Chapter 5
Complex Joint Geometry

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Abstract The finite element method is particularly suited to analyse complex joint geometries. Adhesively bonded joints are increasingly being used in engineering applications where the loading mode, the adherends shape and the material behaviour are extremely difficult to simulate with a closed form approach. A detailed description of finite element studies concerning non-conventional adhesive joints is presented in this chapter. Various types of joints, local geometrical features such as the spew fillet and adherend rounding, three dimensional analyses, hybrid joints and repair techniques are discussed. Special techniques to save computer power are also treated. It is shown that the finite element method offers unlimited possibilities for stress analysis but also presents some numerical problems at sharp edges.

5.1 Introduction

Stress analysis of bonded joints started 70 years ago with the well known Volkersen shear lag model (Volkersen, 1938). Since, various analytical models have been proposed that include the geometrical non-linearity (Goland and Reissner, 1944), the adhesive plasticity (Hart-Smith, 1973), fibre reinforced plastic adherends (Renton

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L.F.M. da Silva, A. Öchsner (eds.), *Modeling of Adhesively Bonded Joints*,

and Vinson, 1975), etc. However, most of the analytical models are for lap joints of regular shape. Closed form analyses are very useful for an initial estimate of the stress distribution and are generally adequate for design purposes. For adhesive joints of complex shape, numerical techniques such as the finite element (FE) method are preferable. Simple lap joints have been modified because of the non-uniform stress distribution along the overlap. The load transfer and shear stress distribution of various single lap joints (SLJ) are schematically represented in Fig. 5.1. It can be seen that there is a stress concentration at the ends of the overlap for the single lap joint with a square end. Modification of the joint end geometry with a spew fillet or a taper spreads the load transfer over a larger area and give a more uniform shear stress distribution.

Adams and co-workers are among the first to have used the FE method for analyzing the stresses in adhesive joints (Adams and Peppiatt, 1974; Crocombe and Adams, 1981; Harris and Adams, 1984; Adams et al., 1986; Adams and Davies, 2002). One of the first reasons for the use of the FE method was to assess the influence of the spew fillet. The joint rotation and the adherends and adhesive plasticity are other aspects that are easier to treat with a FE analysis. The study of Harris and Adams (1984) is one of the first FE analyses taking into account these three aspects.

The increasing use of adhesive bonding with composite materials is another typical example of the great advantage of using finite elements (Adams et al., 1986; Adams and Davies, 2002) since the anisotropic behaviour of these materials complicates drastically any analytical approach.

But it is when irregular geometries are involved that the FE method is an indispensable tool. Any type of geometry can be modelled, i.e., variations in the adherend shape, local adherend rounding, spew fillet, reinforcements, joints with rivets or bolts, etc. In addition, the simulation gives not only one, two or three stress

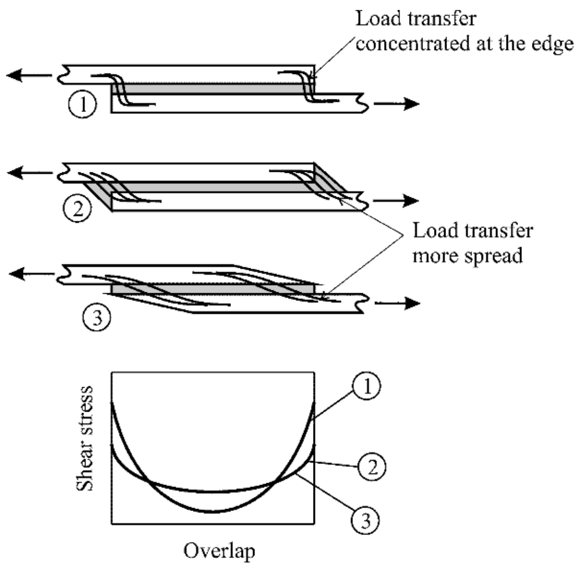


Fig. 5.1 Load transfer and shear stress distribution in single lap joints

components but all the stress components in all members of the joint. Three dimensional (3D) analyses for assessing width effects is another possibility offered by a FE analysis. As the model gets more complicated in terms of geometry and material behaviour, the computation time also increases. Also, if the user wants to change one geometrical feature, a new model has to be created. For these reasons, parametric studies are more difficult with a FE approach than with a closed form approach. However, special techniques are emerging for solving this problem. All these aspects are discussed in this chapter with a mention to the difficult task of dealing with the singular points present in any type of adhesive joint.

5.2 Types of Joints

Adhesive joint studies are generally related to lap joints with flat adherends. The first theoretical analysis of such joints was an analytical model proposed by Volkersen (1938). Since then, many analytical models have been proposed, being the overwhelming majority about lap joints with flat adherends. This is because joints of a more complex geometry are difficult to model using a closed form analysis. Numerical techniques, such as the FE method, can obviously be used for simple geometries such as single or double lap joints with regular flat adherends, but it is for complex geometries that this method of analysis is truly advantageous. Within joints with flat adherends, one can have ‘wavy’ lap joints, ‘reverse bent’ joints, ‘tongue and groove’ joints and scarf joints (see Fig. 5.2). Other types of joints include cylindrical joints, corner joints, T joints and peel joints (see Fig. 5.2). As for analytical models, the majority of the FE analyses are about lap joints with flat adherends. This type of joint is treated in detail in the following sections. However, numerous studies can be found in the literature about other types of joints.

Ávila and Bueno (2004) analyzed a new type of single lap joints where the adherends have a ‘wavy’ configuration along the overlap. An FE analysis and experiments show that great strength improvements can be obtained with this technique. Fessel et al. (2007) also studied this type of joint as well as joints with bent substrates along the overlap. An FE analysis and experiments show great joint strength improvements. Dvorak et al. (2001) show with a FE analysis and experimentally that adhesively bonded tongue-and-groove joints between steel and composite plates loaded are stronger than conventional strap joints.

Nakagawa and Sawa (2001) studied scarf joints using photoelasticity and a 2D FE model. One of the conclusions is that under a static tensile load, an optimum scarf angle exists where the stress singularity vanishes and the stress distributions become flat near the edge of the interface, but under thermal loads, the optimum scarf angle is not found.

Apalak and Davies (1993, 1994) studied different types of corner joints using a linear FE analysis and proposed design guidelines based on the overall static stiffness and stress analysis. Apalak (1999, 2000) developed the previous study taking into account the non-linear geometry. Feih and Shercliff (2005) modelled simple corner joints with composites.

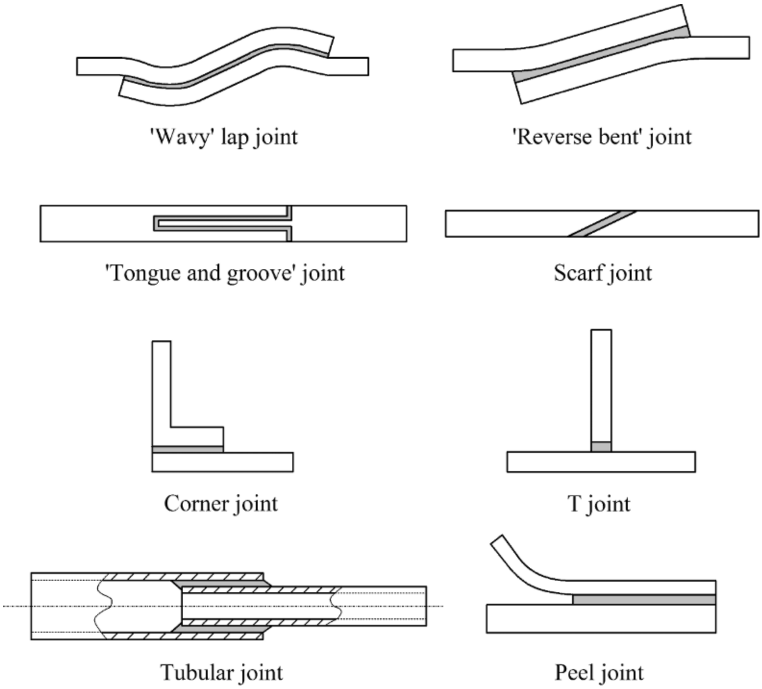


Fig. 5.2 Types of adhesive joints

Apalak (2002) studied the geometrical non-linear response of T-joints. Apalak et al. (2003) studied T-joints in terms of thermal and mechanical loads. da Silva and Adams (2002) used the a FE model to predict the joint strength of T-joints based on the plasticity of the adherend. Marcadon et al. (2006) studied T-joint for marine applications and found that both the overlap length and also the distance between the T plywood and the base have an influence on the tearing strength.

Kim et al. (1992) studied various configurations of tubular joints (single overlap, double overlap, adherends tapering) and found with a FE analysis and experimentally that the double lap configuration is the strongest joint. Kim and Lee (2001, 2004) used FE models to study the effect of temperature on tubular joints. Kim et al. (1999) modelled composite tubular joints. Oh (2007) used a FE computation to analyze tubular composite adhesive joints under torsion. Good agreement was obtained between the predicted joint strength and the available experimental data. Serrano (2001) used a nonlinear 3D FE model to study glued-in rods for timber structures. The model could simulate experimental behaviour.

Peel testing is a very common test for adhesive properties. The problem is highly non-linear in terms of geometry and material and the FE method has been widely used, as for example in the work of Du et al. (2004).

5.3 Adhesive Fillet

Various authors have shown that the inclusion of a spew fillet at the ends of the overlap reduces the stress concentrations in the adhesive and the substrate (Adams and Peppiatt, 1974; Crocombe and Adams, 1981; Adams et al., 1986; Adams and Harris, 1987; Dorn and Liu, 1993; Tsai and Morton, 1995; Lang and Mallick, 1998; Belingardi et al., 2002; Apalak and Engin, 2004; Andreassi et al., 2007; da Silva and Adams, 2007a; Deng and Lee, 2007). The adhesive fillet permits a smoother load transfer and decreases the peak stresses.

Adams and Peppiatt (1974) found that the inclusion of a 45° triangular spew fillet decreases the magnitude of the maximum principal stress by 40% when compared to a square end adhesive fillet. Adams and Harris (1987) tested aluminium/epoxy single lap joints with and without fillet and found an increase of 54% in joint strength for the filleted joint. Adams et al. (1986) tested aluminium/CFRP single lap joints and found that the joint with a fillet is nearly two times stronger than the joint without a fillet. Crocombe and Adams (1981) did similar work but included geometric (overlap length, adhesive thickness and adherend thickness) and material (modulus ratio) parameters. The reduction in peel and shear stresses is greatest for a low modulus ratio (low adhesive modulus), a high adhesive thickness and a low adherend thickness.

Dorn and Liu (1993) investigated the influence of the spew fillet in plastic/metal joints. The study includes a FE analysis and experimental tests and they conclude that the spew fillet reduces the peak shear and peel adhesive stresses and decreases stress and strain concentrations in the adherends in the most critical regions. They also studied the influence of different adhesive and different metal adherends. A ductile adhesive and a more balanced joint (aluminium/plastic instead of steel/plastic) give a better stress distribution.

Tsai and Morton (1995) studied the influence of a triangular spew fillet in laminated composite single lap joints. The FE analysis and the experimental tests (Moire interferometry) proved that the fillet helps to carry part of the load thus reducing the shear and peel strains.

The above analyses are limited to triangular geometry. Lang and Mallick (1998) investigated eight different geometries: full and half triangular, full and half rounded, full rounded with fillet, oval and arc. They showed that shaping the spew to provide a smoother transition in joint geometry significantly reduces the stress concentrations. Full rounded with fillet and arc spew fillets give the highest percent reduction in maximum stresses whereas half rounded fillet gives the less. Furthermore, increasing the size of the spew also reduces the peak stress concentrations.

Andreassi et al. (2007) used a two-dimensional computational fluid dynamics model to study spew fillet formation considering the actual adhesive flow produced during joint assembly. This is an interesting approach that allows tuning the adhesive joint strength by changing the adhesive flow parameters.

The spew fillet is not beneficial in all situations. In effect, the spew fillet tends to generate more thermal stresses than a square end geometry when used at low temperatures (da Silva and Adams, 2007a).

5.4 Adherend Rounding

Adhesive single lap joints may have several singular (infinite) stresses. The stresses tend to infinite at crack tips and at bi-material junctions. Fracture mechanics can be applied to treat the singular points at crack tips. As regards the bi-material singularities, there can be two or three, depending on the joint geometry (see Fig. 5.3). For both situations, there is an abrupt change in slope giving a sharp corner. The stress field in the vicinity of a sharp corner depends on the mesh used. A stress limit criterion will therefore lead to arbitrary results. The first FE analyses 30 years ago used coarse meshes due to computer memory limitations and therefore did not capture the infinite stress at singular points. The joint strength predictions obtained with stress or strain limit based criteria compared well with the experimental results (Harris and Adams, 1984). With the computer power development, finer meshes can be used which lead to a better stress distribution description but also to stresses that tend to infinite at singular points. The stress or strain limit failure criteria give in this circumstance more conservative predictions and are therefore very arguable.

Groth (1988) used a fracture mechanics approach without considering a pre-existing crack. He formulated a fracture criterion based on an equivalent generalised stress intensity factor similar to that in classical fracture mechanics. Comparing it to a critical value, joint fracture may be predicted. However, the critical stress intensity factor needs first to be tuned with an experimental test which makes this approach questionable.

Another approach for dealing with the singularity points is to use a cohesive zone model (CZM). This approach is associated to interface elements and enables to predict crack initiation and crack growth. It is a combination of a stress limit and fracture mechanics approach and relatively mesh insensitive. Many researchers are using this tool with accurate results (Blackman et al., 2003; de Moura et al., 2006; Liljedahl et al., 2007). However, the parameters associated to the CZM require previous experimental 'tuning' and the user needs to know beforehand where the failure is likely to occur. This subject is discussed in detail in Chap. 6.

In practical joints the sharp corners are always slightly rounded during manufacture and the singular points will not necessarily exist (see Fig. 5.4). Adams and Harris (1987) studied the influence of the geometry of the corners. Using a simplified model, the effect of rounding on the local stress was investigated. Rounding the corner removes the singularity. Therefore small local changes in geometry have a significant effect. They also performed an elastic-plastic analysis by calculating the

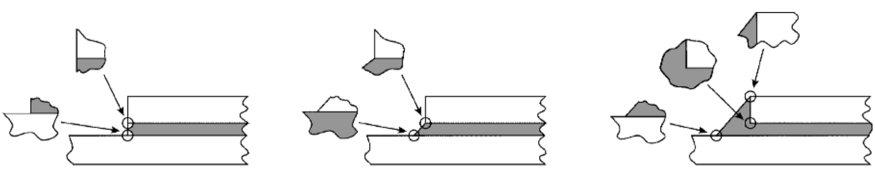


Fig. 5.3 Singular points in adhesive joints

Fig. 5.4 Rounding of adhesive and adherend

plastic energy density. They found that the stress distribution is more uniform with a maximum some distance from the corner. The reason they give is that as the corner is approached, although the normal stress increases, the rigid adherend restrains the adhesive in the transverse direction. The net hydrostatic component is increased and yield is suppressed so that close to the corner there is a reduction in plastic energy density. It should be born in mind that the effects are local: far away from the corner the stress distribution is unaffected. Apart from the simplified model, Adams and Harris (1987) tested three types of aluminium/rubber-toughened epoxy single lap joints so that local changes at the end of the joint could be assessed. They found excellent agreement between the predicted joint strengths, with the modified models, and the experimental values.

5.5 Adherend Shaping

Adherend shaping is a powerful way to decrease the stress concentration at the ends of the overlap. Figure 5.5 presents typical geometries used for that purpose. Some analytical models were proposed to have a more uniform stress distribution along the overlap (Cherry and Harrison, 1970; Adams et al., 1973; Groth and Nordlund, 1991). However, the FE method is more appropriate for the study of adherends shaping. The concentrated load transfer at the ends of the overlap can be more uniformly distributed if the local stiffness of the joint is reduced. This is particularly relevant for adhesive joints with composites due to the low transverse strength of composites. Adams et al. (1986) addressed this problem. They studied various configurations of

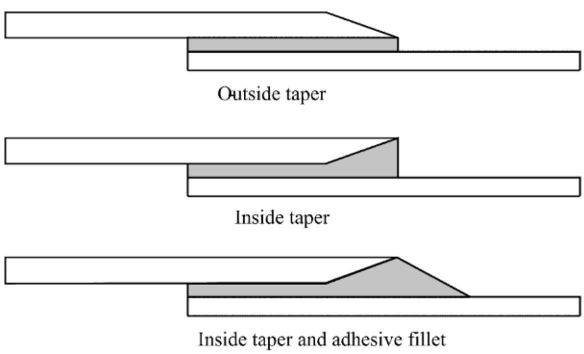


Fig. 5.5 Adherend shaping

double lap joints where the central adherend is CFRP and the outer adherends are made of steel. They found with FE and experiments that the inclusion of an internal taper and an external fillet can triplicate the joint strength. The same geometry was studied at low temperatures and it was found that the thermal stresses reduce substantially the joint strength (da Silva and Adams, 2007a).

Hildebrand (1994) did similar work on SLJs between fibre reinforced plastic and metal adherends. The optimisation of the SLJs was done by modifying the geometry of the joint ends. Different shapes of adhesive fillet, reverse tapering of the adherend, rounding edges and denting were applied in order to increase the joint strength. The results of the numerical predictions suggest that, with a careful joint-end design, the strength of the joints can be increased by 90–150%.

The use of internal tapers in adherends in order to minimize the maximum transverse stresses at the ends of bonded joints has also been studied by Towse (1999), Rispler et al. (2000), Guild et al. (2001), Belingardi et al. (2002) and Kaye and Heller (2002). An evolutionary structural optimisation method (EVOLVE) was used by Rispler et al. (2000) to optimise the shape of adhesive fillets. EVOLVE consists of an iterative FE analysis and a progressive removal of elements in the adhesive which are low stressed.

Other examples of joint end modifications for joint transverse stress reduction but using external tapers are those of Amijima and Fujii (1989), Sancaktar and Nirantar (2003), Kaye and Heller (2005) and Vallée and Keller (2006). Kaye and Heller (2005) used numerical optimization techniques in order to optimize the shape of the adherends. This is especially relevant in the context of repairs using composite patches bonded to aluminium structures (see Sect. 5.9) due to the highly stressed edges.

The FE method is a convenient technique for the determination of the optimum adherend geometry, however the complexity of the geometry achieved is not always possible to realise in practice.

5.6 Other Forms of Geometric Complexity

The FE method can be used to study other complex geometrical features such as voids in the bondline, surface roughness, notches in the adherend, etc. (see Fig. 5.6). Nakagawa et al. (1999) studied the effect of voids in butt joints subjected to thermal stresses and found that stresses around defects in the centre of the joint are more significant than those near the free surface of the adhesive. Lang and Mallick (1999), Olia and Rossettos (1996), de Moura et al. (2006) and You et al. (2007) studied the influence of gaps in the adhesive and found that a gap in the middle of the overlap has little effect on joint strength.

Kim (2003) proposed an analytical model supported by FE modelling to study the effect of variations of adhesive thickness along the overlap. The author showed that the variations found in practice have little effect on the adhesive stresses along the overlap.

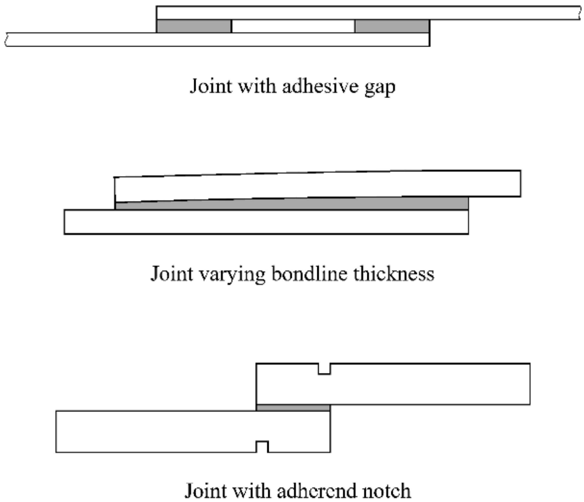


Fig. 5.6 Various forms of geometric complexity

Kwon and Lee (2000) studied the influence of surface roughness on the strength tubular joints by modelling the stiffness of the interfacial layer between the adherends and the adhesive as a normal statistical distribution function of the surface roughness of the adherends. The authors found that the optimum surface roughness was dependent on the bond thickness and applied load.

Sancaktar and Simmons (2000) investigated the effect of adherend notching on the strength and deformation behavior of single lap joints. The experimental results showed a 29% increase in joint strength with the introduction of the notches, which compared well with the FE analysis results. Yan et al. (2007) studied a similar idea, where the notch is in the middle part of the overlap. A FE analysis showed that a more uniform stress distribution along the overlap can be obtained.

5.7 Three Dimensional Analyses

Most of the FE analyses of adhesive joints are two dimensional. Since the width is much larger than the joint thickness, a plane strain analysis is generally acceptable. A generalised plane strain analysis where the joint is allowed to have a uniform strain along two parallel planes in the width direction gives more realistic results. However, for some problems, a two dimensional analysis might lead to erroneous results. Richardson et al. (1993) show that applying the average load on two-dimensional models resulted in errors in the adhesive stresses as high as 20%. The authors propose solutions to limit this problem. Three dimensional effects in the width direction such as lateral straining and the anticlastic bending (see Fig. 5.7) are especially important when analysing composites. Adams and Davies (1996) analysed composite lap joints in three dimensions and showed that the Poisson's ratio effects are significant in the behavior of composite lap joints.

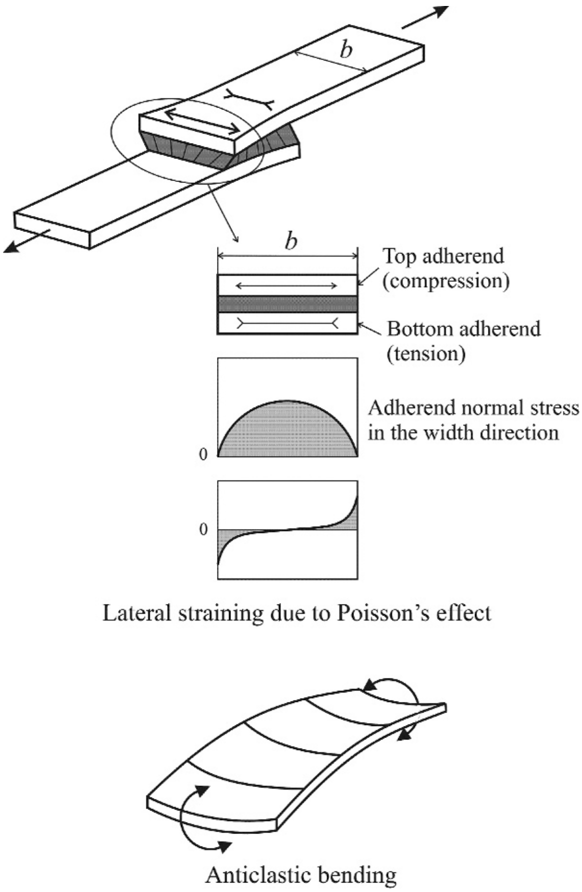


Fig. 5.7 Three dimensional effects in lap joints

The main problem of 3D analyses is the computational time due to the large number of elements. One way of overcoming this problem is to use the submodelling option, where the results from a coarse mesh of the global model are applied as boundary conditions to a much finer mesh of a localised region of particular interest. Bogdanovich and Kizhakkethara (1999) applied this technique to composite double lap joints and concluded that the submodelling approach is an efficient tool although convergence of stresses along certain paths in the joint were not satisfied. They also used the same approach in two dimensions to study the effect of the spew fillet.

5.8 Hybrid Joints

Joints with different methods of joining are increasingly being used. The idea is to gather the advantages of the different techniques leaving out their problems. Another possibility is to use more than one adhesive along the overlap or varying the adhesive and/or adherend properties. All theses cases have been grouped here under a section

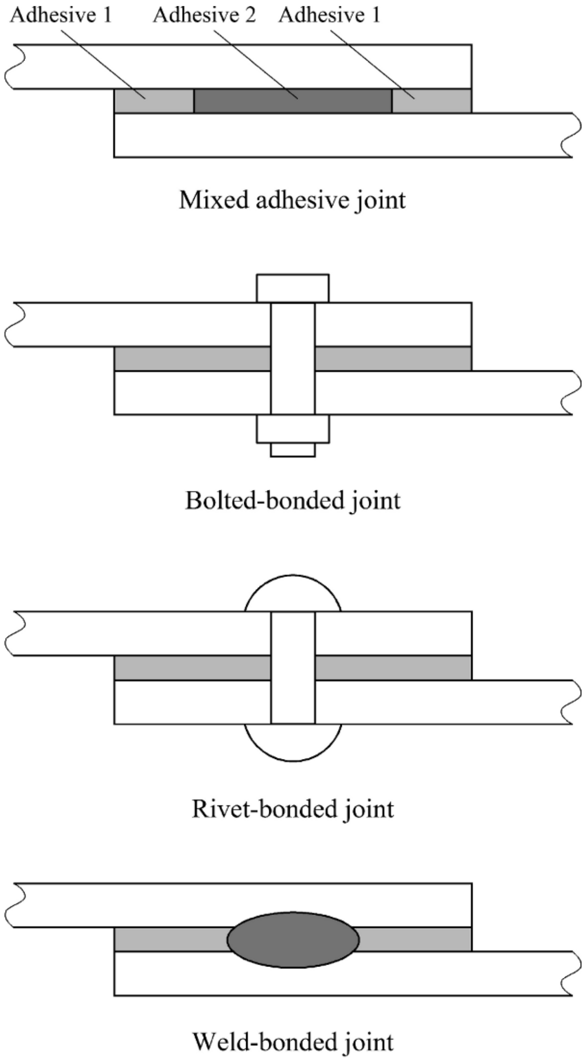


Fig. 5.8 Hybrid joints

called ‘hybrid joints’ (see Fig. 5.8). Such joints are particularly difficult to simulate using analytical models for obvious reasons. The FE method is the preferred tool to investigate the application of such techniques.

5.8.1 Mixed Adhesive Joints

Mixed modulus joints have been proposed in the past (Semerdjiev, 1970; Srinivas, 1975; Patrick, 1976) to improve the stress distribution and increase the joint strength of high modulus adhesives. The stiff, brittle adhesive should be in the middle of the overlap, while a low modulus adhesive is applied at the edges prone to stress concentrations. Sancaktar and Kumar (2000) used rubber particles to toughen the part of the adhesive located at the ends of the overlap and increase the joint strength. The concept was studied with the FE method and proved experimentally. Pires et al. (2003) and Fitton and Broughton (2005) also proved with a FE analysis and experimentally with two different adhesives that the mixed adhesive method gives an improvement in joint performance. Temiz (2006) used a FE analysis to study the influence of two adhesives in double lap joints under bending and found that the technique decreases greatly the stresses at the ends of the overlap. Bouiadjra et al. (2007) used the mixed modulus technique for the repair of an aluminium structure with a composite patch. The use of a more flexible adhesive at the edge of the patch increases the strength performance of the repair. The technique of using multi-modulus adhesives has been extended to solve the problem of adhesive joints that need to withstand low and high temperatures by da Silva and Adams (2007b, 2007c). At high temperatures, a high temperature adhesive in the middle of the joint retains the strength and transfers the entire load while a low temperature adhesive is the load bearing component at low temperatures, making the high temperature adhesive relatively lightly stressed. The authors studied various configurations with the FE method and proved experimentally that the concept works, especially with dissimilar adherends.

5.8.2 Adhesive Joints with Functionally Graded Materials

Functionally graded materials are increasingly being used in various applications including adhesive joints. For example, Apalak and Gunes (2007) studied the effect of a functionally gradient layer between a pure ceramic layer (Al_2O_3) and a pure metal layer (Ni). Gannesh and Choo (2002) studied the effect of spatial grading of adherend elastic modulus on the peak stress and stress distribution in the single-lap bonded joint. The adherends in the overlap length was divided into ten equal regions and material properties assign as a function of the grading. The peak shear stresses could be reduced by 20%. The study previously referred of Sancaktar and Kumar (2000) is effectively a functionally graded adhesive with the use of rubber particles. This is an area that is being intensively studied and where modelling at different scales is essential.

5.8.3 Rivet-Bonded Joints

Liu and Sawa (2001) investigated, using a three-dimensional FE analysis, rivet-bonded joints and found that for thin substrates bonded, riveted joints, adhesive joints and rivet-bonded joints gave similar strengths whilst for thicker substrates the rivet-bonded joints were stronger. They proved this experimentally. Later, the same authors (Liu and Sawa, 2003; Liu et al., 2004) proposed another technique similar to rivet-bonded joints: adhesive joints with adhesively bonded columns. Strength improvements are also obtained in this case. The advantage of this technique of that the appearance is the joint is maintained in relation to an adhesive joint. Grassi et al. (2006) studied through-thickness pins for restricting debond failure in joints. The pins were simulated by tractions acting on the fracture surfaces of the debond crack.

5.8.4 Bolted-Bonded Joints

Chan and Vedhagiri (2001) studied the response of various configurations of single lap joints, namely bonded, bolted and bonded-bolted joints by three-dimensional FE method and the results were validated experimentally. The authors found that for the bonded-bolted joints, the bolts help to reduce the stresses at the edge of the overlap, especially after the initiation of failure. The same type of study was carried out by Lin and Jen (1999).

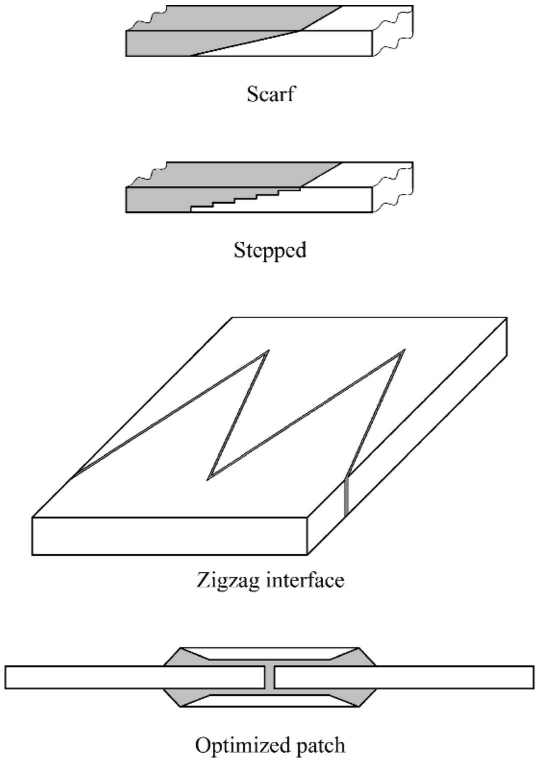
5.8.5 Weld-Bonded Joints

Al-Samhann and Darwish (2003) demonstrated with the FE method that the stress peaks typical of adhesive joints can be reduced by the inclusion of a weld spot in the middle of the overlap. They studied later the effect of adhesive modulus and thickness (Darwish and Al-Samhann, 2004). Darwish (2004) also investigated weld-bonded joints between dissimilar materials.

5.9 Repair Techniques

Adhesively bonded repairs are generally associated to complex geometries and the FE method has been extensively used for the optimization of the repair, especially with composites. The literature review of Odi and Friend (2002) about repair techniques illustrates clearly this point. Typical methods and geometries are presented in Fig. 5.9. Among the various techniques available, bonded scarf or stepped repairs are particularly attractive because a flush surface is maintained which permits a good aerodynamic behavior. Odi and Friend (2004) show that an improved 2D plane stress model is sufficient to get reliable joint strength predictions. Gunnion and Herszberg (2006) studied scarf repairs and carried out a FE analysis to asses the effect of various parameters. They found that the adhesive stress is not much influenced by mismatched adherend lay-ups and that there is a huge reduction in peak stresses with the addition of an over-laminate. Campilho et al. (2007) studied scarf repairs of composites with a cohesive damage model and concluded that the strength of the repair increased exponentially with the scarf angle reduction.

Fig. 5.9 Repair techniques



Bahei-El-Din and Dvorak (2001) proposed new design concepts for the repair of thick composite laminates. The regular butt-joint with a patch on both sides was modified by the inclusion of pointed inserts or a 'zigzag' interface in order to increase the area of contact and improve the joint strength.

Soutis and Hu (1997), Okafor et al. (2005) and Sabelkin et al. (2007) studied numerically and experimentally bonded composite patch repairs to repair cracked aircraft aluminum panels. The authors concluded that the bonded patch repair provides a considerable increase in the residual strength.

Tong and Sun (2003) developed a pseudo-3D element to perform a simplified analysis of bonded repairs to curved structures. The analysis is supported by a full 3D FE analysis. The authors found that external patches are preferred when the shell is under an internal pressure while internal patches are preferred when under an external pressure.

5.10 Special Techniques in Finite Element Simulation

One of the advantages of closed form analyses in relation to the FE method is that parametric studies are much easier to perform. With conventional FE techniques, every time a geometric parameter of the model (adhesive or adherend thicknesses, angle of the spew fillet, etc.) with a different value is considered, a new model is required which is time consuming. However, recent FE libraries include connections that can be described in a parametric form. Similar families of connections need to be meshed only once and the users need only enter the parameters. Actis and Szabó (2003) have used parametric models for the study of bonded and fastened repairs. These models have associated p-type meshing, where convergence of the solution is achieved by increasing the polynomial order of the element rather than increasing the number of elements in the model (known as h-type meshing). More recently, Kilic et al. (2006) present a finite element technique utilizing a global element coupled with traditional elements. The global element includes the singular behaviour at the junction of dissimilar materials.

Other special techniques in FE simulation for the significant reduction of the modelling (mesh generation) and computation time are presented next, which is critical when complex geometries are involved. The main focus is on techniques which do not require any new code at all or are achievable by very simple routines consisting of a few lines of computer code. Advanced topics which require the development of comprehensive new routines, for instance as in the case of new element formulation (e.g. to better approximate singularities), are not considered. The presented techniques are realisable by actual versions of several commercial finite elements codes.

The techniques will be applied to the example of a single lap joint problem taken from Zhao (1991) where the influence of different degrees of rounding on the joint strength was experimentally and numerically investigated. The general problem with geometric dimensions and boundary conditions is shown in Fig. 5.10.

The adherends were made of 3.2 mm thick aluminium sheets (Young's modulus: 70 GPa; Poisson's ratio: 0.33) and the brittle adhesive epoxy resin Ciba MY750 with hardener HY906 (Young's modulus 2.8 GPa; Poisson's ratio: 0.4) was used. A 2D plane strain problem was modelled because of the large joint width and the

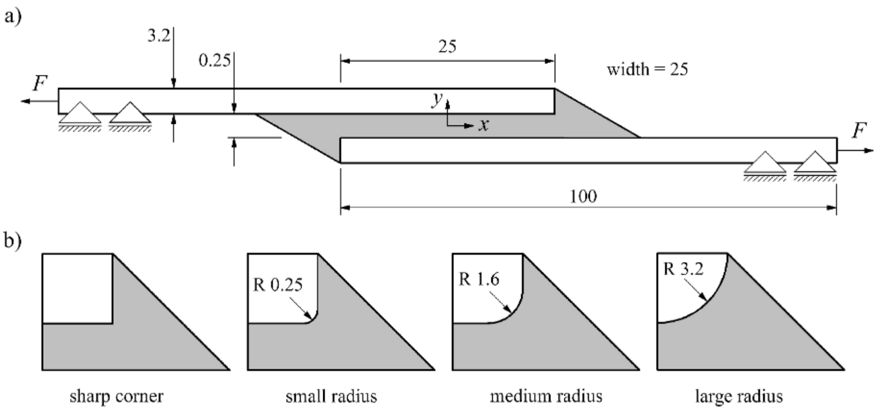


Fig. 5.10 Single lap joint: (**a**) joint geometry and applied boundary conditions (*dimensions in mm*); (**b**) different degrees of rounding (*the grey colour represents the adhesive*) (Zhao, 1991)

ends of the joints were constrained without any rotation to account for the testing conditions. It should be mentioned here that Zhao (1991) used a two steps approach because of limitations in computer power: A first computation was based on a coarse mesh for the entire specimen, followed by a refined mesh in the corner region with the displacement result from the first analysis as the new boundary condition. The following examples which were realised with the commercial code Marc® (MSC Software Corporation, Santa Ana, CA, USA) may be considered as an alternative for such a two step approach.

5.10.1 Consideration of Point Symmetry

Commercial finite element codes allow to consider certain types of symmetry conditions. Common examples are symmetry about an axis (so-called reflective symmetry) or cyclic symmetry (structures with a geometry and a loading varying periodically about a symmetry axis). Point symmetry (or origin symmetry, rotational symmetry by 180°) with respect to the origin of the coordinate system occurs for instance in the case of single lap joints under tensile load (cf. Fig. 5.11a). This type of symmetry is not covered by the standard symmetry conditions which are mentioned above. If one can consider the point symmetric deformation behaviour of a single lap joint (cf. Fig. 5.11b), it is obvious that the mesh size and the resulting system of equations can be reduced by 50% which results in a significant reduction of the calculation time.

Looking at the deformation of a full single lap specimen (cf. Fig. 5.12) in the centre plane, i.e. $x = 0$, one can see that a node at a distance $+a$ from the glue line ($y = 0$) moves under load for the shown arrangement to the negative x and y direction. The movement in the negative y direction is about two orders of magnitude smaller than the displacement in the x direction and difficult to observe in the scale of Fig. 5.12. On the other hand, a node at distance $-a$ from the glue line moves with the same magnitude, however, in the positive coordinate directions. This relationship for a pair of nodes at (0,+ a) and (0,– a) can be expressed by the following constraint condition

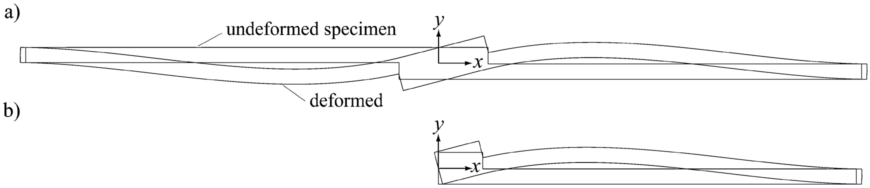


Fig. 5.11 Deformation of a single lap joint under tensile load: (**a**) deformed S-shape of the entire specimen; (**b**) consideration of the S-shape for the half specimen due to appropriate point symmetry condition

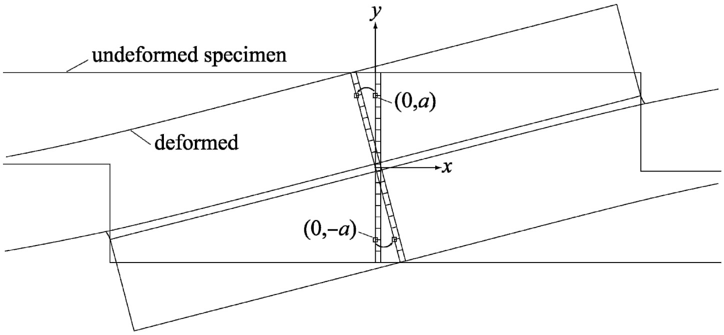


Fig. 5.12 Details of a deformed single lap specimens to illustrate node translation in the centre plane $x = 0$

$$\begin{Bmatrix} u \\ v \end{Bmatrix}_{x=0;y=-a} = \begin{bmatrix} -1.0 & 0 \\ 0 & -1.0 \end{bmatrix} \cdot \begin{Bmatrix} u \\ v \end{Bmatrix}_{x=0;y=+a}, \tag{5.1}$$

where the vector of displacement at node $y = -a$ is referred to as the tied ("slave") node while the node on the right-hand side is referred to as the retained node ("master"). The connecting matrix is called the constrained matrix. Some commercial finite element codes allow the definition of arbitrary homogeneous constraints between nodal displacements by user subroutines (in the case of MSC Marc: UFORMS routine). When such a routine is supplied, the user is simply replacing the one which exists in the program using appropriate control setup. In the considered case of point symmetry, it must be mentioned that the constraint condition must be defined for each pair of nodes in the centre plane $x = 0$.

5.10.2 Connecting Dissimilar Meshes

Modern adhesives can be applied in films of several micron of thickness while the surrounding adherends may extend to a much larger scale in the range of millimetres, centimetres etc. If accuracy in the adhesive layer is requested, several elements must be introduced over the adhesive thickness and this mesh density must be coarsened in order to limit the total number of elements to account for limitations in computer hardware (in particular the random access memory, RAM). In addition, regions with high stress gradients require a fine and regular mesh if these changes should be evaluated. Such constraints may result in a quite complicated mesh which is difficult to generate and a huge system of equations whose solution is time-consuming. In addition, it might be necessary to introduce transition elements which may have poor performance for specific loading conditions, e.g. bending. As an alternative, the approach of *dissimilar* meshes (cf. Fig. 5.13) may be introduced.

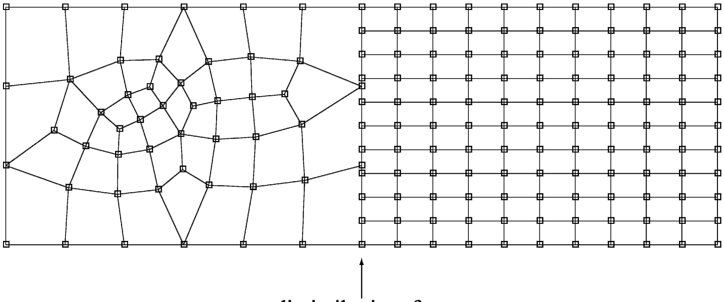


Fig. 5.13 Example of a finite element model composed of two dissimilar meshes (nodes are not coincident at the interface)

The major idea is to generate different types of meshes which are not connected in the classical way, i.e. by common nodes which belong to both touching elements. Regions where high accuracy of the analysis is requested may be composed of a very fine and regular mesh (Fig. 5.13, right part) while other parts can be modelled by coarse and irregular element representations (Fig. 5.13, left part).

One method of connecting these dissimilar meshes is to use interface elements (Schiermeier et al., 2001). Nowadays, actual versions of commercial finite element codes allow such an application in the scope of novel contact options, i.e. to "glue" dissimilar contacting meshes without the need for interface elements. In such a case, by specifying that the glue motion is activated, the constraint equations are automatically written between the two meshes and the contact region is not allowed to separate.

In the case of the corner rounding influence, a basic single lap joint was meshed with a quite coarse mesh (adherend length: 96.8 mm) and several refined inlays (cf. Fig. 5.10b) were separately generated and successively "glued" to the basic joint. Figure 5.14 shows as an example the refined inlay with the sharp corner which is "glued" at the dissimilar interfaces to the basic joint. Similar meshes for the inlays with different degree of rounding were obtained.

To present some numerical results, the normalised von Mises stress along the bond line for the different configurations of corners is shown in Fig. 5.15. It can be clearly seen that the introduction of a rounding decreases the stress peak (as shown in Peppiatt (1974) and Chen (1985)) and thus, results in a higher strength of the adhesive joint. The difference in the peak values are presented in Table 5.1 and compared to experimental findings taken from Zhao (1991). It can be seen that this numerical simulations reveals the same tendency as the experimental values. It must be noted here that the presented numerical results are based on a pure linear elastic analysis and small deformations and an additional consideration of the non-linear material behaviour and appropriate yield conditions of both components can improve the obtained results compared to experimental values.

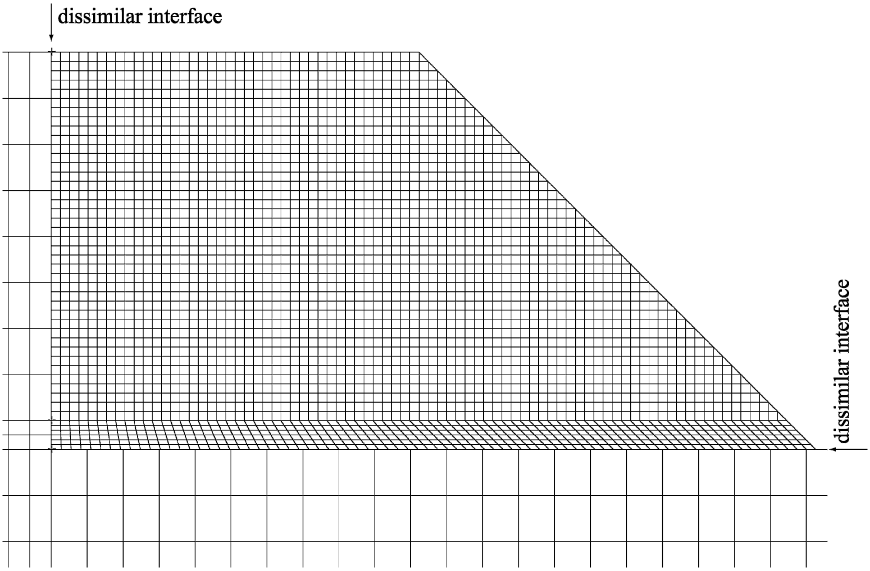


Fig. 5.14 Dissimilar meshes in the case of the sharp corner: the corner region consists of a much finer mesh

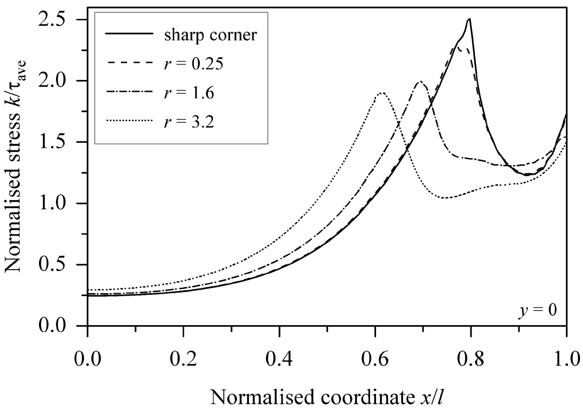


Fig. 5.15 von Mises stress k normalised by average shear stress τ_{ave} along the normalised bond line ($y = 0$). Stress distributions are obtained or the same external load F

Table 5.1 Comparison between joint strength prediction and experiments

Corners	FE (Strength increase in %)	Experiment, Zhao (1991) (Strength increase in %)
sharp	0 (ref.)	0 (ref.)
$r = 0.25$	8.54	16.50
$r = 1.6$	20.55	20.00
$r = 3.2$	24.28	40.15

5.11 Conclusions

This chapter describes studies that deal with adhesive joints of complex geometry using the FE method. The main conclusions are:

1. The FE method can give the stress distribution in the whole bonded structure and is an efficient tool to identify the stress concentrations.
2. For this reason, the FE method is the most adequate to develop and optimize adhesive joints.
3. FE studies of different complexity were discussed: lap joints with irregular shapes, rounding of adherend ends, spew fillet, hybrid joints and repair techniques.
4. The stress concentrations at sharp corners, where the failure is likely to occur, are difficult to handle using traditional stress-strain approaches because the results are mesh dependent.
5. This problem can be solved, to some extend, rounding the edges, using a strength singularity approach or a cohesive zone model. However, even these methods require some kind of experimental 'tuning'.
6. The optimization of joint strength by the use of functionally graded adhesives is one the main challenges in adhesive joints modelling.
7. Parametric studies are more difficult to perform with the FE method due to the re-meshing problem and the computation time. However, recent FE programs include routines for automatic re-meshing. Submodelling is another technique for saving computation time and change geometric parameters in an easier way. This has been performed in the present chapter and validated with experimental results.

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