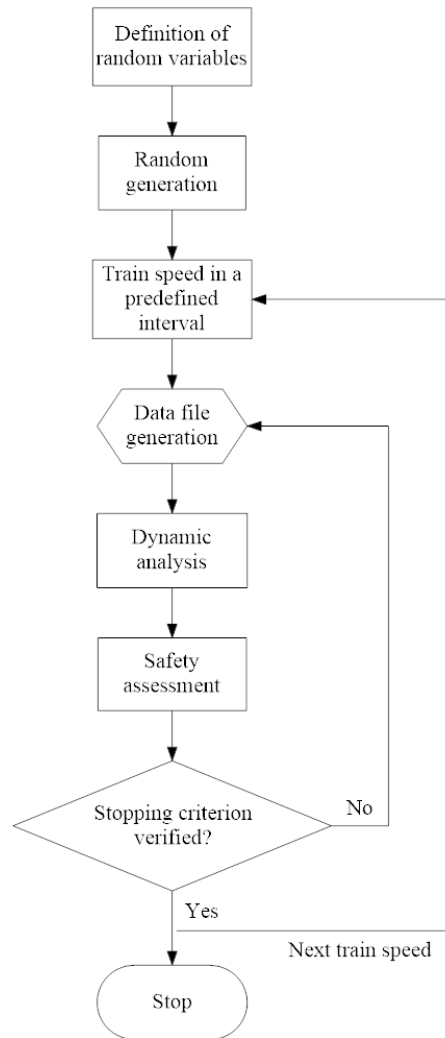


Figure 3.6 – Proposed methodology.

Following the generation of the data files, the dynamic analysis can be performed. An adequate time step needs to be selected for the dynamic analysis and batch processing is used to perform the dynamic analyses for efficiency purposes and to avoid the need for manual intervention.

After the dynamic analyses the results must be processed. Due to the large amount of information an automatic procedure is used. The results processing tool obtains automatically the time history of the dynamic response for a selected bridge section and the maximum dynamic response for that simulation.

To assess the safety of the train-bridge system the probability of failure is estimated using two different techniques. A tail modelling approach based on the extreme value theory that uses the

appropriate functions to model the upper tail of the obtained distribution, as detailed in Section 3.4.1. An Enhanced Simulation technique which uses an approximation procedure based on the estimates of the failure probabilities at moderate levels for the prediction of the far tail failure probabilities, as has been detailed in Section 3.4.2. In order to enhance efficiency and ensure the accuracy of the estimates some criteria have been defined for each methodology and will be discussed in Section 5 where practical examples will be used to demonstrate the purpose, application and advantages of each criterion.

The train-bridge system is considered to be safe if the estimated probability of failure is lower than 10^{-4} , which corresponds to a reference value in JCSS (2001) to assess ultimate limit states for systems where failure has severe consequences.

Modelling of the train-track-bridge system

4.1 Introduction

In the previous chapters the problem that is being studied in this dissertation, as well as the methodologies proposed to achieve the objectives of this thesis, were presented. The current chapter is dedicated to presenting the case study which will be used to test the adequacy, efficiency and accuracy of the proposed methodologies.

A ballasted filler beam bridge was selected as case study as this structural solution is representative of a significant part of the short span bridges that compose the current European high-speed railway network. A similar criterion was used in the selection of the train to be used in the analyses. The TGV double train, which is currently in operation in the European high-speed network, was selected due to having a particularly aggressive configuration for the span length in study.

A thorough description of the case study bridge is presented along with the description of the geometrical and mechanical properties of the train. Based on the properties of both the bridge and the train, a set of random variables is selected and their variability and type of distribution is presented and discussed. Afterwards, the numerical models used to represent both the train and the bridge are presented, an explanation for all the options made during the development of the numerical models is provided and the numerical models are validated. Furthermore, several

particular modelling aspects are discussed in detail in order to provide a better understanding of their influence on the dynamic response of the train-bridge system.

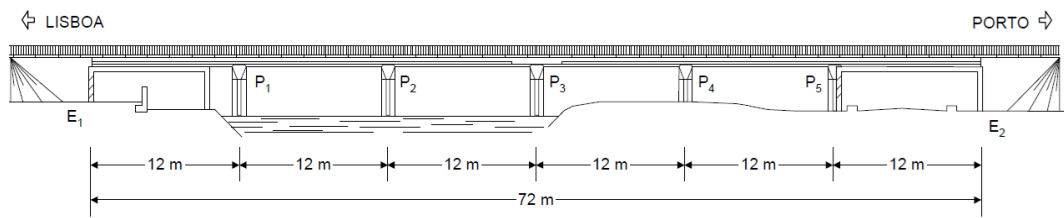
Finally, and due to the few probabilistic studies carried out in this field, it is important to understand which variables have more influence on the dynamic response. Therefore, a sensitivity analysis is carried out in order to identify the variables that govern several aspects of the dynamic response of the train-bridge system.

4.2 Case study

4.2.1 Description of Canelas Bridge

For the purpose of this dissertation Canelas Bridge is selected as case study. Canelas Bridge is located in the Northern line of the Portuguese railway, near Estarreja at the 282.944 km, and is composed of six simply supported spans of 12 m each, with a total length of 72 m. The bridge deck is a composite structure consisting of a concrete slab with embedded rolled steel profiles. This kind of structural system is known as filler beam and is a common structural solution for small span bridges on the European high-speed railway network, especially in France and Germany [Martínez-Rodrigo *et al* (2010); Hoorpah (2005)]. The Canelas Bridge's concrete slab has a height of 0.70 m and has embedded nine HEB 500 profiles. A side view of the bridge used as case study is shown in Figure 4.1.

The bridge supports two ballasted tracks, however, each track is supported by a single half deck due to the existence of a longitudinal expansion joint (see Figure 4.2). Nonetheless, the abutments and columns support both decks. It should also be noted that despite the decks are independent the ballast layer is continuous over both half decks, which can originate some connection between them. This interaction between independent decks due to the continuous ballast layer is a topic that has been studied in recent investigations [Rauert *et al* (2010); Carvalho (2011)] but that is not considered in this dissertation.



a) Schematic representation



b) Longitudinal section

Figure 4.1 – Side view of the Canelas railway Bridge.



Figure 4.2 – Detail of the longitudinal expansion joint.

Furthermore, Canelas Bridge is located in a curve with radii of 892 m and 896 m for the inner and outer tracks, respectively. The grade line of the bridge along its longitudinal profile is horizontal and the rail is located at a 5.75m level. The maximum cant of the track is 0.179 m. The cross section of each deck has a width of 6.20 m and comprises of a concrete slab with a

width of 4.50 m and a height of 0.70m. There is also a cantilever walkway with a width of 1.70 m and a height that varies from 0.30 m to 0.50 m. Nine rolled steel profiles HEB 500 are embedded in the concrete slab with a spacing of 0.475 m. Headed studs with a diameter of 25 mm were welded to the top of the steel profiles in order to improve the bonding with the concrete slab. Cement plates were placed underneath the concrete slab, between the steel profiles, to be used as formwork during the concreting of the slab. It should also be pointed out that each deck has a small ballast retaining wall which forms part of the cantilever with a height of 0.60 m and a width of 0.30 m. Laminated neoprene elastomeric bearings are placed underneath each steel profile, with a total of 9 bearings per half deck in each abutment. The cross section of Canelas Bridge is depicted in Figure 4.3.

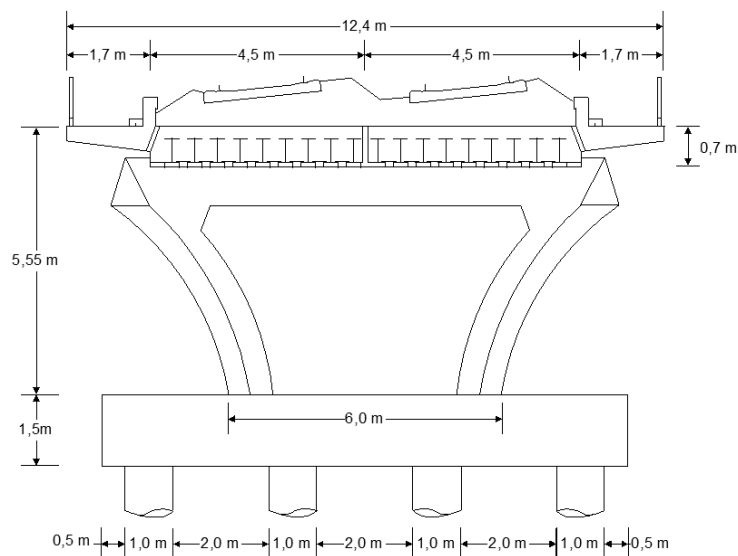


Figure 4.3 – Canelas Bridge cross section.

The reason for selecting of this bridge is that this is a very common structural form used on the European railway network. According to the European Project Sustainable Bridges (2004) composite bridges (steel/concrete or filler beam) represent 14% of the bridges in operation in the European lines, equivalent to more than 30,000 bridges. Furthermore, since the spans of the bridge are within the range of spans for which the European standards indicate that the interaction effects are more significant, it offers the possibility to improve the understanding of such effects on the dynamic behaviour of railway bridges.

4.2.2 Previous research studies on Canelas Bridge

This bridge has already been used as case study in several research investigations. Rodrigues (2004) studied Canelas Bridge at the request of the Portuguese railway network infrastructure management company (REFER) to evaluate the consequences of increasing the line speed over the bridge from 140 km/h to 220 km/h. This investigation involved *in situ* dynamic tests in order to characterise the bridge from a dynamic point of view and to assess its dynamic response under train loading. Structural reliability, train running safety and passenger riding comfort were analysed to assess the bridge response to the increased line speed. Based on the obtained results it was concluded that all the safety criteria were verified for a higher train speed, thus validating the increase of the line speed over the bridge.

Pimentel (2009) also carried out *in situ* tests on Canelas Bridge to determine its dynamic properties. This study aimed to characterise the railway traffic over the bridge and to assess its influence on the dynamic response. Ambient vibration tests were performed in order to identify the modal parameters of the bridge, namely its natural frequencies, modal configurations and damping coefficients. The characterisation of the railway traffic was carried out using a B-WIM (Bridge Weigh-in-Motion) algorithm. This algorithm is based on the strain measurement at three points of the structure, two located on the rail and another on the bridge deck. Such a technique enabled combining the advantages related with the strain measurement at the rail, which allow the identification of the train geometry, with the advantages of the strain measurements at the bridge deck, which allow obtaining the train axle loads.

Silva (2010) developed several three-dimensional finite element numerical models of Canelas Bridge, with varying degrees of complexity. The aim of this study was to assess the influence of the inclusion of the track in the numerical model and also the effect of considering the adjacent span in the dynamic response of the bridge. The numerical models were calibrated using results from experimental tests on the bridge in order to guarantee a realistic representation of the existing structure. Structural safety was assessed for both static, using LM 71 multiplied by an adequate dynamic factor, and dynamic analysis, using the real trains that operate on the European railway network, as prescribed by the current European standards [EN1990 (2002)]. With the increasing complexity of the numerical model the obtained results are less conservative but demanded higher computational capacity and required longer computational times. The author also concluded that accounting for the connection between the decks due to the continuous ballast layer resulted in a significant decrease of the dynamic response, particularly for resonant speeds.

Carvalho (2011) continued the research carried out by Silva (2010), focusing his study on the model updating of Canelas Bridge using optimisation techniques. A genetic algorithm based on the theory of evolution was used for the optimisation in combination with results obtained from ambient vibration tests. Another aspect introduced in this research was the inclusion of elements that reflected the deterioration of the ballast connection between the decks in the numerical model, thus allowing a more realistic representation of the global dynamic behaviour of the bridge. This study concluded that the transversal connection due to the continuous ballast layer is reduced as the ballast in the “connection zone” is highly degraded as a consequence of the successive crossing of trains.

Bonifácio (2012) also studied Canelas Bridge following the aforementioned research by Carvalho (2011). The main focus of this research was centred on the train-bridge interaction effects on the dynamic response of Canelas Bridge. This required the development of numerical models for the trains of both the Alfa-Pendular train, which operates in the Portuguese railway network, and the TGV train, that operates in the European high-speed network. This work concluded that the train-bridge interaction effects are only relevant for resonant speeds, enabling a more realistic analysis. Furthermore, it was not possible to notice significant differences of the dynamic response of the bridge due to the continuous ballast layer when using the moving loads method or the train-bridge interaction approach. This work also analysed the passenger riding comfort and it concluded that the maximum acceleration inside the vehicle tends to increase along the length of the train.

4.3 Numerical models

4.3.1 Track-Bridge numerical models

The Finite Element Method (FEM) was used in the numerical modelling of Canelas Bridge. The bridge was discretised with 2D beam elements using the finite element software FEMIX [Azevedo (2012)]. The numerical model was defined taking into account the design drawings. A single span was modelled and, as the two half slab decks are independent, only a single track was analysed. Furthermore, the numerical model assumed the bridge was in a straight section of a high-speed railway line. Since the structural system is very simple (simply supported bridge) and the goal was to perform a safety assessment (which may require a large number of simulations),

the aim was to develop a numerical model that was also relatively simple, thus enabling to accurately assess the dynamic response of the bridge.

The deck was modelled as a beam positioned at the corresponding centre of gravity. For accuracy reasons [Yang *et al* (2004); Martínez-Rodrigo *et al* (2010)] the bearings were included in the numerical model as springs positioned at the corresponding centre of rotation. Due to the configuration of the bearings (existence of steel plates between each neoprene layer) each layer acts as an individual spring. For this reason, the bearing, as a whole, works as a system of series-connected layers. The bearing stiffness was calculated according to [Manterola (2006)]:

$$K_v = \frac{1}{\sum_{i=1}^n \frac{1}{k_i}} \quad (4.1)$$

$$K_h = \frac{a \cdot b \cdot G}{\sum_{i=1}^n t_i} \quad (4.2)$$

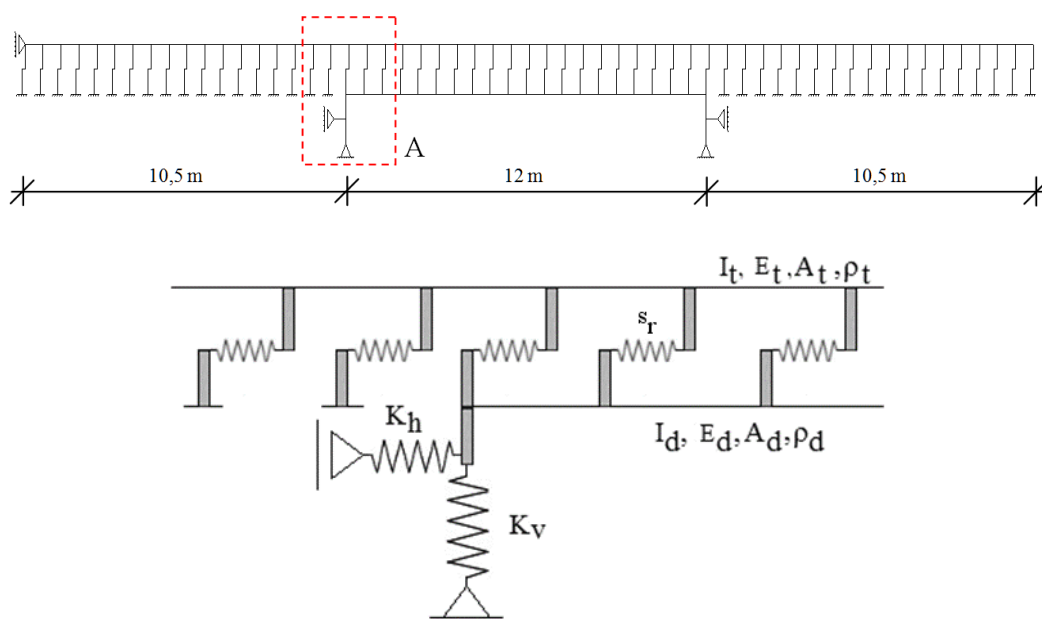
$$k_i = \frac{E \cdot a \cdot b}{t_i} \quad (4.3)$$

$$E = 3 \cdot G \cdot \left(\frac{a}{t}\right)^2 \cdot \gamma \cdot R \quad (4.4)$$

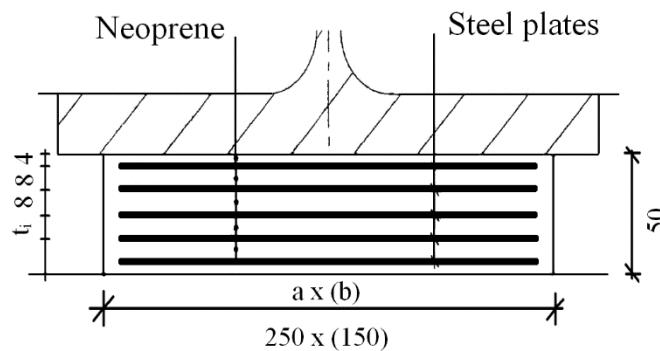
where G is the neoprene shear modulus, E is the neoprene equivalent elasticity modulus, a and b are the smallest and the largest dimensions of the bearings, respectively, t_i is the thickness of each neoprene layer, γ is a coefficient that depends on the relationship between the dimensions of the bearings and R is a coefficient that takes into account the dynamic nature of the loading.

Two different models were developed for the track. The first one was developed at a stage where the interaction effects between the train and the bridge were not taken into consideration [Rocha *et al* (2012)]. At this stage the vertical deformability of the track was neglected and only

the shear behaviour of the track, which enables reproducing the track-bridge composite effect, was taken into consideration. For this reason the connection between the track elements, the deck elements and the bearings was modelled through a series of rigid beams. Although the bridge consists of simply supported spans the rail is continuous and this continuity affects the dynamic response of the track-bridge system. For this reason the track was extended 10.5 m over the length of the bridge in both directions in order to reflect this effect on the numerical model. A schematic representation of this model is represented in Figure 4.4.



a) Schematic representation of the numerical model



b) Bridge bearing [units in mm]

Figure 4.4 – Simplified track-bridge numerical model.

This model is simple and proves to be adequate when analysing the dynamic response through the moving loads method. However, when the train-bridge interaction effects are taken into

account the model led to numerical problems because it neglects the vertical flexibility of the track. This results in an inaccurate assessment of the wheel-rail contact forces and ultimately affects the accuracy of the assessment of the dynamic response.

To overcome the model limitations and to meet one of the objectives of this dissertation (to analyse the influence of the interaction effects on the dynamic response of short span railway bridges), a more adequate track model had to be developed to carry out an adequate analysis of the response of the train-bridge system when accounting for the interaction effects. Different track models with varying degrees of complexity and sophistication can be widely found in the literature. The works of Rigueiro (2007), Vale (2010) and Alves Ribeiro (2012) provide a detailed review of the several models that are typically used to simulate ballasted tracks. A very comprehensive study on the behaviour of ballasted tracks was carried out by Zhai *et al* (2004) and involved both numerical modelling and field experiment tests. One of the simplest ballasted track models simulates the track as a series of continuous springs and dampers that replicate the vertical stiffness and damping properties of the track elements that connect the rail and the structure [Lou (2005)]. Since this model only accounts for the vertical properties of the track, Yang *et al* (2004) proposed including the horizontal properties of the ballast layer through a systems of longitudinal springs and dampers. However, neither of these models accounts for the different properties of the several elements that compose a ballasted track. For this reason, Calçada (1995) divided the track model in two layers of springs and dampers. The top layer connects the rail to the sleepers, which are simulated by lumped masses, and simulates the rails pads. The bottom layer connects the sleepers to the structure and simulates the ballast layer. Zhai *et al* (2004) proposed a more complex model that introduces a third layer in order to take into account the vibrating ballast mass in the numerical model. In this thesis a two layer track model, similar to the one used by Calçada (1995), was adopted. In order to validate its accuracy the obtained results are compared to results obtained using the three layer model proposed by Zhai *et al* (2004) and this is discussed in Section 4.4.3.2.

The rails were modelled as beams with the properties of a UIC 60 rail. The beam representing the rail is connected in parallel to a set of springs and dampers, with stiffness k_{pa} and damping c_{pa} , respectively, simulating the rail pads. Below the spring-dashpot set, a series of lumped masses, with mass M_s , are used to simulate the sleepers. These lumped masses are also connected in parallel to another set of springs and dampers, with stiffness k_b and damping c_b , respectively, simulating the ballast layer (see Figure 4.6). The load transmission from the sleepers to the ballast

follows approximately a cone distribution according to Ahlbeck *et al* (1978). The inclination of the cone, α , is the ballast stress distribution angle. Therefore, the effective acting region of the ballast under each sleeper is the cone region represented in Figure 4.5.

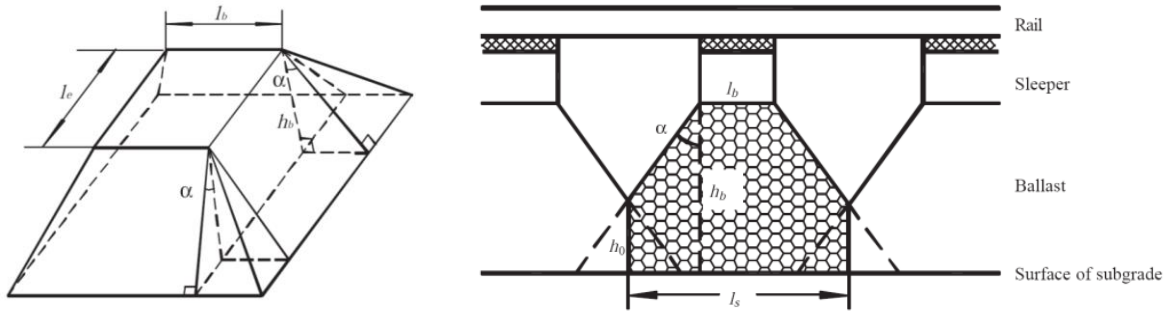


Figure 4.5 – Ballast load distribution according to Ahlbeck, Meacham & Prause (1978).

Consequently, the ballast layer can be modelled as a series of separate spring-dashpot elements placed underneath each sleeper [Zhai *et al* (2004)]. If there is no overlapping of adjacent cone regions of ballast, the vertical stiffness of the ballast layer is given by [Zhai *et al* (2004)]:

$$K_b = \frac{2 \cdot (l_e - l_b) \cdot \tan \alpha}{\ln \left[\left(\frac{l_e}{l_b} \right) \cdot (l_b + 2 \cdot h_b \cdot \tan \alpha) / (l_e + 2 \cdot h_b \cdot \tan \alpha) \right]} \cdot E_b \quad (4.5)$$

Otherwise, when the cone regions overlap, the vertical ballast stiffness is determined by [Zhai *et al* (2004)]:

$$K_b = \frac{K_{b1} \cdot K_{b2}}{K_{b1} + K_{b2}} \quad (4.6)$$

$$K_{b1} = \frac{2 \cdot (l_e - l_b) \cdot \tan \alpha}{\ln \left[(l_e \cdot l_s) / (l_b \cdot (l_e + l_s - l_b)) \right]} \cdot E_b \quad (4.7)$$

$$K_{b2} = \frac{l_s \cdot (l_s - l_b + 2 \cdot l_e + 2 \cdot h_b \cdot \tan \alpha) \cdot \tan \alpha}{l_b - l_s + 2 \cdot h_b \cdot \tan \alpha} \cdot E_b \quad (4.8)$$

where E_b is the ballast elasticity modulus, h_b is the height of the ballast layer, l_s is the sleeper spacing, l_e is the effective supporting length of half sleeper and l_b is the width of the sleeper underside.

Furthermore, the model accounted for the shear behaviour of the ballast layer due to the track-bridge composite effects. The shear behaviour of the ballast was modelled through horizontal springs with stiffness s_r . Since the track platform relative displacements are smaller than the limit of 0.002 m the ballast stiffness was computed in accordance with the elastic range of the bi-linear behaviour proposed by ERRI (1999), which is also currently prescribed in EN1991-2 (2003). Similarly to the previous model, the track continuity effect is also taken into consideration. However, in this case the track length before the bridge also influences the dynamic response of the train-bridge system. The effects of the sudden introduction of the train in the model should be dissipated along the length of the path the train makes prior to entering/crossing the bridge in the numerical model. This is discussed in Section 4.4.3.1 where the results from a parametric study regarding the influence of the track length before the bridge are presented and discussed. A schematic representation of the track-bridge numerical model is shown in Figure 4.6. It should be mentioned that the zoom in Figure 4.6 corresponds to the same section that is composed by an overlap of the two represented elements.

The influence of accounting for the ballast shear behaviour and a comparison between the two layer model adopted in this thesis and the model proposed by Zhai *et al* (2004), in the dynamic response of the train-bridge system are analysed in Section 4.4.3.

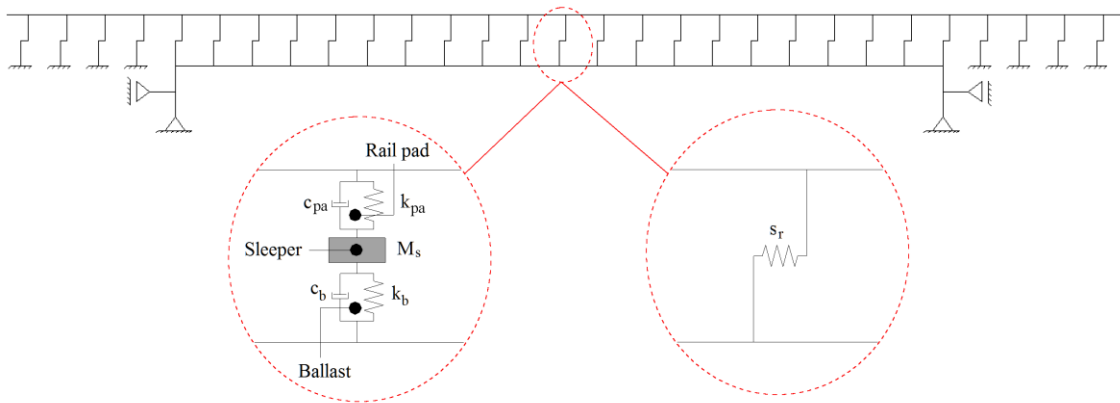


Figure 4.6 – Track-bridge numerical model.

4.3.2 Track irregularities modelling

Track irregularities are a common source of excitation for both the vehicle and the structure, as well as being one of the main sources for the railway noise [Delgado & dos Santos (1997); Khadri *et al* (2013)]. Fryba (1996) defines track irregularities as a deviation of the inside edge of the rail from its ideal geometry. Track irregularities can enhance the bridge dynamic amplifications, originate resonant effects on the vehicles (affecting the passenger riding comfort) or promote the instability of the wheel-rail contact. Furthermore, they can also promote the damage of the several track components, accelerating track degradation and, consequently, increasing maintenance costs.

Track irregularities can be divided into two main groups: isolated irregularities and distributed irregularities. The origin of isolated irregularities can be diverse and may be due to corrosion of the rail, transition zones due to the difference in stiffness of the adjacent sections, the existence of expansion joints, amongst others. The distributed irregularities (analysed in this dissertation) can be caused mainly by the deterioration of the track geometry and/or of the rail or even due to the bridge's displacement [Rigueiro (2007)]. They can be periodic or random and are characterised by the amplitude and the wavelength of the defect. Irregularities that result from the deterioration of the track geometry lead to the change of alignment of the rails (both in the vertical and lateral directions) and can be caused by several reasons such as wear, insufficient maintenance of the permanent way, excessive speed of the trains, longitudinal forces due to the acceleration/braking of the trains, excessive axle loads or differential settlements of the track platform. Fryba (1996) divides this type of irregularities into 4 main groups:

- Elevation irregularities – variation in longitudinal-vertical plane;
- Alignment irregularities – variation in the lateral direction of the horizontal plane;
- Cross level irregularities – variation of the rail elevation along the longitudinal direction against the adjacent rail level;
- Gauge irregularities – variation in the track gauge.

An illustration of these types of irregularities is depicted in Figure 4.7.

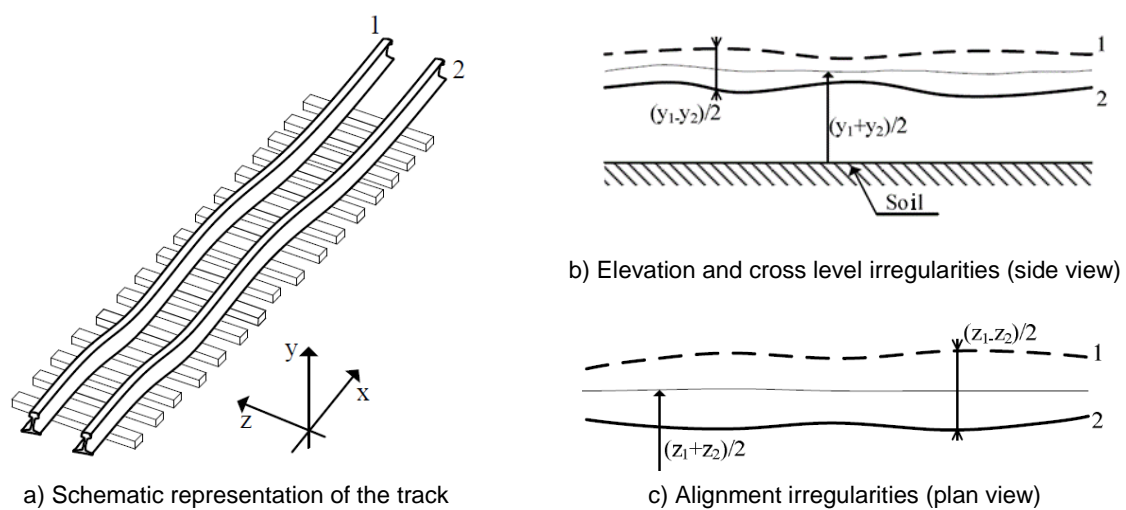


Figure 4.7 – Types of distributed irregularities (adapted from Rigueiro (2007)).

There are two different ways to define the track irregularities: either using values measured experimentally [Xia *et al* (2003); Antolín *et al* (2013)] or through random generation using power spectral density functions [Claus & Schiehlen (1998); Nguyen *et al* (2009)]. Generally the use of experimentally measured track irregularities is limited to the analysis of specific cases. Since the aim of this study is to carry out a safety assessment of the train-bridge system, which requires a significant number of simulations, and due to the limited number of measured track irregularities profiles, the latter option proved to be more suitable. The generation of track irregularities using this method is possible because numerous measurements have shown that track irregularities represent a stationary and ergodic Gaussian random process that may be adequately described by power spectral density functions (PSD) [Claus & Schiehlen (1998)].

Several railway administrations have proposed their own analytical expressions for the PSD functions, based on measured data, for practical application. Thus, each PSD function proposed depends on the maintenance and track quality levels used in each country. In this thesis the power spectral density function adopted by the French administration, SNCF, was selected. This function is expressed by:

$$G(\Omega) = \frac{10^{-6} A}{\left(1 + \frac{\Omega}{\Omega_r}\right)^3} \quad (4.9)$$

where A is a parameter that depends on the track quality and varying between 160 or 550, for a good or bad quality tracks respectively, Ω is the distance frequency and Ω_r is the reference frequency, which takes the value of 0.307 m^{-1} [Frýba (1996)]. Note that SNCF limits the use of Equation (4.9) to wavelengths between 2 m and 40 m. In this study the wavelengths were limited to an interval between 3 m and 25 m, which corresponds to the wavelength range D1 according to EN13484-5 (2008).

The numerical process to generate track irregularities has been described in Hu & Schiehlen (1997). The irregularity profile, $r(x)$, is given by:

$$r(x) = \sqrt{2} \sum_{i=1}^N A_i \cdot \cos(\Omega_i \cdot x - \theta_i) \quad (4.10)$$

where Ω_i is the distance frequency within the wavelength range, A_i corresponds to the amplitude and θ_i is the independent random phase angle that is uniformly distributed in the range between 0 to 2π .

The distance frequency, Ω , corresponds to:

$$\Omega = \frac{2\pi}{\lambda} \quad (4.11)$$

where λ is the wavelength of the irregularity.

Whereas the amplitude, A_i , can be determined through the PSD function, $G(\Omega)$, as follows:

$$A_i = \sqrt{\frac{1}{2\pi} \cdot G(\Omega_i) \cdot \Delta\Omega_i} \quad (4.12)$$

Usually the frequency increment is defined by establishing the lower and upper limits of the distance frequency range, Ω_{min} and Ω_{max} , respectively, and selecting an adequate number of discrete frequencies:

$$\Delta\Omega = \frac{\Omega_{max} - \Omega_{min}}{N} \quad (4.13)$$

The adequate number of frequencies should be carefully selected because if the range of frequencies is large an inadequate value could cause the generated profiles to be dominated by a few frequencies and display patterns, which would make them not random. Taking into account the distance frequency range considered in the current study a set of 1,000 frequencies proved to be sufficient. In addition to this, and in order to guarantee that the frequency range is within the defined limits, the generated profiles are filtered. Two Chebyshev type II filters have been applied: a low pass and a high pass. Two particular aspects have been considered in the design of the filters. The first filter was that the filters ensured that wavelengths smaller than 3 m were removed. The second filter took into account the difficulties of filtering low distance frequency values (has to guarantee that the contribution of wavelengths up to 25 m were taken into consideration) allowing in some cases the contribution of waves with a slightly higher length.

Furthermore, there are several documents that define the quality expected for a railway track. A review of the most relevant documents, which include ETI (2002), UIC518 (2005) and EN13484-5 (2008), was carried out by Vale (2010). In order to guarantee that the generated track irregularity profiles are in agreement with the European standards the values established in

EN13484-5 (2008) must be respected. The values defined in this standard for the alert limit are indicated in Table 4.1 and Table 4.2 and are valid for a track length of 200 m.

Table 4.1 – Standard deviation of the longitudinal level limit.

Train speed (km/h)	Limit (mm)
$V \leq 80$	2,3 – 3,0
$80 < V \leq 120$	1,8 – 2,7
$120 < V \leq 160$	1,4 – 2,4
$160 < V \leq 220$	1,2 – 1,9
$220 < V \leq 300$	1,0 – 1,5

Table 4.2 – Mean to peak value isolated defect limit for the longitudinal level.

Train speed (km/h)	Limit (mm)
$V \leq 80$	12 – 18
$80 < V \leq 120$	10 – 16
$120 < V \leq 160$	8 – 15
$160 < V \leq 220$	7 – 12
$220 < V \leq 300$	6 – 10

In order to validate the quality of the generated irregularities profile one simply needs to invert the generation process. Using the generated irregularities profile the corresponding PSD function can be obtained through the Fourier transformation of the autocorrelation function. Afterwards, the obtained function is compared with the theoretical power spectral density function. The irregularities profile is validated if a match between the two curves is obtained. An example of this comparison for a validated profile is shown in Figure 4.8.

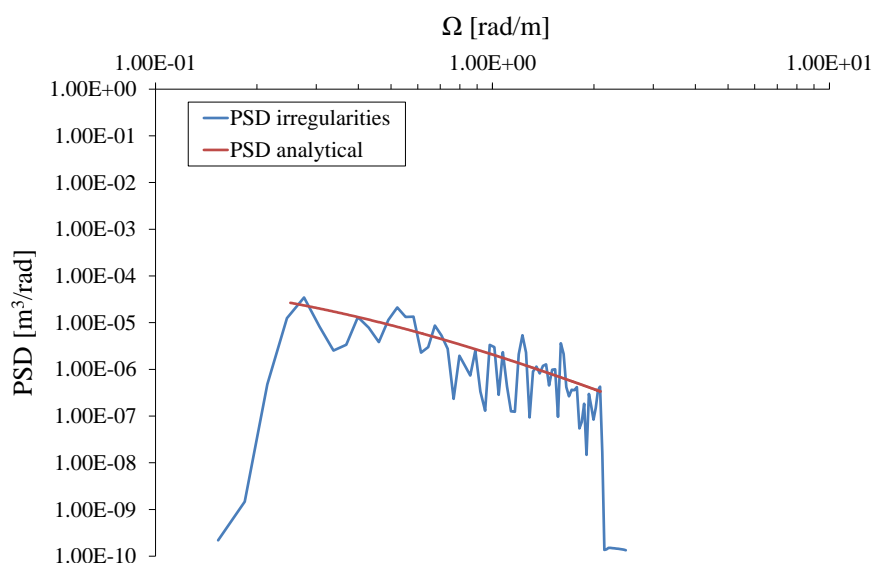


Figure 4.8 – Example of the validation of a generated track irregularities profile.

4.3.3 Basic random variables

As previously indicated two numerical models were developed for the track-bridge system. The initial model was simpler and only intended to analyse the behaviour of the train-bridge system under moving loads models. This represented a preliminary assessment in order to understand and assess the feasibility of a probabilistic approach for the safety assessment of a railway bridge, to understand which variables were more significant for the dynamic response and also to evaluate how the number of variables considered influenced the required computational timings. For this reason, at this preliminary stage the basic variables of the problem to be studied were limited to bridge parameters.

Taking into account the properties of the case study bridge, several bridge parameters that might have relevant nondeterministic properties, of which a variation might lead to a relevant variability on the structural response, were selected at this initial stage of the work. These variables may be divided into three different groups: mass, stiffness and damping. However, at this stage, and due to the importance of damping in the dynamic response of short span railway bridges, particularly for resonant speeds, this parameter was taken as deterministic and a damping coefficient, ζ , of 2% (which was obtained experimentally in previous research works on the structure [Pimentel (2009)]) was adopted. The selected random variables, their simulation identification number as well as their corresponding distribution and variability are defined in

Table 4.3. As it can be noticed from Table 4.3, besides bridge parameters only the contribution of the ballast to the overall weight of the structure was taken into consideration.

Table 4.3 – Random variables, distribution functions and variability for the preliminary assessment.

<i>Variable (simulation)</i>	<i>Distribution</i>	<i>Mean (gaussian)</i>	<i>Std. Deviation (gaussian)</i>
		<i>or</i>	<i>or</i>
		<i>Min. (uniform)</i>	<i>Max. (uniform)</i>
Concrete density weight (1,2)	Gaussian	2.5 t/m ³	0.1 (CV = 4%)
Ballast density weight (3,4)	Uniform	17 kN/m ³	21 kN/m ³
Ballast area (5,6)	Uniform	1.48659 m ²	2.76081 m ²
HEB 500 area (7,8)	Gaussian	Nominal area	0.04 x nominal area
Concrete elasticity modulus (9,10)	Gaussian	36.1 GPa	2.888 (CV = 8%)
Concrete weight (geometrical variation) (11,12)	Uniform	Minimum area	Maximum area
Concrete height (13,14)	Gaussian	Nominal value	10 mm
Concrete width (15,16)	Gaussian	Nominal value	5 mm
Neoprene shear modulus (17,18)	Uniform	0.75 MPa	1.18 MPa
Neoprene elasticity modulus (19,20)	Uniform	420 MPa	600 MPa

Regarding the second model, and having already the experience from this preliminary assessment, the variables that were considered included both bridge parameters as well as track parameters. This was possible because the preliminary assessment showed that a large number of random variables did not affect the efficiency of the analysis in a significant manner and, therefore, a higher number of random variables could be used without compromising the feasibility of the probabilistic approach. It should also be noted that in this model the presence of track irregularities is also accounted for and, as previously indicated, the track irregularity profiles were generated according to the process described in Section 4.3.2. Taken into account the numerical model that was developed (shown in Figure 4.6), the selected random variables for the bridge and the track are presented in Table 4.4 and Table 4.5, respectively. It should be noted that all variables were assumed to be independent.

Table 4.4 – Bridge random variables.

Variable	Distribution	Mean (gaussian)	Std. Deviation (gaussian)
		or Min. (uniform)	or Max. (uniform)
Concrete density weight (γ_c)	Gaussian	2.5 t/m ³	0.1 (CV = 4%)
Concrete elasticity modulus (E_c)	Gaussian	36.1 GPa	2.888 (CV = 8%)
Concrete height (h_c)	Gaussian	Nominal value	10 mm
Concrete width (b_c)	Gaussian	Nominal value	5 mm
HEB 500 area (A_s)	Gaussian	Nominal value	0.04 x nominal area
Neoprene shear modulus (G)	Uniform	0.75 MPa	1.2 MPa
Structural damping (ζ)	Gaussian	2 %	0.3 %

Table 4.5 – Track random variables.

Variable	Minimum	Maximum	Reference
Ballast density weight (γ_b)	1.5 t/m ³	2.1 t/m ³	Fortunato (2005)
Ballast elasticity modulus (E_b)	80 MPa	160 MPa	SUPERTRACK (2005); INNOTRACK (2008)
Ballast height (h_b)	0.30 m	0.60 m	IAPF (2003); UIC (2008)
Ballast load distribution angle (α)	15 °	35 °	Zhai <i>et al</i> (2004)
Sleeper weight (M_s)	220 kg	325 kg	ETI (2002)
Rail pad stiffness (k_p)	100 kN/mm	600 kN/mm	ETI (2002)
Track shear resistance (s_r) (per metre of track)	1.0×10^4 kN/m	3.0×10^4 kN/m	UIC (2001)
Irregularity amplitude (A)	160	275	

With respect to the bridge related basic variables the variability of the parameters related to the concrete were based on the work of Wisniewski (2007) whereas the variability of the neoprene shear modulus was based on Manterola (2006). Regarding the structural damping its variation was defined in accordance with the findings in Rodrigues (2004) and the recommendations from Cantieni (2009).

For the variability of the ballast density weight the results of experimental tests carried out by Fortunato (2005) in the Portuguese Northern Railway Line were taken into consideration. The experimental tests show that the density weight of clean granite ballast varies between 14.8 kN/m³ and 17.7 kN/m³. In the case of contaminated ballast an increase of the density weight is observed reaching a maximum value of 21 kN/m³.

The elasticity modulus of the ballast layer the variation range was defined by taking into consideration the results obtained from two European research projects: INNOTRACK (2008)

and SUPERTRACK (2005). INNOTRACK (2008) observed that the elasticity modulus for ballast made of soils with poor quality is 80 MPa. UIC719 (2008) recommendation suggests using an elasticity modulus of 130 MPa for current tracks where tests are not carried out to determine the characteristics of the ballast layer. SUPERTRACK (2005) identified that some ballasts where the elasticity modulus reached 160 MPa. Given the conclusions obtained by the aforementioned studies, it is possible to notice that the elasticity modulus of the ballast layer displays significant variability. For this reason, in the current work it was assumed that this parameter follows a uniform distribution ranging between the two most extreme values identified by previous research works.

Another parameter that was taken as random was the area of the ballast layer. The recommendation from the Spanish standard IAPF (2003), which suggests a variation of 30% from the values specified by the design drawings, was used. Taking into account the recommendations for the variation of the ballast height produced by UIC (2008), a width of 5 m was defined for the ballast layer and the area variation is introduced by the variation of the height of ballast.

The importance of the track shear resistance has already been highlighted in Section 2. This parameter depends essentially of two main aspects: the fastening system between rail and sleeper and the resistance to move of the rail-sleeper system relative to the bridge deck, which is due to the resistance provided by the ballast layer. Several railway administration companies have defined a plastic resistance, k , of 20 kN/m for unloaded tracks. The German Railway Administration (DB) considers a resistance of up to 60 kN/m for loaded tracks. Currently these limits are already part of EN1991-2 (2003). Therefore, the stiffness of the springs used to simulate the ballast shear resistance take the value of 1.0×10^4 kN/m per metre of track for an unloaded track scenario and 3.0×10^4 kN/m per metre of track for a loaded track scenario. Throughout the simulations the values used for the track shear resistance were randomly generated between these two limits.

Finally, some other track parameters have been taken as deterministic due to the lack of sufficient information about an adequate distribution and because their variability would not significantly affect the results that are being analysed in this study. For the damping of both the ballast layer and the rail pads the value suggested by Zhai *et al* (2004) was used. The ballast damping value suggested by Zhai *et al* (2004) was 5.88×10^{-4} N.s/m and is based on experimental tests using the wheelset-dropping test (a typical test used in the Chinese and Japanese railway

lines). With respect to the rail pad damping the works of Zhai *et al* (2004) and Rigueiro (2007) were taken as reference. It was possible to observe that the values used in both works are very similar. For the purpose of coherency, the value adopted was the value suggested by Zhai *et al* (2004) that adopts a rail pad damping of 7.5×10^{-4} N.s/m.

4.3.4 Dynamic properties of the track-bridge system

In order to understand the dynamic behaviour of the complex coupled train-track-bridge system it is necessary to know the dynamic properties of each subsystem. Therefore, a modal analysis was carried out to identify the main natural frequencies and the corresponding mode shapes for the average scenario, where all the random variables take their mean value.

The results for the model developed for the preliminary assessment are shown in Figure 4.9.

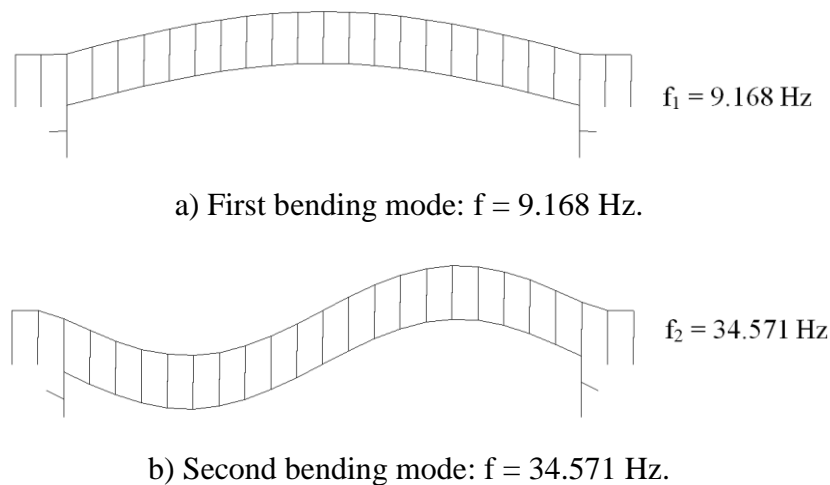
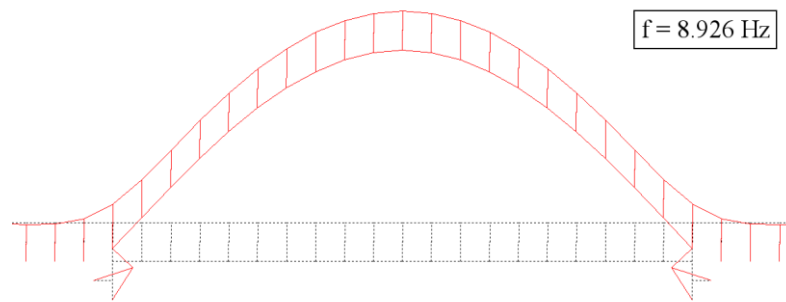


Figure 4.9 – Track-bridge natural frequencies and mode shapes for the preliminary model.

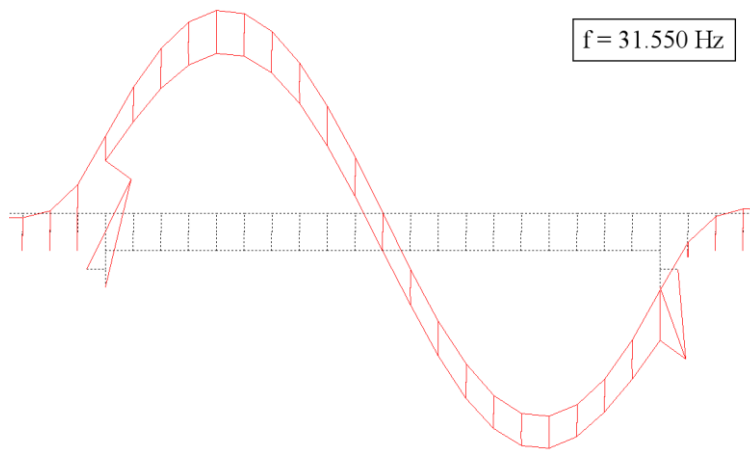
Similarly, the results for the complete track-bridge numerical model are presented in Figure 4.10.

From the comparison between the two models it can be noticed that the first natural frequency is practically unaffected by accounting for the vertical deformability of the track. The second bending mode, due to the mode shape configuration, is more affected and accounting for the

track flexibility results in a reduction of the natural frequency. Both results are consistent and aligned with expectations considering the model assumptions.



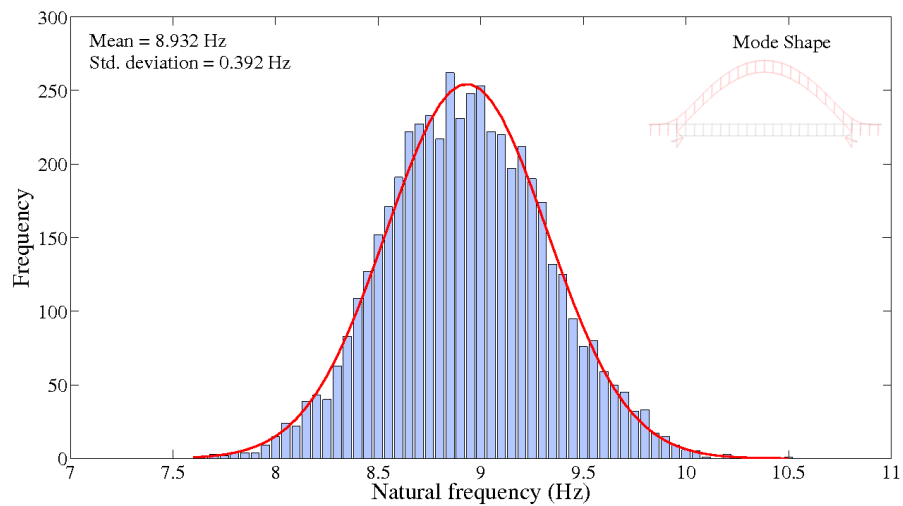
a) First bending mode: $f = 8.926 \text{ Hz}$.



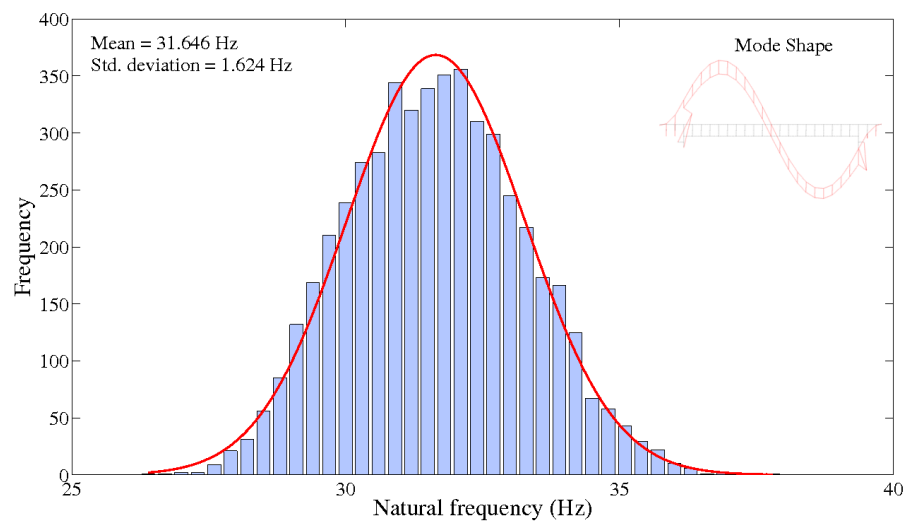
b) Second bending mode: $f = 31.550 \text{ Hz}$.

Figure 4.10 – Track-bridge natural frequencies and mode shapes.

It is also interesting to see how the variability of several parameters affects the natural frequency of the track-bridge system. For this reason, a modal analysis was carried out for a sample of 5,000 Monte Carlo simulations, which is a sample size that enables an accurate assessment of the obtained distribution. The obtained results (with the mean and standard deviation values) are illustrated in Figure 4.11.



a) First bending mode.



b) Second bending mode.

Figure 4.11 – Track-bridge natural frequencies distribution.

The obtained distributions for the two main mode shapes of the bridge approximately follow a normal distribution. Furthermore, and even though the objective was not to model the existing structure, the obtained results for the bridge natural frequencies are in line with results obtained from experimental tests on the existing bridge by previous authors [Rodrigues (2004); Pimentel (2009)].

4.3.5 Train numerical models

The dynamic behaviour of the bridge will be assessed for the crossing of a TGV double high-speed train. This is an articulated train composed by four power cars, four power passenger cars and twelve passenger cars, with a total of 400 m of length and 52 axles. The articulated structure does not include the power cars and only comprises the power passenger cars and the passenger cars. Therefore, the power cars have two independent bogies. In the articulated part of the train the carriages share the bogies as they are located between car bodies and are connected by car body–bogie joints. The power passenger cars have one shared bogie and one independent bogie, whereas the passenger cars have two shared bogies.

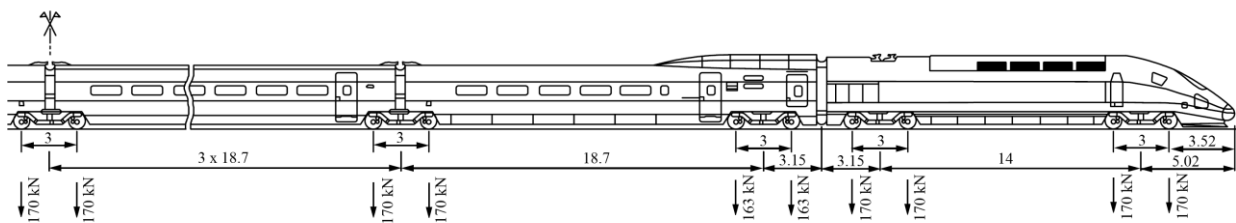


Figure 4.12 – TGV train.

4.3.5.1 Moving loads models

As previously mentioned, at an initial stage the train was modelled as a series of moving loads, meaning that each axle is represented by a point constant-valued load.

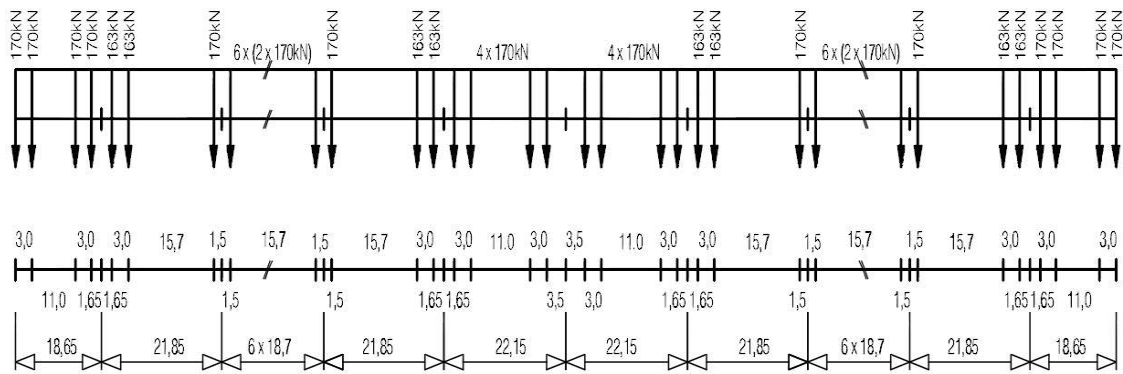


Figure 4.13 – Schematic representation of the moving mass loads model.

As pointed out in Section 2.3.2.3, this approach is one of the most commonly used to assess the dynamic behaviour of bridges due to its simplicity and versatility. However, this type of train model does not take into account the inertial effects of the train masses or the train-bridge interaction effects.

4.3.5.2 Moving masses models

A model that is slightly more sophisticated is the moving mass model, which is based on assuming the train is a moving mass moving over the structure. In this type of model the train-bridge interaction is taken into account as the inertial effects of the train masses are included.

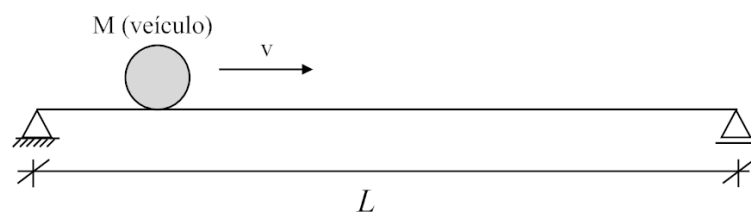


Figure 4.14 – Schematic representation of the moving mass train model.

This type of formulation is not commonly used in modern analysis but its application can be found in the research work of Stanisić & West (1985) and Akin & Mofid (1989). It should be noted that even though this model introduces some advances when compared to moving loads models it does not account for the trains' suspension system which is particularly relevant for

their dynamic behaviour. Furthermore, and similarly to what is verified for the moving load models, this model does not enable the assessment of the passengers riding comfort.

4.3.5.3 Suspended masses models

Another enhancement is the suspended mass model. Compared to the moving mass model these models add the suspension system of the train to the analysis. To do so, a spring-dashpot element is introduced connecting the elements used to simulate the carbody and the bogie (see Figure 4.15).

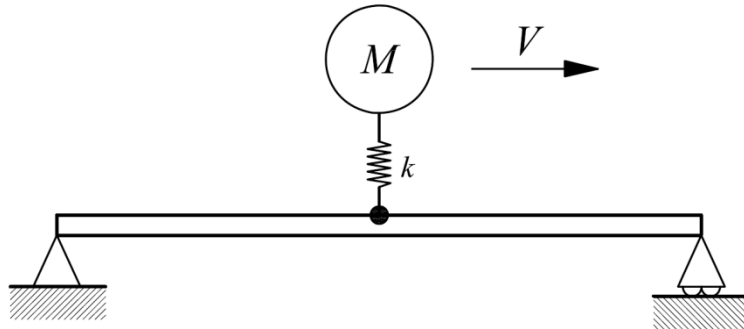


Figure 4.15 – Schematic representation of the suspended mass train model.

Like the moving masses model, this model also accounts for the train-bridge interaction effects and is often selected when this type of analysis is required [Rigueiro (2007)]. Generally, the train axles are independent and the coupling effects caused by the carbodies or the bogies are neglected. This type of train model has been used by Au *et al* (2001) and Yang & Wu (2001) where the authors compared this model with other more complex train models.

4.3.5.4 Complete models

Although the previous numerical model of the train is able to reflect the interaction effects between train and structure, it does not for the dynamic response of the vehicle to be assessed. For this reason and due to the importance of assessing the passenger riding comfort more complex models have been proposed. These models include the car bodies, the bogies, the wheel

axles, the primary and secondary suspensions, the wheel contact and in some cases the connection between adjacent car bodies (articulated trains) or even the seats and passengers.

There are several variations of the type of modelling to adopt, however, the most common typically consists on simulating the car bodies, bogies and wheel axles as rigid bodies with appropriate rotational inertias and masses. The primary and secondary suspensions are modelled as spring-dashpot sets with adequate stiffness and damping and the wheel-rail contact as springs with a suitable stiffness. When included, the seat is normally simulated by a spring-dashpot set with adequate properties and the passenger is simulated by a lumped mass.

The models that can be found in the literature can be either 2D [Zhai *et al* (2001), Goicolea *et al* (2004), Doménech & Museros (2011)] or 3D [Kwark *et al* (2004), Xia & Zhang (2005), Zhang *et al* (2008), Lee & Kim (2010)] and the option chosen is typically based on the type of analysis to be carried out. If the focus is on the analysis of the vertical behaviour of the train-bridge system then the use of 2D models is generally satisfactory. However, if a more complete analysis (where lateral and torsional effects are important) the selection of a 3D model is required.

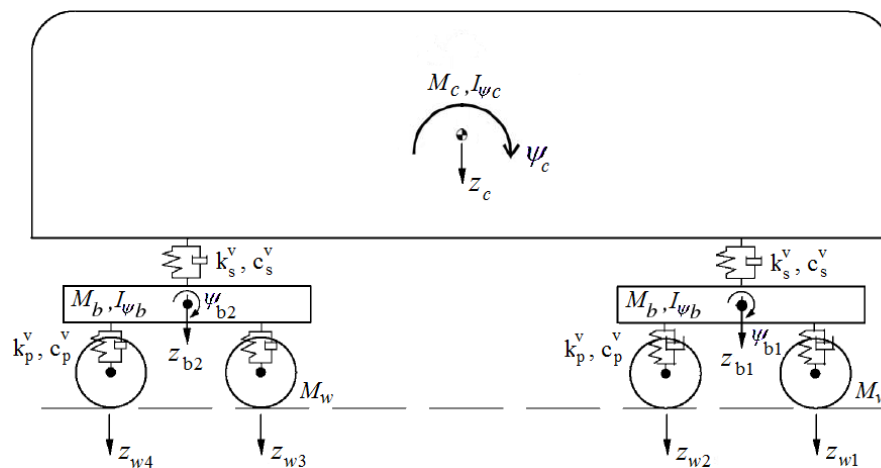


Figure 4.16 – Schematic representation of a 2D numerical model of a train (adopted from Xia & Zhang (2005))

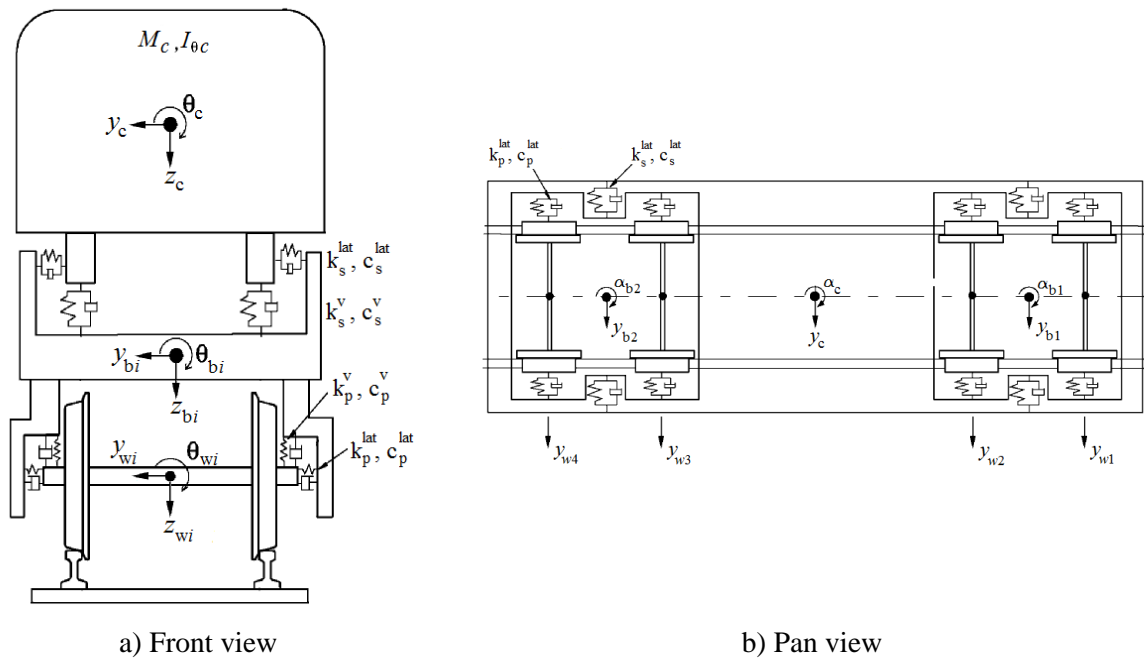


Figure 4.17 – Schematic representation of a 3D numerical model of a train (adapted from Xia & Zhang (2005))

4.3.5.5 Wheel-rail interaction

Several contact models to describe the wheel-rail interaction can be found in the literature. The assumption of nonconformal contact between the two rigid bodies is often used, as the contact area is small when compared with the dimensions of the wheel and the rail [Shabana *et al* (2008)]. In this thesis only vertical interaction is taken into account. The Hertz contact theory, which assumes that the contact area is elliptical (see Figure 4.16), is adopted.

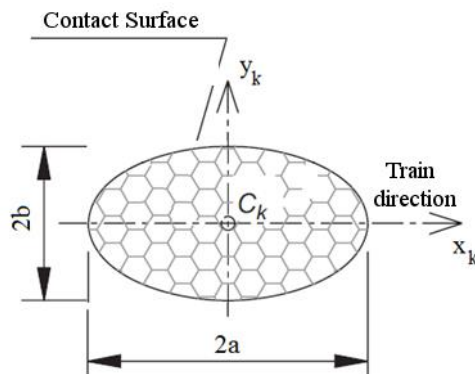


Figure 4.18 – Schematic representation of the Hertz elliptical contact surface.

According to Hertz theory, the normal contact force, F_n , is given by [Shabana *et al* (2008)]:

$$F_n = K_h \cdot \delta^{3/2} = \frac{4 \cdot \beta}{3 \cdot (K_1 + K_2) \cdot \sqrt{A + B}} \cdot \delta^{3/2} \quad (4.14)$$

where δ is the penetration, β is a constant that depends on the ratio A/B , which is related with the geometric properties of the bodies in contact, and K_h is a generalised stiffness coefficient that depends on the material properties of the bodies in contact and on the curvatures of the surfaces at the contact point:

$$K_i = \frac{1 - \nu_i^2}{\pi \cdot E_i} \quad (4.15)$$

$$\frac{n}{m} \approx \left(\frac{A}{B} \right)^{0,63} \quad (4.16)$$

and

$$(m \cdot n)^{3/2} \approx \left(\frac{1 + A/B}{2 \cdot \sqrt{A/B}} \right)^{0,63} \quad (4.17)$$

where E and ν are the Young modulus and the Poisson ratio of the materials, respectively, and m and n are constants that take values which can be consulted in specific abacus [Iwnicki (2006)].

It can be noticed that the Hertz normal contact force equation is non-linear. However, when the penetration is small (general case) and assuming that the bodies in contact have the same material properties, it is possible to establish a linear relationship between the contact force and the penetration [Nguyen *et al* (2009)]:

$$K_{wV} = \frac{3}{2} \cdot F_n^{1/3} \cdot K_h^{2/3} = \frac{3}{2} \cdot F_n^{1/3} \cdot \left[\frac{2}{3} \cdot \frac{E}{1-\nu^2} \cdot (r_w \cdot r_r)^{1/4} \right]^{2/3} \quad (4.18)$$

where K_{wV} is the linearised stiffness coefficient, r_w is the wheel rolling radius and r_r is the head radius of rail cross section.

Given the importance of the wheel-rail contact stiffness on the evaluation of the contact forces and, consequently, on the wheel unloading coefficients, a parametric study is conducted in order to provide a better understanding on how the contact stiffness is influenced by the real variability of the parameters that affect its value. The variability of the Young modulus of the wheels and the rails in addition to the variability of the radius of both elements, is taken into consideration to account for the wear. The Young modulus of both the rail and the wheel are assumed to follow a normal distribution, whereas the radius of both elements are assumed to follow a uniform distribution. The parameters used for each variable are indicated in Table 4.6.

Table 4.6 – Random variables for the parametric analysis of the linearised wheel-rail stiffness coefficient.

Variable	Distribution	Mean (gaussian)	Std. Deviation (gaussian)
		or Min. (uniform)	or Max. (uniform)
Rail Young modulus	Gaussian	205.0 GPa	5.0 GPa
Wheel Young modulus	Gaussian	192.5 GPa	7.5 GPa
Rail head radius	Uniform	200 mm	300 mm
Wheel radius	Uniform	850 mm	1020 mm

The variation of the stiffness coefficient is analysed for a sample of 5,000 Monte Carlo simulations. The obtained probability density function (PDF) is plotted against the PDF of a normal distribution in Figure 4.19.

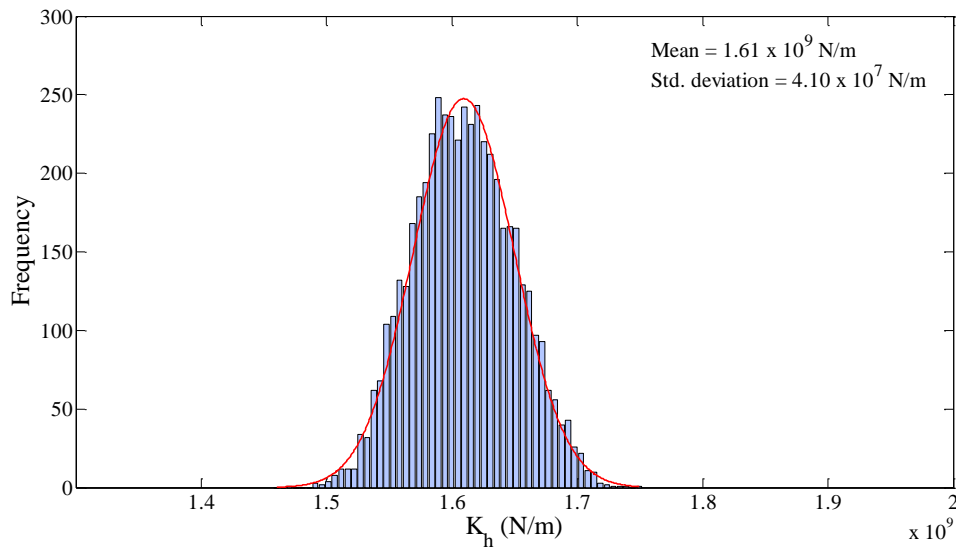


Figure 4.19 – Distribution of the wheel-rail stiffness coefficient.

It can be observed that the linearised wheel-rail stiffness coefficient follows a normal distribution with a mean of 1.61×10^9 N/m and a standard deviation of 4.1×10^7 N/m. The kurtosis and the skewness coefficients tests confirm the assumption of the normal distribution.

4.3.5.6 Train numerical model developed

Bearing in mind the several types of train model referred in the literature and their advantages and drawbacks, at a subsequent stage a FEM model of the TGV train was also developed. A complete 2D numerical model of the TGV train was developed. The car bodies and bogies were simulated by rigid bodies with masses M_c and M_b and rotational inertias I_c and I_b , respectively. The primary and secondary suspensions were simulated by spring-dashpot sets with stiffness k_p and k_s and damping coefficient c_p and c_s , respectively. The wheel sets were simulated by lumped masses, M_e , whereas the wheel-rail contact stiffness was simulated by a spring with stiffness k_h . A schematic representation of this numerical model is shown in Figure 4.20.

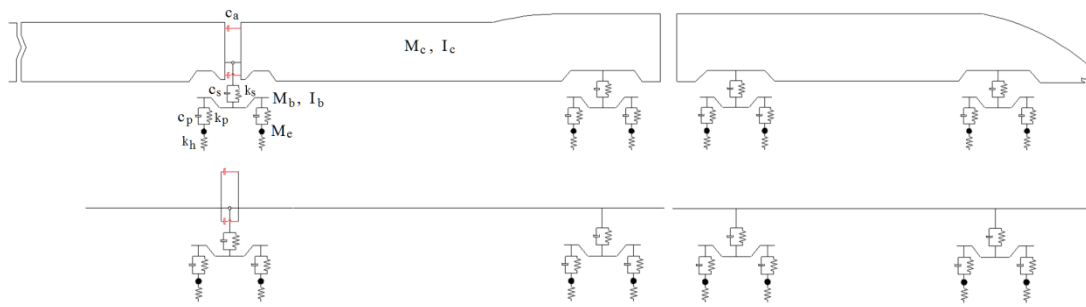


Figure 4.20 – Complete train numerical model.

According to Goicolea *et al* (2002) it is common that the main effects of vehicle interaction with railway bridges are adequately captured using simplified interaction models. Due to the large number of simulations that is expected to be performed, these simplifications can result in a significant reduction of the required computational time to assess the safety of the train-bridge system. Therefore, two simplified train models were developed. The first simplified model simulates the car bodies through lumped masses instead of rigid bodies. The second simplified model eliminates the secondary suspensions and concentrates the car body and bogie masses on lumped mass elements connected to the primary suspension system. The validation of the developed numerical models is analysed in detail on Section 4.4.3.3.

Due to the difficulty of obtaining information from train manufacturers the dynamic properties of the train were defined mostly according to the values presented by ERRI (1999). The variability of suspension parameters is defined in order to guarantee that the frequency of the bogies is within the typical frequency range of 4 to 8 Hz [Museros *et al* (2013)]. Regarding the car body mass, its value is assumed to be constant and the values defined in ERRI (1999) are used. The car body mass used for the power cars, the power passenger and the passenger cars are 51.5 tons, 35.86 tons and 22.525 tons, respectively. Moreover, the variation of the number of passengers on the train is also taken into account. The random variable selected to account for this feature is the occupancy rate of the train.

The random variables selected for the train model are indicated in Table 4.7.

Table 4.7 – Train random variables.

Variable	Distribution	Mean (gaussian)	Std. Deviation (gaussian)
		or Min. (uniform)	or Max. (uniform)
Occupancy rate	Uniform	0 %	100 %
Bogie mass	Uniform	2.32 t	3.48 t
Wheel set mass	Uniform	1.6 t	2 t
Primary suspension stiffness	Uniform	1300 kN/m	3900 kN/m
Primary suspension damping	Uniform	6 kN.s/m	18 kN.s/m
Secondary suspension stiffness	Uniform	290 kN/m	870 kN/m
Secondary suspension damping	Uniform	10 kN.s/m	30 kN.s/m
Wheel-rail contact stiffness	Gaussian	1.61×10^6 kN/m	4.1×10^4 kN/m

To validate the numerical model of the train, its main mode shapes were analysed for the average case scenario and the results are presented in Figure 4.21.

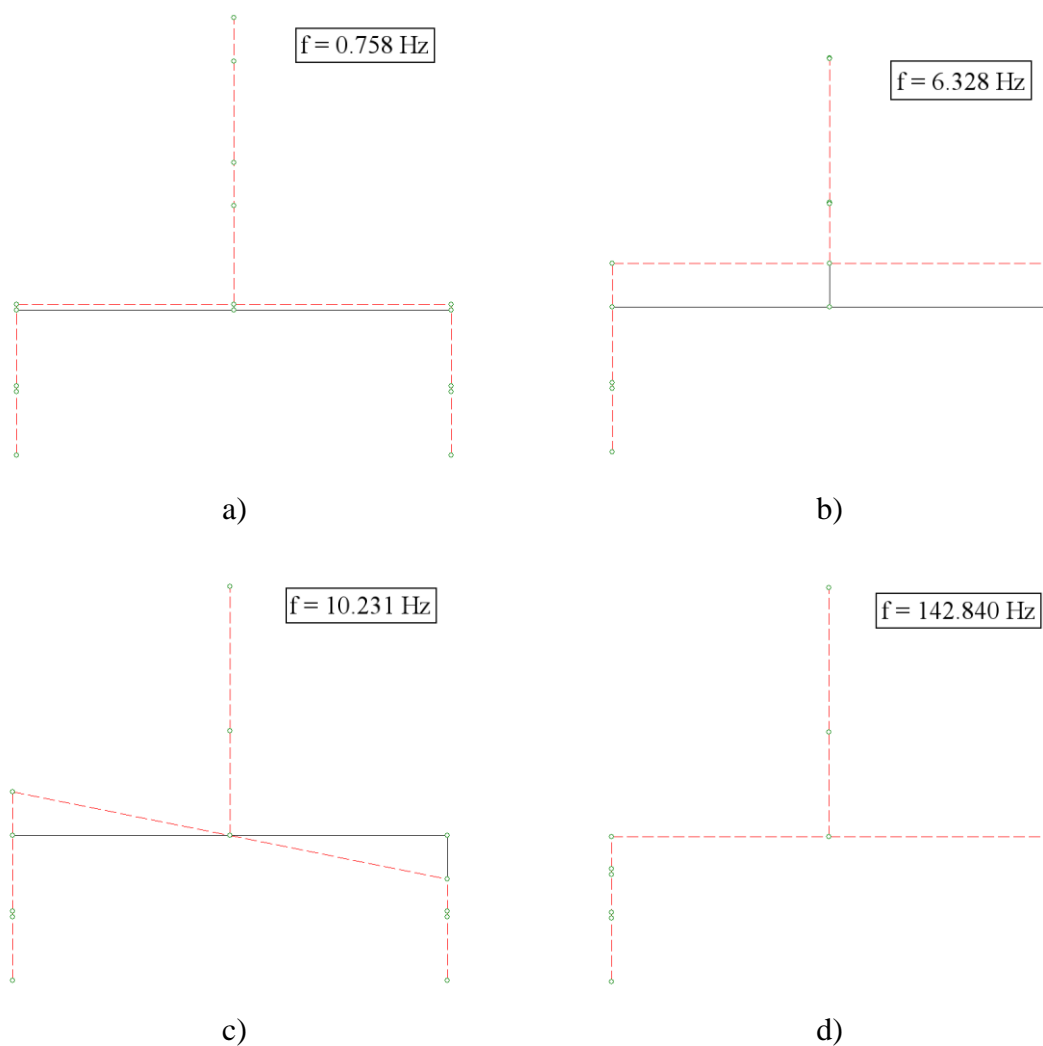
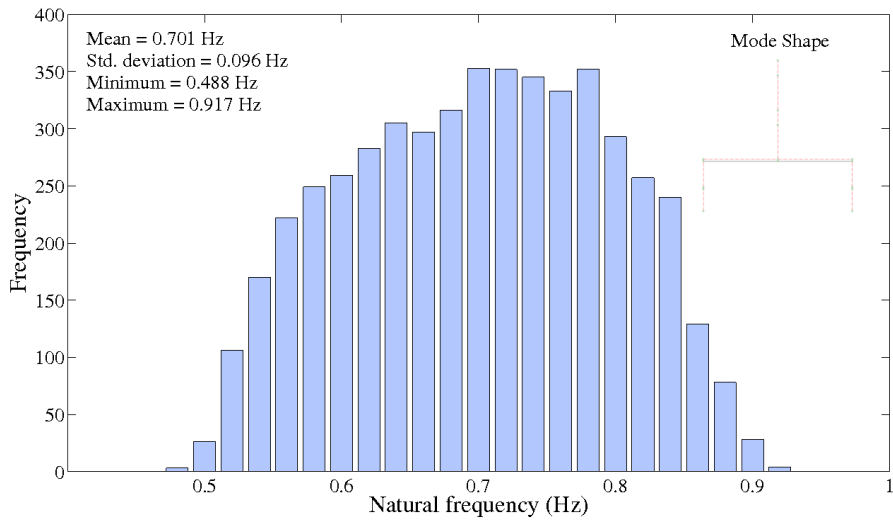


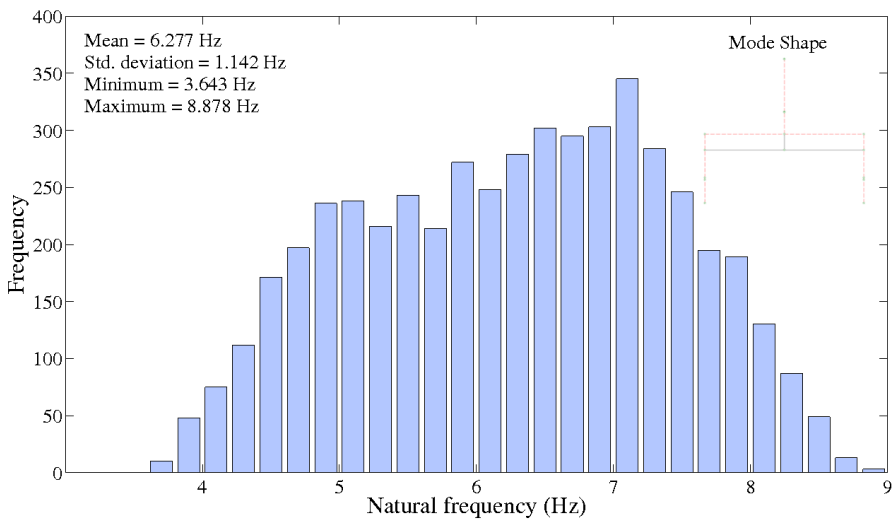
Figure 4.21 – Train natural frequencies and mode shapes.

The obtained results are well within the typical limits referred in the literature which validated the developed model.

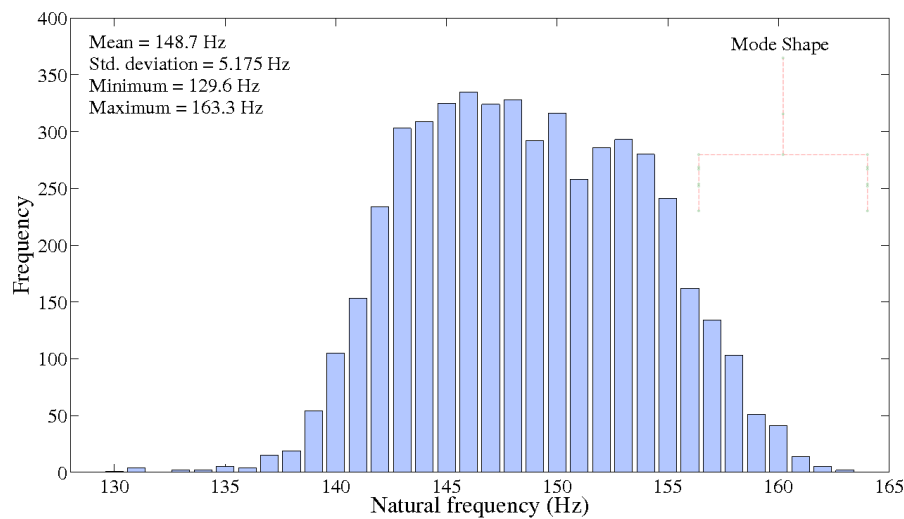
Similarly to what has been done for the track-bridge system a modal analysis was also carried out for a sample of 5,000 Monte Carlo simulations to analyse the type of distribution obtained for the several train mode shapes. The obtained results are illustrated in Figure 4.22 and include the mean and standard deviation values.



a) Car body vibration



b) Bogie vibration



c) Wheel-rail contact vibration

Figure 4.22 – Train natural frequencies distribution.

Contrary to what had been observed for the track-bridge system for the train mode shapes, no clear distribution can be identified. However, it is important to point out that the obtained mode shapes are still within the typical range of frequencies documented in the literature.

4.4 Dynamic response

To provide a better understanding of the dynamic behaviour of the train-bridge system the average case scenario is analysed for different train speeds and the dynamic response is assessed for different sections of the bridge. Several significant modelling aspects are evaluated in detail to understand their effects on the dynamic response of the train-bridge system. Amongst these modelling aspects are the train-bridge interaction effects, the influence of the track irregularities and some other more specific modelling features such as influence of the track and train numerical models and the track extension before the bridge which are discussed in the following sub-sections.

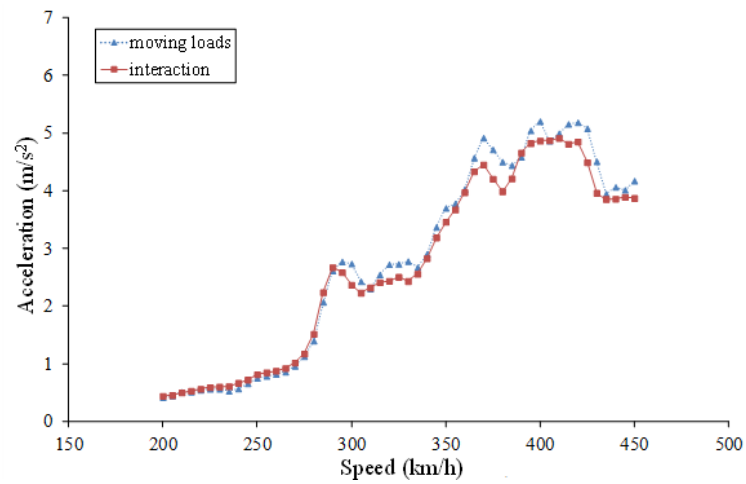
It should be noted that the time step selected for all the analyses is 0.002 s and all the analyses comprise a total of 5,000 steps, for a total of 10 s. The choice for the number of steps was based on the speed range analysed and aimed to guarantee that it allowed sufficient time for the train to cross the bridge and also to capture all significant response in free vibration after the train

leaves the structure. Regarding the selection of the time step value to adopt in the dynamic analysis, the recommendations presented in Section 2.3.2.6 were taken into consideration and a sensitivity study was also carried out. Time steps of 0.02 s, 0.005 s, 0.002 s and 0.001 s were analysed and it was possible to observe that the dynamic response of the train-bridge system can be adequately evaluate with a time step of 0.002 s, which offers a good balance between the accuracy and the computational time.

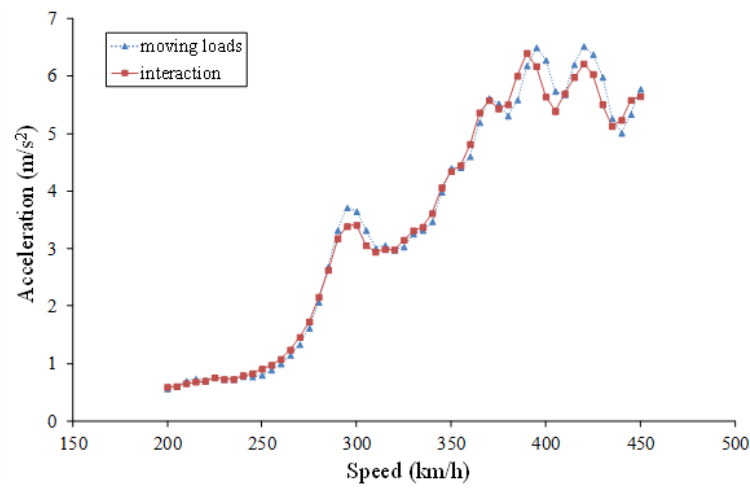
4.4.1 Train-bridge interaction effects

Due to its importance for the dynamic response of short to medium span railway bridges, the first modelling aspect to be analysed was the influence of the train-bridge interaction. The analysis was extended to the full range of speed analysed, which goes from 200 km/h to 450 km/h, in incremental steps of 5 km/h. The maximum acceleration obtained for each speed in different sections of the bridge is shown in Figure 4.23, along with the comparison with results obtained using the moving loads method, which does not take into consideration the interaction effects.

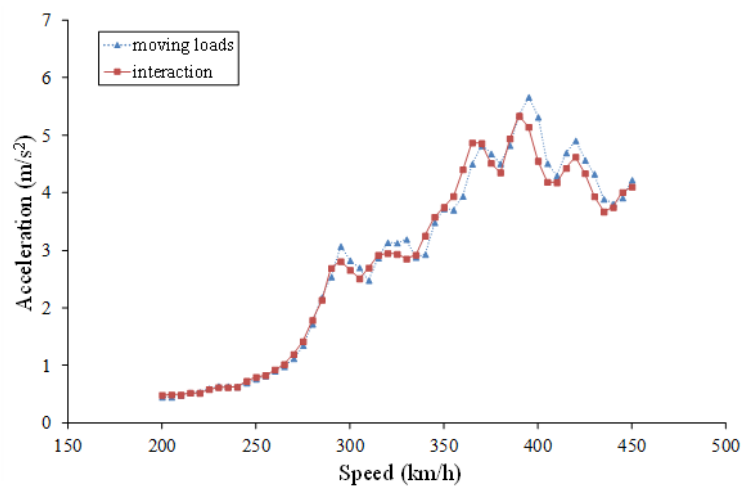
The results illustrated in Figure 4.23 indicate that, in general, accounting for the train-bridge interaction effects tends to reduce the maximum bridge deck acceleration. This difference is mostly noticeable in the resonant peaks. For non-resonant speeds, the differences between the two methods are not significant. However, it should be pointed out is that to be possible to compare the results between the two different methods, the analysis was carried out for a perfect track scenario. Another aspect worth noting was the observation that accounting for the interaction effects seems to originate a slight shift on the obtained dynamic response for speeds greater than 300 km/h. This suggests the existence of some degree of coupling between the two subsystems, in which the train mass contributes to global mass of the system, increasing the modal mass and affecting (in a small scale) the natural frequency of the system, which ultimately results in the differences in the dynamic behaviour of the structure.



a) $\frac{1}{4}$ span section.



b) $\frac{1}{2}$ span section.

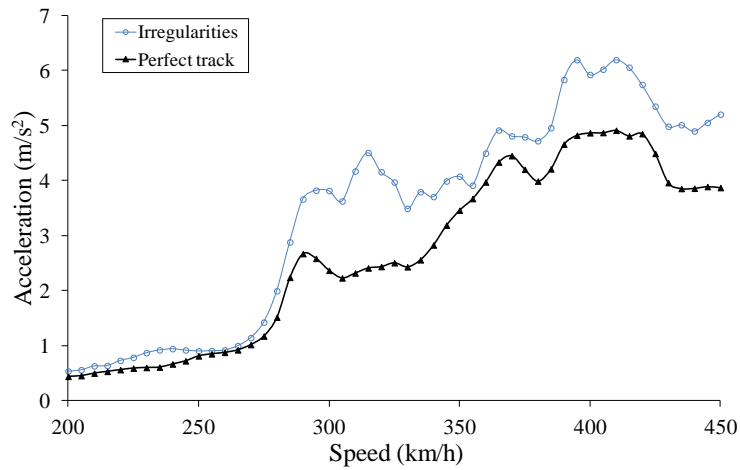


c) $\frac{3}{4}$ span section.

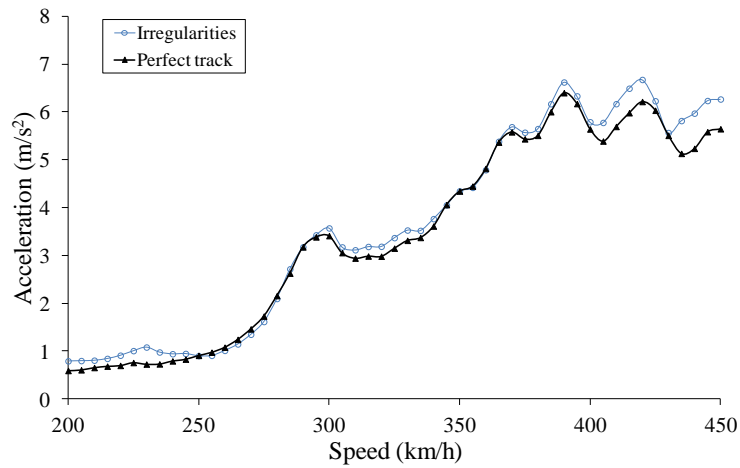
Figure 4.23 – Influence of the train-bridge interaction effects on the dynamic response.

4.4.2 Track irregularities effects

The importance of track irregularities on the dynamic behaviour of the train-bridge system is another aspect that requires adequate understanding. For this reason, a comparison was made between the bridge response obtained for a perfect track and for a track with irregularities and the obtained results are illustrated in Figure 4.24.



a) 1/4 span section.



b) 1/2 span section.

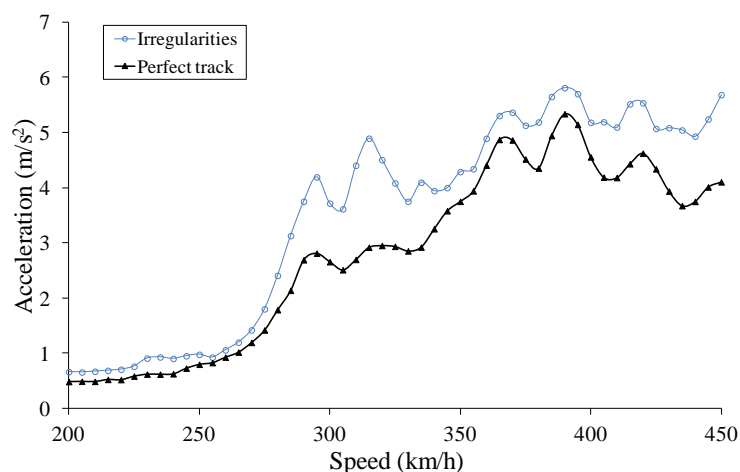
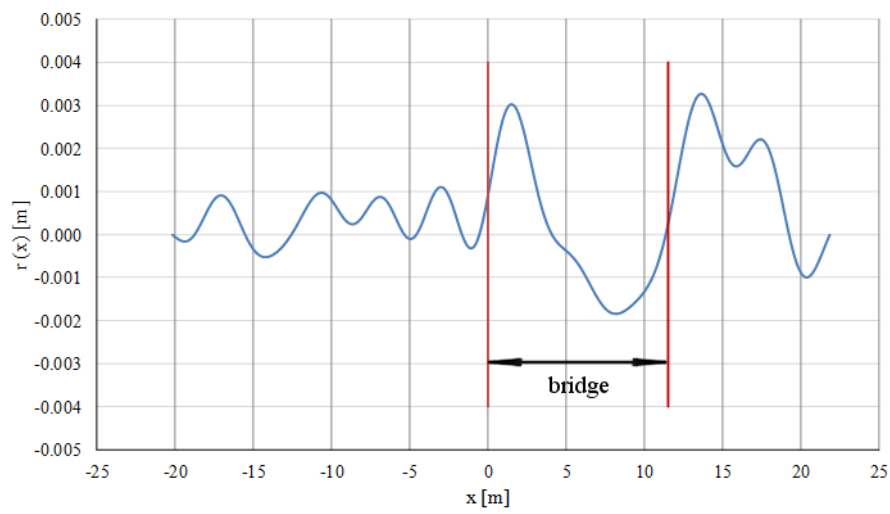
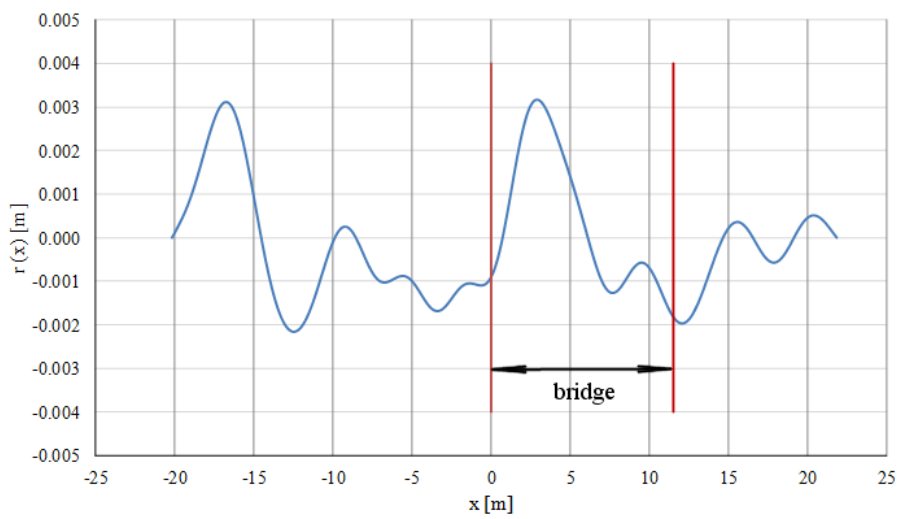
c) $\frac{3}{4}$ span section.

Figure 4.24 – Influence of the existence of track irregularities on the dynamic response.

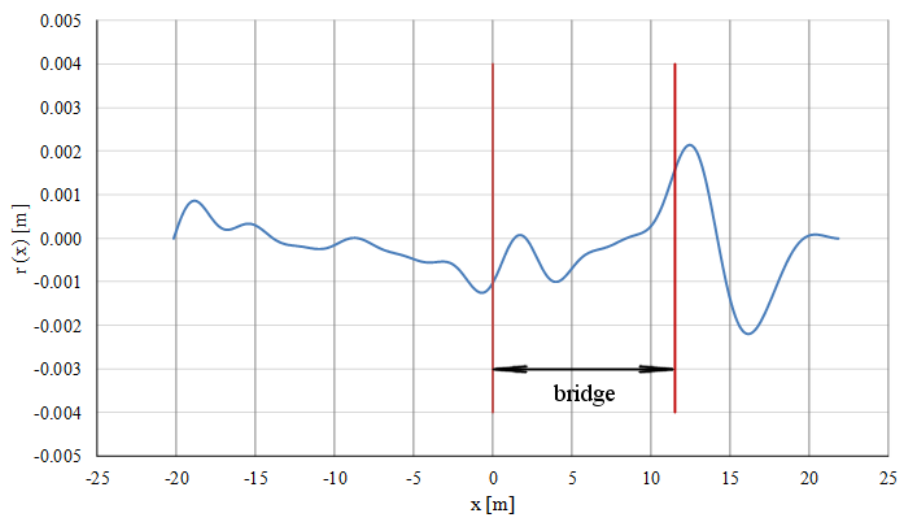
Interesting conclusions can be drawn when comparing the dynamic response obtained for a train running over a perfect track and when rail irregularities are taken into account. At mid span the results are very similar and only a slight increase is observed for the main resonant peaks and for lower train speeds. However, at the quarter span sections significant differences were detected. The significant increase of the dynamic amplification can be explained by the increase of the wheel-rail contact forces originated by the track irregularities. These impact forces excite the second mode shape of the bridge, particularly due to the bearing movement associated with it (see Figure 4.10b), thus originating an important amplification of the dynamic response in these sections. This leads to the conclusion that the behaviour of the quarter span sections is mostly controlled by impact forces that tend to grow proportionally with the train speed. The mid span section is controlled by resonant effects related to the first bending mode of the bridge and therefore is not significantly affected by the presence of track irregularities. In order to clarify this and guarantee that this was not observed only for that specific scenario, further tests were carried out using varying track irregularity profiles. Three different profiles were used and the results for each of them was once again compared to the perfect track scenario. The generated profiles are shown in Figure 4.25, and the obtained results are depicted in Figure 4.26.



a) Irregularities profile 1.

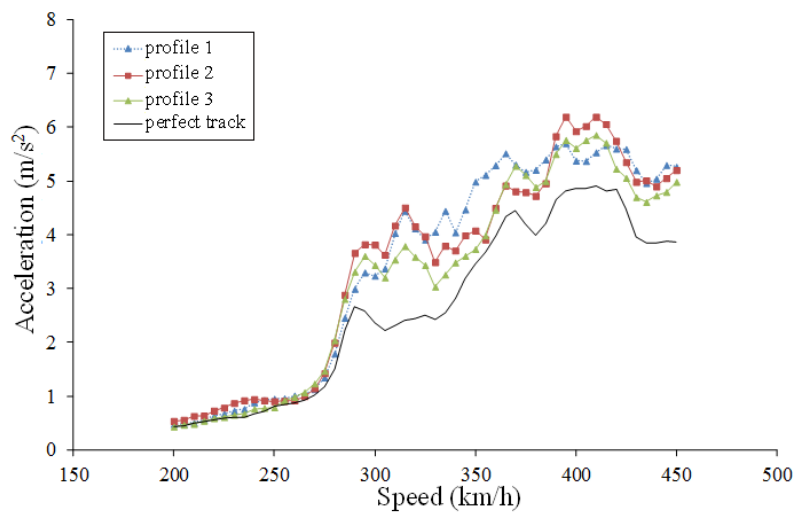


b) Irregularities profile 2.

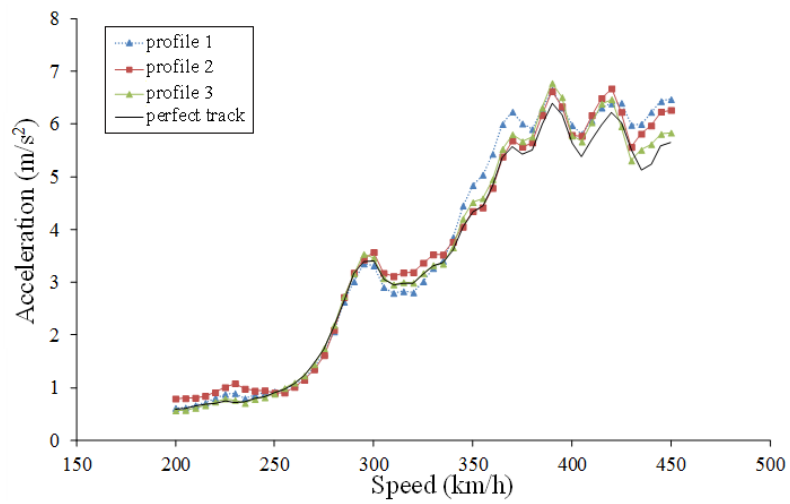


c) Irregularities profile 3.

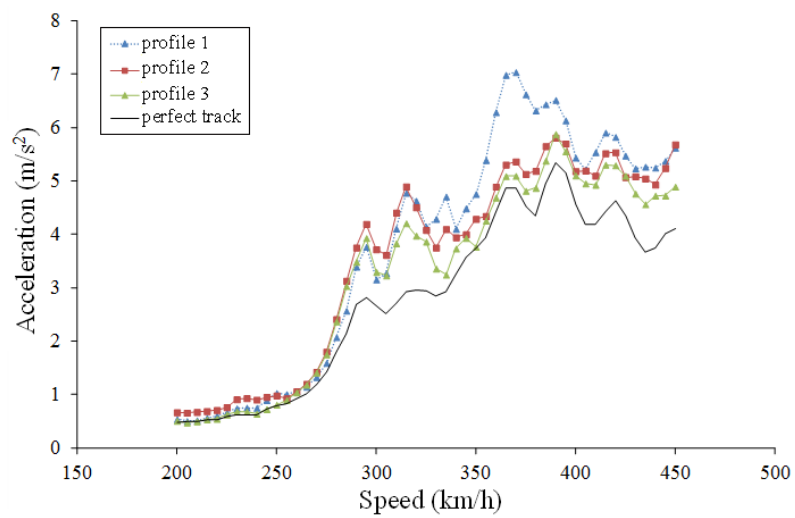
Figure 4.25 –Track irregularities profiles used on the parametric analyses.



a) $\frac{1}{4}$ span section.



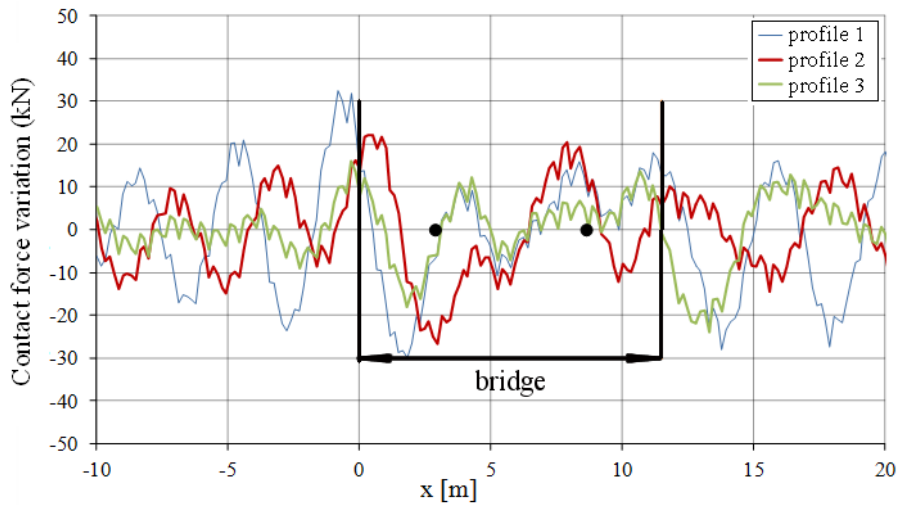
b) $\frac{1}{2}$ span section.



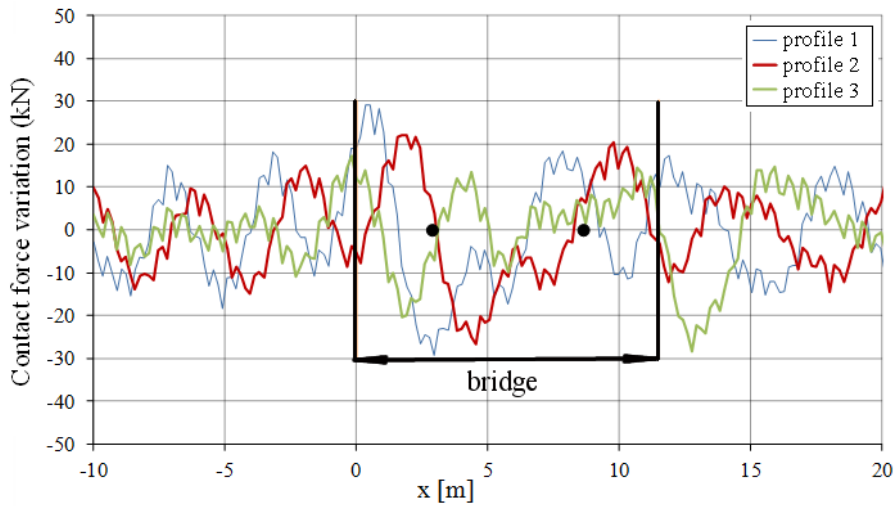
c) $\frac{3}{4}$ span section.

Figure 4.26 – Influence of the track irregularities profile on the dynamic response.

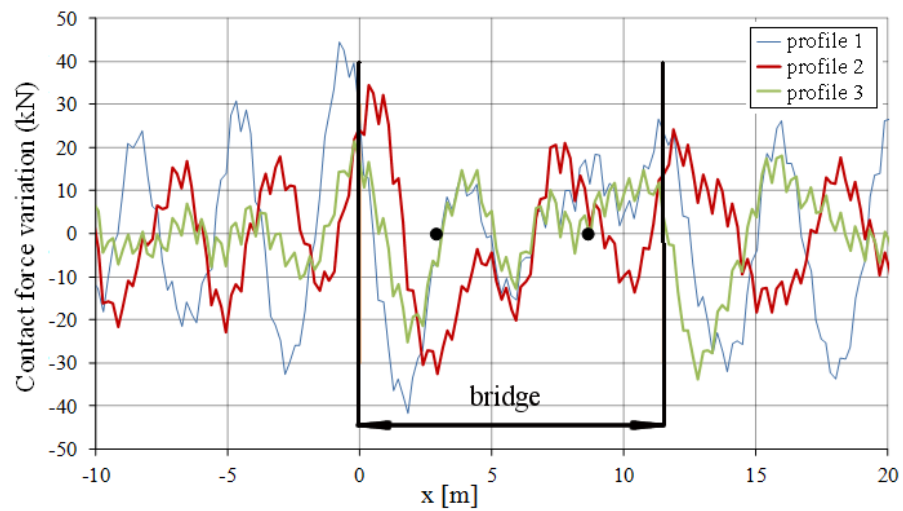
The results confirm what had been previously anticipated by the analysis of a single irregularities profile. It is interesting to observe the variation of the wheel-rail contact forces along the bridge complementing the information obtained from the analysis of the maximum bridge deck accelerations for different sections of the bridge. These results are illustrated for a single train wheel in Figure 4.27 for the different track irregularity profiles and for different train speeds. It should be noted that the black dots identify the quarter span sections.



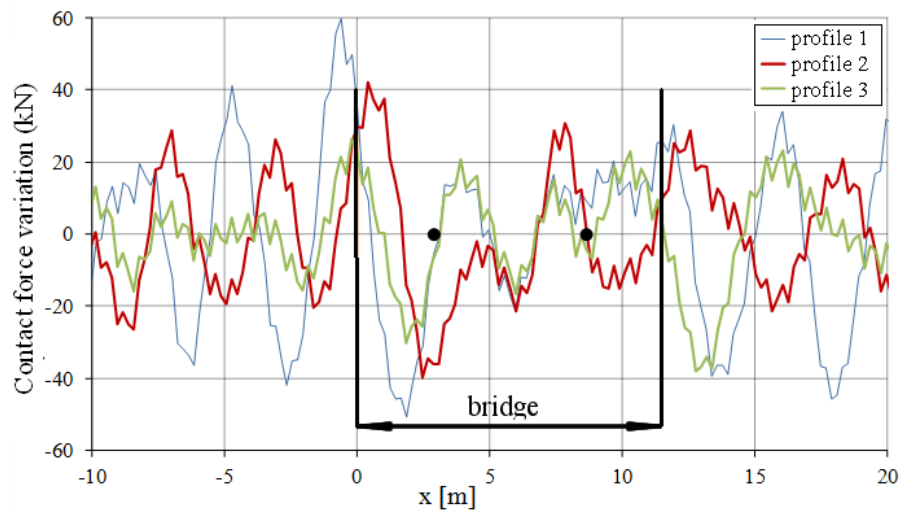
a) $v = 295$ km/h.



b) $v = 315$ km/h.



c) $v = 335$ km/h.



d) $v = 370$ km/h.

Figure 4.27 – Wheel-rail contact force variation.

The introduction of the track irregularity profiles induces significant changes to the wheel-rail interaction forces, as the dynamic response indicated. The different profiles do not show significant differences in terms of magnitude of variation of the contact force but do indicate the influence of the profile geometry. This leads to a very important conclusion that has not been clearly mentioned in previous studies. Several authors point out the importance of accounting for the train-bridge interaction effects specifically owing to the additional damping from the coupling between the train and the bridge. However, the existence of track irregularities significantly affects the wheel-rail contact forces which might induce a significant change in the

dynamic behaviour of the bridge. For bridges with relatively flexible supports (like the one selected as case study) this fact may be even more relevant than the change of methodology as it might alter the critical section, thus greatly affecting the safety assessment of the train-bridge system.

4.4.3 Modelling particularities

Besides the influence of the methodology used to assess the dynamic behaviour and the impact of accounting for track irregularities some other modelling aspects require a careful selection. In this section some of the most relevant aspects considered in the development of the numerical model are discussed.

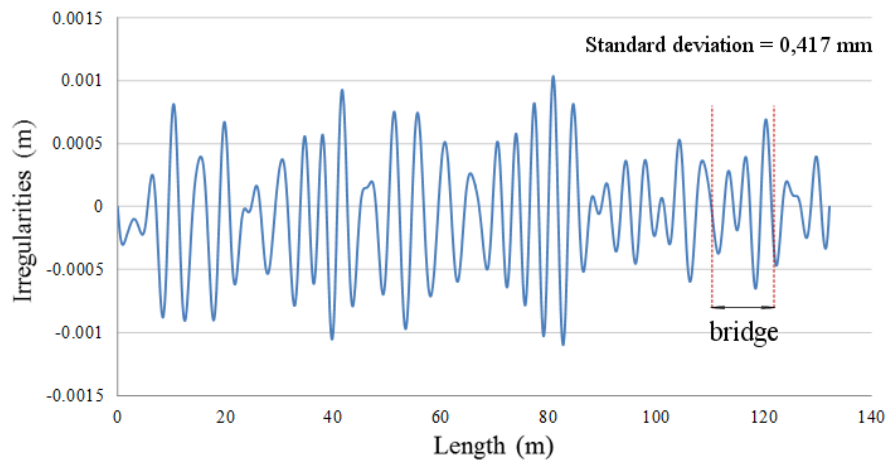
4.4.3.1 Influence of the track length before the bridge

As previously mentioned, the track was extended over the length of the bridge in the numerical model in order to reflect the rail continuity. This was considered in the simpler model and needed to be taken into account in the numerical model where the train-bridge interaction effects are taken into consideration. However, the length of track before the train entering the bridge is even more important in the latter case. An adequate length is required in order to guarantee that the wheel-rail contact forces on the bridge are realistic and to ensure that the effects of the sudden introduction of the vehicle (which are merely a numerical effect and are characterised by the introduction of impact forces) are stabilised and have been dissipated before the train enters the bridge in order to guarantee an accurate assessment of the dynamic behaviour.

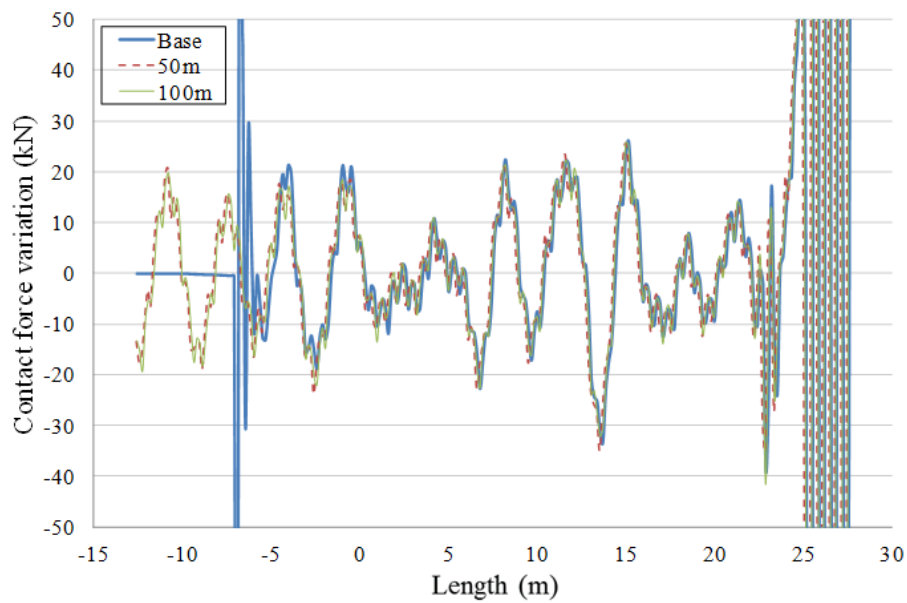
To analyse this, a parametric analysis was carried out. The aim of such an analysis is to define the optimum track length for an accurate assessment. Models with excessive track lengths result in a more time consuming model, which for the purpose of safety assessment analysis may have a more significant impact due to the number of simulations typically associated with such problems.

Initially, three scenarios were analysed: one with the same length as the simpler model (10.5 m), another with a track length of 60 m before the bridge and finally one with a track length of 110 m before the bridge, based on a suggestion by Ribeiro (2004). Three different irregularity profiles were analysed for each scenario but, for the sake of comparison, the geometry was

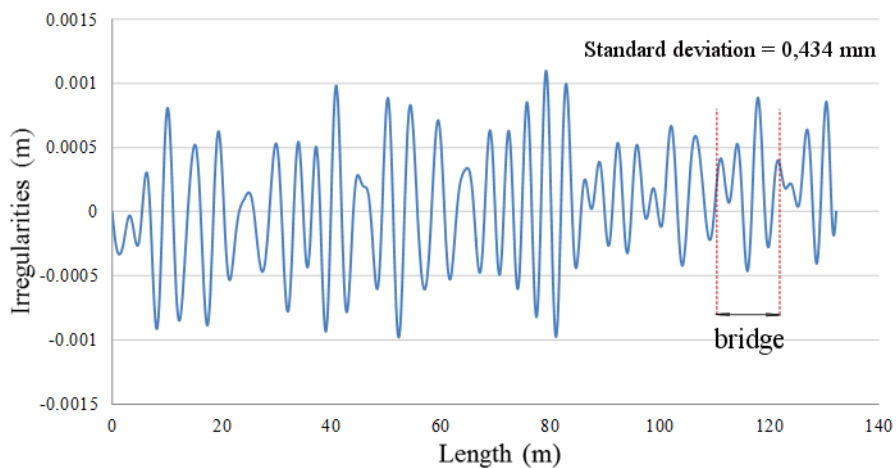
similar on the bridge for each simulation. The comparison is made in terms of wheel rail contact force over the structure, several train speeds are analysed and it should be noted that the results are presented in terms of distance, where the coordinate 0 m indicates the beginning of the bridge. The obtained results are shown in Figure 4.28.



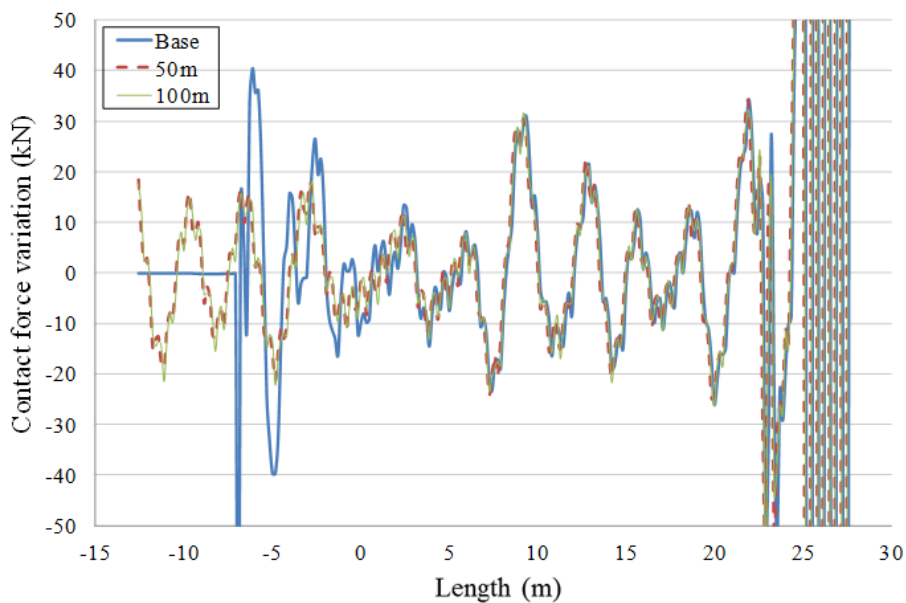
a) Irregularities profile 1.



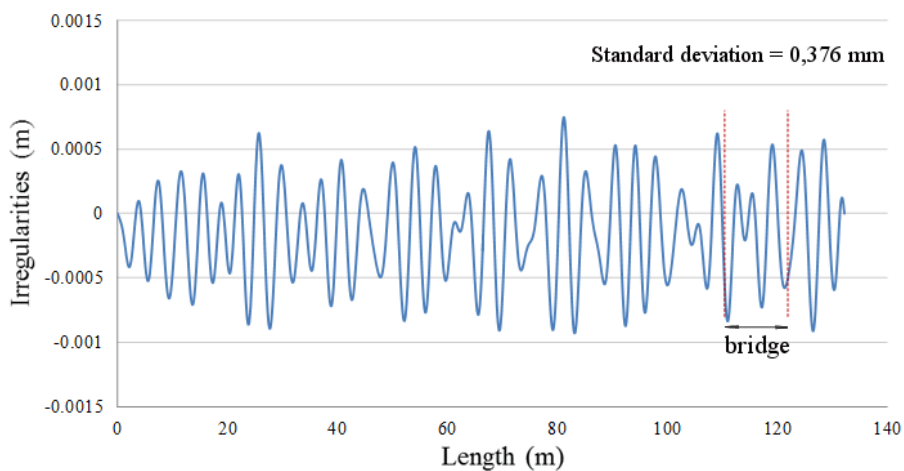
b) Wheel rail contact force for profile 1.



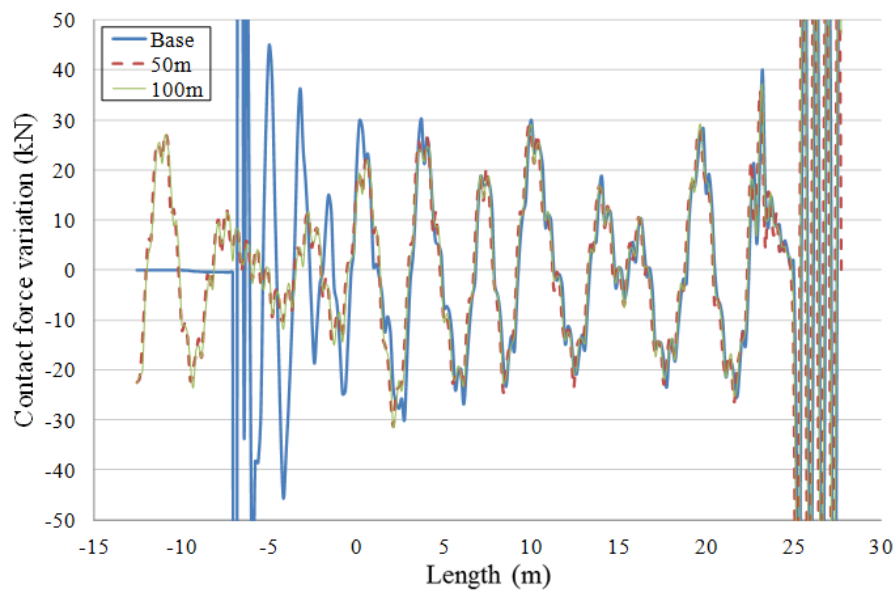
c) Irregularities profile 2.



d) Wheel rail contact force for profile 2.



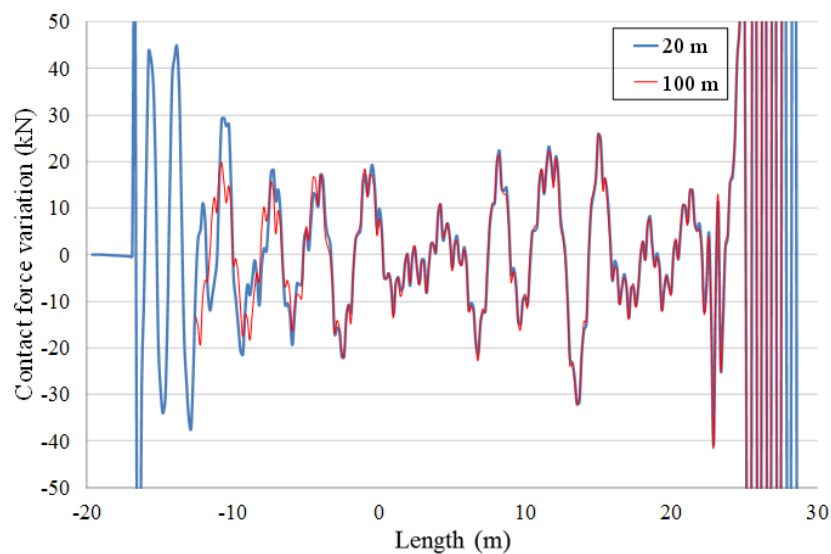
e) Irregularities profile 3.



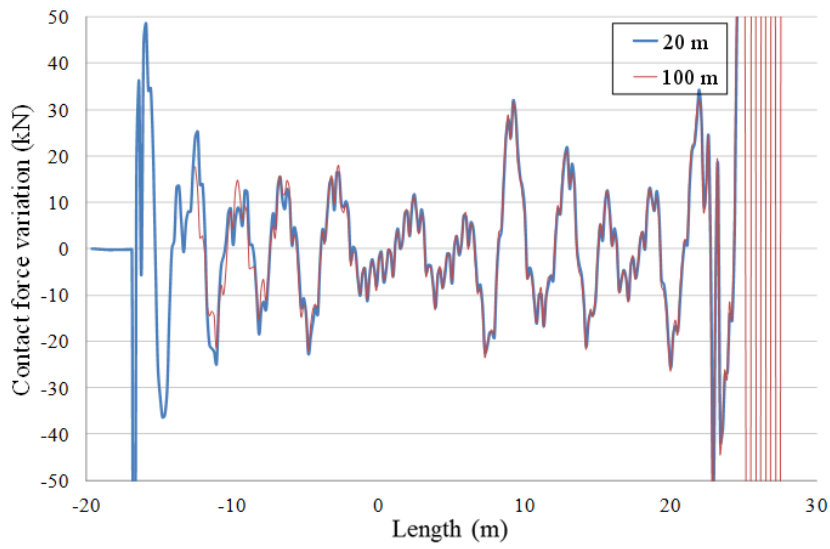
f) Wheel rail contact force for profile 3.

Figure 4.28 – Influence of the track length on the wheel-rail contact forces.

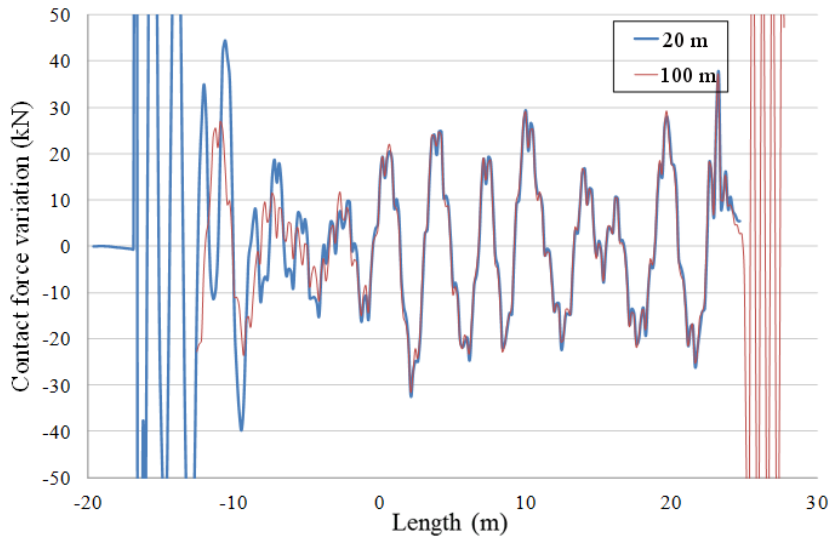
An almost perfect match can be observed for all the three track extensions 2.5 m after the train entered the bridge. This indicates that the 10.5 m length is not sufficient for the accuracy level that is intended. For this reason, a new analysis was carried out using a 20 m track extension. In this case, the results are only compared with the 110 m track extension scenario. The results are illustrated in Figure 4.29.



a) Profile 1.



b) Profile 2.



c) Profile 3.

Figure 4.29 – Assessment of the accuracy using a 20 m track length.

The results obtained using a 20 m track extension are very similar to those obtained using a 110 m extension. Therefore, this is the extension that was selected for the numerical model to be used when assessing the safety of the train-bridge system.

4.4.3.2 Validation of the track numerical model

As previously discussed in Section 2 and also on Section 4.3.1 several track models, with different degrees of complexity, can be found in the literature. The model chosen in this study was the two layer model, used by Calçada (1995). The reason to choose this model is that it is as accurate as other more complex models despite being simpler, which enables it to be more computationally efficient. In order to confirm this a comparison was made with the three layer model proposed by Zhai *et al* (2004). The obtained results are shown in Figure 4.30.

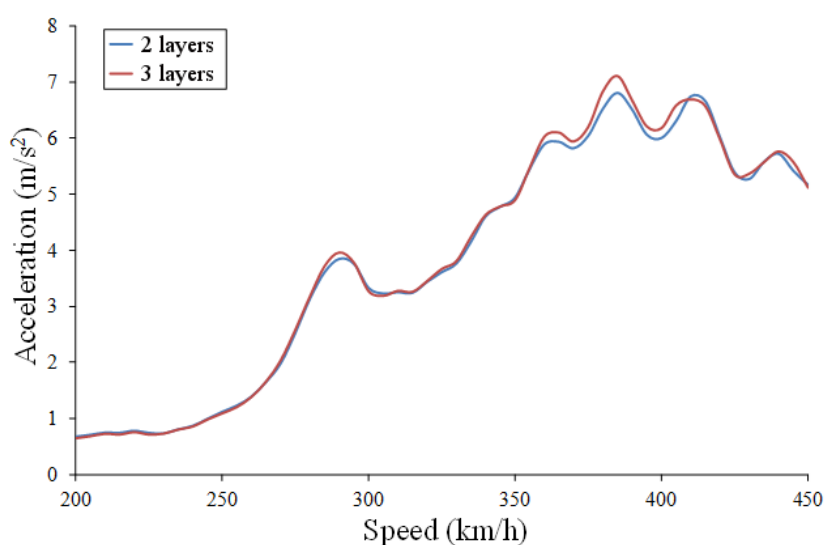


Figure 4.30 – Dynamic response: 2 layer models vs 3 layer model.

The results obtained by both models show a very good agreement with a slightly larger difference being notice for train speeds over 350 km/h. However, the observed differences were minor which validates the results obtained by the two layer model. For this reason, and for the purposes of the present study, the two layer model is considered to be as accurate as the three layer model.

4.4.3.3 Validation of the train numerical model

Another important aspect that can have a significant impact on both the accuracy of the obtained results and the computational time required for the analysis is the train model. As previously mentioned, the work carried out by Goicolea *et al* (2002) demonstrated that the main

effects of vehicle interaction with railway bridges can be adequately captured using simplified interaction models. The two simplifications mentioned in Section 4.3.5.6 were tested to maximise the efficiency of the dynamic analysis in terms of computational timing. In order to validate the simplified model the results obtained using these models were compared with those obtained with the complete model, which was taken as the reference value. The results showed that disregarding the effects of the secondary suspension system leads to some variation to the results obtained when using the complete model. However, the model which simulates the car bodies through lumped masses showed similar results to those obtained with the complete model, as shown in Figure 4.31.

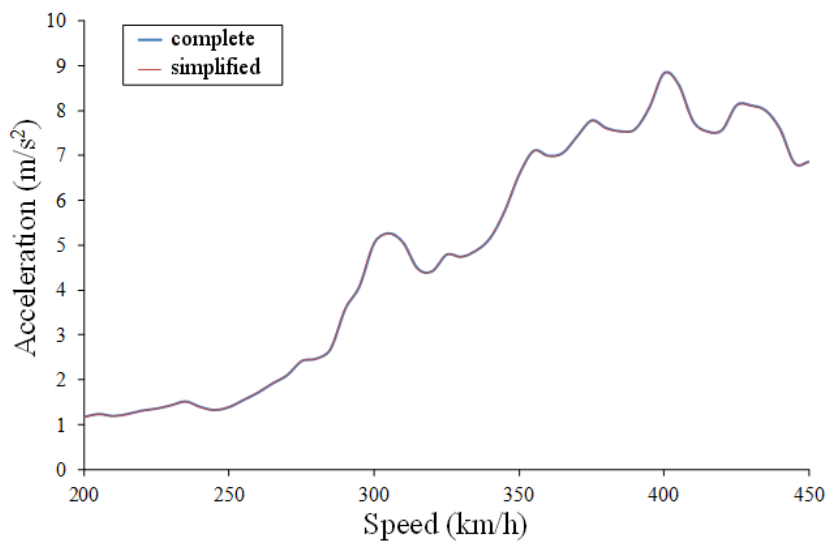


Figure 4.31 – Validation of the simplified train numerical model.

The results depicted in Figure 4.31 validate the use of the simplified model. Furthermore, and since the simplified model is 30% the size of the complete model, this large reduction in the total number of degrees of freedom resulted in a significant reduction of the required computational times thus making this model the most adequate to use in the safety assessment of the train-bridge system.

4.5 Variable screening procedure

In order to understand the influence that each of the basic random variables has on the dynamic behaviour of the train-bridge system, a variable screening procedure (based on a simplified sensitivity analysis) was performed. The goal of such procedure is to identify which variables are relevant to the dynamic response of the bridge and which ones are not, being therefore considered deterministic in the simulation analysis. It should be noted that in this work four distinct response parameters were analysed in the screening procedure: natural frequencies, displacements, accelerations and wheel-rail contact forces.

4.5.1 Description of the methodology

The procedure used on the present work is similar to the procedure used in Henriques (1998). The first stage of the procedure consists of selecting the random variables that allow the definition of the problem: the basic variables. Afterwards, a structural analysis is performed adopting the mean values for all the variables. Next, the bridge dynamic response is computed keeping all the variables with their mean values except for the ‘tested’ variable. The ‘tested’ variable value is modified from its mean value by two times the standard deviation. This step is repeated for all variables allowing for the evaluation of the sensitivity coefficients and importance indicators for each variable. The sensitivity coefficients and the importance indicators are obtained by comparing the difference between the reference results (corresponding to the analysis where all the variables are represented by mean values) and the results obtained for each ‘tested’ variable, as expressed in Equations (4.19) and (4.20), where y denotes the results, while x denotes the basic random variables:

$$b_k = \frac{(\Delta y_k / y_m)}{(\Delta x_k / x_{im})} = \frac{(\Delta y_k / y_m)}{(h \cdot \sigma_{xk} / x_m)} \quad (4.19)$$

$$(b\sigma)_k = b_k \cdot CV \quad (4.20)$$

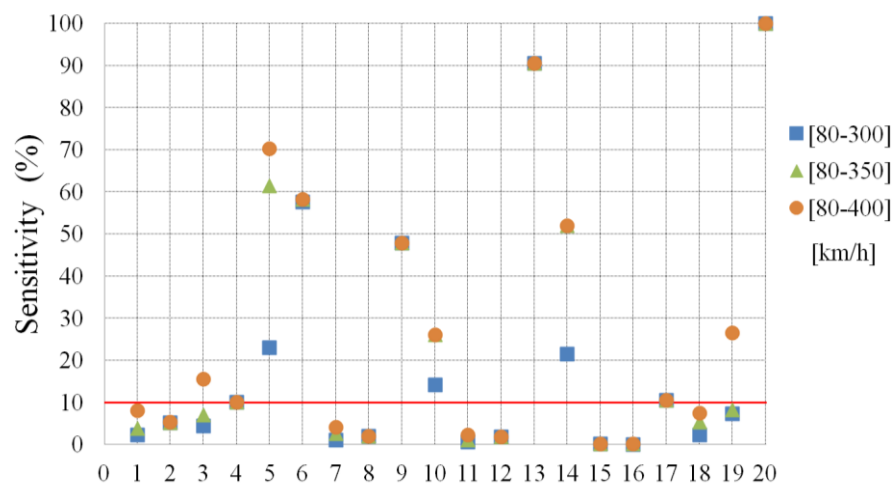
where σ_{xk} is the standard deviation of each variable, CV is the coefficient of variation for each variable and h is a coefficient related with the variation of the mean value of each variable (in this dissertation $h = \text{constant} = 2$). y_m and x_{im} are the reference value of the structural response and the mean value of the ‘tested’ variable, respectively, Δy_k is the difference between the reference results of structural response and the results obtained for each ‘tested’ variable and Δx_{ik} is the difference between the mean value of the ‘tested’ variable and the value used in the analysis.

After analysing these coefficients for all variables, the maximum importance indicator is determined and the relative importance of each variable is established by comparison with the maximum value obtained.

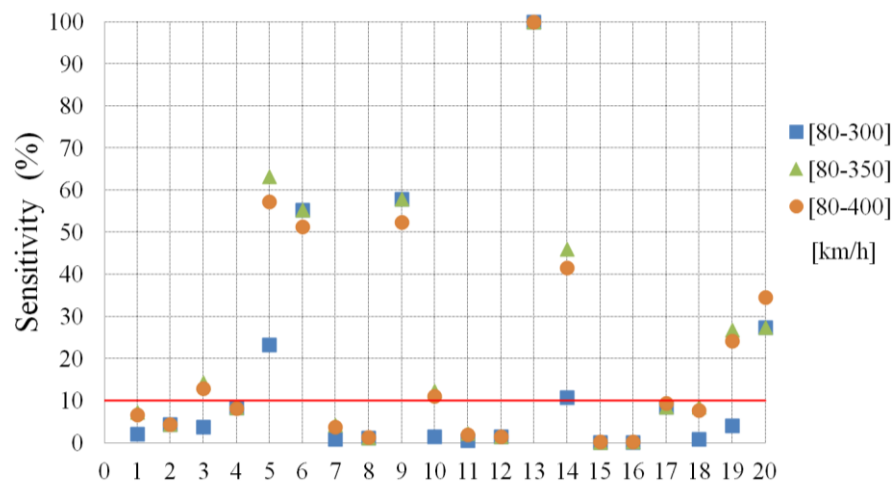
4.5.2 Preliminary approach

At an initial stage of this work the random variables analysed were only related to the bridge parameters, as indicated in Table 4.3. The analysis had two essential goals: the first (and most important) one was the identification of the parameters that displayed a higher influence on the dynamic response; the second was to understand how the number of variables influenced the safety assessment procedure. For this reason, at the preliminary analysis, variables with a relevance smaller than a pre-established value (a 10% value was defined) are considered irrelevant for the variability of the dynamic response of the bridge. In the safety assessment these are taken as deterministic and their mean value is used for the simulation. It should also be pointed out that four distinct response parameters were analysed in the screening procedure: natural frequencies, displacements, accelerations and reactions.

Due to the importance of the train speed on the dynamic response of the bridge, the sensitivity analysis was performed for several train speed ranging from 80 km/h to 450 km/h (speed increased in steps of 5 km/h). Some of the results obtained from the variable screening procedure can be seen in Figure 4.32.



a) Mid-span bridge deck acceleration



b) Mid-span bridge deck displacement

Figure 4.32 – Results of the sensitivity analysis.

For a better readability, only part of the results are presented, allowing for an understanding of the variable selection criteria. It should also be pointed out that despite considering only 10 variables, both the upper and lower bound were analysed, leading to the 20 data sets presented in Figure 4.32, and the variable associated to the reference number in the picture is indicated in Table 4.3. When analysing all four response parameters previously referred, some variables stand out in terms of their influence on the dynamic response of the bridge. Among these variables are the inertia variation due to the variation of the section height, the ballast area, the elasticity modulus of the concrete and the vertical stiffness of the bridge bearings. The variables with less influence on the response (and which will hereafter be considered as deterministic) were the

geometric variation of the rolled steel profiles, the horizontal stiffness of the bridge bearings and the variation of the self-weight of the concrete elements due to the geometrical variations. However, since the last variable cannot be dissociated from the inertia variation, which has a significant relevance on the dynamic response, it was kept as random.

4.5.3 Sensitivity analysis accounting for the train-bridge interaction

At a subsequent stage of this work a similar procedure was carried out whilst accounting for the variability of parameters of the structure, the track and the train. In this stage the train-bridge interaction effects were taken into consideration and this represents a more realistic analysis.

In this stage two particular aspects of the response were analysed: the bridge deck acceleration levels and the wheel unloading rate. These were selected because they are two of the most significant indicators of the running safety of high-speed trains. Unlike what was done in the previous analysis, the sensitivity analysis was limited to adequately selected speed ranges. In the case of the bridge deck acceleration the analysis was limited to the [270 – 300] km/h speed ranges, as this corresponds to the range where the resonant effects are most significant. As for the analysis of the wheel unloading rate, the sensitivity analysis was carried out for the [405 – 435] km/h speed range, as this is the range where the first cases of loss of contact between the wheel and the rail were observed.

4.5.3.1 Bridge deck acceleration

Having already analysed the most significant variables that affect the bridge deck acceleration, it is important to check if adopting a different type of methodology (train bridge interaction *vs* moving loads) leads to different conclusions. The obtained results of the sensitivity analysis for the bridge deck acceleration when accounting for the train-bridge interaction effects are illustrated in Figure 4.33.

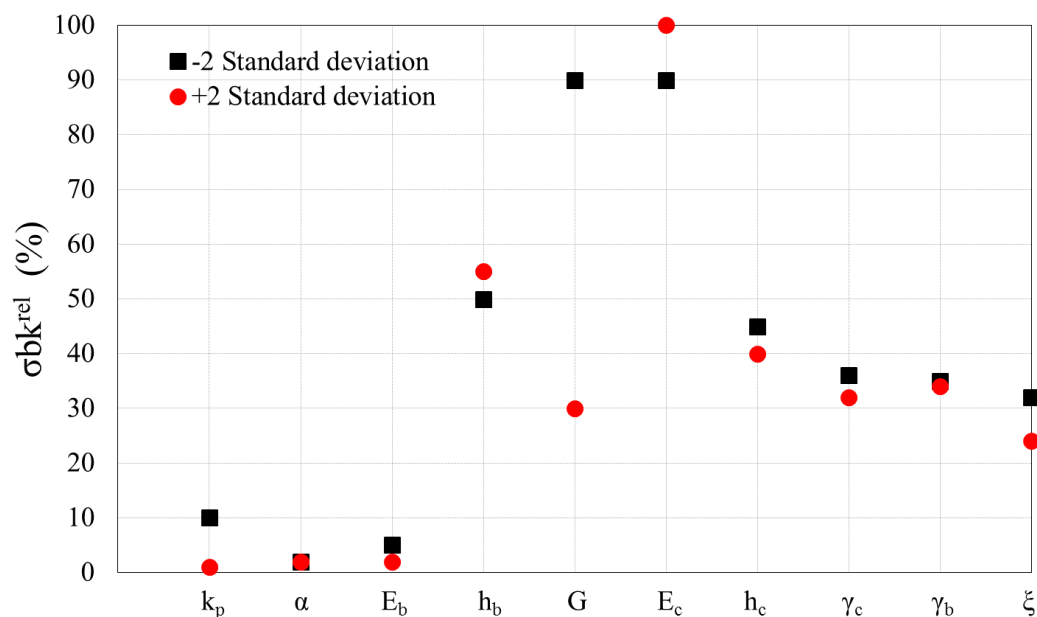


Figure 4.33 – Bridge deck acceleration sensitivity analysis results.

It can be observed that the results are very similar to those obtained in the preliminary stage. As expected, this indicates that the variables identified as key remain the same and that the interaction effects do not introduce any significant changes in the influence displayed by the variables. However, since there are a few new variables that have been introduced in this analysis (and others have been defined in a slightly different manner), the obtained results need some discussion for a better interpretation.

It can be concluded that the variables with a higher influence on the bridge deck acceleration are all structure-related. The variable that displays the highest influence is the elasticity modulus of the concrete, E_c . Other significant variables are the neoprene shear modulus, G , the height of the ballast layer, h_b (which affects the overall weight, and consequently the natural frequency) and of the concrete elements, h_c (which also affects the overall stiffness) and the density weight of both the concrete, γ_c , and the ballast, γ_b . Structural damping, ξ , which was only introduced as a variable in this stage, displays a slightly lower influence than the previous variables but is still significant, with a relative importance of around 30 %. It was also possible to conclude that the track-related variables, in particular those that do not affect the overall mass of the structure, display a very low influence on the bridge deck acceleration.

4.5.3.2 Wheel unloading rate

The results of the sensitivity analysis for the wheel unloading rate are shown in Figure 4.34.

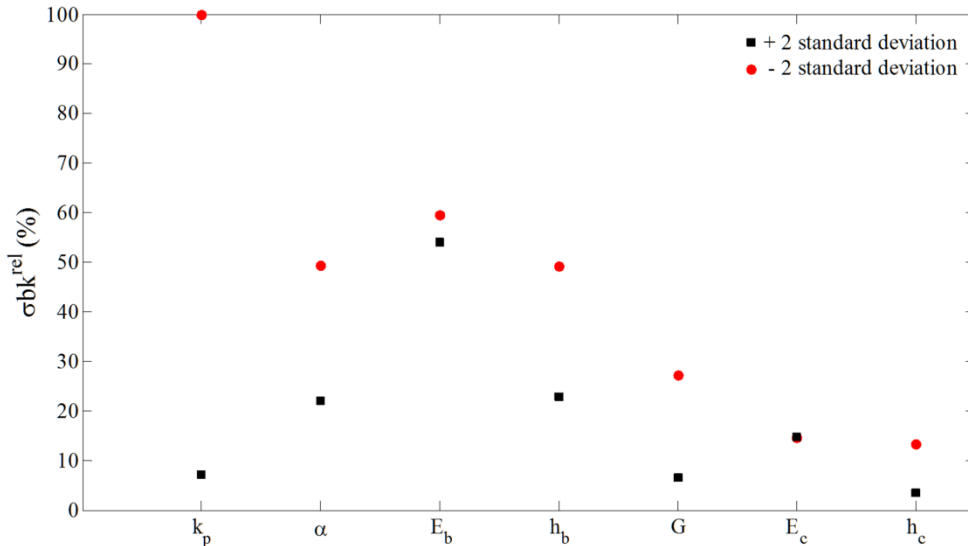


Figure 4.34 – Wheel unloading rate sensitivity analysis results.

With respect to the wheel unloading rate, it can be observed that this parameter is principally affected by track-related variables rather than by the bridge-related variables. The stiffness of the rail pads, k_p , is the variable that most affects the wheel unloading. The ballast elasticity modulus, E_b , the height of the ballast layer, h_b , and the ballast load distribution angle, α , (which are all variables that influence the way that the wheel loads are transmitted to the track) were other track-related parameters that showed a significant influence on the wheel unloading. The bridge-related variables also have some significance, however, compared to the influence of the track-related variables their contribution is significantly lower. The neoprene shear modulus, G , which influences the vertical stiffness of the bridge bearings, was the bridge-related parameter with the highest influence on the wheel unloading rate. The concrete elasticity modulus, E_c , and the variation of inertia due to geometric variation of the concrete section (both related with the stiffness of the bridge) are other variables with some degree of influence on the wheel unloading. The sensitivity analysis results show that the wheel unloading rate is very sensitive to variations of the track properties, as expected.

4.6 Concluding remarks

The case study used to evaluate the efficiency, robustness and accuracy of the proposed probabilistic methodologies was presented in this chapter, along with the numerical models developed to assess the dynamic response. The influence of the dynamic analysis method was also analysed and comparisons were made to understand how accounting for the train-bridge interaction effects and the presence of track irregularities as opposed to the moving loads method or perfect track conditions affect the response of the train-bridge system.

Several modelling aspects were analysed in detail in order to validate the selected models and guarantee that the obtained results are accurate. One of these aspects was the random generation of track irregularities, which included a discussion of the adopted methodology and the validation method used. The track length before the bridge was another aspect that was thoroughly discussed due to its importance for both an accurate assessment of the dynamic response of the train-bridge system and its significant influence on the efficiency of the dynamic analysis. Another relevant aspect is the track numerical model used in the dynamic analysis and the importance of accounting for the shear behaviour of the ballast layer was also demonstrated. Finally, another important feature of the case study bridge was the flexibility of its bearings, which were shown to have a crucial role in the dynamic response of the train-bridge system and demonstrate a key influence on the safety assessment.

The last section of this Chapter was dedicated to a sensitivity analysis, which aimed to provide a better understanding of how accounting for the variability of some parameters affects the dynamic response. The identification of the influence that each of the selected basic random variables has on the dynamic behaviour of the train-bridge was another of the goals of such an analysis. It is possible to conclude that, as expected, the bridge-related variables display a higher influence on the dynamic response of the structure (both for accelerations and displacements), whereas the track-related variables show a higher impact on the wheel unloading rate. The most significant parameters of the bridge dynamic response are the section inertia, the area of the ballast layer (with influences the structural mass), the concrete elasticity modulus and the vertical stiffness of the bridge bearings. With respect to the wheel unloading rate, it could be concluded that the stiffness of the rail pads, the ballast elasticity modulus, the height of the ballast layer and the ballast load distribution angle (which are variables influencing the way that the wheel loads are transmitted to the track) are the governing parameters.

Chapter 5

Track stability assessment

5.1 Introduction

In this Chapter the assessment of the train-bridge system due to track instability is discussed. Following a brief explanation of the safety criterion used to assess track stability, the safety assessment is carried out using the methodologies introduced in Chapter 3. Through the application of the different methodologies to this case study it will be possible to understand how efficient each of them is, what the main aspects that influence both accuracy and efficiency for each method are and finally how they compare in terms of both these indicators.

Two different analyses are carried out and their results are discussed and analysed. At an initial stage the assessment of the train-bridge system is analysed using the moving loads method. This corresponded to the first stage of the work and is divided into a preliminary assessment and a refined analysis to assess the safety [Rocha *et al* (2012)]. At this stage only the variability of the bridge related parameters is taken into consideration and the analysis is limited to the application of the tail modelling approach to estimate the probability of failure of the train-bridge system.

The second part of the Chapter corresponds to the analysis of the safety assessment when accounting for the train-bridge interaction effects, including the variability of parameters related to the bridge, the track and the train were taken into account along with the existence of track irregularities [Rocha *et al* (2014); Rocha *et al* (2015)]. The differences to the previous

stage are highlighted and safety assessment results are analysed and compared for two distinct methodologies: the tail modelling approach and the enhanced simulation approach. The results are compared in terms of accuracy and efficiency and conclusions are drawn with respect to the advantages and drawbacks of each methodology along with the results obtained for each method.

5.2 Safety criterion

The track stability is governed by the vertical acceleration of the bridge deck. As discussed in Section 2.4.2.1 several studies have been carried out in order to establish the vibration levels that originate the instability of the ballast layer. This phenomenon leads to the loss of interlock between ballast grains which results in the loss of the lateral resistance of the track, affecting the running safety of the trains.

The current European standard EN1990-A2 (2005) limits the maximum peak values of the bridge deck acceleration to 3.5 m/s^2 for ballasted track and to 5 m/s^2 for ballastless tracks. This standard also defines that if a dynamic analysis is required to determine the acceleration level of the bridge both real trains and HSLM should be analysed. Frequencies up to 30 Hz or 1.5 times the value of the frequency of the fundamental mode of vibration should be included in the analysis, guaranteeing a minimum of three vibration modes.

However, recent studies carried out by Zacher & Baeßler (2009) (detailed in Section 2.4.2.1) showed that the lateral resistance of the sleeper is only significantly affected when the acceleration level exceeds 7 m/s^2 . These results indicate that the acceleration limit suggested in the European standards to ballasted tracks results from the application of a safety factor of 2. Since this criterion is often the most restrictive aspect of the dynamic response of short to medium span railway bridges, Zacher & Baeßler (2009) propose a reduction of the safety factor from 2 to 1.3, which would translate into limiting the deck acceleration on ballasted tracks to 5.5 m/s^2 and to 7.5 m/s^2 in ballastless tracks.

Another study carried out by Norris (2005) observed that even if some small and localised areas of the deck have very high acceleration levels the ballast layer remains stable as the adjacent ballast provides confinement and prevents local instability. Based on these findings Norris proposed that the deck acceleration limit should be changed to 5 m/s^2 for most of

ballasted bridges and this value could even be increased to 6 m/s^2 for bridges with higher structural damping.

Taking all of this into consideration and bearing in mind that the variability of the several parameters that govern the train-bridge dynamic response are taken as random variables in this dissertation, it was decided not to apply any safety factor to the verification of the track stability. Therefore, failure is assumed when the vertical acceleration of the deck reaches the 7.0 m/s^2 limit. The probability of failure of the train-bridge system is estimated by assessing the probability of the bridge deck reaching this acceleration level.

5.3 Safety assessment – results and discussion

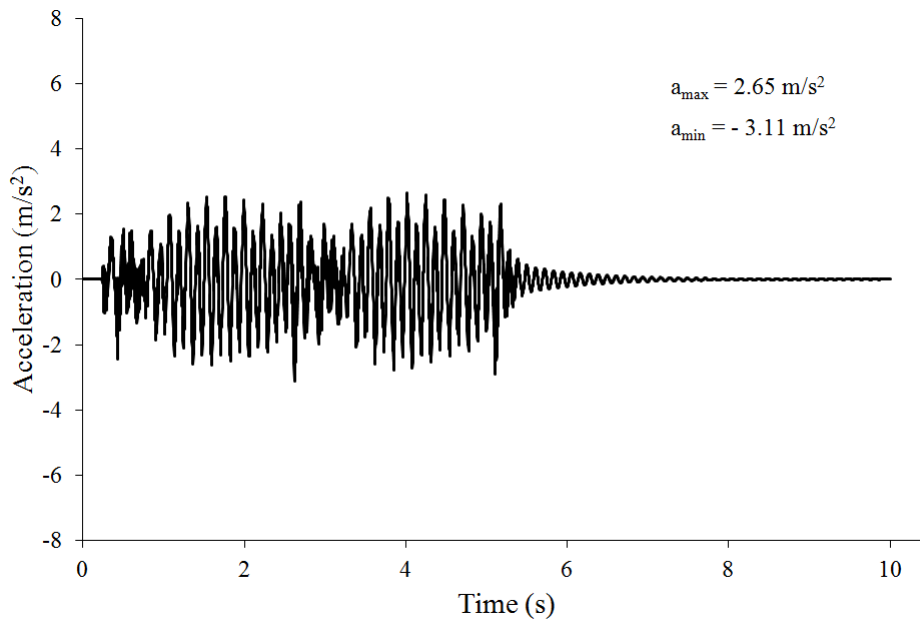
In this section the results from the safety assessment are presented and discussed. Initially, the results obtained from the simplified approach, using the moving loads method, are presented. Subsequently, the results from the analysis that accounts for the train-bridge interaction effects are presented along with the different techniques that have been employed to estimate the probability of failure of the train-bridge system.

5.3.1 Simplified approach – moving loads

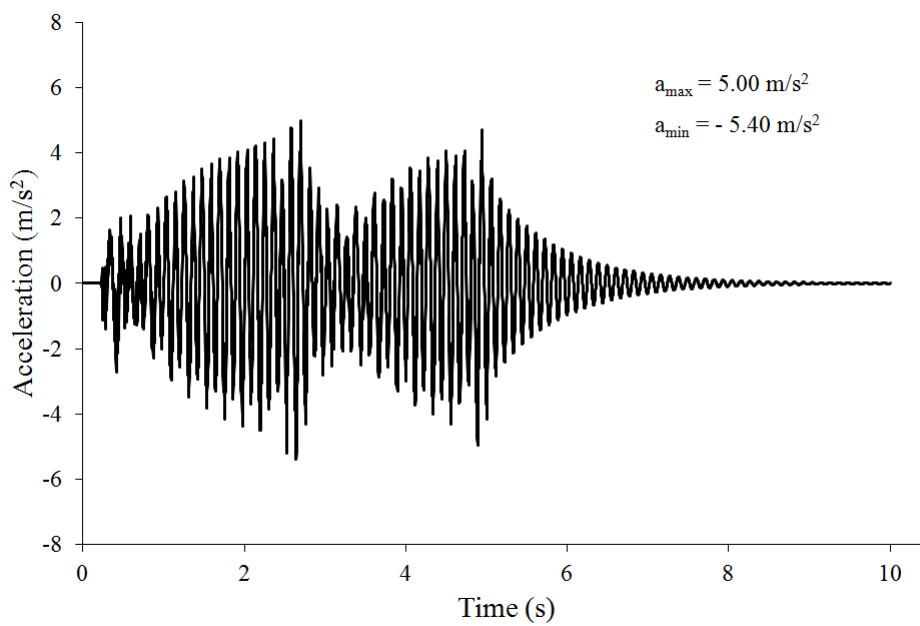
In this preliminary stage the dynamic behaviour of the bridge was analysed for the crossing of the TGV Double train with a speed ranging from 200 km/h to 450 km/h, increasing in steps of 5 km/h. The track-bridge load model used is the one presented in Section 4.3.1 and the moving loads model described in Section 4.4.5.1 is selected to assess the dynamic behaviour.

5.3.1.1 Dynamic response for a deterministic scenario

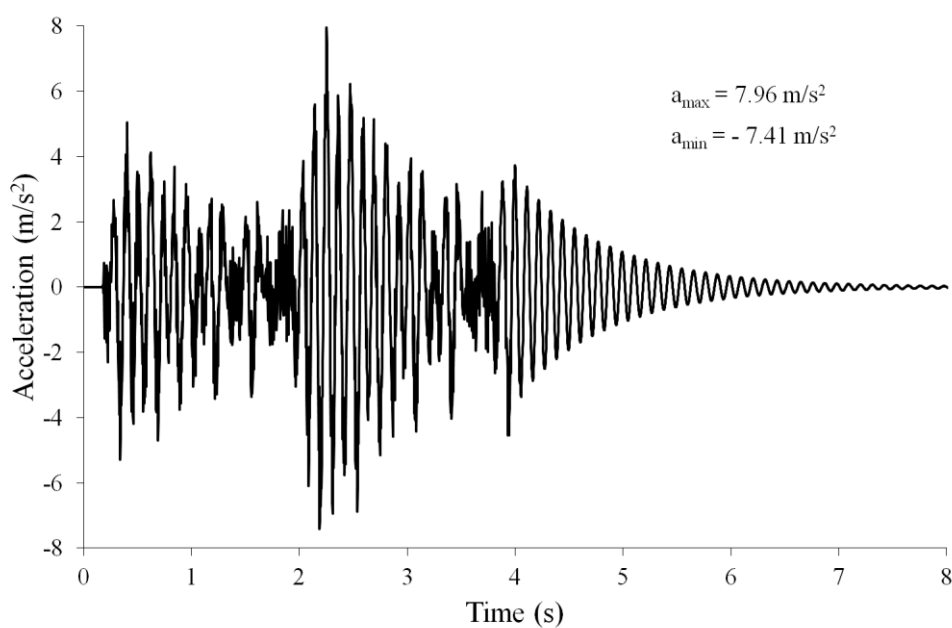
First, the dynamic response of the bridge was analysed, considering that all random variables (defined in Table 4.3) take their mean value and the train that crosses the bridge is characterised according to the moving load model described in Section 4.4.5.1. This helps to understand the dynamic behaviour of the train-bridge system and is useful for the selection of the most adequate type of analysis to be made for the safety assessment. The distinct bridge dynamic responses for different train speeds are illustrated in Figure 5.1.



a) 290 km/h



b) 305 km/h



c) 390 km/h

Figure 5.1 – Bridge dynamic response at mid-span considering a moving loads scenario.

Figure 5.1a illustrates the response for a non-resonant speed, Figure 5.1b shows the obtained response for a resonant speed and in Figure 5.1c an impact response, due to the train speed, can be seen. Although the speed values considered in Figure 5.1a and Figure 5.1b differ only by 15 km/h (290 and 305 km/h, respectively) the difference between their responses is quite visible and reflects how resonance drastically effects affect structural behaviour.

An overview of maximum absolute values of acceleration at mid-span of the bridge for increasing train speeds is provided in Figure 5.2.

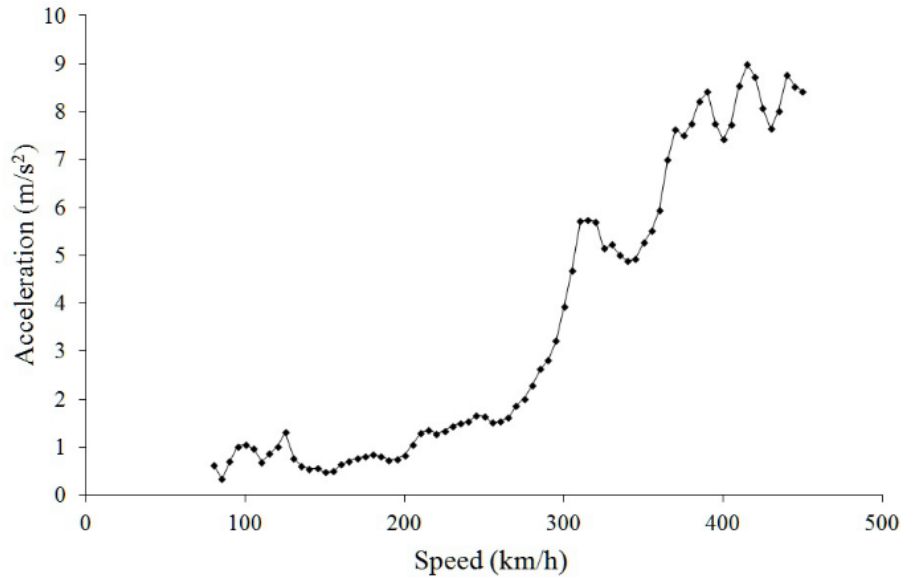


Figure 5.2 – Maximum acceleration values at mid-span of the bridge for a moving loads analysis.

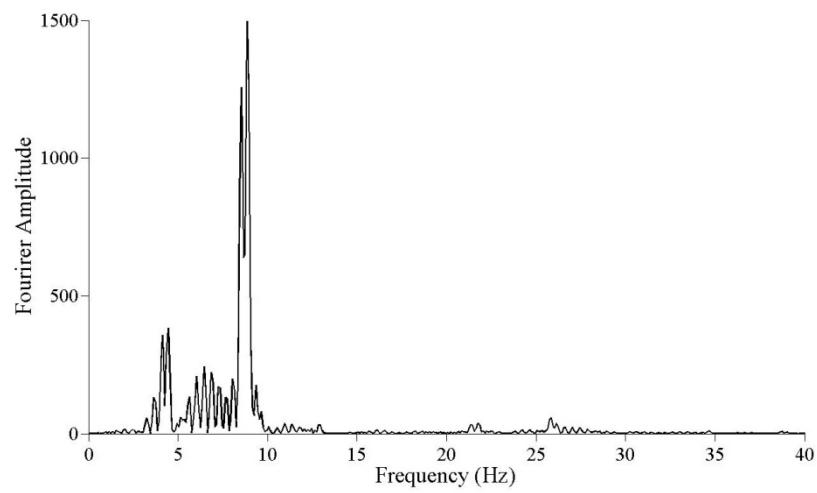
Resonant acceleration peaks can be observed around 310 km/h and 385 km/h. An estimate of resonant speeds can be given by:

$$\mathbf{v}_{res} = n_j \cdot \frac{d}{i} \quad (5.1)$$

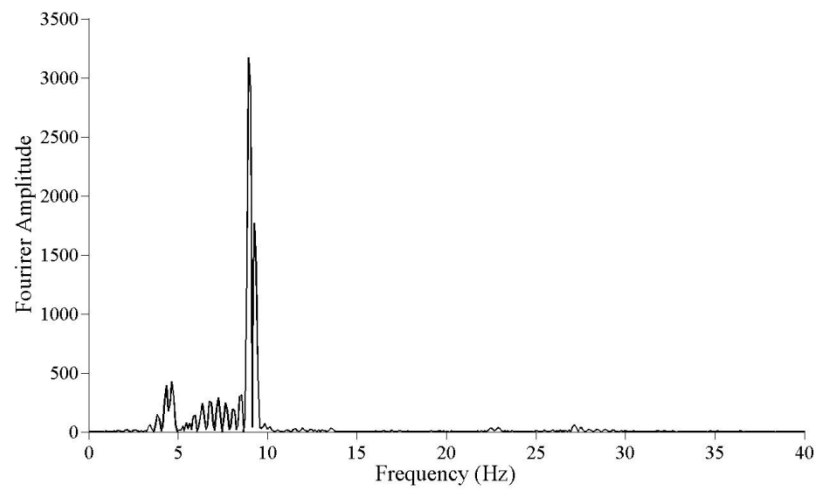
where n_j is a natural frequency of the bridge, d is the regular spacing of groups of axles and i takes the values 1, 2, 3, etc.

Since the first natural frequency of the bridge is 9.168 Hz and the axle distance, d , is 18.7 m, the first expected resonant speed, which occurs taking i equal to 2, is around 308 km/h, which is in compliance with the observed acceleration peaks. The observed behaviour for speeds near 400 km/h is more related with speed effects than with resonance effects. The observed peaks are connected with the second and higher bending modes and for i values over 5. This explains the relatively small size of the observed peaks.

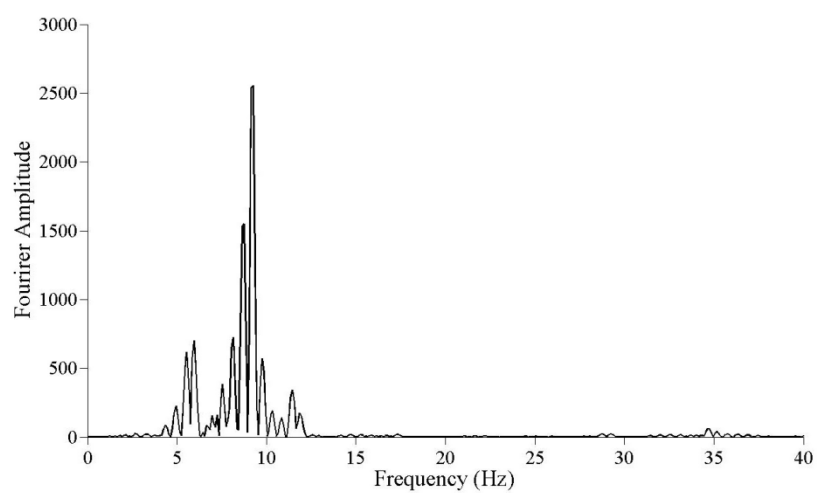
In order to provide a more comprehensive analysis of the dynamic behaviour for different train speeds the FFT response was plotted. These results are shown in Figure 5.3.



a) 290 km/h



b) 305 km/h



c) 390 km/h

Figure 5.3 – Maximum acceleration values at mid-span of the bridge.

Figure 5.3a illustrates a non-resonant response whereas Figure 5.3b shows a typical result for a resonant speed, with the peak located close to 9 Hz, which is the fundamental frequency of the bridge. In the case of Figure 5.3c the FFT results indicate that more frequencies are contributing to the structural vibration, demonstrating that the response is not exclusively due to the fundamental frequency.

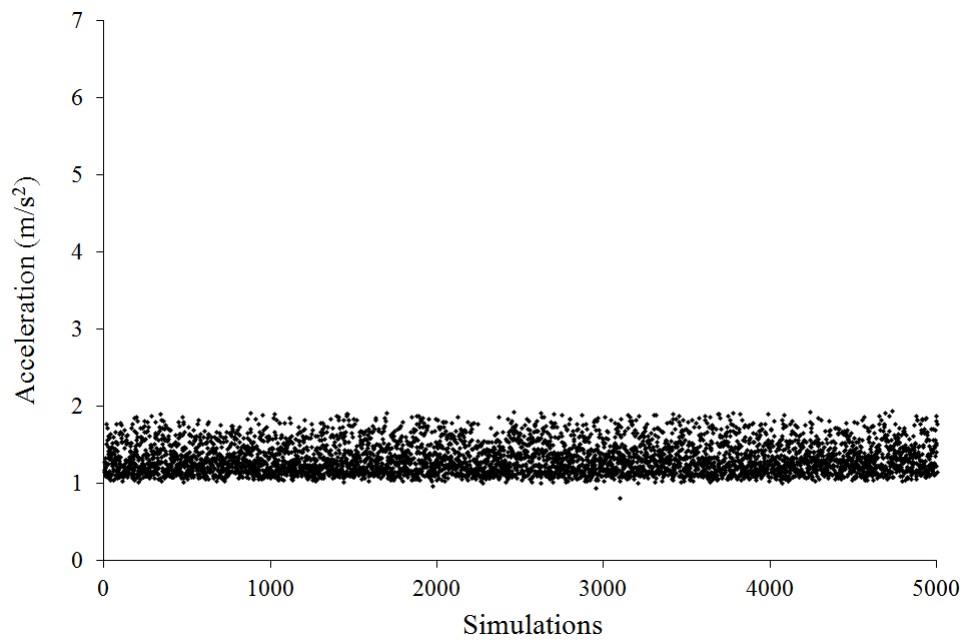
5.3.1.2 Preliminary simulation analysis

Initially, and to provide a better understanding of the problem to be studied, both the Monte Carlo simulation method and the Latin Hypercube simulation were used to provide an overview of the type of response expected. To accomplish this, four distinct sample sizes were used for the Monte Carlo simulation with 5,000, 2,500, 2,000 and 1,000 simulations, respectively. For the case of the Latin Hypercube simulation only two scenarios were analysed, with samples of 500 and 250 simulations. The option for these sample sizes was understood to be sufficiently representative for the estimation of the required safety level within ultimate limit states.

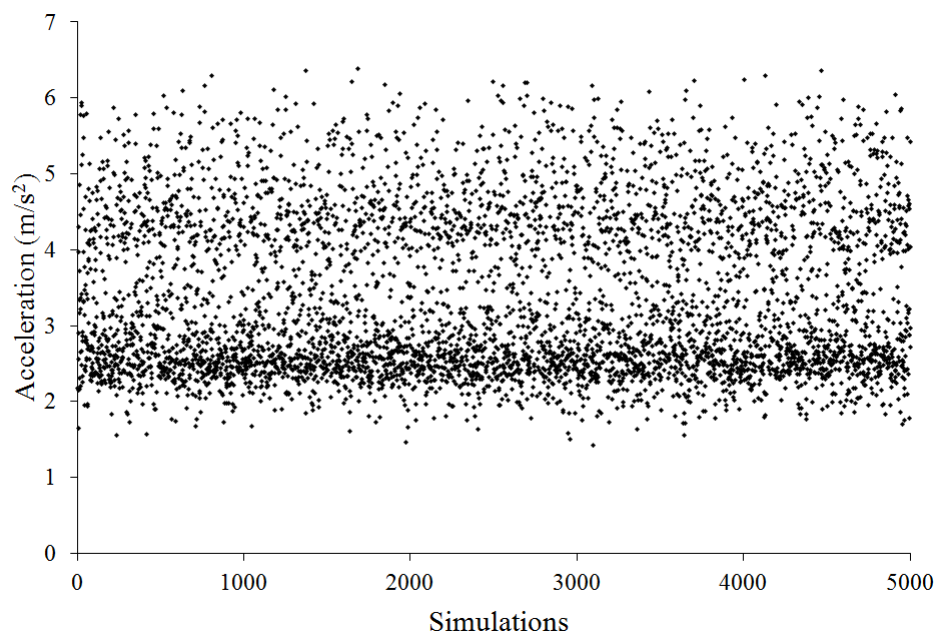
As previously mentioned the dynamic analyses were performed using the FEMIX software with the formulation presented in Neves *et al* (2012), which for a time step of 0.002 s and a total of 5,000 steps, representing a 10 s analysis, taking roughly 30 s to be completed on a computer with an Intel Core i7 920 running at 2.67 GHz.

An example of the results obtained from the simulations is shown in Figure 5.4. The results represent the scenario of 5,000 Monte Carlo simulations and are representative of the results obtained for the other simulation scenarios.

Figure 5.4 shows two of the different scenarios observed in the dynamic response of the bridge. Figure 5.4a shows the results obtained for a non-resonant speed, where small variations of the acceleration values can be observed. On the other hand, in Figure 5.4b a larger scatter of acceleration values can be observed. This is due to the existence of two distinct types of dynamic response: some cases where bridges are undergoing resonant effects, which drastically increases the acceleration values, whereas others are not in resonance and, consequently, the acceleration values are much lower. To make these results even clearer the peak mid-span accelerations for each of the 5,000 Monte Carlo simulations are presented as a histogram in Figure 5.5 for different train speeds.

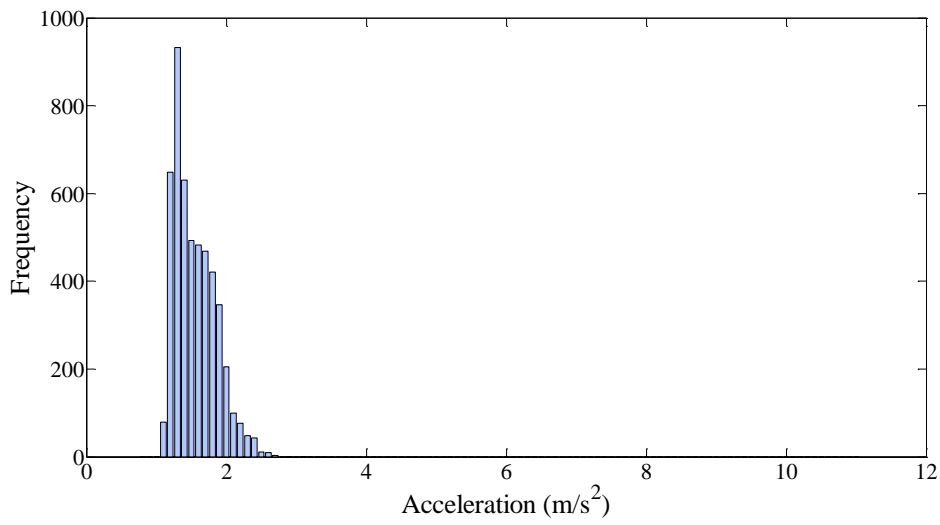


a) 220 km/h

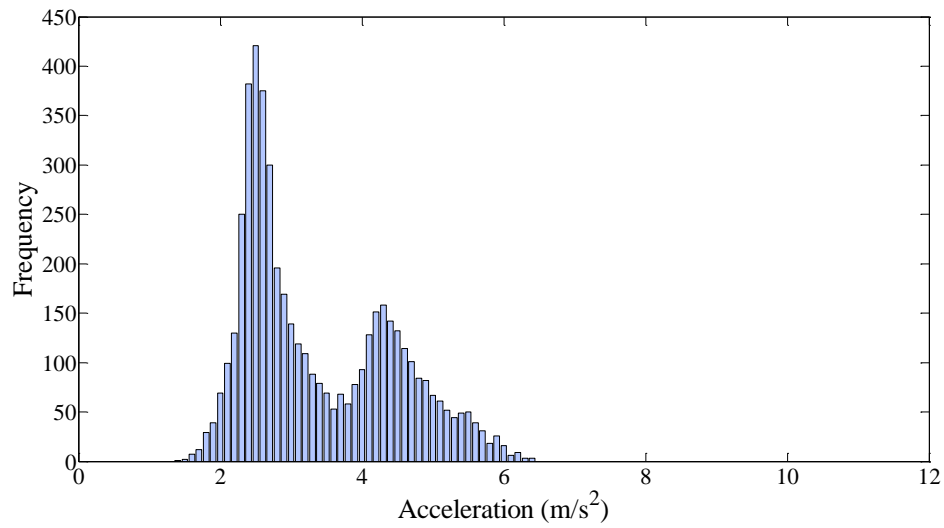


b) 285 km/h

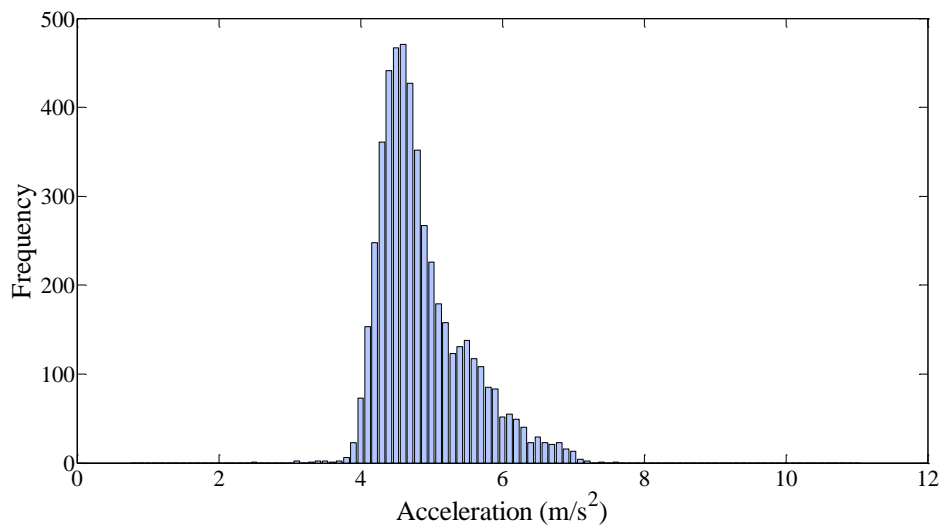
Figure 5.4 – Peak mid-span accelerations for each simulation.



a) 250 km/h



b) 285 km/h



c) 320 km/h

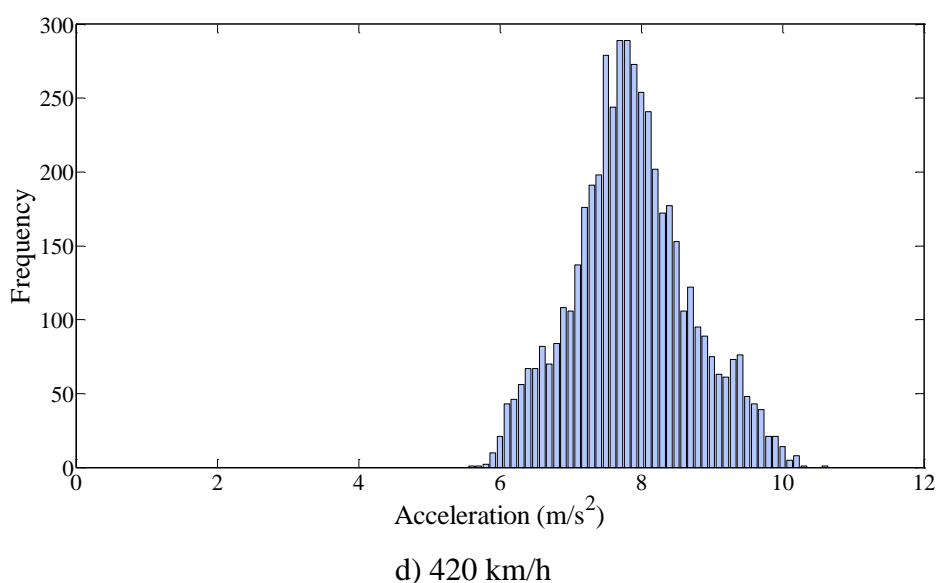


Figure 5.5 – Histogram of the peak mid-span accelerations for each simulation.

Besides highlighting the distinct response for resonant and non-resonant bridges, the results depicted in Figure 5.5 are also particularly useful to show that due to this dual behaviour the structural response does not follow a Gaussian distribution. The existence of skewness is clearly visible in the results along with a combination of different curves (perfectly depicted by Figure 5.5b) particularly in the 285 km/h to 305 km/h speed range, where resonant behaviour is more significant.

In order to complete the information about the obtained results, an overview of the maximum bridge deck acceleration for the 5,000 Monte Carlo simulation for the analysed speed range is illustrated in Figure 5.6. In this figure three curves are represented: one for the average value of the maximum acceleration, another for the average value increased by one standard deviation and also the maximum peak acceleration values obtained.

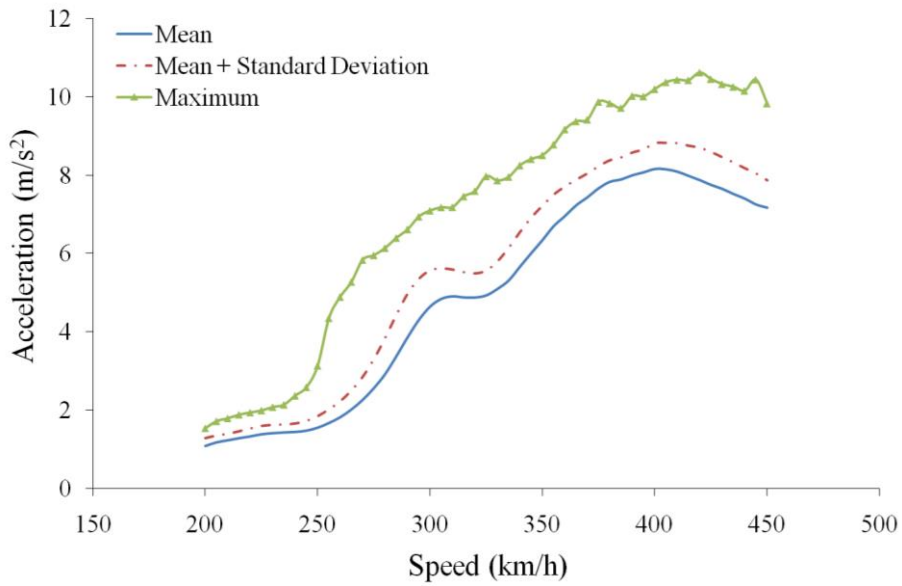


Figure 5.6 – Overview of the mid-span accelerations.

It is also interesting to compare the results obtained for different sample sizes and different simulation methods. For this reason Figure 5.7 shows a comparison for the average and maximum values of the mid-span peak acceleration for the different analysed scenarios.

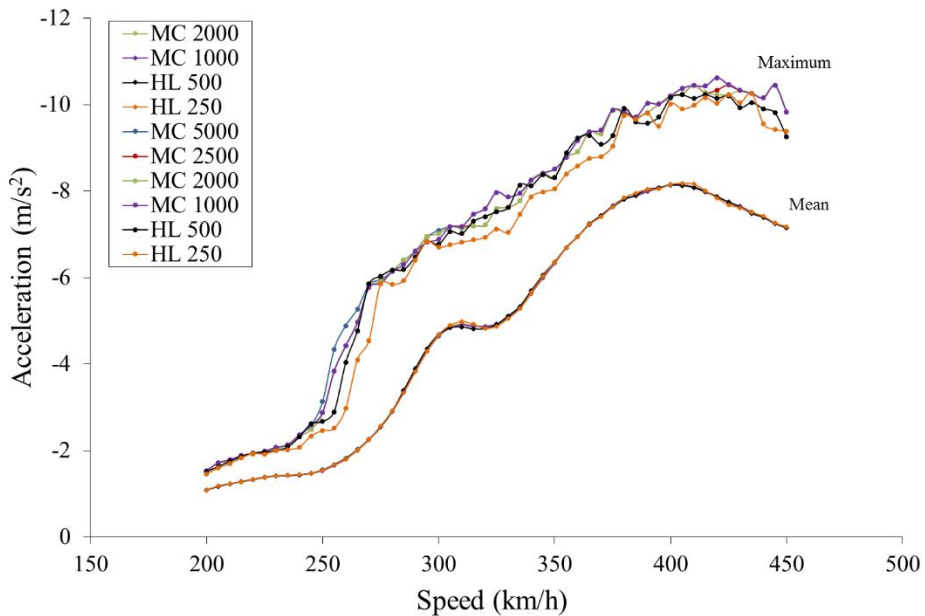


Figure 5.7 – Comparison of the mid-span acceleration for different simulation scenarios.

It can be observed that in terms of mean values (and even standard deviation values, although not depicted in Figure 5.7) there was not a significant difference between different sample sizes and even between different simulation methods. However, when analysing the maximum response values obtained for the different simulation scenarios, a higher scatter is observed. This turns out to be a key issue for the analysis since these are the values that have a higher importance in structural reliability problems. The use of fewer simulations leads to a significant reduction of the number of points near the maximum values, thus affecting the representativeness of the upper end of the structural response distribution. This fact is mostly due to the existence of the two distinct types of structural response (resonant and non-resonant behaviour) and leads to an inadequate evaluation of the safety of the train-bridge system due to the reduced number of extreme points obtained even when using a sample of 5,000 Monte Carlo simulations. It should also be added that since the response is not monotonic this tends to affect the efficiency of the Latin Hypercube method, resulting in an increased number of required simulations. This is illustrated in Figure 5.8, where the evolution of the estimated probability of failure for increasing train speeds is shown for different Monte Carlo simulation scenarios.

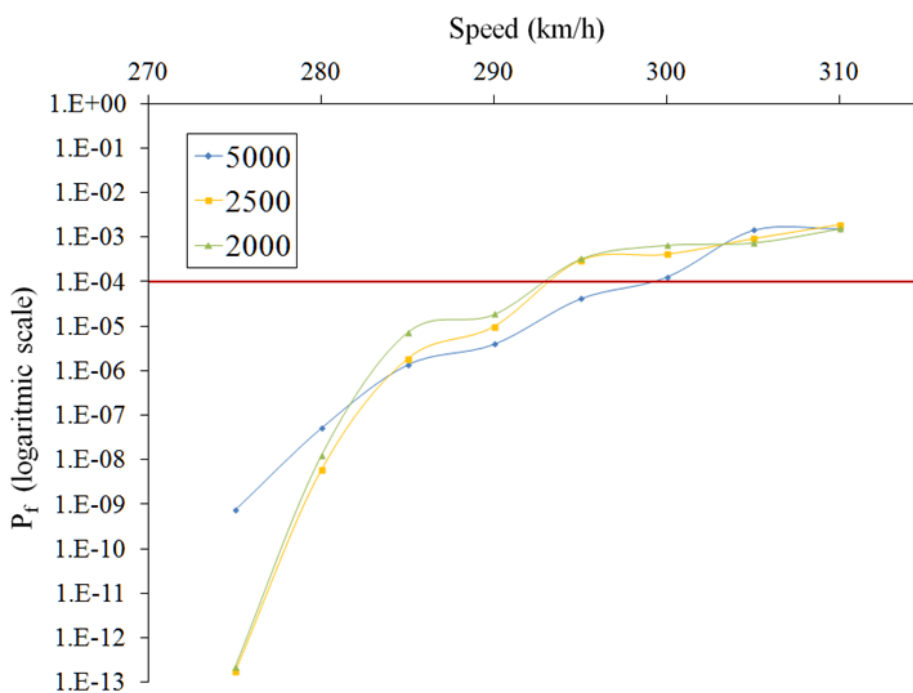


Figure 5.8 – Estimated probability of failure for different Monte Carlo simulation scenarios.

The variation of the results using different sample sizes is significant and shows that the accuracy of the results is insufficient. Therefore, estimating the probability of failure using these results is not adequate and can lead to substantial error. These results, however, can be used as a preliminary assessment and enable the identification of the critical speed range. In order to overcome the accuracy limitation, and having defined the critical speed range through this preliminary analysis, a simulation refinement was carried out for train speeds ranging from 285 km/h to 300 km/h, which is discussed in Section 5.3.1.3.

5.3.1.3 Simulation refinement

The preliminary analysis showed that the number of simulations used to assess the safety of the train-bridge system was insufficient. For this reason, in order to overcome the limitations of that analysis a refinement was made in the critical speed range. Taking into consideration the safety threshold of 10^{-4} defined in this work, it was decided to carry out 100,000 Monte Carlo simulations for train speeds ranging from 285 km/h to 300 km/h. Furthermore, and in order to validate the screening procedure exposed in Section 4.5.2, a sensitivity analysis of the simulation results obtained in this refinement was also performed. This allows validating the sensitivity analysis as well as enabling the identification of the variables that have more influence on the dynamic response of the bridge.

The sensitivity analysis was performed for each variable individually and also for the variables as a whole and both the acceleration and displacements of the bridge were studied. The obtained results of this sensitivity analysis are presented in Table 5.1 and Table 5.2.

The analysis of Table 5.1 and Table 5.2 shows that, apart from inertia variation due to the variation of the concrete section width, all the other parameters have an individual correlation with the response greater than 10%, confirming the results obtained in the variable screening procedure. Similarly to what had been observed in the variable screening procedure, it can be seen that the concrete elasticity modulus, the concrete height and the ballast area are the parameters with higher influence on the dynamic response. It can also be observed that as the speed increases, and becomes closer to resonant speeds, the correlation between the response and the variables decreases. This fact can be explained by the existence of a dual behaviour of the response observed for these speeds (resonant and non-resonant behaviour) which affects the obtained correlation.

Table 5.1 – Correlation between acceleration and basic variables for increasing train speeds.

Variable	Speed (km/h)			
	285	290	295	300
Concrete density weight	-0.197	-0.188	-0.155	-0.083
Ballast density weight	-0.126	-0.113	-0.083	-0.033
Ballast area	-0.378	-0.360	-0.284	-0.130
Elasticity modulus concrete	0.640	0.612	0.533	0.376
Concrete height	0.333	0.324	0.293	0.230
Concrete width	0.046	0.044	0.040	0.031
Elasticity modulus neoprene	0.231	0.231	0.270	0.257
Global analysis	0.880	0.851	0.746	0.534

Table 5.2 – Correlation between displacement and basic variables for increasing train speeds.

Variable	Speed (km/h)			
	285	290	295	300
Concrete density weight	-0.180	-0.164	-0.127	-0.066
Ballast density weight	-0.111	-0.094	-0.063	-0.022
Ballast area	-0.343	-0.308	-0.222	-0.087
Elasticity modulus concrete	0.718	0.692	0.633	0.549
Concrete height	0.388	0.382	0.365	0.337
Concrete width	0.052	0.051	0.050	0.045
Elasticity modulus neoprene	0.083	0.072	0.050	0.021
Global analysis	0.916	0.874	0.780	0.656

In order to estimate the value of p_f two different methods were used. In one method p_f is calculated by simply counting the number of cases where the limit acceleration is exceeded. In the other method p_f was determined by fitting a curve to the upper end of the cumulative distribution function of the mid-span accelerations. It should be mentioned that the regressions

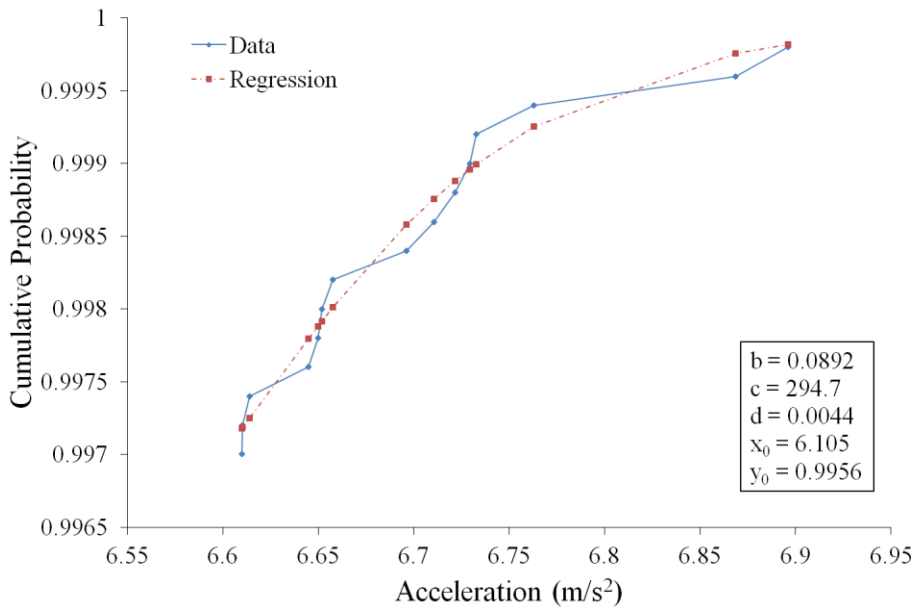
were performed for a different number of points. Three cases with 0.3%, 0.4% and 0.6% of the points from the upper end of the cumulative distribution function were used in the regression and in the end the choice was based on the obtained error. The regression with the smallest error for each speed was selected as the most accurate estimation of the probability of failure.

For this safety assessment several types of regression functions were used, namely sigmoidal and exponential functions. The functions that provided a better regression were the sigmoidal functions which are expressed by:

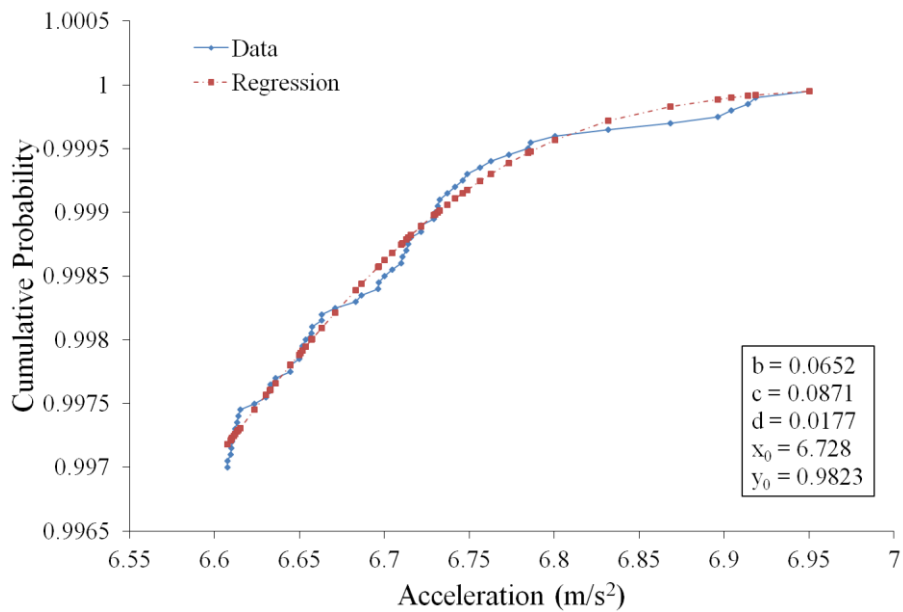
$$P = y_0 + \frac{d}{\left[1 + e^{-\left(\frac{a-x_0}{b}\right)^c} \right]} \quad (5.2)$$

where P is the probability, a the acceleration and b, c, d, x_0 and y_0 are the fitting parameters.

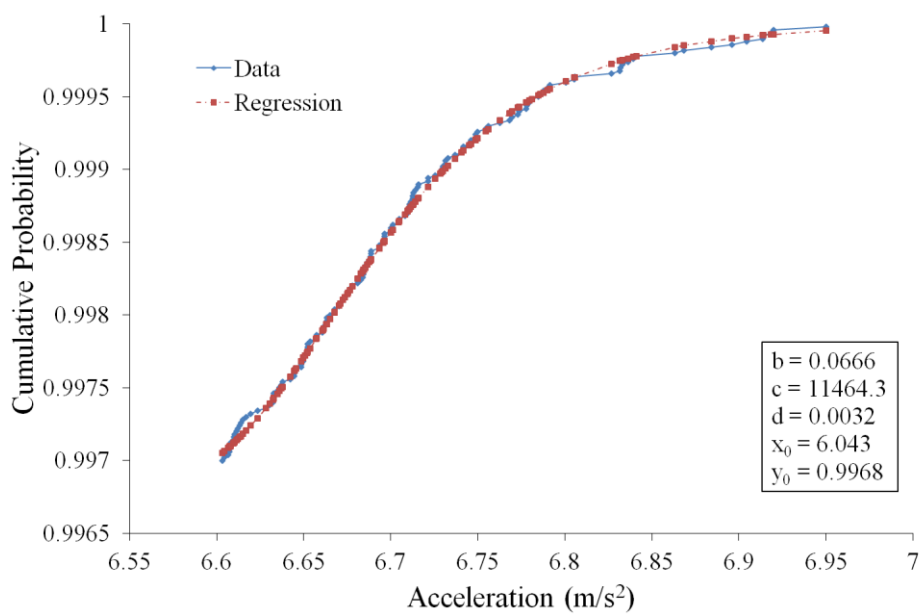
Some of the performed regressions for a speed of 295 km/h and different sample sizes are shown in Figure 5.9.



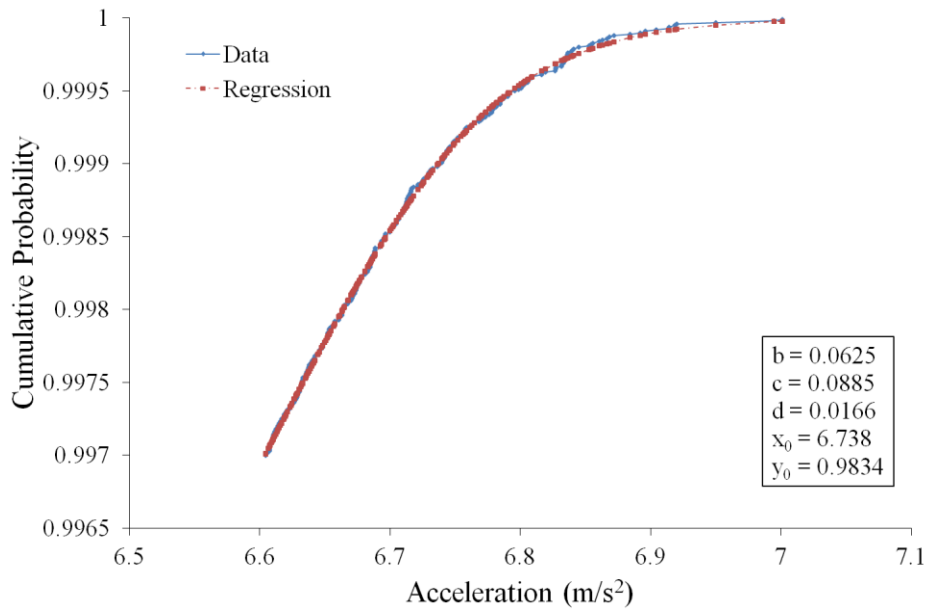
a) 5,000 simulations



b) 20,000 simulations



c) 50,000 simulations



d) 100,000 simulations

Figure 5.9 – Regression function for $v = 295$ km/h (0.3% of the points).

As expected, the increase in the number of simulations leads to a better representation of the upper extremity of the cumulative distribution function, which is, as previously mentioned, the interest zone for safety assessment purposes. This also illustrates and justifies why the accuracy of the results obtained in the preliminary assessment is insufficient.

This procedure was applied to all the train speeds in the refinement range in order to assess the safety of the train-bridge system. The evolution of the estimated probability of failure for increasing train speeds is shown in Figure 5.10.

As previously mentioned, probability values up to 10^{-4} are considered to be acceptable to measure failure due to the instability of the ballast layer. The analysis of Figure 5.10 shows that this value is not exceeded for speeds lower than 295 km/h which, therefore, can be seen as an estimate of the train speed limit to be imposed on the bridge. It is also possible to observe that if a more conservative approach was performed, lowering the limit for the probability of failure to values up to 10^{-5} , the speed limit would not be very different, decreasing only 5 km/h to 290 km/h.

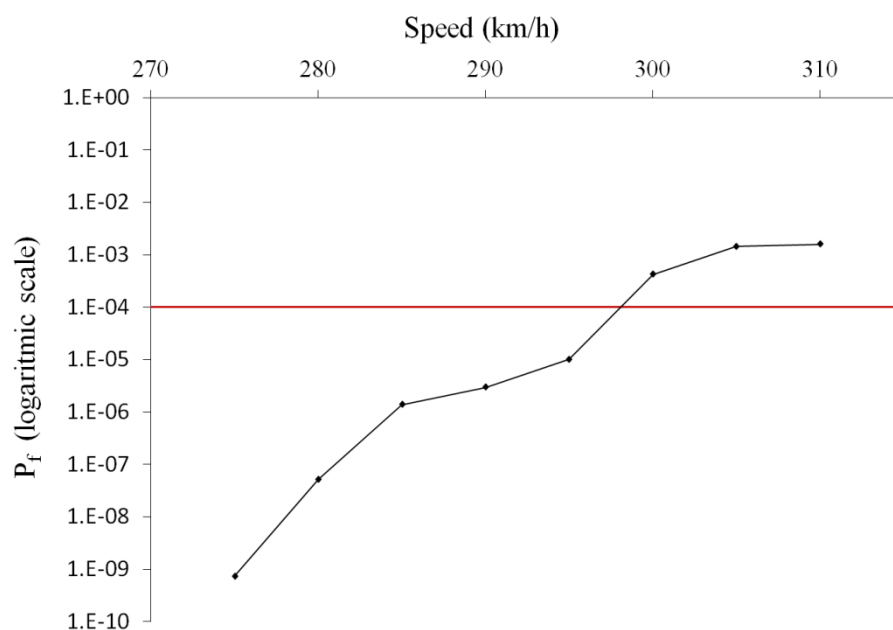


Figure 5.10 – Estimated probability of failure for increasing speeds.

Detailed results of the estimated probability of failure for increasing sample sizes are presented in Table 5.3. It should be noted that these results will only be presented for the speeds where the acceleration exceeded 7 m/s^2 since this is the acceleration limit, as previously mentioned. For this reason, only the results obtained for 295 km/h and 300 km/h are presented, which are the first speeds where the acceleration limit is exceeded. This indicates that for lower speeds the probability of failure cannot be accurately estimated with the number of simulations used, due to lack of precision. However, this is not problematic since the values are much lower than 10^{-4} , and for estimation purposes the precision used is sufficient.

From the analysis of Table 5.3 it is possible to conclude that even for a speed of 295 km/h the number of simulations is not enough for the precision required to obtain the probability of failure associated with this case (smaller than 10^{-5}) using the typical counting method. In Table 5.4 the estimation error is presented. Due to what was just referred, the error analysis is limited to a speed of 300 km/h and the “exact” value is assumed to be obtained for the case of 100,000 simulations using the counting method.

Table 5.3 – Probability of failure for different size sample (value $\times 10^{-4}$).

Number of Simulations	Speed (km/h)			
	295		300	
	count	fit	count	fit
5,000	< 0.10	0.57	< 0.10	0.76
10,000	< 0.10	0.70	3.00	2.32
20,000	< 0.10	0.24	3.50	2.10
30,000	< 0.10	0.23	3.33	1.70
40,000	< 0.10	0.16	3.25	2.16
50,000	< 0.10	0.21	3.40	2.43
60,000	< 0.10	0.16	3.83	2.69
70,000	< 0.10	0.29	3.71	3.02
80,000	0.12	0.29	4.13	3.68
90,000	0.11	0.28	3.89	3.59
100,000	0.10	0.22	4.20	3.92

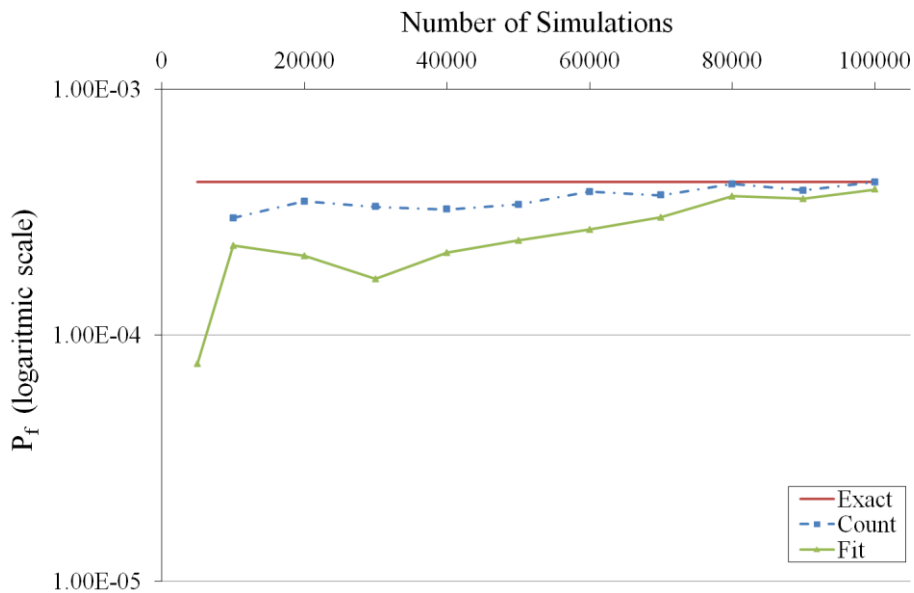


Figure 5.11 – Evolution of the estimated probability of failure for increasing sample sizes (v = 300 km/h).

Table 5.4 – Estimated probability of failure error (value $\times 10^{-4}$) ($v = 300$ km/h).

Number of Simulations	count	fit
5,000	4.20	3.44
10,000	1.20	1.88
20,000	0.70	2.10
30,000	0.87	2.50
40,000	0.95	2.04
50,000	0.80	1.77
60,000	0.37	1.51
70,000	0.49	1.18
80,000	0.08	0.52
90,000	0.31	0.62
100,000	-	0.28

Another interesting observation can be found when calculating the probability of reaching an acceleration of 7 m/s^2 for the first speed where the acceleration limit imposed by the European standards is not satisfied. This occurs for a speed of 255 km/h, which is the lowest speed where accelerations values over 3.5 m/s^2 can be observed. The estimate of reaching an acceleration of at least 7 m/s^2 at mid-span of the bridge leads to a probability of 10^{-8} , which is significantly lower than the limits normally used for ultimate limit states of the track. The comparison between the speed limit obtained when using EN1991-2 (2003) and the probabilistic approach following the limits determined experimentally by Zacher & Baeßler (2009) are indicated in Table 5.5.

Table 5.5 – Speed limit comparison according to EN1991-2 (2003) and the probabilistic approach.

	EN 1991-2 [6]	Probabilistic approach
Speed limit (km/h)	255	295
$P(a = 7\text{m/s}^2)$	$\sim 10^{-8}$	$\sim 10^{-5}$

5.3.1.4 Preliminary conclusions

By analysing the results obtained in this preliminary stage some conclusions can be drawn. First of all, the estimated speed limit obtained was 295 km/h. This limit was obtained considering the typical values used in ultimate limit states, using probability values up to 10^{-4} . It could also be observed that if a more conservative approach was used, and this probability was lowered to values up to 10^{-5} , no significant difference would be obtained, decreasing the speed limit by only 5 km/h to 290 km/h.

Another interesting observation was the comparison between the results obtained when following the guidelines proposed in EN1991-2 (2003) and when adopting a probabilistic approach. The use of a safety factor of 2 leads to a speed limit which is 40 km/h lower than the limit obtained when considering the acceleration limit for the instability of the ballast layer observed in laboratory tests. This is equivalent to the use of a probability of 10^{-8} for the track ultimate limit state.

A final remark should be made regarding the type of response obtained. Two different types of response, resonant and non-resonant behaviour, could be observed for several speeds close to the obtained speed limit. Along with the very low values admissible in Civil Engineering problems, this fact increases the complexity of the studied problem, thus forcing the required number of simulations to increase significantly. For this reason the 5,000 simulations used initially proved to be insufficient and, taking into account that probability values up to 10^{-4} were considered to be acceptable, the number of simulations was increased to 100,000 simulations. Through the error analysis it is possible to observe that a minimum of 60,000 Monte Carlo simulations are recommended. However, this number depends only on the intended accuracy, so other choices may be valid as well. The use of different methodologies to estimate the probability of failure of the train-bridge system may also lead to different results, as will be demonstrated in the next section.

5.3.2 Train-bridge interaction

Due to the importance of the train-bridge interaction effects on the dynamic response of short to medium span railway bridges, this was another aspect that needed to be taken into consideration when assessing the safety of the train-bridge system. The preliminary stage proved extremely useful to provide a better understanding of the dynamic behaviour, the most

significant parameters for the dynamic response and also enable identifying some of the potential issues that are to be faced when trying to carry out this task.

Using the knowledge acquired during that stage, several distinct approaches were tried in order to assess the safety of the train-bridge system in an efficient and accurate manner. Similarly to what was done for the preliminary assessment, a brief description of the dynamic behaviour that is observed when the train-bridge interaction is taken into account is presented. Afterwards, different approaches are presented for the safety assessment and the obtained results are discussed. Both the standard Monte Carlo simulation and the Latin Hypercube sampling method are applied. Furthermore, both simulation methods are combined with two different approaches to enhance efficiency. One is based on the extreme value theory and uses different functions to model the tail of the distribution (described in Section 3.4.1). The other uses an approximation procedure based on the estimates of the failure probabilities at moderate levels to estimate the target probability of failure by extrapolation (described in Section 3.4.2).

5.3.2.1 Dynamic response accounting for the interaction effects

Similarly to what was done for the case of the moving loads analysis, this Section intends to illustrate some of the simulation results obtained when the train-bridge interaction effects are taken into account. Similarly to the moving loads analysis, the dynamic analyses that take into consideration the train-bridge interaction effects were performed using the FEMIX software with the formulation presented in Neves et al (2012), which for a time step of 0.002 s and a total of 5,000 steps, representing a 10 s analysis, takes approximately 40 s to be completed on a computer with an Intel Core i7 920 running at 2.67 GHz.

The first of the results that are presented corresponds to the maximum bridge deck acceleration obtained for increasing train speeds and is shown in Figure 5.12. The black line represents the results for the case where all the variables take their mean value whereas the circles correspond to results obtained from MC simulation, illustrating the variability of the dynamic response for each of the different train speeds analysed.

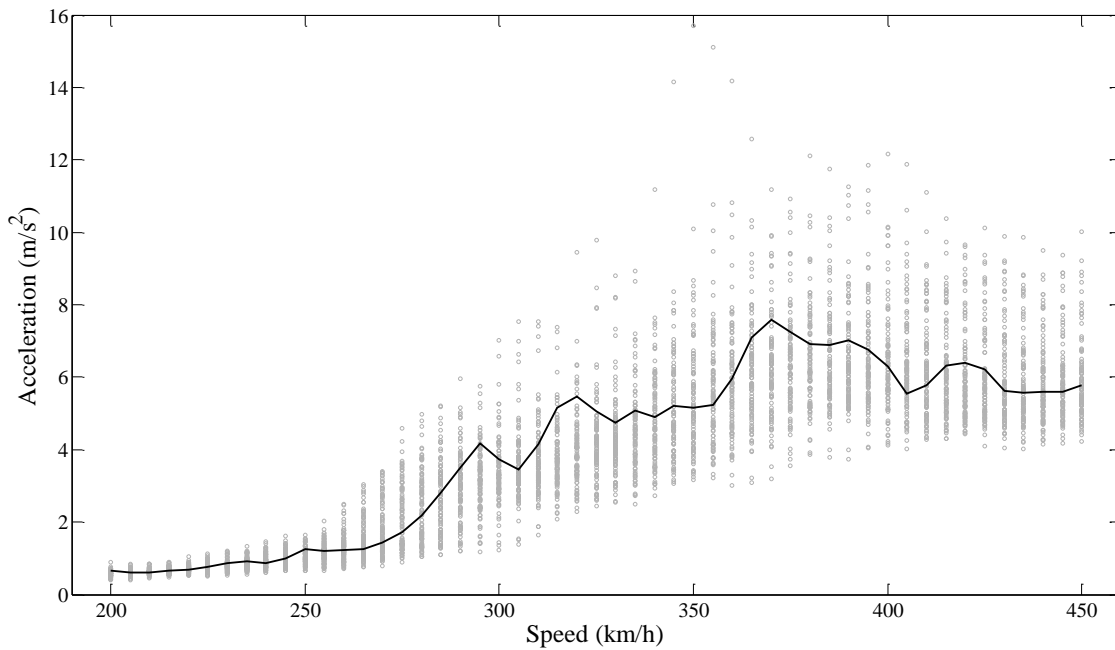
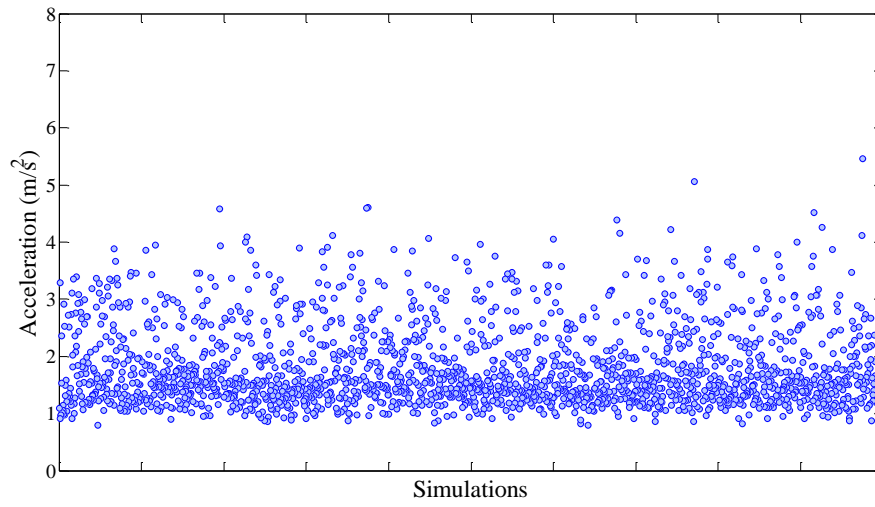


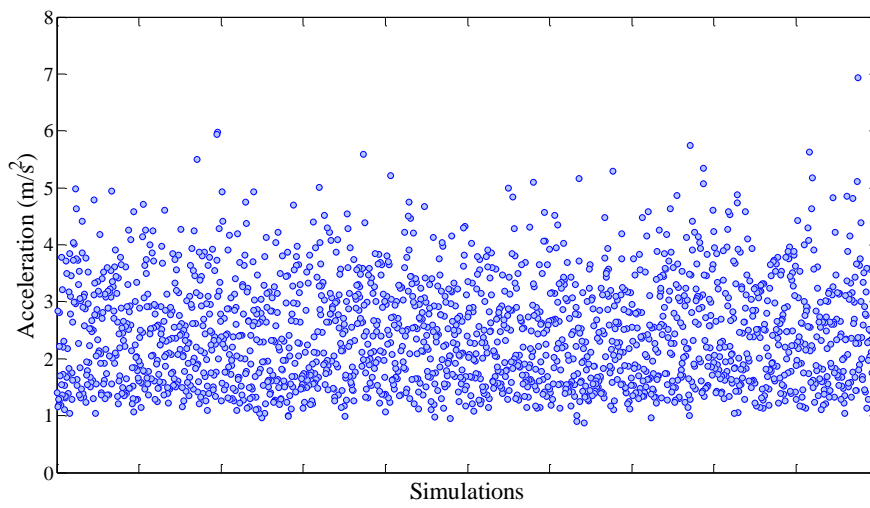
Figure 5.12 – Maximum bridge deck acceleration for increasing train speeds.

It is possible to observe that the variability tends to increase around the speeds where resonant effects can be observed. Furthermore, it is noticed that accounting for the flexibility of the bearings combined with the existence of track irregularities shifted the critical section from the mid-span to the $\frac{3}{4}$ span. The irregularities originate an important increase of the wheel-rail contact forces that lead to the excitation of the bridge second mode shape, resulting in a significant dynamic amplification at the quarter span sections. This indicates that at least for some specific cases the results obtained from a moving loads approach and a train-bridge interaction approach are not directly comparable, particularly if the track irregularities are taken into consideration. It is also possible to notice that for speeds greater than 350 km/h the dynamic response results from impulsive loading due to the passage of the train at high-speed rather than resonant effects.

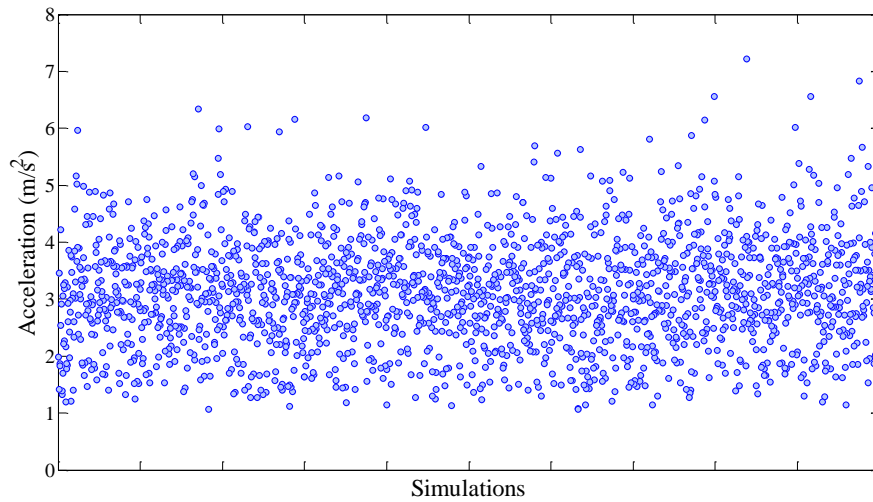
To illustrate the effects of resonance, which are particularly relevant for the track stability safety assessment, the results from the simulations are shown as a scatter plot in Figure 5.13 and as a histogram in Figure 5.14.



a) 270 km/h



b) 280 km/h

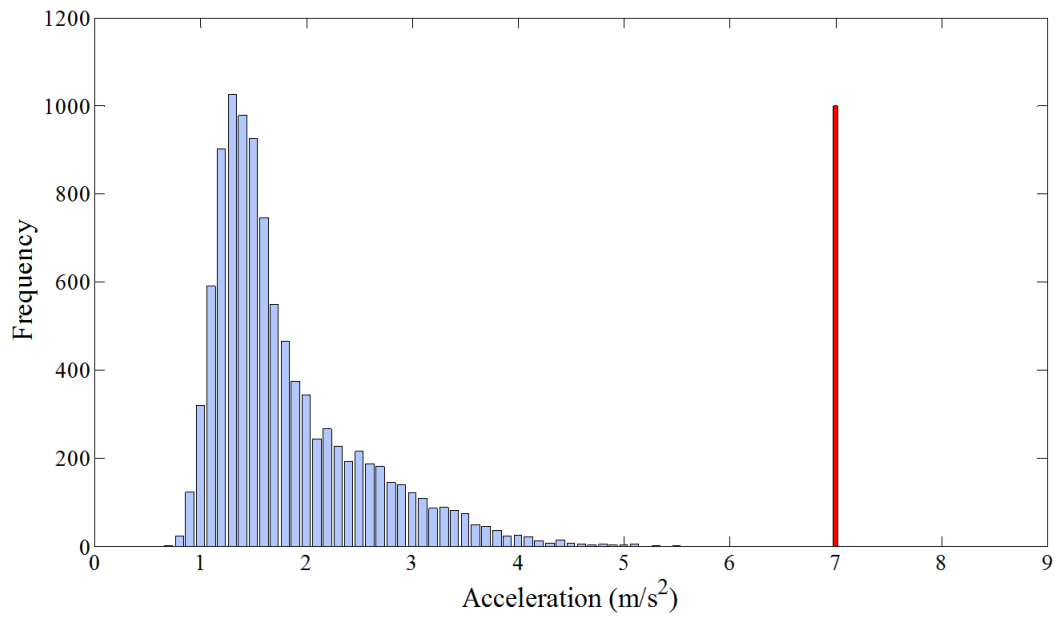


c) 290 km/h

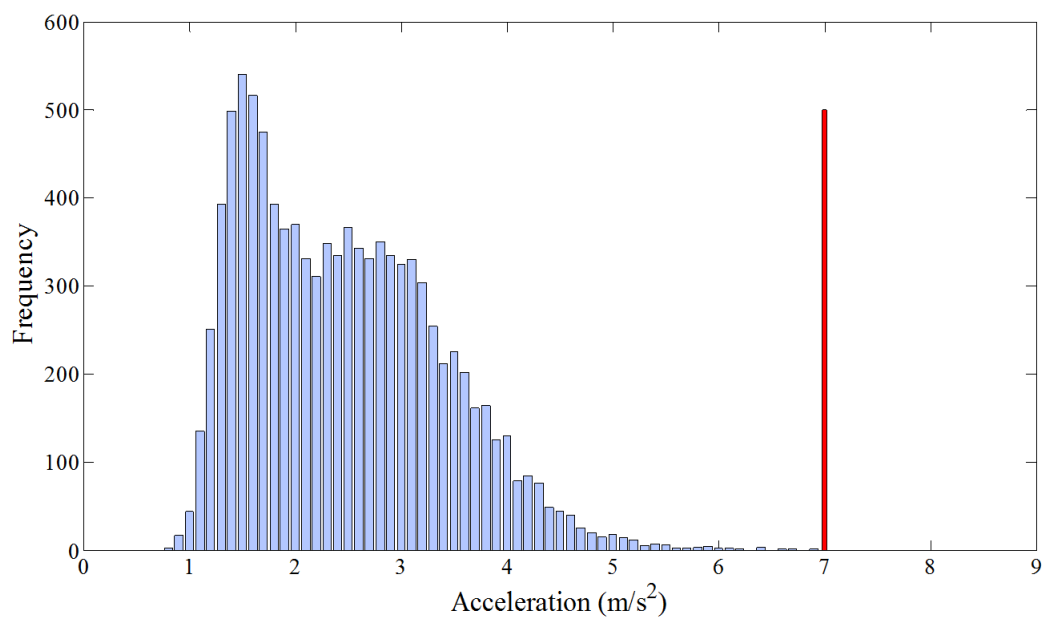
Figure 5.13 – Scatter plot of the maximum bridge deck acceleration for each simulation.

The scatter plots are useful to illustrate how resonance affects the simulation results. As the train speed moves closer to the resonant speeds the maximum bridge deck acceleration tends to increase and the scatter of the response is also higher. For the train speed of 270 km/h the centroid of the results appears to be between 1 and 2 m/s^2 , with few cases far from that range. This changes as the speed increases to 280 km/h where it becomes harder to establish the centroid of the results due to the increase scatter of the response. For the train speed of 290 km/h the scatter is still high but it seems to have increased to the range of 3 to 4 m/s^2 , indicating that most simulations are experiencing resonant effects.

The same results are also plotted as a histogram in Figure 5.14 to show more clearly the existence of two different response curves and how they evolve as the train speed increases.



a) 270 km/h



b) 280 km/h

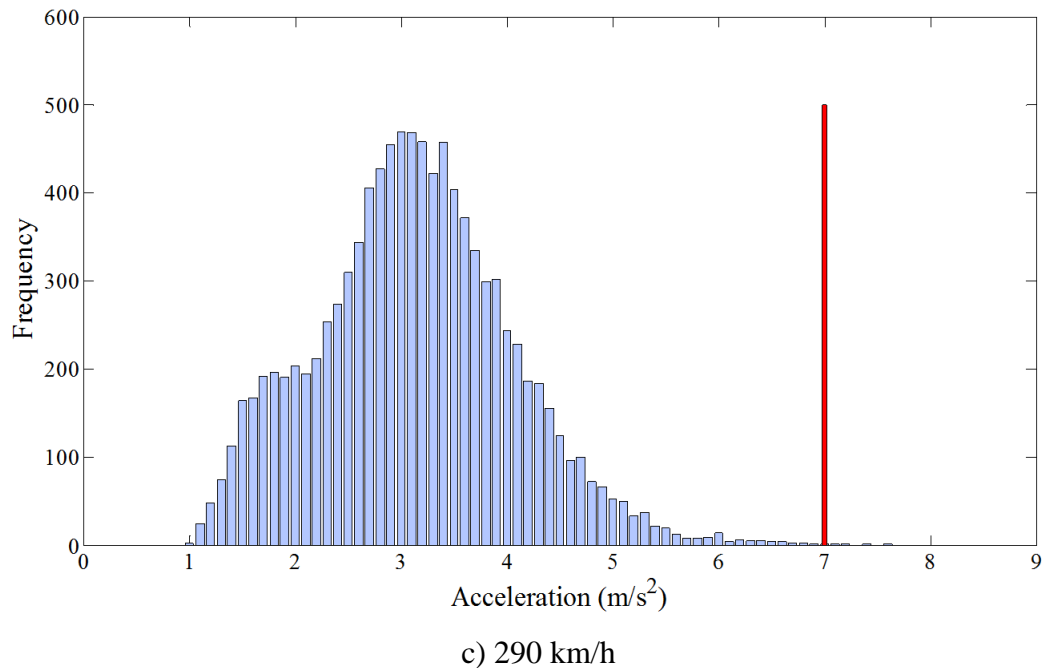
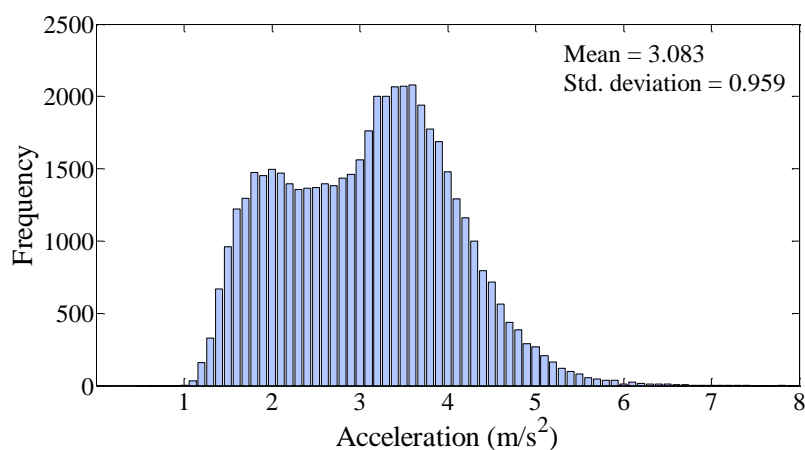


Figure 5.14 – Histogram of the peak bridge deck accelerations.

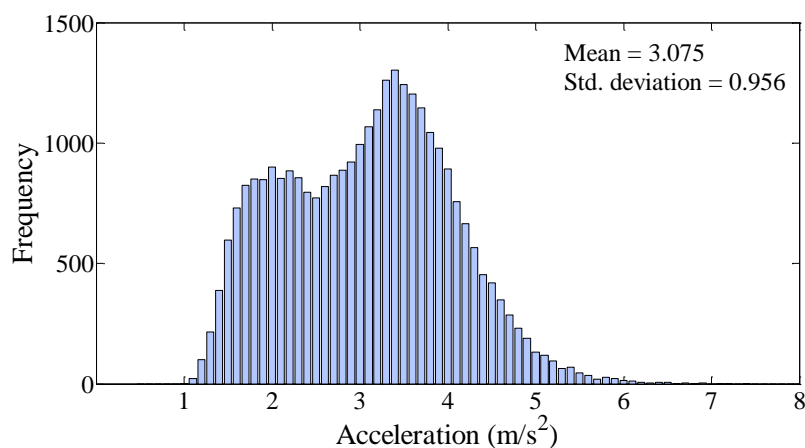
The results obtained for the train speed of 270 km/h resemble those obtained for a lognormal distribution but as the speed increases to 280 km/h two distinct curves can be clearly seen in the histogram indicating the existence of two distinct response types. Finally, the analysis of the histogram for a train speed of 290 km/h shows that most cases are undergoing resonant effects.

Since two different simulation methods are used it is important to confirm that the results obtained for both of them are similar. This validation is shown in Figure 5.15 for a train speed of 285 km/h.

A sample of 50,000 simulations was selected for the Monte Carlo method and a sample of 30,000 simulations was used for the Latin Hypercube method. As expected, the results are practically identical for both simulation methods.



a) 50,000 Monte Carlo simulations



b) 30,000 Latin Hypercube simulation

Figure 5.15 – Comparison of the results using different simulation techniques.

5.3.2.2 Tail modelling approach

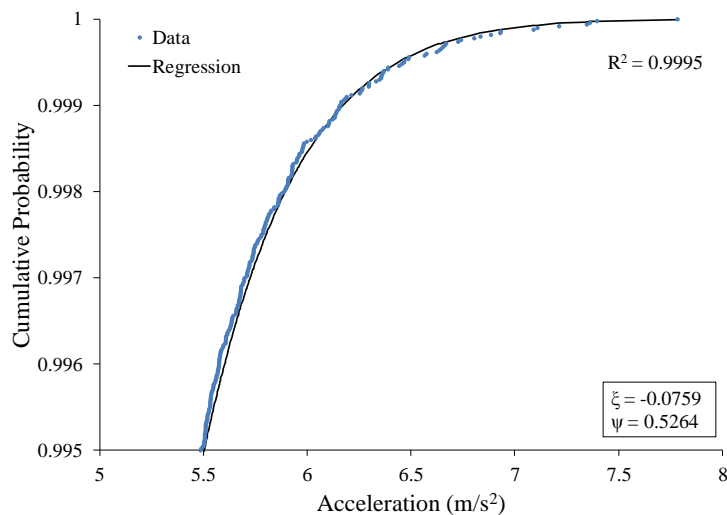
Bearing in mind that structural reliability problems are determined by the tail of the statistical distributions, the computational cost can be significantly reduced if an extrapolation of the Cumulative Distribution Function (CDF) is made using tail modelling techniques. To assess the safety of the train-bridge system the tail of the distribution is modelled using the sigmoid functions that proved to be useful in the preliminary assessment stage along with the traditional Generalised Pareto Distribution.

As stated before, one important aspect that needs to be clearly defined is the accuracy that is desired with respect to the results obtained when estimating the probability of failure. Since the

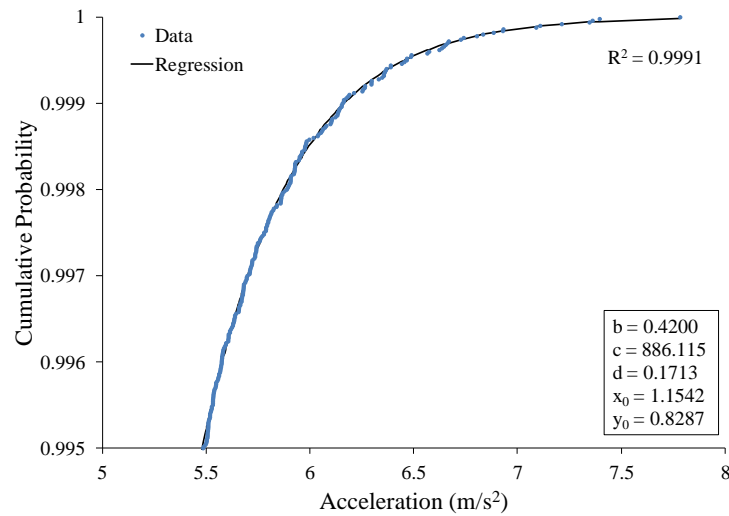
tail modelling approach relies on data points with the largest uncertainty, the stabilisation of the probability of failure estimates may require a large number of simulations.

When using this procedure two different aspects are particularly important: one is the process of modelling the tail, which is significantly dependent on the threshold that is selected and “defines” what is being taken as the tail of the distribution; and the second is the efficient identification of the critical speed range from a safety point of view, in order to direct the computational effort to the speeds where the safety of the train-bridge system is at risk.

With respect to the first point that was mentioned, different thresholds were tested to understand which value should be selected in order to obtain more accurate estimations of the probability of failure from the tail fitting. Threshold values of 0.90, 0.95, 0.98 and 0.99 were analysed. Regarding the GPD fit, the results showed similar accuracy using thresholds of 0.95 and 0.98. The threshold of 0.95 was selected for the safety assessment analysis, as it allows a better representation of the tail for smaller samples sizes. With respect to the sigmoid fit, further thresholds of 0.995 and 0.999 were analysed and it was concluded that the accuracy of the fit increased as the threshold moved closer to one, provided that the CDF tail was adequately represented. Therefore, when using the sigmoid fit instead of defining a specific threshold, 50 points of the tail were considered. Some examples of the performed fits for a sample of 50,000 simulations are illustrated in Figure 5.16.



a) GDP fit



b) Sigmoid fit

Figure 5.16 – Tail modelling examples.

Both methods proved that they provide excellent fits to the data, which is confirmed by the R^2 index values being above 0.99.

Another key aspect to take into account is efficiency. One should bear in mind that the methodology proposed aims to assess the safety of the train-bridge system. Therefore, determining the exact probability of failure is not required for every train speed. For this reason the procedure should direct the computational efforts to the critical train speeds in order to guarantee efficiency. To achieve this, two stopping criteria have been defined: one regarding the stability of the estimated reliability index, β , and the other related to the probability threshold being out of the confidence interval bounds (95%), using the Wald confidence intervals given by [Wald & Wolfowitz (1939)]:

$$[P_{LB}, P_{UB}] = \hat{p}_{im} \pm z_{\alpha/2} \sqrt{\frac{\hat{p}_{im}(1 - \hat{p}_{im})}{N}} \quad (5.3)$$

where \hat{p}_{im} is the estimated probability of failure, N is the sample size, α is confidence level and P_{LB} and P_{UB} are the lower and upper bounds confidence intervals of probability of failure, respectively.

The first criterion aims to analyse the stability of the estimated probability of failure through the analysis of the variation of the reliability index, β , over consecutive sets of simulations. Reliability index, β , and probability of failure, p_f , are related as:

$$p_f = \Phi(-\beta) \quad (5.4)$$

where $\Phi(\bullet)$ is the CDF of the standard normal random variable.

As the number of simulations increases so does the accuracy of the probability of failure estimates. This criterion imposes that the variation of β is smaller than 0.5% during three consecutive increases of the sample size (increase step of 500 simulations). This criterion is useful for speeds where the probability of failure is relatively far from the 10^{-4} limit but the confidence interval bounds exceed this value for a sample size that enables to understand that the estimated probability of failure has stabilised and will not be exceeding the safety threshold. To provide a better understanding of this criterion the dynamic response for a train speed of 280 km/h is analysed as an example. Figure 5.17 shows the evolution of the estimated probability of failure for increasing sample sizes whereas Figure 5.18 shows the evolution of the estimated reliability index for increasing sample sizes.

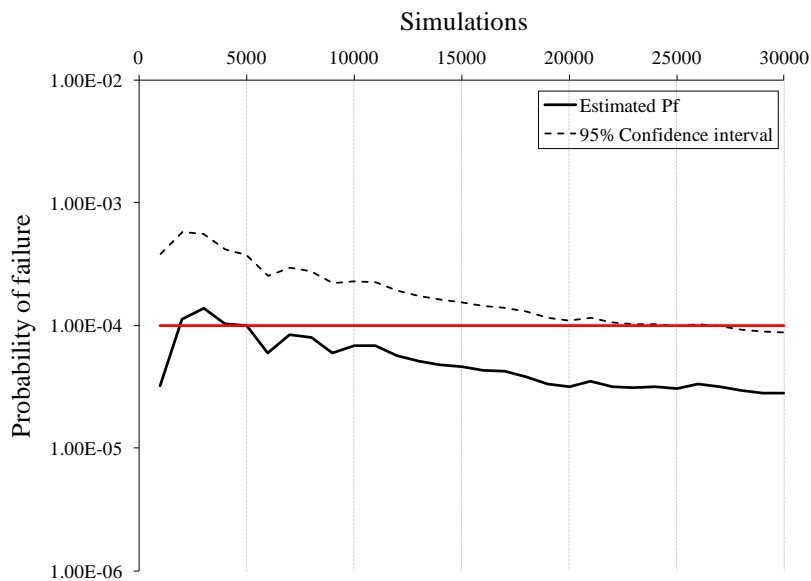


Figure 5.17 – Evolution of the estimated probability of failure at the quarter span section for a train speed of 280 km/h.

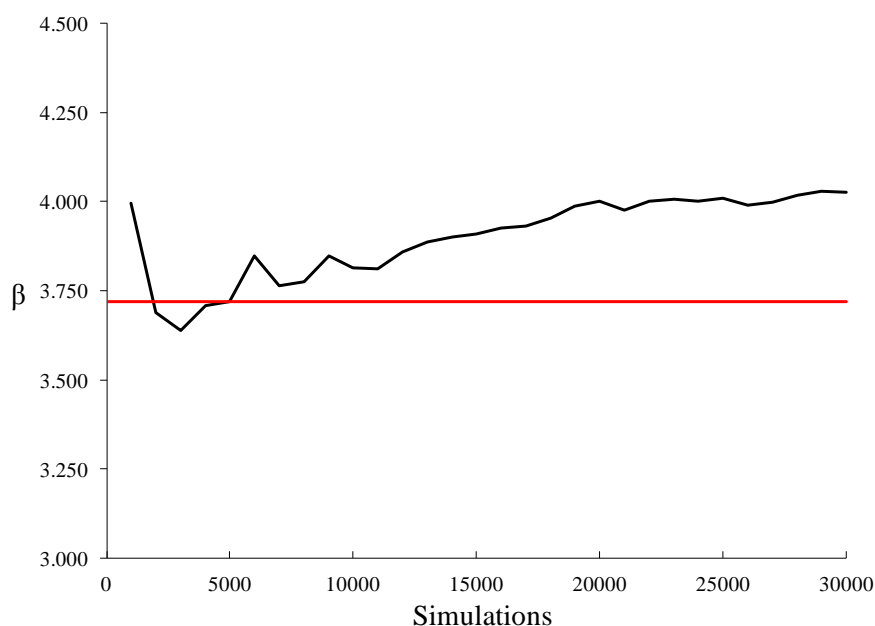


Figure 5.18 – Evolution of the estimated reliability index for increasing sample sizes.

The upper bound of the confidence interval of the estimated probability of failure is over the safety threshold for samples smaller than 27,000 simulations. However, the estimated reliability index stabilises much sooner than this limit (with a slightly increasing tendency of the estimated reliability index). Using the specified criterion fewer than 15,000 simulations are sufficient to rule out this train speed as critical.

The second criterion was based on the confidence of the estimated probability of failure through the Wald confidence intervals, as previously presented. This criterion proved extremely useful for train speeds where the probability of failure is far (an order of magnitude or more) from the 10^{-4} safety limit. Like for the previous criterion an illustrative example of the advantages of using such a criterion is shown for the case of trains crossing the bridge at 270 km/h. The evolution of the estimated probability of failure is shown in Figure 5.19.

The estimated probability of failure is much lower than the safety limit (difference of nearly two orders of magnitude) for this train speed. Although the number of simulations is not enough to accurately predict the probability of failure, the confidence interval of the estimated probability of failure is far below the safety limit after 3,000 simulations.

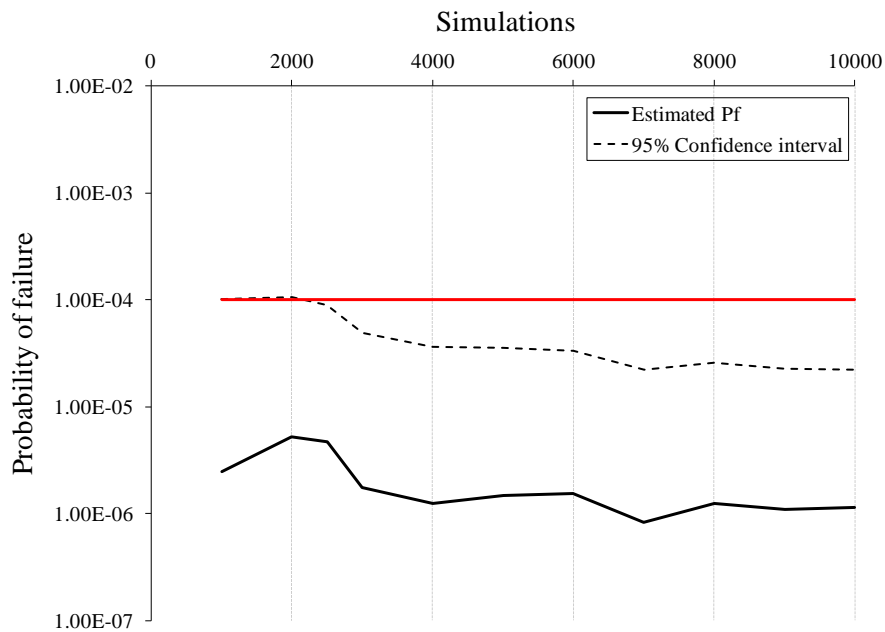


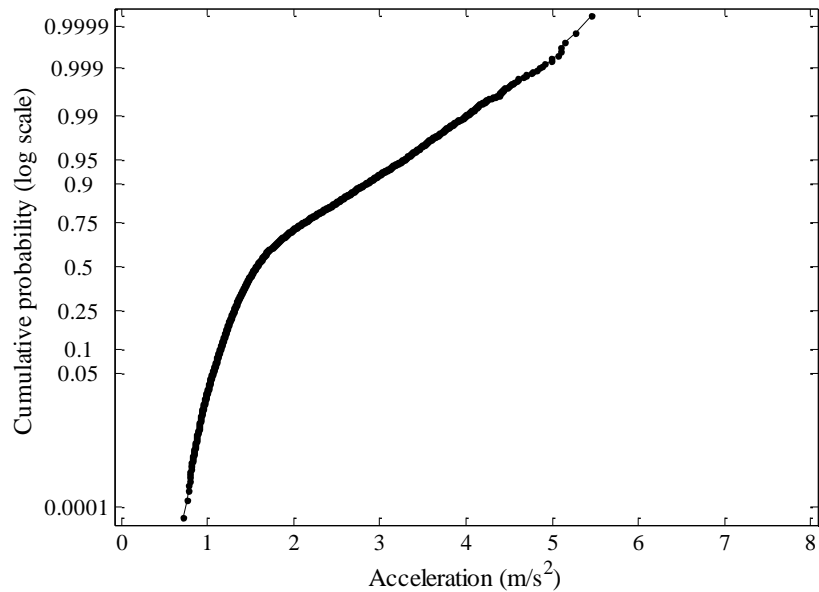
Figure 5.19 – Evolution of the estimated reliability index for increasing sample sizes.

For this reason the required number of simulations can be greatly reduced as the safety of the train-bridge system is not at risk for this train speed. For the current example less than 5,000 simulations would be required to rule out this train speed as critical for the mid-span section. For other cases even fewer simulations may be required. The practical example allows showing that through the application of such a criterion the number of simulations is greatly reduced and the critical speed range can be easily identified. However, the reduction of the number of simulations does not allow an accurate prediction of the probability of failure for these train speeds. Nevertheless, as highlighted earlier, determining the exact probability of failure of the train-bridge system is not required for every train speed since the objective is the safety assessment of the train-bridge system and this is only necessary for the train speeds where the train-bridge system safety is at risk. It should also be added that a minimum of 2,000 simulations are performed for every train speed. This number proved to be reasonable in preventing judgement errors that could lead to a failure in identifying a speed where the safety of the system could be at risk.

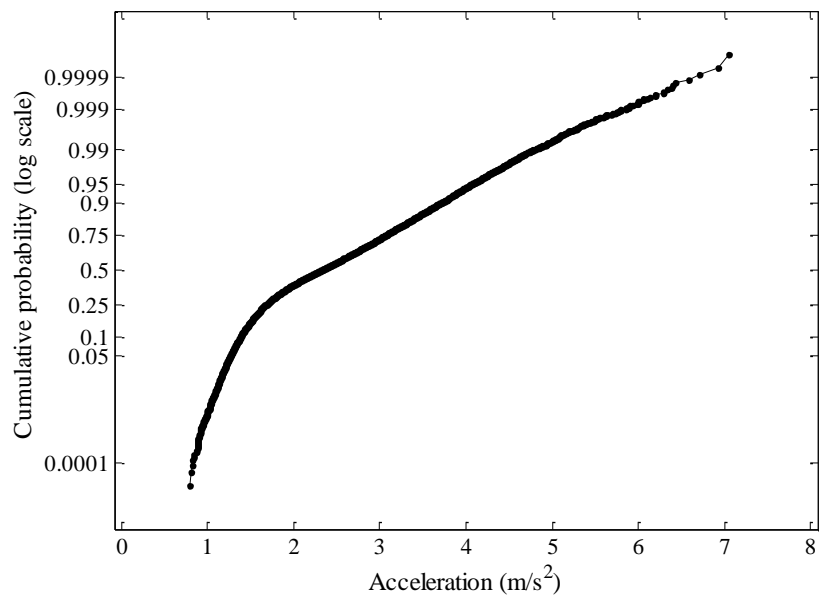
If at least one of these criteria is satisfied then the number of simulations is adequate. Otherwise more simulations need to be carried out until one of the criteria is satisfied. These criteria are extremely useful to direct the computational effort to the speeds where the safety of the train-bridge system is at risk and enable reducing significantly the number of required

simulations. However, for speeds where the safety of the train-bridge system might be at risk, the criteria are disregarded to guarantee accurate estimates of the probability of failure.

To complement some of the results provided in Section 5.3.2.1 the CDF plots of the peak bridge deck acceleration for different train speeds is illustrated in Figure 5.20.



a) 270 km/h



b) 280 km/h

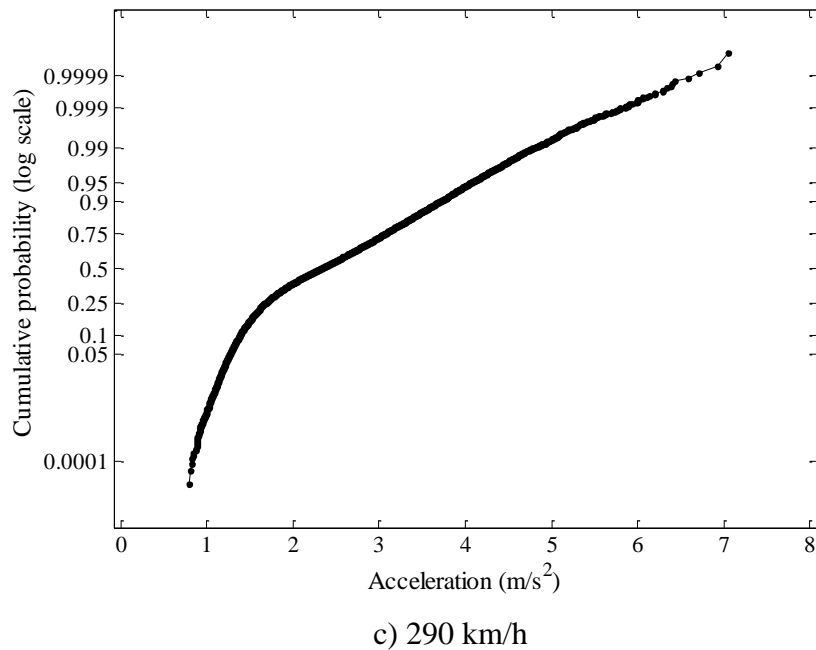


Figure 5.20 – CDF plot of the peak bridge deck accelerations.

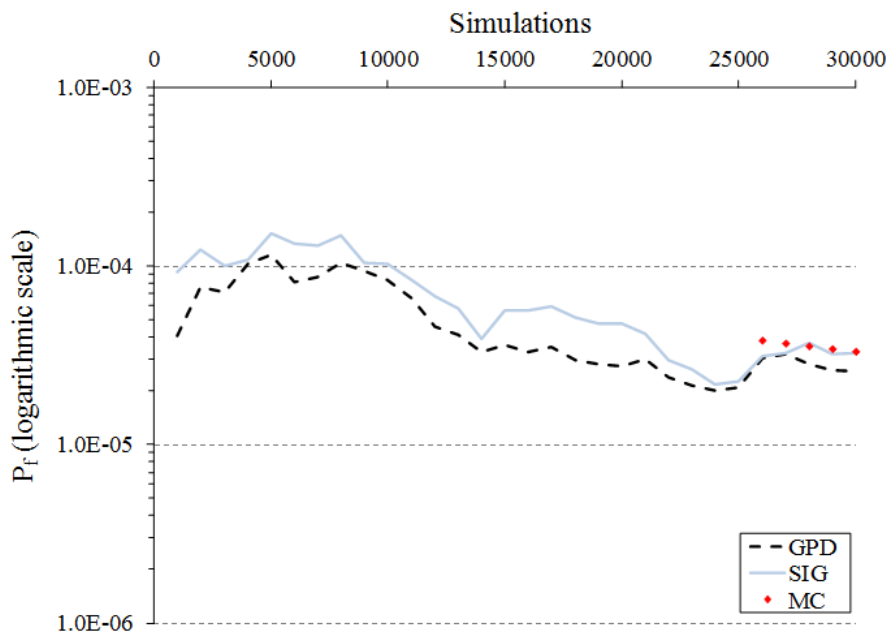
The CDF plots are extremely useful to illustrate one of the main difficulties to be overcome by the tail modelling techniques. It is evident that the CDF curve is relatively stable for moderate to high levels of probability. However, as it approaches the most extreme points there is some perturbation to this stability which arises from the fact that these points are less reliable from a statistical point of view. This is identified as the main drawback of this approach and is also one of the main difficulties in structural safety problems.

In order to estimate the probability of failure the tails of the distribution were modelled using both sigmoid functions and the GPD. Although both methods provide good fits to the data, it is important to confirm that they guarantee accurate estimates of the probability of failure. To do so, using the results from the Monte Carlo simulation, the estimated probability of failure was compared for different train speeds and increasing sample sizes. The estimated p_f is also compared to the results obtained from the Monte Carlo method for further confirmation. The evolution of the estimated probability of failure of the train-bridge system for increasing train speeds is presented in Table 5.6.

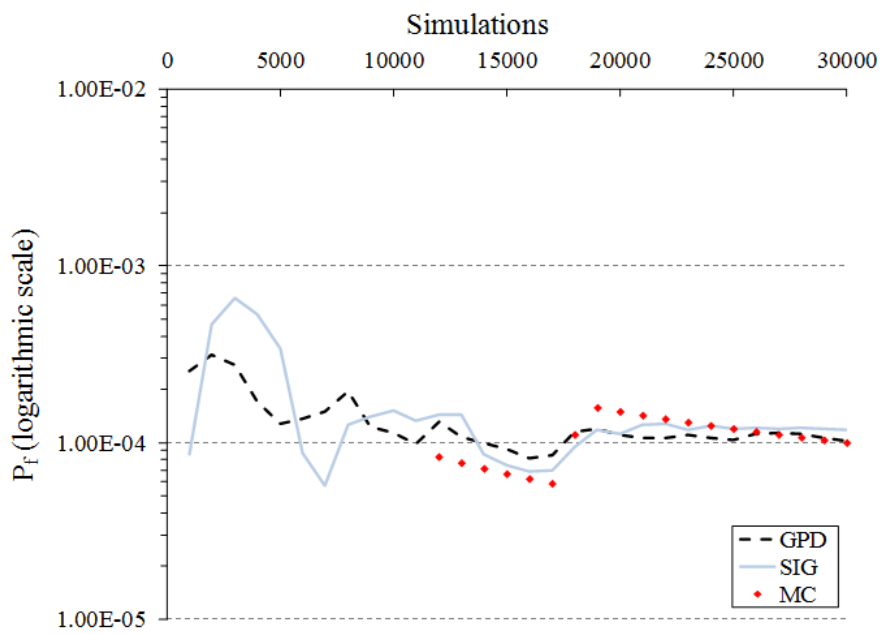
Table 5.6 – Estimated probability of failure for increasing train speeds using the tail modelling approach.

Speed (km/h)		p_f	β
280	MC	3.33E-05	3.988
	GPD	2.76E-05	4.032
	SIG	4.79E-05	3.901
285	MC	1.25E-04	3.662
	GPD	1.06E-04	3.704
	SIG	1.27E-04	3.658
290	MC	4.17E-04	3.341
	GPD	4.73E-04	3.306
	SIG	3.17E-04	3.417
295	MC	1.00E-03	3.090
	GPD	1.06E-03	3.073
	SIG	8.58E-04	3.135
300	MC	1.98E-03	2.881
	GPD	1.99E-03	2.880
	SIG	2.08E-03	2.866
305	MC	4.00E-03	2.652
	GPD	3.70E-03	2.678
	SIG	4.22E-03	2.634

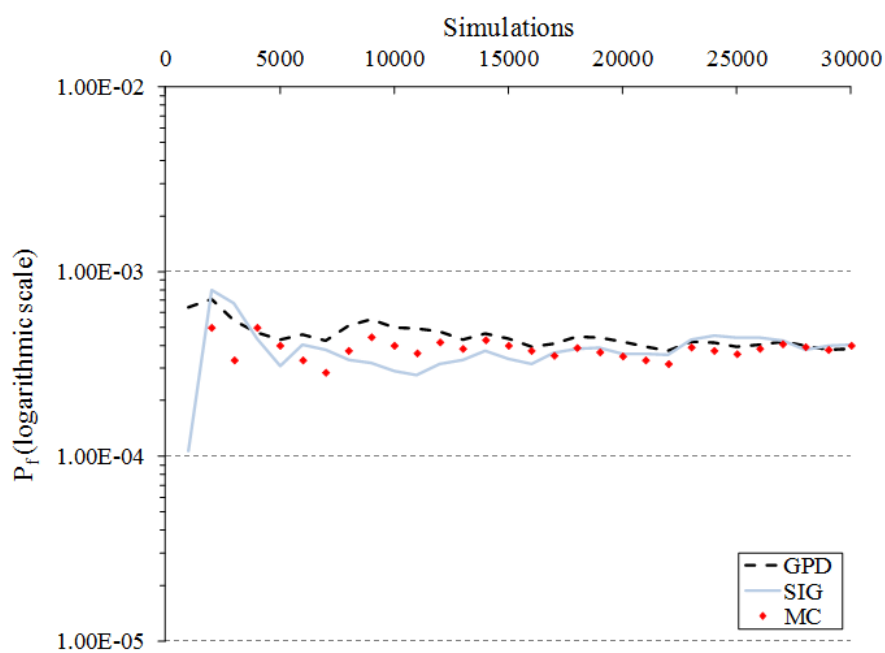
A good agreement between the three estimates is obtained, thus confirming the applicability of the selected methods. However, to assess robustness and efficiency the extrapolation accuracy and the stability of the estimated p_f must also be analysed. To do so, the evolution of the estimated p_f for increasing sample sizes was analysed for different train speeds. Some of these results are shown in Figure 5.21.



a) 280 km/h



b) 285 km/h



c) 290 km/h

Figure 5.21 – Comparison of the evolution of the estimated p_f for increasing sample sizes for the different methods.

The evolution of both fits is very similar in the critical speed range between 280 km/h and 290 km/h. Compared to the MC both fits allow estimating p_f with significantly fewer simulations, particularly when the amount of data above the safety threshold is reduced. For train speeds below 280 km/h long range extrapolation is required as the maximum deck acceleration is far from the safety limit. For these cases GPD is more accurate, whereas the sigmoid fit tends to be conservative. As the estimated probability is far from the safety limit this is not considered to be significant due to the applied stopping criteria. However, if the purpose is to accurately estimate probabilities smaller than 10^{-5} , the use of GPD is recommended. For train speeds above 285 km/h both fits show similar accuracy, even for small sample sizes. This can be explained by the higher number of points in the vicinity of the failure region.

Following the validation of the methodology it is also important to analyse how different simulation methods affect the efficiency and accuracy of the tail modelling approach. Since the GPD approach proved to be more efficient and accurate, only this method was used in the comparison. The results are indicated in Table 5.7 and are also compared with the “crude”

approach of estimating the probability of failure by simply counting the number of cases where the safety threshold is exceeded for both simulation methods.

Table 5.7 – Estimated p_f due to track instability using the tail modelling approach ($\times 10^{-4}$).

Speed (km/h)	MC	LH	GPD-MC	GPD-LH
275	0.20	< 0.1	0.05	0.14
280	0.40	0.50	0.58	0.70
285	1.40	1.33	1.32	1.03
290	5.80	2.67	5.17	4.93
295	10.30	8.33	13.00	7.81

Another very important aspect to analyse is the number of simulations for each train speed required to assess the safety of the train-bridge system as this is an indicator of the efficiency of the methodology. The required number of simulations to accurately assess the probability of failure due to track instability are indicated in Table 5.8.

Table 5.8 – Number of simulations required to accurately assess the probability of failure due to track instability using the tail modelling approach.

Speed (km/h)	SIG-MC	GPD-MC	GPD-LH
275	4000	8000	8000
280	20000	14000	7000
285	22000	15000	25000
290	12000	6000	7000
295	6000	5000	5000

The analysis of the results shows that the efficiency is almost similar for both simulation techniques. It is also observed that, in general, the GPD approach is more efficient than the

sigmoid approach. The GPD-MC proves to be slightly more efficient and the difference to the GPD-LH is 8%, corresponding to 4,000 simulations less on the critical speed range. The lower efficiency is due to the particular train speed of 285 km/h, where the estimated probability of failure is extremely close to the safety threshold of 10^{-4} . Besides being close to threshold, the fact that response is not monotonic also affects LH efficiency. The estimated probability of failure kept shifting from the safe to the unsafe side and ultimately required a larger sample size in order to stabilise above the safety limit. Since the goal was the accurate assessment of the probability of failure this particular speed required 25,000 simulations, corresponding to 48% of the total number of simulations for the analysis of the critical speed range. It should be added that for speeds outside the range presented in Table 5.8 less than 2,000 simulations are required to rule them out as critical, thus indicating that the methodology is very efficient in identifying the critical train speeds.

5.3.2.3 *Enhanced simulation approach*

The other method used to estimate the probability of failure of the train-bridge system is the Enhanced Simulation method. As detailed in Section 3.4.2, this method exploits the regularity of the tail probabilities to set up an approximation procedure based on the estimates of the failure probabilities at more moderate levels for the prediction of the far tail failure probabilities. This reduces the influence of the most extreme values, which are also less reliable from a statistical point of view, and may allow for a more efficient approach when compared to the tail modelling methodology.

The Enhanced Simulation procedure enables quantifying the uncertainties of the estimated probability of failure using its optimisation procedure. This fact is very useful for two particular aspects: it significantly reduces the influence of the sample size and it also provides an extra tool to assess the quality of the failure estimation.

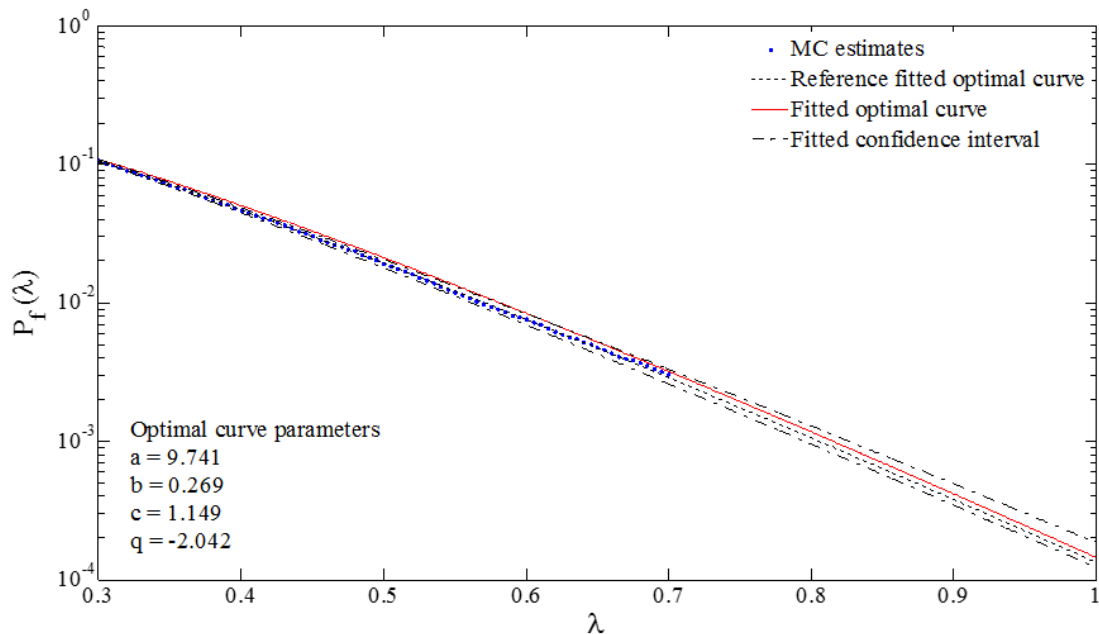
Focusing on the accuracy and robustness of the estimates obtained by this method two different approaches were analysed in order to determine the necessary number of simulations. The first one considers the estimated confidence intervals obtained using a smaller sample size. In order to define the required sample size the estimated probability of failure for a reference sample needed to be within the bounds defined by the estimated confidence interval for that sample size. In the case of MC simulation 50,000 samples were considered whereas in the case

of LH simulation the reference was set for 30,000 samples. These values were identified as sufficient to provide a satisfactory estimate of the p_f taking into account the target reliability level used in this work. In this case the obtained confidence interval band is large but ensuring that the estimated value for the reference sample size is within these limits guarantees an accurate prediction.

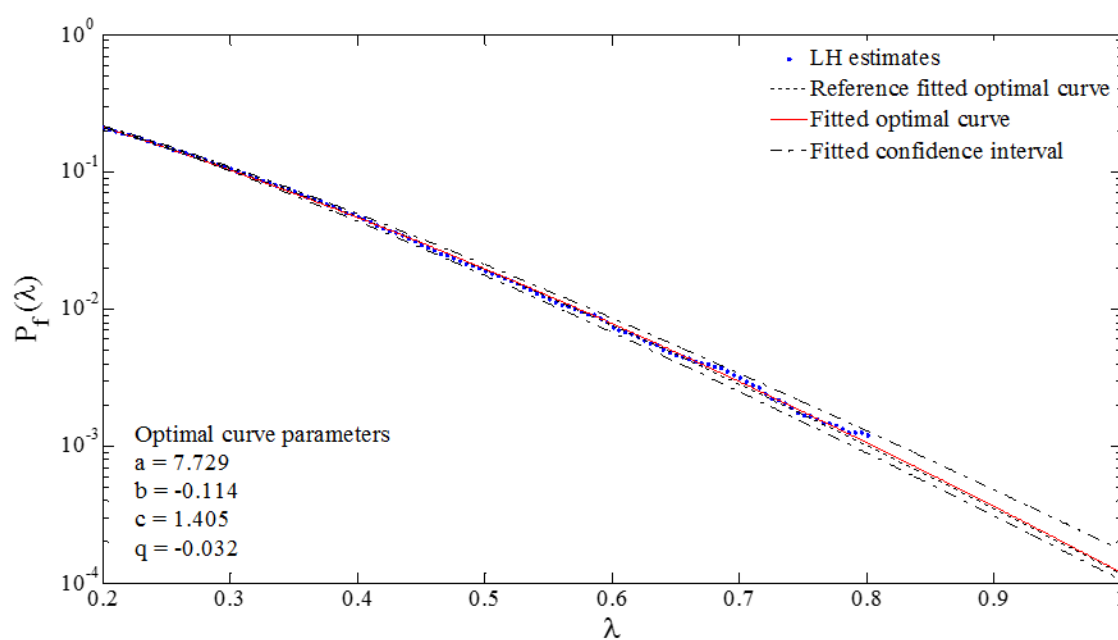
The second approach determines that the estimated probability of failure is accurate enough for a sample size where the estimated value is within the boundaries defined by the confidence intervals predicted for the reference sample scenarios. Using this confidence interval results in a narrower band ensuring that the estimate provided by the smaller sample is accurate.

Bearing in mind that the choice of the most suitable approach should ensure both accuracy and robustness it was decided to use the second criterion in order to define the required sample size. The use of such a criterion leads to the use of a very narrow confidence interval band which guarantees that the estimated probability of failure will be accurate and robust as it needs to be in agreement with the estimates obtained for a significantly larger sample.

To illustrate the robustness of the estimates, an example of the application of the enhanced simulation method, using both MC and LH simulation, for a train speed of 285 km/h is presented in Figure 5.22.



a) Enhanced Monte Carlo



b) Enhanced Latin Hypercube

Figure 5.22 – Assessment of the probability of excessive deck vibration by the enhanced simulation procedure for a train speed of 285 km/h.

Similarly to what was presented for the tail modelling approach, the safety assessment results using the Enhanced Simulation procedure are shown in Table 5.9 and are also compared with the “crude” results. It should also be noted that the estimated confidence interval bounds are also indicated in Table 5.9 as part of the obtained results.

Table 5.9 – Estimated probability of failure due to track instability using the enhanced simulation approach ($\times 10^{-4}$).

Speed (km/h)	MC	LH	ES-MC (C^- ; C^+)	ES-LH (C^- ; C^+)
275	0.20	< 0.1	0.08 (0.06 ; 0.31)	0.08 (0.03 ; 0.22)
280	0.40	0.50	0.44 (0.26 ; 1.47)	0.48 (0.24 ; 1.24)
285	1.40	1.33	1.45 (0.81 ; 3.55)	1.19 (0.82 ; 3.49)
290	5.80	2.67	4.40 (2.94 ; 8.17)	3.76 (2.11 ; 8.24)
295	10.30	8.33	10.80 (7.45 ; 21.1)	10.30 (8.21 ; 14.4)

Again, a good agreement between the results can be observed, regardless of the simulation method used. To compare the efficiency of each simulation method the number of simulations that are required to accurately assess the probability of failure due to track instability using the enhanced simulation procedure are indicated in Table 5.10.

Table 5.10 – Number of simulations required to accurately assess the probability of failure due to track instability using the enhanced simulation approach.

Speed (km/h)	ES-MC	ES-LH
275	3000	9000
280	3000	4000
285	3000	3000
290	3000	3000
295	2000	9000

The results show that for the assessment of the safety due to track instability the use of MC simulation proved to be more efficient. The LH requires twice as many simulations as the MC (28,000 and 14,000 simulations, respectively) for the assessment of the critical speeds. This difference is explained by the multi-modal distribution of the response. When the response is a non-monotonic function the LH tends to lose its efficiency, which was particularly noticed for the trains crossing the bridge at 275 km/h and 295 km/h. In these cases, the efficiency was severely affected and, therefore, the MC proved to be significantly more efficient.

5.3.2.4 Efficiency comparison

As a general comment, it can be noticed that the MC simulation tends to lead to slightly higher estimates of the probability of failure. Regardless of that fact, there is a good agreement between the estimates provided by the different methods.

Comparing the two different LH approaches it can be observed that the ES proves to be more efficient than the GPD approach. Using the ES procedure resulted in reducing the necessary

number of simulations to slightly less than 30% (less 34,000 simulations) for the MC simulation and a little over 50% (less 24,000 simulations) for the LH simulation.

Globally, the safety assessment due to excessive deck acceleration can be accurately analysed with 14,000 simulations by enhanced MC simulation for the critical speed range (275 – 295 km/h). In order to guarantee the track stability, the train speed over the bridge should be limited to 280 km/h.

5.4 Concluding remarks

The assessment of the train-bridge system safety due to track instability led to a few very interesting conclusions. First of all, the fact that the critical speed range from a safety point of view shows the existence of a multi-modal response, due to the existence of two distinct types of behaviour, resonant and non-resonant responses, increases the complexity of the studied problem. The consequence of this is the increase of the necessary number of simulations to accurately estimate the probability of failure, thus increasing the necessary computational times. The fact that the response is not monotonic also impacts the efficiency of some of the selected methodologies, which is shown by the efficiency reduction observed when Latin Hypercube simulation was used.

Secondly, the case study selected showed that, at least for this particular case, the results obtained from a moving loads approach and a train-bridge interaction approach are not directly comparable due to the influence that track irregularities have on the dynamic response of the bridge. It is generally assumed that the moving loads model leads to more conservative results. However, the results obtained as part of the research presented in this dissertation showed a contrasting outcome. The influence of the combination of the bearing flexibility with the increase of the wheel-rail contact forces originated by the track irregularities resulted in a substantial increase of the dynamic amplification, which shifted the general critical section from mid-span (in the moving loads approach) to the quarter span sections (when train-bridge interaction is accounted for). This resulted in an estimated speed limit over the bridge of 295 km/h when using the moving loads approach and a limit of 280 km/h when train-bridge interaction is accounted for. In the preliminary stage it was also interesting to observe that if a more conservative approach was adopted, reducing the safety threshold to 10^{-5} , no significant

difference would be obtained and the speed limit would only decrease 5 km/h to 290 km/h. Furthermore, if the guidelines proposed in EN1991-2 (2003) were adopted, a 40 km/h reduction would be obtained and that would be equivalent to setting the safety threshold at 10^{-8} for the track ultimate limit state.

Finally, it is also interesting to compare the sample sizes required to accurately estimate the probability of failure due to track instability for the different methodologies that have been applied and to notice the efficiency evolution as new approaches have been introduced. It is worth noticing that at an initial stage 100,000 Monte Carlo simulation were performed and after analysing the results a minimum of 60,000 simulations were recommended to accurately estimate the probability of failure for a single train speed. With the development and enhancement of the approach, using the experience acquired in that stage new features were introduced to improve efficiency. As a result the analysis of the whole critical speed range using the sigmoid approach required 64,000 simulations, slightly larger than the value required in the previous stage for a single train speed. The value is even lower if compared to the GPD, reaching 48,000 simulations using the MC method. The introduction of the Enhanced Simulation approach allowed even larger gains, requiring only 14,000 simulations to accurately evaluate the safety of the train-bridge system due to excessive deck vibration, which represents over four times less simulations than the initial value required. This demonstrates that the obtained results, particularly when using the Enhanced Simulation approach, are extremely promising and indicate the feasibility of the application of this type of methodology more frequently due to the reasonable computational costs that are required.

Train running safety assessment

6.1 Introduction

This Chapter is dedicated to the assessment of the train running safety. Like in the previous one, the methodologies presented and discussed in Chapter 3 are applied in order to estimate the probability of failure of the train-bridge system and to establish a speed limit over the bridge in order to guarantee the running safety of the trains.

The several criteria to assess the train running safety have already been discussed in Section 2.4.4, but due to the modelling constraints (only the vertical wheel-rail interaction is taken into consideration and it is assumed that no lateral forces act on the train) in this particular case the analysis is limited to the wheel unloading rate. Two different approaches are used: one where the train is analysed as a whole and the train-bridge system is considered to be unsafe in case a single wheel exceeds the unloading rate threshold; the other one consists on a wheel-by-wheel approach which represents more of an exercise to identify the critical wheels from the safety point of view [Rocha *et al* (2016)].

An overview of the simulation results is also presented and the type of response is compared to that obtained in the analysis of the bridge deck acceleration, discussing the potential consequences of the differences that are observed to the safety assessment.

Finally, in order to assess the train running safety the two most efficient methodologies identified in the track stability assessment are used. The results are presented and again both

methodologies are compared in terms of efficiency and accuracy to identify advantages and drawbacks of each of them and to determine which proves to be the best.

6.2 Safety criterion

Despite the several existing methods to assess the train running safety (previously discussed in Section 2.4.4), in the current dissertation this limit state is assessed through the analysis of the wheel unloading coefficients due to the type of models selected and the constraints associated with a probabilistic analysis. Two different aspects were analysed: firstly, a global analysis is carried out to assess the running safety of the train-bridge system in a realistic perspective, where the train is analysed as a whole and the system is considered to be unsafe in the case of a single wheel exceeds the safety limit; then, simply as an exercise, a wheel-by-wheel analysis is performed and the safety is assessed for each of the train wheels, which enables identifying the critical wheels from the safety point of view.

In the case of the global approach a single value is representative of each simulation and the selection of the representative wheel unloading coefficient is done by either choosing the first value that exceeds the safety limit or, in cases where this limit is not exceeded, the maximum value observed on that simulation. In the wheel-by-wheel approach the representative wheel unloading coefficient for each wheel corresponds to the maximum value obtained on each simulation.

In its most usual form the wheel unloading, η_U , is calculated by:

$$\eta_U = 1 - \frac{Q_{dyn}}{Q_{sta}} \quad (6.1)$$

where Q_{sta} is the static load of each wheel and Q_{dyn} is the dynamic wheel load.

EN14067-6 (2009) limits the unloading to 90% of the static wheel load, corresponding to a 10% safety margin. Since a stochastic analysis is performed and the variability of the train, the track and the bridge is accounted for, no margin was used to assess the train running safety. Thus, in this dissertation the limit for the wheel unloading coefficient is 1. Similarly to what has been

considered in the analysis of the track stability assessment, the train is considered to run safely on the bridge if the estimated probability of failure due to wheel unloading is lower than 10^{-4} , which corresponds to a reference value in JCSS (2001).

6.3 Safety assessment – results and discussion

The safety assessment results are presented and discussed in this section. Initially, an overview of the simulation results is presented and discussed. A comparison between the response obtained in the train running safety analysis and the track stability analysis is also carried out and the influence of the observed differences is analysed and discussed. Next, the assessment of the train running safety is performed using the two methods that proved more efficient in the analysis of the safety due to track instability: the tail modelling approach using the Generalised Pareto Distribution and the Enhanced Simulation procedure. The results obtained for each method are presented and analysed. Finally, through the comparison of the results obtained for each method a discussion on the efficiency of the methodologies is presented.

6.3.1 Simulation results

This Section presents some of the simulation results, providing an overview of the obtained results for the wheel unloading coefficients. A comparison with the results obtained for the analysis of the bridge deck acceleration is also carried out in order to highlight the main observed differences and to discuss how these differences affect the safety assessment.

Similarly to what was done for the bridge deck acceleration, the first results to be presented intend to show how the maximum wheel unloading coefficient evolves with the increase of the train speed. This is shown in Figure 6.1. Again, the solid black line represents the results for the case where all the variables take their mean value whereas the circles correspond to results obtained from MC simulation, illustrating the variability of the dynamic response for each of the different train speeds analysed.

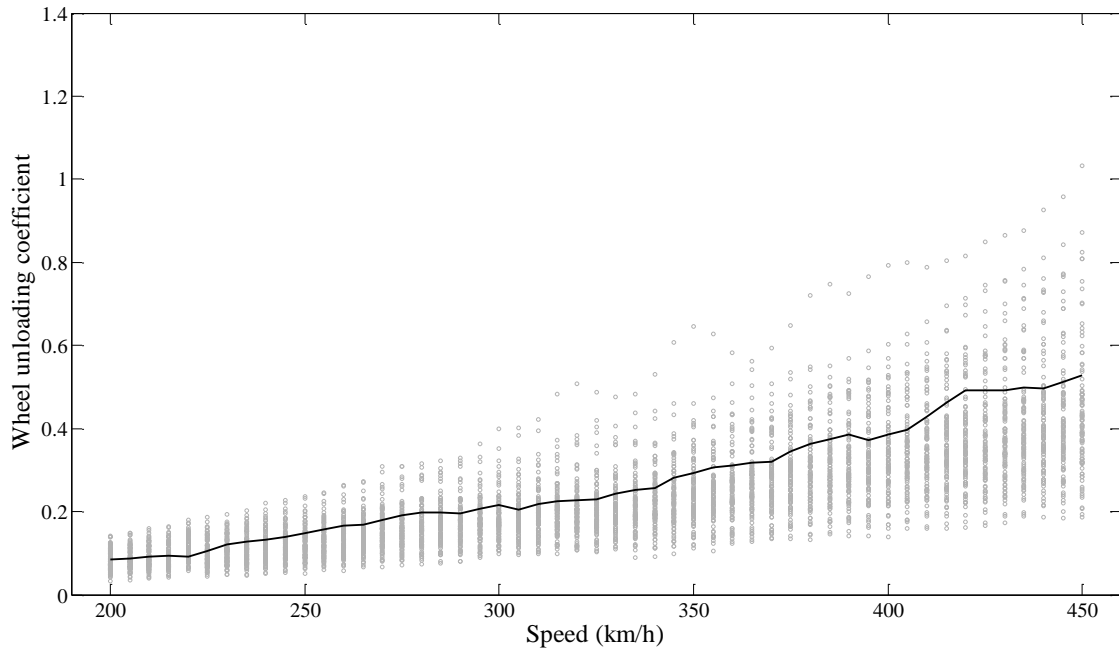


Figure 6.1 – Train speed influence on the maximum wheel unloading rate.

Both the maximum value and the dispersion of the unloading rate show a tendency to increase with speed. However, this is a rather complex phenomenon that depends on a large number of parameters. It is influenced not only by the dynamic response of the bridge (which is affected by both the bridge and track properties) but also by the track irregularities profile. Therefore, establishing a trend line did not prove possible, as the wheel unloading coefficient varies significantly with the change of the dynamic properties of the bridge and the track irregularities profile. However, it can be observed that the track-related variables have more influence on the wheel unloading rate than the bridge-related variables.

In Figure 6.2 the wheel unloading coefficient obtained for each simulation of a sample of 20,000 Monte Carlo simulations is illustrated under a histogram form, for trains crossing the bridge at different speeds.

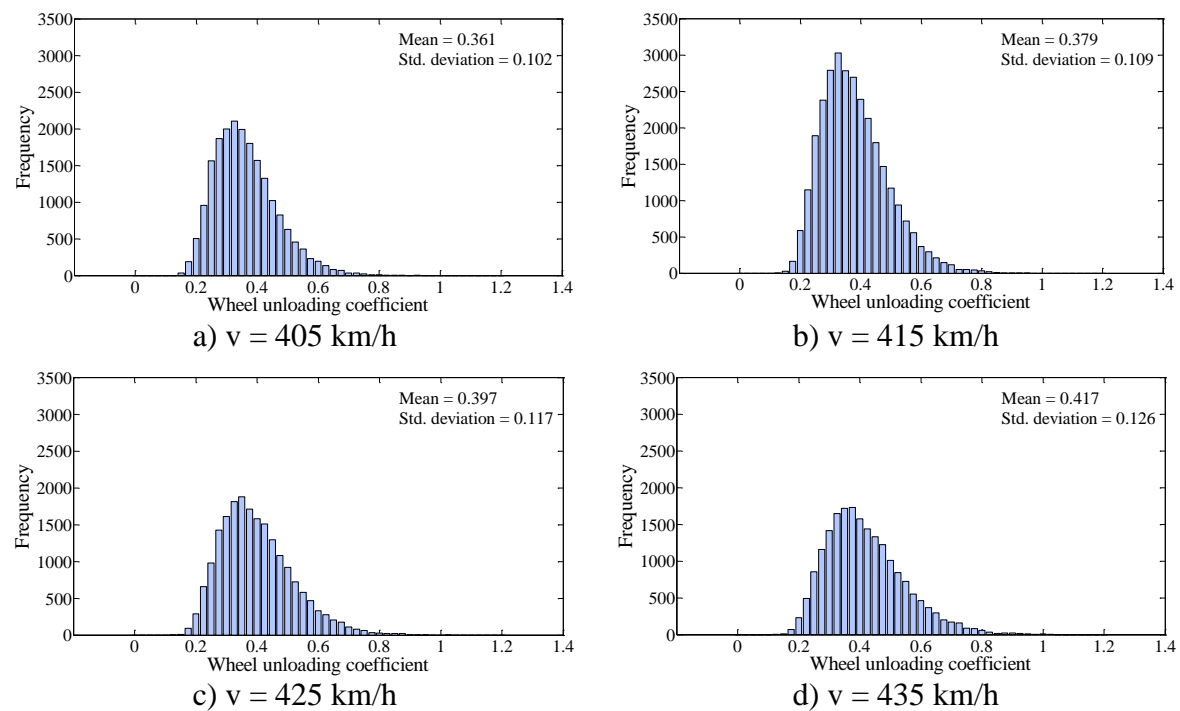


Figure 6.2 – Wheel unloading coefficients for different train speeds.

The results depicted in Figure 6.2 correspond to a global analysis. While the median values do not change significantly with the increase of the train speed, both the mean and standard deviation increase with the increase of the speed. The skewness and kurtosis of the distributions also show significant variations for different train speeds. However, regardless of the train speed, the obtained distribution for the wheel unload coefficient is unimodal unlike what had been observed in the analysis of the bridge deck acceleration (due to the resonant effects, as discussed). For this reason this analysis provides a new possibility to approach the safety assessment as it may be possible to fit a known distribution to the data obtained from the simulations.

In Figure 6.3 the distribution of the wheel unloading coefficient is represented for several wheels of the train crossing the bridge at 425 km/h for a sample of 20,000 Monte Carlo simulations.

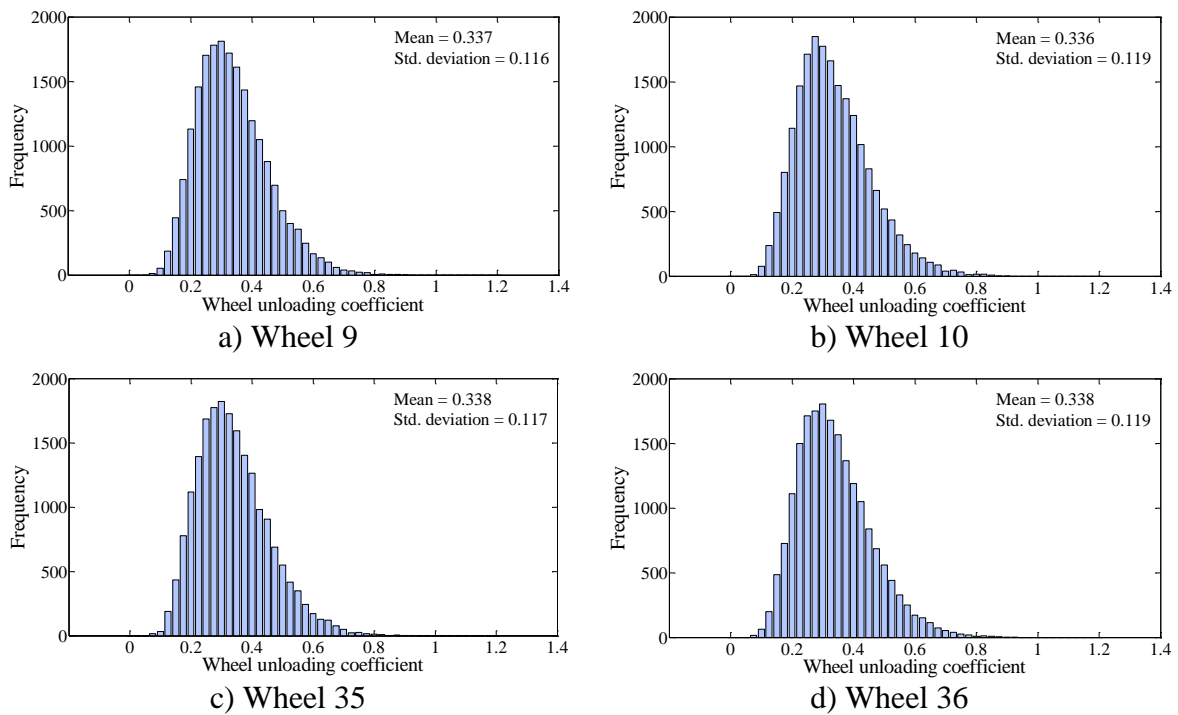


Figure 6.3 – Wheel unloading coefficient distribution for individual wheels.

The results for the several train wheels regarding the distribution are very similar, as can be confirmed by the shape of the curves for each wheel and also by the obtained mean and standard deviation. The only differences occur for the upper tail of the distributions, which govern the estimation of the probability of failure. This is illustrated in Figure 6.4, where the upper tail of the cumulative distribution functions (CDF) obtained for each of the four wheels are represented and the differences can be noted.

Similarly to what was shown for the bridge deck acceleration it is also important to validate the results obtained using different simulation techniques to confirm that they are similar. This is shown in Figure 6.5 where the maximum wheel unloading coefficients for a sample of 50,000 Monte Carlo and 30,000 Latin Hypercube simulations for a train speed of 425 km/h are presented.

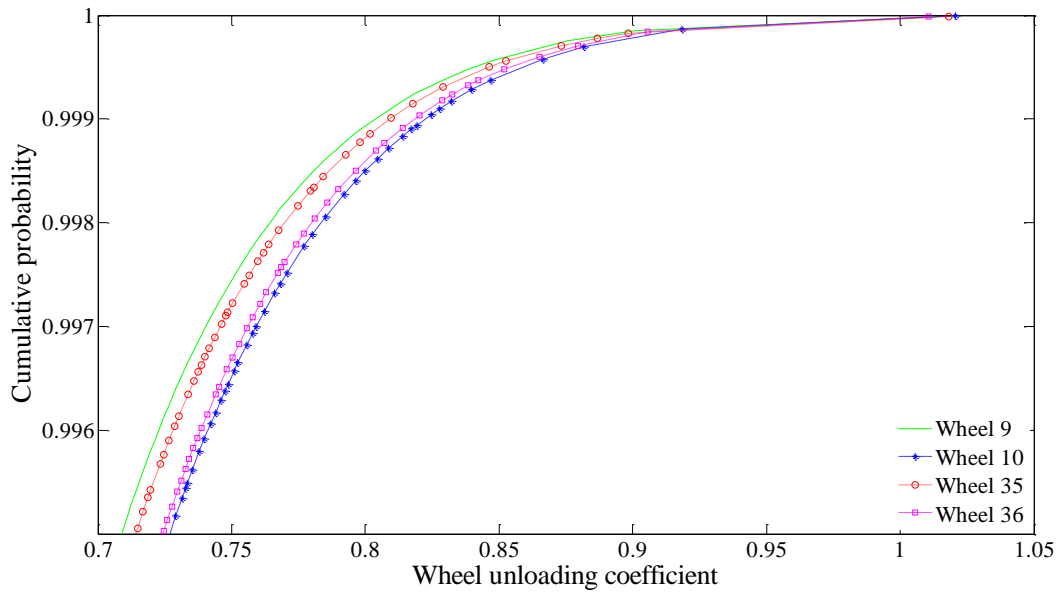


Figure 6.4 – Upper tail of the distributions for the wheel-by-wheel analysis.

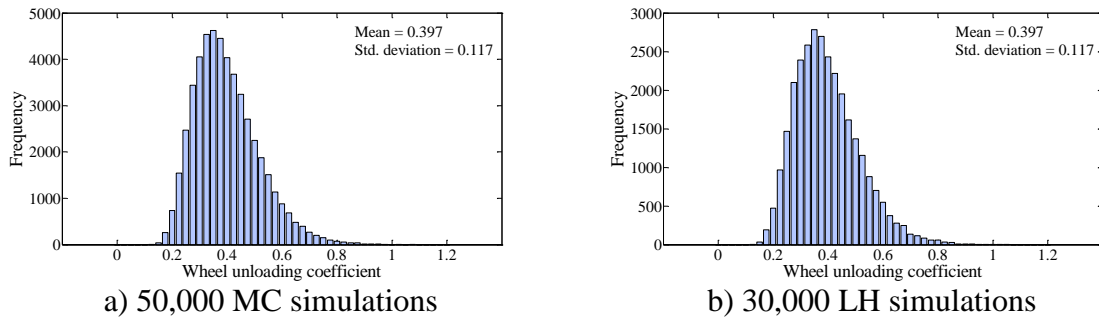


Figure 6.5 – Comparison of the maximum wheel unloading coefficients distribution using different simulation techniques.

Since the simulation results indicate that the obtained distribution is unimodal a new possibility to approach the safety assessment arises as it may be possible to fit a known distribution to the data obtained from the simulations. From a visual analysis the fit of a lognormal distribution seemed to adequately model the data obtained from the simulations and, therefore, was attempted. To assess this possibility tests were carried out in order to confirm that the obtained distribution is lognormal. The Chen-Shapiro test and the Shapiro-Wilk test were performed, as according to Romão *et al* (2010), these are amongst the most suitable tests for asymmetric distributions. However, the results of both tests were similar and led to rejecting the

hypothesis that the wheel unloading coefficient follows a lognormal distribution. For this reason both methods used in the previous Section were applied to assess the running safety of trains due to wheel unloading.

Another interesting observation comes from the quantification of the number of times that each wheel is identified as the one that displays the maximum wheel unloading. An example of the obtained results for train speeds ranging from 405 km/h to 435 km/h is shown in Figure 6.6 for a sample of 20,000 Monte Carlo simulations.

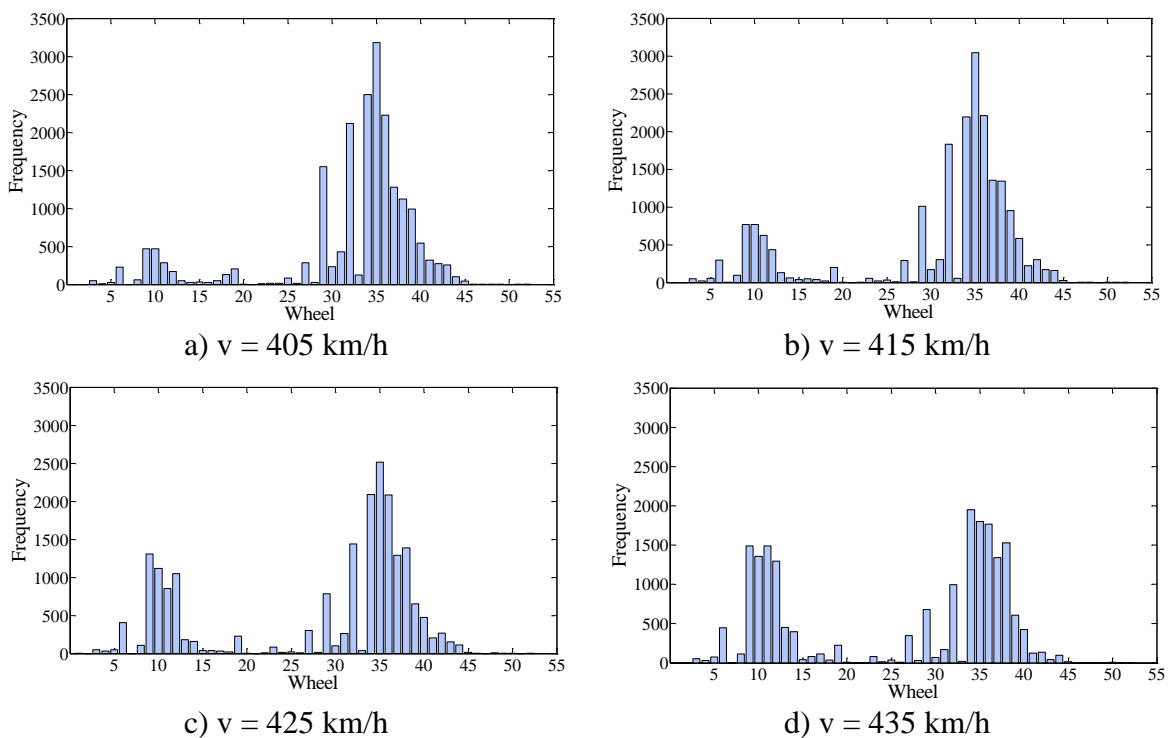


Figure 6.6 – Identification of the wheel with the highest unloading rate.

Again, a trend could be observed regardless of the train speed. Two peaks can be clearly identified: the first is smaller and its centre ranges from wheel 9 to wheel 12; the second peak is the main one and is centred between wheel 34 and wheel 36. The wheel that is more often identified to be the one with the highest unloading is on the second half of the train, shifting from wheel 35 to wheel 34 as the train speed increases. The increase of the train speed also results in the reduction of the influence of the second peak.

6.3.2 Tail modelling approach

Having introduced the results for the wheel unloading rate for different train speeds, providing an overview of how the wheel unloading rate varies along the length of the train and discussing some of the particularities of the safety assessment analysis of the wheel unloading it is now possible to discuss the safety assessment in more detail. The methodologies applied in the assessment of the track stability are used again and the first method to be discussed is the tail modelling approach. Again, the same two different simulation methods used in the previous section are used and the results obtained from them compared and discussed.

An example of the fit of the GPD to the data obtained from the simulations is shown in Figure 6.7 for different train speeds using the Monte Carlo simulation.

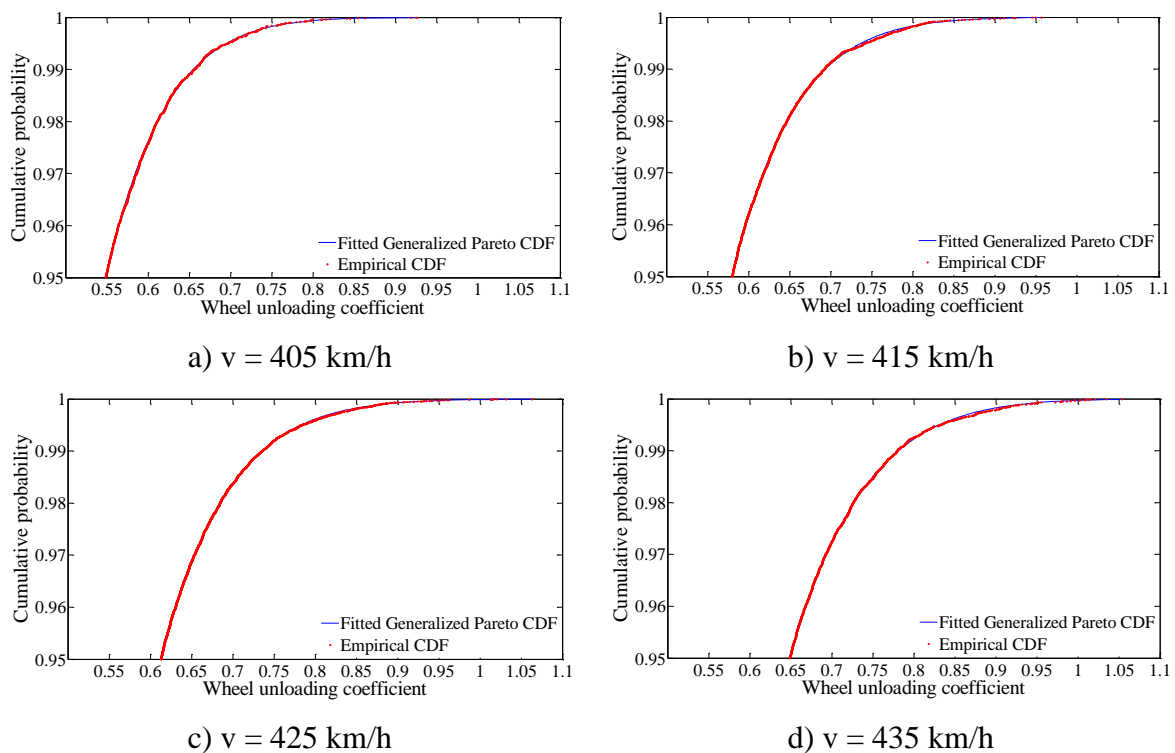


Figure 6.7 – GPD fit to the upper tail of the wheel unloading coefficient distribution.

The fit to the data is very good and this is confirmed by the obtained R^2 index values for the several fits performed, which are always above 0.99, regardless of the train speed that is analysed. Therefore, the accuracy of the estimates is assured.

The results of the estimated probability of failure due to loss of contact between the wheel and the rail are summarised in Table 6.1 and include an estimate of the probability of failure (p_f) by counting the number of failures obtained for both simulation techniques (the results obtained using the counting method are merely indicative, as only 50,000 MC simulations and 30,000 LH simulation were used and an accurate assessment using this approach would require a significantly larger number of simulations).

Table 6.1 – Estimated p_f due to wheel unloading using the tail modelling approach ($\times 10^{-4}$).

Speed (km/h)	MC	LH	GPD-MC	GPD-LH
415	0.40	0.67	0.22	0.57
420	0.80	2.00	0.58	0.96
425	1.60	2.00	1.25	1.13
430	1.80	3.00	1.91	1.83
435	3.00	4.33	3.02	2.41

The required sample size to accurately assess the probability of failure with respect to the train speed for each of the applied simulation methods taking into account the criteria specified in the previous section are presented in Table 6.2.

Analysing the GDP approach in more detail, the results indicate that the combination of the GPD with the Monte Carlo simulation is more efficient than the combination with LH. For the critical speed range the use of MC resulted in reducing the necessary number of simulations to 65% of the number required when using LH, which translates into 20,000 simulations less. The reason for this difference is mainly due to the fit dependency on the most extreme data points. Although the distribution is unimodal the tail is highly irregular. In the particular case of LH simulation the most extreme data only tend to appear for samples larger than 7,000 simulations, as shown in Figure 6.8.

Table 6.2 – Number of simulations required to accurately assess the probability of failure due to wheel unloading using the tail modelling approach.

Speed (km/h)	GPD-MC	GPD-LH
415	9000	9000
420	6000	15000
425	10000	15000
430	5000	9000
435	7000	9000

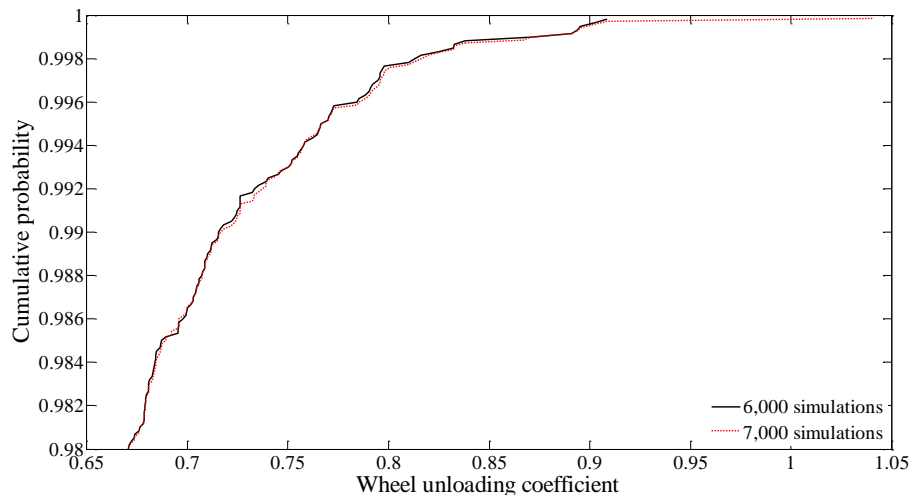


Figure 6.8 – Empirical cumulative distribution functions (CDF) for LH samples with different sizes ($v = 420$ km/h).

This affects the estimated probabilities and, as a consequence, requires larger sample sizes to comply with the variation limit that was used to ensure the accuracy of the estimates. Additionally, the estimated probability of failure for this simulation method was closer to the threshold. This fact also affected the efficiency of the LH method as the estimates were shifting from above and below the threshold and to fulfil the stopping criteria more simulations were required to accurately estimate the probability of failure.

As a merely indicative exercise a wheel-by-wheel safety assessment analysis is performed where the running safety is assessed for each of the train wheels. This makes it possible to identify the critical wheels from the safety point of view. The results obtained from such an analysis are shown in Figure 6.9, where the estimated probability of failure for each of the wheels for a train speed of 435 km/h, which corresponds to the first train speed where at least one of the wheels exceeds the threshold, is presented.

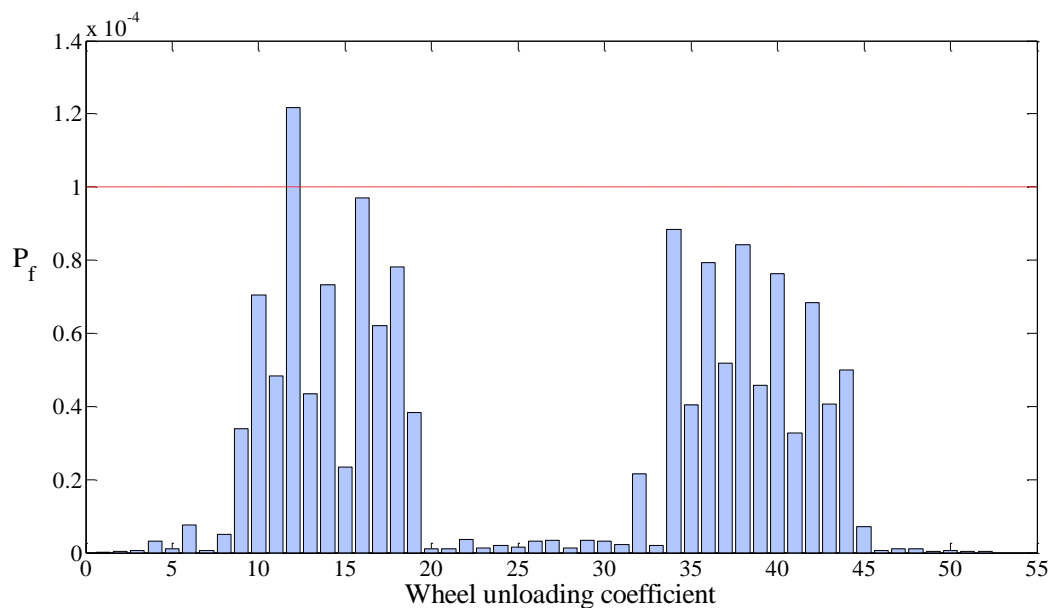


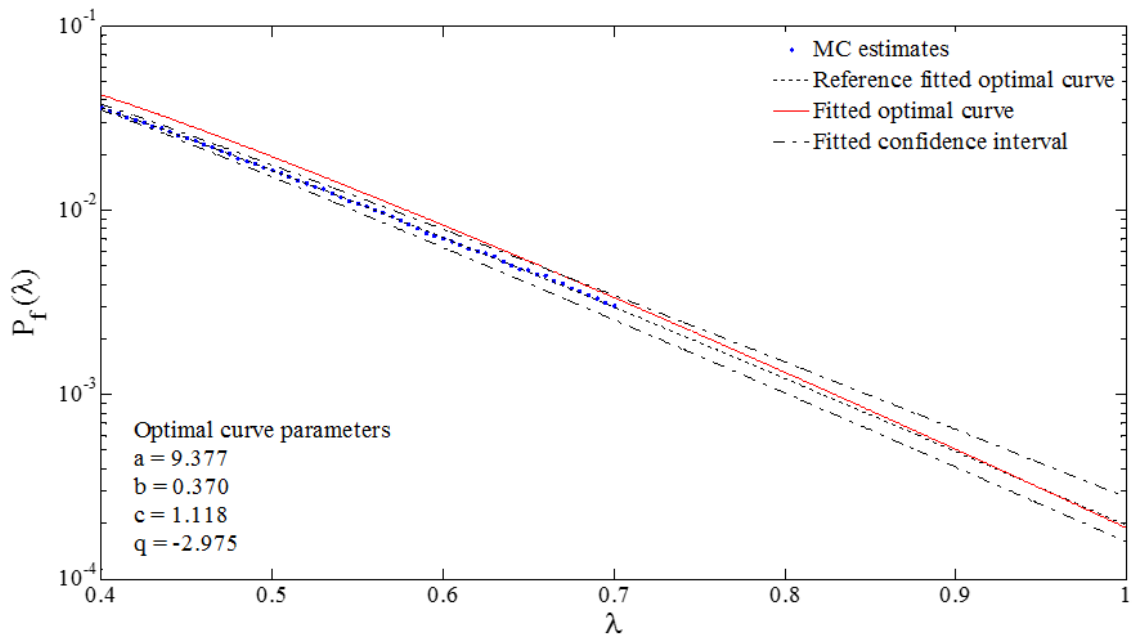
Figure 6.9 – Estimated probability of failure for individual wheels using the GPD approach.

Wheel 12 was identified as the critical one from a safety point of view. However, the wheel that turned out to have the highest unloading the most amount of times for this train speed was wheel 34. This fact is interesting because it shows that for the safety analysis the shape of the upper tail of the distribution (see Figure 6.4), which is highly dependent on the number of cases where the safety limit is exceeded, is more important than the number of times a wheel is identified as the one that displays the highest unloading, which might often occur for situations where the running safety is not at risk.

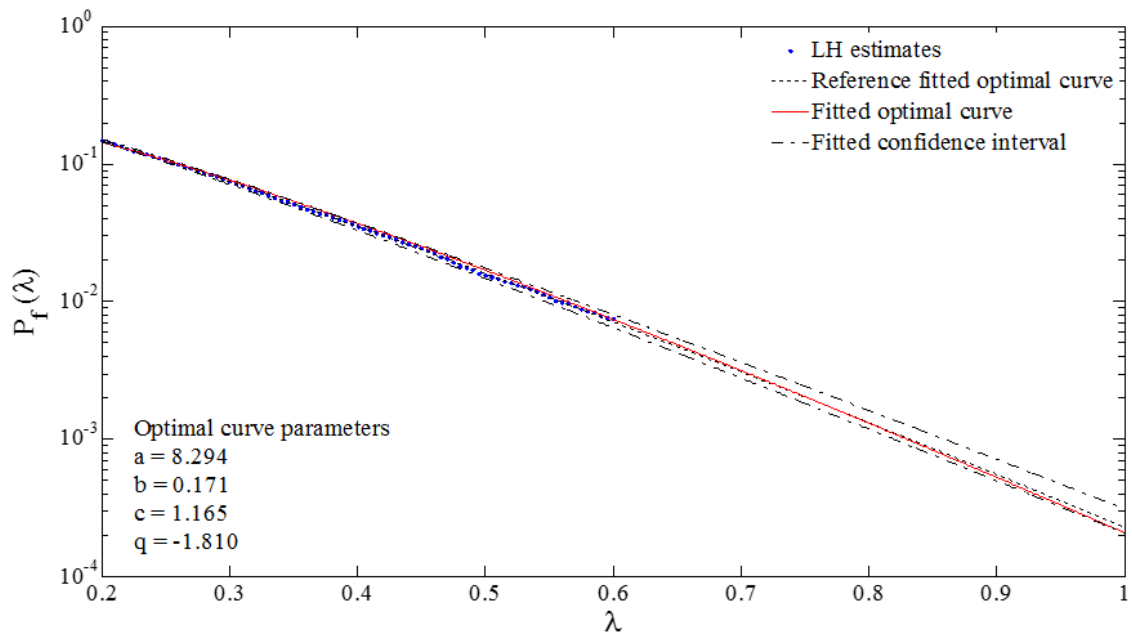
Another interesting observation is that for the same wheelset the second wheel tends to show a higher probability of failure. This can be explained by the increase of vibration associated to the loading of the second axle of the wheelset.

6.3.3 Enhanced Simulation approach

Similarly to what was done for the track stability safety assessment the Enhanced Simulation method is also used. An example of the application of this method for the train speed of 425 km/h is illustrated in Figure 6.10, both for the MC and LH scenarios.



a) Enhanced Monte Carlo



b) Enhanced Latin Hypercube

Figure 6.10 – Assessment of the probability of loss of contact between the wheel and the rail by the ES procedure.

The results of the estimated probability of failure due to loss of contact between the wheel and the rail using the ES method are summarised in Table 6.3.

Table 6.3 – Estimated p_f due to wheel unloading using the enhanced simulation approach ($\times 10^{-4}$).

Speed (km/h)	MC	LH	ES-MC (C^- ; C^+)	ES-LH (C^- ; C^+)
415	0.40	0.67	0.57 (0.26 ; 1.44)	0.27 (0.08 ; 1.07)
420	0.80	2.00	1.03 (0.57 ; 2.36)	1.27 (0.82 ; 3.81)
425	1.60	2.00	1.90 (0.57 ; 5.17)	2.10 (0.98 ; 5.42)
430	1.80	3.00	2.81 (1.10 ; 6.45)	4.67 (2.78 ; 8.94)
435	3.00	4.33	4.51 (2.76 ; 11.2)	7.22 (3.71 ; 14.4)

Similarly to what had been shown in the case of the GPD approach the required sample size to accurately estimate the probability of failure for each of the applied simulation methods is presented in Table 6.4.

Table 6.4 – Number of simulations required to accurately assess the probability of failure due to wheel unloading using the enhanced simulation approach.

Speed (km/h)	ES-MC	ES-LH
415	2000	3000
420	3000	2000
425	3000	2000
430	6000	3000
435	5000	2000

When using the ES method a different conclusion is made regarding the most efficient simulation method when compared to the GPD approach. The combination with LH turned out to be the most efficient method. It is observed that the necessary number of simulations using LH simulations was almost 60% lower (corresponding to 7,000 simulations) than in the case of MC simulation. Since the response is monotonic the use of LH sampling enabled this efficiency increase. This shows that if it is possible to reduce the MC predictions by LH then even further benefits can be obtained by applying the ES method.

6.3.4 Efficiency comparison

As a general comment, there is a good agreement between the estimates provided by the different simulation methods. Unlike what had been observed for the track stability analysis, for the wheel unloading assessment it can be noticed that the LH simulation tends to lead to slightly higher estimates of the probability of failure.

Compared with the GPD results the Enhanced Simulation procedure proves again to be more efficient for both simulation techniques, requiring only nearly 50% of the total number of simulations (18,000 simulations reduction) for the MC and even less, nearly 20% (45,000 simulations reduction), for the LH. The lower dependency of the most extreme data, which show a considerably larger variation and are also less reliable from a statistical point of view, makes the enhanced simulation procedure significantly more efficient than the GPD approach.

Overall, the train running safety assessment can be accurately analysed using only 12,000 simulations in the critical speed range (415 – 435 km/h) by enhanced LH simulation. The analysis shows that in order to ensure the train running safety the speed over the bridge should not exceed 415 km/h.

6.4 Concluding remarks

The safety assessment of the train-bridge system due to wheel unloading also enabled drawing some very interesting conclusions. First of all, the fact that the critical speed range from a safety point of view shows the existence of an unimodal response enables analysing the efficiency and accuracy of the proposed methodologies under significantly different conditions than those that

had been observed in the previous assessment. Despite of that fact, both methods proved once again to be efficient. Since the obtained distribution appeared to follow a lognormal distribution attempts were made to model the data using that type of function. However, adequate tests led to the rejection of this hypothesis and for this reason only the two most efficient methods identified in the previous analysis were applied.

In terms of the number of simulations required to accurately estimate the safety of the train-bridge system it could be observed that there are no significant differences. However, and as could be expected, since the limit state function is less complex for the scenario involving the analysis of the wheel unloading rate, this failure mode required a slightly lower number of simulations.

Since the obtained distribution appeared to follow a lognormal distribution attempts were made to model the data using that type of function as this would probably results in a much more efficient analysis method. However, adequate tests led to the rejection of this hypothesis and prevented the application of such approach.

It was also observed that although the median values do not change significantly with the increase of the train speed, both the skewness and kurtosis of the distributions depend significantly on the train speeds. Furthermore, the existence of some dependency between the bridge dynamic response and wheel unloading rate was observed. However, the wheel unloading is also significantly affected by the track irregularities. Therefore, it can be said that the estimated probability of failure depends on a rather complex combination of both the bridge and track dynamic properties and the irregularities profile.

The wheel-by-wheel analysis, which was simply used as an exercise, showed that for the safety analysis the shape of the upper tail of the distribution is the most important aspect. The shape of the tail is much more dependent on the number of cases where the safety limit is exceeded, rather than the number of times a wheel is identified as the critical one, which might often occur for situations where the running safety is not at risk.

Finally, and comparing the speed limits obtained for the two different failure modes it can be observed that the speed limit to guarantee the running safety is significantly higher than that obtained when analysing the track stability. Therefore, it can be concluded that when no lateral actions are taken into account the safety of the train-bridge system is governed by the deck acceleration, which is related to the track stability.

Conclusions and future developments

7.1 Conclusions

This thesis is focused on assessing the safety of short span railway bridges using robust and efficient probabilistic methodologies. The main objective of the work was the development of a probabilistic methodology that enabled the accurate assessment of the safety of railway traffic over short span bridges while accounting for the variability of the parameters that govern the dynamic behaviour of the train-bridge system. It was also intended to develop adequate numerical models for both the bridge and the train that allowed for an efficient and accurate assessment of the dynamic response. Since the variability of several parameters was taken into account this work also aimed to identify the parameters that govern the dynamic behaviour of this type of bridges and how their intrinsic variability influences the safety of the train-bridge system. Finally, since most of the work previously done in this field is of a deterministic nature, adopting a probabilistic approach was also identified as important because it would help in the understanding of the adequacy of some of the limits defined in the current European standards.

Chapter 2 was dedicated to the analysis and discussion of the dynamic behaviour of railway bridges. An overview of the most significant parameters for the dynamics of railways bridges was presented. Special attention is given to resonance, which is a particular problem in the dynamics of high-speed railway bridges, due to its significance and importance in the behaviour of short to medium span railway bridges. The chapter also includes a review of the most common numerical methods to evaluate the dynamic response of bridges subjected to

train loading, as the selection of an adequate methodology influences the accuracy of the assessment. This review highlights the main advantages and disadvantages of each methodology in terms of the relationship between accuracy and computational costs, as this is a key factor for the selection of the most appropriate method when performing a probabilistic analysis. In the last section of the chapter, the current limit states defined in the European standards to ensure the structural and traffic safety in the high-speed railway network are presented along with a review of some of the most up to date studies and recommendations about this topic. This enables the discussion of possible changes that could be made to the current guidelines in future versions, particularly to the vertical bridge deck acceleration limits.

Since the main objective of this dissertation was to develop an efficient and accurate probabilistic methodology to assess the safety of railway bridges while accounting for the variability of the parameters that govern their dynamic behaviour of the train-bridge system it was fundamental to review the basic concepts of structural reliability. This was presented in Chapter 3 along with some of the most common approaches and methodologies used to address these problems in Civil Engineering. Based on this review and taking into account the complexity of the problem being studied it was concluded that the use of Level III methods is the most appropriate approach. Since the response of the train-bridge system is not likely to be unimodal due to resonance effects the use of response surface methods is also less attractive than the use of simulation techniques. In order to enhance the efficiency of the simulation methods they are combined with other techniques in order to reduce the necessary number of simulations. The methodologies employed are presented in Section 3.4 and the main advantages and drawbacks of each of them are discussed. One of the methods is a tail modelling approach based on the extreme value theory that uses adequate functions to model the tail of the obtained distribution. The other one is an Enhanced Simulation procedure which uses an approximation procedure based on the estimates of the failure probabilities at moderate levels for the prediction of the far tail failure probabilities by extrapolation. These techniques are combined with two simulation methods, the Monte Carlo and the Latin Hypercube, to enhance the efficiency of the assessment.

To test the efficiency and accuracy of the different methodologies that are proposed they are applied to a case study bridge. The selected bridge is a ballasted filler beam structure composed by six simply supported spans of 12 m each. The option for this structure relies on the fact that this is a very common structural solution for small span bridges that compose the current

European high-speed railway network. The case study bridge is presented in detail in Chapter 4. The different numerical models used to represent and study the bridge and the train are also discussed in detail in this chapter along with a description of the methodology used to artificially generate random track irregularity profiles.

Regarding the numerical modelling of the bridge it was possible to conclude that using a track extension of 20 m outside the bridge limits is sufficient to adequately model the dynamic behaviour of the train over the structure, as this extension enables the dissipation of all the dynamic effects that arise from the introduction of the train in the numerical analysis. Additionally, it was also possible to observe that using a two layer track model that accounts for the shear behaviour of the ballast layer is sufficient to obtain accurate results of the response of the train-bridge system. It is necessary to highlight the importance of accounting for the shear behaviour of the ballast layer to accurately assess the dynamic response of the bridge. It was observed that the variation from loaded to unloaded track has a small impact on the dynamic response. However, when the track-bridge composite effect is neglected significant changes can be noted and the dynamic amplification is greatly increased.

It was also observed that a specific aspect of the case study bridge is the flexibility displayed by its bearings. This was particularly important when the train-bridge interaction was taken into consideration and significantly affected the dynamic response of the train-bridge system. Accounting for the track irregularities led to a significant increase of the wheel-rail contact forces. The combination of this increase in the contact forces with the bearing movement, associated with the second vibration mode shape of the bridge, results in an important amplification of the dynamic response in the quarter span sections. The conclusion is that the behaviour of the quarter span sections is mostly controlled by impact forces that tend to grow proportionally to the train speed, whereas the mid span section is controlled by resonant effects related to the first bending mode of the bridge and therefore is not so significantly affected by the presence of track irregularities. This indicates that at least for some specific cases the results obtained from a moving loads approach and a train-bridge interaction approach are not directly comparable, particularly if the track irregularities are taken into consideration. Moreover, in terms of safety assessment these effects changed the critical section from the mid-span to the three quarter span section of the bridge with respect to the analysis of the track stability.

A review of the different train numerical models commonly used in the literature is also presented in Chapter 4 and the model identified to be most suitable and adequate to the safety

analysis is presented and the reasons for its choice explained. As previously highlighted by other authors, this work also confirmed that the main effects of train-bridge interaction can be adequately captured using simplified interaction models. The selection of the most appropriate model is therefore dependent on the problem being studied and on the type of results to be analysed.

The variability of parameters related to the bridge, the track and the train have been taken into account, as well as the existence of track irregularities representing different track maintenance levels. All the random variables selected in this dissertation have been presented and discussed within Chapter 4. A sensitivity analysis was performed in order to understand the influence that each of the basic random variables has on the dynamic behaviour of the train-bridge system. In terms of dynamic response of the bridge, it was observed that the bridge-related variables show a significantly greater influence on the dynamic response than the track-related variables. This is to be expected and it can be shown that the section inertia, the area of the ballast layer (which influences the structural mass), the concrete elasticity modulus and the vertical stiffness of the bridge bearings are the parameters that most affect the variability of the bridge response.

With respect to the wheel unloading rate the opposite is observed and it could be concluded that the wheel unloading is more affected by track-related variables than by the bridge-related variables. The stiffness of the rail pads is the variable that exhibits the highest influence on the wheel unloading rate. The ballast elasticity modulus, the height of the ballast layer and the ballast load distribution angle (which are variables that influence the way that the wheel loads are transmitted to the track) are other track-related parameters that show a significant influence on the wheel unloading. Regarding the bridge-related variables the neoprene shear modulus, which influences the vertical stiffness of the bridge bearings, is the parameter with the highest influence on the wheel unloading rate. The concrete elasticity modulus and the variation of inertia due to geometric variation of the concrete section (both related with the stiffness of the bridge) are other variables with some influence on the wheel unloading. These results indicate that the wheel unloading rate is particularly sensitive to variations of the track properties.

In Chapters 5 and 6 the probabilistic methodologies were applied to the safety assessment of the Canelas Bridge analysing the track stability and the train running safety, respectively. Regarding the track stability assessment a preliminary analysis was carried out using a moving loads approach and taking into consideration only the variability of the bridge parameters. The

main objectives of this preliminary analysis were to test the applicability of simulation methods to assess the safety of the train-bridge system and to identify the parameters governing the dynamic behaviour. With respect to the latter, the results confirmed those obtained in the sensitivity analysis, enabling the validation of the conclusions drawn by that analysis. Regarding the analyses with the simulation methods, they showed that the initial approach was excessively optimistic and that even for the largest sample size scenario of 5,000 Monte Carlo simulations the safety could not be accurately assessed. This could be explained by the existence of the two distinct types of structural response (resonant and non-resonant behaviour) which significantly increased the complexity of the studied problem in the critical speed region. The fact that the response is not monotonic also affects the efficiency of the Latin Hypercube method, resulting in an increase of the number of required simulations, as was shown in subsequent stages of the work. As a result of the insufficient number of extreme points on the upper tail of the distribution, it was not possible to adequately estimate the probability of failure. This demonstrated that a refinement of the simulation in the interest area was required. Taking into account that the safety threshold was set at 10^{-4} it was decided to carry out 100,000 Monte Carlo simulations for train speeds ranging from 285 km/h to 300 km/h to accurately assess the safety of the train-bridge system. As expected, the increase in the number of simulations led to a better representation of the upper extremity of the cumulative distribution function and allowed an accurate estimation of the probability of failure. The error analysis of the results indicated that a minimum of 60,000 Monte Carlo simulations are recommended. However, this number depends on the intended/desired accuracy, so other choices may be valid as well.

The results of the preliminary assessment showed that in order to guarantee the track stability the speed over the bridge should be limited to 295 km/h. Another interesting observation from the preliminary analysis was the fact that if a more conservative approach was adopted, reducing the safety threshold to 10^{-5} , no significant difference would be obtained and the speed limit would only decrease 5 km/h to 290 km/h. Finally, a comparison between the results obtained when following the guidelines proposed in EN1991-2 (2003) and when adopting a probabilistic approach was also carried out. This comparison showed that the use of a safety factor of 2 leads to a speed limit which is 40 km/h lower than the limit obtained when considering the acceleration limit for the instability of the ballast layer observed in laboratory tests, which is equivalent to adopting a safety threshold of 10^{-8} for the track ultimate limit state.

Using the knowledge acquired during the preliminary assessment a more sophisticated and complete analysis was carried out in which the train-bridge interaction effects are taken into consideration along with the variability of parameters related to the bridge, the track and the train, as well as accounting for the existence of track irregularities. The developed probabilistic methodologies were applied and additional criteria were defined in order to guarantee the accuracy of the results and enhance the efficiency. Overall, these criteria prove to be extremely useful to direct the computational effort to the speeds where the safety of the train-bridge system is at risk. The success can be confirmed by the analysis of scenarios where the probability of failure is one order of magnitude apart from the safety threshold. In these cases a reduced number of simulations is required, which results in the identification of the critical trains speeds with a reduced computational cost, achieving one of the desired goals. With respect to the Enhanced Simulation method, accuracy was the key aspect to control and a criterion needed to be established to determine how to manage it. Therefore, the estimated probability of failure is accurate enough for a sample size if its value is within the boundaries defined by the confidence intervals predicted for the reference samples scenarios (50,000 simulations for the MC simulation and 30,000 simulations for the LH method). The use of a very narrow confidence interval band, based on estimates of significantly larger samples, guarantees the robustness and accuracy of the estimated probability of failure. Another indicator of the accuracy and robustness of the results is given by the similarity of the estimates obtained using different probabilistic methodologies and different simulation methods.

Regarding the safety assessment, it was possible to conclude that in order to guarantee the track stability the train speed over the bridge should be limited to 280 km/h. This corresponds to a reduction of 15 km/h from the results obtained from the analysis using the moving loads method. The difference is due to the previously explained impact of the introduction of rail irregularities in the analysis.

In terms of efficiency comparison between the several applied methodologies it can be concluded that the ES approach is the most efficient. Using the ES procedure resulted in reducing the necessary number of simulations to slightly less than 30% (less 34,000 simulations) for the MC simulation and a little over 50% (less 24,000 simulations) for the LH simulation when compared to the GPD approach. The combination of the ES procedure with MC simulation was the most efficient method to assess the track stability safety. This shows that when the response is not monotonic the efficiency of LH simulation is reduced which is

reflected in the increase in the necessary number of simulations when compared to the results obtained using MC simulation. Besides being more efficient it is also important to note that the ES procedure benefits from another important feature which is that it enables the uncertainties of the estimated probability of failure to be quantified. This provides an extra tool to assess the quality of the failure estimation and was used to ensure the accuracy and robustness of the results.

It is also worth mentioning the evolution in terms of efficiency of the different methodologies that have been applied throughout the work presented in this dissertation. At an initial stage 100,000 MC simulations were used to accurately estimate the probability of failure for a single train speed. With the introduction of new methodologies and the enhancement of the approaches this value gradually decreased. Moreover, it was possible to accurately evaluate the safety of the train-bridge system due to excessive deck vibration using only 14,000 simulations for the critical speed range through the application of the ES procedure combined with MC simulation.

To test the efficiency and applicability of the proposed methodologies to different limit states they were also applied to the assessment of the train running safety, which is described in Chapter 6. Taking into account the selected numerical model the running safety of trains was analysed for the case of loss of contact between the wheel and the rail and it was assumed that no lateral forces act on the train. The analysis of the simulation results for the wheel unloading coefficient show that unlike what had been observed for the other limit state, the wheel unloading rate presents a unimodal distribution. This is particularly relevant as it enables the performance of the different methodologies for limit state functions with different degrees of complexity to be assessed.

Two distinct analyses were carried out during the study of the running safety: a global analysis, which enabled assessing the safety of the system and defining a speed limit, and a wheel-by-wheel analysis, used merely to identify the critical wheel from a safety point of view. The global analysis concluded that the train speed must be limited to 415 km/h in order to ensure the running safety of the trains over the bridge. It was also observed that although the median values do not change significantly with the increase of the train speed, both the skewness and kurtosis of the distributions depend significantly on the train speed. Furthermore, the existence of some dependency between the bridge dynamic response and wheel unloading rate was observed. However, the wheel unloading is also significantly affected by the track

irregularities. Therefore, it can be noted that the estimated probability of failure depends on a rather complex combination of both the bridge and track dynamic properties and the irregularities profile.

Compared to the results obtained in previous research stages it can also be observed that the speed limit to guarantee the running safety is significantly higher than that obtained when analysing the track stability. Therefore, it can be concluded that when no lateral actions are taken into account the safety of the train-bridge system is governed by the deck acceleration, which is related to the track stability.

Analysing the efficiency of the different probabilistic methodologies and starting with the GPD approach it is observed that its combination with Monte Carlo simulation is more efficient than the combination with LH. For the critical speed range the use of MC resulted in reducing the necessary number of simulations to 65% of the number required when using LH, which translates into 20,000 simulations less. This is due to the estimated probability of failure for the LH method being very close to the safety threshold for the critical speeds. The estimates were shifting from above and below the threshold and to fulfil the stopping criteria more simulations were required to accurately estimate the probability of failure which ended up affecting the efficiency of the method.

Regarding the ES method the opposite situation is observed and the combination with LH turned out to be the most efficient method. It is observed that the necessary number of simulations using LH simulations was almost 60% lower (corresponding to 7,000 simulations) than in the case of MC simulation. Since the response is monotonic the use of LH sampling enabled this increase in efficiency. This shows that if it is possible to reduce the MC predictions by LH then even further benefits can be obtained if the ES method is applied.

Comparing the two methods, the ES approach proves once again to be the most efficient. Compared to the GPD results the ES procedure proves to be more efficient for both simulation techniques, requiring only nearly 50% of the total number of simulations (18,000 simulations reduction) for the MC and even less, nearly 20% (45,000 simulations reduction), for the LH. The lower dependency of the most extreme data, which show a considerably larger variation and are also less reliable from a statistical point of view, makes the enhanced simulation procedure significantly more efficient than the GPD approach.

The train running safety assessment can be accurately analysed using only 12,000 simulations in the critical speed range (415 – 435 km/h) by enhanced LH simulation and the train speed should be limited to 415 km/h.

Another extremely interesting analysis can arise from the efficiency comparison for both limit states analysed within each of the applied methodologies. Regarding the GPD approach it can be noted that its combination with MC simulation proved to be the most efficient method to assess both the train running safety and the safety due to track instability. However, for the latter failure mode the LH results were significantly influenced by one particular train speed where the probability of failure was extremely close to the safety threshold and by the fact that response is not monotonic. For this reason this speed required almost 50% of the total number of simulations for the analysis of the critical speed range. Overall the results indicate that this approach is much more sensitive to the level of irregularity of the tail of the distribution rather than the complexity of the limit state function.

As for the ES procedure the combination with the LH proved to be the most efficient approach to assess the train running safety, whereas the use of MC simulation proved to be more effective for the safety assessment due to track instability. As previously stated, this shows that in non-monotonic limit state functions the use of LH reduces the efficiency of the ES method.

Globally, the ES procedure proved to be significantly more efficient than the GPD approach. Compared to the GPD procedure the ES required only 32% of the total number of simulations (25,000 simulations less) to assess the train running safety and 29% (34,000 simulations less) for the assessment of the safety due to track instability. The total number of simulations required to accurately assess the safety for each failure mode were 12,000 and 14,000 simulations, respectively. Comparing the number of simulations required to accurately assess the safety of the train-bridge system for each of the limit states analysed, it can be observed that the analysis of the track stability requires 16% more simulations (2,000 simulations) than those required to analyse the train running safety. These results enable the influence of the limit state function on the required number of simulations to be commented on. As it could be expected, the more complex the limit state the higher the number of simulations required to assess the safety of the train-bridge system. It also became evident that if the response is not monotonic the LH affects the efficiency of the ES procedure. Furthermore, it is shown that if the computational costs can be reduced through a refined simulation method then the use of the ES approach will result in even further benefits.

7.2 Future developments

The work presented in this thesis is focused on the development of probabilistic methodologies that accurately assess the safety of high-speed trains running on short span bridges. However, during the course of this research several other lines of investigation were identified as being worthy to study in the future in the author's opinion. Some of these topics are summarised in the following paragraphs:

- a) During the work presented in this thesis the focus was always directed to the analysis of different probabilistic methodologies to analyse the safety of the train-bridge system. Even though the current dissertation analysed two different limit states with different degrees of complexity, the analysis was limited to a single case study. It would be interesting to investigate how the proposed methodologies behave when applied to a different structure. This would be a further validation of the applicability of the methodologies, demonstrate their versatility and confirm their robustness and efficiency.
- b) It would be interesting to apply these methodologies in the assessment of an existing structure. One of the options assumed during this dissertation was the fact that it is intended to deal with the analysis of a structure during the design stage. This is reflected by the large uncertainty of all the parameters related to the structure, the track and even the train. In the author's opinion this line of investigation presents two particularly important goals: one is to analyse how reducing the uncertainty in the parameters that govern the train-bridge dynamic response (possible if a preliminary study of the structure is carried out and some of its material properties are obtained through adequate testing) influences the efficiency of the methodologies; the other is the possibility to address one of the most pressing Engineering challenges in the near future which is the assessment of the capacity of existing structures. This type of probabilistic tool could be extremely useful to the management of the existing railway networks and could prove decisive to the selection of the most appropriate future intervention on the structure (either strengthening or replacement), while reflecting the remaining life of the structure through an adequate selection of the admissible probability threshold. The applicability of this type of tool is not limited to this sort of analysis and can be extended to fit other purposes such as the analysis of the network capacity to be updated for higher train speed or heavier traffic. Therefore, the author believes that the application of the proposed

methodologies to the assessment of existing structures offers several extremely interesting lines of investigation that are worth pursuing in the future.

- c) Another improvement that can be made in the future is the inclusion of lateral interaction in the safety assessment analysis. At this stage the simplifications made were required as the computational costs associated with 3D analysis are still excessively high. However, recent research carried out within the FEUP group by Montenegro (2015), which dedicated his research to the development of a complete train-bridge interaction tool, may overcome this problem in the near future. The combination of the outputs presented in the current thesis with those presented by Montenegro (2015) can enable the probabilistic assessment of the safety of the train-bridge system while accounting for a complete train-bridge interaction. This would offer the possibility to assess the influence that lateral forces acting on trains (such as earthquakes, crosswinds or simply centrifugal forces due to curves) have on the safety of the train-bridge system and would enable identifying the level of force for which the safety of the system starts being governed by the train stability instead of being limited by the bridge dynamic behaviour.
- d) Finally, since all the limit states discussed represent ultimate limit states regarding the train running safety it would also be interesting to apply the probabilistic methodologies in the analysis of serviceability limit states, in particular the passengers riding comfort which is of the utmost importance for the train operating companies. Carrying out an accurate assessment of the passenger riding comfort requires developing more sophisticated models of the train which include modelling the seat and potentially may also include modelling the passenger-seat interface and the passenger itself. However, since these limit states are characterised by a higher admissible probability threshold they required fewer simulations to be analysed, thus enabling the applicability of the methodologies proposed in this thesis. This topic may lead to a better understanding of the influence of the track quality on the riding comfort and may enable the optimisation of the track maintenance programme.

References

A

- Acar, E. (2011) - "*Guided tail modeling for efficient and accurate reliability estimation of highly safe mechanical systems*", Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, Vol. 225, Iss. 1237-1251.
- Andersson, E., Haggstrom, J., Sima, M. and Stichel, S. (2004) - "*Assessment of train-overturning risk due to strong cross-winds*", Journal of Rail and Rapid Transit, Vol. 218, pp. 213-223.
- Ahlbeck, D.R., Meacham, H.C. and Prause, R.H. (1978) - "*The development of analytical models for railroad track dynamics*", in *Railroad Track Mechanics & Technology*, Pergamon Press, Oxford.
- Akin, J. and Mofid, M. (1989) - "*Numerical solution for response of beams with moving mass*", Journal of Structural Engineering, Vol. 115, Iss. 1, pp. 120-131.
- Alves Ribeiro, C. (2012) - "*Transições aterro-estrutura em linhas ferroviárias de alta velocidade : análise experimental e numérica*", PhD Thesis, Universidade do Porto - Faculdade de Engenharia.
- Antolín, P. (2013) - "*Efectos dinámicos laterales en vehículos y puentes ferroviarios sometidos a la acción de vientos transversales*", PhD Thesis, Universidad Politécnica de Madrid, Madrid, Spain.
- Antolín, P., Zhang, N., Goicolea, J.M., Xia, H., Astiz, M.A. and Oliva, J. (2013) - "*Consideration of nonlinear wheel-rail contact forces for dynamic vehicle-bridge interaction in high-speed railways*", Journal of Sound and Vibration, Vol. 32, pp. 1231-1251.
- Au, F.T.K., Cheng, Y.S. and Cheung, Y.K. (2001) - "*Vibration analysis of bridges under moving vehicles and trains: an overview*", Progress in Structural Engineering and Materials, Vol. 3, pp. 299-304.
- Au, F., Wang, J. and Cheung, Y.K. (2002) - "*Impact study of cable-stayed railway bridges with random rail irregularities*", Engineering Structures, Vol. 24, Iss. 5, pp. 529-541.
- Azevedo, A. (2012) - "*FEMIX 4.0 - Finite Element Analysis*", <http://www.alvaroazevedo.com/femix/>.

B

- Baeßler, M., Bronsert, J., Cuéllar, P. and Rücker, W. (2012) - "*The stability of ballasted track supported on vibrating bridge decks, abutments and transition zones*", in Railways 2012 - First International Conference on Railway Technology: Research, Development and Maintenance, Civil-Comp Press, Las Palmas de Gran Canaria, Spain.
- Beachkofski, B. and Grandhi, R. (2002) - "*Improved Distributed Hypercube Sampling*", in American Institute of Aeronautics and Astronautics Paper 1274.
- Bjerager, P. (1990) - "*Methods for structural reliability computations*", in Course on: General principles and applications on Mechanics of Solids and Structures, International Center for Mechanical Sciences, Udine.
- Bonifácio, C. (2012) - "*Comportamento dinâmico de uma ponte ferroviária de pequeno vão para tráfego de alta velocidade*", Tese de Mestrado, Universidade do Porto - Faculdade de Engenharia.
- Boos, D. (1984) - "*Using extreme value theory to estimate large percentiles*", Technometrics, Vol. 1, pp. 33-39.
- Brodig, W.C., Diedrich, F.W. and Parker, P.S. (1964) - "*Structural optimization and design based on reliability design criterion*", Spacecraft, Vol. 1, Iss. 1, pp. 56-61.

C

- Caers, J. and Maes, M.A. (1998) - "*Identifying tails, bounds and end-points of random variables*", Structural Safety, Vol. 20, pp. 1-23.
- Calçada, R. (1995) - "*Efeitos dinâmicos em pontes resultantes do tráfego ferroviário a alta velocidade*", Tese de Mestrado, Faculdade de Engenharia da Universidade do Porto, Porto.
- Cantieni, R. (2009) - "*Application of ambient vibration testing (operational modal analysis) in practice*", European Research Project DETAILS.
- Carlbohm, P. (2000) - "*Carbody and passengers in rail vehicle dynamics*", PhD Thesis, KTH - Royal Institute of Technology, Stockholm, Sweden.
- Carrarini, A. (2006) - "*Reliability based analysis of the crosswind stability of railway vehicles*", PhD Thesis, Technical University Berlin, Berlin, Germany.
- Carvalho, M. (2011) - "*Comportamento Dinâmico de uma Ponte Ferroviária de Pequeno Vão para Tráfego de Alta Velocidade*", Tese de Mestrado, Faculdade de Engenharia da Universidade do Porto, Porto, Portugal.
- Castillo, E. (1988) - "*Extreme value theory in engineering*", Academic Press, San Diego, USA.

-
- Chellini, G. and Salvatore, W. (2007) - "*Updated models for steel-concrete composite HS railway bridges*", in Experimental Vibration Analysis for Civil Engineering Structures - EVACES'07, Porto, Portugal.
- Cho, T., Song, M.K. and Lee, D.H. (2010) - "*Reliability analysis for the uncertainties in vehicle and high-speed railway bridge system based on an improved response surface method for nonlinear limit states*", Nonlinear Dynamics Vol. 59, pp. 1-17.
- Chojaczyk, A.A., Neves, L.C. and Cardoso, J.B. (2012) - "*Fiabilidade de estruturas usando redes neuronais e o método de Monte Carlo*", in 4º Encontro Nacional de Riscos, Segurança e Fiabilidade, Instituto Superior Técnico, Lisboa.
- Chopra, A.K. (1995) - "*Dynamics of structures: theory and applications to earthquake engineering*", Prentice-Hall International.
- Claus, H. and Schiehlen, W. (1998) - "*Modelling and simulation of railway bogie structural vibrations*", Vehicle System Dynamics, Vol. 29, Iss. S1, pp. 538-552.
- Clough, R.W. and Penzien, J. (1993) - "*Dynamics of structures*", McGraw Hill, New York.
- Cornell, C.A. (1969) - "*A probabilistic based structural code*", American Concrete Institute, Vol. 66, Iss. 12, pp. 974-985.
- Costa, P. (2011) - "*Vibrações do sistema via-macizo induzidas por tráfego ferroviário: modelação numérica e validação experimental*", Tese de Doutoramento, Faculdade de Engenharia da Universidade do Porto, Porto, Portugal.
- Cruz, S. (1994) - "*Comportamento dinâmico de pontes ferroviárias em vias de alta velocidade*", Tese de Mestrado, Faculdade de Engenharia da Universidade do Porto, Porto.

D

- Dai, G., Hu, N. and Liu, W. (2010) - "*The recent improvement of high-speed railway bridges in China*", in 34th IABSE Symposium for Bridge and Structural Engineering: Large Structures and Infrastructures for Environmentally Constrained and Urbanised Areas, Venice, Italy.
- Delgado, R.M. and dos Santos, S.M. (1997) - "*Modelling of railway bridge-vehicle interaction on high speed tracks*", Computers & Structures, Vol. 63, Iss. 3, pp. 511-523.
- Deng, J. (2006) - "*Structural reliability analysis fo implicit performance function using radial basis function network*", Solids and Structures, Vol. 43, pp. 3255-3291.
- Dieleman, L. and Fournol, A. (2003) - "*Dynamic behavior of the short railway bridges*", in IABSE - Structures for high-speed railway transportation, Antwerp, Belgium.
- Doménech, A. and Museros, P. (2011) - "*Influence of the vehicle model on the response of high-speed railway bridges at resonance. Analysis of the Additional Damping Method prescribed by Eurocode 1*", in EUROODYN 2011 - 8th International Conference on Structural Dynamics, Leuven, Belgium.

Doménech, A., Museros, P. and Martínez-Rodrigo, M. (2014) - "*Influence of the vehicle model on the prediction of the maximum bending response of simply-supported bridges under high-speed railway traffic*", *Engineering Structures*, Vol. 72, pp. 123-139.

E

EN 1990 (2002), *Eurocode 0 - Basis of structural design*. Comité European de Normalisation (CEN): Brussels, Belgium.

EN 1990-A2 (2005), *Eurocode 0 - Basis of Structural Design - Annex A2: Applications for bridges (normative)*. Comité European de Normalisation (CEN): Brussels, Belgium.

EN 1991-2 (2003), *Eurocode 1 - part 2: Actions on structures - traffic load on Bridges*. Comité European de Normalisation (CEN): Brussels.

EN 13484-5 (2008), *Railway applications - Track - Track geometry quality - Part 5: Geometric quality levels*. European Committee for Standardization (CEN): Brussels, Belgium.

EN 14067-6 (2009), *Railway Applications - Aerodynamics - Part 6: Requirements and Test Procedures for Cross Wind Assessment*. Comité European de Normalisation (CEN): Brussels, Belgium.

ERRI (1999) - "*Rail Bridges for Speeds > 200km/h RP9 - Final Report*", European Rail Research Institute.

ETI (2002), *Directiva 96/48/CE - Especificação Técnica de Interoperabilidade para o Subsistema "Infra-estrutura" do sistema Ferroviário Transeuropeu de Alta Velocidade*. Jornal Oficial das Comunidades Europeias L 245 de 12 de Setembro de 2002.

F

Faber, M.H. (2012) - "*Statistics and Probability Theory: In Pursuit of Engineering Decision Support*", Springer, The Netherlands.

Figueiredo, H. (2007) - "*Dinâmica de pontes mistas aço betão em linhas de alta velocidade*", Tese de Mestrado, Faculdade de Engenharia da Universidade do Porto, Porto, Portugal.

Florian, A. (1992) - "*An Efficient Sampling Scheme: Updated Latin Hypercube Sampling*", *Probabilistic Engineering Mechanics*, Vol. 7, pp. 123-130.

Fortunato, E. (2005) - "*Renovação de plataformas ferroviárias: estudos relativos à capacidade de carga*", Tese de Doutoramento, Departamento de Engenharia Civil, Faculdade de Engenharia da Universidade do Porto, Porto, Portugal.

Fryba, L. (1996) - "*Dynamics of Railway Bridges*", Thomas Telford.

Fryba, L. (2001) - "*A rough assessment of railway bridges for high speed trains*", Engineering Structures, Vol. 23, Iss. 5, pp. 548-556.

G

Goicolea, J.M., Domínguez, J., Gabaldón, F. and Navarro, J.A. (2002) - "*Ressonant effects in short span high speed railway bridges: modelling and design issues*", in Eurodyn 2002, Munich.

Goicolea, J.M., Gabaldón, F., Dominguez, J. and Navarro, J.A. (2004) - "*Dynamic loads in new engineering codes for railway bridges in Europe and Spain*", in Bridges for High-Speed Railways, Faculdade de Engenharia da Universidade do Porto, Porto, Portugal.

Grooteman, F. (2008) - "*Adaptive radial-based importance sampling method for structural reliability*", Structural Safety, Vol. 30, pp. 533-542.

Grooteman, F. (2011) - "*An adaptive directional importance sampling method for structural reliability*", Probabilistic Engineering Mechanics, Vol. 26, pp. 134-141.

H

Haldar, A. and Mahadevan, S. (2000) - "*Reliability assessment using stochastic finite element analysis*", John Wiley & Sons, New York.

Hasofer, A.M. (1996) - "*Parametric estimation of failure probabilities*", in *Mathematical models for structural reliability analysis*, CRC Press, Boca Raton, Florida, USA.

Hasofer, A.M. and Lind, M.C. (1974) - "*An exact and invariant first order reliability format*", ASCE Journal of Engineering Mechanics Division, Vol. 100.

Henriques, A. (1998) - "*Aplicação de novos conceitos de segurança no dimensionamento do betão estrutural*", Tese de Doutoramento, Departamento de Engenharia Civil, Faculdade de Engenharia da Universidade do Porto Porto.

Hoorpah, W. (2005) - "*Dynamics calculations of high-speed railway bridges in France - some case studies*", in Dynamics of High-Speed Railway Bridges, FEUP, Porto.

Hu, B. and Schiehlen, W. (1997) - "*On the simulation of stochastic processes by spectral representation*", On the simulation of stochastic processes by spectral representation, Vol. 12, Iss. 2, pp. 105-113.

Hurtado, J.E. (2007) - "*Filtered importance sampling with support vector margin: A powerful method for structural reliability analysis*", Structural Safety, Vol. 29, pp. 2-15.

I

- IAPF (2003), *IAPF*, in *Instrucción de acciones a considerar en el proyecto de puentes de ferrocarril*. Ministerio de Fomento: Spain.
- INNOTRACK (2008), *Sustainable development, global change and ecosystems. D2.1.3 First phase report on the modelling of poor quality sites*. Czech Technical University, Prague.
- Institut d'aménagement et d'urbanisme - Île-de-France, Retrieved from: <http://www.iau-idf.fr> (accessed on September 2012).
- Ishibashi, T. (2004) - "*Shinkansen structures in Japan*", in Workshop on Bridges for High-Speed Railways, Porto, Portugal.
- Iwnicki, S. (2006) - "*Handbook of Railway Vehicle Dynamics*", CRC Press -Taylor & Francis Group, Boca Raton, FL, USA.

J

- Jacinto, L. (2011) - "*Avaliação da segurança de pontes existentes: Abordagem probabilística Bayesiana*", Tese de Doutoramento, Faculdade de Ciências e Tecnologia da Universidade Nova de Lisboa, Lisboa.
- JCSS (2001), *Probabilistic Model Code*. Joint Committee on Structural Safety, <http://www.jcss.ethz.ch/>, 12th draft.
- James, G. (2003) - "*Analysis of Traffic Load Effects on Railway Bridges*", PhD Thesis, Structural Engineering Division, Royal Institute of Technology, Stockholm, Sweden.
- Johansson, C., Pacoste, C. and Karoumi, R. (2013) - "*Closed-form solution for the mode superposition analysis of the vibration in multi-span beam bridges caused by concentrated moving loads*", *Computers & Structures*, Vol. 119, pp. 85-94.
- Johansson, C., Ní Nualláin, N.A., Pacoste, C. and Andersson, A. (2014) - "*A methodology for the preliminary assessment of existing railway bridges for high-speed traffic*", *Engineering Structures*, Vol. 58, pp. 25-35.

K

- Khadri, Y., Tekili, S., Daya, E.M., Daouadji, A. and Merzoug, B. (2013) - "*Effects of rail joints and train's critical speed on the dynamic behavior of bridges*", *Mechanika*, Vol. 19, pp. 46-52.
- Karpa, O. and Naess, A. (2013) - "*Extreme value statistics of wind speed data by the ACER method*", *Journal of Wind Engineering and Industrial Aerodynamics* Vol. 122, pp. 1-10.

Klasztorny, M. and Langer, J. (1990) - "*Dynamic response of single-span beam bridges to a series of moving loads*", *Earthquake Engineering & Structural Dynamics*, Vol. 19, Iss. 8, pp. 1107-1124.

Kwark, J.W., Choi, E.S., Kim, Y.J., Kim, B.S. and Kim, S.I. (2004) - "*Dynamic behavior of two-span continuous concrete bridges under moving high-speed train*", *Computers & Structures*, Vol. 82, Iss. 4-5, pp. 463-474.

L

Lee, Y.S. and Kim, S.H. (2010) - "*Structural analysis of 3D high-speed train-bridge interactions for simple train load models*", *Vehicle System Dynamics*, Vol. 48, Iss. 2, pp. 263-281.

Lei, X. and Noda, N. (2002) - "*Analyses of dynamic response of vehicle and track coupling system with random irregularity of track vertical profile*", *Journal of Sound and Vibration*, Vol. 258, Iss. 1, pp. 147-165.

Lou, P. (2005) - "*A vehicle-track-bridge interaction element considering vehicle's pitching effect*", *Finite Elements in Analysis and Design*, Vol. 41, pp. 397-427.

Lu, F., Lin, J., Kennedy, D. and Williams, F.W. (2009) - "*An algorithm to study non-stationary random vibrations of vehicle-bridge systems*", *Computers & Structures*, Vol. 87, Iss. 3-4, pp. 177-185.

M

Majka, M. and Hartnett, M. (2008) - "*Effects of speed, load and damping on the dynamic response of railway bridges and vehicles*", *Computers and Structures*, Vol. 86, pp. 556-572.

Manterola, J. (2006) - "*Puentes : apuntes para su diseño, cálculo y construcción*", Colegio de Ingenieros de Caminos, Canales y Puertos.

Martínez-Rodrigo, M., Lavado, J. and Museros, P. (2010) - "*Dynamic performance of existing high-speed railway bridges under resonant conditions retrofitted with fluid viscous dampers*", *Engineering Structures*, Iss. 32, pp. 808-828.

Mckay, M.D., Beckman, R.J. and Conover, W.J. (1979) - "*A comparison of three methods for selecting values of input variables in the analysis of output from a computer code*", *Technometrics*, Vol. 21, pp. 239-245.

Meixedo, A. (2012) - "*Comportamento dinâmico de pontes com tabuleiro pré-fabricado em vias de alta velocidade*", MSc, Universidade do Porto -Faculdade de Engenharia, Porto, Portugal.

Melchers, R.E. (1999) - "*Structural reliability. Analysis and prediction*", John Wiley & Sons, Chichester.

-
- Montenegro, P. (2015) - "*A methodology for the assessment of the train running safety on bridges*", PhD Thesis, Faculdade de Engenharia da Universidade do Porto, Porto.
- Morris, M.D. and Mitchell, T.J. (1995) - "*Exploratory designs for computational experiments*", Journal of Statistical Planning and Inference, Vol. 43, Iss. 3, pp. 381-402.
- Museros, P. (2002) - "*Interacción vehículo-estructura y efectos de resonancia en puentes isostáticos de ferrocarril para líneas de alta velocidad*", PhD Thesis, ETS Ingenieros Industriales, Universidad Politécnica de Madrid, Spain.
- Museros, P. and Alarcon, E. (2005) - "*Influence of the second bending mode on the response of high-speed bridges at resonance*", Journal of Structural Engineering-Asce, Vol. 131, Iss. 3, pp. 405-415.
- Museros, P., Romero, M.L., Poy, A. and Alarcon, E. (2002) - "*Advances in the analysis of short span railway bridges for high-speed lines*", Computers and Structures, Vol. 80, pp. 2121:2132.
- Museros, P., Moliner, E. and Martínez-Rodrigo, M. (2013) - "*Free vibrations of simply-supported beam bridges under moving loads: Maximum resonance, cancellation and resonant vertical acceleration*", Journal of Sound and Vibration, Vol. 332, pp. 326-345.

N

- Nadal, M.J. (1896) - "*Théorie de la stabilité des locomotives, part 2, mouvement de lacet*", Annales des Mines, Vol. 10.
- Naess, A., Leira, B.J. and Batsevych, O. (2009) - "*System reliability analysis by enhanced Monte Carlo simulation*", Structural Safety, Vol. 31, pp. 349-355.
- Neves, S. (2008) - "*Análise Dinâmica com Interação Veículo-Estrutura em Vias de Alta Velocidade*", MSc, Universidade do Porto - Faculdade de Engenharia, Porto, Portugal.
- Neves, S., Azevedo, A. and Calçada, R. (2012) - "*A direct method for analyzing the vertical vehicle-structure interaction*", Engineering Structures, Vol. 34, pp. 414-420.
- Nguyen, D.-V., Kim, K.-D. and Warnitchai, P. (2009) - "*Simulation procedure for vehicle-substructure dynamic interactions and wheel movements using linearized wheel-rail interfaces*", Finite Elements in Analysis and Design, Vol. 45, Iss. 5, pp. 341-356.
- Norris, P. (2005) - "*Recent advances in the understanding of bridge dynamic behaviour on the West Coast main line route modernisation project*", in Dynamics of High-Speed Railway Bridges, Porto, Portugal.
- Nowak, A.S. and Collins, K.R. (2000) - "*Reliability of Structures*", McGraw-Hill, New York, USA.

P

- Pimentel, R. (2009) - "*Caracterização do tráfego ferroviário e dos seus efeitos em pontes de pequeno vão*", Tese de Mestrado, Departamento de Engenharia Civil, Faculdade de Engenharia da Universidade do Porto, Porto, Portugal.
- Pombo, J. (2004) - "*A multibody methodology for railway dynamics applications*", Department of Mechanical Engineering, Instituto Superior Técnico, Lisbon, Portugal.
- Popp, K., Kruse, H. and Kaiser, I. (1999) - "*Vehicle-track dynamics in the mid-frequency range*", *Vehicle System Dynamics*, Vol. 31, Iss. 5-6, pp. 423-464.
- Prud'homme, A. (1967) - "*La résistance de la voie aux efforts transversaux exercés par le matériel roulant*", *Revue Generale des Chemin de Fer*.

R

- Ramu, P. (2007) - "*Multiple tail models including inverse measures for structural design under uncertainties*", PhD Thesis, University of Florida.
- Ramu, P., Kim, N.H. and Haftka, R.T. (2010) - "*Multiple tail median approach for high reliability estimation*", *Structural Safety*, Vol. 32, pp. 124-137.
- Rauert, T., Bigelow, H., Hoffmeister, B. and Feldman, M. (2010) - "*On the prediction of the interaction effect caused by continuous ballast on filler beam railway bridges by experimentally supported numerical studies*", *Journal of Engineering Structures* Vol. 32.
- Ribeiro, D. (2004) - "*Comportamento Dinâmico de Pontes sob Acção de Tráfego Ferroviário a Alta Velocidade*", Tese de Mestrado, Departamento de Engenharia Civil, Faculdade de Engenharia da Universidade do Porto, Porto.
- Ribeiro, D. (2012) - "*Efeitos Dinâmicos Induzidos por Tráfego em Pontes Ferroviárias: Modelação Numérica, Calibração e Validação Experimental*", PhD Thesis, Universidade do Porto - Faculdade de Engenharia, Porto, Portugal.
- Rigueiro, C. (2007) - "*Avaliação dos efeitos dinâmicos em pontes ferroviárias de alta velocidade de pequeno e médio vão*", Tese de Doutoramento, Departamento de Engenharia Civil, Faculdade de Ciências e Tecnologia da Universidade de Coimbra, Coimbra, Portugal.
- Rocha, J.M., Henriques, A.A. and Calçada, R. (2012) - "*Safety assessment of a short span railway bridge for high-speed traffic using simulation techniques*", *Engineering Structures*, Vol. 40, pp. 414-454.
- Rocha, J.M., Henriques, A.A. and Calçada, R. (2014) - "*Probabilistic safety assessment of a short span high-speed railway bridge*", *Engineering Structures*, Vol. 71, pp. 99-111.

-
- Rocha, J.M., Henriques, A.A., Calçada, R. and Rönquist, A. (2015) - "*Efficient methodology for the probabilistic safety assessment of high-speed railway bridges*", *Engineering Structures*, Vol. 101, pp. 138-149.
- Rocha, J.M., Henriques, A.A. and Calçada, R. (2016) - "*Probabilistic assessment of the train running safety on a short span high-speed railway bridge*", *Structure and Infrastructure Engineering*, Vol. 12, pp. 78-92.
- Rodrigues, J. (2004) - "*Identificação Modal Estocástica: Métodos de análise e aplicações em estruturas de engenharia civil*", Tese de Doutoramento, Departamento de Engenharia Civil, Faculdade de Engenharia da Universidade do Porto, Porto, Portugal.
- Romão, X., Delgado, R. and Costa, A. (2010) - "*An empirical power comparison of univariate goodness-of-fit tests for normality*", *Journal of Statistical Computation and Simulation*, Vol. 80, Iss. 5, pp. 545-591.
- Romero, A., Domínguez, J. and Galvín, P. (2011) - "*3D numerical analysis of train-induced vibrations on short bridges considering vehicle-track-structure-soil dynamic interaction*", in EURO DYN 2011 - 8th International Conference on Structural Dynamics, Leuven, Belgium.
- Rubinstein, R.Y. (1981) - "*Simulation and the Monte Carlo method*", John Wiley & Sons, New York.

S

- Schneider, J. (2006) - "*Introduction to Safety and Reliability of Structures*", Report.
- Schuëler, G.I. (2009) - "*Efficient Monte Carlo simulation procedures in structural uncertainty and reliability analysis - recent advances*", *Structural Engineering and Mechanics*, Vol. 32, Iss. 1, pp. 1-20.
- Shin, J.R., An, Y.K., Sohn, H. and Yun, C.B. (2010) - "*Vibration reduction of high-speed railway bridges by adding size-adjusted vehicles*", *Engineering Structures*, Iss. 32, pp. 2839-2849.
- Shinozuka, M. (1972) - "*Monte Carlo solution of structural dynamics*", *Computers & Structures*, Vol. 2, pp. 855-874.
- Silva, L.D. (2010) - "*Comportamento dinâmico de uma ponte ferroviária de pequeno vão para tráfego de alta velocidade*", Tese de Mestrado, Departamento de Engenharia Civil, Faculdade de Engenharia da Universidade do Porto, Porto, Portugal.
- Smith, R.L. (1989) - "*Extreme value analysis of environmental time series: an application to trend detection in ground-level ozone*", *Statistical Science*, Vol. 4, pp. 367-393.
- Sustainable Bridges Project (2004) - "*Assessment for Future Traffic Demands and Longer Lives: Doc. No. WP1-02-T-040601-F-Deliverable D1. 2: "European Railway Bridge Demography"*" (<http://www.sustainablebridges.net>).
- Stanisic, M.M. and West, L. (1985) - "*On a new theory of the dynamic behavior of the structures carrying moving masses*", *Ingenieur-Archiv*, Vol. 55, Iss. 3, pp. 176-185.

Stein, M. (1987) - "*Large Sample Properties of Simulations Using Latin Hypercube Sampling*", Technometrics, Vol. 29, Iss. 2, pp. 143-151.

Stocki, R. (2005) - "*A method to improve design reliability using optimal Latin hypercube sampling*", Computer Assisted Mechanics and Engineering Sciences, Vol. 12, pp. 87-105.

SUPERTRACK (2005), *Sustained performance of railway tracks - Final report. Instrumentation, monitoring and physical modelling of high-speed line*. Norwegian Geotechnical Institute (NGI).

T

Thoft-Christensen, P. and Baker, M.J. (1982) - "*Structural Reliability Theory and Its Applications*", Springer, New York, USA.

Timoshenko, S. and Young, D. (1974) - "*Vibration problems in engineering*", John Wiley & Sons, John Wiley & Sons, New York, USA.

TSI, *Technical specifications for interoperability relating to the infrastructure subsystem*. 2002: Brussels, Belgium.

U

UIC (2008), *Best practice guide for optimum track geometry durability*. UIC Infrastructure Commission - Civil Engineering Support Group.

UIC 518 (2005) - "*Essais et Homologation de Véhicules Ferroviaires du Pont de Veau du Comportement Dynamique - Sécurité - Fatigue de la voie - Qualité de marche*", Union International des Chemins de fer: Paris, France.

UIC 719 (2008), *Earthworks and track bed construction for railway lines*. Union International des Chemins de fer: Paris, France.

UIC 774-3-R (2001). *Track/bridge interaction – Recommendations for calculation*. 2nd Edition, Union International des Chemins de fer: Paris, France.

UIC High-speed, Retrieved from: <http://www.uic.org/highspeed> (accessed on September 2015).

V

Vale, C. (2010) - "*Influência da qualidade dos sistemas ferroviários no comportamento dinâmico e no planeamento da manutenção preventiva de vias de alta velocidade*", Tese de Doutoramento,

Departamento de Engenharia Civil, Faculdade de Engenharia da Universidade do Porto, Porto, Portugal.

Vořechovský, M. and Novák, D. (2003) - "*Statistical Correlation in Stratified Sampling*", in ICASP 9 - 9th International Conference on Applications of Statistics and Probability in Civil Engineering, Millpress, San Francisco, USA.

W

Wald, A. and Wolfowitz, J. (1939) - "*Confidence limits for continuous distribution functions*", The Annals of Mathematical Statistics, Vol. 10, pp. 105-118.

Wei, L. and Griffin, J. (1998) - "*Mathematical models for the apparent mass of the seated human body exposed to vertical vibration*", Journal of Sound and Vibration, Vol. 212, Iss. 5, pp. 855-874.

Weinstock, H. (1984) - "*Wheel climb derailment criteria for evaluation of rail vehicle safety*", in ASME Winter Annual Meeting.

Wisniewski, D. (2007) - "*Safety Formats for the Assessment of Concrete Bridges*", Tese de Doutoramento, Departamento de Engenharia Civil, Universidade do Minho, Guimarães.

X

Xia, H., Zhang, N. and De Roeck, G. (2003) - "*Dynamic analysis of high speed railway bridge under articulated trains*", Computers & Structures, Vol. 81, Iss. 26-27, pp. 2467-2478.

Xia, H. and Zhang, N. (2005) - "*Dynamic analysis of railway bridge under high-speed trains*", Computers & Structures, Vol. 83, Iss. 23-24, pp. 1891-1901.

Xia, H., Zhang, N. and Guo, W.W. (2006) - "*Analysis of resonance mechanism and conditions of train-bridge system*", Journal of Sound and Vibration, Vol. 297, Iss. 3-5, pp. 810-822.

Y

Yang, F. and Fonder, G. (1996) - "*An iterative solution method for dynamic response of bridge-vehicles systems*", Earthquake Engineering & Structural Dynamics, Vol. 25, Iss. 2, pp. 195-215.

Yang, Y., Yau, J. and Hsu, L. (1997) - "*Vibration of simple beams due to trains moving at high speeds*", Engineering Structures, Vol. 19, Iss. 11, pp. 936-944.

-
- Yang, Y., Chang, C. and Yau, J.D. (1999) - "*An element for analysing vehicle-bridge systems considering vehicle's pitching effect*", International Journal for Numerical Methods in Engineering, Vol. 46, Iss. 7, pp. 1031-1047.
- Yang, Y.B. and Wu, Y.S. (2001) - "*A versatile element for analyzing vehicle-bridge interaction response*", Engineering Structures, Vol. 23, Iss. 5, pp. 452-469.
- Yang, Y.B., Lin, C.L., Yau, J.D. and Chang, D.W. (2004) - "*Mechanism of resonance and cancellation for train-induced vibrations on bridges with elastic bearings*", Journal of Sound and Vibration, Vol. 269, pp. 345-360.
- Yang, Y.B., Yau, J.D. and Wu, Y.S. (2004) - "*Vehicle-Bridge Interaction Dynamics: with applications to high-speed railways*", World Scientific Publishing Company.
- Yau, J.D., Wu, Y.S. and Yang, Y.B. (2001) - "*Impact response of bridges with elastic bearing to moving loads*", Journal of Sound and Vibration, Vol. 248, Iss. 1, pp. 9-30.

Z

- Zabel, V. and Brehm, M. (2009) - "*System identification of high-speed railway bridges*", in Weimar Optimization and Stochastic Days 5.0, Dynardo GmbH, Weimar, Germany.
- Zacher, M. and Baeßler, M. (2009) - "*Dynamic behaviour of ballast on railway bridges*", in Dynamics of High-Speed Railway Bridges, Taylor & Francis.
- Zhai, W., Cai, C., Wang, Q., Lu, Z.W. and Wu, X.S. (2001) - "*Dynamic effects of vehicles on tracks in the case of raising train speeds*", Journal of Rail & Rapid Transit, Vol. 215, pp. 125-135.
- Zhai, W., Wang, K. and Cai, C. (2009) - "*Fundamentals of vehicle-track coupled dynamics*", Vehicle System Dynamics, Vol. 47, Iss. 11, pp. 1349-1376.
- Zhai, W., Wang, K. and Lin, J. (2004) - "*Modelling and experiment of railway ballast vibrations*", Journal of Sound and Vibration, Vol. 270, pp. 673-683.
- Zhang, N., Xia, H. and Guo, W.W. (2008) - "*Vehicle-bridge interaction analysis under high-speed trains*", Journal of Sound and Vibration, Vol. 309, Iss. 3-5, pp. 407-425.