A dynamic setting for understanding tax evasion

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Biographical Note

Miguel Borges was born in 17 of August of 1992 in Oporto. He frequented the high school in Santa Maria da Feira where he finished his graduation in 2010.

After that, he frequented an engineering degree for one year but he decided to change for a bachelor degree in management in the end of that year. He entered FEP in 2011 for the management degree and graduated in 2014. Then he joined the Master in Finance, in 2014, and he is currently finishing this master degree.

From January 2016 until April 2016, he worked as an assurance assistant in EY and this was the first working experience in the area. From July 2016 until the present he is working at Adidas group as a retail accountant assistant.

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Abstract

Tax evasion is a much discussed topic in modern finance with some scandals involving well-known firms. Also we know that tax has a great impact in the money collected by government and so the economy and GDP of each country is much affected by taxes. The capability to understand the dynamics of tax evasion and to control tax evasion by the governments can have huge impact in the economy.

We observe that firms have the perception that the context of a country will incentivize them to engage in tax evasion. We studied the dynamics behind tax evasion on firms, finding a gap in literature involving this tax evasion on firms and linking that gap with a real options approach.

We used a real options model with two different stages in order to incorporate several variables that influence the incentives of firms to tax evasion and we also incorporate uncertainty in our model. We reach a critical value for which a firm is incentivized to enter the stage of tax evasion and we make some important conclusions about the dynamics of tax evasion.

Key-words: Real options; Tax evasion.

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1. Introduction

"*When there is an income tax, the just man will pay more and the unjust less on the same amount of income*" – Plato, The Republic, Book I, 343-D

In this dissertation we try to fill the gap between tax evasion related to firms. Despite the several previous works about tax evasion most of them do not refer the importance of this possibility in firms' decisions. Using a real options approach we developed a model that tries to reach a value where a company is incentivized to hide some profits in order to pay fewer taxes. These real options model incorporates several variables that influence the decision of the firm to tax evade like level of cash flows of the firm at the moment, the tax rate of the country, the penalty paid to authorities in case of being caught and the probability of being caught by fiscal authorities or efficiency of fiscal authorities. The uncertainty of cash flows is also incorporated in our real options model, so that way our model is applicable in cases where the cash flows are totally certain or part of them is not.

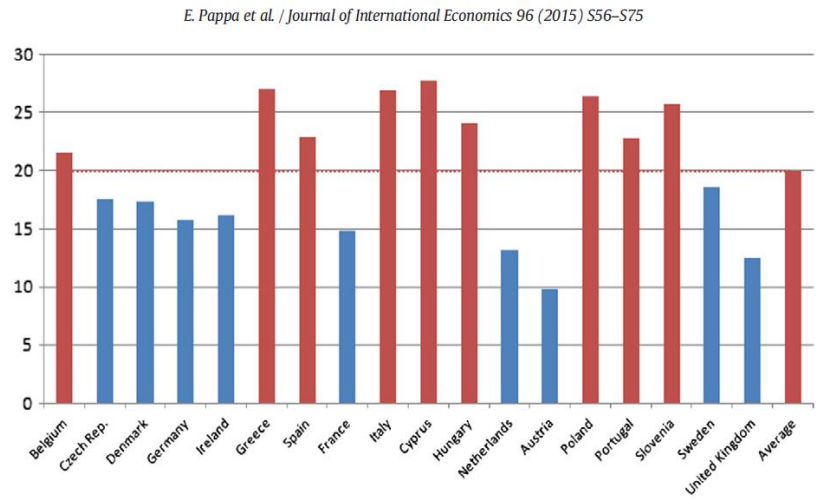
The main aim of this dissertation is to investigate the main drivers of tax evasion by firms. With the model that we developed we were able to anticipate the values for which a company will have incentives to hide some profits by hiding sales or declaring more costs than they had in reality. With comparative analysis we also could see the effects of changing some variables in the critical value for which the manager of the firm would decide to hide profits.

Tax evasion is a practice where individuals or firms tend to intentionally pay fewer taxes than they should, these practice is illegal and there are many ways to do it. Individuals or firms can reduce their tax obligations by underreporting incomes, overstating costs, deductions or credits, by failing to file appropriate tax returns or even by engaging in barter to avoid taxes. These actions can be taken both by individuals or firms in different ways but with the same objective of reducing the level of taxes to be paid. Tax avoidance is sometimes assumed as the same thing as tax evasion but it is not. Tax avoidance is legal and the use of legal means to have tax benefits or lower the tax obligations by the use of deductions, subsidies or credits.

The financial and fiscal crisis along with the fiscal consolidations that some countries took part of, brought some themes to discussion about the macroeconomic problems in the society. However, tax evasion and corruption were not seen as a big issue for the economic growth of countries. For example, recent studies in Spain and Greece showed that tax evasion and corruption increased on those two countries that adopted consolidation policies to improve their economy. The technical staff of Spanish Finance Ministry[1] indicated that shadow economy increased 6.8 p.p between 2008 and 2012, reaching 24,6% of GDP, for example.

In figure 1, we can see how the impact of shadow economy in Europe. We can see that there are some countries where the shadow economy is above 20% of GDP, so this can have a great impact in the economy of a country. In Figure 2 we can see the control of corruption in Europe measured in an Index. Also we can see how corruption affects the economy of countries. Those figures show us how the shadow economy and corruption can influence the economy of a country and distort the incoming of an economy.

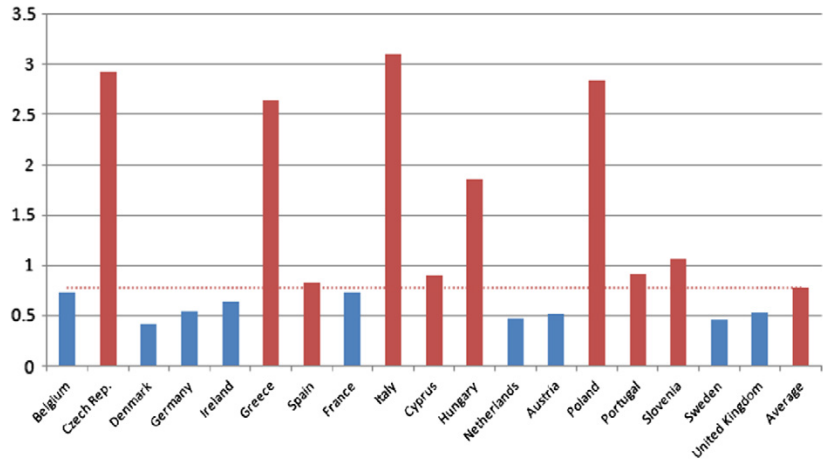
Figure 1 - Shadow Economy in Europe.



Shadow Economy (% GDP), Average over 1999-2010
Source: Schneider and Buehn (2012).

[1] http://www.gestha.es/archivos/actualidad/2014/2014-01-29_InformePrensa_EconomiaSumergida.pdf

Figure 2 - Control of Corruption Index in Europe.



Control of Corruption Index, Average over 1998-2010
Source: World Bank Global Governance Indicators.
Note: The dotted line indicates the average for the countries considered.

Two greek authors also referred the impact of tax evasion in greek recent crisis, they mentioned that the lack of capability of governments to provide public goods with the quality expected according to the level of taxes created a "legitimization" of tax evasion. During the last decades there have been lots of scandals involving big companies and tax evasion or corruption.

Along with the effect of the tax evasion the scandals involving tax evasion and tax avoidance have raised many questions in public opinion and lots of investigations were opened against some firms. One example of that is the FBI investigation concerning UBS switzerland bank. In that case the bank offered their clients offshore financial vehicles in order to evade taxes. The swiss bank was investigated in three diferent countries, Germany, Belgium and France, and already had to pay a penalty.

Several authors tend to agree that more than financial penalties, moral costs are very important to punish a firm or individual that choose to tax evade. Some studies demonstrated that with the same level of taxes and policies, different countries have different levels of tax evasion and the reason pointed out was those moral and reputation costs. Despite that ,financial penalties and policies to increase the probability of caught tax evaders are important and can reduce the tax evasion level.

Using a theme that generates much discussion and having found a gap in literature, we for the first time, developed a real options model to try to understand the dynamics of tax evasion. Linking real options with tax evasion brought us interesting results about how firms react to different inputs and the expected actions that they could take. Our model finds a critical value where the firm, at some moment and some level of cash flows, will have incentives to understate profits and pay fewer taxes. We study the possibility of having different values for multiple variables to explore the possibilities for the firm and see the reaction on the trigger value.

We find that the higher the corporate tax rate, the most a firm will have incentives to tax evade. This is intuitive but with this model we were able to extract some analitical proof of this. As higher the corporate tax rate, the firm will have to pay more to government and that way will have less profits. If the firm has the option to understate profits in someway in order to pay fewer taxes, this part of undeclared profits that will not be taxed will result in a gain for the firm. Also we find out that the more the possibility of understating profits, the sooner a company will have incentives to enter the tax evasion stage. These results of our model confirmed our intuition about this, if a firm has more chance to understating profits they will have higher gains from this activity so they will have bigger incentive to enter this stage. The level of penalty and the probability of being caught or efficiency of fiscal authority are inversely related with the trigger value which means that the higher these two variables, the later a firm will have incentives to tax evasion. These were the expected results because if the fiscal authority is more efficient the firms that choose to tax evade will face higher probability of being discovered and have to pay a penalty along with the moral damage that this will represent for the company. In the same way if the penalty is high enough the firm will have less incentives to enter the tax evasion stage once if the fiscal authorities detect the ilegal activity, the firm will have to pay more to government. One of the most interesteing results was the relation between volatility of cash flows and the trigger value, has higher the volatility is the later a firm will have incentives to tax evade. From our perspective, this makes sense because if a manager could not antecipate with assurance the level of cash flows it will be harder to make a proper decision about enter the stage where the firm is tax evading.

The dissertation is divided in four more chapters. In the next we present the literature review for the topic, then we move for the setup of the model and the comparative statistics. We finish with a numerical example of our model and the conclusions of the dissertation.

2. Literature Review

In 1973, Black and Scholes (1973) have developed a model for valuing both call and put options. They concluded that markets did not reflect the prices predicted by the formula. Until today, this formula is still used to value financial options as well as real options. The term real option was first introduced by Stewart C Myers (1977). This author in his paper compared some assets with call options. The author pointed out some problems with simple models to value corporate borrowing like the one from MM, because they didn't account with the difference between assets in place and future investments. They draw some conclusions like uncertainty is greater with future opportunities, future opportunities can only be available if present investments are made and to have the option in future investments sometimes there is a need to pay more. Also the same author stated that financial theory has been wrong or misapplied in some situations. DCF model is similar to NPV rule and has been used to value growth opportunities with options, abandoning options "out of the money" that needed to be paid to keep or exercising options in wrong timings.

Real options approach should be used to value some growth opportunities that include options, because this approach includes the flexibility on the investment opportunity. (Stewart C. Myers, 1984). Also Trigeorgis (1996) referred the importance of using real options to incorporate flexibility in capital budgeting. The author says that real investments can be viewed as multiple options that interact with competitiveness and strategy. McDonald and Siegel (1986) studied the optimal timing to invest in a project with irreversible costs and where cash flows and investments costs follow a continuous-time stochastic process (gBm process). They concluded that can be valuable to wait until a certain trigger and invest when the cash flows are twice the investment costs, this would be the optimal timing to invest. "Investment under Uncertainty" brought a deeper look to options approach including irreversibility, flexibility and uncertainty in investment decisions. Dixit and Pindyck (1994), developed in this book many models with different variables where they explain the importance of real options to value projects. So, the investor should evaluate the possibility of alternative outcomes regarding the outcome that he is expecting and assess the optimal timing to invest.

Slemrod (2007) wrote a paper where he talks about the presence of tax evasion since the Romans. He also refers to Baumeister (1982) when he stated the vulnerability of self-reports to underreporting. In their paper Tanzi and Shome (1993) referred the universality of tax evasion. The tax gap was ignored in many countries but the governments of several countries are more concerned in reducing tax evasion instead of raising tax rates. Tax evasion could be practiced in different forms like: not declaration of income, under or overreporting of income or costs, smuggle goods or assets and others, as the authors refer. Hines Jr and Rice (1990) developed a paper where they explore the possibility of large US corporations use tax havens to shift their profits and pay lower taxes. This leads to a lower tax collection by the US as the companies choose to reallocate some of the profits to other countries.

Sandmo (2005) gives us an insight about the history of tax evasion. He considers that tax evasion first become to be studied in 1974 by Allingham and Sandmo. The author differentiates tax evasion and tax avoidance. Tax evasion is a violation of the law, but tax avoidance is within the legal framework of the law. In tax evasion the firm or individual have to be worried about being detected but in tax avoidance it was only explored a loophole in tax law in order to pay less tax. In his conclusions, Sandmo referred that under the conclusion of the paper of Becker (1974) the government should choose low probabilities of detection and high penalties. This way the costs of tax administration are lower. The author concludes saying that tax evasion decision is dependent on the behavior of others, as more socially acceptable the tax evasion the more tax evasion the society will have. He also states that tax evasion is not an argument for lower marginal tax rate as the penalty and audit rate are more linked with the decision of tax evasion. In an extreme interpretation of this theory the penalties should be so high that the tax evader goes bankruptcy and the cost of administration would be zero, as the probability of being caught.

Mossin (1968) and Stiglitz (1969) were the first to refer the effect of taxes on individual consumer's choice of portfolio, however they forget about tax evasion as an option. Allingham and Sandmo (1972) incorporated in their paper the possibility of deliberated underreporting in order to avoid taxes. They relate their paper with criminal

activity of tax evasion but also related with analysis of optimal portfolio in economics of uncertainty. In Allingham and Sandmo's model the evaded tax would lead to a penalty to be paid if the authorities caught the tax evasion. These authors developed a model for an individual to choose between tax evade or not. The objective will be the maximization of the income and the individual will face a probability of being caught by authorities and will have to pay a penalty at the undeclared amount. In this case the punishment is to pay the undeclared amount at higher rate than the corporate tax rate of the country. This is a simplification of the penalty in real word, once in the real world there is a moral hazard of being caught in an illegal activity like tax evasion and probably the court will decide the penalty. In other words, it is difficult to preview the penalty to be applied in each case once there is much uncertainty about that, so the authors choose to simplify it.

As Allingham and Sandmo (1972) were more concerned with direct taxes, Marrelli (1984) developed a model related with indirect taxes. He studied the decision of entrepreneur to under-reporting indirect taxes, limiting the model to monopolistic firm. In this case there is a connection between the production and the evasion of taxes. They refer important conclusions: higher tax rates due not lead to higher declarations (Yitzhaki, 1974), they "compare direct versus indirect tax evasion and we show that, for decreasing risk aversion, and indirect tax is evaded as a percentage less than a profit tax of equal yield" and they also state that shifting and tax evasion decisions are independent from each other. As Marrelli (1984) developed a model linking tax evasion with under-reporting of sales taxes, there is also the possibility of over reporting of actual costs.

Wang and Conant (1988) developed a model with the possibility of over reporting the actual production costs. They concluded that the decision of tax evade and production are separable, as concluded previously by Allingham and Sandmo (1972) and Marrelli (1984). The level of tax evasion, on other hand, is related to the variations in tax rate, the penalty rate and probability of detection. The underreporting of actual wage payments was studied by Yaniv (1988). Lin and Yang (2001) developed a dynamic model for tax evasion and they concluded that higher tax rates encourage the

tax evasion in this case of a dynamic setting. This conclusion is highly intuitive and was expressed in a report by the *Economist (*May 28, 1994) where the publication refers that lowering tax rates can lower tax evasion. Also several other authors have tried to build theoretical models to support this theory.

Yaniv (1995) proposed a general model of tax evasion applicable to any type of tax and to any type of evasion, such as under reporting or over reporting. The author assumes that firm wants to maximize their profits. The firm considers the possibility of tax evading part of the total profits, if the firm is detected they have to pay the evaded tax and a penalty that is a multiple. In this paper the author also states the separability between firm production decision and the tax evasion. Panteghini (2000) developed a model that relates tax evasion and flexibility. The author develops a dynamic model characterized by discrete time, risk neutrality and nonzero investment. He includes the effect of "bad news" that represents auditory for tax evading firms. Using real options theory he provides a payoff that incorporates the option to delay the investment. Following Yitzhaki (1974) the penalty is proportional to the tax evaded, given a probability of detection. The author concludes that if an irreversible investment is introduced the decision of investment and production cannot be separated from tax evasion.

In the literature we can observe that tax evasion is more discussed in an individual perspective and not in a firm perspective. Most of the papers are about tax evasion on individuals and not companies. Besides that none of the previous authors used a dynamic setting for study tax evasion, i.e., seeing real options has an option. Our model has the capability of study tax evasion on firms while links real options with that. That way we are able to treat tax evasion has an options for the firms and study the impact of having that option for the firm with different perspectives that the firms has about the market.

3. The model

In this chapter we develop a real options model in order to understand the dynamics of tax evasion. For that purpose we consider two different stages or periods. One where the firm declares all the profits, having the option to modify its behavior. And a second one where the firm is active in tax evasion.

To solve this problem and find the solution to our model we use a backwards procedure standing from the second stage.

3.1. Setup

In a world where taxes are so high, firms have some incentives to tax evade. Managers have the option to understate some profit flows in order to pay fewer taxes and with that have more profits. As usual, we consider that firms want to maximize their profit so it is rational to understate some profits in order to pay fewer taxes. As we referred above, there are two periods in this model. In one period the firm is tax evading and in the other is not. Once a firm decides to tax evade it will be forever until the state discovers it and punishes the firm. This punishment is a defined constant for a question of simplification of the model.

The profit flows of the firm are represented by π and they follow a geometric Brownian motion (gBm), the standard approach defined by Dixit and Pindyck (1994).

$$d\pi = \alpha\pi\, dt + \sigma\pi\, dz \qquad (3.1)$$

where $\pi > 0$, dz is the increment of the Wiener process, α is the instantaneous conditional expected relative change in π, also known as the drift. $\alpha = r - \delta$ ($r > \delta$), where r is the risk-free rate and δ (>0) corresponds to the opportunity cost of deferring and σ is the instantaneous conditional standard deviation.

As mentioned before there are two periods in this model, one with tax evasion and other with no tax evasion. We start by assuming that the company is tax evading

and we use a backwards methodology to reach the differential equation of the period with no tax evasion.

Setting H(X) has the function where the firm is tax evading and it has no flexibility to go back, we reach the following ordinary differential equation (o.d.e):

$$\frac{1}{2}\sigma^2\pi^2 H''[\pi] + \alpha\pi H'[\pi] - rH[\pi] + (1-\gamma)\pi(1-t) + \gamma\pi + \lambda\left(\frac{\pi(1-t)}{r-\alpha} - P - H[\pi]\right) = 0 \quad (3.2)$$

As mentioned before π is the profit flow of the company. t is the tax rate paid by the company according to profit flows and γ is the percentage of profit flows that are evaded by the firm. P represents the punishment when the company is caught by the government in tax evasion. For simplification, we consider that P is constant and it is a fixed amount. This simplification also makes sense because of prescription of fiscal crimes. λ is the probability of the authorities to see that the firm is tax evading and punish the firm because of that. In other words, .λ captures the efficiency of fiscal authorities to detect and punish the company's tax evasion behavior. This efficiency is a perception that the firm has about it.

The equation 3.2 has three different terms. The first term $(1-\gamma)\pi(1-t)$ captures the after tax profit that is currently being declared. As the firm do not declare all the profit flow, only the declared part is subject of the corporate tax rate. The second term $\gamma\pi$ represents the not-taxed profit flow, in other words, this is the gain from not declaring all the profits by the firm. As this part of the profit is not subject to taxes, the gain will be total and not taxes will be retired from the profit flow. The third term $\lambda\left(\frac{\pi(1-t)}{r-\alpha} - P - H[\pi]\right)$ is the expected damage of being caught by fiscal authorities. As we can observe, if the firm is caught will have to pay a penalty for its illegal behavior. Also it is assumed that once the firm is caught it will have to return to full declared profits and the no tax evasion stage.

The solution to this equation is:

$$H(\pi) = A_1\pi^{\eta_1} + A_2\pi^{\eta_2} + \frac{\pi(1-t)}{r-\alpha} + \frac{t\gamma\pi}{r-\alpha+\lambda} - \frac{\lambda}{r+\lambda}P \qquad (3.3)$$

In this stage, we consider that there is no flexibility for the manager. There is no possibility of declaring all the profits now. The manager has no flexibility to regret his decision to tax evade because once the firm is tax evading it will be forever until it gets caught by fiscal authorities. As we assume this condition that the manager cannot go back and the firm stays in the stage of tax evasion, once the manager makes the decision of understate profits, we consider that there is no flexibility. Since there is no flexibility, $A_1 = A_2 = 0$. We reach the following solution, following this assumption:

$$H(\pi) = \frac{\pi(1-t)}{r-\alpha} + \frac{t\gamma\pi}{r-\alpha+\lambda} - \frac{\lambda}{r+\lambda}P \qquad (3.4)$$

Let us move now to the stage where the company declares all the profits but faces the change to engage in tax evasion activity.

When the firm is not tax evading and keeps the option to change to a tax evasion model the ordinary differential equation (o.d.e.) is the following:

$$\frac{1}{2}\sigma^2\pi^2 F''[\pi] + \alpha\pi F'[\pi] - rF[\pi] + \pi(1-t) = 0 \qquad (3.5)$$

In this case the payoff is simple, it is the profit flow (π) less the tax paid according to that profit.

The general solution to the equation 3.5 is the following:

$$F(\pi) = A_1\pi^{\beta_1} + A_2\pi^{\beta_2} + \frac{\pi(1-t)}{r-\alpha} \qquad (3.6)$$

As A_1 and A_2 are constants and arbitrary. Also we know that:

$$\beta_1 = \frac{1}{2} - \frac{\alpha}{\sigma^2} + \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + 2 * \frac{r}{\sigma^2}}; \tag{3.7}$$

$$\beta_2 = \frac{1}{2} - \frac{\alpha}{\sigma^2} - \sqrt{\left(\frac{\alpha}{\sigma^2} - \frac{1}{2}\right)^2 + 2 * \frac{r}{\sigma^2}}; \tag{3.8}$$

Beside that we know that $F(0) = 0$, once that if the profit flow is zero there will be no incentives to tax evasion. If a firm has no profits they will have no opportunity to have gains if they choose to engage in tax evasion activity. This is condition is known as absorption barrier.

To reach the solution the three boundary conditions have to be respected as mentioned before, in this model period with no tax evasion the boundary conditions are the following:

$$F(\pi) = \frac{\pi(1 - t)}{r - \alpha} + \frac{t\gamma\pi}{r - \alpha + \lambda} - \frac{\lambda}{r + \lambda}P \tag{3.9}$$

$$F'(\pi) = \frac{(1 - t)}{r - \alpha} + \frac{t\gamma}{r - \alpha + \lambda} \tag{3.10}$$

The first condition is value matching condition. The second term from the right-hand of the equation 3.6 that represents the extra profit made with the option to evade some profit flows. The company understates some profits in order to pay fewer taxes. As the company has to face the risk of the governments find out and make them pay a penalty, the second part captures efficiency of fiscal authority multiplied by the penalty.

The second condition is smooth pasting condition which assures that the function is continuously differentiable along π.

The value function of our model is defined as following and solution for our model is:

$$F(\pi) = \begin{cases} \frac{\pi(1-t)}{r-\alpha} + \left(\frac{t\gamma\pi^*}{r-\alpha+\lambda} - \frac{\lambda}{r+\lambda}P\right)\left(\frac{\pi}{\pi^*}\right)^{\beta} \; for \; \pi < \pi^* \\ \frac{\pi(1-t)}{r-\alpha} + \frac{t\gamma\pi}{r-\alpha+\lambda} - \frac{\lambda}{r+\lambda}P \; for \; \pi \geq \pi^* \end{cases} \quad (3.6)$$

The trigger is:

$$\pi^* = \frac{\beta}{\beta-1}\frac{r-\alpha+\lambda}{t\gamma}\frac{\lambda}{r+\lambda}P \quad (3.7)$$

In the first branch we have the case where there is no incentive to tax evasion, the trigger is bigger than the profit flow. In the second branch is the opposite, the trigger is equal or smaller than the profit flow. In this case there is incentive to tax evade part of the profits.

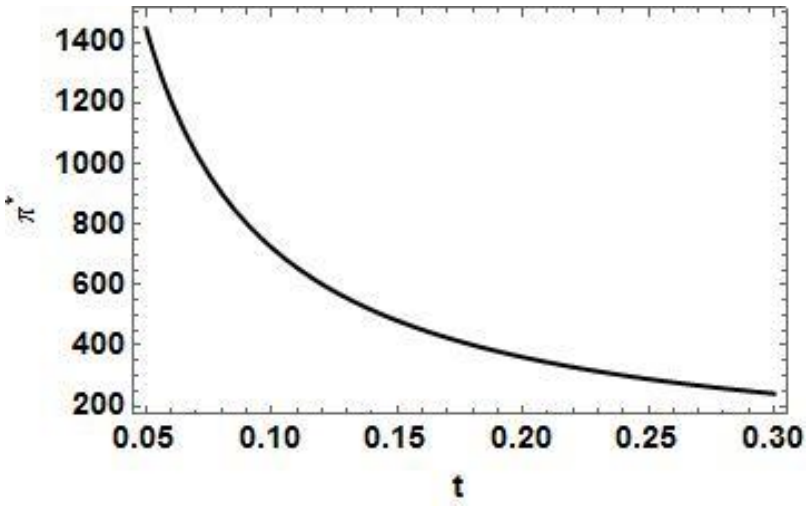
3.2. Comparative statics

In this chapter we present a comparative statistics analysis in order to understand the impact of the main parameters on tax evasion dynamics.

For that purpose we present the derivatives of the trigger value (π^*) in order to three different variables and in figure 3, 4 and 5 we have the relation between the trigger and the different variables assuming the parameters referred in the figures legend.

$$\frac{\partial\pi^*}{\partial t} = -\frac{P\beta\lambda(r-\alpha+\lambda)}{t^2(-1+\beta)\gamma(r+\lambda)} < 0 \quad (3.8)$$

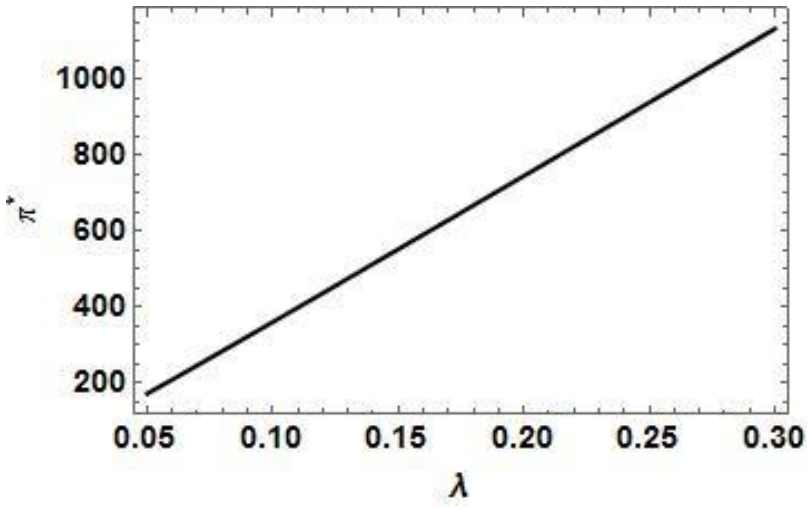
Figure 3 - Sensitivity analysis on tax rate related with cash flows assuming the following parameters: π=100; r=0,04; σ=0,25; α=0.01; **γ**=0.15; **P**=20; λ=0,10



We find that the level of taxes (t) is negatively related with the trigger (π^*). As higher (lower) the level of taxes, lower (higher) will be the trigger. In terms of economy, it makes sense. Because if the taxes are higher the firm has incentives to evade profits earlier than if the level of taxes is lower.

$$\frac{\partial \pi^*}{\partial \lambda} = \frac{P\beta(r^2 - r\alpha + 2r\lambda + \lambda^2)}{t(-1+\beta)\gamma(r+\lambda)^2} > 0 \qquad (3.9)$$

Figure 4 - Sensitivity analysis on probability of being caught by authorities related with cash flows assuming the following parameters: π=100; r=0,04; σ=0,25; α=0.01; **γ**=0.15; **P**=20; t=0,20.

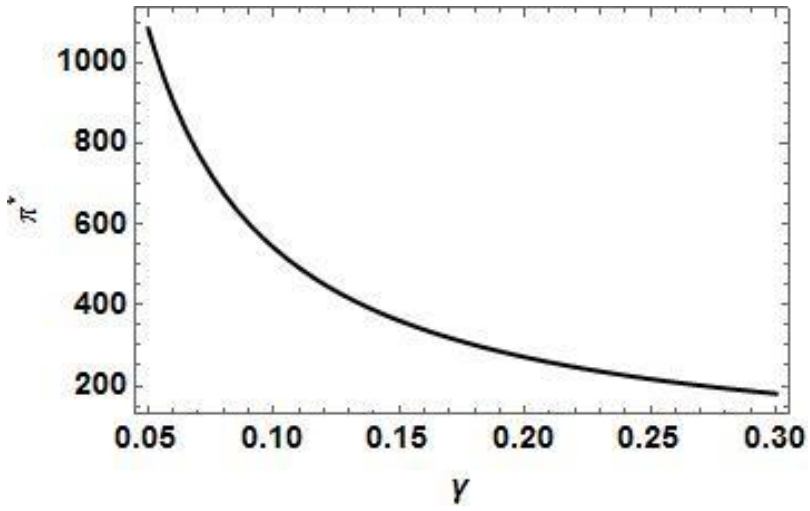


The derivative of π^* the in order to λ is positive. We know that $t(-1+\beta)\gamma(r+\lambda)^2$are positive values, so the denominator is positive. In the numerator, we know that P is positive, r is always bigger than α and λ is positive. We need to prove that $r^2 - r\alpha + 2r\lambda + \lambda^2$ is also positive. In fact we can rearrange this expression to $r(r-\alpha+2\lambda)+\lambda^2$. We know that $r-\alpha$ is always positive, so the expression is positive and it is proved that the signal of the derivative is positive.

The probability of the authorities find out the fiscal evasion of the firm (λ) is positively related with the trigger value. The trigger value will increase (decrease) with the increase (decrease) of the probability of being caught. It is intuitive that if there is more risk the incentives to tax evasion will be lower because there is more probability of penalty, as the authorities find out the tax evasion.

$$\frac{\partial \pi^*}{\partial \gamma} = -\frac{P\beta\lambda(r-\alpha+\lambda)}{t(-1+\beta)\gamma^2(r+\lambda)} < 0 \qquad (3.10)$$

Figure 5 - Sensitivity analysis on level of understated cash flows related with cash flows assuming the following parameters: π=100; r=0,04; σ=0,25; α=0.01; **P**=20; t=0,20; λ=0,10.



We can also see that as higher (lower) the profit understated by the firm is, the lower (higher) is the trigger value. If a firm understates more profits it has more incentives to make tax evasion sooner that if they understate fewer profits.

From this comparative statistics we can make several conclusions. We observe that the raising of corporate tax rate will lead to higher incentives to tax evasion by firms and sooner the companies will have incentives to understate profits. The relation between the level of understated profits and the trigger value is the same of corporate tax rate and the trigger value, it is a positive one. As more the firm has possibility of understating part of the profits and pay fewer taxes, the sooner the firm will have incentives to enter the stage where chooses to engage in tax evasion. The efficiency of fiscal authorities or the probability of being caught by fiscal authorities in a tax evasion stage will have the contrary effect on the trigger value. As higher the efficiency of fiscal authorities is the later the firm will have incentives to enter the stage of tax evasion and they will postpone their decision to engage in this activity.

4. Numerical Example

Let us now present a numerical example. We assume that a firm has the possibility of evading some of the cash flows of the year. The manager has to make a decision whether he wants to evade or not with the risk of being caught and penalized or paying high taxes. If he decides to evade some profits he has to decide the percentage of cash flows to evade and how. We have several parameters to estimate the optimal timing for the manager to evade cash flows, assuming that the parameters have those values presented in the table 1 below. For the probability of being caught we use a low value as it is referred by the literature review. The penalty assumes a value constant as it was referred before and it is a proportional value to the cash flows evaded.

Table 1 - Input variables for the numerical example.

Parameter	Description	Value
π	Profit flow of the firm at moment 0	2000 M€
σ	Cash flows volatility	25%
r	Risk-free Interest Rate	2%
α	Cash flow growth rate	1,5%
t	Tax rate	30,00%
γ	Cash flows evaded	15%
P	Penalty paid to authorities	100
λ	Probability of being caught	10%

Based on the equations (3.12) and (3.13) we find the following output value:

Table 2 - Output values for the numerical example.

Output	Description	Value
π^*	Level of cash flows where is optimal to evade	2116 M€
$\pi^*\gamma$	Cash flows evaded	317 M€

In Table 2 we have the value of the trigger that indicates the optimal timing where the manager is incentivized to evade some percentage of cash flows. In this case, the manager only has incentives to evade cash flows at a level of 2116 million euros. Assuming that the percentage evaded is 15%, the manager would evade 317 million euros. As we see the profit flows are at the moment zero, 2000 million euros, a value close to 2116. When the profit flow achieves a value bigger or equal to 2116 the manager will have incentives to evade. In the real world it is not mandatory to wait for the exact moment when the profit flows achieve 2116, a close value could be sufficient to incentive the manager to evade some cash flows.

Table 3 - Sensitivity analysis of trigger values with different tax rates and probabilities of being caught.

	t					
	2116,029	15%	20%	25%	30%	35%
λ	5%	**1900**	**1425**	**1140**	**950**	**814**
	8%	3055	2291	**1833**	**1527**	**1309**
	10%	4232	3174	2539	2116	**1814**
	15%	6615	4961	3969	3307	2835
	20%	9014	6760	5408	4507	3863

The table 3 above shows us the behavior of the trigger when the tax rate is higher or lower and the same for the probability of being caught. We can see that the increase of the probability of being caught increases the trigger value, what seems logical because if there is more chance of being caught the manager will postpone the tax evasion. On contrary, the increase of the tax rate leads to a decrease in the trigger because the company will have incentives to evade earlier when the taxes are too high.

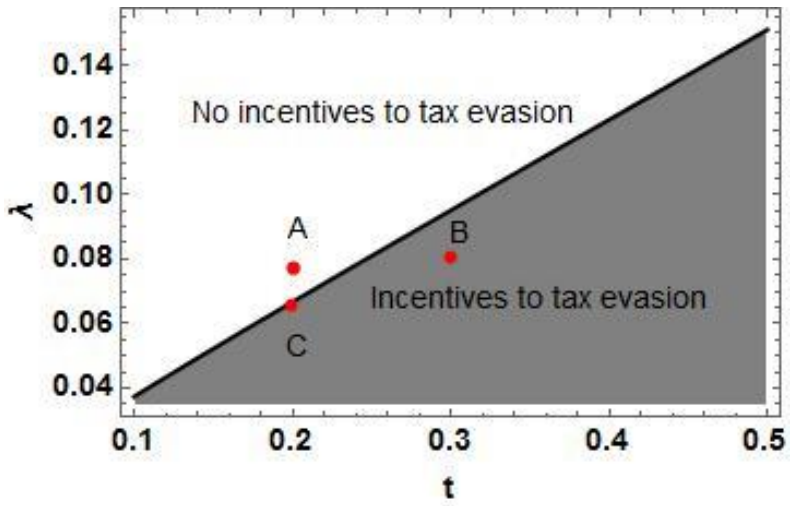
Table 4 - Sensitivity analysis of trigger values with different volatilities and penalty for being caught.

	P					
	2116,029	50	75	100	150	200
σ	15%	**646**	**970**	**1293**	**1939**	2585
	20%	**829**	**1244**	**1659**	2488	3318
	25%	**1058**	**1587**	2116	3174	4232
	30%	**1333**	**1999**	2665	3998	5331
	35%	**1654**	2481	3308	4963	6617

In the table 4 we relate two more variables with the trigger. We see that the volatility, σ, is positively related with the trigger. In other words, when the volatility increases (decreases) the trigger also increases (decreases). So, when the volatility of the cash flows is higher the more the manager waits to evade cash flows. This could happen because if the cash flows are more volatile, the manager would not know if it is worth to evade some part of the cash flows, so he waits more time. The penalty has the same relation with the trigger, when the penalty increases (decreases) the trigger also increases (decreases). It is easy to understand that when the penalty for evading cash flows is higher manager waits more time to evade cash flows because the penalty will affect more the firm, so the cash flows evaded have to be bigger to be worth the risk of evading cash flows.

The highlighted values in the table correspond to values where the profit flow, 2.000M€, is bigger than the trigger that we achieve with those inputs. In those cases the manager is incentivized to evade part of the cash flows.

Figure 6 - Trigger values for which a firm is incentivized to tax evade.

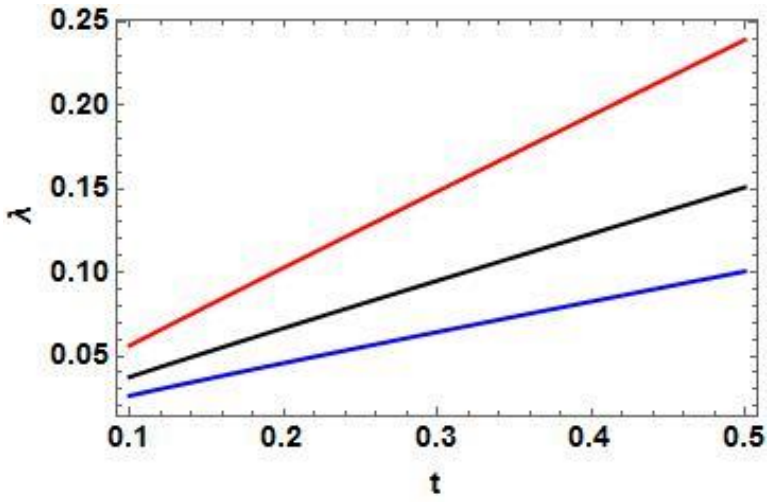


In the figure 6 we have 2 variables, the tax rate (t) and the probability of being caught (λ). The first is represented in the abscissa axis and the second in the ordinates axis. The black line represents the points where the trigger 2116 is reached for different values of the variables. For example, for a value of 20% tax rate the trigger would be achieved if the probability of being caught would be 6,7% (point C), approximately. The region above the line of the trigger represents the region where there are no incentives to evade part of the cash flows. To understand better how the manager can change the decision of evading or not the cash flows we use an example. If we assume that the tax rate is 20% and the probability of being caught is 10%, we are in the region "No incentives tax evasion" where there are no incentives to tax evasion. But, if the probability of being caught changes to 6% the manager will have incentives to evade some part of the cash flows. In this last case the intersection will occur in the region of "Incentives tax evasion".

Also in the figure 6 we can see three represented points: A, B and C. The point A represents the situation where the corporate tax rate is 20% and the efficiency of

fiscal authorities to detect companies that are engaging in tax evasion activities is 8%, this value for the efficiency is the perception the company has about it.We can see that in this case the point is in the region of "No incentives to tax evasion", so the payoff for the company with those parameters is not sufficiently good in order to incentivize the company to engage in tax evasion. However, if the tax rate of the country is changed by the government to 30% we will move to point B or situation B. In that situation we observe that we moved to another region, the region of "Incentives to evasion". Now, the firm has incentives to engage in a tax evasion activity because the level of the corporate tax rate increased and the trigger value decreased. This happens because the gains that the company can now get with the understating of profits are sufficiently high to incentivize the company to do that.

Figure 7 - Impact on the trigger value when the probability of being caught changes.



The figure 7 has the same variables in the axis and the line, but in this case we have 3 different triggers. The black line represents the original trigger with the value of 2116 million euros, the blue represents a trigger with a volatility of 35% and the red represents the trigger if the volatility assumed was 15%. As in the first graph, the region below the lines is the "tax evasion" region where there are incentives to evade cash flows and the region above is the "no tax evasion" region where there are no incentives to evade cash flows for the firm. We can conclude by the graph that a higher volatility represents a smaller region where there are incentives to tax evasion, but a lower volatility leads to a bigger region where the manager is incentivized to evade some cash

flows. Once again, we can see that volatility has negative effect in the trigger value. This is in line with our results in table 4. That way we can say that the higher the uncertainty regarding the cash flows of a firm, the later a firm will have incentives to enter the tax evasion stage. In our perspective this is logical because if a manager has difficulty to preview the cash flows, it will be harder to make the decision of entering the stage of tax evasion or not. Certainty about cash flows will lead to a more precise decision because manager will know if the risk of being in an illegal activity could bring enough gains to compensate those risks taken.

5. Conclusion

Tax evasion is a very hot topic in modern finance nowadays. The impact on GDP of tax is very high and countries' economy are very affected by tax evasion. In order to control this illegal activity, governments can raise the efficiency of fiscal authorities or choose to incentivize firms to tax evade by their policies. For example, higher tax rates do not mean higher gains for governments because some firms may opt for tax evasion. We study this possibility of firms hiding profits from the government and that way pay fewer taxes.

We approach this topic from a different perspective using a real options approach and studying the possibility of firms choosing to hide profits in order to pay fewer taxes. In the literature there is a gap for the relation between tax evasion and firms' decisions with most of the papers being written about tax evasion on individuals and not companies. It was the firm time that tax evasion was linked with firms and real options.

Developing a model that incorporates variables that could incentive a manager of a firm to opt for hiding some profits, was complex but we did incorporate in our model important variables that could lead to that decision or not. This model that we developed brings a dynamic setting to the study of tax evasion on firms. The usage of real options brought us a different understanding about the dynamics of tax evasion and how firms react to differents inputs. Our model has two stages, one where the firms is on tax evasion and a second when there is not. It is also assumed that once the firms enter the stage where they engange in tax evasion there is no possibility of going to the stage with no tax evasion.

This model shows how a process of choosing to hide profits by a firm could be and we can see by our analysis using the derivatives or sensitivity analysis that changes in variables could incetive managers to tax evade. Beside the part of the firm this can also be useful to governments and fiscal authorities once they can see by our model and conclusions how firms could react to changes in tax rate, for example.

In our main findings we can refer that the increase of taxes will lead to a sooner incentive to tax evade by firms or that the more probabibility of getting caught hiding profits the later a firm will choose to tax evade, these two conclusions were confirmed by our analysis as our intuition also leads us to those conclusions. However, we also concluded that the more certain the cash flows are the sooner a manager will have incentives to hide profits. In our analysis of the relation between the trigger value and the volatility we see that they are negatively related, so the lower the volatitily of cash flows the lower the trigger value. The higher the penalty that the firm will have to pay if it gets caught engaging in tax evasion, the later the firm will have incentives to enter this stage and so it will postpone their decision. We also observe that higher possibility of understating profits, i.e., a higher level of understated profits will lead to a lower trigger value and sooner the company will have incentives to enter tax evasion stage.

More than mathematical conclusions, our model brings economical conclusions. Governements have to make several decisions when fighting tax evasion and when they have to collect money from firms or individuals. Taxes are one of the most important forms of collecting money by each country. Our model exposes the possibility of different tax rates leading to different decisions for firms regarding the choice to enter a stage of tax evasion. That way governments may not choose to raise tax rate because this will lead to higher incentives for firms to engage in tax evasion and may represent fewer returns for the country. As we also can see that higher efficiency of fiscal authorities or higher penalties will postpone the incentives of firms to tax evade, governments should also invest in those two possibilities in order to reduce tax evasion. From a firms' perspective we can say that there are some cases when the perception of the firm is that there are incentives to tax evasion and that the gains from undeclared profits compensate the risk of the penalty. Also, we can observe that firms with more certain cash flows will have sooner incentives to engage in tax evasion because they know with more precision the profit flow for that year.

Our model indicates a timing and value optimal where the managers are incentivized to hide profits from the companies and also study the impact of multiple variables in that decision. In the end it also estimates the gains for the firms that could

be made from paying fewer taxes due to tax evasion, which results in losses for the fiscal authorities.

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