A versatile matching algorithm based on dynamic programming with circular order preserving

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ABSTRACT: This paper presents an assignment algorithm with circular order preserving constraint. Given a cost affinity matrix and the desired percentage of correspondences, the algorithm implemented using dynamic programming determines the correspondence of type one-to-one of minimum global cost. Here, it was applied to optimize the global matching between two sets of ordered points that represent the contours of objects previously segmented from images. In the tests performed, we considered affinity matrices previously built based on information on curvature and distance to centroid. The results that have been obtained are better than the ones presented in previous studies, for the cases in which partial deformations or occlusions are involved.

1 INTRODUCTION

As far as Computational Vision is concerned, one of the more frequent and challenging problems is the recognition and alignment of objects represented in images. These tasks are crucial in several applications, such as: tracking of structures along image sequences, visual inspection from images, people recognition/identification from their pose in images, diagnosis in medical imaging, etc. The complexity involved is essentially due to the different projections that objects can assume in images; for instance, due to the existence of varied cameras viewpoints, or even as a result of deformations that the objects may undergo.

There are several methodologies to quantify the similarity between two objects, or between two configurations of an object, from images. One of these techniques is based on the matching of objects' features. Thus, to apply these techniques, one must begin by segmenting features of the objects from the input images, such as points, segments, region, boundaries, surfaces or skeletons. After the segmentation task, costs are typically attributed to each possible match between the objects' features and optimization techniques are frequently used to find the global optimal matching.

Usually, feature points are extracted from the objects to be matched and the affinity among them is quantified in an affinity matrix. Possible examples of techniques used to build these affinity matrices are: spatial information of the intensity gradient, (Lucas & Kanade 1981); modal matching, (Scot & Longuet-Higgins 1991, Shapiro & Brady 1992, Sclaroff & Pentland 1995, Tavares 2000, Carcassoni & Hancock 2003, Bastos & Tavares 2006); shape context, (Belongie et al. 2002); shape signatures, (Otterloo 1991, Cohen & Guibas 1997, Oliveira & Tavares 2007, Oliveira & Tavares, 2009); or probabilistic criteria, (Moisan & Bérenger 2004, Keren 2009).

When the similarity between the objects' feature points is quantified in a cost matrix, the matching problem can be considered as being an optimization problem and assignment algorithms can be used to find the best matching. Examples of approaches of this kind are: linear programming, (Bastos & Tavares 2006); graph search, (Roy & Cox 1998); bipartite graph matching, (Fielding & Kam 2000); concave optimization, (Maciel & Costeira 2003) and dynamic programming, (Scott & Nowak 2006, Oliveira & Tavares 2008). Additionally, non-optimal approaches include, for instance, greedy algorithms, (Wu & Leou 1995) and simulated annealing, (Starink & Backer 1995).

In this paper, we present an assignment algorithm with an order preserving constraint especially designed to match contours' points. The new algorithm is more versatile than the one presented in (Oliveira & Tavares 2008), because the user can define the percentage of matches to be established, while in the previous one it was always considered 100% (that is, the number of points of the contour defined by fewer points).

This paper is organized as follows: First, an explanation of the optimization algorithm is presented. Afterwards, it is presented a study on the quality of the correspondences established using the proposed algorithm when different values of matching percentages are adopted. The last section is dedicated to conclusions and future work perspectives.

2 MATCHING OPTIMIZATION AND REGISTRATION ALGORITHMS

2.1 *Matching optimization*

In Computational Vision, frequently there are matching cases in which some parts of a contour do not have correspondent parts in the contour associated; for instance, in cases of large deformations or partial occlusions. In these cases, the traditional optimization algorithms that force the matching for all points often generate wrong matches. To overcome this, we developed an assignment algorithm that only matches a percentage of the points involved, the ones with greatest affinity. The algorithm includes an order preserving constraint and was implemented using dynamic programming.

Let one suppose that there are two input contours, the template and the source, defined by two sets of ordered points with dimensions *n* and *m*, respectively; an affinity (or matching cost) matrix *C*, where each element, c_{ij}, represents the cost to match point *i* from the template contour with point *j* from the source contour. Without loss of generality, suppose that $n \leq m$ and the goal is to match only *p* points ($p \leq n$).

Adopting the traditional dynamic programming notation, let one define the stage variable *k*, the state variable *s* and the function of minimum cost $f_k(s)$. The value of *k* indicates the correspondence number that is being considered ($k \leq p$). The value of *s* defines each possible correspondence for each value of the stage variable *k*:

$$s \leq (m - p + 1) \times (n - p + 1). \tag{1}$$

Finally, $f_k(s)$ represents the total cost to establish the correspondences $1, 2, ..., k$, considering that the *k*-correspondence is the one defined by the value of *s*.

Notice that $f_k(s)$ is defined using recurrence, thus, each value of $f_k(s)$ also depends on the values previously determined in stages $k = 1,2,...,k-1$. For each stage, we have:

$$f_k(s) = f_{k-1}(r) + c_{ij}, \tag{2}$$

where $r \leq s$ and *i* and *j* depend on the *s* and *k*. Because the points' order must be preserved, both indices of c_{ij} in Equation 2 must be superior to the ones considered in the previous stage (stage $k-1$).

Each value $f_k(s)$ is kept in a table with *p* rows and $(m - p + 1) \times (n - p + 1)$ columns. After calculating all $f_k(s)$ (in total, there are $p \times (m - p + 1) \times (n - p + 1)$ values), the next step is to perform a search in the table built in order to find the matching of minimum cost.

The matching obtained using the process described above preserves the absolute order and is the one that has the minimum cost. However, there are no guarantees that this matching is the matching of minimum cost that preserves the circular order. To solve this problem, the points of template contour are reordered. Thus, *point 2* becomes *point 1*, *point 3* becomes *point 2* and so on, and finally *point 1* becomes *point n*. Then, the new matching of minimum cost is determined. The step to reorder the points and determine the matching of minimum cost is repeated $n-1$ times. Finally, the matching of minimum cost is chosen among all matching of minimum costs found that preserves the successive absolute orders. Therefore, the matching chosen is the one that has the minimum cost that preserves the circular order.

The computational complexity of this global algorithm is:

$$n \times p \times (m - p + 1) \times (n - p + 1). \tag{3}$$

If $p = n$, that is, all points of the template contour are matched, the computational complexity is $n^2 \times (m - n + 1)$ as the algorithm proposed in (Oliveira & Tavares 2008).

To facilitate the understanding of the solution proposed, let us consider the following example.

Example:

Suppose that there is a matching cost matrix *C* of dimension 4×5 (represented by the table considered in Figure 1) and the goal is to match just 3 points. To find the "global" matching of minimum cost, for each absolute order of the points, there are 3 stages (equal to the number of matches pretended) and for each stage there are 6 states, $(5-3+1) \times (4-3+1)$, Equation 1. In the total, it is considered 4 different absolute orders (equal to the number of matrix lines). In Figure 1 can be seen the cost matrix elements used to search and calculate the matching cost for the first and second absolute orders.

Reordering the rows and applying the same formulation, one can determine all the matchings of minimum cost that preserve the 4 absolute orders defined. The matchings obtained and the respective costs for each absolute order, based on the original numeration of the points, are (first row represents the points of the template contour and second row represents the points of the source contour):

$$\underset{\text{cost}=5}{\begin{pmatrix}1&3&4\\2&3&5\end{pmatrix}};\ \underset{\text{cost}=4}{\begin{pmatrix}2&3&1\\2&3&5\end{pmatrix}};\ \underset{\text{cost}=6}{\begin{pmatrix}4&1&2\\2&3&4\end{pmatrix}};\ \underset{\text{cost}=5}{\begin{pmatrix}4&2&3\\2&4&5\end{pmatrix}}.$$

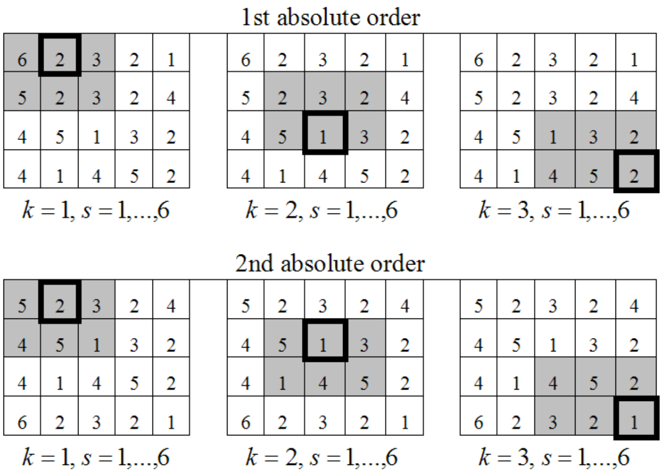

Figure 1: Illustration of the searching for the matching of minimum cost for the first and second absolute orders. This example is based in a cost matrix of dimension 4×5 and it is supposed that are desired just 3 matches. (For each matrix, the cells with gray background represent the possible states s for each stage k and for each absolute order. For each absolute order, the cells with highlighted contour represent the ones selected for the matching of minimum cost and their sum represents the total cost.)

One should notice that if the points are rearranged according on two circumferences, none of the matchings originate crossed correspondences, Figure 2.

If the order constraint was not imposed, the matching of minimum cost will be 3 (lesser than the minimum cost obtained with order constraint), but crossed matches will appear, Figure 3.

2.2 *Registration*

The adopted registration algorithm consists of four steps: 1) extract each contour from the input images; 2) assemble the contours' affinity cost matrix; 3) optimize the matching of the contours' points using the optimization algorithm described; 4) compute the transformation's parameters and align the input images.

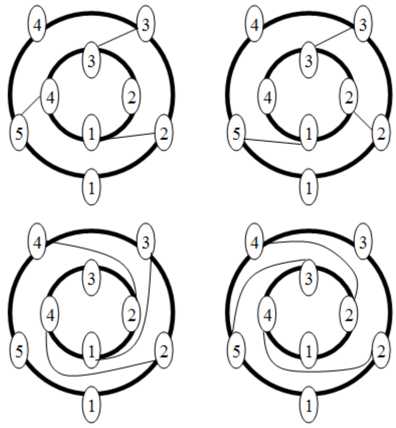

Figure 2: Illustration of the matching for Example considered. On the left side, the matching obtained for the first absolute order and, on the right side, for the second absolute order. (The smaller circumferences represent the template contour and the larger ones represent the source contour; the thin lines represent the correspondences.)

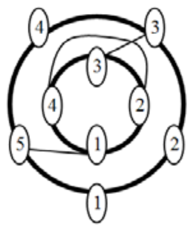

Figure 3: Illustration of the minimum cost matching for the Example considered if the order constraint was not imposed.

To obtain the contours from the images to be aligned, two different approaches were considered. For images of classes *hammer*, *tool* and *hand*, we performed a simple extraction of the boundary points and then applied a contours tracking algorithm to order the extracted points. To extract the contours from the pedobarographic images, in other words, the sets of points that represent constant pressure from the interaction foot/ground, the first step was image binarization followed by a morphological closing operation using a circular structuring element of radius equal to 3 pixels. Then, the boundary points were extracted and a global virtual contour was defined, Figure 4.

Next, following the approach considered in (Oliveira & Tavares, 2009), for each pair of images to be aligned, a contours' affinity cost matrix C is assembled, which describes the similarity between the template and source contours, considering information on contours' curvature and distance of each contours' point to the respective centroid. Thus, each element c_{ij} of C represents the matching cost between point i of the template contour and point j of the source contour; bigger c_{ij} values indicate smaller affinity between the respective points. Next, the optimization algorithm here presented performs a search on matrix C for the global matching of minimum cost given the number of matches pretended and preserving the circular order.

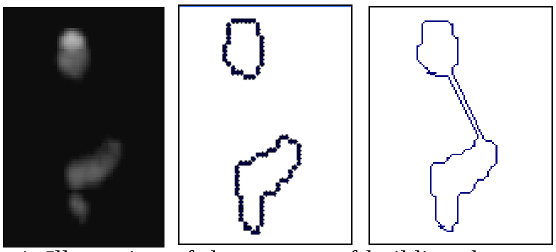

Figure 4: Illustration of the process of building the contour to be considered from each pedobarographic image: On the left side, the original image; on the middle, two sets of boundary points extracted from the left image and, on the right side, the final virtual global contour considered. (In the image on the left side, intensity peak pressure was converted to a gray scale using a linear transformation.)

Afterwards, the geometric transformation that aligns the contours involved and registers the images associated is determined. For classes *hammer*, *tool* and *hand* only similarity transformations (rotations, translations and uniform scaling) were allowed as in (Oliveira & Tavares, 2009). For pedobarographic images only rigid transformations (rotations and translations) were allowed as in (Oliveira et al., in prep.).

3 QUALITY ASSESSMENT

To validate the proposed optimization algorithm when applied to optimize the global matching between the contours of two objects, several experiments were performed. Their main goal was to find the best global correspondence, in terms of minimum global cost, between two contours defined by ordered points and use it to estimate the geometric transformation that best aligns the two input images.

3.1 *Data*

Two sets of data were used. The first set was organized in three classes: *tool* (41 shapes), *hammer* (32 shapes) and *hand* (17 shapes) available in the database "*silhouette database(1032 shapes)*", organized by the Laboratory for Engineering Man/Machine System (LEMS). The second set consists in a set of 30 pairs of peak pressure images from dynamic pedobarography used on a previous study (Pataky et al. 2008, Oliveira et al., in prep.).

3.2 *Registration quality assessment*

Registration quality was assessed on experimentally variable images (i.e. transformation parameters unknown *a priori*) by visual analysis on the matchings and alignments for the images of classes *hammer*, *tool* and *hands*; exclusive-or (XOR), (Pataky et al. 2008), for pedobarographic images; and registration duration (ms) for all images. The value of XOR indicates the percentage of non-zero pixels that overlap zero pixels. Thus, smaller XOR values indicate smaller proportions of non-overlapping pixels and, consequently, better alignment.

To validate the matching optimization algorithm, several experiments were accomplished for different percentage of matches. It was considered percentages between 95% and 100%. Percentages smaller than 95% were not considered as the method used to build the cost matrices is unsuitable for those cases. For each class *hammer* and *tool*, more than 100 matching/alignment experiments were performed.

The current algorithm was implemented in C++, using Microsoft Visual Studio 6 and were tested on a notebook PC with an AMD Turion64 2.0GHz microprocessor, 1.0GB of RAM, and running Microsoft Windows XP.

4 RESULTS

4.1 *Silhouette database images*

Small differences were observed on the matchings established when the matching percentage varies from 95% to 100%. In some cases, a slight improvement on the matching found was enough to significantly enhance the final alignment of the input images, see Figure 5.

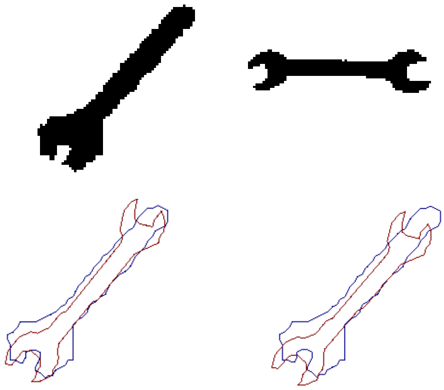

Figure 5: On the top row, two images of the class *tool*. On the bottom row, their contours after alignment (on the left side just 95% of the points were matched and on the right side 100% of the points were matched). (The template contour is represented in blue and the source contour is represented in red.)

For class *hand*, also small differences were observed in major part of the matchings found. However, for images "*hand01*" and "*hand02-1*", the matching found and consequently, the alignment established, were without sense when 100% of the points were matched. However, when just 95% of the points were matched, the matching's quality improves enough to obtain a good alignment from the same ones, Figure 6.

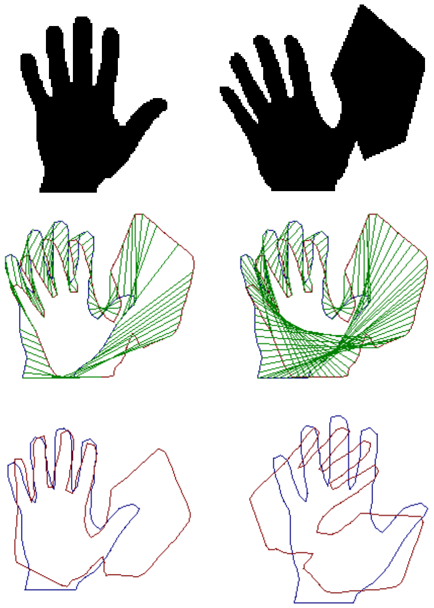

Figure 6: Two images of the class *hand*, the matching and alignment obtained. On the top row, the original input images. On the middle row, the contours on original position and the matching obtained using 95% (on the left side) and using 100% of the points (on the right side). On the bottom row, the alignment obtained using 95% (on the left side) and using 100% of the points (on the right side). (The template contour is represented by blue line, the source contour is represented by red line and the matches found are represented by green line.)

4.2 *Pedobarographic images*

For pedobarographic images, the contours were obtained using a pressure threshold level $\delta = 0.001$ N/cm^2, the minimum value possible for the data used. The pedobarographic images used are of low resolution (45×62 pixels) and therefore, it was possible to analyze each correspondence established and detect the small differences associated to different matching's percentages. In Figure 7 an example of those differences is shown.

By visual analysis was impossible to detect any difference in the alignment of pedobarographic images. However, using XOR as dissimilarity measure, some differences were observed, Figure 8.

In Figure 9 the computational time required for the registration of the pedobarographic images is presented.

5 CONCLUSIONS

For classes *hammer* and *tool*, just small differences were observed between the correspondences found when 95% and 100% of the points were forced to match. However, in some cases significant improvements were obtained in the final alignment when lesser than 100% was considered. For class *hand*, the resultant correspondences and alignments were practically equal when 95% and 100% of the points were matched. However, when the image "*hand02-1*" was used, the alignment found when 100% of the points were matched did not have any sense at all, but it was good when just 95% of the points were matched.

For pedobarographic images, when all points were matched, some wrong matches appear in some situations. However, only point-wise matching was incorrect; image-wise matchings were, as mentioned above, visually indistinguishable. In the case considered in Figure 7, as the heels to be matched present different numbers of points, when all points are forced to be matched, one point from the foot heel was forced to wrongly match with one point of the foot palm.

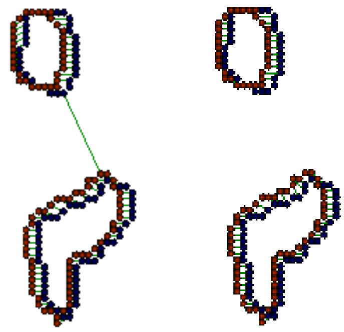

Figure 7: Two examples of global matchings found: On the left side, the matching obtained when all points are forced to be matched, and on the right side the matching obtained when just 99% of the points are forced to be matched. (The template contour is represented by blue points, the source contour by red points and the correspondences found by green lines.)

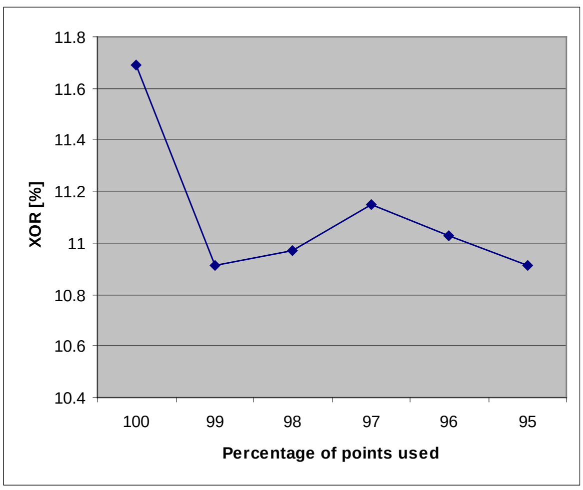

Figure 8: Values of XOR for different matching's percentages.

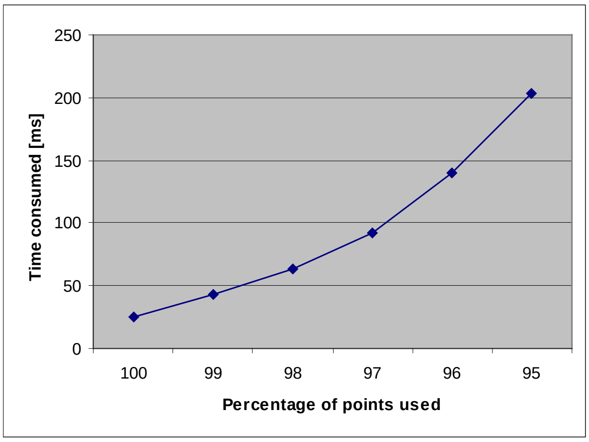

Figure 9: Computational time required for different matching's percentages.

The computational time required increased considerably when the percentage of points forced to be matched diminished. This fact is in agreement with its computational complexity.

The results obtained show that enhanced results can be obtained when not all contours' points are forced to be matched; in particularly, in cases in which there are considerable local deformations.

The cost matrices considered in the experiments performed are suitable for ordered contours with a predominant similarity geometric deformation. Thus, to maintain the matchings' robustness, it is essential that almost all points are forced to match. Because of that, the smaller percentage of points matched considered in this work was 95%. However, for other kinds of cost matrices, it is possible that percentages smaller than 95% originate better and more stable results.

In the near future, the optimization methodology presented in this paper is going to be adopted to match and align organs presented in 2D medical images.

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