FACULDADE DE ECONOMIA DO PORTO

BLURRED LINES: IMPERFECT INFORMATION AND DIFFERENT COSTS OF PRODUCTION IN HOTELLING'S LINEAR CITY

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TESE DE DOUTORAMENTO EM ECONOMIA

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Biographical Note

Ricardo Biscaia was born in December 1987, in Leça da Palmeira, Portugal. After completing the undergraduate course in Economics at FEP in the University of Porto, he entered in the PhD program in Economics at the same university, in 2009. In early 2010, Ricardo joined CIPES – Centre for Research in Higher Education Policies, as a research assistant, where he still works today.

During his path as a student, he specialized in Industrial Organization, Regional Science, and Game Theory, namely, on Hotelling-type models of spatial competition. However, from working at CIPES, Ricardo acquired as well skills in the field of Higher Education.

On his early career as a researcher, he has published in several international refereed journals, namely, *Applied Economics, Higher Education, Journal of Economic Issues* and *Papers in Regional Science*. He has also refereed once for *Papers in Regional Science* and has co-authored a book for the national *Agency for Assessment and Accreditation of Higher Education – A3ES*. He has also several working papers in the FEP system, mainly resulting from the work contained in this manuscript, and his work has been presented by himself or by his co-authors in international scientific meetings, such as the World Congress of the Regional Science Association International, the European Regional Science Association Conference, or the Consortium of Higher Education Researchers.

His most important hobbies are soccer, tennis, bowling, and computer gaming. He gladly welcomes any challenge in any of these occupations.

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Resumo

Esta tese é sobre modelos de competição espacial à la Hotelling. Começamos por fazer uma revisão de literatura detalhada sobre este tópico, com o foco em artigos cuja principal questão de investigação é a localização óptima das empresas. Também fazemos uma análise bibliométrica para compreender a evolução deste campo científico nos anos mais recentes. Depois, contribuímos para a literatura desenvolvendo os nossos próprios modelos, tanto em competição por preços como em competição por quantidades. Nos preços, averiguamos se a localização das empresas muda devido à existência de incerteza no seu custo marginal de produção. Também introduzimos um estágio de investimento como primeiro estágio do jogo, onde este estágio permite às empresas investirem com vista a reduzir o seu custo de marginal de produção. Em competição às quantidades, construímos um modelo para perceber qual a localização óptima escolhida pelas empresas e por um regulador quando as empresas possuem custos marginais de produção distintos. Adicionalmente, introduzimos um modelo com um recurso natural, sendo este essencial para que as empresas produzam os seus bens finais, e verificamos de que forma mudanças no custo unitário de transporte do input afecta as localizações escolhidas pelas empresas.

Concluímos, no caso da competição por preços, que incerteza nos custos pode levar as empresas à aglomeração se a diferença entre as duas diferentes possibilidades de custos marginais de produção forem suficientemente elevados. Concluímos também que a possibilidade das empresas investirem para reduzir os seus custos marginais as leva para um dilema do prisioneiro, onde estas ficariam melhor se não tivessem a hipótese de investir. Nas quantidades, concluímos que as empresas não reagem a diferenças entre os seus custos marginais, na altura de escolher a sua localização. No entanto, um regulador preferiria mudar a localização da empresa ineficiente. No caso do modelo com um recurso natural, as empresas preferem mover-se aglomeradas para perto do recurso natural, à medida que o custo unitário de transporte do input aumenta.

Abstract

This thesis is about spatial competition models *à la* Hotelling. We do a thorough survey of the literature in this topic, focusing on articles whose main concern is the optimal location choice of firms. We also do a bibliometric analysis to assess the evolution of the field in the last years. We then give our own contribution by developing models both in price and quantity competition. In price competition, we test whether the location decisions of firms change, given that there is uncertainty in their marginal cost of production. We also introduce an investment stage as the first stage of the game, which allows firms to invest in order to reduce their marginal costs of production. In quantity competition, we build a model to assess what the optimal location chosen by firms and by a social planner are when firms have different marginal costs. Additionally, we introduce a model with a natural resource that is essential for firms to produce their final goods, and we check how changes in the unit input transportation costs affect firms' locations.

We conclude that, in the case of price competition, cost uncertainty may lead to agglomeration of the firms if the difference between the marginal cost outcomes of both firms is large enough. We conclude as well that the possibility of firms investing in cost-reduction activities leads them to a prisoner dilemma situation, where they would prefer not to have that possibility. For quantity competition, we conclude that firms are unresponsive to changes in their cost structure. However, a social planner would prefer to change the location of an inefficient firm. In the case of the presence of a natural resource model, firms move together towards the location of the input as the unit input transportation cost becomes higher.

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1. Introduction

The current document aims at contributing to the already vast literature on the subject of spatial competition. The main focus is based on the model that was developed by Harold Hotelling (1929) more than eighty years ago, and was subsequently extended by a various number of researchers. Nowadays, it is still a popular way to analyze the interaction between firms in markets in which the spatial component is crucial. However, this framework soon gained importance not only as a spatial component, but also as a microeconomic component: the linear city of Hotelling, which was designed to represent spatial markets, has also gained relevance as a way to model the product space, that is, if a given product has a certain characteristic, the "linear city" may as well be the most appropriate way to model different preferences of consumers regarding the product space, as well as the differentiation positioning of a given product.[1]

This type of models has a huge variety of purposes that can be grouped into two categories. The first one being: scientists explored the Hotelling model because they were interested in justifying the location of firms, when these are competing on a different set of circumstances. If there is one big conclusion that can be taken from studying this literature is that the location choice of firms is highly dependent on the circumstances the firms are competing in. Different circumstances arise due to variables such as: the dimension of the market; the value of the unit transportation cost paid by consumers and/or firms to "move" in the linear city; the nature of these transportation costs, that is, if these are convex, concave, linear, quadratic, linear-quadratic,…; how differently consumers are distributed in the market; the number of firms that are allowed to compete in the market; other agents that may participate in these markets, such as regulation authorities, social planners, agents hired by owners, input suppliers. Many circumstances can change the nature of markets and any of these may have a significant role in justifying the location of firms when pursuing profits and subsequent success in their businesses.

[1] Therefore, it is important to mention that we have tried to write this document with a focus on the geographical argument started by Hotelling. However, in most cases we refer to location, linear city and transportation costs, but we could be referring in a similar way to product differentiation, product space, and product conversion costs, respectively.

The second main purpose of the usage of Hotelling models is to check whether adding a location/horizontal differentiation setting in different situations contributed to change these exact situations. For instance, researchers sought how would considering a market with a spatial component changed, for instance, the conditions for which firms were able to sustain collusion, to address if a spatial market allowed for a larger or smaller number of firms in a given market: if it facilitated monopolists to craft entry barriers, if it changed the relationship between a first-mover and a follower, and many more. Summing up, authors were either interested in using the Hotelling approach to justify the location chosen by firms, or in using the approach as a spatial component that would help justifying different strategies pursued by firms regarding topics other than location. The focus of this work is on the first category: we are interested in understanding why firms act in a given manner towards their location decisions.

The question of "where should firms locate?" has been an interesting subject in the science of Economics. Broadly speaking, location choice of all sorts of economic agents is crucial to the attainment of their objectives in a variety of situations. For instance, one of the earliest location phenomena we all experience in our lives is location choice in the classroom. Seating on the front or back-row location has different consequences regarding how active you are in the classroom, on your grading results, on your behavior in the classroom, and on the teacher-student relationship (see, for instance, Stires (1980)). Moreover, location in the classroom is usually a tool used by the teachers to manage misbehaved students, or students that are not performing so well during the academic year. Many other location decisions affect our everyday lives, some less relevant, as location choice in a theatre, concert, or cinema, or when choosing where to park your car; and others more relevant, like the place you choose to live and how close it is to other amenities, such as the place where you work, the place where your children study, and how close you are from important places like a supermarket, an hospital, or to the town landfill, which in this case you would want to avoid.

When it comes to business, this variable plays a dramatic role. There are many advantages and disadvantages associated with each location choice when choosing where to locate your business, and those perks depend as well on the type of business/industry in question. Two locations that are a few meters distant from each other may have significant differences in terms of visibility, which may determine the difference between the success and the failure of a business. Globalization, the technological developments on the dissemination of information, and the overall lowering of unit transportation costs in both labor and capital might have led, on the one hand, to the lowering importance of location choice in the success of a business. On the other hand, the location possibilities of an agent are also amplified with these changes. Firms can be located, with more or less barriers, within any location in the world, which enhances the heterogeneity of advantages and disadvantages that each location can provide to a certain type of business. Location is a significant factor in determining, to name only a few: the rent paid for land usage, the visibility of the business to consumers, the reputation of the firm, the wages you will be able to pay to your workers, the price you may be able to set for your goods, the working conditions of your staff in terms of environment or accessibility in terms of parking spaces or public transportations, and so on.

The Hotelling framework also has an important characteristic. Models that involve the study of the location choices and were based on the linear city are mainly solved by using Game Theory vocabulary and tools. Most models included competition between two or more firms as the main ingredient behind the location choices of firms, which amplified the importance of location as a strategic variable. Strategy and firms' strategic behavior is, therefore, a key argument on justifying the results found in most papers on the subject.

Therefore, this thesis is based on the Hotelling framework. We use it because we believe it is the better way to have a deeper understanding of the location decision of firms when they face certain conditions. Namely, we have a deeper look on how firms react when their marginal cost of production varies due to different circumstances. Either because there are exogenous differences in the marginal costs of both firms; because of uncertain information in the formation of this costs of production; or because there are vertical relationships in the market, that end up shaping the cost firms have to incur to manufacture their good.

Next follows an outline of the thesis, and a justification/logic behind the choices of the chapters and their content. The following chapter is a critical literature review of the linear city framework and subsequent developments. We started by providing a bibliometric analysis to understand the evolution of the field throughout the years, as well as to identify the most influential authors and the geographical patterns behind the publications in the field. Then, we summarize the location results found in the literature. We would like to reinforce that the focus of the literature review is on articles in which the contribution to understand the location result of firms is substantial. We have also focused more on articles that surveyed duopoly competition instead of competition between more than two firms. Summarizing the literature that included all papers that have used the Hotelling framework would be unfeasible. Still, we think that the review allows the reader to understand the complexity and the deeper attention that these models received in various questions around the economics field, as well as contributing to understand the various different ways the model has been treated. That is possible because the review is separated between models that have core changes in their assumptions, namely models with mill price competition; models with price discrimination; models with quantity competition; models with competition within a non-linear city; models with imperfect information; and experiments that were carried in these models.[2]

The next chapters are our own contribution to the literature on spatial competition. Chapter 3 addresses how imperfect information may change the location patterns of firms in the classic Hotelling framework with quadratic transportation costs. We assume firms do not know exactly what their marginal production cost will be before they choose their location. Firms do know, however, that their marginal cost (and their opponent's) varies between two options that may occur with a given probability. Our most important conclusions for this chapter are that firms may choose to be in the same point of the linear city, contrary to the results found in perfect information for different production costs between firms. In addition, if firms were given the possibility to choose, they would choose to be placed in an imperfect information situation for a significant array of the values of their marginal costs. The reason being that imperfect information provides firms with a credible way of pursuing a riskier behavior, which results in a higher (expected) profit.

[2] A smaller version of the review in Chapter 2 has been accepted for publication in *Papers in Regional Science*, and is listed in the references as Biscaia and Mota (Forthcoming).

Chapter 4 attempts to give an endogenous explanation on why firms would have different production costs in this framework, while retreating to the perfect information case. Both firms are allowed to invest in cost-reducing activities before choosing their location, that is, firms participate in a three-staged R&D-Location-Price game. By allowing firms to have different technological access to R&D investments, firms may have different costs of reducing their marginal cost of production. We conclude that firms still prefer to disperse in the linear city, as predicted in the framework without R&D investments. However, we were not able to solve the problem for all the values of the technological efficiency, due to the fact that the profit function of both firms is a piecewise function, which does not allow the finding of the subgame perfect Nash equilibrium with a single calculation. That is the main reason we have not explored this issue further by considering R&D investments with imperfect information regarding its outcome. Still, we have found that in the case when firms have the same investment technology, they are trapped in a Prisoner's Dilemma with regards to their investment in R&D. This means that firms would be better off by not having the option to invest at all. However, since they do, both are forced to invest in the reduction of their marginal cost in order to defend their demand areas. Nevertheless, since the Hotelling model is bounded, and the demand is inelastic, firms end up having lower operational revenues, since they have the same demand but they charge a lower price for their good. Moreover, firms have to spend money in defending their market position. Therefore, the higher the technological capabilities of both firms, the higher the pressure they have in defending their position, and the lower their profits will be.

Chapter 5 has an important change regarding the two previous chapters, since competition in the last stage is now done in quantities (Cournot) instead of prices (Bertrand), and therefore it is situated in a different strand of literature comparing to the other two chapters. We shifted our focus to Cournot competition, and we attempt to understand the consequences for the firms' location decisions of having different marginal costs in the quantity competition setup. Our conclusion is that the result previously found in the literature, in which both firms prefer to be agglomerated in the center of the linear city, holds for any difference between the marginal costs of production of both firms. In other words, the production costs have no effect on the location result. We proceed on analyzing the robustness of this result if, instead of having firms choosing their own locations, there was a regulator/social planner that would be allowed to choose firms locations, but not their quantities. We conclude that the social planner would still prefer to have the firms located in the city center. However, if one of the firms becomes too inefficient relatively to the other, the central agglomeration result ceases to be equilibrium, but we are unable to find a proper mathematical solution for a new equilibrium. We are led to believe by intuition that the social planner would prefer to put the inefficient firm at the extremes of city, in order to have it producing as fewer quantities as possible, since the efficient firm is able to produce goods at much cheaper cost. We extend the analysis to the case where the social planner, if it wants to, could remove the inefficient firm out of the market. We find that if the firm is too inefficient, the social planner would prefer to keep a monopoly in place, for the sake of the social welfare. The monopolist would naturally locate in the center of the linear city.

Chapter 6 analyses a model in which the different marginal costs of production are endogenous and location-dependent. We assume that in order to produce one unit of the good, firms require a unit of a natural resource input that is controlled by a monopolist who is located in one of the extremes of the linear market. Therefore, when deciding their location, firms have to consider their distance to the input resource, as well as their distance to the demand. We conclude that if the unit input transportation costs are equal to zero, we have the basic case in the literature, which implies firms agglomerating in the city center. However, if these transportation costs start to rise, firms move closer to the location of the natural resource in a quasi-linear fashion (with respect to these transportation costs), while they are still agglomerated with each other. For values of the unit input transportation cost equal to or higher than the unit output transportation cost, firms prefer to agglomerate in the location of the natural resource. We then extend our model to the case where firms' owners delegate the quantity decisions to managers, which aim to maximize the average between the profits and the revenues of the firms. We conclude that such delegation movement is toxic for downstream firms and very beneficial to the upstream monopolist, since the former will demand more quantities comparing to the case with no delegation, which leads to the escalation of the input prices. The location rationale is similar: downstream firms will move, while agglomerated, closer to the natural resource location as soon as its transportation costs rise. However, they do not move in a linear fashion, and therefore delegated firms will be always closer to the center for the same values of unit input transportation costs, when comparing to the case with no delegation.

All the core chapters in this thesis share the same research lines. All of them revolve not only around the linear city of Hotelling, but more specifically, around the marginal costs of production of firms. In a comprehensive way, this thesis aims at verifying, for different market conditions (i.e. competition by prices vs. competition by quantities, endogenous marginal cost differences vs. exogenous marginal cost differences, social planner as a decider vs. firms as a decider, and perfect information vs. imperfect information), what is the effect of having different marginal costs of production in the location outcome of firms. While these articles have this link, the thesis can be fully separated in each of its chapters, as all of them include their own original findings and contributions to the spatial competition literature. We hope that this work can be an important contribution to the literature, in the sense that it can open different research paths for scientists to explore.

2. Literature Review and Recent Developments

2.1. Introduction

Spatial economics is "*concerned with the allocation of scarce resources over space and the location of economic activity*" (Duranton, 2008, p. 1). It may therefore be related to a very broad set of questions, as most economic questions involve space and location issues. However, according to Duranton (2008), the main focus of spatial economics is the location choice of the economic agents. In order to explain how agents choose to locate in certain places, specific modeling problems arise because of the difficulty of inserting location in the framework in a realistic way.

The starting point is the neoclassical paradigm, which assumes perfect competition and constant returns to scale. Accordingly, Debreu (1959) suggests that spatial economics is all about adding a spatial dimension to the goods and agents, meaning that every commodity and agent has different characteristics because they are located in different places, while there are transportation costs of commodities between different locations. In this framework, economic activities will be evenly distributed across a homogeneous space.

However, Starrett (1974) came up with a particular model where the locations are homogenous. Each location, as long as the production and consumption of goods are perfectly divisible and transportation is costly, will satisfy its own needs, reducing its transportation costs to zero, operating as an autarchy. Therefore, the equilibrium results failed to mirror the reality as there is no trade between different locations in the economy: every agent would maximize its utility by interacting only in its location. This finding gave rise to the Spatial Impossibility Theorem, which states that models of competitive equilibrium never involve transportation of commodities, which is counter-factual.

In order to explain the location choices of economic agents and the agglomeration of agents in certain locations, one must relax the core assumptions of the competitive framework. According to Fujita and Thisse (2002), three alternatives emerged and received huge attention in the literature: the assumption of heterogeneity of locations, in which there is an uneven distribution of resources, as in comparative advantage models (e.g. Ricardo, 1963 [1821]; Hecksher-Ohlin, 1991 [1919]) or in pioneering static location models (e.g. Von Thünen, 1966 [1826]; Weber, 1929 [1909]); the externality models, in which the economic activity endogenously generates spillovers that motivate the agglomeration of the agents (e.g. Marshall, 1920; Henderson 1974); and the assumption of imperfect markets, implying that agents have to interact with each other, with location being an important variable, as in spatial competition models (e.g. Hotelling, 1929) or in the monopolistic competition approach (e.g. Lösch, 1954 [1940]; Krugman, 1991).

This review will focus on the development of spatial competition models *à la* Hotelling. Specifically, the main purpose is to study models in which the location choice by the firms plays a major role, instead of those models in which, regardless of the spatial nature of price competition, the location of the firms is fixed.

This topic is extremely appealing, firstly because it mixes Game Theory tools with Regional and Urban Economics in order to explain firms' locations; secondly, because it offers some interesting insights into Industrial Organization, because of firms' strategic interaction and behavior; and finally, because of the huge literature in this research field and the recent insights gained regarding asymmetric information and its application to this subject. As a whole, this topic makes a very solid contribution to micro-economic science.

In subsection 2.2, the roots of spatial competition are reviewed. In subsection 2.3, along with a bibliometric approach to the papers in this area, some of the most important developments in the field are presented, with the focus directed at the optimal location decision. Subsection 2.4 presents the concluding remarks.

2.2. Spatial Competition – The roots

Spatial competition is mainly concerned with the locational interdependence among economic agents under the constraints of imperfect competition. According to Smith (1981), the first major contribution to studying interdependence among firms was by Fetter (1924), who constructed the law of market areas. According to Fetter, consumers compare the prices in both firms and the freight costs needed to buy that product before making their choice and the locations of consumers who are indifferent about buying at either location defines the market boundary of those firms. Some of Fetter's ideas influenced the work of most location theorists in the 1930s, but the most influential paper was that of Hotelling (1929).[1]

Hotelling's model was in fact one of the most significant historical landmarks in the development of Location Theory. In his model there exists a city represented by a line segment, where a uniformly distributed continuum of consumers has to buy a homogenous good in order to survive. Consumers have to pay transportation costs when buying the good, which is to be bought from one of the two firms existing in the city. Within this framework, firms simultaneously choose their locations and afterwards set their prices in order to maximize their profits.

Hotelling was actually more intent on proving the existence of a stable equilibrium in duopoly markets than developing a spatial framework. According to him, the main feature of the paper was the elimination of discontinuities in the demand of each firm, *i.e.* small changes in price would only capture part of the demand existing in the market, which would solve the Bertrand (1883) paradox, in which small changes in price would capture the whole market for one of the firms, leading the firms to an (unrealistic) equilibrium situation with no profits.

Moreover, Hotelling did not think of his framework as a location model, despite mentioning transportation costs. He introduced "distance" between firms as a way of modeling differentiation between the goods produced in each firm, with the goods being homogenous except for the location where they were produced, which is a similar concept of location introduced later by Debreu (1959). However, in the second part of the paper, Hotelling introduced the following question: given the location of a firm, which is the location for the other firm that maximizes its own profits? This question attracted scientific attention to this framework, which was extended in numerous ways in order to answer many different questions within, for instance, location theory (as will be shown later), game theory, industrial organization, social welfare and even mathematical issues such as the existence of equilibrium.

[1] One can notice that the Fetter's law of market areas is present in Hotelling's framework, but Fetter overlooked the issue of the optimal location or even the optimal price decision of the firms and was more concerned about modeling the demand behavior of the market.

In a quite different approach, Chamberlin (1950) introduces the concept of monopolistic competition. This approach arises because of product differentiation, in which firms may combine characteristics of being both in a monopoly and in pure competition, as they possess a somewhat unique product in a competitive market. Product differentiation may refer to many characteristics of the product, including its location. This "middle point" between pure competition and monopoly has new implications for the behavior of the firms when it comes to maximizing their profits. The parallel with the Hotelling framework is evident, as the "linear city" is meant to represent product differentiation throughout the market under study.

This review follows the framework of Hotelling, as the subsequent publications around this framework are more concerned with the agents' location behavior than the developments of Chamberlin, which are used more as a building block for product differentiation; or than the framework of Fetter, which has been relatively forgotten.

2.3. Developments in spatial competition modeling à la Hotelling: a critical review

2.3.1 A bibliometric exercise on research in spatial competition

Before proceeding to the analysis of the main contributions in spatial competition modeling that focus on the location decisions of firms, a numerical study is conducted in order to better understand the temporal development of the field. The analysis begins in 1979, the year that d'Aspremont *et al.* (1979) published what can now be considered a classic paper in the field, and ends in 2012.

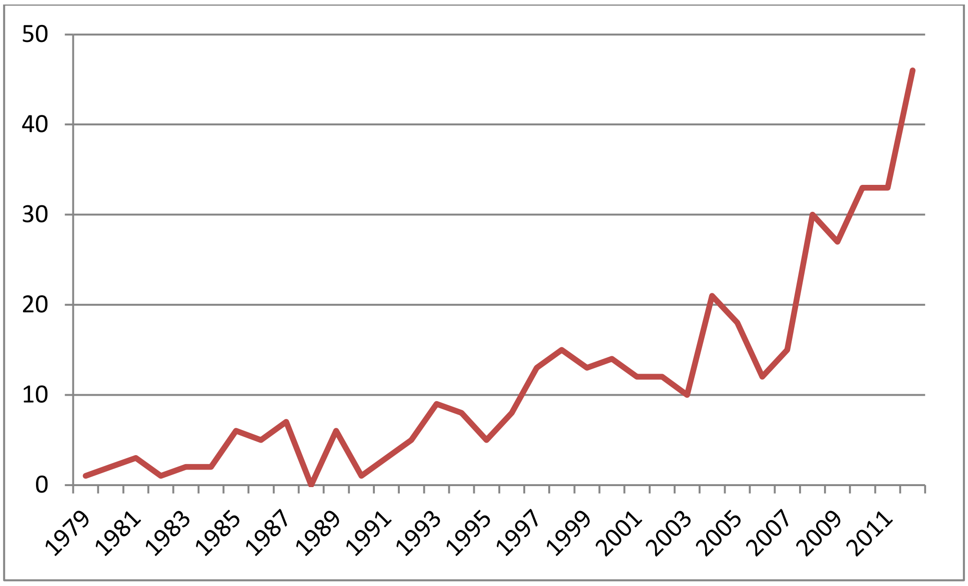
The search engine used was Scopus and only articles in the subject area of "Social Sciences & Humanities" were considered. Document type was filtered to only include peer-reviewed articles and exclude comments, rejoinders, book reviews and *corrigendas*. The database was constructed using the keywords "spatial competition" or "Hotelling" that were sought in the articles' title, keywords and abstract.[2] Finally, in

[2] As an alternative, we searched for the keywords "spatial competition" OR "Hotelling" in whole texts of papers, obtaining a total of 4,543 articles. However, most results were not directly related to the topic under study, and therefore, we chose to search only in titles, abstracts and keywords. Additionally, we also searched for the words 'spatial competition' in articles' titles, abstract and keywords (341 records) and in the whole text (1,383 records), as well as other possibilities ("spatial OR spatially" AND

order to develop a clear description of spatial competition modeling we have excluded any record that is not related with this field, by direct inspection of each article's title and abstract.[3] As a result, the database includes a total number of 398 journal articles published since 1979. Our intention is to give an idea of the development of the field, without intending it to be completely exhaustive.

By analyzing the distribution through time, we can see a gradual increase in publications, suggesting a positive evolution in the field's output (Figure 2.1). However, in relative terms, compared with the total number of peer-reviewed articles in Scopus – Social Sciences and Humanities, that is not the case, with an irregular trend in the importance of spatial competition over time being observed (Figure 2.2).[4]

Figure 2.1 – Number of Articles on Spatial Competition, 1979-2012



This evidence of the importance of the field of spatial competition is not surprising to anyone who is familiar with the literature discussed in the remaining of this section. In fact, spatial competition was a hot topic in the eighties and nineties,

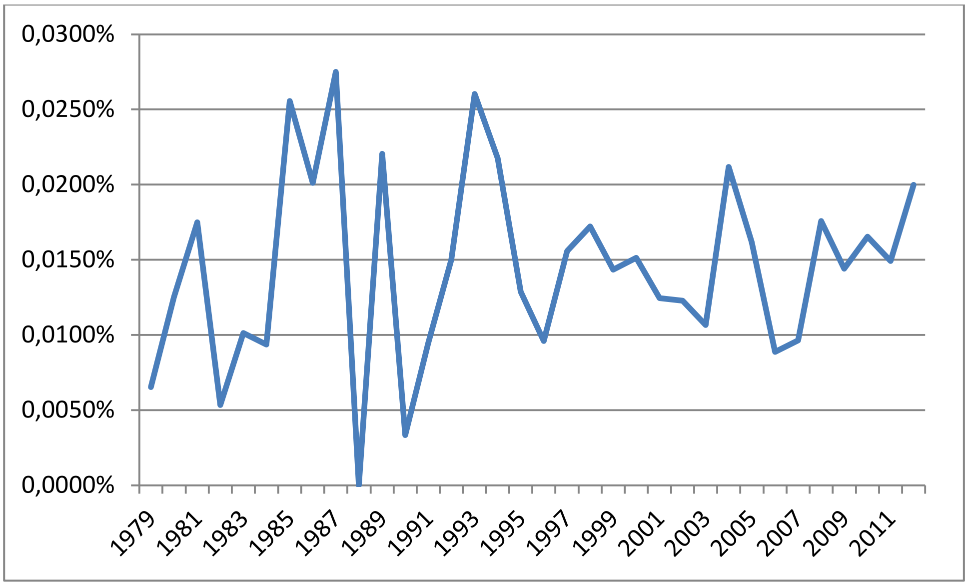
"competition OR competitive" OR "Hotelling" OR 'product differentiation'"; spatial AND competition), and adopted the best option as far as proximity to our subject was concerned.

[3] We had this necessity because Hotelling was also known for a statistical test, a famous rule in the field of exhaustible resources and for the "Hotelling's lemma" in microeconomic theory. Therefore, we excluded these articles to obtain a better assessment of research in spatial competition.

[4] It should be said that Scopus database covers a large set of journals after 1996, but has some limitations in the period before, which might justify the increasing pattern shown in Figure 1, as well as the absence of d'Aspremont *et al.* (1979) in the searched records.
(http://files.sciverse.com/documents/pdf/ContentCoverageGuide-jan-2013.pdf).

when a huge modeling effort was devoted to examining the effects of changing every Hotelling assumption on the subsequent equilibrium conditions.

Figure 2.2 – Published Articles on Spatial Competition (% of total Scopus), 1979-2012



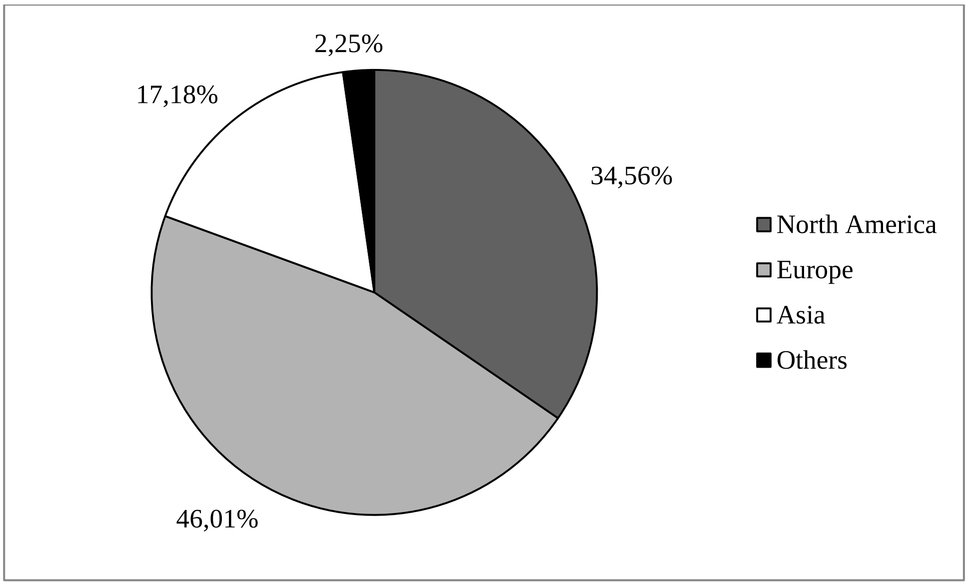
With respect to the authors' efforts regarding the spatial competition modeling, information about the most relevant researchers is displayed in Table 2.1. Noriaki Matsushima and Stefano Colombo are the authors with more articles in this research field, immediately followed by Toshihiro Matsumura and Jacques-François Thisse. In addition, when we take into consideration the average number of citations *per* paper, we may conclude that Jacques-François Thisse and Nicholas Economides are an important part of the most prominent researchers on this topic, together with Takatoshi Tabuchi, Debashis Pal and Jiotirmoy Sarkar[5]. Additionally, information about authors' geographic affiliation (Figure 2.3) reveals the importance of European authors' research into spatial competition.

[5] There are two important setbacks to highlight in our analysis: First, the number of articles is in full counts. This means co-authored articles still count as a full number. This means as well that there are duplicated articles, due to the collaboration of authors in Table 2.1. Examples being Matsumura and Matsushima, or Pal and Sarkar; Second, the indicator "number of citations per article" benefits authors of older papers, since, ceteris paribus, these papers had more time to be cited.

Table 2.1 – Top authors in Spatial Competition, 1979-2012

Author	Number of articles	Citations *per* article
Matsushima, N.	11	9.09
Colombo, S.	10	2.20
Matsumura, T.	8	8.50
Thisse, J. F.	8	32.38
Pal, D.	7	23.43
Braid, R.M.	7	3.71
Hamoudi, H.	6	5.33
Tabuchi, T.	6	21.83
Norman, G.	6	4.50
Straume, O.R.	5	8.40
Lai, F.C.	5	11
Meagher, K.J.	5	4.60
Gupta, B.	5	17.60
Economides, N.	4	32.50
Brekke, K.R.	4	9.25
Grofman, B.	4	1.75
Sarkar, J.	4	23.75
Sanjo, Y.	4	3.75
Lambertini, L.	4	10.25

Figure 2.3 – Authors' geographic affiliation, 1979-2012



In order to assess the quality of the research in spatial competition modeling, a selection of the most frequent journals in this field has been undertaken (Table 2.2). As expected, the vast majority are journals specialized in *Regional and Urban Economics*, besides other journals dealing with *Industrial Organization* or *Public Economics*. However, it is not only specialized journals that are interested in spatial competition, as more general ones also contain articles in this field, with *Economics Letters* and *European Economic Review* amongst those with the most publications in this area of research. Regarding the impact factor of these journals, we can see that approximately half of the journals containing at least 5 articles on spatial competition have an impact factor higher than 1, which implies that a significant number of publications in the field are published in journals that recently have at least a moderate impact.

Table 2.2 – Top journals in Spatial Competition, 1979-2012

Journals	Number of articles	% of total Spatial Competition	Impact factor 2011
Regional Science and Urban Economics	49	12.31%	1.008
Economics Letters	38	9.55%	0.447
International Journal of Industrial Organization	31	7.79%	0.841
Public Choice	13	3.27%	0.913
Economics Bulletin	13	3.27%	N.C
Journal of Economics Zeitschrift Fur Nationalokonomie	11	2.76%	N.C.
Papers in Regional Science	11	2.76%	1.430
European Economic Review	9	2.26%	1.527
Journal of Regional Science	9	2.26%	2.000
Journal of Economics and Management Strategy	8	2.01%	1.093
Journal of Urban Economics	7	1.76%	1.892
Journal of Industrial Economics	7	1.76%	1.040
Games and Economic Behavior	7	1.76%	0.829
Annals of Regional Science	6	1.51%	1.026
Social Choice and Welfare	6	1.51%	0.440
Journal of Economic Theory	6	1.51%	1.235
Economic Theory	5	1.26%	N.C.
Shanghai Jiaotong Daxue Xuebao Journal of Shanghai Jiaotong University	5	1.26%	N.C.
Research in Economics	5	1.26%	N.C.

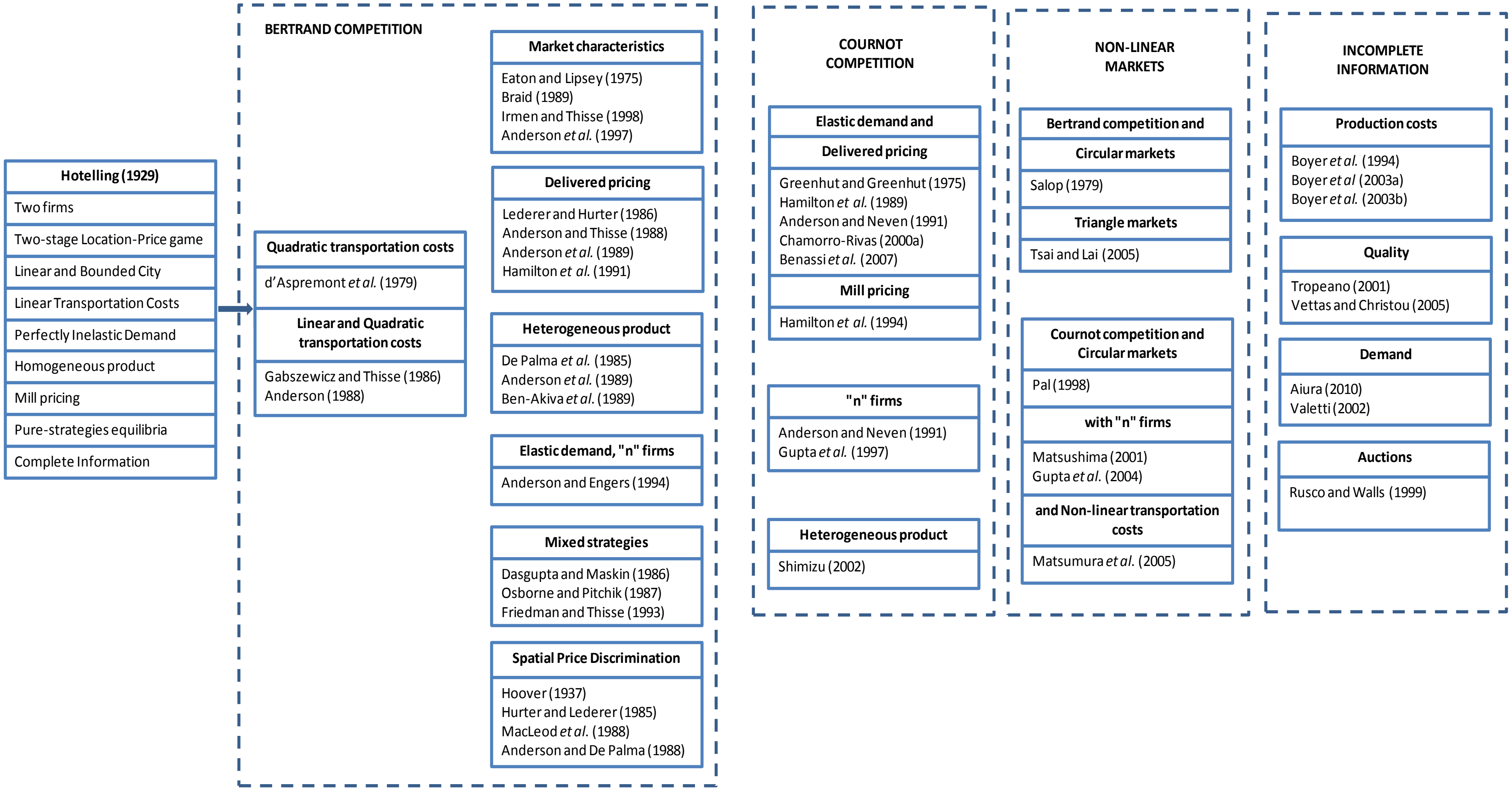
To sum up, the number of articles on spatial competition has been growing at a good pace. In addition, most of these articles have been published in journals with at least "moderate" impact in Economics, that is, journals with an impact factor of the recent issues higher than 1.

After this brief bibliometric overview of the research into spatial competition modeling, this chapter critically reviews the main models for each of the four research paths that we have identified after the work of Hotelling (1929). These paths are ordered according to the greatest frequency of publication, as exemplified in figure 2.4. The first group is Bertrand competition, which immediately follows Hotelling's (1929) model in terms of its assumptions, and shows the highest number of publications; secondly comes Cournot competition, differing from Hotelling's spatial-price competition, focusing on quantity competition in the second stage; in third place are non-linear markets such as circular or triangular markets, diverging from Hotelling's linear city; more recently, models of incomplete information between players have appeared, which extend Hotelling's complete information model. We also expose how the field of Experimental Economics has contributed to this literature, by presenting some of its papers in the review.[6]

Throughout the remainder of the review, the focus is directed on the papers related to the location behavior of the agents, rather than their pricing or quantity behavior. This means that other important articles of "spatial competition *à la* Hotelling", possibly included in the bibliometric search undertaken earlier, are not reviewed.

[6] It should be said that in the bibliometric approach, it is impossible to separate the papers between these different research paths because of the difficulty of finding keywords that are able to do so. For example, comparisons between Cournot and Bertrand competition are very frequent in papers of both research paths. As a result, no single keyword can reliably identify whether a paper contained in the search belongs to a particular research path.

Figure 2.4 – Main research paths in Spatial Competition



2.3.2 Bertrand Competition

2.3.2.1 Mill Pricing

The Hotelling model is an ideal basis for examining the behavior of firms when it comes to their price and location decisions because it allows for easy understanding and it has an appealing logic, and also because of its usefulness in studying firms' interactions. The Hotelling model is based on the following assumptions: two firms are the players in a two-stage location-price game, in which at the first stage, firms must choose their location on a linear and bounded city and at the second stage compete on prices. The good sold by the firms is homogenous except for the location they have chosen in the first stage. Demand is perfectly inelastic; *i.e.*, consumers in that city must buy one unit of the good, while incurring a linear transportation cost when travelling to one of the firms. In the second-stage, firms compete in a mill price setting, *i.e.*, they choose a price for their good, bearing in mind that each consumer takes into account the price plus the transportation costs when deciding from which firm to buy the good. In the mill price setting, a Nash equilibrium in the price stage is defined when both firms simultaneously choose prices (given their previous choice of locations) that maximize their profits, given the price set by the other firm.

With these assumptions, Hotelling concluded that firms would agglomerate at the center of a linear city, thereby laying the foundations for the "Principle of Minimum Differentiation", so called by Boulding (1966). This principle was undisputed and was used as a starting point for research, with its conclusions being studied and extended into many branches of research. However, almost half a century later some scientists started to question this principle, mainly by using the Hotelling model with some different assumptions. The most important conclusion is the one drawn from d'Aspremont *et al.* (1979), which introduced quadratic transportation costs[1]. The introduction of this feature removed the discontinuities verified in the profit and demand functions, which was a problem in the Hotelling model since there were no Nash price equilibrium solutions for all possible locations of the firms. The location decision for the firms in the presence of quadratic transportation costs is to locate at the

[1] Quadratic transportation costs are a realistic assumption when we are thinking of transportation costs different from the physical costs, for instance, consumer tastes.

extremes of the market (principle of maximum differentiation). Firms wish to differentiate more and more in order to relax price competition and thus obtain larger profits.

2.3.2.1.1. Models with the focus on location

Following the paper of d'Aspremont *et al.* (1979), the majority of the models abandon the linear transportation costs assumption, except for the cases where scientists were once again testing the cost functions, such as Gabszewicz and Thisse (1986) and Anderson (1988), who test a transportation cost function with a linear and a quadratic component. They prove that in some cases there is no price equilibrium for fixed symmetric locations and that in most cases no location-price equilibrium exists in the two-stage location game.

The assumption for the bearer of the transportation costs is changed, for instance, in Anderson and Thisse (1988), Anderson *et al.* (1989) and Hamilton *et al*. (1991). Hamilton *et al*. (1991) introduce a model where consumers are allowed to bargain between the two firms. This results in firms choosing the socially optimum locations, 0.25 and 0.75. The bargaining in the model is only possible because firms do not observe the consumers' locations in the city. The other authors reach no specific conclusions regarding location patterns: Anderson and Thisse (1988) and Anderson *et al.* (1989) focus more on the existence of equilibrium than on the location of the firms.

Different Consumer Preferences

In Hotelling's model, firms were interacting in a linear and bounded market, with only one differentiating dimension, and selling homogenous goods. Demand is assumed to be perfectly inelastic, meaning that consumers will always buy one unit of the good, whatever the price (if there is no reservation price). One important step in the literature is in the work of Economides (1984), who abandons the assumption that reservation prices had to be high enough such that all consumers would be covered, in an approach that became known as "the uncovered market model". The author concludes that for these lower reservation prices, firms would move away from the center, and that for even lower reservation prices, firms would prefer to move further to the city extremes in order to obtain local monopolies in each of the sides of the market, leaving some consumers in the city center uncovered. This analysis is extended by

Hinloopen and Van Marrewijk (1999), which conclude that for "intermediate values" of the reservation price, there is a unique equilibrium where both firms locate symmetrically from the city center and cover the entirety of the market. Chirco et al. (2003) extend this analysis by studying the quadratic transportation case. Departing from maximum differentiation, as the reservation price gets low enough, firms start moving closer to the center until the minimum transportation costs point of $x_1 = 0.25$; $x_2 = 0.75$. For lower values, multiple equilibrium locations exist in which firms are local monopolists.

While the linear and bounded market assumptions seem not to be too binding, the others seem quite unreasonable in terms of reality, but are easily understandable. The analysis of the equilibrium of the two-stage game with more than one dimension or with elastic demand proved to be a hard obstacle to overcome, while finding a way to quantify heterogeneity of the goods was not obvious.

Tabuchi (1994) introduce a model with two differentiating dimensions. The author concludes that the Nash equilibrium result involves firms maximizing differentiation in one of the dimensions (the one with a larger dimension, if that dimension is sufficiently larger comparing to the other), but minimizing differentiation in the other. On sequential location, firms differentiate always on the larger market dimension. In an ambitious paper, Irmen and Thisse (1998) extend the Hotelling problem to an n-dimensional market where consumers may weight each dimension differently. They conclude that when a characteristic is sufficiently strong, the situation in which the firms fully differentiate in one characteristic and locate in the center for all the others is a global equilibrium for the usual two-stage game. Therefore, "Hotelling was almost right"[2], in the sense that firms apply the principle of minimum differentiation except for the most important characteristic. The framework of n-dimensions is further extended in Larralde et al. (2009), which introduce heterogeneous logit-distributed consumer tastes similarly to Anderson et al. (1992). The conclusion is that equilibria other than maximum differentiation in all dimensions except on the most important one do arise, and involve partial differentiation in more than one setting, but are less profitable than the equilibrium found by Irmen and Thisse.

[2] That expression is part of the title of the paper written by Irmen and Thisse (1998).

Another change in the demand configuration is developed by Kim and Serfes (2006). The authors assume that it is possible for a consumer to buy the good from both firms, due to brand differentiation (with an example being the newspapers). However, purchasing the good from one firm decreases the utility of consuming the second good. The authors conclude that maximal differentiation is the equilibrium whenever the valuation of the second good bought is low. But when that valuation increases, the equilibrium jumps to agglomeration at the city center. The reason being that for such a high valuation, locating in the middle no longer means a decrease in the demand of both firms, since the majority or all consumers will buy both goods anyway.

Kohlberg (1983) takes the original Hotelling model and adds a waiting time cost to the consumers, which naturally is higher the higher the number of consumers visiting a certain store. Similarly to d'Aspremont et al. (1979), the author verifies that this adaptation eliminates the demand discontinuity problems of Hotelling, but no location patterns are given. This theoretical approach has been continued by Di Cintio (2007) which, in the linear unit transportation costs framework, introduces "crowding" and "congestion" effects per store, that is, the consumer utility varies positively and negatively according to the number of consumers that purchase their goods in a given store, due to crowding and congestion effects. The author concludes that the existence of these effects allows firms to have positive profits in the city center, similarly to goods heterogeneity cases presented in the next paragraph. The resulting location is agglomeration at the city center. This paper received a scientific reply from Ahlin (2008), which exposes that the crowding and congestion effects only solve the demand discontinuity problems of the original Hotelling model if these are big enough, and therefore the model is not valid for lower values of these crowding and congestion effects. Ahlin and Ahlin (2013) further analyze the linear transportation model with congestion costs, assuming consumers pay an extra cost for buying at a certain store, based on the number of quantities purchased there. They conclude that whenever pure-strategy equilibrium in prices exists, locations are as closer to the city center as possible. The greater the congestion costs relatively to transportation costs, the closer firms will be to the center, as a higher number of pure pricing equilibrium strategies exist. The author also concludes that, for quadratic transportation costs, the introduction of congestion costs leaves unchanged the maximum differentiation result.

Some authors address the assumption of homogenous goods by introducing heterogeneity into the model. Three different approaches appeared in the literature: De Palma *et al.* (1985), Anderson *et al.* (1989) and Ben-Akiva *et al.* (1989). De Palma *et al.* (1985) only change the homogeneity of the goods and conclude that when prices are fixed and equal for both firms, agglomeration at the center occurs and the profits for the firms grow with the degree of heterogeneity of the products (when the degree equals zero, we have the Hotelling case). In the two-stage game, agglomeration equilibrium may occur, but only if the degree of heterogeneity is high enough. Anderson *et al.* (1989) test different price schemes for a fixed location by comparing consumer and producer surplus in those cases, but since locations are fixed no conclusions can be drawn regarding location theory. Ben-Akiva *et al.* (1989) introduce a second dimension to the problem by considering brands which are intended to model heterogeneity. When firms play for location and price simultaneously with exogenous brands, agglomeration equilibrium at the center of the city occurs if the heterogeneity in consumer tastes is not too small. This result is very similar to the one found in Irmen and Thisse (1998), since if firms are able to differentiate on brands, they have the incentive to choose the central location because price competition is already softened due to product differentiation.

Anderson and Engers (1994) solve the two-stage location-price game for more than two firms and assume an elastic demand. The conclusion for the case of two firms is that if the demand is perfectly inelastic (Hotelling) or sufficiently inelastic, such firms will still prefer to agglomerate at the center. However, the nature of this game is different from that of Hotelling, as pricing in the second-stage is defined by a social planner.

A strand of literature focuses on "unilateral" models of Hotelling. The main differentiating assumption presented in this strand is that consumers can only travel in one direction of the linear city, which changes significantly the "rules" of the model. The main motivation for the introduction of this assumption is that sometimes it is only possible to travel in one direction, either passengers in one-way roads or in highways, either commodities, with one way pipelines of gas and oil (Kharbach, 2009) or one way river streams with fishermen attempting to catch fish (Lai, 2001), or when dealing with time (Cancian et al. (1995); Nilssen and Sørgard (1998)). Applications in the two stage location-price/quantity game are a recent strand in literature. However, as early as in

1995, Cancian et al. (1995) design a model where consumers could only move to one of the directions in the linear city, and conclude that when maximizing demand there is no location equilibrium for two firms. The authors' work is then extended by Nilssen and Sørgard (1998), who consider the sequential decision of two firms and a second discrete differentiation variable. Lai (2001) study the sequential location decisions of two firms in the directional market, and his work is extended by Sun (2012) to allow for three firms. However, we recall that these models implicitly fix the price decision of firms, turning the location problem into a demand maximization problem, which falls out of the scope of our analysis.

With the exact Hotelling assumptions except for the directional assumption, Kharbach (2009) shows that if the consumers can only walk to the "right" of the linear city, one of the firms would locate at 3/5 of the city, while the other firm would be located at the right extreme. However, Ebina and Shimizu (2012) prove that the solution found by Kharbach is not a subgame perfect Nash equilibrium, since in his solution the firm at the right has a monopoly of consumers, and therefore could charge the highest price possible without losing those consumers, which Kharbach did not consider by assuming the standard "indifferent consumer" way of calculating demand, which is not the most adequate on the unidirectional setting. The authors therefore conclude that this solution is only valid if the ratio between the utility the good gives to the consumers and the unit transportation cost is not too high. A later extension is due to Xefteris (2013), which studies a model in which firms only receive a payoff if their market share is higher than their opponents', but with fixed pricing. The results are generalized for any consumer distribution. The author concludes that there is no pure-strategy equilibrium, but there is a mixed equilibrium where firms locate half of the times in the median consumer, and other half of the times in the extreme to where consumers can travel to. However, this equilibrium is not unique.

Consumer distribution

The main feature of the following authors is to change the assumption that the costumers/consumers were distributed uniformly along the linear city. Shilony (1981) tests, for linear transportation costs, the effect of more general consumer distribution functions on location equilibrium. The author concludes that for any consumer distribution the lack of pricing equilibrium problem found by d'Aspremont et al. (1979) still exists, but another consumer distribution that leads to a price equilibrium result can be always found. Most importantly for our review, the author concludes the agglomeration principle of Hotelling holds whenever equilibrium exists. Neven (1986) tests whether a symmetric to the center consumer density, in which consumers are more concentrated in middle positions of the city, pushes firms out of the extremes. The author concludes that for some uneven distributions, the firms would still prefer to locate in the extremes. However, for larger concavities, firms start moving to the center until the location levels of 0.125 and 0.875 for firm 1 and firm 2, respectively. Anderson *et al.* (1997) change the density of the consumers to a symmetric log-concave function. The conclusion is that if the density function is too concave, asymmetric equilibrium appears in the location decision. Furthermore, if the consumer density function is concentrated more at the center that does not always lead to closer equilibrium locations. Transportation costs make no difference to the equilibrium location. Moreover, with this specification of the density function, there is excess differentiation in the product compared to the social optimum.

Changes in the nature of competition

Hotelling considered the case of only two firms in a two-stage game, deciding first their location and then prices simultaneously with pure strategies. However, the characteristics of this game have also been changed to address different issues or to search for a better overall realistic framework.

Dasgupta and Maskin (1986) prove the existence of mixed-strategy equilibrium for the pricing sub-game for all possible locations of the firms, paving the way for Osborne and Pitchik (1987), who discover that when mixed strategies are allowed only at the second stage, using only pure strategies in the first stage, the symmetric location where firms are located at 0.27 and 0.73 is an equilibrium. This equilibrium is near the social optimum, which are the location of firms that minimize the total transportation costs of the population. However, the transportation costs per unit distance in this model were set as a constant equal to one. Anderson (1988), as previously mentioned, concludes that there is no pure strategy perfect equilibrium for most cases when the transportation cost has a linear and a quadratic component. Nevertheless, allowing for mixed strategies at the price stage, the game becomes well defined, but if the transportation function is not convex enough, symmetric location equilibria must involve mixed strategies in pricing.

Neven (1987) tests sequential decision for the Hotelling model with quadratic transportation costs. For the case where firms are duopolists, the result is the same than in simultaneous decisions, that is, firms differentiate maximally in the city. However, the first firm can deter a second firm from entering if it locates at the middle, and given the fixed costs of entering are high enough. If the second firm enters, both firms will start to move symmetrically to the city center to deter entry from a third firm. Boyer *et al.* (1994) study the case of sequential location decisions within a uniform delivered price setting. In this game with three stages, one firm chooses its location first, followed by the other firm, after which both firms enter into price competition. With transportation costs equal to one and equal marginal costs, firms choose to locate at 2/5 and 4/5, respectively. The same framework, but with the mill pricing setting, is studied by Boyer *et al.* (2003a). If firms have the same marginal costs, the results are the same as those of d'Aspremont *et al.* (1979). However, if one firm has an advantage in its marginal costs, it starts to move progressively to the center, while the firm with the higher marginal costs always chooses the opposing extreme of the market. Gotz (2005) extends the work of Neven (1987) by analyzing entry conditions in market size changes instead of changes in the magnitude of fixed costs, and the location pattern changes a bit. The first entrant can still block a firm by locating at the center, but with a big market size, the third firm is blocked by the sole movement of the first-mover, instead of the symmetric two-firm movement that occurs in the case of Neven.

Lambertini (2002) builds a model where two firms enter a market sequentially, *à la* Hotelling, but take the lag between the entries of both firms as a variable. The main conclusion is that the longer the second firm takes to enter, the closer the first firm will locate to the center, while the second firm will always choose one of the extremes of the market. The first firm prefers to locate in the center because there it can set a higher price and still capture the entire market.

Ago (2008) consider *n* firms on monopolistic competition *à la* Chamberlin (1950), and conclude that all firms agglomerate at the city center independently of the pricing scheme used (between discrimination, and uniform mill and delivered pricing), as long as the transportation costs in the economy are low enough.

Allowing firms to locate outside of the linear city, Tabuchi and Thisse (1995) conclude that firms would prefer to be located outside, in order to further soften price competition, in locations -0.25 and 1.25, respectively. Further results in the unrestricted model are given by Lambertini (1997a). When there is a sequential decision, the first-mover stays at the center, while follower stays at one distance unit from the first-mover. When there is sequential decision in prices, the price leader chooses to locate at the border of the city, while the price follower stays two distance units away from the leader, to the side of the city (for instance, locations 0 and 2 for leader and follower, respectively, in a city with length 1). If firms are alternate leaders, that is, one of them leads in the location stage, while following on the price stage, the equilibrium found is that the location leader stays in one of the extremes of the city, while the price leader stays outside of the city in the opposing city side (e.g. 0 and 4/3 would be the locations for the location and price leader respectively). When firms lead/follow in both stages, the leader ends up locating in the city center, while the "double follower" gets pushed way out of the city, in location 13/6.

Firms' production costs

Some articles focused on changing the assumption that firms had the same marginal costs of production for their good. While others did so in an exogenous approach, focusing on those effects on the location outcome, some literature also justified the sources of cost differentiation either by assuming location-dependent marginal costs, either by allowing firms to invest in cost-reducing activities such as research and development.

An important exogenous approach is the one from Ziss (1993). The author tests what would be the optimal location of firms in the d'Aspremont et al. (1979) setting for any array of marginal costs of production firms could have. The conclusion is that maximum differentiation holds as the optimal location until a certain threshold of the marginal cost difference. Then, no location equilibrium exists as the low cost firm wishes to be as closest as possible to the high cost firm, while the latter would prefer to be as far as possible. An extension to the previous result is in Matsumura and

Matsushima (2009), who allow for mixed strategy equilibrium in locations for the same setting. The authors concludes that when no pure strategy equilibrium exists, there is a single mixed strategy equilibrium, which involves both firms locating in one of the extremes of the linear city with 50% probability. Another interesting extension is due to Meza and Tombak (2009). The authors model a three stage game of timing-location-price, where the timing stage determines which firms will enter first on the market, or whether they enter simultaneously. The authors are interested in checking how exogenous changes in the marginal costs of production affect the location outcome. If the cost differentials are too low, both firms are interested in maximally differentiating; hence firms enter as soon as possible and differentiate. For higher values of the difference, the lower cost firm will always obtain a monopoly if it enters in the middle of the city. For intermediate values of the marginal cost difference, the high-cost firm never enters first (because it would be instantly removed on the next instant by the low-cost firm), therefore the low-cost firm always enters first, which results in maximal differentiation for a larger array of values than what Ziss (1993) predicted; and for a further difference, the high cost firm will always be as far as possible to the low cost firm, while the low cost slowly moves closer to the center, and then after some threshold value finds it more profitable to "jump" to the middle of the city, obtaining a monopoly.

An endogenous approach was taken by Aiura and Sato (2008), which consider location dependent marginal costs, that is, firms have different marginal costs of production depending of their location in the linear the city. The approach is endogenous because the authors place a "natural resource" in the middle of the linear city, which is necessary for firms to make their good. All other assumptions of d'Aspremont et al. (1979) are followed. The conclusion is that if the transportation costs of the raw material are less than three quarters of the costs of consumers' transportation to the shop, the firms move symmetrically from the extremes in the direction of the city center. For higher values of that proportion between costs, another equilibrium result appears, in which one of the firms stays in the city center. However, if the raw material transportation costs are higher than 3 times the consumer transportation costs, firms prefer to agglomerate in the city center.

A different endogenous approach, more directly related with the topic of Industrial Organization, is pursued by Harter (1993), who considers R&D investments in a given industry. Two firms compete in a three-stage game and have a chance of inventing a given product first. In the first stage, firms choose what type of product to invest, and start investing. The second stage occurs after one firm has successfully discovered a product, and therefore entering the market first with the degree of differentiation it has chosen for its product. The non-innovator must choose between maintaining the investment in the same type of product; changing to a different product; or stopping the R&D investment and leaving the market. In the later stage firms choose their prices. However, the author is unable to provide an equilibrium pattern for the case where the two firms keep investing and will therefore enter the market.

In Gerlach et al. (2005), firms invest without costs in a project that has a given chance to succeed. If it succeeds, firms enter the market and are able to produce the good; if not, they stay out of the market. The key ingredient is that firms only know the result of the R&D after deciding where to locate. If the probability of the investment is low, firms locate in the city center. However, as the probability becomes higher, firms gradually move to the outside of the city, until the probability reaches 12/13, and then for higher values, firms maximally disperse.

In a combined approach with a mixed duopoly, Matsumura and Matsushima (2004) consider endogenous marginal costs of production in the typical d'Aspremont et al. setting. The authors introduce an initial stage where firms may reduce their marginal cost by paying an amount of money, and increasing amounts of money are required in order to reduce the marginal cost further. The authors conclude that equivalently to Cremer et al. (1991), having one public firm is sufficient to guarantee that both firms will choose the social optimal locations. This location is characterized by having both firms in ¼ and ¾ when there is no difference in their marginal costs of production, and by having the efficient firm moving towards the center and the inefficient firm moving towards the city extreme, with both firms being always separated by half of the city. Later on, Matsumura and Matsushima (2012a) study a model where firms are allowed to invest in cost-reducing activities in the first-stage. Their focus is not on location, since the resulting location outputs that arise from different marginal costs are already covered in Ziss (1993) and Matsumura and Matsushima (2009), and so they conclude that the optimal location (if unrestricted by city boundaries) leads to excessive R&D investment in terms of social welfare.

Ferreira and Thisse (1996) introduce a setting where firms have different unit transportation costs between them. Though their paper does not focus on the location decision of firms, the authors introduce the idea that firms should be able to invest in their own transportation technologies in the linear city setup.

2.3.2.1.2. Location as a strategic variable

An important share of the published articles use the spatial competition/horizontal differentiation setting not only with the focus of assessing the location outcome of firms, but also with the purpose of checking what is the effect of location as a strategic variable in topics such as: mergers, collusion, delegation, vertical relationships, R&D investments, and so on. We present some of these papers, but our main focus continues to be the location outcome that results from the introduction of any of these previous “ingredients” in Hotelling model.

As for collusion, Chang (1992) adopts and Hotelling approach in which firms are given a certain location, and are given the chance to relocate at a fixed cost *F*. In terms of location, the author concludes that if one of the firms deviates, firms would only relocate to the market extremes if their original positions were too close. Friedman and Thisse (1993) introduce a game in which location is played in the first stage, and then there is a repeated game in which players keep choosing prices for n periods. As the game is repeated, firms play a trigger strategy regarding prices. Allowing only for collusion in the price stage, the subgame perfect Nash equilibrium for this game is agglomeration at the center of the city, with partial collusion being chosen for both firms at the price stage. Hackner (1995) extends the Hotelling case by allowing for endogenous locations and collusion in both stages. He concludes that if the discount factor is low, firms maximally differentiate, but as the discount factor raises firms move symmetrically to the center until they reach the minimizing transportation costs location of 0.25 and 0.75. Correia-da-Silva and Pinho (2011) consider the collusion possibility when there are different marginal costs of production along the linear city, with the cheaper production spots being in the center. When firms are only allowed to collude in the location stage, they locate in the maximum differentiation position at the city extremes for lower differences in the marginal costs of production between points. However, if costs outside the city center become increasingly higher, firms agree to move in the direction of the city center. When in full collusion, firms locate at the social optimal locations for lower values of the difference, and move in the center direction if the difference becomes too high.

In terms of applications to the delegation question, an important work is the one from Barceno-Ruiz and Casado-Izaga (2005). The authors study the classic Hotelling model with quadratic transportation costs and unrestricted locations, with firms delegating the price stage decision to a manager. The authors conclude that delegation puts owners choosing to be located farthest than in the case with no delegation, that is, in -0.75 and 1.75. However, if the managers choose locations, the location results ends up being the same than in the unconstrained model of Tabuchi and Thisse (1995), that is, -0.25 and 1.25. Therefore optimally, firms would prefer to delegate their decision to managers, but only in the price stage, keeping the location stage choice to themselves.[3] Using an uncovered market approach as in Economides, Liang et al. (2011) compare Bertrand and Cournot outcomes when owners choose the location stage and managers sets the prices/quantities, in a linear transportation cost setting. The location outcomes for the Bertrand case are symmetric and range between agglomeration on the city center and 0.25 and 0.75 for each firm, depending on the ratio between reservation price and transportation costs. The higher the latter, the closer firms are to the city center.

A question addressed in Hotelling's model is mixed duopolies, that is, duopolies where one of the firms is private, and other has a public nature, that is, its objective function is not solely profit maximization. Cremer et al. (1991) provide the baseline framework for this case, and conclude that having one public firm is enough for having both firms located at the optimal best: the first and third quartiles in the linear city. Matsumura and Matsushima (2003) analyze the same question with sequential location, and conclude that when the public firm is the leader, the location result is the same as in simultaneous decision. However, when the private firm decides location first, the resulting solution is 1/6 and 1/2 for the public and private firm respectively. Lu and Poddar (2007) extend the previous analysis by considering partial ownership, i.e., a firm

[3] This result is important to the Industrial Organization literature, as it reverses the usual conclusion, which is that firms end up losing if they both choose to delegate their decisions.

can be partially owned by public and private shareholders, which reflects on its objective function. By having a private firm and a partially owned firm, the author concludes that if the share belonging to public shareholders of the partially owned firm is lower than one half, maximum differentiation holds. However, as the public share becomes higher, both firms move symmetrically and in a linear fashion towards the Cremer et al. (1991) result. Kumar and Saha (2008) test exactly the same conditions as Lu and Poddar (2007), but consider a different type of partial ownership. Instead of having the partially owned firm maximizing a weighted combination of social welfare and profits, the authors consider a version proposed by Fershtman (1990), in which the weighting of the response function is considered instead. The results are similar, with firms moving to the Cremer et al. (1991) result for earlier values (one third) of public ownership. Ogawa and Sanjo (2008) consider a mixed duopoly with a multinational firm, that is, the public firm does not consider part of the private firm's output on its social welfare, depending on its share of national capital. The conclusion is that if the share of national capital decreases (note that 100% national capital is equivalent to Cremer et al. (1991)), the public firm starts moving closer to the city center at a quick pace, which pushes the private (multinational) firm away from the center. Additionally, the authors test the sequential game, and conclude that when the public firm leads, the movement of the firms is similar, with the public firm moving towards the center and the private firm moving away from the center with decreases in the national ownership (the 100% case is equivalent to Matsumura and Matsushima (2003)).

Other approach was in the line of vertical relationships, that is, usually there are two types of firms – upstream and downstream firms. The former produces the necessary goods for the latter to make its goods, which are then sold to the final consumer. Matsushima (2004) analyses endogenous downstream firms' location in a linear city model. Upstream firms' locations are exogenous and restricted to symmetry relatively to the city center, and these firms incur quadratic transportation costs in order to sell their good to downstream firms. All other assumptions are equal to d'Aspremont et al. (1979) model. The conclusion is that firms are still maximally differentiated if the input transportation costs are not too high relatively to the output transportation costs and not too far to the location of the closest upstream firm. However, for higher values of the input transportation costs, firms start moving symmetrically towards the center.

When both firms' location choice is endogenous and simultaneously chosen, both firms start by locating in the extremes of the city, but for higher output transportation costs, downstream firms move first in the direction of the city center, then for higher values the upstream firms follow. Matsushima (2009) extends his previous work by studying the effects of integration in the upstream and downstream firms' optimal location decisions. If the input transportation costs are low, no integration occurs and the results are the same as in Matsushima (2004). For increasing values of this parameter, there is in the first place partial integration, and then full integration. For the former case, the integrated firm will always be located at the extreme of its market area, while the non-integrated firms move together towards the city center for higher levels of the input transportation cost; for the latter case of full integration, integrated firms mimic the d'Aspremont et al. (1979) result and stay maximally differentiated whatever the values for the input transportation cost.

Liang and Mai (2006) add vertical subcontracting to the model. They assume, in the d'Aspremont et al. (1979) setting, that there are two vertical integrated firms in the market, but one of the firms produces the input at a higher cost, and therefore subcontracts the other firm to obtain the input cheaply. The input market also has quadratic transportation costs. If the subcontractor gains all the benefits from the subcontracting contract (i.e. has all the bargaining power), the firms maximally disperse if the input transportation costs are relatively low, but may end up agglomerating at 0.125 (given the consignor started in the left-end of the market) as the input transportation costs converge to infinity. When the consignor has full bargaining power, the main difference is that for higher values of the input transportation cost, firms will agglomerate in the extreme where the subcontractor was initially located.[4]

Lai and Tabuchi (2012) introduce inputs in the linear city of Hotelling, and firms have to locate also according to the location of the inputs and their transportation costs, in an approach more related with Weber (1929 [1909]). They conclude that if the output price is fixed and high enough in relation with input transportation costs, both firms will agglomerate at the center. However, if the price is decided in the last stage and inputs

[4] Andaluz (2009) extends this model for vertical differentiation, and therefore falls out of the scope of our study.

are asymmetrically distributed in the city, then the firms disperse symmetrically around the monopoly location, and surprisingly earning the same demand and profits.

Matsushima and Mizuno (2012) consider a setting where three firms are in the linear city in each production stage (three upstream and three downstream firms), but the location of one upstream and downstream is fixed in the city center. The location results imply analyzing various different cases of integration between different firms, and are therefore too many to place in this article.

2.3.2.1.3. Other extensions

This subsection focuses on extensions to the Hotelling model that have had less attention compared to others, but have provided with more explanations for the location decision rationale.

Another topic dealt with is fiscal regimes, that is, the tax that public authorities may be able to set to influence the location decisions of firms. A starting point is provided by Lambertini (1997b), which studies the introduction of taxing/subsidies in the classic d'Aspremont et al. (1979) framework. He concludes it is possible for the regulator to attribute a subsidy/tax dependent on the location that firms choose, such that firms would prefer optimally to move towards the social optimal locations. A different conclusion is reached by Kitahara and Matsumura (2006), which find that both the *ad valorem* and the fixed tax are neutral towards the firms' location outcomes, both in the linear and in the circular market. However, if firms are allowed to have different marginal costs, the *ad valorem* tax may have some effects in the location outcome, in the line of Ziss (1993).

Lai and Tsai (2004) introduce zoning regulation, which means that the social planner may forbid firms from locating at a given area in the linear city. Only one side of the city is zoned, and therefore the location conclusion is that firms do maximally differentiate as well, but the firm on the zoned side is forced to be located closer to the center. This firm will earn more profits than the unconstrained firm. Matsumura and Matsushima (2012b) use the zoning argument combined with delegation to show that restricting the locations (for firms to be inside the linear city only) may not always be optimal for the social welfare.

Bertuzzi and Lambertini (2010) introduce an initial advertising stage and dynamics in the standard Hotelling game, in which firms may advertise the product benefiting both firms in the market in an equal manner.[5] The conclusion is that when the dynamics are unaffected by pricing and location and location is costless, there will never be a pure strategy equilibrium in the dynamic game. However, the authors assume costly reallocation, which allows for the existence of equilibrium. The main conclusion is that if the discount rate for future period earnings and the decay rate of the advertisement investments are larger, the larger is the differentiation between firms. With the rationale being that investments in advertising, when these are cheaper and permanent, are used as a way to increase firms' profits. The higher advertising costs become, the larger is the firm incentive to soften price competition through increasing differentiation.

To conclude this subsection, it is clear the relative importance of linear city mill-pricing competition in the spatial competition literature. The number of articles is fairly larger in this branch comparing to other pricing schemes. The reason is probably because this was the style originally introduced by Hotelling, and therefore these types of studies were regarded highly in the literature. In addition, it is clear to see in the last papers presented in the subsection that the number of applications of the linear city in Industrial Organization related topics is relatively high, and these are not replicated in such scale in other types of spatial competition models *à la* Hotelling. Regarding the location results, the highlight should be done on the multiplicity of results arising from the different models studied. The departure point in most cases seemed to be to find effects that would "face" the incentive to differentiate and shake price competition: an objective which was attained in some of articles presented here.

2.3.2.2 Spatial Price Discrimination Setting

Another frequent way of treating price competition in the linear city model of Hotelling is by introducing the so-called spatial price discrimination. In this setting, firms, instead of fixing a single price in their store, are allowed to set a price for each location in the city. This price will no longer be the price at the store, but is the delivered price, *i.e.* including the transportation costs, which are now incurred by the

[5] This model is based on Piga (1998), but the author fixes firms' location at the market endpoints.

firms. This setting allows firms, when they are monopolists, to fix the maximum price possible in each location, given that each consumer still buys the good. When there is more than one firm, Nash equilibrium in this price sub-game occurs when all the firms participating in the game do not wish to change their delivered price, given the delivered price set by the other firms, for every location in the city.

This setting was introduced by Hoover (1937) and Lerner and Singer (1937), while analyzing the location results of Hotelling and using Hotelling's other assumptions. Hoover (1937) was more cautious in deriving a location result, since it depended too much on the type of industry considered; while Lerner and Singer (1937) argue that the equilibrium locations of a finite number of discriminating firms on a unit interval are the same as the socially optimal locations. Greenhut and Greenhut (1975) studied the profits and prices of firms with different exogenous firm locations. Although not directly studying the two-stage location-price game *à la* Hotelling, this paper had a significant influence in the spatial price discrimination literature. However, in spite of the relevance of these three papers, their results are not entirely related to the two stage location-price game that is the main object of this review.

Moreover, within this setting, the usual focus of researchers is not the two-stage location-price game. Most of the papers present models in which the location of both firms is fixed, and so the focus is the profit and price results of the firms, as well as social welfare for the agents in the linear city. In this subsection, the focus is on the papers that contribute to explaining the location choice of firms, and therefore most articles on spatial price discrimination are not included in this review.

A starting point is Hurter and Lederer (1985). The authors state that the location of the plants that minimizes social costs in a city is an equilibrium result of the game. The reason is that every plant benefits largely from having consumers that live close by, allowing them to price discriminate effectively among those consumers without the "intrusion" of other plants.

An interesting exception is Anderson and de Palma (1988), who study the case where the products are assumed to be heterogeneous regarding consumer tastes. They conclude that, when the degree of heterogeneity is zero, the model is equivalent to Hurter and Lederer (1985)'s model and the location result is the social optimum.

However, as the degree of heterogeneity rises, both firms move away from the city center. But a further increase in the heterogeneity brings the firms closer, and after a given threshold, agglomeration at the center is an equilibrium result of the two-stage location-price game. This non-linear behavior happens because of two opposite effects: when the degree of heterogeneity increases, the concept of market areas becomes blurred, *i.e.* a consumer may prefer the firm located on the left, while another consumer located to the left of the first consumer may prefer the firm to its right. This makes the firms more competitive, and so they prefer to differentiate more in order to lessen price competition. However, as the degree of heterogeneity grows sufficiently, firms gain more monopoly power, as changes in prices become less important in defining each firm's demand, which causes firms to locate closer to each other. This last effect is similar to the one observed by De Palma *et al.* (1985) and Ben-Akiva *et al.* (1989) for the mill-pricing literature.

The paper of Anderson *et al.* (1989) provides a comparison between the profits and prices in different pricing schemes for the firms when products are exogenously heterogeneous in the eyes of the consumers. Although the locations are fixed, one can conclude that when in a duopoly, the profits for the firms and total social surplus are both higher in a mill pricing than in a price discrimination setting for any value of products' heterogeneity.

Although they focus more on finding the equilibrium conditions in a circular market setting (a disc), Lederer and Hurter (1986) prove that it is impossible for two identical firms that are price discriminating to be located at the same point in the market, since this leads to zero profits for both firms. In the same line of research, MacLeod *et al.* (1988) allow the firms to choose the number of stores to build in a linear city. After concluding that there is a Nash Equilibrium at the price sub-stage for every possible location of the plants, the location that minimizes social costs still remains an equilibrium result of the game. In terms of the number of firms entering the market, the authors are inconclusive, stating that: "In addition, we find that equilibrium may generate the socially optimal level of product variety, but may also produce more or less product variety than is socially optimal" (MacLeod *et al.*, 1988, p. 444).

In a short paper, Gupta (1992) examines whether firms still locate at the socially optimum places in the case of sequential entry. The paper states that, in the case of two firms, the first mover will locate closer to the center (0.4) while the second mover has to settle at a greater distance from the central location (0.8). Pires (2005) tests whether the second-best social optimum (that is, when the social planner controls only the location of the firms, but not the pricing), still emerges as an equilibrium result of the location-price game. The author concludes that when demand is perfectly inelastic or when firms practice first-degree price discrimination, the second-best location is achieved. Else, firms will disperse more than the social optimum.

Braid (2008), after a good summary of the results in the literature, models a two-stage location-price game in which consumers have an exogenous preference (other than location) over the goods of the firms. The author concludes that firms choose to locate in the socially optimum locations for the model.

Similarly to the mill pricing case, some work has been done in order to find the location consequences of studying the unidirectional Hotelling model. The starting point is Colombo (2009), who allows for price discrimination in the second stage of the game. The author concludes that one of the firms locates in the middle, while the other locates in the endpoint where the consumers can only travel to. Colombo (2011) extends this analysis by introducing elastic demand, concluding that one of the firms locates in the same endpoint, while the other stays in the center if the unit transportation cost is zero, but moves progressively to the endpoint with the increase in the unit transportation costs.

The mergers phenomenon is analyzed by Rothschild (2000) in a spatial price discrimination model of three firms, which two of them can potentially merge. However, the location analysis is too complicated to be resumed in a single result. Nevertheless, firms have the incentive to merge and to relocate after the merger. Rothschild et al. (2000) analyze a similar question, but they treat the merger option exogenously, with a merger happening between two firms with a probability p. If the merged firms split their incremental profits resulting from merger, and as the probability of both firms merging increase, they will locate closer to each other and farther from the firm that did not merge.

Following the example of Liu and Serfes (2007), who treat collusion in the price discrimination setting, but fixing the location of the firms, Colombo (2010a) analyzes collusion for endogenous location choices, and concludes that the firms' location, given that they are symmetrically located with respect to the city center, does not interfere with the decision of colluding when firms are colluding on discriminatory prices.

For the literature of mixed duopolies in the spatial price discrimination setting, the main contribution comes from Heywood and Ye (2009a), which found that simultaneous location in these duopolies does not change the location and welfare result comparing to the two private firms competing case. However, on sequential location, having the public firm as follower increases welfare, as it pushes all other private firms to more symmetric locations in the linear city. The same authors (Heywood and Ye, 2009b) also analyze the case where there is a foreign firm in the market, similarly to Ogawa and Sanjo (2008) in the mill pricing case. Introducing a foreign firm in a duopoly has no effect in the equilibrium locations chosen by both private (foreign) and public firm.

Some attempts have been made to introduce vertical relationships in the spatial price discrimination setting. An early approach is due to Gupta et al. (1994), who consider an upstream monopolist serving *n* downstream firms, which have to choose their locations in the linear market. The conclusion is that in the case of two firms, their optimal locations will be 1/2 and 5/6. Beladi et al. (2008) test a model where downstream firms cannot make all varieties demanded by the consumers: each firm makes a unique variety, and a common variety, but upstream firm supplies an input that suits all varieties. In result, firms locate symmetrically to the center and are as distant to it the higher the fraction of consumers that wants to buy the common variety, the higher the price paid for the input and the lower the fraction of consumers that prefers to buy the unique variety. Beladi et al. (2010) test the exact same model but with sequential location decision between the downstream firms. The conclusion is that the first-mover manages to get closer to the center than the other firm, and therefore snatching higher profits.

In terms of tax effects, two opposing results appear in this framework. On the one hand, Cazado-Izaga (2010) extends the Kitahara and Matsushima (2006) result in mill pricing to conclude that both fixed and *ad valorem* tax are neutral regarding firms' locations in the Hurter and Lederer (1985) example. On the other hand, Colombo (2010b) extends the framework to allow for elastic demand in each point of the market (and with uniform delivered pricing instead of perfect discrimination pricing), and removes the neutrality results of the model. The conclusion is that 1) the higher the tax rate the lower is the equilibrium distance between the firms; and 2) both *ad valorem* and fixed taxing have equivalent results on the location equilibrium.

To conclude this subsection, we can state that the location results for the game when firms are allowed to price discriminate against the consumers, according to literature, tends to be around the socially optimum values. This seems to happen because of two effects: on the one hand, each firm is interested in locating as far as possible from its opponents, allowing a better price discrimination against the consumers that are, in a sense, exclusive to the firm; on the other hand, what keeps the firms from settling at the extremes of the market is that they are responsible for paying the transportation costs of the good. Therefore, firms want to locate in a place that minimizes their transportation costs when transporting the goods to the potential demand. These two effects lead to the straightforward conclusion that in a duopoly in which the linear city model is symmetric in all its characteristics, *i.e.* in which neither side or firm has an advantage, firms share the market evenly and locate in the middle of their market areas, which coincides with the socially optimum result that minimizes the transportation costs of the economy.

2.3.3 Cournot Competition

This review will deal with the two-stage location game in which firms compete à la Cournot (in quantities), instead of competing à la Bertrand (using prices), at the second stage. The assumption of competition in quantities is usually less realistic than competition in prices when we think about competition among firms. The price of a good is an important determinant of the demand for it in most cases, while the quantities placed in a market seem to be a more indirect determinant of demand. However, in modeling non-spatial duopoly cases, the Bertrand (1883) model produces less realistic results than the Cournot (1897 [1838]) model (Mas-Collel et al., 1995, p. 394). In some industries, however, competition in quantities is a better assumption than competition in prices: the Cournot assumption is more appropriate for markets where quantity is less flexible than price at each market point (Anderson and Neven, 1991; Pal and Sarkar, 2002), and also when there are significant lags between the production decision and the price setting (Hamilton *et al.*, 1994). It is not a surprise, then, that some authors have decided to analyze this kind of location games.

It should be stressed that the way Cournot spatial competition is modeled has some significant differences relative to the Bertrand competition case. In the two-stage location-quantity game, firms select their location simultaneously and then choose the quantities supplied. However, at the second stage, instead of setting a quantity for the whole market and waiting for the consumers to travel to their store (as in the Bertrand case), each firm chooses to supply a quantity for each location in the city (similar to the spatial price discrimination setting, applied to quantities), which implies that the combination of quantities chosen by each firm in each location determines the price of the good in each location. Thus, a Nash equilibrium is defined at this second stage when for all locations in the city, all the firms set a quantity such that there is not a single firm that wishes to change its quantity delivered, given the quantities delivered by other firms (*i.e.* there must be a Nash equilibrium in all the locations of the city).

As the reader may have noticed, the agents that pay transportation costs within this framework are the firms, as they have to take the good to each location in the city. This framework can be better understood if we think that firms compete in a typical Cournot setting in every location of the city, with their "marginal costs" equal to the price of the good plus the cost of delivering the good to the chosen location. The profits of the firms will be the sum of the resulting profits in all locations of the city.

In terms of results, this different framework has new implications. In Bertrand competition with a mill-price setting, firms have their own market areas based on the existence of an indifferent consumer. In quantity competition, firms compete in every location of the city in a typical Cournot setting. Therefore, instead of having a "market area", both firms may sell their homogenous good everywhere in the city, which seems to provide a more realistic result. Additionally, the assumption of inelastic demand must be dropped, since competition in quantities in this type of demand would result in corner solutions in which the price would be infinite, somewhat analogous to the zero-profit condition in Bertrand competition (Hamilton *et al.*, 1989).

The starting point for this framework is considered to be the work of Greenhut and Greenhut (1975), who adapt the setting of spatial price discrimination, allowing for more than one firm competing in the market. Although not directly based on the Hotelling framework, firms select quantities when interacting with each other. This paper derived the profile of the delivered price schedule, paving the way for future studies into spatial Cournot Competition.

The baseline case used in this subsection will be that of Hamilton *et al.* (1989), who compare the case of price and quantity competition. The authors conclude that in the framework of quantity competition, for all values in which there exists a solution, firms will always agglomerate in the central location of the city. This is in contrast with the case of price competition, in which firms never agglomerate for any feasible range of values for transportation costs, given exactly the same assumptions.

Anderson and Neven (1991) extend these results by studying the equilibrium conditions of this two-stage location game. Ensuring that the reservation price is high enough such that in all locations every consumer buys from both firms, they conclude that when the demand is linear and transportation costs are convex, there is a unique equilibrium in the game, where both firms locate at the center of the market. Furthermore, for any changes in the demand or cost transportation functions, any location equilibrium in this game must involve symmetric locations between firms.

Later, Chamorro-Rivas (2000a) relaxes the assumption of high reservation prices and found that for lower reservation prices, the agglomeration equilibrium at the center ceases to be unique, although it is still an equilibrium result. For even lower reservation prices, Benassi *et al.* (2007) find that the central agglomeration location is no longer an equilibrium result. The unique equilibrium found is a dispersed symmetric equilibrium. Therefore, agglomeration does not hold when the reservation price (transportation costs) is too low (or high).

Hamilton *et al.* (1994) examine the two-stage game of location and quantities with Cournot competition where consumers pay the linear transportation costs. In this framework, there is no pure strategy equilibrium in quantities for all possible locations of the two firms (see Hamilton *et al.* (1994), p. 913, for a very intuitive graphical explanation). However, considering only the case for symmetric firm locations, the authors solve the two-stage game and conclude that firms locate very near to the center, where low values for transportation costs pertain, even if at the second stage mixed strategies are played.

The following three papers, in line with the Bertrand competition strand, change some assumptions regarding firms' and consumers' conditions for operating in the market. Mayer (2000) introduces the assumption of different production costs throughout the city, meaning that the location of the firms also matters in relation to the cost structure for the production of the goods. The main result is that if the global convexity of the production cost distribution holds, there is an agglomeration equilibrium result between the minimum cost location and the center. Depending on the cost distribution of the city, firms face a trade-off between the demand effect and the diminution of the marginal cost of production. However, they may still agglomerate even if it is not at the central location. Gupta *et al.* (1997) change the distribution of consumers in the city using a consumer density function, in a similar way to Anderson *et al.* (1997), in the case of price competition. They conclude that in the case of two firms, non-agglomeration cannot occur if the population density is sufficiently "thick" for all points of the city. Also, the agglomeration equilibrium found is unique. Shimizu (2002) introduces product differentiation into the Hamilton *et al.* (1989) framework. However, the main location result does not change and the central agglomeration equilibrium is unique for any relationship between the products of both firms, that is, if these are substitutes, complements or independent between them.

Extensions also appear in the case of competition within *n* firms. Anderson and Neven (1991) conclude that all firms agglomerate at the center, given linear demand and linear transportation costs, while Gupta *et al.* (1997) prove that agglomeration is the unique equilibrium if the non-uniform consumer density is not too "thin" along the linear city.

Pal and Sarkar (2002) introduce the interesting case whereby two firms compete by having more than one store, *i.e.* they can choose more than one location in the city. The main conclusion is that if the two firms have the same number of stores and the demand is high in relation to transportation costs, both firms choose their monopoly locations, thus partially agglomerating in the city. The results for the case where firms have a different number of stores vary significantly depending on the numbers involved.

The unidirectional Hotelling model has also some adaptations to the quantity competition case. An interesting case is the work by Sun (2010), who considers the unidirectional case in a circular market, and allows firms to choose to which direction they prefer to supply their product, since contrary to the Hotelling case, firms support the transportation costs to the consumer. The author concludes that when both firms can only travel in the same direction the equilibrium involves an equidistant location. On the other hand, if both firms can travel in opposite directions, then they agglomerate in any point of the city. If firms are allowed to choose to which way they can travel, the only solution in the three-stage game, or even in the two-stage game where firms choose their travelling direction and their location simultaneously, is that firms agglomerate and choose to distribute their product in different directions. Colombo (2011) considers the linear city model, and concludes that for lower values of the unit transportation cost, both firms agglomerate in the extreme of the market to where the consumers can walk to. As the unit transportation costs gets higher, one of the firms moves progressively until being one third away from the extreme where the other firm is located. Building on Colombo, Andree (2011) extends the model by introducing product differentiation in the style of Shimizu (2002). The result is similar than Colombo, and firms agglomerate for lower levels of the transportation costs, with one of the firms progressively dispersing from the extreme as unit transportation costs arise above a certain threshold. The main conclusion is that the introduction of product differentiation facilitates agglomeration, since it raises the value of unit transportation cost needed for dispersion to occur. Later, Colombo (2013) relaxes the assumption of "unidirectionality" by considering that firms pay different transportation costs to travel for different sides of the market, an assumption followed as well by Nilssen (1997) out of the context of the two-stage game. The author concludes that for the circular city, an equidistant equilibrium is the unique solution independently of the transportation costs for each direction. For the linear case, the author concludes that the standard agglomeration result occurs, but firms move away to the center in the direction where travelling is more expensive, in order to save on the transportation costs.

In terms of mergers and collusion, Norman and Pepall (2000) analyze the merger possibility of two firms in the Anderson and Neven (1991) framework. The authors conclude that the merger of two firms can be profitable due to the relocation possibility of firms.[6] The merged firms coordinate their locations such that one sells only in the left side of the market, while the other serves the right side. The optimal decision of firms is then to locate symmetrically to the center, but closer to the market extreme, due to the larger competition happening in the central locations. Tomé and Chamorro-Rivas (2007) analyze the collusion possibility when an infinite game is played in the second-stage, similarly to Friedman and Thisse (1993) in price competition. In terms of location outcomes, if firms collude, they split the market and position themselves in 0.25 and 0.75 respectively. However, attaining collusion is harder when the locations are defined endogenously.

Regarding the problem of delegation, Liang et al. (2011) analyze the Cournot case for an uncovered market with inelastic demand (therefore, in a different approach than the one of Anderson and Neven) The location outcomes for the Cournot case are equal to the Bertrand case: they are symmetric and range between agglomeration on the city center and 0.25 and 0.75 for each firm, depending on the ratio between reservation price and transportation costs. The higher the latter, the closer firms are to the city center.

In terms of endogenous production costs of firms, Wang and Chen (2008) introduce the hiring of workers by firms and analyze the equilibrium conditions with wage bargaining. For lower values of the transportation costs, firms agglomerate in the middle of the city. However, as transportation costs become higher, firms have the incentive to move away from the center in a symmetric fashion, in order to sell fewer quantities, which drive the wages of workers down.

In the zoning literature, Chen and Lai (2008), in a similar way to Lai and Tsai (2004), extend the literature by analyzing the effects of zoning regulations on the optimal decisions of firms. Firms are forbidden to be located in a central area which is symmetric to the city center. In addition, the authors assume that the firms do not have

[6] Remember that in the classic non-spatial Cournot approach, where it is assumed that there no efficiency gains from a merger and firms are assumed to be equal, a merger between two firms is only profitable if these firms are alone in the market (e.g. Szidarovsky and Yakowitz (1982) and Salant et al. 1983), which is the so-called “merger paradox”.

to sell the products in all points of the market, similarly to Chamorro-Rivas (2000a). The authors conclude that firms will locate at the endpoints of the zoning area, that is, as closer to the center as possible.

To sum up regarding quantity competition, one can say that in these models *à la* Hotelling, the conclusions are similar to those relating to the non-spatial quantity competition when comparing with the price competition setting in Industrial Organization: Cournot competition has less realistic assumptions, such as the delivered price setting and the competition in quantities itself, which is less realistic than competition in prices; however, the results are more fitting to reality, as the agglomeration result may be obtained more easily, and it is a fact that firms sell everywhere in the city, in contrast with the less realistic result of a "market area" for each firm. We can see that fewer assumptions from the Hotelling model in the location-quantity game are changed throughout time compared with the location-price game. This is one proof that the literature on price competition is more developed and that it is the result of the high attention that location theorists have paid to this kind of competition, seeking to solve the Bertrand paradox.

2.3.4 Alternative market specifications

One of the lines of research that followed Hotelling (1929) abandons the assumption of a simple linear market while remaining in the two-stage location-price framework. The most significant "deviation" from this assumption is the circular market framework developed by Salop (1979). In this subsection we also include quantity competition in the second stage, whenever it is developed in an alternative market specification.

Why should one work on circular markets? First of all, it is interesting to analyze the results for the location of firms, given that there are no extremes in the market. One can see that in the circular market, no location is *a priori* better than any other, which is not the case in linear market models (Gupta *et al.*, 2004). Secondly, there are some markets that may be better represented by a circular market, for instance, time-dependent markets, such as television companies who must choose time slots to broadcast their programs (Gupta *et al.*, 2004), or travelling companies who have to choose the schedule for their travelling services (buses, trains, airplanes).

Salop (1979) varies the Hotelling framework by assuming that consumers are located on a circle rather than on a line segment, although his paper is not the first to assume a circular city model (see Vickrey et al., [1999 (1964)] or Eaton and Lipsey (1975) for an early reference). The choice of this city specification is due to allowing "*the "corner" difficulties of the original Hotelling model to be ignored*" (Salop 1979, p. 142). This paper does not undertake an analysis of the two-stage location game, because it takes location as given. However, it is important as a starting point for all the subsequent two-stage game analysis in circular markets.

In the context of spatial price discrimination, Lederer and Hurter (1986) conclude that when firms have identical marginal costs and transportation rates, the agglomeration result cannot be equilibrium. Moreover, when firms are not identical, the equilibrium involves both firms being located on the opposite side of the diameter of the circle when the market is given by a disc. Within mill-pricing competition, the "base" result comes from Kats (1995), who concludes that in the circular market with linear transportation costs, firms optimally locate in opposing locations in the circular city.

In a short paper, Pal (1998) introduces the circular market into the two-stage location game in order to prove that Cournot competition does not yield spatial agglomeration in all situations. He concludes that, in equilibrium, two (or more) firms will locate equidistantly from each other on the city circle, which is a maximum differentiation result. Matsushima (2001) extends the conclusions to the case of n firms and proves the existence of partial agglomeration equilibrium, that is, half of the firms agglomerate at a point and the other half agglomerate at the diametrically opposite point of the circular city, given that the number of firms in the market is even. Gupta *et al*. (2004) take an important step in the study of circular markets, by identifying multiple equilibrium locations for a given number of firms, in which the findings of Pal (1998) and Matsushima (2001) are included. The highlight of the results is the existence of a huge number of equilibrium locations, though none of them involves agglomeration of all firms at the same point. An interesting result is that in the case of an even number of firms, all equilibrium situations yield equal profits and equal consumer surpluses. Matsumura and Shimizu (2006) conclude that independently of any profile of the transportation cost functions as long as it is increasing, the location equilibrium for two firms will always be the equidistant equilibrium location. Yu (2007) found that the location equilibrium in the case of spatial Cournot discrimination and spatial Bertrand discrimination are equal, meaning that if firms are discriminating sales per market point, it is indifferent for the location outcome whether they are competing in quantities or in prices.

Chamorro-Rivas (2000b) extends the analysis for two firms that can have more than one plant. In the case of two firms and two plants, the conclusion is that in equilibrium, the plants locate in each quarter of the market, with each firm setting its plants at diametrically opposite points.

Yu and Lai (2003) investigate what is the location decision of firms in the circular market depending or the degree of substitutability between products sold between two firms. The conclusion is that when firms are substitutes (for any degree of substitutability), the location equilibrium is having two firms located in the opposing side of the circle, while for any degree of complementarity, the optimal result is agglomeration at any city point. When there is not any relationship between the products, naturally any location pattern is location equilibrium. Similarly to Chamorro-Rivas (2000b), the authors extend their framework allowing firms to have more than one store in the city, and conclude that for two stores for any degree of substitutability the stores disperse one in each quarter of the circle, while partial agglomeration of two stores occurs in opposing locations, for the case of complementarity.[7]

De Frutos et al. (2002) study the consequences of the introduction of non-linear transportation costs in the circular framework, and conclude that for any convex transportation cost function there is a concave function for which the two stage location-price game is strategically equivalent. Matsumura *et al*. (2005) extend the previous framework by assuming as well nonlinear transportation costs. However, the paper considers the existence of four isolated markets in the city rather than a continuum of consumers. The main objective was to assess which equilibrium (Pal, 1998 *vs.* Matsushima, 2001) was the more robust, by checking its existence, given different configurations of the transportation cost function. It is shown that in the case of simultaneous entry, the location pattern identified by Pal is always an equilibrium, while the one identified by Matsushima only occurs if the transportation costs are not

[7] A quick summary of the location results in this strand of the literature is in Table 1 of Yu and Lai (2003, p. 577).

“too concave or too convex”. In the case of sequential entry, the location pattern of Pal is the unique equilibrium if the transportation costs are non-linear. Therefore, dispersion equilibrium seems more robust than partial agglomeration equilibrium.

There are a number of variations of the Salop circular city model. For instance, Brueckner *et al.* (2002) distribute the firms and the skills of the workers in a circular city, adapting the framework to the study of labor markets; and Arakawa (2006) applies the framework to studying the location problem of shopping centers, following an application of Henkel *et al*. (2000) which uses the well-known model of monopolistic competition in a spatial framework.

Regarding collusion and mergers in spatial competition models, in a rather complete approach, Posada and Straume (2004) analyze how firms relocate after the existence of a merger, or after the partial collusion possibility in either location or price stage. The authors assume that three firms are located equidistantly, and have to pay a cost to relocate their business. For a market with linear transportation costs, a merger between two of the firms results in a relocation of the merged firms towards the firm that did not merge. However, if firms collude only in the price stage, if there is relocation, the merged firms will move away from the firm that did not merge, while the reverse happens when firms are only able to collude in the location stage.

Similarly to the linear market case of Aiura and Sato (2008), Karlson (1985) introduce earlier a way to turn endogenous the marginal costs of production of firms by setting a natural resource in the circular city model of Salop, but allowing for elastic demand at every point. The author concludes that if no natural resource existed, firms would maximally differentiate due to the assumption of elastic demand. The existence of the natural resource brings firms closer, as the higher the importance of the natural resource in the model, that is, the higher the transportation cost required to transport the resource, the closer firms will locate to the natural resource and therefore, the closer they will be. Wang and Chen (2008) introduce wage bargaining in a quantity competition setting, but the conclusion is that it does not change the equilibrium locations found by Pal (1998).

The zoning topic, brought up by Lai and Tsai (2004) in Bertrand competition has also been extended to the circular market case by Hamoudi and Risueño (2012). The authors consider the classic two stage location price but add a first stage where a social planner chooses the area where firms are allowed to be located in. The unique location chosen by firms under this setting is one firm in each of the extremes of the zoned area. The social planner, knowing that firms will choose this location setting, will decide to zone half of the city, meaning firms will maximally differentiate. However, if the social planner places a weight higher than 4/7 on the firms' profit, the planner will only allow firms to be located in a single point in the city, therefore firms will be agglomerated.

In short, the conclusions arising from the assumption of circular markets are quite different from those found in the previous sections of this review. The main differences are that while a unique equilibrium was easier to find in a linear market setting, multiple equilibrium locations often arise in a circular market. Moreover, agglomeration of all firms in one location is rarely an equilibrium outcome in circular markets, in which most commonly partial agglomeration arises.

Turning to other market specifications, a different type of market is studied by Braid (1989), who examines the two-stage location-price game along intersecting roadways, and concludes that there is no equilibrium in the first stage of the game, for any number of firms, given that transportation costs are linear. Braid (2013) extends the model for n finite roads intersecting at a central location, but with the quadratic transportation costs assumption. There is one firm exogenously fixed in the intersection, but there is one additional firm for each line of the market, and it can only choose to locate there. The author concludes that the location of the firms in the line depends only on the number of firms, with the rationale that a higher number of firms changes the pricing incentives of the firm located in the middle. The higher the number of intersecting lines (and therefore, firms), the closer firms in the line locate to the center. The social maximizing location is between the resulting locations when there are 2 and 3 firms and intersecting lines, so firms deviate more from the social optimum the larger the number of intersecting lines.

Another possibility is to consider triangular consumer densities in the one dimensional models. Tabuchi and Thisse (1995) show that if the model of d'Aspremont et al. (1979) is extended to allow firms to locate outside the unit interval, then the symmetric location pattern (-1/4, 5/4) is obtained, but if the model is further extended to allow a symmetric triangular consumer density with a peak in the middle of the unit interval, then there are two possible location patterns, both asymmetric: (-0.272, 0.680) and (0.320, 1.272). In a slightly different approach, Mulligan (1996) assumes a market in which consumers are located uniformly in three different lines that form a triangle. His main important finding regarding the location behavior of firms is that the market configurations are critical to the equilibrium result, and therefore more realistic configurations should be sought instead of the classic linear or circular approaches. Tsai and Lai (2005) also assume a triangular market, in which firms are restricted to choose their location in only one of the lines, denoted as the "main street". However, similarly to Tabuchi and Thisse, firms are also allowed to locate outside of the interval that forms the "main street" line. For a symmetric triangle, the authors conclude that firms locate in the location pattern (-1/4, 5/4), similarly to the uniform distribution case and contrary to the findings of Tabuchi and Thisse for a symmetric triangle.

Huang (2009) models a case where firms locate in a linear city but demand is located in a parallel line, with the distance between those being an exogenous variable. The conclusion is that the original demand discontinuity problem of the Hotelling framework with linear transportation costs can be solved if the distance between the lines becomes sufficiently big. For intermediate values of this distance, firms depart from the center and choose symmetric equilibriums with intermediate differentiation, but for higher values the location equilibrium becomes maximum differentiation.

To conclude this section, the location results from alternative market specifications are naturally very dependent on the type of market specification assumed. The circular market is the most popular variant, and for some reason, quantity competition seemed to be the type of approach that was more explored in the circular literature. Other alternative approaches were usually sought when authors were intending to analyze very specific characteristics of a given market. Another interesting fact is that an important share of these articles are short notes with less than 10 pages, and published in journals that accept this type of articles, such as *Economic Letters*.

2.3.5 Incomplete Information

Now we shall turn to a more recent strand in the literature, dedicated to studying the location equilibrium of firms in cases where the agents do not have perfect information. As is known, the assumption of perfect information is quite unrealistic, as firms usually do not know the precise cost structure of the other firms or even the tastes of the consumers regarding their product and other competitors' products. The literature on this subject differs depending on the type of lack of information assumed.

In some of the following models, location is usually observed by all the firms and therefore it is used by the incumbent or by the first mover as a signal to the other firms of its cost structure or the quality of its good, previously determined by "nature". This type of games may be defined as signaling games (Macho-Stadler and Perez-Castrillo, 2001).

Boyer *et al.* (1994) study the case of sequential location decisions within a delivered price setting, where two firms choose their locations and afterwards their prices in a context of asymmetric production costs. Firm 1 first chooses its location having either equal or lower marginal costs with a given probability. Asymmetric information arises because firm 2 does not know the marginal costs of firm 1 before choosing its location. Therefore, location for firm 1 is used as a signaling mechanism for its cost structure.

When the difference between the marginal costs is low, a unique refined separating Perfect Bayesian equilibrium (PBE) exists, with firm 1's location being closer to the center compared to the case of complete information when it has both low and high costs. However, when the difference between the efficiency of high and low cost firms becomes too marked, the only refined PBE is pooling, and the incumbent finds it more profitable to locate in the same place independently of its cost efficiency. Its position in the linear city will depend on the beliefs of firm 2 that firm 1 is a lower cost firm.

Later, Boyer *et al* (2003a) develop a similar model, but with a mill pricing setting. In this case, there is a unique separating equilibrium if firm 1's possible disadvantage is not great enough or if it's possible advantage is very significant, which implies that high-cost firm 1 locates at the extreme and the low cost firm moves progressively to the center as its possible advantage is great, while firm 2 locates at the other extreme. If the relative advantage is not too big (for either of the sides), there is a unique pooling equilibrium at the extremes of the city for both firms.

In a similar context, Boyer *et al.* (2003b) study the case where there is an incumbent who might have a high or low marginal cost and an entrant who has to decide if it will enter the market. However, the entrant does not know the true cost of the incumbent, which allows the latter to use location as a signaling mechanism. Agglomeration equilibrium never occurs, for both delivered and mill price settings. This happens because in the pooling equilibrium, the incumbent chooses a central location, preventing the entry of the second firm, while for the separating equilibrium, whenever the incumbent chooses a location closer to the center it is because it is a low-cost firm, thus pushing the entrant to the other extreme of the market.

The following models also deal with lack of information, but have a different modeling perspective, other than the signaling game explained by Macho-Stadler and Perez-Castrillo (2001).

Vettas and Christou (2005), allowing for vertical differentiation, study Hotelling's two-staged location game. Two firms know the existing quality difference between them, but do not know who has the better quality. In the first stage they decide on their locations, while in the second they know the relationship between both qualities and so compete in prices. If there is no quality difference between the firms, the results for the location game are the same as those shown by d'Aspremont *et al.* (1979). As the quality difference grows, firms tend to draw close to the center. This mechanism occurs because firms compete in prices, which implies that the equilibrium prices when the firms are agglomerated are exactly the quality difference for the firm with the higher quality and zero for the other firm. Therefore, there is an incentive to agglomerate if this quality difference (keeping the transportation cost constant) improves, because the possible monopoly profits are very high in the case of a firm with better quality.

In a paper by Harter (1996), uncertainty is in the demand location. Firms have to decide their location knowing only that demand is randomly located: The linear city is [0, 2], but the consumers of the good are only located in [θ, θ+1], with θ belonging to the interval between 0 and 1. An additional assumption is that firms that have chosen to locate out of the linear city will automatically earn 0 profits. In a sequential location game, the first firm will locate close to center (1.01), while the second firm chooses the middle of the larger market side (0.50). In a similar setting, Aiura (2010) studies the equilibrium locations of three firms when location is decided sequentially among them. That is, the game has three stages, at the first stage of which firm 1 chooses its location and so on until all the three firms have chosen their locations. Prices are fixed, which implies that maximizing profits is equal to maximizing demand. The linear city is similar to the one in Harter (1996), so there is uncertainty in the location of demand. The asymmetric information problem arises because firms do not know θ when choosing their locations. However, the subsequent firms can observe the demand of those that have already chosen their location, thus updating their beliefs about θ. The Perfect Bayesian Equilibrium result is that firm 1 locates in the center, firm 2 also locates in the center and firm 3 unambiguously chooses to locate infinitesimally to the left or right of both firms. The rationale is very intuitive: firm 1 chooses the value that is expected to capture the maximum demand possible in the future. Firm 2 chooses the same as firm 1 in order not to provide firm 3 with any kind of information. Firm 3, since it does not know anything about the true location of θ, will randomly choose to capture one of the two sides of the market. Therefore, agglomeration equilibrium at the center of the city occurs in this interesting case.

Although the following papers do not mimic the classic problem of information presented in microeconomics, the problem identified in these articles is very similar to a signaling problem. Meagher and Zauner (2004) model a different type of demand uncertainty, with the parameter that defines demand following a normal distribution instead of a uniform one. Bonein and Turolla (2009) extend the type of demand uncertainty in Meagher and Zauner to account for sequential location decisions, with only one of the firms having perfect information about the demand location. In both cases, the results are derived with respect to the normal distribution parameters, and are therefore hard to summarize in our work.

In opposition to the previous problems, Schultz (2004) considers a case where the uncertainty comes from the consumers, which are unsure about the degree of the horizontal differentiation that the goods sold by the firms, that is, consumers are not certain about the location of the firms before they buy the good. The author concludes that the higher the degree of consumers that do not know the exact locations of the firms, the farther firms will locate from the city center (given they depart from the out-of-the city result by Tabuchi and Thisse (1995) in an unrestricted model). Therefore, uncertainty benefits firms, but it worsens the consumer situation and the total social welfare. In a similar model, Schultz (2005) analyses the effect of this uncertainty in the collusion possibilities of the firms in a Hotelling model. However, locations are fixed in the city extremes. Krol (2012) introduces uncertainty in the location of demand as well, but additionally firms do not know the unit transportation costs incurred by the consumers to purchase the good. This leads firms to make an expectation about how their profits will be, and the authors use a minmax criterion, in which firms combine linearly the best and the worst possible outcome. The conclusion is that if firms are too pessimistic, an increase in demand uncertainty leads to a decreasing differentiation between firms. Moreover, if firms were allowed to choose their "degree of pessimism", firms would choose maximum pessimism, which would lead them to locate optimally in closer locations than they would under other degrees of pessimism.

The following model by Valletti (2002) is a typical case of adverse selection. The consumer has private information before the purchase of the good and therefore the firm has to design different goods and prices for each type of consumer (Macho-Stadler and Perez-Castrillo, 2001). Valletti (2002) builds a model where consumers are distributed within a linear city but there is also a vertical component, determined by the quality of the good. In each location, there are two types of consumers: the ones who prefer a high quality product and those who prefer the low quality one. Therefore, the two-stage location game played by the duopoly firms is slightly different from the Hotelling location-price game. In the first period firms choose their location but in the second period they offer discriminatory contracts, as is usual in the case of principal-agent problems. The conclusions regarding the locations in the two-stage location game depend on the ratio between the high quality and the low quality goods demanded by the consumers. However, firms' locations will always be close to the socially optimal levels for any value of this ratio. The main changes that different values for the ratio induce are in the distribution of the surpluses between the firms and the consumers.

To conclude, the agglomeration results in this literature seem to depend heavily on the type of asymmetric information assumed. Models without the standard specification of asymmetric information are able more easily to find conditions for agglomeration of the firms.

Within a different framework, Rusco and Walls (1999) develop an auction model, in which two firms located at the extremes of the market compete for the purchase of some good, which is randomly located somewhere within Hotelling's linear city. The game has two stages: in both stages, firms participate in an auction in order to acquire the good. The main feature of the model is that the firm that wins the first stage will have an expected lower utility in the second stage auction. The imperfect information issue arises because firms do not know where the second auction will take place, which will condition their behavior at the first stage, since if they lose the first auction they will have a relative advantage over their opponent in winning the second auction. Although this approach does not reach any conclusions regarding the location of the firms, its interesting framework may be developed in order to explain the location behavior of firms when participating in an auction.

To conclude this section, and in our opinion, the literature is not yet fully developed on this topic. Imperfect information is a realistic assumption when competing in a duopoly. Competing firms are usually very motivated in gaining the upper hand relatively to its opponents, and therefore imperfect information may arise because, on the one hand, the secrecy of firms actions is an important topic, and firms must act without knowing the exact conditions in which its opponents operates, or in which conditions the quality/variety of its own product relatively to the product/variety of the opponent is perceived by the consumers; on the other hand, duopoly firms have the capacity and are pushed to invest in R&D to obtain different or better varieties, and there is a natural uncertainty regarding the output of R&D investments. How this uncertainty relates with the optimal location of firms is already being answered in the articles summarized in this subsection. However, we feel that there is still a long way to go in terms of finding how different forms of uncertainty, in markets that present very different conditions and specificities, factor in the firms' choice of optimal location.

2.3.6 Experimental approaches

An important and recent approach has been the one related with experimental economics. In this type of empirical testing, instead of dealing with databases and subsequent econometric estimations to verify the validity of a theory or a theoretical result, scientists analyze the reactions of economic agents in an environment close to a laboratory, since the rules of game and subsequent outputs are explained to the experimental subjects, whose reactions are compared with the reactions that are predicted in theory. In models *à la* Hotelling, typically the former empirical analysis is not used, due to the difficulty of finding a real-world variable that represents product differentiation, especially horizontal differentiation. Therefore, economic experiments appeared in this framework essentially because of the lack of real-world variables (Barreda-Tarrazona et al., 2011).

A first approach in spatial competition is due to Brown-Kruse et al. (1993). The authors test what would be the location outcome of the experiments in a duopoly, with prices fixed, and with the game being repeated for a finite number of stages (the game could end after each stage with a probability of 0.125), to allow for a profitable collusion between the players. In addition, the market points had a linear demand, which allowed for an increasing penalty to the firms of being distant from a certain market point. An interesting ingredient is that in some cases the experiments were allowed to communicate between them before choosing their location, which allowed testing if the opportunity to communicate led to more or less cases of collusion. The experimental results were that when there was no communication, the players chose very often to stay around the city center, which is the safer position since it has the highest "worst income possible" of all locations. However, when both players choose the city center, the payoff is the minimum. The results when communication is allowed was significantly different, as players often found out the maximum output they could earn by playing this game, which was location on the quartiles of the market.

Brown-Kruse and Schenk (2000) do a similar approach than in the previous paper, but the main difference is in the distribution of consumers in the linear city, in which now non-uniform distributions are tested, with consumers being either concentrated in the city center or more concentrated in the extremes, alongside with the usual uniform distribution. Then, the authors also test the experimental results of a simple 2x2 decision matrix that sums up the Pareto and Nash equilibrium options, to understand if a simpler decision environment changes the experimental results. The most important results were that symmetric play was observed between the experimental subjects, but the joint profit maximization option was harder to achieve. Simplifying the decision array of the subjects helped them in achieving a collusion solution, but again the communication between the subject rose as the most important facilitator for cooperation. Another approach was done by Collins and Sherstyuk (2000), who consider three firms in the market, and inelastic demand in each point, in an assumption closely related to the Hotelling original model. However, the price decision is still exogenous, and between different sessions the players changed, removing chances of collusive behavior between the players. The experimental results were somewhat in line with the theory, with the players locating between the first and third quartiles very often, which corresponds to the mixed equilibrium result found by Shaked (1982) in this three-player game. However, there were a few differences, namely with the dispersion values of the location points chosen, that were higher in the experimental case. The authors test some reasons for this behavior, and concluded that risk-aversion had a role, since the solutions of the experiment were closer to a risk-averse theoretical prediction than to a risk-neutral one. Huck et al. (2002) extend the analysis to a four firms setting with the same assumptions from Hotelling. The theoretical Nash equilibrium is firms partially agglomerating in the first and third quartile, as in Eaton and Lipsey (1975). The objective was to test if the given 4 subjects, during 50 consecutive rounds and having perfect information about what happened in previous rounds, were capable of converging to the equilibrium which is not very intuitive. The authors conclude that there was concentration of answers around the first and second quartiles, but a smaller concentration around the city center was verified. Across time, the distance between the players did not show a decreasing trend, suggesting lack of long-run convergence to the Nash Equilibrium.

An important breakthrough in this strand of literature is in the consideration of a characteristic closer to the two-stage game *à la* Hotelling, which is the main focus of this revision. Further experiments took in account that the distance between both firms affects the price they can set, and therefore the way that it affects their profits. Camacho-Cuena et al. (2005) consider that the experimental subjects may choose their location and then, given the locations that were previously chosen, the price sub-game is repeated either two or five times, in order to assess the effects of different location rigidities on the output level. The assumptions are as in Hotelling, with a significant difference being the fact that the consumers are not uniformly distributed in the city, but instead choose their locations after the sellers. The theoretical model, calibrated with the assumptions tested in the experiment, predicts that firms would locate sufficiently apart from each other. The authors conclude that longer periods of the price game resulted in a location closer to the center, therefore in a more defensive position, which resulted from the fact that the subjects could not change their location often.

In an experiment that was designed specifically to test the "principle of minimum differentiation", Barreda-Tarrazona et al. (2011) simulate the two-stage location price game of Hotelling in a city with seven discrete points, and with the possibility of players choosing price levels such that the market may become uncovered. Given the parameters set, the collusive equilibrium would be locating (approximately) on the first and third quartiles and setting the highest price possible to cover all the market, while the non-collusive equilibrium would imply the same locations but with lower prices. The results were clear: The players preferred to locate closer to the center than what was predicted, possibly due to the demand incentive. Players were then kept with their location decision for 5 price rounds, but were unable to find the collusive solution, because they were so closely located that a price war emerged. However, for the cases where a level of differentiation was attained, the price behavior followed what was predicted by theory, with price competition being relaxed, players were able to set higher prices and therefore profits. In addition, in almost all the cases the market was always covered.

To conclude, this section shows the interesting nature of the Hotelling framework. The field of Experimental Economics excels in showing how simple assumptions like the profit/utility maximization rationality are not always easily achieved by players/human beings. The world presents too many variables for any player to be able to maximize entirely the outcome of his daily actions, as a mathematician does. Of course, these actions become even harder to understand and attain when there is competition between players, as it is in the case of competition games as the one developed by Hotelling. In fact, in the articles we have summarized before, players struggled in many cases to achieve the location decisions that theory predicts they would do, and that is the main contribution of this strand in the literature, which definitely calls for a deeper understanding of the human behavior when humans are being placed in similar conditions as theoretical models invite to: as Hotelling players.

2.4. Conclusion

After the appearance of the Hotelling (1929) model and the important findings of d'Aspremont *et al*. (1979), scientists had access to a simple and successful means of introducing a spatial component into the modeling behavior of economic agents. This review has focused on the developments that were intended to explain the equilibrium locations of the firms, mainly when competing in a duopoly. However, many successful variants of this framework were used to justify spatial price discrimination and different market specificities, to furnish two examples.

In the 80s, this field became a hot topic for research. There are numerous applications of the Hotelling model, which mainly focus on changing the framework assumptions. The field developed significantly with the successful modeling experience of Hamilton *et al*. (1989), which allowed for competition in quantities.

More recently, Pal (1998) combined the circular framework of Salop (1979) in order to study the location decision of firms. In addition, the development of the asymmetric information framework in microeconomics and its successful adaptation to the context of spatial competition again led to the extension of the field. The approaches extended to the field of Experimental Economics, where several papers have been made based on Hotelling's theoretical approaches. However, these last approaches did not receive similar attention.

After a brief look at the numerical exercise done in section 3.1, it would seem that most of the important features that justify the spatial behavior of firms have already been explored. The future of the field depends on the researchers' capacity to find an (even more) interesting and innovative way of studying spatial competition. There is a high proportion of spatial competition models *à la* Bertrand or *à la* Cournot, compared with the most recent assumptions presented in this review. In that sense, we think that future researching efforts in spatial modeling should be made in the incomplete information setting. Furthermore, researchers could intensify the relationship between spatial competition and Industrial Organization. For example, spatial competition may provide a more complete answer in relation to vertical differentiation/integration of duopoly firms or to the R&D investment decisions by firms, in line with the seminal work of d'Aspremont and Jacquemin (1988).

In terms of how the next chapters contribute to the above-mentioned topics in the literature, all chapters have a concern on how firms will react given that they may face different marginal costs than their opponent. Chapter 3 contributes in the line of Ziss (1993), with firms having different marginal cost of production between them, but with the ingredient that they are uncertain about what their marginal cost is before choosing, and therefore contributing to the literature of incomplete information. Chapter 4 is more connected with Industrial Organization topics, with firms being able to choose endogenously their marginal cost of production before setting their location and prices. Chapters 5 and 6 are in the framework of quantity competition. Chapter 5 assesses what happens when firms have different marginal costs between them, while chapter 6 contributes in the literature of vertical relationships, with firms needing a localized input to make their output goods.

3. Firms Location Decision under Marginal Cost Uncertainty

3.1. Introduction

Quite often firms have to decide their exact location without having full knowledge about their costs, even considering that previously they carried out detailed market and technical studies to support their location decisions. Only after locating the plant and starting the production process, firms understand their true production costs. This is the situation that we study in this work. We analyze the behavior of two competitors in the framework of Hotelling (1929) but we depart from the classical two-staged location-price game by introducing uncertainty regarding firms' costs. We assume that firms are only fully aware of the costs between the two stages of the game, that is, firms only know their true marginal costs of production only after deciding where to locate in the linear city.

There are innumerous conditions that have significant impact on costs, and that firms only learn after setting up their plant in a given location. As an example of such condition we have, for instance, a correct assessment of costs needed to enter in the market, the consumers' behavior towards the product; the type of relationship with suppliers and with rival firms; or the technological capacities. In spite of the various determinants, cost uncertainty in this chapter is referred to as being caused by technology features. Before setting up their plant firms are expecting certain productive behavior as a result of investments made in technology and capacity. However, only after locating the plant and starting the production firms fully understand the effects of these investments on their marginal production costs.

We analyze a simple type of cost uncertainty. We assume that firms can only have two possible marginal costs: low marginal cost (which corresponds to a successful investment) or high marginal cost (which corresponds to unsuccessful investment). The unsuccessful marginal cost is equal for both firms, while the successful marginal costs are allowed to be different. This hypothesis is justified, for instance, when firms are expecting the results from cost-reducing R&D investments. If these investments fail, firms maintain their high marginal cost. However, if R&D investments are successful and a more efficient technology becomes available for production, the firm will have lower costs. We allow firms to have different costs, in case both firms have successful R&D investments. This represents the fact that firms may have invested differently before and/or that they have a different technology advantage *a priori*.

In addition, we assume that firms can only be located at the extremes of the linear city. This assumption is adequate to represent the situation of industrial firms, which may be forced to be located in the suburbs, away from central positions in the city. That may happen either because those industries are specifically forbidden to be located closest to the populations for example, due to their pollution, noises and increased fire risk. Or because of the higher rents in urban central locations, that inhibits firms to locate there, since these industrial firms usually require significant extensions of land to be able to work. Other example could be found in the distribution sector, as big suppliers' markets are usually located in the outskirts of cities, supplying their produce to retailers who are distributed closer to central locations in the city.

Therefore, the main question we want to answer is if firms that are uncertain about their marginal costs and are forced to be located in the extremes of the cities, have any incentives to choose different locations, or if they prefer to locate close to each other. We analyze this question comparing our results with the perfect information situation developed by Ziss (1993),who concluded that, if there is equilibrium, firms choose to locate in different extremes of the city.

Our main contribution is that agglomeration is an optimal outcome when there is uncertainty about costs. This conclusion is in clear contrast with the results obtained under perfect information (Ziss, 1993) as in this case firms are either dispersed throughout the linear city or are located randomly, depending on their marginal cost difference. Our results highlight the importance of considering imperfect information about costs when studying the location decision of firms. Firm agglomeration is a frequent phenomenon in several industries[1], and our model presents an explanation for this reality. We conclude that the agglomeration result happens mainly because firms are able to risk agglomerating and face a typical Bertrand competition setting in which they could conquer all the market and obtain high profits. The dispersion solution would

[1] To find empirical evidence on firms' agglomerative behavior in many industries, see Ellison *et al*. (2010) and the references therein.

be safer to their business, but a firm with a cost advantage loses both on demand and profits if dispersed, contrary to if it is agglomerated with its opponent.

This chapter is organized as follows: subsection 3.2 fits the paper in the literature; subsection 3.3 presents the timing and the assumptions of the model; subsection 3.4 solves the two-stage location-price game; subsection 3.5 discusses the implications of the results, while subsection 3.6 displays how the location results of the model change with the probability of a successful marginal cost; subsection 3.7 concludes.

3.2. Theoretical Background

The model of Hotelling (1929) is a well-known approach when it comes to justify the location choices of firms. The original Hotelling model assumes that two firms play a two-stage location-price game. In the first stage, firms simultaneously choose the location in a linear and bounded city, and in the second stage they simultaneously choose the prices. Firms offer a homogenous product, except for the location, and have the same cost structure. Demand is perfectly inelastic, that is, each consumer must buy one unit of the product. In order to buy the product consumers bear a unit transportation cost, which is linear with respect to the distance traveled. Hotelling concludes that firms optimally agglomerate in the center of the city. This is the basis for the "Principle of Minimum Differentiation", called so by Boulding (1966).

However, the original model of Hotelling had some tractability problems due to demand discontinuity, which implies that no price equilibrium exists for all the possible locations of the firms in the city. An important approach to the Hotelling model was made by d'Aspremont et al. (1979). By introducing quadratic (instead of linear) unit transportation costs with respect to the distance travelled, the authors conclude that the result of central agglomeration disappeared, with firms optimally locating in each of the extremes of the city. This result became known as the "Principle of Maximum Differentiation". The introduction of quadratic transportation costs also eliminates the tractability problems of the Hotelling model, allowing the existence of price equilibrium for all possible locations of the firms.

The location behavior raised by Hotelling and d'Aspremont et al. attracted the interest of the academic world. Using the same model in a more regional and urban economics approach, some scientists attempted to explain the conditions in which firms agglomerate in cities, while other scientists were more concerned about the horizontal differentiation problem, approaching this model from the interests of Industrial Organization. The field expanded significantly, and more important publications appeared about this issue. Most of these publications focused on changing the assumptions of the original Hotelling model and then concluding about the new location decisions of the firms, as it is shown in the previous chapter.

The most direct reference for the present work is Ziss (1993), who derives the results for the two-stage location-price game of Hotelling by allowing for different marginal costs between the two firms. Ziss concludes that when the difference between the marginal costs is small, firms prefer to locate in different extremes of the city. However, when the difference is higher than a given threshold, the low cost firm prefers to locate as close as possible to the high cost firm which, in turn, wishes to locate as far as possible from the other firm. This leads to the absence of location equilibrium. Additionally, if the difference between both marginal costs is high enough, the low cost firm drives the high cost firm out of the market.

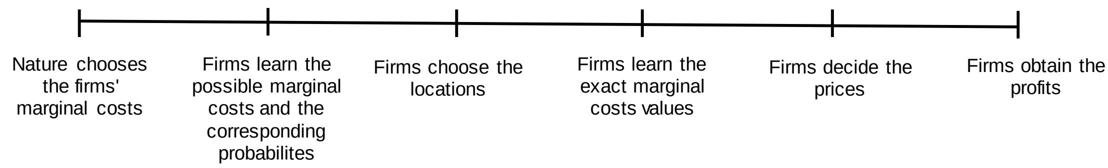
Matsumura and Matsushima (2009) extend the location-price game of Ziss (1993) allowing for mixed strategies in the location stage. The authors conclude that when no pure strategy equilibrium exists, there is a single mixed strategy equilibrium, which involves both firms locating in one of the extremes of the linear city with 50% probability. Meza and Tombak (2009) design a model with different marginal costs of production, but they introduce timing as a first stage in the model, that is, firms are allowed to choose in which period of time they enter in the market, and manage to extend the pure equilibrium possibilities for a larger array of cost differentiation values, where firms still maximally differentiate.

On other approaches similar to this chapter, Boyer et al. (2003) study the location-price decisions when firms choose locations sequentially and then choose prices simultaneously. Additionally, those authors assume that the first mover has perfect information, while the second mover does not know if the opponent firm has a low or high marginal cost. Although the nature of imperfect information is different from our model, the approach of Boyer et al. (2003) signals the importance of imperfect information when analyzing firms' location decisions. Vettas and Christou (2005) consider a model with imperfect information where firms know there is a quality difference between both firms but they do not know who is in advantage. The authors conclude that the larger the quality, the closer firms will locate to the city center, with an analogous rationale to ours: Firms risk their luck to be able to be in a strong monopoly situation in 50% of the times.

3.3. The Model

We consider two firms that compete in a two-staged location price game. Before taking any decision firms learn the possible values of the marginal costs and the corresponding probability. Then, at the first stage, firms simultaneously choose the locations between one of the two extremes of the linear and bounded city of Hotelling. Before simultaneously deciding the prices firms learn the exact value of the marginal costs, as detailed in Figure 3.1.

Figure 3.1 – Timing of the game



We consider a very simple type of imperfect information. At the beginning of the game firms know that the marginal cost of each firm can be unsuccessful or successful. We assume that the unsuccessful marginal cost is equal for both firms, while the successful marginal cost might have different values represented by c_1 and c_2 for firm 1 and 2, respectively. The probability of each firm having the successful cost is equal for both firms and is represented by $\upsilon = 50\%$.

The remaining assumptions are the standard in the Hotelling (1929) model with the extension proposed by d'Aspremont et al. (1979). Firms offer a homogenous good except for the location. Consumers are uniformly distributed across the linear city. Each consumer buys one unit of good, as the reservation price of the consumers is assumed to be high enough so that the market is always fully covered. Consumers incur a transportation cost to buy the good, which is quadratic with respect to distance.

Firms' location are represented by x_1 and by x_2, for firm 1 and firm 2 respectively, where x_1 is the distance from the left extreme of the market and x_2 is the distance from the right extreme. Hence, when $x_1 + x_2 = 0$, firms are fully differentiated and when $x_1 + x_2 = 1$, firms are agglomerated in any point of the market. We assume, without loss of generality, that firm 1 is never located to the right of firm 2 ($x_1 \leq 1 - x_2$). Moreover, we assume that the unit transportation cost is one, the city length is also one and the firms are risk neutral. Note that we assume that x_1 and x_2 can only be equal to 0 or 1, as firms must be located in the extremes of the market.

3.4. Solving the model and results

The assumption of imperfect information implies that firms are not entirely sure what their marginal cost is when they choose where to locate in the linear city. Each firm has a successful cost with a probability given by $v = 50\%$, and the probabilities of occurrence of a successful marginal firm for the firms are uncorrelated. This means that there are four possible outcomes in the model, after the location decision. Either both firms are successful or unsuccessful, or one firm is successful and the other is not. Then the probability of each outcome is equal to 25%. We assume that the unsuccessful outcome implies a marginal cost equal to 10. This is without any loss of generality, since as we will prove in later on, that due to the inelastic demand assumption, the demand and consequently the optimal location decision depends solely on the difference between the marginal costs of production of both firms, and not on their absolute value. The successful outcome is set between 0 and 10, and before choosing the location, each firm knows its own opponent's successful marginal cost.

To compute the profit of each firm, first we have to determine their demand. As usual in the Hotelling models with price competition the demand of each firm is derived directly from the location of the indifferent consumer. The indifferent consumer is defined by the consumer which is indifferent between choosing to purchase the good between one of the two possible stores. The consumer located in point x obtains utility from the purchase of the good from the firm i in the following manner: $U_i(x) = V - p_i - (x - x_i)^2$, where V is the utility given by the consumption of the good, and it is assumed to be too high such that the market is always covered, and p_i is the price set by firm i. Consumers browse all the firms in the market and choose to buy the good that gives them the highest utility. Therefore, each firm has its own market area, that is, an area where all the consumers buy in their store. The boundary of this area is given by the location of the indifferent consumer, which is given by:

$$x = \frac{1 - 2x_2 - x_1^2 + x_2^2 - p_1 + p_2}{2 - 2x_1 - 2x_2}$$

Therefore, the demand (market area) of firm 1 is given by x while the demand of firm 2 is given by $1 - x$.

However, when calculating the profit functions for each firm we must ensure that the value for the indifferent consumer's location does not become lower than 0 or higher than 1. Hence, the profit function for each firm is given by:

$$\Pi_i(x_i, x_j, p_i, p_j) = \begin{cases} p_i - c_i & \text{if} \quad p_i < \underline{\theta} \\ \frac{(p_i - c_i)(1 - 2x_j - x_i^2 + x_j^2 - p_i + p_j)}{2 - 2x_i - 2x_j} & \text{if} \quad \underline{\theta} \leq p_i \leq \overline{\theta} \\ 0 & \text{if} \quad p_i > \overline{\theta} \end{cases} \qquad \text{with i =1, 2}$$

and $\underline{\theta} = p_j - x_i^2 + 2x_i + x_j^2 - 1$, $\overline{\theta} = p_j - x_i^2 - 2x_j + x_j^2 + 1$. Both the indifferent consumer function and the profit function are only valid for $x_1 < 1 - x_2$. When $x_1 = 1 - x_2$, which means that the firms have the same location, the profits functions are equal to the ones in Bertrand (1883) model.

In the first stage firms choose simultaneously their location and in the second stage they set simultaneously the price for their good. The game is solved by backward induction and the concept of equilibrium sought is Sub-game Perfect Nash Equilibrium (SPNE).

3.4.1. Price Stage

As firms have perfect information when arriving at the price stage, they choose the price that maximizes the profit function considering their true marginal costs, which are already known in this stage. Therefore, we solve the maximization problem for both firms under the four possible cases: both firms are successful, firm 1 is successful and firm 2 is not, firm 2 is successful and firm 1 is not and both firms are unsuccessful. These four cases are denoted by (S,S), (S,U), (U,S) and (U,U), respectively. The general expression for the prices that solves the maximization profit problem is given by:[2]

$$p_i(x_i, x_j) = 1 - \frac{2}{3}x_i - \frac{4}{3}x_j - \frac{1}{3}x_i^2 + \frac{1}{3}x_j^2 + \frac{2}{3}\overline{c_i} + \frac{1}{3}\overline{c_j} \tag{3.1}$$

In order to find the optimal price policies we just need to correctly replace $\overline{c_i}$ and $\overline{c_j}$ by 10, c_1 or c_2, depending on the outcome revealed between both stages. However, the above expression is only valid when both firms have a positive market share. When one of the firms becomes monopolist, it sets the highest possible price such that its opponent remains out of market. Replacing the price policies in (3.1), the profit function for each firm becomes:

$$\Pi_i(c_i, c_j) = \begin{cases} -\hat{c}_i - x_i^2 + x_j^2 + 2x_i - 1 & \text{if} \quad \hat{c}_i < \underline{\theta} \\ \frac{(2x_i + 4x_j + x_i^2 - x_j^2 + \hat{c}_i - 3)^2}{9(2 - 2x_i - 2x_j)} & \text{if} \quad \underline{\theta} \le \hat{c}_i \le \overline{\theta} \\ 0 & \text{if} \quad \hat{c}_i > \overline{\theta} \end{cases} \tag{3.2}$$

Where $\hat{c}_i$ is the marginal cost difference expressed by $c_i - c_j$ and $\underline{\theta}$ and $\overline{\theta}$, are the same than before, but after replacing the optimal price decision these change to $-x_i^2 + 4x_i + x_j^2 + 2x_j - 3$ and $-x_i^2 - 2x_i + x_j^2 - 4x_j + 3$, respectively.

Hence, for each marginal cost outcome, the profit's expressions and the thresholds of the different branches change. Nevertheless, since firms already have perfect information when setting the prices, the results for this stage become simpler after knowing the optimal location decision and the marginal cost.

3.4.2. Location Stage

Only after the location decision firms learn their own and their opponent's final marginal cost. This means that when choosing the location, firms have to consider an expected profit function dependent on each of the four possible outcomes. Note that

[2] See appendix 3.1 for the detailed derivation of the prices that maximize the profit under the four possible cases, and the definition of the general expression for price results.

when both firms have unsuccessful marginal cost, their marginal costs are equal and we are exactly in the case of d'Aspremont et al. (1979). On the other hand, when both firms have the successful marginal cost, we have the case of Ziss (1993). The expected profit function for firm *i* is given by:[3]

$$\mathrm{E}(\Pi_i) = (0.25)(\Pi_i(c_i, c_j)) + (0.25)(\Pi_i(c_i, 10)) + (0.25)(\Pi_i(10, c_j)) + (0.25)(\Pi_i(10,10))$$

As both firms are only allowed to agglomerate at one of the extremes of the city or fully disperse, we have to compare the profits obtained in each situation. Without loss of generality let us assume that firm 2 is located at the right extreme of the linear city. Then, firm 1 prefers to agglomerate if:

$$\mathrm{E}[\Pi_1(x_1 = 1, x_2 = 0)] > \mathrm{E}[\Pi_1(x_1 = 0, x_2 = 0)]$$

The expected profit functions are obtained combining the thresholds of the four possible outcomes.

The expected profit function for the agglomeration case has a simpler specification as it corresponds to the Bertrand case allowing different marginal costs. For firm 1 (and similarly for firm 2) the expected profit function is given by:

$$\mathrm{E}(\Pi_1(c_1, c_2)) = \begin{cases} (0.25)(-\hat{c}_1) + (0.25)(10 - c_1) + (0.25)(0) + (0.25)(0) \; if \; \hat{c}_1 < 0 \\ (0.25)(0) + (0.25)(10 - c_1) + (0.25)(0) + (0.25)(0) \;\; if \;\; \hat{c}_1 \geq 0 \end{cases}$$

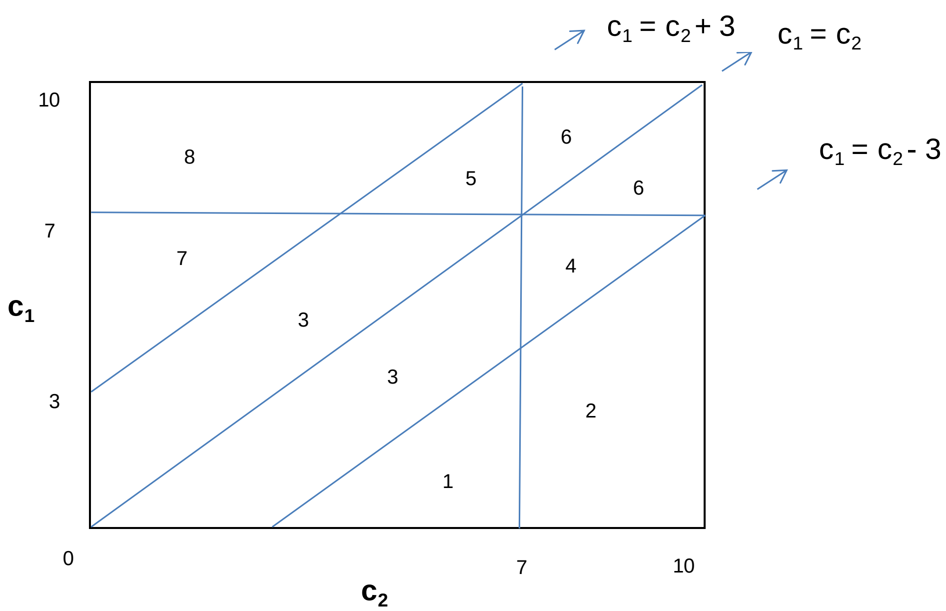
Simplifying we obtain: $\mathrm{E}(\Pi_1(c_1, c_2)) = \begin{cases} \frac{1}{4}c_2 - \frac{1}{2}c_1 + \frac{5}{2} & \text{if} \quad \hat{c}_1 < 0 \\ \frac{5}{2} - \frac{1}{4}c_1 & \text{if} \;\; \hat{c}_1 \geq 0 \end{cases}$

Notice that this expected function is obtained replacing in (3.2) the variables x_i and x_j by 1 and 0 respectively. Then, we have the following results regarding firm 1 profit: when both firms are successful, firm 1 has profit equal to $-(c_1-c_2)$ if firm 1 is more efficient than firm 2, or has 0 otherwise; when firm 1 is successful and firm 2 is not, firm 1 has profit equal to $10 - c_1$; when firm 2 is successful and firm 1 is not, firm 1 has zero profit; finally, when both firms are unsuccessful firm 1 has zero profit.

[3] Note that by considering that firms' profit is similar to firm's utility, we are implicitly assuming that firms are risk neutral. That is, firms maximize their expected profit and they do not care about the volatility of their final result.

The dispersion case is a bit more complicated, as the profit function is a piecewise, and changes between eight different pieces. That happens since different values of differences in the marginal costs of both firms imply different demand, and consequently different profit functions, due to the nature of demand in the Hotelling model. The eight branches are represented in figure 3.2.

Figure 3.2 – Branches of the Expected Profit Functions for both firms



The interpretation of the eight branches is the following: when both firms are successful in the reduction of their marginal costs it is important to distinguish the cases when the marginal cost difference is lower than 3, because in these cases firms share the market, or when is higher than 3. Under this last case the firm with the lower marginal cost stays with all the market. Branches 3, 4, 5 and 6 correspond to cases with successful marginal cost differences lower than 3; branches 7 and 8 correspond to cases with successful cost differences higher than 3, with firm 1having the high cost and, finally, branches 1 and 2 correspond to cases with cost differences higher than 3, with firm 2 having the high cost.

The existence of the eight pieces in the expected profit function for dispersion is due to the combination of all 4 cases with the number of branches that exists in each of the cases, i.e., whenever the relationship between the successful marginal costs is such that one of the firms becomes monopolist, the expression for the profit changes.[4] Each of the 8 pieces, due to do the aggregation of all possible cases, has a different expression for the expected profit function (in this case of dispersion). Nevertheless, the expected profit function is continuous with respect to both successful marginal costs values of the firms. Table 3.1 presents the results for each of the four outcomes in each branch of the expected profit function for firm 1.

Table 3.1 – Profits of firm 1 for the different branches in the case of dispersion

Restrictions	(S,S)	(S,U)	(U,S)	(U,U)
Branch 1	$((c_2 - c_1) - 1)$	$9 - c_1$	0	$\frac{1}{2}$
Branch 2	$((c_2 - c_1) - 1)$	$9 - c_1$	$\frac{1}{2}(-\frac{1}{3}c_2 + \frac{7}{3})^2$	$\frac{1}{2}$
Branch 3	$\frac{1}{2}(\frac{1}{3}(c_1 - c_2) - 1)^2$	$9 - c_1$	0	$\frac{1}{2}$
Branch 4	$\frac{1}{2}(\frac{1}{3}(c_1 - c_2) - 1)^2$	$9 - c_1$	$\frac{1}{2}(-\frac{1}{3}c_2 + \frac{7}{3})^2$	$\frac{1}{2}$
Branch 5	$\frac{1}{2}(\frac{1}{3}(c_1 - c_2) - 1)^2$	$\frac{1}{2}(\frac{1}{3}c_1 - \frac{13}{3})^2$	0	$\frac{1}{2}$
Branch 6	$\frac{1}{2}(\frac{1}{3}(c_1 - c_2) - 1)^2$	$\frac{1}{2}(\frac{1}{3}c_1 - \frac{13}{3})^2$	$\frac{1}{2}(-\frac{1}{3}c_2 + \frac{7}{3})^2$	$\frac{1}{2}$
Branch 7	0	$9 - c_1$	0	$\frac{1}{2}$
Branch 8	0	$\frac{1}{2}(\frac{1}{3}c_1 - \frac{13}{3})^2$	0	$\frac{1}{2}$

In order to assess the decision of firm 1 we have to compare the expected profit in agglomeration and in dispersion for each branch. However, these 8 branches that arise for the dispersion case have to be complemented with the two different branches

[4] Appendix 3.2 shows in detail the relationships that led to the expected profit function with 8 branches.

that we had on the agglomeration case: When the successful marginal cost difference is higher or is lower than 0. Therefore, we conclude that there are ten different pieces in the expected profit function of firms within the relevant values for the successful marginal costs $c_i \in [0,10]$. Note that the agglomeration thresholds divide the branches 3 and 6 of the expected profit function in the dispersion case into two regions, in order to separate the case $c_1 < c_2$ from $c_1 > c_2$.These branches are already presented in Figure 3.2. Notice also that the thresholds of all the branches are independent of the probability of firms having a successful marginal cost v, so these are valid for any probability v we may want to consider.

For each of the ten branches, there are different expressions for the expected profit function. Our next step is to identify the location decision of firm 1 for each of the regions by comparing the expected profit from the two alternative decisions (agglomeration or dispersion). Table 3.2 presents the conclusions of this comparison. Notice that both expected profit functions (for agglomeration and dispersion) are continuous throughout their entire domain.

Table 3.2 – Expected profit and optimal choice of firm 1

Branch	Dispersion	Agglomeration	Solution
1	$\frac{c_2}{4}-\frac{c_1}{2}+\frac{17}{8}$	$\frac{1}{4}c_2-\frac{1}{2}c_1+\frac{5}{2}$	Disp < Agg
2	$\frac{c_2}{4}-\frac{c_1}{2}+\frac{1}{8}(\frac{c_2}{3}-\frac{7}{3})^2+\frac{17}{8}$	$\frac{1}{4}c_2-\frac{1}{2}c_1+\frac{5}{2}$	Disp < Agg
3.1	$\frac{1}{8}(\frac{1}{3}(c_2-c_1)+1)^2-\frac{c_1}{4}+\frac{19}{8}$	$\frac{5}{2}-\frac{1}{4}c_1$	Disp < Agg
3.2	$\frac{1}{8}(\frac{1}{3}(c_2-c_1)+1)^2-\frac{c_1}{4}+\frac{19}{8}$	$\frac{1}{4}c_2-\frac{1}{2}c_1+\frac{5}{2}$	Disp < Agg

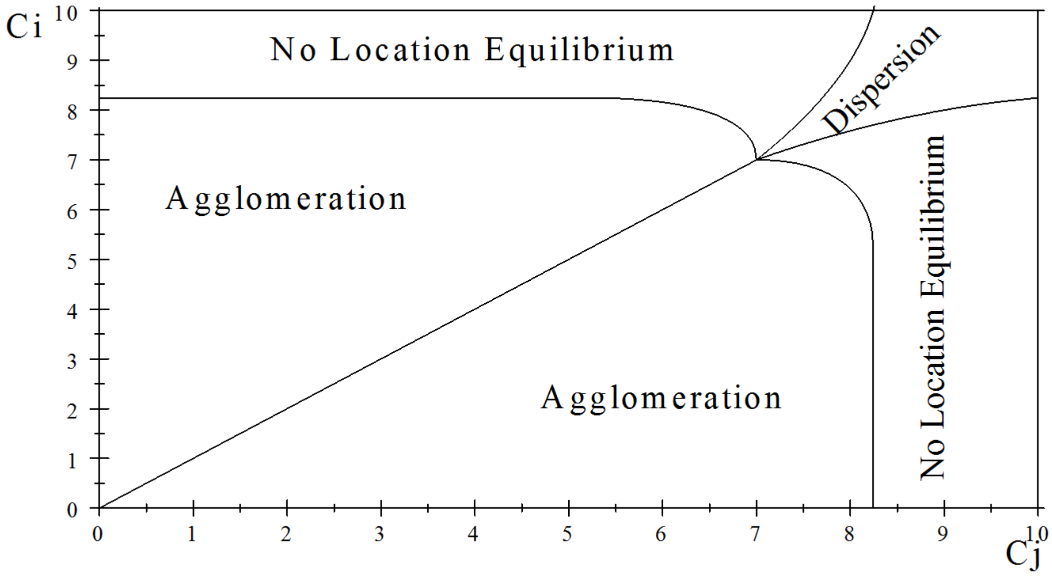
4	$\frac{1}{8}(\frac{c_2}{3}-\frac{7}{3})^2-\frac{c_1}{4}+$ $\frac{1}{8}(\frac{(c_2-c_1)}{3}+1)^2+\frac{19}{8}$	$\frac{1}{4}c_2-\frac{1}{2}c_1+\frac{5}{2}$	Disp < Agg
5	$\frac{1}{8}(\frac{(c_2-c_1)}{3}+1)^2+$ $\frac{1}{8}(\frac{c_1-13}{3})^2+\frac{1}{8}$	$\frac{5}{2}-\frac{1}{4}c_1$	Disp < Agg if: $c_1<\frac{7+c_2}{2}+\frac{\sqrt{-(c_2+5)(c_2-7)}}{2}$
6.1	$\frac{1}{8}(\frac{c_2-7}{3})+\frac{1}{8}(\frac{c_1-13}{3})+$ $\frac{1}{8}(\frac{(c_2-c_1)}{3}+1)^2+\frac{1}{8}$	$\frac{5}{2}-\frac{1}{4}c_1$	Disp > Agg
6.2	$\frac{1}{8}(\frac{c_2-7}{3})+\frac{1}{8}(\frac{c_1-13}{3})+$ $\frac{1}{8}(\frac{(c_2-c_1)}{3}+1)^2+\frac{1}{8}$	$\frac{1}{4}c_2-\frac{1}{2}c_1+\frac{5}{2}$	Disp < Agg if: $c_1<\frac{c_2-2}{2}+\frac{\sqrt{3}\sqrt{-c_2^2+16c_2-36}}{2}$
7	$\frac{19}{8}-\frac{1}{4}c_1$	$\frac{5}{2}-\frac{1}{4}c_1$	Disp < Agg
8	$\frac{1}{8}(\frac{c_1-13}{3})+\frac{1}{8}$	$\frac{5}{2}-\frac{1}{4}c_1$	Disp < Agg if: $c_1<8.2426$

After identifying the decision of firm 1 the same analysis is done for firm 2. Due to symmetry the results for firm 2 are identical to the ones obtained to firm 1.[5] Finally, to identify the equilibria we compare the decisions of the firms in each piece of the function. If both firms prefer agglomeration or dispersion, then that would be the equilibrium. On the contrary, if they prefer different location patterns, it means no

[5] The results for firm 2 are exactly the same than displayed in table 3.2 due to symmetry. The only necessary changes are between c_1 and c_2 and vice-versa, and to adapt to the symmetric regions: Region 1 is symmetric to region 8; Region 2 is symmetric to region 7; Region 4 is symmetric to region 5; Region 3.1 is symmetric to 3.2; Regions 4.1 and 4.2 are symmetric between them.

location equilibrium is found. Figure 3.3 synthetizes the conclusions on the location decision, which are fully detailed on appendix 3.3.

Figure 3.3 – Location Equilibrium with imperfect information and *v*=0.5



From Figure 3.3 we identify three different results: i) when both firms may not obtain a significant success in cost reduction, and therefore both c_i and c_j are not very far from 10, firms will disperse; ii) when the two firms have very different successful costs, one being close to 10 and the other not, no location equilibrium is found; iii) when both firms may achieve a significant success in cost reduction, and therefore, the difference between the successful and the unsuccessful marginal costs is large for both firms, agglomeration is an equilibrium result.

The absence of location equilibrium occurs if one of the firms wants to be as close as possible to the opponent (because it has a lower successful marginal cost) and the other wishes to be the farthest (because it has a higher successful marginal cost), similarly to what happens in the perfect information model of Ziss (1993).

Some properties of the final location results are easily understandable in Figure 3.3. First, the results are symmetric with respect to the $c_i = c_j$ line. This is an expected result, as firms have no other relative advantage than the marginal costs difference. Secondly, the continuity of the expected profit functions implies that the four regions in the Figure 3.3 are closed sets. Thirdly, the d'Aspremont et al. (1979) case is also represented in the Figure 3.3 by the point (10;10), that is, when the unsuccessful marginal cost is equal to the successful one for both firms. Firms are dispersed in this case. Fourthly, there is a line, defined by $c_i = c_j < 7$, when both firms are indifferent between agglomerating and dispersing. In this position, the benefit of agglomerating, which is the 25% possibility of obtaining a strong monopoly, is equal to the benefit of dispersing, which is the softening of price competition, allowing the practice of higher prices when both firms have equal marginal cost conditions.

3.5. Discussion of the results

3.5.1 Location

The differences between the perfect and imperfect information case are striking. With imperfect information the agglomeration result is an optimal location outcome for both firms, while in perfect information firms would only either disperse or not find location equilibrium. So, what motivates firms to agglomerate when the variability of their marginal costs gets high? Firms wish to agglomerate due to the possibility of having a monopoly, which occurs when firm *i* achieves a significant cost reduction, and firm *j* does not (this happens with probability of 25%). In this case firms have a great difference between marginal costs in that outcome, which, when agglomerated, allows them to have high profits. This behavior for both firms in the model explains the existence of the agglomeration equilibrium. The question can be better illustrated with an example with different situations regarding the successful marginal costs. The comparison of the profits for both firms in each of the four possible outcomes when $(c_i, c_j) = (8{,}8)$ and $(c_i, c_j) = (6{,}6)$ is presented in Table 3.3. The case when $(c_i, c_j) = (8; 8{,}5)$ and $(c_i, c_j) = (6; 6{,}5)$ is presented in Table 3.4. Since we are assuming $v = 0.5$, the probability of occurrence of each outcome is 25%.

Table 3.3 – Profit of both firms by outcome for equal low marginal costs, with $v = 0.5$

(c_i, c_j)	Outcome	(S,S)	(S,U)	(U,S)	(U,U)	Expected Profit
(6,6)	Dispersion	1/2	3	0	1/2	1
(6,6)	Agglomeration	0	4	0	0	1
(8,8)	Dispersion	1/2	25/18	1/18	1/2	11/18
(8,8)	Agglomeration	0	2	0	0	1/2

When the successful marginal costs of both firms are equal, we find that dispersion dominates the agglomeration outcome. For the case $(c_i = c_j < 7)$, when firms have a similar outcome (S,S) or (U,U), dispersion is more profitable than agglomeration since firms are able to soften price competition and share the market equally. However, this difference in profits in the dispersion case is exactly compensated by the higher profits obtained by the firms in the case of agglomeration, when the outcome is the most favorable for firm *i* (see the first two lines of Table 3.3). Therefore, in this case, the expected profit of dispersion and agglomeration is equal. In the case of $c_i = c_j > 7$, when the outcomes are equal, the explanation of the profits is similar to the previous case. However, due to the softening of price competition, prices are higher in the dispersion case, which allows both firms to obtain a higher profit in spite of having less demand. Note also that the less fortunate firm still earns profits when fully differentiated from its rival.

The threshold value of this relationship between dispersion and agglomeration is given by 7 for a particular reason: this is the threshold marginal cost (given that the unsuccessful marginal cost is 10) below which the successful cost firm takes the unsuccessful cost firm out of the market when firms are located in opposing extremes. This value has been previously identified by Ziss (although we assume a given city length and unit transportation costs, both equal to 1). Therefore, when both firms have a successful marginal cost below 7, in the case of firms having different outcomes, the successful firm captures the whole market. This implies that the derivative of the expected profit function with respect to the difference in the marginal costs is equal for both agglomeration and dispersion cases, for $(c_i = c_j < 7)$.

Table 3.4 – Profit of both firms by outcome for different successful marginal costs, with $v = 0.5$.

(c_i, c_j)	Outcome	(S,S)	(S,U)	(U,S)	(U,U)	Expected Profit
(6;6,5)	Dispersion Firm i	49/72	3	0	1/2	301/288
(6;6,5)	Agglomeration Firm i	1/2	4	0	0	9/8
(8;8,5)	Dispersion Firm i	49/72	25/18	1/18	1/2	21/32
(8;8,5)	Agglomeration Firm i	1/2	2	0	0	5/8
(6;6,5)	Dispersion Firm j	25/72	5/2	0	1/2	241/288
(6;6,5)	Agglomeration Firm j	0	7/2	0	0	7/8
(8;8,5)	Dispersion Firm j	25/72	9/8	1/18	1/2	73/144
(8;8,5)	Agglomeration Firm j	0	3/2	0	0	3/8

For the case $(c_i, c_j) = (6; 6{,}5)$, both firms prefer to agglomerate. This happens because compared to the dispersion case, the gain in the profits when the outcome is favorable for firm *i* or *j* ((S,U) or (U,S), depending on the firm) is higher than the losses caused by all other outcomes, even for the firm that has higher successful marginal cost. However, for the case $(c_i, c_j) = (8; 8{,}5)$, both firms prefer to disperse. When firms are dispersed and the difference between the successful and unsuccessful marginal costs is small, the demand effect is still in operation, which makes up for the difference in the higher profits of agglomeration when firms have a favorable outcome.

3.5.2 Prices

The comparison between the price policies is only interesting for the case when there is location equilibrium for both cases of perfect and imperfect information. Differences in the price policies between both cases arise because of the probability of non-occurrence of successful marginal costs for both firms and because of the different location patterns chosen in the previous stage.

Regarding location patterns when in a monopoly, the difference between the prices in perfect and imperfect information is because the monopolist has to set a lower price when dispersed in order to cover the whole market compared to the agglomeration situation. That difference between the prices is constant and equal to 1. On the other hand, in a duopoly when firms are agglomerated, price is always equal to the higher marginal cost existent between the two firms. In the case of dispersion, the optimal pricing policy can be rewritten in terms of their own marginal cost and of the difference between the marginal costs of both firms, that is:

$$p_i = c_i - \frac{1}{3}(\hat{c}_i) + 1, \text{for } |\hat{c}_i| < 3$$

For the low-cost firm, we find that the price set when firms are dispersed is higher than when firms are agglomerated if:

$$c_i - \frac{1}{3}(\hat{c}_i) + 1 > c_j \Leftrightarrow \hat{c}_i > -\frac{3}{2}$$

Since we are only comparing pure-strategy equilibrium results, that is, the cases when there is equilibrium in perfect competition, $|\hat{c}_i| < (6 - 3\sqrt{3})$, this result means that the prices when firms are dispersed are always higher than prices when firms are agglomerated.

However, the main determinant of the pricing policy is the probability of the outcomes. As stated before, perfect information is the particular case of imperfect information when the outcome is always the successful marginal cost for both firms, that is, when $v = 1$. All other outcomes imply higher marginal costs for at least one of the firms, a fact that is expected to bring higher prices in the market. Table 3.5 displays the pricing policy for the same marginal costs set in Table 3.4 and for $v = 0.5$.

Table 3.5 – Pricing policies of both firms by outcome for different low marginal costs

(c_i, c_j)	**Outcome**	**(S,S)**	**(S,U)**	**(U,S)**	**(U,U)**	**Average Price**
(6;6,5)	Perfect Information Firm i	43/6				43/6
(6;6,5)	Imperfect Information Firm i	39/6	60/6	N/M	60/6	53/6
(8;8,5)	Perfect Information Firm i	55/6				55/6
(8;8,5)	Imperfect Information Firm i	55/6	58/6	63/6	66/6	121/12
(6;6,5)	Perfect Information Firm j	44/6				44/6
(6;6,5)	Imperfect Information Firm j	N/M	N/M	60/6	60/6	60/6
(8;8,5)	Perfect Information Firm j	54/6				54/6
(8;8,5)	Imperfect Information Firm j	54/6	62/6	60/6	66/6	121/12

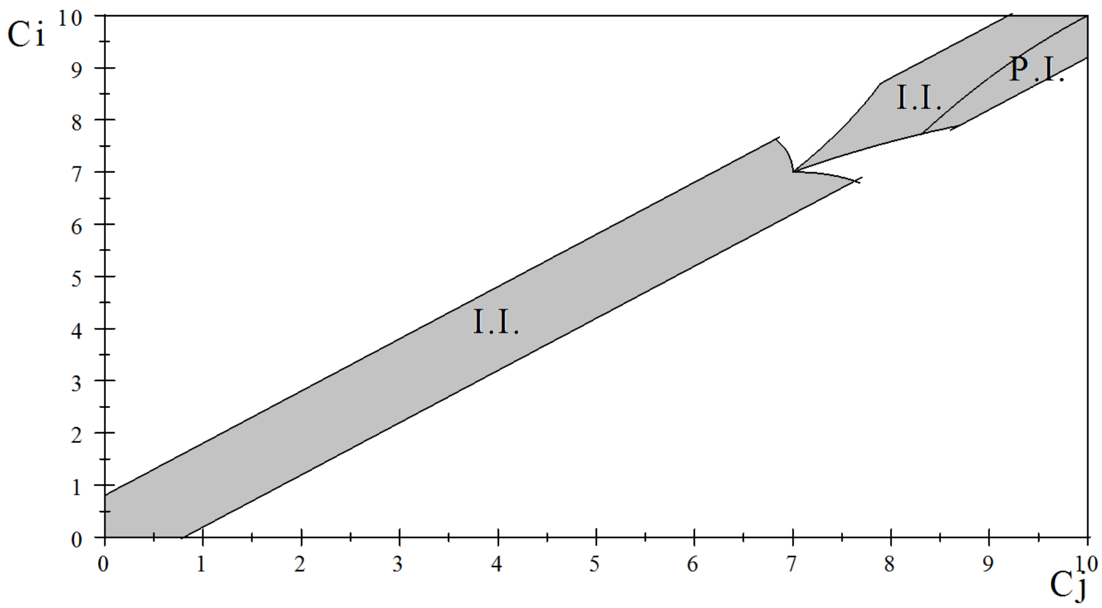
N/M – Not in the Market

We can see that the perfect information case results in average lower prices for the market. This happens mainly because of the nature of the perfect information assumed in this model, which determines that firms have lower marginal costs when in perfect information, allowing for the setting of lower prices. On the other hand, there is another effect changing the prices, which results from changes in the location patterns of the firms. Firms now agglomerate for lower values of successful marginal costs and agglomeration itself forces firms to practice lower prices in the market. Note that this effect only occurs when firms decide to agglomerate, that is, when $c_i, c_j < 7$. However, the dominant effect proves to be the assumed nature of perfect information.

3.5.3 Profits

The comparison of profits can only be analyzed when there is location equilibrium in both perfect and imperfect information cases. This equilibrium occurs only in the shaded area shown in Figure 3.5. This area is clearly defined in Appendix 3.3. In the upper region, both firms choose to disperse with perfect and imperfect information. In the lower region, in perfect information firms disperse and in imperfect information firms prefer to agglomerate, as is seen in Figures 3.2 and 3.4.

Figure 3.4 – Set of parameter values when there is location equilibrium for both cases, and profit comparisons for firm *i*.



(I.I. = Imperfect Information; P.I. = Perfect Information)

In the lower region, both firms always prefer to be in imperfect information, in spite of having a higher expected marginal cost compared to the perfect information case. This happens because in the lower region, there is always a 25% (when $v = 0.5$) possibility of both firms having a very profitable monopoly, since the difference between the marginal costs of both firms becomes very high in each firm's most favorable outcome.

By comparing the profits between both cases we find that in the upper region, when in imperfect information firms have a higher probability of having higher profits. Firm *i* prefers to be in imperfect information when:

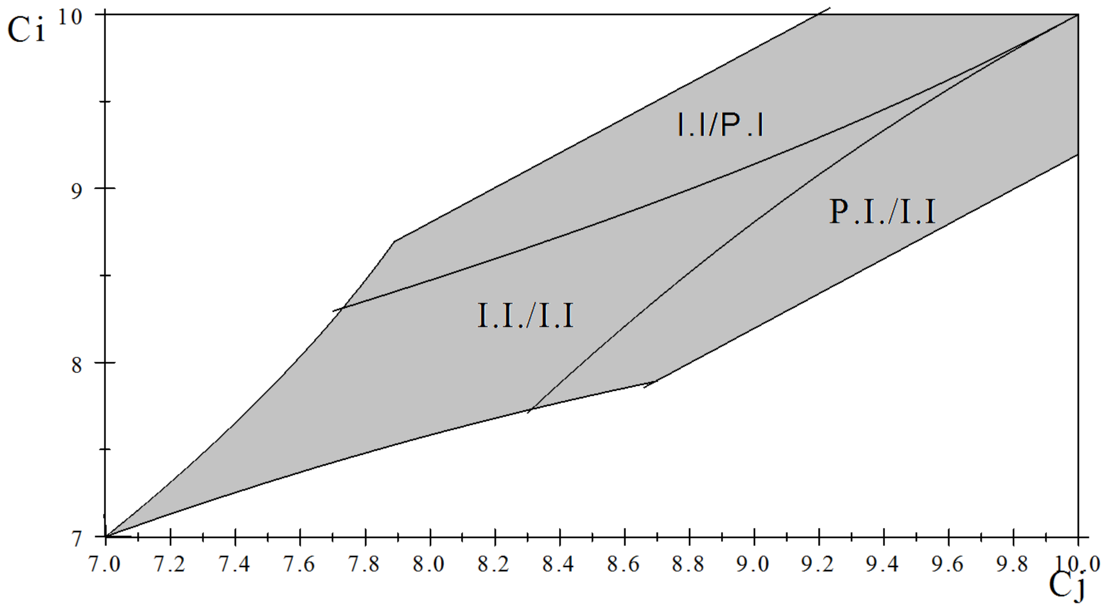
$$\frac{1}{8}\left(\frac{1}{3}c_j - \frac{7}{3}\right)^2 + \frac{1}{8}\left(\frac{1}{3}c_j - \frac{1}{3}c_i + 1\right)^2 + \frac{1}{8}\left(\frac{1}{3}c_i - \frac{13}{3}\right)^2 + \frac{1}{8} > \frac{1}{18}\left(c_i - c_j - 3\right)^2 \Leftrightarrow$$

$$c_i > \frac{3}{2}c_j - \frac{1}{2}\sqrt{5c_j^2 - 88c_j + 416} - 2$$

Note, however, that firm's *j* preferences are symmetric with respect to the line $c_i = c_j$. This means that there is no situation where both firms would like to be in perfect information. The reason is rather obvious: when a firm has a disadvantage in terms of its successful marginal cost, being in imperfect information would allow the firm to have a cost advantage with some probability relative to its rival, which makes up for the possibility of negative outcomes where the cost disadvantage becomes higher than in perfect information. When the low marginal costs between firms are similar, both firms would prefer to be in imperfect information.

In the upper region, firms are dispersed, independently of being in perfect or imperfect information. The differences in the profits arises because when firm *i* has a lower marginal cost, it prefers to be in perfect information where it is sure it will have a good profit. However, as its marginal cost becomes relatively higher, firm *i* prefers imperfect information as the mix of the 4 outcomes becomes more profitable, as discussed above. Figure 3.5 presents a close-up of the upper shaded region of the final result for both firms, where it is better shown that the perfect information outcome is never a preferred outcome for both firms at the same time.

Figure 3.5 – Profit comparison for both firms in the upper region $c_i, c_j \in [7,10]$



(I.I = Imperfect Information; P.I. = Perfect Information)

The result that firms have a higher profit when in imperfect information is a puzzling one. In this model, it occurs because the absence of knowledge of the firms relatively to their own and their opponent's marginal costs induces firms to risk more in terms of their location, agglomerating more often and leading them to a higher expected profit.

Firms have higher expected profit under imperfect information than under perfect information, because under the first hypothesis it is possible that one firm obtains a stronger cost reduction than the rival. Hence, the firm would have a cost advantage, which explains why the agglomeration decision might appear under imperfect information. This is a very interesting result as it contributes to explaining the willingness to invest in uncertain cost reducing activities (such as R&D). Firms invest, because they have the expectation of obtaining a high cost advantage, which is not possible under the perfect information scenario considered here.

3.6. Effects of changes in firms' probabilities of successful cost outcomes

This section discusses what would happen in the second-stage of the game for changes in the value of the probability of occurrence of successful marginal costs for both firms. Figure 3.3 and all the subsequent analysis is based on $v = 0.5$. Also, when $v = 1$, the model turns out to be the perfect information case of Ziss, while when $v = 0$, we have the d'Aspremont et al. (1979) model, in which the location result is dispersion for all the values of c_i and c_j, since the successful marginal costs are never going to occur and firms end up with equal marginal costs. To perform this analysis, we had to compare agglomeration and dispersion expected profits for all the 10 different branches of the profit functions, since it is not possible to solve the game for all possible values of v with a single mathematical expression, exactly due to the existence of this piecewise expected profit function.

Departing from $v = 0.5$ and from Figure 3.3, a small increase in this parameter increases the probability of both firms having a successful marginal cost outcome at the expense of a decrease of the probability of both (S,U) and (U,S) outcomes and a larger decrease in the probability of (U,U) outcome. If the successful marginal costs are different between firms, one of the firms will most probably be at a disadvantage compared with the rival and will prefer to disperse more often, while the firm with an advantage will prefer to agglomerate more often. This will increase the possibilities of absence of location equilibrium. Concerning agglomeration vs. dispersion, firms will more often have similar outcomes between them ((S,S) + (U,U) > (S,U) + (U,S)), which makes firms disperse more often. Summing up, the areas of agglomeration are expected to decrease and the areas of dispersion and "no location equilibrium" are expected to increase with a small increase of v when departing from $v = 0.5$. The results become progressively closer to the conclusion of Ziss (1993).

Similarly, a small decrease in parameter v raises the odds of firms being located in the (U,U) outcome. This result induces both firms to disperse more often, which has two consequences: the firms will be at the no location equilibrium situation more often, as the firm with the higher successful marginal cost will disperse more often in the cases when the advantaged firm still prefers to agglomerate; the second consequence is similar to the previous case, as firms are expected to have more similar outcomes than before, which pushes firms to disperse rather to agglomerate. Therefore, the resulting pattern is going to be very similar to the case of an increase in parameter v. The exception is that since dispersion occurs more often than agglomeration, the area of "no location equilibrium” got lower at the expense of the area of dispersion. Hence, we conclude that $v = 0.5$ maximizes the region of agglomeration. This happens because that value for the parameter maximizes the probability of both firms of having a successful marginal cost while the other firm has an unsuccessful marginal cost, which is the outcome that gives the most profit for agglomerated firms. As an example, the results for v=0.4 and v=0.75 are displayed in Figures 3.6 and 3.7.

Figure 3.6 - Location Equilibrium with imperfect information, for v = 0.4

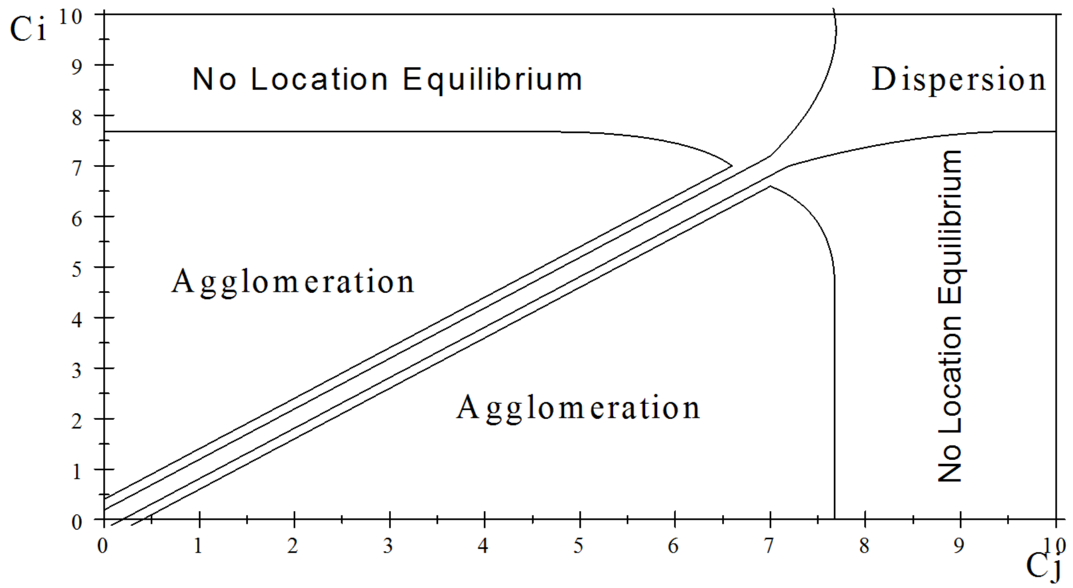
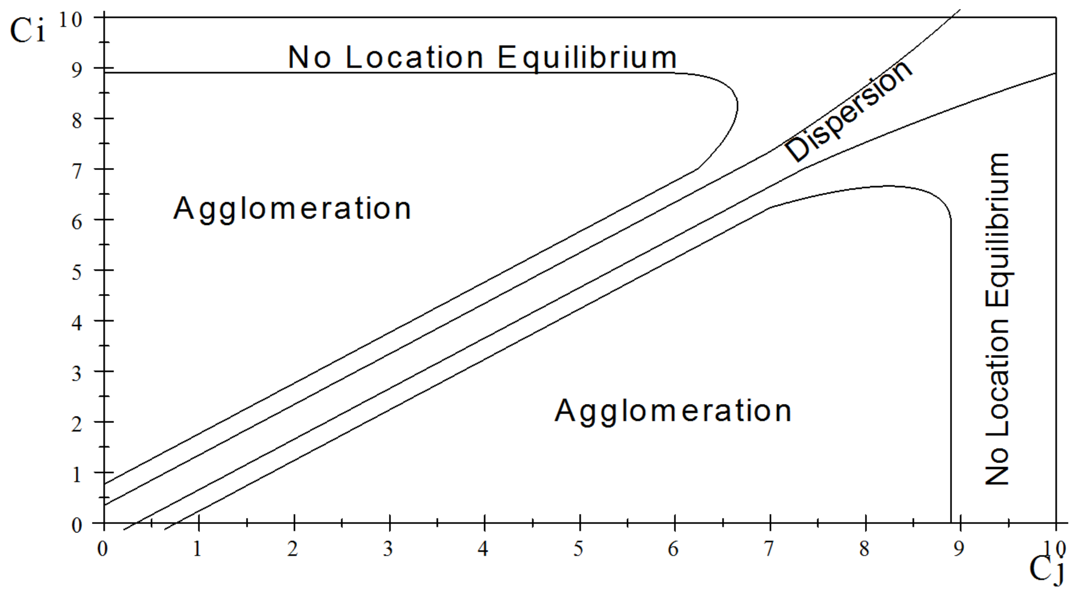


Figure 3.7 – Location Equilibrium with imperfect information, for v = 0.75



Recall the perfect information case result of Ziss (1993), in which both firms disperse if the difference between both marginal costs is not high, and there is not an equilibrium solution otherwise. When departing from v=1 and slowly decreasing the probability of successful marginal cost for both firms, we have that the introduction of the smallest uncertainty (for instance, v=0.9999) already allows for an agglomerative equilibrium. This occurs in the case when both firms have a successful marginal cost lower than 7, but when these successful marginal costs are very different between the firms. Note that when v=1, the higher successful marginal cost firm has no demand when dispersing or agglomerating, so its profits are always equal to 0. However, a decrease in v opens a small possibility of monopoly for that firm, that happens when its marginal cost is successful and the other firm is not. Then, agglomeration maximizes that outcome possibility, while in all other outcomes its profits are equal to 0 anyway. So, both firms prefer to agglomerate whenever $|\hat{c}_i| > 3$ and $(c_i = c_j < 7)$. This shows how fragile the perfect information result may be regarding uncertainty in the production costs of firms.

In addition, when departing from the lower values of v the result of d'Aspremont et al. holds, that is, both firms wish to disperse independently of the value of their successful marginal cost until v reaches the value of $2 - \sqrt{3}$, which is approximately 0.26795. As v rises above that value, when the difference between the successful marginal costs is sufficiently large, the lower successful marginal cost firm starts preferring to agglomerate, which leads to a "no location equilibrium" area. However, when v reaches 1/3, the higher successful marginal cost firm wishes to agglomerate as well due to the increasing possibility of having a monopoly of its own if the marginal cost outcome is favorable to it (S,U in the case of firm *i*).

These values (v=1/3 and v=1) are the only discontinuities in the location decision function, as the regions of agglomeration, dispersion and no location equilibrium react continuously to the remaining changes in the values of the parameter v. Overall, we conclude that agglomeration for both firms may occur in this setting when $v \in [1/3; 1[$, with the highest area of agglomeration occurring when $v = 0.5$.

3.7. Conclusion

In this paper, we introduce imperfect information in the framework of Hotelling (1929), d'Aspremont et al. (1979) and Ziss (1993), in the sense that firms are unaware of which marginal cost they will have before choosing their location. We have opted for a simplifying case of imperfect information as firms have two possible outcomes for their marginal cost: an unsuccessful marginal cost, equal for both firms, and a successful marginal cost not necessarily equal for both firms. We analyze closer the case when each firm has a 50% probability of having a successful marginal cost and we assume that firms can only be located in the extremes of the market.

The main conclusion drawn from this work is that agglomerative location equilibrium becomes possible when the successful marginal cost of both firms is sufficiently different from their unsuccessful marginal cost, and when the probability of both firms having a successful marginal cost is between 1/3 and 1. This result is not possible in the perfect information models of d'Aspremont et al. (1979) and Ziss (1993). The agglomeration result happens mainly because firms are able to risk agglomerating and face a typical Bertrand (1883) competition setting in which they could possibly conquer all of the market rather than dispersing, when it is necessary to set lower prices in order to conquer the same demand.

This paper proves that imperfect information matters to the analysis of the two-stage location equilibrium game *à la* Hotelling, a claim also made by Boyer et al. (2003). Moreover, lack of information is an important issue for competing firms, as uncertainty arises in some important processes that they may face in many different areas of their organization. For instance, estimation of demand; estimation of the impact of a certain marketing strategy; uncertainty about worker productivity; uncertainty about innovation outcomes; uncertainty about the conditions that other firms in the market face… and many more.

In order to better understand the results of this model, future research should focus on making endogenous the uncertainties of the model, for instance, considering an investment in marginal cost reduction, considering a suppliers' market or considering different assumptions for the demand. This would allow for a better understanding of how different types of uncertainties would affect the market and firms' decisions.

4. R&D Investments in Spatial Competition

4.1. Introduction

This paper studies the behavior of two competitors in the framework of Hotelling (1929). We depart from the classical two-staged location-price game by introducing a R&D investment stage in the first period. Firms are allowed to invest in order to reduce the marginal cost of production of their goods. This marginal cost is used later to compete in the market, where firms have to choose the location of their store in the linear city and the price that they will set for their good.

The introduction of this investment in R&D is an interesting feature for the analysis of behavior in the Hotelling model. Firms are given another decision variable to affect their results. Moreover, this decision variable adds to the fitness of the model to the reality: In the original model and in many of its extensions, the marginal costs of production are assumed as given, which, in spite of simplifying significantly the analysis, is a very strong assumption. Even after considering the nature of the good and some other exogenous costs for its production (like, for instance, the cost of raw materials), firms are an important determinant in setting the cost of their own good. For instance, by possessing a good organizational structure, by amplifying their capacity or by investing in R&D, firms may be able to reduce their marginal cost. This capacity of influencing the marginal costs tends to be higher the larger is the firm, because larger firms have, overall more control on a set of important factors such as the quality of an organizational structure; the bargaining power with their suppliers and consumers; the financial power to perform investments in capacity and R&D; and so on. Therefore, this question of endogenous marginal costs is even more realistic when applied to duopoly firms, which is the case in this model.

However, the hypothesis of perfect information does not fit entirely to the R&D case: Firms allocate considerable amounts of staff and funds in order to perform research without precisely knowing if there is going to be any innovation. Even if the innovation occurs, firms may not know if that innovation will be significant to the production process, either in terms of the quality of the product or in terms of its production costs. Therefore, uncertainty is inherent to the innovation process and should be considered when modeling investments in R&D.

We could model imperfect information by assuming, similarly to the previous chapter, that after investing firms have a fixed probability that their investment is successful and their marginal cost is reduced to the intended objective cost. However, if the investment is unsuccessful, it means that firms will remain with the same marginal costs. In either case, firms lose their investment cost, which is considered as a fixed cost. However, we were not able to proceed to the imperfect information analysis. We have found mathematical problems that prevented us to entirety solve the investment stage even for the perfect information case. Therefore, we did not proceed on the calculations with uncertainty in investment.

Nevertheless, we have concluded that with perfect information, when firms are equally efficient regarding the effects of the investment on the marginal cost reduction, they enter a prisoner's dilemma situation, where at the Pareto optimal firms would not invest. However, they enter in some sort of "investment race", and the Nash equilibrium implies investing and incurring in lower profits than in the no-investment situation. Given the inelastic demand and the high reservation price assumptions of the Hotelling model, firms earn absolutely nothing more for having a lower marginal cost than the initial situation, since what determines profits in these conditions is the difference of marginal costs of production between both firms.

This chapter is organized as follows: Subsection 4.2 fits the paper in the literature. Subsection 4.3 details the assumptions of the model, while subsection 4.4 details the perfect information case. Subsection 4.5 shows the Prisoner's dilemma result, while subsection 4.6 concludes.

4.2. Theoretical Background

The model of Hotelling (1929) is the main framework for this study. In the original Hotelling model, two firms must choose at first stage their location in a linear and bounded city, and in the second stage they compete in prices. The good sold by the firms is homogenous, except for the location they have chosen in the first stage, and the firms have the same cost structure. Demand is perfectly inelastic and consumers incur in a linear transportation cost in order to buy the good. Hotelling concludes that firms would optimally agglomerate in the center of the city. A crucial extension to the model of Hotelling was the one done by d'Aspremont et al. (1979). By introducing quadratic transportation costs (instead of linear) with respect to distance, the result of central agglomeration disappeared with firms optimally locating in each of the extremes of the city. After the work by d'Aspremont et al., the field expanded significantly and more important publications appeared about this issue. Most of these publications focused on changing the assumptions of the original Hotelling model and then concluding about the new location decisions of the firms, as it can be seen in a detailed review on the chapter 2 of this manuscript.

The most direct reference of this work is the paper from Ziss (1993). The author derives the results for the two-stage location-price game of Hotelling allowing for different marginal costs between the two firms. Ziss concludes that when the difference between the marginal costs is small firms prefer to locate in different extremes of the city. However, as the difference is higher than a given threshold, the low-cost firm prefers to locate as closer as possible to the high-cost firm that, in an opposing fashion, wishes to locate the farthest possible from the other firm. This leads to the absence of location equilibrium in pure strategies. However, if the difference between both marginal costs is high enough, the low-cost firm drives the high-cost firm out of the market. Later on, Matsumura and Matsushima (2009) extend the Location-Price game of Ziss (1993), but allowing for mixed strategies in the location stage. They conclude that there is a mixed-strategy for all cost differentials in the game. For the cost differentials where no pure strategy equilibrium exists, the mixed equilibrium involves each firm choosing to locate in each extreme of the market 50% of the times.

In terms of endogenous marginal costs, Matsumura and Matsushima (2004) present an appealing model about mixed duopolies: Two firms, one of public and other of private nature, compete in a three-stage R&D-Location-Price game, similarly to this paper, in a linear market *à la* Hotelling. The public firm wishes to maximize social welfare, while the private firm wishes to maximize its own profits. They conclude that the private firm incurs in an excessive strategic cost-reducing investment, although the locations chosen by both firms are equal to the ones chosen by a social planner. In a very similar approach, Matsumura and Matsushima (2012a) introduce a model where in the first-stage firms invest in cost-reducing activities, but they focus only on smaller differences between the marginal costs of production of both firms.

A different type of R&D investments coupled with uncertainty is the model from Harter (1993). Firms choose what type of horizontally differentiated good they will manufacture, but entry only occurs when the investment is complete, which means firms may have different time periods for entering. In a discrete variation, Gerlach et al. (2005) model the case where firms have a given probability of entering the market or not, but they have to choose their degree of differentiation before entering (or not). However, these approaches are not related with cost-reducing investments, with is the focus of this chapter.

4.3. The Model

The basic assumptions of the model are as following: Two firms compete in a three-stage R&D-Location-Price game. In the first stage, firms choose simultaneously the marginal costs that they are going to use in the subsequent stages of the game. The investment function is equal to the one assumed in Matsumura and Matsushima (2004), which is given by $I(c_i) = \alpha_i(C - c_i)^2$, where C is the initial marginal cost of both firms and c_i the marginal cost firms have chosen to be their final one. Since there is a clear relationship between the investment and the cost reduction, in this framework to choose the final marginal cost is equivalent to choose the amount of funds to invest. One can see that the investment cost function has the desired and realistic properties: A lower final marginal cost implies larger investment costs for the relevant set of parameters ($\frac{\partial I}{\partial c_i} < 0$, for $C > c_i$) and lowering marginally the marginal cost implies an increasing investment effort for the firms ($\frac{\partial^2 I}{\partial^2 c_i} > 0$).

Also, firms are assumed to have no initial advantage over the other except for the parameter α_i, which represents the efficiency of firms when investing in R&D. A lower α_i implies higher efficiency when investing in the R&D function, which means that if $\alpha_i < \alpha_j$ there exists an advantage for firm i.

After choosing the level of investment, firms choose simultaneously their locations within the linear and bounded city of Hotelling, which is assumed to have length equal to one. The location variables are represented by x_1 and x_2 for firm 1 and firm 2 respectively, where x_1 is expressed as the distance from the left extreme of the market and x_2 is the distance from the right extreme. Note that under these specifications of distance, when $x_1 + x_2 = 0$, firms are fully differentiated and when $x_1 + x_2 = 1$, firms are agglomerated in any point of the market. We assume also, without loss of generality, that firm 1 is never located to the right of firm 2, which is equivalent to impose that $x_1 \leq 1 - x_2$. Finally, in the third-stage, firms simultaneously choose prices, which are represented by p_1 and p_2, respectively, for firm 1 and firm 2. The remaining assumptions are equal to Hotelling (1929), with the proposed extension by d'Aspremont et al. (1979): The goods produced by the firms are homogenous to the eyes of the consumers except for the location that firms choose. Consumers are uniformly distributed across the linear city and are obliged to buy one unit of good in order to survive. Therefore, the reservation price of the consumers is assumed to be high enough such that the market is always fully covered by the existing firms. Consumers incur in a transportation cost to buy the good that is quadratic with respect to distance. Without loss of generality, we assume that the unit transportation cost is equal to one and that the initial marginal cost of both firms is set to 10, similarly to what we did in the previous chapter.

4.4. Solving the model and results

In the perfect information case, firms participate in the three stage game without any uncertainty regarding the outcome of the investment in R&D: If firms invest a given quantity I, they are certain that they will obtain the corresponding new marginal cost, given by c_i. We solve the game using backward induction.

The demand of each firm is derived directly from the location of the indifferent consumer. Typically in price competition in models *à la* Hotelling, each firm has its own market area, that is, each firm has an area in which all consumers located there buy in their store. The boundary of that area is given by the point x where the indifferent consumer is located. Firm 1 will have a demand equal to x, while firm 2 will have a demand equal to $1 - x$. The indifferent consumer is located in:

$$x = \frac{1 - 2x_2 - x_1^2 + x_2^2 - p_1 + p_2}{2 - 2x_1 - 2x_2}$$

However, one must consider, when drawing the profit functions for each firm that the indifferent consumer location can never be lower than 0 or higher than 1. The profit function for each firm is represented by Π_i with i = 1, 2 and is given by:

$$\Pi_i = \begin{cases} (p_i - c_i) - \alpha_i(10 - c_i)^2 & \text{if} \quad p_i < \underline{\theta} \\ \dfrac{(p_i - c_i)(1 - 2x_j - x_i^2 + x_j^2 - p_i + p_j)}{2 - 2x_i - 2x_j} - \alpha_i(10 - c_i)^2 & \text{if} \quad \underline{\theta} \leq p_i \leq \overline{\theta} \\ -\alpha_i(10 - c_i)^2 & \text{if} \quad p_i > \overline{\theta} \end{cases}$$

For i=1, 2 and $\underline{\theta} = p_j - x_i^2 + 2x_i + x_j^2 - 1$, $\overline{\theta} = p_j - x_i^2 - 2x_j + x_j^2 + 1$. The upper part of the expression corresponds to the case where firm i has all the demand, while the lower part corresponds to the situation where the firm is unable to participate in the market, due to its price being high enough comparing with the one from the rival. In addition,, this indifferent consumer function and the profit function are only valid for the case of $x_1 < 1 - x_2$. When $x_1 = 1 - x_2$, firms are in the same location. In this case, the idea of the indifferent consumer stops making sense and the resulting profits are similar to the ones in Bertrand (1883).

4.4.1 Price Stage

The price behavior when one firm sells the good for all the market is obvious: this firm wants to set the highest price that allows it to keep the entire market. The "threshold" price increases with the distance from the opponent's store, *ceteris paribus*. When a firm does not have any consumer, the price is equal to the marginal cost.

The interesting case is when both firms have their own market areas. After differentiating the profit function with respect to the price for each firm and after equalizing to zero, we obtain the best-response functions in terms of pricing. Combining the expressions for both firms we get, for each firm:

$$p_i(x_i, x_j, c_i, c_j) = 1 - \frac{2}{3}x_i - \frac{4}{3}x_j - \frac{1}{3}x_i^2 + \frac{1}{3}x_j^2 + \frac{2}{3}c_i + \frac{1}{3}c_j$$

Replacing both prices in the profit function, we get:

$$
\Pi_i(x_i,x_j,c_i,c_j)=\begin{cases}-1+2x_i-x_i^2+x_j^2+c_j-c_i-\alpha_i(10-c_i)^2 & \text{if} \quad \hat{c}_i<\underline{\theta}\\ \dfrac{(-3+2x_i+4x_j+x_i^2-x_j^2+c_i-c_j)^2}{9(2-2x_i-2x_j)}-\alpha_i(10-c_i)^2 & \text{if} \quad \underline{\theta}\le\hat{c}_i\le\overline{\theta}\\ -\alpha_i(10-c_i)^2 & \text{if} \quad \hat{c}_i>\overline{\theta}\end{cases}
$$

Where $\hat{c}_i$ is the marginal cost difference expressed by $c_i - c_j$ and $\underline{\theta}$ and $\overline{\theta}$, are the same than before, but after replacing the optimal price decision these change to $-x_i^2+4x_i+x_j^2+2x_j-3$ and $-x_i^2-2x_i+x_j^2-4x_j+3$, respectively.

4.4.2 Location Stage

To solve for the location stage, the usual methodology of deriving the profit function with respect to the decision variable and equalizing to zero is not the most adequate, mainly for two reasons: The resulting values and equations are too complicated to withdraw direct conclusions and the optimal result does not take into consideration the natural boundaries for the location of the firms, that is $x_1 \le 1-x_2$ and $0 \le x_1, x_2 \le 1$. Therefore, we have assumed just like in the previous chapter that both firms are restricted to choose only between locating in one of the extremes of the linear city. This means that in terms of location strategy, firms can only be agglomerated or fully dispersed relatively to their opponent.

Therefore, the problem of the optimal location is reduced to finding whether it is more profitable for both firms to locate between two possible choices. By the straight comparison of both profits, we find that firm *i* prefers to agglomerate when:

$$
\Pi_i(x_i=1,x_j=0)>\Pi_i(x_i=0,x_j=0)\Leftrightarrow
$$
$$
\Leftrightarrow -\hat{c}-\alpha_i(10-c_i)^2>\frac{(-3+\hat{c})^2}{18}-\alpha_i(10-c_i)^2\Leftrightarrow \hat{c}<-(6-3\sqrt{3})
$$

The resulting value is the one already obtained by Ziss (1993), but considering fixed length and transportation costs. Note that this result is valid because for a lower difference in the marginal costs, the low-cost firm does not wish to leave its own extreme.

However, for $\hat{c}<-(6-3\sqrt{3})$, the low-cost firm does not wish to be dispersed with the high-cost firm, since that would imply lower profits. The high-cost firm always wants to be far from the low-cost firm, as it benefits in two ways: It increases its demand it increases the price of its good, due to the softening of price competition. Therefore, in this case there is no location equilibrium in pure strategies, as one of the firms prefers to be as close as possible to the opponent, while the other firm prefers to be as far as possible.

Contrary to Ziss (1993), who considers the possibility of entry, we assume that the high-cost firm has to remain in the city even if it has negative profits. In this situation the negative profits arrive only from the investment costs taken in the first-stage. The choice of keeping the high-cost firm in the market has a useful purpose: to limit the maximum price that the monopolist can set, which makes the analysis of profit maximization more rigorous. If a second firm did not exist, the monopolist could set a price equal to the reservation cost of consumers, which is assumed to be sufficiently high. Therefore, the existence of a potential entrant keeps the monopolist aware, allowing for prices not to be that high and allowing the value of the "premium" for achieving a monopoly situation at a reasonable level.

In order to proceed with the analysis we consider a mixed strategy for the location stage. Using part of the result of Matsumura and Matsushima (2009), we assume that when there is no pure location equilibrium, firms play a mixed strategy that implies that half of the times firms agglomerate in one of the extremes and in other half of the times, they locate in the opposing extremes of the market. So, the optimal location pattern is given by:

$$x_i = \begin{cases} (1:0.5;0:0.5) & ,\text{for } \hat{c} < -\left(6-3\sqrt{3}\right) \\ 0 & ,\text{for } -\left(6-3\sqrt{3}\right) < \hat{c} < \left(6-3\sqrt{3}\right) \\ (1:0.5;0:0.5) & ,\text{for } \hat{c} > \left(6-3\sqrt{3}\right) \end{cases}$$

Therefore, the profits of the firms, after replacing for the optimal location decision, are given by equation 4.1:

$$\Pi_i = 0.5(-1-\hat{c}_i) - 0.5\hat{c}_i - \alpha_i(10-c_i)^2 \quad ,\text{for } \hat{c}_i < -3$$

$$\Pi_i = 0.5\frac{(-3+\hat{c}_i)^2}{18} - 0.5\hat{c}_i - \alpha_i(10-c_i)^2 \quad ,\text{for } -3 < \hat{c}_i < -(6-3\sqrt{3})$$

$$\Pi_i = \frac{(-3+\hat{c}_i)^2}{18} - \alpha_i(10-c_i)^2 \quad , \text{for } -(6-3\sqrt{3}) < \hat{c}_i < (6-3\sqrt{3}) \quad (4.1)$$

$$\Pi_i = 0.5\frac{(-3+\hat{c}_i)^2}{18} - 0.5(0) - \alpha_i(10-c_i)^2 \quad , \text{for } (6-3\sqrt{3}) < \hat{c}_i < 3$$

$$\Pi_i = 0.5(0) + 0.5(0) - \alpha_i(10-c_i)^2 \quad , \text{for } \hat{c}_i > 3$$

In all branches of the profit function except for the middle one, it is represented an expected profit , since half of the times the firms are agglomerated and in the other half they are fully dispersed, which results from the mixed strategy played (Matsumura and Matsushima, 2009). The first and last branches of the profit function correspond to the case where only one of the firms sells the good for the entire market, independently of the final location outcome. The interval for the difference of marginal costs is also identified by Ziss (1993), and it is the case when the high-cost firm, even when fully differentiated relatively to its opponent, is not able to obtain any demand. In all the cases, firms still have to pay the investment cost.

4.4.3 Investment Stage

In the third-stage, firms have to decide the amount of money to invest in cost-reducing activities, given the future decisions on location and pricing. As mentioned before, the investment cost function depends on a technology parameter α_i. For our analysis, we have fixed the parameter of firm 2 to $\alpha_2 = 0.4$, which still allows us to assess the behavior of firms when they have an advantage/disadvantage in terms of their marginal cost of production. Therefore, this parameter is set without loss of generality. Note that the game is fully solved with respect to the parameter α_1, which represents the efficiency of firm 1 when engaging in cost-reducing activities.

However, we faced some problems in finding the optimal decision of both firms, since we are optimizing on a piecewise function in a problem with two variables. Then, a direct calculation does not allow us to find the sub-game perfect Nash equilibrium. Therefore, we could not solve the model for all values of the technology parameter.

We started by analyzing what is the optimal investment decision of firms in each branch of the profit function. For the middle branch of equation 4.1., both firms are dispersed and their profit is given by:

$$\Pi_i = \frac{(-3+\hat{c}_i)^2}{18} - \alpha_i(10 - c_i)^2 \quad (4.2)$$

When firm 1 manages to obtain a significant cost advantage, firms play the mixed strategy and their profits are given by the second and fourth branches of equation 4.1:

$$\Pi_1 = 0.5\frac{(-3+c_1-c_2)^2}{18} - 0.5(c_1 - \mathrm{c}_2) - \alpha_1(10 - c_1)^2 \quad (4.3)$$

and:

$$\Pi_2 = 0.5\frac{(-3+c_2-c_1)^2}{18} - 0.5(0) - \alpha_2(10 - c_2)^2 \quad (4.4)$$

Respectively, when firm 1 is monopolist, its profits are given by the first branch of equation 4.1:

$$\Pi_1 = 0.5(-1 - (c_1 + c_2) - 0.5(c_1 - \mathrm{c}_2) - \alpha_1(10 - c_1)^2 \quad (4.5)$$

While firm 2 only pays its investment costs. We only show these three options (middle branch; firm 1 with an advantage; firm 1 as a monopolist) since for $\alpha_2 = 0.4$, firm 2 never achieves a sufficient marginal cost advantage whatever the technology parameter for firm 1..

Solving the investment stage, firms maximize their profits given they can choose the optimal value of investment, which determines the marginal cost of production of their good. For the branch where both firms disperse, firms maximize their profits by choosing the following marginal costs:

$$c_1 = \frac{1320a_1 - 107}{132a_1 - 9} \text{ and } c_2 = \frac{1320a_1 - 80}{132a_1 - 9}$$

Given the location choice of both firms, note that these values are only valid when $\hat{c}_1 > -(6 - 3\sqrt{3})$. Therefore, replacing c_1 and c_2 is the restriction, we can see this solution is only valid when $a_1 > \frac{3\sqrt{3}-9}{44\sqrt{3}-98}$, that is, approximately 0.17457.

From the case where the profits of both firms are given by equations 4.3 and 4.4, the investment choice of both firms obtained from the maximization of these profits is given by:

$$c_1 = \frac{5880a_1 - 371}{588a_1 - 18} \text{ and } c_2 = \frac{5700a_1 - 155}{588a_1 - 18}$$

Since these values are only valid if $-3 < \hat{c}_1 < -(6 - 3\sqrt{3})$, after replacing these solutions and solving with respect to α, we have that the solution is only valid for:

$$\frac{5}{36} < a_1 < \frac{3\sqrt{3} - 18}{98\sqrt{3} - 206}$$

That is, approximately $0.13889 < a_1 < 0.35312$.

For the branch where firm 1 holds the monopoly situation (equation 4.5), the optimal marginal costs chosen by firm are given by $c_1 = \frac{20a_1 - 1}{2a_1}$ and $c_2 = 10$, which is valid only if $a_1 < 0.16667$.

A careful analysis of the above results reveals that the branches relatively to the technology parameter of firm 1 are conflicting. When $0.17457 < a_1 < 0.35312$ or when $0.13889 > a_1 > 0.16667$, the adequate branch is not unique since for these values of a_1 more than one optimal branch solution is found.

Then, the direct result for the marginal cost that we obtain from solving each of the branches separately might not be a Nash Equilibrium. Each firm has other investment possibilities that are not covered in a single branch option. For instance, when we calculate the profits for each branch, we are implicitly assuming that the profit of the firms will always be given by that expression, which is not always the case. In more practical terms, imagine that we are maximizing for the branch where there is a moderate disadvantage by firm 2 (that is, the branch where $-3 < \hat{c}_i < -(6 - 3\sqrt{3})$). For some values of the investment value chosen by firm 1, which is given by ($c_1 = \frac{5880a_1 - 371}{588a_1 - 18}$), firm 2 would most likely prefer to invest a bit more in order to decrease the advantage of firm 1 to a value lower than $(6 - 3\sqrt{3})$, which would imply a different profit outcome than the one that was subject to the maximization problem. In other words, and following closely the definition of Nash Equilibrium, the value found in the maximization case of this branch for firm 2 ($c_2 = \frac{5700a_1 - 155}{588a_1 - 18}$) is not necessarily the optimal marginal cost value chosen by the firm given all its investment possibilities, which a piecewise function cannot cover entirely.

Therefore, we inspect what would be the optimal choice of a firm given the choice of the other firm, by comparing the branch possibilities that we have. We did the computation for the profits in equation 4.2 where both firms have demand and are dispersed in the linear city. Unfortunately, our analysis is limited due to the fact we cannot test all possible pair of values of c_1 and c_2. We test the values that were chosen by firms in the unrestricted case. So, the choice of marginal cost for firm 2 when firm 1 chooses:

$$c_1 = \frac{1320a_1 - 107}{132a_1 - 9}$$

Is summarized in Table 4.1. Similarly, we summarize in the same table the choice of marginal cost for firm 1 when firm 2 chooses its optimal reaction in the restricted case, which was given by:

$$c_2 = \frac{1320a_1 - 80}{132a_1 - 9}$$

After checking the best responses for each case, we verified for which values of α the responses were valid. Then we compared, in case of conflicting solutions (two or more valid solutions for each α) which options would the firm prefer.

Table 4.1 – Best-response functions of firm 1 and 2 given their investment option in the first branch when $c_1 = \frac{1320a_1 - 107}{132a_1 - 9}$ and $c_2 = \frac{1320a_1 - 80}{132a_1 - 9}$.

	Optimal Response	Valid if	Chosen if
Firm 1	$c_1 = \frac{1320a_1 - 107}{132a_1 - 9}$	$a_1 > 0.17457$	$a_1 > 0.21759$
	$c_1 = \frac{47520a_1^2 - 6054a_1 + 188}{4752a_1^2 - 456a_1 + 9}$	$a_1 < 0.26589$	$0.13942 < a_1 < 0.21759$
	$c_1 = \frac{20a_1 - 1}{2a_1}$	$a_1 < 0.15035$	$a_1 < 0.13942$
Firm 2	$c_2 = \frac{1320a_1 - 80}{132a_1 - 9}$	$a_1 > 0.17457$	$a_1 > 0.17457$
	$c_2 = \frac{2112a_1 - 396\sqrt{3}a_1 + 27\sqrt{3} - 161}{132a_1 - 9}$	All a_1	$0.14653 < a_1 < 0.17457$
	$c_2 = \frac{62700a_1 - 4190}{6468a_1 - 441}$	$a_1 < 0.19605$	$\frac{1}{9} < a_1 < 0.14653$
	$c_2 = 10$	$a_1 < \frac{1}{9}$	$a_1 < \frac{1}{9}$

Therefore, we can see that, contrary to what the initial analysis suggests, the choice of marginal costs, given by:

$$(c_1 = \frac{1320a_1 - 107}{132a_1 - 9}; c_2 = \frac{1320a_1 - 80}{132a_1 - 9})$$

is a Nash Equilibrium for this stage only when $a_1 > 0.21759$, that is, when both firms consider that their current choice is the best given the current choice of the opponent. Consider, for instance, that for a lower value of a_1 firm 1 would choose a different marginal cost given the choice of firm 2. Finding the optimal investment result for all other values of a_1 would not be feasible, given that there is no direct way to find a generic optimal response function for any of the firms. In this example, since firm 1 would choose a different value, its optimal choice would be different. That would imply also that the optimal reply of firm 2 would change, which would change again the optimal response of firm 1. Finding an optimal response for each a_1 would imply tracing all these iterations, which we consider that as an unfeasible procedure. Therefore, we only solve the model when $a_1 > 0.21759$.

This impossibility in finding the Nash equilibrium for the investment game for all the possible values of firm efficiency led us to conclude that pursuing the case when there is imperfect information in the R&D stage would not be that interesting, since we would be unable to make a direct comparison with the perfect information case.

4.5. Prisoner's Dilemma in R&D in the Hotelling framework

Even though we were not able to fully solve the previous three-stage model presented before given that one of the technology parameters was set, it is still possible to solve the model if we consider two similar firms in an Hotelling market, which means considering two firms that have equal technology parameters. This simplification leads firms to choose the same marginal cost of production, since it eliminates all the problems related to analyze optimization problems in piecewise functions, which leads both firms to have their profit functions as expressed in equation 4.2., since and where both firms are always dispersed for any level of the technology parameter. A similar assumption is followed by Matsumura and Matsushima (2012a), since they only consider the possibility of smaller cost differences between firms, in order to work only with the case where both firms are dispersed.

Given that the investment functions are equal, firms maximize profit (equation 4.2) with respect to final production cost. Note that by this point, firms have symmetric profit functions with respect to the decision variables. Firms invest in production cost reduction until the marginal benefit equals the marginal cost of investing. Table 4.2 shows the optimal production cost calculation.

Table 4.2 – Investment Best-Response Functions and Final Marginal Production Cost for firm *i*

Marginal Benefit of Investment	Marginal Cost of Investment	Best-Response Function	Final Production Cost Decision
$\frac{1}{9}(c_i - c_j) - \frac{1}{3}$	$2\alpha(c_i - 10)$	$c_i = \frac{180\alpha - c_j - 3}{18\alpha - 1}$	$c_i = \frac{60\alpha - 1}{6\alpha}$

Replacing the marginal production costs of both firms in equation 4.2, firms' profits become only dependent on the efficiency parameter α, which solves the model.

$$\Pi_i = \frac{1}{2} - \frac{1}{36\alpha} \tag{4.6}$$

In this equation (4.6), the first term refers to the firms' operational profits while the second term represents the investment costs. A striking result emerges as firms' profits decrease with higher levels of efficiency in their investment. It would be expected that, since firms would have access to better ways of reducing their production cost, the profits would be higher. However, the operational profit remains the same, while the investment costs increase when α decreases.

In the Hotelling model when transportation costs are quadratic and production costs are symmetric, firms share the market and earn profits equal to one half times the unit transportation costs. Our model replicates this result for every value of α, since firms are dispersed and choose the same production cost. However, since investment is totally neutral towards firms' profits, these would be better off if they could agree to not invest on that resource. This resembles the classical prisoner's dilemma game, which is exactly the result for the investment stage. In this simplified version, firms would have the choice to invest (choosing the optimal value defined above) or not. In the Nash Equilibrium of this game, both firms invest, despite a Pareto optimal solution that would occur if they do not invest. Table 4.3 exhibits that example, which is only valid for $\alpha > 1/18$, since for lower values of α the investing firm achieves a monopoly situation.

Table 4.3 – The Prisoner's Dilemma game in the investment stage (for $\alpha > \frac{1}{18}$)

Investment Firm j / Investment Firm i	$c_j = 10$	$c_j = \frac{60\alpha - 1}{6\alpha}$
$c_i = 10$	$(\frac{1}{2}, \frac{1}{2})$	$(\frac{(18A-1)^2}{648\alpha^2} - 0, \frac{(18A+1)^2}{648\alpha^2} - \frac{1}{36\alpha})$
$c_i = \frac{60A - 1}{6A}$	$\frac{(18A+1)^2}{648\alpha^2} - \frac{1}{36\alpha}, \frac{(18A-1)^2}{648\alpha^2} - 0)$	$(\frac{1}{2} - \frac{1}{36\alpha}, \frac{1}{2} - \frac{1}{36\alpha})$

What can explain this unexpected result? The efficiency of the investment in terms of the Hotelling model plays a similar role to the unit transportation costs in terms of firms' profits: although it is common sense that the society and firms may benefit from lower unit transportation costs, in the Hotelling framework these costs are the source of differentiation between firms, which makes the local monopolies stronger, so they can set higher prices to their segment of consumers without fearing the opposition of the other firm. So, the existence of an investment function hurts firms in the sense that their local monopolies are threatened due to the possibility of investment by the rival. Firms must defend themselves from that threat which, in the end, yields them the same operational profit, since they end up dispersed, with the same marginal cost, and therefore, same demand.

4.6. Concluding Remarks

In this chapter, and in line with the previous chapter, we have tried to make endogenous the marginal cost decision of both firms, in order to understand the implications on firms' location, pricing, and resulting profits. Furthermore, we analyzed the effects of uncertainty in the investment decisions, by having a probability of non-occurrence of a decrease in the marginal cost of production. However, our efforts were stopped by the mathematical unfeasibility of fully solving a 2-variable optimization problem of a piecewise function.

After fixing the technology parameter of one of the firms, we could only find a Nash Equilibrium in the investment stage for values where the technology parameter of the remaining firm was not too big. In this case, firms disperse and the relationship between their profits and demand depends (naturally) on the relationship between their technology parameters: for $a_1 > 0.40$, firm 1 would have a higher marginal cost and therefore a lower profit, with the reverse happening when $a_1 < 0.40$.

Then, we reported a curious result found when considering firms with the same investment capacity. Instead of increasing profits as expected, an improvement in the technology parameter of both firms reduces firms' profits. This happens because firms invest as they fear that the opponent conquers their demand. Consequently, both firms defend their positions, even though they would be better off by not investing at all, and end up having the same operational profits. We feel that this result may be justified by the inelastic demand assumption usually taken in the Hotelling model, which prevents the market to expand itself beyond new customers.

5. Cost Inefficiency and Optimal Market Structure in the Spatial Cournot Framework

5.1. Introduction

Most studies about firms' behavior and market structure assume that firms are equal in many characteristics. Although this is a very natural assumption, since it allows researchers to isolate the effects of firms' asymmetries from the effect one is trying to study, in truth firms do not always face the same conditions, especially when they have significant market power as in the case of a duopoly. When a small number of firms compete for the same market, they usually follow strategies to differentiate themselves in order to increase profits. Differentiation strategies can rely on the production process; R&D investments; advertising campaigns; distribution channels; product quality; product variety, and many more.

In this paper, we adapt the spatial quantity competition setting allowing firms to have different marginal costs of production. The difference in marginal cost may be interpreted as a summary of a great variety of differences between firms, which may benefit one firm over the other. Under this framework our main question is: how firms react in terms of the location decisions when facing a production cost (dis)advantage? Additionally we analyze the optimal decisions of the social planner when facing firms with different technologies of production. We allow the social planner to control the locations of the stores of both firms, as well as to remove the inefficient firm out of the market if its presence hurts the global social welfare of the economy. This paper tests the robustness of Anderson and Neven (1991) results when firms face different marginal costs, as well as the conclusions of Matsumura and Shimizu (2005) regarding social welfare. In addition, this paper provides a starting point for analyzing more complex questions as the ones that happens when different marginal costs are endogenous. Examples for which the modeling of different marginal costs is suitable are the analysis of location-dependent marginal costs in the city; the analysis of R&D investment decisions of firms; the analysis of models where firms are uncertain about their marginal costs; the study of incumbent vs. entrant problems with different marginal costs; and so on.

Traditionally, quantity competition models are more adequate for markets in which the productive capacity decision of each firm is inflexible. When adding space, this model is suitable when the allocation of output by each firm to the different market points is also relatively inflexible, with energy markets being a good example of a market that fits in this setting (Anderson and Neven, 1991).

Therefore, we test what is the optimal reaction of firms when these have exogenous differences in their marginal costs of production. Additionally, we consider what would be the optimal choice of a social planner to maximize the social welfare, when he is allowed to change firms' location, but not the optimal quantities sold. We also study the case where the social planner is allowed to remove one of the firms out of (or symmetrically, forbidding one of the firms from entering on) the market. We compare this situation with the non-spatial case.

We conclude that the central agglomeration result known in the literature holds for any difference in the marginal costs of both firms, which comes in opposition with the result found in price competition, where no pure equilibrium in locations is found if the marginal costs become too big, with the only pure equilibrium being total dispersion between firms (Ziss, 1993). Regarding social welfare, we conclude that central agglomeration result only holds if the difference between the marginal costs of both firms is not too high. Also, when allowing the social planner to prevent the high cost firm from entering the market, we conclude that this firm would not be allowed to enter if the marginal cost difference becomes too high. Additionally, we conclude that for low values of the marginal cost difference, the optimal location pattern implies that firms are always located in the center of the city, either in monopoly or in duopoly.

We conclude as well that the conditions that lead the social planner to remove the inefficient firm do not change significantly with the introduction of the space dimension. That is, when removing an inefficient firm, the social planner must be more worried about the degree of inefficiency of the firm than with level of transportation costs in the economy.

This chapter is organized as follows. Subsection 5.2 provides the theoretical background for the model. Subsection 5.3 presents the model and solves the two-stage location quantity game problem of profit maximization for each firm. Subsection 5.4 includes the analysis of the problem of the maximization of social welfare. Subsection 5.5 concludes.

5.2. Theoretical Background

The idea of competition between two firms when in the first place firms choose the location in a linear city and then compete in prices was developed by Hotelling (1929). This two-stage game attracted the interest of many scientists, since it provides a relatively simple tool to introduce space (either physical or product space) in competition between firms. Hotelling concluded that firms would agglomerate in the center of the city, which became widely known as the "Principle of Minimum Differentiation".

However, 50 years after the original paper by Hotelling, a paper by d'Aspremont et al. (1979) proved that by introducing quadratic transportation costs in the previous framework, firms would optimally be located in the extremes of the linear segment. This new feature also allowed for better mathematical tractability, since under this assumption the demand and profits become continuous functions with respect to the price decisions of the firms. The mathematical tractability, combined with the intriguing result of that game led to a significant expansion of the field in the 1980s and the 1990s. Many papers were published in important journals in Economics, either generalized or specialized in Regional and Urban Economics and in Industrial Organization during that time. This conclusion is withdrawn from the chapter 2 of this dissertation.

The introduction of competition in quantities in the linear city framework is due to Greenhut and Greenhut (1975). Each firm chooses to deliver a given quantity to separable markets that are located throughout every point in the city. Some years later, Hamilton et al. (1989) and Anderson and Neven (1991) replicate the two-stage game by Hotelling using quantity instead of price competition in the second stage. Hamilton et al. (1989) conclude that when transportation costs are linear, firms choose to agglomerate at the city center. Anderson and Neven (1991) extend this result allowing for different transportation configurations, and concluded that central agglomeration is the optimal result if the transportation costs are convex. This equilibrium result is also extended for *n* firms if transportation costs are linear.

Since the appearance of Cournot two-stage game, some important extensions and results followed. An important extension is Gupta et al. (1997), which abandon the uniform distribution of consumers across the city and studied the location equilibrium allowing for different types consumers' distribution. The authors conclude that both agglomeration and dispersed equilibrium may arise. For the case of duopoly, if the consumer density is sufficiently thick in all points of the city, then central agglomeration is the only equilibrium. A related approach is due to Shimizu (2002), who studies the degree of complementarity/substitutability between the goods sold by both firms, and for the linear city case concludes that the agglomeration result remains unchanged independently of the relationship between both goods.

Matsumura and Shimizu (2005) focused on the social welfare results of the locations decisions. They conclude that, when in a duopoly, agglomeration at the center can be an optimal location concerning social welfare, given that consumer density is sufficiently thick in all points of the market. Chamorro-Rivas (2000) and Benassi et al. (2007) lowered the reservation costs of the consumers, unrestricting the model in the sense that every firm does not have to supply every point of the market. Chamorro-Rivas (2000) conclude that a second equilibrium solution arises, and therefore central agglomeration is no longer a unique equilibrium. Benassi et al. (2007) find that for even lower reservation costs than Chamorro-Rivas, central agglomeration ceases to be equilibrium and firms disperse in a symmetric pattern.

The question on how firms behave if they have different marginal costs is yet to be answered conveniently in the competition by quantities setting. In Bertrand, this problem was addressed by Ziss (1993), who conclude that the principle of maximum differentiation holds if the difference between the marginal costs of both firms is not too big. After a certain threshold, firms are not able to find location equilibrium, since the low-cost firm wishes to be as close as possible to the opponent, while the other firm wishes to be as far as possible. An important extension is the work of Matsumura and Matsushima (2009), which extends the location-price game of Ziss but allowing for mixed strategies in the location stage. For the cost differentials where no pure strategy equilibrium exists, the mixed equilibrium involves each firm choosing to locate in each extreme of the market 50% of the time.

5.3. The Model

The model follows all assumptions detailed in Anderson and Neven (1991), which in turn are similar to the ones in Hotelling (1929) except for the ones that allow competition in quantities to be tractable. In our model two firms compete in a two-stage location-quantity game in a linear city, assumed to have length equal to 1. In the first-stage both firms choose simultaneously their location in the city, and then, in the second stage, they compete in quantities across all markets in the linear segment. The goods sold by both firms are homogenous and therefore are perfect substitutes. Transportation costs are supported by the firms and the unit transportation cost is assumed to be linear with respect to the travelled distance. Both firms are able to discriminate between consumers, since firms control transportation, or in other words, the unit transportation costs between consumers are assumed to be high enough such that they are unable to make a profit by selling goods between them. The difference from the framework of Anderson and Neven (1991) is that we allow firms to have different marginal costs of production. The linear market therefore consists in a continuum of market points which are assumed to have uniform density. In each point there is a market with an inverse demand function for the good given by $P = (100 - Q)$.

5.3.1. Quantity Stage

In the first stage, firms decide simultaneously how many goods they sell to each point in the market. Firms are allowed to discriminate between different markets in the city by providing different quantities for each market point. In each point, the calculations are equivalent to a non-spatial Cournot, with the marginal cost of each firm being its unit transportation cost to that specific point plus the marginal cost of production. Therefore, the profit function for firm *i* is given by:

$$\Pi_{i,x}(q_i, q_j, x_i, x_j) = (100 - (q_i + q_j) - t|x - x_i| - c_i)q_i$$

With q representing the quantities produced in this point; t the unit transportation cost; x representing the location variable; c representing the marginal cost of production; and with i=1,2, and j representing the opponent firm. Maximizing both firms' profits with respect to quantities, which is the second-stage variable, we have that the optimal quantities supplied by each firm i to a given point x is given by:

$$q^{*}_{i,x}(q_{j,x}, x_i) = \frac{100 - q_{j,x} - t|x_i - x| - c_i}{2}$$

Solving both best-response functions, we have the optimal decision of firm i depending on the marginal costs and location decisions of both firms, and the market point for which both firms are selling the product.

$$q^{*}_{i,x}(x_i, x_j) = \frac{100 - 2c_i + c_j - 2t|x_i - x| + t|x_j - x|}{3}$$

With the SOC always verified for feasible values of the each variable:

$$\frac{\delta^2(\Pi_{i,x}(q_i, q_j, x_i, x_j))}{\delta^2 x_i} < 0 \Leftrightarrow -2 < 0$$

And:

$$\left(\frac{\delta^2\Pi_{i,x}}{\delta^2 x_i} * \frac{\delta^2\Pi_{j,x}}{\delta^2 x_j}\right) - \left(\frac{\delta^2\Pi_{i,x}}{\delta x_j \delta x_i} * \frac{\delta^2\Pi_{j,x}}{\delta x_i \delta x_j}\right) > 0 \Leftrightarrow (-2)*(-2) - ((-1)*(-1)) > 0 \Leftrightarrow 3 > 0$$

The total profit for each firm is given by the sum of the profits of all market points. The profits in a given market point is given by the following expression:

$$\Pi_{i,x}((x_i, x_j)) = \frac{(100 - 2c_i + c_j - 2t|x_i - x| + t|x_j - x|)^2}{9}$$

The sum of profits in all the points is given by;

$$\Pi_i(x_i, x_j) = \int_0^1 \left(\frac{(100 - 2c_i + c_j - 2t|x_i - x| + t|x_j - x|)^2}{9}\right)dx \qquad (5.1)$$

However, equation (5.1) cannot be computed directly due to the existence of the absolute value in the expression, due to transportation costs. Therefore, the integral expressions have to be separated before being computed, as done similarly by Anderson and Neven (1991).We are able to separate the integrals due to the assumption that $x_1 \leq x_2$, that is, firm 1 is never located "at the right" of firm 2. With this, we know exactly the sign of the transportation costs in each of the possible cases: For the market points between 0 and x_1, we have, for firm 1:

$$\Pi_1(x_1, x_2) = \int_0^{x_1} \left(\frac{(100 - 2c_1 + c_2 - 2t(x_1 - x)) + t(x_2 - x))^2}{9}\right) dx$$

Between x_1 and x_2, the sum of profits in all market points is given by:

$$\Pi_1(x_1, x_2) = \int_{x_1}^{x_2} \left(\frac{(100 - 2c_1 + c_2 - 2t(-x_1 + x)) + t(x_2 - x))^2}{9}\right) dx$$

While for the market points between x_2 and 1, the sum of profits of firm 1 is given by:

$$\Pi_1(x_1, x_2) = \int_{x_2}^{1} \left(\frac{(100 - 2c_1 + c_2 - 2t(-x_1 + x)) + t(-x_2 + x))^2}{9}\right) dx$$

Then, after summing up the 3 integrals (and doing a similar procedure for the profits of firm 2), we obtain the profits of both firms. These are dependent on firms' location choices, firms' marginal costs and the value of unit transportation costs. The profit results of both firms lose their symmetric property after this integral transformation. Therefore, firm's 1 profits are given by:

$$\begin{aligned}\Pi_1(x_1, x_2) &= \left(\frac{4}{27}t^2\right)x_1^3 + \left(\frac{8tc_1 - 400t - 4tc_2 - 4t^2x_2 + 4t^2}{9}\right)x_1^2 + \\ &+ \left(\frac{400t + 4t^2x_2^2 - 8tc_1 + 4tc_2 - 4t^2x_2 - 2t^2}{9}\right)x_1 + \frac{t^2x_2^2 + t^2x_2 - 4tc_1x_2^2 + 4tc_1x_2 + 2tc_1}{9} + \\ &+ \frac{2tc_2x_2^2 - 2tc_2x_2 - tc_2 + 200tx_2^2 - 200tx_2 - 100t + 4c_1^2 - 4c_1c_2 - 400c_1 + c_2^2 + 200c_2}{9} + \\ &+ \frac{10000}{9} + \frac{t^2 - 4t^2x_2^3}{27}\end{aligned}$$

While for firm 2 the profits are given by:

$$\Pi_2(x_1,x_2)=(-\frac{4}{27}t^2)x_2^3+(\frac{8tc_2-4tc_1-400t+4t^2x_1}{9})x_2^2+$$
$$+(\frac{400t-4t^2x_1^2+4tc_1-8tc_2-4t^2x_1-2t^2}{9})x_2+\frac{t^2x_1^2+t^2x_1+2tc_1x_1^2-2tc_1x_1-tc_1}{9}+$$
$$+\frac{4tc_2x_1-4tc_2x_1^2+2tc_2+200tx_1^2-200tx_1-100t+c_1^2-4c_1c_2+200c_1+4c_2^2-400c_2}{9}+$$
$$+\frac{10000}{9}+\frac{4t^2x_2^3+t^2}{27}$$

5.3.2. Location Stage

To determine the optimal location for each firm, we apply the first-order conditions to each firm's objective function, with respect to the location variable. The derivatives of both firms' profits are given by:

$$\frac{\partial\Pi_1(x_1,x_2)}{\partial x_1}=-\frac{2t}{9}(t+4c_1-2c_2+400x_1-4tx_1+2tx_2-8c_1x_1+4c_2x_1-2tx_1^2-$$
$$2tx_2^2+4tx_1x_2-200)$$

$$\frac{\partial\Pi_2(x_1,x_2)}{\partial x_2}=-\frac{2t}{9}(t-2c_1+4c_2+400x_2+2tx_1-4tx_2+4c_1x_2-8c_2x_2+2tx_1^2$$
$$+2tx_2^2-4tx_1x_2-200)$$

After equalizing these expressions to zero, we solve the system of equations with respect to the location variables, and we obtain the optimal location decision irrespective of the optimal location decision of the other firm. We arrive at the following result, for which the second-order conditions are satisfied.[1]

$$(x_1=\frac{1}{2};x_2=\frac{1}{2})$$

Hence we arrive at Proposition 5.1.

[1] By solving the first-order conditions and verifying the second-order conditions, other solutions were found. However, these either did not respect the constraint that firm 1 must not be located at the right of firm 2 (for which the corresponding profits are not correct, due to the specification of the integral we have built) or did not respect the fact that firms must be located inside the linear city. These results were therefore not considered.

Proposition 5.1: Independently of the value of the marginal costs for both firms, and given that the dimension of the market is big enough such that both firms have no negative profit in all points of the market, central agglomeration is the unique equilibrium for the two-stage location-quantity game.

Proof: Result of the first and second-order conditions of both firms' profit maximization.

After replacing the optimal location in the profit functions of both firms, these retain their symmetric property. The profit of firm *i* becomes:

$$\Pi_i = \frac{t^2 + 12tc_i - 6tc_j - 600t + 48c_i^2 - 48c_i c_j - 4800c_i + 12c_j^2 + 2400c_j + 120000}{108} \quad (5.2)$$

The first result appearing in this model is that the location decision of both firms is unaffected by the marginal costs values of both firms. This result is interesting, and goes against the initial intuition that a firm with high marginal costs would decide to move away from the opponent to one of the sides of the market, in order to conquer more demand in a small region of the city instead of locating in the center. However, analyzing the profit derivative (Equation 5.1) of the high-cost firm with respect to its own location decision after fixing the location of the other firm in $x_2 = 0.5$, which is given by:

$$\frac{\delta \Pi_1(x_1, x_2)}{\delta x_1}(x_2 = 0.5) = \frac{t(2x_1 - 1)(3t + 8c_1 - 4c_2 + 2tx_1 - 400)}{9}$$

We find that the incentives to agglomerate at the center are similar to the ones in Anderson and Neven (1991), that is, by deviating from the center firms increase the transportation costs in a large part of the market and decrease the transportation costs in a short part of the market, leading to a decrease in the profits. The different marginal costs only change the penalty for moving away from the center: If the marginal costs are too different, the high-cost (low-cost) firm has a lower (higher) penalty from moving away from the center. This occurs because having such a disadvantage (advantage), firms sell less (more) quantities to all the points in the market, and so the transportation costs decrease (increase) when moving to non-central positions. This penalty is always positive for all feasible values of the model.

A second conclusion emerges from the comparison with the price competition model. We verify that not only that the difference between the marginal cost matters, but also that the absolute value of the marginal costs has an important role. In Bertrand competition, due to the assumption of inelastic demand and sufficiently high reservation price, having zero or huge marginal costs of production is similar in terms of profits for both firms as long as the difference between them remains the same. In Cournot competition, a more realistic result arises; the difference is not the only determinant of profits, since having a lower marginal cost expands the markets in all points of the city, leading to higher profits for both firms (given that the difference between both is kept constant). Note that the profits of the firms cannot be written in terms of the difference between marginal costs, as it is possible in price competition (e.g. see Ziss (1993) or Matsumura and Matsushima (2009)).

We have to make the remark that this location result depends crucially on the assumption that firms are obliged to sell in every point of the market. If that would not be the case, and the market dimensions in each point were not too big, the most important effect in justifying the location decision of both firms, which is the minimization of transportation costs, would be weaker, and firms could focus on selling to smaller market niches.

5.4. Social Welfare

5.4.1. Duopoly

In Bertrand studies of the classic Hotelling model, the social welfare analysis is confined to analyzing the optimal location decision of firms. Due to the existence of inelastic demand, changes in price affect only the distribution of the welfare between firms and consumers, but not its total value, and therefore the only social welfare maximizing problem is related with the minimization of the transportation costs. When competing in quantities, however, the social welfare is not only sensitive to locations and to the difference in the marginal costs, but also to price policies. A price decrease from one firm leads not only to an additional advantage over the other firm, but also to an expansion of the markets, since demand has some price elasticity.

5.4.1.1. Maximization of social welfare

We seek a second-best solution, that is, we consider that the social planner can intervene only in the first-stage, choosing the location of the firm's plants in order to maximize the social welfare of the city. The measure of social welfare considered is the sum of consumer surplus with the profits of both firms. The profits are equivalent to the producer surplus since there are no fixed costs. The consumer surplus is calculated similarly to the profits, that is, by summing the consumer surplus of all market points throughout the city.

Here we set the marginal cost of the efficient firm to 0. We do so after concluding that the value of marginal costs is not indifferent as in the case of Bertrand. Nevertheless, this assumption is made without loss of generality, as changes can be only found in the magnitude of the resulting values. We start by calculating the consumer surplus, which will later be added with the firms' profits, totaling the objective function of the social planner, the social welfare. Then, after finding the optimal solution for the social planner, we seek a further explanation for that behavior, by computing the optimal solution when the social planner is maximizing only the firms' profits or only the consumer welfare. Then, we check if the social planner is interested in removing the less efficient firm from the market imposing a monopoly instead of a duopoly market structure. The consumer surplus in each market point is given by the following expression:

$$CS_x(x_1, x_2) = \frac{(c_1 - 200 + t|x_1 - x| + t|x_2 - x|)^2}{18} \tag{5.3}$$

Then, consumer surplus is calculated using a definite integral, similarly to the one we have presented in the previous subsection: The integral must be separated in three parts, which correspond to three different functions for the consumer surpluses in a given point, since the absolute value expression changes given the positioning of firm 1, firm 2 and the market point we are calculating. After summing the consumer surplus with the profits, which are represented by equations 5.1 and 5.3 and after replacing c_2

with 0, we have the expression for the total social welfare of the city, dependent on the dimension of each market, the unit transportation cost and the location choice of both firms. This expression is given by:

$$SW(x_1,x_2)=(\frac{7t}{27})tx_1^3+(\frac{22c_1-800-14tx_2+11t}{18})tx_1^2+(\frac{400+7tx_2^2-11c_1-7tx_2-2t}{9})tx_1+$$
$$+\frac{-2t^2x_2-7tc_1x_2^2+7tc_1x_2+2tc_1-400tx_2^2+400tx_2-400t-400c_1+40000}{9}+$$
$$+\frac{11t^2x_2^2+11c_1^2}{18}+\frac{-7t^2x_2^3+4t^2}{27}$$

Matsumura and Shimizu (2005) conclude that, in the case of a uniform distribution of the consumers, the locations that maximize social welfare are the same that firms choose in the location stage, that is, agglomeration at the city center. In the previous section we proved that the difference between marginal costs of production is irrelevant to the optimal location decision. However, is central agglomeration still optimal when concerning social welfare? Considering the problem of maximizing social welfare with respect for both location variables, we solve the system of equations that satisfy the first-order conditions, given by:

$$\frac{\delta SW}{\delta x_1}=-\frac{t}{9}(2t+11c_1+800x_1-11tx_1+7tx_2-22c_1x_1-7tx_1^2-7tx_2^2+14tx_1x_2-400)=0$$

$$\frac{\delta SW}{\delta x_2}=-\frac{t}{9}(2t-7c_1+800x_2+7tx_1-11tx_2+14c_1x_2+7tx_1^2+7tx_2^2-14tx_1x_2-400)=0$$

By solving these first-order conditions, we verify if the second-order conditions are fulfilled. We arrive at two candidates for maximal. However, for one of the candidates we have found that both the restrictions $x_1 \leq x_2$ and the fact that both firms should be inside the linear city were not respected, therefore we only kept one candidate, which is central agglomeration. The second-order conditions for this candidate are given by:

$$\frac{\delta^2 SW}{\delta^2 x_1}<0 \Leftrightarrow \frac{t}{9}(11t+22c_1-800)<0 \Leftrightarrow t<\frac{800}{11}-2c_1$$

$$\frac{\delta^2 SW}{\delta^2 x_2} < 0 \Leftrightarrow \frac{t}{9}(11t - 14c_1 - 800) < 0 \Leftrightarrow t < \frac{800 + 14c_1}{11}$$

And:

$$(\frac{\delta^2 SW}{\delta^2 x_1} * \frac{\delta^2 SW}{\delta^2 x_2}) - (\frac{\delta^2 SW}{\delta x_2 \delta x_1} * \frac{\delta^2 SW}{\delta x_1 \delta x_2}) > 0 \Leftrightarrow$$

$$\Leftrightarrow \frac{4t^2}{81}(18t^2 + 22tc_1 - 4400t - 77c_1^2 - 1600c_1 + 160000) > 0 \Leftrightarrow$$

$$\Leftrightarrow t < \frac{1100}{9} - \frac{\sqrt{1507c_1^2 - 19600c_1 + 1960000} - 11c_1}{18}$$

Of the three restrictions the second-order conditions impose, only the last is binding since if the third is verified, all others are verified. After representing the condition in order of the marginal cost of firm 1, we reach Proposition 5.2:

Proposition 5.2: Central Agglomeration maximizes social welfare if and only if $c_1 < c_1^{'}$ with $c_1^{'} = \frac{11t - 800 + \sqrt{1507t^2 - 356400t + 12960000}}{77}$, that is, if the inefficient firm is not too inefficient.

Proof: Result of the first and second-order conditions of the total welfare maximization.

The second-order conditions for the central agglomeration solution hold only if $c_1 < c_1^{'}$, that is, if the disadvantage of firm 1 becomes too high, the central agglomeration result ceases to be optimal from the social welfare point of view.[2] Hence, central agglomeration only holds as an optimal result if the inefficient firm is not too inefficient. For higher values of the marginal cost of the inefficient firm, no solution is found.[3]

[2] There is another solution that satisfies the second-order conditions for all the feasible value of the parameters, but the location pattern either falls out of the linear city or to the constraint $x_1 > x_2$, which does not reflect the true profits of the firms, due to the way the definite integral is constructed.

[3] Since we are maximizing a continuous function in a compact set, there must be a solution. However, we aren't able to find it for all values for c_1. We think that the solution involves, naturally, locating the inefficient firm in one of the extreme of the market, such that it sells less quantities possible. However, we are not able to prove that so far.

After replacing the central agglomeration result in the social welfare result, it becomes:

$$SW = \frac{2t^2 + 6tc_1 - 1200t + 33c_1^2 - 2400 + 240000}{54} \tag{5.3}$$

To better understand this location decision by the social planner, we separate the analysis of the optimal decision of the social planner when maximizing each of the components of the total social welfare. First we consider the consumer surplus and secondly the profits of both firms. This way, we intend to analyze what is the effect of the different components in the optimal decision taken by the social planner. Note that the maximization of the producer surplus is equivalent to considering both firms colluding in the location stage.

5.4.1.2. Consumer Surplus

The total consumer surplus, which is obtained after summing the consumer surplus of all the market points using the definite integral similarly to what was used when determining the profits of firms, is given by:

$$CS(x_1, x_2) = (-\frac{t^2}{27})x_1^3 + (\frac{tc_1 - 200t + t^2 x_2}{9} + \frac{t^2}{18})x_1^2 + (\frac{200t - tx_2^2 - tc_1 + t^2 x_2 - t^2}{9})x_1 + \\ + \frac{-t^2 x^2 + tc_1 x_2^2 - tc_1 x_2 + tc_1 - 200tx_2^2 + 200tx_2 - 200t - 200c_1 + 20000}{9} + \frac{t^2 x_2^2 + c_1^2}{18} + \\ + \frac{t^2 x_2^3 + 2t^2}{27}$$

We compute the derivative of the consumer surplus and we equalize to zero, therefore satisfying the first-order conditions. Two solutions meet the first-order conditions, namely $(x_1 = \frac{1}{2}; x_2 = \frac{1}{2})$ and $(x_1 = \frac{t + c_1 - 200}{2t}; x_2 = \frac{t + c_1 - 200}{2t})$. The second-order conditions confirm that the first solution is satisfied for all the values of the parameters, while the second solution is never satisfied. Therefore, we arrive at Proposition 5.3.

Proposition 5.3: Irrespectively of the marginal cost of production of the inefficient firm, consumer surplus is maximized when both firms agglomerate at the center.

Proof: First and second-order conditions of the consumer welfare maximization problem.

A similar result is presented in Matsumura and Shimizu (2005), with central agglomeration being an optimal result to consumer welfare independently of the consumer distribution in the city. The intuition of this result is that the efficiency of transportation of both firms is the highest when located at the center, so that they provide goods all over the city with lower costs overall when located there, which maximizes the quantities sold. This component of the social welfare does not contribute to the explanation on why the second-order conditions of the maximization problem cease to verify for a large difference in the marginal costs of both firms, since the consumer welfare maximization is an agglomerative force to the decision of the social planner. Then, the dispersion force must be the maximization of firms' profits.

5.4.1.3 Producer Surplus

When maximizing the producer surplus, we have that the central agglomeration result appears again as a candidate for a maximum. The producer surplus is given by the sum of the profits of both firms after they have decided on the production of their quantities. The expression is:

$$\Pi_1(x_1,x_2)+\Pi_2(x_1,x_2)=\frac{8t^2}{27}x_1^3+\left(\frac{10tc_1-200t-8t^2x_2+5t^2}{9}\right)x_1^2+\left(\frac{200t+8t^2x_2^2}{9}+\right.$$
$$\left.+\frac{-10tc_1-8t^2x_2-t^2}{9}\right)x_1+\frac{5t^2x_2^2-t^2x_2+2t^2-8tc_1x_2^2+8tc_1x_2+tc_1-200tx_2^2}{9}+$$
$$+\frac{200tx^2-200t+5c_1^2-200c_1+20000}{9}+\frac{-8t^2x_2^3+2t^2}{27}$$

After seeking for the first-order conditions for the maximization with the location variables, similarly to the maximization of the total of the social surplus, we arrive at two candidates for maximum/minimum. However, in one of these solutions we have found, both the restrictions $x_1 \le x_2$ and the fact that both firms should be inside the linear city were not respected, therefore we only kept the first solution, which was central agglomeration. The second order-conditions are given by:

$$\frac{\delta^2(\Pi_1(x_1,x_2)+\Pi_2(x_1,x_2))}{\delta^2 x_1} < 0 \Leftrightarrow \frac{t}{9}(10t+20c_1-400) < 0 \Leftrightarrow t < 40-2c_1$$

$$\frac{\delta^2(\Pi_1(x_1,x_2)+\Pi_2(x_1,x_2))}{\delta^2 x_2} < 0 \Leftrightarrow \frac{t}{9}(10t-16c_1-400) < 0 \Leftrightarrow t < 40+\frac{8}{5}c_1$$

And:

$$(\frac{\delta^2(\Pi_1+\Pi_2)}{\delta^2 x_1} * \frac{\delta^2(\Pi_1+\Pi_2)}{\delta^2 x_2}) - (\frac{\delta^2(\Pi_1+\Pi_2)}{\delta x_2 \delta x_1} * \frac{\delta^2(\Pi_1+\Pi_2)}{\delta x_1 \delta x_2}) > 0 \Leftrightarrow$$
$$\Leftrightarrow \frac{4t^2}{81}(9t^2+10tc_1-2000t-80c_1^2-400c_1+40000) > 0 \Leftrightarrow$$
$$\Leftrightarrow t < \frac{1000}{9} - \frac{\sqrt{5}\sqrt{149c_1^2-1280c_1+128000}-5c_1}{9}$$

Only the last restriction is binding, given all the others are satisfied when the last one is. Organizing the restriction with respect to the marginal cost of the inefficient firm, we arrive at proposition 5.4.

Proposition 5.4: Central agglomeration maximizes producer surplus if $c_1 < c_1^{''}$ with

$$c_1^{''} = \frac{5t-200+\sqrt{5}\sqrt{149t^2-32400t+648000}}{80}.$$

Proof: first and second-order conditions of the producer surplus maximization problem.

Hence, only if the difference between marginal costs of both firms is not that big, the social planner would be interested in agglomerating the firms in the city center. Even after considering that the dimension of the market is big enough such that both firms serve the market, the central agglomeration result may not be a maximum for firms' profits.[4] This result happens because when one firm becomes too inefficient, the social planner prefers to remove that firm from the city central position such that it produces the fewest quantities possible. [5]

[4] The problem found in finding a second solution for the remaining feasible values of the model is similar to the total social welfare case.

[5] Note that the optimal solution found for the social planner is a second best, since the social planner cannot control the quantities produced by each firm. If that was the case, then the optimal solution (after a

If the inefficient firm stays at the center, it produces quantities to all locations in the city at a lower margin than the efficient firm. Moreover, it forces the efficient firm to lower its output, which reduces the sum of their profits. However, we are not able to find what would be the optimal choice of the social planner for higher values of the inefficient firm's marginal cost. Our guess, based on intuition, is that it would be better to push the inefficient firm to one of the market extremes, therefore letting the efficient firm enjoy higher profits on the market end that is farther from the inefficient firm; and most importantly, pushing the inefficient firm implies having the efficient firm producing higher quantities comparing with the case where both firms are located at the center, which is naturally more efficient given the lower cost of production of this firm.

5.4.1.4. Discussion

The previous result sheds some light on the result for social welfare as a whole: what motivates the social planner to make firms leave the central location is the component associated with the profits of the firms and not the consumer surplus. Analyzing the effects of changing the marginal cost of the inefficient firm on total social welfare, given that firms are agglomerated at the city center, allows a better explanation of the intuition behind this result.

$$\frac{\delta SW}{\delta c_1}\left(x_1=\frac{1}{2}, x_2=\frac{1}{2}\right)=\frac{t+11c_1-400}{9}$$

So, if c_1 becomes sufficiently big ($c_1 > \frac{400-t}{11}$), the derivative becomes positive, which means that a decrease in the marginal cost of the inefficient firm decreases social welfare. This strange result happens because at a certain point, the firm is so inefficient that by producing the good it lowers at a higher extent the profits of the efficient firm. Imagine now a small decrease in the marginal cost of the inefficient firm. It has two positive effects. A first effect is that the inefficient firm is able to produce more goods, which leads to an expansion of its profit. A second effect is an expansion of the total quantity and a reduction of the price in all markets, which expands the consumer surplus. However, there is also a negative effect: The efficient firm is forced

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certain threshold for the difference in the marginal costs between both firms) would be asking the inefficient firm to stop its activity, therefore letting the efficient firm producing all quantities.

to reduce its output, due to the expansion of the opponent's quantities. After $c_1 > \frac{400-t}{11}$, the negative effect becomes stronger than the sum of the positive effects. Nevertheless, this result is never attained for the solution range we were able to solve, since the central agglomeration location ceases to be optimal for a lower threshold value of $c_1 < \frac{5t - 200 + \sqrt{5}\sqrt{149t^2 - 32400t + 648000}}{80}$. Nonetheless, it shows how detrimental it is for the overall welfare to maintain the inefficient firm in the city center.

5.4.2. Optimal Market Structure

In Industrial Organization, it is typically assumed that having more than one firm is positive in terms of social welfare, since it expands consumer surplus more than the contraction in the producer surplus. This result is often associated with the analysis of non-spatial Cournot markets, when firms have the same marginal costs. However, as shown in the previous section, the presence of an inefficient firm might lead to lower producer surplus, which compromises the social welfare purposes. Is it possible that for too high levels of inefficiency the social planner prefers to remove the inefficient firm from the city, allowing for a monopoly of the most efficient firm? To answer this question, we compare the social welfare in the monopoly case with the one obtained a duopoly situation. Then, this analysis is equivalent to a three-stage model, where in the first stage the social planner chooses the market structure between monopoly and duopoly, in the second stage chooses the optimal location of firm(s) and in the third stage firms choose their quantities. Once again, we seek a second-best solution, since the social planner only controls the first two stages of the game.

Naturally, we assume that the monopolist is the efficient firm and therefore its marginal cost of production is set to 0, as in the previous subsection. By standard calculations we determine the quantities supplied by the monopolist to each market point in the city. Then, the profits and consumer surplus for each market point are added using a definite integral. The resulting expression is the social welfare, which depends on the location choice, dimension of the market and transportation costs. The social planner then chooses the optimal location for the monopolist, such that it maximizes social welfare. This location is again the central location ($x_m = 0.5$), since it minimizes the transportation costs.[6] The social welfare in the case of the monopolist firm is given by:

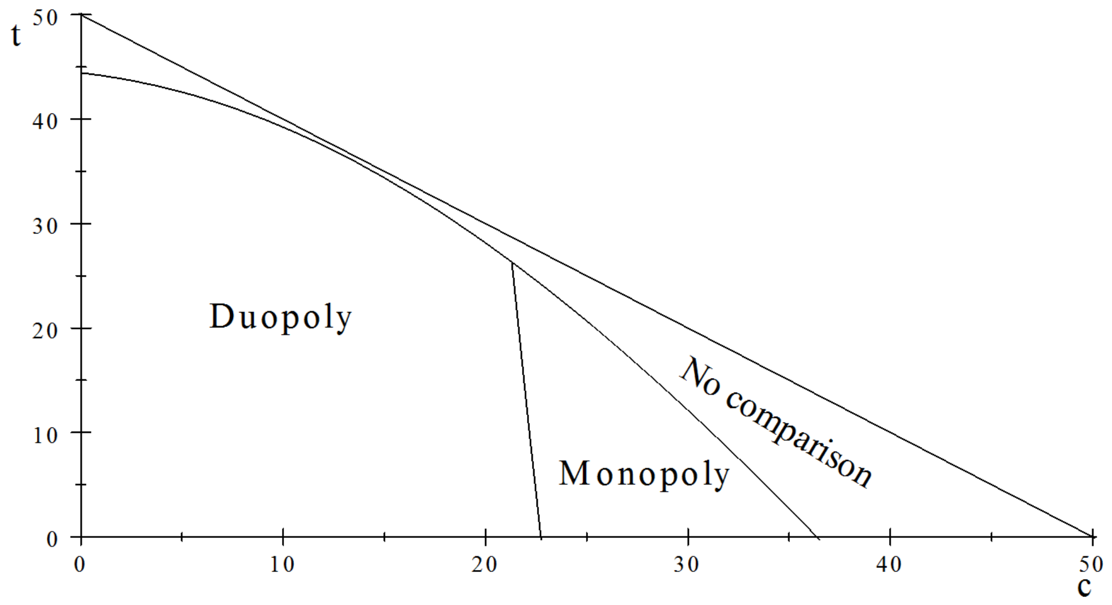
$$SW = \frac{t^2}{32} - \frac{75}{4}t + 3750$$

The second order conditions are always met. We have to compare this value with the social welfare in the case of a duopoly, which is expressed in equation 5.3. By doing the difference between both expressions, we conclude that the social planner prefers to let only the efficient firm operating in the market when $c_1 > c_1^{''''}$, with $c_1^{''''} = \frac{-12t - 4800 + \sqrt{3}\sqrt{-7t^2 - 5400t + 1080000}}{132}$, that is, when c_1 becomes sufficiently high. Since we have not found the solution for the duopoly situation outside the central agglomeration one, we cannot compare that situation with the one from the monopoly. However, we are led to believe that the more inefficient the second firm is, the better it would be for the monopoly situation in terms of social welfare. Still, we rigorously complete Figure 5.1 with only what we have proved so far.

In the previous subsection, we have determined that the central agglomeration solution holds while $c_1 < c_1^{'} = \frac{11t - 800 + \sqrt{1507t^2 - 356400t + 12960000}}{77}$. Also, we have to consider the restriction that both firms serve the entirety of the market, given by $c_1 < 50 - t$. Figure 5.1 shows the relationship between the three expressions and the market structure solution of the three-stage game.

[6] Notice that if the monopolist maximizes his profits the choice of location would be the same.

Figure 5.1 – Solution of the Entry Stage



We can see that when both firms are obliged to serve all points in the market, there are three different regions of equilibrium outcomes. When the marginal cost of the inefficient firm is low, duopoly is the preferred situation for the social planner. As the marginal cost rises, monopoly becomes the preferred outcome. For higher values of the marginal cost, we are not able to find the optimal values of location in the duopoly case, so the comparison between both cases is not possible.

Proposition 5.5: For the values where we have found a solution for the social welfare maximization problem in a duopoly situation, if the marginal cost of the inefficient firm becomes higher than $c_1^{'''}$ with $c_1^{'''} = \frac{-12t - 4800 + \sqrt{3}\sqrt{-7t^2 - 5400t + 1080000}}{132}$, the regulator prefers to remove it from the market.

Proof: Straight comparison of the social welfare results for the monopoly and duopoly situation.

Proposition 5.5 also occurs in the case of a non-spatial Cournot. In that case, the social planner prefers to remove the inefficient firm if $c_1 > \frac{250}{11}$. This threshold can also be found in our results after replacing t=0. The effect of transportation costs in this threshold is decreasing, that is, an increase in t lowers the threshold value for the inefficient firm to be removed, so the higher the transportation costs, the lower is the contribution of an inefficient firm to the social welfare. Summing up from the perspective of the social planner, the higher the transportation costs in the economy, the harder it is to sustain an inefficient firm in the market. However, the effect of the transportation costs is relatively small compared to the effect of the marginal cost of the inefficiency firm. That result is clear in figure 5.1, when looking at the condition in which the social welfare of both monopoly and duopoly are equal: The threshold is almost vertical, meaning that a change in the transportation cost does not affect that much the amount of inefficiency needed for the social planner to remove the inefficient firm. Therefore, we can conclude that in the spatial Cournot framework developed by Anderson and Neven (1991), the difference between the spatial and the non-spatial case is not very significant. Transportation costs have only a small effect in the firms' profits and in the relationships between both firms. Moreover, the assumption that both firms have to sell in all points of the market (which is an assumption that mathematically is of the highest importance, allowing the use of an integral to quickly sum up all the market points) is very restrictive in terms of the results, since firms are not allowed to have local monopolies and are therefore "forced" to be in the city center in order to save on overall transportation costs. However, breaking this assumption is not a straightforward task in mathematical terms, with Benassi et al. (2007) being the best example.

5.5. Conclusion

In this paper, we analyze the two-stage quantity-location game when firms have different production costs. We solve the profit maximization problem by both firms, as well as the social planner problem both when it controls the firms' locations and when, additionally, it decides whether the inefficient firm can enter the market.

We conclude that the central agglomeration result of Anderson and Neven (1991) and Hamilton et al. (1989) still holds for any difference in the marginal costs of both firms. In terms of social welfare, we conclude that the central agglomeration result found by Matsumura and Shimizu (2005) only holds if the difference between the marginal costs of both firms is not too high. The dispersion force comes from the producer surplus, since the consumer welfare is always maximized with firms agglomerating in the center of the city. When allowing the social planner to prevent the inefficient firm from entering the market, we conclude that when we are able to find a solution for duopoly, the optimal location outcome implies that firms are always located in the center of the city, either in monopoly or in duopoly. Moreover, we conclude that the social planner prefers a monopoly when one of the firms is sufficiently inefficient, and that higher unit transportation costs facilitate (at a small scale comparing with the inefficiency levels of the other firm) the existence of monopolies for the social planner.

Future research should focus on explaining the reasons why firms would differentiate in terms of marginal costs. That could occur, for instance, when analyzing location-dependent marginal costs in the city, analyzing R&D investments decisions of firms, analyzing models where firms are uncertain about their marginal costs, studying incumbent vs. entrant problems with different marginal costs, studying supplier/retailer relationships, and many more.

6. Location Decisions in a Natural Resource Model of Cournot Competition

6.1. Introduction

In this chapter we are interested in assessing the location choice of two competing firms that need to acquire an essential input which is set in a specific location. This input is costly to transport from its extraction/location point to the transforming industry. Moreover, we assume that the sale of the natural resource is controlled by a monopolist that is not related with any of the two firms, which introduces another component to the strategy of the location decision process. As an example of industries that share part of these location problems, one can think of products that are dependent on other commodities, such as iron or wood, whose final goods have to be transported to cities after being produced. Another interpretation would be of a location resource that can only be acquired through one transportation breaking point that is being controlled by an intermediary. For instance, acquired raw materials stored in ports, whose seller faces a local monopoly towards transforming industries.

We have further extended the model to allow the owner of the downstream firms to delegate the quantity competition decisions to a manager. Delegation is a relevant topic in the firms' strategic behavior, since it allows the owner to follow a different strategy other than profit maximization with credibility – that is, hiring a manager with different objectives commits the owner to a policy that the opponent knows that is credible. The most notable example on the importance of delegation as a way to increase the firms' profits is shown in a typical Cournot framework, where one firm using delegation can outperform a rival firm that does not use it, by being more aggressive and supplying a higher quantity, mimicking the Stackelberg duopoly result. The owner offers an incentive for the manager to be more aggressive, and that results in a higher profit for the firm (e.g. Vickers, 1985; Fershtman and Judd, 1987). However, if both firms are allowed the possibility to use delegation, they use it and the result is a prisoner's dilemma in which the firms choose to get a manager, and get worse off in terms of profits, supplying a higher quantity comparing to the normal competition case.

We deal with the question of location using the linear city framework created by Hotelling (1929). We use an adaptation of the model created to analyze competition by quantities, developed by Hamilton et al. (1989) and Anderson and Neven (1991).

We conclude that when the unit input transportation cost is equal to zero, firms optimally locate in the city center, but as soon as this cost rises, firms move in a quasi-linear fashion towards the extreme of the city, where the input resource is located in. Firms stay in the natural resource position when the unit input transportations costs are higher or equal than the output ones. Moreover, firms are always located in the same position, therefore we conclude that firms are agglomerated whatever the value of the unit input transportation cost. In our setting, the upstream firm earns higher profits than the downstream firms jointly, due to its monopoly position and due to the fact it does not pay transportation costs. Furthermore, if the social planner is allowed to choose the locations of the firms, the chosen locations are practically similar to the ones chosen by the firms.

For the delegation results, we conclude that similarly to the no delegation case, firms move from the city center to the location of the natural resource, but at a non-linear fashion with respect to the unit input transportation costs: for the same cost value, firms are closer to the natural resource. Additionally, downstream firms have lower profits, while the upstream firm hugely benefits from this change, mainly due to an increase in the input quantities demanded.

This chapter is organized as follows: the next subsection presents the theoretical background of the article; subsection 3 details the assumptions of the model and solves the game attached to our problem; subsection 4 analyzes the results of the model; and subsection 5 concludes.

6.2. Theoretical Background

The original game involving a location stage in the linear city concept is due to Hotelling (1929). In a two-stage game, firms first decide their location in the linear city and then both firms set the prices simultaneously. Hotelling concluded that firms would locate in the city center, given his assumptions about the market. Fifty years later, d'Aspremont et al. (1979) revolutionized the field by assuming quadratic instead of linear transportation costs, which eliminated discontinuities in demand that the original Hotelling model had. D'Aspremont and others concluded that firms would prefer to be located one at each extreme of the market, in order to soften price competition. This original result, allied with the new mathematical tractability of the model, originated a significant expansion in the field through the late 80s and 90s, in which many authors tried to restore the original minimum differentiation result from Hotelling, and as it is shown in the chapter 2 of this manuscript.

Amongst the immense literature on the subject, some papers introduce valuable insights regarding location theory. Ziss (1993) allowed for different marginal costs of production, and concluded that if one firm has a significant advantage over the other, location equilibrium ceases to exist. Anderson et al. (1997) changed the assumption of uniform distribution of consumers in the linear city, and concluded that firms may have asymmetric location configurations, even if the distribution of consumers is symmetric towards the center and firms are homogenous. Irmen and Thisse (1998) considered a market with n dimensions in which firms can differentiate, and conclude that if one dimension is sufficiently more important than the others, then firms choose to differentiate only on that dimension in order to soften price competition. Firms decide to remain homogenous on all other dimensions, in a result that mirrors both Hotelling and d'Aspremont solutions. In a paper that has a similar idea to ours, Aiura and Sato (2008) consider a natural resource that is present in the middle of the linear city, and which firms have to transport to their location. Lai and Tabuchi (2012) consider the possibility that firms may need more than input to produce the good, and study duopoly competition when inputs are exogenously distributed around the city. However, these models do not consider price competition, and ignore the possibility that the rights for the exploitation of the natural resource may be owned by one or more firms, so their results are not comparable with ours.

This chapter is about quantity competition in the spatial setting. Anderson and Neven (1991) firstly formulated a two-stage game similar to Hotelling, but the price stage is replaced by a quantity stage, and some assumptions were changed in order for this framework to be tractable. The authors conclude that when the demand is linear, and transportation costs are convex and are not very high, such that firms are able to sell the product in all points of the market, agglomeration in the city center occurs. Gupta et al. (1997) change consumer density functions as well, and conclude that agglomeration occurs if the population density is sufficiently "thick" in all market points of the city. Mayer (2000) extends this analysis by introducing different production costs along the city, and concludes that when the convexity of the production cost function holds, firms agglomerate between the minimum cost location and the city center. This article by Mayer has some similarities to ours, as we develop further on.

Although we do not investigate directly the vertical relationships between firms (that is, the possibility of integration or foreclosure, see Rey and Tirole (2007) for a review), we do consider the existence of an upstream firm that sells inputs to downstream firms. We introduce that analysis in the spatial framework of Hotelling which, to our knowledge, is limited to several papers[1]: An early approach is by Gupta et al. (1994), in which an upstream firm serves *n* downstream firms, which have to choose their location in the linear city. Matsushima (2004) analyses the location decision of downstream firms by fixing the upstream firm's location at the extremes; Liang and Mai (2006) consider the case with two inputs and two downstream producers, and study the possibility of subcontracting, given that one of the upstream firms produces cheaper than its rivals; Beladi et al. (2008; 2010) formulate vertical relationships but in a specific downstream market where each firms makes a variety that cannot be made by other firm, and find the optimal location decisions of firms both in the simultaneous and in the sequential case; Matsushima (2009) analyses the effects of integration in the location outcome of firms, and extends the endogenous location decision to both upstream and downstream firms; Kouranti and Vettas (2010) compare the location outcomes depending on when the upstream and downstream firms choose their locations, and conclude that when upstream firms choose their first, firms location becomes closer to the center which intensifies competition; Matsushima and Mizuno (2012) conclude that firms after integrating locate farther from the opponent, and that larger firms are more likely to integrate than smaller ones. However, all these approaches involve price competition between firms.

[1] We are referring to articles in which the location choice of downstream firms is endogenous, not those papers who consider horizontal differentiation but in which location of the firms is exogenously determined (e.g. Colangelo (1995); Chen (2001); Hackner (2003)).

Mukherjee and Zanchettin (2012), on the other hand, analyses quantity competition instead of price competition. However, the authors do not work on the Hotelling framework of differentiation, and model product differentiation as the degree of substitutability. So, our paper is a rather novel approach to the existence of upstream firms towards the equilibrium location outcome of the downstream firms.

Moreover, our problem is also related to the classical problem of industrial locations researched by Weber (1929). Firms have to decide where to locate given the position of the raw materials and the markets. However, our problem differs in the sense that the market is a continuum of points instead of a single location, and the raw material's extraction is controlled by a monopolist.

Regarding delegation, in the literature there are usually three types of incentive contracts between the owner and the manager that are analyzed in this context. The incentive contract used first, and probably the most used in the literature is the linear combination between firms' profits and revenues, firstly introduced by Vickers (1985) and used by, for instance, in Fershtman (1985); Fershtman and Judd (1987); Sklivas (1987); Scymanski (1994); Ishibashi (2001); Huck et al. (2004) and Hoernig (2012). Another type of incentive contract combines the profits of the firm with its relative performance in the market, that is, a combination between the profits of the firms and the profits of its opponents. Aggarwal and Samwick (1999) formulate a model using these incentives, along with Miller and Pazgal (2001; 2002; 2005). Other type of incentive contract used is related with the market share, since it combines the profits of the firms with the market share the firm obtains. This contract is used in Jansen et al. (2007), as well as in Ritz (2008).

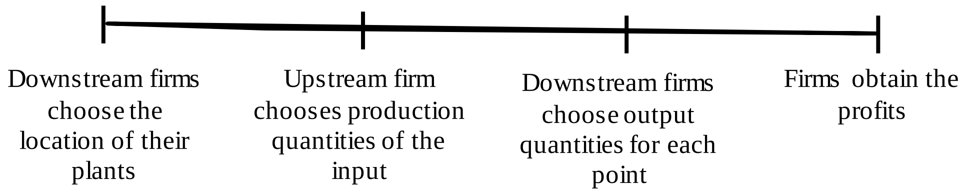
A few attempts have been made regarding the introduction of delegation in spatial competition models. Barceno-Ruiz and Casado-Izaga (2005) delegate the price decisions in the standard d'Aspremont framework, and conclude that delegation is profitable for both firms, as it results on further differentiation comparing to the non-delegation case. Other approaches include Liang et al. (2011) and Matsumura and Matsushima (2012). While in the model present in the former, firm owners delegated the latest stage with linear transportation costs for both Bertrand and Cournot, and concluding that the locations depended crucially on the reservation price of consumers, the latter did not focus on the optimal location decision problem of firms.

6.3. The Model

There are two downstream firms which compete in a market that is spatially differentiated. The market is composed by a continuum of markets distributed evenly in a linear city of length [0,L], and we assume that L=1 without loss of generality. Each location in the linear city is assumed to have an inverse demand function, which is linear, and is defined by P=10-Q, similarly to Anderson and Neven (1991). In order to being able to produce one unit of the good, downstream firms must acquire one unit of a natural resource (fixed coefficients technology). We assume that the natural resource is located in the extreme of the linear market, as it would be the case when thinking of raw materials such as wood or iron. We assume that one upstream firm, not related with the downstream firms, managed to get the full extraction rights of this resource, having therefore a monopoly position. This firm is located in the same place as the natural resource, that is, in the extraction point.

Downstream firms have to transport the raw resource from the extraction point to their production plant in order to be able to produce the output. After the good is produced, these firms transport the goods throughout the market points of the linear city, in order to sell it to the consumers. The transportation costs are assumed to be linear with respect to the different points of the product space, and the unit transportation cost of the natural resource is given by t, while the unit transportation cost (given by T) of the output good is fixed to 1. The value of output transportation costs is constrained by the dimension of the market, since it allows both firms to sell in all points of the city, independently of any combination of locations that may arise, which is an assumption that is common in the literature of Cournot Spatial Competition (see Anderson and Neven (1991) for an example). Both downstream firms sell in all the markets if the sum of both unit transportation costs are relatively small, that is, if $T + t < \frac{2}{7}A$, where t and T are the transportation costs and A the dimension of the market.[2] Downstream firms are assumed to bear both transportation costs, that is, the transportation costs of bringing the natural resource to the production plant, and the transportation costs of the distribution of goods through the markets in the city. All three firms are assumed, without loss of generality, to have no marginal costs of either extracting the natural resource, or transforming the natural resource into an output good. The timing of the game follows.

Figure 6.1 – Timing of the game



In the first stage, the downstream firms choose simultaneously their location x_1 and x_2 in the linear city, which are restricted to be inside of it, that is, x_1 and $x_2 \in [0,1]$. We assume without loss of generality that firm 1 will never choose a location "to the right" of firm 2, that is, $x_1 \geq x_2$. In the second stage, the upstream firm chooses the quantities to sell of their input good. We assume, similarly to Clementi (2011), that the input price is formed due to a mechanism, in which "the downstream firms submit an aggregate input demand schedule to a walrasian auctioneer, while simultaneously the input provider submits his aggregate supply schedule. This auctioneer matches the input demand and supply and finds the market clearing price" (Clementi, 2011, p.6). Finally, in the third stage, firms choose simultaneously their quantity schedule, that is, firms choose the quantity they are going to supply to each market in the city. We seek a Perfect Subgame Nash Equilibrium, and we solve the game by backward induction.

6.3.1 Output Quantities Stage

First, we calculate the optimal quantity decision in each point of the city. Downstream firms will have the following profit in point x:

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[2] To obtain this condition, we test what are the minimum values for t, T and A for which a firm located in x=1 sells the good at the point x=0, where the input firm and the rival are located. If the firm can sell a positive quantity at x=0, then it can sell in all points of the market, since x=0 is the worst condition that a firm could face to sell its product. This condition is obtained similarly in the baseline case of Anderson and Neven (1991).

$$\prod_{i,x}(q_{i,x}, q_{j,x}, x_i) = (10 - q_{i,x} - q_{j,x} - I - (|x_i - x|) - tx_i)q_{i,x} \tag{6.1}$$

Where $\prod_{i,x}$ is the profit of firm *i* in point *x*, $q_{i,x}$ and $q_{j,x}$ are the quantities chosen by the firm and its opponent for point x, respectively, I is the input price set by the upstream firm, and the following parts of the equation 6.1 represent the transportation costs: The first part is the transportation cost of the output between the location in which the firm set its plant and the consumer in point x; while the second part is the transportation cost of the raw material to the location chosen by the firm in the first stage. Summing up, the unitary profit in each point is given by the price of the good minus the input price and both transportation costs.

Firm *i* maximizes its profit by choosing the optimal quantities for each point. After satisfying the first and second-order conditions of both firms, these quantities are given by:

$$q*_{i,x}(x_i, x_j) = \frac{|x_j - x|}{3} - 2\frac{|x_i - x|}{3} + \frac{10 - I - 2tx_i + tx_j}{3} \tag{6.2}$$

Where $q*_{i,x}$ is the optimal quantity chosen by firm *i* on market point *x*. To obtain all the quantities supplied by the downstream firms, we have to sum the quantities offered to all the points. However, due to the existence of the absolute value for the output transportation costs, the integral has to be separated in three different parts, one for each of the combinations of points that are "at the left" of firm 1; "between" firm 1 and 2 and "at the right" of firm 2, in order to remove the absolute value from the integral expression (e.g. see Anderson and Neven, 1991).

$$Q_i(x_i, x_j) = \int_0^{x_1}(q*_{i,x})dx + \int_{x_1}^{x_2}(q*_{i,x})dx + \int_{x_2}^{1}(q*_{i,x})dx$$

After computing the integral, we obtain the following total quantity for firm i.

$$Q*_i(x_i, x_j) = \frac{2(x_i - x_i^2) + (x_j^2 - x_j)}{3} + \frac{(tx_j - 2tx_i - I)}{3} + \frac{19}{6} \tag{6.3}$$

6.3.2 Input Quantities Stage

If we sum the quantities demanded for both firms, we get the demand function of the upstream firm, depending on the location choice of the downstream firms, of the input transportation costs and of the input price. Solving the equation in order of the input price, we obtain the inverse demand function of the input, which is given by:

$$I = \frac{x_1 + x_2 - x_1^2 - x_2^2 - tx_1 - tx_2 - 3Q_u + 19}{2} \qquad (6.4)$$

Where Q_u is the quantity demanded of the input, obtained by summing both downstream firms' demand. The profit function of the upstream firm is simply given by multiplying the input price by the quantities sold, since the upstream does not pay any production or transportation cost. Maximizing it with respect to the quantity (FOC and SOC respected), the optimal quantity chosen by the upstream firm is:

$$Q_u* = \frac{x_1 + x_2 - x_1^2 - x_2^2 - tx_1 - tx_2 + 19}{6} \qquad (6.5)$$

Replacing Q_u* in the above inverse demand function (equation 6.4) we get the input price with respect to the input transportation cost and the location choice of both firms. Replacing the input price on the firms' demanded quantities (equation 6.3), we observe that this input price clears the market, since the quantities supplied equals the quantities demanded.

The profit of the upstream firm is therefore given by:

$$\Pi_u = \frac{(x_1 + x_2 - tx_1 - tx_2 - x_1^2 - x_2^2 + 19)^2}{24} \qquad (6.6)$$

6.3.3 Location Stage

After knowing what their input price will be, firms are now left with the decision of choosing where to locate in the linear city. The steps to solve the problem involve very long mathematical expressions. After calculating the firms' profit in each market point, the total profit of each firm is obtained by summing all market points using the three-step integral that was presented in the previous subsection. Then, the profit functions are maximized regarding the location variables x_1 and x_2.

The solution is given by:

$$x_1 = x_2 = \frac{7t+9}{18} + \theta + \frac{91t^2 + 42t - 771}{\theta}$$

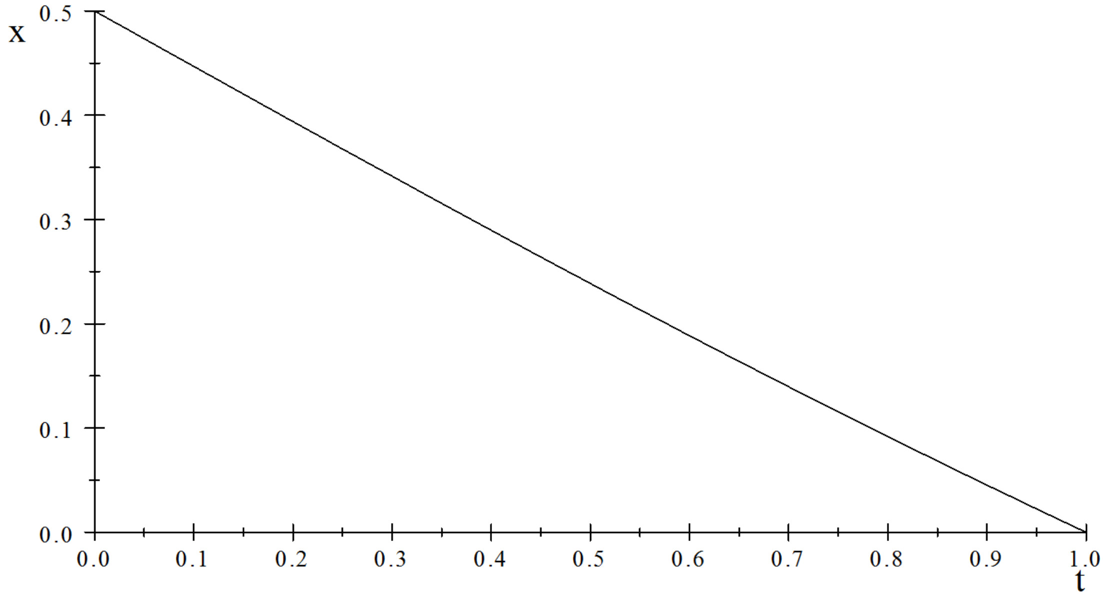
Where θ is an amount that depends non-linearly on t.[3] This solution satisfies the second-order conditions, and is better understandable in Figure 6.2. The location is equal to both firms, meaning that firms agglomerate whatever the cost of input transportation.

Proposition 6.1: Given that there is an upstream firm selling the input at the extreme of the market and that downstream firms bear all transportation costs in an extension of Anderson and Neven (1991), firms choose to agglomerate, independently of the value of the unit transportation cost.

Proof: Solution of the maximization problem of optimal location for the firms.

Figure 6.2 – Optimal location decision for both firms

[3] The value of θ is displayed in the appendix 6.1.



From figure 6.2 it is clear that as the unit input transportation costs increase, firms start to locate closer to the raw material location. If there are no input transportation costs, and firms have to pay only the cost for the acquisition of the input and the transportation cost of the output, both firms choose to locate in the middle of the linear city, since it is the one that minimizes transportation costs for a firm that is capable of selling in all points of the market (e.g. Anderson and Neven, 1991).[4]

This location result is in line with the work of Mayer (2000). The author extends the framework of Anderson and Neven (1991) by considering different production costs in different points of the city. Mayer (2000) concludes that the firms will agglomerate somewhere between the location that minimizes production cost (in our case, in the extreme where the input firm is located) and the location that minimizes transportation cost (the middle of the city). In fact, our model can be seen as an extension to Mayer, in the sense that it gives an endogenous explanation for the occurrence of different production costs that may occur in the city.

[4] Note however, that in spite of what it looks like in Figure 6.2, the optimal location is not linear with respect to the input transportation cost.

6.4. Discussion of the results

6.4.1 Location

So why does this happen to the optimal location choice of both firms? To better understand what pushes the firms to the edge of the market we proceed to the decomposition of different effects on firms' location decision.

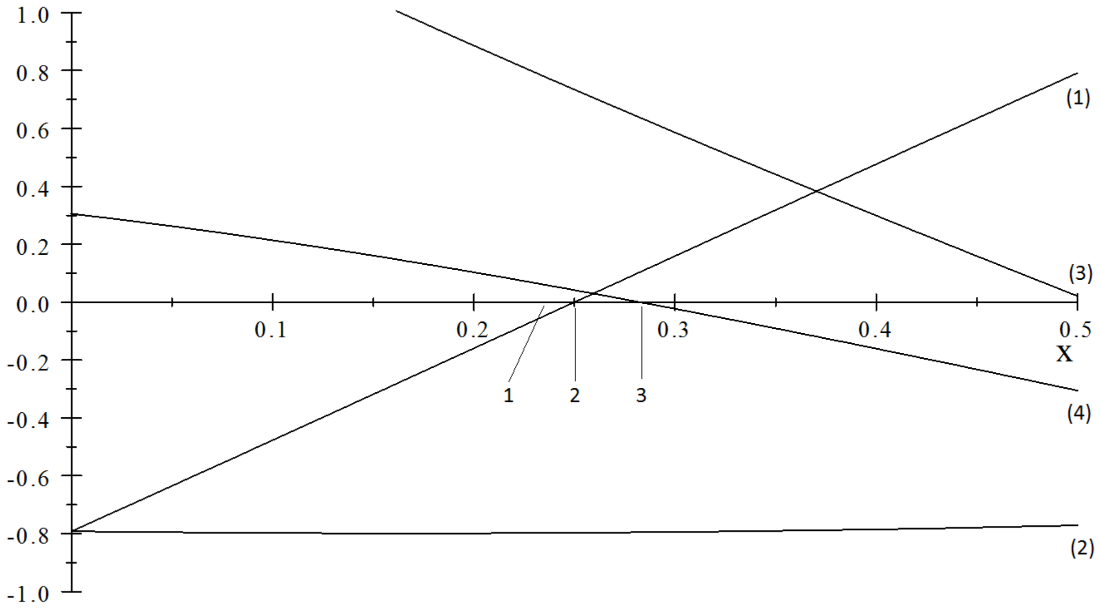
We divide the profits of the downstream firms between four components: 1) the input acquisition cost 2) The input transportation cost 3) The output transportation cost, and 4) the sales revenue. We represent the costs as their contribution to the profit function, and therefore these are negative throughout the entire domain. We do the analysis knowing what the optimal decisions of firms are at the input and output quantities stage, to focus only on the location consequences. Moreover, since we already know beforehand that both firms will choose to locate in the same point of the market (proposition 6.1), we set $x_1 = x_2$ for this first part of the analysis.

We will look at the specific case where the unit input transportation cost is fixed (t=0.5). Then, we decompose the profit in its four components (each depending on x), and we calculate the derivative with respect the location for each case. These derivatives are presented on Table 6.1 and Figure 6.3.

Table 6.1 – The Four Components of the profit function at t=0.5

Component	Derivative of the profit component with respect to Location
Input Purchasing Cost(1)	$-\frac{1}{3}x^3+\frac{1}{4}x^2+\frac{25}{8}x-\frac{19}{24}$
Input Transportation Cost (2)	$\frac{1}{4}x^2-\frac{1}{12}x-\frac{19}{24}$
Output Transportation Cost (3)	$-\frac{2}{3}x^3+\frac{5}{4}x^2-\frac{7}{2}x+\frac{37}{24}$
Sales Revenue (4)	$\frac{2}{3}x^3-\frac{7}{6}x^2-\frac{29}{36}x+\frac{11}{36}$

Figure 6.3 – The derivatives of each component of the profit function of firm i with respect to location



(1) Input Purchasing cost; (2) Input Transportation cost; (3) Output transportation cost; (4) sales revenue

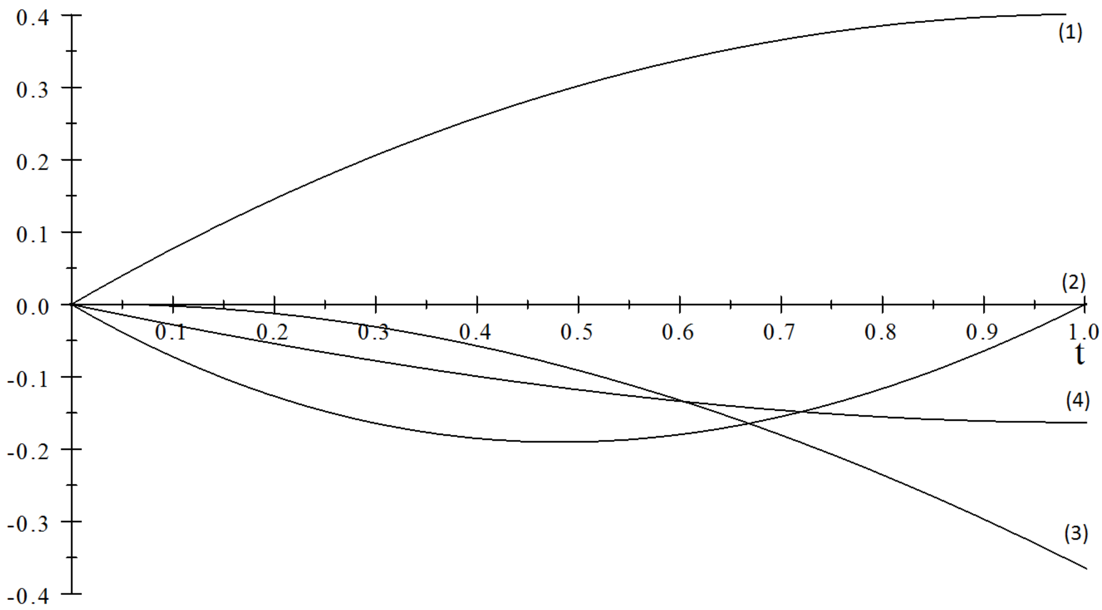
For t = 0.5 the optimal solution for the location of both firms is $x \cong 0.23865$ (point 1 in Figure 6.3). We can see that the four effects have very different magnitudes. At the optimal solution, which is represented by point 1, it is shown the pressure that both input and output transportation costs exert over the location of the firms, that is, when the firm is located in point 1, deviating from this point would have a bigger effect in terms of their output and input transportation costs, compared to the input purchasing costs and output sales. By moving towards its right, both firms are expected to save from output transportation costs at the expense of the input transportation costs. The effects of the input purchasing and of the sales revenue are smaller at the equilibrium point. Note that the sales revenue is not maximized at the city center as one would initially expect but it is maximal for a location near the left extreme (point 3 in Figure 6.3). This happens because the input purchasing price varies as well with location, which harms the sales possibilities of both firms in the downstream market. Note also the nature of the input purchasing costs: Given firms' current optimal position, these costs would be at their maximum at point 2. However, the costs decrease both at the left and at the right locations. This means that the downstream firms are located closer to the point where the upstream firm would choose them to be (input purchasing costs are equal to upstream profits).

The above analysis assumes t=0.5 . However, for different values of t the effects have the same nature although with significant changes in their magnitude: As t increases the input purchasing costs curve goes up, meaning that there is a pressure for firms to go to the center. The input transportation costs curve goes down, meaning that the incentive to locate closer to the upstream firm increases. The output transportation costs curve does not change, and the sales revenue curve goes down. The combination of these effects results in the outcome shown in figure 6.2.

6.4.2 Transportation Costs

An important perspective of this problem is to investigate the effects of changing the transport cost on the different components along the optimal solution path. If we fix the optimal location of firms for every value of the unit input transportation cost we get a different picture of the problem. Figure 6.4 presents the variation of the four components relatively to the point where the unit input transportation costs are equal to zero. Therefore, the figure details what happens to the four components as the unit input transportation cost increases: as soon as the unit input transportation cost increases, firms "travel" closer to the left extreme, and both transportation costs increase – The output costs increase since the firm is now on average more distant from its consumers, in spite of selling less quantities overall; the input costs increase directly due to the increase in its unit transportation costs. However, when the unit input costs become nearly half of the unit output costs, the total input transportation costs start to decrease, as the firms become relatively closer to natural resource location. The output transportation cost increases in an increasing fashion because further movements of the firms to the extreme of the market increases at an increasing rate the average distance to the consumers in the city, in spite of the decrease in total quantity sold. It is left to say that further increases in the unit input transportation costs do not bring any effects to any of the firms' decisions, leaving the results of the model unchanged.

Figure 6.4 – Profit variation of the 4 components along the optimal path



((1) Input Purchasing cost; (2) Input Transportation cost; (3) Output transportation cost; (4) sales revenue)

The sales revenue and the input purchasing costs evolve differently as well. The sales revenue decreases, and there are two effects determining that change: A stronger, negative effect which is the dislocation from the city center, which leads to an average sale of less quantities in each market point. And a positive, weaker, but surprising effect, which is the lowering of both firms "unit production costs", given by the sum of the input price with the input transportation price per unit (as shown in figure A.6.1 in the Appendix). These results lead us to investigate what happens to the input purchasing costs (or equivalently, to the upstream profit): We verify that these costs get lower with the increase of the unit transportation cost, since two negative effects occur: Downstream firms purchase less quantities and the input price decreases.[5] We can conclude when summing the 4 components, that the profit decreases with an increase in the unit input transportation costs.

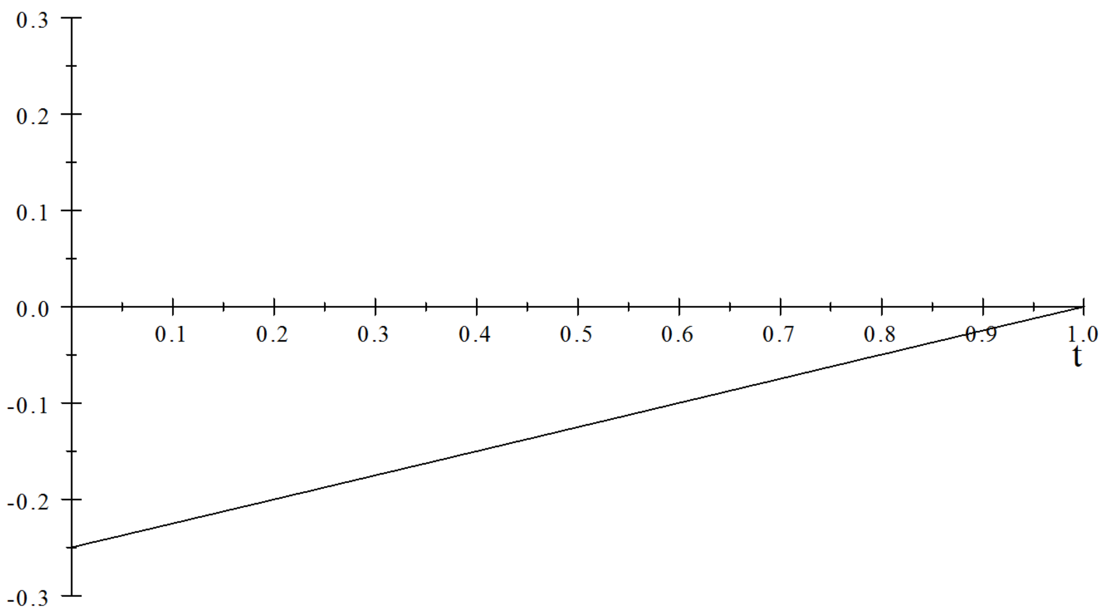
[5] Remember that costs are represented as a negative function. When the variation is negative, this means that the costs are increasing, and vice-versa.

Another interesting relationship is found in the consequences of an increase in the unit input transportation costs (along the optimal path) on the input price set by the upstream firm. After the occurrence of an increase in the unit input transportation cost, there are two effects affecting the input price determination: 1) the marginal cost of the downstream firm, which is an ambiguous effect, depending on the product between the unit input transportation cost itself and the distance to the natural resource. This effect is negative to the input prices for lower values of the unit transportation cost, but becomes positive after a certain threshold; and 2) the demand for the output good, which becomes lower since the firms become farther from the market center, which decreases the demand for the input good. The second effect is always larger than the first, which leads to a decrease in the input market price at a slower rate with the increase in the unit input transportation cost.

Proposition 6.2: Given the conditions of our model, the input price I is decreasing with an increase in the unit input transportation costs, until these become irrelevant ($t>1$).

Proof: We are unable to do the proof analytically due to the complicated mathematical expressions resulting from the location result. We present the solution numerically for all the values of parameter t. Figure 6.5 shows clearly that the derivative of the input price with respect to the unit input transportation costs is negative, except for the case where $t=1$.

Figure 6.5 – Derivative of the input price with respect to the unit input transportation costs



Then we conclude that our model is a particular endogenous case of the model of Mayer (2000), since the production costs in each location are a function of the location of both downstream firms, and can be divided in two parts: The first part is the input price, which is determined by the quantities that both firms are able to sell in the downstream market, as well as is determined on the value of the unit input transportation cost itself. The second part is the input transportation cost, which is paid by downstream firms, which depends on the unit input transportation cost, and the distance of the own firm relatively to the supplier.

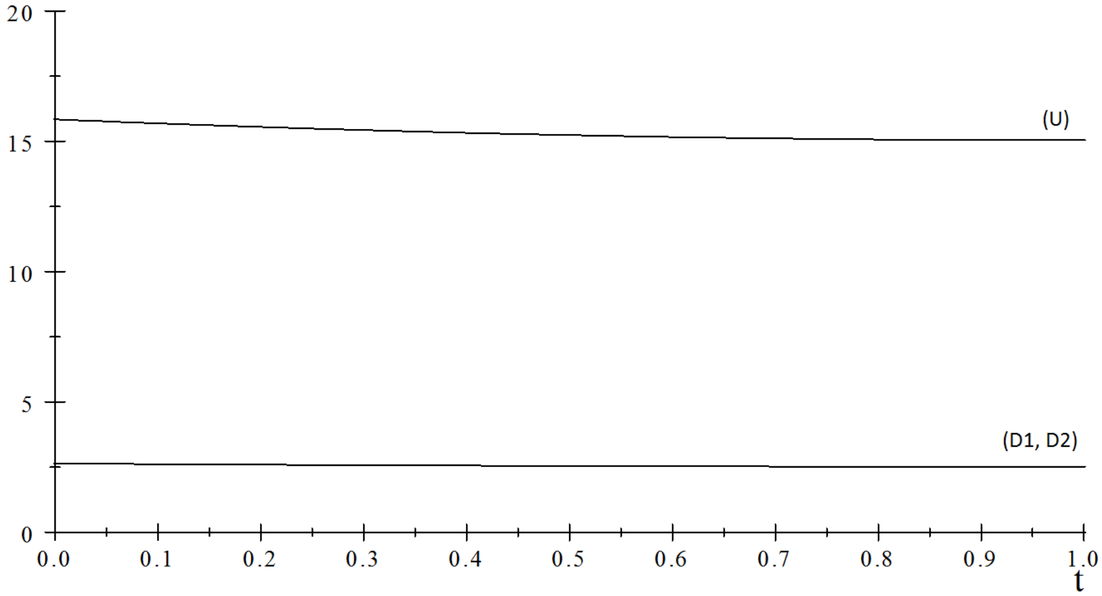
6.4.3 Profits

In this subsection we compare what happens to the profit of the firms if they were not allowed to change their location easily. Therefore, we separate the analysis between two cases: The short, and the long-run. In the former, firms are not allowed to change their location after seeing a change in the unit input transportation costs. Setting the location of the firms is a process that implies having a good amount of irreversible costs, which obviously impedes firms to adjust quickly to short-run changes. Therefore, the difference between short and long-run is that firms are unable to change their location setting in the short-run. We do this analysis to assess if there are significant differences in this framework between being and not being able to change the location of the firms, in terms of their profits.

6.4.3.1 Long run analysis

Naturally, the profit of all three firms decreases when facing higher unit input transportation costs (Figure 6.6). The upper line indicates the profit for the upstream firm, while the lower line indicates the profit of each of the downstream firms (equal for both firms). We conclude a bit surprisingly that the upstream firm suffers more with the increase of the input transport cost than the downstream firms, whose profit remains relatively unaltered. The surprise comes from the fact that the downstream firms are the ones bearing these transportation costs, so they could have been more affected by those.

Figure 6.6 – Long-Run Profit of the upstream and downstream firms depending on the input transportation cost.



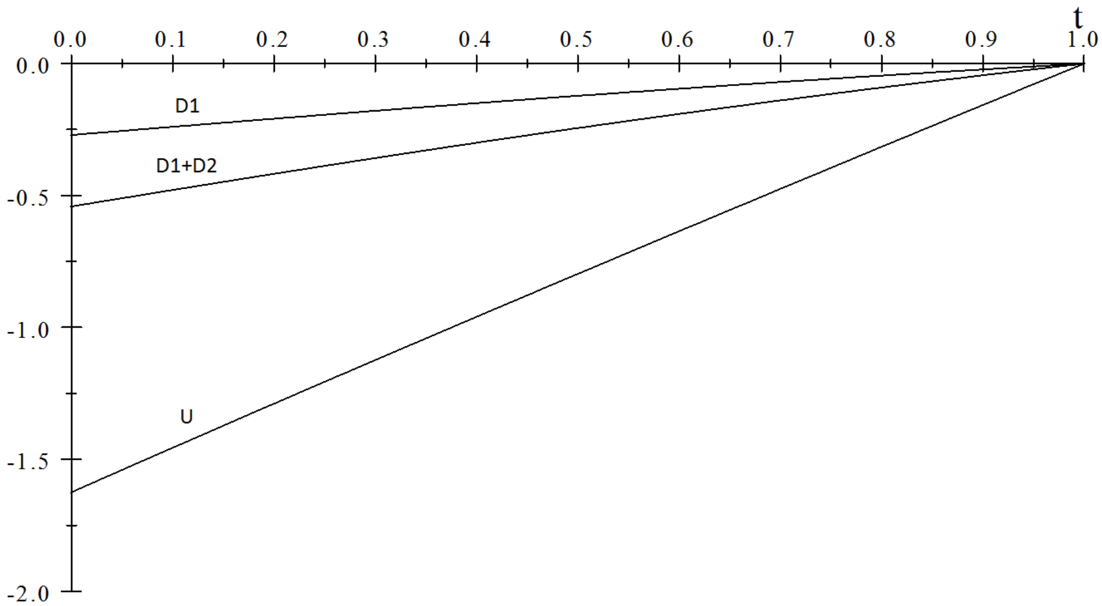
(D1 = Downstream Firm 1; D2=Downstream Firm 2; U = Upstream firm)

This result happens due to the effect detailed in the previous section: The upstream firm loses profits because the input price decreases at a slower rate, while the quantity sold in the input market also decreases. Downstream firms, on the other hand, lose profits because they are progressively farther from the majority of its consumers and because of the increase in the total transportation costs. The difference in the decrease happens mainly because the overall quantities sold in the market decrease (remember, the input quantities sold equal the output quantities), and the profit margin the upstream firm is larger, since it is monopolist on the market. Moreover, downstream firms benefit from the abovementioned reduction of the input price, which reduces the effect the total transportation costs have on their profit.

Proposition 6.3: Given an increase in the unit input transportation cost, the upstream profit decreases more than the profit of each downstream firm.

Proof: Similarly to the previous proposition, the proof we present is based in the simulation for all possible values of the unit input transportation cost. Figure 6.7 compares both derivatives, and shows that the upstream firm is the one that loses more profit with an increase in the transportation cost.

Figure 6.7 – Comparison between the derivatives of the upstream/downstream profits with respect to the unit input transportation costs.



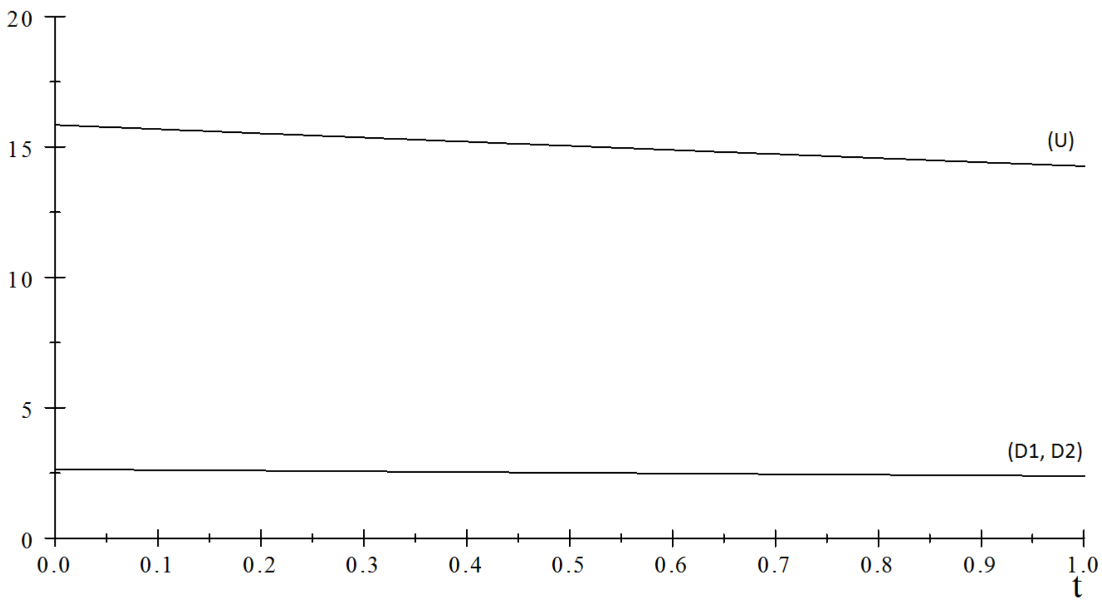
(D1 = Downstream Firm 1; D1+D2=Sum of both downstream firms' profits 2; U = Upstream firm)

However, Figure 6.6 clearly displays one of the main weaknesses of the Cournot framework: The effects of the change in the input transportation costs on the profits of the firms are very small, or close to irrelevant. We can see that both firms do not have their position in the market at risk, or do not even suffer too much if they do not follow the optimal location decision. This happens because of the assumption that the output transportation costs cannot be too big in relation to the market size. Then the effect of moving towards the extreme of the linear city has on the firms' profits is low. Abandoning this assumption proves to be a very complicated task (e.g. Chamorro-Rivas, 2000; Benassi et al., 2007).

6.4.3.2 Short run analysis

Location is, by its nature, something that is very expensive to change in the short-run, due to the high fixed costs that are associated with that change. We implicitly assume that the unit input transportation cost does not change in the short-run, since firms are able to see this cost before choosing the location of their plant. However, if we assume that both downstream firms cannot change their location (departing from the case where there were no unit input transportation costs: $x_1 = x_2 = 0.5$, the effects on the profits of the firms are amplified, as shown in figure 6.8.

Figure 6.8 – Short-Run Profit of the upstream and downstream firms depending on the input transportation cost.



The interpretation of this situation goes as follows: Assume that initially there were no input transportation costs, or there was a natural resource in the center of the city. However, for some unexpected reason, the unit input transportation costs were raised, without downstream firms having the possibility to change their location. Naturally the profits decrease for all the firms, but the effect is not much different from the long-run case, except, of course, for values of the unit transportation cost higher than 1, in which further decreases have an effect that does not exist in the short-run case.

We can see, under this assumption, that the profit is not too sensitive to changes in the transportation costs or changes in the location of firms. That is one of the reasons why profits are not usually analyzed in the context of spatial models: Smaller changes in transportation costs may bring different outcomes, but the consequences of not changing location itself in the profits are very slim. In other words, the crucial determinant of the amount of profits firms get is the dimension of the output good demand.

6.4.4 Social Planner

Next, we consider what would be the solution for the social planner if he controlled the downstream firms' location. The solution we seek is therefore a second-best, since we only allow the planner to control the first stage of the abovementioned game.

Knowing what the output and input quantities are going to be in future stages, as previously shown in equations 6.2 and 6.5, the social planner is left to maximize the total surplus (TS) of this market, that is, the sum of the profits (of the upstream firm and the downstream firms) and the consumer surplus (CS).

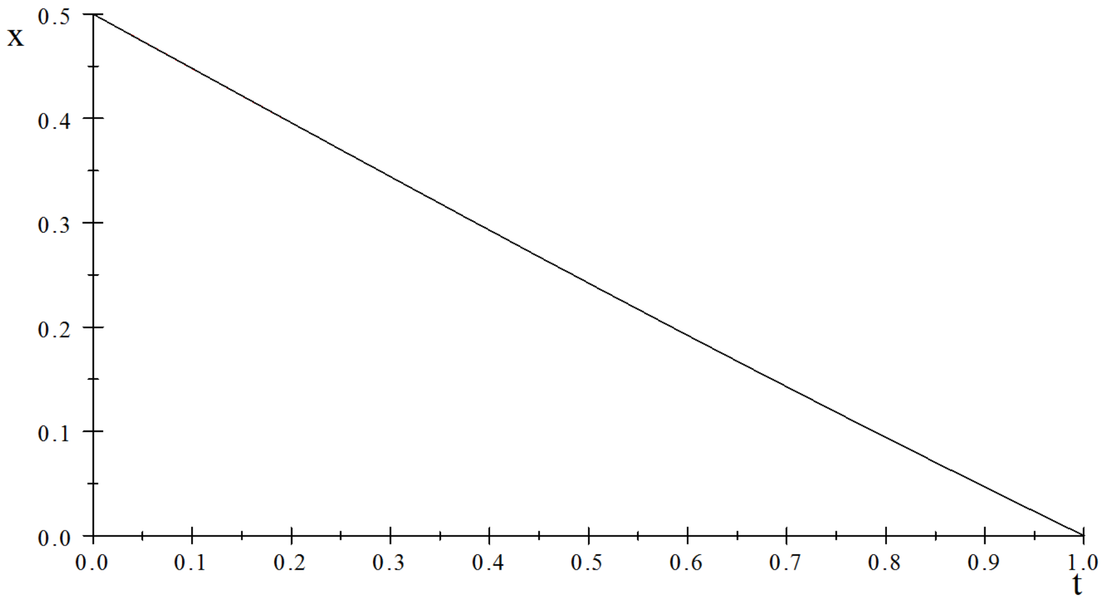
$$TS = \Pi_U + \Pi_1 + \Pi_2 + CS$$

To find the consumer surplus of all consumers, we need to calculate the consumer surplus in all points x and then, using an integral similar to the one that is used in previous subsections, sum the CS of all the points x. This integral can be broken in three parts due to the existence of two different absolute values. The profits of the three firms are the same used on previous calculations. The solution for location is given by:

$$x_1 = x_2 = \frac{5t+3}{6} + \phi + \frac{35t^2 + 10t - 187}{36\phi}$$

Where ϕ is a value that depends non-linearly on t. [6] Figure 6.9 shows the result of both centralized and uncentralized equilibrium regarding firms' location. We can see that, a bit surprisingly, the social planner equilibrium is very similar to the one found before, and pratically undistinguishable in Figure 6.9. This leads us to conclude that firms, when thinking about maximizing their own profit, are choosing a location very close to what it would be the location chosen by a central planner.[7] This result goes in line with the one found by Matsumura and Shimizu (2005). In addition, the social planner also chooses the same location for firm 1 and 2. Note that this result arises from the fact that both firms, by assumption, are obliged to sell on every point of the market.[8]

Figure 6.9 – Location Choice for Firms and Social Planner



This conclusion happens because firms and the social planner have similar rationales for their objectives, which is the minimization of transportation costs. Firms intend to minimize their transportation costs in order to provide cheaperly different goods in each market point. That way, they are able to provide more quantities of each good, therefore maximizing their profit. The social planner, on the other hand, is

[6] The value for this parameter is displayed in the appendix. In addition, the second-order conditions are met.
[7] Note that, in spite of being very close in Figure 6.9, the locations chosen in both cases are not exactly equal.
[8] It is unsure though whether the social planner would prefer a different solution. If the sole purpose was the minimization of the input and output transportation costs, then probably the social planner would distribute both firms along the product space.

interested in having the highest quantities in every market point, and that is only possible if firms find a way to minimize their transportation costs, such that they can be competitive in every point.[9]

The negligeble difference between the private and social choices is justified by the worriness that the social planner has with individual values of the consumer surplus and upstream firm's profits. By staying closer to the center, consumers will get larger quantities of goods, and this means as well that the upstream firm will have higher demand for its input. However, these effects are very small compared to the importance of a correct location choice for both the quantities sold in the downstream markets and therefore, the profits that the downstream firms will receive. The importance of this effect, which is an objective for both firms and regulator, justifies the proximity of the result.

Since the social planner only controls the location stage, the results towards the profit of the three firms are similar as it was analyzed in the previous section. The total surplus does not differ significantly between both cases, even though the one resulting from social planner maximization is naturally superior.

6.5. The case of Delegation

Here we have introduced two delegated firms in the vertical model we have detailed before. Our objective was not to see whether firms would prefer to be delegated or not, but yet to analyze how location, quantities and profits results would change in the presence of a downstream market with two delegated firms, with these firms having managers that are more aggressive than the owners. We have therefore assumed that both owners offered their managers an incentive contract that was weighted as 50% for the profit that firm had, and 50% to its revenues, therefore following the first type of incentive contracts detailed in subsection 6.2.

We keep the same timing of the game. The only difference is that in the last stage, the manager chooses the quantities to offer in each point of the linear city. We

[9] Note that the social planner is interested in minimizing transportation costs not only because they are a source of inefficiency (in classical price competition models with horizontal differentiation this is the only source of inefficiency the social planner faces), but also because better placed firms are capable of selling more quantities, contributing to total surplus in every point of the market.

have assumed that the owner would keep the choice of location, since it is a variable more associated with the medium/long-run. We solve the model by backward induction and therefore we start by computing the optimal quantities chosen by the manager in the last stage. The objective function of the manager combines the profits and the revenues for each point in the market. Therefore, for the market point located in x, the objective function of the manager of firm *i* (*i* = 1, 2) is given by:

$$O_{i,x} = (20 - q_{i,x} - q_{j,x} - I - (|x_i - x|) - tx_i)\frac{q_{i,x}}{2}$$

This objective function results from the average between the profits and the revenues. Similarly to the non-delegation case, Firm *i* maximizes its objective function by choosing the optimal quantities for each point. After satisfying the first and second-order conditions of both firms, the quantities are given by:

$$q_o *_{i,x} (x_i, x_j) = \frac{|x_j - x| - 2|x_i - x| - I - 2tx_i + tx_j + 20}{6}$$

Comparing to the optimal quantity chosen by the firms' owners, we conclude that the quantity chosen by both managers is higher than the quantities chosen in the non-delegation case. This is due to the more aggressive behavior of the manager stimulated by the incentive to obtain extra revenue. After summing the quantities for all points, using an integral in a similar way used in the non-delegation case, the quantity for each firm *i* is given by:

$$Q_o *_i (x_i, x_j) = \frac{2(x_i - x_i^2) + (x_j^2 - x_j) + (tx_j - 2tx_i - I)}{6} + \frac{13}{4}$$

Summing the quantities for both firms, we obtain the input market demand function faced by the upstream firm. The inverse demand function is therefore obtained by rearranging the demand respectively to the input price, which yields:

$$I = \frac{x_1 + x_2 - x_1^2 - x_2^2 - tx_1 - tx_2 - 3Q_{ou} + 39}{2}$$

Where Q_{ou} is the quantity produced by the upstream firm, which differs from the quantity Q_u in the non-delegation case. Note that the input price in both cases is very similar, except that in the delegation case it is higher by 10. To obtain the profit of the upstream, we just multiply the input price for the quantity sold, which leaves the upstream firm with the decision of choosing the optimal quantity $Q_{ou}*$ that maximizes its profit. After satisfying the first and second-order conditions, the quantity chosen is given by:

$$Q_{ou}*(x_1,x_2)=\frac{x_1+x_2-x_1^2-x_2^2-tx_1-tx_2+39}{12}$$

Notice that the upstream firm produces a higher quantity, which is an expected result given that the demand for their product has increased. Compared to the non-delegation case, the upstream firm sells more at a higher price, which is in line with the usual result when a firm faces an increase of the demand for its good. The profit of the upstream firm is given by:

$$\Pi_{ou}(x_1,x_2)=\frac{(x_1+x_2-tx_1-tx_2-x_1^2-x_2^2+39)^2}{48} \tag{6.7}$$

Given that both the location decisions and the unit transportation costs are bounded between 0 and 1, by comparing equations 6.6 and 6.7 for the case of non-delegation and delegation respectively, we verify that delegation benefits greatly the upstream monopolist, who takes full advantage of the increasing demand for its product.

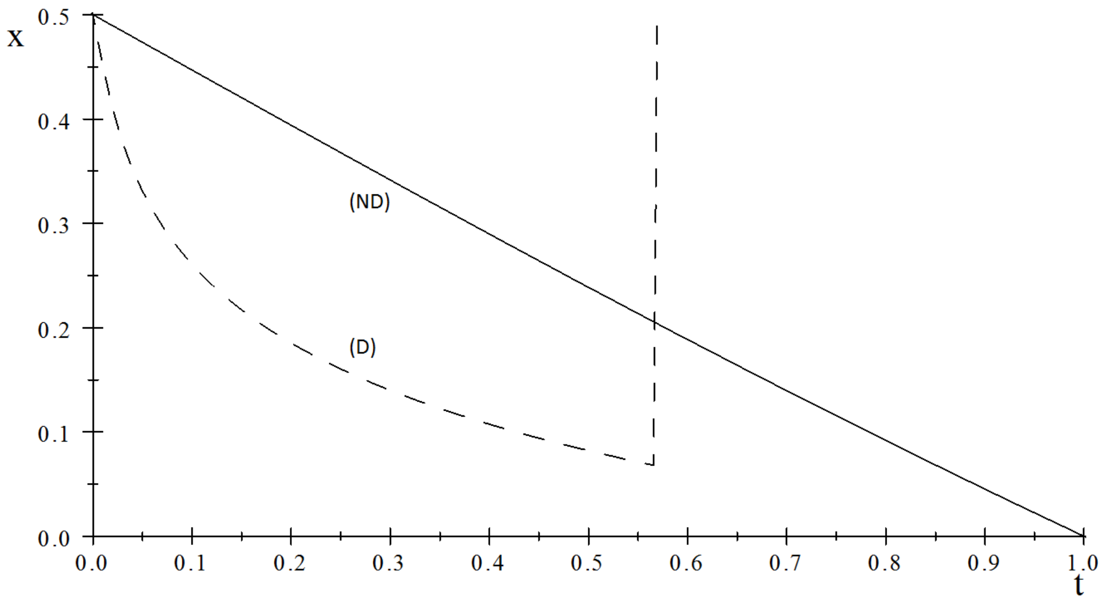
After replacing the input price and the quantities chosen in the profit of both firms, we obtain the profit of the firm. The owners look to locate their firms in order to maximize their profit. The location outcome we found is the following:[10]

$$x_1=x_2=\frac{11t+15}{30}+\sigma+\frac{231t^2+110t-15}{900\sigma}$$

[10] Now all that is left is for owners to choose the location of their plants. However, for the current model, we have not been able to find the optimal solution for all possible values for the unit input transportation costs, given that the only solution found (respecting both first-order and second-order conditions) fails to be inside the linear city for the entirety of the domain. Even though a maximum must exist given that we are optimizing inside a compact set.

With the value of σ shown in more detail in the appendix 6.1. Figure 6.10 depicts the location results for the delegation and non-delegation cases. We can observe that the location chosen by the owner (for the values for which the solution is valid) is closer to the location of the input production in the case of delegation, given the same unit input transportation cost. Note however that, since there must exist a maximum for this problem, we can assume that it follows the same trend of the valid part of the solution: Both firms keep approaching the input location in a decreasing fashion, until the unit input and output transportation costs are equal. Similarly to the non-delegation case, both firms choose to be located in the same position, irrespectively of the value of the unit transportation cost.

Figure 6.10 – Location result with and without delegation



(D = Delegation; ND= No Delegation

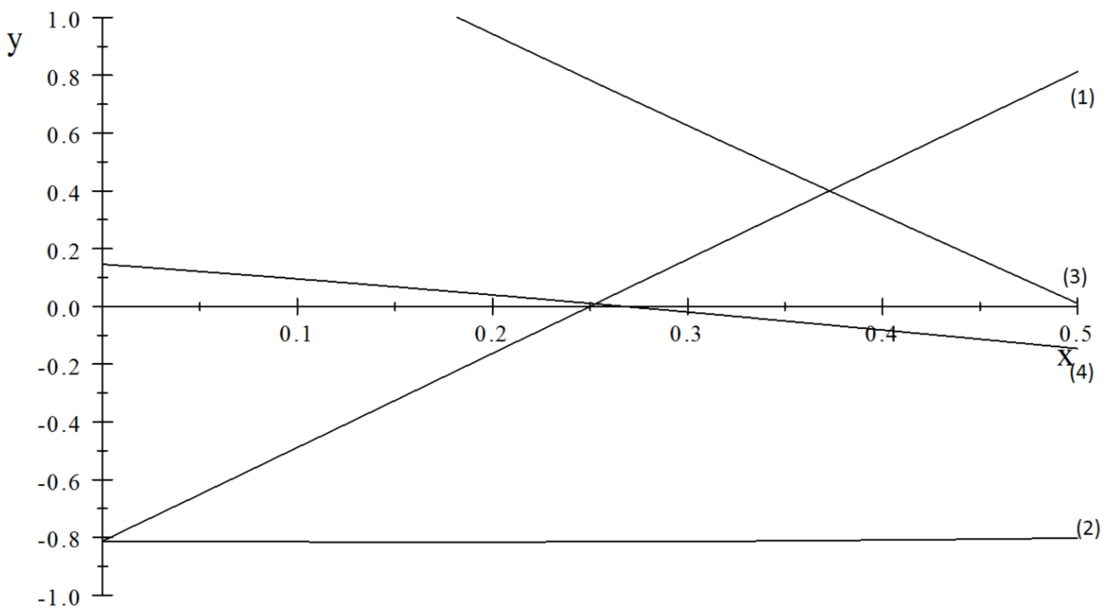
To better understand the rationale of this result, we separate the profits in the 4 components: The 1) Input purchasing cost, 2) Input transportation cost, 3) Output transportation costs, and 4) sales revenue. We fix the unit input transportation costs to t=0.5, to establish a comparison with the non-delegation case. We also assume, since we have verified that it happens throughout all domain, that the firms are agglomerated, that is, $x_1 = x_2$.

Table 6.2 – The Four Components of the profit function at t=0.5

Component	Derivative of the profit component with respect to Location
Input Purchasing Cost (1)	$-\frac{1}{6}x^3+\frac{1}{8}x^2+\frac{155}{48}x-\frac{13}{16}$
Input Transportation Cost (2)	$\frac{1}{8}x^2-\frac{1}{24}x-\frac{13}{16}$
Output Transportation Cost (3)	$-\frac{1}{3}x^3+\frac{5}{8}x^2-\frac{41}{12}x+\frac{77}{48}$
Sales Revenue (4)	$-\frac{1}{6}x^3-\frac{7}{24}x^2-\frac{23}{48}x+\frac{7}{48}$

Figure 6.11 shows clearly the difference between the delegation and the non-delegation case. The derivatives differ in their value, but they are all very similar between the input location and the city center. The only exception is the sales revenue component, in which the differences are more striking comparing to the previous case. We can see the curve is now steeper, meaning that the sales revenue component has a lower effect in determining the optimal location position, or in other words, that the total of sales revenue is less sensitive to changes in location comparing to the non-delegation case. This variation is in line with the location pattern found: When the effect of sales revenue is to "push" the firm to the city center, this effect is now weaker, so firms move rapidly to the input location. However, when this effect is reversed, "pushing" the firm to input location, this effect is also weaker in the delegation case, and accordingly the location result would (if the location pattern within the linear city is similar for higher values of t) move slower to the spot where the input is produced.

Figure 6.11 - The derivatives of each component of the profit function of firm i with respect to location



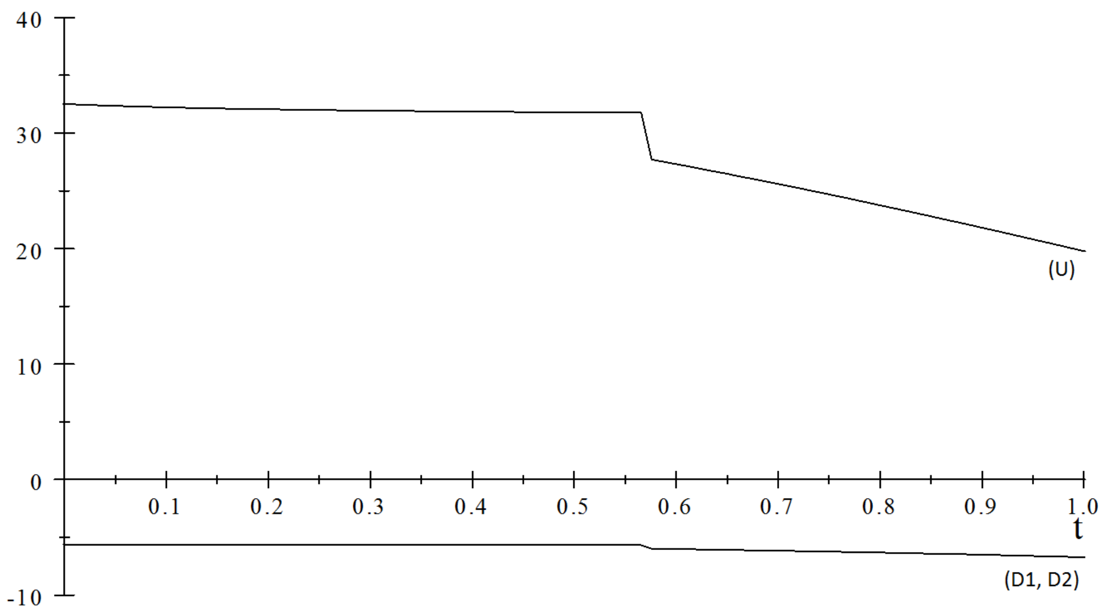
(1) Input Purchasing cost; 2) Input Transportation cost; 3) Output transportation cost; 4) sales revenue)

In addition, the similar configuration of the four curves indicates that most likely, the optimal location decision of firms within the domain is similar to what we have described earlier: The location decision curve continues in a similar fashion until it reaches the point where both firms locate in the same place than the upstream producer.

Even though we have not made endogenous the delegation choice by both firms, as well as the parameter for the incentive contract, we can see in Figure 6.12 that delegation is hazardous to both firms, in a way that they do not even have positive profit.[11] Most likely (though not confirmed in our work), the prisoner dilemma that exists in the non-spatial Cournot case is replicated here, and both firms have to choose a higher value for their incentive parameter than what they would if they were allowed to cooperate.

[11] Note, however, that we do not have the optimal solution when t is higher than approximately 0.57. We assume that the behavior of the true location solution, and the subsequent profits, are similar to the non-delegation case, which would imply that an increase in the transportation costs does not change much the profits of the firms.

Figure 6.12 – Long-Run Profit of the upstream and downstream firms depending on the input transportation cost.



However, the penalty for not cooperating is higher in this case due to the presence of the upstream firm, since the higher the quantities demanded, the higher the price for the input will be. The managerial "quest" for increasing revenues greatly increases the purchasing costs of the downstream firms, which is the main reason for the negative results. The negativity arising from these results is exacerbated due to the fact that the upstream firm is a monopolist in the input market. If the input market faced perfect competition, then the consequences would only arise from the increase of the quantity demanded, and there would not be any consequences on the pricing. So, the higher the number of firms in the market (assuming similar firms), the lower are the consequences in terms of the profit for these downstream firms. In line with this explanation, the upstream firm's profit is now higher than in the non-delegation case, and the difference that existed between both businesses is now higher in this case.

6.6. Conclusion

In the context of spatial competition, few articles have analyzed the implications of a vertical relationship in forming the marginal costs that firms have to face. In this chapter, the presence of an input that is required for firms to be in the market, and the subsequent problems of acquiring and transporting the good for the business to be successful, along with the strategic duopoly interaction makes this model suitable to explain the behavior of industries that are very dependent on vertical relationship to be successful.

In the framework of spatial competition by quantities developed by Anderson and Neven (1991), we conclude that the transportation costs are crucial to the location decision of both firms, in an almost linear relationship between the location chosen and the unit input transportation costs. However, by analyzing the resulting profits and the social welfare, input transportation costs seem to have a minor role. Additionally, both firms agglomerate, independently of the unit transportation costs. This happens because of the strategic substitutiability nature of quantity competition, which makes firms concentrate more on being better located relatively to the demand than relatively to its opponent. This is the reason why we find that the location outcome chosen by a social planner is close to the solution chosen by the firms themselves: The main concern in both cases is with the transportation costs, and this is the most important driver for the location decision.

We also conclude that an increase in unit input transportation costs cripples more the profit of the upstream monopolist than the profit of the downstream firms, even though the latter supports the transportation costs. The reason is that the downstream firms sell less in the downstream market, which means a decrease of demand in the upstream market, leading to a decrease in the quantities sold, and in the price as well.

Regarding the delegation case, we can see that the optimal location pattern is different, even though firms are still agglomerated for all possible values of unit input transportation costs. Downstream firms move at a non-linear rate to the location of the input good, staying closer to the upstream firm comparing to the non-delegation case. This change happens due to a higher pressure from the “sales revenue” profit component. In terms of profits, we conclude that the gap between upstream and downstream firms becomes higher, with the former highly profiting due to the latter’s increased quantity purchase. Similarly to the non-spatial case, both firms using delegation is negative for their profits, but it is left to answer in this framework if there is a “delegation arms race” that forces owners to employ managers while giving them high incentives for revenue maximization at the expense of their own profits.

More important than its results, this chapter may be the starting point for an interesting analysis of the consequences of vertical relationships in the spatial competition literature. Even though the framework of Anderson and Neven (1991) has a very interesting nature, this model may not be the most suitable to the development of this vertical analysis, given the (mathematical) requirement that both firms must sell in all market points. Breaking this assumption would induce competition between firms, which would have been forced to choose their location with the concern of a better coverage of the market respectively to their opponent. However, leaving this assumption implies finding deep mathematical problems that would probably undermine a good analysis of vertical relationship's implications on the downstream firms' profits, quantities sold and location decisions.

7. Conclusion

The main objective of this dissertation was to contribute to the literature on spatial competition models, namely on achieving a better understanding on why firms pick a given location in the market to develop their activities. Moreover, the location variable itself is regarded as a strategy tool that firms have at their disposal to better manage the relationship with its rival on a given market.

We started by showing the relevance of the spatial competition literature. From Chapter 2 it is clear that this field, i.e. the Hotelling framework and its subsequent developments are still important in the Economics science as a whole, with many published articles on the most important journals in the Industrial Organization and Regional Economics fields, as well as in economics journals that do not focus on a specific topic within its scientific domain. Then, our literature review proved the existing complexity around these models, and how many different factors may be a justification for the optimal location decision that firms have to take, even in the simplest form of markets where a city is linear and when there are only two firms competing.

In our original contribution to the literature on spatial competition models, we have extended the base frameworks of price (d'Aspremont et al, 1979) and quantity (Anderson and Neven, 1991) competition with a spatial component. In terms of price competition, we have proved in chapter 3 that uncertainty regarding costs of production may have firms choosing a different location setting than they would if the information was available since the start of the game. We think that the assumption that there is cost uncertainty before firms choose their location is very realistic, and therefore that this result makes an important contribution to the literature. We have simplified the choice of location of firms, but the effects that lead firms to agglomerate would still be verified in the full location choice setting: firms may agglomerate because of the possibility of having a monopoly in the market. On chapter 4, we focused on justifying why would firms have a different marginal cost, and we introduced cost-reducing activities in the original two-stage location then price game. To our dismay, we could not find an equilibrium solution for all values of the investment game due to the fact that the profit function is a piecewise function, whose branches depend on the amount of demand that firms may have. However, we find that when firms have the same efficiency, they get trapped in a prisoner's dilemma result for the investment, that is, firms would prefer to not have investment possibilities.

For the quantity competition case, which is based on the work of Anderson and Neven (1991), in chapter 5 we found that allowing firms to have different marginal costs of production does not have any effect in their location equilibrium result. We also find that a social planner would prefer to remove the inefficient firm out of the market if it becomes too inefficient relatively to its opponent. Moreover, if the inefficient firm has to stay on the market, the central agglomerative solution ceases to be equilibrium. This happens because the profit margins of the efficient firm are higher, and the inefficient firm by staying in the center produces more quantities and steals part of this profit margin. Therefore, our guess, based on intuition, is that the inefficient firm would be moved to one of the extremes of the market in order to produce as little as possible.

In chapter 6, we introduced a natural resource as an essential input for two competing firms, and we conclude that, the larger the transportation costs of that resource, the closer both firms would move to the input location, which resembles the classic problem of Weber (1929). We also prove that delegation is harmful in that case, since downstream firms would pay an increasing cost for their raw material due to the larger amount of input quantities ordered. Therefore, for the same values of the unit input transportation cost, firms' owners choose to be farther to the city center, comparing to the case of no delegation. We suspect that the delegation possibility acts as a prisoner's dilemma as well: if both firms had to choose between delegating and not delegating, they would choose delegation to protect themselves from the opponent, but would end up being worse than if they did not choose to delegate.

After all this work, our feeling is that the frameworks by d'Aspremont et al. (1979) and Anderson and Neven (1991) have assumptions that hindered part of the analysis and the impact that our contributions could have in the literature. Regarding the price competition framework, the assumption of inelastic demand, i.e. the fact that the consumers located in each market point can only buy the same quantity, irrespectively of the price that is offered to them, hinders the analysis that involves different marginal costs of production between firms and therefore, cost-reducing activities. One important motivation for firms to reduce their marginal costs is not only the fact that they can steal the market share from their opponents, but also that they could make their product affordable for different layers of their market, and therefore their total quantities sold would not only increase with a decrease in their marginal costs of production due to the business they have stolen from their opponent, but also because lower marginal costs would allow for lower pricing, and therefore new consumers could start buying the market good. The latter effect is not a possibility in the original d'Aspremont et al. framework. This problem is very clear in our results in the Prisoner Dilemma analysis in chapter 4. Firms overinvested in R&D only to see their operational revenues unchanged, due to the inability of attracting new customers to their stores after the decrease of their good prices, following an equal decrease in the marginal costs of production. The inelastic demand assumption does not capture the whole benefits of investing in cost-reducing R&D that could happen in most markets. Therefore, finding a better way to insert more realistic R&D settings in the Hotelling model, or finding an alternative way of expressing price competition with spatial components that could allow a correct assessment of R&D investments could be a road for future research.

Concerning the quantity competition framework, the restricting assumption is the one that obliges the market to be big enough such that both firms sell in every market point. This assumption implicitly removes any possibility that firms could be monopolists in few points of the linear city, This assumption has huge effects regarding the optimal location decision of firms. If firms have to sell in every point of the linear city market, what is the point for them in locating outside of the city center, where their transportation costs are minimized? If it is impossible to achieve a local monopoly, why would firms want to be located closer to one of the edges of the market? But the most important consequence of this assumption is the fact that the competition mechanism between firms is not very important. The consequences for the own location decision when a rival firm changes its location are not very significant. This is especially clear in chapter 5, in the result where irrespectively of their marginal costs of production, firms will always locate in the city center. It would be expected that in the case where one of the firms has a significant advantage over the other, the firm possessing a disadvantage would have the incentive to move away from the city center in order to diminish that disadvantage in some points of the market, achieving better results only in some points of the city. However, by definition, the difference in the marginal costs cannot be too big, because then the inefficient firm would not be able to sell in all points of the market, contradicting the model's assumption itself. Moreover, the effect in terms of magnitude of the value of unit transportation costs in this chapter is too low, suggestion that the location variable is relatively unimportant to the firms' profits and decision comparing to the effect of firms' marginal costs of production. We feel that, both in the case of price and quantity competition, a correct analysis of the differences in marginal costs of production is hindered by these assumptions.

In terms of policy implications, we feel that our work is somewhat limited, and some avenues for future research could exist in the open questions we leave regarding these policy implications. After adding ingredients and assumptions to the basic Hotelling model, it is hard to find markets that fit in the exact assumptions of the markets we have tested here in this work. Nevertheless, some policy insights can be drawn (with some carefulness) from our work. Even though we have not proved some of them, these could be a starting point for future work. On the d'Aspremont et al. (1979) model, the social welfare analysis is confined to the analysis of where should firms locate in order to minimize the overall transportation costs of the economy. In the model with cost differentials, the case is not that simple. Since firms have different marginal costs of production, the most efficient firm should produce most of the quantities in order to save on the economy's production costs. So, location is not the only determinant of maximum social welfare. Firms end up dispersed for some values of the difference between them, but similarly to the non-differential case, firms should be pushed to more central positions in order to minimize transportation costs. However, the most efficient firm should optimally be closer to the center than its opponent.

In the quantity competition framework, the expected conclusion is that the social planner should not have much to worry about. Firms usually pick location decisions that are already optimal to the point of view of the social planner, mainly because they share the common goal of choosing an efficient location relatively to the transportation costs of the economy. In chapter 5, however, we have concluded that a firm that is not competitive enough should be removed from the market, or at least it should focus in market niches such that the efficient firm produces the bulk of the quantities. This is not in line with the firms' location outcome, in which they would prefer to be located at the city center all the times. In chapter 6, however, the conclusions regarding the social optimum are closer to the prediction of the literature. Firms' optimal location choice, given that the markets are big enough such that firms are obliged to sell in all points of the city, when there is a resource located in the extreme city, are very similar to the locations that the social planner would choose, meaning that there is not much room for a social planner to act in this type of market.

Appendix

Appendix 3.1: Optimal prices of the second stage of the game

For the case when both firms are successful, the profit function of each firm is given by:

$$\Pi_i(x_i, x_j, p_i, p_j) = \begin{cases} p_i - c_i & \text{if} \quad p_i < \underline{\theta} \\ (p_i - c_i)(1 - 2x_j - x_i^2 + x_j^2 - p_i + p_j) & \text{if} \quad \underline{\theta} \le p_i \le \overline{\theta} \\ 0 & \text{if} \quad p_i > \overline{\theta} \end{cases}$$

For i=1,2 . Using the first order condition of profit maximization (with second order conditions verified) for the middle branch, firms set a price schedule. For the case where one of the firms has an advantage such that it gets all the demand, the monopolist optimally sets the maximum price possible such that its opponent remains out of the market. Therefore, the prices that maximize the profits are the following:

$$p_1 = \begin{cases} c_1 & \text{if} \quad \hat{c}_1 < \underline{\theta} \\ \dfrac{-x_1^2 - 2x_i + x_2^2 - 4x_2 + 2c_1 + c_2 + 3}{3} & \text{if} \quad \underline{\theta} \le \hat{c}_1 \le \overline{\theta} \\ c_2 - 1 + 2x_1 - x_1^2 + x_2^2 & \text{if} \quad \hat{c}_1 > \overline{\theta} \end{cases}$$

$$p_2 = \begin{cases} c_1 - 1 + 2x_2 + x_1^2 - x_2^2 & \text{if} \quad \hat{c}_1 < \underline{\theta} \\ \dfrac{x_1^2 - 4x_1 - x_2^2 - 2x_2 + c_1 + 2c_2 + 3}{3} & \text{if} \quad \underline{\theta} \le \hat{c}_1 \le \overline{\theta} \\ c_2 & \text{if} \quad \hat{c}_1 > \overline{\theta} \end{cases}$$

Where $\hat{c}_1$ is the marginal cost difference expressed by $c_1 - c_2$ and $\underline{\theta}$ and $\overline{\theta}$, after replacing the optimal price decision, change to $-x_i^2 + 4x_i + x_j^2 + 2x_j - 3$ and $-x_i^2 - 2x_i + x_j^2 - 4x_j + 3$, respectively.

Replacing the optimal prices in the profit function, the profit for firm i (i=1,2) becomes:

$$\Pi_i(x_i,x_j)=\begin{cases} c_j-c_i-1+2x_i-x_i^2+x_j^2 & if \quad \hat{c}_i<\underline{\theta} \\ \dfrac{(x_i^2+2x_i-x_j^2+4x_j+c_i-c_j-3)^2}{9(2-2x_i-2x_j)} & if \quad \theta\le\hat{c}_i\le\overline{\theta} \\ 0 & if \quad \hat{c}_i>\overline{\theta} \end{cases}$$

Similarly to this first case where both firms are successful, we present the other three cases.

When firm 1 is successful and firm 2 is unsuccessful, the profits are given by:

$$\Pi_1(x_1,x_2,p_1,p_2)=\begin{cases} p_1-c_1 & if \quad p_1<\underline{\theta} \\ (p_1-c_1)(1-2x_2-x_1^2+x_2^2-p_1+p_2) & if \quad \theta\le p_1\le\overline{\theta} \\ 0 & if \quad p_1>\overline{\theta} \end{cases}$$

$$\Pi_2(x_1,x_2,p_1,p_2)=\begin{cases} p_2-10 & if \quad p_2<\underline{\theta} \\ (p_2-10)(1-2x_1-x_2^2+x_1^2-p_2+p_1) & if \quad \theta\le p_2\le\overline{\theta} \\ 0 & if \quad p_2>\overline{\theta} \end{cases}$$

Similarly to the previous case, the optimal price schedule is given by:

$$p_1=\begin{cases} c_1 & if \quad c_1-10<\underline{\theta} \\ \dfrac{-x_1^2-2x_1+x_2^2-4x_2+2c_1+13}{3} & if \quad \theta\le c_1-10\le\overline{\theta} \\ 9+2x_1-x_1^2+x_2^2 & if \quad \hat{c}_1-10>\overline{\theta} \end{cases}$$

$$p_2=\begin{cases} c_1-1+2x_2+x_1^2-x_2^2 & if \quad c_1-10<\underline{\theta} \\ \dfrac{x_1^2-4x_1-x_2^2-2x_2+c_1+23}{3} & if \quad \theta\le c_1-10\le\overline{\theta} \\ 10 & if \quad \hat{c}_1-10>\overline{\theta} \end{cases}$$

After replacing the prices the optimal profit functions become:

$$\Pi_1(x_1,x_2)=\begin{cases} 9+2x_1-x_1^2+x_2^2-c_1 & if \quad c_1-10<\underline{\theta} \\ \dfrac{x_1^2+2x_1-x_2^2+4x_2+c_1-13}{9(2-2x_1-2x_2)} & if \quad \underline{\theta}\le c_1-10\le\overline{\theta} \\ 0 & if \quad c_1-10>\overline{\theta} \end{cases}$$

$$\Pi_2(x_1,x_2)=\begin{cases} 0 & if \quad c_1-10<\underline{\theta} \\ \dfrac{x_2^2+2x_2-x_1^2+4x_1-c_1+7}{9(2-2x_1-2x_2)} & if \quad \underline{\theta}\le c_1-10\le\overline{\theta} \\ c_1+x_1^2-x_2^2+2x_2-11 & if \quad c_1-10>\overline{\theta} \end{cases}$$

For the case where firm 1 is unsuccessful, and firm 2 is successful, the profit functions are given by:

$$\Pi_1(x_1,x_2,p_1,p_2)=\begin{cases} p_1-10 & if \quad p_1<\underline{\theta} \\ (p_1-10)(1-2x_2-x_1^2+x_2^2-p_1+p_2) & if \quad \underline{\theta}\le p_1\le\overline{\theta} \\ 0 & if \quad p_1>\overline{\theta} \end{cases}$$

$$\Pi_2(x_1,x_2,p_1,p_2)=\begin{cases} p_2-c_2 & if \quad p_2<\underline{\theta} \\ (p_2-c_2)(1-2x_1-x_2^2+x_1^2-p_2+p_1) & if \quad \underline{\theta}\le p_2\le\overline{\theta} \\ 0 & if \quad p_2>\overline{\theta} \end{cases}$$

Similarly to the previous case, the optimal price schedule is given by:

$$p_1=\begin{cases} 10 & if \quad 10-c_2<\underline{\theta} \\ \dfrac{-x_1^2-2x_1+x_2^2-4x_2+1c_2+23}{3} & if \quad \underline{\theta}\le 10-c_2\le\overline{\theta} \\ c_2-1+2x_1-x_1^2+x_2^2 & if \quad 10-c_2>\overline{\theta} \end{cases}$$

$$p_2=\begin{cases} 9+2x_2+x_1^2-x_2^2 & if \quad 10-c_2<\underline{\theta} \\ \dfrac{x_1^2-4x_1-x_2^2-2x_2+2c_2+13}{3} & if \quad \underline{\theta}\le 10-c_2\le\overline{\theta} \\ c_2 & if \quad 10-c_2>\overline{\theta} \end{cases}$$

After replacing the optimal price schedule, the profit functions become:

$$\Pi_1(x_1,x_2)=\begin{cases} 0 & if \quad c_2-10<\underline{\theta} \\ \dfrac{x_1^2+2x_1-x_2^2+4x_2+c_1-13}{9(2-2x_1-2x_2)} & if \quad \underline{\theta}\le c_2-10\le\overline{\theta} \\ c_2+x_2^2-x_1^2+2x_1-11 & if \quad c_2-10>\overline{\theta} \end{cases}$$

$$\Pi_2(x_1,x_2)=\begin{cases} 9+2x_2-x_2^2+x_1^2-c_2 & if \quad c_2-10<\underline{\theta} \\ \dfrac{x_2^2+2x_2-x_1^2+4x_1+c_2-13}{9(2-2x_1-2x_2)} & if \quad \underline{\theta}\le c_2-10\le\overline{\theta} \\ 0 & if \quad c_2-10>\overline{\theta} \end{cases}$$

For the case where both firms are unsuccessful, the profits of each firm is given by:

$$\Pi_i(x_i,x_j,p_i,p_j)=\begin{cases} p_i-10 & if \quad p_i<\underline{\theta} \\ (p_i-10)(1-2x_j-x_i^2+x_j^2-p_i+p_j) & if \quad \underline{\theta}\le p_i\le\overline{\theta} \\ 0 & if \quad p_i>\overline{\theta} \end{cases}$$

For i=1,2. The optimal price chosen by each of firms is given only by one branch, since independently of their successful marginal cost firms will always have their actual marginal cost equal to 10.

$$p_1 = \frac{-x_1^2 - 2x_1 + x_2^2 - 4x_2 + 33}{3}$$

$$p_2 = \frac{x_1^2 - 4x_1 - x_2^2 - 2x_2 + 33}{3}$$

Replacing the optimal price in the profit function, the profit for firm i (i=1,2) becomes:

$$\Pi_i(x_i, x_j) = \frac{(x_i^2 + 2x_i - x_j^2 + 4x_j - 3)^2}{9(2 - 2x_i - 2x_j)}$$

Appendix 3.2. Expected profit function of firm 1 in case of dispersion

To obtain the profit function in the case of dispersion, it is enough to replace $x_1=x_2=0$ in the above profit functions for all four cases. That includes replacing the thresholds for each branch. We only present the results for firm 1 because the result for firm 2 comes from symmetry, and the functions could be easily constructed from the previous section.

1st case: (S,S)

$$\Pi_i = \begin{cases} c_2 - c_1 - 1 & if \quad c_1 < c_2 - 3 \\ \dfrac{(c_1 - c_2 - 3)^2}{18} & if \quad c_2 - 3 \le c_1 \le c_2 + 3 \\ 0 & if \quad c_1 > c_2 + 3 \end{cases}$$

2nd case: (S,U)

$$\Pi_1 = \begin{cases} 9 - c_1 & if \quad c_1 < 7 \\ \dfrac{(c_1 - 13)^2}{18} & if \quad c_1 \ge 7 \end{cases}$$

3rd case: (U,S)

$$\Pi_1 = \begin{cases} 0 & if \quad c_2 < 7 \\ \dfrac{(7 - c_2)^2}{18} & if \quad c_2 \ge 7 \end{cases}$$

4th case: (U,U)

$$\Pi_1 = \frac{1}{2}$$

From the weighted combination of these four cases, different expressions for the profit function arise depending on the values of c_1 and c_2. These are the regions shown in Figure 3.2.

Appendix 3.3

From the direct comparison of agglomeration and dispersion cases (like shown in Table 3.2), we can conclude whether firms 1 or 2 prefer to disperse or agglomerate. From the joint analysis of both results, we concluded whether the result would be: Agglomeration, Dispersion or No location equilibrium (in the case both firms have different opinions on their location decision). Note also that the decision regarding both firms is symmetric, which allows the direct comparison between different regions, since every region has a symmetric region.

Table A.3.1 – Location choices of each firm and resulting location outcome

Firm 1	Region (Firm 1; Firm 2)	Firm 2	Result
Disp < Agg	1 and 7	Disp < Agg	Agglomeration
Disp < Agg	2 and 8	Disp < Agg if $c_2 < 8.2426$	Agg. if $c_2 < 8.2426$ No Eq. if $c_2 > 8.2426$
Disp < Agg	3.2 and 3.1	Disp < Agg	Agglomeration
Disp < Agg	3.1 and 3.2	Disp < Agg	Agglomeration
Disp < Agg	5 and 4	Disp < Agg if $c_1 < c_2 + \sqrt{-c_2^2 + 8c_2 + 2} - 3$	Agg. if $c_1 < c_2 + \sqrt{-c_2^2 + 8c_2 + 2} - 3$ No Eq. if $c_1 > c_2 + \sqrt{-c_2^2 + 8c_2 + 2} - 3$

Disp < Agg if $c_1 < \frac{c_2+7}{2} + \frac{\sqrt{-(c_2+5)(c_2-7)}}{2}$	4 and 5	Disp < Agg	Agg. if $c_1 < \frac{c_2 + \sqrt{-(c_2+5)(c_2-7)}+7}{2}$ No eq. if $c_1 > \frac{c_2 + \sqrt{-(c_2+5)(c_2-7)}+7}{2}$
Disp > Agg	6.2 and 6.1	Disp < Agg if $c_1 < \frac{c_2+13}{2} - \frac{\sqrt{3}\sqrt{-c_2^2+6c_2+19}}{2}$	No eq. if $c_1 < \frac{c_2}{2} - \frac{\sqrt{3}\sqrt{-c_2^2+6c_2+19}}{2} + \frac{13}{2}$ Disp. if $c_1 > \frac{c_2}{2} - \frac{\sqrt{3}\sqrt{-c_2^2+6c_2+19}}{2} + \frac{13}{2}$
Disp < Agg if $c_1 < \frac{c_2}{2} + \frac{\sqrt{3}}{2}\sqrt{-c_2^2+16c_2-36} - 1$	6.1 and 6.2	Disp > Agg	No eq. if $c_1 < \frac{c_2}{2} + \frac{\sqrt{3}}{2}\sqrt{-c_2^2+16c_2-36} - 1$ Disp. if $c_1 > \frac{c_2}{2} + \frac{\sqrt{3}}{2}\sqrt{-c_2^2+16c_2-36} - 1$
Disp < Agg	7 and 1	Disp < Agg	Agglomeration
Disp < Agg if $c_1 < 8.2426$	8 and 2	Disp < Agg	Agg. If $c_1 < 8.2426$ No eq. if $c_1 < 8.2426$

The results displayed in the last column are summarized in Figure 6.3.

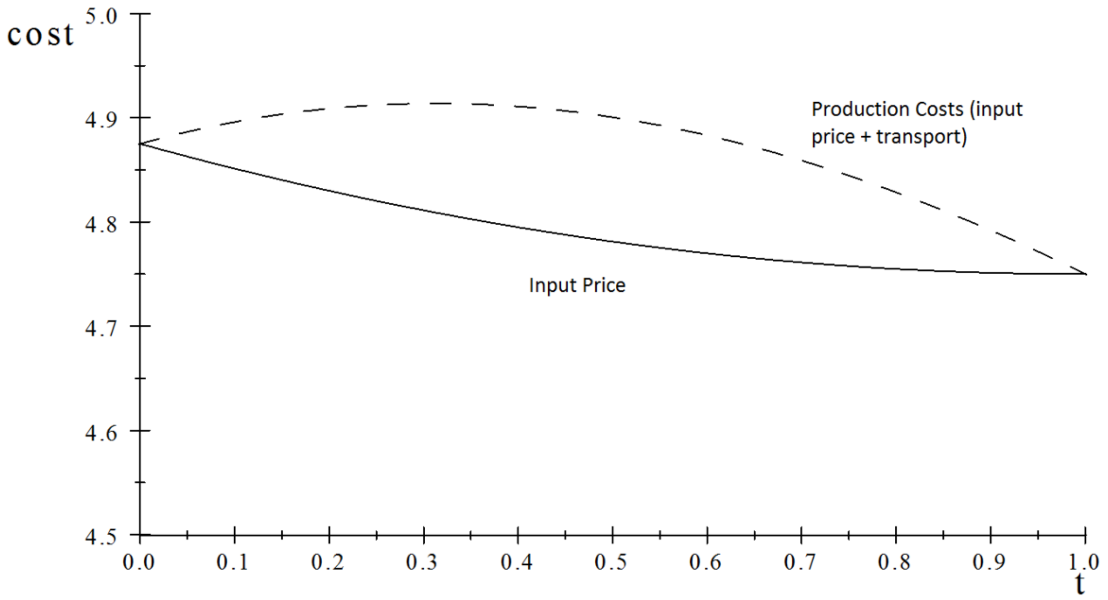
Appendix 6.1

$$\theta = \sqrt[3]{\sqrt{-\frac{1715t^6}{419904} + \frac{9947t^5}{629856} - \frac{383033t^4}{1259712} - \frac{2401t^3}{3888} + \frac{7725809t^2}{1259712} - \frac{462343t}{209952} + \frac{16974593}{1259712}} + \frac{98t^3}{729} + \frac{14t^2}{81} - \frac{266t}{81}}$$

$$\emptyset = \sqrt[3]{\sqrt{-\frac{2856t^6}{46656} + \frac{625t^5}{2592} - \frac{73625t^4}{15552} - \frac{38875t^3}{11664} + \frac{527585t^2}{15552} - \frac{174845t}{7776} + \frac{6539203}{46656}} + \frac{25t^3}{27} + \frac{5t^2}{9} - \frac{95t}{9}}$$

$$\sigma = \sqrt[3]{\sqrt{-\frac{97163t^6}{29160000} + \frac{62557t^5}{4860000} - \frac{8591t^4}{648000} - \frac{291599t^3}{7290000} + \frac{56483t^2}{3240000} - \frac{11t}{108000} + \frac{1}{216000}} + \frac{1573t^3}{13500} + \frac{143t^2}{900} - \frac{13t}{100}}$$

Figure A.6.1 – Input price and unit production costs for the optimal path.



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