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Integrated Production and Distribution Planning of Perishable Goods

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"Quand il m'arrive quelque chose, je préfère être là."
L'Étranger, Albert Camus

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Integrated Production and Distribution of Perishable Goods

Abstract: The pressure of reducing costs in supply chains forces companies to take an integrated view of their production and distribution processes. In perishable goods besides the cost issue there is an important freshness concern that shall not be disregarded. This challenging logistic problem involves several tightly interrelated production planning, scheduling, distribution and routing problems. Even when considered as independent from the other ones, each of the mentioned problems is of large combinatorial complexity. This thesis aims to understand the potential impact of perishability in the production and distribution planning both in a decoupled and integrated framework. The contribution of this research is aligned in three axis. First, realistic and integrated mathematical programming models are proposed to tackle these problems. These models gradually incorporate the consumer purchasing behaviour of perishable goods. Second, new state-of-the-art hybrid optimization algorithms are developed. These algorithms combine exact and metaheuristic methods, such as evolutionary algorithms to solve problems of proved difficulty. And, finally, managerial insights are given based on the numerical experiments performed in real-world and random-generated instances. These insights aim to deliver advices to practitioners related to the management of different supply chain processes of perishable goods.
Keywords: supply chain planning, perishability, mixed-integer programming, metaheuristics

Planeamento Integrado da Produção e Distribuição de Produtos Perecíveis

Resumo: A pressão de redução de custos nas cadeias de abastecimento obriga as empresas a ter uma visão integrada dos seus processos de produção e de distribuição. Em bens perecíveis, além da questão do custo, há uma preocupação relacionada com a frescura dos produtos que não deve ser desconsiderada. Este desafiador problema logístico envolve diversos sub-problemas de planeamento fortemente inter-relacionados, tais como, escalonamento, distribuição e roteamento. Mesmo quando considerados independentes uns dos outros, cada um dos problemas mencionados é de elevada complexidade combinatória. Esta tese tem como objetivo compreender o impacto potencial da perecibilidade no planeamento da produção e da distribuição, tanto num quadro desacoplado como num quadro integrado. A contribuição desta investigação está alinhada em três eixos. Primeiro, modelos realistas e integrados de programação matemática são propostos para resolver estes problemas. Estes modelos incorporam gradualmente o comportamento de compra do consumidor de bens perecíveis. Segundo, novos algoritmos de otimização híbridos são desenvolvidos. Esses algoritmos combinam métodos exatos e meta-heurísticos, tais como algoritmos evolutivos, para resolver problemas de grande dificuldade. E, finalmente, *guidelines* sobre as melhores práticas de gestão destes processos da cadeia de abastecimento são sugeridas com base nas experiências numéricas realizados em instâncias do mundo real e instâncias geradas aleatoriamente.

Palavras-chave: planeamento da cadeia de abastecimento, perecibilidade, programação inteira-mista, meta-heurísticas

Planification Intégrée de la Production et Distribution de Denrées Périssables

Résumé: La pression de la réduction des coûts sur les chaînes d'approvisionnement oblige les entreprises à adopter une vision intégrée de leur processus de production et de distribution. Pour les denrées périssables en plus de la question des coûts, la fraîcheur est une préoccupation non-négligeable. Elle entraîne un certain nombre de défis logistiques et de production qui sont étroitement liés entre eux. Même en considérent indépendamment, chacun des problèmes évoqués est d'une grande complexité combinatoire. Cette thèse vise à comprendre l'impact potentiel de la nature périssable des produits dans la planification de la production et de la distribution, à la fois dans un cadre découplé et intégré. La contribution de cette recherche est orientée en trois axes. Tout d'abord, des modèles réalistes de programmation mathématique sont proposés pour résoudre ces problèmes. Ces modèles intègrent progressivement le comportement d'achat des consommateurs de denrées périssables. Deuxièmement, des nouveaux algorithmes de pointe en matière d'optimisation sont développés. Ces algorithmes combinent des méthodes exactes et métaheuristiques, tels que les algorithmes évolutionnaires pour résoudre les problèmes dont la difficulté est avérée. Enfin, cette thèse cherche à fournir des conseils sur la gestion des différents processus de la chaîne d'approvisionnement de denrées périssables, sur la base des expériences numériques effectuées en conditions réelles.
Mots-Clefs: planification de la chaîne d'approvisionnement, périssable, programmation linéaire en nombres entiers, métaheuristiques

Contents

Chapter 1

Thesis Motivation and Framework

The supply chain planning of perishable goods is ruled by the dynamic nature of its products. Throughout the planning horizon, the characteristics of these products go through significant changes. The root cause for these changes may be related to, for example, the physical nature of the product or the value that the customer lends it. Without acknowledging the perishable nature of the products, one may incur in avoidable spoilage costs (for example, in the case of meat products) or, on the other hand, sell the product before it is close enough to its best state (for example, in the case of cheese products). In this thesis we are concerned with perishable products that start losing their properties after being produced.

Fleischmann et al. (2008) define planning as the activity that supports decision-making by identifying the potential *alternatives* and making the best decisions according to the *objective* of the planners. Let us look into the challenges of engaging in this planning activity in the context of perishable goods.

In order to identify the *alternatives* it is important to understand the decisions that the decision maker wants to make. It is common to organize the supply chain planning according to two dimensions: the supply chain process and the hierarchical level. The scope of this research is framed by the production and distribution supply chain processes in the tactical and operational decision levels. Hence, literature problems, such as lot-sizing, scheduling and vehicle routing will be addressed here. Traditionally, and for the case of non-perishable products, the usage of inventory to decouple the production and distribution processes is possible. Nevertheless, it is acknowledged that some advantages may be leveraged from an integrated planning approach. On the other hand, with the inclusion of the perishability phenomenon, which results in a (most of the times) decaying value of the stored and transported products, there is a need to further investigate the pertinence of an integrated planning approach. In practice, especially for highly perishable goods, such as a take-away pizza or regular catering, it is common to schedule both processes simultaneously in a Just-in-Time fashion. A key contribution of this thesis is to give quantitative and theoretical insights of the integrated planning for perishable goods. To achieve such target, a systematic approach is used that starts by analysing the planning of each of these processes separately. Then, having enough knowledge about the interaction of perishability in the decoupled case, an integrated perspective of the whole problem comprising these two (production and distribution) supply chain processes is taken.

The second part of Fleischmann et al. (2008) definition of planning relates to the *objectives* of the planners. The literature in supply chain planning tackles most of the problems with single objective models. The objective is usually related either to an operational measure, such as makespan, or to some monetary measure, such as cost or profit. Nevertheless, it is also acknowledged that to show to decision makers the trade-offs of the choices measured by different dimensions, a multi-objective approach needs to be undertaken. In this thesis, we intend to use a multi-objective approach in a more instrumental way, in order to understand the relation between cost (probably, the most used objective) and freshness, which is an indicator we consider to be crucial for this kind of products. Therefore, from our perspective and taking a supply chain oriented view, we think that besides avoiding the products spoilage, there may exist a substantial intangible gain from delivering fresher products to customers. Such considerations are closely related to the consumer purchasing behaviour of perishable goods that should worry any planner in a (food) company with a supply chain orientation. Thus, the second gap that is closed in this thesis is the introduction of consumer purchasing behaviour related issues into the production and distribution planning of perishable goods, in order to leverage the related intangible gains. These gains are mostly directed towards customer satisfaction, customer loyalty and intertwined spillover effects.

One thing that is not mentioned in Fleischmann et al. (2008) definition of planning is the difficulty of solving these planning and scheduling problems. In fact, this depends, of course, on the type of problems solved. In this thesis, however, the hardness of the problems poses several challenges that need to be addressed. In this sense, we aim at understanding the suitability of hybrid solution methods, combining meta-heuristics with exact algorithms in order to solve the arising coupled/decoupled supply chain planning problems. Therefore, despite being a secondary contribution of this thesis, we aim at showing the suitability of this type of solution methods to solve supply chain planning problems dealing with perishability. Moreover, several contributions are also made to the research field on hybrid solution methods.

In the remainder of this chapter we will pose the research questions mainly related with the supply chain planning of perishable goods (Section 1.1). The second part of the chapter (Section 1.2) gives guidance to the reader about the organization and subjects of the thesis and the (un)published work that serves as basis for this document.

1.1 Research objectives

This research intends to give new insights into the current literature by formulating novel mathematical programming models extending the existing approaches regarding production planning, distribution planning and integrated approaches dealing with perishable products. These models need to be able to handle the complexities and specificities of perishable goods industries (namely the synchronization between stages, sequence dependent setups, deterioration of products and customers with tight requirements) turning them very realistic. Both production and distribution planning problems may be NP-hard and, therefore, computationally intensive to

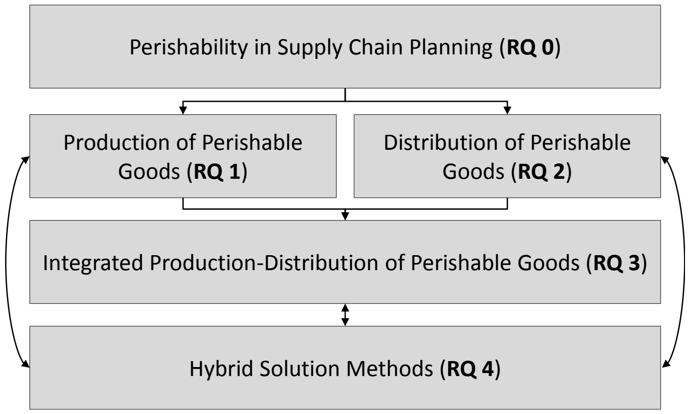

Figure 1.1: Research canvas and research questions.

solve. There is a strong need for approaches that are capable of finding satisfactory solutions to these complex problems in short computational times. To develop solution methods for the arising problems, new concepts will be necessary, and we seek to obtain breakthroughs on state-of-the-art solution techniques incorporating ideas from exact methods, heuristic and meta-heuristic algorithms. These methods will be tested not only on artificial instances, but also on real-world data. Moreover, this research will potentially improve the supply chain planning processes of industries dealing with perishable products. We find the integration of scientific breakthroughs within companies to be fundamental, as there is a significant gap between theory and practice regarding decision support for supply chain planning tasks.

In Figure 1.1 a visual representation of the research questions (RQs) that this thesis is intended to answer and their relationship is presented.

All supply chain planning problems, either in the production process or in the distribution process, will be modelled taking into account the perishability phenomenon explicitly.

Research question 0:

How to define perishability in light of supply chain planning? Do different types of perishability enforce different modelling techniques?

The classifications proposed so far for perishability do not seem to address completely the complexities of this phenomenon, and the linkage of perishabilty and supply chain modelling is very circumvented to inventory management. Therefore, to have this research question answered, a new and comprehensive classification and definition for perishability is needed. Furthermore, a deep understanding between the classification and different modelling techniques is of great value in guiding future works in the area.

After understanding the dynamics of perishability, each of the core supply chain processes (production and distribution) will raise the following research questions.

Research question 1:

How to comprehensively address perishability in production planning for perishable goods?

The outcome related with this question will be a set of formulations that model the principal characteristics of complex production systems dealing with perishable goods. These models need to account for different features such as sequence dependent setup times and costs, recipe structures or different production strategies (make-to-order and make-to-stock). Moreover, consumer purchasing behaviour of perishable goods will be gradually taken into account in the developed formulations.

Research question 2:

How to comprehensively address perishability in distribution planning for perishable goods?

This research question shares a very parallel path with research question 1. In this case, we focus on planning problems such as the vehicle routing problem. Customers of perishable products are usually very demanding both in terms of product freshness and service quality. Therefore, the models developed within this research question will take into account both various distribution scenarios and preferably product freshness. We expect to be able to draw several conclusions regarding the influence of different possible distribution scenarios in the perishable state of products.

As a natural sequence of the above mentioned research questions, comes research question 3.

Research question 3:

What is the impact of using an integrated approach over a decoupled one for supply chain planning problems dealing with perishable goods?

As a scientific outcome of this research question it will be possible to understand if the integrated planning of perishable products entails more advantages than the integrated planning for non-perishable goods. Furthermore, we hope to expand the base study so that besides comparing the use of a decoupled approach vis-à-vis an integrated one between perishable and non-perishable products, we can also compare the advantages of the integrated approach among different kinds of perishable goods.

In order to be able to answer the three previous research questions we need to solve very hard combinatorial problems with efficiency and efficacy. Thus, the final research question stems naturally from this need.

Research question 4:

Are innovative hybrid solution methods suitable to solve hard mixed-integer models related with supply chain planning problems of perishable goods?

The outcome of this research question will be materialized through a set of hybrid solution methods that will try to share as most as possible in common regarding its structure and, simultaneously, deliver good results for the different supply chain planning problems tackled. We will be able to contribute to the literature by showing the application of hybrid ideas mixing meta-heuristics with mathematical programming in solving hard supply chain planning problems.

1.2 Thesis structure and synopsis

This is a cumulative thesis. Hence, several articles are aligned to answer the questions previously presented in this chapter. Despite the cumulative structure of the thesis, we clustered papers in a specific subject under a given chapter. In the remainder of this section we briefly point out the main subjects and contributions dealt with in each of the following chapters and the references where this research was originally published or submitted.

In Chapter 2 we review planning models that handle perishability issues in production and distribution planning. It has a threefold contribution to the literature. First of all, it fills a gap by proposing a unified framework for classifying perishability. Since the community started to worry about perishability issues, a lot of different definitions have been put forward. In this research there is an attempt to unify these different perspectives by proposing a more comprehensive classification based on three dimensions: authority limits, customer value and physical deterioration. Second, this chapter aims to provide a general review of references for researchers in the field of production and distribution planning dealing with perishability. These two contributions will be crossed by categorizing the reviewed literature according to the proposed framework for classifying perishability within the different types of planning problems. Third, this comprehensive review will allow us to indicate new areas for further research. The power of supply chain management comes from the ability to take an integrated look at closely related processes. When one or more of these links is subject to perishability, specific modelling issues have to be accounted for. Hence, this review points out the existing gaps in modelling these aspects. The reference that serves as basis for this chapter is:

- *Pedro Amorim, Herbert Meyr, Christian Almeder and Bernardo Almada-Lobo. Managing Perishability in Production-Distribution Planning: a discussion and review. Flexible Services and Manufacturing Journal, pages 1-25, 2011. DOI: 10.1007/s10696-011-9122-3.*

In Chapter 3, the main complexities related to the production planning problems of perishable products are addressed. We consider a general setting within the fast moving food consumer goods in which, generally, a large numbers of products are produced from a few initial product recipes. Most of these production systems fall in the category of make-and-pack environments. The focus is on the packaging stage where the divergent product structure (i.e., a low number of raw materials leading to a high number of final products) shows its impact by imposing to the planner the need to deal with a great amount of stock keeping units to lot-size and schedule. Hence, general production system characteristics of these industries are modelled encompassing characteristics such as sequence dependent changeover times and costs, hybrid production strategies and demand uncertainty. Simultaneously, a further specification of these models is conducted, based on the evidence that consumers of perishable goods look for visual and other cues of freshness, such as the printed expiry dates, . The main contributions of this chapter are aligned in three distinct axis. Firstly, a thoroughly description of modelling features important for production systems dealing with perishable goods is provided. Secondly, regarding hybrid methods, we propose a hybridization of a multi-objective evolutionary algorithm with a mixed-integer programming (MIP) solver and a hybridization of a truncated path-relinking with a MIP solver. Both of these solution methods take advantage of the special structure of the underlying production planning problems dealing with perishable goods. Thirdly, the computational experiments that were run on random-generated and real-world instances gave important managerial insights related to the production planning of perishable goods. This chapter is based on the following references:

- *Pedro Amorim, Carlos H. Antunes and Bernardo Almada-Lobo. Multi-Objective Lot-Sizing and Scheduling Dealing with Perishability Issues. Industrial Engineering and Chemistry Research, 50, 3371-3381, 2011.*
- *Pedro Amorim, Carlos H. Antunes, Bernardo Almada-Lobo. A dual mutation operator to solve the multi-objective production planning of perishable goods. Operations Research / Computer Science Interfaces Series, pages 1-22, 2012. (Accepted)*
- *Pedro Amorim, Alysson M. Costa and Bernardo Almada-Lobo. Influence of Consumer Purchasing Behaviour on the Production Planning of Perishable Food. 28pp, 2012.*
- *Pedro Amorim, Alysson M. Costa and Bernardo Almada-Lobo. A Hybrid Path-Relinking Method for Solving a Stochastic Lot Sizing and Scheduling Problem. Proceeding of the fourth international workshop on model-based Metaheuristics'12, September 17-20, Angra dos Reis, Brazil, pages 1-12, 2012.*

Chapter 4 studies the impact of products' perishability in the vehicle routing problem. We consider highly perishable food products that can lose an important part of their value in the distribution process. Hence a multitude of products are considered to be delivered to a set of customers with many requirements. As we disregard the production process in this chapter, we consider that a Just-in-Time strategy is in place, assuring a very good freshness state before starting the delivery. We propose a multi-objective model that decouples the minimization of the distribution costs from the maximization of the freshness state of the delivered products. This model is instrumental to examine the relation between different distribution scenarios and the cost-freshness trade-off. The main conclusions point out that, first, there is an evident trade-off between the mentioned objectives; second, time windows have a strong impact on the freshness levels of products delivered, hence, large time windows lead to less spoilage; finally, regarding customer typology no conclusions could be taken. In this experience, small size instances adapted from the vehicle routing problem with time windows are solved with an ε-constraint method and for large size instances a multi-objective evolutionary algorithm is implemented. In the second article of the chapter, a real-world problem from a food distribution company is solved. Through the analysis of its characteristics, it is classified as a *heterogeneous fleet site dependent vehicle routing problem with multiple time windows*. Since this specific problem has never been solved we chose to use a very general search procedure that has proven to deliver very good results for different vehicle routing related problems – the ALNS (Adaptative Large Neighbourhood Search) framework. The results show an average cost reduction for the company of about 17% in the distribution task at peak seasons. These savings are mainly achieved through a better capacity utilization of the vehicles and a reduction on the distance travelled to visit all customers. Hence, two research papers are embedded in this chapter:

• *Pedro Amorim and Bernardo Almada-Lobo. The Impact of Food Perishability Issues in the Vehicle Routing Problem. 30pp, 2012.*
• *Pedro Amorim, Sophie Parragh, Fabrício Sperandio, Bernardo Almada-Lobo. A rich vehicle routing problem dealing with perishable food: a case study. TOP, 2012. (Accepted with minor revision)*

In Chapter 5, we extend the integrated production and distribution planning to tackle highly perishable products. Firstly, we explore through a multi-objective framework the advantages of integrating these two intertwined planning problems when using direct deliveries. We formulate models for the case where perishable goods have a fixed and a loose shelf-life (i.e. with and without a best-before-date, respectively) and for the coupled and decoupled planning approaches. After solving illustrative instance with several different methods, the results show that the economic benefits derived from using an integrated approach are much dependent on the freshness level and nature of the delivered products. Secondly, we study the joint production and distribution planning, which details the complete routing needed to visit all customers. This problem has only been addressed by batching the orders, disregarding the sizing of the lots in the production process. In order to investigate this issue two exact models are presented for this problem. The first model includes batching decisions and the latter lot-sizing decisions. The value of considering lot-sizing versus batching is further investigated per type of scenario. In a scenario a key instance attribute is varied such as the type of customer time windows. Results point out that lot-sizing is able to deliver better solutions than batching. The added flexibility of lot-sizing can reduce production setup costs and both fixed and variable distribution costs. Moreover, the savings derived from lot-sizing are leveraged in scenarios with customer oriented time windows and production systems with non-triangular setups. The following research papers are the basis of this chapter:

• *Pedro Amorim, Hans-Otto Günther and Bernardo Almada-Lobo. Multi-Objective*

Integrated Production and Distribution Planning of Perishable Products. International Journal of Production Economics, 138, 89-101, 2012.
• *Pedro Amorim, Marcio B. Filho, Franklina Toledo and Bernardo Almada-Lobo. Lot Sizing versus Batching in the Production and Distribution Planning of Perishable Goods. 14pp, 2012.*

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Chapter 2

Perishability Classification and Modelling of Production and Distribution Planning Problems Dealing with Perishable Goods

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Managing perishability in production-distribution planning: a discussion and review

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Abstract Managing perishability may represent a remarkable problem in supply chain management of a varied set of industries. In fact, perishability can influence, for example, productivity or customer service and it may happen to occur in one or more processes throughout the supply chain. In this paper a review on planning models that handle perishability issues in production and distribution is conducted. The contribution of this paper is three-fold. First, a new framework for classifying perishability models based on multiple process features is presented. Second, it draws the community attention to the importance of managing perishability in many different industries' supply chains by showing its relevance and by reviewing the literature related to production and distribution planning. Finally, it points towards research opportunities so far not addressed by the research community in this challenging field.

Keywords Perishability · Production-distribution planning · Survey

1 Introduction

In many kinds of industries ranging from discrete manufacturing to process industry, raw materials, intermediate goods and final products are often perishable.

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This fact enforces specific constraints on a set of different supply chain processes, such as, procurement, production planning and inventory management, as well as on distribution.

For example, in the yoghurt industry, perishability is found in every link of the supply chain: from the raw milk that enters the dairy factories and has to be processed within strict time limits, to the intermediate products that are highly perishable and, finally, the different final products which are all stamped with a best-before-date (BBD) fixing its shelf-life. Looking at the beer industry, before bottling and pasteurizing the final products, beer can only stay a fixed amount of time in buffer tanks before it perishes. Taking a broader view of perishability, in companies producing and distributing daily newspapers, perishability is induced by the actuality of the product that forces a very accurate integration of the supply chain processes. More examples like these can be drawn from other processed food industries, chemical industries, blood banks, or from the agri-food business.

It is rather difficult to identify or characterize a general perishable supply chain, since perishability is a phenomenon that may happen to occur in a wide variety of situations and it has a rather fuzzy definition. Wee (1993) defined perishability as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of a commodity that results in decreasing usefulness from the original one.

Although managing perishability presents some of the most important challenges in supply chain management (Shulman 2001), the literature had overlooked this important issue for a long time. In fact, the major body of literature on perishability is concerned about inventory management. Some current trends seem to point towards an increasing interest in the management of supply chains of perishable goods. From a consumer point of view, nowadays, buyers are worried about having as much information as possible, especially with products that may have an impact on health, such as food products which are highly perishable. These expectations demand increasing traceability and higher production standards for perishable products. To incorporate such trends, a critical regard to the current practices and plans of these supply chains needs to be taken. For example, producers will need to review the use of inventory as a buffer to hedge against demand variability and move towards more integrated approaches. In increasingly competitive markets, producers need to pay special attention to opportunities to achieve efficiency and consumer satisfaction gains. This takes a higher dimension in supply chains of perishable goods where products may get spoiled incurring companies in avoidable costs. Moreover, product freshness is highly related to customer satisfaction, so a good management of final products' perishability may entail strong competitive advantages. On a strategic level, perishability may also play an important role by forcing better relations and more integration between the supply chain network of organizations. For example, most of the food industries rely on third party logistic providers (3PL) to perform the distribution of their products. A good relation between those companies shall result in additional gains. At the other end of the supply chain, one can think of a vendor-managed inventory relating a supplier of a perishable raw product and a company that processes this product. This should potentially lead to less spoilage and, hence, less costs.

In this paper, the focus is kept on handling perishability explicitly through analytical planning models. So far, reviews only concerned about perishability were most of them done in the field of inventory management such as Nahmias (1982), Raafat (1991), Goyal and Giri (2001) and Karaesmen et al. (2009). Thus, our review distinguishes from these ones by looking at different and important supply chain planning problems that may benefit from tackling perishability explicitly. There is also the review performed in Pahl et al. (2007) that is only focused in production planning and inventory. However, this review is distinct from ours because no classification or framework is used to analyse each paper. Furthermore, it does not distinguish between papers dealing explicitly or implicitly with this phenomenon. There are other reviews worth mentioning concerned not only about perishability, but that also have a relation with it. Ahumada and Villalobos (2009) review models for the agri-food business where products may be perishable or not, but their focus is on procurement and harvesting planning and the only goods they are interested in are crops. Finally, Akkerman et al. (2010) review works done in the field of food distribution where different characteristics are identified as key issues such as quality, safety and sustainability. Hence, their scope is broader in the subjects dealt and they are not only concerned about explicitly modelling these characteristics.

Our work has a threefold contribution. First of all, this paper fills a gap by proposing a unified framework for classifying perishability. Since the community started to worry about perishability issues, a lot of different definitions have been put forward. In this work there is an attempt to unify these different perspectives by proposing a more comprehensive classification based on three dimensions. Second, this paper aims to provide a general review of references for researchers in the field of production and distribution planning dealing with perishability. Third, this comprehensive review will allow us to indicate new areas for future research. The power of supply chain management comes from the ability to take an integrated look at closely related processes. When one or more of these links is subject to perishability, specific modelling issues have to be accounted for. Hence, this review points out the existing gaps in modelling these aspects.

The remainder of the paper is organized as follows. In Sect. 2, perishability is formally treated, its characteristics and modelling approaches are exposed, and a unified framework for its classification is presented. Sect. 3 consists of an organized review of the works dealing with the problem of modelling perishability in production and distribution planning. These works will be examined in light of the proposed framework. The paper ends in Sects. 4 and 5 where the main conclusions and research opportunities are pointed out, respectively.

1.1 Perishability classification: a new unified framework

Throughout the years a lot of different, complementary and, sometimes, contradictory classifications were proposed to deal with perishability. In Ghare and Schrader (1963), the authors classify the deteriorating properties of inventory with three categories: (1) direct spoilage, e.g., vegetables, flowers and fresh food, etc.; (2) physical depletion, e.g., gasoline and alcohol, etc.; (3) decay and obsolescence, for instance in radioactive products and with the loss of value in inventory, e.g., newspapers and uranium. Nahmias (1982) distinguishes two classifications of perishability: (1) fixed lifetime: items' lifetime is specified beforehand and, therefore, the impact of the deteriorating factors is taken into account when fixing it. In fact, the utility of these items degrades during their lifetime until the good perishes completely and has no value to the customer, e.g., milk, yoghurt and blood in inventory, etc. (2) random lifetime: there is no specified lifetime for these items. Hence, the lifetime of these goods can be modelled as a random variable according to a given probability distribution. Examples of products in this category are fruits, vegetables and flowers. In another work, Raafat (1991) first defines decay or deterioration as "any process that prevents an item from being used for its intended original use" and names the examples of spoilage (e.g., foodstuff), physical depletion (e.g., evaporation of volatile liquids), and decay (e.g., radioactive substances). These examples are strictly related to the ones mentioned by Ghare and Schrader (1963). Afterwards, Raafat (1991) gives a categorization of perishability depending on the relation between time and value of the inventory: (1) utility constant: its utility remains the same as times goes by until the end of the usage period, e.g., liquid medicine; (2) utility increasing: its utility increases as time passes, e.g., some cheeses or wines; (3) utility decreasing: its utility decreases as time passes, e.g., fruits, vegetables and other fresh foods, etc. More recently, Lin et al. (2006) state that deterioration can be classified as: (1) age-dependent on-going deterioration and (2) age-independent on-going deterioration, with the assumption that the aging process starts just after production. Meat, vegetables and fruits are examples of goods subject to age-dependent on-going deterioration. Volatile liquids such as gasoline and alcohol, radioactive materials, and agri-food products are examples of age-independent on-going deteriorating items. In these items it is hard to define a dependency between age and perishability since these products can be stored indefinitely though they suffer natural attrition while being held in inventory degrading its condition. Finally, Ferguson and Koenigsberg (2007) emphasize the related utility loss and distinguish two kinds of products: (1) with functionality deteriorating over time, e.g., fruits, vegetables, or milk; (2) without functionality deteriorating, but customers perceived utility deteriorates over time, e.g., fashion clothes, high technology products with a short life cycle, newspapers.

There is another concept very related with perishability and deterioration which is shelf-life. Shelf-life is defined as the period of time after the manufacture of a product during which it is of satisfactory quality (Kilcast and Subramaniam 2000). It is the length of time that a given item can remain in a saleable condition on a retailer's shelf. Shelf-life does not necessarily reflect the physical state of a product, since many products deteriorate only a while after their shelf-life finishes, however, it may reflect its marketable life (Xu and Sarker 2003).

The enumeration of perishability's classifications shows that these categories overlap among each other and, furthermore, they are highly tailored for a specific propose. Either they are specially concerned about the customer and then reflecting the utility of the good (Raafat 1991), or concerned about the physical state of the product itself (Ghare and Schrader 1963), or, for example, looking more into the mathematical modelling point of view of perishability (Nahmias, 1982). In fact, these classifications were neither always used to classify the papers dealing with perishability nor to define the applicability of the proposed models. Most of the times, different classifications based on the mathematical properties of the modelling approaches were used afterwards (cf. Raafat 1991), thus, neglecting the previous classifications concerned with perishability and loosing, consequently, the linkage to the underlying perishability phenomenon expressed by the model. A perishable good that illustrates the difficulty of having a clear and univocal classification, with the existing classifications, is the yoghurt. Yoghurt has a fixed shelf-life expressed by its BBD, when it perishes (after the BBD) its value is close to zero and, finally, from the customer's point of view it has been proven empirically by Tsiros and Heilman (2005) that the willingness to pay for it decreases over the shelf-life of the product. Hence, although its utility decreases, it has a fixed lifetime and its functionality does not deteriorate over time.

In light of the above discussion, we propose a unified framework to classify perishability and, thus, make clear the contribution of past and future works on this current and interesting research field. Table 1 shows the proposed framework with corresponding examples for the different categories. This framework intends to allow for an exhaustive classification of perishability in conceptual forms through relating different perspectives of the same phenomenon. This means that it may be used to understand the conceptual scope of the perishable phenomenon we want to model. For supply chain planning purposes it is important to link the conceptual form with a mathematical representation. This means that while in other reviews concerned about perishability issues the focus was on classifying mathematical models that were able to cope with perishability, in this review we rather want to understand how the mathematical model approach grasps certain aspects of perishability by clustering it in our framework. This classification will be helpful in Sect. 4 when literature gaps are identified and in building a kind of roadmap linking the specific perishability issues with mathematical modelling techniques.

The proposed framework for classifying perishability is composed of three classifying dimensions:

1. Physical Product Deterioration,
2. Authority Limits and
3. Customer Value.

Table 1 Framework for classifying perishability

	Authority Limits			
	Fixed		**Loose**	
Physical Product Deterioration: Yes	e.g. Human Blood	e.g. Yoghurt	e.g. Cheese, Gasoline	e.g. Fruits, Radioactive Materials
Physical Product Deterioration: No	*Not-realistic*	e.g. Daily Newspapers	*Not-perishable*	e.g. Haute-Couture Fashion Clothes
	Constant	**Decreasing**	**Constant**	**Decreasing**
	Customer Value			

It is quite straightforward that these dimensions are linked with three different perspectives of the same phenomenon: Product, Authority and Customer, respectively. The added value of this framework comes from the fact that when relating the different perspectives we are able to cluster the perishability phenomenon in a more accurate way then just by looking at a single dimension. This framework might be applied to all different forms of product perishability, either when it manifests itself through the changing of the physical state or not. Moreover, it can be applied to models dealing with any process(es) of the supply chain. Therefore, it differentiates from the former classifications by its flexibility and by the fact that it can be applied to any supply chain planning problem.

Looking down to each dimension independently as if we were to classify perishability with just one of them, the *Physical Product Deterioration* process dimension reflects if the good is actually suffering physical modifications or not. Thus, either the product is physically deteriorating which is the case, for example, with every fresh food product, by spoilage, by decay or by depletion; or the perishable nature of the product comes from another dimension. This is the case, for example, of a daily newspaper that will not suffer any physical deterioration from one day to the other, but it does loose its value. This dimension is quite stable once the scope of the model is defined since it only depends on the product itself, so it is a matter of understanding the physical phenomenon related to perishability. The dimension concerned with the *Authority Limits* represents the external regulations or conventions that influence directly the perishability phenomenon. The reasons why these limits are set can come from various different sources such as concerns about customer safety or well-being as well as for the matter of keeping the customer better informed. This dimension is quite interesting from a modelling point of view because the influence of authorities may reduce the stochasticity of the perishability phenomenon when a lifetime is fixed. For example, in blood banks since human lives cannot be jeopardized, authorities establish tight fixed shelf-lives to blood stock. On the other hand, the fruits sold at retailers have a much more random behaviour concerning their physical state and their lifetime is rather loose in definition making it difficult to know when the product perishes. Nevertheless, even when authority limits are fixed they may not constrain the selling time of an item but they give a clear and determinant information to the customer that the product has perished. Hence, even if in a stand a newspaper from the day before would be for selling, customers would attribute no value to it (because due to the printing date they would know that the news are outdated). Finally, the perceived *Customer Value* has a correspondence to the willingness to pay for a certain good. This willingness to pay may be dynamic in the case when customers attribute a decreasing value to an item as time goes by or it can be static when customers give the same value throughout the lifetime of the product. Thus, the *Customer Value* has a tremendous impact on the operational decisions. A decreasing value enforces efficient operations that deliver the product as fast as possible. A constant *Customer Value* gives more flexibility in batching operations and generates benefits by economies of scale. It is important to clarify that the word customer refers to any user of the perishable good and not only to the final consumer. Gasoline is a good example of a product to which customers attribute the same value during its lifetime since it is guaranteed that its performance will be stable until its expiry date which is usually very long. But, for example, vegetables as soon as they start to look somehow not completely fresh there is an automatic decreasing of customer value.

Crossing the three dimensions to classify perishability, *Physical Product Deterioration*, *Authority Limits* and *Customer Value*, gives a tighter insight on the perishability phenomenon affecting a product and consequently points towards the important mathematical modelling issues that should be considered. To understand the applicability of the framework let us consider, for example, the supply chain planning problem of production and distribution of fresh milk. Using only *Physical Product Deterioration*, we could say that this product after production is undergoing a physical deterioration process (spoilage) so its lifetime is supposed to be rather short. When looking into the *Authority Limits* dimension we would say it is fixed since there is a BBD stamp on this kind of products indicating the marketable period after production of each item. Finally, through a *Customer Value* perspective we would say that it is decreasing since customer will prefer packages with a later BBD comparing with others having an earlier one. Hence, to tackle explicitly the production and distribution planning problem of fresh milk it would be mandatory to have an integrated look at the three different perspectives to grasp the true perishable phenomenon. The mathematical model would need to capture these identified features to be able to control all issues that perishability entails in this case. Hence, an option would be to model a set of constraints limiting the amount of time an item can stay in stock tackling the fixed authority limit and then, for the decreasing customer value attribute, one could model different degrees of freshness for the distributed product and give a value to them maximizing the sum in the objective function.

Based on this discussion we are in position of giving our own definition of perishability and related concepts.

> A good, which can be a raw material, an intermediate product or a final one, is called “perishable” if during the considered planning period at least one of the following conditions takes place: (1) its physical status worsens noticeably (e.g. by spoilage, decay or depletion), and/or (2) its value decreases in the perception of a(n internal or external) customer, and/or (3) there is a danger of a future reduced functionality in some authority’s opinion.

Thus, we distinguish between a worsening or stable physical status, a constant or decreasing customer value, and fixed or loose authority limits, depending on whether the authorities react on such a phenomenon or not.

In order to understand the impact of perishability for different planning horizons in practical production and distribution planning, let us consider three different scenarios that show the different influences that this phenomenon may yield. First, for highly perishable products, such as bakery products or pre-cooked meals, where the planning horizon is very short (one day), the integration of production and distribution is most of the times mandatory. Second, in regular perishable products, such as yoghurt, where the planning is usually one or several weeks, there is limited storage between production and distribution. Nevertheless, in this scenario, integrated lot sizing and scheduling covering also the usual master production scheduling functions is essential. Third, in a scenario with products showing a considerable but limited shelf-life, e.g. beverages with best before periods of six months, the remaining shelf life is a critical issue due to retailers' requirements. But generally, for operational planning no specific attention must be given to shelf-life in the related fast moving consumer goods industries because high inventory turnover is a major concern anyway.

It is important to note that we only concentrate on the negative effects of aging. For instance, if the perceived customer value of a product is not decreasing, we do not further differentiate whether it might even be increasing. We just denote it as being *constant* concerning customer value, because usually only a decreasing value has a noticeable impact on the operations. Further note in Table 1 that according to our definition goods which are physically stable, have a constant customer value, and have fixed or loose authority limits are either not realistic (patterned area of the classification framework) or not perishable (shadow area of the classification framework), respectively. The latter case induces us not to consider them when reviewing the papers.

2 Modelling perishability in production-distribution planning

Most of the literature regarding perishable goods is focused on inventory management, pricing and reverse logistics. The work of Chen et al. (2009) acknowledges that papers discussing other areas of perishable supply chains are rather scarce. Nevertheless, it is widely accepted that perishability in real world may enforce special constraints and different objectives throughout different problems in supply chain planning models.

In this section we perform a review of the work done in modelling perishability for production and distribution planning. Hence, we will cover supply chain planning problems in the production process such as lot sizing and scheduling, as well as in the distribution process such as vehicle routing and final goods replenishment. Therefore, problems related with the procurement and sales functions that may still benefit from tackling explicitly perishability will be out of our scope.

In procurement these problems can be, for example, in ordering raw materials that may be highly perishable such as milk, or in estimating the value of information derived from the cooperation between supply chain partners dealing with deteriorating goods. In the sales functions, the importance of considering perishability can be found in determining where to locate the decoupling point that may not only depend on lead times and variability, but also on the shelf-life of the products (Van Donk 2001). Further, pricing models for deteriorating products is also a very active field of research (cf. Abad 2003).

The core of the review is organized under three main subsections: production planning tasks, distribution planning tasks, and integrated approaches. Moreover, we will cluster works dealing with the same planning problems in each of the following subsections and clarify the specific subject dealt with in each paper. Note that we are only concerned with papers tackling perishability explicitly and, therefore, the papers that just mention perishability or tackle it in a non-analytical way will not be put against our classifying framework.

As said before, the unified framework aims at conceptually characterizing the perishability of the supply chain planning problems at hands. To be more precise in identifying literature gaps and understanding the links between the conceptual phenomenon of perishability and the analytical modelling tools used we will also provide information about the mathematical details employed to handle perishability. Classically, mathematical supply chain models are dichotomized as being either stochastic or deterministic (cf. Beamon, 1998). In the readers' interest, we will further exploit this division by differentiating if the stochastic properties pertain to the perishability itself or to some exogenous factor such as travel times or customers' demand. In the summarizing Tables 3, 5, and 7 we explicitly state if a model considers *Perishability Stochastic* or some *Other Stochastic* feature, which might be on the supply side (*Supp*.), on the production part (*Prod*.), on the distribution part (*Travel times*), or on the customers' demand (*Dem*.). There is another important modelling characteristic that should be regarded when modelling the perishability phenomenon that is concerned about how the good's quality is formulated (*Quality Tracing*). Hence, either the model is able to grasp the *continuous* deterioration of the good, or it just differentiates between being fresh or spoiled (*boolean*). With this refinement we aim at establishing important relations between the dimensions of perishability and the available modelling tools.

2.1 Production planning tasks

In **lot sizing** we need to determine the size of lots to be produced while trading off the changeover and stock holding costs. Afterwards, these lots are scheduled according to the planner's preferences, while taking into account the available capacity. Hence, production planning models may have control over production quantities and sequences, as well as influence directly the inventory quantities. Moreover, the tactical models differ from the operational ones in the level of aggregation and concerns of the planners that are incorporated in the models. Perishability is in many cases a very important issue concerning the tactical and operational levels of production planning. It may enforce several constraints such as upper bounds on lot sizes and consequently the need of **scheduling** more often a given family of products increasing the difficulty of sequencing. In Table 2 the main objective of the articles is exposed and in Table 3 the reviewed articles are crossed against the proposed framework.

In the field of **lot sizing**, Hsu (2003) defines cost functions for every single period, i.e., the inventory and backorder costs are accounted for in a period-by-period fashion. He considers explicitly stock deterioration through the possibility of losing inventory from period to period. Abad (2000) deals with finding an optimal lot size when partial backordering and lost sales are the case.

There are industries, like the food or the pharmaceutical industry, where reworkable defective products are perishable and rather common (Flapper et al., 2002). Teunter and Flapper (2003) study a production system in which the cost and/or time of reworking a defective good increase as a function of the stocking time.

Table 2 Summary of revised literature in production planning

Model	Main objective of the paper
Lot sizing	
Abad (2000)	Determine optimal lot size with partial backordering and lost sales
Hsu (2003)	Economic Lot Size model for a perishable product where the costs of holding stocks in each period depend on the age of inventories. The problem is solved using dynamic programming
Teunter and Flapper (2003)	Find the Economic Lot Size for a perishable reworkable product
Wang et al. (2009)	Find the economic production quantity for integrated operation-traceability planning for perishable food management
Scheduling	
Arbib et al. (1999)	Flow line production scheduling where perishability may happen on either the beginning or ending of the process with the objective of maximizing system productivity
Chowdhury and Sarker (2001), Goyal and Viswanathan (2002), Sarker and Chowdhury (2002)	Analyse the effect on spoilage by reducing either cycle time, or production rate, or cycle time and production rate simultaneously. Take into consideration the raw materials also
Soman et al. (2004)	Analyse the effect of shelf-life in food industries by neither allowing a decrease in production rates nor backordering
Yao and Huang (2005)	Determine an ELSP for several products subject to exponential deterioration under a Power-of-Two policy
Lin et al. (2006)	Determine an ELSP for several products subject to exponential deterioration under a common cycle production-inventory policy
Gawijnowicz (2007)	Schedule deteriorating jobs on machines with different characteristics with the objective of minimizing completion time
Lot sizing and scheduling	
Neumann et al. (2002)	Solving batching and batch scheduling in process industries where intermediate products may be perishable
Lütke Entrup et al. (2005)	Develop lot sizing and scheduling MIP models that integrate shelf-life
Cai et al. (2008)	Develop a lot sizing and scheduling model to deal with highly perishable raw materials and a rigid deadline for product manufacturing simultaneously
Pahl and Voß (2010)	Extend discrete lot sizing and scheduling models to include deterioration constraints
Pahl et al. (2011)	Extend discrete lot sizing and scheduling models with sequence-dependent setup times and costs to include deterioration constraints
Amorim et al. (2011)	Use a hybrid formulation embedded in a multi-objective framework to maximize freshness as a second objective

Table 3 Cross the proposed framework with the revised literature of production planning

Model	Authority limits	Product deterioration	Customer valuer	Perishability stochastic	Quality tracing	Other stochastic
Lot sizing						
Abad (2000)	L	Y	C	Y	b	–
Hsu (2003)	L	Y	D	N	c	–
Teunter and Flapper (2003)	L	Y	D	N	c	–
Wang et al. (2009)	F	Y	D	N	b	–
Scheduling						
Arbib et al. (1999)	F	–	C	N	b	–
Chowdhury and Sarker (2001)	F	–	C	N	b	–
Goyal and Viswanathan (2002)	F	–	C	N	b	–
Sarker and Chowdhury (2002)	F	–	C	N	b	–
Soman et al. (2004)	F	Y	C	N	b	–
Yao and Huang (2005)	L	Y	C	Y	b	–
Lin et al. (2006)	L	Y	C	Y	b	–
Gawijnowicz (2007)	L	Y	C	N	b	–
Lot sizing and scheduling						
Neumann et al. (2002)	L	Y	C	N	b	–
Lütke Entrup et al. (2005)	F	Y	D	N	c	–
Cai et al. (2008)	F	Y	C	N	b	Prod.
Pahl and Voß (2010)	F	–	C	N	c	–
Pahl et al. (2011)	F	–	C	N	c	–
Amorim et al. (2011)	F	Y	D	N	c	–

F fixed, *L* loose, *Y* yes, *N* no, *C* constant, *D* decreasing, *b* boolean, *c* continuous

The increase can be due to either perishability or technological obsolescence of the product. Thus, the limited product life-cycle is a plausible reason. The production system under study has only one stage and one product.

Concerning the importance of information, Wang et al. (2009) focused on integrating the optimization of production batch size and traceability. Traceability is growing in importance in many sectors ranging from the pharmaceutical to the food industry. The authors developed a model that integrates traceability concerns and operational indicators to attain both product quality and minimum impact of product recall. It takes into account production setup cost, inventory holding cost, raw material cost, product spoilage cost, and recall cost in order to find the most economical way of making the necessary decisions.

With regard to the integration of perishability in production **scheduling** approaches, most research deals with adding a shelf-life constraint to the Economic Lot Scheduling Problem (ELSP), which is concerned about obtaining a cyclic schedule for several products, for a single resource and under the assumption of a constant demand rate (e.g., Elmaghraby 1978; Cooke et al. 2004). Soman et al. (2004) provide a review of the major contributions. Chowdhury and Sarker (2001), Goyal and Viswanathan (2002), and Sarker and Chowdhury (2002) work on three possibilities: changing the production rate, changing the cycle time and changing production rate and cycle time simultaneously with respect to production scheduling and raw material ordering. Soman et al. (2004) state that in case of high capacity utilization as it can be found, for example, in the food industry the production rate should not be reduced due to quality problems that may arise with this adjustment. Yao and Huang (2005), sparked by the fact that most of the ELSP models do not consider multiple continuously deteriorating items, proposed a new model with this extension. In this line, Lin et al. (2006) consider an ELSP with multiple products subject to exponential deterioration, where all of them are under the same production cycle policy. However, each product has a different deterioration rate and demand.

In a more operationally oriented way, Gawijnowicz (2007) studies a parallel machine scheduling problem with deteriorating products where the objective is to minimize the total completion time of jobs subject to a certain machine capacity. Arbib et al. (1999) consider a production scheduling problem for perishable products, which is studied under two independent aspects: the relative perishability of products and the feasibility of the completion time. Tadei et al. (1995) develop a partitioning algorithm coupled with local search techniques for production planning and scheduling in the food industry. In this work, although the authors acknowledge that goods are perishable, neither the model nor the solution procedure take into account this crucial fact, therefore, this paper is not shown in Tables 2 and 3.

Marinelli et al. (2007) propose a solution approach for a capacitated **lot sizing and scheduling** problem with parallel machines that share the same buffers. This problem is typical in the yoghurt industry at the packaging stage. The problem is formulated as a capacitated lot sizing problem with setup considerations and a two steps optimisation algorithm that decomposes the problem into a lot sizing problem and a scheduling problem is proposed. This model does not account for perishability explicitly, instead it makes use of a make-to-order production strategy. With a similar approach, but focused on batch processing, Neumann et al. (2002) decompose production planning and scheduling for continuous production systems into batching and batch scheduling problems. The batching problem translates the primary demand for products into individual batches, where the objective is to minimize the workload. First, this batching problem is formulated as a nonlinear mixed-integer program, but then it is transformed into a linear mixed-integer program of a still moderate size through variable substitution. The batch scheduling problem allocates the batches to the processing units, workers, and intermediate storage facilities, where the objective is to minimize the makespan. The batch scheduling problem is formulated as a resource-constrained project scheduling problem. In this work some intermediate perishable products cannot be stored eliminating the buffer between activities. Pahl and Voß (2010) and Pahl et al. (2011) extend well known discrete lot sizing and scheduling models by including deterioration and perishability constraints. In the special case of yoghurt production, Lütke Entrup et al. (2005) develop three mixed-integer linear models that incorporate shelf-life issues into production planning and scheduling of the packaging stage. Amorim et al. (2011) differentiate, in a multi-objective framework, between costs and freshness. Hence, the result of the lot sizing and scheduling problem is a Pareto front trading off these two dimensions. Cai et al. (2008) develop a model and an algorithm for the production of seafood related products. Due to a deadline constraint related with the expedition of the production and the raw material perishability, the production planning has to determine three decisions: the products to be produced; the quantity of each product type; and the production sequence. It is interesting to notice that here the perishability is mainly focused on the raw materials.

Almost all of the production planning papers reviewed are based on the assumption that customers attribute a constant value for the perishable product which reflects to a certain extent a myopic view of the supply chain. Moreover, most of the papers do not consider any stochastic element neither related to the perishability nor to other external factors.

2.2 Distribution planning tasks

Distribution planning tasks take into account tactical decisions, such as delivery frequencies or fleet dimensioning. A manufacturer producing a perishable good for a retailer may need to increase the frequency of deliveries in order to achieve a better customer service regarding product freshness, which in turn may influence fleet dimensioning. Also in a mid-term planning horizon, the **allocation** of perishable products between manufacturing plants and distribution centres may also be impacted by the different shelf-lives of products. On a more operational level, transport planning that defines distribution quantities between echelons of the supply chain and outbound **vehicle routings** are planning problems that can also be affected by perishability. For example, it can be important to acknowledge that the temperature/time of the distribution has an impact on the lifetime of the perishables goods forcing some constraints on the travel duration or making it important to include cooling costs in the objective function.

Replenishment problems are also included in this section since most of the inventory management papers do not distinguish whether a procurement department orders material from an external supplier or whether a distribution warehouse orders finished goods from a production plant in a sort of in house process. Hence, we will focus on the problem of refilling the outbound warehouses (**replenishment**). In fact, inventory management plays a significant role in controlling the raw materials, work-in-processes and finished goods. Poor inventory management throughout the supply chain may lead to an excessive amount of capital fixed in inventory. Inventory in the form of safety stocks (either in raw material or final products) is built to hedge against uncertainty in demand, production process and supply. When dealing with perishable products this financial trade-off is compounded with the possibility that the stock perishes and then loses all value. The presented papers related with this planning problem do not aim to be fully exhaustive in covering all works, since, as said before, there are comprehensive reviews in this area. However, they will be important to show the impact of the perishability framework in classifying this important task.

Tables 4 and 5 summarize work on distribution planning with respect to perishability. One type of the reviewed research is concerned with maximising business profits through the **allocation** of perishable inventories in an operational process according to their shelf-life (Lin and Chen 2003). In this line of research, the shelf-life of the good acts as a constraint to a delivery planning decision. Hence, some research employs a concept called "product value" that represents the utility and/or quality of a good that will be used on the decision process related with the operational planning. Li et al. (2006) propose an inventory allocation model for fresh food products based on real-time information that is provided through RFID. The objective of this dynamic planning approach is to optimize the profits of a retailer.

Among the recent publications on **replenishment** models, Goyal (2003) studies an economic order quantity (EOQ) extension model in which demand, production

Table 4 Summary of revised literature in distribution planning

Model	Main objective of the paper
Allocation	
Lin and Chen (2003)	Develop an optimal control mechanism for the allocation of orders and distribution quantities to prioritized suppliers and retailers
Li et al. (2006)	Inventory planning and allocation based on on-line perishability information through RFID
Replenishment	
Manna and Chaudhuri (2001)	Develop an EOQ model taking into account time dependent demand for items with a time proportional deterioration rate
Kar et al. (2001)	Propose an inventory model for several continuously deteriorating items, sold from two shops—primary and secondary shops
Goyal (2003)	Develop a model for the production-inventory problem in which the demand, production and deterioration rates of a product vary with time and shortages of a cycle are allowed to be backlogged partially
Rau (2003)	Present a multi-echelon inventory model for a deteriorating item in order to derive an optimal joint total cost from an integrated perspective
Yang and Wee (2003)	Discuss the integration of inventory systems for deteriorating items in supply chains
Chen and Chen (2005)	Investigate the effect of joint replenishment and channel coordination for an inventory system in a multi-echelon supply channel where the lot sizes can differ between shipments
Minner and Transchel (2010)	Develop an inventory control model which considers service level constraints and two different issuing policies to satisfy demand
Vehicle routing	
Hsu et al. (2007)	Extend a VRPTW by considering randomness of the perishable food delivery process. Special attention was devoted to inventory and energy costs
Osvald and Stirn (2008)	Extend a VRPTW by considering time-dependent travel-times and perishability as part of the overall distribution costs

Table 5 Cross the proposed framework with the revised literature of distribution planning

Model	Authority limits	Product deterioration	Customer value	Perishability stochastic	Quality tracing	Other stochastic
Allocation						
Lin and Chen (2003)	F	Y	C	N	b	Dem., Sup.
Li et al. (2006)	L	Y	D	N	c	–
Replenishment						
Manna and Chaudhuri (2001)	L	Y	C	Y	b	Dem., Prod.
Kar et al. (2001)	L	Y	D	Y	b	–
Goyal (2003)	L	Y	C	Y	b	Dem., Prod.
Rau (2003)	L	Y	C	Y	b	–
Yang and Wee (2003)	L	Y	C	Y	b	–
Chen and Chen (2005)	L	Y	D	Y	b	Dem., Prod.
Minner and Transchel (2010)	F	Y, N	D, C	N	c	Dem.
Vehicle routing						
Hsu et al. (2007)	L	Y	D	Y	c	Travel time
Osvald and Stirn (2008)	F	Y	D	N	c	Travel time

F fixed, *L* loose, *Y* yes, *N* no, *C* constant, *D* decreasing, *b* boolean, *c* continuous

and deterioration rates of a product vary with time. This work starts by considering an infinite planning horizon, but afterwards it looks into the case of a finite planning horizon and solves it near to optimality. Before, Manna and Chaudhuri (2001) have also performed a similar work. Minner and Transchel (2010) develops a period review inventory policy to meet given service level requirements while considering a good that perishes after a fixed number of periods. They consider two different demand situations. In the first situation, demand is always satisfied using the oldest units first. In the second scenario customers always request the freshest units available.

An emerging area of research in inventory management is focused on dealing with a multi-product and/or multi-echelon supply chain, which produces, distributes, and then sells perishable products. Kar et al. (2001) propose an inventory model for several continuously deteriorating products that are sold in two shops under the same management that has to deal with constraints on investment and total floor-space area. First, the products are purchased and received in lots at the primary shop. Then, the fresh goods are separated from the deteriorated ones. The fresh units are sold at the primary shop and the deteriorated are transported for sale at the second shop. Rau (2003) as well as Yang and Wee (2003) extend the inventory models to a multi-echelon supply chain by integrating suppliers, producers and buyers. Both of them consider a single product, but while Rau (2003) tackles only one buyer, Yang and Wee (2003) consider multiple ones. Later, Chen and Chen (2005) have extended these works by considering multiple products in a two-echelon supply chain with variable demand over the planning horizon. In this work the authors were able to investigate the effects of joint replenishment and channel coordination on cost savings in the supply chain.

Especially in the **routing problems**, most of the papers reviewed concerned with perishability do not explicitly model it. Therefore, although we briefly describe them, they are not put against our framework in the corresponding tables.

Tarantilis and Kiranoudis (2001) analyse the distribution of fresh milk. They formulated the problem as an heterogeneous fixed fleet vehicle routing problem. In Tarantilis and Kiranoudis (2002) a real-world distribution problem of fresh meat is presented. They modelled the problem as an open multi-depot vehicle routing problem. Faulin (2003) presents an implementation of a hybrid algorithm procedure that uses heuristics and exact algorithms in the solution of a vehicle routing problem (VRP). This VRP is extended with constraints enforcing narrow time-windows and strict delivery quantities, which is normal in the agribusiness industry.

In none of the above models the specific degradation of quality during transport is taken into account. Osvald and Stirn (2008) develop a heuristic for the distribution of fresh vegetables, with perishability as a critical factor. The problem is formulated as a vehicle routing problem with time windows (VRPTW) and time-dependent travel times. The model considers the impact of perishability as part of the overall distribution costs. Hsu et al. (2007) consider the randomness of the perishable food delivery process and present a stochastic VRPTW model to obtain routes, loading, vehicles and departure and delivery times at the distribution center. The objective function takes into consideration inventory costs due to deterioration of perishable food and energy costs occurring in the cooling of the transportation vehicles.

The distribution of ready mixed concrete (RMC), which is a very perishable product, has received considerable attention from the research community. Nevertheless, in this case practice says that the distribution process is so tense in terms of time that the perishability process is dealt by imposing very narrow, hard time-windows and by enforcing a strictly uninterrupted supply of concrete. These constraints make the problem very hard to solve shifting the community attention to the solution method. From the vast body of literature in this field we mention some references for readers interested in this subject. Matsatsinis (2004) presents an approach for designing a decision-support system that is able to do the routing of a fleet of trucks to distribute RMC. He concentrates on the decision-support system, while routing is done using heuristics. Naso et al. (2007b) implemented a hybrid approach combining a constructive heuristic with a genetic algorithm. Their fleet of vehicles is homogeneous in terms of their capacity. Naso et al. (2007a) take into account even more realistic assumptions, such as plant capacities and variable truck speeds that lead to a non-linear mathematical model. Recently, Schmid et al. (2010) developed a hybrid solution procedure based on a combination of an exact algorithm and a Variable Neighbourhood Search for a heterogeneous fleet also distributing RMC.

Looking at the research in the replenishment field over time it is noticeable that current research is having more a supply management perspective through focusing on multi-echelon issues and on the importance of information sharing regarding stocks between supply chain partners. The paucity of papers tackling explicitly perishability in routing planning tasks is in line with the findings of the related review by Akkerman et al. (2010). Nevertheless, the two papers about VRP shown in Tables 4 and 5 are important examples of how to incorporate this phenomenon in modelling.

Thus, in the distribution planning papers for all categories reviewed there are papers using stochastic models. The stochasticity is present both in the perishability process and in other characteristics such us the travel time and demand. On the other hand, papers dealing with perishable products having fixed shelf-life are rather scarce.

2.3 Integrated approaches

The models reviewed in this subsection are the ones which, in fact, have a more supply chain management oriented perspective by attempting to integrate different functions of the supply chain (Min and Zhou 2002). In numerous articles the advantages of integrating traditionally decoupled decision models into integrated ones have been shown (Chen and Vairaktarakis 2005; Park 2005). These advantages, however, can be leveraged when dealing with perishable products. From a modelling perspective this is undoubtedly a very challenging field.

In Table 6 for each article of this section a small resume is done and in Table 7 the papers are classified according to the framework presented in Sect. 2 Joint production and distribution planning seems to be mandatory in a lot of processing industries such as in the production and delivery of RMC (Garcia and Lozano 2004, 2005) and in several industrial adhesive materials (Armstrong et al. 2007). Those products are subject to very short lifetimes. Hence, after production they should be delivered to the customer immediately. For these industries, the production and distribution processes are intertwined with no or little buffer time separating them. Other examples of such an integration of processes can be found in the newspaper industry (Van Buer et al. 1999) and snail-mail dispatching (Wang 2005). In the papers mentioned so far, perishability is not modelled explicitly. The explanation lies in the fact that real-world problems are so constrained for these products that the advantages inherent to tracking perishability are inhibited.

Eksioglu and Mingzhou (2006) address a production and distribution planning problem in a general two-stage supply chain with dynamic demand. The model takes into account that the final product subject to perishability and, therefore, its shelf-life is restricted. Furthermore, strong assumptions are made, such as unlimited capacity. They formulate this problem as a network flow problem with a fixed charge cost. Based on the EOQ, Yan et al. (2010) developed an integrated production-distribution model for a deteriorating good in a two-echelon supply chain. The objective of the model is to minimize the total aggregated costs. Some restrictions concerning perishability are imposed. For example, the supplier's production batch size is limited to an integer multiple of the delivered quantity to the buyer. In a more operational perspective, Chen et al. (2009) propose a nonlinear mathematical model to tackle both production scheduling and the VRPTW for perishable food products in the same formulation. The customer demand is assumed to be stochastic and perishable goods deteriorate as soon as they are produced. The

Table 6 Summary of revised literature in integrated approaches

Model	Main objective of the paper
Federgruen et al. (1986)	Present an integrated allocation and distribution model for a perishable product to be distributed to a set of locations with random demands
Eksioglu and Mingzhou (2006)	Develop an MIP model for a production and distribution planning problem in a dynamic, two-stage supply chain for a product with deterioration in inventory
Rong et al. (2009)	Integrate in an MIP model the possibility to trace product quality throughout the processes of production and distribution
Chen et al. (2009)	Develop a nonlinear mathematical model to consider production scheduling and vehicle routing with time windows for perishable food products
Yan et al. (2010)	Extend the basic EOQ to integrate these two processes taking into account perishability of inventories

Table 7 Cross the proposed framework with the revised literature of integrated approaches

Model	Authority limits	Product deterioration	Customer value	Perishability stochastic	Quality tracing	Other stochastic
Federgruen et al. (1986)	F	Y	C	N	b	Dem.
Eksioglu and Mingzhou (2006)	F	Y	C	N	c	–
Rong et al. (2009)	L	Y	C	N	c	–
Chen et al. (2009)	F	Y	D	N	c	Dem.
Yan et al. (2010)	L	Y	C	N	b	–

F fixed, *L* loose, *Y* yes, *N* no, *C* constant, *D* decreasing, *b* boolean, *c* continuous

objective of this model is to maximize the expected total profit of the supplier, considering that the value of the goods delivered depends on their freshness value. The decision variables relate to the production quantities, to the time to start production and the vehicle routes. The solution approach couples the constrained Nelder-Mead method and a heuristic for the VRPTW. Recently, Rong et al. (2009) developed an MIP model where a single product quality is modelled throughout a multi-echelon supply chain. Their model uses the knowledge of predictive microbiology in forecasting shelf-life specially based on the temperature of transportation and stocking. Their objective function reflects this reality by taking into account the incremental cooling costs necessary to achieve a longer shelf-life.

Hwang (1999) develops an allocation-distribution model for determining optimal cycles of food supply and inventory allocation in order to focus the effort in reducing starvation of a region in which there is famine. The proposed VRP incorporating inventory allocation aims at minimizing the amount of pain and starving people instead of using the common objective of minimizing travel distance. Once again this work does not explicitly consider food perishability. Federgruen and Zipkin (1984) consider a single-period problem in which the quantity of product available in the inventory is limited. Their work was one of the first dealing with the integrated allocation and distribution problem in the same framework. They present a heuristic solution based on the decomposition of the main problem into a non-linear inventory allocation problem and a number of Traveling Salesman problems that relates to the number of vehicles available. Federgruen et al. (1986) extended this previous work to the case in which the good under consideration is subject to perishability. In this problem the product units in the system are classified either as being “old” when they perish in the present period or as being “fresh” when they have a shelf-life that is larger than the current period. In order to reduce the number of spoiled units, several effective distribution policies are considered in the study and the conclusion points towards a hybrid choice of them.

Like in the production planning models most of the integrated approaches focus on customers which attribute a constant willingness to pay for a perishable product. Models that explicitly keep track of the age/quality of the produced products seem to be gaining increasing importance in this field.

3 Identification of literature gaps

To identify the research gaps based on the literature review given in Sect. 3 two visual maps were built to support the analysis. Each of these maps couples the relevant supply chain planning problem with the proposed framework for perishability classification. The first map, Fig. 1, refers to the decoupled processes of production and distribution, and the second map, Fig. 2, concerns the possible integration forms that these processes may be subject to. For each and every possible conjugation of the perishability classification in one of the relevant supply chain planning problems, either the perishability process in question has been somehow explored in the literature for the corresponding planning tasks (Explored) or there is a potential research gap that might need to be addressed (Gap).

To fill in Fig. 1 we relied directly on Tables 3 and 5 making a straight linkage with the perishability classification and the supply chain planning task. To fill in Fig. 2 only Table 7 was used.

Note that we have left out of Figs. 1 and 2 the sub-dimension related with products without physical deterioration. The fundamental reason for this is the fact that there seems to be almost no literature in production-distribution planning that deals explicitly with this type of perishability phenomenon. As noted before the papers dealing with RMC or newspapers, which represent products belonging to this category, enforce such tight time constraints in the models that a further, explicit modelling of the perishability phenomenon is not necessary. However, in pure inventory management problems this may be a very interesting topic since the spoilage of a product may represent the spoilage of the whole inventory. To better understand this reasoning consider, for example, the selling of fruits and monthly magazines in a supermarket. If one of the fruits spoils, the inventory manager has less to worry about than if one of the magazines ends its shelf-life because this means the deterioration of all magazines from that month simultaneously. Thus,

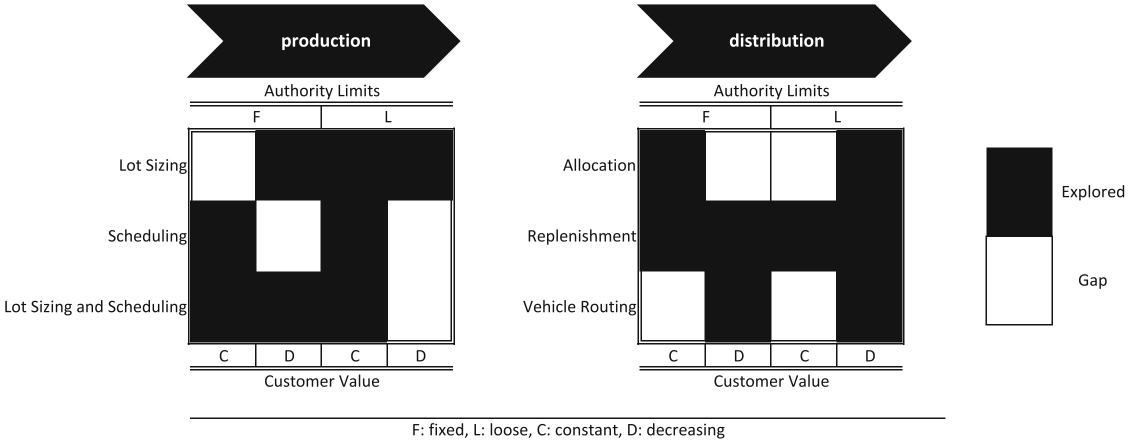

Fig. 1 Visual representation of literature gaps for decoupled supply chain processes

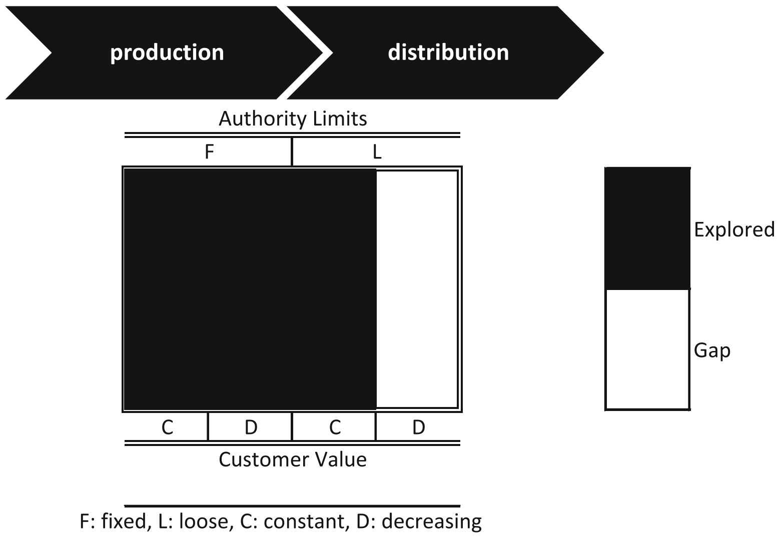

Fig. 2 Visual representation of literature gaps for integrated supply chain processes

inventory management seems to have an extended importance for products that are not subject to physical deterioration.

Regarding production models, Lütke Entrup et al. (2005) is a good example of a planning model that incorporates the relevant features of perishability within its scope, modelling the customer decreasing value and the fixed authority limits in the yoghurt production. However, more work is needed in modelling customer decreasing value especially for products with loose lifetimes, since this happens to be the case of many real-world perishable products. Probably the biggest challenge of filling this gap is to introduce stochastic elements in problems that traditionally are modelled with a deterministic setting. Further, in distribution, there is a planning task, transport planning, for which we have not found any paper. Therefore, there is neither a section devoted to it nor it appears in Fig. 1. Nevertheless, there is also a gap to fill in order to understand, for example, what transportation modes offered by the 3PL between full truck-load (FTL) and less than full truck-load (LTL) should be contracted to better tackle the freshness aspect. In replenishment a lot of work is done and models incorporate very complex features (Goyal 2003).

Regarding integrated models, the main issue of current research is the lack of applicability in real cases due to very strong assumptions (Eksioglu and Mingzhou 2006). Therefore a gap exists for models accounting for capacity limitations and also for multiple deteriorating products. However, efforts in almost every form of perishability exist except for products with customer decreasing value and loose authority limits such as vegetables.

Production and distribution planning models that address these gaps are expected to fill needs in industries and vis-a-vis final consumers.

Concerning the linkage between the modelling techniques used and the correspondent conceptual perishability phenomenon there are two conclusions to be noted. First, there is a strong relationship between products subject to a loose authority and the use of stochastic formulations in the planning problems. However, this link is even more straightforward for the inverse situation of fixed authority limit and deterministic formulations. According to Goyal and Giri (2001) when the lifetime of a product is fixed then the correspondent formulation should be deterministic, and when the exact lifetime cannot be determined in advance, then lifetime is assumed to be a random variable. According to our review this statement is too bold since, for example, with the help of predictive microbiology it may be possible to model products subject to loose authority with deterministic formulations (Rong et al. 2009). The second relationship regards customer value and the way quality is traced. In most of the cases, if customers decrease their willingness to pay throughout the lifetime of the product, it is rather important to have a continuous tracing of the product's quality. Furthermore, when tracing continuously the product's quality it is easier to incorporate the freshness value in the objective function instead of just penalizing in case the product actually perishes.

4 Conclusions

First and foremost, this review and discussion showed that efficient production and distribution planning is growing in importance in the field of perishing goods judged by the number of papers reviewed. By reviewing the literature dealing with classifications of perishability, it was pointed out that the classifications used so far were rather disperse and not suitable to be used in different contexts. This issue was overcome by proposing a new transversal and unified framework for classifying perishability (Sect. 2). This classification has been used successfully in categorizing all the reviewed works that dealt explicitly with the perishability phenomenon.

Depending on the process analysed, there are always a considerable number of papers that, although claiming to address a perishability issue, do not consider it explicitly. This reflects the extended difficulty that modelling this issue may enforce on the mathematical models and solution approaches. However, only when considering it explicitly the decision maker is able to control the influence of perishability completely. Perishability is a rather large and diffuse phenomenon so it is important to look at each specific planning problem independently in order to be able to express it correctly. It seems quite difficult to find a single way of modelling all the different forms and influences that perishability may have in a certain process(es). Nevertheless, some hints on the linkage between the conceptual level of perishability and the mathematical modelling techniques were given. Mainly it was identified that products subject to a loose authority are suitable to be modelled with stochastic formulations and when the product's quality is of paramount importance, it is necessary to use variables to continuously track the product's freshness state.

To conclude, this review classified production and distribution planning models dealing with goods subject to physical deterioration in light of a new framework. Throughout the review we pointed out specific problems that perishability may enforce in such models and how mathematical modelling techniques can address the panoply of different perishability forms. In the end we were able to identify possible research gaps and challenges that may be very important to address in scientific and practical terms. Hence, we expect that the research activities in this area will increase significantly in the near future.

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Chapter 3

Perishability Impact on Production Planning

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Multi-Objective Lot-Sizing and Scheduling Dealing with Perishability Issues

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S *Supporting Information*

ABSTRACT: The recent evidence demonstrating the importance of perishables in terms of store choice and shopping experience makes these products a very interesting topic in many different research areas. Nevertheless, the production planning research has not been paying the necessary attention to the complexities of production systems of such items. The evidence that consumers of perishable goods search for visual and other cues of freshness, such as the printed expiry dates, triggered the development of a multi-objective lot-sizing and scheduling model taking this relevant aspect into account by considering it explicitly as an objective function. A hybrid genetic algorithm based on NSGA-II was developed to allow the decision maker a true choice between different trade-offs from the Pareto front. Computational experiments were based on a case study, reported in the literature, concerning a diary company producing yogurt.

INTRODUCTION

With an attentive look, one may realize that all products do deteriorate depending only on the considered planning horizon. For some products, such as books and furniture, perishability only plays a role in a planning horizon that is too long in practical managerial terms. On the other hand, in yogurt production, the time horizon for operational planning, which is usually a week, is sufficiently long compared to the useful life of the item, so that the influence of perishability plays an essential role in this planning task. To guide our discussion and for the sake of understandability, we will recurrently use the example of the yogurt production process, which is quite interesting regarding its production system challenges. Nevertheless, generality will be guaranteed to allow a roll out of the proposed models to other industries dealing with perishables that are close to the scope of this problem.

The literature commonly recognizes that managing perishability may represent a remarkable problem in production and service systems in a varied set of industries. The interest among researchers in this field has been sparked primarily by problems of blood bank management and dates back to the pioneering work of Millard.[1] In food industries, and in particular in processes requiring maturation, the fresh material to be transformed (e.g., meat, milk, vegetables, etc.) can be stored up to a precise time after which the maturation process must necessarily begin. Once this period is over, the product is packed and preparation/expiry dates (differing from each other by a fixed interval) are printed onto the package. This limits both the duration of the initial storage and the period between production and selling dates, forcing these issues to be one major interest of the producer. There are also cases in which perishability issues are not as clear. For instance, in the publishing trade, the delivery of newspapers to commuters is subject to severe time constraints. Here, the newspapers behave as perishable products with a fixed lifetime, as highlighted in Buer et al.[2] These expiry dates are also known as Best Before Dates (BBD) which are defined as the end of the period, under any stated storage conditions, during which the product will remain fully marketable and retain any specific qualities for which tacit or express claims have been made. It was proved empirically by Tsiros and Heilman[3] that the willingness to pay for products stamped with a BBD, which to some extent reflects the degree of freshness, decreases over the shelf life of the product. Yogurt is an idiosyncratic example of the group of products having these characteristics. It has a fixed shelf life expressed by its stamped BBD. From the customer's point of view, its value is continuously decreasing throughout its lifetime, and when it perishes, its value is close to zero. This defines the scope within perishable products where this paper is actually making its contribution.

Most literature regarding perishable goods is in the fields of pricing, return policies, and inventory control to a retailer (see refs 4−7). Chen et al.[8] acknowledge that papers discussing production scheduling of perishable goods are relatively rare, and papers discussing simultaneous lot sizing and scheduling are even rarer. Still, perishability is in several cases a very important issue concerning the tactical and operational level of production planning. For example, by considering perishability in lot sizing, upper bounds on the lots quantities may be enforced. Consequently, there is a need for scheduling more often the same product, which increases the difficulty of sequencing. On a more tactical level, when demand is affected by seasonality and the production facility has a tight capacity, it may be mandatory to produce seasonal stocks. Nevertheless, for perishable products,

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these seasonal stocks need to be balanced with the possibility of spoilage, making it important to consider other options, such as working overtime in peak seasons.

Revising the literature on production planning of perishable products, Gawijnowicz[9] studies the parallel machine scheduling problem with deteriorating jobs, where the objective is to minimize the total completion time of jobs or total machine workload. Arbib et al.[10] consider a three-dimensional matching model for perishable production scheduling, which is studied under two independent aspects: the relative perishability of products and the feasibility of launching/completion time. Tadei et al.[11] present a partitioning algorithm and local search techniques for aggregate planning and scheduling in the food industry. In this work, although the authors acknowledge that goods are perishable, neither the model nor the solution procedure takes into account this crucial fact. Marinelli et al.[12] propose a solution approach for a capacitated lot sizing and the scheduling real problem with parallel machines and shared buffers, arising in a packaging company producing yogurt. The problem has been formulated as a hybrid continuous setup and capacitated lot-sizing problem A two-stage optimization heuristic based on the decomposition of the problem into a lot-sizing problem, and a scheduling problem has been developed. This model accounted for perishability by using a make-to-order production strategy. With a similar approach, but focused in batch processing, Neumann et al.[13] decompose detailed production scheduling for batch production into batching and batch scheduling. The batching problem converts the primary requirements for products into individual batches, where the workload is to be minimized. They formulate the batching problem as a nonlinear mixed-integer program and transform it into a linear mixed-binary program of moderate size. The batch scheduling problem allocates the batches to scarce resources such as processing units, workers, and intermediate storage facilities, where some regular objective function like the makespan is to be minimized. The batch scheduling problem is modeled as a resource-constrained project scheduling problem, which can be solved by a truncated branch-and-bound algorithm. In this work, intermediary perishable products cannot be stored. Pahl and Voss[14] extend well-known discrete lot-sizing and scheduling models by including deterioration and perishability constraints. In the special case of yogurt production, Entrup et al.[15] develop three mixed-integer linear programming models that integrate shelf life issues into production planning and scheduling of the packaging stage. In this work, the decreasing value of the produced products is modeled by a linear function that represents the price retailers need to pay as a function of product freshness. Cai et al.[16] develop a model and an algorithm for the production of seafood products. Tacking into account a deadline constraint and the raw material perishability, the manufacturer determines the product types to be produced, the machine time to be allocated for each product type, and the sequence to process the products selected. It is interesting to note that here the perishability is mainly focused on the raw materials.

On the basis of this review, when developing production models for the type of perishable products treated in this work, four different ways of tackling the perishability phenomenon were used. First, on a more strategic level, it is possible to impose a make-to-order strategy for all products, so that is likely that no products will spoil and they will be delivered with good freshness standards. However, this production strategy may not be possible for consumer good products with hundreds of stock keeping units (SKUs), tight capacities, and with high setup costs because it would result in prohibitive costs and low customer satisfaction regarding delivery lead times. Second, it is possible to enforce constraints on the number of macro-periods that a product can stay in stock or just control the number of spoiled products and penalize it in the objective function. Either way, one would obtain a solution that satisfies these constraints but that may oversee other solutions that for similar costs would obtain fresher products. The third way of taking into consideration product perishability is by differentiating holding costs depending on the shelf life. So items with a shorter shelf life are given higher holding costs, and items with a longer shelf life are given lower holding costs. As a drawback, it is not controlled when a product expires and, moreover, to assign values to so many different holding costs may be very difficult and inaccurate. Finally, it is possible to attribute a value to the different degrees of freshness that a product has when delivered. This way, we are explicitly controlling the perishability process because we know for every delivered product the corresponding monetary value of the remaining shelf life, but on the other hand, we need to attribute a value to the different degrees of freshness. This economic value may be very difficult to obtain because retailers will not reflect directly this quality difference on the price paid for the products, even if it is clear that both retailers and final customers attribute an intangible positive value to product freshness.

Hence, to the best of our knowledge, this is the first work that deals with simultaneous lot sizing and scheduling of perishable products using a multi-objective framework. Several authors (e.g.,[10,17]) acknowledge that to comprehensively solve either a scheduling or a lot-sizing and scheduling problem dealing with perishability, a multi-objective approach should be taken. In fact, designing a multi-objective approach coupled with some of the existing ways of controlling perishability may circumvent most of the drawbacks of existing approaches. Our proposal is to separate the economic production tangible costs from the customer intangible value of having fresher products in two distinct objective functions. The first one aims at minimizing the total production costs, and the second one maximizes the average freshness of delivered products. These two objectives are certainly conflicting because achieving a higher freshness of products delivered has to be done at the expense of higher production costs, for example, through the splitting of some lots. Coupled with the multi-objective formulation, we keep track of spoiled inventory and penalize it in the economic objective function as a disposal cost. Therefore, we acknowledge the complete different nature of the two complementary objectives and the difficulty to attribute different monetary values to different degrees of perishability. As a result, the decision maker will be offered a trade-off between freshness of delivered products and total costs. This trade-off is represented by a set of solutions that do not dominate one another regarding both objectives (nondominated or Pareto optimal front).

Another important issue is related with the production strategy used. According to Soman et al[18], companies from consumer good products should work in a hybrid make-to-order/make-to-stock (MTO-MTS) strategy. This is motivated by a huge increase in product variety and shorter lead-time requirements of the customers. For example, in the yogurt industry where there is limited capacity, producing a very large number of products on pure MTS basis is not viable because of highly variable demand, which results in high safety stocks, and the perishable nature of the products. Also, pure MTO is difficult to implement because of the large number of relatively long, costly setups that characterizes these type of industries. To conclude, on one hand, there is a need to be flexible and react to customer demand, and on the other hand, there is a goal to restrict changeovers and produce economically. Following this concept, we first develop a MTO model, which has been the strategy more used to deal with perishable products so far and then move toward a more complete model based on a hybrid MTO-MTS strategy. In this latter model, we are not concerned about the classification of a given product according to its production strategy. This means that there is a decision to be made on a strategic level to decide whether a product is to be produced in MTO or in MTS.[19]

At the core of the proposed lot-sizing and scheduling formulations lies the general lot-sizing and scheduling problem (GSLP) that has been proved to be NP-complete even when we are just concerned about finding a feasible solution.[20] Hence, we do not expect that exact methods are able to cope with the computational difficulties of this problem. The computational complexity and the multi-objective nature of the problem led us to use a population-based meta-heuristic approach to exploit the model and characterize a nondominated front. We have adapted the non-dominated sorting genetic algorithm (NSGA-II)[21] that is able to find a well-spread set of solutions to deal with our problem through a hybridization with a commercial mixed integer linear programming solver.

The remainder of the paper is organized as follows. The next section is devoted to the description of the yogurt packaging problem. In Section 3, two models are formulated for the multi-objective lot-sizing and scheduling problem with perishable products: one using a MTO strategy (MO-LSP) only, and the other one using a hybrid MTO-MTS strategy (MO-LSPI). This section ends with an illustrative example. In Section 4, a brief review of multi-objective optimization with genetic algorithms is done, and the algorithm proposed is present. In Section 5, the results of computational experiments are provided. Some conclusions are drawn in Section 6.

■ THE YOGURT PACKAGING PROBLEM

Yogurt is arguably one of the most popular consumer goods. It is a semi-solid fermented product made from standardized milk mixed with a symbiotic blend of yogurt culture organisms.[22] Several types of yogurt exist; the two main types are set and stirred yogurt.[23] While set yogurt is incubated and fermented in the retail cups, stirred yogurt is fermented before packaging. As all fresh products, yogurt has a relatively short shelf life. The shelf life of yogurt produced under normal conditions is about 8−10 days when satisfying the normal storage conditions. However, following the trend to concentrate production capacities, to extend the markets supplied and to increase the product portfolio, many manufacturers have increased the shelf life of their products up to 3 or 4 weeks, mainly by means of aseptic packaging technology.

Yogurt production could be considered as a particular case of a batch or semi-continuous production process. In the literature, a production environment where a continuous production stage is followed by a packaging stage is called “make-and-pack” production.

Lot sizing and scheduling constitute the major challenges in this type of production environment. One of the main features of batch processes is that large numbers of products are produced from a few initial product recipes. The same description holds for yogurt production. Thus, in industrial yogurt production, there is a wide variety of products that differ in features like the fat content, whey used to produce the mixture, added flavor, special ingredients (e.g., chocolate flakes and fruit), size of the container, or language on the label. The yogurt production process is well reported in the literature; for additional details, the readers are referred to Entrup et al.[15] or Kopanos et al.[24] In this work, we are only concerned with the packaging stage where the divergent product structure (i.e., a low number of raw materials leading to a high number of final products) shows its impact by imposing to the planner the need to deal with a great amount of SKUs to lot size and schedule.

As pointed out by Nakhla[25] regarding scheduling operations at a yogurt production line, an empirical law is that the succession of products must follow an increasing fat level. For example, if skimmed milk is used to produce unflavored yogurt, it should be produced before full-fat milk in order to reduce setup time and the wasted quantity. Similarly, plain yogurt is processed before flavored. The opposite would require significant cleaning time and costs in order to make sure that no flavor or colors would be transferred to the plain yogurt that follows. This intrinsic characteristic of yogurt production based on recipes may be observed in many other consumer goods industries and pushes the planners toward the concept of product family. This production environment is much related to the block planning concept, also called production wheel policy, which will be later explained and used in our formulation.

Because of the great diversification that has been imposed by the market and the rather short-life of yogurt, many different products must be produced on a daily basis, which increases complexity in making scheduling decisions. Efficient scheduling paths become harder to find when additional constraints on products, machines, or time are enforced. Furthermore, the filling stage of the yogurt production process is characterized by sequence-dependent setups that require an integrated view of lot-sizing and scheduling levels. As it is empirically understandable, we do not only need to know when to produce but also how much to produce in order to account for perishability in a weekly rolling planning horizon.

Because yogurt production is characterized by a high complexity and yogurt has a great impact on the fresh diary industry, which is the most important fresh food sector, it is believed that good production models can be easily rolled out to other industries dealing with similar perishables.

■ MATHEMATICAL FORMULATION

In this section, we develop multi-objective mixed-integer programming models for two different strategic scenarios: pure MTO and hybrid MTO-MTS environments. Hence, in the first model, all products are produced on the basis of certain demand orders. In the second model, some products are produced on the basis of the same information, and others are produced on the basis of uncertain demand forecast for the planning horizon considered. Because these multi-objective formulations are novel, an illustrative example is presented to better understand the impact of the models. The proposed formulations are essentially leveraged by three different lot-sizing and scheduling concepts addressed in the literature.

First, we rely on the general lot sizing and scheduling for parallel lines (GLSPPL) formulation[26] that uses a mixture of large and small time buckets. This two-level time structure is crucial to attain two distinct objectives of our problem. On one hand, with the macro-time structure, we are able to control our main external factor, which is perishability/freshness, along with the more traditional external dynamics, such as demand and inventory. On the other hand, the micro-time structure allows the necessary flexibility to handle the difficult production issues arising in perishable industries, helping to incorporate issues as sequence-dependent setups.

Second, we make use of the simple plant location (SPL) reformulation[27] to model production quantity decision variables. This reformulation of the production quantities allows us to know which day is referred to as the production day. Hence, every demand element has associated a number of production quantities all having a production day tag. With this approach, it is rather easy to measure the freshness of all products delivered and model this attribute explicitly in the objective function.

Third, we take into account the concept of block planning developed in Gunther et al.,[28] especially suited for the consumer goods industry in which the yogurt description fits. As discussed before in Section 2, in yogurt production there are natural sequences within a recipe. Therefore, a block corresponds to a recipe, and within a recipe, the sequence of products is set *a priori*, so the only decision to be made for each block, besides the lot size, is to produce a given product or not. Nevertheless, there is still an important sequencing problem of blocks on a production line. Taking advantage of the inner production characteristics of the problem makes it more understandable to the decision maker; because the definition of the recipes is something very familiar to him, it enlarges its application potential and reduces the overall complexity.

This mixed formulation results in a model suitable to deal with the operational production planning of perishable products related to yogurt because it models product freshness explicitly with the help of SPL reformulation. The intrinsic recipe structure is translated through the block planning concept, and the necessary production flexibility in sequencing blocks and in obtaining lot-sizes is achieved by using GLSPPL. Finally, the multi-objective framework puts in evidence the trade-off between production costs and product freshness, linking these concepts together.

The two mathematical models are formally defined below, which demonstrate how these concepts interact. An illustrative example is also presented.

Make-to-Order Formulation. All product variants $k = 1,...,K$ based on the same recipe form a block; therefore, a product can be assigned to one block only. Blocks $j = 1,...,N$ are to be scheduled on $l = 1,...,L$ parallel production lines over a finite planning horizon consisting of macro-periods $d = 1,...,T$ with a given length. The scheduling takes into account that the setup time and cost between blocks is sequence dependent. The sequence of products for a given recipe is set *a priori* because of the natural constraints in these kind of industries. Hence, when changing the production between two products that are variants of the same recipe, solely a minor setup is needed, which is not dependent on the sequence but on the product to be produced.

A macro-period, in this case a day, is divided into a fixed number of nonoverlapping micro-periods with variable length. Because the production lines can be scheduled independently, this is done for each line separately. S_{ld} denotes the set of micro-periods s belonging to macro-period d and production line l. All micro-periods are put in order $s = 1,...,S^l$, where S^l represents the total number of macro-periods in line l defined implicitly by S_{ld}.

The length of a micro-period is a decision variable, which is expressed by the production of several products of one block produced in the micro-period on a line. A sequence of consecutive micro-periods, where the same block is produced on the same line, defines the lot of a given block, and the quantity of the products from that recipe produced during these micro-periods defines the size of the lot. Therefore, a lot may aggregate several products from a given block and may continue over several micro- and macro-periods. Moreover, a lot is independent of the discrete time structure of the macro-periods. The number of micro-periods of each day defines the upper bound on the number of blocks to be produced daily on each line.

As a consequence of the fixed number S_{ld}, a lot may contain idle micro-periods with null production. If, after an idle micro-period, the same block is produced on the same line again, the setup state is conserved, i.e., no further setup is necessary.

Consider the following indices, parameters, and decision variables.

Indices

l = parallel production lines
i,j = blocks
k = products
d,h = macro-periods
s = micro-periods

Parameters

K_j = set of products belonging to block j
$|K_j|$ = number of products belonging to block j
S_{ld} = set of micro-periods s within macro-period d and line l
Cap_{ld} = capacity (time) of production line l available in macro-period d
a_{lk} = capacity consumption (time) needed to produce one unit of product k on line l
c_{lk} = production costs of product k (per unit) on line l
u_k = shelf life duration of product k after completion of its production (time)
m_{lj} = minimum lot size (units) of block j, if produced on line l
$scb_{lij}(stb_{lij})$ = sequence-dependent setup cost (time) of a changeover from block i to block j on line l
$scp_{lk}(stp_{lk})$ = sequence-independent setup cost (time) of a changeover to product k on line l
d_{kd} = certain demand for product k in macro-period d (units)
y_{lj0} = equals 1, if line l is set up for block j at the beginning of the planning horizon (0 otherwise)

Decision Variables

w_{khd} = fraction of demand of product k produced in macro-period h for meeting demand in macro-period d, $\forall d \geq h \wedge d \leq h + u_k$
q_{lks} = quantity of product k produced in micro-period s on line l
p_{lks} = setup state: $p_{lks} = 1$, if line l is set up for product k in micro-period s (0 otherwise)
y_{ljs} = setup state: $y_{ljs} = 1$, if line l is set up for block j in micro-period s (0 otherwise)
z_{lijs} = takes on 1, if a changeover from block i to block j takes place on line l at the beginning of micro-period s (0 otherwise)

We note that variable w_{khd} is only instantiated for $d \geq h \wedge d \leq h + u_k$ to ensure that demand fulfilled in period d is produced beforehand with units that have not perished yet. In the pure MTO setting, we assume that the initial inventory at the beginning of the planning horizon is null, and that no stock is kept at the end of it. Nevertheless, intermediate units might be held when the w_{khd} has $d>h$.

The multi-objective MTO lot sizing and scheduling of perishable goods (MO-LSP) reads:

MO-LSP

$$\min \sum_{l,i,j,s} scb_{lij} z_{lijs} + \sum_{l,k,s} (scp_{lk} p_{lks} + c_{lk} q_{lks}) \tag{1}$$

$$\max \sum_{k,h,d} (h + u_k - d) w_{khd} \tag{2}$$

subject to:

$$\sum_{h} w_{khd} = 1 \quad \forall\ k, d: \quad d_{kd} > 0 \tag{3}$$

$$\sum_{h} w_{khd} = 0 \quad \forall\ k, d: \quad d_{kd} = 0 \tag{4}$$

$$\sum_{l, s \in S_{lh}} q_{lks} = \sum_{d} d_{kd} w_{khd} \quad \forall\ k, h \tag{5}$$

$$\sum_{k \in K_j} p_{lks} \leq y_{ljs} |K_j| \quad \forall\ l, s, j \tag{6}$$

$$q_{lks} \leq \frac{\mathrm{Cap}_{ld}}{a_{lk}} p_{lks} \quad \forall\ l, k, d, s \in S_{ld} \tag{7}$$

$$\sum_{i,j,s \in S_{ld}} stb_{lij} z_{lijs} + \sum_{k,s \in S_{ld}} (stp_{lk} p_{lks} + a_{lk} q_{lks}) \leq \mathrm{Cap}_{ld} \quad \forall\ l, d \tag{8}$$

$$\sum_{j} y_{ljs} = 1 \quad \forall\ l, s \tag{9}$$

$$\sum_{k \in K_j} q_{lks} \geq m_{lj} (y_{ljs} - y_{lj,s-1}) \quad \forall\ l, j, s \tag{10}$$

$$z_{lijs} \geq y_{li,s-1} + y_{ljs} - 1 \quad \forall\ l, i, j, s \tag{11}$$

$$w_{khd}, q_{lks}, z_{lijs} \geq 0; \ p_{lks}, y_{ljs} \in \{0, 1\} \tag{12}$$

In the first objective (eq 1), production related costs are minimized, namely, sequence-dependent setup costs between blocks, sequence-independent setup costs of products, and production line costs. This objective function is common in the literature for other lot-sizing and scheduling variants, and it only takes into account purely economic issues. Nevertheless, this first objective function has an important detail to consider. Because we are splitting the major from the minor setups, we are considering them both for the first product to be produced in a given block. For example, in yogurt production, when changing from one kind of yogurt to another, a major setup might correspond to cleaning the lines and linking the new yogurt tank, while the minor setup may correspond to setting up the machine to fill the yogurt in the new packages. These two operations can not be done in parallel.

In the second objective (eq 2), the mean freshness of products to be delivered is implicitly maximized. The BBD of a given product can be estimated through the proposed model by $h + u_k$, where h accounts for the completion date of the product and u_k for the shelf life of that specific product. Subtracting the delivery date of the product lot, we have an estimation of the available time of product k on the retailer's shelves. Therefore, from this objective to the mean freshness, we only needed to divide the objective value by the number of occurrences in the demand matrix $[d_{kd}]$. This cardinality, for a given input set data, is constant and easily computed. Therefore, the maximization of the current objective corresponds to the maximization of the mean freshness of delivered products. Figure 1 shows a scheme of how this objective function works for one demand order, d_{kd}.

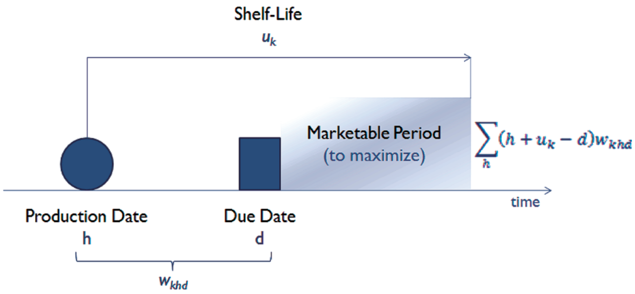

Figure 1. Schematic representation of the objective related with maximizing the freshness of the delivered products.

Remark that, given the definition of w_{khd}, the term $h + u_k - d$ is always non-negative. This second objective is closely related to a more common one, in pure scheduling problems, related with the just-in-time philosophy through the minimization of earliness. Earliness of a job j is given by $E_j = \max(0, d_j - D_j)$, where d_j is the due date of job j, and D_j is the completion time of job j. Tardiness is not allowed in our model $D_j \leq d_j$, hence $E_j = d_j - D_j$. By a trivial reasoning, $\min E_j = \min (d_j - D_j) = \max (D_j - d_j)$. Replacing this notation with the one presented in the model and ignoring the shelf life, u_k, which is a fixed parameter for each product, makes clear the close relation between those objectives (maximization of freshness and minimization of earliness). Nevertheless, it is important to consider u_k because it allows a weighting of this function on the basis of the durability of product k.

Looking now at the constraints to which this problem is subjected, each day demand is met with specific production done until that day without backlogging by eq 3 and eq 4.

Equation 5 forces the total production for each day to meet demand from that day onward in the current planning horizon. Moreover these constraints link parallel production lines together.

Equations 6 and 7 ensure that a product can only be produced if both the correspondent block and product are set up. Furthermore, with eq 9, just one block can be produced on a given line and in a given micro-period. Limited capacity in the lines is to be reduced by setup times between blocks, setup times between products, and also by the time consumed producing products (eq 8). Equation 10 introduces minimum lot sizes for each block. Finally, the connection between setup states and changeover indicators for blocks is established by eq 11.

Hybrid Make-to-Order/Make-to-Stock Formulation. The basic definition provided in the MTO model is valid for the hybrid MTO-MTS formulation but needs to be expanded in some aspects. Recall that in the MO-LSP formulation, we do not allow carrying stock over the next planning horizon. Thus, both initial and final inventory are zero, but in this formulation MTS products have this restriction relaxed. Therefore, there is a necessity to expand the time dimension backward in order to account for stock built in previous planning horizons that can be used to fulfill current demand. The length that needs to be considered is related with the product with the longest shelf life.

One shall consider an integer multiple X of past planning horizons that is enough to cover the longest shelf life, i.e., $X = \lceil(\max u_k)/(T)\rceil$, hence $t = -XT+1,...,0,1,...,T$. Let $T^- = \{-XT+1,...,0\}$ and $T^+=\{1,...,T\}$, thus, the domain of t is equivalent to $[T] = T^- \cup T^+$.

The indices, parameters, and decision variables are almost the same used before in the MTO formulation. Therefore, we only refer to the modifications performed.

Indices

t,d,h,b = macro-periods: $t \in [T]$; $d,h \in T^+$; $b \in T^-$

Parameters

ψ^{MTS} = set of products produced in a make-to-stock strategy
ψ^{MTO} = set of products produced in a make-to-order strategy
ϕ_k = spoilage cost of product k in inventory (per unit), $k \in \psi^{MTS}$
I^+_{kh} = maximum inventory of product k to hold in macro-period h (units), $k \in \psi^{MTS}$
I^-_{kh} = minimum inventory of product k to hold in macro-period h (units), $k \in \psi^{MTS}$
R^*_{kb} = stock of product k at the beginning of the planning horizon that was produced in macro-period b (units), $\forall b \in T^-, k \in \psi^{MTS}$
$\tilde{d}_{kd}$ = forecast demand for product k in macro-period d (units), $k \in \psi^{MTS}$

Decision Variables

B_{kd} = quantity of stock of product k that spoils in macro-period d (units), $k \in \psi^{MTS}$
R_{kd} = production to stock of product k produced in macro-period d to be used in the next planning horizon (units), $d \geq T - u_k \wedge d \geq 0$, $k \in \psi^{MTS}$
w_{ktd} = fraction of demand of product k produced in macro-period t for meeting demand in macro-period d, $\forall d \geq t \wedge d \leq t + u_k$. If $k \in \psi^{MTS}$ then $t \in [T]$; if $k \in \psi^{MTO}$ then $t \in [T^+]$

We note that variable R_{kd} is only instantiated for $d \geq T - u_k \wedge d \geq 0$ to avoid building stock to use in the next planning period when it would perish still in the current one. Similarly to what was done in the pure MTO formulation, w_{ktd} is only instantiated for $d \geq t \wedge d \leq t + u_k$ to ensure that demand fulfilled in period d is produced beforehand with units that have not perished yet. But, in this case, the domain is extended for the MTS products to cover the stock that has been made to stock beforehand in the last planning horizons.

The multi-objective hybrid MTO-MTS lot-sizing and scheduling of perishable goods (MO-LSPI) reads:

MO-LSPI

$$\min \sum_{l,i,j,s} scb_{lij} z_{lijs} + \sum_{l,k,s} (scp_{lk} p_{lks} + c_{lk} q_{lks}) + \sum_{k,d} \phi_k B_{kd} \quad (13)$$

$$\max \sum_{k \in \psi^{MTS},t,d} (t + u_k - d) w_{ktd} + \sum_{k \in \psi^{MTO},h,d} (h + u_k - d) w_{khd} \quad (14)$$

subject to:

$$\sum_d \tilde{d}_{kd} w_{kbd} \leq R^*_{kb} \quad \forall k \in \psi^{MTS}, b \quad (15)$$

$$B_{kd} \geq R^*_{kb} - \sum_h \tilde{d}_{kh} w_{kbh} \quad \forall k \in \psi^{MTS}, b, d = b + u_k + 1 \quad (16)$$

$$\sum_t w_{ktd} = 1 \quad \forall k \in \psi^{MTS}, d: \tilde{d}_{kd} > 0 \quad (17)$$

$$\sum_h w_{khd} = 1 \quad \forall k \in \psi^{MTO}, d: d_{kd} > 0 \quad (18)$$

$$\sum_t w_{ktd} = 0 \quad \forall k \in \psi^{MTS}, d: \tilde{d}_{kd} = 0 \quad (19)$$

$$\sum_h w_{khd} = 0 \quad \forall k \in \psi^{MTO}, d: d_{kd} = 0 \quad (20)$$

$$\sum_{l,s \in S_{lh}} q_{lks} = \sum_d \tilde{d}_{kd} w_{khd} + R_{kh} \quad \forall k \in \psi^{MTS}, h \quad (21)$$

$$\sum_{l,s \in S_{lh}} q_{lks} = \sum_d d_{kd} w_{khd} \quad \forall k \in \psi^{MTO}, h \quad (22)$$

$$I^-_{kh} \leq \sum_{d > h, t \leq h} \tilde{d}_{kd} w_{ktd} + \sum_{d \leq h} R_{kd} \leq I^+_{kh} \quad \forall k \in \psi^{MTS}, h \quad (23)$$

Equations 6−11

$$B_{kd}, R_{kd}, w_{ktd}, q_{lks}, z_{lijs} \geq 0; \; p_{lks}, y_{ljs} \in \{0, 1\} \quad (24)$$

The first objective function (eq 13) is the same as in the first model, except for the addition of the last term, which represents the costs of inventory spoilage.

In eq 14, the mean freshness of products to be delivered both in MTS and MTO production is maximized. Hence, this objective is rather similar to eq 2, but in this case, besides maximizing the freshness of MTO products, we also maximize the freshness of products produced in a MTS strategy. All the other reasoning made before regarding this objective in the former model applies to this one.

Equation 15 ensures that the initial stock of all products produced before the actual planning horizon can be spent in the current planning horizon to fulfill demand. Nevertheless, there is a special concern with perishability, and therefore, only products that have not perished are allowed to be used. Equation 16 is used to calculate the quantity of perished inventory.

Each day, demand is only to be met with specific production done until that day in the MTO case (eq 17), and for the MTS products, it can also be met with stock left from the past planning horizons (eq 18). Equations 19 and 20 are just needed to ensure that production variables w_{ktd} are zero when the demand of a period d is null.

For MTS products, total production for each day is either to be used to meet future demand in the current planning horizon or to constitute final stock to use in the next planning horizons, as stated by eq 21. For MTO products, total production for each day is only used to meet demand of the current planning horizon (eq 22). Moreover, these two constraints link parallel production lines together. In eq 23, we set the daily minimum and maximum inventory for each product produced to stock. In fact, using the

Table 1. Data for the Illustrative Example

		parameters					R_{kb}		demand				
strategy	blocks	u	ϕ	a	c	m	−1	0	1	2	3	4	5
MTO	A	2	10	1	1	5	–	–	–	40	–	–	–
MTS	B	2	10	1	1	5	5	5	30	30	–	–	40
MTS	C	2	10	1	1	5	5	5	10	–	10	–	10
MTS	D	2	10	1	1	5	5	5	10	20	30	–	–

Table 2. Setup Matrix for the Illustrative Example (Cost and Time)

	block			
	A	B	C	D
A	_	0.5	0.75	1
B	1	_	0.5	0.75
C	1.25	1	_	0.5
D	1.5	1.25	1	_

SPL reformulation to model the production variables, there is no need to explicitly define an inventory decision variable. The stock in a macro-period h is equal to the production done until macro-period h to be used in a future period in the current planning horizon, in addition to the production to final stock until macro-period h. The definition of these bounds is quite important, both regarding consumer satisfaction and the quantity of spoiled products, but this is not in the scope of our research.

Equation 6−11 from the first model have the same meaning as before.

Illustrative Example. Because this is the first time that a multi-objective formulation is proposed for this problem, it is of interest to show by an illustrative example the conflicting nature of both objectives. The example will be applied to MO-LSPI, which can be regarded as an extension of the MO-LSP, where products can be produced either to stock or to order.

In the example (based on the one presented in Pahl and Voss[14]), there are four products to be scheduled on one production line. Each of these products belongs to a different block, and therefore, there is always setup time and cost to consider when changing from one product to another. The number of micro-periods per macro-period is set at the constant value of 4, allowing the production of all products in a macro-period. The capacity of the line is 70 units per macro-period. At the beginning of the planning horizon, the line is set for product A. Tables 1 and 2 give the remainder data for the example.

To understand that the two objectives (minimization of costs and maximization of freshness) are conflicting and, hence, the individual optimal solutions are very different and somehow complementary, three cases were tested. In Case 1, the problem is solved considering only the single objective function in eq 13 (minimization of costs), and in Case 2, we are only concerned with eq 14 (maximization of mean freshness). Case 3 is an example of an intermediate solution obtained by optimizing a random weighted linear sum of both objectives.

The optimal results obtained for each case using CPLEX 12.1 are displayed in Table 3. From its analysis, the trade-off is clear between production costs and freshness of products delivered. The two extreme solutions (solutions 1 and 2) show well the conflicting behavior of the objective function values. When

Table 3. Results of Illustrative Example

			macro-periods					objective function	
			1	2	3	4	5	cost	freshness
solution 1	production	A	40	–	–	–	–	203.00	1.123
		B	29.5	20.5	40	–	–		
		C	–	–	10	–	10		
		D	–	48.75	1.25	–	–		
solution 2	production	A	22	18	–	–	–	259.25	1.933
		B	26.5	30	–	–	40		
		C	10	–	10	–	10		
		D	10	20	30	–	–		
	spoiled	A	–	-	-	–	–		
		B	–	5	1.5	–	–		
		C	–	5	5	–	–		
		D	–	5	5	–	–		
solution 3	production	A	16.25	23.75	–	–	–	226.25	1.809
		B	20	30	–	–	40		
		C	5	–	10	–	10		
		D	10	15	30	–	–		
	Spoiled	A	–	–	–	–	–		
		B	–	–	–	–	–		
		C	–	5	–	–	–		
		D	–	5	–	–	–		

increasing freshness from solution 1 to 2, the costs have also to be increased. With solution 3, it is interesting to note that with a small increase in costs, with respect to the optimal value, a much higher freshness of delivered products can be achieved. To make this point clearer, we can state that the percentage of remaining shelf life of the products delivered in solution 1, at the lowest cost, is around 56.15%, and in solution 2, is about 96.64%, at the expense of a cost increase. These figures are obtained dividing the value of the solution for the second objective by the shelf life of the products, which is the same for the four products.

A point shall be made in relation to the scale of the problem. Because, in this case, the different production combinations are rather limited, we expect that with larger instances, the quantity of nondominated solutions that expose the trade-off between total costs and product freshness will be significant.

■ SOLVING THE MULTI-OBJECTIVE LOT SIZING AND SCHEDULING DEALING WITH THE PERISHABILITY PROBLEM

Like in the single-objective literature, where the researchers are much divided between scheduling and lot sizing separately, the multi-objective literature suffers from the same problem. In fact, papers that take an aggregate view of these two inter-related problems simultaneously are very few. Because we are interested in tackling simultaneously the lot-sizing and scheduling problems, we will only review general relevant issues of multi-objective optimization and papers directly related with multi-objective scheduling problem (MOSP).

Recently, two extensive reviews on this topic were performed.[29,30] Loukil[31] distinguishes five main approaches in the literature concerning MOSPs: (1) hierarchical approach, where

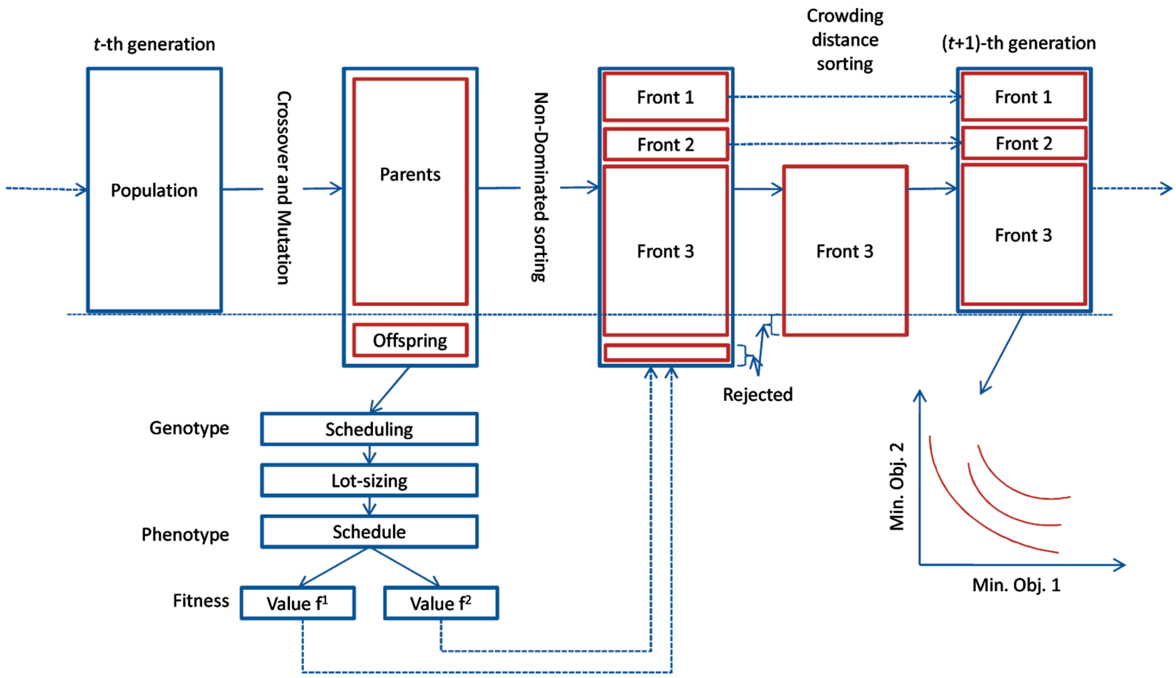

Figure 2. Outline of the hybrid multi-objective GA to solve the MO-LSP and MO-LSPI.

the objectives considered are ranked in a priority order and optimized in this order, (2) utility approach, where an utility function or weighting function is used to aggregate the objectives in a single one, (3) goal programming, where all the objectives are taken into account as constraints, which express some satisfying levels, and the aim is to find a solution which provides a value as close as possible of the predefined goal for each objective, (4) interactive approach, where at each step of the procedure, the decision maker express his preferences in regard to one (or several) solutions proposed so that the method will progressively converge to a satisfying compromise between the objectives, and finally, (5) the Pareto approach, where the aim is to generate, or to approximate in case of an heuristic method, the complete set of nondominated solutions.

To understand how the Pareto approach works, consider a set of solutions for a problem with multiple objectives. By comparing each solution to every other solution, those solutions dominated by any other for all objectives are flagged as inferior. For multi-objective problems, determining a good set of Pareto front solutions (representative of different trade-offs between the objective functions in distinct regions of the search space) provides maximum information about the optimization problem,[32] and therefore, this is the approach we will use in solving our problem.

From a varied set of methods possible to use in order to obtain a Pareto front, multi-objective evolutionary algorithms (MOEAs) have became a trend of the related study for MOSP and similar problems, as discussed in Deming.[33] In fact, MOEAs offer a series of highly relevant advantages in this area.[34,35] They are capable of finding good approximations of the optimal Pareto front in NP-complete problems, as is the case with our problem, at the expense of a reasonable amount of computational time. Hence, they offer a good trade-off between efficiency and effectiveness. The parallel nature of their search method allows them to handle complex and difficult search spaces. Moreover, they offer a choice of potential solutions from the population at any stage in the optimization process and allow them to work with incomplete or inexact data through adequate representations of the individuals. In addition, MOEAs have shown to be effective methods for multi-objective optimization of complex real-life applications and to find nonconvex solution spaces (contrary to more traditional methods based on the aggregation of objectives). In the literature, a number of MOEAs have been suggested, such as NSGA-II, SPEA, SPEA2, and PESA (for a comparison of these algorithms the readers are referred to Khar et al.[36]). In the review performed by Deming[29], a trend toward hybrid multi-objective algorithms was identified. In this hybrid approach, standard multi-objective algorithms are combined with other techniques, such as integrating local search procedures or solvers to deal with specific subproblems. To solve our problem, we have used a hybridization of a MOEA and a commercial mixed integer linear solver. The underlying MOEA is the non-dominated sorting genetic algorithm (NSGA-II), which has proven to perform rather well in comparison with other MOEAs.[36] The readers are referred to Deb et al.[21] for the details of NSGA-II. In Figure 2, an outline of the proposed hybrid algorithm to solve MO-LSP and MO-LSPI is shown.

There are five important components that are needed to completely describe any GA:[37] the representation format used by the GA; the operators of crossover, mutation, and selection; the fitness evaluation function; the various parameters like the population size, the number of generations, the probability of applying the operators, etc.; and a method for generating the initial population. Concerning our approach, it is also important to include a section that discusses infeasibility/constraint handling. The parameters component will be discussed and analyzed in Section 5.

Representation of an Individual. The usual representation of a solution in a GA for the lot-sizing and scheduling problem is through a string of paired values for each scheduling period in the planning horizon.[38] The first value indicates the type of product, and the second value indicates the number of units of that product type to be produced, i.e., the lot size. Because of our hybrid approach, an individual is composed of a string of single values. The value of each chromosome represents the block to be scheduled in each micro-period of a given machine. Therefore there are $\sum_l S^l$ chromosomes in each individual. For each individual, the lines are put in numerical order, so at every position it is possible to know exactly the micro-period and the machine accessed. This yields an incomplete representation of the individual, as it only determines the value of y_{lks}. The value of the other variables will be obtained afterward through the mixed integer linear solver.

Genetic Operators. Three kinds of genetic operators are implemented as follows.

Crossover. Because it is usual in these industries to have dedicated lines, a multitude of recipes/products may only be produced on certain lines. The one-point crossover was used in such way that the changes done by this operator will respect this condition.

Mutation. The mutation operator used is similar to the bitwise mutation, but in this case, a chromosome is randomly selected. Its value is changed into a value corresponding to a block that is allowed to be produced on that line.

Selection/Reproduction. After the offspring population is created, the whole population is sorted according to nondominance. The new parent population is formed by adding solutions from the first front until the size exceeds a defined parameter of the size of population. Thereafter, the solutions of the last accepted front are sorted according to the crowding operator, in order to have a well-dispersed set of solutions, and the rest of the new population is filled.

Fitness of an Individual. The hybridization of the NSGA-II is done in this step. Each individual representing a sequence of blocks to be scheduled on each line is fed into the mix-integer linear programming model fixing the values of all y_{lks} and, consequently, a significant number of p_{lks} also. Notice that y_{lks} and p_{lks} are the only binary variables of our mathematical models. From this moment onward, the problem is much easier for the commercial tool to solve because optimal lot sizes can be computed on the basis of the fixed block sequence of the individual, while minimizing a random weighted linear combination of both objectives. Hence, each time an individual is fed into the solver a new random pair of weighting coefficients is generated in order to combine the two objectives transforming the model into a single objective. This random weighting aims at providing a good dispersion of results throughout the Pareto front. After defining the optimal lot sizes, the values of the two objectives are fed back to the GA that will use both values to rank the individuals and compute the Pareto fronts.

Initializing a Population. A population is initialized by generating random values for each chromosome. Nevertheless, special attention is paid to be sure that the possible values are in compliance with the possible blocks/products that can be produced on a line. This way, in the first step, we are sure the individuals are feasible with respect to allowed sequences in every machine.

Infeasible Individuals. Infeasible individuals are not automatically disqualified from the population nor are they repaired to create feasible individuals. Instead, the amount of infeasibility is defined as an additional objective that must be minimized. Justification for treating the infeasibility in this way is provided in Shaw et al.[34] and Oliveira Santos et al.[39] This approach has been suggested by and Beasley[40] and Surry et al.,[41] as a method of avoiding the process of defining a more complex but constantly

Table 4. Setting of GA Parameters

population size	100
number of generations	100
crossover rate	0.9
mutation rate	0.00625

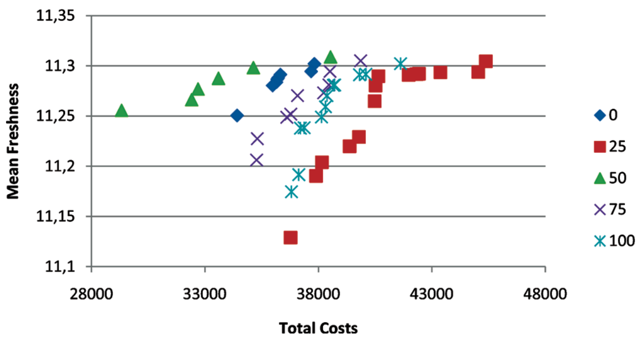

Figure 3. Results for the five strategic scenarios for instance 1.

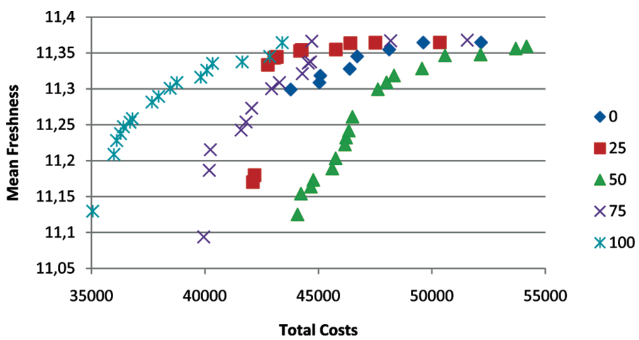

Figure 4. Results for the five strategic scenarios for instance 2.

feasible representation, specifically designed operators or a repair method. Furthermore, because the optimal solutions may lie close to the infeasible region, this approach let these frontier regions to be well explored. Therefore, when trying to calculate the fitness of an individual regarding both objectives, if it is verified that the individual is infeasible, then a new objective is set $\min \sum_i v_i$, where v_i is the amount of violation of constraint i. To measure this violation slack, variables are introduced, and their value is to be minimized, expecting that the individual will turn feasible in a future generation when subject to crossover and mutation.

■ COMPUTATIONAL EXPERIMENTS

The complete set of data provided by Kopanos et al.[24] for the case study of a lot-sizing and production scheduling problem of a multi-product yogurt production system in a real life dairy plant was extended for testing the proposed models and algorithm. This complex case study consists of a plant with four dedicated packaging lines. It produces 93 different products, which can be grouped in 23 product families (recipes). The setups allowed in the data set between product families were so constrained that the sequences were already defined making the problem rather easy to solve because the sequencing task was not performed. In our experiments, we allow any of the forbidden sequences of the case study by adding a small penalty for performing such sequences. We believe that this is close to the real world problem,

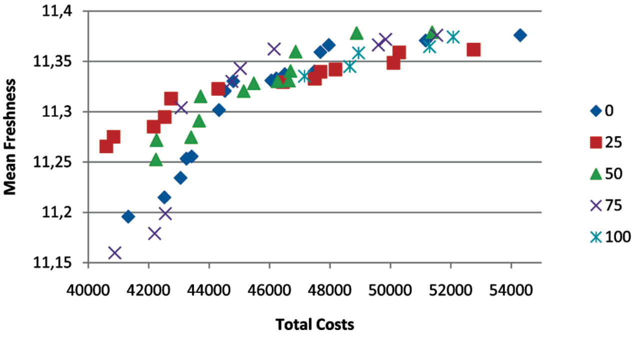

Figure 5. Results for the five strategic scenarios for instance 3.

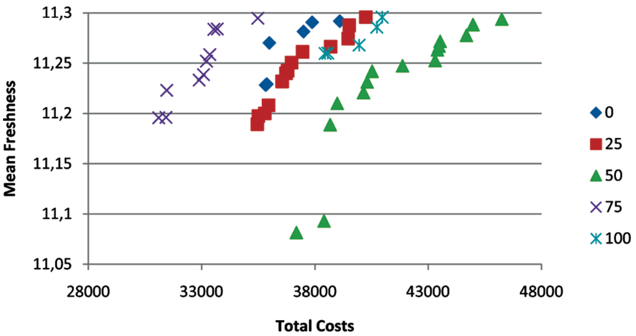

Figure 6. Results for the five strategic scenarios for instance 4.

where besides predefined sequences, it might be worth doing a somehow different sequencing between product families in order to take advantages of other key performance indicators, such as freshness. The setup costs and times between products of the same recipe are assumed to be negligible. This case study did not consider freshness explicitly, so there was a need to define the shelf life of each product, which was set between 10 to 15 days. The demand is disaggregated by product, and the considered planning horizon is 5 days. Regarding the design of the experiment taking into account the production strategy, five scenarios were considered varying the ratio of MTO/MTS products: 0% MTO, 25% MTO, 50% MTO, 75% MTO, and 100% MTO. The products produced in a MTS strategy had an initial stock of 15% of the total demand over the planning horizon. In this case study there are two different demand instances, which in our tests were completed with two more instances. Hence, we aim at understanding the behavior of the algorithm in reacting to different dynamic scenarios, as well as at observing the different trade-offs for different instances and strategic scenarios.

The problems were solved by the proposed hybrid multi-objective genetic algorithm, where the setting of parameters used is shown in Table 4. These values were found through exhaustive tests.

Figures 3− 6 show the results for the five strategic scenarios for each of the four demand instances, based on the supramentioned case study. The labels on the figures correspond to each of the five strategic scenarios: 0% MTO, ..., 100% MTO.

The computation time required for a complete run is about 30 min using a PC with 1 GB of RAM and an Intel Dual Core with 1.66 GHz running on Windows XP. All the implementations were done in C++, and the commercial mixed integer linear solver used was CPLEX 12.1. For a matter of comparison, when using CPLEX alone to solve the complete problem for just random aggregations of the two objectives, the best solutions reported after 30 min have integrality gaps of about 20%. With our approach, we are able to evaluate hundreds of solutions spread in the Pareto fronts and report in the end of a run multiple nondominated solutions for the decision maker.

Looking at the results for the four instances, it is clear that the strategic choice of which products to produce in a certain policy (MTO or MTS) has a direct impact on the set of possible solutions representing the trade-off between total costs and product freshness. This is a intermediate conclusion that adds a small contribution to what is in itself a research field concerned about this strategic decision.[19] The different results show that the Pareto fronts can take many forms in the solution space, even nonconvex. This confirms the choice that was taken regarding a solution method that is able to cope with such complexities, which is the case of the hybrid MOEA used.

The Pareto fronts, independent of the production strategy, prove to give valuable information to the decision maker regarding the important trade-off of freshness versus cost. Hence, having access to a pool of solutions that lead to different outcomes concerning total costs and product freshness, the decision maker is able to make a more informed decision, opting, for example, not for the solution which minimizes total costs, but for the one that only with a small cost increase is able to deliver fresher products.

■ CONCLUSION

In this paper, the simultaneous lot sizing and scheduling of perishable products problem was tackled. The main contribution of this paper lies on the multi-objective framework proposed to solve this problem overcoming the identified disadvantages of former modeling approaches. Hence, besides avoiding the assignment of fictitious costs, which are less realistic, the multi-objective approach offers the decision maker a tool to trade-off explicitly total costs against product freshness, which is quite related to customer satisfaction. Two different models were formulated. In the first one, a MTO strategy, which is being used in the literature to deal with this problem, was implemented. The second model extends the first one by considering a hybrid MTO-MTS strategy found to be important in this type of perishable industries.

A hybrid genetic algorithm based on NSGA-II was developed to solve both models, and it was tested in variations of instances reported in the literature. The results confirmed the conflict between the objectives and the advantages that the decision maker can have through having access to trade-off information between two immiscible performance indicators.

Future work shall be devoted to increase the algorithm efficiency and efficacy in order to achieve a better spread of the solutions in less computational time. It would also be interesting to test the proposed models in other perishable industries to fully grasp the adaptability of the proposed framework. Other further directions of research can focus on the incorporation of perishable factors in strategic issues, such as the production strategy, or in a more tactical level to study the impact of perishability in master production planning.

■ ASSOCIATED CONTENT

S Supporting Information. Complete test data for the computational experiments based on the referred literature. This material is available free of charge via the Internet at http://pubs.acs.org/.

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A dual mutation operator to solve the multi-objective production planning of perishable goods

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Abstract This work proposes a dual mutation operator that uses dual information to solve the multi-objective lot-sizing and scheduling problem arising in perishable goods production planning. This operator takes advantage of the hybrid structure of a multi-objective evolutionary algorithm, which combines the well-known Non-Dominated Sorting Genetic Algorithm and a mixed-integer linear programming solver. Chromosomes are coded as strings of integer values fixing the production sequence. For each feasible individual, the reduced costs of the relaxed binary variables coming from the coding scheme are used to guide the mutation process. To assess the performance of this operator, a set of randomly generated instances based on a methodology reported in the literature is tested and evaluated with multi-objective performance metrics. The results indicate that the operator is able to achieve consistently better solutions in terms of an approximation to the Pareto solutions compared to the same solution procedure without the dual mutation.

1 Introduction

Increasing attention has been paid to tackle product perishability in supply chain planning and methods able to deal with models which explicitly consider it are needed [1, 4]. The effects of final product perishability in production planning and

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scheduling impact various performance indicators, such as the inventory levels, the number of setups performed or the customer satisfaction.

Simultaneous lot-sizing and scheduling is mandatory to be executed whenever the underlying production process has sequence-dependent setups [2]. This is the case in many production systems of food consumer goods, such as the yoghurt production [24]. When considering lot-sizing and scheduling as separate problems, several works recognize the importance of a multi-objective approach to tackle conflicting objectives. This is not the case in the works dealing with these problems in an integrated manner. However, in case freshness needs to be accounted for when dealing with perishable products, a multi-objective framework is an advantageous modelling technique with significant benefits for supporting the decision maker [3]. In fact, besides avoiding the assignment of fictitious costs related to freshness loss, which are less realistic, the multi-objective approach offers the decision maker a tool to trade-off explicitly total costs against product freshness.

When solving a multi-objective lot-sizing and scheduling problem, it is important to take into account three features of the problem structure. First and foremost, single-objective lot-sizing and scheduling has been proven to be NP-complete, even for finding a feasible solution [9]. The multi-objective version will be at least as hard as the single-objective version. Second, in a multi-objective problem two important characteristics of the Pareto-front should be satisfied: a well-spread set of solutions and a front that is as near as possible to the optimal. Third, we should be aware that lot-sizing and scheduling problems have mainly two types of variables: continuous variables related to the size of the lots to be produced and binary variables related to the scheduling of those lots. In this paper we show how a multi-objective evolutionary algorithm (MOEA) [6] coupled with a mixed-integer linear programming (MIP) solver can be used to solve this problem by exploring these three features. Additionally, we focus on the potentialities of this hybridization by proposing a dual mutation operator that takes advantage of the information extracted from the MIP solver in order to guide the search [14]. This operator acts on feasible individuals assigning tailored mutation probabilities to each gene based on the reduced costs of the relaxed binary variables coded in the chromosomes. Thus, we aim at exploring each individual by guiding its path through the solution space.

The remainder of the paper is organized as follows. The next section is devoted to the literature review on similar problems and solution methods. Section 3 outlines the problem definition and its mathematical formulation. In Section 4, a detailed description of the MOEA and the dual mutation operator is given. In Section 5, the results of computational experiments are provided. Finally, in Section 6 conclusions are drawn and future research directions are pointed out.

2 Literature review

The present work intersects two research areas that have been identified as being still in their infancy: the production planning and scheduling of perishable goods and the development of hybrid multi-objective algorithms that combine metaheuristics and exact methods.

In the recent paper [20] and in the review [4], the key papers in the field of production planning and scheduling of perishable goods are enumerated and analysed. [25] studies the production planning in the yoghurt industry. The problem is formulated as a special case of the capacitated lot-sizing and scheduling problem with parallel machines, which includes shared buffers containing the same recipe. The objective function is typical for production planning problems and aims to minimize inventory, production and setup costs. In order to solve this hard combinatorial problem, a two-step heuristic that decomposes the problem into a lot-sizing and a scheduling problem is developed. Still regarding the planning and scheduling of the packaging stage in the yoghurt industry, [24] presents three mixed-integer linear models that incorporate shelf-life issues. These models use the concept of families/blocks to simplify the scheduling decisions. All models are solved recurring to a state-of-art MIP solver. [28] and [29] extend well known discrete lot-sizing and scheduling models, such as the general lot-sizing and scheduling problem by including deterioration and perishability constraints. The impact of minimum lot-sizes on the number of spoiled products is well documented. [3] tackles the production planning of perishable products using a multi-objective framework, trading-off costs and freshness. Two models were proposed depending on the production strategy and a number of real-life dairy instances are solved. The solution method used consists of a hybrid multi-objective evolutionary algorithm (MOEA). The works [19, 22, 21, 20] propose general MIP formulations for food processing industries ranging from yoghurt to ice-cream plants. These models have multi-stage considerations and several industry related constraints, such as natural setup sequences, sequence-dependent changeover times and costs, and the possibility of production overtimes. The latter papers focus also on the development of valid inequalities that improve the performance of solvers in finding solutions for real-life instances. More focused on scheduling, the work [11] presents various heuristics to solve the problem arising at the packing lines in dairy industries.

Concerning hybrid multi-objective algorithms, the work [8] reviews procedures that combine metaheuristics and exact methods. The authors acknowledge that despite the fact that in single-objective optimization the amount of cross-fertilization work has been rising interestingly, "it is astonishing to observe that this way to proceed is marginal in the multi-objective context". Nevertheless, among the few papers taking advantage of this hybridization, different motivations can be found underlying the innovative search procedures. In [10] the idea is to reduce the decision search space of the multi-objective knapsack problem with an exact procedure and afterwards running an adapted tabu search on a potentially more efficient region. The cuts applied ensure that no optimal solutions are eliminated and, therefore, in this solution method the exact methods help in enhancing the metaheuristic performance. Contrarily, [31] proposes a solution procedure for the multi-objective assignment problem in which a population based heuristic using path-relinking is used to accelerate an implicit enumeration scheme. Concerning the multi-objective knapsack problem, [13] combines heuristic search strategies that are used to detect potentially good regions in the decision search space, which are then explored by an exact method. Following the same reasoning of using exact methods to solve subproblems to optimality, [17] addresses the bi-objective covering tour problem using a MOEA combined with a branch-and-cut algorithm.

The literature reviews helps to make clearer the contribution of this work. First, it consolidates the importance of more work in the production planning and scheduling of perishables goods. Second, it proposes a dual mutation operator that takes advantages of the hybridization between the MOEA and the exact solver. This field of research is incipient and, therefore, needs actual proofs of the improvement in the results that can be derived from investing in hybrid methods capable of making the most of metaheuristics and exact methods.

3 Problem definition

This paper considers the lot-sizing and scheduling problem of perishable products. The main characteristic of perishable products is their relative short shelf-life. Our focus is on the semi-continuous production process of perishable consumer goods. Their production environment is characterized by a continuous production stage, followed by a packaging stage, which is known in the literature as "make-and-pack" production.

Lot-sizing and scheduling constitutes one of the major challenges in this type of production environment, in which a large number of products is produced from a few initial product recipes. Thus, in these settings, there is a wide variety of products that differ in small features, such as the size of the container, the language on the label, or the flavour. In this work we are only concerned with the packaging stage where the divergent product structure (i.e., a low number of raw materials leading to a high number of final products) shows its impact by imposing to the planner the need to deal with a great amount of stock keeping units to size and schedule.

In this kind of consumer goods industries, recipes are usually subject to empirical sequencing rules based on colours, flavours or temperatures that push the planners towards the concept of product family. This planning feature is much related to the block planning concept, also called production wheel policy, which will be partially used in our formulation [15]. Hence, if we consider, for example, the recipe of high fat yoghurt that has a setup time dependent on the recipe that was produced before, the product sequencing within this recipe is pre-defined and always starts with the plain yoghurt and ends with the chocolate flavour.

Due to the great diversification that has been imposed to these goods by the market and their rather short-life, many different products must be produced on a daily basis, which increases the complexity in making scheduling decisions. Moreover, in order to account for perishability, it is important to simultaneously know when and how much to produce in a weekly rolling-planning horizon.

3.1 Mathematical model

All product variants $k = 1, ..., K$ based on the same recipe form a block; therefore, a product can be assigned to one block only. Blocks $j = 1, ..., J$ are to be scheduled on $l = 1, ..., L$ parallel production lines over a finite planning horizon consisting of macro-periods $d, h = 1, ..., D$ with a given length (in this case a day). The scheduling takes into account the sequence dependent setup time and cost between blocks. The sequence of products for a given block is set *a priori* due to natural constraints in this kind of industries. Hence, when changing the production between two products that are variants of the same block/recipe solely a minor setup is needed, which is not dependent on the sequence but on the product to be produced.

A macro-period is divided into a fixed number of non-overlapping micro-periods with variable length. Since the production lines can be independently scheduled, this division is done for each line separately. S_{ld} denotes the set of micro-periods s belonging to macro-period d and production line l. All micro-periods are put in order $s = 1, ..., S^l$, where S^l represents the total number of micro-periods on line l implicitly defined by S_{ld} ($S^l = \sum_d |S_{ld}|$).

The length of a micro-period is a decision variable that depends on the production amount of several products of the same block. A sequence of consecutive micro-periods, where the same block is produced on the same line, defines the lot of a given block. Therefore, a lot may aggregate several products from a given block and may continue over several micro and macro-periods. Moreover, a lot is independent of the discrete time structure of the macro-periods. The number of micro-periods of each day defines the upper bound on the number of blocks to be produced daily on each line. As a consequence of the parameter S_{ld}, a lot may contain idle micro-periods with null production. In case, the same block is produced after an idle micro-period, on the same line, the setup state is conserved. Note that this lot-sizing and scheduling structure is based on the general lot-sizing and scheduling structure for parallel lines [26].

Consider the following indices, parameters and decision variables.

Indices

l	parallel production lines
i, j	blocks
k	products
d, h	macro-periods
s	micro-periods

Parameters

K_j	set of products belonging to block j
$\lvert K_j \rvert$	number of products belonging to block j
S_{ld}	set of micro-periods s within macro-period d and line l
$[d_{kd}]$	number of non-zero occurrences in the demand matrix
Cap_{ld}	capacity (time) of production line l available in macro-period d
a_{lk}	capacity consumption (time) needed to produce one unit of product k on line l
c_{lk}	production costs of product k (per unit) on line l
u_k	shelf-life duration of product k after completion of its production (time)
m_{lj}	minimum lot-size (units) of block j if produced on line l
$sc^b_{lij}(st^b_{lij})$	sequence dependent setup cost (time) of a changeover from block i to block j on line l
$sc^p_{lk}(st^p_{lk})$	sequence independent setup cost (time) of a changeover to product k on line l
d_{kd}	demand for product k in macro-period d (units)
y_{lj0}	equals 1, if line l is set up for block j at the beginning of the planning horizon (0 otherwise)

Decision Variables

w_{khd}	fraction of demand of product k produced in macro-period h for meeting demand in macro-period d, $h \leq d \leq h+u_k$
q_{lks}	quantity of product k produced in micro-period s on line l
p_{lks}	setup state: $p_{lks} = 1$, if line l is set up for product k in micro-period s (0 otherwise)
y_{ljs}	setup state: $y_{ljs} = 1$, if line l is set up for block j in micro-period s (0 otherwise)
z_{lijs}	takes on 1, if a changeover from block i to block j takes place on line l at the beginning of micro-period s (0 otherwise)

We highlight that variable w_{khd} is only instantiated for $h \leq d \leq h+u_k$ to ensure that demand fulfilled in period d is produced beforehand with units that have not perished yet. It is assumed that the inventory at the beginning of the planning horizon is null, and that no stock is kept at the end of it. Nevertheless, intermediate units might be held when variable w_{khd} is positive for $d > h$.

The lot-sizing and scheduling of perishable goods (MO-LSP) may be formulated as a multi-objective MIP model as follows:

MO-LSP

$$\min f^1 = \sum_{l,i,j,s} sc^b_{lij} z_{lijs} + \sum_{l,k,s} (sc^p_{lk} p_{lks} + c_{lk} q_{lks}) \tag{1}$$

$$\max f^2 = \frac{1}{[d_{kd}]} \sum_{k,h,d} \frac{h+u_k-d}{u_k} w_{khd} \tag{2}$$

subject to:

$$\sum_{h} w_{khd} = 1 \quad \forall k,d : d_{kd} > 0 \tag{3}$$

$$\sum_{h} w_{khd} = 0 \quad \forall k,d : d_{kd} = 0 \tag{4}$$

$$\sum_{l,s \in S_{lh}} q_{lks} \geq \sum_{d} w_{khd} d_{kd} \quad \forall k,h \tag{5}$$

$$\sum_{k \in K_j} p_{lks} \leq y_{ljs} |K_j| \quad \forall l,j,s \tag{6}$$

$$q_{lks} \leq \frac{Cap_{ld}}{a_{lk}} p_{lks} \quad \forall l,k,d,s \in S_{ld} \tag{7}$$

$$\sum_{i,j,s \in S_{ld}} st^{b}_{lij} z_{lijs} + \sum_{k,s \in S_{ld}} (st^{p}_{lk} p_{lks} + a_{lk} q_{lks}) \leq Cap_{ld} \quad \forall l,d \tag{8}$$

$$\sum_{j} y_{ljs} = 1 \quad \forall l,s \tag{9}$$

$$\sum_{k \in K_j} q_{lks} \geq m_{lj}(y_{ljs} - y_{lj,s-1}) \quad \forall l,j,s \tag{10}$$

$$z_{lijs} \geq y_{li,s-1} + y_{ljs} - 1 \quad \forall l,i,j,s \tag{11}$$

$$(w_{khd},\, q_{lks},\, z_{lijs}) \geq 0,\, (p_{lks},\, y_{ljs}) \in \{0,1\} \tag{12}$$

The first objective is related to the minimization of production costs and it has an intrinsic combinatorial nature, while the second objective aims at maximizing the freshness of the products delivered and has a linear structure. The freshness objective takes advantage of the simple plant location reformulation used in the production variables that determine explicitly when a product is produced and delivered. Taking into account the shelf-life information for each product, the goal is to maximize the average percentage remaining shelf-life of all requests. In this expression, different remaining shelf-lives between products are all normalized between 0 and 1. With equations (3) and (4) the demand for all products is fulfilled and with constraints (5) the demand requirements are translated into production quantities. Constraints (6) and (7) ensure that in order to produce a certain product, the necessary setup for it and for the corresponding recipe has to be performed, respectively. Moreover, at each moment only products from the same recipe may be produced (9) and they are subject to a minimum batch size (10). Constraints (8) limit the use of the capacity in each macro-period and, finally, constraints (11) are responsible for tracking the changeover between blocks. For more details about the mathematical model the readers are referred to [3].

4 Multi-objective hybrid genetic algorithm

To solve this problem we use an algorithm similar to the one presented in [3], which consists of a hybridization of a MOEA and an MIP solver. This was the first time that an MO-LSP model had been proposed. The underlying MOEA is the Non-Dominated Sorting Genetic Algorithm (NSGA-II), which has proven to perform rather well in comparison to other MOEAs [18]. The readers are referred to [7] for the details of NSGA-II. In this work, we further exploit the potentialities of such hybridization by developing a new dual mutation operator that acts on feasible individuals. It is based on the individual values of the reduced costs from the variables related to the chromosome representation. Generally, standard mutation operators have a myopic view about the differences on the information coded across the population. With this operator this limitation is overcome by giving tailored mutation probabilities for each gene. In Figure 1, an outline of the proposed hybrid algorithm is shown with the dual mutation operator used to solve the MO-LSP (the meaning of f^3 is clarified in Section 4.5).

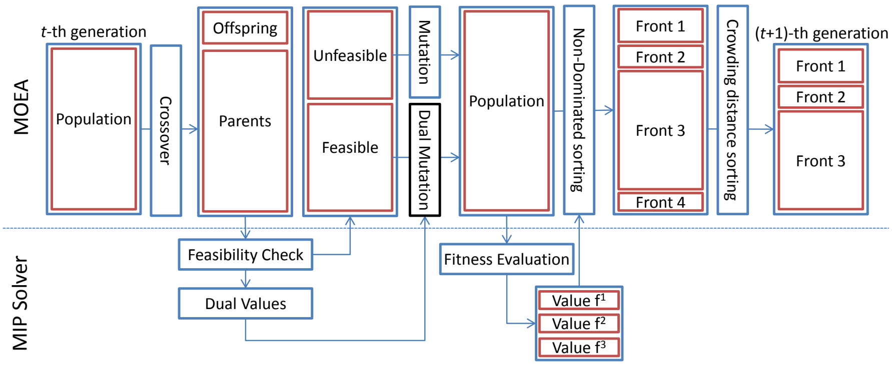


Fig. 1 Hybrid genetic multi-objective algorithm with dual mutation

There are five important components that are needed to completely describe any genetic algorithm (GA) [12]: the representation format used by the GA; the genetic operators; the fitness evaluation function; the various parameters, such as the population size, the number of generations, the probability of applying the operators, etc.; and a method for generating the initial population. As far as our approach is concerned, it is also important discuss about infeasibility/constraint handling issues to increase the flexibility of the solution space search. The parameters will be described in Section 5.3.

4.1 Representation of an individual

The usual representation of a solution in a GA for a lot-sizing and scheduling problem is a string of paired values for each scheduling period in the planning horizon [30]. The first value indicates the type of product and the second value the number of units of that product type to be produced, i.e., the lot-size. Due to our hybrid approach, an individual is composed of a string of single values. The value of each gene represents the block to be scheduled in each micro-period of a given machine. Therefore, there are $\sum_l S^l$ genes in each chromosome. The lines are put in numerical order so that at every position it is possible to return exactly the micro-period and the machine accessed. This yields an incomplete representation of the individual, as it only determines the value of the y_{lks} variables. The values of the remaining variables are obtained in a subsequent step by means of the MIP solver through an aggregated scalar objective function.

4.2 Genetic operators

Four types of genetic operators are implemented as follows.

4.2.1 Crossover and standard mutation

Since it is usual in these industries to have dedicated lines, sets of blocks / products may only be produced on certain machines. Hence, one needs to ensure that the changes performed by the crossover operator respect this condition. Here, the one-point crossover is used and, consequently, when combining the genetic material of the two parents, we are sure that the exchanges occur for the same lines. Therefore, we cut the chromosomes by a given micro-period, which belongs to a line and a macro-period, and obtain the child population based on these cut points.

Regarding the standard mutation operator, it is similar to the bitwise mutation but, in this case, a gene is randomly selected, which represents a block produced in a given micro-period, and its value is changed into a value corresponding to a block that is allowed to be produced on that line. This standard mutation operator is only applied on infeasible individuals after the execution of a feasibility check.

4.2.2 Dual mutation

This mutation operator only acts on feasible individuals. As input it uses the reduced costs, $\bar{y}_{ljs}$, of the y_{ljs} variables. Hence, if the MIP associated with a certain individual yields a feasible solution, the algorithm fixes all integer variables of the solution to the values obtained from the solution representation and the MIP solver. After, the corresponding linear programming (LP) model is solved in order to obtain $\bar{y}_{ljs}$. Note that $\bar{y}_{ljs}$ can be obtained by solving the following equations:

$$\bar{y}_{ljs} = co_{ljs} - \sum_{ct \in M} ro_{ct,ljs} sp_{ct} \quad \forall l, j, s$$

where co_{ljs} represents the implicit coefficients of y_{ljs} in the aggregated objective function, M is the set of all constraints ct in which y_{ljs} appears, sp_{ct} is the shadow price associated with constraint ct and $ro_{ct,ljs}$ is the coefficient of y_{ljs} in constraint ct. Thus, if we think of co_{ljs} as marginal revenues, the reduced cost $\bar{y}_{ljs}$ represents the net marginal revenue associated with variables y_{ljs} in the LP relaxation.

$\bar{y}_{ljs}$ can take three possible types of values: (i) positive, if the entry of such variable to the base would increase the objective function value; (ii) negative, if the entry of such variable would decrease the objective function value; or (iii) 0, if the variable is already a basic variable of the solution (as fixed through the chromosome representation) or the entry of such variable would not change the objective value. Recall that these straightforward conclusions are valid for the LP relaxation. The objective functions in the related MIP are not influenced in such a direct way because y_{ljs} are binary variables.

Let us now focus on the meaning of these values regarding our specific problem and define α, β, γ as the weighting coefficients associated with each of $\bar{y}_{ljs}$ possible value (positive, negative and 0, respectively). The probability of mutation to the respective variable is proportional to these waits. Since both objective functions are coded in the algorithm as minimizations, in case $\bar{y}_{ljs} > 0$ we want to avoid that the associated y_{ljs} variables take the value of 1 in individuals of a future generation. Therefore, the value of α should remain close to zero in order to prevent such situation. Looking deeper at the case where $\bar{y}_{ljs} < 0$, we should note that concerning the cost minimization function, an increase of one unit in the lower bound of any y_{ljs} would almost never result in a negative reduced cost as it would probably augment or maintain this objective function value through the execution of more setups. We highlight that inventory costs are not incorporated in the objective function f^1. Therefore, the usual trade-off between holding and setup costs does not play a role here. Thus, negative values of $\bar{y}_{ljs}$ will be linked with the objective function related to the maximization of freshness. Therefore, β is related to the contribution of the dual mutation operator in attaining better values of freshness. Finally, when $\bar{y}_{ljs} = 0$, the impact of having the corresponding y_{ljs} as a basic variable is not clear, but it might turn out to be beneficial in a future generation. Consequently, γ should have a significant value so that diversification is accounted for.

The transformation that each feasible individual undertakes from the dual mutation operator is depicted in Figure 2. The weights in this example are set as follows: $\alpha = 0$, $\beta = 0.5$ and $\gamma = 1$. The probabilities are calculated by dividing the weight of a given block over the sum of all weights.

To understand the possible path of a gene let us consider, for example, gene 2 (that corresponds to the second micro-period of the first macro-period) that is setting up the machine for producing block **3** close to the beginning of the planning horizon. First, the reduced costs for each block that can be set up are calculated. The reduced cost related to block 4 is positive and, therefore, the final probability

Old Individual	j	2	3	5	1	1	4	4	4	2	5
	1	-20	0	10	0	0	-5	10	0	20	-10
	2	0	0	0	20	0	0	-5	0	0	-10
y_{ljs}	3	0	0	0	-5	0	-20	0	10	10	0
	4	-10	10	1	20	-5	0	0	0	10	-10
	5	5	-20	0	5	10	20	5	5	20	0
	1	0.5	1	0	1	1	0.5	0	1	0	0.5
	2	1	1	1	0	1	1	0.5	1	1	0.5
Weights	3	1	1	1	0.5	1	0.5	1	0	0	1
	4	0.5	0	0	0	0.5	1	1	1	0	0.5
	5	0	0.5	1	0	0	0	0	0	0	1
	1	0.17	0.29	0.00	**0.67**	**0.29**	0.17	0.00	**0.33**	0.00	0.14
	2	**0.33**	0.29	0.33	0.00	0.29	0.33	0.20	0.33	**1.00**	**0.14**
Probability	3	0.33	0.29	0.33	0.33	0.29	0.17	0.40	0.00	0.00	0.29
	4	0.17	0.00	0.00	0.00	0.14	**0.33**	**0.40**	0.33	0.00	0.14
	5	0.00	**0.14**	**0.33**	0.00	0.00	0.00	0.00	0.00	0.00	0.29
New Individual		2	5	5	1	1	4	4	1	2	2

Fig. 2 Schematic steps of the dual mutation operator. The probabilities randomly selected to perform the corresponding family mutation are in bold.

assigned to it is 0. With regard to block 5, its negative reduced cost would probably induce an increase of the freshness value. The probability of 0.14 of this block is computed from the following expression: $0.5/(1+1+1+0.5)$. In the end, this gene mutates to either block 1, 2, or 3 with equal probability of 0.29 and to block 5 with probability of 0.14 (in Figure 2 the result is a mutation to block **5**).

4.2.3 Selection/reproduction

After the offspring population is created, the whole population is sorted according to the non-dominance criterion. In the end of this process every individual belongs to a front. The new parent population is formed by adding solutions from the best fronts until the number of individuals selected reaches a pre-defined parameter of the size of the population. Thereafter, the solutions of the last front (the worst front accepted from the non-dominance ranking) are sorted according to the crowding operator. This operator conducts an internal ranking of the last front based on the distance of each individual to its closest neighbours. Individuals with the *farthest* neighbours are then selected until the rest of the new population is filled. Clearly, this operator favours a well-dispersed set of solutions.

4.3 Fitness of an individual

The hybridization of the NSGA-II with the MIP solver is also done in this step. Each individual representing a sequence of blocks to be scheduled on each line is fed into the MIP model by fixing the values of the y_{lks} variables and, consequently, a significant number of p_{lks} as well. Notice that y_{lks} and p_{lks} are the only binary variables of our mathematical model. The sub-problem is much easier for the MIP solver since optimal lot-sizes can be computed based on the fixed block sequence of the individuals while minimizing a weighted linear combination of both objectives. Each time an individual is fed into the solver, the two objectives are randomly combined with a weighted sum leading to a scalar objective function. This random weighting aims at providing a good dispersion of results throughout the Pareto-front, since with fixed weights the solution tends to converge quickly to a certain region of the solution space and be entrapped in it. The objective function values are then used to rank individuals according to non-dominance and to compute the Pareto-fronts. If the individual turns out to be infeasible, then a feasibility recovery procedure is applied, as explained in Section 4.5.

4.4 Initializing a population

A population is initialized by randomly generating values for each chromosome. Nevertheless, special attention is paid to be sure that the possible values are in compliance with the set of blocks/products that can be produced on a line. This way, we ensure from the beginning that individuals respect the allowed sequences on every machine. Moreover, diversity within the population is also guaranteed.

4.5 Infeasible individuals

Infeasible individuals are neither automatically disqualified from the population, nor are they repaired to create feasible individuals. Instead, the amount of infeasibility is defined as an additional objective that must be minimized. Justification for treating the infeasibility in this way is provided in [27]. This approach has been suggested by [5], as a way of avoiding the definition of a more complex, but constantly feasible representation, specifically designed operators, or a repair method. Furthermore, since the optimal solutions may lie close to the infeasible region, this approach allows these frontier regions to be well explored.

Therefore, when trying to calculate the fitness of an individual regarding both objectives, in case the individual is infeasible, a new objective is set as $\min f^3 = \sum_{ct} v_{ct}$, where v_{ct} is the amount of violation of constraint ct (cf. Figure 1). In order to measure this violation, slack variables are introduced and their values are to be minimized expecting that the individual will turn feasible in a future generation by means of crossover and mutation.

5 Computational experiments

In this section, the computational performance of the algorithm with and without the dual mutation operator is assessed. To test the proposed method, a C++ implementation with the MIP solver CPLEX 12.1 is run.

5.1 Data generation

A total of 27 instances were systematically generated, following a methodology very similar to [16]; therefore, L was set to 1. For all products $a_{lk} = 1$ and the machine is set up for block 1 in the beginning of the planning horizon. The number of blocks J is 5, 10 and 15, and each block has only one product k. The number of macro-periods D is 5, 10 and 20. The number of micro-periods within a macro-period S_{ld} is set to the value of J allowing all products to be produced in each macro-period with minimum lot-sizes m_{lj} of 1 unit. A demand matrix with 15 products (rows) and 20 macro-periods (columns) is randomly generated, where each element d_{kd} is in the interval [40, 60] of the uniform distribution. For the setup times between blocks (st^b_{lij}) the interval [2,10] is used (except for the case where $i = j$, where the setup is 0). For the setup of products it was considered that both the respective time and cost as well as production costs are null. Shelf-lives u_k are generated for all 15 products for each possible planning period length choosing randomly from the interval [1, D]. Hence, for an instance with J blocks and D macro-periods we used the data given in the first J rows and the first D columns of the demand matrix, the first J rows and columns of the setup time, and the first J entries of the shelf-life vector built for D macro-periods. Then, as in [16], the concept of common random number is present in our experiments. The setup cost sc^b_{lij} for a changeover from block i to block j is computed as:

$$sc^b_{lij} = f_{sc} st^b_{lij} \quad \forall l, i, j.$$

Although in [16] the parameter f_{sc} is varied between 50 and 500, in our case, due to the structure of our algorithm, the impact of this parameter in the efficiency is null. The relation between setup times and costs does not influence the performance of our algorithm at all, because the sequencing decision is explicitly defined from the individual representation. Hence, this parameter is fixed to 50 for every instance. The capacity per macro-period Cap_{ld} is determined according to:

$$Cap_{ld} = \frac{\sum_k d_{kd}}{U}, \quad \forall l, d,$$

where the capacity utilization U is 0.4, 0.6 or 0.8. It is important to notice that the utilization of capacity is only an estimate, as setup times do not influence the computation of Cap_{ld}. In summary we have:

$$|5,10,15| \times |5,10,20| \times |50| \times |0.4,0.6,0.8| = 27 \text{ instances.}$$

In Table 1 the size of each instance that is part of the computational study is scrutinized in terms of variables and constraints of the MIP formulation.

Table 1 Size of the instances in terms of the Number of Binary variables, Number of Continuous variables and Number of Constraints.

J	15			10			5		
D	20	10	5	20	10	5	20	10	5
No. of Binaries	9000	4500	2250	4000	2000	1000	1000	500	250
No. of Continuous	75151	36826	18226	24101	11551	5651	4051	1776	826
No. of Constraints	23405	11695	5840	10610	5300	2645	2815	1405	700

5.2 Evaluation metrics

In order to be able to evaluate the quality of the non-dominated Pareto-front computed with the introduction of the dual mutation operator, we rely on two performance measures: one unary metric, the hypervolume [32] and the first-order empirical attainment functions [23].

The hypervolume metric represents the volume of the objective space that is dominated by a solution set. To calculate this value a reference point is needed. For minimization problems, its value is set to exceed the maximal values for each objective. Hence, in our experiments it is found automatically with the best extreme points for each objective found for each instance. When using this metric to compare the performance of two or more solution methods, the one giving solutions with the largest covered hypervolume is considered the best.

In addition, the first-order empirical attainment functions (EAFs) are used to represent the probabilistic performance of MOEAs by measuring the attainment of a reference set based on the generated solutions using multiple runs. Therefore, different curves are plotted, such as the minimum attainment surface and the maximum attainment surface, based on all runs. We also plot an average attainment surface representing the surface which was attained in 50% of the runs.

5.3 Parameter tuning

The setting of the parameters is the following: 100 generations, 100 individuals, a crossover rate of 0.9 and a mutation rate (for standard and dual mutation) of $\frac{2}{D \cdot J}$. These values were tuned through exhaustive preliminary tests and recommendations in the literature. The specific values of the dual mutation operator were set to: $\alpha = 0$, $\beta = 0.1$ and $\gamma = 1$.

The reason behind the value of $\alpha = 0$ was already given. Regarding the other values we should refer to the two objective functions. Clearly, the cost minimization is harder to solve than the freshness maximization due to the combinatorial nature of the former. Thus, we bring more weight on the dual mutation parameter that can improve this harder objective. It is important to note that if α, β and γ are set to the same values, the dual mutation is equal to the standard mutation operator since the probabilities to mutate each gene to any possible block are the same.

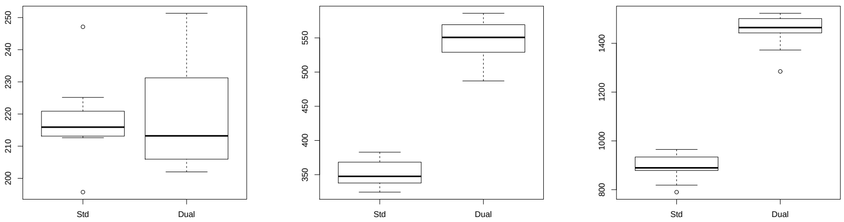
5.4 Experimental results

To account for the randomness of the MOEA, 10 runs were executed for each instance. To illustrate the results for all instances, the results for the hypervolume indicator and the differences between the first-order EAFs of the two algorithm variants are given for the instances with 60% capacity utilization in Figures 3, 4, 5 and 6, 7, 8, respectively. The *Std* results refer to the variant where the standard mutation operator (cf. Section 4.2.1) acts on both feasible and infeasible solutions. The *Dual* results are obtained when the dual mutation operator is activated on feasible individuals (cf. Section 4.2.2), letting the standard mutation operator to act on infeasible ones.

Note that the EAF plots have both objectives as minimization ones. Moreover, all figures have the instance identified in the form of JxD. In Table 2 median values for all instances with and without dual mutation are given. Moreover, the Kruskal-Wallis non-parametric statistical test is used for assessing if the hypervolume results differed significantly between algorithms and the respective $p - value$ is computed (significant values are in bold).

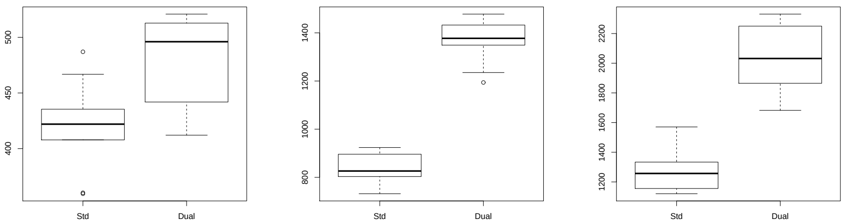
First, it should be noticed that the algorithm is able to solve all instances of this difficult problem, even instances that could not be solved in the original paper where they were introduced. The running times of the algorithm increase with the number of blocks J and macro-periods D. Hence, it runs from 5 to 30 minutes in order to attain 100 generations. For higher values of the utilization coefficient U it becomes harder to spot feasible solutions. It should be highlighted that the difference between the running times of the MOEA with and without the dual mutation is only of a few seconds.

The hypervolume indicator gives strong evidence that the dual operator is able to get better solutions in terms of Pareto-front optimality. This superiority seems to increase with the number of micro-periods. When setting the parameters of the



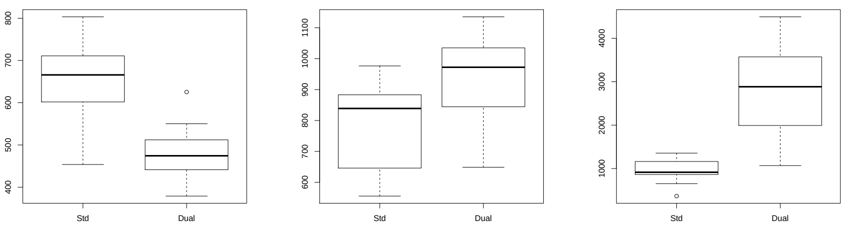
(a) 5x5. (b) 5x10. (c) 5x20.

Fig. 3 Boxplots of the hypervolume indicator for instances with 5 products and 60% of capacity utilization.



(a) 10x5. (b) 10x10. (c) 10x20.

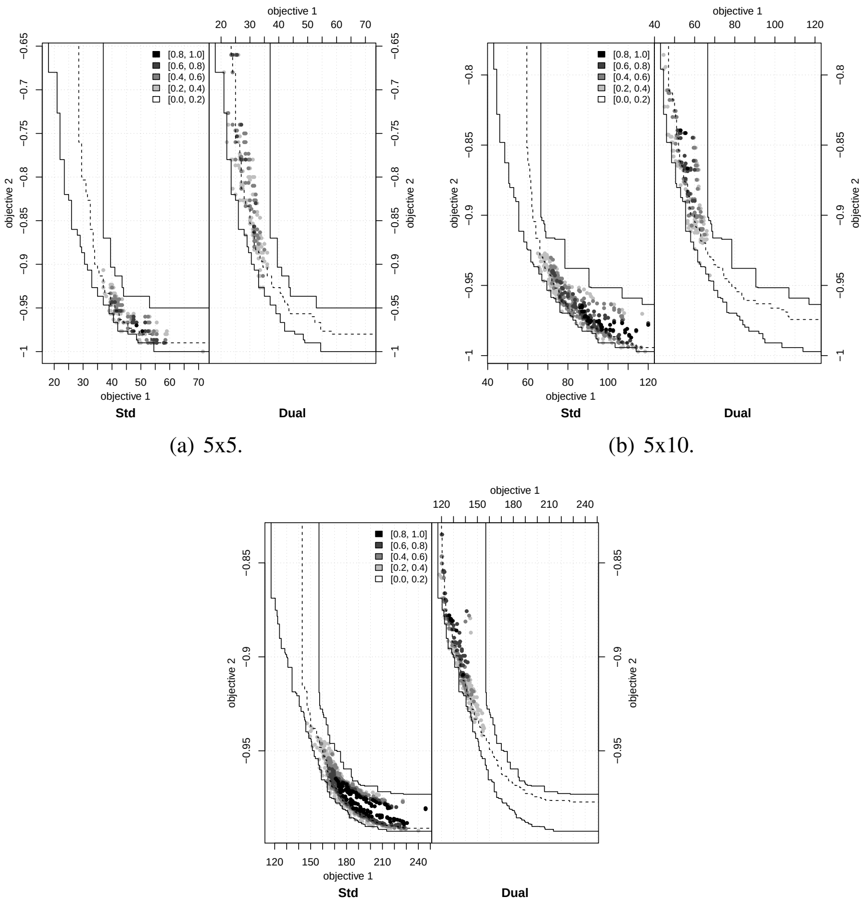
Fig. 4 Boxplots of the hypervolume indicator for instances with 10 products and 60% of capacity utilization.



(a) 15x5. (b) 15x10. (c) 15x20.

Fig. 5 Boxplots of the hypervolume indicator for instances with 15 products and 60% of capacity utilization.

dual operator it was referred that the effort was given upon the cost minimization



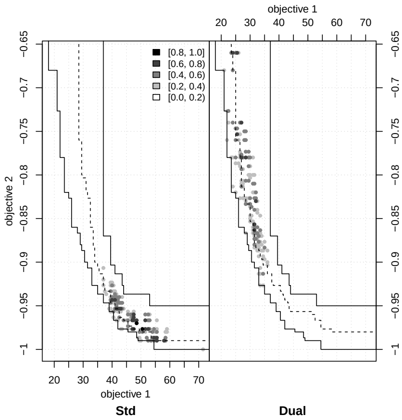
(a) 5x5. (b) 5x10.

(c) 5x20.

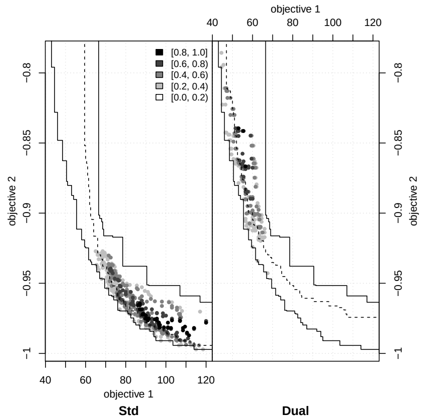
Fig. 6 EAF plots for instances with 5 products and 60% of capacity utilization.

objective. The EAF plots show that the dual operator is able to achieve indeed lower costs. On the other hand, the EAF plots evidence that the best solutions in terms of freshness are obtained by the MOEA without dual mutation, suggesting that β values could be increased to that end.

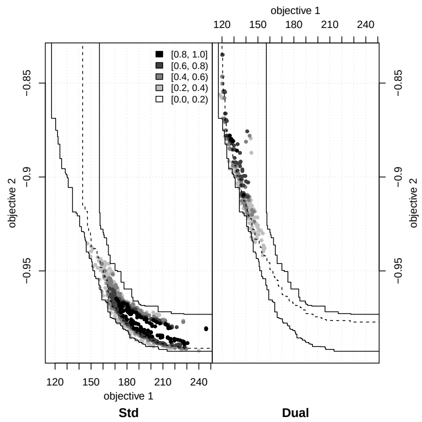
Figure 9 shows the impact of increasing β values in order to find solutions closer to the freshness optimum. Notice that the difference between the algorithm variants with and without dual mutation is almost negligible for $\beta = 2$. However, in the variant with dual mutation, the average attainment surface is much closer to the maximum attainment surface.



(a) 10x5.



(b) 10x10.

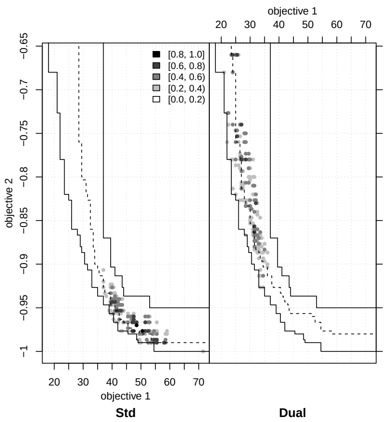


(c) 10x20.

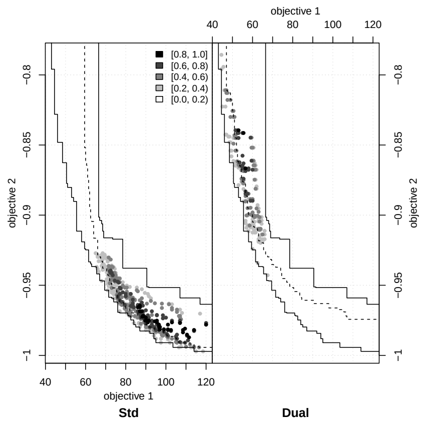
Fig. 7 EAF plots for instances with 10 products and 60% of capacity utilization.

6 Conclusions and future research

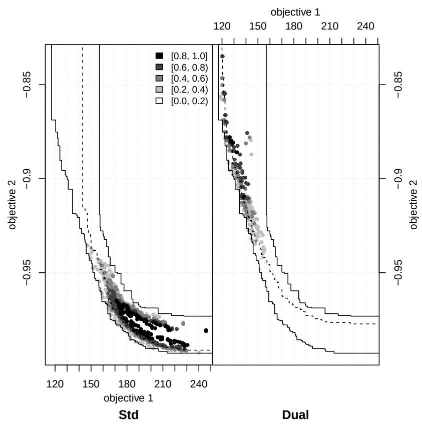
The main contribution of this paper lies in the exploration of hybrid methods to solve complex combinatorial multi-objective problems, such as the multi-objective lot-sizing and scheduling of perishable goods. Specifically, a new dual mutation operator is proposed to give tailored mutation probabilities to each individual based on the reduced costs of the encoded variables coming from the LP relaxation. This operator enhances consistently the performance of the base MOEA through a better and directed exploration of the search space, ignoring mutation paths that do not seem promising. The rationale behind this dual mutation operator is replicable for



(a) 15x5.



(b) 15x10.



(c) 15x20.

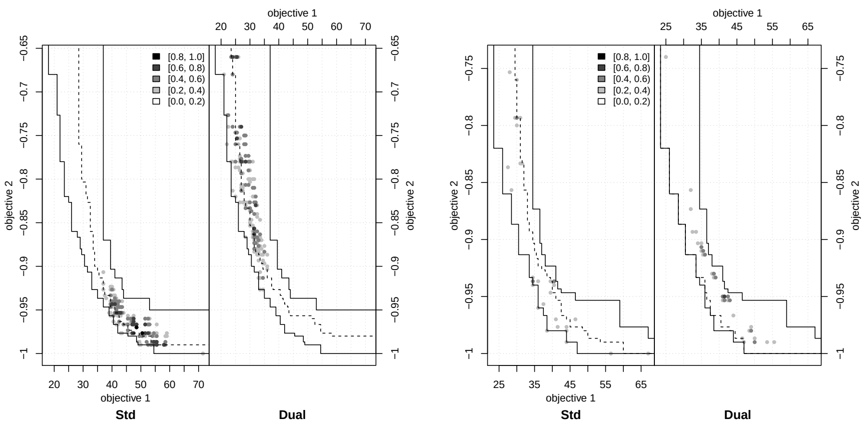
Fig. 8 EAF plots for instances with 15 products and 60% of capacity utilization.

other solution approaches in problems where the presence of LP relaxations makes possible to embed the use of dual values to guide the search.

Future work shall be devoted to the on-line adaptative setting of the dual mutation parameters and to testing this operator in multi-objective problems with a similar structure to the one presented in this paper. Moreover, the comparison of the performance in other MOEAs is of great interest.

Table 2 Hypervolume medians for the two versions of the MOEA and $p-values$ of the Kruskal-Wallis statistical tests.

		$U = 0.4$			$U = 0.6$			$U = 0.8$		
J	D	Std	Dual	$p-value$	Std	Dual	$p-value$	Std	Dual	$p-value$
5	5	174.62	250.96	**0.000**	215.92	213.21	0.762	347.52	550.71	**0.000**
5	10	572.81	825.31	**0.000**	347.52	550.71	**0.000**	621.66	632.20	0.496
5	20	1038.66	1874.71	**0.000**	889.47	1464.54	**0.000**	1247.58	1306.87	0.013
10	5	457.94	440.88	0.406	422.00	496.04	**0.010**	616.32	535.42	**0.008**
10	10	1043.31	952.26	**0.001**	826.38	1377.33	**0.000**	616.32	1066.71	**0.000**
10	20	1535.19	2092.29	**0.002**	1257.10	2031.76	**0.000**	1676.61	1426.47	**0.000**
15	5	511.58	580.56	0.112	665.88	474.35	**0.002**	357.38	180.45	**0.000**
15	10	982.60	812.58	0.013	838.96	972.30	0.023	752.55	579.73	0.034
15	20	516.54	589.03	**0.000**	914.79	2884.26	**0.000**	516.54	589.03	**0.000**



(a) 5x5 and $U = 0.6$ with $\beta = 0.1$.

(b) 5x5 $U = 0.6$ with $\beta = 2$.

Fig. 9 EAF plots for illustrating the importance of dual mutation parameters in achieving different areas of the Pareto-front.

Acknowledgements

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Influence of Consumer Purchasing Behaviour on the Production Planning of Perishable Food

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Abstract

In this paper, we assess the impact of consumer purchasing behaviour on the production planning of perishable food products for companies operating in the fast moving consumer goods using direct store delivery. We build on previous marketing studies related to the effects of expiry dates, in order to derive mathematical formulae, which express the age dependent demand for different categories of perishable products. These demand expressions take into account both customer willingness to pay and product quality risk. A deterministic and a stochastic production planning model, which incorporate the customer's eagerness to pick up the fresher products available, are presented. Results point out that not considering the decreasing customer willingness to pay has an important impact both on the profit losses and on the amount of spoiled products. On the other hand, it was concluded that neglecting the fact that customers pick up the fresher products and the assumption that all products have the same production quality risk have a reduced impact on profit losses.

Keywords: Food Perishability, Production Planning, Consumer Purchasing Behaviour, Demand Uncertainty

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1. Introduction

Since the beginning of the 80s, retailers have been gaining increasing power over suppliers. One of the most important measures derived from this trend is the *direct store delivery* (Otto et al., 2009). In this business practice, producers are responsible for all supply chain processes downstream of production until reaching the final customer. Hence, producers are responsible for selling the products directly at the point of sales and they assure themselves the distribution of the goods bypassing the retailers' warehouses. Moreover, all merchandising activity is also under control of the producers. The reported advantages for retailers are related to sales increase due to more intense merchandising activity and out-of-stocks reduction. Furthermore, through bypassing the retailers' warehouses, hours of administrative and physical handling of suppliers' goods are saved. For producers, the benefits come less from a reduction in direct out-of-pocket costs than from being more able to contact directly with their final customers. This enables companies to adjust promotional activities faster or even to observe and react promptly to market desires for new products.

According to a joint study of the University of Regensburg and SAP about direct store delivery, named Global Direct Store Delivery Analysis (GDA)[1], 24 out of the 30 larger worldwide fast moving consumer goods companies and 8 out of the 10 larger worldwide food companies adopt direct store deliveries. In fact, the advantages for the aforementioned producers concerning the adoption of direct store delivery are leveraged when the products handled are perishable (Amorim et al., 2012). This happens to be the case in many food industries. The distribution configuration flowing from direct store delivery is usual a 2 or 3-Tier network linking the production facility to the retailer. Hence, either products flow directly from facilities to retail stores or there is an intermediate storage decoupling these two stages. In case products are very perishable a 2-Tier configuration is preferred in order to decrease the amount of freshness lost during distribution. Thus, production models that account for perishability using a more supply chain oriented approach and focusing on the consumer purchasing behaviour are expected to achieve better overall results when companies use this type of lean practices.

The discussion about the impact of the perishability phenomenon is gain-

[1]http://www.sdn.sap.com/irj/scn/go/portal/prtroot/docs/library/uuid/d09e75f9-44e0-2910-269c-9093b5973b7e?QuickLink=index& overridelayout=true

ing importance, throughout different issues, both in the Marketing and Operations Research communities. Marketing and consumer behaviour related studies have focused on understanding the effects of product freshness in consumption (Sen & Block, 2009), on investigating how customers react to stock-outs of very perishable products (Woensel et al., 2007) and on the effect of expiration dates in the purchasing behaviour (Tsiros & Heilman, 2005). In the Operations Research community the recent reviews of Karaesmen et al. (2008), Akkerman et al. (2010) and Amorim et al. (2011b) covering the researches done in different supply chain planning problems refer to the importance of incorporating perishability explicitly in formulating these problems.

In this study we consider the perishability definition proposed in Amorim et al. (2011b): "A good, which can be a raw material, an intermediate product or a final one, is called 'perishable' if during the considered planning period at least one of the following conditions takes place: (1) its physical status worsens noticeably (e.g. by spoilage, decay or depletion), and/or (2) its value decreases in the perception of a(n internal or external) customer, and/or (3) there is a danger of a future reduced functionality in some authority's opinion.".

The focus of this paper is on the fast moving food consumer goods that are subject to physical spoilage. Examples of these products are found in the catering, dairy and processed food industries. The production systems related to these industries involve complex setup sequences that are often decided by specialized planners according to natural constraints. These characteristics together with high inventory rotation levels force the collapse of traditional tactical and operational planning levels (Bilgen & Günther, 2009). Within this scope we consider that the producer (that ultimately sells perishable food products to final customers) has no control over the pricing of the products, which is assumed to be fixed within the considered planning horizon. For the readers interested in combining lot sizing and pricing decisions, there are numerous studies on this topic, such as those of Abad (2001); Chen & Chen (2005); Abad (2003); Chang et al. (2006). In this setting, we propose mathematical models that are able to differentiate between different functions of the age dependent demand and/or between products with or without a stamped *best-before-date*. Our aim is to bridge the gap between consumer purchasing behaviour and production planning of perishable products by addressing the producers' problem arising from an increasing control over the downstream supply chain. This is indeed the case of many food industries that use direct store delivery and of companies that produce and sell in the same establishment (such as bakeries, for example). Hence, we extend the production planning formulations dealing with perishable products by incorporating the consumer purchasing behaviour. This is done by adjusting products' demand and inventory depletion to reflect the consumer's attitude towards perishability. We consider that demand is influenced by two distinct factors: the decreasing consumer willingness to pay for products with an increasing age and the different demand shapes that are related to the product quality risk. Moreover, we acknowledge that customers, having the opportunity to choose between equivalent products with different ages, will pick the fresher ones.

In the remainder of this article, we first review the papers dealing with production planning of perishable goods and analyse how perishability has been incorporated. Then, Section 3 attempts to describe analytically key issues that have been discovered in the consumer purchasing behaviour theory about customer willingness to pay and perceived product quality risk when dealing with perishable food goods. In Section 4, a deterministic mathematical model and a stochastic one that considers demand uncertainty are presented. After, a computational study is conducted in Section 5 to evaluate the impact of incorporating consumer behaviour in the production planning of perishable goods. Finally, in Section 6 the main conclusions and future work are discussed.

2. Production Planning dealing with Perishability Issues

There are only a few papers focusing on production planning problems for perishable goods. However, these studies are very recent and growing in number in the last years, showing the increasing interest that this topic has been receiving. In this section, only literature closely related to our problem description is reviewed. The readers interested in more general tactical / operational production planning research are referred to the reviews of Drexl & Kimms (1997); Karimi et al. (2003); Zhu & Wilhelm (2006); Jans & Degraeve (2008); Buschkühl et al. (2008).

Marinelli et al. (2007) formulate a situation arising in a packaging company producing yoghurt as a hybrid continuous setup lot sizing problem with shared buffers. The authors decompose a relaxed version of the overall problem into a lot sizing problem on tanks (buffers, which store the product recipe) and a scheduling problem on production lines. This decomposition is only possible because the authors neglect setup times (for both tanks and lines) and setup costs (for the lines). They account for perishability by imposing a make-to-order strategy, which is rather hard to follow in the fast moving consumer goods industries. Also in the case of yoghurt production, Lütke Entrup et al. (2005) develop three models that incorporate shelf-life issues into production planning and scheduling of the packaging stage. Their study uses the block planning concept that delivers a practical means for solving such planning problems (Günther et al., 2006). They consider in the objective function a decreasing value for the perishable goods throughout the course of their shelf-life. However, the authors acknowledge that the proposed approach is hard to be implemented in practice since retailers are not willing to pay to producers a price that is based on the remaining shelf-life of the products delivered. Finally, Doganis & Sarimveis (2007) also develop a mixed-integer programming for a very similar problem. Pahl & Voß (2010) and Pahl et al. (2011) extend well known discrete lot sizing and scheduling models, such as the general lot sizing and scheduling problem (Fleischmann & Meyr, 1997), by including deterioration and perishability constraints. They confirm the importance of including such constraints that may reduce the solution search space. These results are in line to what is highlighted in Clark et al. (2011). In Amorim et al. (2011a) a multi-objective framework with one objective related to production costs and another to freshness is used. This problem is solved through a hybrid multi-objective evolutionary algorithm and the result of the lot sizing and scheduling problem is a Pareto front trading off these two key performance indicators. The conclusions point out that just by considering perishability in one objective function, a much higher freshness standard may be achieved at the expense of an additional small cost.

In all the aforementioned papers, demand is seen as an external, dynamic and deterministic parameter that has not a real connection to the perishable nature of the product. This link is evidenced in this work by bringing in the research on consumer purchasing behaviour related to food products into production planning models. Moreover, we investigate the influence of demand uncertainty in such setting. Therefore, further insights on the impact of perishable food goods on production planning decisions are expected.

3. Modelling Consumer Purchasing Behaviour

In this section, the demand expressions that are function of the age of the products, which will feed the production planning formulations, are derived. The basis of our modelling is the consumer purchasing behaviour for perishable products described in Tsiros & Heilman (2005). To the best of our knowledge, this is the only consumer purchasing behaviour study that thoroughly investigates the effects of perishability on the purchasing pattern of customers across different perishable products. In this study, the authors conclude that customer willingness to pay (WTP) decreases throughout the course of the products' shelf-life. Moreover, this decrease follows a linear function for products with a low product quality risk (PQR), while the WTP follows an exponential negative function for products with a high PQR. In their sample the authors consider lettuce, milk, carrots and yoghurt as low PQR products, and beef and chicken as high PQR products. Note that PQR is defined as the expected negative utility associated with the product as it reaches its expiry date, and WTP is the maximum price a customer is willing to pay for a given product in a given point in time. It is important to highlight that the customer WTP for a perishable product does not have to be strictly related to the organoleptic condition of its content.

In order to understand how demand varies with an increasing age of the product for a fixed list price, two relations need to be explicitly understood. First, how the WTP (price) varies with an increasing age and, second, what is the relation between demand and price. Let us denote the function describing the behaviour of price p for each age a as $p = f(a)$ and assume that the first order derivative of this function is independent of the demanded quantity. Further, consider a demand function for a product with age 0 (fresher state) $d^0 = h^0(p)$, where d denotes demand, which also has the same first order derivative across the functions for each different age. Hence, given that f is a monotonic decreasing function, the respective h^a function for a product with age a is obtained by shifting function h^0 by $f(0) - f(a)$. We are now in position of defining the demand in terms of price and age as:

$$d(p, a) = h^0(p + f(0) - f(a)). \quad (1)$$

In Figure 1 a graphical representation of the aforementioned functions is provided.

Within the range of price/demand considered in medium-term production planning and following the same reasoning of the vast majority of the

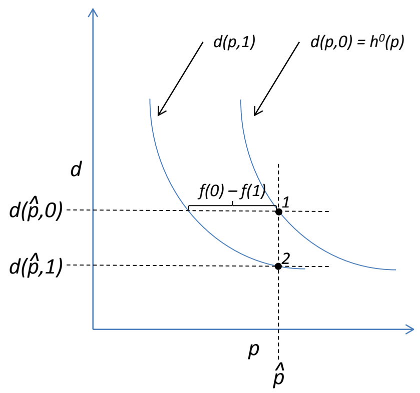

Figure 1: Graphical interpretation of the mathematical relation between function f, d and h^0. Point 1 gives the demand for a list price $\hat{p}$ of a product with 0 units of age and point 2 for a product with 1 unit of age.

economic studies aiming at finding the price elasticity of demand for different products, we consider a constant value for elasticity that will only depend upon the product being considered. Thus the constant elasticity demand function $h^0(p)$ reads:

$$h^0(p) = Cp^{\epsilon}, \tag{2}$$

where C stands for a constant and ϵ for the absolute elasticity. Therefore,

$$d(p, a) = C(p + f(0) - f(a))^{\epsilon}. \tag{3}$$

Assuming that the demand is known for each product in its fresher state (d^0) for a given list price ($\hat{p}$), which corresponds to the usual setting in production planning problems, constant C can be written as:

$$C = \frac{d^0}{\hat{p}^{\epsilon}}.$$

Replacing this expression in equation (3) gives:

$$d(p, a) = d^0 (\frac{p + f(0) - f(a)}{\hat{p}})^{\epsilon}. \tag{4}$$

To fully describe the demanded quantity as function of the age of the product for a given list price, one only needs to describe the f functions, which are empirically studied in Tsiros & Heilman (2005). Based on the observations of the WTP for different products at different ages in this study, we are able to derive mathematical functions describing such behaviour. In order to analyse a wider range of products than those considered in that study, we propose a new function, establishing a concave WTP profile, that is used for products with a low PQR, such as bread. The linear and convex demand profiles proposed by Tsiros & Heilman (2005) are used for products with medium and high PQR, respectively. All these functions have a similar behaviour as they are monotonically decreasing, having its maximum value for the product with a maximum freshness (p^0) and a value of 0 at the end of shelf-life u. The closeness of the WTP to 0 monetary units as the product reaches its shelf-life is controlled by parameter α, which is independent of the function shape (linear, concave or convex). Hence, this parameter that varies between 0 and 1 will represent the customer sensibility to the decaying freshness of the product. In case $\alpha = 0$, the customer attributes a constant value to the perishable product, and if $\alpha > 0$, then the customer gives an increasing importance to product freshness until the point when $\alpha = 1$ that corresponds to a customer that towards the end of the shelf-life will be willing to pay 0 monetary units for the product.

Equations (5)-(7), as well as Figure 2 represent the linear, concave and convex WTP shapes.

$$\textbf{Linear } f_{linear}(a) = p^0 - \frac{\alpha p^0 a}{u-1} \tag{5}$$

$$\textbf{Concave } f_{concave}(a) = p^0 - \frac{\alpha p^0 a}{u-1}\left(\frac{a}{u-1}\right) \tag{6}$$

$$\textbf{Convex } f_{convex}(a) = p^0 - \frac{\alpha p^0 a}{u-1}\left(2 - \frac{a}{u-1}\right) \tag{7}$$

In the figures, all functions consider a WTP for the product at its fresher state p^0 to be equal to 100 and shelf-life u equal to 6. Note that in every function the price is just represented until age 5 since at age 6 the products spoil and they can no longer be sold. A concave demand function means that the customers get more sensible towards the end of the shelf-life and its WTP for each age of the product is always above the linear function. On the other hand, when the WTP function is convex the WTP drops very fast as soon as the product is produced. In this case the curve is always below the linear function. This is actually the case in processed fish, since customers

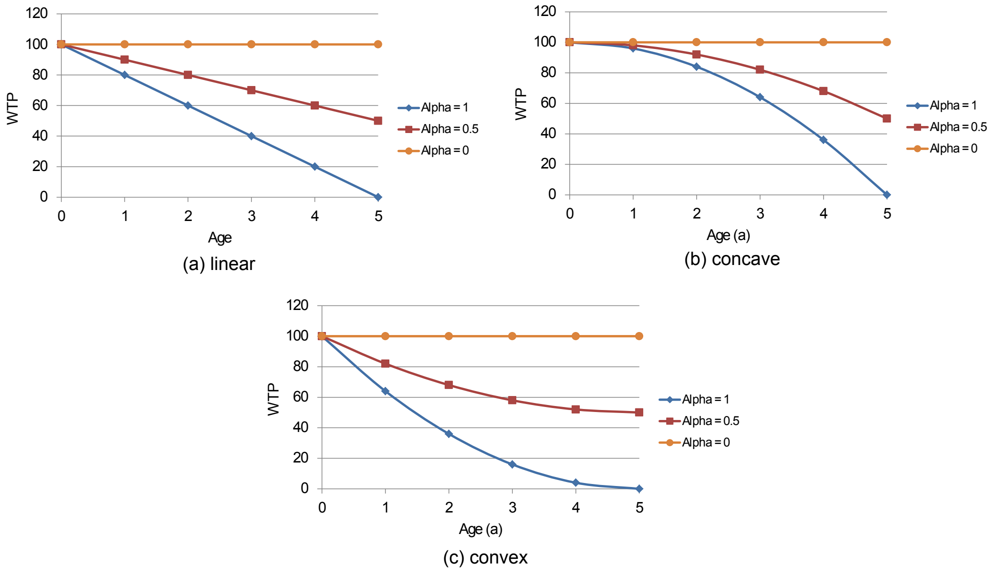

Figure 2: Impact of different α when customers WTP function is (a)linear, (b)concave, (c)convex.

are aware of the increased risk of consuming it in a less fresh state. The linear shape represents products to which consumers attribute a medium risk, such as yoghurt.

Finally, the demand functions for different ages can be described by replacing $f(a)$ functions in $d(p, a)$ functions. Equations (8)-(10) represent the linear, concave and convex demand shapes for list price $\hat{p}$.

$$\textbf{Linear } d_{linear}(\hat{p}, a) = d^0\left(\frac{\hat{p} + \frac{\alpha p^0 a}{u-1}}{\hat{p}}\right)^{\epsilon} \tag{8}$$

$$\textbf{Concave } d_{concave}(\hat{p}, a) = d^0\left(\frac{\hat{p} + \frac{\alpha p^0 a}{u-1}\left(\frac{a}{u-1}\right)}{\hat{p}}\right)^{\epsilon} \tag{9}$$

$$\textbf{Convex } d_{convex}(\hat{p}, a) = d^0\left(\frac{\hat{p} + \frac{\alpha p^0 a}{u-1}\left(2 - \frac{a}{u-1}\right)}{\hat{p}}\right)^{\epsilon} \tag{10}$$

In order to illustrate the behaviour of the demand curves the examples of lettuce and beef products are provided in Figures 3 and 4, respectively.

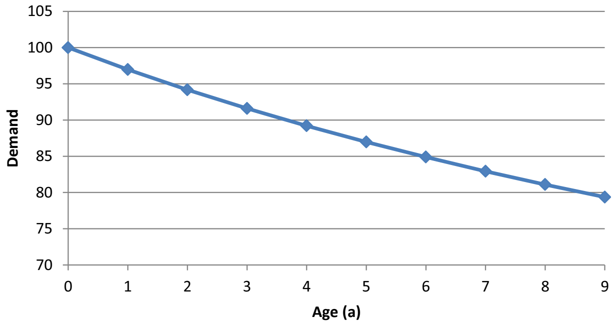

Figure 3: Example of the demand for lettuce over its shelf-life of 10 days, starting at 100 units. According to Tsiros & Heilman (2005) these products have a medium PQR, $\alpha = 0.62$, $\hat{p} = 2.49$ and $p^0 = 2.86$; and according to Andreyeva et al. (2010) these products have $\epsilon = -0.58$.

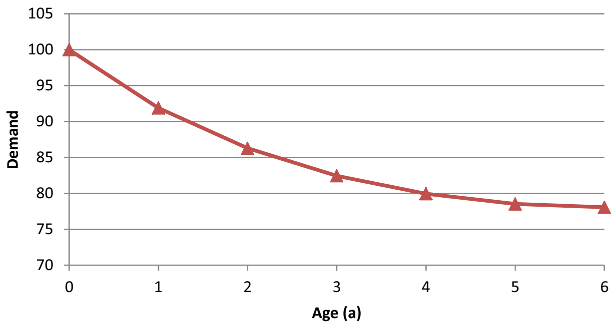

Figure 4: Example of the demand for beef over its shelf-life of 7 days, starting at 100 units. According to Tsiros & Heilman (2005) these products have a high PQR, $\alpha = 0.52$, $\hat{p} = 2.68$ and $p^0 = 2.52$; and according to Andreyeva et al. (2010) these products have $\epsilon = -0.75$.

4. Deterministic and Stochastic Mathematical Models

In this section, we formulate a deterministic production planning model to deal with perishable food goods that considers consumer purchasing behaviour. This formulation is then extended to incorporate demand uncertainty and a two-stage stochastic programming with recourse model is proposed.

Consider products $k = 1, ..., K$ that are produced based on a certain recipe forming a block $i, j = 1, ..., N$. There is only one recipe to produce each product and, therefore, a product is assigned to one block only. Hence, for each block j there is a set $[K_j]$ of products k related to it. Blocks are to be scheduled on parallel production lines $l = 1, ..., L$ over a finite planning horizon consisting of periods $t = 1, ..., T$ with a given length. This length is related to the company practice of measuring external elements, such as demand (thus, periods correspond to days, weeks or months in most of the cases). The production sequence is defined *a priori* obeying to natural sequences and having in mind product families. These tight production conditions are frequent in the fast moving consumer goods industries and reflect the technological and batch requirements that these industries face. Hence, we rely on the block planning approach that predefines the sequence of the blocks and products beforehand, minimizing the setup times and costs according to the planner expertise (Günther et al., 2006). Consider, for example, the production of beverages. If one is to switch over to a similar package where the label is the only changing element, then a minor setup is to be performed. However, if the beverage to be produced also changes, then a major setup is required.

Consider the following indices, parameters, and decision variables that are used both in the deterministic and stochastic formulations.

Indices

$l \in [L]$	parallel production lines
$i, j \in [N]$	blocks
$k \in [K_j]$	products
$t \in [T]$	periods
$a \in [A]$	ages (in periods)

Parameters

C_{lt}	capacity (time) of production line l available in period t
a_{lk}	capacity consumption (time) needed to produce one unit of product k on line l
c_{lk}	production costs of product k (per unit) on line l
$\bar{p_k}$	opportunity cost of producing product k as it spoils
u_k	shelf-life duration of product k right after being produced (time)
m_{lj}	minimum lot size (units) of block j when produced on line l
$\bar{s}_{lj}(\bar{\tau}_{lj})$	setup cost (time) of a changeover to block j on line l
$\underline{s}_{lk}(\underline{\tau}_{lk})$	setup cost (time) of a changeover to product k on line l
p_k	price of each product k sold
p_k^0	willingness to pay for product k in its fresher state
α_k	customer's sensibility to the ageing of product k
β_k	spoilage randomness for product k
ϵ_k	price elasticity of demand for product k

Production Related Decision Variables

q_{lkt}	quantity of product k produced in period t on line l
p_{lkt}	equals 1, if line l is set up for product k in period t (0 otherwise)
y_{ljt}	equals 1, if line l is set up for block j in period t (0 otherwise)

Figure 5 exemplifies the relation between blocks and products in our production planning approach.

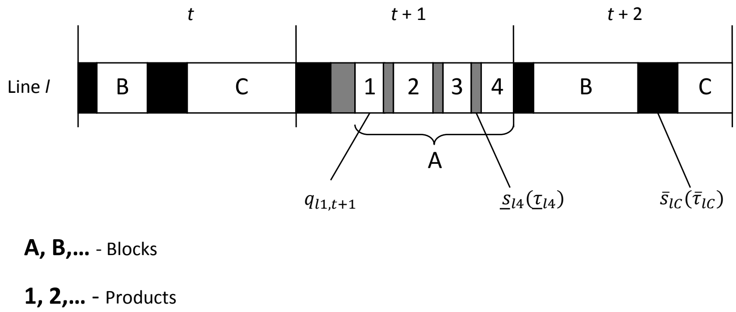

Figure 5: Relation between blocks, produts, major and minor setups.

4.1. Deterministic Demand

To formulate the production planning model for perishable products with deterministic demand (PP-P-D) we need to define the decision variables related to demand fulfilment. First, the parameter d_{kt}^a (demand for product k

with age a in period t) is calculated in a preprocessing step based on one of the described functions (8) - (10) (cf. Section 3) depending on the specific PQR and the parameters defined. The decision variables that shall be added to the already defined ones are:

w_{kt}^a	initial inventory of product k with age a available at period t, $a = 1, ..., \min\{u_k, t-1\}$
ψ_{kt}^a	fraction of the maximum demand for product k delivered with age a at period t, $a = 1, ..., \min\{u_k - 1, t-1\}$
θ_{kt}^a	equals 1, if inventory of product k with age a is used to satisfy demand in period t (0 otherwise), $a = 1, ..., \min\{u_k - 1, t-1\}$

Note that these three set of decision variables are only instantiated for certain domains to ensure that no perished products are kept in stock or used to fulfil demand. It is also assumed that initial and final inventories are empty. Hence, it is important to differentiate between the dynamic set $[A_w] = \{a \in \mathbb{Z}^+ | a \leq \min\{u_k, t-1\}\}$ and $[A_{\psi,\theta}] = \{a \in \mathbb{Z}^+ | a \leq \min\{u_k - 1, t-1\}\}$ depending on the related decision variables. The utility of these sets will be clearer in the development of the constraints.

Figure 6 represents the relation between the demand fulfilment decision variables for an example with product k having production only in day t. The demand is completely fulfilled in this period and partially met in period $t+1$ with a stock of age 1.

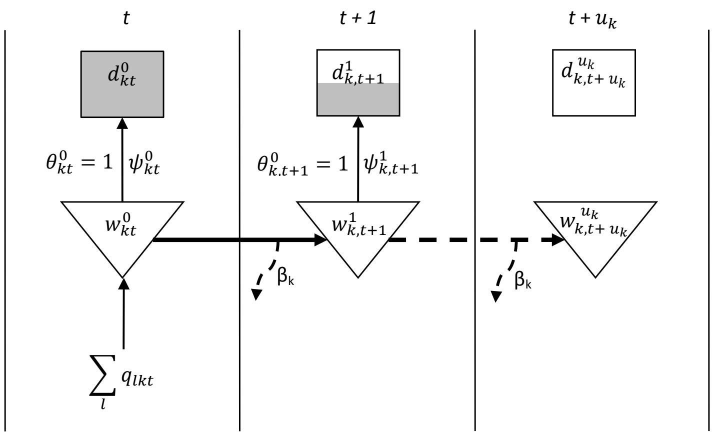

Figure 6: Relation between demand fulfilment variables.

The deterministic model reads:

PP-P-D

$$\max \sum_{k,t,a} p_k\, \psi^a_{kt} d^0_{kt} - \sum_{l,j,t} \bar{s}_{lj}\, y_{ljt} - \sum_{l,k,t} (\underline{s}_{lk}\, p_{lkt} + c_{lk}\, q_{lkt}) - \sum_{k,t,a} \bar{p}_k (1-\beta_k)\, (w^a_{kt} - \psi^a_{kt} d^0_{kt}) \tag{11}$$

subject to:

$$\sum_{a \in [A_{\psi,\theta}]} \psi^a_{kt} \leq 1 \quad \forall k \in [K], t \in [T] \tag{12}$$

$$\psi^a_{kt} d^0_{kt} \leq d^a_{kt} \quad \forall k \in [K], t \in [T], a \in [A_{\psi,\theta}] \tag{13}$$

$$\psi^a_{kt} \leq \theta^a_{kt} \quad \forall k \in [K], t \in [T], a \in [A_{\psi,\theta}] \tag{14}$$

$$w^{a-1}_{kt} - \psi^{a-1}_{kt} d^0_{kt} \leq (1 - \theta^a_{kt}) M \quad \forall k \in [K], t \in [T], a \in [A_{\psi,\theta}] \setminus \{0\} \tag{15}$$

$$w^a_{kt} = (w^{a-1}_{k,t-1} - \psi^{a-1}_{k,t-1} d^0_{k,t-1}) \beta_k \quad \forall k \in [K], t \in [T+1], a \in [A_w] \setminus \{0\} \tag{16}$$

$$\sum_l q_{lkt} = w^0_{kt} \quad \forall k \in [K], t \in [T] \tag{17}$$

$$p_{lkt} \leq y_{ljt} \quad \forall l \in [L], j \in [N], k \in [K_j], t \in [T] \tag{18}$$

$$q_{lkt} \leq \frac{C_{lt}}{a_{lk}} p_{lkt} \quad \forall l \in [L], k \in [K], t \in [T] \tag{19}$$

$$\sum_j \bar{\tau}_{lj}\, y_{ljt} + \sum_k (\underline{\tau}_{lk}\, p_{lkt} + a_{lk}\, q_{lkt}) \leq C_{lt} \quad \forall l \in [L], t \in [T] \tag{20}$$

$$\sum_{k \in [K_j]} q_{lkt} \geq m_{lj} y_{ljt} \quad \forall l \in [L], j \in [N], t \in [T] \tag{21}$$

$$\psi_{kt}^a,\ w_{kt}^a,\ q_{lkt} \geq 0;\ p_{lkt},\ y_{ljt},\ \theta_{kt}^a \in \{0, 1\} \tag{22}$$

Objective function (11) maximizes the profit of the producer over the planning horizon. From the sum of the revenue of each unit sold, major setup costs between blocks, minor setup costs between products and variable production costs are subtracted. Moreover, the cost of spoiled products is also subtracted and valued through the opportunity cost $\bar{p}_k$. This opportunity cost corresponds to the revenue yielded by the best alternative that could have been produced and sold instead of producing product k that got spoiled. However, it may also be regarded, in a more quantifiable manner, as a disposal cost for each unit of perished inventory that has to be properly discarded.

Equations (12) forbid the sum of all sold products of different ages to exceed the total demand for the product in the fresher state. Then, equations (13) do not allow the quantity of sold products of a given age to be above the demand curve derived from the customer WTP. However, with only these two constraints, it is assumed that the seller is able to assign the optimal inventory quantities with different ages to customers in order to maximize profit. With constraints (14) and (15) this situation is avoided by mimicking the more instinctive consumer purchasing behaviour related to perishable food products that will drive customers to pick products with the highest degree of freshness. Thus, constraints (14) turn the value of θ_{kt}^a to 1, whenever inventory of a given product k in period t with age a is used to satisfy demand. The value of this variable θ_{kt}^a is used in equations (15) to ensure that an older inventory can only be used after depleting the fresher inventory. Note that parameter M denotes a big number. Constraints (16) establish the inventory balance constraints, which are modelled here in a propagation form, updating the age of the inventories throughout the planning horizon. The products in inventory may either have a fixed lifetime expressed by a stamped best-before-date (milk and meet) or a random lifetime in case no stamp is found (fruits and vegetables). The quantity throughout the time of these latter products is commonly modelled as subject to exponential decay. In constraints (16) this spoilage phenomenon is controlled by parameter β_k. On one hand, if $\beta_k = 1$ the product in inventory will only spoil by the end of its shelf-life. On the other hand, if $\beta_k < 1$, then inventory is subject to exponential decay. The lower the β_k value the higher the amount of spoilage from one period to the subsequent.

Equations (17) establish that the production done in a certain period on all lines for a given product is equal to the available stock of that same product with age 0. These constraints link the production flow and the demand fulfilment requirements. Constraints (18) and (19) ensure that in order to produce a certain product, the necessary setups for the correspondent recipe and for the product are performed, respectively. Moreover, each block is subject to a minimum lot size (21). Finally, constraints (20) limit the use of the capacity with setups and production in each period.

4.2. Uncertain Demand

In this section we extend the previous deterministic model by incorporating demand uncertainty, which is very typical in food products. These products are subject to a very intense promotion activity with uncertain outcomes and companies in this business are very keen to launch new products to the market (Lütke Entrup, 2005). It is important to highlight that in the deterministic setting, unless minimum lot sizes are of a very significant size when compared to the demand orders, there will never be any spoiled product, for the case of products with a fixed lifetime ($\beta_k = 1$). In fact, the production quantities are such that after fulfilling the predicted demand no inventory may last until the end of its shelf-life, depending on the minimum batch size requirements. However, in real-world problems, consumer goods industries of food products face enormous challenges in reducing the amount of spoilage derived from not selling produced products.

In order to cope with the demand uncertainty we propose a two-stage stochastic programming with recourse model. Stochastic programming models with recourse are adequate when decisions may be decoupled in two separate stages: first stage decisions are taken before the uncertainty is unveiled and second stage decisions after uncertainty has occurred. Thus, the second stage decisions may be able to correct some imprecisions coming from the myopic decisions in the first stage. For an introductory study on stochastic programming the readers are referred to Birge & Louveaux (2011). Justification for treating uncertainty with this methodology is given, for example, in Sodhi (2005). This work deals with the supply chain planning problem in a real-world electronics company that has an uncertain demand.

In this problem, the first stage decisions are related to the production planning and scheduling of blocks and products, and the second stage decisions are responsible for managing the ongoing perishable inventory and demand fulfilment. The motivation for such division comes from the fact that most of the producers decide on the production plans before actually knowing the accurate demand. For example, in the yoghurt production where the final packing stage is connected to the bulk recipe stage that has significant lead times, the production planning and sequencing is done with weeks of advance. Afterwards, the inventory at the retailers or distribution centers is used to hedge against the demand variability.

The stochastic demand is modelled through a set $[V]$ of discrete scenarios $v = 1, ..., V$, each of them with an associated probability ϕ^v, such that $\phi^v > 0$, $\forall v$ and $\sum_v \phi^v = 1$. Hence, in order to formulate the stochastic model, we further need to extend the domains of the demand parameter and of every demand fulfilment variable defined in the beginning of Section 4.1 by adding a new index v related to the probable scenarios.

The two-stage stochastic programming with recourse model tackling the production planning problem for perishable products with uncertain demand (PP-P-U) may be formulated as follows:

PP-P-U

$$\max \sum_v \phi^v [\sum_{k,t,a} (p_k \, \psi_{kt}^{av} \, d_{kt}^{0v} - \bar{p}_k (w_{kt}^{u_k,v} + (1 - \beta_k)(w_{kt}^{av} - \psi_{kt}^{av} \, d_{kt}^{0v})))] \\ - \sum_{l,j,t} \bar{s}_{lj} \, y_{ljt} - \sum_{l,k,t} (\underline{s}_{lk} \, p_{lkt} + c_{lk} \, q_{lkt}) \qquad (23)$$

subject to:

$$\sum_{a \in [A_{\psi,\theta}]} \psi_{kt}^{av} \leq 1 \quad \forall k \in [K], t \in [T], v \in [V] \qquad (24)$$

$$\psi_{kt}^{av} d_{kt}^{0v} \leq d_{kt}^{av} \quad \forall k \in [K], t \in [T], a \in [A_{\psi,\theta}], v \in [V] \qquad (25)$$

$$\psi_{kt}^{av} \leq \theta_{kt}^{av} \quad \forall k \in [K], t \in [T], a \in [A_{\psi,\theta}], v \in [V] \qquad (26)$$

$$w_{kt}^{a-1,v} - \psi_{kt}^{a-1,v} d_{kt}^{0} \leq (1 - \theta_{kt}^{av}) M \quad \forall k \in [K], t \in [T], a \in [A_{\psi,\theta}] \setminus \{0\}, v \in [V] \qquad (27)$$

$$w_{kt}^{av} = (w_{k,t-1}^{a-1,v} - \psi_{k,t-1}^{a-1,v} d_{k,t-1}^{0v})\beta_k \quad \forall k \in [K], t \in [T+1], a \in [A_w] \setminus \{0\}, v \in [V] \tag{28}$$

$$\sum_l q_{lkt} = w_{kt}^{0v} \quad \forall k \in [K], t \in [T], v \in [V] \tag{29}$$

(18)-(21)

$$\psi_{kt}^{av},\, w_{kt}^{av},\, q_{lkt} \geq 0;\, p_{lkt},\, y_{ljt},\, \theta_{kt}^{av} \in \{0,1\} \tag{30}$$

In the stochastic problem the objective function differs from the one in the deterministic model (11) not only because it accounts for the different demand scenarios, but also because it tries to minimize the spoilage coming from products that reach the end of the shelf-life without being sold. These products are obtained in a straightforward manner with the proposed formulation since they correspond to the inventory that reaches an age of u_k (given by $w_{kt}^{u_k,v}$). Recall that in case this spoilage term related with products reaching their shelf-lives had been included in the deterministic objective function (11), its value would have been null (if the minimum lot sizes were kept small enough compared with the demand orders).

Equations (24)-(29) have the same meaning as equations (12)-(17) described in the deterministic model, except that they were extended to deal with the different demand scenarios. The remaining constraints related to the production planning are exactly the same as the ones already described in the deterministic model (cf. Section 4.1).

5. Computational Study

The computational study aims at understanding the impact of the consumer purchasing behaviour on the production planning of perishable food goods through several perspectives. Hence, we focus on:

1. the importance of considering age dependent demand as theorized in the consumer purchasing behaviour literature;
2. the importance of considering customer's eagerness to pick up the fresher products available;

3. the impact of neglecting different PQR, assuming a medium PQR for every product;
4. the amount of spoiled products due to not considering age dependent demand.

In order to perform a sensitivity analysis we create instances by varying the parameter α_k, which is related to customer's sensibility to the ageing process of the product and the product mix that yields different proportions of products with different PQR. The generated instances are solved to optimality with a mixed-integer programming solver.

5.1. Data Generation

The analysis is performed using real data for the consumer purchasing behaviour and random data for the production system related parameters. To generate the parameters related to production stage we follow a methodology similar to Haase & Kimms (2000). Therefore, L was set to 1 and for all products $a_{lk} = 1$, $m_{lj} = 1$. We also consider that all products are packaged and stamped with a best-before-date ($\beta_k = 1$). The number of blocks J is 6 and each block has 4 products ($K = 24$). The number of periods T is 20. To generate the demand for products in the fresher state a matrix with 24 rows (products) and 20 columns (periods) is randomly generated, where each element d^0_{kt} is in the interval [40, 60] of the uniform distribution with a probability of 0.75 and of 0 with a probability of 0.25. The setup costs and times between blocks ($\bar{s}_{lj}$ and $\bar{\tau}_{lj}$) are randomly chosen in the interval [5,10]. For the setup of products ($\underline{s}_{lj}$ and $\underline{\tau}_{lj}$) the interval [1,4] is used and all blocks have products with the same product setup values. The capacity per period C_{lt} is determined according to:

$$C_{lt} = \frac{\sum_k d^0_{kt}}{U}, \quad \forall l, t,$$

where the capacity utilization U is 0.6. It is important to notice that the utilization of capacity is only an estimate, as setup times do not influence the computation of C_{lt}.

The real data related to customer's purchasing behaviour for different perishable food goods is based on Tsiros & Heilman (2005) and on Andreyeva et al. (2010). This data is given in Table 1. Notice that $\bar{p}_k$ is obtained by multiplying p_k by 0.5 and c_{lk} is obtained by multiplying it by 0.1. Moreover,

Table 1: Data related to consumer purchasing behaviour.

Block	u_k	p_k	$\bar{p}_k$	p_k^0	α_k	PQR	ϵ_k	c_{lk}
1 - Lettuce	10	2.49	1.245	2.86	0.62	Medium	-0.58	0.25
2 - Milk	14	2.7	1.35	4.05	0.82	Medium	-0.59	0.27
3 - Chicken	7	2.99	1.495	2.78	0.50	High	-0.68	0.30
4 - Carrots	21	1.69	0.845	3.12	0.78	Medium	-0.58	0.17
5 - Yogurt	21	0.62	0.31	1.14	0.77	Medium	-0.65	0.06
6 - Beef	7	2.68	1.34	2.52	0.52	High	-0.75	0.27

remark that each of the six blocks to be produced has a directed correspondence to a real good (lettuce, milk,...) and within this block four products are considered corresponding to different packaging. Despite the fact that in reality a production line that packages such different food products is hard to find, preference was given to use real-world data for the consumer purchasing behaviour that reflect the focus of this study.

To set the stochastic model, we have defined 3 scenarios, all with the same probability $\phi^v = 1/3$. In the average scenario the demand for the products at their fresher state d_{kt}^0 is exactly the same as the values which are used to feed the deterministic model. The other two scenarios have a demand for the fresher products that corresponds to an increase and a decrease of 30% on the average values of d_{kt}^0. To obtain the complete set of data of the demand for all possible product ages we use expressions (8) and (10) for products with medium and high PQR, respectively.

Regarding the sensitivity analysis, we vary the average value of α_k (Alpha) by increasing (Alpha+) / decreasing (Alpha-) it by 25% and by testing three different product mixes. The first mix has to fulfil demand for all products (Mixed), the second only for the products belonging to blocks with a medium PQR (Lettuce, Milk, Carrots and Yoghurt) (Medium) and the third for products with a high PQR (Chicken and Beef) (High).

5.2. Results and Discussion

In order to streamline the flow of this section, the results are organised by the four different perspectives presented in the beginning of Section 5.

5.2.1. Importance of considering age dependent demand

This perspective analyses the impact on profit due to neglecting the consumer purchasing behaviour of perishable food goods for companies using direct store delivery. To test this impact we compare the results obtained by solving the deterministic model (Section 4.1) both when considering and not considering consumer behaviour while deciding the production related decision variables. Hence, for the first approach we solve the whole model already considering the age dependent demand. In the second approach we first optimize the production related decisions for the case where customers have a constant WTP and, afterwards, with the production decision variables fixed, we solve again releasing the demand fulfilment variables for the case where the actual customer behaviour is unveiled. Figure 7 plots the percentage objective function lost due to not considering the real consumer purchasing behaviour.

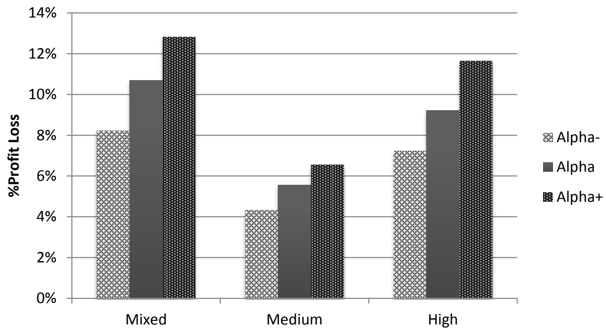

Figure 7: Results for the first perspective analysed.

Overall, neglecting the age dependent demand resulted in a significant profit loss. As it is expected more sensible customers yield higher losses and products with lower perceived risk, such as vegetables have a lower impact on the profit. Thus, if producers are able to reduce the perceived quality risk, this may have an important impact on the potential revenue. Most of these gains are achieved through an augmentation of the total demand throughout the product's shelf-life. It is also important to note that there is a considerable interaction between PQR and customer exigency. Hence, as customers get more sensible to perishability, the impact of product perceived risk tends to augment. Therefore, for example, if the producer is serving a retailer with very demanding customers, it is very important to decrease as much as possible the product quality risk (besides delivering very fresh products). In case of package food goods, such as yoghurt, this can be achieved by filling the product in glass containers instead of plastic ones (Dyllick, 1989).

5.2.2. *Importance of considering customer's eagerness to pick up the fresher products*

As mentioned before, besides incorporating consumer purchasing behaviour through developing a set of expressions able to describe the demand through the shelf-life of products, we introduce (15) in the production planning formulation to describe the customer behaviour of picking the fresher product available. Hence, in order to understand the importance of considering such behaviour in the production planning, we follow a similar approach to the one presented in the previous section. Therefore, we compare the profit for the solution which includes the mentioned customer behaviour with the solution where the production decision variables are optimized without considering such behaviour. Figure 8 plots the percentage objective function lost due to considering the fact that customers pick up the fresher products.

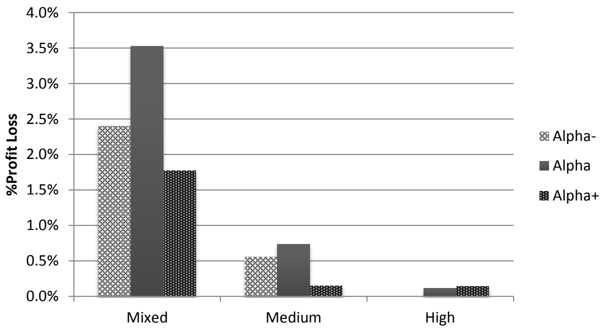

Figure 8: Results for the second perspective analysed.

Results point out that the profit loss of considering this consumer behaviour is less significant than the loss coming from not considering the age dependent demand. In fact, while acknowledging the decreasing WTP we are implicitly assuming consumers preference for fresher products. This will drive production plans towards a leaner strategy and, therefore, implicitly incorporate this characteristic of consumers. Hence, these results point out that despite the fact these constraints have a small impact on the profit loss, the solution structure may differ considerably as only the fresher products may be used to satisfy demand. Moreover, these constraints yield a greater reduction of the solution space for instances having products with longer shelf-lives (Medium PQR). This is reflected in the higher profit losses.

5.2.3. Impact of assuming a medium PQR for every product

With this analysis we aim at understanding the error coming from assuming a medium PQR for all products independently of their nature. The motivation for looking to such perspective comes from the fact that in practical terms it may be rather hard to implement the different equations (8)-(10) for all different products. The method used to obtain results follows the approach used in the previous sections. Hence, the results obtained in case production quantities and timings are established assuming a medium PQR are compared to the case where production planning already incorporates the correct demand profile throughout the age of the products.

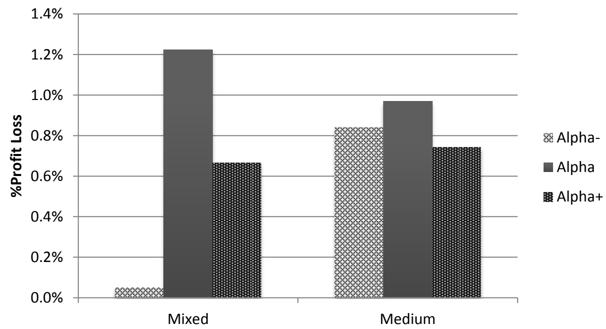

Figure 9: Results for the third perspective analysed.

In all instance, a profit loss inferior to 1.3% was obtained. This indicates that differentiating between different PQR is not as important as differentiating between the remaining inputs for the age dependent demand. Our conclusion is that, in practical applications, planners of food products should focus in understanding both the initial WTP and the sensibility that customers have towards a decreasing shelf-life.

5.2.4. Amount of spoiled products due to not considering age dependent demand

Finally, we analyse the influence of age dependent demand in the amount of spoiled products. As said before, a solution of the deterministic model will hardly yield a solution with any spoiled product. Therefore, to assess this impact the stochastic model is used. We compare the solution obtained when the first-stage variables are optimized for a constant demand throughout the shelf-life of the products, with a solution in which both first and second-stage variables take into account the age dependent demand.

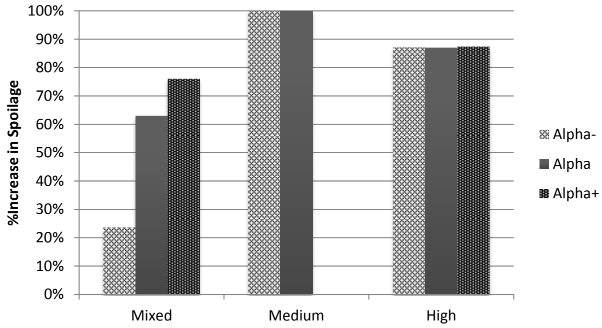

Figure 10: Results for the fourth perspective analysed.

The results indicate that the amount of spoilage is severely impacted by the acknowledgement of a decreasing demand throughout the age of the product. The potential savings in product spoilage ascends to a complete reduction in the spoiled inventory. In an era of strong environmental awareness both in the civil society and in companies this is a crucial indicator to be taken into consideration.

6. Conclusions and Future Work

In this study we first develop a set of age dependent demand functions for products with different PQR based on researches analysing consumer purchasing behaviour for perishable food products. We propose a deterministic model for the production planning of perishable goods that accounts both for decreasing WTP and customers' eagerness to choose the product in a fresher state. This deterministic model is extended to a stochastic one dealing with demand uncertainty, which is a common characteristic of the fast moving food consumer goods markets. The computational study focuses on a sensitivity analysis where the main parameters related to the novelties introduced are varied. Results pointed out that extending food production planning models to deal with an age dependent demand is of great importance both in terms of profit and product spoilage.

Future work should focus on different extensions of this problem. First, it is interesting to study the impact of modelling β_k as a random variable (Nahmias, 1982). Second, demand uncertainty gives further motivation to investigate this problem under a risk management perspective. Recent studies started to exploit the multi-objective problem of considering risk management in production planning (Tometzki & Engell, 2010). The production planning of perishable goods is, indeed, a very promising field to go further in this direction. Finally, it is of most interest to develop a proper solution method that is able to solve larger instances of this problem, especially for the stochastic formulation. As Birge & Louveaux (2011) highlight, standard methods (as the ones embedded in mixed-integer programming solvers) do not take advantage of the problem structure of the two-level stochastic models with recourse.

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A Hybrid Path-Relinking Method for Solving a Stochastic Lot Sizing and Scheduling Problem

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Abstract

Path-relinking has been used to help solving deterministic problems by exploring the neighbourhood of elite solutions in an intelligent way. In this paper, we present an algorithm that combines a mixed-integer linear solver with a truncated path-relinking method in order to solve a stochastic lot sizing and scheduling problem dealing with perishable products. This supply chain planning problem may be seen as a two-stage stochastic integer problem with complete recourse and first stage integer variables. In the first stage the decision maker decides about the production sequence and the production quantities. Afterwards, the uncertain demand is unveiled and the second stage decisions concerned about inventory usage are taken. The key idea of this method is to take advantage of the possible scenario-based decomposition in an innovative way, which can be generalized to problems with a similar structure. Therefore, path-relinking is used to combine optimised solutions from different scenarios in pursuing good stochastic solutions. Computational results show a clear advantage of the proposed method in solving this stochastic problem when compared to a state-of-the-art mixed-integer solver, especially for the medium and larger instances.

Keywords: Path-Relinking, Mixed-Integer Solver, Stochastic Programming, Lot Sizing and Scheduling, Demand Uncertainty

1. Introduction

Within supply chain planning tasks, the lot sizing and scheduling problem is responsible for determining the size of each production lot and the sequence in which these

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lots are produced in a medium to short term planning horizon. Stochastic lot sizing and scheduling problems appear when parameter's uncertainty is taken into account. Lot sizing and scheduling problems are known to be NP-hard and, therefore, provably optimal solutions are difficult to obtain even for medium-sized instances. Moreover, the stochastic version of this deterministic problem, where demand is unknown in advance, turns the problems to be solved even harder. We focus on the stochastic lot sizing and scheduling problem of perishable products (S-LSP-PP) arising in the food consumer goods industries that use *direct store delivery*. The main impact of this delivery practice is an increasing responsibility of producers for all downstream processes of the supply chain until reaching the final customer. This responsibility entails a heavier burden in case the companies produce perishable products that have a limited shelf-life, inhibiting the abusive use of intermediate storage to hedge against demand uncertainty. This problem is presented in more detail in Amorim et al. (2012).

The S-LSP-PP can be classified as a two-stage stochastic integer problem with complete recourse and first stage integer variables. In this problem structure, first stage decisions are responsible for the production planning, with lot sizing and scheduling decisions, whereas second stage variables allocate the inventory and production output to the uncertain demand. This work aims to propose a hybrid solution procedure to solve the S-LSP-PP, which can be generalized to other problems with a similar model structure. This algorithm combines a truncated path-relinking method and a mixed-integer programming-based method. The truncated path-relinking starts the same path of a standard path-relinking, but stops the search in between the starting and guiding solutions. We take advantage of the inherent structure of this kind of problems in order to develop a hybrid heuristic approach. This procedure combines deterministic integer solutions coming from each demand scenario in order to approach the global optimum to the stochastic model.

Stochastic mixed-integer problems have been used to formulate numerous planning problems and a multitude of solution approaches are available to solve them. In the review of Bianchi et al. (2008) an exhaustive coverage of exact methods and metaheuristics used to solve such problems is performed. Basically, almost every traditional metaheuristic and exact method has been adapted to solve these hard problems, such as tabu search (Costa & Silver, 1998), ant colony optimization (Gutjahr, 2003) or branch and bound (Gutjahr & Strauss, 2000). Among all these solution methods, some take advantage of the scenario sampling structure that characterizes a large portion of these problems. Our hybrid path-relinking positions itself along with these methods that use a scenario-based decomposition. A paradigmatic example of this kind of algorithms is the Progressive Hedging that was first proposed by Lokketangen & Woodruff (1996). This method considers a set of representative scenarios that grasp the stochasticity in the second-stage parameters. Each of these scenarios is solved by means of a deterministic model that captures the related sub-problem. By the end of this first step, a pool of solutions optimizing independently the scenarios is obtained. Afterwards, through an averaging procedure of all considered solutions, a compromise is obtained for all possible uncertain outcomes. Several improvements and enhancements of the base algorithm have been suggested and the readers are referred to Watson & Woodruff (2010) for an updated overview of this method. Our proposed method distinguishes itself from the mentioned ones by handling through a different methodology, path-relinking, the combination of individual scenario solutions.

The remainder of this paper is as follows. In Section 2, a mathematical description of the S-LSP-PP is given. The formulation will be used afterwards by the hybrid path-relinking described in Section 3. In Section 4 the computational results are reported. Finally, Section 5 resumes the main findings and points out paths for future research.

2. Formal Problem Description

Consider products $i, j = 1, ..., N$ that are to be scheduled on $l = 1, ..., L$ parallel production lines over a finite planning horizon consisting of macro-periods $t = 1, ..., T$ with a given length. The changeover time and cost between products on a line is dependent upon the sequence.

A macro-period is divided into a fixed number of non-overlapping micro-periods with variable length. Since the production lines can be independently scheduled, this division is done for each line separately. Let $[S_{lt}]$ denote the set of micro-periods $s = 1, ..., S_{lt}$ belonging to macro-period t and production line l. The number of micro-periods of each macro-period defines the upper bound on the number of products to be produced on each line. The length of a micro-period is linked to the decision variable accounting for the quantity of products produced. A product lot may continue over several micro and macro-periods since setup carry-over is considered. Thus, a lot is independent of the discrete time structure of the macro-periods. Note that this lot sizing and scheduling time structure is based on the general lot sizing and scheduling structure for parallel lines (Meyr, 2002).

The demand for each product j at its fresher state in macro-period t (d_{jt}^{0v}) is stochastic and obtained through the sampling of discrete scenarios $v = 1, ..., V$. Each of these scenarios has an associated probability ϕ^v, such that $\phi^v > 0$, $\forall v$ and $\sum_v \phi^v = 1$. In Amorim et al. (2012), the authors study the influence of different consumer purchasing behaviours reflected in the demand profile over the age of the perishable products (until reaching the shelf-life, u_j). The present study focuses on a linear demand shape for customers that are rather sensitive to product freshness. With equation (1) the demand parameter for product j with age a in period t according to scenario v (given by d_{jt}^{av}), is calculated.

$$d_{jt}^{av} = d_{jt}^{0v} - \frac{0.5 d_{jt}^{0v} a}{u_j - 1}. \tag{1}$$

Consider the following indices, parameters, and decision variables.

Indices

l	parallel production lines
i, j	products
t	macro-periods
s	micro-periods
a	age (in macro-periods)
v	scenarios

Parameters

$[S_{lt}]$	set of micro-periods s within macro-period t and line l
C_{lt}	capacity (time) of production line l available in macro-period t
a_{lj}	capacity consumption (time) needed to produce one unit of product j on line l
c_{lj}	production costs of product j (per unit) on line l
p_j	price of each product j sold
$\bar{p}_j$	cost incurred for each product j spoiled
u_j	shelf-life duration of product j right after being produced (time)
m_{lj}	minimum lot size (units) of product j when produced on line l
$s_{lij}(\tau_{lij})$	sequence dependent setup cost (time) of a changeover from product i to product j on line l
y_{lj0}	equals 1, if line l is set up for product j at the beginning of the planning horizon (0 otherwise)

First Stage Decision Variables

q_{ljs}	quantity of product j produced in micro-period s on line l
y_{ljs}	equals 1, if line l is set up for product j in micro-period s (0 otherwise)
z_{lijs}	equals 1, if a changeover from product i to product j takes place on line l at the beginning of micro-period s (0 otherwise)

Second Stage Decision Variables

w_{jt}^{av}	inventory of product j with age a available at macro-period t in scenario v
ψ_{jt}^{av}	quantity of product j with age a delivered at macro-period t in scenario v

We denote a given set $\{1, 2, ..., M\}$ as $[M]$. Further note that variables w_{jt}^{av} and ψ_{jt}^{av} are only instantiated for a certain domain to ensure that no perished product is kept in stock or used to fulfil demand. Hence, the dynamic set $[A_w] = \{a \in \mathbb{Z}^+ | a \leq \min\{u_j, t-1\}\}$ and the set $[A_\psi] = \{a \in \mathbb{Z}^+ | a \leq \min\{u_j - 1, t-1\}\}$ are used depending on the corresponding decision variable. Without loss of generality, we assume that both inventory at the beginning and at the ending of the planning horizon are null.

S-LSP-PP

$$\max \sum_{j,t,a,v} \phi^v (p_j \psi_{jt}^{av} - \bar{p}_j w_{jt}^{u_j,v}) - \sum_{l,i,j,s} s_{lij} z_{lijs} - \sum_{l,j,s} c_{lj} q_{ljs} \tag{2}$$

subject to:

$$\sum_a \psi_{jt}^{av} \leq d_{jt}^{0v} \quad \forall j \in [N], t \in [T], v \in [V] \tag{3}$$

$$\psi_{jt}^{av} \leq d_{jt}^{av} \quad \forall j \in [N], t \in [T], a \in [A_\psi], v \in [V] \tag{4}$$

$$w_{jt}^{av} = (w_{j,t-1}^{a-1,v} - \psi_{j,t-1}^{a-1,v}) \quad \forall j \in [N], t \in [T+1], a \in [A_w] \setminus \{0\}, v \in [V] \tag{5}$$

$$\sum_{l,s \in [S_{lt}]} q_{ljs} = w_{jt}^{0v} \quad \forall j \in [N], t \in [T], v \in [V] \tag{6}$$

$$q_{ljs} \leq \frac{C_{lt}}{a_{lj}} y_{ljs} \quad \forall l \in [L], j \in [N], t \in [T], s \in [S_{lt}] \tag{7}$$

$$\sum_{i,j,s \in [S_{lt}]} \tau_{lij} z_{lijs} + \sum_{j,s \in [S_{lt}]} a_{lj} q_{ljs} \leq C_{lt} \quad \forall l \in [L], t \in [T] \tag{8}$$

$$\sum_j y_{ljs} = 1 \quad \forall l \in [L], t \in [T], s \in [S_{lt}] \tag{9}$$

$$q_{ljs} \geq m_{lj}(y_{ljs} - y_{lj,s-1}) \quad \forall l \in [L], j \in [N], t \in [T], s \in [S_{lt}] \tag{10}$$

$$z_{lijs} \geq y_{li,s-1} + y_{ljs} - 1 \quad \forall l \in [L], i, j \in [N], t \in [T], s \in [S_{lt}] \tag{11}$$

$$\psi_{jt}^{av}, w_{jt}^{av}, q_{ljs}, z_{lijs} \geq 0; \; y_{ljs} \in \{0, 1\} \tag{12}$$

Objective function (2) maximizes the planning horizon profit by subtracting to the revenue of the sold products the spoilage, the setup and the variable production costs. The quantity of spoiled products that reach the end of the shelf-life without being sold is obtained in a straightforward manner with the proposed formulation since it corresponds to the inventory that reaches an age of u_j (it is given by $w_{jt}^{u_j,v}$).

Equations (3) do not allow the quantity of sold products of a given age to be above the demand curve derived from the customer willingness to pay. Equations (4) force the sum of all sold products of different ages not to exceed the total demand for the product with the fresher state for each scenario. Hence, a strong assumption is made in this formulation regarding the perfect control of the retailer over the inventory available to customers. The discussion about other inventory policies is out of the scope of this work.

Constraints (5) establish the inventory balance constraints that are modelled here in a propagation form that updates the age of the inventories throughout the planning horizon for each demand scenario and discounts the products sold. Equations (6) establish that the production done in a certain macro-period on all lines for a given product is equal to the available stock of that same product with age 0 (maximum freshness). These constraints link the production planning with the demand fulfilment requirements and, thus, the first and the second stage decision variables of the stochastic model. Constraints (7) ensure that in order to produce a certain product, the necessary setup is performed. Moreover, at each moment only one product may be produced on a certain line (9) and each product lot is subject to a minimum lot size (10). Constraints (8) limit the use of the capacity with setups and production in each macro-period and, finally, constraints (11) are responsible for tracking the changeover between products. Note that the integrality condition of variables z_{lijs} is not necessary.

3. The Hybrid Path-Relinking Method

Path-relinking is a method that explores the neighbourhood of elite solutions in a systematic way. This method was proposed by Glover & Laguna (1993) and readers are referred to Resende et al. (2010) to a deeper understanding of this method and its variants, such as forward, backward and mixed path-relinking. The key reasoning behind path-relinking is that good solutions should share a similar structure among them. Thus, in the basic version of path-relinking, two solutions are chosen at each time and the solution elements of one of them are changed through a path that finalizes when the starting solution converts into the target one.

In this paper, we reframe the idea of path-relinking to be applied in stochastic problems represented by different probability scenarios. As mixed-integer linear solvers do not take advantage of the structure of this kind of problems, we tackle heuristically this issue by using a hybridization of path-relinking with a solver that independently explores the deterministic scenarios. Our reasoning is that an optimal solution should potentially lie somewhere in between different scenario solutions. Hence, our first step is to optimize individually the deterministic problem associated with each scenario. Afterwards, the most promising solutions are combined via path-relinking and the intermediate solutions are evaluated by the stochastic objective function. Figure 1 shows the main outline of the method where the first step is performed within a branch-and-bound scheme.

Throughout the algorithm execution two different types of calls to the mixed-integer solver are executed: $solveSCE(v)$ and $solveEVA(\vec{y}_v)$. In $solveSCE(v)$ a scenario v is explored by the solver without regarding the stochastic nature of the problem. The output is the vector $\vec{y}_v$ of integer decision variables obtained from the best solution found. Routine $solveEVA(\vec{y}_v)$ solves the linear problem of the stochastic model derived from fixing

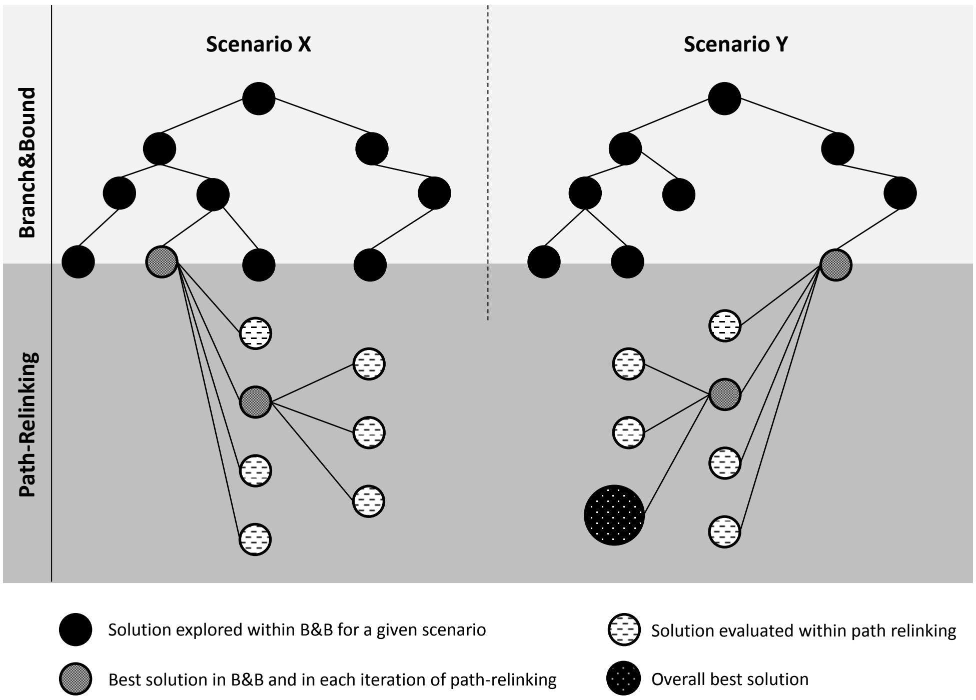

Figure 1: Outline of the hybrid path-relinking.

the first-stage integer variables with the values of $\vec{y}_v$. In the end it returns the stochastic objective function value.

Algorithm 1 describes the pseudo-code for the hybrid path-relinking where these calls are used. First, a feasible solution is found for every deterministic demand scenario within a certain time limit ($solveSCE(v)$). These deterministic solutions are then evaluated in the stochastic scenario ($solveEVA(\vec{y}_v)$). The two best solutions found within all scenarios are then explored within a truncated path-relinking strategy connecting the best solution to the second best. Basically, at each iteration an element of the starting solution is changed into one of the target solution until reaching a degree of resemblance set by the $Stop$ criterion. To calculate this criterion another parameter is used ($DimParam$) that varies between 0 and 1. Hence, the cardinality of the set containing the different positions in which the two considered integer solutions are different is calculated. The position i of the vector of integer decision variables $\vec{y}_v$ is denoted by $y_{i,v}$. This cardinality is then multiplied by $DimParam$ limiting the proportion of resemblance between the two solutions. Therefore, if $DimParam$ has the value of 1, than the truncated path-relinking converts to the standard path-relinking.

Algorithm 1: Pseudo-code for the hybrid path-relinking

```
for v = 1 to V do
    y⃗_v := solveSCE(v);
max := arg max_{v∈[V]} {solveEVA(y⃗_v)};
max2 := arg max_{v∈[V]\{max}} {solveEVA(y⃗_v)};
δ⃗ = {i = 1, ..., |y⃗_v| : y_{i,max} ≠ y_{i,max2}};
Stop = |δ⃗| × DimParam;
while |δ⃗| > Stop do
    for i ∈ δ⃗ do
        y⃗_max = {..., y_{i-1,max}, y_{i,max2}, y_{i+1,max}, ...};
        Obj := solveEVA(y⃗_max);
        if Obj > ObjMax then
            ObjMax = Obj;
            i_max = i;
    δ⃗ = δ⃗ \ {i_max};
    if ObjMax > ObjBest then
        ObjBest = ObjMax;
Output: ObjBest
```

4. Computational Study

In this section the performance of the hybrid path-relinking method is assessed through computational experiments. To test the proposed method we run a C++ implementation of the algorithm with the mixed-integer programming solver CPLEX 12.2 on a PC with 4 GB of RAM and an Intel i5 with 2.67 GHz running on Windows 7.

4.1. Data Generation

A total of 18 instances were systematically generated, following a methodology similar to the one proposed by Haase & Kimms (2000); therefore, L was set to 1. For all products $a_{lj} = 1$, $c_{lj} = 0.5$, $p_j = 2$ and $\bar{p}_j = 2$. Moreover, the machine is set up for product 1 at the beginning of the planning horizon. The number of products N is 5, 10 and 15. The number of macro-periods T is 5, 10 and 20. The number of micro-periods within a macro-period ($|S_{lt}|$) is set at the value of N allowing for all products to be produced in each macro-period with minimum lot-sizes (m_{lj}) of 1 unit. For the setup times between products (τ_{lij}) the interval [2,10] was used for the 15 products (except for the case where $i = j$, where the setup is 0). Shelf-lives (u_j) were generated for all 15 products for each possible planning period length choosing randomly from the interval [1, T]. The setup cost s_{lij} for a changeover from product i to product j on line l is computed as: $s_{lij} = 50\tau_{lij}$. Each element of the initial demand matrix (d_{jt}^{0v}) for the average scenario with 15 products (rows) and 20 macro-periods (columns) was randomly generated on the interval [40, 60]. Afterwards, using equations (1) the complete demand for all ages is created (d_{jt}^{av}). For the possible demand realizations 5 different scenarios were generated by multiplying the average demand elements by 0.6, 0.8, 1.2 and 1.4. We assume that every scenario has the same probability of occurrence and we test two cases: i) all 5 scenarios and ii) only the intermediate scenario and the less extreme ones (multiplying the average demand by 0.8 and 1.2). Finally, the capacity per macro-period Cap_{lt} is determined according to:

$$C_{lt} = \frac{\sum_j d_{jt}^{0v}}{U}, \quad \forall l, t, v,$$

where the capacity utilization U is equal to 0.8. It is important to notice that the utilization of capacity is a rough estimative, as setup times do not influence the computation of C_{lt}. In summary there are:

$$|5, 10, 15| \times |5, 10, 20| \times |3, 5| = 18 \text{ instances.}$$

4.2. Parameters Tuning

The proposed hybrid path-relinking method only requires a few parameters to be set. The time limit for solving each scenario (*solveSCE*) depends on the number of products handled in each instance. Hence for instances with 5, 10 and 15 products, *solveSCE* runs for 30, 60 and 180 seconds, respectively. Routine *solveEVA* runs the evaluation of each solution until optimality. For the parameter *DimParam*, which is responsible for controlling how truncated the path-relinking is,the value of 0.7 is adopted.

Table 1: Results of the computational study

Instance	Path Relinking		Cplex			
$NxTxV$	**Solution**	**Gap**	**Solution**	**Gap**	**Running Time**	**UB Diff.**
5x5x5	1933.36	14%	1918.05	15%	178	0.80%
5x10x5	4496.41	9%	4519.17	8%	206	-0.50%
5x20x5	8346.66	13%	8465.46	11%	266	-1.40%
					216.67	**-0.37%**
10x5x5	3963.86	21%	3761.99	27%	384	5.37%
10x10x5	8620.64	15%	8185.85	21%	670	5.31%
10x20x5	15879.03	20%	14926.09	28%	2059	6.38%
					1037.67	**5.69%**
15x5x5	5771.56	26%	4174.62	75%	1365	38.25%
15x10x5	11870.01	26%	10645.59	40%	2196	11.50%
15x20x5	20878.68	37%	18670.28	53%	5255	11.83%
					2938.67	**20.53%**
5x5x3	1059.82	20%	1070.22	19%	112	-0.97%
5x10x3	2550.74	14%	2622.72	11%	146	-2.74%
5x20x3	4774.37	21%	4797.97	20%	205	-0.49%
					154.33	**-1.40%**
10x5x3	2097.5	38%	1996.46	45%	262	5.06%
10x10x3	4808.97	24%	4255.96	40%	449	12.99%
10x20x3	8845.34	32%	8293.97	41%	1706	6.65%
					805.67	**8.23%**
15x5x3	3039.13	46%	2504.94	77%	754	21.33%
15x10x3	6409.11	40%	5444.13	65%	851	17.73%
15x20x3	11647.11	50%	9444.59	85%	6211	23.32%
					2605.33	**20.79%**

4.3. Results

To assess the performance of our algorithm, we compared it to the performance of CPLEX. The hybrid path-relinking ran with the specified parameters for every instance and the execution times of each solution were fed back to CPLEX, in order to obtain the best solution found in the same execution time. The final results for the 18 instances are presented in Table 1. For each instance, the best solution (**Solution**), the integrality gap (**Gap**) that is calculated with the lower bound given by CPLEX and the execution time (**Running Time**) are presented for each solution method (**Path Relinking** and **CPLEX**). Furthermore, the relative difference between the solutions of both methods is computed (**UB Diff.**). The calculation of this indicator is obtained by subtracting the CPLEX solution value to the Path Relinking and dividing this difference by the Path Relinking solution value. Finally, averages for the instances clusters are presented in bold.

Results point out that the proposed method outperforms CPLEX for all the instances with 10 and 15 products. Whereas, for instances with 5 products, for the same execution time, CPLEX is able to find for the majority of the instances better integer solutions. It seems also that the advantage of the proposed method increases with the size of the instances. In fact, for instance with 15 products, the hybrid path-relinking is able to improve on average by more than 20% the solution obtained by CPLEX.

5. Conclusions and Future Work

The main contribution of this work lies on the exploration of hybrid methods to solve complex stochastic problems, such as the stochastic lot sizing and scheduling of perishable goods. Specifically, we propose a novel hybridization of a truncated path-relinking with a mixed-integer solver that takes advantages of the special structure of this problem. From a reasonable upper bound for each demand scenario separately, a path-relinking method works on the most promising solutions from the stochastic point of view. Since we are dealing with a two-stage stochastic model with only linear second stage variables, each iteration evaluation of the path-relinking is rather fast. Results show the increased advantage of this method over CPLEX alone.

Future work is to be performed in three directions. Improve the decision on which starting and guiding solutions to use, reduce the need of the MIP solver in order to improve scalability and, finally, assess the performance of other path-relinking strategies embedded in this solution scheme.

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Chapter 4

Perishability Impact on Distribution Planning

The Impact of Food Perishability Issues in the Vehicle Routing Problem

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Abstract

Highly perishable food products can lose an important part of their value in the distribution process. We propose a novel multi-objective model that decouples the minimization of the distribution costs from the maximization of the freshness state of the delivered products. The main objective of the work is to examine the relation between distribution scenarios and the cost-freshness trade-off. Small size instances adapted from the vehicle routing problem with time windows are solved with an ϵ-constraint method and for large size instances a multi-objective evolutionary algorithm is implemented. The computational experiments show the conflicting nature of the two objectives.

Keywords: Routing, Food Perishability, Freshness, Multi-objective optimization

1. Introduction

Product perishability may manifest itself in a whole set of difference forms. Products subject to perishability range from daily newspapers that lose their value soon after the day they are reporting, to flowers that look wilted sometimes even before reaching stores, or blood used for transfusions. This last example sparked the study of the perishable inventory (Millard, 1960). Decision models valid for this broad range of perishable products

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should capture their specific nature distinguishing, for example, between products with and without *best before dates* (Amorim et al., b). Highly perishable products have an important role in the operational distribution process, particularly in the vehicle routing planning task. Examples of this kind of products are fruits, vegetables and prepared meals. In this category of goods, the quality changing imposed by the perishability phenomenon is noticeable by the entity receiving the products during the planning horizon. Hence, in this paper, the decreasing value that customers attribute to a decreasing freshness state is acknowledged.

Let us focus on the prepared meals segment to exemplify the particularities of this vehicle routing problem that delivers highly perishable goods. Consider a company specialized in gourmet prepared meals based on a very busy city that services daily hundreds of customers. Moreover, this company runs an own fleet of refrigerated trucks to perform the distribution of the products. The meals are ordered on the day before and customers are very demanding in terms of expected delivery time and freshness of the food received. Hence, if a customer orders duck with sauce, he is expecting that when he receives the meal, it will seem as the sauce was just made and poured above the duck. On the other hand, if the customer orders an assortment of cheeses, he will be less sensible to how much time the product stayed in the truck before reaching him.

This operational distribution planning task fits into the vehicle routing problem (VRP) class of problems. In specific, we are dealing with a VRP with time windows (VRPTW) that has to consider the perishable nature of the products delivered. This hard problem (Savelsbergh, 1985) is to be modelled using a multi-objective framework in which distribution costs are minimized and the freshness of the products delivered to the customers is maximized simultaneously. The first objective reflects explicitly the need of reducing operation costs and the second one expresses the intangible customer value stemming from product freshness, which the company wants to grasp when designing their routes.

In order to investigate the impact of product perishability in the distribution process a set of empirical hypotheses are postulated, relating distribution scenarios and the cost-freshness trade-off. The customers' typology, time windows width and perishability intensity are varied through these distribution scenarios. To test these hypotheses an ϵ-constraint method is employed to solve exactly the well-known Solomon instances (Solomon, 1983) with 25 customers. Afterwards, the findings are validated for the instances with 100 customers using a multi-objective evolutionary algorithm (MOEA) as the exact methods fail to generate feasible solutions.

The remainder of this paper is as follows. In the next section a brief literature review is performed, and the mathematical formulation that models this problem is then presented in Section 3. Section 4 formulates the hypotheses that establish the possible influences of distribution scenarios in the cost-freshness trade-off. In Section 5 the methodology used to test the hypothesis for small and large instances is described. Afterwards, in Section 6 the results obtained through the computational experiments are shown. Finally, the Conclusions section resumes the main findings of this work and gives some hints for future research.

2. Literature Review

The VRP field of research is very vast and proficuous. The same applies for the VRPTW extension. In this review, the focus will be kept on papers dealing with the VRP for perishable goods. The readers are referred to Laporte (2007) for more general VRP problems.

Some literature focus on different distribution problems related with perishable food products but without considering explicitly the degradation of quality (losing of freshness) during transportation. In fact, these models could be most of the times applied to products without a perishable nature. The work of Tarantilis & Kiranoudis (2001) concentrates on the distribution of fresh milk and formulates the problem as a heterogeneous fixed fleet VRP. The same authors (Tarantilis & Kiranoudis, 2002) solve a real-world distribution problem of fresh meat as a multi-depot VRP. Faulin (2003) implements a hybrid algorithm procedure that uses a combination of heuristics and exact algorithms to find a solution to a VRP with constraints enforcing narrow time windows and strict delivery quantities. According to the author, these delivery scenarios are usually the case in the agribusiness industry.

Concerning the articles modelling perishability explicitly, Osvald & Stirn (2008) extend a heuristic proposed by the first author in a previous work to solve the problem of distributing fresh vegetables in which perishability represents a critical factor. The problem is formulated as a VRPTW with time-dependent travel times. The objective function minimizes the distance and time travelled, the delay costs for servicing late a customer and the costs related to perishability. In this model, the perishability costs are calculated by multiplying the load transported in each arc by the time needed to do

Table 1: Comparison of the VRP related works dealing with perishability

Author	**VRP-Type**	**Number of Obj.**	**Perishability Behaviour**	**Products**	**Solution Method**	**Instances**
Chen et al. (2009)	STW	1	Random	Multiple	Heuristic	Solomon (1983)
Osvald & Stirn (2008)	HTW; TD	1	Deterministic	Single	Heuristic	Solomon (1983)
Hsu et al. (2007)	STW; TD	1	Random	Single	Heuristic	Tailor-Made
This paper	HTW	2	Deterministic	Multiple	MOEA	Solomon (1983)

STW: soft TW, HTW: hard TW, TD: time dependent travel time

it. Hsu et al. (2007) consider the randomness of the perishable food delivery process and present a stochastic VRPTW model that is further extended to consider time-dependent travel times. The objective function of this work is very similar to that of Osvald & Stirn (2008), but the calculation of costs due to perishability is done in a stochastic manner. The authors attribute probability density functions to determine the chances of having spoiled products due to the opening of the vehicle door and to the travel time. The problem is solved by a heuristic procedure. Chen et al. (2009) integrate production scheduling with the VRPTW for perishable food products. In the distribution part, they consider a value decay on the products distributed that will influence the price paid by the retailer to the transporter. This model has a stochastic nature by defining the demand through a probability density function. Afterwards, the integrated model is solved in an iterative scheme in which the VRP part is solved by a constructive heuristic followed by an improvement one.

In Table 1 we compare our work against the closest papers in the literature, in terms of modelling characteristics (type of VRP considered, number of objectives, perishability behaviour and number of products), solution methods and instances tested.

From the literature review, it is clear that incorporating the perishability factor explicitly in the formulations seems to be of great advantage (Akkerman et al., 2010) since the customers' point of view is also taken into account. In our work, a multi-objective framework is used to tackle this phenomenon and, hence, to give to the decision maker a whole set of equally efficient solutions, evidencing the trade-off between supply chain optimization and customer service related to the freshness aspect. Furthermore, in the experiment design, the goal is to provide new insights into the relation between distribution scenarios and the aforementioned trade-off.

3. Mathematical Formulation

This section aims to present a formal definition of the Multi-Objective Vehicle Routing Problem with Time Windows dealing with Perishability (MO-VRPTW-P). The formulation and notation is based on the VRPTW formulation proposed in Cordeau et al. (2001).

The MO-VRPTW-P trades-off the optimal design of routes and the freshness state of the delivered products. A set K of identical fixed capacity vehicles indexed by $k = 1, ..., m$ initially located at a depot are available to deliver perishable food goods to a set N of customers $i, j = 1, ..., n$ through a set of arcs A. The number of vehicles m is enough to always guarantee a feasible solution. The VRPTW structure can be defined on a direct graph $G = (V, A)$ with $V = N \cup \{0, n + 1\}$, where the depot is simultaneously represented by the two vertices 0 and $n + 1$ and, therefore, $|V| = n + 2$. Each possible arc (i, j) has an associated time and cost that is related to the euclidean distance of the vertices that it connects to. Each customer has a demand that needs to be satisfied for a certain number of products. Without loss of generality, these products are of identical size, and they have different deterministic shelf-lives. It is assumed that as soon as the vehicle departs the depot, all products that it is carrying are at their maximum freshness. Moreover, customers want their requests available within a strict time window and they need a certain time to be served. From a modelling point of view we will just account for the shelf-life of the products within a customer order that deteriorate the most. Thus, by maximizing the freshness of the products delivered, it is ensured that the worst case is tackled for each customer and, therefore, all the other products still have some remaining shelf-life when delivered.

A feasible solution for this problem implies a collection of routes that correspond to paths starting at vertex 0 and ending at vertex $n + 1$. These routes have to ensure that each customer is visited exactly once satisfying simultaneously its demand and time window. Furthermore, it is not admissible that any of the products delivered to a customer is spoiled and the cumulative demand that each vehicle serves can not exceed its capacity.

Consider the following indices, parameters, and decision variables.

Indices

k	vehicles
i, j	vertices
(i, j)	arcs

Parameters

C	vehicle capacity
s_i	service time of customer i
$c_{ij}(t_{ij})$	travel cost (time) from customer i to customer j
a_i	starting time of time window of customer i
b_i	finishing time of time window of customer i
d_i	demand of customer i
sl_i	shelf-life of the most perishable product contained in the request of customer i

Decision Variables

x_{ij}^k	equals 1 if arc (i, j) is crossed by vehicle k, 0 otherwise
w_i^k	time at which vehicle k starts servicing vertex i, 0 if vertex i is not visited by vehicle k
fr_i	freshness level for customer i request upon delivery

In the preprocessing of the arcs, besides eliminating arcs (i, j) due to temporal reasons (in case $a_i + s_i + t_{ij} > b_j$), and to capacity reasons (in case $d_i + d_j > C$), arcs are also eliminated due to perishability reasons (in case $s_i + t_{ij} > sl_j$).

Let $\delta^+(i) = \{j : (i, j) \in A\}$ and $\delta^-(j) = \{i : (i, j) \in A\}$ denote the set of successors and predecessors of i and j, respectively. The mathematical formulation for the MO-VRPTW-P can be stated as follows:

MO-VRPTW-P

$$\min f^1 = \sum_{k \in K} \sum_{(i,j) \in A} c_{ij} x_{ij}^k \tag{1}$$

$$\max f^2 = \frac{1}{N} \sum_{i \in N} fr_i \tag{2}$$

subject to:

$$\sum_{k\in K}\sum_{j\in\delta^+(i)} x_{ij}^k = 1 \quad \forall i \in N \tag{3}$$

$$\sum_{j\in\delta^+(0)} x_{0j}^k = 1 \quad \forall k \in K \tag{4}$$

$$\sum_{i\in\delta^-(j)} x_{ij}^k - \sum_{i\in\delta^+(j)} x_{ji}^k = 0 \quad \forall k \in K, j \in N \tag{5}$$

$$\sum_{i\in\delta^-(n+1)} x_{i,n+1}^k = 1 \quad \forall k \in K \tag{6}$$

$$x_{ij}^k(w_i^k + s_i + t_{ij} - w_j^k) \le 0 \quad \forall k \in K, (i,j) \in A \tag{7}$$

$$a_i \sum_{j\in\delta^+(i)} x_{ij}^k \le w_i^k \le b_i \sum_{j\in\delta^+(i)} x_{ij}^k \quad \forall k \in K, i \in V \tag{8}$$

$$\sum_{i\in N}\sum_{j\in\delta^+(i)} d_i x_{ij}^k \le C \quad \forall k \in K \tag{9}$$

$$fr_i \le \frac{w_0^k + sl_i - w_i^k + G(1 - \sum_{j\in\delta^+(i)} x_{ij}^k)}{sl_i} \quad \forall k \in K, i \in N \tag{10}$$

$$\sum_{j\in\delta^+(i)} x_{ij}^k(w_0^k + sl_i - w_i^k) \ge 0 \quad \forall k \in K, i \in N \tag{11}$$

$$x_{ij}^k \in \{0,1\}; \quad fr_i, w_i^k \ge 0. \tag{12}$$

The objective function (1) minimizes the total routing cost, which is the usual objective in the VRP related problems. The objective function (2) maximizes the average freshness of all requests by standardizing the remaining shelf-life of the most deteriorating product of each customer (see expression (10)). The value of this objective varies between 0% and 100%, where 100% corresponds to the maximum possible freshness. We assume that when a vehicle leaves the depot the freshness of the products inside is at its maximum. Naturally, we are assuming that all the upstream supply chain planning is based on the distribution plans and follows the Just-in-Time (JIT) philosophy. Thus, in case customer i is serviced by vehicle k, the freshness of the

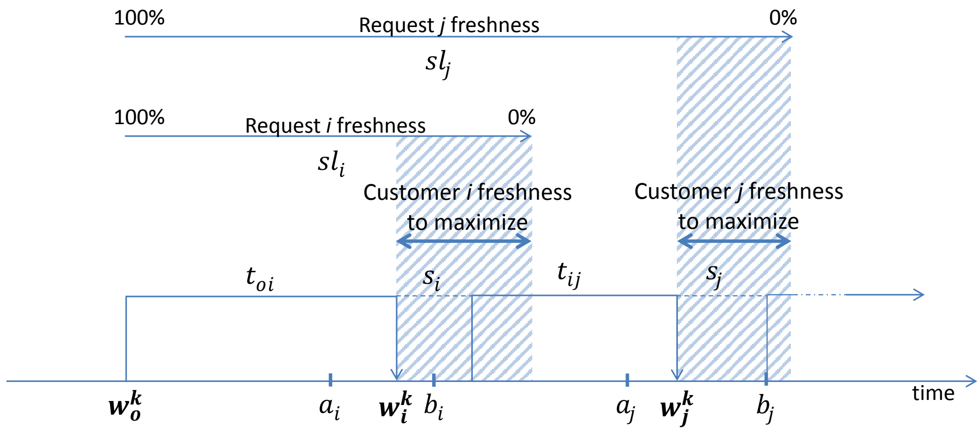

Figure 1: Representative scheme of the freshness objective function.

product delivered to the customer is obtained by subtracting to the maximum shelf-life, sl_i, the time it takes to start servicing customer i, given by the term $w_i^k - w_0^k$. Dividing this value by sl_i the freshness is converted to a value that is between 0% and 100%. In the second case, when the customer i is not serviced by vehicle k, constraints (10) are loose (in these constraints $G = b_0 - a_0$). In Figure 1, a visual representation of this objective function is given.

Constraints (3) state that each customer is visited exactly once, while constraints (4)–(6) ensure that each vehicle is used and that flow conservation is satisfied at each customer vertex. The consistency of the time variables w_i^k is ensured through constraints (7), while the time windows are imposed by (8). Again, these last constraints also enforce w_i^k to be 0 in case customer i is not visited by vehicle k. Regarding the vehicle capacity, constraint (9) enforces it to be respected. Finally, constraints (11) ensure that customer requests are satisfied with products that are not spoiled (i.e. that still have some degree of freshness).

Remark 1 Formulation (1)–(12) is nonlinear because of constraints (7) and (11). The first constraint can be linearised as follows:

$$w_j^k \geq w_i^k + s_i + t_{ij} - M_{ij}(1 - x_{ij}^k) \quad \forall k \in K, (i, j) \in A, \tag{13}$$

where $M_{ij} = \max\{0, b_i + s_i + t_{ij}\}$ is a constant. We have changed the calculation of this parameter from the original formulation of Cordeau et al. (2001). In that formulation M_{ij} is equal to $\max\{0, b_i + s_i + t_{ij} - a_j\}$. However, for the case where a vehicle does not pass neither at i nor at j then this parameter would force $a_j \leq b_i$ since both w_i^k and w_j^k would be 0 due to constraints (8). Clearly, this equation does not hold for every instance and so the overall model becomes ill defined.

The linearisation of constraints (11) is as follows:

$$w_0^k + sl_i - w_i^k \geq 0 \quad \forall k \in K, i \in N. \tag{14}$$

4. Relations Between Distribution Scenarios and the Cost-Freshness Trade-Off

From the modelling of the MO-VRPTW-P we predict that to obtain an increase in the freshness of the products delivered, a higher distribution cost needs to be incurred. Moreover, the behaviour of the relation between freshness and distribution cost probably depends on various factors, such as customers typologies, time windows width that each customer require to be served and the perishability intensity of the food products delivered.

In order to organize and to guide the computational experiments needed to understand the relation between the cost-freshness trade-off and distribution scenarios, a set of intuitive hypotheses were raised covering the aforementioned factors influencing the problem under study. Figure 2 guides the following hypotheses.

The first two more general hypotheses are as follows:

H 1. *For a given scenario, the distribution cost incurred to satisfy an increasing freshness level increases in a non-decreasing fashion.*

H 2. *For a given scenario, the number of vehicles used to satisfy an increasing freshness level increases in a non-decreasing fashion.*

The third hypothesis is related to the general influence of the perishability intensity:

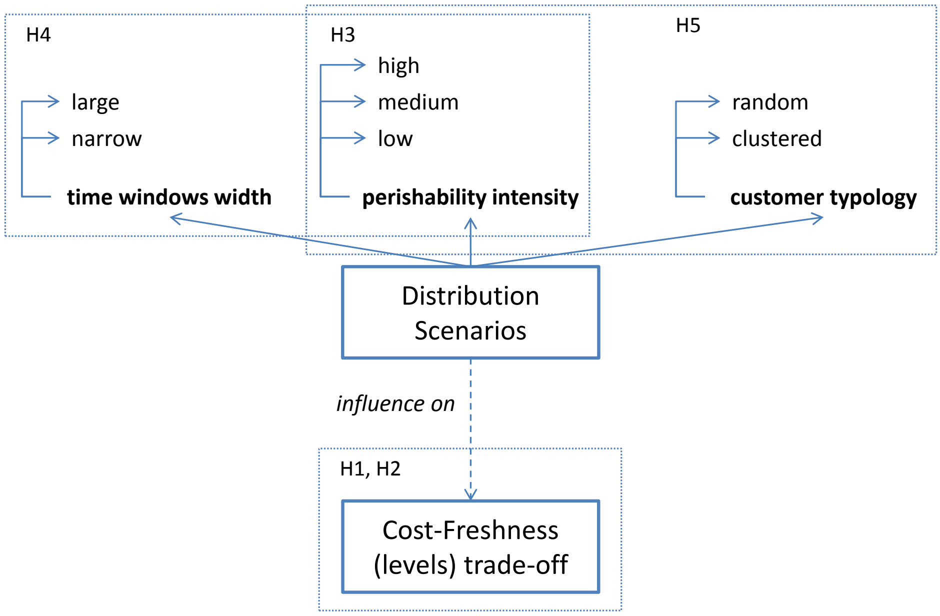

Figure 2: Hypotheses framework for studying the influence of perishability scenarios in the cost-freshness trade-off.

H 3. *For each freshness level, the distribution cost incurred in a scenario where products are subject to low perishability is less or equal to a scenario where products are subject to high perishability (everything else held constant).*

The next hypothesis focuses on the influence of time windows regarding the relation between distribution costs and freshness:

H 4. *On average, when increasing the perishability intensity from one scenario to another (everything else held constant), the distribution cost to satisfy the same freshness level will be higher in a setting with customers having narrow time windows compared with customers having large time windows.*

Finally, the last hypothesis reflects our intuition about the influence of customers spatial organization on the studied interaction.

H 5. *On average, when increasing the perishability intensity from one scenario to another (everything else held constant), the distribution cost to satisfy the same freshness level will be higher in a setting with customers randomly dispersed compared with customers organized in clusters.*

5. Methodology

Before looking into the details of the algorithms used to solve this problem, we emphasize that the main goal of this paper is to understand the relationship between distribution cost and freshness for different scenarios in this operational supply chain planning problem. Nevertheless, to conduct the study it is mandatory to solve with efficacy and a reasonable efficiency the underlying hard problem.

In order to test the hypotheses we need to be able to ensure that each solution found is very close to the Pareto optimal front. To achieve that, an exact method is employed to solve this problem to optimality for small size instances. Afterwards, the results are generalized for large instances of a size closer to real-world problems, by employing an approximate method.

5.1. Small Size Instances

In order to solve the small size instances, an ϵ-constraint method is used by adapting the algorithm proposed by Berube et al. (2009) to solve bi-objective combinatorial problems with integer solutions. The properties of this exact method will be important in the computational experiments to assess the relative cost increase for each freshness level when increasing perishability intensity.

To understand how this method works let us first define $P_1(\epsilon_2)$ as:

$$\min f^1$$

subject to:

$$\begin{gathered}(3) \text{ - } (6)\\(8) \text{ - } (10)\\(13) \text{ - } (14)\end{gathered}$$

$$f^2 \geq \epsilon_2 \tag{15}$$

$$x^k_{i,j} \in \{0, 1\}; \quad fr_i, w^k_i \geq 0, \tag{16}$$

where f^1 and f^2 denote the expressions of the objective functions (1) and (2), respectively.

Second, we shall define z^2_u and z^2_l as the theoretical highest and lowest possible values for the freshness objective (f^2). z^2_u is attained in case a vehicle is allocated to each customer and there is no waiting time at the customer site:

$$z^2_u = \frac{1}{N} \sum_{i \in N} \frac{sl_i - t_{0i}}{sl_i}. \tag{17}$$

As for z^2_l, we assume that the vehicle departs as soon as possible and serves each client at the latest possible point in time:

$$z^2_l = \frac{1}{N} \sum_{i \in N} \frac{\max(0; a_0 + sl_i - b_i)}{sl_i}. \tag{18}$$

Algorithm 1 shows the procedure to get the Pareto front F for the MO-VRPTW-P using the ϵ-constraint method.

Algorithm 1 Pseudo-code to find the Pareto front with the ϵ- constraint method

---

$\epsilon_2 = z^2_u$
while $\epsilon_2 \geq z^2_l$ **do**
  Solve $P_1(\epsilon_2)$ with exact solver
  $F \leftarrow Solution of P_1(\epsilon_2)$
  $\epsilon_2 = \epsilon_2 - 0.05$
end while
return F

5.2. Large Size Instances

To solve the MO-VRPTW-P for larger instances we have adapted an approach based on that presented in Amorim et al. (a), which proved to be successful in solving a lot-sizing and scheduling problem also concerned with perishable products. This approach hybridizes a MOEA with a mixed

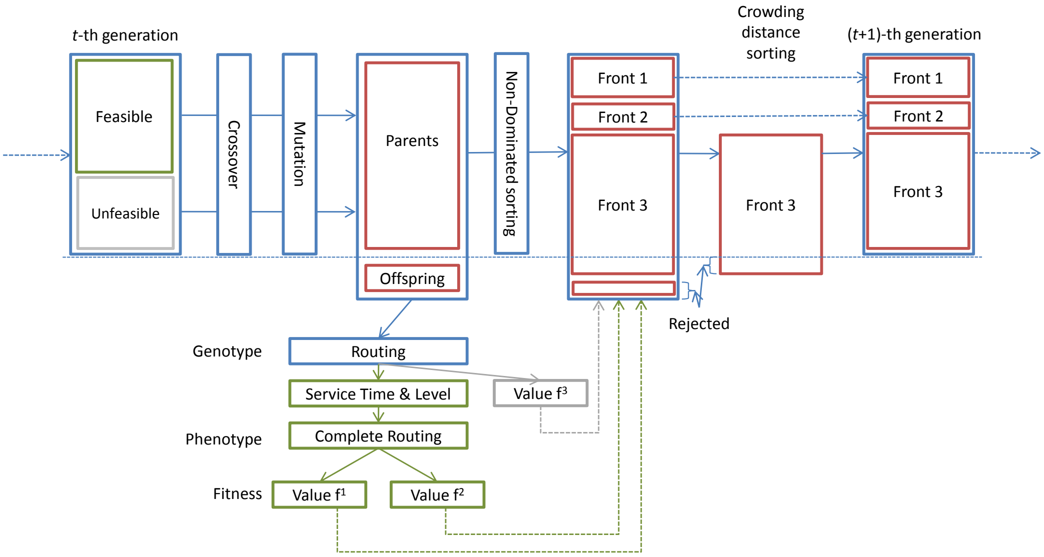

Figure 3: Outline of the hybrid multi-objective GA to solve the MO-VRPTW-P.

integer linear solver. The underlying MOEA is the Non-dominated Sorting Genetic Algorithm (NSGA-II), which has been successfully used in a panoply of different problems and with competitive results compared to other MOEAs (Khare et al., 2003). The readers are referred to Deb et al. (2000) for the specific details of NSGA-II. In Figure 3, the outline of the used algorithm is depicted.

In order to completely understand the solution method, the main building blocks of the GA are detailed (Goldberg, 1989): the coding scheme used; the crossover, mutation, and selection operators; the fitness evaluation functions; the parameters: the population size, the number of generations, the probability of applying operators, among others; and a method generating the initial population. Since infeasible chromosomes are allowed in our approach, it is also important to have a section devoted to infeasibility handling. The parameters are detailed in the Computational Experiments section.

The operations performed in the main building blocks are based on the work of Ombuki et al. (2006). In this work the authors were able to develop a MOEA to solve the VRPTW that proved to be very competitive in terms of solution quality when comparing to the results of other works. Hence, for the sake of completeness, we will briefly describe our adaptation of the algorithm. However, for a complete explanation the readers are referred to

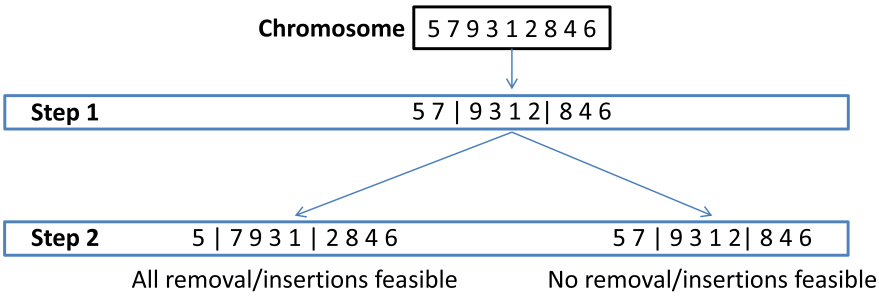

Figure 4: Representation of a Chromosome and Routing Scheme.

the cited references (Amorim et al., a; Ombuki et al., 2006).

a) Representation of a Chromosome and Routing Scheme The encoding of the chromosomes is of utmost importance for the success of any GA. In this algorithm each chromosome has a length equivalent to the number of customers in a given instance. Each gene contains a customer and the gene sequencing represents the order in which they are visited. So a chromosome is a collection of routes containing all clients. The only decision left is the assignment of a sequence of genes to a given route. For that end, a two-step routing scheme translates the input chromosome into a cluster of routes.

The first step is responsible to assign each customer to a route that appears in the chromosome by the order that they are, ensuring feasibility regarding the vehicle capacity and the customers' time windows. In the second step, a removal and insertion operator sweeps the solution found in step 1 and tries to reallocate the last customer visited in each route as the first customer visited in the following one. If this movement proves to be better in terms of cost then it is performed, otherwise the chromosome is kept as in step 1. We note that perishability concerns are not taken into account yet. In Figure 4 these steps and encoding scheme are illustrated for an instance with 9 customers.

b) Initializing a Population The initial population is created in two different ways. 90% of the chromosomes are generated based on random permutations of the total customer nodes. The remaining 10% are generated with the nearest customer insertion scheme.

c) Genetic Operators The first two genetic operators are derived from

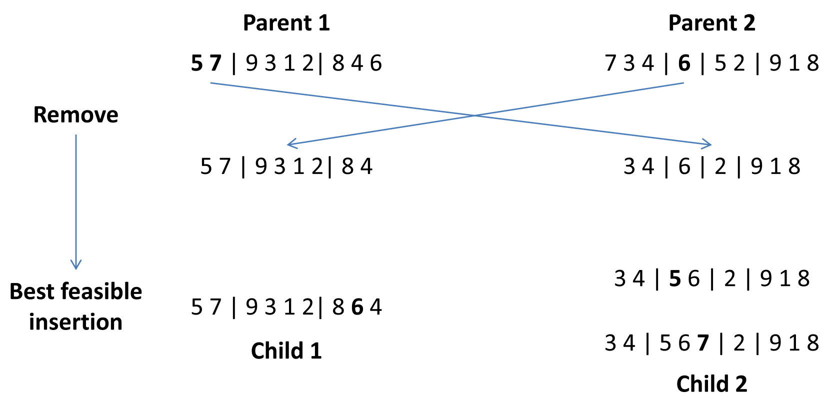

Figure 5: Best Cost Route Crossover example.

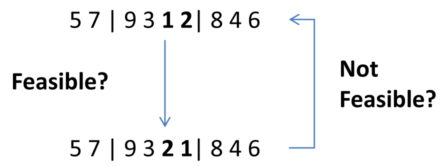

Figure 6: Constraint Route Inversion Mutation example.

the work of Ombuki et al. (2006) and the last one comes from the NSGA-II framework.

• *Crossover* The Best Cost Route Crossover is implemented. This operator consists of two distinct steps. Firstly, a route from each parent is randomly chosen and the respective customers are removed from the opposite parent. Secondly, the best possible insertion of the removed customers is performed while ensuring feasibility. The order of the missing customers to insert is chosen randomly. If there is no feasible insertion, then a new route is created. Figure 5 shows an example of this operator for an instance with 9 customers.

• *Mutation* The Constraint Route Inversion Mutation operator is designed to help the search escaping from local optima with the minimum possible number of infeasibilities created in the procedure. Figure 6 shows how this operator works. After choosing randomly a route from a chromosome, two consecutive customers are selected and inverted. If this inversion results in a feasible chromosome, then it is maintained, otherwise it is rejected.

• *Selection/Reproduction* After the offspring population is created, the whole population is sorted according to nondominance ranking. The new parent population is formed by adding solutions from the better fronts until the number of chromosomes exceeds the size of the population. The solutions of the last accepted front are sorted according to the crowding operator, in order to have a well-dispersed set of solutions, which are added until the population size is reached (see Figure 3). The crowding operator ranks the chromosomes of the last front based on their relative solution distance through a measure estimating a solution perimeter around its neighbours.

d) Fitness of a Chromosome and Infeasible Chromosomes The fitness of each chromosome in our solution method is dependent on two distinct states: feasible or infeasible. Infeasibility may only occur regarding the spoilage state of the products delivered, since our representation always entails a chromosome satisfying the vehicles capacity and the customers' time windows. Hence, if the chromosome is feasible we should be able to evaluate its fitness by means of the two objective functions: cost and freshness. The cost function can be calculated directly from the representation just by using the routing scheme already explained to cluster the genes in routes. As for the freshness objective, we propose and study two alternatives that influence the way unfeasibility is handled. However, one thing is common to both approaches: infeasible chromosomes are neither automatically disqualified from the population nor repaired to create feasible solutions in order to promote the search into the infeasibility domain (Oliveira Santos et al., 2010). Instead, the amount of infeasibility is defined as an additional objective (f^3) that must be minimized. Simultaneously, the values of the first two objectives are set to values that are worse than those of any other feasible chromosome. The two alternatives are presented bellow:

• *LP solver* In this first approach we use the information coded in the chromosomes to solve a LP model that takes into account the time windows and capacity feasibility of the input routes. Let the parameters $\widehat{X_{ij}^k}$ equals 1 in case arc (i, j) is crossed by vehicle k, and zero otherwise. These parameters are fixed for each chromosome. Moreover, we take advantage of the chromosome information to set the correct bounds for the w_i^k variables. This means that if customer i is not visited by vehicle k, then w_i^k is set to zero. Otherwise, its bounds are limited according to the time window of the corresponding customer (i.e. $a_i \leq w_i^k \leq b_i$). The following LP is solved for each chromosome in order to find the maximum freshness for a given set of fixed routes.

Sub MO-VRPTW-P ($\widehat{X}$)

$$\max \frac{1}{N} \sum_{i \in N} fr_i \tag{19}$$

subject to:

$$w_i^k + s_i + t_{ij} - w_j^k \leq M_{ij}(1 - \widehat{X_{ij}^k}) \quad \forall k \in K, (i,j) \in A \tag{20}$$

$$fr_i \leq \frac{w_0^k + sl_i - w_i^k + G(1 - \sum_{j \in \delta^+(i)} \widehat{X_{ij}^k})}{sl_i} \quad \forall k \in K, i \in N \tag{21}$$

(14)

$$fr_i, w_i^k \geq 0. \tag{22}$$

This approach seems to be worthwhile in case the model is to be applied into a real-world scenario where more unpredictable constraints may appear, for example, having compartments for different types of products in each vehicle. Thus, by coding the related auxiliary constraints in the LP model a whole set of related problems can be solved.

When the chromosome is infeasible, constraints (14) are relaxed by introducing slack variables v_i and the respective new model is solved attempting to minimize $f^3 = \sum_i v_i$ (c.f. Figure 3). This violation v_i represents the amount of time that spoiled products passed their shelf-lives in the current solution. By preferring chromosomes with better freshness conditions (even if with some spoiled products) we expect that in the following generations the operators guide the search into the freshness feasible domain.

• *Alternative objective function* In order to avoid to solve recursively an LP model each time a chromosome is evaluated, this section presents a less flexible, but potentially faster procedure to evaluate the freshness-related objective.

Recall that a chromosome already fixes routes that comply with the customer time windows, capacity and demand. Nevertheless, even for a chromosome that is able to deliver every product without spoilage, it is rather hard to compute directly the freshness objective because there are several degenerated solutions for the same value of freshness in a given set of routes. In order to overcome this situation, we take advantage of the fact that for a fixed set of routes, the controllable loss in product freshness comes from the waiting time to serve a customer.

Let us define model MINWAIT($\widehat{X}$) that tries to minimize the waiting time of the vehicle, setting simultaneously the departure of the vehicle from the depot as late as possible.

MINWAIT($\widehat{X}$)

$$\min \sum_{k\in K}\sum_{i\in N}\sum_{j\in\delta^-(i)} \max\{0; (a_i - (w_j^k + s_j + t_{ji})\}\widehat{X_{ji}^k} - \sum_{k\in K} w_0^k \qquad (23)$$

subject to:

$$(20) \text{ - } (21)$$

$$(14)$$

$$w_i^k \geq 0. \qquad (24)$$

Proposition 1. *For a given input set of routes (given by $\widehat{X}$), maximizing freshness is equivalent to simultaneously minimizing waiting time and maximizing the departure time at which each vehicle leaves the depot.*

We show the proposition by proving that models Sub MO-VRPTW-P($\widehat{X}$) and MINWAIT($\widehat{X}$) lead to the same optimal solution. Since the solution domains of both models are equivalent, one just needs to be able to transform the objective function of Sub MO-VRPTW-P($\widehat{X}$) into that of MINWAIT($\widehat{X}$). First, we make use of parameter $\widehat{X_{ij}^k}$ to rewrite the objective function (19) as follows:

$$\max \frac{1}{N}\sum_{k\in K}\sum_{i\in N}\frac{(w_0^k + sl_i - w_i^k)\sum_{j\in\delta^+(i)}\widehat{X_{ij}^k}}{sl_i} \qquad (25)$$

By removing some scalars and constants, and taking into account the balance of the flows, (25) can be further reduced into:

$$\min \sum_{k \in K} \sum_{i \in N} ((-w_0^k + w_i^k) \sum_{j \in \delta^-(i)} \widehat{X_{ji}^k}). \tag{26}$$

Rewriting w_i^k as a function of its predecessor, allows us to replace (26) by the expression:

$$\min \sum_{k \in K} \sum_{i \in N} \sum_{j \in \delta^-(i)} ((-w_0^k + \max\{a_i; w_j^k + s_j + t_{ji}\}) \widehat{X_{ji}^k}). \tag{27}$$

It is easy to see that (27) is equivalent to (23).□

MINWAIT($\widehat{X}$) can be solved by a simple heuristic (see Algorithm 2) that finds the appropriate w_i^k by minimizing waiting time and, thus, maximizing freshness. In the description of the algorithm $p' = g(p)$ denotes a function that returns customer i that is in position p of route k. Basically, this algorithm defines, sequentially, for each route the w_i^k values that imply a service to the customers as late as possible in order to avoid waiting times. In case a entering customer needs to be serviced earlier than the solution is pointing to, a feedback mechanism synchronizes the customers upstream to be sure that the customer inserted is serviced on time. Algorithm 2 returns the freshness value in case a chromosome is feasible, and the amount of infeasibility incurred, otherwise.

6. Computational Experiments

In order to perform the computational study about the influence of the distribution scenarios in the MO-VRPTW-P, we have adapted the instances developed in Solomon (1983) for the VRPTW. Four different instances types are considered: C1, C2, R1 and R2. In these instances, the travel cost and time of each arc is equal to its length. Type C problem type refers to clustered customers whose time windows were generated based on a known solution, whereas type R to customers located randomly generated over a square. Then, sets of type 1 contain narrow time windows and small vehicle capacity. Sets of type 2 have large time windows and large vehicle capacity. Therefore, the solutions of type 2 problems usually yield very few routes and significantly more customers per route.

Algorithm 2 Pseudo-code to evaluate freshness in *Alternative objective function*

for $k = 1 \to NumberOfRoutes$ **do**
  $w^k_{1'} = b_{1'}$
  $w^k_0 = w^k_{1'} - t_{01'}$
  for $p = 2 \to NumberOfCustomersOfRoute(k)$ **do**
    $w^k_{p'} = w^k_{(p-1)'} + s_{(p-1)'} + t_{(p-1)',p'}$
    if $w^k_{p'} \leq a_{p'}$ **then**
      $w^k_{p'} \leftarrow a_{p'}$
    else
      if $w^k_{p'} \geq b_{p'}$ **then**
        $w^k_{p'} \leftarrow b_{p'}$
        for $u = p - 1 \to 0$ **do**
          $w^k_{u'} \leftarrow w^k_{(u+1)'} - s_{u'} - t_{u',(u+1)'}$
        end for
      end if
    end if
  end for
  for $p = 1 \to NumberOfCustomersOfRoute(k)$ **do**
    $CustFresh_p \leftarrow (w^k_0 + sl_{p'} - w^k_{p'})/sl_{p'}$
    if $CustFresh_p < 0$ **then**
      $f^3 \leftarrow f^3 + CustFresh_p$
      $FlagViolation = TRUE$
    else
      $Freshness \leftarrow Freshness + CustFresh_p$
    end if
  end for
end for
if $FlagViolation = FALSE$ **then**
  return $Freshness/NumberOfCustomers$
else
  return f^3
end if

The extension of these sets is performed in order to grasp the changes related to different perishability intensity scenarios, attributing to each customer order a shelf-life. Three different perishability intensity scenarios are studied corresponding to Low, Medium and High perishability. The reference point used to set the different scenarios is the end of the time window at the depot that corresponds to the end of the planning horizon. Hence, the Low scenario is set in such a way that the product with the lowest shelf-life is able to last throughout the planning horizon; the Medium means that the shelf-life of the products lasts about 75% of the planning horizon and the High corresponds to 50%.

First we report results for the small size instances (25 customers) and, afterwards, for the large size instances (100 customers).

6.1. Small Size Instances

We have run Algorithm 1 on the 40 instances from Solomon for the two extreme perishability scenarios (Low and High), resulting in 80 instances of 25 customers. The ϵ-constraint method was applied to each instance with a maximum number of 20 iterations, chancing iteratively the freshness level with a 5% step (0%, 5%,...,95%,100%). Each MIP had a 10 minute time limit and was solved on a Cluster IBM eServer 1350 equipped with CPLEX 12.1. Out of the 80 instances we were able to obtain 34 instances with an optimality gap bellow 10%. These are the instances used to confirm the hypothesis and on which the statistical tests were performed.

Concerning the first hypothesis, **H 1**, which relates the conflicting nature of the objectives, Figure 7 shows the Pareto front for one of the instances that is representative of the behaviour observed for the other instances. Clearly, in order to achieve a higher freshness level, higher distribution costs are incurred, confirming the hypothesis. In **H 2** we intended to assess the impact of the number of vehicles on achieving better freshness conditions of the products delivered. Hence, for every step increase of the freshness level, the number of vehicles used was calculated. Again, Figure 7 is representative of the behaviour of all instances and it is noticeable the increasing usage of vehicles to satisfy a higher freshness level. In fact, to increase from 60% to 90% the freshness level, it is necessary to multiply by four the number of vehicles.

Derived from the third hypothesis, the Pareto front from the Low perishability scenario should weakly dominate the one from the High perishability

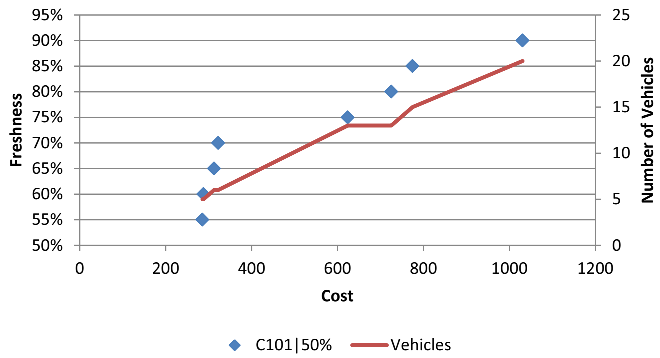

Figure 7: Pareto front for instance C101 with 50% of perishability and vehicles used.

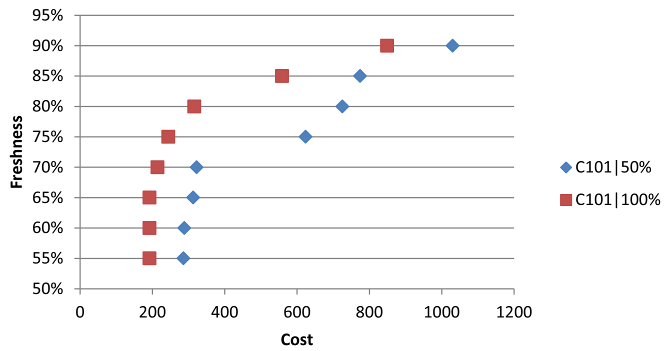

Figure 8: Pareto fronts for instances C101 with 50% and 100% of perishability intensity.

scenario. In testing **H 3** it was observed that within the 283 pairs of solutions analysed (a pair is found in the same instance comparing the cost for the same freshness level for perishability intensities of 100% and 50%), the cost for achieving the same freshness level is systematically higher when perishability intensity was about 50% compared with 100%. Figure 8 shows an example of the dominance of the Low perishability scenario over the High one for a representative instance. In Table 2 the average relative cost increase for all instances considered when shifting from Low to High perishability is presented. This measure is crucial to test the last two hypotheses.

To test **H 4** - **H 5** we statistically assess the respective average differences calculated a priori. Table 5 shows these results. In order to understand the validity of **H 4** the difference between instances with narrow time windows

Table 2: Percentage average cost increase for shifting from Low to the High perishability intensity.

Instance	% Average Cost Increase
C101	58%
C102	74%
C105	59%
C106	61%
C107	62%
C201	17%
C202	43%
C205	11%
C206	18%
C207	12%
R101	16%
R102	32%
R105	25%
R106	99%
R111	123%
R201	12%
R202	46%

Table 3: Results for **H 4 - H 5**

	Test	Average Difference	p-value
H 4	C1-C2	43%	0.00
	R1-R2	34%	0.00
H 5	R1-C1	0%	0.49
	R2-C2	-9%	0.11

Table 4: Parameters of the MOEA

Population size	300
Number of generations	350
Crossover Rate	0.9
Mutation rate	0.1

(type 1) and large time windows (type 2) was calculated for the two typologies (C and R). The results indicate a higher cost increase for type 1 customers than for type 2, when the perishability intensity augments. This difference is statistically significant for both types of topologies. However, regarding **H 5** no conclusions can be drawn. For both types of time windows width no significant different is found across the customer typologies.

6.2. Large Size Instances

In this section the computational experiments are extended to instances with 100 customers. Furthermore, in this section considers the three perishability scenarios (Low, Medium and High).

All the instances are solved by the MOEA, which incorporates Algorithm 2 to compute the freshness of each chromosome. Based on preliminary tests and on the literature recommendations, we have defined the setting of the MOEA parameters shown in Table 4. The computation time required for a complete run of an instance is about 10 minutes on a PC with 1 GB of RAM and an Intel Dual Core with 1.66 GHz running on Windows XP. All the implementations were done in C++.

In Figures 9, 10, 11 and 12 the aggregate results (solutions for all instances) are illustrated for the C1, R1 and C2, R2 instances, respectively. Space restrictions prevent us from showing all disaggregated results. These graphs show unequivocally the trade-off between delivering fresher products

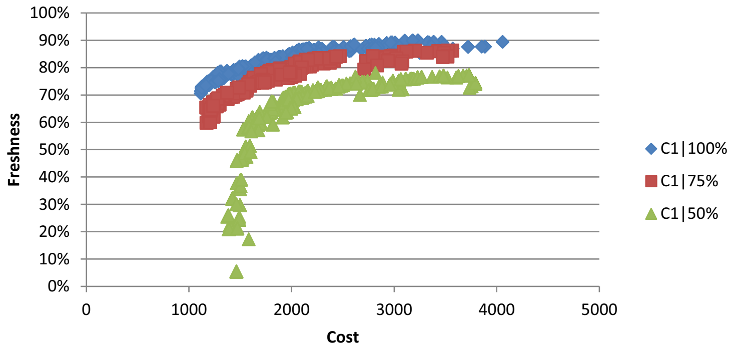

Figure 9: Pareto fronts of C1 instances for the 3 levels of perishability intensity.

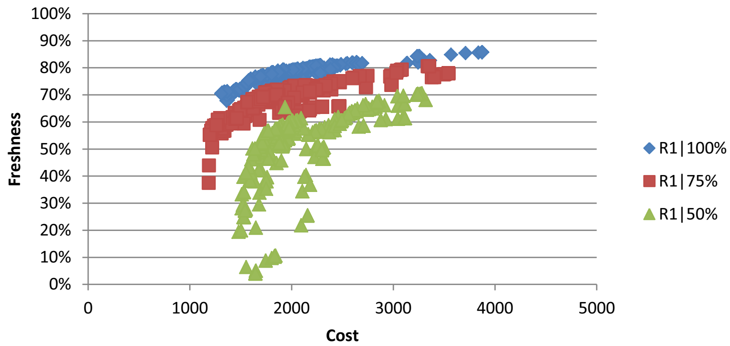

Figure 10: Pareto fronts of R1 instances for the 3 levels of perishability intensity.

and the routing cost involved in those solutions. Products subject to High perishability (50%) emphasize this trade-off and to deliver fresher products more costs have to be incurred (validating hypothesis **H 1**). Furthermore, as it is intuitive, products with larger shelf-lives relief the cost increase needed to achieve better freshness standards (**H 3**).

To test hypothesis **H 4** and **H 5** the same reasoning as for the small instances is applied after rounding freshness levels. The only difference is that for the large instances three perishability intensity scenarios are tested and, therefore, the statistical tests were performed for convenient pairs of intensities (Between High and Medium intensities and between Medium and Low intensities). Table 5 summarizes the results that support the conclusions found for the small instances. Once again no conclusions can be taken

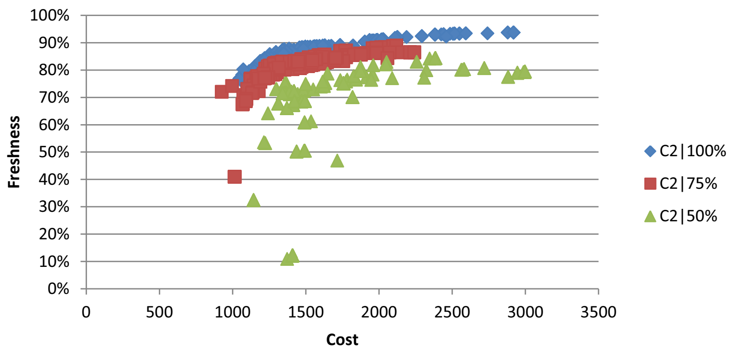

Figure 11: Pareto fronts of C2 instances for the 3 levels of perishability intensity.

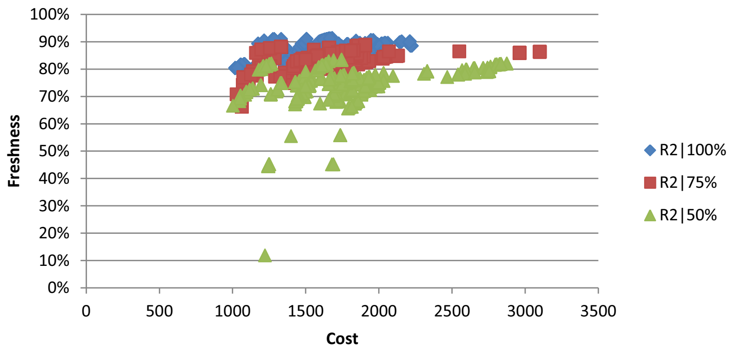

Figure 12: Pareto fronts of R2 instances for the 3 levels of perishability intensity.

Table 5: Parameters of the MOEA

	Test	Comparison Interval	Average Difference	p-value
H 4	C1-C2	100%—75%	16%	0.00
		75%—50%	3%	0.32
	R1-R2	100%—75%	27%	0.00
		75%—50%	40%	0.00
H 5	R1-C1	100%—75%	2%	0.38
		75%—50%	29%	0.01
	R2-C2	100%—75%	-20%	0.12
		75%—50%	-24%	0.02

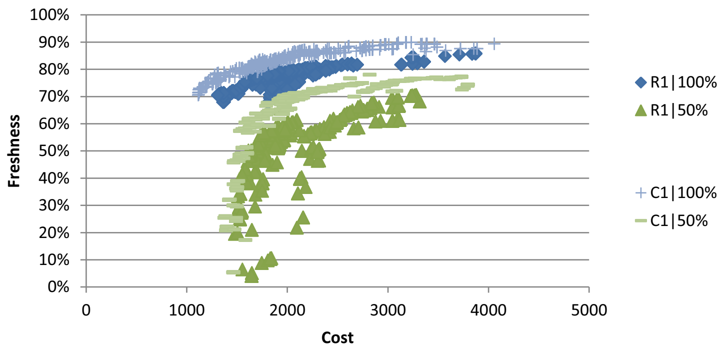

Figure 13: Pareto fronts of C1 and R1 instances having 100% and 50% levels of perishability.

regarding the difference between serving customers located randomly or in clusters. Concerning time windows width, the previous conclusions were validated despite one of the tests giving statically insignificant (C1-C2 for Medium and High perishability intensity).

Finally, Figures 13 and 14 show the aggregate results, comparing now customer typologies. These results point out that the freshness level attained when customers are organized in clusters is considerably higher than when geographically dispersed, despite the fact that the average cost increase when subject to a higher perishability intensity does not differ from random to clustered customers (**H 5**).

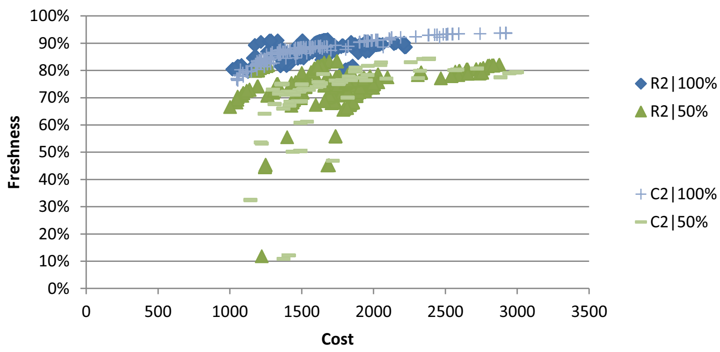

Figure 14: Pareto fronts of C2 and R2 instances having 100% and 50% levels of perishability.

7. Conclusions

In this paper, a new formulation for the vehicle routing problem dealing with perishability issues was proposed based on a multi-objective framework. The model is decoupled into two separate objectives: the minimization of operational tangible costs and the maximization of the freshness intangible value.

Based on this formulation, a set of experiments was designed to understand the relationship between different distribution scenarios and the cost-freshness trade-off. Four out of five hypotheses that were formulated proved to be true testing the 25 customer instances adapted from Solomon for the VRPTW. The main conclusions point out that, first, there is an evident trade-off between the mentioned objectives; second, time windows have a strong impact on the freshness levels of products delivered, hence, large time windows lead to less spoilage; finally, regarding customer typology no conclusions could be taken.

Two versions of a MOEA were proposed to solve instances with 100 customers. While the first alternative hybridizes the algorithm with a LP solver, the second alternative speeds up the MOEA performance by making use of a tailored algorithm to obtain the freshness value. Graphical solutions were presented and the conclusions drawn from the smaller instances were partially validated.

Future work shall be devoted to test the proposed model on real-world instances. Moreover, it could be interesting to understand the effect of dropping the JIT assumption and integrate this model with production scheduling decisions.

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A rich vehicle routing problem dealing with perishable food: a case study

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Abstract This paper presents a successful application of operations research techniques in guiding the decision making process to achieve a superior operational efficiency in core activities. We focus on a rich vehicle routing problem faced by a Portuguese food distribution company on a daily basis. This problem can be described as a *heterogeneous fleet site dependent vehicle routing problem with multiple time windows*. We use the adaptative large neighbourhood search framework, which has proven to be effective to solve a variety of different vehicle routing problems. Our plans are compared against those of the company and the impact that the proposed decision support tool may have in terms of cost savings is shown. The algorithm converges quickly giving the planner considerably more time to focus on value-added tasks, rather than manually correct the routing schedule. Moreover, contrarily to the necessary adaptation time of the planner, the tool is quite flexible in following market changes, such as the introduction of new customers or new products.

Keywords Vehicle Routing Problem · Adaptative Large Neighbourhood Search · OR in Industry · Decision Support Systems

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1 Introduction

In Portugal, the farm to fork associated food industries generate a total sales value of over €10.6 billion and a total services value of €150 million, corresponding to 7.2 percent of the national gross domestic product[1]. Within this value chain, half of the amount comes from the production and the other half from the distribution activities. Distribution companies face several typical problems, at different hierarchical decision levels. For example, on a tactical level, the set of logistics providers to select and with which kind of contract, or on which days the clients should be visited, which is stipulated upon a distribution calendar contract; on a more operational level, there is the daily problem of designing the routes to serve customers previously assigned to that day based on their demand orders. This last problem is the focus of this case-study.

This work studies a real-life problem faced by a Portuguese food distribution company that supplies a wide range of food solutions for a diverse range of clients. The north filial (the one we are working with) has in their portfolio around 1570 active clients spread in the north of Portugal (Figure 1). Their market ranges from primary schools, to prisons, banks and wholesalers. A food solution is defined as a service that provides a quick response to customer orders for a set of food products. These food products may be of various types and in many different quantities. They vary from pallets of beer to small baskets with rice, milk and potatoes. In fact, the number of products in portfolio is in the order of thousands.

The distribution business of this company is affected by high seasonality throughout the year. This situation could demand significant fixed and maintenance costs related to the fleet management. These costs would be hardly diluted in the operational efficiency, since the company would need to have a fleet dimensioned for the peak season that would result in a low return on assets. To overcome such scenario and increase the operational flexibility, the company decided to contract with a pool of third party logistics providers upper and lower levels on the number of vehicles of various kinds (in size and compartments) that have to be available to the company within a twelve hour window. In practice, this means that having customer orders fixed the day before, the company is able to create the routing plan and know precisely how many and which type of vehicles are needed from the providers.

Through a set of meetings with the company's employees and executives, five main practical motivations to carry out this work are found. First, planned routes are almost fixed from day to day and only small adjustments are performed as the planner sees post-processing opportunities. Second, whenever the senior planner is not at work, the company's plans suffer a considerable quality decrease. Third, this planner has other functions in the company that are seriously jeopardized by the tremendous amount of time (4 hours per day on average) that he spends improving the generated plans. Forth, whenever

[1] Source: National Institute of Statistics

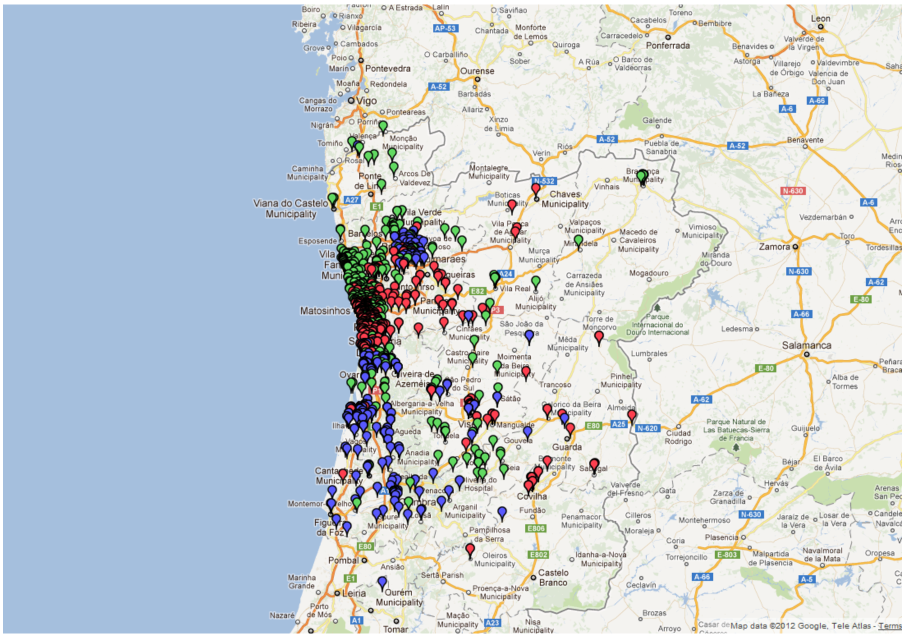

Fig. 1 Geographical location of customers in the north filial

there is a disruptive happening in the amount of clients to be dealt with, the planner needs some weeks to adapt to the new situation and, meanwhile, the plans are not of the same quality. Fifth, there is a need to cut distribution costs.

The operational routing problem, which the planner has to deal with, can be seen as an extension of the vehicle routing problem (VRP). The VRP is a problem in which a set of vehicles with the same capacity (homogeneous fleet) is initially located at a depot. These vehicles shall visit a set of customers exactly once and both their locations and demands are known. The objective is to minimize the total travel costs and/or vehicles used to visit all customers. The demand fulfilled in each route can not exceed the vehicle's capacity and all routes have to end at the depot. The VRP is a NP-hard problem (Savelsbergh, 1985) and to solve real-world instances approximate solution methods, such as metaheuristics have to be used. The routing problem faced by the company relaxes several of the assumptions imposed by the traditional VRP and includes some specific characteristics, making the overall problem more demanding.

The company needs to fulfil customers' demand on a daily basis for an assortment of products of different temperature requirements, categories and quantities. In perishable food distribution (Amorim et al, 2011), products have different temperature requirements to avoid spoilage during routing and to conserve the organoleptic proprieties of the products. Basically, the products can be split into three categories: dry, cold (fresh) and frozen. Therefore, vehicles equipped with compartments that can be set to different temperatures are employed. Consequently, the first generalization of the traditional VRP concerns the homogeneity of the fleet. In our problem, trucks can be divided according to their different temperature compartments in dry and refrigerated. On one hand, refrigerated trucks are able to carry dry, fresh and frozen foods (as no problem arises from transporting dry products at lower temperatures) and they are more expensive to operate. On the other hand, dry trucks can only transport food that has a stable behaviour at an ambient temperature. For the sake of competitiveness, food distribution companies, such as the one under study are able to choose upon different types of vehicles regarding their temperature capabilities. However, this is not the only factor inducing a heterogeneous fleet. Since customers are not only very heterogeneous in terms of business activities but also in terms of their accessibility conditions, there is a need for vehicles with different sizes. There are customers located on the side of the motorway, but others are in the very inner city center or in rural areas difficult to access. This means that large trucks may be forbidden to service a certain customer due to the impossibility of accessing the delivery site. Therefore, the company's fleet may be catalogued according to the respective temperature compartments and to the size of the trucks.

The different product categories yielding different temperature requirements enforce an extra relaxation of the traditional VRP regarding the imposition that each customer may only be visited exactly once, by one truck (no split-deliveries). In our case study, if a customer demands both dry and fresh/frozen products, then he may be serviced either by a sole refrigerated truck carrying all products or by a combination of two trucks where one carries the fresh/frozen products and the other the dry demand.

There is one more characteristic worth of mention that distinguishes our case study problem from the traditional VRP. Customers serviced during the day have multiple hard time windows to be serviced. Most of the times either customers demand to be serviced early in the morning (for example, hospitals) or they demand not to be serviced at lunch time (hence, a time window in the morning and another in the afternoon). There are also customers that demand to be serviced only at night. In practice the routing for such customers may be done independently from the day customers, since for servicing the night customers the trucks need to go back to the warehouse.

According to the proper literature nomenclature, this problem corresponds to a *heterogeneous fleet site dependent vehicle routing problem with multiple time windows* (HF-SD-VRP-MTW).

Our paper has a threefold contribution. First, a state-of-the-art metaheuristic (the adaptative large neighboorhod search) is adapted to solve a problem that, to the best of our knowledge, has never been solved in an integrated manner. Second, we show the impact of efficient operations research techniques in considerably lowering operational costs for business core activities. Third, we assess and understand the vehicle routing business practice of our case study and disclose other opportunities to integrate such operational research techniques in similar environments.

The remainder of our paper is as follows. In the next section the literature about related problems is reviewed. In Section 3, a formal mathematical description of the problem at hands is given and in Section 4 we describe the algorithm used. Thereafter, in Section 5 our results are compared with the ones obtained by the company's software. Finally, some conclusions are drawn in Section 6.

2 Literature Review

In this section we review the literature on food distribution and on VRPs that share common features with our HF-SD-VRP-MTW.

In Jansen et al (1998) the authors investigate the importance of multi-compartment distribution for catering companies through simulation. The profile of the customers' demand is very close to our case with a split in dry, fresh and frozen products. However, routing is not part of their research. The authors conclude that multi-compartment distribution gets more economical as the number of customers serviced decrease. Mullaseril et al (1997) deal with the problem of distributing food in a cattle ranch. The problem is formulated with a set of split delivery capacitated rural postman problems with time windows since different feeds have to be distributed in different trucks because no compartments exist to separate them. Tarantilis and Kiranoudis (2001), concentrating on the distribution of fresh milk, formulate the problem as a heterogeneous fixed fleet VRP. In Tarantilis and Kiranoudis (2002) a real-world distribution problem of fresh meat is solved as a multi-depot VRP. Faulin (2003) implements a hybrid method that uses a combination of heuristics and exact algorithms to find a solution of a VRP with constraints enforcing narrow time windows and strict delivery quantities. According to the authors, these delivery scenarios are usually the case in the agribusiness industry. Osvald and Stirn (2008) extend a heuristic proposed in a previous work to solve the problem of distributing fresh vegetables in which perishability represents a critical factor. The problem was formulated as a vehicle routing problem with time windows (VRP-TW) and time-dependent travel times. The objective function minimizes the distance and time travelled, the delay costs for servicing late a customer and the costs related with perishability. Hsu et al (2007) consider the randomness of the perishable food delivery process and present a stochastic VRP-TW model that is further extended to consider time-dependent travel times. The problem is then solved by a heuristic procedure. Chen et al (2009) integrate production scheduling with the VRP-TW for perishable food products. The price paid by the retailer to the transporter varies as the product more or less spoils. Afterwards, the integrated model is solved with an iterative scheme in which the production part is solved using the Nelder-Mead method and the distribution part is solved by a constructive heuristic followed by an improvement one. Ambrosino and Sciomachen (2006) describe a case-study that is rather similar to ours. The company that the authors study is a food company that has to deliver perishable food through the national highway.

Their fleet is homogeneous and able to carry dry, fresh and frozen products. They formulate the problem as an asymmetric capacitated vehicle routing problem with split deliveries and use a cluster first-route second heuristic to solve it.

Through the review on the food distribution literature it is noticeable that most of the features of our problem were tackled, although in a separated manner. However, the site dependent and the multiple time windows extensions are yet to be tackled.

Nag et al (1998) are the first to study the site dependent vehicle routing problem (SD-VRP). In this work several simple heuristics are developed. Chao et al (1999) continue the work on the SD-VRP by proposing a new heuristic that is both tested on previous and new instances. Cordeau and Laporte (2001) show that the SD-VRP can be converted into a periodic vehicle routing problem (P-VRP) and they provide results obtained using a tabu search heuristic for the P-VRP presented in a previous paper. The vehicle routing problem with multiple time windows (VRP-MTW) appears to be one of the VRP extensions with less work devoted to it. Doerner et al (2008) develop exact and approximate algorithms for the pickup of perishable goods (blood) motivated by a real case of the Austrian Red Cross. In this problem customers have multiple interdependent time windows. More recently, Bitao and Fei (2010) develop an ant colony algorithm coupled with local search to solve a problem with the same features.

Summarizing, although most of the current research on the VRP focuses on extensions to it, there is no single work portraying all together the characteristics of our problem.

3 Problem Statement and Mathematical Formulations

The notation and formulations used in this section are based on the VRP-TW formulation proposed by Cordeau and Laporte (2001).

In our company's problem, a set K of different fixed capacity vehicles $k = 1, ..., m$, initially located at a depot, are available to deliver perishable food goods to a set N of customers $i, j = 1, ..., n$ through a set of arcs A. The problem is defined on a directed graph $G = (V, A)$, with $V = N \cup \{0, n+1\}$, where the depot is simultaneously represented by the two vertices 0 and $n+1$ and, therefore, $|V| = n + 2$. Each possible arc (i, j) has an associated time and distance. Since we have as input real data based on the Portuguese road network (with highways and national roads) the triangular inequality does not hold in terms of distance, however, it holds in terms of time. Hence, travelling from i to j is always faster, than passing by customer c in between. But travelling from i to j is not necessarily shorter, than passing intermediately through customer c. Each customer has a demand that needs to be satisfied for a certain number of products that may be dry, fresh or frozen. Moreover, customers want their requests available within hard time windows that can be more than one throughout the day and they need a certain time to be served that is dependent on the demand.

A feasible solution for this problem implies a collection of routes that correspond to paths starting at vertex 0 and ending at vertex $n+1$. These routes have to ensure that each customer is visited exactly once by one of the vehicles allowed for the service, satisfying simultaneously its demand and time windows. Furthermore, the cumulative demand of all customers that each vehicle serves can not exceed its capacity.

In order to have such a definition for a feasible solution and to decrease the complexity of the mathematical formulation, three preprocessing steps are performed. Firstly, service times are assumed to be function of the demand of each client. Hence, for each 100 kg that have to be delivered at a customer, 5 minutes are needed. As the demand is known before performing the routing, all service times can be calculated beforehand. Secondly, regarding the site dependency characteristic of the problem some adjustments may also be performed. Let Kt_i denote the set of vehicles that are able to serve i regarding its demand temperature requirements and Kl_i denote the set of vehicles that are able to serve i regarding its accessibility conditions. In the preprocessing it is possible then to find $K_i = Kt_i \cap Kl_i$ as the set of vehicles able to serve i. Thirdly, because vehicles can either transport only dry products or products of all temperature requirements, the company is able to gain some extra flexibility in the search space and, thus, a potential improvement in the objective function, by allowing a customer to be serviced by two vehicles if he requires dry and fresh and/or frozen products. This is a particular case of split delivery where one allows a client to be serviced by different trucks depending on the demanded temperature requirements. However, each delivery has to contain all the demand for a certain temperature requirement. Our procedure to tackle this situation in the preprocessing is as follows: if customer i has demand for both dry and either fresh and/or frozen products, then this customer is split into i' and i''. These two new dummy customers have the same location as i. The demand of i' adds up for all dry products and the demand of i'' aggregates the demand for fresh and frozen products. Service times for these dummy customers are calculated based on the split demand. Customer i' has no special requirements regarding vehicle temperature compartments, i.e. $K_{i'} = Kl_i$. However, for i'' we have $K_{i''} = Kt_{i''} \cap Kloc_i$. Thus, i' and i'' can be serviced at the same time by the same vehicle if it carries the dry products along with the fresh and frozen ones. Alternatively, it may be serviced by different trucks: one carrying the dry products and the other the remaining products.

The goal of the company is to minimize total costs. These costs correspond to variable travel costs, renting vehicle costs and driver costs. Regarding driver costs these are calculated based on the route duration. The logistic provider delivers each vehicle with a driver that can drive up to the regulated 8 hours. If the total time of a route surpasses such legal limit, a new driver has to accompany the main one yielding an extra cost.

We are now able to define the mathematical formulations for the HF-SD-VRP-MTW. We use the following indices, parameters, and decision variables.

Indices

k	vehicles
i, j	vertices after preprocessing
v	time windows

Sets and Parameters

K_i	set of vehicles able to serve vertex i
TW_i	set of time-windows on vertex i
C^k	vehicle's k capacity
s_i	service time of customer i
$td_{ij}(tt_{ij})$	travel distance (time) from customer i to customer j
vc^k	variable travel cost associated with vehicle k
fc^k	daily fixed cost for subcontracting vehicle k
a_i^v	starting time of time window v at customer i
b_i^v	finishing time of time window v at customer i
d_i	demand of customer i
dd	cost of having two drivers for the same vehicle k
ah	allowed hours for each driver to work

Decision Variables

x_{ij}^k	takes on 1, if arc (i, j) is used by vehicle k (0 otherwise)
w_i^k	time at which the vehicle k starts servicing vertex i
u_i^v	takes on 1, if customer i is visited in time-window v (0 otherwise)
e^k	takes on 1, if vehicle k requires an extra driver (0 otherwise)

Let $\delta^+(i) = \{j : (i, j) \in A\}$ and $\delta^-(j) = \{i : (i, j) \in A\}$ denote the set of successors and predecessors of i and j, respectively. The model F_{day} for finding the optimal routes for customers serviced during the day is as follows:

$$F_{day} = \min \sum_{(i,j)\in A} \sum_{k\in K_i\cap K_j} vc^k\, td_{ij}\, x_{ij}^k + \sum_{k\in K} fc^k(1 - x_{0,n+1}^k) + \sum_{k\in K} dd\, e^k \quad (1)$$

subject to:

$$w_{n+1}^k - w_0^k \leq ah(e^k + 1) \quad \forall k \in K \quad (2)$$

$$\sum_{k\in K_i} \sum_{j\in\delta^+(i)} x_{ij}^k = 1 \quad \forall i \in N \quad (3)$$

$$\sum_{j\in\delta^+(0)} x_{0j}^k = 1 \quad \forall k \in K \tag{4}$$

$$\sum_{i\in\delta^-(j)} x_{ij}^k - \sum_{i\in\delta^+(j)} x_{ji}^k = 0 \quad \forall j \in N, k \in K_j \tag{5}$$

$$\sum_{i\in\delta^-(n+1)} x_{i,n+1}^k = 1 \quad \forall k \in K \tag{6}$$

$$w_i^k + s_i + tt_{ij} - w_j^k \leq b_0^0(1 - x_{ij}^k) \quad \forall (i,j) \in A, k \in K_i \cap K_j \tag{7}$$

$$\sum_{v\in TW_i} u_i^v a_i^v \leq w_i^k \leq \sum_{v\in TW_i} u_i^v b_i^v \quad \forall i \in V, k \in K_i \tag{8}$$

$$\sum_{v\in TW_i} u_i^v = 1 \quad \forall i \in V \tag{9}$$

$$\sum_{i\in N} \sum_{j\in\delta^+(i)} d_i x_{ij}^k \leq C^k \quad \forall k \in K \tag{10}$$

$$x_{ij}^k, u_i^v, e^k \in \{0,1\}; \quad w_i^k \geq 0. \tag{11}$$

Objective function (1) minimizes the total cost involved in the daily routing. Since the fleet is completely outsourced it is possible to measure accurately the cost of the routing plan. Depending of the vehicle there is a variable cost related with the distance travelled, a fixed cost related with each vehicle used and, finally, a cost for having trips that are long enough to require two drivers.

In equation (2) the need for the extra driver is assessed through the allowed hours ah that each driver can work. Constraints (3) ensure that each customer is visited exactly once. Equations (4)-(6) establish the flow of each vehicle. Hence, every vehicle has to leave the depot and return to it by passing through the customers it is designed to serve. A vehicle can only start servicing a customer after having finished servicing the previous customer and after the time spent on traveling from the previous to the current customer (7). In equation (8) the start of the customer service is forced to be in between one of the multiple customer's time windows. However, at each customer only one of the possible time windows may be used (9). Constraints (10) ensure that the different vehicle capacities are respected. To be more accurate both weight and volume constraints should exist in such model. However, due to the lack of reliable data, only weight requirements are considered (a practice also done by the company).

In order to model the problem of serving the night customers, a new parameter nc to account for the extra-cost of having a driver at night has to be defined. In terms of the solution space, this problem is rather the same as the one for the day customers except for the fact that the night customers do not have multiple time windows and trips require no extra drivers. The night model (F_{night}) is formulated as:

$$F_{night} = \min \sum_{(i,j)\in A} \sum_{k\in K_i \cap K_f} vc^k td_{ij} x_{ij}^k + \sum_{k\in K} nc(1 - x_{0,n+1}^k) \qquad (12)$$

subject to:

(3) - (7), (10)

$$a_i \leq w_i^k \leq b_i \quad \forall i \in V, k \in K_i \qquad (13)$$

$$x_{ij}^k \in \{0,1\}; \quad w_i^k \geq 0. \qquad (14)$$

The objective function of F_{night} differs considerably from the one of F_{day}. In (12) we aim at minimizing the travel costs for the different vehicles, but the fixed costs are no longer dependent on the vehicle (since it was already paid for in the day shift) but just upon the night usage.

One could have opted for a single model to tackle the complete routing problem involving day and night customers. However, through the splitting of the models we achieve a considerable reduction in the problem size without jeopardizing optimality. Only in the case where F_{day} does not contain the best trucks to be used at night, the decoupled models would not guarantee the same optimal solution as an integrated approach. However, due to the reasonably low heterogeneity of the fleet and the difference of magnitude between the number of day and night customers (day >> night) this situation will never occur.

4 Solution Method

In order to solve various types of vehicle routing problems, a wide variety of solution methods have been developed. In our case, we want a flexible metaheuristic that is able to incorporate all the specificities of our problem and delivers reasonable results in short computation time. Gendreau and Potvin (2010) acknowledge that the adaptative large neighbourhood search (ALNS) framework introduced in Ropke and Pisinger (2006b) is able to obtain equal or better results for a wide variety of routing problems compared to existing algorithms. In Pisinger and Ropke (2007) very good results are reported for different classes of the VRP. Within the extensions tested some characteristics are coincident with features of our problem. The relevant problems are the VRP, the VRP-TW, and the SD-VRP.

Our algorithm is based on the ALNS developed by Kovacs et al (2011) for service technician routing and scheduling problems, incorporating site-dependent aspects due to skills and skill level requirements of the tasks. For specific details the readers are referred to this manuscript. For the sake of self-containedness we present here only the key blocks and ideas and the modifications we made with respect to the original algorithm.

Algorithm 1 outlines the ALNS framework. First, a feasible solution s is generated. This solution may easily be generated because we allow some customers not to be serviced (*unassigned customers* included in set N^0) at a given penalty cost. In every iteration, a destroy-repair heuristic pair (d, r) is chosen based on its respective score and weight obtained in previous iterations. Second, the destroy operator of the chosen pair is used to remove customers of the incumbent solution and places them into the set of unassigned customers. Third, the repair operator r takes unassigned customers and inserts them into the routes. If the new solution s' meets the acceptance criteria, then it replaces s. If it is better than the best solution found so far, it replaces s^*. Finally, scores and weights (ψ_{dr} and ρ_{dr}) are updated and the algorithm proceeds to the next iteration. This is repeated until the stopping criterion is met.

Algorithm 1 Pseudo-code of the ALNS framework

```
generate a feasible solution s
set s* := s
repeat
    choose a destroy-repair heuristic pair (d, r) based on adaptative weights ρ_dr
    generate s' from s applying (d, r)
    if s' better than s* then
        set s* := s'
        set s := s'
    else if s' complies with the acceptance criteria then
        set s := s'
    end if
    update scores ψ_dr and weights ρ_dr
until stopping criterion is met
return s*
```

In the next subsections, the different destroy and repair operators, the used acceptance criteria, and how a destroy-repair heuristic pair is chosen are briefly presented.

4.1 Destroy Operators

Kovacs et al (2011) use four destroy operators, namely a random removal, a worst removal, a related removal, and a cluster removal operator. They are all based on operators introduced by Ropke and Pisinger (2006b) and Pisinger and Ropke (2007). In every iteration the number of customers u to be removed from the different routes is chosen randomly from the interval $\left[0.1|N - N^0|, 0.4|N - N^0|\right]$.

The *random removal* operator removes u customers randomly from their routes. The *worst removal* operator removes u customers from the different routes biasing the selection towards customers that are not well inserted in terms of their distances from their current direct predecessor and successor customer locations. The *related removal* operator removes related requests. Since we consider vehicle capacities, we do not resort to the same relatedness measure as Kovacs et al (2011) but we use the relatedness measure of Pisinger and Ropke (2007), combining distance, time and load terms,

$$R_{ij} = \alpha td_{ij} + \beta|w_i - w_j| + \gamma|d_i - d_j|.$$

Their respective weights are set to $\alpha = 9$, $\beta = 3$, $\gamma = 4$ (Pisinger and Ropke, 2007). Finally, the *cluster removal* operator removes customers forming a cluster. Each selected route is split into two clusters through the computation of a minimum spanning where the longest arc is removed. Entire clusters are removed until the number of removed customers $\geq u$.

4.2 Repair Operators

Following Kovacs et al (2011), we use six different insertion heuristics in terms of repair operators: a greedy insertion heuristic, four regret heuristics, and a sequential insertion heuristic. The *greedy heuristic* repeatedly inserts a customer from the set of unassigned customers at the cheapest feasible position. This is repeated either until all customers have been inserted or no more customers can be inserted maintaining feasibility. *Regret heuristics* improve the greedy heuristic described above, by integrating look ahead information when selecting customers to insert. Let Δ_i^k denote the change in the objective value for inserting customer i at its best position in its k-cheapest route. In each iteration, the regret heuristic chooses the next customer i to be insert as follows:

$$i := \arg\max_{i \in N^0} \{ \sum_{k=2}^{\min(q,m)} (\Delta_i^k - \Delta_i^1) \}$$

depending on the chosen value of q. We use $q \in \{2, 3, 4, m\}$. The parameter m denotes the number of routes currently available for insertion. The *sequential insertion* heuristic corresponds to the I1 heuristic of Solomon (1987). It estimates the benefit coming from servicing a customer on the selected route rather than being serviced on a single customer tour.

Our objective function does not only incorporate distance based costs. The insertion of an additional customer into an existing route may also increase the number of necessary drivers. Therefore, we approximate the actual insertion costs by adding the costs for an additional driver in the case where the current duration of the route plus the additional time needed to service the respective customer exceeds the maximum duration of a single driver.

Furthermore, instead of a single time window at each customer location, we consider multiple time windows. Therefore, in order to check the feasibility of an insertion, we sequentially check time window feasibility at each customer that is serviced after the prospective insertion position of the new customer with respect to all available time windows.

4.3 Acceptance Criteria

As in Ropke and Pisinger (2006a), the destroy and repair operators are embedded into a simulated annealing framework. Hence, a solution s' is accepted if it is better than s. If s' is worse, s' replaces s with a probability of $e^{(f(s')-f(s))/\hat{t}}$. The parameter $\hat{t}$ denotes the current *temperature*.

4.4 Choosing a destroy-repair heuristic pair (d, r)

Instead of using separate scores and weights for each destroy and each repair operator, we use scores and weights for pairs of operators as in Kovacs et al (2011). Every combination out of the set of destroy and repair operators is used. The probability for choosing an operator pair is proportional to ρ_{dr} for each destroy-repair pair (d, r). To understand the computation of the weights, let us consider n_d and n_r as the respective number of destroy and repair heuristics implemented. First, the probability ϕ_{dr} of choosing a given pair is calculated as follows:

$$\phi_{dr} = \frac{\rho_{dr}}{\sum_{d'=1}^{n_d} \sum_{r'=1}^{n_r} \rho_{d'r'}}.$$

One pair is chosen in every iteration of the ALNS algorithm using roulette wheel selection. Based on the scores obtained, the weights are adjusted dynamically during the search. In the beginning, the weights ρ_{dr} of all heuristic pairs are set to one and the scores ψ_{dr} are set to zero. At the end of every iteration the scores ψ_{dr} of the employed heuristic pair (d, r) are updated as follows: $\psi_{dr} + \sigma_1$, if the destroy-repair heuristic pair gives a solution that improved the global best solution s^*; $\psi_{dr} + \sigma_2$, if the destroy-repair heuristic pair gives a solution that was not visited before and improved the incumbent solution s; $\psi_{dr} + \sigma_3$, if the destroy-repair heuristic pair gives a solution that was not visited before and was accepted as the new incumbent solution s, although it was worse; otherwise the value rest the same. Following Ropke and Pisinger (2006b) and Kovacs et al (2011), the parameters are set to $\sigma_1 = 33$, $\sigma_2 = 9$, and $\sigma_3 = 13$. Every 100 iterations, the weights are updated based on the current scores and the scores are reset to zero.

5 Experimental Analysis and Comparison with Company's Practice

The company of our case study relies on a routing software to partially obtain their routes. This routing software is connected to the company's ERP and it receives all orders for the next day. It disposes of a database which contains the fixed customer data, such as geographical location, vehicle typologies and allowed time windows. To obtain a routing plan for the next day, the software is run for about 10 minutes and retrieves a solution that is then refined by the company's dispatching expert. The software uses as main optimization input a set of predefined routes defined by the planner based on his expertise.

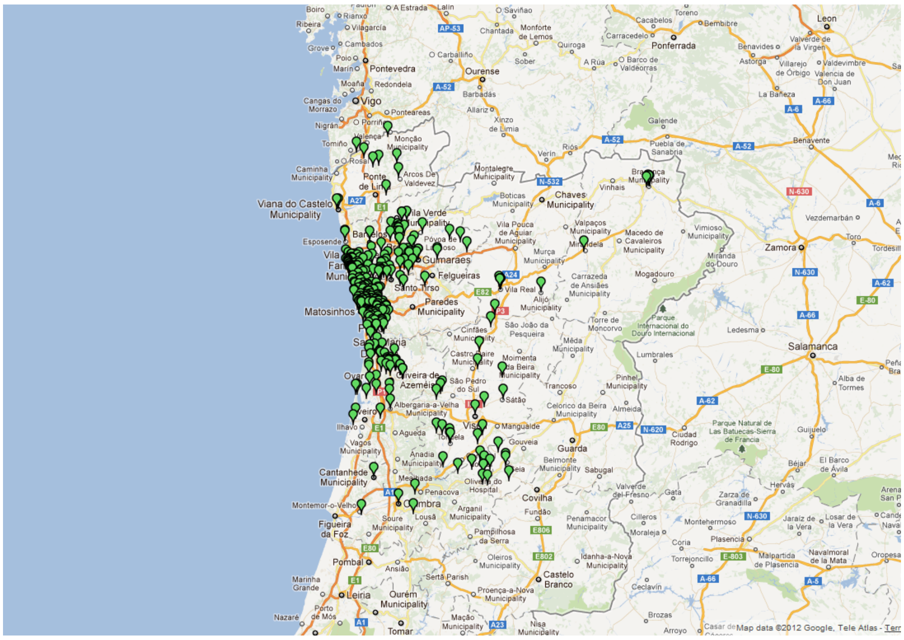

Fig. 2 Geographical location of customers in the first instance

Regarding our solution method, a main data problem regarding the travel times and distances was found. Since the routing software only had Cartesian coordinates for the customers and, unfortunately, we could not access its distance matrix, we recurred to Google Maps in order to calculate both travel times and distances. In that sense, a C++ program that uses the Google Maps API was created in order to build the complete distance matrix. To feed the program the actual addresses of each client are used and we adopted as preference prioritizing motorways over small streets. The remaining data was simply compiled, because the company has the distribution operation completely outsourced and, therefore, vehicles costs, extra drivers, etc. were easily obtained and left no gap for subjective interpretations of the final results. This together with the use of real distances was very important in convincing the company of the validity of the generated plans.

In order to compare the impact of our automatically generated solutions with the plans of the company we used two peak days from the high season. The first instance contains 350 customers to be serviced (see Figure 2) and the second one 366 (see Figure 4).

In Tables 1 and 2, the detailed results for the first instance according to the company's plans and to the plans generated by the ALNS (in about 10 minutes) are presented, respectively. Three key operational indicators are worth of notice. Firstly, the average capacity utilization rises in the generated plans from 74% to 86%. Secondly, although each route takes more time to be completed in our plan, the average distance travelled per route decreases.

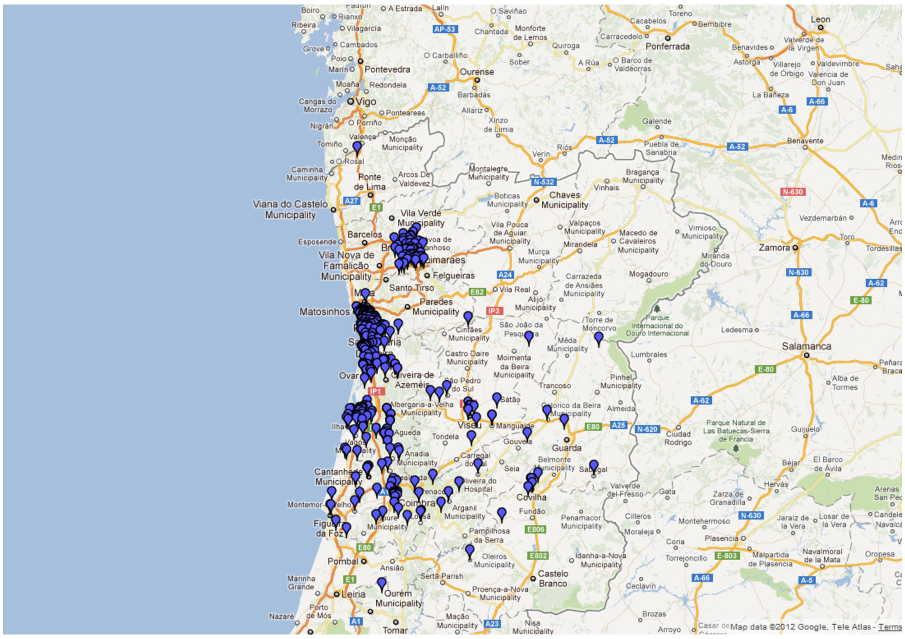

Fig. 3 Geographical location of customers in the second instance

Finally, a better utilization of the drivers' 8-hour working windows is notorious. In Figure 4 a graph plotting the different costs for both plans is given. The cost advantage that the automated routing generates is clear from the graph. From the three costs adding to the total cost: fixed vehicle costs, variable vehicle cost and extra driver cost, it is the variable vehicle cost that contributes the most to the obtained cost reduction. This fact relates very much to the gain of operational efficiency already mentioned regarding the decrease on the kilometres travelled by each truck.

In Figure 5 the total costs for both instances and both plans are plotted. Within these two instances the cost relation between the generated and the company's plans is stable. Overall, the ALNS plans were able to raise the vehicle utilization from 75% to 89% allowing for a decrease in the number of vehicles used. Since the company's plans were based on loosely fixed routes, demand consolidation in order to augment vehicle capacity utilization was harder to perform. The total distance travelled per day is reduced considerably in the order of 1200 kilometres. This is a very important achievement since variable vehicle costs account for most of the bill. Furthermore, with the increasing prices of oil derivatives the tendency is to see these costs rising in new contracts. Finally, we can expect that in the peak season the daily out-of-pocket savings can ascent to 1200 euros. By the end of the year these savings may have an interesting impact on the company's operating income, considering also the fact that the south filial of the company faces the same problem with a similar number of customers and a similar routing methodology.

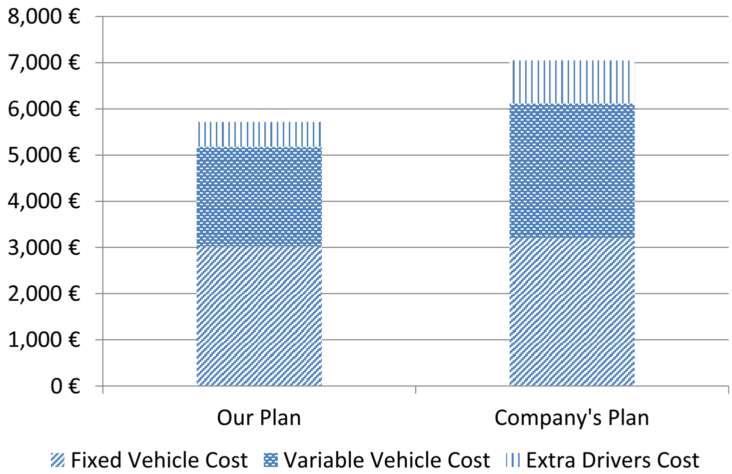

Fig. 4 Comparison of both plans for all cost factors in the first instance.

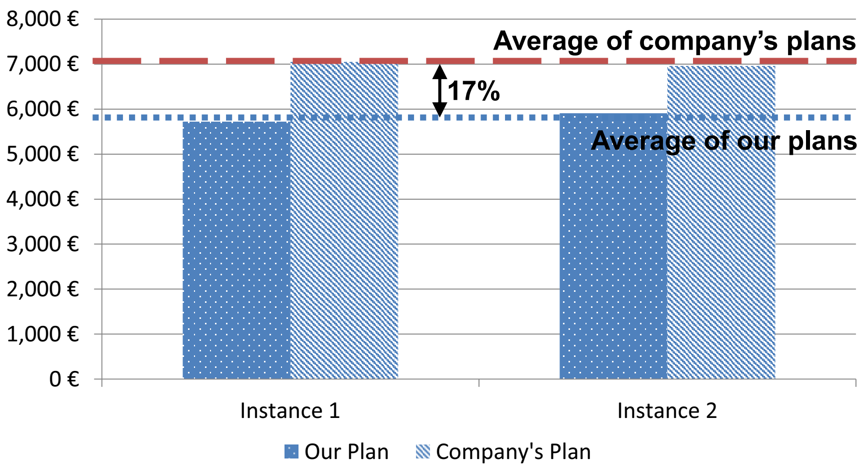

Fig. 5 Comparison of both plans for the two instances.

Table 1 Detailed company's plan aggregated by route for the first instance.

Route	**No. of Customers**	**Vehicle Capacity (Kg)**	**Charge (Kg)**	**Utilization**	**Travel Time**	**Distance Travel (Km)**	**Fixed Veh. Cost (€)**	**Variable Veh. Cost (€)**	**Drivers Cost (€)**
1	22	8700	6821	78%	6:23	183	138	93	0
2	12	6500	5131	79%	16:05	503	167	317	78
3	14	6500	6051	93%	12:54	403	167	254	78
4	15	6500	5208	80%	11:21	327	167	206	78
5	9	6500	3721	57%	7:18	286	178	109	0
6	12	6500	5012	77%	9:28	224	138	114	78
7	12	6500	5241	81%	8:28	204	167	129	78
8	17	6500	5201	80%	9:08	241	167	152	78
9	18	6500	5464	84%	9:11	222	167	140	78
10	20	6500	4666	72%	9:14	145	167	91	78
11	15	6500	3129	48%	9:02	122	178	47	78
12	12	3950	3706	94%	17:11	704	135	422	78
13	13	3900	1587	41%	10:20	442	135	265	78
14	19	3900	2939	75%	5:37	69	135	42	0
15	23	3900	3560	91%	6:46	61	135	37	0
16	25	3950	3063	78%	6:11	163	135	98	0
17	23	4200	3200	76%	7:21	151	165	83	0
18	29	5450	3361	62%	8:22	177	165	97	78
19	26	3950	2472	63%	6:31	107	135	64	0
20	12	3950	3489	88%	6:43	215	135	129	0
21	2	2500	487	19%	2:14	47	135	28	0
Average	17	5398	3977	74%	8:50	238	153	139	44
						TOTAL COST			**€7050**

Overall, the ALNS plans are able to reduce the consolidated costs per vehicle, which is a very important indicator for the company's top management, by almost 20% (from 350 to 286). Most of the cost reduction is achieved through a much better routing that consolidates more demand and delivers every product in a lower total distance. This kind of plan is hard to grasp and unveil by using common sense analysis.

6 Conclusions

In this work we present a complex vehicle routing problem faced by a food distribution company. Through analysing its characteristics we classified it as a *heterogeneous fleet site dependent vehicle routing problem with multiple time windows.*

Table 2 Detailed plan generated by the metaheuristic aggregated by route for the first instance.

Route	No. of Customers	Vehicle Capacity (Kg)	Charge (Kg)	Utilization	Travel Time	Distance Travel (Km)	Fixed Veh. Cost (€)	Variable Veh. Cost (€)	Drivers Cost (€)
1	8	6500	4996.88	77%	7:11	58	167	37	0
2	19	6500	6316.22	97%	11:06	206	167	130	78
3	19	6500	6298.05	97%	12:22	349	167	220	78
4	17	6500	5236.53	81%	14:19	418	167	263	78
5	7	6500	5629.71	87%	7:57	191	167	120	0
6	14	6500	6310	97%	12:09	276	167	174	78
7	24	3950	3478.43	88%	7:55	113	135	68	0
8	21	3950	3125.56	79%	7:57	76	135	46	0
9	19	3950	3436.37	87%	7:38	143	135	86	0
10	25	3950	3713	94%	7:40	77	135	46	0
11	23	3950	3836	97%	7:50	258	135	155	0
12	29	3950	2686	68%	7:54	97	135	58	0
13	18	3950	3857.22	98%	7:58	127	135	76	0
14	15	3950	3603.98	91%	6:43	53	135	32	0
15	20	3950	3389.69	86%	7:45	100	135	60	0
16	11	800	723.09	90%	3:57	115	103	48	0
17	15	5400	4576.97	85%	10:17	217	165	119	78
18	12	4200	3391.43	81%	5:44	61	165	34	0
19	17	6500	4382.13	67%	19:29	430	178	163	78
20	17	6500	5589.07	86%	16:19	651	178	247	78
Average	18	4898	4229	86%	9:30	201	150	109	27
					TOTAL COST				**€5722**

Vehicle routing problems are known to be NP-hard and to solve them metaheuristics are most of the times used. Since this specific problem has never been solved we chose to use a very general search procedure that has proven to deliver very good results for different vehicle routing related problems – the ALNS framework.

The results show an average cost reduction for the company of about 17% in the distribution task at peak seasons. These savings are mainly achieved through a better capacity utilization of the vehicles and a reduction on the distance travelled to visit all customers. Our tool aims to dote the company with an automatic decision support system that is independent of the expertise of the user. Moreover, it can react to any new major development, such as the introduction of more customers.

This tool does not aim at replacing the planner of the company but instead it aims at providing a much better starting point for adjustments. Hence, the planner is released to do more value-added tasks.

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Chapter 5

Reaching an Integrated Production and Distribution Planning of Perishable Goods

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Multi-objective integrated production and distribution planning of perishable products

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ABSTRACT

Integrated production and distribution planning have received a lot of attention throughout the years and its economic advantages are well documented. However, for highly perishable products this integrated approach has to include, further than the economic aspects, the intangible value of freshness. We explore, through a multi-objective framework, the advantages of integrating these two intertwined planning problems at an operational level. We formulate models for the case where perishable goods have a fixed and a loose shelf-life (i.e. with and without a best-before-date). The results show that the economic benefits derived from using an integrated approach are much dependent on the freshness level of products delivered.

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1. Introduction

Rapidly deteriorating perishable goods, such as fruits, vegetables, yoghurt and fresh milk, have to take into account the perishability phenomenon even for the operational level of production and distribution planning, which has a timespan ranging from 1 week to 1 month. Usually these products start deteriorating from the moment they are produced on. Therefore, without proper care, inventories may rapidly get spoiled before their final use making the stakeholders incur on avoidable costs. The customers of these products are aware of the intense perishability they are subject to, and they attribute an intangible value to the relative freshness of the goods (Tsiros and Heilman, 2005). To evaluate freshness customers rely on visual cues which may differ among the broad class of perishable products. Nahmias (1982) dichotomized deteriorating goods in two categories according to their shelf-life: (1) fixed lifetime: items' lifetime is pre-specified and therefore the impact of the deteriorating factors is taken into account when fixing it. In fact, the utility of these items may decrease during their lifetime, and when passing its lifetime, the item will perish completely and become of no value, e.g., milk, inventory in a blood bank, and yoghurt, etc. and (2) random lifetime: there is no specified lifetime for these items. The lifetime for these items is assumed as a random variable, and its probability distribution may take on various forms. Examples of items that keep deteriorating with some probability distribution are electronic components, chemicals, and vegetables, etc.

When the shelf-life is fixed the most common visual cue that customers rely on is the best-before-date (BBD). The BBD can be defined as the end of the period, under any stated storage conditions, during which the product will remain fully marketable and retain any specific qualities for which tacit or express claims have been made. In this case, customers will adapt their willingness to pay for a product based on how far away the BBD is. On the other hand, when the expiry date of a product is not printed and then the shelf-life is loose, customers have to rely on their senses or external sources of information to estimate the remaining shelf-life of the good. For example, if a banana has black spots or if flowers look wilted, then customers know that these products will be spoiled rather soon.

In the case of loose shelf-life, especially in the fresh food industry, manufacturers can make use of predictive microbiology to estimate the shelf-life of these kinds of products based on external controllable factors, such as humidity and temperature (Fu and Labuza, 1993). To make concepts clearer, shelf-life is defined as the time period for the product to become of no value for the customer due to the lack of the tacit initial characteristics that the product is supposed to have. Thus, in our case, this period starts on the day the product is produced. The determination of shelf-life as a function of variable environmental conditions has been the focus of many research activities in this field and a considerable number of reliable models exist, such as the Arrhenius model, the Davey model and the square-root model. These models take into account the knowledge about microbial growth in decaying food goods under different temperature and humidity conditions.

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Regarding production and distribution planning, many authors have shown the economic advantages of using an integrated decision model over a decoupled approach (Martin et al., 1993; Thomas and Griffin, 1996). These advantages are believed to be leveraged when the product suffers a rapid deterioration process and hence pushes towards a more integrated view of these intertwined problems. For perishable goods the final products inventory that is usually used to buffer and decouple these two planning decisions have to be questioned since customers distinguish between different degrees of freshness and there is an actual risk of spoilage. In this work, we want to study the potential advantages of using an integrated approach for operational production and distribution planning of perishable goods compared with a decoupled one. These advantages will be analyzed through the economic and product freshness perspective. We focus on highly perishable consumer goods industries, with a special emphasis on food processing, which have to cope with complex challenges, such as the integration of lot sizing and scheduling, the definition of setup families considering major and minor setup times and costs, and multiple non-identical production lines (Bilgen and Günther, 2010). We are also interested in understanding if these potential advantages differ among the two distinct perishable goods classes that we have mentioned before: with fixed shelf-life and with loose shelf-life. For both the cases, since we are interested in rapidly deteriorating goods, we consider a customer who prefers products with a higher freshness level. To tackle explicitly this customer satisfaction issue we embedded our integrated operational production and distribution planning problem in a multi-objective framework distinguishing two very different and conflicting objectives of the planner. The first objective is concerned with minimizing the total costs over the supply chain covering transportation, production, setup and spoilage costs. The second one aims at maximizing the freshness of the products delivered to distribution centers and, therefore, maximize customers' willingness to pay.

The remainder of the paper is organized as follows. The next section is devoted to the literature review of related topics. In Section 3, two integrated and two decoupled models are formulated for the multi-objective production and distribution planning of the perishable products problem. First we will focus on products with a fixed shelf-life and then with a loose one. In Section 4, an illustrative example to understand the impact of the models is developed and analyzed. The paper is concluded in Section 5.

2. Literature review

Taking into account that perishability in supply chains has received increased attention both in practice and in academic research it urges to continue the development of models that incorporate explicitly this phenomenon (Ahumada and Villalobos, 2009; Akkerman et al., 2010). Since in this section we only review literature directly connected to our research and explore concepts and strategies used to model this problem, readers are referred to Karaesmen et al. (2011) for an extensive review on papers managing perishable inventories. First we look into what has been done in production and distribution modeling from a decoupled and integrated point of view with a special focus on perishability and consumer goods industries. Then, based on this paper a discussion about the multi-objective approach used and some other important modeling concepts follows. Hence, the contribution of our work in this research field is clarified.

For tackling the operational production planning of perishable goods, Marinelli et al. (2007) propose a solution approach for a real world capacitated lot sizing and scheduling problem with parallel machines and shared buffers, arising in a company producing yoghurt. The problem has been formulated as a hybrid continuous setup and capacitated lot sizing problem. To solve this problem a two-stage heuristic procedure based on the decomposition of the problem into a lot sizing and a scheduling sub-problem has been developed. This model accounted for perishability by using a make-to-order production strategy. With a similar approach, but focused on batch processing, Neumann et al. (2002) decompose detailed production scheduling for batch production into batching and batch scheduling. The batching problem converts the primary requirements for products into individual batches, where the workload is to be minimized. They formulate the batching problem as a nonlinear mixed-integer program and transform it into a linear mixed-binary program of moderate size. The batch scheduling problem allocates the batches to scarce resources such as processing units, workers, and intermediate storage facilities, where some regular objective function like the makespan is to be minimized. The batch scheduling problem is modeled as a resource-constrained project scheduling problem, which can be solved by a truncated branch-and-bound algorithm. In this work some intermediate perishable products cannot be stored eliminating the buffer between related activities. Pahl and Voß (2010) extend well-known discrete lot-sizing and scheduling models by including deterioration constraints. Also for the special case of yoghurt production, Lütke Entrup et al. (2005) develop three mixed-integer linear programming models that integrate shelf-life issues into production planning and scheduling of the packaging stage. In this work product freshness is modeled as a linear function hoping that retailers could pay the difference between different deterioration stages. Cai et al. (2008) develop a model and an algorithm for the production of seafood products. Due to a deadline constraint and the raw material perishability, the manufacturer determines three decisions: the product types to be produced; the machine time to be allocated for each product type; and the sequence to process the products selected. It is interesting to notice that here the perishability is focused on the raw materials.

Especially suited for lot-sizing and scheduling in the consumer goods industry, where natural sequences in sequencing products can be found in order to minimize changeover time and to ensure quality standards, there is the concept of block planning. A block represents a sequence, set *a priori*, of production orders of variable size, where each production order corresponds to a unique product type. Hence, each product type occurs at a given position in a block (Günther et al., 2006). This concept will be very useful in reducing the complexity and augmenting the applicability of our models in the next section.

There are two papers worth of reference when modeling explicitly routing for perishable products. Osvald and Stirn (2008) develop a heuristic algorithm for the distribution of fresh vegetables in which perishability represents a critical factor. The problem is formulated as a vehicle routing problem with time windows (VRPTW) and with time-dependent travel times. The model considers the impact of perishability as a part of the overall distribution costs. Hsu et al. (2007) consider the randomness of the perishable food delivery process and construct a stochastic VRPTW model to obtain optimal delivery routes, loads, fleet dispatching and departure times for delivering perishable food from a distribution center. They take into account inventory costs due to deterioration of perishable food and energy costs for cold storage vehicles.

Regarding transportation in the consumer goods industry there is a significant trend in outsourcing this function to third party logistics service providers (3PL) (Lütke Entrup, 2005). This decision enables consumer goods manufacturers to move towards a more flexible cost structure and, hence, the manufacturer is able to choose generally between Less-than-truckload (LTL), where the shipper only pays a price according to the capacity used, and full-truckload where the shipper pays a fixed cost per load (Günther and Seiler, 2009). The temperature of the transportation, in the case of perishable goods, especially food, has to be in accordance with the requirements of the perishable goods transported, for instance, chilled or frozen. In our case we will consider that all products have the same temperature requirements and hence can be grouped together in every shipment.

Looking at the literature on integrated production and distribution models dealing with perishable products, Eksioglu and Mingzhou (2006) address a production and distribution planning problem in a dynamic, two-stage supply chain. Their model considers that the final product is perishable and has a limited shelf-life. Furthermore, strong assumptions are made, such as, unlimited capacity. They formulate this problem as a network flow problem with a fixed charge cost. Based on the economic order quantity, Yan et al. (2011) develop an integrated production–distribution model for a deteriorating item in a two-echelon supply chain. Their objective is to minimize the total system cost. Some restrictions concerning perishability are imposed. For example, the supplier's production batch size is limited to an integer multiple of the discrete delivery lot quantity to the buyer. In a more operational perspective, Chen et al. (2009) propose a nonlinear mathematical model to consider production scheduling and vehicle routing with time windows for perishable food products. The demand at retailers is assumed to be stochastic and perishable goods will deteriorate once they are produced. Thus, the revenue of the supplier is uncertain and depends on the value and the transaction quantity of perishable products when they are carried to retailers. The objective of this model is to maximize the expected total profit of the supplier. The production quantities, the time to start production and the vehicle routes are determined in the model iteratively through a decomposition procedure. The solution algorithm is composed of the constrained Nelder–Mead method and a heuristic for the vehicle routing with time windows. Recently, Rong et al. (2011) developed an MIP model where the quality of a single product is modeled throughout a multi-echelon supply chain. Their model uses the knowledge of predictive microbiology in forecasting shelf-life based on the temperature of transportation and stocking. Their objective function reflects this reality by taking into account the incremental cooling costs necessary to achieve a longer shelf-life. This work has a straight linkage to the models developed for the loose shelf-life case of this paper.

Based on this paper, when developing production and/or distribution models applied to the type of perishable products treated in this work, a considerable number of different approaches could be considered. We can impose a make-to-order strategy for all products so that it is likely that no products will spoil and they will be delivered with good freshness standards. It is also possible to enforce constraints on the number of periods that a product can stay in stock or just control the number of spoiled products and penalize it in the objective function. A third way of taking into consideration products' perishability is by using differentiating holding costs depending on the shelf-life. So items with a shorter shelf-life are given higher holding costs and items with a longer shelf-life are given lower holding costs. Finally, from a customer value point of view, it is possible to either attribute a value to the different degrees of freshness that a product has when delivered, or to assign a demand function that varies in function of items' remaining shelf-life. This way we are explicitly controlling the perishability process since we know for every delivered product the corresponding monetary value of the remaining shelf-life and, therefore, we may focus on maximizing profit. However, most of these alternatives imply an exhaustive differentiation of costs, freshness values and demand functions among products. This can be very inaccurate in the consumer goods industries that need to handle hundreds of stock keeping units. Thus, without demeriting these approaches, to tackle the perishability phenomenon in planning tasks, such as production and distribution, it is necessary to employ another method to understand the complementary effects of supply chain costs and product perishability. In order to follow our goal of understanding the impact of integrating the analysis of production and distribution planning, both in economic and in freshness terms, the use of a single objective function would hinder the important trade-off between these two conflicting objectives. Some authors have already given hints about the importance of using a multi-objective framework to fully explore the perishability phenomenon (e.g. Arbib et al., 1999; Lütke Entrup, 2005). Nevertheless, to the best of our knowledge, this is the first work that addresses the integrated production and distribution planning of perishable products in a multi-objective framework.

3. Problem statement and model formulations

The production and distribution planning problem considered in this paper consists of a number of plants $p=1,\ldots,P$ having dedicated lines which produce multiple perishable items with a limited capacity to be delivered to distribution centers. It is relevant to understand the importance of the design choice of having such a complex supply chain instead of just considering one plant and multiple distribution centers. As said before, we focus on perishable consumer goods industries which are known for demanding increasing flexibility in the supply chain planning processes. Thus, to consider a network of production plants which can add increased flexibility and reliability to hedge against the complex dynamics of such industries is crucial. Therefore, although we are tackling an operational level of decision making for these two planning tasks we assume a central organizational unit that makes decisions which are followed directly at a local level. The length of the planning horizon for such planning problem ranges from 1 week to 1 month.

All product variants $k=1,\ldots,K$ belonging to the same family form a block. Therefore a product can only be assigned to one block. Blocks $j=1,\ldots,N$ are to be scheduled on $l=1,\ldots,L$ parallel production lines over a finite planning horizon consisting of macro-periods $d=1,\ldots,T$ with a given length. The scheduling takes into account that the setup time and cost between blocks is dependent on the sequence of production (major setup). The sequence of products in a block is set *a priori* due to natural constraints in this kind of industries. Hence, when changing the production between two products of the same block only a minor setup is needed that is not dependent on the sequence, but only on the product to be produced.

In order to consider the initial stock that might be used to fulfil current demand it is important to have an overview of the inventory built up in each macro-period due to perishability concerns. The length of the horizon that needs to be considered is related to the product with the longest shelf-life. One shall consider an integer multiple X of past planning horizons that is enough to cover the longest shelf-life, i.e. $X=\lceil \max \tilde{u}_k/T \rceil$, where $\tilde{u}_k$ is a conservative value for shelf-life of product k, hence $t=-XT+1,\ldots,0,1,\ldots,T$. Let $T^-=\{-XT+1,\ldots,0\}$ and $T^+=\{1,\ldots,T\}$, thus the domain of t is equivalent to $[T]=T^-\cup T^+$.

A macro-period is divided into a fixed number of non-overlapping micro-periods with variable length. Since the production lines can be scheduled independently, this is done for each line separately. S_{ld} denotes the set of micro-periods s belonging to macro-period d and production line l. All micro-periods are put in order $s=1,\ldots,S^l$, where S^l corresponds to the total number of micro-periods of line l. It is important to notice that each line is assigned to a plant. The length of a micro-period is a decision variable, expressed by the production of several products of one block in the respective micro-period on a line and by the time to set up the block in case it is necessary. A sequence of consecutive micro-periods, where the same block is produced on the same line, defines the size of a lot of a block through the quantity of products produced during these micro-periods. Therefore, a lot may aggregate several products from a given block and may continue over several micro and macro-periods. Moreover, a lot is independent of the discrete time structure of the macro-periods. The number of micro-periods of each day defines the upper bound on the number of blocks to be produced daily on each line.

There is no inventory held at production plants. Thus, at the end of each day the production output is delivered to distribution centers $c = 1, \ldots, DC$, which have an unlimited storage capacity. The delivery function is assured by a 3PL, and we assume that it charges a flat rate per pallet transported between a plant and a DC. Moreover, it is assumed that the 3PL is able to cope with whatever distribution planning was decided beforehand and, hence, there is no capacity restriction for transportation. This 3PL takes care of all sorts of other decisions besides the quantities to be transported and it complies with the tightest recommendations and regulations on transportation of perishable goods. The distances between production plants and distribution centers are small enough so that the product is delivered on the same day it is produced. Therefore, the decrease of freshness during the transportation is considered to be negligible compared with the decrease at the storage process. The small distance assumption is quite realistic in supply chains of highly perishable goods where the distribution centers are not very far away from the production plants. However, this poses a limitation for the application of this model when this assumption is not verified. For our purposes these assumptions shall not pose a problem since we are still considering directly the most important cost drivers for transportation services: distance, quantity and service level. The demand for an item in a macro-period at a distribution center is assumed to be dynamic and deterministic.

The problem is to plan production and distribution so as to minimize total cost and maximize mean remaining shelf-life of products at the distribution centers over a planning horizon.

In Fig. 1 a graphical interpretation of the problem dealt with in this paper is presented. It represents the product flow from plants to DCs and the correspondent demand satisfaction. To understand that this is just a layer of the supply network a shadowed replication was drawn behind the main scheme. The figure depicts a weekly planning horizon and the inventory that is carried linking the consecutive planning horizons. Special care was taken when representing the major setups showing that changing between different blocks triggers setups that are dependent on the sequence and, on the other hand, minor setups are fixed for a given product to be produced (in this case they are all equal).

In the remainder of this section two cases are formulated: (1) when the shelf-life is fixed beforehand, and (2) when the shelf-life is indirectly a decision variable influenced by the environmental setting. For each of these cases an integrated and a decoupled production and distribution planning model is presented in order to compare the two different approaches afterwards. The environment variables considered for the loose shelf-life case are quite related to the fresh food industry. Hence, for the sake of correctness, we should consider that we are focusing on this specific perishable industry. In a latter phase the appropriated generalizations will be introduced.

Consider the following indices, parameters and decision variables that are applicable to both studied cases:

Indices

l	parallel production lines
i,j	blocks
k	products
t,d,h,b	macro-periods: $t \in [T]$; $d,h \in T^+$; $b \in T^-$
s	micro-periods
c	distribution centers (DCs)
p	production plants

Parameters

L_p	set of lines at plant p
K_{lj}	set of products belonging to block j and line l
$\lvert K_{lj} \rvert$	number of products belonging to block j and line l
S_{ld}	set of micro-periods s within macro-period d for line l
$[d_{kdc}]$	number of non-zero occurrences in the demand matrix
Cap_{ld}	capacity (time) of production line l available in macro-period d
a_{lk}	capacity consumption (time) needed to produce one unit of product k on line l
c_{lk}	production costs of product k (per unit) on line l
m_{lj}	minimum lot-size (units) of block j if produced on line l
$scb_{lij}(stb_{lij})$	sequence dependent setup cost (time) for a change-over from block i to block j on line l

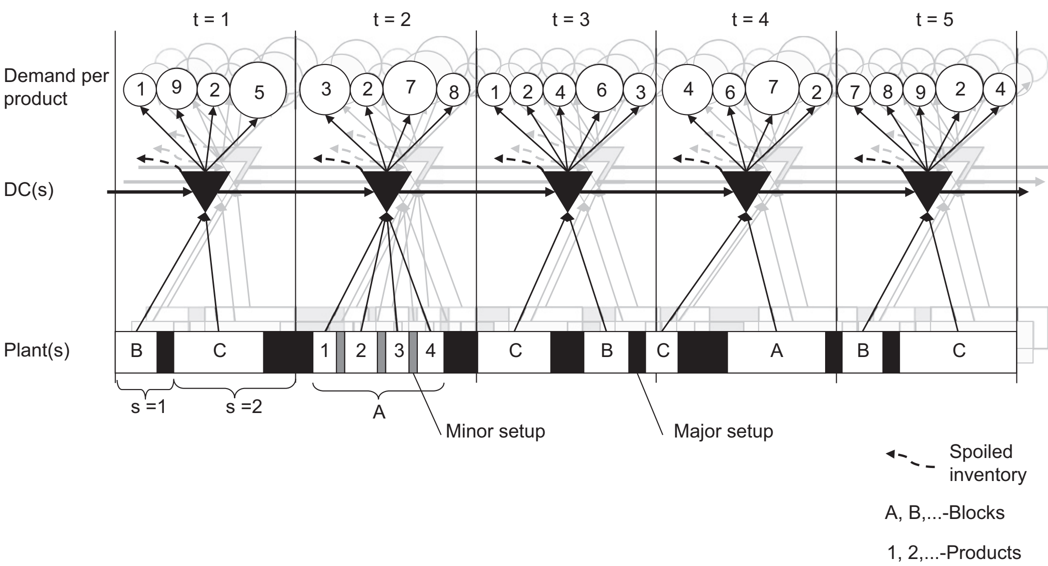

Fig. 1. Graphical interpretation of the problem statement.

$scp_{lk}(stp_{lk})$ sequence-independent setup cost (time) for a changeover to product k on line l
ϕ_k cost associated with the spoilage of one unit of product k in inventory
tc_{pc} cost for transporting one item from production plant p to DC c
d_{kdc} demand for product k in macro-period d at DC c (units)
y_{lj0} equals 1, if line l is set up for block j at the beginning of the planning horizon (0 otherwise)
I^-_{khc} minimum inventory of product k to be held in macro-period h at DC c (units)
R^*_{kbc} stock of product k at the beginning of the planning horizon that was produced in macro-period b at DC c (units), $\forall b \in T^-$

Decision variables

B_{kdc} quantity of stock of product k that spoils in macro-period d at DC c (units)
R_{kdc} quantity of stock of product k produced in macro-period d to be used in the next planning horizon at DC c (units)
w_{ktdc} fraction of demand in macro-period d of product k produced in macro-period t at DC c
q_{lks} quantity of product k produced in micro-period s on line l (units)
p_{lks} setup state: $p_{lks}=1$, if line l is set up for product k in micro-period s (0 otherwise)
y_{ljs} setup state: $y_{ljs}=1$, if line l is set up for block j in micro-period s (0 otherwise)
z_{lijs} takes on 1, if a changeover from block i to block j takes place on line l at the beginning of micro-period s (0 otherwise)
x_{khpc} quantity of product k produced in macro-period h shipped from production plant p to DC c (units)

3.1. Case 1: fixed shelf-life

In the case where shelf-life is fixed, there is only need of adding an extra parameter to the list already defined, namely

u_k shelf-life duration of product k after completion of its production (macro-periods)

When we have a fixed shelf-life it is possible to admit that the variable w_{ktdc} is only instantiated for $d \geq t \wedge d \leq t+u_k$ to ensure that demand fulfilled in period d is produced beforehand with units that have not perished yet. Following the same reasoning variable R_{kdc} is only instantiated for $d \geq T-u_k$.

3.1.1. Integrated model

The integrated production and distribution planning of perishable goods with fixed shelf-life (PDP-FSL) may be formulated as a multi-objective mixed-integer model: **PDP-FSL**

$$\min \sum_{l,i,j,s} scb_{lij} z_{lijs} + \sum_{l,k,s} (scp_{lk} p_{lks} + c_{lk} q_{lks}) + \sum_{k,h,p,c} tc_{pc} x_{khpc} + \sum_{k,d,c} \phi_k B_{kdc} \tag{1}$$

$$\max \left(\sum_{k,t,d,c} \frac{t+u_k-d}{u_k} w_{ktdc} \right) \frac{1}{[d_{kdc}]} \tag{2}$$

$$\text{subject to}: \quad \sum_{k \in K_{lj}} p_{lks} \leq y_{ljs} |K_{lj}| \quad \forall l,s,j \tag{3}$$

$$q_{lks} \leq \frac{Cap_{ld}}{a_{lk}} p_{lks} \quad \forall l,k,d,s \in S_{ld} \tag{4}$$

$$\sum_{i,j,s \in S_{ld}} stb_{lij} z_{lijs} + \sum_{k,s \in S_{ld}} (stp_{lk} p_{lks} + a_{lk} q_{lks}) \leq Cap_{ld} \quad \forall l,d \tag{5}$$

$$\sum_j y_{ljs} = 1 \quad \forall l,s \tag{6}$$

$$\sum_{k \in K_{lj}} q_{lks} \geq m_{lj}(y_{ljs} - y_{lj,s-1}) \quad \forall l,j,s \tag{7}$$

$$z_{lijs} \geq y_{li,s-1} + y_{ljs} - 1 \quad \forall l,i,j,s \tag{8}$$

$$\sum_{l \in L_p, s \in S_{lh}} q_{lks} = \sum_c x_{khpc} \quad \forall k,h,p \tag{9}$$

$$\sum_p x_{khpc} = \sum_d w_{khdc} d_{kdc} + R_{khc} \quad \forall k,h,c \tag{10}$$

$$\sum_d w_{kbdc} d_{kdc} \leq R^*_{kbc} \quad \forall k,c,b \tag{11}$$

$$\sum_t w_{ktdc} = 1 \ \forall k,d,c : \quad d_{kdc} > 0; \tag{12}$$

$$\sum_t w_{ktdc} = 0 \ \forall k,d,c : \quad d_{kdc} = 0; \tag{13}$$

$$I^-_{khc} \leq \sum_{d>h, t \leq h} w_{ktdc} d_{kdc} + \sum_{d \leq h} R_{kdc} \quad \forall k,h,c \tag{14}$$

$$B_{kdc} \geq R^*_{kbc} - \sum_h w_{kbhc} d_{khc} \forall k,b,d = b+u_k+1,c \tag{15}$$

$$B_{kdc},\ R_{kdc},\ w_{ktdc},\ q_{lks},\ x_{khpc},\ z_{lijs} \geq 0; \quad p_{lks},\ y_{ljs} \in \{0,1\} \tag{16}$$

In the first objective (1) total costs are minimized, namely: production costs, transportation costs and spoilage costs. The production costs include: sequence dependent setup costs between blocks (major setup), sequence independent setup costs of products (minor setup) and variable production line costs. This objective function aggregates the measurable economic importance throughout the considered supply chain.

In the second objective (2) the mean fractional remaining shelf-life of products to be delivered is maximized. Hence, the remaining shelf-life of a product is expressed by $t+u_k-d$, where t accounts for the completion date of the product, u_k for the shelf-life of the product and d for the delivery date of such order. This quantity reflects the available time of product k at DC to satisfy retailers' demand. Dividing this quantity by the initial shelf-life we obtain the fractional remaining shelf-life. Since we are concerned about multiple items with different shelf-lives, by doing this operation we somehow normalize the impact of shelf-life variation among products. To obtain the mean fractional remaining shelf-life we need to divide the quantity relative to all orders by the number of occurrences in the demand matrix $[d_{kdc}]$. This cardinality, for a given input set data, is constant and easily computed. It is important to note that this objective takes values between 0% and 100%. To further understand the behavior of such objective function let us now consider a scenario where a client demands two different products. One of the products is very perishable (1 unit of time) and the other is subject to low perishability (10 units of time). If, after producing and delivering them, the very perishable one has 2 days to be sold and the other one has 5 days, they will contribute equally to this objective function.

This multi-objective approach for modeling the integrated production and distribution planning for perishable goods has an interesting aspect to consider regarding inventory costs. When maximizing freshness in the second objective we are already trying to minimize stocks since we will try to produce as late as possible (note that tardiness is not allowed). Hence, if we had also included inventory costs in the first objective we would be somehow duplicating the inventory carrying cost effect. There is also a reasoning that comes more from a practical point of view to justify the disregarding of inventory carrying costs in operational planning of perishable goods. Let us consider the holding cost H_t to carry one unit (palette) of inventory from one period t (day) to period $t+1$. A fair assumption is that the holding cost consists entirely of interest on money tied up in inventory. Hence, if i is the annual interest rate and we consider periods as days (operational planning) then $H_t = ic_k/365$, where c_k is the average production cost of producing a palette of product k. Looking, for instance, at the yoghurt production where to produce a pack of 4×125 g cups would cost about €0.5 and a palette can hold 1056 packs, then for a 5% interest rate, one would have a holding cost per period of about €0.07 per pallet. This cost seems not significant enough compared with the costs of major and minor setups, as well as when compared with the different variable production costs between lines.

Regarding the constraints that bound this model, constraints (3) and (4) ensure that a product of a given block can only be produced if both the block and the product are set up. Furthermore, with (6) just one block can be produced on a given line and in a given micro-period. Limited capacity in the lines is to be reduced by setup times between blocks, setup times between products and also by the time consumed in production (5). Constraint (7) introduces minimum lot-sizes for each block. The connection between setup states and changeover indicators for blocks is established by (8).

Constraint (9) ensures that total production in each macro-period at every production plant is to be distributed among the different DCs. Moreover, this constraint links parallel production lines together.

Products arriving from different production plants at a DC are used to meet future demand in the current planning horizon or to constitute final stock to use in the next planning horizons, as stated by (10).

Constraint (11) ensures that the initial stock in each DC of products produced before the actual planning horizon can be spent in the current planning horizon to fulfil demand. Nevertheless, there is a special concern with perishability and therefore only products which have not perished are allowed to be used. Constraint (15) is used to calculate the quantity of perished inventory.

Each day demand is to be met without backlogging with specific production done until that day and with stock left from the past planning horizons (12). Eq. (13) is just needed to ensure that production variables w_{ktdc} are zero when the demand at a DC in a period d is null.

In (14) minimum inventory levels for each product in stock are set per macro-period and per DC. In fact, with the formulation chosen there is no need to explicitly define an inventory decision variable. The stock of product k in a macro-period h is equal to the production done until macro-period h to be used in a future period in the current planning horizon in addition to the production to stock until macro-period h for each DC. The definition of these bounds, made in an upper hierarchical decision level, is quite important both regarding consumer satisfaction and the quantity of spoiled products. Nevertheless, in terms of storage capacity at the DCs probably this is not the most realistic and important setting for practitioners. For perishable consumer products, the storage capacity per temperature zone, also known as climate zone, is the most common. In this case, the storage space is divided in zones according to the temperature requirements of different products, for instance, chilled, frozen and ambient (for a comprehensive discussion of this topic please refer to Broekmeulen, 2001). However, in this work we are more focused on the customer satisfaction at an operational level, and hence looking specifically at every product inventory level to be sure that the demand variability, so common in the consumer goods industry, as well as seasonality is well taken into account.

Finally, (16) defines the domain of the decision variables.

3.1.2. Decoupled production and distribution model

A decoupled model is designed to mimic a two-stage procedure commonly found in industry due to practical organizational reasons. In the first stage, the production quantities of products in each production plant over the planning horizon, q_{lks}, are determined. Then in the second stage, the delivery quantities, x_{khpc}, are found in function of product availability at each production plant. To allow an analytical comparison between the integrated and decoupled approach the two formulations bellow (PP-FSL and DP-FSL) will need to fulfil the same production requirements as the integrated approach. This will happen when in the integrated approach one does not allow carrying stocks from one planning horizon to the other. Therefore, production requirements are the same as final demand requirements.

The production planning of perishable goods with fixed shelf-life (PP-FSL) may be formulated as a multi-objective mixed-integer model: **PP-FSL**

$$\min \sum_{l,i,j,s} scb_{lij} z_{lijs} + \sum_{l,k,s} (scp_{lk} p_{lks} + c_{lk} q_{lks}) \tag{17}$$

$$\max \left(\sum_{k,h,d} \frac{h+u_k-d}{u_k} \sum_{c} w_{khdc} \right) \frac{1}{[d_{kdc}]} \tag{18}$$

subject to : (3)−(8)

$$\sum_{l,s \in S_{lh}} q_{lks} \geq \sum_{d,c} w_{khdc} d_{kdc} \quad \forall k,h \tag{19}$$

$$\sum_{h,c} w_{khdc} = 1 \quad \forall k,d,c \quad \sum_{c} d_{kdc} > 0; \tag{20}$$

$$\sum_{h,c} w_{khdc} = 0 \quad \forall k,d,c \quad \sum_{c} d_{kdc} = 0; \tag{21}$$

$$w_{khdc},\ q_{lks},\ z_{lijs} \geq 0;\ p_{lks},\ y_{ljs} \in \{0,1\} \tag{22}$$

In the objective function (17) the cost minimization only reflects production costs. The objective function (18) aims at maximizing also the mean fractional remaining shelf-life of products to be delivered. However, it considers an aggregation of demand by DC because when we are just planning production it is not known which plant will serve a certain demand order at a DC. Furthermore, at this stage we cannot take directly into account the possible initial stock at the DCs with a certain freshness level.

Constraints (3)–(8) from the PDP-FSL model are to be included in this one. Constraint (19) ensures that total production in each macro-period at every production plant is enough to cover the aggregated future demand, while constraints (20) and (21) have the same meaning as (12) and (13), respectively, but on an aggregated level.

Given the production quantities, $\overrightarrow{Q} = (q_{lks})_{l,k,s}$, from the production sub-problem, the distribution planning of perishable goods with fixed shelf-life (DP-FSL) may be formulated as a multi-objective linear model: **DP-FSL** ($\vec{Q}$)

$$\min \sum_{k,h,p,c} tc_{pc}x_{khpc} + \sum_{k,d,c} \phi_k B_{kdc} \tag{23}$$

$$\max \left(\sum_{k,t,d,c} \frac{t+u_k-d}{u_k} w_{ktdc} \right) \frac{1}{[d_{kdc}]} \tag{24}$$

$$\text{subject to}: \quad (9)-(15)$$

$$B_{kdc},\ R_{kd},\ w_{ktdc},\ x_{khpc} \geq 0 \tag{25}$$

In the objective function (23), the cost minimization encompasses transportation and spoilage costs. The objective function (24) aims at maximizing the mean fractional remaining shelf-life of products to be delivered at each DC.

Constraints (9)–(15) from the PDP-FSL model have the same meaning as before.

3.2. Case 2: loose shelf-life

In the case where the shelf-life of the perishable products is not fixed, for example, by a stamp tagging the BBD, but rather depends on different external factors (cf. Section 1), it is important to define several other indices, parameters and decision variables to model the production and distribution planning. As discussed in Section 2, we will rely on the fact that manufacturers can estimate the shelf-life of products throughout the supply chain based on external factors using the knowledge of predictive microbiology.

We will consider that the goods produced have the same temperature requirements and hence the temperature control is done per DC. This idea follows what has been said in relation to temperature transportation requirements and product grouping at distribution centers in Section 2.

Indices

ρ discrete temperatures allowed for storing products respecting legal requirements

Parameters

$\Delta_{\rho k}$ fraction of shelf-life decrease of product k when spending a macro-period in stock at temperature ρ

$\Theta^*_{\rho bc}$ temperature state selected in macro-period b at DC c, $\forall b \in T^-$

$\phi_{\rho c}$ cost of keeping DC c at temperature ρ

$\tilde{u}_k$ conservative estimation of the shelf-life duration of product k after completion of its production (macro-periods) based on the worst storage conditions

Decision variables

$\Theta_{\rho dc}$ temperature state: $\Theta_{\rho dc}=1$, if DC c is at temperature ρ in macro-period d (0 otherwise)

u^+_{ktdc} fraction of remaining shelf-life of product k produced in macro-period t for meeting demand in macro-period d at DC c

u^-_{ktdc} fraction of violated shelf-life of product k produced in macro-period t if it was meeting demand in macro-period d at DC c

$\overline{u}^+_{ktdc}$ spoilage state: $\overline{u}^+_{ktdc}=1$, if product k produced in macro-period t for meeting demand in macro-period d at DC c is not spoiled (0 otherwise)

$\overline{u}^-_{ktdc}$ spoilage state: $\overline{u}^-_{ktdc}=1$, if product k produced in macro-period t for meeting demand in macro-period d at DC c is spoiled (0 otherwise)

Unlike the former case, in Case 2 it is not possible to bound decision variable domains based on the precise shelf-life because shelf-life is itself, indirectly, a decision variable. The modeling of this case grasps this characteristic by making shelf-life dependent on the deterioration characteristics of the product itself expressed by $\Delta_{\rho k}$ and on the environment (temperature) conditions expressed by $\Theta_{\rho tc}$. Nevertheless, it is possible to limit the decision variables based on a conservative estimation of the product's shelf-life $\tilde{u}_k$. Hence for w_{ktdc} we have $d \geq t \wedge d \leq t+\tilde{u}_k$ and R_{kbc} is only instantiated for $d \geq T-\tilde{u}_k$. The use of $\tilde{u}_k$ will be helpful to model the freshness objective in the decoupled production model dealing with loose shelf-life (PP-LSL).

3.2.1. Integrated model

The integrated production and distribution planning of perishable goods with loose shelf-life (PDP-LSL) may be formulated as a multi-objective mixed-integer nonlinear model: **PDP-LSL**

$$\min \sum_{l,i,j,s} scb_{lij}z_{lijs} + \sum_{l,k,s}(scp_{lk}p_{lks} + c_{lk}q_{lks}) + \sum_{k,h,p,c} tc_{pc}x_{khpc} + \sum_{k,d,c}\phi_k B_{kdc} + \sum_{\rho,d,c}\Theta_{\rho dc}\phi_{\rho c} \tag{26}$$

$$\max \left(\sum_{k,t,d,c} u^+_{ktdc} w_{ktdc} \right) \frac{1}{[d_{kdc}]} \tag{27}$$

$$\text{subject to}: \quad (3)-(14)$$

$$B_{kdc} \geq R^*_{kbc}\overline{u}^-_{kbdc} - \sum_{h<d} w_{kbhc} d_{khc} \forall k,b,d,c \tag{28}$$

$$u^+_{khdc} + u^-_{khdc} = 1 - \sum_{t>h}^{d}\sum_{\rho}\Theta_{\rho tc}\Delta_{\rho k} \quad \forall k,h,d,c \tag{29}$$

$$u^+_{kbdc} + u^-_{kbdc} = 1 - \sum_{t>b}^{0}\sum_{\rho}\Theta^*_{\rho tc}\Delta_{\rho k} - \sum_{t=1}^{d}\sum_{\rho}\Theta_{\rho tc}\Delta_{\rho k} \quad \forall k,b,d,c \tag{30}$$

$$u^-_{ktdc} \geq -\overline{u}^-_{ktdc}(d-t)\max_{\rho}\Delta_{\rho k} \tag{31}$$

$$u^+_{ktdc} \leq \overline{u}^+_{ktdc} \quad \forall k,t,d,c \tag{32}$$

$$\overline{u}^+_{ktdc} + \overline{u}^-_{ktdc} \leq 1 \quad \forall k,t,d,c \tag{33}$$

$$w_{ktdc} \leq \overline{u}^+_{ktdc} \quad \forall k,t,d,c \tag{34}$$

$$\sum_{\rho}\Theta_{\rho dc} = 1 \quad \forall d,c \tag{35}$$

$$B_{kdc},\ R_{kd},\ w_{ktdc},\ q_{lks},\ x_{khpc},\ z_{lijs},\ u^+_{kbdc} \geq 0;\ \ u^-_{kbdc} \leq 0; \quad p_{lks},\ y_{ljs},\ \Theta_{\rho tc},\ \overline{u}^+_{kbdc},\ \overline{u}^-_{kbdc} \in \{0,1\} \tag{36}$$

In objective function (26) production, transportation and spoilage costs are taken into account similarly to the case of fixed shelf-life. Beyond these costs, the energy cost of keeping the DCs at a certain temperature is also included. Objective function (27) aims at maximizing the mean fractional remaining shelf-life of products to be delivered. In this case the remaining shelf-life of a certain quantity used to fulfil an order is explicitly expressed by u^+_{ktdc}. Due to the uncertain nature of the shelf-life of each product, this second objective renders the multi-objective model nonlinear and non-convex, contrarily to the former case (fixed shelf-life,) which did not have nonlinearities.

Concerning the constraints this problem is subject to, constraints (3)–(14) are the same as in PDP-FSL and concern the production stage.

Constraint (28) is a nonlinear constraint used to calculate the quantity of perished inventory.

Constraints (29)–(33) define the whole set of possible fractions of remaining, u^{+}_{kbdc}, and violated, u^{-}_{kbdc}, shelf-lives. Constraints (31) and (32) ensure that when a product perishes then $\overline{u}^{-}_{ktdc}$ takes value 1 and when a product has still some remaining shelf-life then $\overline{u}^{+}_{ktdc}$ takes value 1, respectively. For the "big M" constraints (31), a tight value for "M" is given. It is calculated for every combination of d,t,k after getting the maximum value of $\Delta_{\rho k}$ for a given k and then multiplying it by the difference between the demand and the production day. Note that for (32) there is no need to define the value of "M" since the constraint is already tight. Constraint (33) is used to ensure that a product either perishes or still has some freshness, since these are completely disjoint physical states. Hence, constraints (29) and (30) define the amount of remaining or violated fractional shelf-life for products produced in the current and in the last planning horizon, respectively. For each combination of k,h,d,c (k,b,d,c) either u^{+}_{khdc} (u^{+}_{kbdc}) takes a positive value if the deterioration process has not yet made the product spoiled or u^{-}_{khdc} (u^{-}_{kbdc}) takes a negative value representing the violation of the shelf-life. These values are defined based on the product inner characteristics and on the profile of temperature storage.

Constraint (34) makes sure that only products with some remaining shelf-life are used to satisfy demand and constraint (35) only lets one temperature to be chosen in each macro-period for each DC.

Finally, constraints (36) define the variable domains. We note that $u^{-}_{kbdc} \leq 0$.

In Fig. 2 the behavior of a certain product k that has a loose shelf-life is depicted. After production, the product has 100% of remaining shelf-life and, as stated before, throughout the transportation process it is able to conserve that freshness. Afterwards, in the distribution center, if the temperatures are set constantly at $\rho = 3$ or 4 then the products will perish before being delivered to the retailers. The contrary happens when $\rho = 1$ or 2. The dashed line represents the case when the products are stored in the beginning with $\rho = 3$ but, during their storage, the temperature is cooled down to $\rho = 1$ ensuring that the products have some remaining shelf-life before they are delivered.

3.2.2. Decoupled production and distribution model

As done before, here we present a decoupled model designed to mimic a two-stage procedure for the two planning problems of production and distribution planning. The overall reasoning developed before for the fixed shelf-life case applies to this one as well.

The production planning of perishable goods with loose shelf-life (PP-LSL) can be formulated as a multi-objective mixed-integer model: **PP-LSL**

$$\min \sum_{l,i,j,s} scb_{lij} z_{lijs} + \sum_{l,k,s} (scp_{lk} p_{lks} + c_{lk} q_{lks}) \tag{37}$$

$$\max \left(\sum_{k,h,d} \frac{h + \tilde{u}_k - d}{\tilde{u}_k} \sum_{c} w_{khdc} \right) \frac{1}{[d_{kdc}]} \tag{38}$$

$$\begin{aligned} &\text{subject to:} \quad (3)-(8) \\ &(19)-(21) \\ &w_{khdc},\ q_{lks},\ z_{lijs} \geq 0; \quad p_{lks},\ y_{ljs} \in \{0,1\} \end{aligned} \tag{39}$$

In the objective function (37) the cost minimization only reflects production costs. The objective function (38) aims at maximizing the mean fractional remaining shelf-life of products to be delivered but it considers an aggregation of demand by DC. However, this objective value is calculated based on a rough estimation of the remaining shelf-life at delivery time using $\tilde{u}_k$ because in the decoupled approach we do not know at what temperatures the storage will be done afterwards. With this measure we are able to compute an upper bound of the mean remaining shelf-life of products delivered.

Constraints (3)–(8) belong to the PDP-FSL model and regard production restrictions, whereas constraints (19)–(21) from PP-FSL are concerned about satisfying the demand in an aggregated fashion.

Given the production quantities, $\overrightarrow{Q}$, from the production subproblem, the distribution planning of perishable goods with loose shelf-life (DP-FSL) can be formulated as a multi-objective nonlinear model: **DP-FSL ($\overrightarrow{Q}$)**

$$\min \sum_{k,h,p,c} tc_{pc} x_{khpc} + \sum_{k,d,c} \phi_k B_{kdc} + \sum_{\rho,d,c} \Theta_{\rho dc} \phi_{\rho c} \tag{40}$$

$$\max \left(\sum_{k,t,d,c} u^{+}_{ktdc} w_{ktdc} \right) \frac{1}{[d_{kdc}]} \tag{41}$$

$$\begin{aligned} &\text{subject to:} \quad (9)-(14) \\ &(28)-(35) \\ &B_{kdc},\ R_{kd},\ w_{ktdc},\ x_{khpc},\ u^{+}_{kbdc} \geq 0; \quad u^{-}_{kbdc} \leq 0; \\ &\Theta_{\rho tc},\ \overline{u}^{+}_{kbdc},\ \overline{u}^{-}_{kbdc} \in \{0,1\} \end{aligned} \tag{42}$$

In the objective function (40) the cost minimization encompasses transportation, spoilage, and cooling costs. The objective function (41) aims at maximizing the fractional remaining shelf-life of products to be delivered at each DC as it was done in objective function (27).

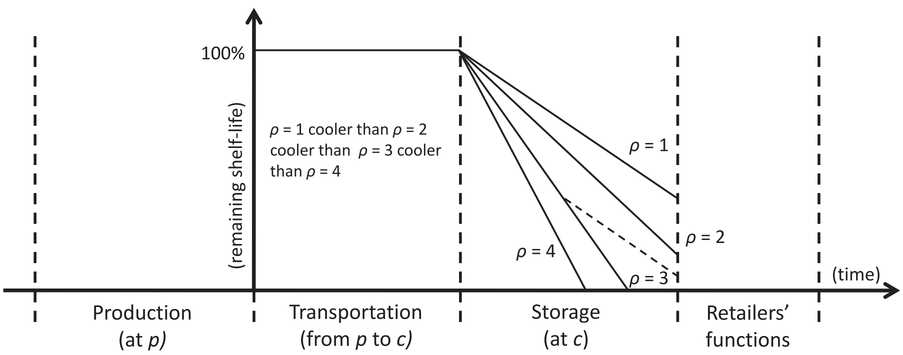

Fig. 2. Behavior of a product with loose shelf-life regarding remaining lifetime.

Constraints (9)–(14) and (28)–(35) belong to the PDP-FSL and PDP-LSL models, respectively. The first group is concerned about the distribution function of perishable goods and the second one about the determination of the remaining shelf-life of products delivered.

4. Illustrative example

To understand the trade-off present in the two models (fixed and loose shelf-life) regarding total costs and product freshness as well as the differences between an integrated over a decoupled approach for production and distribution planning an illustrative example was developed.

In this instance there are four products to be scheduled and produced on two production lines that are located in two different production plants. Each of these products belongs to a different block and therefore there is always sequence-dependent setup time and cost to consider when changing from one product to another. Moreover, although the first line is able to produce every product, the second one is not able to produce all of the products. The production lines are considered similar and, therefore, variable production costs are neglected. The number of micro-periods per macro-period was set at the constant value of four allowing the production of all products in a macro-period. The capacity of each line is the same in all macro-periods and every production plant (100 units). The planning horizon is 10 days (macro-periods) and the shelf-life of products varies considerably among them, from highly perishable ones (1 day) to others which can last throughout the entire planning horizon. Demand has to be satisfied in two different DCs and products can be transported between any pair *production plant–DC*. Initial stock was set to zero in both DCs. In case shelf-life is not fixed, there are three different temperature levels possible to be chosen at each DC influencing its duration. Finally, a sensitivity analysis regarding the perishability impact was conducted and different scenarios where shelf-lives and decay rates are varied were analyzed (Table 4). In Tables 1–3 the remainder data of the illustrative example is given.

The remainder of this section shows the results when solving the illustrative example for both cases: fixed and loose shelf-life.

Table 1
Demand data for illustrative example.

DC	Product	Macro-period									
		1	2	3	4	5	6	7	8	9	10
1	1	0	40	0	0	0	0	0	0	48	0
2	1	0	0	40	0	0	0	0	0	0	48
1	2	30	30	0	0	0	40	48	36	36	0
2	2	40	30	30	0	0	0	0	48	36	36
1	3	10	0	10	0	0	10	12	12	0	12
2	3	10	0	10	0	0	10	12	12	0	12
1	4	10	20	30	0	0	0	0	12	24	36
2	4	0	10	20	30	36	0	12	0	0	24

Table 2
Transportation and temperature related costs data for illustrative example.

DC	Production plant		Temperatures		
	1	2	1	2	3
1	0.03	0.1	2	2.5	3.25
2	0.15	0.05	1.75	2.25	3

Table 3
Changeover times and costs data for illustrative example.

Block *i*	Block *j*	Line *l*	stb_{lij}	scb_{lij}
1	2	1	5	5
1	4	1	10	10
1	2	2	2.5	2.5
1	3	2	5	5
1	4	2	7.5	7.5
2	1	1	10	10
2	4	1	7.5	7.5
2	1	2	7.5	7.5
2	3	2	2.5	2.5
2	4	2	5	5
3	1	2	10	10
3	2	2	7.5	7.5
3	4	2	2.5	2.5
4	1	1	15	15
4	2	1	12.5	12.5
4	1	2	12.5	12.5
4	2	2	10	10
4	3	2	7.5	7.5

Table 4
Shelf-lives and decay rates for the different scenarios.

Scenario	Temperatures	Products							
		Fixed shelf-life				Loose shelf-life			
		1	2	3	4	1	2	3	4
	1					0.111	0.111	0.125	1.000
Base	2	9	9	8	1	0.100	0.100	0.111	0.500
	3					0.091	0.091	0.100	0.333
	1					0.500	0.333	0.500	1.000
High perish.	2	2	3	2	1	0.333	0.250	0.333	0.500
	3					0.250	0.200	0.250	0.333
	1					0.111	0.111	0.125	0.167
Low perish.	2	9	9	8	6	0.100	0.100	0.111	0.143
	3					0.091	0.091	0.100	0.125

To complement the explanations of the following sections about how the different problems were solved readers should refer to Fig. 3.

4.1. Results case 1: fixed shelf-life

In this section results for the case where the shelf-life is fixed are presented. Since the models developed are either multi-objective mixed-integer or multi-objective linear it was possible to obtain optimal solutions for this small instance with the help of CPLEX. For the integrated approach, merely by aggregating both objectives with varying weights and solving the resulting single-objective model to optimality it was possible to determine a variety of solutions from which the Pareto-optimal front could be constructed. On the other hand, to obtain the results for the decoupled approach, in order to compare them with the ones of the integrated approach, a more complex method is needed. First, solutions on the Pareto-optimal front for the production sub-problem were determined. Afterwards, for each of the production solutions a new Pareto-optimal front was computed for the distribution sub-problem. To compute the solution of the coupled problem it is necessary to obtain the values for the two objectives for each pair of connected production–distribution solutions. For the first objective, concerning total costs, the summation of each of the sub-problems' first objective was considered. Then, for the second objective, concerning the mean remaining shelf-life of the products delivered, the retrieved value of the distribution sub-

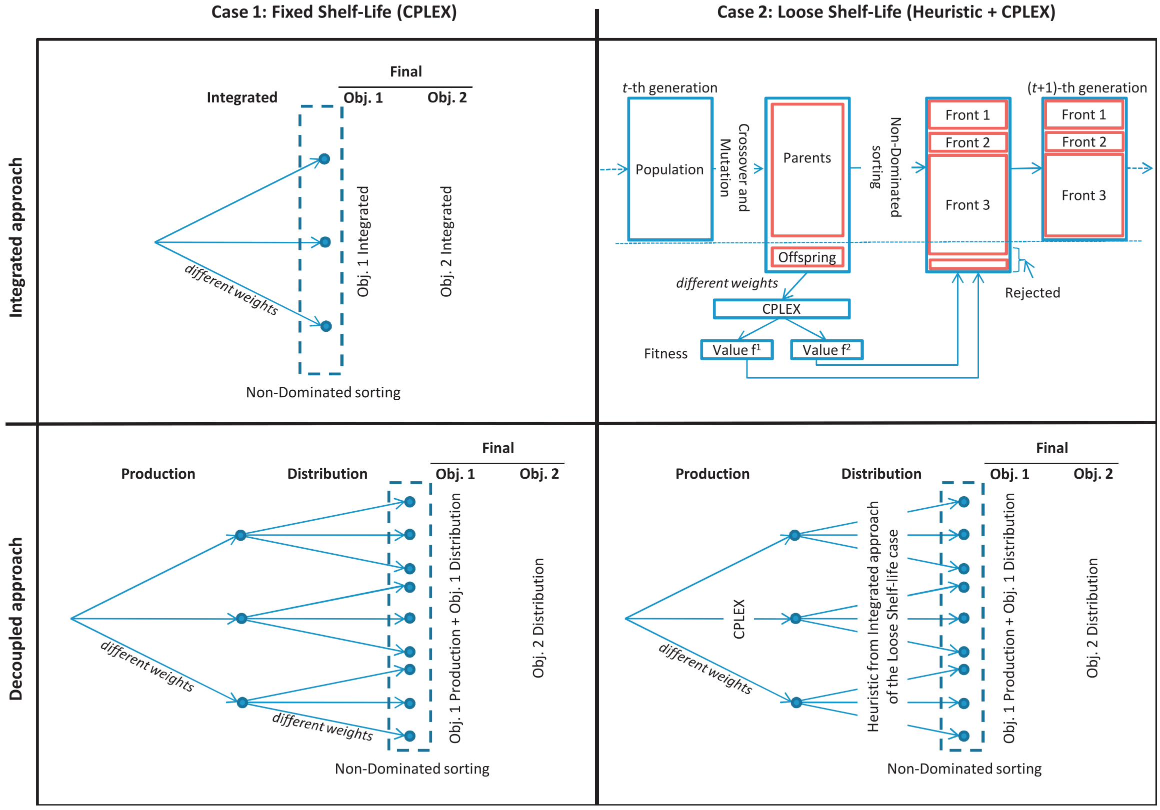

Fig. 3. Matrix showing a visual representation of the solving strategies for each *Case–Approach* combination.

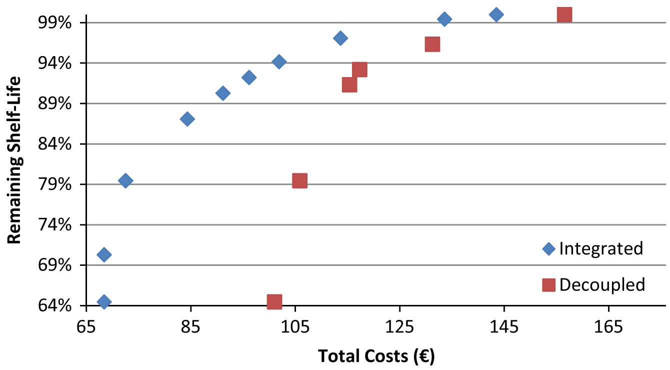

Fig. 4. Pareto-optimal fronts of the illustrative example when using an integrated and a decoupled approach (Case 1).

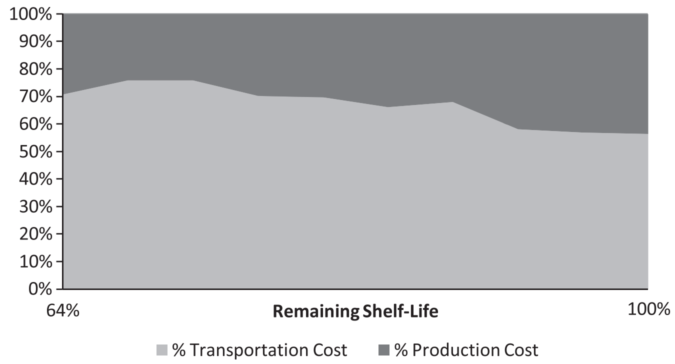

Fig. 5. Relative importance regarding total cost of production and transportation cost using an integrated approach (Case 1).

problem is the one that represents the coupled solution value. In fact, ultimately, it is the distribution planning that fixes the product delivery freshness. To obtain the integrated approach Pareto-front we needed about 30 min CPU running time while for the decoupled approach, since the problems are less complex, half of the computational time was enough. The two left quadrants of Fig. 3 represent these solution procedures.

In Fig. 4 the Base scenario solutions of the Pareto-optimal fronts for both the integrated and decoupled approach are presented.

It is rather clear from the comparison of the Pareto fronts that the integrated approach strongly dominates the decoupled one. Both curves have a similar behavior, which means that for the lower values of freshness just a small increase in costs fosters significantly the remaining shelf-life of delivered products. Nevertheless, when we are approaching a strict Just-in-Time (JIT) accomplishment of the demand, touching very high freshness standards, the costs start to increase in a more important way. Furthermore, it is interesting to notice that the savings in costs when using an integrated approach over a decoupled one tend to fade when we aim at an increased freshness. This may be explained by the fact that to achieve very high freshness standards almost no inventory is allowed since we are working under a JIT policy, this will constrain so much the solution space that the integrated and coupled solutions are rather the same.

Figs. 5 and 6 concern the relative importance of production and transportation costs for both the integrated and the decoupled approach in the Base scenario.

The relative importance of the costs in both approaches is rather independent of the freshness of the products delivered. Only when approaching a very high performance by delivering products with a remaining shelf-life around 100% it is possible to see that the two graphs converge (with an increase of the % Production Costs). When comparing the two approaches, it seems that having a larger view over the information in the supply chain (integrated approach), while

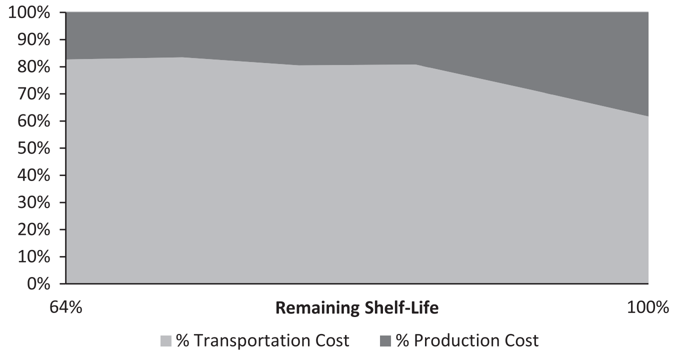

Fig. 6. Relative importance regarding total cost of production and transportation cost using a decoupled approach (Case 1).

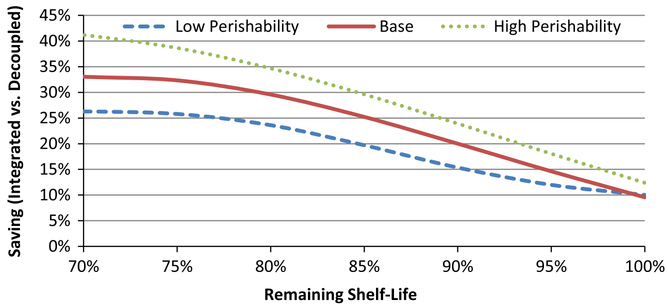

Fig. 7. Total percentage saving when using an integrated approach over a decoupled one for three scenarios (Case 1).

taking operational decisions of production and distribution planning, leads to better overall costs by means of avoiding the greedy behavior of locally optimizing production and then adjust distribution. The decoupled approach has to emphasize quite heavily the costs of the transportation process in order to compensate the potential mistakes of the myopic production planning. The integrated approach increases the share of the production costs in the total costs in order to have a global decrease afterwards.

Finally, in Fig. 7 we perform a sensitivity analysis regarding the perishability settings. The percentage saving of using an integrated approach over a decoupled one is plotted for the three scenarios. In order to calculate the saving, both Pareto fronts (integrated and decouple approach) were estimated through a second-order polynomial regression which has a good fit to the experimental data with all R^2 above 90%.

The potential savings of using an integrated approach over a decoupled one are rather considerable for the fixed-shelf-life case and, independently of the scenario, the behavior over the remaining shelf-life is quite similar. For the scenario with highly perishable products the savings can ascend up to 42% when aiming at 70% of remaining shelf-life.

When comparing the three scenarios it is observable that the advantages of using an integrated approach are leveraged by the degree of perishability the goods are subject to. In fact, when we are planning using a decoupled approach and the products are subject to intense perishability, the myopic mistakes incurred in production planning will be hardly corrected by the distribution process because the buffer between those activities is reduced by the small amount of time that goods can stay stored. Therefore, the advantages of using an integrated approach are boosted considerably for this scenario. On the other hand, when dealing with products with low perishability the buffer enables the possibility of correcting the potential production mistakes and the integrated approach has less comparative advantage.

4.2. Results case 2: loose shelf-life

In this section we focus on the case where the shelf-life is loose. Since in this case there are models which are multi-objective mixed-integer nonlinear it was not straightforward to obtain directly any solution with the help of exact methods as done before. Therefore, a simple hybrid genetic heuristic was developed to solve this problem. This heuristic bases itself on the fact that when fixing $\Theta_{\rho tc}$ the model is no longer non-linear because the fractional remaining shelf-life u^{+}_{ktdc} and also $\overline{u}^{-}_{kbdc}$ are then known.

Hence, a population of individuals representing the temperature states, $\Theta_{\rho tc}$, is randomly created ensuring feasibility. At each generation these values are fed into CPLEX and optimal solutions for the single-objective model resulting from aggregating both objective functions with varying complementary random weights are obtained. The population is then subject to simple cross-over and mutation operators. In the end all the individuals are ranked according to non-domination (Deb et al., 2000) and a Pareto front is obtained. More details about this heuristic can be found in Amorim et al. (2011). The number of different combinations for $\Theta_{\rho dc}$ is $(DC.T)^{|\rho|}$, where $|\rho|$ stands for the cardinality of possible temperatures. Therefore, for this small example, we would have already 8000 possible solutions that would need to be evaluated until optimality could be achieved for each of the varying weightings given to both objective functions. To obtain the results for the decoupled approach this heuristic had to be inserted in the distribution planning sub-problem in a scheme very close to the one used in the decoupled approach for fixed shelf-life (refer to the right quadrants of Fig. 3). Before, the production planning sub-problem is solved with CPLEX since it has no nonlinearities. To unveil the near-optimal Pareto front of the integrated approach we needed about 50 min CPU running time since this problem was considerably harder to solve than the one with fixed shelf-life. Once again to obtain the Pareto front for the decoupled approach less computational effort is needed.

In Fig. 8 the results of the Pareto fronts for both the integrated and decoupled approach are presented. These solutions concern the Base scenario.

As it happened with the results of Case 1 the Pareto front related to the integrated approach strongly dominates the one corresponding to the decoupled approach. It is interesting to note that our simple heuristic was able to capture the non-convexities of both Pareto fronts. The other reasoning made before for Case 1 regarding the behavior of the fronts also applies to this case.

As done before for Case 1, in Fig. 9 the results of the sensitivity analysis to understand the effect of different perishability settings are presented. The percentage saving of using an integrated approach over a decoupled one is plotted for the three scenarios.

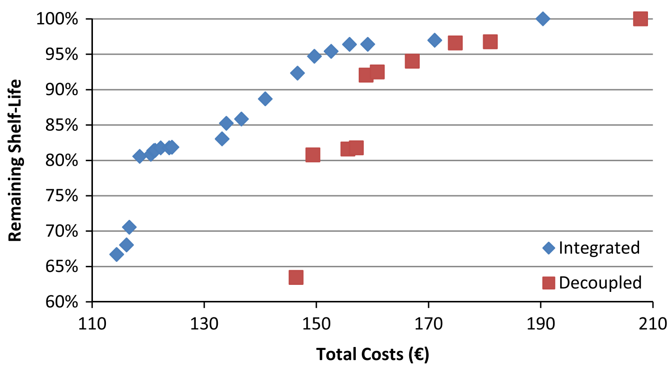

Fig. 8. Pareto-optimal fronts for the illustrative example when using an integrated and a decoupled approach (Case 2).

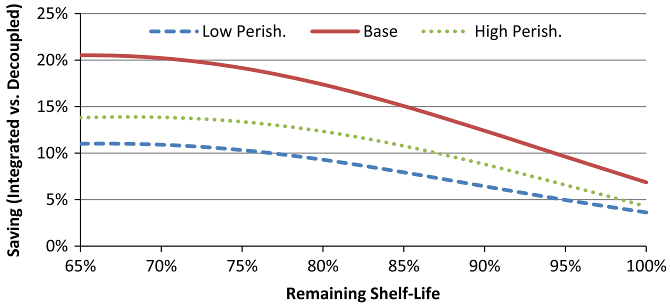

Fig. 9. Relative importance regarding total cost of production and transportation cost using a decoupled approach (Case 2).

Unlike Case 1 the savings are not as bold and the maximum saving ascends to 20% for an average remaining shelf-life of about 65% in the Base scenario, which is still rather remarkable. Nevertheless, the behavior of both saving curves (from Case 1 and Case 2) is very similar. The explanation for the difference in the amount of savings between the fixed and loose shelf-life case may lie in the fact that for the loose shelf-life case the distribution process has much more freedom to influence both costs and especially product freshness. Hence, for the decoupled approach even after the production process has fixed the production quantities, the distribution process is still able to compensate potential mistakes through the decisions on temperature of storage.

Looking at the differences between the three scenarios it is interesting to notice that in this case the reasoning is not as straightforward as in Case 1. Here, the two extreme scenarios have a similar behavior for different reasons. The scenario with products subject to low perishability has a rather humble saving when using an integrated approach for the same reasons as in Case 1. Hence, since the time buffer between production and distribution is rather large the advantages of using an integrated approach are hindered. In the scenario having products with a high perishability the explanation for the relative low saving is related to the possibility of correcting freshness problems coming from a myopic production planning in the decoupled approach through controlling the temperature of storage in the distribution planning. When products are highly perishable a small decrease in the storage temperature will entail a strong percentage augmentation of shelf-life. Hence, if in a product with 7 days of shelf-life we are able to augment it to 8 days through storing it at cooler temperature, then the percentage increasing of shelf-life is not very significant. But, if the product is highly perishable, then an absolute increase of 1 day will reflect a strong percentage increase. Therefore, the scenario with products subject to intermediate perishability (Base scenario) is the one which gains more from an integrated approach.

5. Conclusions

In this paper, we have discussed the importance of integrating the analysis for a production and distribution planning problem dealing with perishable products. The logistic setting of our operational problem is multi-product, multi-plant, multi-DC and multi-period. We have developed models for two types of perishable products: with fixed shelf-life and with loose shelf-life, always taking into account that customers attribute decreasing value to products while they are aging until they completely perish. The novel formulations allow a comprehensive and realistic understanding of these intertwined planning problems. Furthermore, the loose-shelf-life model was able to incorporate the possibility of dealing with the underlying uncertainty of a random spoilage process with the help of predictive microbiology. To understand the impact of the integrated approach in both the economic and the freshness perspective a multi-objective framework was used. Since the models for the loose shelf-life case were not possible to solve with standard solvers, even for a small example, a simple heuristic was developed for these cases.

Computational results for an illustrative example show that the Pareto front of the integrated approach strongly dominates the Pareto front of the decoupled one for both classes of perishable products. The economic savings that this coupled analysis entail is smoothed as we aim to deliver fresher products. Nevertheless, in the fixed shelf-life case for a 70% mean remaining shelf-life of delivered products we may reach savings around 42%. The explanation regarding the fact that the gap between the integrated and the decoupled approach tends to smooth for very high freshness standards may be due to the reason that in the latter case no inventory is allowed since we are working completely under a JIT policy, turning the problem at hand so constrained that the integrated and coupled solutions are rather the same. The multi-objective framework proved to be essential to draw these multi-perspective conclusions.

This work should be perceived as an exploratory research in this challenging field. Future work should focus on understanding the impact of the integrated approach at the tactical planning level and how these potential benefits and awareness of the cost-freshness relationship can be applied in industrial environments. More in detail, it is important to go deeper into the freshness objective function and evaluate what other indicators besides the mean value of the fractional remaining shelf-life can be modeled and what are the differences in the plans that they yield.

Acknowledgment

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Lot Sizing versus Batching in the Production and Distribution Planning of Perishable Goods

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Abstract Joint production and distribution planning at the operational level has received a great deal of attention from researchers. In most industries these processes are decoupled by means of final goods inventory that allow for a separated planning of these tasks. However, for example, in the catering industry, an integrated planning framework tends to be favorable due to the perishable nature of the products that forces a make-to-order strategy. So far this problem has only been addressed by batching the orders, disregarding the sizing of the lots in the production process. Two exact models are presented for this problem. The first model includes batching decisions and the latter lot sizing decisions. The value of considering lot sizing versus batching is further investigated per type of scenario. Results point that lot sizing is able to deliver better solutions than batching. The added flexibility of lot sizing can reduce production setup costs and both fixed and variable distribution costs. Finally, the savings derived from lot sizing are leveraged by customer oriented time windows and production systems with non-triangular setups.

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1 Introduction

Strategic, tactical and operational integration of the production and distribution processes is reported as being able to deliver better results for companies than a decoupled approach (Park, 2005; Sarmiento and Nagi, 1999). Very often this integration is driven by a management decision, rather than by an actual need of the underlying processes. However, when the final products are not allowed to be stocked due to, for example, freshness reasons this integration scenario becomes imperative. Within these three decision levels, it is on the operational one where more research needs to be conducted (Chen, 2009), since actual models fail to be accurate and detailed enough for the real-world problems.

The motivation for studying the operational integrated production and distribution problem comes from very practical industry situations when it is not possible or advisable to keep final inventory decoupling these two processes. In this case, companies are forced to engage in a make-to-order production strategy. Therefore, the production for a certain demand order may only start after the order arrival. The examples found in practice are related to the computer assembly industries, the food-catering, the industrial adhesive materials or the ready-mixed concrete. The importance of a holistic vision of these processes is driven by very demanding customers requiring a product that cannot wait a long time to be delivered after production. These products, having a very short lifespan, will be called hereafter as perishable. Hence, the considered operational integrated production and distribution problem relates to the decisions on how to serve a set of customers with demand for different products. The planner has to simultaneously decide on the production planning and vehicle routing, in a setting where inventory is not allowed.

Regarding the production process, the definitions proposed by Potts and Van Wassenhove (1992) are followed, where batching is defined as the decision of whether or not to schedule similar jobs contiguously and lot sizing refers to the decision of when and how to split a production lot of identical items into sublots. The modelling of our problem considers a complex production system that is accurately synchronized with the distribution process to allow for as much flexibility as possible. Therefore, no specific industry constraints are modelled, but instead the formulation is as general as possible. Several parallel production lines with sequence dependent setups are taken into account. Moreover, the demand from different customers for a set of products has to be delivered within strict time windows on different routes that have to be determined together with the production planning.

The research community has tackled this operational integrated production and distribution problem by batching orders of customers as if lot sizing decisions were never to yield a better solution in this single period scenario. This is clearly not the case in the production planning literature where the importance of considering lot sizing and scheduling simultaneously is consensual for the multi-period setting (for example Almada-Lobo et al., 2010). To the best of our knowledge, the incorporation of lot sizing decisions in the operational production and distribution problem has never been analysed. Therefore, a major objective of this work is to evaluate whether lot sizing decisions may deliver better results than batching when this integrated problem approaches real-world complexity. After proving that lot sizing should be considered in this problem setting, the aim shifts towards understanding the difference of the solutions found with this extra flexibility and the scenario conditions that leverage the benefits of lot sizing versus batching.

The remainder of this work is organized as follows. The next section reviews the literature on the operational integrated production and distribution problem. Section 3 describes the considered problem and proposes two mathematical formulations for the operational production and distribution problem of perishable goods: one considering batching and the other lot sizing. In Section 4, the results of the computational study are presented and the impact of considering lot sizing versus batching is assessed. Finally, the paper is concluded in Section 5 with the main findings and ideas for future work.

2 Literature Review

The literature in integrated production and distribution problems is vast and, therefore, only the papers very related to the scope of this work will be reviewed here. Our problem statement refers to the gap pointed out, in the review of Chen (2009), about operational integrated models dealing with *multi-customer batch delivery problems with routing*.

The research community has tackled this integrated production and distribution problem by batching orders in the production process. In Chen and Vairaktarakis (2005) orders are delivered right after their production completion time. The authors model a single product to be scheduled on the production line(s) and an unlimited number of vehicles, with a fixed capacity, which perform the routing. Two objective functions are considered for a variety of related problems. This work also investigates the value of integration, comparing the use of a decoupled versus an integrated approach. They conclude that the improvement is more significant when the goal is to minimize the average delivery time than the maximum delivery time. In Geismar et al. (2008) product perishability is taken into account and there is a single production facility with a constant production rate. The routing process is performed by a single, capacitated vehicle that may return to the facility, therefore, performing multiple trips during the planning period. The objective is to determine the minimum makespan of the integrated production and distribution for a given set of customers. Armstrong et al. (2007) solve a related problem with a single product subject to a fixed lifespan that is also delivered by a single vehicle, but, in this case, there is no possibility of performing multiple trips. Moreover, the sequence of production and distribution is fixed and forced to be the same. Finally, Chen et al. (2009) present a model that considers stochastic demand for multiple products subject to perishability. The production environment does not consider setups between products and the delivery function is assured by a set of capacitated vehicles, however, the vehicle operating costs are disregarded.

Again, none of the aforementioned papers on the operational integrated production and distribution planning include lot sizing decisions. However, on pure production scheduling, the advantages of lot sizing over batching for a leaner environment have been proven. Santos and Magazine (1985); Wagner and Ragatz (1994); Low and Yeh (2008) show how lot sizing can reduce lead time in the scheduling of machines. Moreover the impact of setup times is investigated. Nieuwenhuyse and Vandaele (2006) proves that lot sizing improves the reliability of the deliveries in a system accounting for production and direct deliveries to customers.

Based on this literature review the contribution of this paper is clearer. First, it investigates the potential performance improvement that lot sizing decisions may add to the operational production and distribution planning. Second, previous studies are extended by considering a more general production system with sequence-dependent costs and times between products.

3 Problem Statement and Mathematical Formulations

In this section, the problem statement is given as well as two mathematical formulations for this problem. The first formulation models the operational integrated production and distribution problem that only considers batching of orders (I-BS-VRPTW) and the second formulation extends the first one by considering the sizing of the lots (I-LS-VRPTW). Both models are then compared.

The operational integrated production and distribution planning problem considered in this work consists of a set M of parallel lines $l = 1, ..., m$ with limited capacity that produce a set P of items $i, j = 1, ..., p$ to be delivered to a set N of customers $c, d = 1, ..., n$ through a set A of arcs (c, d). The delivery is assured by a set K of identical fixed capacity vehicles indexed by $k = 1, ..., m$ initially located at a depot. Hence, the routing can be defined on a direct graph $G = (V, A)$, $V = N \cup \{0, n+1\}$, where the depot is simultaneously represented by the two vertices 0 and $n+1$, and, therefore, $|V| = n + 2$.

Some of the products may be perishable while others last substantially beyond the considered planning horizon. Furthermore, the utilization of equipment, such as ovens in the food-catering, makes the changeover between different products dependent on the sequence. Hence, products are to be scheduled on the parallel production lines over a finite planning horizon that ranges up to the time of the last scheduled delivery.

The distribution is performed using several vehicles serving multiple customers on different routes. There exists a variable cost dependent on the total distance travelled and a fixed cost for each vehicle used. It is assumed that there are no fleet constraints such that any distribution plan can be executed. This assumption is realistic since reference contracts are usually established assuring that there always exists a fleet of sufficient size available. The two models determine the routing taking into account the vehicle capacity, and the time and cost to travel from one customer/depot to another. A customer order may aggregate several products that have to be delivered within strict time windows with a single delivery (i.e., split deliveries are not allowed). Moreover, it is assumed that demand is dynamic and deterministic.

The challenge is to model the production and distribution problem that minimizes total cost of the supply chain covering these processes over the short planning horizon.

The main advantage of these models comes from the accurate synchronization of the two planning processes. While at the tactical level the integrated production and distribution planning has the possibility to assume that at the end of the period, after production, one will start the delivery process to all customers that is not possible at the operational level. At this level one needs to go one step further and be sure that the customer orders production times are accurately traced so that as soon as a customer has his order completed, the vehicle servicing him may depart. However, the departure only takes place after the last customer's (serviced by the same vehicle) order has been produced.

Consider the following indices, parameters and decisions variables needed to formulate the operational production (with batching or lot sizing decisions) and distribution models.

Parameters

Cap_l	available capacity (= latest completion time) of production line l
$CapV$	vehicle capacity
s_c	service time of customer c
$ct_{cd}(tt_{cd})$	cost (time) of travelling from customer c to d
$\bar{f}t$	fixed cost associated with each vehicle k
$[a_c, b_c]$	time window for customer c
sl_j	shelf-life of product j (time)

Decision Variables

f_c	completion time of the production of customer c's order
x^k_{cd}	equals 1, if arc (c, d) is used by vehicle k (0 otherwise)
w^k_c	starting time at which vertex c is serviced by vehicle k

3.1 Integrated Batch Scheduling and Vehicle Routing Problem (I-BS-VRPTW)

This formulation is based on the work of Mendez et al. (2000). For each pair product-customer (j, c), a job h is associated in case customer c has a positive demand for product j. Let G denote the set of these jobs ($G = \{1, ..., g\}$).

In order to formulate the integrated problem considering batching decisions, the following additional parameters and decision variables are needed to be added to the aforementioned ones.

Parameters

$cp_{lh}(tp_{lh})$	production cost (time) on line l of job h
$scb_{lh'h}(stb_{lh'h})$	sequence dependent setup cost (time) on line l of a changeover from job h' to job h
$scb0_{lh}(stb0_{lh})$	sequence dependent setup cost (time) on line l if job h is the first scheduled (depends on the initial setup state of the line)
dem_h	quantity demanded for job h (units)

Decision Variables

R_{lh}	equals 1, if job h is produced on line l (0 otherwise)
$R0_{lh}$	equals 1, if job h is the first to be produced on line l (0 otherwise)
RN_{lh}	equals 1, if job h is the last to be produced on line l (0 otherwise)
$V_{h'h}$	equals 1, if job h is scheduled after h' (0 otherwise)
Ct_h	completion time of job h

The batch scheduling coupled with the vehicle routing problem with time windows (I-BS-VRPTW) may be formulated as follows:

I-BS-VRPTW

$$\min \sum_{l,h',h} scb_{lh'h}V_{h'h} + \sum_{l,h} scb0_{lh}R0_{lh} + \sum_{l,h} cp_{lh}R_{lh} + \bar{f}t\sum_{k}(1 - x^k_{0,n+1}) + \sum_{k}\sum_{c,d} ct_{cd}x^k_{cd} \quad (1)$$

subject to

$$\sum_{h} R0_{lh} = 1 \quad \forall l \quad (2)$$

$$R0_{lh} \leq R_{lh} \quad \forall l, h \quad (3)$$

$$\sum_{h} RN_{lh} = 1 \quad \forall l \quad (4)$$

$$RN_{lh} \leq R_{lh} \quad \forall l, h \quad (5)$$

$$\sum_{l} R_{lh} = 1 \quad \forall h \quad (6)$$

$$R_{lh'} + V_{h'h} \leq R_{lh} + 1 \quad \forall l, h', h \quad (7)$$

$$\sum_{l} R0_{lh} + \sum_{h'} V_{h'h} = 1 \quad \forall h \quad (8)$$

$$\sum_{l} RN_{hl} + \sum_{h'} V_{hh'} = 1 \quad \forall h \quad (9)$$

$$Ct_h \geq Ct_{h'} + \max_{l}\{Cap_l\}(V_{h'h} - 1) + \sum_{l}(tp_{lh} + stb_{lh'h})R_{lh} \quad \forall h', h \quad (10)$$

$$Ct_h \geq \sum_{l}(tp_{lh} + stb0_{lh})R0_{lh} \quad \forall h \quad (11)$$

$$Ct_h \leq \max_{l}\{Cap_l\} + (Cap_l - \max_{l}\{Cap_l\})R_{lh} \quad \forall l, h \quad (12)$$

$$f_c \geq Ct_{(j,c)} \quad \forall c, (j, c) \in G \quad (13)$$

$$Ct_{(j,c)} - tp_{l,(j,c)} + sl_j - \sum_{k} w^k_c \geq 0 \quad \forall l, c, (j, c) \in G \quad (14)$$

$$w^k_0 \geq f_c - \max_{l}\{Cap_l\}(1 - \sum_{d} x^k_{cd}) \quad \forall k, c \quad (15)$$

$$\sum_{k}\sum_{d} x^k_{cd} = 1 \quad \forall c \quad (16)$$

$$\sum_{d} x^k_{0d} = 1 \quad \forall k \quad (17)$$

$$\sum_{c} x^k_{cd} - \sum_{c} x^k_{dc} = 0 \quad \forall k, d \quad (18)$$

$$\sum_{c} x^k_{c,n+1} = 1 \quad \forall k \quad (19)$$

$$w^k_d \geq w^k_c + s_c + tt_{cd} - M_{cd}(1 - x^k_{cd}) \quad \forall k, c, d \quad (20)$$

$$a_c \sum_{d} x^k_{cd} \leq w^k_c \leq b_c \sum_{d} x^k_{cd} \quad \forall k, c \quad (21)$$

$$\sum_{(j,c)\in G} dem_{(j,c)} \sum_{d} x^k_{cd} \leq CapV \quad \forall k \quad (22)$$

$$f_c,\, Ct_h,\, w^k_c \geq 0; \quad R_{lh},\, R0_{lh},\, RN_{lh},\, V_{h'h},\, x^k_{cd} \in \{0, 1\} \quad (23)$$

Objective function (1) minimizes supply chain related costs, namely: sequence dependent setup costs, variable productions costs, and fixed and variable transportation costs.

Constraints (2) - (6) assign each job h to a line either in the beginning, in the end or in the middle of the scheduling sequence. Constraints (7) ensure that consecutive jobs are assigned to the same line. Equations (8) establishes that a job is either assigned in the beginning of the scheduling or preceded by other job. Similarly, equations (9) imposes that a job is assigned at the end of the scheduling or precedes other job. For tracing the completion time of each job, constraints (10) and (11) are used. Note that in (10), $\max\{Cap_l\}$ denotes the latest possible completion time due to the capacity limitations of the lines. Also, these constraints are responsible for the job scheduling. Job completion time must not exceed the available capacity of the line which is (12) assigned (12). To define f_c that tracks the customer order finishing time, constraints (13) are used. To account for perishability, (14) assures that the delivery is performed while products still have some lifetime.

In (15) the link between production and the vehicle departing times is established. This synchronization ensures that a vehicle only departs after the completion of the production for all customers visited along the vehicle's route. Constraints (16)-(22) refer to the distribution process. Each customer is visited exactly once by (16), while constraints (17)-(19) ensure that each vehicle is used once and that flow conservation is satisfied at each customer vertex. The consistency of the time variables w_c^k is ensured through constraints (20), while time windows are imposed by (21). Regarding the vehicle capacity, constraints (22) enforce it to be respected. Finally, the domain of the variables is limited by (23).

3.2 Integrated Lot Sizing and Scheduling and Vehicle Routing Problem (I-LS-VRPTW)

Due to production planning modelling reasons, the planning horizon is divided in the lot sizing formulation into a fixed number of non-overlapping slots, indexed by s, of variable length. Since the production lines can be independently scheduled, this partition is done for each line separately ($s \in S_l$). The length of a production slot is a decision variable that is a function of the production quantity of a certain product on a line and of the time to set up the machine for this product in case it is required. A sequence of consecutive production slots, where the same product is produced on the same line, defines the size of the lot of a product. Therefore, a lot may span over several slots. The number of production slots of a certain line defines the upper bound on the number of setups and deliveries to be performed during the planning horizon.

Contrarily to the more tactical lot sizing and scheduling formulations that integrate the delivery process (Boudia et al., 2007), this model considers a continuous time scale since the external factors, such as demand are pulled from the customer desires, expressed in its time window boundaries. It is interesting to notice that the slot structure of the mathematical formulation related to the production planning resemble the micro-period time structure of the general lot sizing and scheduling problem (Fleischmann and Meyr, 1997).

Consider the additional parameters and decision variables.

Parameters

$cp_{lj}(tp_{lj})$	production cost (time) of product j (per unit) on line l
$scb_{lij}(stb_{lij})$	sequence dependent setup cost (time) of a changeover from product i to product j on line l
y_{lj0}	equals 1, if line l is set up for product j at the beginning of the planning horizon
dem_{jc}	demand for product j at customer c (units)

Decision Variables

q_{ljs}^c	quantity of product j produced in slot s on line l to serve customer c
y_{ljs}	equals 1, if line l is set up for product j in slot s (0 otherwise)
z_{lijs}	equals 1, if a changeover from product i to product j takes place at the beginning of slot s on line l (0 otherwise)
str_{ls}	starting time of production slot s on line l
λ_{ljs}^c	equals 1, if there is production of product j for customer c in production slot s on line l (0 otherwise)
F_j^c	starting time of the lifespan of product j for customer c

The lot sizing and scheduling coupled with the vehicle routing problem with time windows (I-LS-VRPTW) is formulated as follows:

I-LS-VRPTW

$$\min \sum_{l,i,j,s} scb_{lij} z_{lijs} + \sum_{l,j,s,c} cp_{lj} q_{ljs}^c$$
$$+\bar{f}t \sum_{k} (1 - x_{0,n+1}^k) + \sum_{k} \sum_{c,d} ct_{cd} x_{cd}^k \quad (24)$$

subject to

$$\sum_{l,s} q^c_{ljs} = dem_{jc} \quad \forall j, c \tag{25}$$

$$\sum_{c} q^c_{ljs} \leq \frac{Cap_l}{tp_{lj}} y_{ljs} \quad \forall l, j, s \tag{26}$$

$$\sum_{j} y_{ljs} = 1 \quad \forall l, s \tag{27}$$

$$\sum_{i,j,s} stb_{lij} z_{lijs} + \sum_{j,s,c} tp_{lj} q^c_{ljs} \leq Cap_l \quad \forall l \tag{28}$$

$$z_{lijs} \geq y_{li,s-1} + y_{ljs} - 1 \quad \forall l, i, j, s \tag{29}$$

$$str_{l1} = 0 \quad \forall l \tag{30}$$

$$\begin{aligned} str_{ls} \geq\ & str_{l,s-1} + \sum_{i,j} stb_{lij} z_{lij,s-1} \\ & + \sum_{j,c} tp_{lj} q^c_{lj,s-1} \quad \forall l, s > 1 \end{aligned} \tag{31}$$

$$q^c_{ljs} \leq Cap_l \lambda^c_{ljs} \quad \forall l, j, s, c \tag{32}$$

$$\begin{aligned} f_c \geq\ & -Cap_l(1 - \sum_{j} \lambda^c_{ljs}) + str_{ls} + \sum_{i,j} stb_{lij} z_{lijs} \\ & + \sum_{j,d} tp_{lj} q^d_{ljs} \quad \forall l, s, c \end{aligned} \tag{33}$$

$$\begin{aligned} F^c_j \leq\ & Cap_l(1 - \lambda^c_{ljs}) + str_{ls} \\ & + \sum_{i} stb_{lij} z_{lijs} \quad \forall l, j, s, c \end{aligned} \tag{34}$$

$$F^c_j + sl_j - \sum_{k} w^k_c \geq 0 \quad \forall j, c : dem_{jc} > 0 \tag{35}$$

(15) - (22)

$$\begin{aligned} & q^c_{ljs},\ z_{lijs},\ str_{ls},\ f_c,\ F^c_j,\ w^k_c \geq 0; \\ & \qquad y_{ljs},\ \lambda^c_{ljs},\ x^k_{cd} \in \{0, 1\} \end{aligned} \tag{36}$$

In the objective function (24) the same costs are minimized as in batching related formulation.

Looking now at the constraints that this problem is subject to, demand is to be satisfied with production that may come from different lines (25). Constraints (26) ensure that a product can only be produced if there exists a setup for it and constraints (27) limit to one the number of products to be simultaneously produced on each line. Limited capacity in the lines is to be used by setup times and the time consumed producing products (28). The connection between setup states and changeover indicators for products is established by (29). In order to define f_c that tracks the customer order finishing time in constraint (33), the starting time of each production slot is traced with (30) and (31). Requirements (32) determine the customers for which the production in a given slot is devoted to. It is worth mentioning that this production may satisfy demand from several customers. Constraints (34) and (35) account for product perishability similarly to equations (13) and (14). Note, that the model formulation allows for the production of the same product for different customers in a single slot. In such a case f_c and F^c_j are considering only the end and the start of the time slot, but not the exact time of production products for a certain customer. But this can always be avoided by producing the same product of different customer orders in separate (possibly subsequent) slots without additional cost or capacity needs ($scb_{lii} = stb_{lii} = 0$). Constraints (15)-(22) from the previous model are also used in this one.

The domain of variables is stated in (36) and the remaining constraints come from the integrated model with batching decisions (I-BS-VRPTW).

3.3 Relation Between both Models

The meaning of the main decision variables of both formulations is graphically presented in Figure 1. It is easy to see that both solutions of this illustrative example are equivalent, as the two jobs of I-BS-VRPTW are not split in the I-LS-VRPTW.

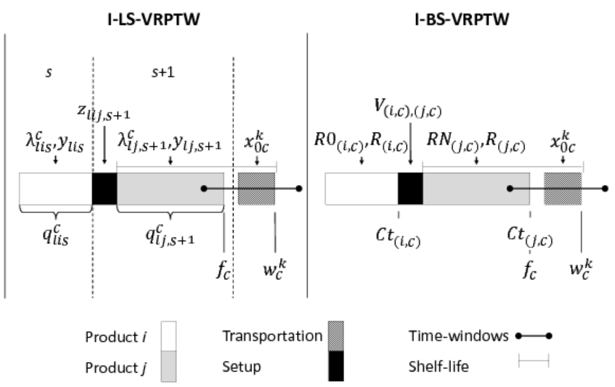


Fig. 1 Comparing the decision variables of I-BS-VRPTW and I-LS-VRPTW.

In the following theorem it is shown that the optimal solution to I-LS-VRPTW is at least as good as the optimal solution to I-BS-VRPTW. Let $\nu(\cdot)$ denote the optimal values of underlying optimization problems.

Theorem 1 *We have* $\nu(I - LS - VRPTW) \leq \nu(I - BS - VRPTW)$.

Proof We prove the statement by showing that I-BS-VRPTW is a special case of I-LS-VRPTW and therefore any feasible solution to I-BS-VRPTW is also feasible to I-LS-VRPTW. Let model $\overline{f}_{LS}$ be derived from I-LS-VRPTW by adding to the latter the following constraints: $\sum_{l,s\in S_l} \lambda^c_{ljs} = 1$, for every j in N and c in C, and $\sum_{c,j} \lambda^c_{ljs} = 1$, for every l and s in S_l. These conditions mean that demand for a given pair product j-customer c is produced in just one lot, and that each production slot can only be allocated to pair $j - c$. Now, we show the equivalence between I-BS-VRPTW and $\overline{f}_{LS}$. Let $Q^*(f_c, Ct_h, w^k_c, R_{lh}, R0_{lh}, RN_{lh}, V_{h'h}, x^k_{cd})$ be a feasible solution to I-BS-VRPTW. Each job h entails a product j to be produced and delivered to a customer c. Consider in the following a given line l. Each job of I-BS-VRPTW relates to one production slot of $\overline{f}_{LS}$. The sequence $(h_1, h_2, \ldots, h_g)$ can be easily transformed into the sequence $(j_1 - c_1, j_1 - c_2, \ldots, j_p - c_n)$, where the quantity of each product produced in each slot (q^c_{ljs}) equals the amount of demand of the respective job. In case job h in I-BS-VRPTW is produced in the s-th position of the sequence, its completion time (C_h) is equivalent in $\overline{f}_{LS}$ to the finishing time of the s-th slot where the respective product j is produced to supply the same customer c (i.e. $C_h = str_{ls} + \sum_{i,j} stb_{lij} z_{lijs} + \sum_{j,c} tp_{lj} q^c_{ljs}$). Moreover, the starting time of the lifespan of product j for customer c (F^c_j) in $\overline{f}_{LS}$ is equivalent to the term $C_h - tp_{lh}$ of the respective job in I-BS-VRPTW. Clearly, Q fulfils the constraints related to the production part of $\overline{f}_{LS}$, from (25) to (34). The routing-related requirements are the same in both formulations. This clearly shows that ν(I-LS-VRPTW)$\leq \nu(\overline{f}_{LS}) \leq \nu$(I-BS-VRPTW).

4 Computational Study

This section aims at quantifying the impact of considering lot sizing versus batching and analysing the solution changes that this extra production flexibility yields. To this end, a set of instances have been systematically generated with different parameters. Due to the strong NP-hardness of the operational integrated production and distribution problem, it is not possible to find even integer solutions to medium-size instances with mixed-integer programming (MIP) solvers. Therefore, we rely on small instances across the computational experiments to benchmark both formulations and respective solutions. Next it is reported how the test instances are generated. Afterwards, the computational results are presented and, finally, some examples comparing the improvements of the lot sizing over the batching solutions are analysed.

4.1 Data Generation

The instance generators used by Haase and Kimms (2000), Armstrong et al. (2007) and Viergutz (2011) are integrated since, to the best of our knowledge, there are no reported instances for the settings of this problem. A total of 120 small instances were generated. The impact of different key parameters on the lot sizing importance is verified by varying: the *number of perishable products*, the *length of the shelf-life*, the *setup time and cost structure*, the *tightness of the time windows* and the *orientation of the time windows.*

For the sake of compactness, the description of parameters' generation is exposed only for I-LS-VRPTW. However, the data conversion for I-BS-VRPTW is straightforward. The number of lines m is set to 1 and for all products $tp_{lj} = 1$ and $cp_{lj} = 0$. In the beginning of the planning horizon the machine is set up for product 1. There are 3 items ($p = 3$) to be produced for 5 customers ($n = 5$). The number of production slots S_l is set to $p \times n$ in order to ensure that all necessary setups and deliveries may be performed. 75% of the demand dem_{jc} is generated from the uniform distribution in the interval $U[40, 60]$ and the remaining 25% is set to 0. The *number of perishable products* (PP) can be 1 or 2 out of 3 items. In order to define the *length of the shelf-life* of perishable products (sl_j), parameter SL is multiplied by the average quantity of a demand order. This parameter SL can be 3 or 5.

The *setup time and cost structure* may obey or not to the triangular inequality. In case setups obey to triangular inequality, in the optimal solution the production of the same product will never take place twice in the same period. On the contrary, setups not obeying to the triangular inequality, which are frequent in the food industry with the use of cleaning products, may result in optimal solutions in which the same product is set up more than once in the same period (favoring the lot sizing). For the instances with triangular setup times (TS) between products stb_{lij}, $U[6, 10]$ is used for all pairs (except for the case where $i = j$, where the setup is 0). The instances not obeying to such inequality (NTS) have setup times chosen randomly from $U[1, 5]$. The setup costs scb_{lij} of a changeover from product i

to j are computed as:

$$scb_{lij} = 25.0 stb_{lij} \text{ and } scb_{lij} = 66.67 stb_{lij},$$

for triangular and non-triangular setups, respectively. Both setup structures have an average setup cost of 200 units. The line capacity Cap_l is determined according to:

$$Cap_l = \frac{\sum_{jc} dem_{jc}}{0.6}.$$

It is important to notice that the utilization of capacity (0.6) is an estimate only, as setup times do not influence the computation of Cap_l.

For the computation of the travel times tt_{cd} and costs ct_{cd}, which are assumed to be the same, all customers are positioned randomly in a square of locations from (0,0) to (100,100). The Euclidean distance is then calculated between all pairs of customers (assuming that travel times are equal to the travel distances) fixing the depot at the point (50,50). The number of available vehicles is set to n and the cost of using each vehicle $\bar{f}t$ is set to 250. This value was set after preliminary computational experiments to reflect the industry practice in relation to the vehicle variable costs. The capacity of the vehicle is computed through the expression

$$CapV = 0.5 \sum_{jc} dem_{jc}.$$

The last parameters are the time windows of each customer (parameters a_c and b_c), which are calculated by four different methods that change the *tightness* and *the orientation of the time windows*. These methods are described in Appendix A. With regard to the *tightness*, instances with standard (S) and loose (L) time windows are considered. Concerning their *orientation*, instances with time windows oriented by production requirements (P) and by customers' demand (C) are considered.

By varying the aforementioned parameters, 24 types of instances are generated. For each of them, 5 random instances are considered. All the 120 instances were tested for feasibility purposes on the I-BS-VRPTW model with a commercial solver. In case a solution had not be found, then a new instance was generated until feasibility was achieved.

4.2 Computational Results

All computational experiments were performed on an Intel Core i5 processor, with 2.80 GHz CPU and 8GB of random access memory under Linux Ubuntu 10.04, 64 bit. CPLEX version 12.2 from IBM was used as the MIP solver. The data generator described in Section 4.1 was used to obtain the instance set. The computational time to solve each MIP is limited to 3600 seconds. As the I-BS-VRPTW was solved to optimality by CPLEX within a maximum/average running time of 126.97/6.07 seconds, these solutions were used as a starting point for the I-LS-VRPTW (i.e. they were injected into its branch-and-bound tree).

In this subsection, two different gaps are used to evaluate the results. The first gap, called gap_{LB}, denotes the relative difference between the upper bound (UB_L) and the lower bound (LB_L) obtained by CPLEX for the I-LS-VRPTW. The second gap, called gap_{sol}, refers to the relative difference of solutions between the I-LS-VRPTW (UB_L) and I-BS-VRPTW (UB_B). These gaps are calculated as:

$$gap_{LB} = \frac{UB_L - LB_L}{UB_L} \text{ and } gap_{sol} = \frac{UB_B - UB_L}{UB_L}.$$

Tables 1 reports the integrality gap gap_{LB} of I-LS-VRPTW for all the instances. The sign "-" means that CPLEX was able to solve to optimality the respective instance within one hour. This was the case for 31 out of 120 instances. For the remaining problems, CPLEX reported an average gap_{LB} of 17.0%, achieving the maximum of 37.7% for one instance of the class $PP = 2$, $SL = 3$, $PP - S - TS$. The instances with more perishable products ($PP = 2$) and with smaller shelf-lives ($SL = 3$) presented on average a greater gap_{LB}. Moreover, customer oriented time windows (C) results in more problems not solved to optimality.

Table 2 provides the solution improvement gap_{sol} of I-LS-VRPTW over I-BS-VRPTW for the same set of instances. Here, a sign "-" means that the I-BS-VRP-TW solution was not improved by I-LS-VRP-TW model, within the time limit. The cause behind the solution improvements also presented in the same table. In general, the cost decrease on the solution of I-LS-VRPTW may yield five main changes in relation to the solution of I-BS-VRPTW:

- St-(+): number of setup operations;
- Sc-(+): total setup cost;
- Seq: setup sequence;
- Dist-(+): distance travelled;
- V-(+): number of used vehicles.

The signs - (+) mean a decrease (increase) of the indicator of the respective change. Notice that, contrarily to the case of triangular setup structure, the case of non-triangular setups may allow for setup inclusions (St+) that result in setup cost reduction (Sc-). Therefore, Sc- is omitted for triangular setups when the related changes are due to St- or Seq.

I-LS-VRPTW obtained better solutions for 35 out of 120 instances. In 22 instances, both formulations reported the same provably optimal solution, while for

Table 1 Integrality gaps (gap_{LB}) for I-LS-VRPTW (lot sizing).

PP	SL	#	P-S-TS	P-L-TS	P-S-NTS	C-S-TS	C-L-TS	C-S-NTS
1	3	1	-	5.8%	-	19.4%	2.6%	12.5%
		2	11.5%	-	8.1%	24.8%	5.4%	-
		3	15.0%	7.9%	12.5%	19.2%	18.5%	6.6%
		4	5.8%	-	7.7%	20.0%	-	22.1%
		5	25.9%	20.5%	19.2%	27.4%	1.3%	11.6%
	5	1	-	1.4%	-	19.6%	0.0%	11.6%
		2	11.6%	7.5%	-	24.9%	7.4%	12.4%
		3	0.2%	-	-	19.6%	9.4%	9.6%
		4	-	-	-	13.4%	-	23.6%
		5	-	-	-	24.7%	4.9%	22.3%
2	3	1	7.6%	5.9%	7.5%	17.4%	3.0%	16.1%
		2	24.9%	29.6%	27.1%	25.8%	33.8%	31.6%
		3	17.2%	23.5%	20.1%	0.0%	27.0%	-
		4	25.4%	28.3%	28.2%	25.3%	26.4%	27.5%
		5	37.7%	33.4%	31.7%	16.9%	25.6%	23.3%
	5	1	18.2%	9.0%	4.8%	9.6%	-	6.0%
		2	-	-	-	18.3%	-	15.9%
		3	-	-	-	28.9%	20.2%	20.4%
		4	9.7%	-	-	22.4%	9.4%	14.4%
		5	6.5%	-	-	24.8%	14.8%	8.2%

PP - Number of Perishable Products, SL - Length of the Shelf-life, # - Instance Number, P - Production Oriented Time Windows, C - Customer Oriented Time Windows, S - Standard Time Windows, L - Loose Time Windows, TS - Triangular Setup Structure, NTS - Non Triangular Setup Structure

the remaining 63 it is still unknown whether it may pay-off to use the lot sizing formulation (for which I-LS-VRPTW was not solved to optimality). The maximum gap_{sol} is 20.0% caused by the reduction of setup operations. The average gap_{sol}, for instances with positive gaps, is 6.5%. The main cause of cost decrease, when lot sizing is allowed, is the reduction of setup operations, which was responsible for 21 out of the 35 instances improved. Customer oriented time windows (C) has leveraged the number of solutions improved by lot sizing. Loose time windows (L) permitted more changes related to distribution decisions. Moreover, non-triangular setups (NTS) increased the number of instances improved by I-LS-VRPTW.

4.3 Solution Examples

In this subsection, illustrative examples of instances in which the I-LS-VRPTW overcomes the I-BS-VRPTW are shown. In each example, two Gantt charts are used to represent graphically the solutions. The top chart represents the Gantt chart of the I-BS-VRPTW solution and the bottom illustrates the I-LS-VRPTW solution. Customers are arranged according to their time windows boundaries and vertically at the Gantt chart, from customer 1 to 5. Products 1, 2 and 3 are denoted by light grey, dark grey and dotted bars, respectively. Setup operations are in black colour bars. The shelf-lives of perishable products are represented by thin white bars starting at the beginning of the production process. The time windows boundaries are indicated by two vertical lines delimiting delivery operations. The travel time from the depot (or customer) to a customer is represented by 45 degree downward hatch box and the opposite operation, from customer to depot, by a 45 degree upward hatch bar. On the right side of the graphs, the route (R) number to which the customer belongs to is denoted. Moreover, the jobs that were split are pointed out by an arrow in the respective I-LS-VRPTW graphical solution.

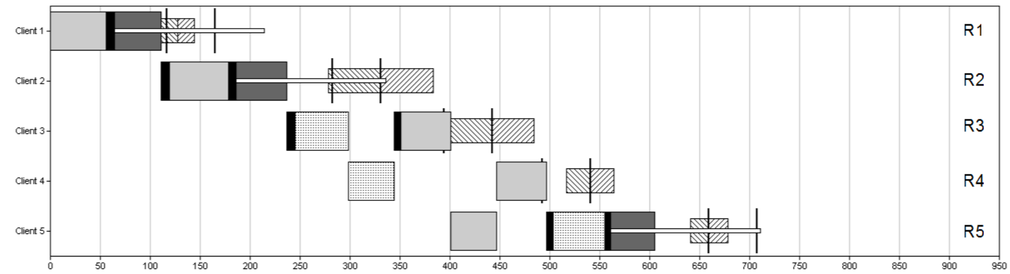
In example 1 of Figure 2 lot sizing can improve the solution of I-BS-VRPTW by reducing setup operations (St-). In the I-BS-VRPTW solution the setup sequence is $(1, 2, 1, 2, 3, 1, 3, 2)$. With the lot sizing flexibility, it is possible to better use the shelf-life limitation of product 2 for customer 2 and rearrange the production sequence by sizing the lot of product 1 for customer 2. Thus, the new setup sequence is $(1, 2, 1, 3, 2, 1)$, which entails two less setup operations, one for product 2 and one for product 3. The delivery operations are the same for both solutions.

Example 2 (Figure 3) is similar to example 1, but instead of reducing the number of setup operations, lot sizing has enabled a modification of the setup sequence (Seq), resulting in a lower solution cost. This example shows the importance that lot sizing can have when setup costs do not obey to the triangular inequality. It is noticeable that the changeover from product 1 to 2 is more economic if product 3 is produced in between. The lot sizing operation allows for such setup sequence, while the products are still delivered without getting spoiled. Moreover, by sizing the lot of product 1 for cus-

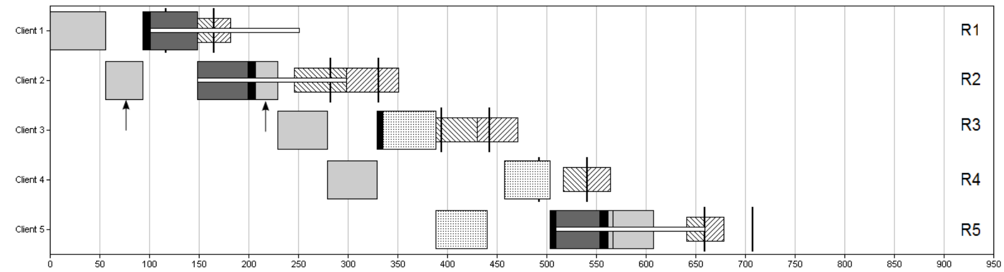
Table 2 Gaps between batching and lot sizing solutions.

PP	SL	#	P-S-TS	P-L-TS	P-S-NTS	C-S-TS	C-L-TS	C-S-NTS
1	3	1	-	-	-	2.9% (St-)	-	6.1% (Seq)
		2	-	-	-	11.2% (St-)	1.7% (Dist-)	1.3% (Dist-)
		3	-	-	-	-	-	-
		4	-	3.6% (V-,Dist-,St+)	-	15.3% (St-)	8.7% (St-)	9.0% (Seq)
		5	-	3.4% (Dist+,St-)	6.8% (St-)	2.7% (St-)	8.7% (Dist+,St-)	-
	5	1	-	-	-	1.0% (Seq)	-	-
		2	-	-	-	-	-	2.9% (St+,St-)
		3	-	6.3% (V-,Dist-,St+)	-	-	2.3% (St-)	6.0% (St-)
		4	-	-	-	-	-	-
		5	-	-	3.9% (Dist+,St-)	-	-	-
2	3	1	9.3% (St-)	-	2.4% (St+,Sc-)	8.1% (St-)	9.3% (St-)	15.3% (Seq)
		2	-	-	-	-	-	-
		3	-	-	20.0% (St-)	13.3% (St-)	-	-
		4	-	-	-	-	-	2.7% (St+,Sc-)
		5	-	-	2.4% (St+,Sc-)	16.3% (St-)	-	-
	5	1	-	-	0.9% (Dist-)	-	2.5% (St-)	9.1% (Dist+,St-)
		2	-	-	-	-	-	3.4% (St-)
		3	-	-	-	-	-	-
		4	-	-	4.9% (St-)	-	-	-
		5	-	-	3.4% (St+.Sc-)	-	-	-

PP - Number of Perishable Products, SL - Length of the Shelf-life, # - Instance Number, P - Production Oriented Time Windows, C - Customer Oriented Time Windows, S - Standard Time Windows, L - Loose Time Windows, TS - Triangular Setup Structure, NTS - Non Triangular Setup Structure



(a) I-BS-VRPTW



(b) I-LS-VRPTW

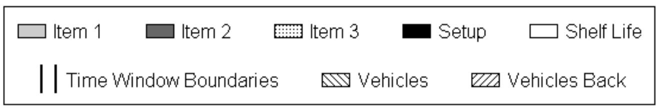
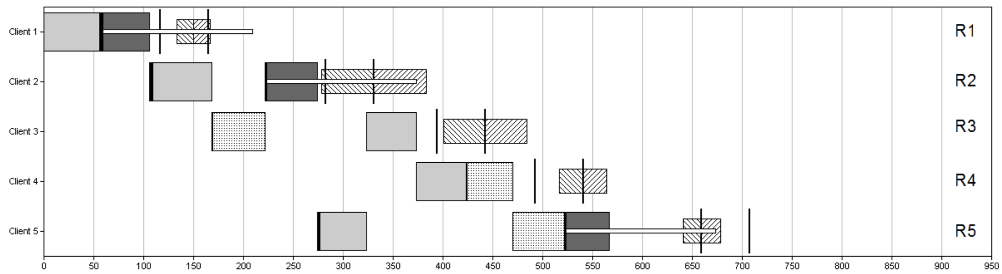
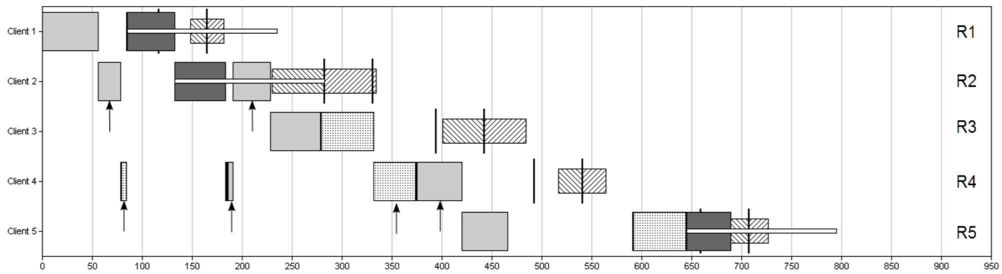


Fig. 2 I-BS-VRPTW and I-LS-VRPTW solutions to instance PP=1, SL=3, #=4, C-S-TS (St-).



(a) I-BS-VRPTW



(b) I-LS-VRPTW

Fig. 3 I-BS-VRPTW and I-LS-VRPTW solutions to instance PP=1, SL=3, #=4, C-S-NTS (Seq).

tomer 2 it was possible to reduce one setup for product 2.

In the example of Figure 4, the difference between batching and lot sizing solutions is once again related to the reduction of setups. However, in this case, the delivery operations were also changed (Dist+, St-). The splitting of job (3,3) - product 3 for customer 3 - allowed a single batch production of product 2. This production change yields a different routing maintaining the same number of vehicles. Hence, the reduction of the setup costs counterweights the increase of the distance travelled.

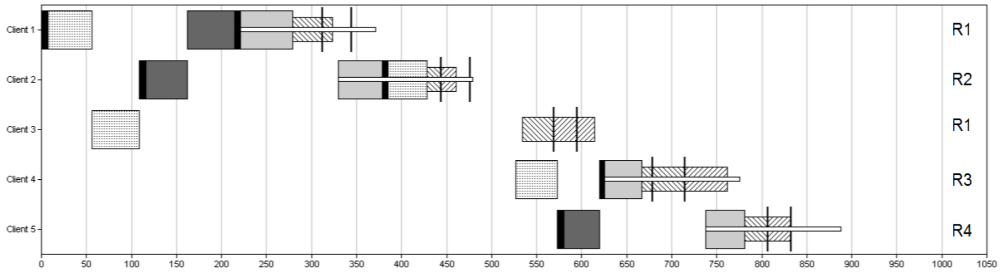
Figure 5 shows an instance where the travel costs decrease due to the routing change provided by lot sizing (Dist-) and the setup costs remain unchanged. The batching solution uses a vehicle for supplying customers 1 and 4 and another for customers 2 and 3. When lot sizing is allowed, customers 1 and 3 are part of the same vehicle's route while customers 2 and 4 belong to other.

Finally, Figure 6 illustrates the improvement of a batching solution by means of the reduction of one vehicle (V-, Dist-, St+). With the splitting of job (1,2) - product 1 for customer 2, it is possible to serve customers 1 and 4 along the same route. It is interesting to note that in this solution, the usage of customers' time windows up to the boundary. In the batching solution, only customers 3 and 4 share a vehicle's route, while all the others are supplied by different vehicles. On the other hand, the lot sizing solution only uses three vehicles, also reducing the travel costs. However, more setup operations are needed increasing the total setup costs (that does not surpass the distribution costs decrease). In the batching solution, the setup sequence is $(1,2,3,2)$, against $(1,3,2,1,3,2)$of the lot sizing model.

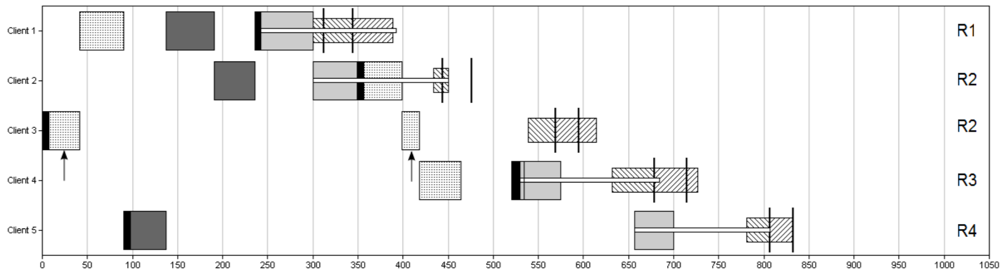
5 Conclusions and Future Work

In this paper, we have analysed the importance of considering sizing the lots (or in other words, splitting the jobs) besides pure batching at the operational production and distribution planning. The logistic setting of our operational problem encompasses multiple perishable products subject to sequence dependent changeovers, which have to be delivered in a certain route by one of the available vehicles. We have developed models for accurately integrating both lot sizing and batching with the vehicle routing problem with time windows. In order to understand the impact of the extra flexibility coming from the possibility of splitting the lots, an experiment varying different key parameters is designed and the solutions between the batching and lot sizing models are compared.

Computational results for the set of systematically generated instances show that lot sizing is able to decrease the integrated production and distribution costs on very different types of instances. Both customer ori-

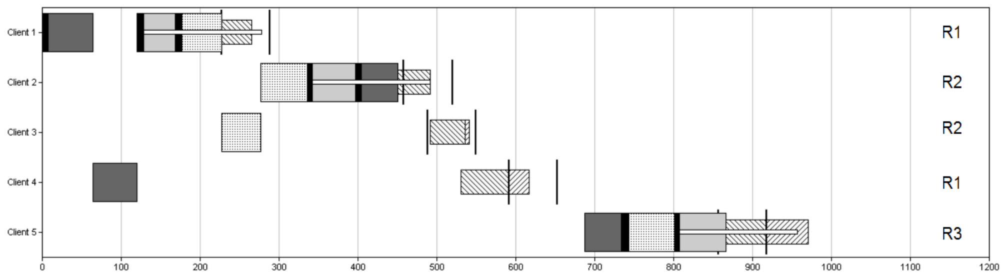


(a) I-BS-VRPTW

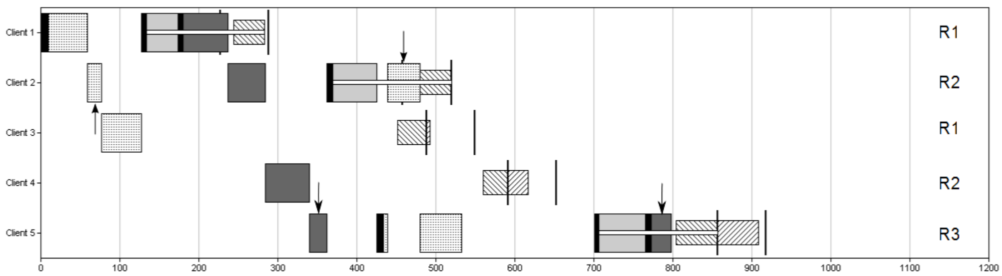


(b) I-LS-VRPTW

Fig. 4 I-BS-VRPTW and I-LS-VRPTW solutions to instance PP=1, SL=3, #=5, P-L-TS (Dist+, St-).

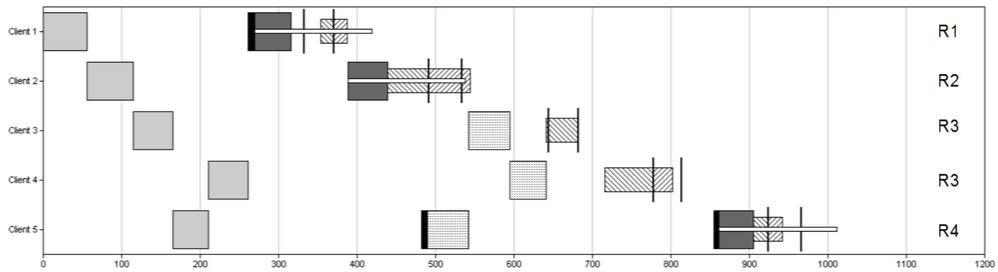


(a) I-BS-VRPTW

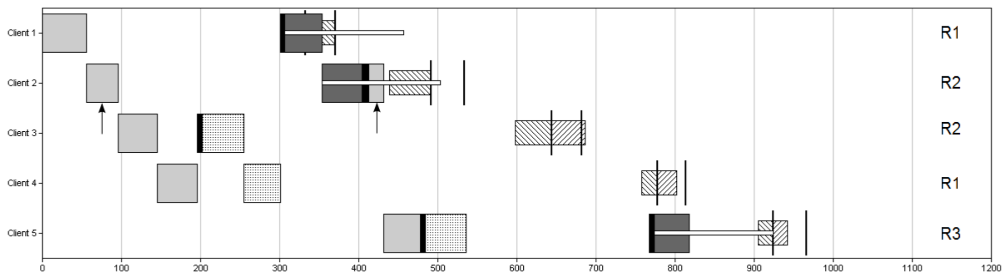


(b) I-LS-VRPTW

Fig. 5 I-BS-VRPTW and I-LS-VRPTW solutions to instance PP=1, SL=3, #=2, C-L-TS (Dist-).



(a) I-BS-VRPTW



(b) I-LS-VRPTW

Fig. 6 I-BS-VRPTW and I-LS-VRPTW solutions to instance PP=1, SL=3, #=4, P-L-TS (V-, Dist-, St+).

ented time windows and production environments with non-triangular setups seem to favour the importance of considering lot sizing in this operational problem. Several mechanisms to improve the batching solution were found by the lot sizing model. The lot sizing solution could achieve a better performance by: reducing the number of setups, changing the sequence, reducing setup costs, reducing the number of vehicles and/or the total travelled distance.

This work should be perceived as an exploratory research in this challenging field. Future work should focus on strengthening the I-LS-VRPTW formulation and on developing an efficient solution method to solve this challenging and important problem.

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Appendix A: Time Windows Generation

For the generation of time-windows data, an auxiliary parameter τ (that estimates the length of a vehicle tour) needs to be defined in two steps. First, a greedy nearest neighbourhood procedure finds a path for all customers without considering time windows. The distance of the solution found is then multiplied by 0.5 in order to account for the necessary expected vehicles (recall that a vehicle is able to carry half of the total demand), defining τ. Let us now define μ_{tw} as the mean width of the time windows that equals to 0.1τ. Two different methods are responsible for varying the *orientation of time windows*: production (P) or customer (C) oriented. To generate these time windows the algorithm proposed in Viergutz (2011) is adapted and described in Algorithm 1.

The customer's time windows generated by Algorithm 1 are production (P) oriented, in the sense that the first time window just starts after the necessary time to complete half of the total demand. The second method generates customer (C) oriented time windows and yields a profile in which parameters a_c and b_c are now defined according to the demand of each customer. Algorithm 2 describes the generator of time windows instances C. In the description, the maximum setup time is denoted by $maxStb$ and the value of the average demand element by $avDem$.

Algorithm 1 Pseudo-code to generate production (P) oriented time windows

```
aux ← 0.5 ∑_{jc} dem_{jc}
auxWidth ← 2/5 μ_{tw}
auxGap ← 2/5(∑_{jc} dem_{jc}/n)
for c = 1 → n do
   a_c ← aux
   auxLow ← max{0, μ_{tw} − auxWidth/2}
   twWidth ← RANDOM(auxLow, auxLow+auxWidth)
   b_c ← a_c + twWidth
   auxLow ← max{1, ∑_{jc} d_{jc}/n − auxGap/2}
   Gap ← RANDOM(auxLow, auxLow + auxGap)
   aux ← aux + Gap
end for
```

Algorithm 2 Pseudo-code to generate customer (C) oriented time windows

```
aux = 0
auxWidth ← 2/5 μ_{tw}
for c = 1 → n do
   for j = 1 → p do
      if d_{jc} > 0 then
         aux ← aux + d_{jc} + maxStb
      end if
   end for
   auxLow ← max{0, μ_{tw} − auxWidth/2}
   a_c ← aux + tt_{0c} − auxLow
   b_c ← a_c + auxLow + avDem * 0.5
end for
```

In order to vary the *tightness of time windows*, the standard (S) tightness of the time windows calculated in Algorithms 1 and 2 is relaxed to achieve loose (L) time windows. Hence, the L time windows are calculated by postponing by 20% the time windows calculated with the previous two methods. Consequently four different types of time windows may be generated, considering the tightness and the orientation of the time windows.