# The staff scheduling problem: a general model and applications

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## Abstract

The scheduling of employees is a complex and time-consuming task. It is complex because it involves assigning the right people to the right job at the right time. It is conditioned by several legal, work and other organizational rules. And it often copes with conflicting objectives, such as the minimization of the labor costs or the workforce size and the satisfaction of the employees preferences, for example. It is time-consuming because it is a periodic task and it is still manually performed in most organizations.

The research work described in this thesis deals with the development of methods for the automatic scheduling of employees, in particular for their simultaneous assignment to working shifts and days-off. The work focuses on the design of a general integer programming (IP) model that, following an optimization approach, can be easily adapted and solve a wide set of different real-world problems. An innovative formulation of the sequence and consecutiveness constraints gives the model the flexibility to accommodate variable features of the problems. A cyclic approach ensures the generation of equitable and predictable work schedules.

The application of the general model is illustrated with three real-world case studies and a collection of benchmark instances available in the literature. Computational results demonstrate the good performance of the model, achieving optimal solutions for the majority of the problems in useful time.

A constructive heuristic is also developed for solving one of the case-studies. Based on a set of simple calculations, the proposed procedure reveals to be an efficient alternative to the IP optimization approach for solving the practical problem considered. The good performance achieved with tests on a set of larger computer generated instances confirms the robustness of this approximate approach.

Although its apparent pertinency to the activity sector, staff scheduling problems in hospitality management have been quite unnoticed by the research community. This thesis dedicates a chapter to this topic, namely to the assessment of the potential of hospitality management as an application area for staff scheduling problems and of possible resolution approaches.

## Resumo

O escalonamento de pessoal é uma tarefa complexa e fortemente consumidora de recursos. É complexa porque envolve a afetação das pessoas certas ao trabalho certo no momento certo. É condicionada por diversas regras de natureza legal, laboral ou organizacional. Lida normalmente com objetivos divergentes, tais como a mimimização de custos ou a dimensão da equipa de trabalho e a satisfação das preferências dos trabalhadores, por exemplo. Consome recursos porque é feita periodicamente e ainda de modo manual, em muitas organizações.

O trabalho de investigação descrito nesta tese aborda o desenvolvimento de métodos para o escalonamento automático de pessoal, em particular com a sua afetação simultânea a turnos de trabalho e dias de descanso. O trabalho centra-se no desenvolvimento de um modelo geral de programação inteira que, seguindo uma abordagem de otimização, pode ser facilmente adaptado e resolver um conjunto alargado de diferentes problemas reais. A formulação inovadora das restrições de sequência e consecutividade confere ao modelo a flexibilidade necessária para acomodar características variáveis dos problemas. Uma abordagem cíclica assegura a geração de horários equilibrados e previsíveis.

A aplicação do modelo geral é ilustrada através de três casos de estudo baseados em problemas reais e de um conjunto de instâncias de *benchmark* disponíveis na literatura. Os resultados computacionais demonstram o bom desempenho do modelo, obtendo as soluções ótimas para a maior parte dos problemas em tempo útil.

Uma heurística construtiva foi também desenvolvida para um dos casos de estudo. Baseado num conjunto de cálculos simples, o procedimento proposto revela ser uma alternativa eficiente à abordagem de otimização para a resolução do problema prático considerado. O bom desempenho conseguido com testes em instâncias de maior dimensão comprova a robustez deste método aproximado.

Apesar da sua aparente pertinência para o sector de atividade, os problemas de escalonamento de pessoal na área de gestão da hospitalidade não têm merecido a devida atenção por parte da comunidade académica. Esta tese dedica um dos seus capítulos a este tópico, nomeadamente à avaliação do potencial da gestão da hospitalidade como uma área de aplicação para os problemas de escalonamento de pessoal e de possíveis abordagens de resolução.

## Acknowledgments

This thesis is the culmination of the work accomplished during the most demanding phase of my personal life. During these four years, I felt my physical and emotional endurance being strained to the limit. If I succeeded and reached this far I owe it to the people that supported me in this journey, to whom I here address my sincere acknowledgments.

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# Chapter 1

# Introduction

## 1.1 Motivation

Staff scheduling is a common problem to most organizations, either from the service sector or industrial plants. Basically, it seeks to assign employees to tasks, work shifts or rest periods, taking into account organizational and legal rules, employees' skills and preferences, demand needs, and other applicable requirements. It is therefore a complex problem and a top concern for human resource management (Enz (2009)). Even nowadays, it is still done manually in several activity sectors, consuming time and resources that could be used more efficiently with automatic scheduling generators. Thompson (2003) points out three reasons for caring about staff scheduling: the time spent developing a schedule by hand leaves the manager less time for managing the employees and interacting with the customers; a schedule that better satisfies employees' preferences increases the on-job-performance and consequently the productivity and the service quality; in a good schedule work is assigned in the most effective manner, leading to a cost reduction due to over and understaffing and an increase in profitability. It is not only a matter of reducing costs, but also a matter of finding a solution that better fits cost minimization, compliance with work and legal rules, satisfaction and well-fare of employees. The design of schedules should take therefore into account objective factors such as labour costs, applicable legislation, organizational rules and demand needs but also other sensitive dimensions like flexibility, stability, predictability or fairness. It is generally acknowledged the significative impact that these last attributes can have on the productivity and engagement of an employee (Glass and Knight (2010)). Stressful factors such as short periods of rest and long periods of work, inadequate distribution between rest and work periods or non-standard working shifts, for example, can negatively affect the mental and physical health of employees (Totterdell (2005)).

Although the staff scheduling problem has been intensively explored in the literature, studies usually focus on solving very particular problems that derive from practical needs. Models are usually developed for specific applications and their adaptation to other cases implies significative reformulation. It is generally considered by researchers that cyclic scheduling approaches are inflexible because they impose a rigid schedule, not adjustable to unpredictable changes. Workload balance is usually tackled as a non-mandatory or soft constraint of the problem. When dealing with real-life problems, the trend has been to use approximate solution approaches rather than optimization methods. This is mainly due to their high complexity and size. However such approximate procedures are, by nature, tailored for specific problems.

This research has a twofold motivation. From a business perspective, it aims to contribute to the increase in both productivity and profitability of a company. The development of an automatic scheduling procedure and the adequate design of the schedules contributes to these goals. From an academic point of view, this work aims to provide a wide-range approach that is able to find optimal solutions for different real-life problems. It intends to be innovative, combining an original formulation of the sequence and consecutiveness constraints with a flexible cyclic scheduling approach.

## 1.2 Research approach

The main objective of the research work described in this thesis is to develop an optimization model that can be easily adjusted to address different real-life staff scheduling problems, from different application areas. This goal imposes a preliminary investigation into the current literature on staff scheduling problems in order to understand the problem in depth and to justify the relevance of the proposed approach. An additional output of this literature review is the insight into the particular application of these problems to hospitality management operations, which is an almost unexplored combination.

Stimulated by three real case-studies, the research concentrates on solving the problem of simultaneously assigning employees to work shifts and days-off in each of the three applications. The problems have similar work environments based on a 24-hour continuous operation and work shifts with fixed starting-times and lengths. The workforce is single-skilled in two of the problems, but in one of the case-studies multi-skilled employees are grouped in teams and the scheduling is made for each team, which is a novel modelling aspect. While in one of the cases the staff is composed only by full-time employees, the other two problems consider different types of labor contracts. Constraints common to all problems concern daily demand requirements, sequences of work shifts and days-off and consecutive number of work shifts/days-off. Long weekends-off periodicity and planned absences are occasionally tackled in different case-studies.

Each one of the three problems has a different sequence of shifts and days-off that must be followed and the workload must be evenly distributed between all the employees. These two conditions represent the main modelling challenges of this work. The way they are dealt with in the proposed formulation intends to be a worthy contribution to the research literature. The sequence and consecutiveness constraints are formulated in a very innovative way that gives the model an increased flexibility to tackle any pattern of work shifts and days off. The workload balance is ensured by the cyclic scheduling approach, through a hard constraint. To counteract the inflexibility often assigned to cyclic scheduling, it is used to successfully solve problems that are typically addressed with acyclic approaches, namely problems with a heterogeneous workforce and fluctuating demand levels.

Instead of setting the planning horizon as an input of the problem, as is the common practice in the literature, we study several planning periods in order to choose the planning horizon that better fits the goals of the problem. We explore the integration of periods with different lengths into a longer planning horizon. This is another original contribution of this research.

The developed integer programming (IP) general model is successfully applied to the three case-studies with minor adjustments, mainly parameterizations. In order to demonstrate its consistency and reinforce its flexibility, the model is also adapted to solve a collection of benchmark instances.

The study of a heuristic approach aims to enrich the contribution of this research with a comparison between an optimization and an approximate method for solving one of the real-world case-studies. The heuristic procedure is based on simple calculations and assumptions that derive from the analysis of the problem's input data. Although it is built for a particular problem, the heuristic demonstrates a consistent performance when applied to a set of larger computer generated instances, which result from the variation of some of the parameters, revealing to be a viable alternative to the optimization method.

## 1.3 Thesis outline

The thesis is organized around 9 chapters, besides the present introductory chapter.

Chapter 2 presents a comprehensive overview on the staff scheduling problem, its main features, variants and most common applications. Some relevant modeling issues are addressed and the related literature is reviewed. The aim is to provide essential background on the topic.

Chapter 3 is devoted to a study on hospitality management, with the purpose of understanding the concept of hospitality and exploring the potential of this activity sector as an application area for staff scheduling problems.

Chapters 4, 5, 6, 7 and 8 concern the developed optimization approach. The general IP model is firstly introduced in Chapter 4. The next three chapters illustrate the application of the general IP model to three practical case studies, one from an industrial plant and two from services. Chapter 5 reports a long-term staff scheduling problem in a glass production unit. Then, the general model is adjusted to the problem of scheduling a set of care takers in a continuous care unit (Chapter 6). Chapter 7 concerns the problem of nurse scheduling in a portuguese hospital. In Chapter 8, the general model is adjusted to a set of benchmark instances. The application of the model is extended to a range of problems with larger size, testing the consistency of the model's performance.

Chapter 9 presents a constructive heuristic to solve the glass unit problem. A comparison between this approach and the previous optimization approach is carried out.

The last chapter (Chapter 10) sums up the accomplished research work and suggests future developments.

# Chapter 2

# Staff scheduling problems

The aim of this chapter is to provide some important background information on staff scheduling problems to a reader who is not deeply familiar with the subject. The first section presents the staff scheduling problem in detail, describing the sub-problems and the application areas that have been more explored in the literature. Next, an overview on the main modeling issues is included. The last section of the chapter goes through some of the most relevant related works published in the literature of staff scheduling, from surveys and general studies to more specific research papers.

## 2.1 Defining the problem

Wren (1996) defines scheduling as “the allocation, subject to constraints, of resources to objects being placed in space-time, in such a way as to minimize the total cost of some set of the resources used” and rostering as “the placing, subject to constraints, of resources into slots in a pattern. One may seek to minimize some objective, or simply to obtain a feasible allocation. Often the resources will rotate through a roster. (...) Once shifts have been produced showing the daily work of personnel, these shifts are placed into a roster to show which shifts are worked by individuals on particular days”.

A shift usually corresponds to a block of work periods to be performed consecutively, with or without short meal or rest breaks. In the same work, Wren classifies rostering, as well as timetabling and sequencing, as special cases of scheduling, which in turn refer to both the generic scheduling problem and also to some of its specific types. Despite this differentiation, he recognizes that these terms tend to be generally used in a nonrigid way. A quick look at published articles confirms an inconsistency in the use of the expressions rostering and scheduling. Nevertheless, it is not abusive to state that rostering is typically associated with the allocation of people (human resources) while the objects of scheduling may vary from human resources, to vehicles, machines, or examinations to jobs. Several designations can be found in the literature to refer to the general problem of allocating human resources to work schedules. Those include staff, workforce, labour, employee or personnel scheduling problem. For the purposes of this research work staff scheduling and rostering are treated as synonym and the first expression is adopted.

The staff scheduling problem in any organization embraces basically the following challenges: determining the demand requirements, designing the most suitable work basic blocks (shifts, duties, pairings, etc.), arranging those blocks into lines of work or schedules, and assigning the staff elements to the schedules.

As in many other planning problems, these involve decisions that are not independent from each other and can be seen in a timeline perspective, from long to short-term planning, from strategic to tactical and operational decision-making and therefore temporal dependencies between them shall be considered, as illustrated by Fig. 2.1. Although there are situations where some of these decisions do overlap in time and the problems are tackled simultaneously, most of the cases explored in the literature focus only on part of the decision-making process. The most common sub-problems in-

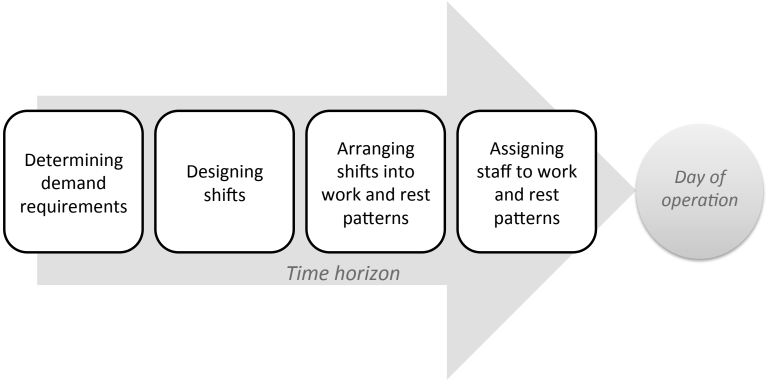

Determining demand requirements
Designing shifts
Arranging shifts into work and rest patterns
Assigning staff to work and rest patterns
Day of operation
Time horizon

Figure 2.1: Main decisions in the staff scheduling problem in a timeline perspective.

clude, among others: staffing, demand modeling, shift scheduling, days-off scheduling, tour scheduling, crew scheduling, crew rostering, staff assignment, rotating or cyclic workforce scheduling. There are many variations of these sub-problems, with different features and complexity, according to the application area or industry sector. An overview of their main features is presented next.

Demand is the trigger to any activity. Without demand, there is no point in providing a service or producing a product. Demand levels can be defined with basis on the number of patient arrivals to a hospital emergency unit, on the number of calls arriving to a call center or even on client orders received at an industrial plant, in a determined time interval. Demand modeling consists in determining demand levels, translating them into the amount of work that needs to be performed and evaluating the corresponding staff requirements for each planning period, for each shift or for each task. This step is an important part of the process and it is often tackled at a higher level of more strategic planning decision-making, together with the recruitment or staffing process, where not only the number of employees to hire is considered but also their skills and types of contract. A generic illustration of the demand modeling output can be seen in Fig. 2.2, where the number of employees for each of the working shifts (Morning, Afternoon and Night) is determined.

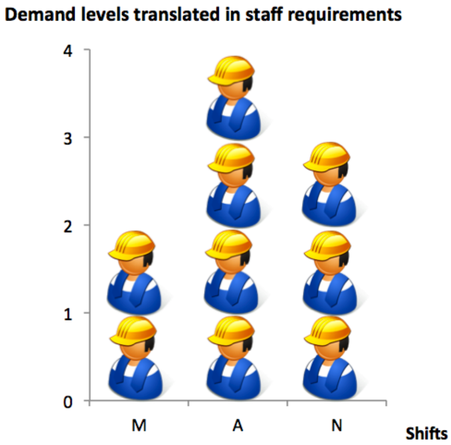


Figure 2.2: Example of demand modelling.

In some service operations, where customer arrivals are usually random and fluctuate throughout the planning horizon, forecasting, queueing theory and simulation techniques are widely used to determine demand levels and the respective staff requirements. On the other hand, in activity sectors such as transportation, demand is modeled based on the requirements of a predefined list of individual tasks to be performed by an employee (driver). Demand modeling in nurse rostering, for example, is based on the number of staff required for each shift, which must be in compliance with predefined service ratios (ex: nurse/patient). In hotels only a part of the demand, corresponding to confirmed reservations, can be known beforehand. The remaining demand determination must be based on historic information and forecasting techniques. Of course, also the component of daily check-ins must be considered. The case studies addressed in this research work do not consider the demand modeling phase, since demand levels are considered to be known in advance and are therefore input data.

One of the most explored staff scheduling sub-problems in the literature is the tour scheduling problem. It combines both the shift and the days-off scheduling problems (Fig. 2.3). The shift scheduling problem involves designing the work stretches of time that will be performed by an employee, usually on a daily basis, and also determining which shift will be performed in each of the days of the planning horizon. A shift is characterized by a start and a finish time and is subject to work and legal rules that limit, for example, its minimum and maximum length or the number and placement of breaks during the shift. Shifts can be fixed when, for example, all employees work daily on one of the three 8-hour shifts with 1 hour meal break, or vary in terms of starting-times, length or breaks' placement, for each employee and for each day. This flexibility is very important in some dynamic work environments, in order to minimize staff costs, but it significantly increases the dimension and complexity of the problem to solve. Other issues conditioning the shift scheduling problem may include a mandatory or preferable sequence of consecutive shifts to be followed, forbidden shift sequences, demand coverage constraints and minimum rest periods between shift changes. Several variations of this problem may, therefore, be considered. The days-off scheduling problem, as the name implies, is focused on determining the most suitable rest days in the planning horizon for an employee. Obviously, this implies defining simultaneously both the days-off and the work days. This problem is pertinent, for example, when the cost of different days-off patterns is different and the objective is to minimize the total labour cost. Such case is studied by Alfares (1998).

The tour scheduling problem is typical of organizations that work around the clock, 24 hours a day, 7 days a week. This is the case of several service sectors, such as hospitals, police stations or other emergency services but it is also present in some types of production systems, such as the glass manufacturer that is addressed in Chapter 5. Figure 2.4 shows an example of the output of a tour scheduling problem with staff assignment. Staff

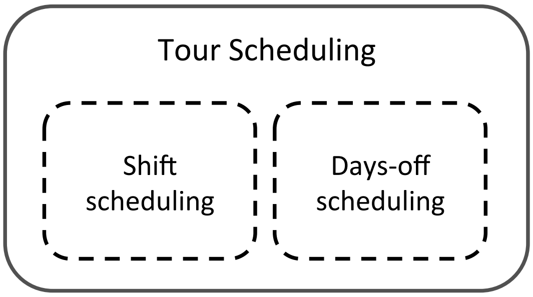
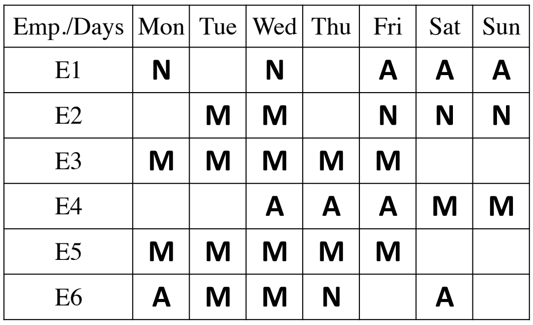


Figure 2.3: The tour scheduling problem.

assignment can take place in the last phase of the process or it can be done while constructing the lines of work, specially when employees have different scheduling constraints. In the problems studied in this work, shift and days-off scheduling, as well as staff assignment are tackled simultaneously. The blank cells (Fig. 2.4) represent the days-off.

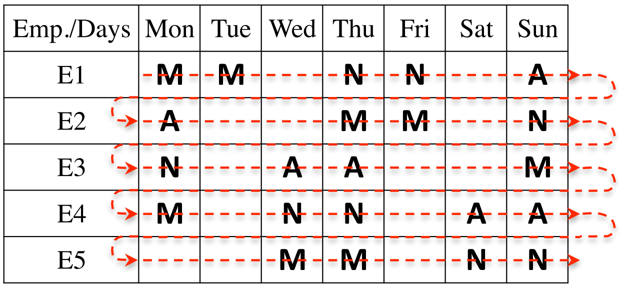


| Emp./Days | Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|---|---|---|---|---|---|---|---|
| E1 | N |  | N |  | A | A | A |
| E2 |  | M | M |  | N | N | N |
| E3 | M | M | M | M | M |  |  |
| E4 |  |  | A | A | A | M | M |
| E5 | M | M | M | M | M |  |  |
| E6 | A | M | M | N |  | A |  |

Figure 2.4: Example of tour scheduling.

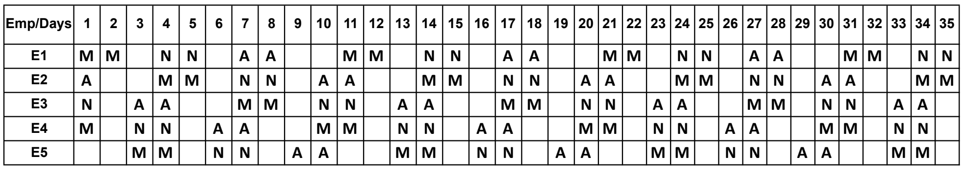
Crew scheduling and crew rostering are equivalent to the shift and tour scheduling problems respectively, but applied to transportation systems. In these systems the demand is usually determined on the grounds of a set of previously defined tasks. There is also a geographical or spacial dimension to consider, usually associated to each task, which can be the trips between two consecutive stops (buses, railways) or flight legs (airlines) that will be combined into roundtrips or pairings. See Kohl and Karisch (2004) for a recent survey on airline crew rostering problem types, modeling and optimization.

In situations where the demand patterns repeat on a regular basis, it is possible to have all the employees assigned to the same line of work, but with a time lag between them. It is denominated a cyclic or workforce rotating scheduling problem. Figure 2.5 presents a typical representation of a weekly cyclic schedule for 5 employees, and for a planning horizon of 5 weeks. Following the arrows of the dashed red line, it is possible to foresee the complete schedule for employee 1 (E1), for the whole planning horizon. In week 2, E1 will take the same line of work that E2 took in week 1 and in week 3 the same line of work that E3 took in week 1, etc. The same happens with the other employees. In week 2, E2 will take the schedule that E3 took in week 1, E3 will take the schedule that E4 took in week 1 and E5 will take the schedule that E1 worked on week 1. In Fig. 2.6, another representation of the same schedule is presented, now extensively showing the 35 days of the planning horizon. It is possible to verify that the first line of work in this solution, for E1, corresponds to the sequence indicated by the dashed red line in Fig. 2.5.



| Emp./Days | Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|---|---|---|---|---|---|---|---|
| E1 | M | M |  | N | N |  | A |
| E2 | A |  |  | M | M |  | N |
| E3 | N |  | A | A |  |  | M |
| E4 | M |  | N | N |  | A | A |
| E5 |  |  | M | M |  | N | N |

Figure 2.5: Example of cyclical scheduling (1).



| Emp/Days | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E1 | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |
| E2 | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |
| E3 | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |
| E4 | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  |
| E5 |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  |

Figure 2.6: Example of cyclical scheduling (2).

A typical application of this problem is the bus or train driver scheduling problem, where timetables usually repeat on a weekly basis. In the opposite situation are call centers, where demand fluctuates every week and so schedules are typically acyclic. Cyclic scheduling has the advantages of providing an equal distribution of shifts and days-off among all employees and of providing stability, since employees know their schedule some time in advance and can plan their lives according to their future availability. On the other hand, they lack flexibility, being much less adjustable to last-minute changes.

## 2.2 Modeling the problem

Different work environments imply different requirements and, consequently, staff scheduling problems with distinct features naturally arise. Some of the main differentiating dimensions that have been explored in the literature are:

- the adopted planning period, which can range from few days to several weeks or months up to one year, or can be user-defined;
- the operating hours of the organization, which can work in a 24-hours continuous or in a less than 24-hours discontinuous operation;
- the workforce, which can be composed of employees with: single or mixed contract types (full-time/part-time), different skills, distinct productivity levels, different availability and/or individual personal preferences; employees substitutability rules, based on hierarchy or on specific skills for example, may be considered;
- shift flexibility: working shifts can be fixed or can vary in terms of starting-time, length, placement and/or duration of breaks; overlap between shifts may be considered.

The problems addressed by this research work have a 24-hours continuous multi-shift environment, with all shifts having fixed starting-times, lengths and breaks placement. Therefore, shift flexibility is not considered. In terms of workforce, the problems vary: the glass unit and the hospital consider only a fixed full-time set of workers while the continuous care unit considers both full and part-time work, which can vary according to the demand requirements. Mixed skills are not considered but in the hospital case study workers have different contract types that must be taken into account when modeling the problem. The planning period is not fixed. Several periods were tested in order to find the one that allowed for a better solution, i.e., a better schedule.

When modeling the problem, constraints and objectives also vary according to the problem's features. Types of constraints that are often found in the literature include:

- coverage: minimum/maximum number of assignments required/allowed per shift or per week day or other planning interval;
- consecutiveness or sequence: mandatory by law or preferred by the employees, ex. maximum/minimum number of consecutive working/rest days, compulsory patterns of working shifts and days-off (stints), day(s) off after a night shift, etc.;
- weekends: weekends off periodicity, compensation for weekends assignments by week days-off, long weekends;
- workload balance: even distribution of shifts/days-off between all the employees.

Constraints can be treated either as hard constraints, if their satisfaction is mandatory, or otherwise as soft constraints,. The non-fulfillment of soft constraints is often penalized, for example in the objective function when using mathematical programming models or in the evaluation function in a metaheuristic approach.

The models proposed in this work consider all these types of constraints as hard constraints. Emphasis is yet given to the formulation of sequence constraints that, to the best of our knowledge, has not been proposed before in the literature. The workload balance is a main concern for all the problems addressed. Weekend-off periodicity constraints were considered in the glass industry case study and the hospital model was extended in order to account for planned absences.

Types of objectives that are often used to model the staff scheduling problem include:

- to minimize total labour costs;
- to maximize the percentage of the contractual work hours assigned or to minimize the percentage of the unassigned hours;
- to minimize workforce size;
- to minimize the gap between assignments and demand (under or over coverage);
- to minimize the gap between assignments and employees' preferences;
- to balance the workload between employees;
- to maximize employees satisfaction.

Although having different objective functions, all the problems studied in this work share the goal of achieving a balanced and fair schedule for all workers. In the glass unit problem this is directly formulated in the objective function. In the continuous care unit problem, the objective function seeks to minimize the part-time requirements. In the hospital problem, the objective function looks for the minimization of the deviation between assigned and contracted hours.

Integer Programming (IP) has been one of the most used techniques in the literature to model the staff scheduling problem. Most of the IP formulations are based on the set covering model introduced by Dantzig (1954). An example is the following model for a tour scheduling problem, proposed by Alfares (2004).

$$\text{Minimize } W = \sum_{j=1}^{J} x_j$$

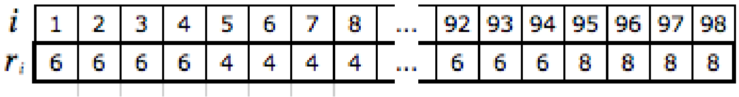
subject to:

$$\sum_{j=1}^{J} a_{ij} x_j \geq r_i, \qquad i = 1, 2, ..., I$$

$$x_j \geq 0 \text{ and integer}, \;\; j = 1, 2, ..., J$$

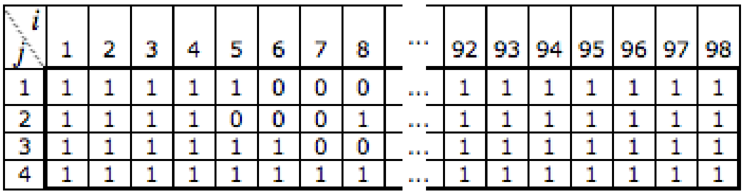
In this formulation the objective is to minimize the number of employees assigned to all $J$ tours. The planning horizon is a week, while originally in Dantzig's model it was a day, representing the decision variable $x_j$ the number of employees assigned to weekly tour $j$. The coefficients $a_{ij}$ take the value 1 if time period $i$ is a work period for tour $j$, otherwise equal 0. The minimum required labour demand is represented by $r_i$ and $I$ is the number of time periods to be scheduled over the week.

Considering, for example, an operating day from 7 a.m. to 2 p.m. and time periods ($i$) of 30 minutes, we would have 98 time periods ($I$) to be scheduled over the 7 days of the weekly planning horizon. Figures 2.7 and 2.8 illustrate this problem with the correspondent staff needs ($r_i$) for each planning time period and the matrix of coefficients $a_{ij}$, for $J = 1,...,4$ tours.



| $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | ... | 92 | 93 | 94 | 95 | 96 | 97 | 98 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $r_i$ | 6 | 6 | 6 | 6 | 4 | 4 | 4 | 4 | ... | 6 | 6 | 6 | 8 | 8 | 8 | 8 |

Figure 2.7: Example of a weekly scheduling demand requirement.



| $j$ \ $i$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | ... | 92 | 93 | 94 | 95 | 96 | 97 | 98 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | ... | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 2 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | ... | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 3 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | ... | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 4 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | ... | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

Figure 2.8: Example of the weekly scheduling $a_{ij}$ matrix data. $a_{ij}$ take the value 1 if time period $i$ is a work period for tour $j$, and 0 otherwise.

The model associates to each tour (defined by a shift and break periods combination) an explicit decision variable $x_j$. In problems with high shift flexibility, having different shift start, finish or break times, different shift lengths, etc., this formulation associates a separate integer variable to each variation of each of these features and therefore, the number of variables can increase in such a way it is very difficult or even impossible to get an optimal solution. To overcome this drawback some authors have worked on the problem formulation in order to reduce the model size using, for example, implicit modeling. This technique associates each decision variable to a shift-type or a tour-type. A shift type can be a possible combination of shift starting time, shift length and break window (interval of time in which a break can start), for example. Additional constraints are introduced in order to ensure the correct placement of breaks. A tour-type can have fixed starting-times for every day of the tour or variable starting-times. In this last situation, a start-time band can be defined, which is a range in which shift start-times can vary within a single tour. When start-time bands contain shifts with the same coverage of periods they are named overlapping start-time bands. Implicit modeling has proven to be particularly important in those staff scheduling problems that deal with variable shift starting-times and with breaks placement. For detailed information and practical applications of this technique along the last decades we refer to Bechtold and Jacobs (1990), Brusco and Johns (1996), Aykin (2000), Isken (2004), Addou and Soumis (2007) and Rekik et al. (2010).

In order to overcome the complexity of solving large-size set covering problems, some authors have explored network flow formulations (Balakrishnan and Wong (1990), Çezik et al. (2001), Moz and Pato (2004)). In a network flow model, the source node can correspond, for example, to the beginning of the first day and the sink node to the end of the last day of the planning period. Each node corresponds, then, to the end of a day and to the beginning of the following day. Each work shift or rest period is represented, in the same example, by an arc. Each path from the source to the sink node represents an alternative feasible pattern of work and rest periods, which satisfies the sequence and maximum/minimum consecutive shift/days-off blocks constraints. This option allows for a simple visual representation of every feasible tour, which can be significantly advantageous in problems with many sequence constraints. There are other alternative representations, though, that have been adopted by different authors.

While coverage requirements are typically formulated as hard constraints, workload balance and sequence constraints are often treated as soft constraints, i.e., constraints that can be violated, though at a defined cost added to the objective function. Goal programming or multi-objective techniques are used to incorporate these constraints into the scheduling models. Deviations from desired patterns of shifts, patterns of working and rest days, ratio between number of night and day shifts or other requirements are penalized in the objective function, which seeks the minimization of the sum of the weighted deviations (see, for example, the work of Topaloglu and Ozkarahan (2004), Azaiez (2005), Burke et al. (2010b) or Castillo et al. (2009)). With these formulations the user can analyze the impact of giving different weights to each of the goals. This sensitivity analysis can be very helpful in supporting the decision of choosing the most convenient solution from a set of feasible solutions.

Côté et al. (2009) divide mathematical programming formulations into three categories: compact assignment, explicit set covering and implicit set covering formulations. The last two have already been briefly presented in this section. Compact assignment formulations “use decision variables to assign activities to each employee at each period” of time. It is our conviction that the models proposed in this work can be classified as compact assignment formulations as will be explained in detail in Chapters 4 to 8. Our formulation uses binary variables to assign the working and rest shifts to each employee in each of the days of the planning period. It cannot be defined as a common assignment problem since, exception made for the glass unit problem, there is not a one to one assignment relationship. Although each employee can be assigned to only one shift, either a work or a rest shift, the same shift can be allocated to more than one employee. Demand coverage, consecutiveness and sequence restrictions are addressed as hard constraints. Workload balance is also tackled by hard constraints, imposing a cyclic scheduling approach to the model.

## 2.3 Reviewing related works in the literature

### 2.3.1 Surveys and general works

The developments on staff scheduling problems, their applications, models and solution methods reported in the literature, have been collected and reviewed by several authors over the last four decades.

Ernst et al. (2004a) present one of the most comprehensive surveys of the staff scheduling problem. More than 700 published papers are classified according to: the problem type (or sub-problem) addressed, the solution approach and the application area. In order to classify the sub-problems, Ernst et al. propose a framework based in several categories, which include, by order of representativeness: crew scheduling, tour scheduling, flexible demand, workforce planning, crew rostering, shift scheduling, cyclic rostering, days-off scheduling, shift demand, task based demand, demand modeling, task assignment, shift assignment, among others. Some of these sub-problems have already been described in 2.1. For a detailed description of all the categories see Ernst et al. (2004b).

In a very recent work, Van den Bergh et al. (2012) review 291 articles published from 2004 onwards. Papers are categorised according to four main topics: 1) personnel characteristics (contract type, skills, individual/team) decision delineation and shifts definition (overlap, start-time, length); 2) constraints (hard/soft, coverage, time-related, fairness and balance), performance measures (different costs) and flexibility (related to constraints); 3) solution method and uncertainty incorporation (uncertainty of demand, arrival and capacity) and 4) application area and applicability of research. A list of the journals with more than 3 publications on personnel scheduling is also included. All manuscripts are listed and categorised in 16 detailed tables, allowing for a straightforward usage of the information. Some relevant findings about the reviewed papers can be highlighted. The coverage constraint is a key constraint, with almost 75% of the authors defining it as a hard constraint. When considered, the balance constraint is modelled as a soft constraint by almost all the researchers. The consecutiveness and sequence constraints are tackled as soft or hard according to the origin of the imposition, whether if it is a legal setting or a preference scenario, for example. In terms of solution methods, mathematical programming approaches and metaheuristics lead the choices of the authors. In a innovative perspective, this survey work also addresses the integration of uncertainty in the decision-making process as well as the applicability of the staff scheduling research in the real-world setting.

Transportation systems, nurse scheduling and call-centers are among the most explored application areas of the staff scheduling problem in the literature. Within the transportation sector, the airline crew scheduling and the bus driver scheduling appear as the most studied problems. Surveys on the airline crew scheduling can be found in Arabeyre et al. (1969), in Etschmaier and Mathaisel (1985) and more recently in Gopalakrishnan and Johnson (2005). For an overview of advances in the bus driver scheduling problem see Wren and Rousseau (1995) and Wren (1998). Reference review studies in nurse scheduling are the works of Warner (1976), Silvestro and Silvestro (2001) and Burke et al. (2004). A tutorial and state-of-the art on telephone call centers is presented in Gans et al. (2003).

In a tour scheduling scope survey, Alfares (2004) reviews over 70 papers published between 1990 and 2001, comparing mathematical models and classifying the studies, according to the solution methods adopted, in ten categories: manual solution, IP, implicit modeling, decomposition, goal programming, working set generation, LP-based solution, construction/improvement, metaheuristics and other methods (network-flow models, expert systems, heuristics, etc.). During the period considered in the survey, metaheuristics (mainly simulated annealing) were the most used techniques, followed by constructive/improvement methods, decomposition, manual solution and IP. However, when considering only the second half of the survey period, the trend seems to be more favorable to the use of metaheuristics, IP and manual solutions rather than to the other methods. In an era of technology advances, it is quite surprisingly that manual solutions appear as one of the most popular methods, but the truth is that staff scheduling is still done manually in some activity sectors, like hospital wards, for example. Laporte (1999) suggested the manual design of cyclical schedules, arguing that IP formulations are too rigid to be applicable to real-world problems.

In an earlier work, Baker (1976) reviews mathematical programming formulations for the shift and the days-off scheduling problems with cyclic demand patterns. Baker highlights the importance of demand modeling as a crucial stage within the shift and the days-off problems. Although they were typically treated separately, Baker suggests the development of an integrated model for both problems, since they share a common context and a dependency in terms of staff requirements. In the same work, Baker discusses the trend of the researchers to simplify real problems, treating demand in a deterministic way, even when the problem has probabilistic features. Application areas of staff scheduling problems tackled in this survey include mainly service activities as baggage handlers, bus drivers, telephone operators or toll collectors.

Considering the complexity of the staff scheduling problem and its variants, it is easy to foresee the difficulty in finding a homogeneous problem classification approach among the several surveys published in the literature. Every author proposes its own definitions scheme, which makes it harder for the comparison of problems and the evaluation of achieved results. In a recent work, De Causmaecker and Vanden Berghe (2011) overcome this gap, proposing a framework for the classification of staff scheduling problems in services. It considers three categories: personnel environment, which includes different types of personnel constraints and skills; work characteristics, which refers to coverage constraints and shift types; and optimisation objectives. Such a classification system allows the benchmarking of problems, the evaluation of the instances in terms of hardness and complexity and also the comparison of solution approaches.

In a conceptual work, Warner (1976) identifies an interesting set of indicators to measure schedules' performance in terms of: coverage, quality, stability, flexibility, fairness and cost. *Coverage* measures how close the solution fits the demand requirements. The *quality* of a roster indicates how well the schedule matches the employee's request or wish, while *fairness* is a measure of how the employee feels about his/her schedule when compared to the schedules of the other employees. *Stability* is related to predictability and *cost* is a measure of resource consumption in developing the schedule.

### 2.3.2 Specific works

Several variants of the staff scheduling problem can be found in the literature, concerning the different model features that were created due to practical needs of the problems, the modeling options and the solving methods. Our specific literature review focuses on those variants that share common features with our problems (mixed contract types, sequence and consecutiveness constraints, workload balance, variable demand) and that is, somehow, work of acknowledged relevancy. In order to give the reader an overview of the published related work, some additional references are also included. It is not intended to make an exhaustive literature review, though. In the present sub-section, studies are organized according to two approaches: non-cyclical and cyclical. The latter is the one adopted in our case studies. The main features and the adopted solution methods are described for each case. The most common modeling techniques have already been overviewed in Section 2.2 and some related studies were then pointed out.

Examples of non-cyclical optimization approaches to solve problems considering a mixed workforce are the works of Bard et al. (2003), Eitzen et al. (2004) or Rong (2010). All of them use IP techniques.

Bard et al. (2003) address the tour scheduling problem of a postal service company, which includes full and part-time staff as well as variable shift starting-times. Bard et al. decompose the scheduling problem in smaller problems, which are easier to solve. In a first phase, staff levels and shifts are determined by an IP model, where the objective is to minimize the weekly total cost of the workforce. While the weekly cost of a shift for full-time staff is fixed, the weekly cost of a part-time shift varies since it can have several different lengths. In a following post-processing phase, the days-off problem and the assignment of breaks in each shift are addressed in parallel. A constructive algorithm is used to solve the days-off problem, while the placement of breaks is tackled with a network flow formulation and solved by the CPLEX solver with OPL Studio. A final VBA procedure is called to build the weekly schedule for each employee.

Eitzen et al. (2004) develop a set of three IP based methodologies (column generation, column subset and branch-and-price) for solving a multi-skilled, non-hierarchical workforce scheduling problem of a power station unit. The workforce contains a fixed set of 48 full-time and 8 part-time employees, which are grouped according to their skills. The unit works in a three fixed 8-hour shift scheme (day, afternoon and night) and with a demand forecasted for a 12 weeks period. Schedules are built for 2 week-cycles. Emphasis is given to ensuring equity between schedules of the employees with the same skills, which is achieved by means of a score levels assignment. Each employee is given a different score for a day, night or weekend shifts. The cumulative score of the past schedules for an employee is used to assign to him/her the most convenient shifts in the current planning period. The equity for an employee is the cumulative score over the planning horizon and it is different from group to group. The three solution methodologies are compared for a set of instances, varying the size of the workforce, the number of skills and the demand levels. The problems are solved with CPLEX. All the three methods revealed limitations when looking for an optimal solution given the large dimension of the problems. Eitzen et al. (2004) use the fact of having to deal with a multi-skilled workforce and a fluctuating demand to justify the use of an acyclic approach rather than a cyclic one. We believe we can refute this theory with the hospital case study, described in Chapter 7.

Staff mixed skills and weekend off requirements are explored in the work of Rong (2010). Two IP formulations are developed that simultaneously determine workforce size with different employee types (skills) and assign workers to jobs satisfying a fluctuating demand across the hours of the day and across the days of the month. The objective is to minimize the total workforce cost. The models differ in the way that lunch breaks assignment is addressed: a general IP formulation assigns lunch break hours according to worker types and a binary IP formulation that assigns lunch break hours explicitly to individual workers. Although it leads to a problem with a larger size, the second approach has a simpler model structure. Models were solved with CPLEX and results show that the binary approach is more efficient than the general one. A novel framework is proposed by Rong that introduces a 0-1 matrix for the worker type-skill representation. This matrix accommodates both hierarchical and non-hierarchical workforce scheduling. Hierarchical rules have also been the scope of the work of Ulusam Seçkiner et al. (2007).

Beaulieu et al. (2000) propose a goal programming approach to solve the scheduling problem of physicians in a hospital emergency room in Montreal. The proposed model uses binary assignment decision variables, but also considers succession and deviation variables. Succession variables are used to formulate the constraints related to the sequences of working shifts and days-off. For each sequence rule imposed to the model a different type of succession variables is defined and several constraints are added. This formulation is useful to address this particular problem but it may not be of easy generalization and adaptation to different shift sequence requirements, namely in different application areas. Deviation variables represent deviations from the goals defined for the constraints that ensure the fairness or balance of the schedule, such as the number of working hours a physician must work per week or the distribution of some night and evening shifts among the physicians. The objective function seeks the minimization of a weighted sum of the goals. Beaulieu et al. tried to solve the model using branch-and-bound techniques but that has proven to be impracticable given the large size of the problem. Some constraints had to be relaxed or eliminated from the model in order to get a feasible solution, although of poor quality. An iterative procedure was then adopted to improve this first solution: firstly, the violated rules are identified and the corresponding constraints are added to the model; secondly, branch-and-bound is used to find a new schedule, better than the initial solution. A good quality schedule was generated in less time than the time needed by the real (human) planner of the hospital.

When an optimal solution is not mandatory, heuristics and metaheuristics, such as tabu search (Burke et al. (1999, 2001)) and genetic algorithms (Aickelin and White (2004)), as well as constraint programming (Abdennadher and Schlenker (1999)) are alternative approaches that have been widely used to address consecutiveness and workload balance constraints.

Brusco and Johns (1996) propose a general set covering formulation to model the discontinuous tour scheduling problem considering both part-time and full-time employees with variable levels of cost and productivity. To solve the problem, a mixed IP heuristic is presented. In a more recent work, Thompson and Goodale (2006) also address the problem of employees with different productivity levels but use a nonlinear representation of the problem, incorporating the stochastic nature of the demand in service operations (customer arrivals). Thompson and Goodale use simulated annealing based heuristics to solve their problem.

Brucker et al. (2008) propose a decomposition approach based on a two-stage adaptive construction procedure to solve the nurse scheduling problem in a fixed 4-shift environment. Three types of constraints are defined: sequence, schedule and roster related. Sequence constraints define the sequences of shifts for each nurse, according to his/her skills. Schedule constraints are associated with all the rules that limit the construction of the schedule. Roster constraints are essentially coverage constraints. The first stage of the proposed procedure consists in constructing shift sequences for nurses by only considering the sequence constraints. In a second stage, the schedule for one nurse as well as the roster for all nurses are iteratively built, based on the sequences obtained in the previous phase and considering now the schedule and roster rules. The novelty of the developed procedure is to separately account for the problem's constraints, considering first the sequence and schedule and roster constraints after. Although the first stage calls for an exhaustive enumeration of all shift sequences, the achieved results are promising and the method has proven to be efficient.

A randomized greedy procedure is proposed in Carrasco (2010) to balance the workload in a long-term (annual) planning horizon. This work is one of the few exceptions that tackle the balance constraints as hard. Employees' preferences are not considered in that case, which decreases the complexity of the problem.

A novel heuristic approach combining mathematical programming with local search procedures is proposed in the recent work of Constantino et al. (2011), where the objective is to balance employees' satisfaction levels, measured in terms of assigned versus preferred working shifts in a specific day. The combination or hybridization of different techniques is becoming popular, since it can explore the features of all the used methods. Examples of hybrid approaches are described in Qu and He (2009), Valouxis and Housos (2000) or Sellmann et al. (2000).

Hyperheuristics are a more recent technique that uses a high-level strategy to manage a set of low-level heuristics (or parts of heuristics). Instead of evaluating a space of solutions for a given problem, this method evaluates a set of heuristics, at each stage of the solution construction process. A deep insight on this topic can be found in Burke et al. (2010a). Examples of the application of hyperheuristics to nurse scheduling and to a home care scheduling problem are reported in Burke (2003) and Misir et al. (2010), respectively.

Many of the works mentioned so far tackle the problem of determining the optimal workforce size before or simultaneously with the shift scheduling or the days-off scheduling. And they are all acyclic scheduling problems. In fact, as far as we could perceive, problems with variable workforce size have been widely studied in the literature of acyclic staff scheduling. However, that is not the case of the problems we address in this work, which are tackled with a cyclic approach. All of them have a fixed set of full-time employees and the continuous care unit has an additional set of part-time employees, which are requested according to the demand requirements.

In problems of non-cyclic nature, cyclical approaches are avoided because of their apparent inflexibility to deal with unexpected changes in schedules (absences, etc.), but they guarantee the balance and fairness of the schedule, in terms of workload distribution and days-off, and they are predictable. Cyclic scheduling problems have been studied by some authors. Exhaustive enumeration of all feasible patterns (or sequences) of working shifts/days and days-off is often a common method in the construction of cyclical schedules to overcome sequence restrictions, which are typically a factor of complexity to most of the models. Making use of his wide practical experience, Laporte (1999) argues that cyclical scheduling is more of “an art than a science”, suggesting that in order to get workable solutions, some of the problem’s rules must be violated.

Chan et al. (2001) propose a constraint programming approach to solve a cyclic scheduling problem considering an annual planning horizon. In addition to common work rules and legal constraints, annual leaves are also included in this case. Work cycles are not just repeated along the planning horizon, but rather relaxed (extended or shortened) to allow for days-off. The constraints developed in this approach were embedded in a more complete software application that has been successfully implemented in real work context, producing annual schedules for 150 employees. Another constraint programming algorithm is proposed by Laporte and Pesant (2004).

Beaumont (1997) uses a multi-objective mixed integer formulation to model the days-off scheduling problem in a long-term planning horizon (47 and 48 weeks cycle). Constraints are imposed on consecutive working and off days and on the weekly mean workload. The objective function is a weighted sum of three components: the preference of employees for long work periods and long breaks, the balance of the workload among employees in a 30-day period and the management decision of having a number of employees on duty on each day of the week proportional to the demand on that day. The decision variables defined are binary variables that indicate whether a specific day is a workday or a day-off. This is a simpler problem than the ones considered in our work, since the assignment of shifts to working days and to each employee is not considered. The model was solved with a CPLEX solver. Three schedules were generated for each cycle, considering different goal weights, to be analyzed by the client.

Alfares (1998) addresses the days-off scheduling problem with five working days and two days-off cycles. The problem is decomposed in two stages. In a first phase, an expression to calculate the minimum workforce size is determined. In a later phase, that value is included as a constraint in the linear programming model of the problem, which is a relaxation of the IP model, ensuring an optimal integer solution. This approach has the advantage of being applicable to problems with different days-off pattern costs.

A decomposition two-phase framework is also developed by Balakrishnan and Wong (1990), who propose a network flow formulation to solve a cyclic scheduling problem with fixed shifts. The optimal solution is found using a shortest path based technique. A novel approach is presented by Hao and Lai (2004), who solve a cyclic scheduling problem for airport ground staff with a neural network methodology. Experiments revealed encouraging results when compared with the solutions obtained by simulated annealing, tabu search and genetic algorithms.

Heuristics and metaheuristics based methods have also been used to solve the cyclic scheduling problem, as for example in the work of Mora and Musliu (2004) and Musliu (2006). Mora and Musliu (2004) propose a generic algorithm based methodology while Musliu (2006) explores the tabu-search potentialities to develop and compare a set of heuristic procedures to automatically generate cyclic schedules. In the last mentioned work, Musliu uses a benchmark data set to compare results, which is available in http://www.dbai.tuwien.ac.at/staff/musliu/benchmarks. These examples are used to analyze the performance of our formulation, as will be described in detail in Chapter 8.

## 2.4 Summary

This chapter introduced the staff scheduling problem: main concepts, features and applications. The aim was not only to provide background on the topic, but also to situate the problems addressed by this research work. An overview of modeling aspects was presented, with emphasis on IP techniques. The related literature was reviewed, focusing on those works that shared features with the problems studied in our work. This analysis revealed an existing trend to develop IP models for specific applications and justified the opportunity to build a general model that could be easily adapted to solve different problems. This model should be flexible to accommodate complex but relevant constraints, such as employee preferences and the equity of the staff schedules. The main challenge was to formulate such a general model using IP techniques and apply it to different real-life problems, solving them to optimality.

# Chapter 3

# Hospitality management

This chapter is dedicated to the description of hospitality management as a potential application area of staff scheduling problems. The first section introduces the concept of hospitality and gives an overview on how hospitality management is discussed in the research literature. A reference to the contextualization of hospitality activities in the Portuguese setting is included. Afterwards, some insights on the staff scheduling problem applied to hospitality management operations are presented. Firstly, its main features are pointed out and an attempt to approximate it to applications in other areas that have been already extensively studied in the literature is made. This exercise is followed by a literature review of the related works. To close the chapter, a final outlook on the results of the research work described in this chapter is given.

## 3.1 Hospitality management

Hospitality is not a recent activity. In the social sense of the concept it dates from ancient times, where many societies had traditions of travelers protection and welcoming. King (1995) overviews historical and sociological roots of hospitality and proposes a model emphasizing the importance of relationships between individuals (hosts, guests/ customers, employees) in any hospitality context, whether it takes place in a private or in a commercial setting.

Hospitality and hospitality management have been the scope of many research articles, essentially in the social sciences field, where the discussion has been focused on defining a common, generically accepted, definition and on the development of a framework to be the basis of an independent academic discipline.

Although still being often merged with tourism and leisure sector activities, hospitality services are a growing activity sector in a society where customer's satisfaction and well-being run the market. They usually include hotels, restaurants and other sort of lodging, food and drinks services providers. Due to the specifications of the kind of service provided, hospitality management has to deal with complex variables and constraints. An unpredictable customer demand, a multiskilled workforce, different staff labour contracts' demands, employees satisfaction and costs minimization are just some of the conditioning issues that an organization has to deal with in order to achieve a flexible, profitable and high quality service provision.

A survey undertaken by Enz (2009), in cooperation with the Center for Hospitality Research of Cornell University, identified human resources management as the subject of most concern for hotel managers, above other aspects such as economic or environmental problems, and regardless of the geographical location. The study was based on the statement of 243 experienced hotel executives from six countries. This highlights the importance and worldwide relevance of human resource management to a hospitality organization. Staff scheduling are typical problems to solve within this area. There is however a big lack of published articles focusing on these problems applied to the hospitality sector, as realized by Ernst et al. (2004a), in opposition to other application areas such as hospitals, transportation or call centers.

One of the reasons for the lack of research articles focusing on staff scheduling problems in hospitality is perhaps the lack of a consensual and generically accepted definition of the activity itself. Etymologically, the word hospitality, in Latin *hospitalities*, has its origin in *hospes* or *hospitis* (genitive), which means foreigner or guest. Dictionary definitions include "cordial and generous reception of or disposition toward guests" ("hospitality", The American Heritage Dictionary of the English Language) and "kindness in welcoming strangers or guests" ("hospitality", Collins Essential English Dictionary). It is synonym of hospitableness and widely used to define welcoming host-guest relationships, being thus traditionally associated with cultural and social values of each community.

In the industrial context, the term hospitality has been adopted mainly in the English-speaking countries to refer to the activity of hotels, restaurants and other sort of lodging, food and drinks services' providers, whether it takes place in a public/ commercial or in a private/ social context. Lashley (2008) argues that this framework can be understood as an effort to "create a more favorable impression" of these activities, promoting a further hospitable commercial activity and letting the profit provision motivation remain in the background. While British researchers have traditionally based the discussion on this definition, American academics tend to use a broader meaning of hospitality, associating these activities with others under the tourism field, such as travel, leisure or entertainment.

In a first essay, hospitality management would then be intuitively defined as the management of those hospitality activities. In accordance, Brotherton and Wood (2008) write that hospitality management is a generically used expression to easily replace other labels such as “hotel management”, “restaurant management” or “catering management”, but consequently none or few reflection has been given to the genuine meaning or nature of hospitality. They state that hospitality research has been characterized throughout the years by an unsystematic and scattered analysis, rendering a meaningful synthesis very hard to achieve.

In the academic community, researchers have been seeking out the development of the specialist discipline of hospitality management that would embody a theoretical framework and link it to the industry sector, but the lack of a consensual definition of hospitality has effectively been a barrier both to research progress (Jones (1996), Taylor and Edgar (1996)) and to the creation of a robust and mature branch of knowledge. The discussion has been driven by some authors into the field of cultural and social sciences (Brotherton (1999), Hemmington (2007), Jones (2004), Lashley (2008)), incorporating in the debate the importance of studying hospitality from a wider perspective rather than the commercial one. The contribution of authors from different fields of research and their vision’s diversity could potentially be a major value but it could also be understood as a reflex of a fragmented and unstructured hospitality research.

King (1995) introduces a hospitality model based on the interaction of social “rituals” in the commercial operation, associated with the process of the guest arrival, welcoming and departure. The author defines hospitality as a host-guest relationship between individuals, taking place in a commercial or private setting, whose success is assessed by the clear perception of the guest needs and their genuine satisfaction by the host. This perspective underlies an unconditional moral duty of hospitable behavior that can, at the edge, merge the meanings of hospitality and hospitableness, which Brotherton (1999) contests, arguing that hospitableness has a much broader scope than hospitality activities. In fact, hospitable concerns are a competitive advantage in any activity where there is a “service” relationship with the customers, whether it is from the hospitality sector or not.

Believing that hospitality is a time evolving phenomenon, i.e, that hospitality' characteristics change over time, Brotherton (2006) presents a conceptual model for hospitality comprising four dimensions: spatial, behavioral, temporal and physical. These dimensions help to analyze the extent of hospitality in terms of place of occurrence, motivational aspects, time and material features involved. In this conceptual model, the nature, incidence and forms of hospitality in a particular society in any given time period, expressed by domestic or commercial hospitality behavior, are a function of the human and natural resources available, which in turn are conditioned by the economic, socio-cultural, politico-legal and technological conjuncture. The author tried to operationalize this model through case studies (Brotherton and Wood (2008)) in two hotels and later in two fast food restaurants, where guests/customers where asked to participate through an interview, associating words that best fitted their notion of hospitality. Although this exercise did not produce statistically significant results in terms of the influence of social factors (like age, gender, occupancy, etc.), it did provide inputs for understanding guests' perception of the meaning of hospitality that still needs to be further explored.

The comprehensive approach of studying the commercial hospitality activity from a wider social sciences perspective has indeed been quite controversial, as it turned out to happen after the publication of the book “In search of hospitality: theoretical perspectives and debates” by Lashley and Morrison (2000). The referred work presents the nature of hospitality from several views, from Anthropology to Marketing, and proposes an integrated “three-domains approach”: the private, the social and the commercial domains. The main idea of this conceptualization is to consider and evaluate the effect of the social and cultural dimensions of hospitality in the commercial or business activity, despite their blurred boundaries. The book also defends the existence of hospitality management as an independent activity, apart from any other management activity.

Slattery (2002) is one of the researchers who is most critical of this approach, arguing that it overestimates the social side in relation to the economic one and "excludes the hospitality industry context". His classification model of hospitality industry is based on the place where activities effectively take place: Free-Standing Hospitality Business (hotels, restaurants, bars), Hospitality in Leisure Venues (casinos, cinemas, health clubs), Hospitality in Travel Venues (airports, bus stations, trains, ferries) and Subsidiary Hospitality (workplaces, health care, education). He thus considers that confining hospitality to lodging, food and drinks activities falls short since hospitality necessarily undertakes the management of several other sort of associated leisure activities, in order to respond to the increasing complexity of customer demand.

In his review, Jones (2004) identifies five main hospitality schools of thought: science model, management, studies, relationship and systems, attesting that the state of hospitality research is not yet consolidated and there is a lack of consensus concerning its definition. Even though this diversity of thoughts persists, the management perspective was recognized to be in a dominant position in relation to other emerging views. But even from a management point of view the author finds three different approaches, with their main divergence in the focus of the research. While the traditional point of view considers hospitality to be a sub-discipline inside the main management disciplines, a different conviction uses hospitality as an application of the main discipline and a third perspective assumes a "multidisciplinary approach" studying hospitality from several different main management subjects.

In a recent article, Ottenbacher et al. (2009) analyze the pedagogical and research implications of defining the hospitality discipline. Based on a services marketing perspective, the authors defend a taxonomical classification, considering hospitality as a field supported by the economic output of a group of six related industries: lodging, food services, leisure, travel, attractions and conventions. Each of these independent industries takes, in turn, "input from hospitality either directly or indirectly for its survival and success." The article suggests the need of exploring separately each one of these activities, which are often ignored in the literature, recognizing the diversity of their constitutive market segments.

In the Portuguese context a translation for the concepts of hospitality or hospitality management is still missing and consequently there is not a consolidated research activity focused in this thematic area, or at least with an acknowledged published work. A few exceptions include for instance the work on hotel management efficiency using Data Envelopment Analysis (Barros and Mascarenhas (2005), Barros et al. (2008)). A hospitality association was created - Hospitality Management Institute (HMI (2008)), as a result of the cooperation between Turismo de Portugal, ISCTE (Instituto Superior de Ciências do Trabalho e da Empresa), Universidade do Algarve and ESHTE (Escola Superior de Hotelaria e Turismo do Estoril), sponsored by the Portuguese Government and the National Strategic Council for Education and Training in Tourism, that aims to promote advanced management training and to support applied research in tourism.

Portuguese hotel and restaurant industries have traditionally been considered as a part of the tourism sector, for statistics, economic indicators and sectorial strategies, as well as several other service providers connected to touristic services, such as travel agencies, touristic operators or leisure activities promotors. There are many different associations: Portuguese Hotel Association (AHP), Association of Hotels and Tourism Enterprises of the Algarve (AHETA), Association of Restaurants and Associated Industries of Portugal (ARESP), Portuguese Association of Congress Companies (APECATE), Portuguese Association of Travel Agencies (APAVT), among others.

In Portugal there are promising perspectives to the hospitality activities as the tourism sector is strategically regarded as a priority for the Portuguese economy, due to its ability to create jobs and wealth and due to its recognized international competitive advantage. Stakeholders are committed to the development of the sector, and so is the Government, as is clearly assumed in the National Strategic Plan for Tourism, where the ambitious objectives defined aim to reach, by 2015, 20 million tourists and €15 billion of revenues, what will represent in terms of economic impact over 15% of GDP and 15% of national employment (Ministry for Economy and Innovation (2011)).

## 3.2 Staff scheduling in hospitality management

The staff scheduling problem in hospitality services shares common features with other service activities. It is quite noticeable, for instance, its similarity with the nurse rostering problem, so deeply explored in the literature. Considering hospitality in its narrowest sense, as defined in the previous section of this chapter, which includes mainly lodging units and restaurants, both problems seek to assign a set of employees to a set of working days, shifts and rest periods in order to satisfy demand levels, taking into account work rules, employees' skills, availability and preferences. The staff scheduling problem both in hotels and hospitals is typically characterized by an around the clock operation, 7 days per week and a fluctuating demand. The use of different contract types (e.g. part-time) is therefore a common and necessary practice.

Of course the place where it takes place and the set of conditions under which it can be found make the approaches to the rostering problem to be different in a hospital or in a hotel. The nurse rostering problem takes place in a hospital unit - a ward, where usually the different skill categories of the nurse function (e.g. head nurse, regular nurse, caretaker) need to be taken into account. The considered shift types are usually the conventional 7 or 8 hour shifts: early, late and night. Demand is usually determined with basis on desired service levels (e.g. nurse/patient ratios) or/and forecasting techniques and staff levels are defined for each shift and skill category. Workload distribution follows a daily pattern, usually ignoring weekends. Work rules are strict in terms of shift sequence, maximum/minimum number of consecutive assignments for each shift, periodicity of rest days, etc.

In addition to its lodging core operation, which involves several different functions (receptionist, concierge, doorkeeper, cleaning staff, maintenance operator, administrative staff) a hotel usually includes other activities such as restaurant, bar, leisure spaces, etc. The staff scheduling problem in a hotel may also be applied to a single functional area, for example the scheduling of the reception staff or the cleaning staff. There are situations, however, where staff is multifunctional and so an integrated approach is more appropriate, increasing the complexity of the problem to solve. It requires a high flexibility in terms of shift length, starting and finish times and needs to manage a bigger diversity of employee contract types as well as multiple functions. In what concerns work rules, just like in the case of hospitals, hotels and restaurants are very conditioned by sectorial union agreements or contracts, namely in terms of working and rest periods. In the current globalization context, multinational hospitality organizations, and specially hotels, must follow the motto: “think globally, act locally”, meaning that although having general rules, common to all their units, each unit must have its organization and practice adapted to the local context where they are placed in. Different cultures and different habits usually mean different needs. In the hotels case, the majority of the reclaimed service levels are strategically imposed by its category (number of stars). Staffing needs must be determined based on the historic data, on guest arrival forecasts, on a slack for daily late arrivals but also based on the desired service levels.

In a similar way to the case of nurse rostering, important schedule’s characteristics to take into account in hotels are: coverage, quality, stability, flexibility, fairness and cost (Warner (1976)). These concepts have already been described in detail in section 2.3. In particular, quality, fairness, stability and flexibility are those characteristics that are connected to employees’ preferences and are, therefore, very important to guarantee a motivated and productive workforce. This is a critical issue in any customer oriented activity, and specially in services where there is such a deep interaction between employee/host and customer/guest, as is the case of hospitality operations. The perception of customer needs and their satisfaction is very dependent on the performance of every employee. Therefore, employees’ welfare must be safeguarded. One adoptable approach to the rostering problem in hospitality organizations, and in hotels in particular, is the tour scheduling problem.

Applications of staff scheduling problems have been explored in many activity sectors in the literature, with emphasis on hospitals and transportation systems (Ernst et al. (2004a)). Hospitality, in turn, has had much less attention of researchers working in the quantitative field, with very few published articles, referring mainly to restaurants (Glover (1986), Love and Hoey (1990), Loucks and Jacobs (1991), Thompson (1996), Eveborn and Rönnqvist (2004) and Choi et al. (2009)).

In an attempt to justify the relevance of staff scheduling for hospitality managers, Thompson proposes a four-stage method for the hospitality industry, which he presents in a series of four articles: forecasting demand (Thompson (1998a)), translating those forecasts into staff requirements (Thompson (1998b)), scheduling staff (Thompson (1999a)) and monitoring the schedule in real-time (Thompson (1999b)).

In the third paper, Thompson (1999a) proposes a methodology for developing work schedules in hospitality organizations that seeks to balance both the organization's and the employee's goals. A comparison of two traditional approaches to staff scheduling, one by Dantzig (1954) and the other by Keith (1979), is presented and their limitations are pointed out. Two new methods are proposed, under two different perspectives: economic and service standards, with the objective of achieving the highest schedule's economic outcome and optimal service standards respectively. The main difference between these methods and the classic ones is that employee requirements are no longer independently set for each planning period but are now instead taken into account both in the determination of demand levels and on the actual scheduling process. Assuming that a surplus employee cannot have the same cost or bring the same benefit/value no matter in which period he is added, it is possible to maximize the level of service provided or to develop the best schedule from an economic point of view, as defined by the organization. An emphasis is given therefore to the importance of satisfying employees' preferences, not disregarding their availability and skills and to the advantage of considering this information in the shifts development process. Thompson defends the use of a heuristic procedure in order to reach a good schedule in a reasonable amount of time in opposition to trying to find optimal solutions, which is typically too time consuming. The recommended planning horizon is one or two weeks, mainly due to the typical difficulties in predicting service demand more than two weeks in advance.

Although presenting it as the outcome of a long work experience, Thompson does not apply this work to a practical case study. It would be interesting and certainly of great value to see the application of this approach in a hotel unit.

Choi et al. (2009) applied Thompson's framework to a restaurant, where the workforce is composed of 30 full-time and 19 part-time employees. Staffing and scheduling problems are both addressed in a weekly planning horizon. Based on past experience, the management defined a ratio of full-time vs part-time workers of 6:4, which they assumed to ensure the desired service levels. The developed IP model assumes that full-time workers have higher productivity levels than part-timers, but also imply higher costs. The objective of the problem is therefore to minimize the overall labor costs while ensuring the appropriate service levels. Results demonstrated an increase in the overall efficiency of the scheduling system, achieving a reduction in the labor costs of overstaffing and also in the opportunity costs of understaffing. This work emphasizes the problem of the high turnover costs associated with hospitality organizations. An efficient scheduling can contribute to the engagement of the employees and motivate an increase in the workforce retention rate. The model had some simplifications though, not considering for example employees' preferences or availability, which can also be very relevant to the retention in a job.

Over and understaffing are also addressed in Eveborn and Rönnqvist (2004). A scheduling software system is developed - SCHEDULER, which is composed by several optimization modules. The main model of the system is an IP set partioning formulation which is solved using a branch and price approach. The problem's features considered include legal constraints and employee's preferences. The system provides an interface where the user can select different weightings between costs and preferences. The fairness of the schedules is measured through a constraint that limits the sum of penalty points associated with each schedule. Although the authors mentioned that hotels and restaurants are among the organizations where the SCHEDULER was implemented, those cases are not reported in the study.

Staff scheduling problems in fast-food restaurants have typical features: the workforce is composed of a set of full and part-time workers, with different skills and availability. The assignment of employees to tasks or shifts must cover the requirements of a demand that fluctuates through the hours of the day. The objective considered is usually to minimize over and understaffing costs. Glover (1986), Love and Hoey (1990) and Loucks and Jacobs (1991) solve this problem using three different approaches, respectively: a tabu-search based procedure, a network flow technique and a goal-programming formulation solved with a constructive heuristic.

## 3.3 Final reflections

Although hospitality and hospitality management are subjects that have been quite explored by social science researchers, they are not commonly referred in the operations research literature, or more precisely in the scheduling operations literature. But the truth is that there is a point in exploring rostering and scheduling problems in this area. First of all, because as in any other activity in the services industry, the importance of staff expenditure is typically very significant in the total operating costs. Secondly, because the quality and efficiency of the service provided by a hotel or a restaurant have direct impact on its customers' satisfaction, as in few other service activities. The social dimension of hospitality, which has been in the hospitality research agenda in the last decades, increases the complexity of staff rostering problems in this activity area. It is no longer only a matter of assuring the required employees' technical skills, but also of guaranteeing that they have the right personal competences to interact with customers, to understand and satisfy their needs. The staff must be motivated and engaged. Staff scheduling systems shall therefore account for the workforce well-fare, considering employees' preferences in terms of work and rest days, weekends off and holidays, shifts assignment, shifts change, shifts starting and finishing times flexibility, compatibility or incompatibility with other staff elements, etc. Possible approaches to the staff scheduling and rostering problem in hospitality management, or its sub-problems, may be inspired by the work that has been comprehensively done both in tour scheduling and nurse rostering. As exposed before in this chapter, nurse rostering and hospitality are two activity areas with many similarities concerning rostering issues. Examples of the few divergences between them include the seasonality, the weekly and daily cycles operation inherent to hospitality activities, that contrast with the Winter/Summer seasonal workload distribution of hospitals. Thompson (1999a) gives a very important contribution to staff scheduling and rostering in hospitality management. It should have triggered the interest of researchers in this area, namely in the development of quantitative approaches, but the truth is that it didn't, according to the latest reviews on this subject that have been analyzed. This work aims to be a recall, as there is still a lot to be done. Future work may be based on the adaptation of tour scheduling, nurse rostering or even shift scheduling models and solution methods to hospitality operations. Schedules should be flexible enough to be easily adaptable to actual workplace environments changes and social aspects should be considered.

# Chapter 4

# General Model

This chapter presents the model developed for a general staff scheduling problem. A set of features, which are relevant and common to many variants of the problem are considered. Those are described in Section 4.1. Next, Section 4.2 introduces and explains the proposed IP formulation. Finally, in Section 4.3 we highlight some special features of the model that, to the best of our knowledge, represent a novel and valid contribution to this field of research.

## 4.1 Problem description

The general model was developed for the staff scheduling problem of an organization that works continuously, 24 hours a day. The day is divided in $nS$ working shifts. The model considers a set of $nT$ teams of homogeneous (single skilled and full-time) employees, that must be assigned to either a work or a break shift, in each of the $nD$ planning period days. Daily shift demand levels must be satisfied, meaning that the model must guarantee a required number of teams working in each shift on each day. Work rules include a minimum and a maximum number of consecutive working days for each team, as well as a predefined sequence of working shifts to be respected.

Each shift change must have a break or non-working day in between. The objective is to minimize and to level the number of days each team works in each shift, in order to balance the workload.

## 4.2 Mathematical model

The following notation was defined:

### Indices

$d$ $\in \{1, \ldots, nD\}$, day;

$t$ $\in \{1, \ldots, nT\}$, team;

$s$ $\in \{1, \ldots, nS\}$, working shift;

$s'$ $\in \{1, \ldots, 2 \times nS\}$, extended shift.

Shifts $s'' \in \{nS + 1, \ldots, 2 \times nS\}$ are non-working shifts that carry the information on the last working shift of the team;

$n(s')$ is the extended shift that follows the extended shift $s'$ in a given sequence;

For example, considering 3 working shifts $\{1, 2, 3\}$ and 3 non-working shifts $\{4, 5, 6\}$ a possible sequence could be 1-4-2-5-3-6-1-4-..., as defined in Table 4.1.

| $s'$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $n(s')$ | 4 | 5 | 6 | 2 | 3 | 1 |

Table 4.1: Example of a possible sequence of shifts

The indices $t$ and $d$ should take values in a circular list. The list for index $d$ should for instance be $\{1, \ldots, nD-1, nD, 1, \ldots, nD-1, nD, \ldots\}$. For implementation purposes index $d$ should be replaced by $[(d-1) \mod (nD)]+1$ and index $t$ should be replaced by $[(t-1) \mod (nT)]+1$.

**Parameters**

| | |
|---|---|
| $nT$ | number of teams; |
| $nS$ | number of shifts; |
| $nD$ | number of days in the planning period; |
| $demand_s$ | daily demand for each working shift $s$; |
| $maxD$ | maximum number of consecutive working days; |
| $minD$ | minimum number of consecutive working days. |

**Decision variables**

$$x_{ts'd} = \begin{cases} 1 & \text{if team } t \text{ is assigned to shift } s' \text{ on day } d \\ 0 & \text{otherwise} \end{cases}$$

**Decision variables (auxiliary)**

$$b_{tdm} = \begin{cases} 1 & \text{if team } t \text{ works at least } minD \text{ consecutive days,} \\ & \text{starting on day } d+m-1 \\ 0 & \text{otherwise} \end{cases}$$

**Objective function**

$$\min \max_{ts} \sum_{d} x_{tsd} \tag{4.1}$$

**Linearized objective function**

$$\min Z \tag{4.2}$$

**Constraints**

$$\forall_{ts} \sum_{d} x_{tsd} - Z \leq 0 \tag{4.3}$$

$$\forall_{td} \sum_{s'} x_{ts'd} = 1 \tag{4.4}$$

$$\forall_{sd} \sum_{t} x_{tsd} \geq demand_s \tag{4.5}$$

$$\forall_{td} \sum_{q=0}^{maxD} \sum_{s} x_{ts(d+q)} \leq maxD \tag{4.6}$$

$$\forall_{td} \sum_{m=1}^{minD} b_{tdm} - \sum_{s} x_{ts(d+minD-1)} \geq 0 \tag{4.7}$$

$$\forall_{td}\ \forall_{m=1}^{minD} \sum_{s} \sum_{q=m}^{m+minD-1} x_{ts(d+q-1)} - minD \times b_{tdm} \geq 0 \tag{4.8}$$

$$\forall_{ts'd}\ x_{ts'd} - x_{ts'(d+1)} - x_{tn(s')(d+1)} \leq 0 \tag{4.9}$$

$$\forall_{ts'dm}\ x_{ts'd}, b_{tdm} \in \{0, 1\} \tag{4.10}$$

The objective function seeks the minimization of the maximum number of days that a team works in each shift. It levels the working days of each team, leading to a solution in which each team works the same number of days in each shift. The linearization of (4.1) results in the linear objective function expressed in (4.2), where $Z$ represents the maximum number of days that a team works in each shift, and also in Equations 4.3.

Equations (4.4) state that each day every team has exactly one shift assigned, either a working shift or a break shift.

Equations (4.5) are coverage constraints, making sure that each shift daily requirements are fulfilled.

Equations (4.6) ensure that no team works more than $maxD$ consecutive days. For each day $d$ a window of length $maxD + 1$ is opened and at least one of the corresponding $x_{tsd}$ must be 0, independently of the working shift $s$.

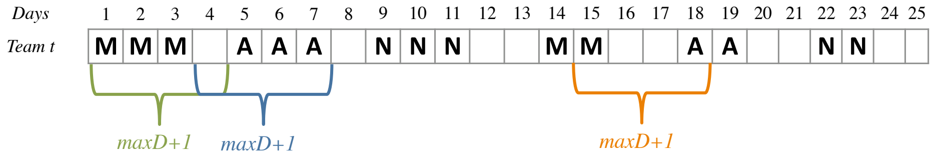


Figure 4.1: Illustration of Eqs. (4.6)

Equations (4.7) and (4.8) guarantee that each team works at least $minD$ consecutive working days. The second term on the left-hand-side of Eq. (4.7) sums up the working days $x_{ts(d+minD-1)}$ within a window of width $minD$, starting at $d$. If all $x_{ts(d+minD-1)}$ are zero no constraint is imposed to the variables $b_{tdm}$. However, if at least one $x_{ts(d+minD-1)} = 1$ then at least one of the variables $b_{tdm}$ must be equal to 1. When the variable $b_{tdm}$ equals zero, the corresponding Eq. (4.8) is fulfilled. However if Eq. (4.7) imposes that a variable $b_{tdm}$ equals one, then the first term on the left-hand-side of Eq. (4.8) has to sum-up at least $minD$, i.e. the team has to work at least $minD$ consecutive days. The meaning of $m$ is that if a team works one day within a window of width $minD$, then it has to work at least $minD$ consecutive days, starting at $m = 1$ or $m = 2$ or $\ldots$ or at $m = minD$. Figure 4.2 illustrates this process for shift M.

Equations (4.9) ensure that the required shift sequence is followed. The basic sequencing requirement is defined over the working shifts that follow the sequence: $1, 2, 3, 1 \ldots$, but, as there are breaks between the working shifts, the breaks must carry the memory of the last working shift. This is

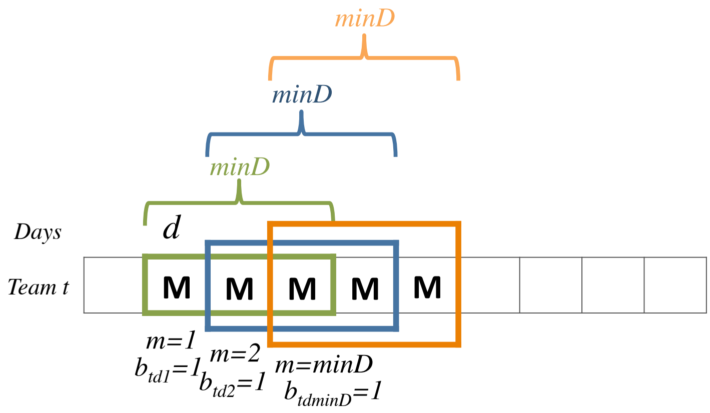


Figure 4.2: Illustration of Eqs. (4.7) and (4.8)

obtained through the "extended shift" $s'$. For instance, if a team has an extended shift $s' = 4$ assigned, it means that the team is having a breaking shift after a working shift 1. If the same shift is assigned on days $d$ and $d+1$, then the corresponding Eq. (4.9) is satisfied independently of the value of $x_{tn(s')(d+1)}$. However if the shift ends, i.e., a different shift is assigned on days $d$ and $d+1$, then the next possible shift is imposed by the vector of indices $n(s')$ and the Eq. (4.9). Figure 4.3 illustrates the application of Eqs. (4.9) for the sequence of shifts defined in the example of Table 4.1.

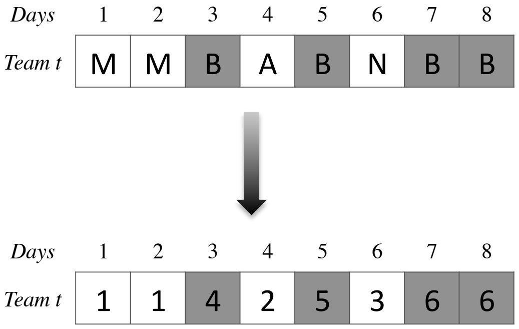


Figure 4.3: Illustration of Eqs. (4.9)

## 4.3 Special features

Emphasis must be given to the wide scope and flexibility introduced with the formulation of the sequence shift restriction (Eqs. 4.9). Any desired sequence pattern of working shifts and days-off can be imposed through the proper definition of the vector of indices $n(s')$.

The limits on the maximum and minimum number of consecutive days for each shift enable the distinction between the length of the working and rest periods, but also between the work shifts' length itself. Some activities have work rules that impose different maximum allowable numbers of consecutive working shifts, for instance night vs day shifts. But those parameters, together with the shift sequence constraints, also allow to control the periodicity of days-off, as well as the length of the tour or sub-period or sub-cycle of the planning horizon. It is possible to impose a schedule with sub-cycles of equal length (if it is a divisor of the planning period) or give the model flexibility to construct sub-cycles with different lengths.

The flexible application of these features is demonstrated in the case studies that are described in the next chapters.

# Chapter 5

# Application of the general model to a glass production unit

The general model presented in Chapter 4 was first adapted to the real-life problem of a glass industry. This chapter describes that experience and is organized as follows. Section 5.1 introduces the facility, the work environment and the features of this particular problem. Then, in Section 5.2 the adjustments that were made to the general model are described, followed by the achieved computational results, which are indicated in Section 5.3. An illustration of the developed solutions is shown in Section 5.4. Section 5.5 sums up the contents of this chapter, emphasizing some important outcomes of the work that was carried out.

## 5.1 Problem description

The facility produces glass bottles using two furnaces, with four lines each. The workforce was distributed in 4 teams but the management wanted to test the scenario of having a higher number of teams. They were convinced this change would increase the scheduling flexibility and provide a more equitable schedule to all employees, in terms of workload and rest periods distribution. Each team is a group of 36 employees that is in charge of all the manufacturing operations during a shift. Occasional changes due to absences of any employee are usually fulfilled within his own team.

The facility works 365 days per year, 24 hours a day, in three different eight-hour shifts: M - morning (from 5:00 a.m. to 1:00 p.m), A - afternoon (from 1:00 p.m. to 9:00 p.m.) and N - night (from 9:00 p.m. to 5:00 a.m.). All teams are considered to be homogeneous in terms of skills and contract types, everyone works full-time. Shift daily demand is fixed and known in advance: exactly 1 team for each shift. A mandatory shifts sequence must be respected (M-N-A) and there must be a rest shift (B) between each working shift change (M-B-N-B-A-B). There is a predefined minimum and maximum number of allowed consecutive working days.

## 5.2 Mathematical model

In order to adapt the general model to this new problem, the following adjustments were made.

### Indices

$n(s')$ is the extended shift that follows the extended shift $s'$ in a given sequence;

Considering the 3 working shifts {M,A,N} as $\{1, 2, 3\}$ and the 3 non-working shifts $\{4, 5, 6\}$, the sequence M-B-N-B-A-B is now defined as shown in Table 5.1.

### Parameters

$\delta D$ offset between the working cycles of the teams (in number of days).

| $s'$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $n(s')$ | 4 | 5 | 6 | 3 | 1 | 2 |

Table 5.1: Sequence of shifts for the glass production unit

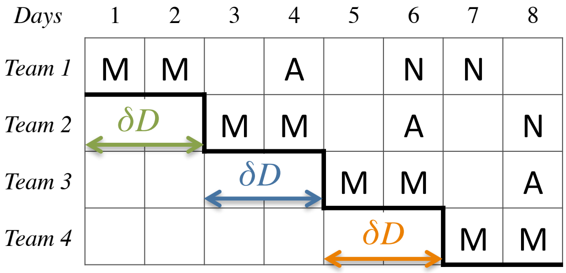
### Constraints

$$\forall_{sd} \quad \textstyle\sum_t x_{tsd} = 1 \tag{5.1}$$

$$\forall_{t=1}^{nT-1} \forall_{s'd} \quad x_{ts'd} - x_{(t+1)s'(d+\delta D)} = 0 \tag{5.2}$$

Equations (5.1) make sure that the daily requirements are fulfilled.

Equations (5.2) impose that all teams have the same schedule, but starting with a time lag of $\delta D$ days between them. With this additional constraint the objective function could be replaced by a constant number but the computational experiments showed that keeping the objective function lowers the computational time. Figure 5.1 illustrates these constraints.



| *Days* | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| *Team 1* | M | M |  | A |  | N | N |  |
| *Team 2* |  |  | M | M |  | A |  | N |
| *Team 3* |  |  |  |  | M | M |  | A |
| *Team 4* |  |  |  |  |  |  | M | M |

Figure 5.1: Illustration of Eqs. (5.2)

With the introduction of Eqs. (5.2), other constraints can be simplified, reducing the size of the model. This is the case of the sequence constraints and the maximum/minimum number of consecutive days constraints. There is no need to apply them to all teams, it is enough to ensure that it is fulfilled for team 1 and the offset constraint guarantees that it satisfied for the remaining teams. Therefore, Eqs. (4.6), (4.7), (4.8) and (4.9) were adjusted to, respectively:

$$\forall_d \qquad \sum_{q=0}^{maxD} \sum_s x_{1s(d+q)} \leq maxD \tag{5.3}$$

$$\forall_d \qquad \sum_{m=1}^{minD} b_{1dm} - \sum_s x_{1s(d+minD-1)} \geq 0 \tag{5.4}$$

$$\forall_d \, \forall_{m=1}^{minD} \qquad \sum_s \sum_{q=m}^{m+minD-1} x_{1s(d+q-1)} - minD \times b_{1dm} \geq 0 \tag{5.5}$$

$$\forall_{s'd} \qquad x_{1s'd} - x_{1s'(d+1)} - x_{1n(s')(d+1)} \leq 0 \tag{5.6}$$

This approach was adopted in all the other applications of the general model, as will be described next in this work (Chapters 6, 7 and 8).

The same analysis can be done with respect to the demand constraints (Eqs. 4.5). Theoretically, there is no need to apply them to all the $nD$ days of the planning period, but only to the first $\delta D$ days, since the offset constraints ensure the fulfilment of the demand requirements for the remaining days. However, after testing the referred simplifications, only the first group of adjustments brought improvements to the model's performance, reducing significantly the resolution times. Therefore, the simplification of the demand constraints was not considered.

## 5.3 Computational experiments

The model was coded in OPL Studio version 6.3 and solved using the CPLEX 12.1.0 solver on a server machine powered by 2 Intel® Xeon® processors of 2,4 GHz and 1,39 GHz, and with 2 GB RAM. Tests were performed considering 4 and 5 teams of employees and 3 working shifts. The maximum ($maxD$) and minimum ($minD$) number of consecutive working days were set to 4 and 2, respectively. A set of different planning periods and offsets combinations were tested. The experience showed that this problem is extremely tight in terms of relationships between parameters. There is no place for large parameters variations and variables are strongly connected, what makes the space of feasible solutions very limited. The only combinations that always guarantee the existence of a feasible solution are those that verify the following condition:

$$\delta D = \frac{nD}{nT}$$

The offset parameter ($\delta D$) must be equal to the ratio between the number of days of the planning period ($nD$) and the number of teams ($nT$). Putting it simple, this means that the offset parameter must be the one that divides the planning period in a number of sub-periods (or sub-cycles) equal to the number of teams. Table 5.2 reports the computational time results and model size, in terms of number of decision variables and constraints, for a set of different planning periods for the 5-team scenario.

These variations were tested in order to evaluate the robustness of the model but also to analyze the solutions built for different planning periods in order to find the one that better fitted the objectives of the company. As expected, the performance of the model, in terms of running time, gets worse as the problem size increases. For 330 and 365 days the model did not generate any feasible solution within 60000 seconds (1.7 hours).

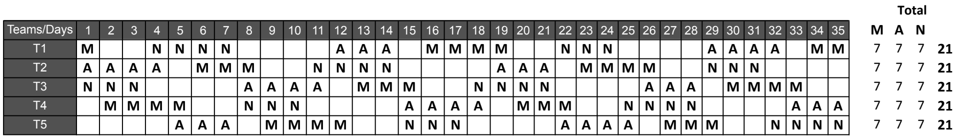
## 5.4 Solutions

The solution found for a 35 days (5 weeks) planning period was achieved in 0.57 seconds (Fig. 5.2). This schedule can either be repeated as many times as the company wants, or be integrated with schedules of other lengths, in order to build a longer (yearly) plan. For the particular purposes of the glass unit, the summer holidays special period was also taken into account in a subsequent phase. Considering a planning period of 16 days, with only

| $nD$ | $\delta D$ | No. Decision Variables | No. Constraints | Time (sec.) |
|---:|---:|---:|---:|---:|
| 25 | 5 | 750 | 1115 | 0.48 |
| 30 | 6 | 900 | 1335 | 0.45 |
| 35 | 7 | 1050 | 1555 | 0.57 |
| 60 | 12 | 1800 | 2655 | 1.55 |
| 70 | 14 | 2100 | 3095 | 26.86 |
| 90 | 18 | 2700 | 3975 | 3.32 |
| 150 | 30 | 4500 | 6615 | 384.63 |
| 180 | 36 | 5400 | 7935 | 110.84 |
| 210 | 42 | 6300 | 9255 | 332.56 |
| 270 | 54 | 8100 | 11895 | 18026.00 |
| 330 | 66 | 9900 | 14535 | - |
| 365 | 73 | 10950 | 16075 | - |

Table 5.2: Model size and computational times for a 5-team schedule

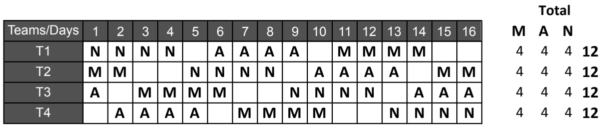
4 working teams (the fifth is on holidays), a summer schedule was developed (Fig. 5.3) that was integrated in the 5-team schedule with some calendar adjustments. This 4-team solution was achieved in 0.4 seconds.



| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | Total | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Teams/Days | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | M | A | N | |
| T1 | M | | | N | N | N | N | | | | | A | A | A | | M | M | M | M | | | N | N | N | | | | | A | A | A | A | | M | M | 7 | 7 | 7 | 21 |
| T2 | A | A | A | A | | M | M | M | | | N | N | N | N | | | | | A | A | A | | M | M | M | M | | | N | N | N | | | | | 7 | 7 | 7 | 21 |
| T3 | N | N | N | | | | | A | A | A | A | | M | M | M | | | N | N | N | N | | | | | A | A | A | | M | M | M | M | | | 7 | 7 | 7 | 21 |
| T4 | | M | M | M | M | | | N | N | N | | | | | A | A | A | A | | M | M | M | | | N | N | N | N | | | | | A | A | A | 7 | 7 | 7 | 21 |
| T5 | | | | | A | A | A | | M | M | M | M | | | N | N | N | | | | | A | A | A | A | | M | M | M | | | N | N | N | N | 7 | 7 | 7 | 21 |

Figure 5.2: 5-team schedule for the glass production unit ($nD$=35 days and $\delta D$=7)

Figure 5.4 shows the annual schedule that was built with the integration of both schedules of 35 and 16 days. An adjustment was made in the rotation order of the 4 teams in the summer period in order to ensure the feasibility of the shifts patterns. The 3-day blocks that are filled in purple



| | | | | | | | | | | | | | | | | | | Total | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Teams/Days | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | M | A | N | |
| T1 | N | N | N | N | | A | A | A | A | | M | M | M | M | | | 4 | 4 | 4 | **12** |
| T2 | M | M | | | N | N | N | N | | A | A | A | A | | M | M | 4 | 4 | 4 | **12** |
| T3 | A | | M | M | M | M | | | N | N | N | N | | A | A | A | 4 | 4 | 4 | **12** |
| T4 | | A | A | A | A | | M | M | M | M | | | N | N | N | N | 4 | 4 | 4 | **12** |

Figure 5.3: 4-team schedule for the glass production unit ($nD$=16 days and $\delta D$=4)

correspond to long-weekends. This problem had an additional constraint that imposed the assignment of at least one long-weekend, for each team, in every 5 weeks cycle. This constraint was overcome by means of a simple calendar synchronization with the developed schedule.

The offset parameter gives a cyclic dimension to the schedule, since every team/employee is allocated to exactly the same sequence of working shifts and days-off, but with a time lag of $\delta D$ days between each other. This feature allows an even distribution of workload among employees, as is evidenced by Table 5.3, which sums up the annual number of working days for each team, for each shift and in aggregate. In this case, any unexpected adjustment to the schedule would be done within each team of employees, having little or no impact in the long-term schedule.

| Teams/Shifts | M | N | A | Total |
|---|---|---|---|---|
| T1 | 75 | 72 | 75 | 222 |
| T2 | 72 | 76 | 72 | 220 |
| T3 | 75 | 73 | 73 | 221 |
| T4 | 72 | 72 | 76 | 220 |
| T5 | 74 | 75 | 72 | 221 |

Table 5.3: Annual number or work-days for each team

The developed model was embedded in a decision support system fitted to the needs of the planning department of the glass production unit. The system has a set of control flags, allowing for the user to enable all or only part of the constraints, as well as some quality indicators of the solution.

## 5.5 Conclusions

This chapter described the application of the general IP model to the real-life problem of a glass production unit. The main adjustment was the introduction of the offset constraints (Eqs. 5.2), which ensure the workload balance between the teams and allow for a reduction in the overall number of the remaining constraints, improving the model's performance. A new sequence of shifts and days-off was imposed, by simply defining a corresponding vector of indices $n(s')$. This demonstrates the flexibility of the approach developed for the sequence constraints. Experiments focused on the evaluation of different optimal solutions achieved for different planning periods and allowed for the selection of those that better suited the company's goals, namely in terms of holiday distribution along the year. An integrated long-term scheduling solution was proposed, through the replication of two different planning periods, one for the winter months and another for the summer period. Computational times revealed the high efficiency of the model for this particular application, which was embedded in a more complex decision support system to be used for the glass industry management.

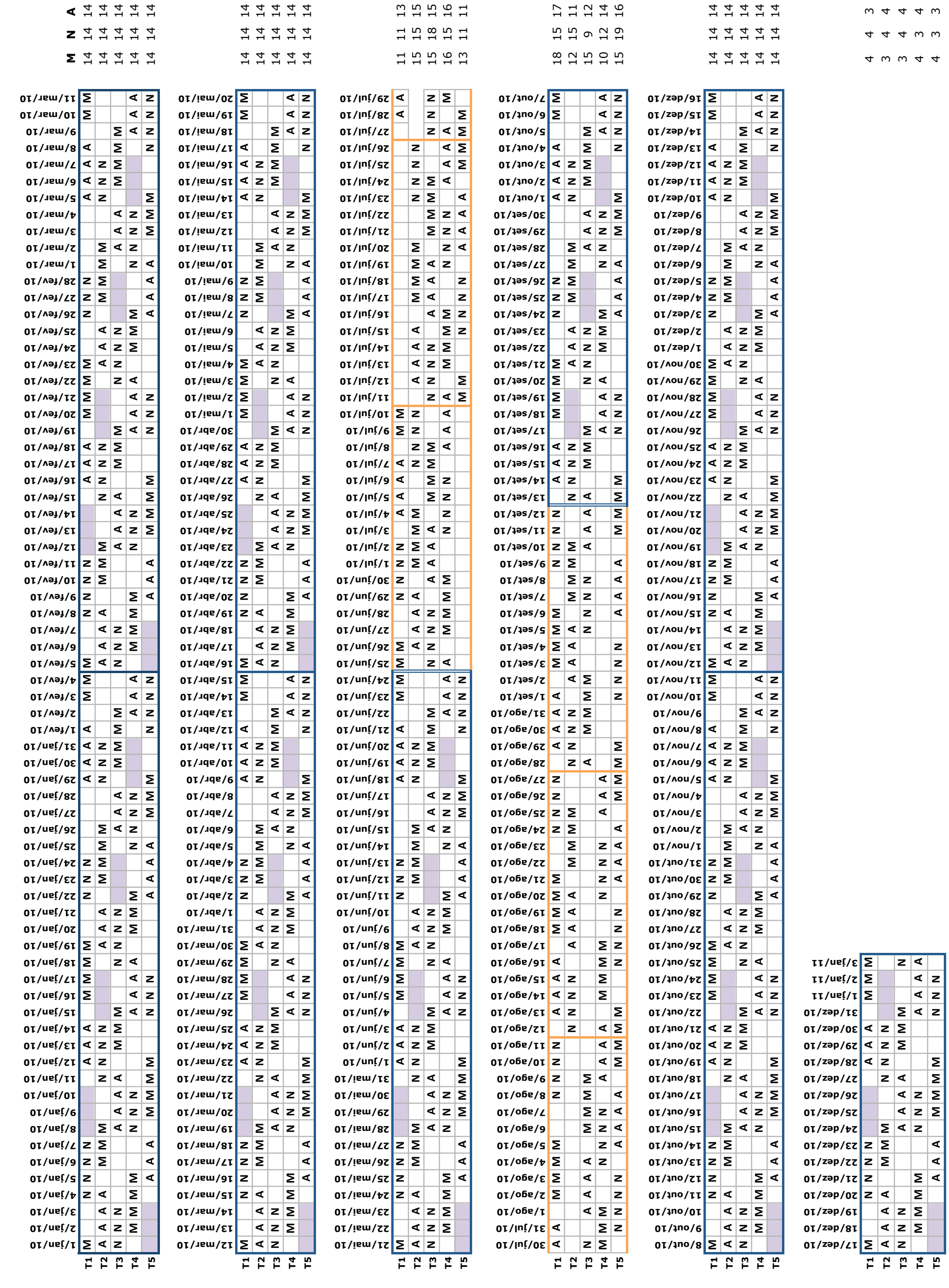


Figure 5.4: Annual schedule for the glass production unit

# Chapter 6

# Application of the general model to a continuous care unit

The general model described in Chapter 4 was in a second phase adapted to the real-world problem of a continuous care unit. This chapter presents that work and is organized as follows. Section 6.1 introduces the service type, the work environment and the features of this particular problem. Then, in Section 6.2 the adjustments that were made to the general model are explained, followed by the achieved computational results, which are presented in Section 6.3. A proposed solution is shown in Section 6.4. Section 6.5 resumes this chapter, highlighting some important outcomes of the developed research work.

## 6.1 Problem description

The organization provides private lodging and nursing home services, directed mainly to the elderly. This work addresses the scheduling of only part of the workforce that is composed by care takers, which are employees with no specific qualifications that are responsible for the daily basic needs of the guests/patients, such as personal hygiene.

The unit works continuously, around the clock, 24h a day, in a multi-shift working scheme: M - morning (from 8:00 a.m. to 2:00 p.m.), A - afternoon (from 2:00 p.m. to 8:00 p.m.), N - night (from 8:00 p.m. to 0:00 a.m.) and D - after-night (from 0:00 a.m. to 8:00 a.m.). In opposition to the previous case study, the demand is now different for each shift and the maximum and minimum number of consecutive working (or rest) days is now indexed to each shift. Again, no daily meal breaks are considered, as well as weekends-off restrictions. Employees' preferred sequence of shifts and breaks (B) must be assured (M-A-N-D-B) and preference is given to a balanced schedule between employees. The workforce is considered single skilled but is now heterogeneous in terms of contract types. It combines a fixed workforce of 49 full-time, permanent, employees with a variable pool of part-time workers. The objective of the model is to minimize the part-time requirements, assuming that full-time contracted hours must be as fully assigned as possible. Part-time workers are not subject to any constraints.

## 6.2 Mathematical model

In order to adapt the general model to this new problem, the following adjustments were made.

### Indices

$n(s')$ is the extended shift that follows the extended shift $s'$ in a given sequence;

Considering 4 working shifts $\{1, 2, 3, 4\}$ and 1 non-working shift $\{5\}$ the new sequence M-A-N-D-B is now defined as shown in Table 6.1.

| $s'$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $n(s')$ | 2 | 3 | 4 | 5 | 1 |

Table 6.1: Sequence of shifts for the continuous care unit

$maxD_{s'}$ maximum number of consecutive working (shifts 1 to 4) and rest (shift 5) days for each shift;

$minD_{s'}$ minimum number of consecutive working (shifts 1 to 4) and rest (shift 5) days for each shift;

$ptCost_s$ hourly cost of a part-time employee working in shift s;

$h_s$ number of working hours of shift s.

**Parameters**

$\delta D$ offset between the working cycles of the teams (in number of days).

**Objective function** Minimization of the cost with part-time work.

$$\min \quad \textstyle\sum_d \sum_s ptCost_s \times h_s \times (demand_s - \sum_t x_{tsd}) \tag{6.1}$$

**Constraints**

$$\forall_{sd} \sum_t x_{tsd} \leq demand_s \tag{6.2}$$

Equations (6.2) state that each day, the number of full-time employees assigned to every working shift is less than or equal to the demand. The difference between the assigned and the demanded work will be assured by part-time workers.

The cyclic approach introduced in Chapter 5 was also adopted in this formulation. Therefore, the model constraints include Eqs. (5.2), (5.3), (5.4), (5.5) and (5.6). See Section 5.2 for a detailed description of each one of the equations.

Equations (5.3), (5.4), (5.5) were adjusted to consider extended shifts, which implied the replacement of the parameters $maxD$ and $minD$ by the indexed parameters $maxD_{s'}$ and $minD_{s'}$. These represent minor adjustments, but in order to allow for an easier reading the new Eqs. (6.3), (6.4) and (6.5) are presented next.

$$\forall_{s'd} \qquad \textstyle\sum_{q=0}^{maxD_{s'}} x_{1s'(d+q)} \leq maxD_{s'} \tag{6.3}$$

$$\forall_{s'd} \qquad \textstyle\sum_{m=1}^{minD_{s'}} b_{1dm} - x_{1s'(d+minD_{s'}-1)} \geq 0 \tag{6.4}$$

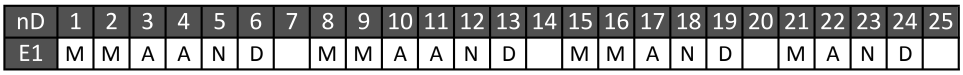
$$\forall_{s'd} \, \forall_{m=1}^{minD_{s'}} \qquad \textstyle\sum_{q=m}^{m+minD_{s'}-1} x_{1s'(d+q-1)} - minD_{s'} \times b_{1dm} \geq 0 \tag{6.5}$$

## 6.3 Computational experiments

The model was coded in OPL Studio version 6.3 and solved using the CPLEX 12.1.0 solver on a server machine powered by 2 Intel® Xeon® processors of 2,4 GHz and 1,39 GHz, and with 2 GB RAM. The number of employees ($nT$) is 49 and the working shifts ($nS$) are now 4. The daily shift demand ($demand_s$) is 20 M, 17 A, 11 N and 11 D. Tests were conducted for planning periods of 25, 28 and 30 days, considering different combinations of the parameters $minD_{s'}$, $maxD_{s'}$ and $ptCost_s$. The choice of the values of these parameters took into account the desired length of the sub-periods, the assurance of the minimum of one day-off every 7 days, and also that the working hours assigned to each employee should fall below 160h in a 30-day period in order to respect labor contracts. The reasoning made in the previous case study, concerning the value of the offset parameter, does not make sense in this problem, since the planning period is now shorter than the number of employees. Therefore, the offset was set to 1, as it achieved satisfactory results.

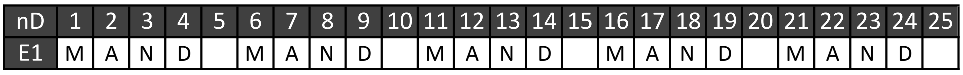
In terms of dimension, the number of decision variables of this problem varies between 6125 for $nD$=25 and 7350 for $nD$=30 and the number of constraints reaches the maximum of 8250 for $nD$=30 and $maxD_1 = maxD_2 = 3$, $maxD_3 = maxD_4 = maxD_5 = 1$.

Table 6.2 reports the computational results for a set of different values of input parameters. The column "solution pattern" contains the schedule for one employee for the whole planning period considered, as illustrated in the next examples. Figures 6.1 and 6.2 show the schedule for E1 in a $nD = 25$ days scenario.



| nD | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E1 | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | N | D |  | M | A | N | D |  |

Figure 6.1: Schedule of E1 for $nD$=25 days; $maxD_1 = maxD_2 = 2$ and $maxD_3 = maxD_4 = maxD_5 = 1$; $minD_1 = minD_2 = minD_3 = minD_4 = minD_5 = 1$; $ptCost_1 = ptCost_2 = ptCost_3 = ptCost_4 = 1$.



| nD | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E1 | M | A | N | D |  | M | A | N | D |  | M | A | N | D |  | M | A | N | D |  | M | A | N | D |  |

Figure 6.2: Schedule of E1 for $nD$=25 days; $maxD_1 = maxD_2 = maxD_3 = maxD_4 = maxD_5 = 1$; $minD_1 = minD_2 = minD_3 = minD_4 = minD_5 = 1$; $ptCost_1 = ptCost_2 = ptCost_3 = ptCost_4 = 1$.

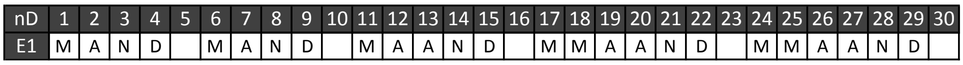
The ability to control the solution pattern with the variation of input parameters is noticeable. For this scenario, for example, it is possible to get a balanced solution, with sub-periods of equal length, with a reduction of $maxD_{s'}$, forcing the model to assign exactly 1 day to each shift. In this solution the number of sub-periods increases from 4 in Fig. 6.1 to 5 in Fig. 6.2, meaning that the number of breaks or days-off of full-time employees will also increase, as well as the requirements for part-time service (higher/poorer solution value).

The results of the tuning of $maxD_{s'}$ and $minD_{s'}$ can also be checked for a 30 days planning horizon. In this case, fixing to 2 the minimum number

| $nD$ | $maxD_s$ | $minD_s$ | $ptCost_s$ | Solution Pattern | PT Req. |
|---|---|---|---|---|---|
| | s = M A N D B | s = M A N D B | s = M A N D | (sub-periods) | (hours) |
| 25 | 2 2 1 1 1 | 1 1 1 1 1 | 1 1 1 1 | MMAANDB-MMAANDB-MMANDB-MANDB | 2676 |
| 25 | 1 1 1 1 1 | 1 1 1 1 1 | 1 1 1 1 | MANDB-MANDB-MANDB-MANDB-MANDB | 2970 |
| | | | | | |
| 28 | 3 3 1 1 1 | 1 1 1 1 1 | 1 1 1 1 | MMMAANDB-MMAANDB-MMAAANDB-MANDB | 2856 |
| 28 | 2 2 1 1 1 | 1 1 1 1 1 | 1 1 1 1 | MMAANDB-MMAANDB-MMAANDB-MMAANDB | 2856 |
| 28 | 2 2 1 1 1 | 1 1 1 1 1 | 1 1 3 3 | MAANDB-MANDB-MAANDB-MANDB-MAANDB | 3150 |
| 28 | 2 2 1 1 1 | 1 1 1 1 1 | 1 5 1000 1000 | MAANDB-MANDB-MAANDB-MAANDB-MANDB | 3150 |
| | | | | | |
| 30 | 3 3 1 1 1 | 1 1 1 1 1 | 1 1 1 1 | MMMANDB-MMMAANDB-MMMANDB-MMAAANDB | 2976 |
| 30 | 2 2 1 1 1 | 1 1 1 1 1 | 1 1 1 1 | MANDB-MANDB-MAANDB-MMAANDB-MMAANDB | 3270 |
| 30 | 2 1 1 1 1 | 2 1 1 1 1 | 1 1 1 1 | MMANDB-MMANDB-MMANDB-MMANDB-MMANDB | 3270 |
| 30 | 1 1 1 1 1 | 1 1 1 1 1 | 1 1 1 1 | MANDB-MANDB-MANDB-MANDB-MANDB-MANDB | 3564 |
| 30 | 3 3 1 1 1 | 1 1 1 1 1 | 1 1 1000 1000 | MANDB-MANDB-MANDB-MANDB-MANDB-MANDB | 3564 |

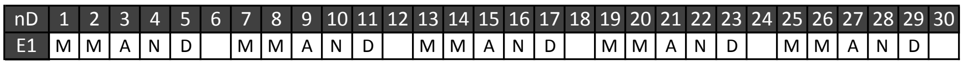
Table 6.2: Model computational parameters and results for the continuous care unit

of consecutive days of shift M, results in a balanced solution but with the same number of sub-periods and, therefore, with the same solution value. Figures 6.3 and 6.4 illustrate this example.



| nD | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E1 | M | A | N | D |  | M | A | N | D |  | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  |

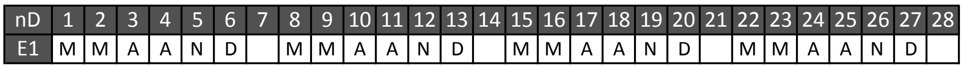
Figure 6.3: Schedule of E1 for $nD$=30 days; $maxD_2 = maxD_2 = 2$ and $maxD_3 = maxD_4 = maxD_5 = 1$; $minD_1 = minD_2 = minD_3 = minD_4 = minD_5 = 1$; $ptCost_1 = ptCost_2 = ptCost_3 = ptCost_4 = 1$.



| nD | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E1 | M | M | A | N | D |  | M | M | A | N | D |  | M | M | A | N | D |  | M | M | A | N | D |  | M | M | A | N | D |  |

Figure 6.4: Schedule of E1 for $nD$=30 days; $maxD_1 = 2$ and $maxD_2 = maxD_3 = maxD_4 = maxD_5 = 1$; $minD_1 = 2$ and $minD_2 = minD_3 = minD_4 = minD_5 = 1$; $ptCost_1 = ptCost_2 = ptCost_3 = ptCost_4 = 1$.

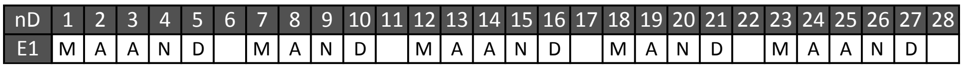
The influence of the $ptCost_s$ parameter can be verified in the 28 days planning horizon case. The increase from 1 (Fig. 6.5) to 3 (Fig. 6.6) units, results in a higher number of sub-periods and therefore, in a higher number of days-off of full time employees and higher part-time needs, leading to a worse solution.



| nD | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E1 | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  |

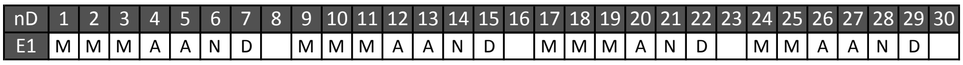
Figure 6.5: Schedule of E1 for $nD$=28 days; $maxD_1 = maxD_2 = 2$ and $maxD_3 = maxD_4 = maxD_5 = 1$; $minD_1 = minD_2 = minD_3 = minD_4 = minD_5 = 1$; $ptCost_1 = ptCost_2 = ptCost_3 = ptCost_4 = 1$.

The same happens in the $nD$=30 case, where an increase from 1 (Fig. 6.7) to 1000 (Fig. 6.8), in the hourly cost of the part-time night and after-night shifts, achieves a solution with 2 additional sub-periods, which means more part-time requirements and consequently a worse solution value. In terms of execution times, each run took less than 4 seconds.



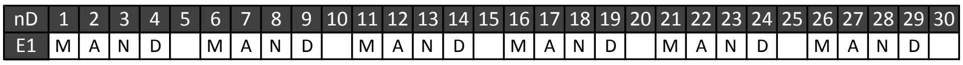
| nD | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E1 | M | A | A | N | D |  | M | A | N | D |  | M | A | A | N | D |  | M | A | N | D |  | M | A | A | N | D |  |

Figure 6.6: Schedule of E1 for $nD$=30 days; $maxD_1 = maxD_2 = 2$ and $maxD_3 = maxD_4 = maxD_5 = 1$; $minD_1 = minD_2 = minD_3 = minD_4 = minD_5 = 1$; $ptCost_1 = ptCost_2 = 1$ and $ptCost_3 = ptCost_4 = 3$.



| nD | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E1 | M | M | M | A | A | N | D |  | M | M | M | A | A | N | D |  | M | M | M | A | N | D |  | M | M | A | A | N | D |  |

Figure 6.7: Schedule of E1 for $nD$=30 days; $maxD_1 = maxD_2 = 3$ and $maxD_3 = maxD_4 = maxD_5 = 1$; $minD_1 = minD_2 = minD_3 = minD_4 = minD_5 = 1$; $ptCost_1 = ptCost_2 = ptCost_3 = ptCost_4 = 1$.



| nD | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E1 | M | A | N | D |  | M | A | N | D |  | M | A | N | D |  | M | A | N | D |  | M | A | N | D |  | M | A | N | D |  |

Figure 6.8: Schedule of E1 for $nD$=30 days; $maxD_1 = maxD_2 = 3$ and $maxD_3 = maxD_4 = maxD_5 = 1$; $minD_1 = minD_2 = minD_3 = minD_4 = minD_5 = 1$; $ptCost_1 = ptCost_2 = 1$ and $ptCost_3 = ptCost_4 = 1000$.

## 6.4 Solutions

Figure 6.9 shows the schedule for $nD$=28 days, $maxD_1 = maxD_2 = 2$, $maxD_3 = maxD_4 = maxD_5 = 1$, $minD_1 = minD_2 = minD_3 = minD_4 = minD_5 = 1$ and $ptCost_1 = ptCost_2 = ptCost_3 = ptCost_4 = 1$. With this solution, which was obtained in 2.6 seconds, the number of required part-time shifts is 476, corresponding to a total of 2856 hours. According to this schedule, each employee has 144 hours of work assigned in a 28 days period. In fact, this is very close to the target of 160 hours in a 30-days period.

## 6.5 Conclusions

The problem of the continuous care unit highlighted the challenge of adapting the general IP model to a service environment and more specifically to the hospitality sector. The introduction of part-time workers called for an adjustment in the objective function, which now sought to minimize part-

| nD | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E1 | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A |
| E2 | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A |
| E3 | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M |
| E4 | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M |
| E5 | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  |
| E6 |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |
| E7 | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N |
| E8 | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A |
| E9 | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A |
| E10 | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M |
| E11 | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M |
| E12 | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  |
| E13 |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |
| E14 | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N |
| E15 | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A |
| E16 | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A |
| E17 | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M |
| E18 | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M |
| E19 | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  |
| E20 |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |
| E21 | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N |
| E22 | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A |
| E23 | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A |
| E24 | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M |
| E25 | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M |
| E26 | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  |
| E27 |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |
| E28 | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N |
| E29 | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A |
| E30 | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A |
| E31 | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M |
| E32 | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M |
| E33 | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  |
| E34 |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |
| E35 | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N |
| E36 | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A |
| E37 | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A |
| E38 | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M |
| E39 | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M |
| E40 | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  |
| E41 |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |
| E42 | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N |
| E43 | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A |
| E44 | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A |
| E45 | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M |
| E46 | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M |
| E47 | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  |
| E48 |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |
| E49 | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N | D |  | M | M | A | A | N |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | Total |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | **shifts** | **hours** |
| **Assigned hours** | M | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 392 | 2352 |
|  | A | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 14 | 392 | 2352 |
|  | N | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 196 | 784 |
|  | D | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 7 | 196 | 1568 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | **1176** | **7056** |
| **Part-time requirements** | M | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 168 | 1008 |
|  | A | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 3 | 84 | 504 |
|  | N | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 112 | 448 |
|  | D | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 4 | 112 | 896 |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | **476** | **2856** |

Figure 6.9: Schedule for the continuous care unit ($nD$=28 days)

time requirements. The cyclic approach introduced in Chapter 5 was also adopted in order to ensure a balanced solution. An adjustment in the demand constraints (Eqs. 4.5, 6.2) was made in order to consider an upper bound for the number of assigned full-time workers. The number of consecutive days-off was now limited to a minimum and a maximum value, requiring an extension of Eqs. (5.3), (5.4) and (5.5) to consider extended shifts (index $s'$). A new different sequence of shifts and days-off was imposed, by simply defining a new vector of indices $n(s')$. Again, the flexibility of the approach developed for the sequence constraints was demonstrated. Experiments focused on the evaluation of different optimal solutions achieved for three planning periods: 25, 28 and 30 days. Computational results prove the efficiency of the model. Several scenarios were tested in a few seconds and their impact in terms of different indicators were analyzed. A sensitivity analysis to the variation of the parameters $maxD_{s'}$, $minD_{s'}$ and $ptCost_s$ was made in Section 6.3, demonstrating the potentials of the proposed formulation. The tuning of those parameters allows for differentiating the length of each block of working shifts and days-off and therefore for controlling the length of the patterns of working shifts and days-off. A schedule built for a period of 28 days was presented. Although it was not implemented in real context, the model was adjusted to consider the real features and restrictions of the problem, as reported by the continuous care unit. All the input data for the experiments were also provided by the organization.

# Chapter 7

# Application of the general model to a hospital

This chapter describes the application of the general IP model introduced in Chapter 4 to a hospital unit. This is a real problem of nurse scheduling in the general surgery service of a Portuguese hospital that was addressed before in Antunes and Moz (2011). Section 7.1 presents the work environment and the features of this particular problem. The adjustments that were made to the general model are explained in Section 7.2. Next, the achieved computational results and proposed solutions are presented in Section 7.3. Section 7.4 closes this chapter.

## 7.1 Problem description

As in any hospital, this general surgery unit works continuously 24 hours a day. Three work shifts are considered: M - morning (from 8:00 a.m. to 4:00 p.m.), A - afternoon (from 3:30 p.m. to 11:00 p.m.) and N - night (from 11:30 p.m. to 8:30 a.m.). There are also 2 rest shifts to take into account: D and B. In each week (7 days), at least two of the days must be assigned to shifts D and/or B and the nurses cannot work more than 6 consecutive days. The preferred sequence of shifts is M-M-T-T-D-N-N-B. The daily requirements of each working shift are limited by a minimum and a maximum number of nurses, as presented in Table 7.1.

| Shift / Day | Mon | Tue | Wed | Thu | Fri | Sat | Sun |
|---|---|---|---|---|---|---|---|
| M | 12/13 | 12/12 | 13/13 | 12/13 | 13/13 | 8/9 | 8/9 |
| T | 7/8 | 8/8 | 8/8 | 8/8 | 8/9 | 7/8 | 7/8 |
| N | 6/7 | 6/7 | 6/7 | 6/7 | 6/7 | 6/7 | 6/7 |

Table 7.1: Minimum/maximum no. of nurses required daily for each shift

The workforce is composed by 42 nurses with 5 different contract types, distributed as presented in Table 7.2.

| Type of contract | No. of nurses | No. of contracted hours/ week |
|---|---|---|
| 1 | 5 | 42 |
| 2 | 31 | 40 |
| 3 | 2 | 33 |
| 4 | 2 | 32 |
| 5 | 2 | 31 |

Table 7.2: Types of contracts

The objective is to assign a number of hours to each nurse as close as possible to the contracted ones.

## 7.2 Mathematical model

In order to adapt the general model to this new problem, the following adjustments were made.

**Indices**

$n(s')$ is the extended shift that follows the extended shift $s'$ in a given sequence;

Considering 3 working shifts $\{1,2,3\}$ and 2 non-working shifts $\{4,5\}$ the preferred sequence M-M-A-A-D-N-N-B can be represented as displayed in Table 7.3.

| $s'$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $n(s')$ | 2 | 4 | 5 | 3 | 1 |

Table 7.3: Sequence of shifts for the hospital problem

k $\in \{1,2,3,4,5\}$, type of nurse (or contract);

$maxD_{ks'}$ maximum number of consecutive working (shifts 1 to 3) and rest (shifts 4 and 5) days for each shift and for each type of nurse $k$;

$minD_{ks'}$ minimum number of consecutive working (shifts 1 to 3) and rest (shifts 4 and 5) days for each shift and for each type of nurse $k$;

$p_s$ number of hours of each working shift $s$;

$a_t$ balance of worked vs contracted hours of nurse $t$ in the previous planning period;

$h_t$ number of contracted working hours of nurse $t$ in the planning period $nD$; depends on the type of contract;

$dMin_{sd}$ minimum number of nurses required in each shift $s$, in each day $d$;

$dMax_{sd}$ maximum number of nurses required in each shift $s$, in each day $d$;

w time window (days).

**Objective function** Minimization of the difference between contracted and assigned working hours.

$$\min \quad \sum_t |[(h_t - a_t) - \sum_s \sum_d (p_s \times x_{tsd})]| \tag{7.1}$$

**Constraints**

$$\forall_{sd} \quad \sum_t x_{tsd} \geq dMin_{sd} \tag{7.2}$$

$$\forall_{sd} \quad \sum_t x_{tsd} \leq dMax_{sd} \tag{7.3}$$

Equations (7.2) and (7.3) state that each day, the required number of working nurses for each shift is ensured.

$$\forall_{td} \quad \sum_{i=0}^{w-1} \sum_{s'=4}^{5} x_{ts'(d+i)} \geq 2 \tag{7.4}$$

Equation (7.4) imposes a minimum of two non-working days (D or B) in each window $w$.

The cyclic approach introduced in Chapter 5 was once more adopted. The model constraints include Eqs. (5.2), (5.3), (5.4), (5.5) and (5.6). See Section 5.2 for a detailed description.

Equations (5.3), (5.4), (5.5) were adjusted to consider the indexed parameters $maxD_{ks'}$ and $minD_{ks'}$. These parameters can be set to different values in order to evaluate different patterns for the sequence of shifts of the different types of contracts. These represent minor adjustments, but in order to allow for an easier reading the new Eqs. (7.5), (7.6) and (7.7) are presented next.

$$\forall_{ks'd} \qquad \sum_{q=0}^{maxD_{ks'}} x_{1s'(d+q)} \leq maxD_{ks'} \tag{7.5}$$

$$\forall_{ks'd} \qquad \sum_{m=1}^{minD_{ks'}} b_{1dm} - x_{1s'(d+minD_{ks'}-1)} \geq 0 \tag{7.6}$$

$$\forall_{ks'd} \, \forall_{m=1}^{minD_{ks'}} \quad \sum_{q=m}^{m+minD_{ks'}-1} x_{1s'(d+q-1)} - minD_{ks'} \times b_{1dm} \geq 0 \tag{7.7}$$

The scheduling constraints that ensure a maximum of 6 consecutive work days for each nurse are imposed by the tuning of the parameters $maxD_{ks'}$ and $minD_{ks'}$.

## 7.3 Computational experiments and solutions

The model was coded in OPL Studio version 6.3 and solved using the CPLEX 12.1.0 solver on a server machine powered by 2 Intel® Xeon® processors of 2,4 GHz and 1,39 GHz, and with 2 GB RAM. In this problem the workforce is composed by 42 nurses with different contract types. For implementation purposes, the 42 nurses were divided in 5 groups, as shown in Fig. 7.4.

| $t$ | Type of contract |
|---|---|
| 1. . . 5 | 1 |
| 6. . . 36 | 2 |
| 37. . . 38 | 3 |
| 39. . . 40 | 4 |
| 41. . . 42 | 5 |

Table 7.4: Association of index $t$ to the type of contract

Since each type of contract is linked to a number of contracted hours, each nurse is thus initially connected to a number of contracted hours per planning period. This approach led to a lower number of decision variables than the alternative of indexing each decision variable to a nurse. In opposition to the previous case studies, as there are nurses with different contracted hours, it was not reasonable to impose the same offset to all the schedules. Therefore, each group of nurses of the same type had its own offset. In terms of size, this model dealt with 5880 decision variables and 952 constraints. Just to give an idea of the influence of the offset constraints in the simplification of the model, if these constraints were not considered the overall number of constraints would be 37 times higher, increasing from 952 to 35392. Figure 7.1 shows a solution obtained for this case study in 1170.9 seconds (20 minutes). In this case, $maxD_{ks'} = 5$ for $k$=1,. . . ,5 and $s'$=1,. . . ,4; $maxD_{15} = maxD_{25} = 5$ and $maxD_{35} = maxD_{45} = maxD_{55} = 7$; $minD_{ks'} = 1$, for $k$=1,. . . ,5 and $s'$=1,. . . ,5.

The formulation of the parameters $maxD_{ks'}$ and $minD_{ks'}$ allows us to control the relation between the number of work and rest days for each type of nurse. In this case, it makes sense to allow for more breaks for those nurse types that have less contracted work hours ($maxD_{35} = maxD_{45} = maxD_{55} = 7$).

This solution does not consider planned absences or holidays. Although this constraint has not been treated in the previous applications of the IP model, we decided to introduce it in the hospital case study since it was addressed in the solution proposed in the original work of Antunes and Moz (2011) and, therefore, it makes the comparison between approaches easier and more realistic. In order to consider this scenario, the decision variables corresponding to each planned day-off were initially set to 0 and the offset constraints could not be applied to the teams that had planned days-off. Sequence and maximum/minimum consecutive days constraints were also adjusted individually for each of those teams in order to exclude the days-off. Figure 7.2 shows the solution obtained when considering the planned days-off for this particular month. As expected, the execution time increased, as the model gets more constrained, and it takes now 2256.3 seconds (about 38 minutes) to obtain this solution.

| Nurse/nD | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | Assigned hours | Contracted hours | Deviation | Current balance |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | M | A | D | N | N | B | M | M | A | D | N | B | M | M | A | D | D | N | N | N | B | M | A | D | N | N | N | B | 168 | 168 | 0 | -2.3 |
| 21 | B | M | A | D | N | N | B | M | M | A | D | N | B | M | M | A | D | D | N | N | N | B | M | A | D | N | N | N | 168 | 168 | 0 | -0.5 |
| 31 | N | B | M | A | D | N | N | B | M | M | A | D | N | B | M | M | A | D | D | N | N | N | B | M | A | D | N | N | 168 | 168 | 0 | -0.2 |
| 41 | N | N | B | M | A | D | N | N | B | M | M | A | D | N | B | M | M | A | D | D | N | N | N | B | M | A | D | N | 168 | 168 | 0 | -3 |
| 51 | N | N | N | B | M | A | D | N | N | B | M | M | A | D | N | B | M | M | A | D | D | N | N | N | B | M | A | D | 168 | 168 | 0 | -1 |
| 12 | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | N | B | B | M | M | M | A | D | N | B | 157 | 160 | -3 | 4.1 |
| 22 | B | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | N | B | B | M | M | M | A | D | N | 157 | 160 | -3 | 1.1 |
| 32 | N | B | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | N | B | B | M | M | M | A | D | 157 | 160 | -3 | 0.9 |
| 42 | D | N | B | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | N | B | B | M | M | M | A | 157 | 160 | -3 | -4.9 |
| 52 | A | D | N | B | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | N | B | B | M | M | M | 157 | 160 | -3 | -11.1 |
| 62 | M | A | D | N | B | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | N | B | B | M | M | 157 | 160 | -3 | 0.1 |
| 72 | M | M | A | D | N | B | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | N | B | B | M | 157 | 160 | -3 | -2.6 |
| 82 | M | M | M | A | D | N | B | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | N | B | B | 157 | 160 | -3 | 0.3 |
| 92 | B | M | M | M | A | D | N | B | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | N | B | 157 | 160 | -3 | -2.8 |
| 102 | B | B | M | M | M | A | D | N | B | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | N | 157 | 160 | -3 | -4.5 |
| 112 | N | B | B | M | M | M | A | D | N | B | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | 157 | 160 | -3 | -3.3 |
| 122 | D | N | B | B | M | M | M | A | D | N | B | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | A | A | 157 | 160 | -3 | -6.8 |
| 132 | A | D | N | B | B | M | M | M | A | D | N | B | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | A | 157 | 160 | -3 | 2.2 |
| 142 | A | A | D | N | B | B | M | M | M | A | D | N | B | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | 157 | 160 | -3 | -6.1 |
| 152 | M | A | A | D | N | B | B | M | M | M | A | D | N | B | M | M | A | D | N | B | M | M | A | A | D | N | B | M | 157 | 160 | -3 | -5 |
| 162 | M | M | A | A | D | N | B | B | M | M | M | A | D | N | B | M | M | A | D | N | B | M | M | A | A | D | N | B | 157 | 160 | -3 | 1.1 |
| 172 | B | M | M | A | A | D | N | B | B | M | M | M | A | D | N | B | M | M | A | D | N | B | M | M | A | A | D | N | 157 | 160 | -3 | -5.6 |
| 182 | N | B | M | M | A | A | D | N | B | B | M | M | M | A | D | N | B | M | M | A | D | N | B | M | M | A | A | D | 157 | 160 | -3 | -6.1 |
| 192 | D | N | B | M | M | A | A | D | N | B | B | M | M | M | A | D | N | B | M | M | A | D | N | B | M | M | A | A | 157 | 160 | -3 | -0.7 |
| 202 | A | D | N | B | M | M | A | A | D | N | B | B | M | M | M | A | D | N | B | M | M | A | D | N | B | M | M | A | 157 | 160 | -3 | -5.8 |
| 212 | A | A | D | N | B | M | M | A | A | D | N | B | B | M | M | M | A | D | N | B | M | M | A | D | N | B | M | M | 157 | 160 | -3 | -2.7 |
| 222 | M | A | A | D | N | B | M | M | A | A | D | N | B | B | M | M | M | A | D | N | B | M | M | A | D | N | B | M | 157 | 160 | -3 | -1 |
| 232 | M | M | A | A | D | N | B | M | M | A | A | D | N | B | B | M | M | M | A | D | N | B | M | M | A | D | N | B | 157 | 160 | -3 | 2.7 |
| 242 | B | M | M | A | A | D | N | B | M | M | A | A | D | N | B | B | M | M | M | A | D | N | B | M | M | A | D | N | 157 | 160 | -3 | -12.2 |
| 252 | N | B | M | M | A | A | D | N | B | M | M | A | A | D | N | B | B | M | M | M | A | D | N | B | M | M | A | D | 157 | 160 | -3 | -5.7 |
| 262 | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | N | B | B | M | M | M | A | D | N | B | M | M | A | 157 | 160 | -3 | -6.2 |
| 272 | A | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | N | B | B | M | M | M | A | D | N | B | M | M | 157 | 160 | -3 | 2.3 |
| 282 | M | A | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | N | B | B | M | M | M | A | D | N | B | M | 157 | 160 | -3 | -4.5 |
| 292 | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | N | B | B | M | M | M | A | D | N | B | 157 | 160 | -3 | -0.2 |
| 302 | B | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | N | B | B | M | M | M | A | D | N | 157 | 160 | -3 | -6.6 |
| 312 | N | B | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | N | B | B | M | M | M | A | D | 157 | 160 | -3 | -3.4 |
| 13 | D | N | B | M | M | M | A | D | D | N | B | M | A | D | D | D | N | B | M | M | M | A | D | N | B | M | A | D | 134 | 132 | 2 | 1.6 |
| 23 | D | D | N | B | M | M | M | A | D | D | N | B | M | A | D | D | D | N | B | M | M | M | A | D | N | B | M | A | 134 | 132 | 2 | 1.5 |
| 14 | N | B | M | M | A | D | D | D | N | B | M | M | A | D | N | N | B | M | M | A | D | D | D | N | B | M | A | D | 136 | 128 | 8 | 13 |
| 24 | D | N | B | M | M | A | D | D | D | N | B | M | M | A | D | N | N | B | M | M | A | D | D | D | N | B | M | A | 136 | 128 | 8 | 4.2 |
| 15 | A | D | N | B | B | B | B | B | B | M | A | A | D | N | B | B | M | M | A | D | N | B | M | A | D | N | B | M | 117.5 | 124 | -6.5 | -5.1 |
| 25 | M | A | D | N | B | B | B | B | B | B | M | A | A | D | N | B | B | M | M | A | D | N | B | M | A | D | N | B | 117.5 | 124 | -6.5 | -5 |
| M | 12 | 12 | 13 | 14 | 13 | 11 | 12 | 13 | 13 | 13 | 13 | 13 | 12 | 12 | 13 | 13 | 13 | 13 | 13 | 12 | 11 | 12 | 13 | 14 | 12 | 13 | 11 | 10 | | | | |
| A | 7 | 8 | 8 | 8 | 9 | 8 | 7 | 7 | 8 | 9 | 10 | 10 | 10 | 8 | 7 | 8 | 9 | 9 | 9 | 8 | 7 | 7 | 8 | 8 | 9 | 8 | 10 | 8 | | | | |
| N | 9 | 8 | 7 | 6 | 7 | 7 | 7 | 6 | 6 | 6 | 6 | 6 | 6 | 7 | 7 | 6 | 6 | 6 | 7 | 8 | 9 | 8 | 6 | 7 | 7 | 7 | 9 | 8 | | | | |

Figure 7.1: Schedule for the hospital case study without planned absences

In both solutions, the deviation between assigned and contracted hours transferred from the previous month was set from real data. Column “Deviation” data refers only to the present month’s deviation and column “Current balance” shows the actual deviation balance after the present month’s assignment. We consider this last column in order to compare our IP solution with the one achieved by Antunes and Moz (2011) and also with the real schedule that was made by hand by the head nurse of the hospital. We will name them IP, Antunes and Real solutions respectively. Table 7.5 shows some statistical data for the deviation balance in the three solutions.

| Deviation balance | Solutions | | |
|---|---|---|---|
| (hours) | IP | Antunes | Real |
| Maximum | 13.8 | 8 | 19.4 |
| Median | 4.7 | 4.8 | 5.5 |
| Average | 4.8 | 4.7 | 5.8 |

Table 7.5: Statistic analysis of the solutions

The mathematical model proposed by Antunes and Moz (2011) limits the maximum deviation balance of each nurse to 8 hours. In the Antunes solution, this value is reached in the schedules of two of the nurses. In the Real solution this 8 hours-value is exceeded in 12 situations, which corresponds to approximately 29% of the workforce, and the maximum deviation is 19.4 hours. In our IP solution, there are 8 nurses (19%) with a deviation balance above 8 hours, but the maximum value is 13.8. Nevertheless, the IP solution obtains a deviation below 4 hours for 45% of the nurses, against 31% of the Antunes solution and 43% of the Real solution. Although the average and the median values are not very distant, the fact is that high deviations are harder to manage and should be avoided. In our approach, we had to make a trade-off between the resolution time and the minimization of the sum of

| Nurse/nD | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | Assigned hours | Contracted hours | Deviation | Planned absences | Current balance |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 11 | M | M | A | D | D | N | B | M | M | A | D | N | N | B | M | A | A | D | N | N | B | M | M | A | D | N | N | B | 163.5 | 168 | -4.5 | 0 | -6.8 |
| 21 | B | M | M | A | D | D | N | B | M | M | A | D | N | N | B | M | A | A | D | N | N | B | M | M | A | D | N | N | 163.5 | 168 | -4.5 | 0 | -5 |
| 31 | N | B | M | M | A | D | D | N | B | M | M | A | D | N | N | B | M | A | A | D | N | N | B | M | M | A | D | N | 163.5 | 168 | -4.5 | 0 | -4.7 |
| 41 | M | M | M | A | A | D | D | B | B | B | B | B | B | B | M | M | M | M | M | A | A | A | A | D | N | N | B | B | 129 | 168 | -39 | 43 | 1 |
| 51 | N | N | N | B | B | B | B | M | M | M | M | M | A | A | A | A | A | B | B | B | B | B | B | B | B | B | B | B | 107.5 | 168 | -60.5 | 67 | 5.5 |
| 12 | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | 165 | 160 | 5 | 0 | 12.1 |
| 22 | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | 165 | 160 | 5 | 0 | 9.1 |
| 32 | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | 165 | 160 | 5 | 0 | 8.9 |
| 42 | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | 165 | 160 | 5 | 0 | 3.1 |
| 52 | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | 165 | 160 | 5 | 0 | -3.1 |
| 62 | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | 165 | 160 | 5 | 0 | 8.1 |
| 72 | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | 165 | 160 | 5 | 0 | 5.4 |
| 82 | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | 165 | 160 | 5 | 0 | 8.3 |
| 92 | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | 165 | 160 | 5 | 0 | 5.2 |
| 102 | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | 165 | 160 | 5 | 0 | 3.5 |
| 112 | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | 165 | 160 | 5 | 0 | 4.7 |
| 122 | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | 165 | 160 | 5 | 0 | 1.2 |
| 132 | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | 165 | 160 | 5 | 0 | 10.2 |
| 142 | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | 165 | 160 | 5 | 0 | 1.9 |
| 152 | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | 165 | 160 | 5 | 0 | 3 |
| 162 | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | 165 | 160 | 5 | 0 | 9.1 |
| 172 | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | 165 | 160 | 5 | 0 | 2.4 |
| 182 | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | 165 | 160 | 5 | 0 | 1.9 |
| 192 | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | 165 | 160 | 5 | 0 | 7.3 |
| 202 | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | 165 | 160 | 5 | 0 | 2.2 |
| 212 | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | 165 | 160 | 5 | 0 | 5.3 |
| 222 | A | D | N | B | M | M | M | A | D | N | B | M | M | A | A | D | N | B | M | M | M | A | D | N | B | M | M | A | 165 | 160 | 5 | 0 | 7 |
| 232 | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | 0 | 160 | -160 | 160 | 5.7 |
| 242 | B | B | B | B | B | B | B | B | B | B | B | B | B | B | M | A | A | D | D | D | D | N | N | N | N | B | M | A | 78.5 | 160 | -81.5 | 87 | -3.7 |
| 252 | A | A | D | N | N | B | B | M | A | D | N | N | N | B | B | M | M | M | M | A | D | B | B | B | B | B | B | B | 120 | 160 | -40 | 43 | 0.3 |
| 262 | A | A | A | A | A | D | D | N | N | N | N | B | B | B | B | M | M | M | M | A | D | D | D | N | B | M | A | D | 142.5 | 160 | -17.5 | 23 | 2.3 |
| 272 | M | A | A | A | B | B | B | B | B | B | B | B | B | B | B | B | B | B | M | A | A | D | N | B | M | M | A | A | 94.5 | 160 | -65.5 | 66 | 5.8 |
| 282 | B | M | A | D | N | N | B | M | A | A | D | D | D | N | N | N | B | M | A | D | N | B | M | M | M | A | D | N | 155.5 | 160 | -4.5 | 7.5 | 1.5 |
| 292 | N | B | M | M | A | D | N | B | M | A | D | N | N | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | 79 | 160 | -81 | 92 | 13.8 |
| 302 | B | B | B | B | B | B | B | B | M | M | A | A | D | D | D | N | N | N | N | N | B | B | B | M | M | M | A | D | 112.5 | 160 | -47.5 | 51 | -0.1 |
| 312 | B | B | M | M | M | A | D | D | N | N | N | N | B | B | M | A | A | A | A | D | D | N | B | M | M | M | A | D | 151 | 160 | -9 | 16 | 6.6 |
| 13 | D | D | D | N | N | N | N | B | B | M | M | M | A | A | D | D | D | N | N | B | B | M | A | A | A | A | D | D | 137 | 132 | 5 | 0 | 4.6 |
| 23 | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | B | 0 | 132 | -132 | 132 | -0.5 |
| 14 | M | A | D | N | B | B | B | M | A | A | A | A | D | D | D | N | N | B | B | B | B | M | A | D | N | N | N | B | 129 | 128 | 1 | 0 | 6 |
| 24 | D | N | B | B | M | A | D | N | B | M | M | M | A | D | N | B | M | A | A | D | D | D | N | B | B | M | A | D | 125.5 | 128 | -2.5 | 7.5 | 1.2 |
| 15 | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | D | D | N | B | B | M | M | M | A | D | D | D | D | 123.5 | 124 | -0.5 | 0 | 0.9 |
| 25 | D | D | N | B | M | M | A | A | D | N | B | M | M | A | A | D | D | D | N | B | B | M | M | M | A | D | D | D | 123.5 | 124 | -0.5 | 0 | 1 |
| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| M | 12 | 12 | 13 | 12 | 13 | 10 | 8 | 12 | 12 | 13 | 12 | 13 | 9 | 8 | 12 | 12 | 13 | 12 | 13 | 9 | 9 | 12 | 12 | 13 | 12 | 13 | 9 | 8 | | | | | |
| A | 7 | 8 | 8 | 8 | 8 | 7 | 7 | 7 | 8 | 9 | 8 | 8 | 9 | 9 | 7 | 8 | 9 | 8 | 8 | 8 | 7 | 7 | 8 | 8 | 8 | 8 | 10 | 7 | | | | | |
| N | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 7 | 6 | 7 | 7 | 6 | 6 | 6 | 6 | 6 | 7 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | 6 | | | | | |

Figure 7.2: Schedule for the hospital case study considering planned absences

these deviations (translated by the objective function). In order to get an optimal solution in a reasonable time, a lower bound to the objective function was imposed. After testing several values, the best compromise was found for the solution of Fig. 7.2, with an objective function of 200.

The limits on the daily shift requirements $dMaxD_{sd}$ and $dMinD_{sd}$ have also a significant influence on the model's performance. The closer to real data these parameters are set, the tighter the model gets and the longer it takes to reach a solution. In our approach, only the $dMinD_{sd}$ real data was imposed. The limits on $dMaxD_{sd}$ had to be relaxed in order to get a solution in a reasonable amount of time. Looking again into the solution of Fig. 7.2, the information on the daily assigned hours can be checked in the rows below the schedule, for each one of the working shifts. A comparison of these figures with the initial demand requirements proves that the minimum daily requirements are satisfied, whereas the maximum limits are violated in 6 different days: one extra morning shift on day 6, one extra afternoon shift on days 10, 13, 14 and 17, and two afternoon shifts in excess on day 27. This drawback does not seem very representative when we look at the real solution, where the number of similar violations is much higher, reaching 23 situations, mostly affecting the afternoon and night shifts. Nevertheless, we do not have enough information to evaluate the impact of these violations in the real setting.

## 7.4 Conclusions

This chapter described the application of the general IP model to the problem of nurse scheduling in a Portuguese hospital (Antunes and Moz (2011)). This problem differs from the others already described in the previous two chapters (Chapters 5 and 6) in two main features: the workforce composition and the objective. The workforce is now composed of 5 groups of nurses, which are grouped according to their contract types. The objective is to minimize the gap between assigned and contracted hours. The cyclic approach introduced in Chapter 5 was adopted in order to ensure a balanced solution within each group of nurses. Demand constraints (Eqs. 4.5, 7.2 and 7.3) were adapted in order to consider minimum and maximum daily requirements for each shift. Equations (5.3), (5.4) and (5.5) to each contract type (index $k$) made it possible to impose different limits on the number of consecutive working/rest days to each group of nurses. In this problem, it was more reasonable to allow the groups with less contracted hours to have more days-off assignments than the others. A new sequence of shifts and days-off that met the preferences of the nurses was imposed, by simply defining a new vector of indices $n(s')$. Although it was not considered in this problem, we could define different sequence patterns to each group of nurses by simply indexing the sequence constraints (Eqs. 5.6) to each type of contract. This example illustrates the flexibility and potential of the proposed formulation. Experiments focused on finding a trade-off between the value of the optimal solution and the computational time. In order to compare our solution with the one proposed by Antunes and Moz (2011) and the real solution manually developed by the head nurse of the hospital, we adjusted the model to consider the absences planned for the present month. An optimal solution was built in 38 minutes, considerably more than the 0.38 seconds taken by the optimal solution proposed by Antunes and Moz (2011), which was specifically tailored to this problem, but much less than the 8 hours the head nurse needed to develop it by hand. Results are encouraging, demonstrating that our formulation can also accommodate restrictions on planned absences, such as holiday or training.

# Chapter 8

# Benchmark instances

In order to evaluate the flexibility and wide scope of the developed formulation and to compare results with other approaches, experiments were carried out on a collection of 20 rotating workforce scheduling problems available in http://www.dbai.tuwien.ac.at/staff/musliu/benchmarks and presented in Musliu (2006). This chapter describes the application of the general model introduced in Chapter 4 to those benchmark problems. Section 8.1 presents the features of the problems. Next, the adjustments that were made to the general formulation are explained in Section 8.2, giving special attention to the sequence constraints adaptation. Section 8.3 shows the computational results and a comparison with other methods. Solutions are discussed in Section 8.4, followed by some concluding remarks that end the chapter.

## 8.1 Problem description

In these instances, the number of employees vary from 7 to 163. The number of standard shifts is either 2 (day and afternoon) or 3 (day, afternoon and night). The length of working and days-off blocks is now limited by a minimum and a maximum number of consecutive days, as well as the length of each sequence of days assigned to the same shift. In the previous case studies only this last situation is considered. The main difference from these problems to the previous ones is the existence of a set of forbidden shift sequences, instead of a predefined sequence to follow. The forbidden sequences that are imposed are of two types: (N-D, N-A and A-D) or (N-B-N, A-B-D, N-B-A and N-B-D). Naturally, the second type is considered only if a single day-off is allowed. Otherwise, only the first type is taken into account. Problems have either both types of forbidden sequences or only type 1.

## 8.2 Mathematical model

The formulation used to solve these problems followed the adaptations made for the glass unit problem (Chapter 5). We refer to Section 5.2 for further details. This section is therefore focused on new adjustments: limits on the length of working and days-off blocks and, especially, a new perspective in the sequence constraints. Following the same reasoning used in the previous case studies, it is possible to define allowable sequences for each given type of forbidden sequences.

Consider the 3 working shifts {D,A,N} as $\{1, 2, 3\}$ and the 3 non-working shifts as $\{4, 5, 6\}$. Unlike the previous case studies, this is not the only sequence allowed. For the first type of forbidden sequences: N-D, N-A and A-D, the allowed sequences can be represented as shown in Table 8.1. $n(s')$ is the extended shift that can follow the extended shift $s'$ in a given sequence.

From Table 8.1, it is possible to realize that only the shifts that follow the shifts 2 and 3 must be constrained. All the remaining shifts can be followed by any of the other shifts, inclusive by themselves. The sequence constraints in the general mathematical model are therefore replaced by the following constraints:

| $s'$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| | 4 | 5 | 6 | 1 | 1 | 1 |
| | 2 | 3 | | 2 | 2 | 2 |
| $n(s')$ | 3 | | | 3 | 3 | 3 |
| | 5 | | | 5 | 4 | 4 |
| | 6 | | | 6 | 6 | 5 |

Table 8.1: First type of allowable sequences

$$\forall_{td}\ x_{t2d} - x_{t2(d+1)} - x_{t5(d+1)} - x_{t3(d+1)} \leq 0 \tag{8.1}$$

$$\forall_{td}\ x_{t3d} - x_{t3(d+1)} - x_{t6(d+1)} \leq 0 \tag{8.2}$$

Considering now both types of forbidden sequences: N-D, N-A, A-D, N-B-N, A-B-D, N-B-A and N-B-D, the allowed sequences are represented in Table 8.2.

| $s'$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| | 4 | 6 | 4 | 1 | 1 | 2 |
| | 2 | 3 | | 2 | 2 | 3 |
| $n(s')$ | 3 | | | 3 | 3 | 4 |
| | 5 | | | 5 | 4 | 5 |
| | 6 | | | 6 | 6 | |

Table 8.2: Allowable sequences

Note that, in this case, shift 3 (N) can only be followed by another shift 3 or by a minimum of two consecutive days-off (B). To simplify the model and in order to satisfy this limitation, shift 4 is now defined as the non-working shift that follows shift 3 and, in the model's parameterization, the minimum number of consecutive days ($minD'_s$) indexed to shift 4 is set to 2. The sequence constraints in the general mathematical model are now restricted to the constraints on shifts 2, 3 and 6, defined as follows:

$$\forall_{td}\ x_{t2d} - x_{t2(d+1)} - x_{t6(d+1)} - x_{t3(d+1)} \leq 0 \tag{8.3}$$

$$\forall_{td}\ x_{t3d} - x_{t3(d+1)} - x_{t4(d+1)} \leq 0 \tag{8.4}$$

$$\forall_{td}\ x_{t6d} - x_{t6(d+1)} - x_{t2(d+1)} - x_{t3(d+1)} - x_{t4(d+1)} \leq 0 \tag{8.5}$$

For the problems with only two shifts {1,2}, the only forbidden sequence is A-D, and so the allowed sequences are represented in Table 8.3.

| $s'$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| $n(s')$ | 2 | 4 | 2 | 1 |
| | 3 | 3 | 4 | 3 |
| | 4 | | 1 | 2 |

Table 8.3: Allowable sequences for nS=2

The sequence constraints in the general mathematical model are replaced by the following constraints:

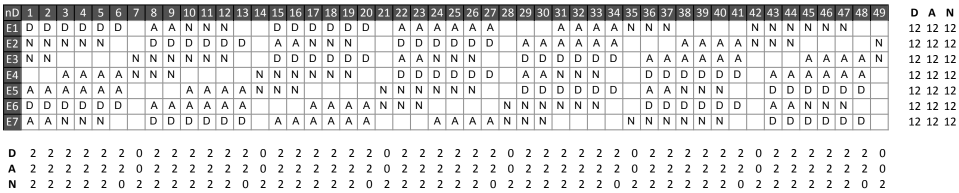
$$\forall_{td}\ x_{t2d} - x_{t2(d+1)} - x_{t4(d+1)} - x_{t3(d+1)} \leq 0 \tag{8.6}$$

## 8.3 Computational experiments

Like in the previous case studies, this model was also coded in OPL Studio version 6.3 and solved using the CPLEX 12.1.0 solver on a server machine powered by 2 Intel® Xeon® processors of 2,4 GHz and 1,39 GHz, and with 2 GB RAM. Tests were performed for the 20 instances and results are shown in Table 8.4. The offset considered for all the examples is 7 days, so the planning periods are defined by $7 \times nT$. The optimization model found the optimal solution for 12 problems, with the number of employees varying from 7 to 29. For the remaining problems, signed with "-", the optimization model did not reach any feasible solution within 20000 seconds of running time. This happened for all the instances with more than 27 employees. The model is able to reach an optimal solution for problem 16, for example, while for problem 7 it does not find any feasible solution. Although both problems have the same number of employees, 29, the shift daily demand, the limits on the length of shift sequences, working-day and days-off blocks differ from one problem to another. It is evident that the combination of these parameters has a decisive influence on the complexity of each problem and consequently on the model's performance. In Musliu (2006), the best resolution times for these instances are achieved by a tabu search based heuristic (MC-T). In the same work, Musliu compares the resolution times of MC-T with the ones achieved by a commercial software, First Class Scheduler (FCS). The resolution times achieved by Musliu with FCS are also reported in the last column of Table 8.4, where "-" signs the problems to which FCS did not find any solution within 1000 seconds of running time. As expected, comparison of resolution times shows that the heuristic based method outperforms the optimization approach in all instances. Nevertheless, when comparing the resolution times obtained by the IP model and FCS, we can conclude that the former has better times for 7 of these instances than the latter.

## 8.4 Solutions

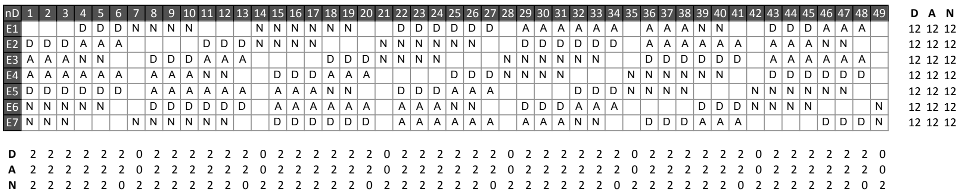
The comparison of the values of the objective functions proves that the solutions obtained by the MC-T method are all optimal (have the same value), in spite of the different pattern composition of the schedules. As an example, Fig. 8.1 and Fig. 8.2 show the solutions obtained for Ex. 6 with our IP optimization model and with MC-T respectively. The optimal value of the objective function is 12 for both solutions.



| nD | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E1 | D | D | D | D | D | D |  | A | A | N | N | N |  |  | D | D | D | D | D | D |  | A | A | A | A |
| E2 | N | N | N | N | N |  |  | D | D | D | D | D | D |  | A | A | N | N | N |  |  | D | D | D | D |
| E3 | N | N |  |  |  |  | N | N | N | N | N | N |  |  | D | D | D | D | D | D |  | A | A | N | N |
| E4 |  |  | A | A | A | A | N | N | N |  |  |  |  | N | N | N | N | N | N |  |  | D | D | D | D |
| E5 | A | A | A | A | A | A |  |  |  | A | A | A | A | N | N | N |  |  |  |  | N | N | N | N | N |
| E6 | D | D | D | D | D | D |  | A | A | A | A | A | A |  |  |  | A | A | A | A | N | N | N |  |  |
| E7 | A | A | N | N | N |  |  | D | D | D | D | D | D |  | A | A | A | A | A | A |  |  |  | A | A |
| D | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 |
| A | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 |
| N | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 |

| nD | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | D | A | N |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E1 | A | A |  |  |  | A | A | A | A | N | N | N |  |  |  |  | N | N | N | N | N | N |  |  | 12 | 12 | 12 |
| E2 | D | D |  | A | A | A | A | A | A |  |  |  | A | A | A | A | N | N | N |  |  |  |  | N | 12 | 12 | 12 |
| E3 | N |  |  | D | D | D | D | D | D |  | A | A | A | A | A | A |  |  |  | A | A | A | A | N | 12 | 12 | 12 |
| E4 | D | D |  | A | A | N | N | N |  |  | D | D | D | D | D | D |  | A | A | A | A | A | A |  | 12 | 12 | 12 |
| E5 | N |  |  | D | D | D | D | D | D |  | A | A | N | N | N |  |  | D | D | D | D | D | D |  | 12 | 12 | 12 |
| E6 |  |  | N | N | N | N | N | N |  |  | D | D | D | D | D | D |  | A | A | N | N | N |  |  | 12 | 12 | 12 |
| E7 | A | A | N | N | N |  |  |  |  | N | N | N | N | N | N |  |  | D | D | D | D | D | D |  | 12 | 12 | 12 |
| D | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 |  |  |  |
| A | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 |  |  |  |
| N | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 |  |  |  |

Figure 8.1: IP solution for Ex6



| nD | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E1 |  |  |  | D | D | D | N | N | N | N |  |  |  | N | N | N | N | N | N |  |  | D | D | D | D |
| E2 | D | D | D | A | A | A |  |  |  |  | D | D | D | N | N | N | N |  |  |  | N | N | N | N | N |
| E3 | A | A | A | N | N |  |  | D | D | D | A | A | A |  |  |  |  | D | D | D | N | N | N | N |  |
| E4 | A | A | A | A | A | A |  | A | A | A | N | N |  |  | D | D | D | A | A | A |  |  |  |  | D |
| E5 | D | D | D | D | D | D |  | A | A | A | A | A | A |  | A | A | A | N | N |  |  | D | D | D | A |
| E6 | N | N | N | N | N |  |  | D | D | D | D | D | D |  | A | A | A | A | A | A |  | A | A | A | N |
| E7 | N | N | N |  |  |  | N | N | N | N | N | N |  |  | D | D | D | D | D | D |  | A | A | A | A |
| D | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 |
| A | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 |
| N | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 |

| nD | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | D | A | N |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| E1 | D | D |  | A | A | A | A | A | A |  | A | A | A | N | N |  |  | D | D | D | A | A | A |  | 12 | 12 | 12 |
| E2 | N |  |  | D | D | D | D | D | D |  | A | A | A | A | A | A |  | A | A | A | N | N |  |  | 12 | 12 | 12 |
| E3 |  |  | N | N | N | N | N | N |  |  | D | D | D | D | D | D |  | A | A | A | A | A | A |  | 12 | 12 | 12 |
| E4 | D | D | N | N | N | N |  |  |  | N | N | N | N | N | N |  |  | D | D | D | D | D | D |  | 12 | 12 | 12 |
| E5 | A | A |  |  |  |  | D | D | D | N | N | N | N |  |  |  | N | N | N | N | N | N |  |  | 12 | 12 | 12 |
| E6 | N |  |  | D | D | D | A | A | A |  |  |  |  | D | D | D | N | N | N | N |  |  |  | N | 12 | 12 | 12 |
| E7 | A | A |  | A | A | A | N | N |  |  | D | D | D | A | A | A |  |  |  |  | D | D | D | N | 12 | 12 | 12 |
| D | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 |  |  |  |
| A | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 |  |  |  |
| N | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 | 2 | 2 | 2 | 2 | 2 | 0 | 2 |  |  |  |

Figure 8.2: Benchmark solution for Ex6

## 8.5 Conclusions

In this chapter a set of 20 benchmark instances was used to demonstrate the consistency and wide scope application of the general model proposed in Chapter 4. These instances vary in size, shift daily requirements and also in the types of shift sequences. The general model was adjusted in order to accommodate a new perspective on the sequence constraints. Instead of having a single allowable sequence of shifts and days-off, the new problems imposed a set of forbidden sequences. This means allowing for multiple feasible sequences. The adaptation of the sequence constraints (Eqs. 4.9) was explained in detail in Section 8.2. Once more, the versatility of the proposed formulation is evident from this application. The optimization model found the optimal solution for 12 problems, some of them of very large dimension, which corresponds to a success rate of 60% of the total number of problems considered. The solutions were compared with the ones achieved by Musliu (2006). Although the exact matching of the objective function values, the schedules developed by both methods (IP and MC-T) differ in the patterns of shifts and days-off assigned. An example for Ex.6 is shown in Section 8.4.

| | | | | Time(sec.) | | |
|---|---|---|---|---|---|---|
| Ex. | $nD$ | $nT$ | $nS$ | IP | MC-T | FCS |
| 1 | 63 | 9 | 3 | 7.94 | 0.07 | 0.90 |
| 2 | 63 | 9 | 3 | 2.90 | 0.07 | 0.40 |
| 3 | 119 | 17 | 3 | 907.40 | 0.42 | 1.90 |
| 4 | 91 | 13 | 3 | 1.59 | 0.11 | 1.70 |
| 5 | 77 | 11 | 3 | 2.47 | 0.43 | 3.50 |
| 6 | 49 | 7 | 3 | 1.23 | 0.08 | 2.00 |
| 7 | 203 | 29 | 3 | - | 52.79 | 16.10 |
| 8 | 112 | 16 | 3 | 7.60 | 0.74 | 124.00 |
| 9 | 329 | 47 | 3 | - | 15.96 | - |
| 10 | 189 | 27 | 3 | - | 0.60 | 9.50 |
| 11 | 210 | 30 | 3 | - | 13.15 | 367.00 |
| 12 | 140 | 20 | 2 | 310.00 | 1.17 | - |
| 13 | 49 | 7 | 3 | 255.95 | 0.87 | - |
| 14 | 91 | 13 | 3 | 73.14 | 0.76 | 0.54 |
| 15 | 448 | 64 | 3 | - | 159.04 | - |
| 16 | 203 | 29 | 3 | 1923.00 | 0.54 | 2.44 |
| 17 | 231 | 33 | 2 | 29.64 | 2.16 | - |
| 18 | 371 | 53 | 3 | - | 6.83 | 2.57 |
| 19 | 840 | 120 | 3 | - | 75.83 | - |
| 20 | 1141 | 163 | 3 | - | 71.38 | - |

Table 8.4: Computational times for the benchmarking instances using the IP model, MC-T and FCS

# Chapter 9

# Heuristic approach

An optimization approach is typically limited in terms of performance for real-life large dimension problems. The challenge of developing a heuristic procedure naturally arose as a means of systematically overcoming that handicap and to compare results. In this chapter we propose a constructive heuristic to solve the problem of the glass unit addressed in Chapter 5. Section 9.1 describes the heuristic procedure, explaining the initial assumptions and the developed algorithms. Computational results for the glass unit problem are shown in Section 9.2 and a comparison of the performance of both approaches, heuristic and optimization, is carried out. In order to analyze the consistency of the heuristic, a set of computer generated instances was also used, varying in size with the number of teams and the number of shifts. The solutions generated for the glass unit problem are presented in Section 9.2. Section 9.3 draws some conclusions and reflexions for future extensions of this work.

## 9.1 Heuristic

### 9.1.1 Initial assumptions

From our previous experience with the optimization model, described in Chapter 5, we realized that a key issue that guarantees the existence of feasible solutions is the availability of a sufficient number of breaks, to allow for an adequate sequence of working days and days-off for each team. This break availability condition is on the basis of the proposed heuristic. When the condition is valid, for a set of input parameters, the heuristic defines a feasible schedule for the first team, which will be afterwards replicated for the remaining teams, with a time lag or offset days in between. In the development of the first team's schedule, working-day blocks are assigned following the sequence of the shifts in which the teams must work. The length of the working blocks is limited by the minimum and maximum numbers of allowable consecutive working days. It may occur, therefore, that all working blocks have the same length, i.e. the same number of consecutive days, or that blocks have different lengths. Between each two consecutive working blocks, i.e., between each change of working shifts, there must be at least one break day. The heuristic assures that blocks of working days plus breaks have always the same length. Therefore, in the case of a schedule with working-day blocks with different lengths, more than one break is assigned after the working blocks with shorter lengths, in order to have a block of working days plus breaks with the same length as the block with the maximum length plus one break. The heuristic tests different combinations of working blocks until a feasible solution is reached. These procedures are next explained in detail.

### 9.1.2 Algorithm 1 - *checkAvailableBreaks*

The first step of the heuristic is the verification of the available breaks condition, described in Algorithm 1 - *checkAvailableBreaks*, where:

$nD$ is the number of days in the planning period;

$nT$ is the number of teams;

$nS$ is the number of working shifts;

$minD$ is the minimum required number of consecutive working days;

$maxD$ is the maximum allowed number of consecutive working days;

$w_d$ is the number of working days of the planning period to be assigned to each team, for each shift;

$n_{blocks}$ is the number of blocks of working days, for each team and for each shift;

$maxBlock$ is the block of working days with the maximum length to be considered;

$F_{tot}$ is the total number of available breaks in the planning period, for each team;

$F_{min}$ is the minimum number of required breaks in the planning period, for each team, considering the required number of working-day blocks.

The algorithm first calculates the values for $F_{tot}$ and $w_d$. If, in each day, there are 3 shifts to be assigned to exactly 3 teams, the remaining 2 teams are necessarily assigned to breaks. So, 2 breaks are available in each day. These 2 breaks multiplied by the number of days in the planning period ($nD$) and divided by the number of teams ($nT$) result in the total number of available

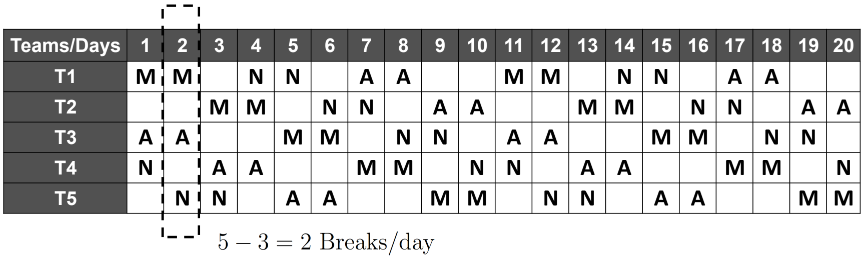
**Algorithm 1** *checkAvailableBreaks*

```
$F_{tot} \leftarrow nD * (nT - nS)/nT$
$w_d \leftarrow (nD - F_{tot})/nS$

for $i = 0 \rightarrow (minD + i)$ do
    if $w_d$ is a multiple of $(minD + i)$ then
        $n_{blocks} \leftarrow w_d/(minD + i)$
        $F_{min} \leftarrow n_{blocks} * nS$
        if $F_{min} \leq F_{tot}$ then
            $solutionExists$
            $maxBlock \leftarrow (minD + i)$
            $equalBlocks$
            Exit
        end if
    end if
end for
  $testBlocksCombination()$

if $solutionExists$ then
    Exit
else
    $noFeasibleSolutionAssured$
    Exit
end if
```

breaks per team, $F_{tot}$. This figure represents the maximum number of breaks the procedure has available to assign to each team, and must be enough to ensure the mandatory minimum number of breaks to assign between shift changes. Taking $F_{tot}$ out of $nD$ and dividing it by the number of shifts ($nS$) we get the number of working days, in each shift, to assign to each team, $w_d$. Figures 9.1, 9.2 and 9.3 illustrate the reasoning of these calculations for $nD = 20$ days, breaks are represented by the blank cells. Algorithm 1 proceeds by checking the possibility of assigning only working blocks with equal length: $(minD + i)$. Looking into the same example of $nD = 20$ days, with $minD = 2$ days and $maxD = 4$ days, the number of days in each of the working blocks is achieved for $i = 0$, thus $w_d/2 = 2$. When that is not feasible, the function $testBlocksCombination()$ is called and combinations of working blocks with different lengths are evaluated. This is the case illustrated in Fig. 9.4, for a planning period of 25 days. In this scenario, $w_d = 5$ days, which is not divisible for any $minD + i$, for any $i$. Therefore, a combination of blocks of $minD$ and $minD + 1$ days, 2 days and 3 days respectively, is used. If no combination of blocks satisfies the condition $F_{min} \leq F_{tot}$, then the problem may not have a feasible solution. Consequently, the following statement can be made:

> *If the input parameters selected verify the condition: $F_{min} \leq F_{tot}$, then it has a feasible solution. Otherwise, it may have or not a feasible solution.*

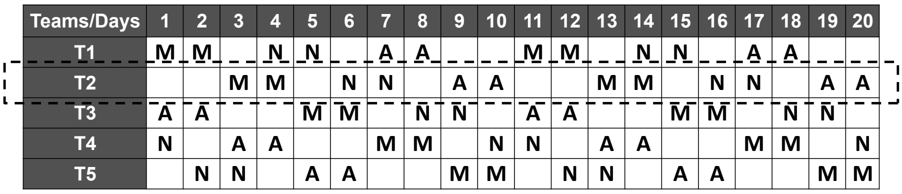
This condition is thus sufficient but not necessary.



| Teams/Days | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  |
| T2 |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |
| T3 | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  |
| T4 | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N |
| T5 |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |

$5 - 3 = 2$ Breaks/day

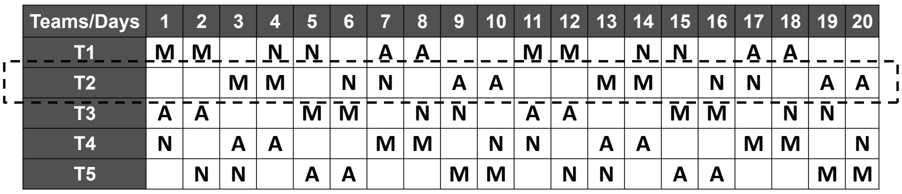
Figure 9.1: Example: calculation of the number of available breaks/day.



| Teams/Days | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  |
| T2 |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |
| T3 | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  |
| T4 | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N |
| T5 |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |

$$\frac{2 \times 20}{5} = 8 \text{ Breaks/team}$$

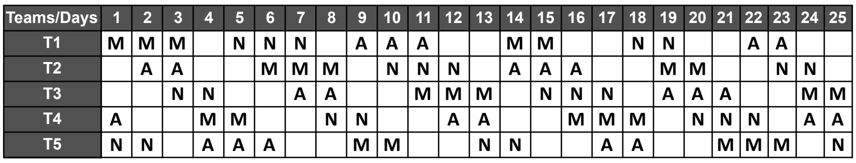
Figure 9.2: Example: calculation of the number of available breaks/team.



| Teams/Days | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  |
| T2 |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |
| T3 | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N | N |  |
| T4 | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |  | N |
| T5 |  | N | N |  | A | A |  |  | M | M |  | N | N |  | A | A |  |  | M | M |

$$w_d = \frac{20-8}{3} = 4 \text{ Working days/shift/team}$$

Figure 9.3: Example: calculation of the number of working days/shift/team.



| Teams/Days | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| T1 | M | M | M |  | N | N | N |  | A | A | A |  |  | M | M |  |  | N | N |  |  | A | A |  |  |
| T2 |  | A | A |  |  | M | M | M |  | N | N | N |  | A | A | A |  |  | M | M |  |  | N | N |  |
| T3 |  |  | N | N |  |  | A | A |  |  | M | M | M |  | N | N | N |  | A | A | A |  |  | M | M |
| T4 | A |  |  | M | M |  |  | N | N |  |  | A | A |  |  | M | M | M |  | N | N | N |  | A | A |
| T5 | N | N |  | A | A | A |  |  | M | M |  |  | N | N |  |  | A | A |  |  | M | M | M |  | N |

Figure 9.4: Example: solution for nD=25 days.

### 9.1.3 Algorithm 2 - *generateSchedule*

If the condition is met we propose a constructive heuristic to build a feasible solution, as described in Algorithm 2 - *generateSchedule*. It starts by checking the relation between the input parameters $nD$ and $nT$. The procedure only proceeds when $nD$ is a multiple of $nT$. Then the function *checkAvailableBreaks* is called in order to verify if there are enough available breaks to guarantee the existence of a feasible solution, as explained before. After this condition is fullfiled, the schedule is generated, either with only working blocks with the same length, through the function *generateEqualBlocksSchedule* or combining working blocks with different length, calling the function *generateCombinationBlocksSchedule*.

### 9.1.4 Algorithm 3 - *generateEqualBlocksSchedule*

The pseudo code of *generateEqualBlocksSchedule* is described in Algorithm 3. The idea is to build a feasible solution for the first team and then replicate it for the remaining $(nT - 1)$ teams. In this scenario, all working

**Algorithm 2** *generateSchedule*

```
if $nD$ is not a multiple of $nT$ then
    Msg: "Change input parameters"
end if
$checkAvailableBreaks$

if solutionExists then $initializeSchedule$
    if $equalBlocks$ then
      $generateEqualBlocksSchedule$
    else
      $generateCombinationBlocksSchedule$
    end if
else if $noFeasibleSolutionAssured$ then
    Msg: "$F_{min} > F_{tot}$, the problem may not have a feasible solution!"
end if
```

blocks have the same length of $maxBlock$ days and are separated by one break. The procedure begins by assigning the first block of the first working shift to all teams, with a time lag, $\delta D$, equal to $minD$ days between them, if $maxBlock \leq minD$ or, otherwise, equal to $nD/nT$ days. It proceeds by assigning the first blocks of the remaining shifts to the first team, separated by a break. In order to satisfy the shift daily demand, which forces each working shift to be assigned to only one team in each day of the planning period, the second sub-period can only begin when the first shift is available again, and for $maxBlock$ consecutive days. This means that the second working block of the first shift can only be assigned to team 1 in a slot with, at least, $maxBlock$ consecutive days where there isn't any team assigned to that shift. The first sub-period is then replicated for the first team until the whole planning period is fulfilled, resulting in the schedule for the first team. This schedule is replicated for all the remaining $(nT - 1)$ teams, with a time lag of $\delta D$ days between them.

**Algorithm 3** *generateEqualBlocksSchedule*

```
  Use working blocks of $maxBlock$ days

  Assign the first working block of the first shift to team 1

  Use a time lag: $\delta D = minD$ or $nD/nT$ days to assign the first block of
  the first shift to all remaining $(nT - 1)$ teams

  Assign the first blocks of the remaining working shifts to team 1, separat-
  ing each block with one break

  Insert breaks at the end of the last shift block to wait until the first shift
  is available again, in order to close the first sub-period

  Replicate the first sub-period to the first team as many times as necessary
  to fulfill the planning period

  Replicate the schedule of the first team to the remaining $(nT - 1)$ teams,
  with a time lag of $\delta D = nD/nT$ days
```

### 9.1.5 Algorithm 4 - *generateCombinationBlocksSchedule*

Algorithm 4 presents the procedure *generateCombinationBlocksSchedule*. The process is similar to the one followed in the previously described function *generateEqualBlocksSchedule* procedure, but in this case, the working blocks can have different lengths: $maxBlock$ $(maxBlock - 1)$ and/or $(maxBlock - 2)$ days. The time lag $\delta D$ to be used between teams' schedules is $nD/nT$ days. The procedure first assigns the working blocks of $maxBlock$ days, then of $(maxBlock - 1)$ days and ends up with the assignment of blocks with $(maxBlock - 2)$ days, if applicable. An important aspect to take into account is the number of breaks to insert between working blocks, that must be 1 between $maxBlock$ blocks, 2 between $(maxBlock - 1)$ blocks and 3 between $(maxBlock - 2)$ blocks. This makes all blocks of (working days plus breaks) to have the same length, equal to $(maxBlock + 1)$ days. Again, we must ensure that no working shift is assigned to more than one team in each day, as already stated in Algorithm 3. This is achieved by inserting

**Algorithm 4** *generateCombinationBlocksSchedule*

```
Use a combination of working blocks of $maxBlock$, $(maxBlock-1)$ and/or $(maxBlock-2)$ days, if applicable
for $i = 0 \rightarrow 2$ do
    Assign the first $(maxBlock-i)$ working block of the first shift to team 1
    Use a time lag: $\delta D = nD/nT$ to assign the first $(maxBlock-i)$ block of the first shift to all remaining $(nT-1)$ teams
    Assign the first $(maxBlock-i)$ block of the remaining working shifts to team 1, separating each block with $(i+1)$ breaks
    Insert breaks at the end of the last shift block to wait until the first shift is available again, in order to close the first sub-period
end for
Replicate the first sub-period to the first team as many times as necessary to fulfill the planning period
Replicate the schedule of the first team to the remaining $(nT-1)$ teams, with a time lag of $\delta D = nD/nT$ days
```

breaks at the end of the last working block of the first sub-period until the first shift is available again for the second sub-period assignment. The first sub-period is then replicated to the first team as many times as necessary in order to complete the whole planning period, resulting in the schedule for the first team. This schedule is replicated to all the remaining $(nT-1)$ teams, with a time lag of $\delta D$ days between them.

## 9.2 Computational experiments and solutions

The heuristic was coded using VBA for Excel 2011 and ran on a server machine powered by 2 Intel® Xeon® processors of 2,4 GHz and 1,39 GHz, and with 2 GB RAM. Tests were made for both 4 and 5 teams of employees and planning periods ranging up to 365 days. Several combinations of input parameters: number of days in the planning period ($nD$) and time lag, or offset days ($\delta D$), were considered. Feasible solutions were consistently found, except for periods of 125 and 275 days, where the heuristic didn't find any combination of working-day blocks to build a feasible solution. When comparing with the solutions generated by the IP model, it is possible to conclude that the heuristic always reaches the optimal solution, in what concerns to the value of the objective function. However, the composition of the schedule is not the same in the solutions obtained by both methods. This can be explained by the fact that the IP model does not impose the same schedule structure that is naturally imposed by the heuristic procedure. Although the schedules are not exactly equal, the quality of the solutions in terms of work balance is not compromised since all teams work the same number of days, in each shift, along the planning horizon. Two examples are shown in Figs. 9.1 - 9.3 and 9.4, for planning periods of 20 and 25 days respectively.

Table 9.1 presents computational times for a set of different combinations of $nD$ and $\delta D$, which verify the integer relationship between them.

The heuristic reaches a feasible solution in less than 0.5 seconds for planning periods up to 180 days, and in less than a minute for 330 and 365 days, while the IP model takes longer times as the length of the planning period increases. For 330 and 365 days the execution of the IP model was stopped after 60000 seconds, without finding any feasible solution. A comparison between the computational times taken by both methods reveals that the heuristic consistently outperforms the optimization model.

The definition of the planning period is not a very explored issue in the literature, since it is closely related to demand forecast periods and it is

| Planning period | Offset | Execution time (sec.) | |
|---|---|---|---|
| (days) | (days) | Heuristic | IP |
| 25 | 5 | 0.29 | 0.48 |
| 30 | 6 | 0.32 | 0.45 |
| 35 | 7 | 0.27 | 0.57 |
| 60 | 12 | 0.39 | 1.55 |
| 70 | 14 | 0.37 | 26.86 |
| 90 | 18 | 0.39 | 3.32 |
| 150 | 30 | 0.43 | 384.63 |
| 180 | 36 | 0.47 | 110.84 |
| 210 | 42 | 0.52 | 332.56 |
| 270 | 54 | 0.69 | 18026.00 |
| 330 | 66 | 0.83 | – |
| 365 | 73 | 3.92 | – |

Table 9.1: Computational results for 5 teams

.

often an input parameter. But the initially set planning period may not be the one that gives the best solution and so, it is pertinent to study which is the "ideal" planning period for a specific instance. This experience was acquired in the tests carried out in this work. We tested planning periods from 15 to 365 days, with a 5 day-interval. The planning periods that better fitted the goals of this problem were 35 days for the winter period, using 5 teams, and 16 days for the summer period, using 4 teams, considering in this last case that the fifth team was on holidays. These solutions are shown in Figs. 9.5 and 9.6.

In order to analyze the robustness of the heuristic, additional tests were carried out, considering the variation of other input parameters, namely the number of teams ($nT$) and the number of shifts ($nS$). Given the constraint

| | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | Total | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Teams/Days | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 | 31 | 32 | 33 | 34 | 35 | M | A | N | |
| T1 | M | M | M | M | | N | N | N | N | | A | A | A | A | | | | | M | M | M | | | N | N | N | | | A | A | A | | | | | 7 | 7 | 7 | **21** |
| T2 | A | A | A | | | | | M | M | M | M | | N | N | N | N | | A | A | A | A | | | | | M | M | M | | | N | N | N | | | 7 | 7 | 7 | **21** |
| T3 | | | N | N | N | | | A | A | A | | | | | M | M | M | M | | N | N | N | N | | A | A | A | A | | | | | M | M | M | 7 | 7 | 7 | **21** |
| T4 | | | | | M | M | M | | | N | N | N | | | A | A | A | | | | | M | M | M | M | | N | N | N | N | | A | A | A | A | 7 | 7 | 7 | **21** |
| T5 | N | N | | A | A | A | A | | | | | M | M | M | | | N | N | N | | | A | A | A | | | | | M | M | M | M | | N | N | 7 | 7 | 7 | **21** |

Figure 9.5: Solution for nD=35 days.

| | | | | | | | | | | | | | | | | | | Total | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Teams/Days | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | M | A | N | |
| T1 | M | M | M | M | | N | N | N | N | | A | A | A | A | | | 4 | 4 | 4 | **12** |
| T2 | A | A | | | M | M | M | M | | N | N | N | N | | A | A | 4 | 4 | 4 | **12** |
| T3 | N | | A | A | A | A | | | M | M | M | M | | N | N | N | 4 | 4 | 4 | **12** |
| T4 | | N | N | N | N | | A | A | A | A | | | M | M | M | M | 4 | 4 | 4 | **12** |

Figure 9.6: Solution for nD=16 days.

that imposes that, on each day, each working shift must only be assigned to exactly one team, there is a ratio between $nT$ and $nS$ that must be kept reasonable. It is not realistic to significantly increase the number of teams without simultaneously increasing the number of shifts because, in that situation, there would be too many teams without any working shift assigned on each day and the problem would become too simple and senseless. Table 9.2 presents the computational results achieved for the 48 instances, and that were created having this reasoning in mind. The instances consider the division of each 8-hour daily working shift (M, A and N) in several work stations. In this scenario, $nS$ refers to the number of work stations. Each team of employees must cover all work stations, following the predefined sequence of work shifts: M-N-A, and a break day between each shift change must still be guaranteed. Whenever a team returns to the same work shift, it moves on to the next work station. Figure 9.7 illustrates an example of this process for 9 work stations, 3 for each work shift.

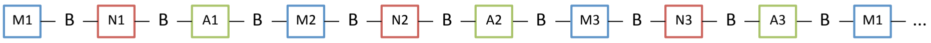


Figure 9.7: Example: sequence for 9 work stations ($nS$=9).

Following this reasoning, the values for $nS$ were set considering always multiples of 3, up to 30. $nT$ varies from 4 to 50 teams and $nD$ from 16 up to 152 days. In terms of the ratio between $nS$ and $nT$, tests were made for $nS/nT = 0.60, 0.70, 0.75$ and 0.80. However, not all of these values can be verified for all $nS$. For example, for $nS = 3$ it is not possible to have $nS/nT = 0.8$, since $3/4 = 0.75$ and it is not possible to have simultaneously $nS = 3$ and $nT = 3$, since there must be breaks to interpose between the working blocks. The closer the ratio $nS/nT$ gets to 1, the tighter the problem becomes, as there are few breaks compared to the number of teams. Again, it is possible to see the robustness and consistency of the heuristic results, which fall below 5 seconds, even for a large number of teams. The IP results, on the contrary, reveal an unsteady performance and, for instances with more than 30 teams, it does not find any feasible solution at all, within 60000 seconds of running time.

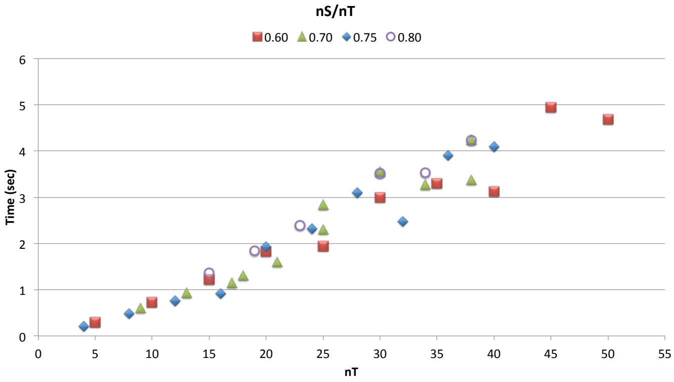


Figure 9.8: Computational results of the heuristic for different values of the ratio $nS/nT$ according to the variation of $nT$.

Figures 9.8 and 9.9 illustrate the performance of the heuristic, in terms of execution times, for the four different values of the ratio $nS/nT$ that were tested, in relation to the variation of $nT$ and $nS$ respectively. It is clear that the execution times smoothly increments with the number of teams, or the number of shifts, increases. As the maximum tested value for $nS$ was 30, the scenarios with 45 and 50 teams are only viable for $nS/nT = 0.60$, which explains why the two single red squares on the right side of the graph

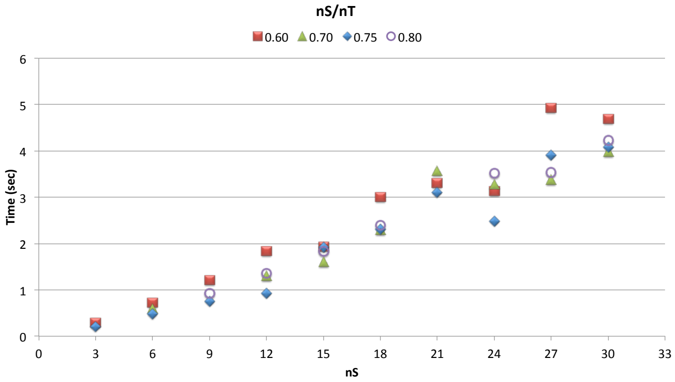


Figure 9.9: Computational results of the heuristic for different values of the ratio $nS/nT$ according to the variation of $nS$.

of Fig. 9.8 have no correspondence in the other three data series. It is not useful to graphically compare the results of the heuristic with the results of the IP model because the scales are not compatible, as shown by Table 9.2. Again, although both methods reach solutions with the same value of the objective function, i.e., the maximum number of days that a team works in each shift, the composition of the schedule is not the same in the solutions obtained by the heuristic and the IP model.

| No. | Planning | No. | Execution time (sec.) | |
|---:|---:|---:|---:|---:|
| shifts | period (days) | teams | Heuristic | IP |
| 3 | 16 | 4 | 0.20 | 0.36 |
| 3 | 36 | 4 | 0.25 | 0.89 |
| 3 | 25 | 5 | 0.29 | 0.48 |
| 3 | 30 | 5 | 0.32 | 0.45 |
| 3 | 35 | 5 | 0.27 | 0.57 |
| 6 | 32 | 8 | 0.39 | 1.88 |
| 6 | 24 | 8 | 0.48 | 1.31 |
| 6 | 36 | 9 | 0.59 | 2.16 |
| 6 | 30 | 10 | 0.72 | 1.94 |

Continued on next page

| No. | Planning | No. | Execution time (sec.) | |
|---|---|---|---|---|
| shifts | period (days) | teams | Heuristic | IP |
| 6 | 40 | 10 | 0.67 | 2.66 |
| 6 | 50 | 10 | 0.56 | 5.38 |
| 6 | 60 | 10 | 0.78 | 8.42 |
| 9 | 48 | 12 | 0.67 | 7.92 |
| 9 | 36 | 12 | 0.75 | 3.58 |
| 9 | 39 | 13 | 0.93 | 63.00 |
| 9 | 30 | 15 | 1.22 | 3.95 |
| 12 | 60 | 15 | 1.36 | 19.38 |
| 12 | 64 | 16 | 1.23 | 2718.00 |
| 12 | 48 | 16 | 0.92 | 20.70 |
| 12 | 51 | 17 | 1.14 | 9.22 |
| 12 | 36 | 18 | 1.30 | 8.88 |
| 12 | 40 | 20 | 1.83 | 9.89 |
| 15 | 76 | 19 | 1.84 | 2666.30 |
| 15 | 80 | 20 | 1.41 | 7095.00 |
| 15 | 60 | 20 | 1.92 | 1435.00 |
| 15 | 63 | 21 | 1.60 | 35.00 |
| 15 | 50 | 25 | 1.94 | 21.86 |
| 18 | 92 | 23 | 2.39 | 219.02 |
| 18 | 72 | 24 | 2.31 | 7132.10 |
| 18 | 96 | 24 | 1.83 | 29024.00 |
| 18 | 100 | 25 | 2.30 | – |
| 18 | 75 | 25 | 2.84 | 1558.40 |
| 18 | 60 | 30 | 3.00 | 53.66 |
| 21 | 84 | 28 | 3.10 | 4252.70 |
| 21 | 90 | 30 | 3.56 | – |

Continued on next page

| No. | Planning | No. | Execution time (sec.) | |
|---:|---:|---:|---:|---:|
| shifts | period (days) | teams | Heuristic | IP |
| 21 | 70 | 35 | 3.30 | 7086.50 |
| 24 | 120 | 30 | 3.52 | 14164.00 |
| 24 | 96 | 32 | 2.48 | – |
| 24 | 102 | 34 | 3.27 | – |
| 24 | 80 | 40 | 3.13 | – |
| 27 | 108 | 36 | 3.91 | – |
| 27 | 136 | 34 | 3.53 | – |
| 27 | 114 | 38 | 3.38 | – |
| 27 | 90 | 45 | 4.94 | – |
| 30 | 152 | 38 | 4.23 | – |
| 30 | 120 | 40 | 4.09 | – |
| 30 | 129 | 43 | 3.98 | – |
| 30 | 100 | 50 | 4.69 | – |

Table 9.2: Computational results for a set of combinations of the ratio $nS/nT$.

## 9.3 Conclusions

In this chapter a new constructive heuristic was proposed for solving the staff scheduling problem of the glass manufacture unit introduced in Chapter 5. The developed procedures were described and a comparison of the results achieved by both heuristic and optimization approaches was presented, highlighting a consistent outperformance of the heuristic over the optimization approach, with all results falling below 5 seconds.

An important and novel contribution of this work is the approach introduced with Algorithm 1 - *checkAvailableBreaks*. With some simple calculations over the problem's input data, it is possible to foresee the existence of a feasible solution. Although the proposed heuristic was developed for this specific instance, it is flexible enough to account for variations in some of the problem's parameters and constraints. That is the case of the sequence of working shifts, which can be any that the user defines, as well as the minimum and maximum number of consecutive working days. With slight adjustments, the number of breaks to interpose between working blocks can also be redefined. Nevertheless, the proposed procedure has limitations when considering its direct applicability to other instances. It imposes the existence of at least one break day between each change of working shifts, so it does not allow for the possibility of having different shifts on consecutive days. The shift daily demand, for example, is a very strong constraint of this particular problem since it is on the basis of the calculation of the number of available breaks, as it was previously explained, which is a conditioning factor for the existence of a feasible solution. In problems with more than one team working simultaneously on the same shift, or with different daily requirements for each shift, this constraint would have to be reformulated and would imply deeper adjustments in the procedures proposed, but the base reasoning would be the same. As stated before, the "working-shifts sequence" approach is a strength of this work, for problems where a unique predefined sequence must be followed. But it can also be a limitation because, in problems where a set of sequences should be avoided, the heuristic may not respond accordingly, unless a preferred sequence can be chosen.

As a conclusion, we believe that the achieved results are promising and encouraging of further extensions of the heuristic in order to consider, for example, different shift daily demands or forbidden sequences of shifts.

# Chapter 10

# Conclusions

## 10.1 Contributions of this work

In this thesis, we proposed an optimization method for simultaneously assigning work shifts and days-off to each employee. A general IP model was developed and applied, with slight adjustments, to three real-world problems: a glass production unit, a continuous care unit and a hospital. A set of benchmark instances was also used in order to evaluate the model's performance when solving larger problems and to compare results with other methods. Two main goals of the model were to ensure a balanced and equitable schedule between all employees, in terms of workload, and also to respect a predefined sequence of work shifts and days-off, either following work rules or employees' preferences. The first goal was achieved, initially, through the levelling of the number of days that each team works in each shift, as imposed by the objective function defined for the general model and in a next phase, through the imposition of equal schedules to all employees, with a time lag, of a predefined number of days, between them. This feature gives a cyclic dimension to the schedule. The second condition was achieved through the formulation of an array of indices that, together with the definition of a maximum and a minimum number of consecutive days, enable the imposition of any desired pattern of work shifts and days-off. This pattern can be the same for all employees or can be defined according to contract types, skills, employees preferences, etc. This original formulation also makes it possible to control the periodicity of days-off, as well as the length of the tour or sub-period of the planning horizon. The definition of the planning period is not a very explored issue in the literature, since it is closely related to demand forecast periods and it is often an input parameter. But the initially set planning period may not be the one that better fits the problem's features and so it is pertinent to study which is the "ideal" planning period for a specific instance. This experiment was carried out. Even though constraints on the periodicity of long-weekends and on planned absences were not an initial issue, the model was able to handle them as well, as shown in two of the case-studies.

The model developed in this work demonstrated to be general and flexible, with several degrees of freedom and with the capacity of being easily applied to different real-life staff scheduling problems, but at the same time with a cyclic feature that ensures the equitableness and predictability of the schedule. The cyclic approach, often considered to be inflexible and not easily adjustable, proved to be flexible enough to successfully solve problems that are typically addressed with acyclic scheduling, namely with heterogeneous staff and fluctuating demand levels. This is a new insight and represents a novel contribution to the academic literature.

From a company's point of view, the use of the automatic scheduling model proposed in this research work can represent a powerful tool for increasing both the efficiency and the effectivity of the staff scheduling process, leading to higher profitability and productivity. However, the implementation of such a solution into practice is not always easy, it deeply depends on the involvement of the company in the whole development process.

The developed heuristic approach represents an alternative method for solving one of the real-world problems studied in this thesis, allowing also for a comparative evaluation of the optimization model's performance. An original contribution of the heuristic is that with some simple calculations over the problem's input data, it is possible to foresee the existence of a feasible solution. This reduces the solution search space.

Additionally, the comprehensive study on the staff scheduling problem and the insight into hospitality management operations constitute two assets for researchers looking for background on these topics.

As a conclusion, we proposed a generic, novel and valuable approach to staff scheduling. We developed generic methodologies, showed their flexibility and solved a set of different problems. We challenged the potential of cyclic scheduling and proved it can be flexible. We developed an innovative formulation of sequence and consecutiveness of shifts. And we believe this research work can add value to a company by leading to cost reduction and an increase in the productivity.

## 10.2 Future research directions

In order to consolidate our findings, future research could address the application of the proposed IP model to more real-world problems, from different activity sectors. Hospitality management is a promising area that should be more explored, namely hotels (housekeeping staff) and restaurants.

Concerning the problems' features, all the problems studied in this work had fixed shifts. It would be interesting to extend the IP model to consider the case of variable shifts, in terms of starting-times, length or even the placement of breaks.

One of the drawbacks that are usually pointed out in the literature to cyclic scheduling approaches is their difficulty in handling non predicted absences. In Chapter 5 absentees were replaced within their team. We have also shown how the IP model can ocasionally accommodate planned absences (Chapter 7). A systematic way of addressing this constraint could be worthy of further research.

Although the proposed heuristic was developed for a specific problem, it proved to be flexible enough to account for variations in some of the problem's parameters and constraints. The achieved results encourage further extensions of the procedure in order to consider, for example, different shift daily demands or forbidden sequences of shifts.

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