Electromagnetic properties of the $\Delta(1232)$ and decuplet baryons in the self-consistent SU(3) chiral quark-soliton model

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We examine the electromagnetic properties of the $\Delta(1232)$ resonance within the self-consistent chiral quark-soliton model. In particular we present the Δ form factors of the vector-current $G_{E0}(Q^2)$, $G_{E2}(Q^2)$ and $G_{M1}(Q^2)$ for a momentum-transfer range of $0 \leq Q^2 \leq 1\,\mathrm{GeV}^2$. We apply the symmetry-conserving quantization of the soliton and take $1/N_c$ rotational corrections into account. Values for the magnetic moments of all decuplet baryons as well as for the $N-\Delta$ transition are given. Special interest is also given to the electric quadrupole moment of the Δ.

I. INTRODUCTION

The hadron spectrum can be ordered by flavor-$SU(3)$ multiplets where the low lying baryons are assigned to either an octet or decuplet with spin 1/2 and 3/2, respectively. The main focus of this work is the hyper-charge +1 state of the decuplet, the Δ. Eventhough the Δ is the first excitation of the proton and rather isolated from other resonances, due to its short life time many of its properties are not yet experimentally determined with accurate precision. This is reflected in the poor experimental knowledge of the magnetic moment of the Δ which is listed by the Particle Data Group as $\mu_{\Delta^{++}} = 3.7 \sim 7.5\mu_N$ and $\mu_{\Delta^+} = (2.7^{+1.0}_{-1.3}(\text{stat.}) \pm 1.5(\text{syst.}) \pm 3(\text{theor.}))\,\mu_N$, where $\mu_N = e/2M_N$ is the nucleon magneton [1]. The former value is extracted from the reaction $\pi^+ p \to \pi^+ p\gamma$, e.g. [2, 3], and the latter one from the process $\gamma p \to p\pi^0\gamma'$ [4]. The study of the transition process of the nucleon to the Δ can be used to gain additional information about the $N\Delta$ system. This process is characterized by a magnetic dipole and an electric quadrupole transition moment which are in [5] extracted as $\mu_{N\Delta} = 3.46 \pm 0.03\,\mu_N$ and $Q_{N\Delta} = -(0.0846 \pm 0.0033)\,e\mathrm{fm}^2$, respectively. Appart from the Δ, experimental data on electromagnetic properties of decuplet baryons only exist for the magnetic moment of the Ω^- baryon $\mu_{\Omega^-} = (-2.02 \pm 0.05)\,\mu_N$ [1].

On the theoretical side, the Δ was investigated within many different frameworks. In the case of $SU(6)$ symmetry the Δ magnetic moment is predicted to be $\mu_\Delta = Q_\Delta \mu_p$, with Q_Δ being the charge of the Δ and μ_p the magnetic moment of the proton, which yields a value of $\mu_{\Delta^{++}} = 5.58\,\mu_N$ [6]. Other approaches include quark models [7, 8, 9, 10, 11, 12, 13], large N_c and soliton models [14, 15, 16], lattice QCD calculations [17, 18, 19, 20], QCD sum rules and chiral perturbation theory [21, 22, 23, 24, 25, 26]. Very recently lattice QCD calculations of electromagnetic form factors of the Δ up to a momentum-transfer of $Q^2 \leq 2.5\,\mathrm{GeV}^2$ were presented in [27]. In addition, large N_c relations which connect the magnetic moments of the octet and the electric quadrupole moments of the $N\Delta$ transition to the moments of the Δ are found in [28, 29, 30].

In the present work we investigate the electromagnetic form factors of the $\Delta^+(1232)$ in the framework of the self-consistent chiral quark-soliton model (χQSM) assuming iso-spin symmetry. In particular we calculate the charge (G_{E0}), electric quadrupole (G_{E2}) and magnetic dipole (G_{M1}) form factors of the Δ^+ up to a momentum-transfer

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of $0 \leq Q^2 \leq 1\,\mathrm{GeV}^2$. We also present values for the magnetic moments of all decuplet baryons as well as for the $N-\Delta$ transition. In the χQSM baryons are seen as certain $SU(3)$ rotations of a classical soliton, having therefore the same origin. The quantization of these rotations allows only $SU(3)$ multiplets with zero triality, hence the octet and decuplet appear naturally. Because of this, the χQSM is able to describe various observables of various baryons within the same set of parameters. These parameters are fixed by reproducing mesonic experimental data, letting the constituent quark mass to be the only free paramter in the baryon sector. Since we can not take an exact form of the momentum-dependent constituent quark mass we use the value of $M = 420\,\mathrm{MeV}$ which is known to reproduce very well the experimental data [31, 32, 33, 34, 35]. The regularization behavior of the momentum-dependence is mimicked by the proper-time regularization. The cut-off parameter and the averaged current quark mass are then fixed for a given M to the pion decay constant f_π and m_π, respectively. The model parameters used in the present work are the same as in previous works [34, 35, 36, 37, 38, 39, 40, 41, 42], no additional readjusting for different observables were done. Given that, the χQSM, with model-parameters fixed in the meson-sector and natural inclusion of octet and decuplet baryons, provides a unique framework with predictive power.

In the past the χQSM was applied successfully to the octet baryon (axial) vector form factors [32, 33, 34, 35, 36, 37, 39], parton- and antiparton-distributions [43, 44, 45, 46, 47, 48, 49, 50]. Furthermore, the χQSM was also applied to observables of the anti-decuplet pentaquarks [40, 41, 42, 51, 52, 53, 54, 55]. The vector current of decuplet baryons at $Q^2 = 0$ were investigated in various versions of the χQSM in the past: in the self-consistent χQSM [56, 57], in the χQSM version formulated in the infinite momentum frame [58] and in the so-called *model independent* χQSM version [55]. Both self-consistent χQSM calculations in the literature, which presented the decuplet magnetic moments, were prior to the symmetry-conserving quantization of the χQSM [59] which is explicitly applied in this work and ensures the realization of the Gell-Mann-Nishijima relation in the model.

The outline of this work is as follows. In the section II we give the general, model-independent expressions for the observables in question. The given formulae at the end of this section are suitable for calculation in the χQSM. Section III then describes how these expressions are treated in the model. Final results for the self-consistent χQSM are given in section IV. We summarize the work in section V and give more detailed expressions in the appendix.

II. GENERAL FORMALISM

Our aim is to investigate the $\Delta(1232)$ electromagnetic form factors and compare them to nucleon electromagnetic form factors and the $N-\Delta$ magnetic transition moment in the self-consistent $SU(3)$ χQSM. For that, we will summarize in this section the relevant model-independent definitions of these quantities. The form factors are defined through the baryon matrix-element of the vector-current where the virtual photon couples to the NN, $N\Delta$ and $\Delta\Delta$ systems.

A. The $\gamma^* NN$ Vertex

The baryon matrix element of the vector-current, $V^{\mu\chi}(0) = \overline{\Psi}(0)\gamma^\mu\Psi(0)$, between nucleon states is parametrized by two form factors $F_1(Q^2)$ and $F_2(Q^2)$

$$\langle N(p',s')|V^\mu(0)|N(p,s)\rangle = \overline{u}(p,s)\Big[F_1(Q^2)\gamma^\mu + iF_2(Q^2)\frac{\sigma^{\mu\beta}q_\beta}{2M_N}\Big]u(p,s)\ , \tag{1}$$

with $q = p' - p$, $Q^2 = -q^2$, $u(p,s)$ as the nucleon-spinor of mass M_N and third-spin component s. In the Breit-frame the Sachs form factors are defined as

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M_N^2}F_2(Q^2) \qquad\qquad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)\ , \tag{2}$$

which are projected out by the operations

$$G_E(Q^2) \ = \ \int \frac{d\Omega_q}{4\pi}\langle N(p',\frac{1}{2})|V^0(0)|N(p,\frac{1}{2})\rangle\ , \tag{3}$$

$$G_M(Q^2) \ = \ 3M_N \int \frac{d\Omega_q}{4\pi}\,\frac{q^i\epsilon^{ik3}}{i\mid\vec{q}^{\,2}\mid^2}\langle N(p',\frac{1}{2})|V^k(0)|N(p,\frac{1}{2})\rangle\ , \tag{4}$$

where we have in the Breit-frame $Q^2 = \vec{q}^{\,2}$. The right-hand side of these equations can be evaluated in the χQSM.

B. The $\gamma^* N\Delta$ Vertex

We take the rest-frame of the final Δ with momentum $p' = (M_\Delta, 0)$ and mass M_Δ. The incoming nucleon has the momentum $p = (E_N, -\vec{q})$ and energy E_N. For the $\gamma^* N\Delta$-Vertex we use the decomposition of [60, 61]. The baryon-matrix element is written by using the Rarita-Schwinger spinors $u^\alpha(p,s)$ for the Δ as

$$\begin{aligned}\langle\Delta(p',\frac{1}{2})|V_\mu(0)|N(p,\frac{1}{2})\rangle &= i\sqrt{\frac{2}{3}}\,\overline{u}^\beta(p',\frac{1}{2})\;\Gamma_{\beta\mu}\;u(p,\frac{1}{2}) &(5)\\ \Gamma_{\beta\mu} &= G_M^{N\Delta}(Q^2)\mathcal{K}^M_{\beta\mu} + G_E^{N\Delta}(Q^2)\mathcal{K}^E_{\beta\mu} + G_C^{N\Delta}(Q^2)\mathcal{K}^C_{\beta\mu}\;, &(6)\end{aligned}$$

with the magnetic dipole ($G_M^{N\Delta}$), electric quadrupole ($G_E^{N\Delta}$) and Coulomb quadrupole ($G_C^{N\Delta}$) form factors. The corresponding structures are

$$\begin{aligned}\mathcal{K}^M_{\beta\mu} &= \frac{-3(M_\Delta + M_N)}{[(M_\Delta+M_N)^2+Q^2]2M_N}\epsilon_{\beta\mu\sigma\tau}P_\sigma q_\tau &(7)\\ \mathcal{K}^E_{\beta\mu} &= -\mathcal{K}^M_{\beta\mu} + \frac{6}{4M_\Delta^2|\vec{q}|^2}\epsilon_{\beta\sigma\nu\gamma}P_\nu q_\gamma \epsilon_{\mu\sigma\alpha\delta}p'_\alpha q_\delta i\gamma^5\frac{M_\Delta+M_N}{M_N} &(8)\\ \mathcal{K}^C_{\beta\mu} &= 3\Delta^{-1}(q^2)q_\beta\Big[q^2P_\mu - q\cdot Pq_\mu\Big]i\gamma^5\frac{M_\Delta+M_N}{M_N}\;, &(9)\end{aligned}$$

where the momenta are defined as $P = \frac{1}{2}(p'+p)$, $q = p' - p$ and $\Delta^{-1}(q^2) = 4M_\Delta^2|\vec{q}|^2$. We are interested in the magnetic transition moment of the $N\to\Delta$ process. We will use again the projector $3\int\frac{d\Omega_q}{4\pi}\frac{q^i\epsilon^{ik3}}{i|\vec{q}|^2}$ for which the term $\mathcal{K}^C_{\beta k}$ vanishes.
Applying the projector on the baryon matrix-element leads to

$$\begin{aligned}&3\int\frac{d\Omega}{4\pi}\frac{q^i\epsilon^{ik3}}{i|\vec{q}^2|}\langle\Delta(p',\frac{1}{2})|V_k(0)|N(p,\frac{1}{2})\rangle\\ &= \sqrt{\frac{E_N+M_N}{2M_N}}2(M_\Delta+M_N)\Big[\frac{M_\Delta}{M_N}\frac{G_M^{N\Delta}(Q^2)-G_E^{N\Delta}(Q^2)}{[(M_\Delta+M_N)^2+Q^2]} + \frac{1}{2M_N}\frac{G_E^{N\Delta}(Q^2)}{E_n+M_N}\Big]\;. &(10)\end{aligned}$$

The electromagnetic $N\to\Delta$ transition is dominated by the form factor $G_M^{N\Delta}$ (exp. $G_E^{N\Delta}/G_M^{N\Delta} = (-2.5\pm0.5)\%$ [1]. Neglecting the $G_E^{N\Delta}(Q^2)$ contribution and taking the point $Q^2=0$ we have

$$3\int\frac{d\Omega}{4\pi}\frac{q^i\epsilon^{ik3}}{i|\vec{q}^2|}\langle\Delta(p',\frac{1}{2})|V_k(0)|N(p,\frac{1}{2})\rangle|_{Q^2=0} = \sqrt{\frac{E_N(0)+M_N}{2M_N}}\frac{M_\Delta}{M_N}\frac{2}{(M_\Delta+M_N)}G_M^{N\Delta}(0)\;. \quad (11)$$

The magnetic transition moment is given by [61]

$$\begin{aligned}\mu_{N\Delta} &= \sqrt{\frac{M_\Delta}{M_N}}\,G_M^{N\Delta}(0)\,\mu_N\;, &(12)\\ Q_{N\Delta} &= -\frac{6}{M_N}\frac{2M_\Delta}{M_\Delta^2-M_N^2}\sqrt{\frac{M_\Delta}{M_N}}\,G_E^{N\Delta}(0)\;. &(13)\end{aligned}$$

Although we will denote the quadrupole moment in units of fm^2 in this paper, it is understood that the electric quadrupole moment is expressed in units of $e\mathrm{fm}^2$, with e as the electric charge.
These above equations can be investigated in the χQSM.

C. The $\gamma^*\Delta\Delta$ Vertex

The baryon matrix element of the vector-current, $V^\mu(0) = \overline{\Psi}(0)\,\gamma^\mu\,\Psi(0)$, between Δ-states is parametrized by four form factors

$$\langle\Delta(p',s')|V^\mu(0)|\Delta(p,s)\rangle \;=\; -\overline{u}^\alpha(p',s')\Big\{ \quad \gamma^\mu\Big[F_1^* g_{\alpha\beta} + F_3^*\frac{q_\alpha q_\beta}{(2M_\Delta)^2}\Big] + i\frac{\sigma^{\mu\nu}q_\nu}{2M_\Delta}\Big[F_2^* g_{\alpha\beta} + F_4^*\frac{q_\alpha q_\beta}{(2M_\Delta)^2}\Big] \quad \Big\} u^\beta(p,s) \;. \tag{14}$$

The electric charge and quadrupole form factors G_{E0}, G_{E2} and magnetic dipole and octupole form factors G_{M1},G_{M3} are defined in the Breit-frame by

$$G_{E0}(Q^2) \;=\; (1+\frac{2}{3}\tau)\Big[F_1^* - \tau F_2^*\Big] - \frac{1}{3}\tau(1+\tau)\Big[F_3^* - \tau F_4^*\Big] \tag{15}$$

$$G_{E2}(Q^2) \;=\; \Big[F_1^* - \tau F_2^*\Big] - \frac{1}{2}(1+\tau)\Big[F_3^* - \tau F_4^*\Big] \tag{16}$$

$$G_{M1}(Q^2) \;=\; (1+\frac{4}{5}\tau)\Big[F_1^* + F_2^*\Big] - \frac{2}{5}\tau(1+\tau)\Big[F_3^* + F_4^*\Big] \tag{17}$$

$$G_{M3}(Q^2) \;=\; \Big[F_1^* + F_2^*\Big] - \frac{1}{2}(1+\tau)\Big[F_3^* + F_4^*\Big] \;, \tag{18}$$

with $\tau = \frac{Q^2}{4M_\Delta^2}$. We will concentrate in this work on the form factors G_{E0}, G_{E2} and G_{M1} and postpone the discussion on G_{M3} for future work. The zeroth-component of the matrix-element Eq.(14) for both Δ having a third-spin component of $+3/2$ reads

$$\langle\Delta(p',\frac{3}{2})|V^0(0)|\Delta(p,\frac{3}{2})\rangle \;=\; G_{E0}(Q^2) - \tau\frac{2}{3}\sqrt{\frac{4\pi}{5}}Y_{20}(\Omega_q)G_{E2}(Q^2) \;, \tag{19}$$

where the projections on G_{E0} and G_{E2} are given by

$$G_{E0}(Q^2) \;=\; \int\frac{d\Omega_q}{4\pi}\langle\Delta(p',\frac{3}{2})|V^0(0)|\Delta(p,\frac{3}{2})\rangle \tag{20}$$

$$G_{E2}(Q^2) \;=\; -\int d\Omega_q\sqrt{\frac{5}{4\pi}}\frac{3}{2}\frac{1}{\tau}\langle\Delta(p',\frac{3}{2})|Y_{20}^*(\Omega_q)V^0(0)|\Delta(p,\frac{3}{2})\rangle \;. \tag{21}$$

Using the projector $3\int\frac{d\Omega}{4\pi}\frac{q^i\epsilon^{ik3}}{i|\vec{q}^2|}$ on the Δ-matrix element Eq.(14) gives

$$3\int\frac{d\Omega}{4\pi}\frac{q^i\epsilon^{ik3}}{i|\vec{q}^2|}\langle\Delta(p',\frac{3}{2})|V^k(0)|\Delta(p,\frac{3}{2})\rangle \;=\; \frac{1}{M_\Delta}\Big[[1+\frac{4}{5}\tau][F_1^*+F_2^*] - \tau\frac{1+\tau}{2}\frac{4}{5}[F_3^*+F_4^*]\Big] \;=\; \frac{1}{M_\Delta}G_{M1}(Q^2) \;. \tag{22}$$

The magnetic moment of the Δ is given by [61]

$$\mu_\Delta = \frac{M_N}{M_\Delta}G_{M1}(0)\mu_N \;, \tag{23}$$

and the electric quadrupole moment by

$$Q_\Delta = \frac{1}{M_\Delta^2}G_{E2}(0) \;. \tag{24}$$

We will also denote Q_Δ, like $Q_{N\Delta}$ in the section before, in units of fm^2. The projectors which in the nucleon case project on the electric and magnetic form factors, project in the Δ case on the electric charge and magnetic dipole form factors. We will investigate Eqs.(20,21,22) in the χQSM.

III. FORM FACTORS IN THE CHIRAL QUARK-SOLITON MODEL

We will now briefly describe how equations like Eqs.(3,4,11,20,21,22) are evaluated in the SU(3) χQSM. For details we refer to Ref.[31, 32, 33]. The main part of the form factors come from the baryonic matrix element

$$\langle B'(p')|\mathcal{J}^{\mu\chi}(0)|B(p)\rangle = \langle B'(p')|\Psi^\dagger(0)\mathcal{O}^{\mu\chi}\Psi(0)|B(p)\rangle, \quad (25)$$

where the explicit form of the operator $\mathcal{J}^{\mu\chi} = \Psi^\dagger(0)\mathcal{O}^{\mu\chi}\Psi(0)$ (χ being a flavor index) are given by the projector in question

$$\begin{aligned}
\mathcal{J}^{\mu\chi} &\rightarrow 1 \text{ for the rotational Hamiltonian} && (26)\\
\mathcal{J}^{\mu\chi} &\rightarrow \int \frac{d\Omega}{4\pi}\langle B'(p')|\Psi^\dagger(0)\gamma^0\gamma^0\Psi(0)|B(p)\rangle \text{ for } G_E && (27)\\
\mathcal{J}^{\mu\chi} &\rightarrow \int d\Omega_q\langle B'(p')|\Psi^\dagger(0)\gamma^0\gamma^0 Y^*_{20}(\Omega_q)\Psi(0)|B(p)\rangle \text{ for } G_{E2} && (28)\\
\mathcal{J}^{\mu\chi} &\rightarrow \int \frac{d\Omega}{4\pi}\langle B'(p')|\Psi^\dagger(0)\gamma^0[\vec{q}\times\vec{\gamma}]_z\Psi(0)|B(p)\rangle \text{ for } G_M,\ G_M^{N\Delta},\ G_{M1}\ . && (29)
\end{aligned}$$

The matrix-element Eq.(25) will be treated in the path-integral formalism with the following effective partition function of the quark and chiral fields Ψ and $U(x)$, respectively:

$$\begin{aligned}
\mathcal{Z}_{\chi\text{QSM}} &= \int \mathcal{D}\psi\mathcal{D}\psi^\dagger\mathcal{D}U \exp\left[-\int d^4x\Psi^\dagger iD(U)\Psi\right] = \int \mathcal{D}U\exp(-S_{\text{eff}}[U])\ , && (30)\\
S_{\text{eff}}(U) &= -N_c\text{Tr}\ln iD(U)\ , && (31)\\
D(U) &= \gamma^4(i\partial\!\!\!/ - \hat{m} - MU^{\gamma_5}) = -i\partial_4 + h(U) - \delta m\ , && (32)\\
\delta m &= \frac{-\overline{m}+m_s}{3}\gamma^4\mathbf{1} + \frac{\overline{m}-m_s}{\sqrt{3}}\gamma^4\lambda^8 = M_1\gamma^4\mathbf{1} + M_8\gamma^4\lambda^8, && (33)
\end{aligned}$$

where the Tr represents the functional trace, N_c the number of colors, D the Dirac differential operator in Euclidean space and $\hat{m} = \text{diag}(\overline{m}, \overline{m}, m_s) = \overline{m} + \delta m$ the current quark mass matrix of the average of the up- and down-quark mass and strange quark mass, respectively. We assume iso-spin symmetry. The SU(3) single-quark Hamiltonian $h(U)$ is given by

$$\begin{aligned}
h(U) &= i\gamma^4\gamma^i\partial_i - \gamma^4 MU^{\gamma_5} - \gamma^4\overline{m}\ , && (34)\\
U^{\gamma_5}(x) &= \begin{pmatrix} U^{\gamma_5}_{\text{SU}(2)}(x) & 0\\ 0 & 1\end{pmatrix}\ , && (35)\\
U^{\gamma_5}_{SU(2)} &= \exp(i\gamma^5\tau^i\pi^i(x)) = \frac{1+\gamma^5}{2}U_{SU(2)} + \frac{1-\gamma^5}{2}U^\dagger_{SU(2)}\ , && (36)
\end{aligned}$$

where we use Witten's embedding of the SU(2) field $U(x)_{SU(2)} = \exp(i\tau^i\pi^i(x))$ into the SU(3). The $\pi^i(x)$ denote the pion-fields. We use the factor of N_c in Eq.(31) in the large N_c limit to integrate the chiral-field in Eq.(30) with the saddle-point approximation. For that we have to find the pion field that minimizes the action in Eq.(31). Generally the following Anstze for the chiral-field $U(x)$ and the baryon state $|B\rangle$ in Eq.(25) are made:

$$\begin{aligned}
U_{\text{SU2}} &= \exp[i\gamma_5\hat{n}\cdot\vec{\tau}P(r)] \quad \text{and} \quad |B(p)\rangle = \lim_{x_4\to-\infty}\frac{1}{\sqrt{\mathcal{Z}}}e^{ip_4x_4}\int d^3\vec{x}\, e^{i\vec{p}\cdot\vec{x}}\, J^\dagger_B(x)\,|0\rangle\ , && (37)\\
\text{with} \quad & J_B(x) = \frac{1}{N_c!}\Gamma_B^{b_1\cdots b_{N_c}}\,\varepsilon^{\beta_1\cdots\beta_{N_c}}\,\psi_{\beta_1 b_1}(x)\cdots\psi_{\beta_{N_c}b_{N_c}}(x)\ . && (38)
\end{aligned}$$

The first equation assumes the SU(2) field U to have the most symmetric form, a hedgehog form, with the radial pion profile function $P(r)$ while the last two take the baryon state as an Ioffe-type current consisting of N_c valence quarks. The matrix $\Gamma_B^{b_1\ldots b_{N_c}}$ carries the hyper-charge Y, isospin I, I_3 and spin J, J_3 quantum numbers of the baryon and the b_i and β_i denote the spin-flavor- and color-indices, respectively.
Applying the above treatments to the baryonic matrix element Eq.(25) yields:

$$\begin{aligned}
\langle B_2(p_2)|\mathcal{J}^{\mu\chi}(0)|B_1(p_1)\rangle &= \frac{1}{\mathcal{Z}}\lim_{T\to\infty} e^{-ip_2^4\frac{T}{2}+ip_1^4\frac{T}{2}}\int d^3\vec{x}'d^3\vec{x}e^{i\vec{p}_1\cdot\vec{x}-i\vec{p}_2\cdot\vec{x}'}\\
&\times\int \mathcal{D}U\mathcal{D}\psi^\dagger\mathcal{D}\psi J_{B'}\left(\frac{T}{2},\vec{x}'\right)\mathcal{J}^{\mu\chi}(0)J^\dagger_B\left(-\frac{T}{2},\vec{x}\right)\ \exp\left[-\int d^4x\,\psi^\dagger iD(U)\psi\right]\ . && (39)
\end{aligned}$$

Finding the minimizing chiral-field configuration U_c, the soliton, corresponds to determine its profile function $P_c(r)$. This is done by setting $\mathcal{J}^{\mu\chi}(0) = 1$ in Eq.(39). For large Euclidean times, $T \to \infty$, the expression is proportional to the nucleon correlation function from which we can obtain the χQSM expression for the nucleon mass. Solving numerically the equation of motion coming from $\delta S_{eff}/\delta P(r) = 0$ (minimizing the χQSM nucleon energy) in a self-consistent approach determines the function $P_c(r)$.
Rotations and translations of the soliton also minimize the effective action and are written as

$$U(\vec{x}, t) = A(t)U_c(\vec{x} - \vec{z}(t))A^\dagger(t) \ , \tag{40}$$

where $A(t)$ denotes a time-dependent SU(3) matrix and $\vec{z}(t)$ stands for the time-dependent translation of the center of mass of the soliton in coordinate space. Sofar, we considered only the classical version of the χQSM which has to be quantized. Suitable quantum numbers are now obtained by quantizing the rotational zero-mode. A detailed formalism can be found in Refs.[31, 33].
The Dirac operator of Eq.(32) written in terms of the soliton U_c and its zero-modes acquires the form:

$$D(U) = T_{z(t)}A(t)\left[D(U_c) + i\Omega(t) - \dot{T}^\dagger_{z(t)}T_{z(t)} - i\gamma^4 A^\dagger(t)\delta m A(t)\right]T^\dagger_{z(t)}A^\dagger(t), \tag{41}$$

where the $T_{z(t)}$ denotes the translational operator and the $\Omega(t)$ represents the soliton angular velocity defined as

$$\Omega = -iA^\dagger\dot{A} = -\frac{i}{2}\mathrm{Tr}(A^\dagger\dot{A}\lambda^\alpha)\lambda^\alpha = \frac{1}{2}\Omega_\alpha\lambda^\alpha. \tag{42}$$

The standard way to proceed is to treat all three terms $\Omega(t)$, $\dot{T}^\dagger_{z(t)}T_{z(t)}$ and δm perturbatively by assuming a slow rotating and moving soliton and by regarding δm as a small parameter. Generally we expand Eq.(41) to the first order in $\Omega(t)$, δm and to the zeroth-order in $\dot{T}^\dagger_{z(t)}T_{z(t)}$.
After introducing the collective baryon wave function on the level of Eq.(39) as

$$\psi^{(\mathcal{R};YII_3)}_{(\mathcal{R}^*;Y'JJ_3)}(A) := \lim_{T\mapsto\infty}\frac{1}{\sqrt{\mathcal{Z}}}e^{-p^{4\prime}T/2}\int d^3\vec{u}' e^{i\vec{p}'\cdot\vec{u}'}(\Gamma^{b_1\ldots b_{N_c}}_B)^*\Pi^{N_c}_{l=1}\left[\varphi^\dagger_{v,b_l}(\vec{u}')A^\dagger\right] \ , \tag{43}$$

and expanding the occuring fermionic determinant and product of propagators and quantizing the soliton rotation, we obtain the following collective Hamiltonian [62]:

$$H_{coll} = H_{\rm sym} + H_{\rm sb} \ , \tag{44}$$

where $H_{\rm sym}$ and $H_{\rm sb}$ represent the SU(3) symmetric and symmetry-breaking parts, respectively,

$$H_{\rm sym} = M_c + \frac{1}{2I_1}\sum_{i=1}^{3}J_iJ_i + \frac{1}{2I_2}\sum_{a=4}^{7}J_aJ_a, \tag{45}$$

$$H_{\rm sb} = \frac{1}{\overline{m}}M_1\Sigma_{SU(2)} + \alpha D^{(8)}_{88}(A) + \beta Y + \frac{\gamma}{\sqrt{3}}D^{(8)}_{8i}(A)J_i \ . \tag{46}$$

The M_c denotes the mass of the classical soliton and I_i and K_i are the moments of inertia of the soliton [31], of which the corresponding expressions can be found in Ref.[63] explicitly. The components J_i denote the spin generators and J_a correspond to the generalized $SU(3)$ spin-generators. The $\Sigma_{\rm SU(2)}$ is the SU(2) pion-nucleon sigma term. The $D^{(8)}_{88}(A)$ and $D^{(8)}_{8i}(A)$ stand for the SU(3) Wigner D functions in the octet representation and the Y is the hypercharge operator. The parameters α, β, and γ in the symmetry-breaking Hamiltonian are

$$\alpha = \frac{1}{\overline{m}}\frac{1}{\sqrt{3}}M_8\Sigma_{SU(2)} - \frac{N_c}{\sqrt{3}}M_8\frac{K_2}{I_2}, \quad \beta = M_8\frac{K_2}{I_2}\sqrt{3}, \quad \gamma = -2\sqrt{3}M_8\left(\frac{K_1}{I_1} - \frac{K_2}{I_2}\right). \tag{47}$$

The collective wave-functions of the Hamiltonian in Eq.(44) can be found as SU(3) Wigner D functions in representation $\mathcal{R}$:

$$\langle A|\mathcal{R}, B(YII_3, Y'JJ_3)\rangle = \Psi^{(\mathcal{R};YII_3)}_{(\mathcal{R}^*;Y'JJ_3)}(A) = \sqrt{\dim(\mathcal{R})}\,(-)^{J_3+Y'/2}\,D^{(\mathcal{R})*}_{(Y,I,I_3)(-Y',J,-J_3)}(A). \tag{48}$$

The Y' is related to the eighth component of the angular velocity Ω. During the quantization process Y' is constrained to be $Y' = -N_c/3 = -1$. In fact, this constraint allows us to have only SU(3) representations with zero triality.
The $H_{\rm sb}$ mixes the representations for the collective baryon states and are treated by first-order perturbation by

Table I: Moments of inertia and mixing coefficients for $M = 420\,\mathrm{MeV}$.

I_1 [fm]	I_2 [fm]	K_1 [fm]	K_2 [fm]	$\Sigma_{\pi N}$ [MeV]	$c_{\overline{10}}$	c_{27}	a_{27}	a_{35}
1.06	0.48	0.42	0.26	41	0.037	0.019	0.074	0.018

$$|B_{\mathcal{R}}\rangle = |B^{\rm sym}_{\mathcal{R}}\rangle - \sum_{\mathcal{R}'\neq\mathcal{R}} |B_{\mathcal{R}'}\rangle \frac{\langle B_{\mathcal{R}'}|\, H_{\rm sb}\, |B_{\mathcal{R}}\rangle}{M(\mathcal{R}') - M(\mathcal{R})} \ . \tag{49}$$

From this we obtain the collective wave functions for the baryon octet and decuplet with inclusion of wave function correction proportional to the strange quark mass as (other wave function corrections are listed in the appendix)

$$|N_8\rangle \ = \ |8_{1/2}, N\rangle + c_{\overline{10}}\sqrt{5}|\overline{10}_{1/2}, N\rangle + c_{27}\sqrt{6}|27_{1/2}, N\rangle \ , \tag{50}$$

$$|\Delta_{10}\rangle \ = \ |10_{3/2}, \Delta\rangle + a_{27}\sqrt{\frac{15}{2}}|27_{3/2}, \Delta\rangle + a_{35}\frac{5}{\sqrt{14}}|35_{3/2}, \Delta\rangle \ , \tag{51}$$

with

$$c_{\overline{10}} = -\frac{I_2}{15}\Big(\alpha + \frac{1}{2}\gamma\Big) \qquad\qquad c_{27} = -\frac{I_2}{25}\Big(\alpha - \frac{1}{6}\gamma\Big) \tag{52}$$

$$a_{27} = -\frac{I_2}{8}\Big(\alpha + \frac{5}{6}\gamma\Big) \qquad\qquad a_{35} = -\frac{I_2}{24}\Big(\alpha - \frac{1}{2}\gamma\Big) \ . \tag{53}$$

Turning now to the general expression Eq.(39) for a certain operator $\mathcal{J}^{\mu\chi}(0)$ which we can now write in the form

$$\langle B'(p')|\psi^\dagger(0)\mathcal{O}^{\mu\chi}\psi(0)|B(p)\rangle \ = \ \int \mathcal{D}A \int d^3z\ e^{i\vec{q}\cdot\vec{z}}\Psi^*_{B'}(A)\mathcal{G}^{\mu\chi}(\vec{z})\Psi_B(A)e^{S_{eff}} \ , \tag{54}$$

$$= \ \int d^3z\ e^{i\vec{q}\cdot\vec{z}}\,\langle B'|\mathcal{G}^{\mu\chi}(\vec{z})|B\rangle \ . \tag{55}$$

We have used again the saddle-point approximation and expanded the Dirac operator with respect to Ω and δm to the linear order and $\dot{T}^\dagger_{z(t)}T_{z(t)}$ to the zeroth order, everything contained in the expression $\mathcal{G}^{\mu\chi}(\vec{z})$. The $\mathcal{D}A$ and d^3z arise from the zero-modes due to summing over all U_c configurations which minimize the χQSM action. The expression $\mathcal{G}^{\mu\chi}(\vec{z})$ contains the specific form factor parts originating from the explicit choice of $\mathcal{J}^{\mu\chi}(0)$. The expansion in Ω and δm provides the following structure of the form factors in the χQSM:

$$G_{E,M}(Q^2) \ = \ G^{(\Omega^0,m_s^0)}_{E,M}(Q^2) + G^{(\Omega^1,m_s^0)}_{E,M}(Q^2) + G^{(m_s^1),{\rm op}}_{E,M}(Q^2) + G^{(m_s^1),{\rm wf}}_{E,M}(Q^2) \ , \tag{56}$$

where the first term corresponds to the leading order (Ω^0, m_s^0), the second one to the first $1/N_c$ rotational correction (Ω^1, m_s^0), the third to the linear m_s corrections coming from the operator, and the last one to the linear m_s corrections coming from the wave function corrections, respectively.
In the χQSM Hamiltonian of Eq.(34) the constituent quark mass M would in general be momentum dependenet, introducing a natural regularization-scheme for the divergent quark loops in the model. However, the inclusion of a momentum dependent constituent quark mass is not straight forward and in the present framework the standard way to proceed is to take the quark mass as a free, constant parameter and to introduce an additional regularization scheme. The value of $M = 420\,\mathrm{MeV}$ is known to reproduce very well experimental data [31, 32, 33, 34, 35] together with the proper-time regularization. In the meson-sector the cut-off parameter and the $\overline{m}$ are then fixed for a given M to the pion decay constant f_π and m_π, respectively. Proceeding to the baryon-sector does not include any more new parameters. Throughout this work the strange current quark mass is fixed to $m_{\rm s} = 180\mathrm{MeV}$. We want to emphasize that all these model parameters are the same as in previous works [34, 35, 36, 37, 38, 39, 40, 41, 42], no additional readjusting for different observables were done. The numerical results for the moments of inertia and mixing coefficients are summarized in Tab.I for $M = 420$ MeV. In case of the form factors we apply the symmetry conserving quantization as found in [59].

A. The $\gamma^* NN$ Vertex in the χQSM

We now give final expressions for Eqs.(3,4) evaluated in the χQSM on the ground of Eq.(54). References are [31, 32, 33]. The projector contracts the Lorentz-index and an average over the momentum transfer orientation gives raise to spherical Bessel-functions $j_{0,1}(|\vec{q}||\vec{z}|)$. In the Breit-frame we have $Q^2 = |\vec{q}|^2$. The electric and magnetic form factors are obtained by choosing in Eq.(39) $\mathcal{J}^\mu(0)$ as

$$\mathcal{J}^\mu(0) \stackrel{E}{\to} \Psi^\dagger \, \gamma^0\gamma^0 \, \Psi \ , \tag{57}$$

$$\mathcal{J}^\mu(0) \stackrel{M}{\to} \Psi^\dagger \, \gamma^0 z^i \gamma^j \epsilon^{ij3} \, \Psi \ , \tag{58}$$

according to Eqs.(3,4).
The electric and magnetic form factors in the χQSM read:

$$G_E(Q^2) = \frac{1}{2}G_E^{\chi=3}(Q^2) + \frac{1}{2\sqrt{3}}G_E^{\chi=8}(Q^2) \quad , \quad G_M(Q^2) = \frac{1}{2}G_M^{\chi=3}(Q^2) + \frac{1}{2\sqrt{3}}G_M^{\chi=8}(Q^2) \tag{59}$$

with the expressions

$$G_E^\chi(Q^2) \;=\; \int d^3z \, j_0(|\vec{q}||\vec{z}|) \, \int dA \langle B'|A\rangle \mathcal{G}_E^\chi(\vec{z})\langle A|B\rangle \ , \tag{60}$$

$$G_M^\chi(Q^2) \;=\; M_N \int d^3z \frac{j_1(|\vec{q}||\vec{z}|)}{|\vec{q}|\vec{z}|} \, \int dA \langle B'|A\rangle \mathcal{G}_M^\chi(\vec{z})\langle A|B\rangle \ . \tag{61}$$

The electric and magnetic densities are given by

$$\begin{aligned}
\mathcal{G}_E^\chi(\vec{z}) \;=\; & D_{\chi 8}^{(8)}\sqrt{\frac{1}{3}}\mathcal{B}(\vec{z}) - \frac{2}{I_1}D_{\chi i}^{(8)} J_i \mathcal{I}_1(\vec{z}) - \frac{2}{I_2}D_{\chi a}^{(8)} J_a \mathcal{I}_2(\vec{z}) \\
& - \frac{2}{\sqrt{3}} M_1 D_{\chi 8}^{(8)} \mathcal{C}(\vec{z}) - \frac{2}{3} M_8 D_{88}^{(8)} D_{\chi 8}^{(8)} \mathcal{C}(\vec{z}) \\
& + 4\frac{K_1}{I_1} M_8 D_{8i}^{(8)} D_{\chi i}^{(8)} \mathcal{I}_1(\vec{z}) + 4\frac{K_2}{I_2} M_8 D_{8a}^{(8)} D_{\chi a}^{(8)} \mathcal{I}_2(\vec{z}) \\
& - 4 M_8 D_{8i}^{(8)} D_{\chi i}^{(8)} \mathcal{K}_1(\vec{z}) - 4 M_8 D_{\chi a}^{(8)} D_{8a}^{(8)} \mathcal{K}_2(\vec{z}) \ ,
\end{aligned} \tag{62}$$

and

$$\begin{aligned}
\mathcal{G}_M^\chi(\vec{z}) \;=\; & -\sqrt{3} D_{\chi 3}^{(8)} \mathcal{Q}_0(\vec{z}) - \frac{1}{\sqrt{3}}\frac{1}{I_1} D_{\chi 8}^{(8)} J_3 \mathcal{X}_1(\vec{z}) + \sqrt{3}\frac{1}{I_1} d_{ab3} D_{\chi b}^{(8)} J_a \mathcal{X}_2(\vec{z}) + \sqrt{\frac{1}{2}}\frac{1}{I_1} D_{\chi 3}^{(8)} \mathcal{Q}_1(\vec{z}) \\
& + \frac{2}{\sqrt{3}}\frac{K_1}{I_1} M_8 D_{83}^{(8)} D_{\chi 8}^{(8)} \mathcal{X}_1(\vec{z}) - 2\sqrt{3}\frac{K_2}{I_2} M_8 D_{8a}^{(8)} D_{\chi b}^{(8)} d_{ab3} \mathcal{X}_2(\vec{z}) \\
& + 2\sqrt{3}\Big[M_1 D_{\chi 3}^{(8)} + \frac{1}{\sqrt{3}} M_8 D_{88}^{(8)} D_{\chi 3}^{(8)}\Big]\mathcal{M}_0(\vec{z}) \\
& - \frac{2}{\sqrt{3}} M_8 D_{83}^{(8)} D_{\chi 8}^{(8)} \mathcal{M}_1(\vec{z}) + 2\sqrt{3} M_8 D_{\chi a}^{(8)} D_{8b}^{(8)} d_{ab3} \mathcal{M}_2(\vec{z}) \ .
\end{aligned} \tag{63}$$

Since M_1 and M_8 are proportional to m_s only the first lines of the above expressions remain in case of flavor-$SU(3)$ symmetry. The expressions $\mathcal{B}(\vec{z}), ..., \mathcal{M}_2(\vec{z})$ are given in the appendix. The Wigner D-functions depend on the rotation A, e.g. $D_{\chi 3}^{(\chi)} = D_{\chi 3}^{(\chi)}(A)$ and expressions like

$$\int dA \, \langle B'|A\rangle \, D_{\chi 3}^{(8)}(A) \, \langle A|B\rangle \tag{64}$$

are evaluated as described in the appendix. The value for the nucleon mass M_N in front of Eq.(61) is taken as the value given by the classical soliton mass, i.e. by the mass of the nucleon in the χQSM, which is by a factor of 1.36 heavier than the experimental mass [31].

B. The $\gamma^* N\Delta$ Vertex in the χQSM

We now investigate Eq.(11) in the χQSM. In order to evaluate the left hand side of Eq.(11) in the χQSM we had to take lim $N_c \to \infty$

$$\lim_{N_c\to\infty} 3\int \frac{d\Omega}{4\pi}\frac{q^i\epsilon^{ik3}}{i|\vec{q}^2|}\langle\Delta(p',\frac{1}{2})|V_k(0)|N(p,\frac{1}{2})\rangle|_{Q^2=0} = \lim_{N_c\to\infty}\sqrt{\frac{E_N(0)+M_N}{2M_N}}2\frac{M_\Delta}{M_N}\frac{G_M^{N\Delta}(0)}{(M_\Delta+M_N)} \ , \tag{65}$$

$$\mu_{N\Delta} = \lim_{N_c\to\infty}\sqrt{\frac{M_\Delta}{M_N}}G_M^{N\Delta}(0)\,\mu_N \ . \tag{66}$$

In the whole χQSM approach we do not take any N_c^{-2} and also not all N_c^{-1}corrections into account, e.g. corrections coming from the translational zero-mode in Eq.(41) or vabriations of the classical soliton U_c were not considered. According to this we could rewrite the factors of the right hand side of Eq.(66) as follows:

$$E_N = M_N + \frac{\vec{p}^2}{2M_N} + \mathcal{O}(N_c^{-2}) \tag{67}$$

$$\sqrt{\frac{E_N(0)+M_N}{2M_N}} = 1+\mathcal{O}(N_c^{-2}) \tag{68}$$

$$\frac{M_\Delta}{M_N} = \frac{M_N+\frac{3}{2I_1}}{M_N} = 1+\frac{3}{2I_1M_N} = 1+\mathcal{O}(N_c^{^{-2}}) \tag{69}$$

$$\frac{2}{M_\Delta+M_N} = \frac{1}{M_N}\frac{1}{1+\frac{3}{2I_1M_N}} = \frac{1}{M_N}+\mathcal{O}(N_c^{-2}) \tag{70}$$

$$\sqrt{\frac{M_\Delta}{M_N}} = 1+\mathcal{O}(N_c^{^{-2}}) \ . \tag{71}$$

The expressions of Eq.(66) then reads

$$\lim_{N_c\to\infty} 3\int \frac{d\Omega}{4\pi}\frac{q^i\epsilon^{ik3}}{i|\vec{q}|^2}\langle\Delta(p',\frac{1}{2})|V_k(0)|N(p,\frac{1}{2})\rangle|_{Q^2=0} = \frac{1}{M_N}G_M^{N\Delta}(0) \ , \tag{72}$$

$$\mu_{N\Delta} = G_M^{N\Delta}(0)\,\mu_N \ . \tag{73}$$

The χQSM expression is then given by

$$G_M^{N\Delta}(0) = \frac{1}{2}G_M^{N\Delta\chi=3}(0)+\frac{1}{2\sqrt{3}}G_M^{N\Delta\chi=8}(0) \ , \tag{74}$$

$$G_M^{N\Delta\chi}(0) = M_N\int d^3z\frac{j_1(|\vec{q}||\vec{z}|)}{|\vec{q}||\vec{z}|}|_{Q^2=0}\int dA\langle\Delta(\frac{1}{2})|A\rangle\mathcal{G}_M^{\chi}(\vec{z})\langle A|N(\frac{1}{2})\rangle \ , \tag{75}$$

where the density $\mathcal{G}_M^{\chi}(\vec{z})$ is the same as in Eq.(61) since the projectors in Eqs.(4,11) are the same. The only $1/N_c$ correction which is taken into account on the level of Eq.(54) are those originating from $\mathcal{G}(\vec{z})$ but not from the expression $e^{i\vec{q}\cdot\vec{z}}$. This is connected to the fact that we just expand Eq.(41) to the zeroth-order in $\dot{T}^{\dagger}_{z(t)}T_{z(t)}$. In case of the rest-frame of the Δ we have for $\vec{q}^2$ the expression

$$\vec{q}^2 = (M_\Delta-E_N)^2+Q^2 = Q^2+\mathcal{O}(N_c^{-2}) \tag{76}$$

$$|\vec{q}| = \sqrt{Q^2}+\mathcal{O}(N_c^{-2}) \ . \tag{77}$$

This means in the present formalism the $|\vec{q}|$ entering in Eq.(74) is actually $\sqrt{Q^2}$. Applying the above large N_c arguments means, we neglect *all* $1/N_c$ corrections beside those coming from the rotational frequency (Ω) expansion of Eq.(41). After having done this, we put $N_c = 3$ in order to get finite numerical numbers.

C. The $\gamma^*\Delta\Delta$ Vertex in the χQSM

For the Δ electromagnetic form factors we use again the Breit-frame with $Q^2 = \vec{q}^2$ and

$$G_{E0}(Q^2) = \frac{1}{2}G_{E0}^{\chi=3}(Q^2)+\frac{1}{2\sqrt{3}}G_{E0}^{\chi=8}(Q^2) \ , \tag{78}$$

$$G_{E2}(Q^2) \;=\; \frac{1}{2}G_{E2}^{\chi=3}(Q^2) + \frac{1}{2\sqrt{3}}G_{E2}^{\chi=8}(Q^2) \;\;, \tag{79}$$

$$G_{M1}(Q^2) \;=\; \frac{1}{2}G_{M1}^{\chi=3}(Q^2) + \frac{1}{2\sqrt{3}}G_{M1}^{\chi=8}(Q^2) \;\;. \tag{80}$$

The projector of the electric charge form factor of the Δ is the same as for the nucleon case, hence we can use Eq.(60) with

$$G_{E0}^{\chi}(Q^2) = \int d^3z\, j_0(|\vec{q}||\vec{z}|)\langle\Delta(\frac{3}{2})|\mathcal{G}_E^{\chi}(\vec{z})|\Delta(\frac{3}{2})\rangle \;\;. \tag{81}$$

The Δ magnetic dipole form factor Eq.(22) and magnetic moment have the pre-factors

$$\frac{1}{M_\Delta} \;=\; \frac{1}{M_N + \frac{3}{2I_1}} = \frac{1}{M_N}\frac{1}{1+\mathcal{O}(N_c^2)} \;\;, \tag{82}$$

$$\frac{M_N}{M_\Delta} \;=\; \frac{M_N}{M_N}\frac{1}{1+\mathcal{O}(N_c^2)} \;\;, \tag{83}$$

and give therefore the expressions in the χQSM

$$G_{M1}^{\chi}(Q^2) \;=\; M_N\int d^3z\frac{j_1(|\vec{q}||\vec{z}|)}{|\vec{q}||\vec{z}|}\langle\Delta(\frac{3}{2})|\mathcal{G}_M^{\chi}(\vec{z})|\Delta(\frac{3}{2})\rangle \;\;, \tag{84}$$

$$\mu_\Delta \;=\; G_{M1}(0)\,\mu_N \;\;. \tag{85}$$

The densities $\mathcal{G}_E^{\chi}(\vec{z})$ and $\mathcal{G}_M^{\chi}(\vec{z})$ are the same as in Eqs.(60,61) since the projectors in Eqs.(3,20) and Eqs.(4,22) are the same, respectively.
The projector on G_{E2} is different. The electric quadrupole form factor reads in terms of Eq.(54)

$$G_{E2}^{\chi}(Q^2) \;=\; -\int d\Omega_q\sqrt{\frac{5}{4\pi}}\frac{3}{2}\frac{1}{\tau}\int dz^3 e^{i\vec{q}\cdot\vec{z}}\langle\Delta(\frac{3}{2})|[Y_{20}^{*}(\Omega_q)\mathcal{G}^{0\chi}(\vec{z})]|\Delta(\frac{3}{2})\rangle \;\;, \tag{86}$$

which gives after performing the integral over $d\Omega_q$

$$G_{E2}^{\chi}(Q^2) \;=\; 6\sqrt{5}M_\Delta^2\int dr\; r^4\,\frac{j_2(k\cdot r)}{k^2r^2}\;\int d\Omega_z\langle\Delta(\frac{3}{2})|\;[\sqrt{4\pi}Y_{20}(\Omega_z)\mathcal{G}^{0\chi}(\vec{z})]\;|\Delta(\frac{3}{2})\rangle \;\;, \tag{87}$$

with $r=|\vec{z}|$ and $k=|\vec{q}|$. The expression $[\sqrt{4\pi}Y_{20}(\Omega_z)\mathcal{G}^{0\chi}(\vec{z})] = \mathcal{G}_{E2}^{0\chi}(\vec{z})$ shall illustrate the χQSM form factor density which we obtain when we choose the operator $\mathcal{J}^{\mu}(0)$ in Eq.(39) as

$$\mathcal{J}^{\mu}(0) \overset{E2}{\to} \Psi^{\dagger}\;\sqrt{4\pi}Y_{20}(\Omega_z)\gamma^0\gamma^0\;\Psi \;\;, \tag{88}$$

according to Eq.(21).
Since G_{E2} is extracted out from the zeroth-component of the vector-current the Lorentz-structure is the same as for the form factor G_E. Hence, we can construct the G_{E2} χQSM form factor density from the expression for G_E. For the form factor G_{E2} we will not take any m_s-corrections coming from the operator into account and start from the $SU(3)$-expression of G_E which reads

$$G_E^{\chi}(Q^2) \;=\; \int d^3z\, j_0(|\vec{q}||\vec{z}|)\;\int dA\langle B'|A\rangle\mathcal{G}_E^{\chi}(\vec{z})\langle A|B\rangle \;\;, \tag{89}$$

with the density

$$\begin{aligned}
\mathcal{G}_E^{\chi}(\vec{z}) \;&=\; D_{\chi 8}^{(8)}\sqrt{\frac{1}{3}}\mathcal{B}(\vec{z}) - 2\{\frac{J_i}{2I_1}, D_{\chi j}^{(8)}\}\mathcal{I}_1^{ij}(\vec{z}) - 2\{\frac{J_a}{2I_2}, D_{\chi a}^{(8)}\}\mathcal{I}_2(\vec{z}) \\
\frac{1}{N_c}\mathcal{B}(\vec{z}) \;&=\; \phi_v^{\dagger}(\vec{z})O\phi_v(\vec{z}) - \frac{1}{2}\sum_n \mathrm{sign}(\varepsilon_n)\phi_n^{\dagger}(\vec{z})O\phi_n(\vec{z}), \\
\frac{1}{N_c}\mathcal{I}_1^{ij}(\vec{z}) \;&=\; \frac{1}{2}\sum_{\varepsilon_n\neq\varepsilon_v}\frac{1}{\varepsilon_n-\varepsilon_v}\langle v|\tau^i|n\rangle\phi_n^{\dagger}(\vec{z})O\tau^j\phi_v(\vec{z}) + \frac{1}{4}\sum_{n,m}\mathcal{R}_3(\varepsilon_n,\varepsilon_m)\langle n|\tau^i|m\rangle\phi_m^{\dagger}(\vec{z})O\tau^j\phi_n(\vec{z}) \\
\frac{1}{N_c}\mathcal{I}_2(\vec{z}) \;&=\; \frac{1}{4}\sum_{\varepsilon_{n^0}}\frac{1}{\varepsilon_{n^0}-\varepsilon_v}\langle n^0|v\rangle\phi_v^{\dagger}(\vec{z})O\phi_{n^0}(\vec{z}) + \frac{1}{4}\sum_{n,m^0}\mathcal{R}_3(\varepsilon_n,\varepsilon_{m^0})\phi_{m^0}^{\dagger}(\vec{z})O\phi_n(\vec{z})\langle n|m^0\rangle \;\;.
\end{aligned}$$

The choice of $\mathcal{J}^\mu(0)$ defines the operator O in the densities $\mathcal{B}, \mathcal{I}_1, \mathcal{I}_2$ which in case of the form factor G_E is $O = \gamma^0\gamma^0 = 1$ and in case of G_{E2} it is $O = \sqrt{4\pi}Y_{20}(\Omega_z)$. The density $\mathcal{B}$ originates from the zeroth-order Ω^0 in the rotation-velocity expansion of Eq.(41) whereas $\mathcal{I}_1, \mathcal{I}_2$ are the first rotational Ω^1-corrections. The Ω^1 corrections are also referred to as $1/N_c$ corrections. In case of the operator $O = \sqrt{4\pi}Y_{20}(\Omega_z)$ the corresponding densities $\mathcal{B}(\vec{z})$ and $\mathcal{I}_2(\vec{z})$ are zero.
The final expression in the χQSM for the form factor G_{E2} reads

$$G^{\chi}_{E2}(Q^2) \;=\; \frac{12}{I_1\sqrt{2}}\, M^2_\Delta \langle B'|\Big[3D^{(8)}_{\chi 3}J_3 - D^{(8)}_{\chi i}J_i\Big]|B\rangle \int dr\; r^4\, \frac{j_2(|\vec{q}|r)}{|\vec{q}|^2 r^2}\,\mathcal{I}_{1E2}(r) \;\;, \tag{90}$$

with the density

$$\frac{6}{N_c}\mathcal{I}_{1E2}(r) \;=\; \sum_{n\neq v}\frac{1}{\varepsilon_n - \varepsilon_v}\,(-)^{G_m}\,\langle A^v, G^v||\tau_1||A^n, G^n\rangle\langle A^n, G^n||r\rangle\{\sqrt{4\pi}Y_2\otimes\tau_1\}_1\langle r||A^v, G^v\rangle \tag{91}$$

$$+\frac{1}{2}\sum_{n,m}\mathcal{R}_3(\varepsilon_n,\varepsilon_m)\,(-)^{G_n-G_m}\,\langle A^n, G^n||\tau_1||A^m, G^m\rangle\langle A^m, G^m||r\rangle\{\sqrt{4\pi}Y_2\otimes\tau_1\}_1\langle r||A^n, G^n\rangle \;\;, \tag{92}$$

where the sum over the third grand-spins of the basis sates in App.VI E are already taken. The whole G^{χ}_{E2} form factor originates from the rotational corrections and therefore scales as $1/N_c$ and vanishes in the large N_c limit.
The same density also occures in the χQSM expression for the $N-\Delta$ transition form factor ratios $R_{EM} = -G^{N\Delta}_E(0)/G^{N\Delta}_M(0) = E2/M1$ and $R_{SM} = C2/M1 \sim G^{N\Delta}_C(0)/G^{N\Delta}_M(0)$ in [38]. The final results of that χQSM $SU(3)$ analysis are $E2/M1 = -1.4\%$ and $C2/M1 \approx -1.8\%$ for which we can write

$$0.78 \approx \frac{E2/M1}{C2/M1} = \frac{E2}{C2} = \frac{1}{3}\frac{\int dr\,\frac{\partial}{\partial r}\Big[rj_2(|\vec{q}|r)\Big]\,\mathcal{I}_{1E2}(\vec{x})}{\int dr\, j_2(|\vec{q}|r)\,\mathcal{I}_{1E2}(\vec{x})} \;\;, \tag{93}$$

by using the formulae presented in [38]. Inserting the density $\mathcal{I}_{1E2}(r)$ of this work reproduces the 0.78. In addition we can also reproduce the values for M^{E2} presented in [64] by using the expressions of that work with the density $\mathcal{I}_{1E2}(r)$ of this work.

IV. RESULTS AND DISCUSSION

We now present and discuss the final results of this work. We have calculated the electromagnetic form factors G_{E0}, G_{E2} and G_{M1} of the $\Delta(1232)$ and compare them to the form factors G_E and G_M of the nucleon. We also consider the magnetic transition moment of the process $N \to \Delta$ and give numerical values for all other decuplet magnetic moments. All results are achieved by using the self-consistent SU(3) χQSM. In this formalism the constituent quark mass M is the only free parameter with standard value $M = 420$ MeV. Numerical parameter are fixed as described in Sec.III and are exactly the same as in the works [34, 35, 36, 37, 38, 39, 40, 41, 42]. With the numerical parameters of Tab.I, the χQSM yields masses of the octet and decuplet baryons in unit of MeV as Ref.[42]:

$$M_N \;=\; 1001(939),\;\; M_\Lambda = 1124(1116),\;\; M_\Sigma = 1179(1189),\;\; M_\Xi = 1275(1318) \;\;, \tag{94}$$

$$M_\Delta = 1329(1232),\;\; M_{\Sigma^*} = 1431(1385),\;\; M_{\Xi^*} = 1533(1530),\;\; M_\Omega = 1635(1672) \;\;,$$

where the numbers in the parentheses are the experimental values of the Particle Data Group [1]. The χQSM values were obtained by first calculating the hyper-charge splittings with Eq.(44) and afterwards starting from the experimental octet mass center, $M_8 = (M_\Lambda + M_\Sigma)/2 = 1151.5$ MeV.
In general for the observables investigated in this work a change of the constituent quark mass between the values $M = (400 \sim 450)$ MeV affect the numerical values of the observables by 4%. We therefore present only final results for $M = 420$ MeV.
We will first discuss the values of the form factors at the point $Q^2 = 0$ and afterwards their Q^2 dependence up to $Q^2 = 1\,\mathrm{GeV}^2$.
The magnetic moments are obtained from Eqs.(59,74,80)

$$G_M(0) = \frac{1}{2}\Big[G^{\chi=3}_M(0) + \frac{1}{\sqrt{3}}G^{\chi=8}_M(0)\Big] \;\;, \tag{95}$$

w_1	w_2	w_3	w_4	w_5	w_6
-12.94 (with $\mathcal{M}_0$) -13.64 (without $\mathcal{M}_0$)	7.13	5.16	-1.31	-0.78	0.07

Table II: Magnetic parameters for Eq.(98). The parameters are for a constituent quark mass of $M = 420$ MeV and a mass of $M_N^{\chi QSM} = 939 \cdot 1.36$ MeV in Eq.(61) as described in the text. The density $\mathcal{M}_0$ is proportional to m_s.

μ/n.m.	Δ^{++}	Δ^{+}	Δ^{0}	Δ^{-}	Σ^{+}_{10}	Σ^{0}_{10}	Σ^{-}_{10}	Ξ^{0}_{10}	Ξ^{-}_{10}	Ω^{-}
this work	4.85	2.35	-0.14	-2.63	2.47	-0.02	-2.52	0.09	-2.40	-2.29
χQSM '98 [57]	4.73	2.19	-0.35	-2.90	2.52	-0.08	-2.69	0.19	-2.48	-2.27

Table III: Magnetic moments of the decuplet in the self-consistent χQSM for $M = 420$ MeV. All numbers are given with inclusion of flavor-$SU(3)$ symmetry breaking effects. The flavor-$SU(3)$ symmetric value of this work is given by $\mu_{B^{10}} = 2.47\ Q_{10}\,\mu_N$. The χQSM Ω^- magnetic moment agrees well with the experimental value given by the Particle Data Group of $\mu_{\Omega^-} = (-2.02 \pm 0.05)\,\mu_N$ [1]. The mass factor of Eq.(61) is $M_N^{\chi QSM} = 939 \cdot 1.36$ MeV as described in the text.

for which we can rewrite Eqs.(63) in the following simple form:

$$G_M^{\chi}(0) = \int dA\ \langle B'|A\rangle \left[\hat{\mathcal{G}}_M^{\chi} + \hat{\mathcal{G}}_M^{\chi(opc)}\right] \langle A|B\rangle \tag{96}$$

$$\hat{\mathcal{G}}_M^{\chi} = w_1 D^{(8)}_{\chi 3} + w_2 d_{pq3} D^{(8)}_{\chi p} \hat{J}_q + w_3 \frac{1}{\sqrt{3}} D^{(8)}_{\chi 8} \hat{J}_3 \tag{97}$$

$$\hat{\mathcal{G}}_M^{\chi(opc)} = w_4 \frac{1}{\sqrt{3}} d_{pq3} D^{(8)}_{\chi p} D^{(8)}_{8q} + w_5 (D^{(8)}_{\chi 3} D^{(8)}_{88} + D^{(8)}_{\chi 8} D^{(8)}_{83}) + w_6 (D^{(8)}_{\chi 3} D^{(8)}_{88} - D^{(8)}_{\chi 8} D^{(8)}_{83})\ . \tag{98}$$

All magnetic constants in this work can be reproduced (within accuracy) by using the values of Tab.II and the matrix-elements of App.VI F. In the case of flavor-$SU(3)$ symmetry only the paramters w_1, w_2 and w_3 contribute whereas w_4, w_5 and w_6 are m_s corrections coming from the operator; wave function corrections contribute via $|B\rangle$ with the paramters w_1, w_2 and w_3. Since the right hand-sides of Eqs.(61,74,84,90) are model-equations we also take the model-value for the nucleon mass which is by a factor of 1.36 larger than the experimental value, $M_N^{\chi QSM} = 939 \cdot 1.36$ MeV.
As in [57] we can write the magnetic moments of the decuplet baryons in flavor-$SU(3)$ symmetry by the simple formula

$$\mu_{B^{10}} = -\frac{1}{12}(w_1 - \frac{1}{2}w_2 - \frac{1}{2}w_3)\ Q_{10}\ J_3\ \mu_N\ , \tag{99}$$

where Q_{10} is the charge of the decuplet baryon and J_3 its third-spin component. The numerical value of this equation, given later (Eq.(100)), is close to the *model independent* analysis in [57] and comparable to the one in [55]. The χQSM analysis of [56, 57] gave in flavor-$SU(3)$ a decuplet magnetic moment of $2.23 \cdot Q_{10}\mu_N$. Eventhough the numerical value of the present work is close, there are differences in its determination. As explicitly mentioned in [57] the so-called symmetry conserving quantization (SCQ) technique [59] was not applied and the magnetic moment of $2.23 \cdot Q_{10}\mu_N$ is normalized to the experimental nucleon mass in Eq.(60). The SCQ has as a consequence that it decreases μ_B, like g_A^3 in [35] compared to [65] , but the normalization to the nucleon mass as it comes out in the self-consistent χQSM enhances μ_B. The final numerical value for the decuplet flavor-$SU(3)$ magnetic moment with $J_3 = 3/2$, application of SCQ and normalization to the soliton nucleon mass, $M_N^{\chi QSM} = 939 \cdot 1.36$ MeV, is

$$\mu_{B^{10}}^{\chi QSM} = 2.47 \cdot Q_{10}\ \mu_N\ , \tag{100}$$

by using the values of Tab.II. Our final results for the magnetic moments by including flavor-$SU(3)$ breaking effects are summarized in Tab.III. The m_s corrections of this work are more moderate compared to the results in [57]. This is also a consequence of the SCQ. The SCQ has a significant impact on the parameter w_1, therefore alters the ratio of the wave function to operator corrections in this work compared to [57]. For the wave function corrections, numerically the factor a_{27} is dominant and the magnetic moment corrections originating from it are sensitive to w_1. However, in general the m_s corrections in this work are maximal 8% beside the neutral baryons . The m_s corrections in this work have the same sign as in [56] which is not always the case by comparing with [57].

Magnetic moments for the nucleon, the $N - \Delta$ and Δ^+ are discussed in more detail in Tab.IV. Since the χQSM uses the large N_c approximation, to some extent the large N_c relations of [28] should be fulfilled. The relations given

$\mu[\mu_N]$	Ω^0	Ω^{0+1}	$\Omega^{0+1}+\delta m_s^1$	large N_c rel.	exp.
μ_p	1.25	2.46	2.44		2.79
μ_n	−0.93	−1.63	−1.68		−1.91
$\vert\mu_{\Delta N}\vert$	1.38	2.56	2.72	$\mu_{\Delta N}=\frac{1}{\sqrt{2}}(\mu_p-\mu_n)=2.91$	3.46 ± 0.03
μ_{Δ^+}	1.16	2.47	2.35	$\mu_{\Delta^+}\approx\frac{3}{5}(\mu_p-\mu_n)=2.47$	$2.7\pm1.15^{(stat)}\pm1.5^{(syst)}$

Table IV: Magnetic moments of the nucleon, the N-Δ transition and the Δ^+ in the self-consistent χQSM for $M=420$MeV. The second column corresponds to the leading order in rotation whereas the third and forth columns are linear rotational and m_s corrections, respectively. The last column are experimental data taken from [1, 4, 5, 66] with the uncertainty of $\mu_{\Delta^+}=(2.7^{+1.0}_{-1.3}(stat.)\pm1.5(syst.)\pm3(theo.))\,\mu_N$. The normalization in Eq.(61) is taken as $M_N^{\chi QSM}=939\cdot1.36$ MeV for all given observables as described in the text. The values for the large N_c relations are given by using the χQSM values where in case of the μ_{Δ^+} the μ_{Δ^0} contribution is omitted.

in that paper are exact up to the order $\mathcal{O}(N_c^{-2})$. In the present approach of the χQSM there are two reasons why this relations should not be exactly fulfilled. First, in order to achieve numerical values the transition back to $N_c=3$ is done. Second, not all N_c^{-1} corrections are taken into account, e.g. corrections from the translational zero-mode are not considered. Generally, also for other decuplet magnetic moments in the χQSM of Tab.III the large N_C relations of [28]

$$\mu_{\Delta^{++}}-\mu_{\Delta^-} = \frac{9}{5}(\mu_p-\mu_n)+\mathcal{O}(N_c^{-2}) \ , \quad (101)$$

$$\mu_{\Delta^{+}}-\mu_{\Delta^0} = \frac{3}{5}(\mu_p-\mu_n)+\mathcal{O}(N_c^{-2}) \ , \quad (102)$$

$$\mu_{\Sigma^+_{10}}-\mu_{\Sigma^-_{10}} = \frac{3}{2}(\mu_{\Sigma^+}-\mu_{\Sigma^-})+\mathcal{O}(N_c^{-2}) \ , \quad (103)$$

$$\mu_{\Xi^0_{10}}-\mu_{\Xi^-_{10}} = -3(\mu_{\Xi^0}-\mu_{\Xi^-})+\mathcal{O}(N_c^{-2}) \ , \quad (104)$$

are satisfied up to 7%.

In case of the $N-\Delta$ transition and the Δ form factors we made use of large N_c arguments in Eqs.(71,83) for several mass-ratio factors, which lead to the values, also presented in the Tab.IV and Tab.V, in the self-consistent χQSM of

$$G_M^{N\Delta}(0)=2.72 \qquad \mu_{N\Delta}=2.72\ \mu_N \ , \quad (105)$$

$$G_{M1}^{\Delta^+}(0)=2.35 \qquad \mu_{\Delta^+}=2.35\ \mu_N \ . \quad (106)$$

Keeping these mass-ratio factors, which are over-all factors, yields

$$G_M^{N\Delta}(0)=2.30 \qquad \mu_{N\Delta}=2.72\ \mu_N \ , \quad (107)$$

$$G_{M1}^{\Delta^+}(0)=3.09 \qquad \mu_{\Delta^+}=2.35\ \mu_N \ . \quad (108)$$

The first treatment would correspond to neglecting *all* $1/N_c$ corrections beside the rotational corrections while keeping the pre-factors would correspond to keeping some more $1/N_c$ corrections but neglecting all model-based $1/N_c$ corrections besides the rotational ones.
We will discuss now the Δ^+ electric and magnetic form factors G_{E0} and G_{M1} for $Q^2\leq1\,\mathrm{GeV}^2$.
The results of the self-consistent χQSM calculations for the electric and magnetic form factors G_E^p, G_M^p, $G_{E0}^{\Delta^+}$ and $G_{M1}^{\Delta^+}$ are best reproduced by a dipole type form factor

$$G_{E,M}(Q^2)=\frac{G_{E,M}(0)}{\left(1+\frac{Q^2}{\Lambda^2_{E,M}}\right)^2} \ . \quad (109)$$

In Tab.V we present the fitted parameter which reproduce the proton and Δ^+ electric and magnetic form factors of Fig.1. In case of the lattice results [27] an exponential type form factor for G_{M1}

$$G_{M1}(Q^2)=G_{M1}(0)\ e^{-Q^2/\Lambda^2_{M1}} \ , \quad (110)$$

parametrizes best the lattice results. We compare our results in Tab.V with those of [27].

	$\Lambda_E^2/(\mathrm{GeV}^2)$	$G_M^p(0)$	$\Lambda_M^2/(\mathrm{GeV}^2)$	$\Lambda_{E0}^2/(\mathrm{GeV}^2)$	$G_{M1}^{\Delta^+}(0)$	$\Lambda_{M1}^2/(\mathrm{GeV}^2)$
χQSM	0.614	2.438	0.716	0.585	2.354 $[3.089]^{1)}$	0.736^{dip} $[0.490]^{\mathrm{exp}}$
Quenched Wilson				1.101	2.635	0.978^{exp}
Dyn. $N_f = 2$ Wilson				1.161	2.344	1.022^{exp}
Hybrid				1.126	3.101	0.895^{exp}
Experiment	0.523	2.793				

Table V: Table of proton and Δ^+ parameters for the dipole (dip) and exponential (exp) form factor fits Eqs.(109,110). The numbers in parentheses corresponds to 1) using normalizaion of $M_\Delta^{\chi QSM} = 1232 \cdot 1.36$ MeV in Eq.(84) 2) using an exponential type form factor. The self-consistent χQSM calculation for G_{M1} is best reproduced by a dipole type form factor while the numbers for Λ_{M1}^2 in case of the lattice results are for an exponential type form factor.

The charge and magnetic dipole form factors of the decuplet baryons in case of flavor-$SU(3)$ symmetry can be written as

$$G_{E0}(Q^2) \;=\; Q_{10} \times \int d^3z\, j_0(|\vec{q}||\vec{z}|)\Big[\frac{1}{24}\mathcal{B}(\vec{z}) + \frac{5}{8}\frac{\mathcal{I}_1(\vec{z})}{I_1} + \frac{1}{4}\frac{\mathcal{I}_2(\vec{z})}{I_2}\Big] \;,$$

$$G_{M1}(Q^2) \;=\; Q_{10} \times J_3 \times \frac{M_\Delta}{12}\int d^3z \frac{j_1(|\vec{q}||\vec{z}|)}{|\vec{q}||\vec{z}|}\Big[\sqrt{3}\mathcal{Q}_0(\vec{z}) - \frac{1}{2}\frac{\mathcal{X}_1(\vec{z})}{I_1} + \frac{\sqrt{3}}{2}\frac{\mathcal{X}_2(\vec{z})}{I_2} - \sqrt{\frac{1}{2}}\frac{\mathcal{Q}_1(\vec{z})}{I_1}\Big] \;,$$

with Q_{10} as the charge of the decuplet baryon and its third-spin component J_3 and M_Δ the normalization of the magnetic form factor. In case of the neutral decuplet baryons the entire form factors for G_{E0} and G_{M1}, even for $Q^2 > 0$, are only due to strange-quark mass corrections.
For the proton the experimental value of the charge radius is $[\langle r_E^2\rangle^P]^{1/2} = 0.8750 \pm 0.0068\,\mathrm{fm}$ ($\langle r_E^2\rangle^P \approx 0.766\,\mathrm{fm}^2$) [1]. The charge radii of the proton and Δ^+ of G_E and G_{E0} in the self-consistent χQSM with $M = 420$ MeV are, respectively

$$\langle r_E^2\rangle_P \;=\; 0.768\,\mathrm{fm}^2 \qquad \langle r_E^2\rangle_P^{SU(3)} = 0.770\,\mathrm{fm}^2 \;, \tag{111}$$

$$\langle r_E^2\rangle_{\Delta^+} \;=\; 0.794\,\mathrm{fm}^2 \qquad \langle r_E^2\rangle_{\Delta^+}^{SU(3)} = 0.813\,\mathrm{fm}^2 \;, \tag{112}$$

and the magnetic radii for $G_M(Q^2)$ and $G_{M1}(Q^2)$ are

$$\langle r_M^2\rangle_P \;=\; 0.656\,\mathrm{fm}^2 \qquad \langle r_M^2\rangle_P^{SU(3)} = 0.665\,\mathrm{fm}^2 \;, \tag{113}$$

$$\langle r_M^2\rangle_{\Delta^+} \;=\; 0.634\,\mathrm{fm}^2 \qquad \langle r_M^2\rangle_{\Delta^+}^{SU(3)} = 0.658\,\mathrm{fm}^2 \;, \tag{114}$$

where the index $SU(3)$ indicates the value in case of flavor-$SU(3)$ symmetry. The above radii are calculated by differentiating the χQSM form factor expression, i.e. explicitly integrating the χQSM form factor densities. Alternatively one could calculate the radii by using the dipole fit due to $\langle r_{E,M}^2\rangle = 12/\Lambda_{E,M}^2$ for which the values only differ by max 1%.
In Fig.1 we compare the final χQSM results for the Δ^+ form factors G_{E0} and G_{M1} with those of the lattice calculation [27]. The χQSM form factors drop faster with increasing Q^2. In case of the χQSM it is known that the Q^2 dependence of the experimental data of the electric and magnetic form factors for both nucleons are very well reproduced [31]. In the lattice work [67] the nucleon iso-vector form factor $F_1^{p-n}(Q^2)$ for pion-masses ranging from $m_\pi = 775$ MeV down to $m_\pi = 359$ MeV was calculated. It was found that the form factor becomes steeper by lowering the pion-mass. Still for a value of $m_\pi = 359$ MeV the results of [67] are above the experimental values. The minimal value of m_π in Ref.[27] for the form factors G_{E0} and G_{M1} of the Δ^+, Fig.1, is $m_\pi = 353$ MeV and also do not fall off as fast as the χQSM results. This can also be seen in the fact, that the lattice results are best reproduced by an exponential type form factor while the χQSM are more of a dipole type form factor. The Δ magnetic moment is presented in the range of $\mu_{\Delta^+} = (1.58 \sim 1.91)\,\mu_N$ in the pion mass range $m_\pi \approx (353 - 400)$ MeV. The value of the present χQSM calculation is $\mu_{\Delta^+} = 2.35\,\mu_N$.
Recently, a first dynamical lattice QCD calculation [20] of the Δ and Ω^- magnetic dipole moments was also performed using a background field method. The calculation for Ω^- was done at the physical strange quark mass, with the result $\mu_{\Omega^-} = -1.93(8)\,\mu_N$ in very good agreement with the experimental number. The Δ has been studied at smallest pion mass value $m_\pi = 366$ MeV with the result $\mu_{\Delta^+} = 2.40(6)\,\mu_N$.

We will now discuss the results for the Δ^+ electric quadrupole form factor G_{E2}. In Fig.2 we present the final results and compare them with the recent lattice calculations in [27]. As already mentioned in Sec.III C the form factor

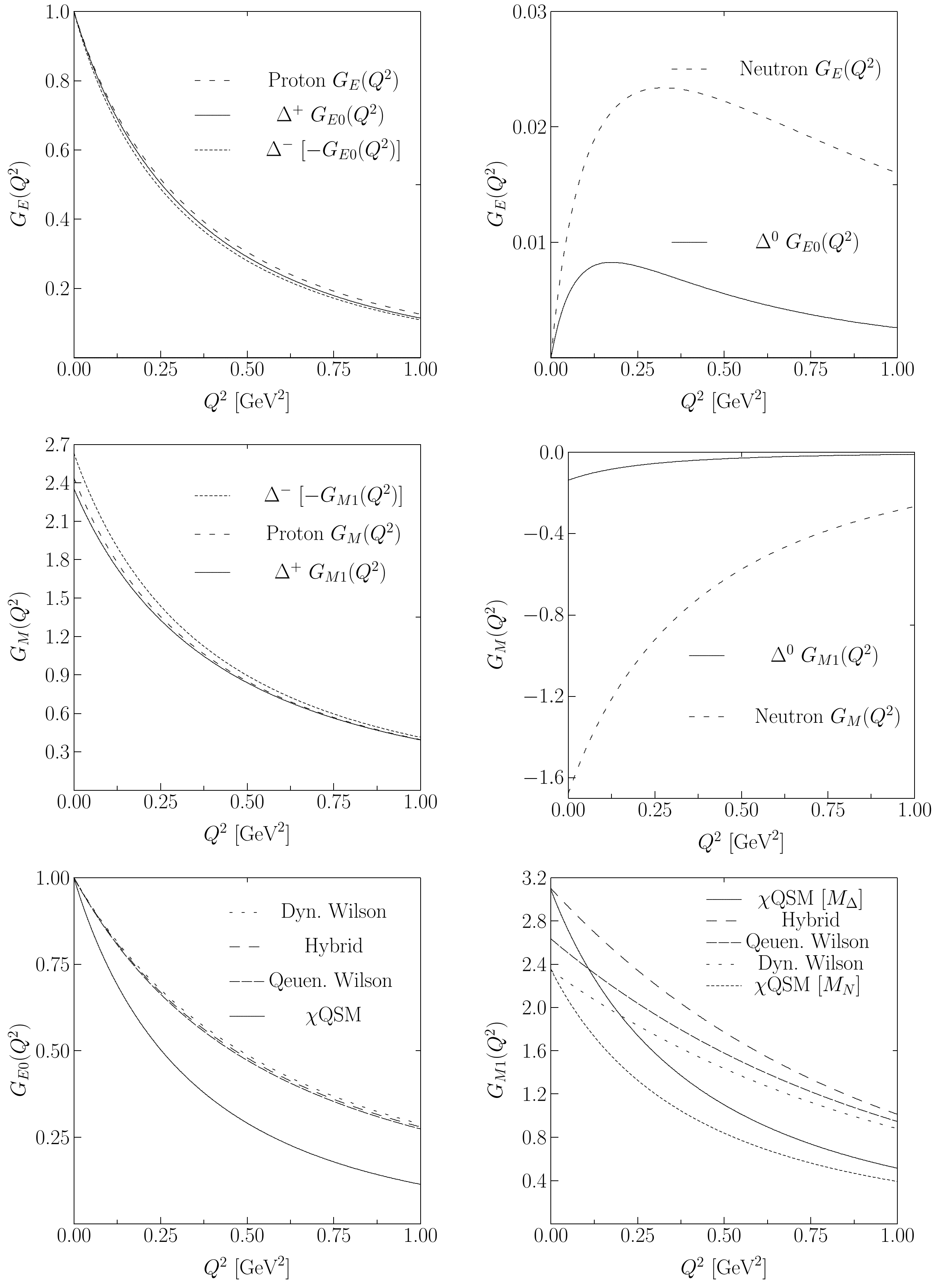

Figure 1: Electric and magnetic form factors of the Δ^+, Δ^0, Δ^- and the nucleon in the self-consistent χQSM compared to lattice results. The form factors of the Δ^{++} are roughly by an overall factor of 2 larger than those for the Δ^+ and are not explicitly shown. The $G_{E0}^{\Delta^0}$ and $G_{M1}^{\Delta^0}$ for $Q^2 > 0$ are entirely due to m_s corrections and therefore smaller compared to the neutron G_M. For all magnetic form factors $M_N^{\chi QSM} = 939 \cdot 1.36$ MeV is used in Eq.(61) beside the χQSM graph in the lower-right picture where we also take $M_\Delta^{\chi QSM} = 1232 \cdot 1.36$ MeV and indicate the normalization by $[M_{N(\Delta)}]$. In the last two figures we compare our final results for the Δ^+ form factors G_{E0} and G_{M1} with those of the lattice results in [27].

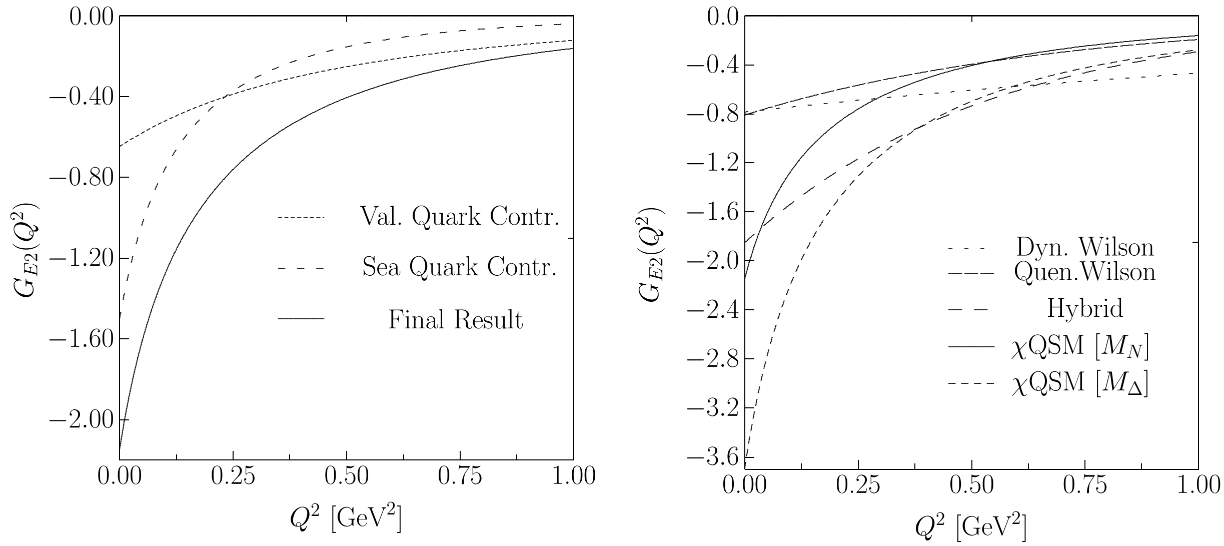

Figure 2: The electric Δ^+ quadrupole form factor G_{E2} in the self-consistent χQSM and comparison to the lattice results of [27]. The left picture shows the form factor decomposed into its valence and sea quark contributions while the right picture compares the final result with those of the lattice calculation. In the right picture we once took $M_N^{\chi QSM} = 939 \cdot 1.36$MeV and once $M_\Delta^{\chi QSM} = 1232 \cdot 1.36$MeV for the mass in Eq.(90).

	χQSM	Quenched Wilson	Dynamical Wilson	Hybrid
$G_{E2}^{\Delta^+}(0)$	-2.145	-0.810	-0.784	-1.851
$\Lambda_{E2}^2/(\mathrm{GeV}^2)$	0.369^{dip} $[0.268]^{\mathrm{exp}}$	0.696^{exp}	1.938^{exp}	0.542^{exp}

Table VI: Table for fit parameters of the form factor G_{E2}. The indices "dip" and "exp" corresponds to fitting with a dipole or exponential type form factor Eqs.(109,110). A dipole type form factor reproduces the self-consistent χQSM calculation more accurate than an exponential fit.

G_{E2} in the χQSM is only due to rotational corrections which are seen as $1/N_c$ corrections. In the large N_c limit the χQSM leads to a vanishing form factor. In the left panel of Fig.2 we decomposed the form factor into its contributions coming from the valence and sea quarks. The sea contribution gives the most sizeable part of the form factor. This behavior is also seen in Ref.[64] where the electric quadrupole moment $Q_{N\Delta}$ was investigated in the $SU(2)$ χQSM. The density $\mathcal{I}_{1E2}(r)$ also contributes to the $N\Delta$ transition in [64]. The Fig.2 shows the same behavior of valence and sea quark contributions for G_{E2} as Fig.1 in Ref.[64] for the quantity $Q_{N\Delta}$. In case of the χQSM we had to introduce a regularization scheme for the sea quark contribution which was the proper-time regularization. The fact that the sea quarks give the dominant part of the form factor could result in a sensibility of the χQSM G_{E2} to the applied regularization scheme. An analogous situation is met, and well known, in case of the $\Sigma_{\pi N}$ form factor in [68, 69]. In this work we do not investigate the regularization dependence of the form factor G_{E2} and give all final results for applying the proper-time regularization.

For the parametrization of this form factor we prefer a dipole type fit Eq.(109). In Tab.VI we summarize the parameters which reproduce the self-consistent χQSM calculation and compare them to the results of the lattice calculation of [27]. In case of the electric quadrupole form factor the lattice results are more divergent. Again the χQSM result falls off faster in the region $0 \leq Q^2 \leq 0.50\,\mathrm{GeV}^2$ compared to all three lattice results but compares well to the quenched Wilson and hybrid action results for $0.50\,\mathrm{GeV}^2 \leq Q^2 \leq 1\,\mathrm{GeV}^2$, respectively.

In the Ref.[29] a relation in the large N_c limit is found which connects the quadrupole moment of the $N-\Delta$ transition $Q_{N\Delta}$ to the quadrupole moment Q_Δ of the Δ

$$Q_{\Delta^+} = \frac{2\sqrt{2}}{5} Q_{p\Delta^+} + \mathcal{O}(N_c^{-2}) \ . \tag{115}$$

The Ref.[5] extracted the value of

$$Q_{N\Delta} = -(0.0846 \pm 0.0033)\,\mathrm{fm}^2 \ , \tag{116}$$

which gives with the above large N_c relation

$$Q_{\Delta^+} = (-0.048 \pm 0.002)\,\mathrm{fm}^2 \ .$$

The final result of this work in the self-consistent χQSM is

$$Q^{\chi QSM}_{\Delta^+} = \frac{G_{E2}(0)}{M^2_\Delta} = -0.0509\,\mathrm{fm}^2 \ , \tag{117}$$

which agrees well to the above estimation. From the left panel in Fig.2 we see that for the electric quadrupole moment, propotional to $G_{E2}(0)$, the sea quark contribution dominates the valence quark contribution. Furthermore, one can expect the sea quark contribution to have a broader spatial distribution than the one for the valence quarks. This in turn leads to a steeper Q^2 dependence of the contribution to G_{E2} of sea quarks as compared with valence quarks. This is evidenced in the present calculation as shown in Fig.2.

In the χQSM work [64] the authors presented an electric quadrupole transition moment of $Q_{N\Delta} = -0.020\,\mathrm{fm}^2$. Also for this quantity the main contribution comes from the sea quarks. The small value of $Q_{N\Delta} = -0.020\,\mathrm{fm}^2$ in [64] is in contrast to $Q_{N\Delta} = -(0.0846 \pm 0.0033)\,\mathrm{fm}^2$ from [66] and the relative large electric Δ quadrupole moment $Q^{\chi QSM}_{\Delta^+} = -0.0509\,\mathrm{fm}^2$ of this work. We can reproduce with the density $\mathcal{I}_{1E2}(r)$ of this work the values given in [64]. The discrepancy of the above numbers could be due to a possible breakdown of the approximation $k \cdot R \ll 1$ performed in [64], with k being the photon-momentum at $Q^2 = 0$ of the $\gamma^* N\Delta$ process and R being the nucleon charge radius. This remains to be investigated in future studies.

In the work [70] the Δ^+ electric quadrupole moment is estimated to $Q^{imp\,(exc)}_{\Delta^+} = -0.032\,\mathrm{fm}^2\,(-0.119\,\mathrm{fm}^2)$ by using a constituent quark model with once configuration mixings and no exchange current and once with an exchange current but no configuration mixing, respectively. A recent light cone QCD sum rule calculation [71] obtained an electric quadrupole moment of $Q_{\Delta^+} = -(5.8 \pm 1.45)10^{-4}\,\mathrm{fm}^2$. Our value of $Q^{\chi QSM}_{\Delta^+} = -0.0509\,\mathrm{fm}^2$ is more comparable to the constituent quark model results.

V. SUMMARY

In the present work we investigated in the framework of the self-consistent $SU(3)$ χQSM the electromagnetic form factors of the vector current for the decuplet baryons. We explicitly take the symmetry conserving quantization, linear $1/N_c$ rotational as well as linear strange-quark mass corrections into account. Earlier self-consitent $SU(3)$ χQSM results only calculated the decuplet magnetic moments and did not apply the symmetry conserving quantization. Numerical parameters of the model are fixed in the meson-sector as described at the end of Sec.III. The only free parameter of the χQSM for the baryon-sector is then the constituent quark mass. All these parameters were fixed by previous studies and were also used in the present work. No additional readjusting is done. With these parameters, the general way to calculate observables in the model is to determine the eigenvalues of the χQSM hamiltonian numerically by using a self-consistent pion-field profile, the soliton. These eigenvalues are then used for determining all observables in the χQSM.

In particular we calculated the form factors G_{E0}, G_{M1} and G_{E2} for the Δ^+ up to a momentum-transfer of $Q^2 \leq 1\mathrm{GeV}^2$ and magnetic moments for all decuplet baryons and the $N-\Delta$ transition. In general all χQSM form factors are best reproduced by a dipole type fit.

Experimental data for decuplet magnetic moments are available for the Δ^{++} with $\mu_{\Delta^{++}} = 3.7 \sim 7.5\mu_N$ [1], the Δ^+ with $\mu_{\Delta^+} = (2.7^{+1.0}_{-1.3}(\mathrm{stat.}) \pm 1.5(\mathrm{syst.}) \pm 3(\mathrm{theor.}))\,\mu_N$ [4] and for the Ω^- with $\mu_{\Omega^-} = (-2.02 \pm 0.05)\mu_N$. The present work yields values of $\mu_{\Delta^{++}} = 4.85\,\mu_N$, $\mu_{\Delta^+} = 2.35\,\mu_N$ and $\mu_{\Omega^-} = -2.29\,\mu_N$ which is in good agreement with the experimental ones. The $N-\Delta$ magnetic transition moment was extracted in [5] as $\mu_{N\Delta} = 3.46 \pm 0.03\,\mu_N$ whereas this work yields a value of $\mu_{\Delta N} = 2.72\,\mu_N$. Other χQSM results for decuplet magnetic moments are summarized in Tab.III.

The final results for the magnetic dipole and electric charge form factors are presented in Figs.1. In the χQSM the Δ^+ radii of these form factors, $\langle r^2_E \rangle = 0.794\,\mathrm{fm}^2$ and $\langle r^2_M \rangle = 0.634\,\mathrm{fm}^2$, are comparable to the ones of the proton, $\langle r^2_E \rangle = 0.768\,\mathrm{fm}^2$ and $\langle r^2_M \rangle = 0.656\,\mathrm{fm}^2$, keeping in mind that we take for both baryons the same classical soliton configuration. The experimental value for the proton electric radius is $\langle r^2_E \rangle \approx 0.766\,\mathrm{fm}^2$.

We also presented the electric quadrupole form factor of the Δ^+. The value $G_{E2}(0)$ is directly proportional to the Δ electric quadrupole moment for which we found a value of $Q_\Delta = -0.0509\,\mathrm{fm}^2$. The electric quadrupole moment and the electric quadrupole form factor appear in the model entirely as $1/N_c$ corrections arising form the expansion in the rotation velocity of the soliton. Hence, in the large N_c limit the model leads to a vanishing form factor and moment. In addition a decomposition into the valence and sea quark contribution of the electric quadrupole form factor, Fig.2, shows that the main contribution originates from the sea quarks. Furthermore, one can expect the sea quark contribution to have a broader spatial distribution than the one for the valence quarks. This in turn leads to a steeper Q^2 dependence of the contribution to G_{E2} of sea quarks as compared with valence quarks which is explicitly seen in the present calculation.

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VI. APPENDIX

A. Model Independent Quantities

We use the Breit-frame in which the incoming p and outgoing p' momenta are defined as

$$p' = (E, \frac{\vec{q}}{2}) \ , \ p = (E, -\frac{\vec{q}}{2}) \ , \ q = (0, \vec{q}) \ , \ Q^2 = -q^2 = \vec{q}^2 \ , \ q = |\vec{q}|(0, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta) \tag{118}$$

with $\vec{q}^2 = 4(E^2 - M^2)$. We use the Rarita-Schwinger spin-3/2 spinor

$$u^\alpha(p,s) = \sum_{\lambda,s'} C^{\frac{3}{2}s}_{1\lambda\frac{1}{2}s'} \ e^\alpha(p,\lambda) \ u(p,s') \qquad \text{with} \qquad u(p,s) = \sqrt{\frac{E+M}{2M}} \begin{pmatrix} \phi_s \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+M}\phi_s \end{pmatrix} \ .$$

The spin-1 vector $e^\alpha(p,\lambda)$ is defined by with $\lambda = \pm 1, 0$

$$e^\alpha(p,\lambda) = \Big(\frac{\hat{e}_\lambda\cdot\vec{p}}{M}, \hat{e}_\lambda + \frac{\vec{p}\cdot(\hat{e}_\lambda\cdot\vec{p})}{M(p^0+M)}\Big) \text{ with } \hat{e}_{+1} = \sqrt{\frac{1}{2}}\begin{pmatrix} -1 \\ -i \\ 0 \end{pmatrix} \ , \ \hat{e}_0 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \ , \ \hat{e}_{-1} = \sqrt{\frac{1}{2}}\begin{pmatrix} 1 \\ -i \\ 0 \end{pmatrix} \ . \tag{119}$$

The final and initial Δ states for the used third-spin components read

$$u^\beta(p,+\frac{3}{2}) \ = \ u(p,+\frac{1}{2}) \ e^\beta(p,+1) \tag{120}$$

$$u^\beta(p,+\frac{1}{2}) \ = \ \sqrt{\frac{2}{3}} \ u(p,+\frac{1}{2}) \ e^\beta(p,0) + \sqrt{\frac{1}{3}} \ u(p,-\frac{1}{2}) \ e^\beta(p,+1) \ . \tag{121}$$

For the zeroth-component of the vector current, $\langle\Delta(\frac{3}{2})|V^0|\Delta(\frac{3}{2})\rangle$ we obtain by using the Breit-frame

$$\overline{u}(p',s')\gamma^0 u(p,s) \ = \ \delta_{s's} \qquad ; \ e^{*\alpha}(p',1)g_{\alpha\beta}e^\beta(p,1) = -1 - \frac{2}{3}\tau + (3\cos^2\theta - 1)\frac{\tau}{3} \ , \tag{122}$$

$$\overline{u}(p',s')\sigma^{0\nu}q_\nu u(p,s) \ = \ -i\frac{q^2}{2M}\delta_{s's} \ ; \ e^{*\alpha}(p',1)q_\alpha q_\beta e^\beta(p,1) = 4M^2\tau\frac{1+\tau}{3}[1 - \frac{1}{2}(3\cos^2\theta - 1)] \ , \tag{123}$$

with $\tau = Q^2/(4M^2)$.
For the spatial-component of the vector current $\langle\Delta|V_k|N\rangle$we obtain by using the rest-frame of the Δ

$$\epsilon_{\beta k\sigma\tau}P_\sigma q_\tau \ = \ M \ \delta^{\beta b} \ \epsilon^{bks}q^s \ , \tag{124}$$

$$\epsilon_{\beta\sigma\nu\gamma}P_\nu q_\gamma \epsilon_{k\sigma\alpha\delta}p'_\alpha q_\delta = \epsilon_{\beta\sigma\nu\gamma}P_\nu q_\gamma \epsilon_{k\sigma 0\delta}Mq_\delta \ = \ M^2 \ \delta^{\beta b} \ [\delta^{bk}\vec{q}^2 - q^b q^k] \ . \tag{125}$$

B. χQSM Electric Densities

The electric densities of Eq.(62) are

$$
\begin{aligned}
\frac{1}{N_c}\mathcal{B}(\vec{z}) &= \phi_v^\dagger(\vec{z})\phi_v(\vec{z}) - \frac{1}{2}\sum_n \mathrm{sign}(\varepsilon_n)\phi_n^\dagger(\vec{z})\phi_n(\vec{z}), \\
\frac{1}{N_c}\mathcal{I}_1(\vec{z}) &= \frac{1}{2}\sum_{\varepsilon_n\neq\varepsilon_v}\frac{1}{\varepsilon_n-\varepsilon_v}\langle v|\tau^i|n\rangle\phi_n^\dagger(\vec{z})\tau^i\phi_v(\vec{z}) + \frac{1}{4}\sum_{n,m}\mathcal{R}_3(\varepsilon_n,\varepsilon_m)\langle n|\tau^i|m\rangle\phi_m^\dagger(\vec{z})\tau^i\phi_n(\vec{z}), \\
\frac{1}{N_c}\mathcal{I}_2(\vec{z}) &= \frac{1}{4}\sum_{\varepsilon_{n^0}}\frac{1}{\varepsilon_{n^0}-\varepsilon_v}\langle n^0|v\rangle\phi_v^\dagger(\vec{z})\phi_{n^0}(\vec{z}) + \frac{1}{4}\sum_{n,m^0}\mathcal{R}_3(\varepsilon_n,\varepsilon_{m^0})\phi_{m^0}^\dagger(\vec{z})\phi_n(\vec{z})\langle n|m^0\rangle, \\
\frac{1}{N_c}\mathcal{C}(\vec{z}) &= \sum_{\varepsilon_n\neq\varepsilon_v}\frac{1}{\varepsilon_n-\varepsilon_v}\phi_v^\dagger(\vec{z})\phi_n(\vec{z})\langle n|\gamma^0|v\rangle + \frac{1}{2}\sum_{n,m}\langle n|\gamma^0|m\rangle\phi_m^\dagger(\vec{z})\phi_n(\vec{z})\mathcal{R}_5(\varepsilon_n,\varepsilon_m), \\
\frac{1}{N_c}\mathcal{K}_1(\vec{z}) &= \frac{1}{2}\sum_{\varepsilon_n\neq\varepsilon_v}\frac{1}{\varepsilon_n-\varepsilon_v}\langle v|\gamma^0\tau^i|n\rangle\phi_n^\dagger(\vec{z})\tau^i\phi_v(\vec{z}) + \frac{1}{4}\sum_{n,m}\langle n|\gamma^0\tau^i|m\rangle\phi_m^\dagger(\vec{z})\tau^i\phi_n(\vec{z})\mathcal{R}_5(\varepsilon_n,\varepsilon_m), \\
\frac{1}{N_c}\mathcal{K}_2(\vec{z}) &= \frac{1}{4}\sum_{\varepsilon_{n^0}}\frac{1}{\varepsilon_{n^0}-\varepsilon_v}\phi_v^\dagger(\vec{z})\phi_{n^0}(\vec{z})\langle n^0|\gamma^0|v\rangle + \frac{1}{4}\sum_{n,m}\mathcal{R}_5(\varepsilon_n,\varepsilon_{m^0})\phi_{m^0}^\dagger(\vec{z})\phi_n(\vec{z})\langle n|\gamma^0|m^0\rangle.
\end{aligned}
$$

The vectors $\langle n|$ are eigenstates of the χQSM Hamiltonian $h(U)$ which are a linear combination of the eigenstates $\langle n^0|$ of the Hamiltonian $H(1)$ [72].

C. χQSM Magnetic Densities

The operator for the magnetic form factors in the χQSM is $O_1 = \gamma^0[\vec{z}\times\vec{\gamma}]_3 = \gamma^5[\vec{z}\times\vec{\sigma}]_{10}$ and the magnetic densities of Eq.(63) are

$$
\begin{aligned}
\frac{1}{N_c}\mathcal{Q}_0(\vec{z}) &= \langle v||\vec{z}\rangle\{O_1\otimes\tau_1\}_0\langle\vec{z}||v\rangle + \sum_n\sqrt{2G_n+1}\langle n||\vec{z}\rangle\{O_1\otimes\tau_1\}_0\langle\vec{z}||n\rangle\mathcal{R}_1(\varepsilon_n), \\
\frac{1}{N_c}\mathcal{X}_1(\vec{z}) &= \sum_{\varepsilon_n\neq\varepsilon_v}\frac{1}{\varepsilon_n-\varepsilon_v}(-)^{G_n}\langle v||\vec{z}\rangle O_1\langle\vec{z}||n\rangle\langle n||\tau_1||v\rangle \\
&\quad + \frac{1}{2}\sum_{n,m}\mathcal{R}_5(\varepsilon_n,\varepsilon_m)(-)^{G_m-G_n}\langle n||\tau_1||m\rangle\langle m||\vec{z}\rangle O_1\langle\vec{z}||n\rangle, \\
\frac{1}{N_c}\mathcal{X}_2(\vec{z}) &= \sum_{\varepsilon_{n^0}}\frac{1}{\varepsilon_{n^0}-\varepsilon_v}\langle n^0||\vec{z}\rangle\{O_1\otimes\tau_1\}_0\langle\vec{z}||v\rangle\langle v\mid n^0\rangle \\
&\quad + \sum_{n,m^0}\mathcal{R}_5(\varepsilon_n,\varepsilon_{m^0})\sqrt{2G_m+1}\langle m^0||\vec{z}\rangle\{O_1\otimes\tau_1\}_0\langle\vec{z}||n\rangle\langle n\mid m^0\rangle, \\
\frac{1}{N_c}\mathcal{Q}_1(\vec{z}) &= \sum_{\varepsilon_n}\frac{\mathrm{sign}(\varepsilon_n)}{\varepsilon_n-\varepsilon_v}(-)^{G_n}\langle n||\vec{z}\rangle\{O_1\otimes\tau_1\}_1\langle\vec{z}||v\rangle\langle v||\tau_1||n\rangle \\
&\quad + \frac{1}{2}\sum_{n,m}\mathcal{R}_4(\varepsilon_n,\varepsilon_m)(-)^{G_m-G_n}\langle n||\vec{z}\rangle\{O_1\otimes\tau_1\}_1\langle\vec{z}||m\rangle\langle m||\tau_1||n\rangle, \\
\frac{1}{N_c}\mathcal{M}_0(\vec{z}) &= \sum_{\varepsilon_n\neq\varepsilon_v}\frac{1}{\varepsilon_n-\varepsilon_v}\langle v||\vec{z}\rangle\{O_1\otimes\tau_1\}_0\langle\vec{z}||n\rangle\langle n|\gamma^0|v\rangle \\
&\quad - \frac{1}{2}\sum_{n,m}\mathcal{R}_2(\varepsilon_n,\varepsilon_m)\sqrt{2G_m+1}\langle n|\gamma^0|m\rangle\langle m||\vec{z}\rangle\{O_1\otimes\tau_1\}_0\langle\vec{z}||n\rangle, \\
\frac{1}{N_c}\mathcal{M}_1(\vec{z}) &= \sum_{\varepsilon_n\neq\varepsilon_v}\frac{1}{\varepsilon_n-\varepsilon_v}(-)^{G_n}\langle n||\gamma^0\tau_1||v\rangle\langle v||\vec{z}\rangle O_1\langle\vec{z}||n\rangle \\
&\quad - \frac{1}{2}\sum_{n,m}\mathcal{R}_2(\varepsilon_n,\varepsilon_m)(-)^{G_m-G_n}\langle n||\gamma^0\tau_1||m\rangle\langle m||\vec{z}\rangle O_1\langle\vec{z}||n\rangle,
\end{aligned}
$$

$$\frac{1}{N_c}\mathcal{M}_2(\vec{z}) = \sum_{\varepsilon_{n^0}} \frac{1}{\varepsilon_{n^0}-\varepsilon_v}\langle v||\vec{z}\rangle\{O_1\otimes\tau_1\}_0\langle\vec{z}||n^0\rangle\langle n^0|\gamma^0|v\rangle$$
$$-\sum_{n,m^0}\mathcal{R}_2(\varepsilon_n,\varepsilon_{m^0})\sqrt{2G_m+1}\langle m^0||\vec{z}\rangle\{O_1\otimes\tau_1\}_0\langle\vec{z}||n\rangle\langle n|\gamma^0|m^0\rangle.$$

D. Regularization Functions

The regularization functions are defined as:

$$\mathcal{R}_1(\varepsilon_n) = -\frac{1}{2\sqrt{\pi}}\varepsilon_n\int_{1/\Lambda^2}^{\infty}\frac{du}{\sqrt{u}}e^{-u\varepsilon_n^2}, \tag{126}$$

$$\mathcal{R}_2(\varepsilon_n,\varepsilon_m) = \int_{1/\Lambda^2}^{\infty}du\frac{1}{2\sqrt{\pi u}}\frac{\varepsilon_m e^{-u\varepsilon_m^2}-\varepsilon_n e^{-u\varepsilon_n^2}}{\varepsilon_n-\varepsilon_m}, \tag{127}$$

$$\mathcal{R}_3(\varepsilon_n,\varepsilon_m) = \frac{1}{2\sqrt{\pi}}\int_{1/\Lambda^2}^{\infty}\frac{du}{\sqrt{u}}\Big[\frac{1}{u}\frac{e^{-\varepsilon_n^2 u}-e^{-\varepsilon_m^2 u}}{\varepsilon_m^2-\varepsilon_n^2}-\frac{\varepsilon_n e^{-u\varepsilon_n^2}+\varepsilon_m e^{-u\varepsilon_m^2}}{\varepsilon_m+\varepsilon_n}\Big], \tag{128}$$

$$\mathcal{R}_4(\varepsilon_n,\varepsilon_m) = \frac{1}{2\pi}\int_{1/\Lambda^2}^{\infty}du\int_0^1 d\alpha e^{-\varepsilon_n^2 u(1-\alpha)-\alpha\varepsilon_m^2 u}\frac{\varepsilon_n(1-\alpha)-\alpha\varepsilon_m}{\sqrt{\alpha(1-\alpha)}}, \tag{129}$$

$$\mathcal{R}_5(\varepsilon_n,\varepsilon_m) = \frac{1}{2}\frac{\mathrm{sign}\varepsilon_n-\mathrm{sign}\varepsilon_m}{\varepsilon_n-\varepsilon_m}, \tag{130}$$

$$\mathcal{R}_6(\varepsilon_n,\varepsilon_m) = \frac{1-\mathrm{sign}(\varepsilon_n)\mathrm{sign}(\varepsilon_m)}{\varepsilon_n-\varepsilon_m}. \tag{131}$$

E. Reduced Matrix Elements for $\{\sqrt{4\pi}Y_2\otimes\tau_1\}_1$

We use the basis of [72] where the iso-spin τ and total angular momentum j is coupled to the grand-spin $G=\tau+j$ ($j=l+s$)

$$|0\rangle = |l=G\ ;\ j=G+\frac{1}{2}\ ;\ GG_3\rangle\ , \tag{132}$$

$$|1\rangle = |l=G\ ;\ j=G-\frac{1}{2}\ ;\ GG_3\rangle\ , \tag{133}$$

$$|2\rangle = |l=G+1\ ;\ j=G+\frac{1}{2}\ ;\ GG_3\rangle\ , \tag{134}$$

$$|3\rangle = |l=G-1\ ;\ j=G-\frac{1}{2}\ ;\ GG_3\rangle\ . \tag{135}$$

The reduced matrix elements for the operator $\{\sqrt{4\pi}Y_2\otimes\tau_1\}_1$ in the density $\mathcal{I}_{1E2}(r)$ Eq.(90) are with the notation $\langle n||\{\sqrt{4\pi}Y_2\otimes\tau_1\}_1||m\rangle$:

$$A^0(G) = (-)(G+2)\sqrt{\frac{2G}{(2G+1)(G+1)}}$$

$$B^0(G) = (-)3\sqrt{\frac{1}{2(2G+1)}}$$

$$C^0(G) = (G-1)\sqrt{\frac{(2G+2)}{(G)(2G+1)}}$$

$$A^1(G) = (-)G(2G+4)\sqrt{\frac{1}{(2G+1)(2G+2)(2G+3)}}$$

$$B^1(G) = 3\sqrt{\frac{(G+2)}{2(2G+1)(2G+3)}}$$

$$C^1(G) = (-)3\sqrt{\frac{G(G+1)(2G+4)}{(2G+1)(2G+3)}}$$

$$D^1(G) = (-)3\sqrt{\frac{G}{2(2G+1)(2G+3)}}$$

$G^m = G^n$	$\|0(G)\rangle$	$\|1(G)\rangle$	$\|2(G)\rangle$	$\|3(G)\rangle$
$\langle 0(G)\|$	$A^0(G)$	$B^0(G)$	0	0
$\langle 1(G)\|$	$B^0(G)$	$C^0(G)$	0	0
$\langle 2(G)\|$	0	0	$A^0(G)$	$B^0(G)$
$\langle 3(G)\|$	0	0	$B^0(G)$	$C^0(G)$

$G^m = G^n + 1$	$\|0(G+1)\rangle$	$\|1(G+1)\rangle$	$\|2(G+1)\rangle$	$\|3(G+1)\rangle$
$\langle 0(G)\|$	0	0	$B^1(G)$	$A^1(G)$
$\langle 1(G)\|$	0	0	$C^1(G)$	$D^1(G)$
$\langle 2(G)\|$	$B^1(G)$	$A^1(G)$	0	0
$\langle 3(G)\|$	$C^1(G)$	$D^1(G)$	0	0

F. Matrix-Elements

The baryon matrix-elements, such as $\langle B'|D^{(8)}_{\chi 3}|B\rangle$, are evaluated by using the $SU(3)$ group algebra [73, 74]

$$\langle B'_{\mathcal{R}'}|D^{n}_{\chi m}(A)|B_{\mathcal{R}}\rangle = \sqrt{\frac{\mathrm{dim}\mathcal{R}'}{\mathrm{dim}\mathcal{R}}}(-1)^{\frac{1}{2}Y'_s+S'_3}(-1)^{\frac{1}{2}Y_s+S_3} \times \sum_{\gamma}\begin{pmatrix} \mathcal{R}' & n & \mathcal{R}_\gamma \\ Q' & \chi & Q \end{pmatrix}\begin{pmatrix} \mathcal{R}' & n & \mathcal{R}_\gamma \\ -Y'_s S' - S'_3 & m & -Y_S S - S_3 \end{pmatrix} , \tag{136}$$

with $Q = YII_3$. $(\cdots)$ denote the $SU(3)$ Clebsch-Gordan coefficients.
The wave function corrections Eq.(49) for the other decuplet baryons are

$$|B_{10}\rangle = |10_{3/2}, B\rangle + a^B_{27}|27_{3/2}, B\rangle + a^B_{35}|35_{3/2}, B\rangle , \tag{137}$$

with the mixing coefficients

$$a^B_{27} = a_{27}\begin{pmatrix} \sqrt{15/2} \\ 2 \\ \sqrt{3/2} \\ 0 \end{pmatrix}, \quad a^B_{35} = a_{35}\begin{pmatrix} 5/\sqrt{14} \\ 2\sqrt{5/7} \\ 3\sqrt{5/14} \\ 2\sqrt{5/7} \end{pmatrix} , \tag{138}$$

in the bases $[\Delta, \Sigma^*_{10}, \Xi^*_{10}, \Omega]$.

1. *Magnetic Part*

We take the abbreviation $d_{ab3}D^{(8)}_{\chi b}J_a = dD_\chi J$ and the matrix-element for the magnetic form factors of the decuplet baryons read as follows with $|B_{10}\rangle = |B_{10}(Y, I_3, S_3)\rangle$:

Leading order:

$\langle\Delta|D^{(8)}_{33}|\Delta\rangle = \langle\Sigma_{10}|D^{(8)}_{33}|\Sigma_{10}\rangle = \langle\Xi_{10}|D^{(8)}_{33}|\Xi_{10}\rangle = \langle\Omega|D^{(8)}_{33}|\Omega\rangle =$	$-I_3\frac{1}{6}S_3$
$\langle\Delta|D^{(8)}_{38}S_3|\Delta\rangle = \langle\Sigma_{10}|D^{(8)}_{38}S_3|\Sigma_{10}\rangle = \langle\Xi_{10}|D^{(8)}_{38}S_3|\Xi_{10}\rangle = \langle\Omega|D^{(8)}_{38}S_3|\Omega\rangle =$	$I_3\frac{1}{4}\sqrt{\frac{1}{3}}S_3$
$\langle\Delta|dD_3J|\Delta\rangle = \langle\Sigma_{10}|dD_3J|\Sigma_{10}\rangle = \langle\Xi_{10}|dD_3J|\Xi_{10}\rangle = \langle\Omega|dD_3J|\Omega\rangle =$	$I_3\frac{1}{12}S_3$
$\langle\Delta|D^{(8)}_{83}|\Delta\rangle = \langle\Sigma_{10}|D^{(8)}_{83}|\Sigma_{10}\rangle = \langle\Xi_{10}|D^{(8)}_{83}|\Xi_{10}\rangle = \langle\Omega|D^{(8)}_{83}|\Omega\rangle =$	$-Y\frac{1}{4}\sqrt{\frac{1}{3}}S_3$
$\langle\Delta|D^{(8)}_{88}S_3|\Delta\rangle = \langle\Sigma_{10}|D^{(8)}_{88}S_3|\Sigma_{10}\rangle = \langle\Xi_{10}|D^{(8)}_{88}S_3|\Xi_{10}\rangle = \langle\Omega|D^{(8)}_{88}S_3|\Omega\rangle =$	$Y\frac{1}{8}S_3$
$\langle\Delta|dD_8J|\Delta\rangle = \langle\Sigma^*_{10}|dD_8J|\Sigma^*_{10}\rangle = \langle\Xi^*_{10}|dD_8J|\Xi^*_{10}\rangle = \langle\Omega|dD_8J|\Omega\rangle =$	$Y\frac{1}{8}\frac{1}{\sqrt{3}}S_3$

Wavefunction corrections:

$$\langle\Omega|D^{(8)}_{33}|\Omega\rangle = \langle\Omega|D^{(8)}_{38}J_3|\Omega\rangle = \langle\Omega|dD_3J|\Omega\rangle = 0$$

$$\langle\Omega|D^{(8)}_{83}|\Omega\rangle = -a^B_{35}S_3\sqrt{\frac{1}{105}} \quad \langle\Omega|D^{(8)}_{88}S_3|\Omega\rangle = S_3a^B_{35}\frac{5}{2}\sqrt{\frac{1}{35}} \quad \langle\Omega|dD_8J|\Omega\rangle = -\frac{5}{2}S_3\sqrt{\frac{1}{105}}a^B_{35}$$

	Δ		Δ
$D^{(8)}_{33}$	$I_3 S_3 \left[-a^B_{27}\frac{5}{9}\sqrt{\frac{1}{30}} - a^B_{35}\frac{1}{15}\sqrt{\frac{1}{14}}\right]$	$D^{(8)}_{83}$	$S_3 \left[a^B_{27}\frac{5}{6}\sqrt{\frac{1}{10}} - a^B_{35}\frac{1}{2}\sqrt{\frac{1}{42}}\right]$
$D^{(8)}_{38} J_3$	$I_3 S_3 \left[-a^B_{27}\frac{5}{6}\sqrt{\frac{1}{10}} + a^B_{35}\frac{1}{2}\sqrt{\frac{1}{42}}\right]$	$D^{(8)}_{88} J_3$	$S_3 \left[a^B_{27}\frac{15}{4}\sqrt{\frac{1}{30}} + a^B_{35}\frac{5}{4}\sqrt{\frac{1}{14}}\right]$
$d D_3 J$	$I_3 S_3 \left[-\frac{5}{18}\sqrt{\frac{1}{30}}a^B_{27} - \frac{1}{6}a^B_{35}\sqrt{\frac{1}{14}}\right]$	$d D_8 J$	$S_3 \left[\frac{5}{12}\sqrt{\frac{1}{10}}a^B_{27} - \frac{5}{4}\sqrt{\frac{1}{42}}a^B_{35}\right]$
	Σ^*_{10}		Σ^*_{10}
$D^{(8)}_{33}$	$I_3 S_3 \left[-a^B_{27}\frac{1}{6} - a^B_{35}\frac{1}{6}\sqrt{\frac{1}{35}}\right]$	$D^{(8)}_{83}$	$S_3 \left[a^B_{27}\frac{1}{3}\sqrt{\frac{1}{3}} - a^B_{35}\sqrt{\frac{1}{105}}\right]$
$D^{(8)}_{38} S_3$	$I_3 S_3 \left[-a^B_{27}\frac{3}{2}\sqrt{\frac{1}{3}} + a^B_{35}\frac{5}{2}\sqrt{\frac{1}{105}}\right]$	$D^{(8)}_{88} S_3$	$S_3 \left[a^B_{27}\frac{1}{2} + a^B_{35}\frac{5}{2}\sqrt{\frac{1}{35}}\right]$
$d D_3 J$	$I_3 S_3 \left[-\frac{1}{12}a^B_{27} - \frac{5}{12}\sqrt{\frac{1}{35}}a^B_{35}\right]$	$d D_8 J$	$S_3 \left[\frac{1}{6}a^B_{27}\sqrt{\frac{1}{3}} - \frac{5}{2}\sqrt{\frac{1}{105}}a^B_{35}\right]$
	Ξ^*_{10}		Ξ^*_{10}
$D^{(8)}_{33}$	$I_3 S_3 \left[-a^B_{27}\frac{7}{9}\sqrt{\frac{1}{6}} - a^B_{35}\frac{1}{3}\sqrt{\frac{1}{70}}\right]$	$D^{(8)}_{83}$	$S_3 \left[a^B_{27}\frac{1}{6}\sqrt{\frac{1}{2}} - a^B_{35}\frac{3}{2}\sqrt{\frac{1}{210}}\right]$
$D^{(8)}_{38} S_3$	$I_3 S_3 \left[-a^B_{27}\frac{7}{6}\sqrt{\frac{1}{2}} + a^B_{35}\frac{5}{2}\sqrt{\frac{1}{210}}\right]$	$D^{(8)}_{88} S_3$	$S_3 \left[a^B_{27}\frac{3}{4}\sqrt{\frac{1}{6}} + a^B_{35}\frac{15}{4}\sqrt{\frac{1}{70}}\right]$
$d D_3 J$	$I_3 S_3 \left[-\frac{7}{18}a^B_{27}\sqrt{\frac{1}{6}} - \frac{5}{6}\sqrt{\frac{1}{70}}a^B_{35}\right]$	$d D_8 J$	$S_3 \left[\frac{1}{12}\sqrt{\frac{1}{2}}a^B_{27} - \frac{15}{4}\sqrt{\frac{1}{210}}a^B_{35}\right]$

Operator corrections $D^{(8)}_{88}D^{(8)}_{83} = D^{(8)}_{83}D^{(8)}_{88}$

	Δ	Σ^*_{10}	Ξ^*_{10}	Ω
$D^{(8)}_{88}D^{(8)}_{33}$	$-S_3 I_3 \frac{5}{126}$	$-S_3 I_3 \frac{1}{42}$	$-S_3 I_3 \frac{1}{126}$	0
$D^{(8)}_{83}D^{(8)}_{38}$	$-S_3 I_3 \frac{5}{126}$	$-I_3 \frac{1}{42}$	$-S_3 I_3 \frac{1}{126}$	0
$D^{(8)}_{8a}D^{(8)}_{3b}d_{ab3}$	$-S_3 I_3 \frac{11}{126}\sqrt{\frac{1}{3}}$	$-S_3 I_3 \frac{5}{42}\sqrt{\frac{1}{3}}$	$-S_3 I_3 \frac{19}{126}\sqrt{\frac{1}{3}}$	0
$D^{(8)}_{83}D^{(8)}_{88}$	$S_3 \frac{1}{28}\sqrt{\frac{1}{3}}$	$S_3 \frac{1}{42}\sqrt{\frac{1}{3}}$	$-S_3 \frac{1}{28}\sqrt{\frac{1}{3}}$	$-S_3 \frac{1}{7}\sqrt{\frac{1}{3}}$
$D^{(8)}_{8a}D^{(8)}_{8b}d_{ab3}$	$S_3 \frac{5}{84}$	$-S_3 \frac{1}{63}$	$-S_3 \frac{5}{84}$	$-S_3 \frac{1}{14}$

2. Electric Part

The Δ state $|\Delta\rangle$ is explicitly $|\Delta\rangle = |\Delta(I_3, S_3)\rangle$ and the matrix elements for the electric form factor read:

$$
\begin{aligned}
\langle\Delta|D^{8}_{38}|\Delta\rangle &= I_3\Big[\frac{1}{4}\sqrt{\frac{1}{3}} - a^{\Delta}_{27}\frac{5}{6}\sqrt{\frac{1}{10}} + a^{\Delta}_{35}\frac{1}{2}\sqrt{\frac{1}{42}}\Big] \\
\langle\Delta|D^{(8)}_{3i}J_i|\Delta\rangle &= I_3\Big[-\frac{5}{8} - a^{\Delta}_{27}\frac{25}{12}\sqrt{\frac{1}{30}} - a^{\Delta}_{35}\frac{1}{4}\sqrt{\frac{1}{14}}\Big] \\
\langle\Delta|D^{(8)}_{3a}J_a|\Delta\rangle &= I_3\Big[-\frac{1}{4} + a^{\Delta}_{27}\frac{5}{6}\sqrt{\frac{1}{30}} + a^{\Delta}_{35}\frac{1}{2}\sqrt{\frac{1}{14}}\Big] \\
\langle\Delta|D^{(8)}_{88}|\Delta\rangle &= \frac{1}{8} + a^{\Delta}_{27}\frac{15}{4}\sqrt{\frac{1}{30}} + a^{\Delta}_{35}\frac{5}{4}\sqrt{\frac{1}{14}} \\
\langle\Delta|D^{(8)}_{8i}J_i|\Delta\rangle &= -\frac{15}{16}\sqrt{\frac{1}{3}} + a^{\Delta}_{27}\frac{25}{8}\sqrt{\frac{1}{10}} - a^{\Delta}_{35}\frac{15}{8}\sqrt{\frac{1}{42}} \\
\langle\Delta|D^{(8)}_{8a}J_a|\Delta\rangle &= -\frac{3}{8}\frac{1}{\sqrt{3}} - a^{\Delta}_{27}\frac{5}{4}\sqrt{\frac{1}{10}} + a^{\Delta}_{35}\frac{15}{4}\sqrt{\frac{1}{42}}
\end{aligned}
$$

$$
\begin{aligned}
\langle\Delta|D^{(8)}_{8i}D^{(8)}_{3i}|\Delta\rangle &= I_3\frac{13}{84}\sqrt{\frac{1}{3}} \\
\langle\Delta|D^{(8)}_{8a}D^{(8)}_{3a}|\Delta\rangle &= -I_3\frac{5}{42}\sqrt{\frac{1}{3}} \\
\langle\Delta|D^{(8)}_{88}D^{(8)}_{38}|\Delta\rangle &= -I_3\frac{1}{28}\sqrt{\frac{1}{3}} \\
\langle\Delta|D^{(8)}_{8i}D^{(8)}_{8i}|\Delta\rangle &= \frac{17}{56} \\
\langle\Delta|D^{(8)}_{8a}D^{(8)}_{8a}|\Delta\rangle &= \frac{15}{28} \\
\langle\Delta|D^{(8)}_{88}D^{(8)}_{88}|\Delta\rangle &= \frac{9}{56}
\end{aligned}
$$

VII. REFERENCES

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