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Optimization of municipal solid waste collection routes based on the containers' fill status data

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Master in Informatics and Computing Engineering

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Abstract

Fraunhofer Portugal Research Center for Assistive Information and Communication Solutions is currently developing a system to monitor the fill status of waste containers. The introduction of a waste container fill status monitoring system in the city of Porto, Portugal, gives rise to several opportunities. For example, it allows the development of a detailed analysis of the city's waste generation distribution and the optimization of waste collection routes.

This document describes the architecture design of the information system to store and retrieve data regarding the containers' status. Furthermore, it provides a description of several algorithms that can be used to obtain efficient collection routes. This optimization problem is modeled as the *Capacitated Vehicle Routing Problem*. To address this problem, two approaches were analyzed; the first involves solving the associated *Asymmetric Traveling Salesman Problem* — in which vehicle capacity constraints are ignored — followed by clustering the resulting tour into feasible routes. This approach is called *route-first-cluster-second*. The second approach relies on the usage of a construction heuristic by Clarke and Wright.

Regarding the optimization of the *Asymmetric Traveling Salesman Problem* solution, this study compares several techniques: two construction heuristics — *greedy* and *repetitive nearest neighbor* — and three meta-heuristics — *hill climbing*, *genetic algorithms* and *MAX-MIN ant system*. Additionally, *MAX-MIN ant system* was subjected to a parameter sensibility analysis.

Results show that *MAX-MIN ant system* achieves more efficient routes when the number of ants is higher, although it increases the algorithm's running time. When dealing with a scenario in which there is a limited time-frame, it is recommended that a low number of ants is used. The algorithm was also shown to be very sensitive to changes in parameter β, which indicates if an ant should give more importance to the distance between two vertices or to the pheromone levels in that arc. This analysis suggests that β should be close to 20.

When evaluating the performance of the presented techniques applied to the *Capacitated Vehicle Routing Problem*, *MAX-MIN ant system* produced, in average, more efficient routes than the other approaches.

Resumo

O Centro de Pesquisa para Soluções de Informação e Comunicação Assistiva da Fraunhofer Portugal está a desenvolver um sistema de monitorização do estado de enchimento dos contentores de lixo. A introdução deste sistema na cidade do Porto dá origem a várias oportunidades. Por exemplo, torna-se possível fazer uma análise detalhada da distribuição da geração do lixo na cidade. Este projecto permite, também, implementar um sistema de optimização das rotas de recolha do lixo.

Este documento descreve o desenho da arquitectura do sistema de informação que permitirá armazenar — e disponibilizar — a informação referente ao estado dos contentores. O documento oferece também uma descrição de vários algoritmos que podem ser utilizados para obter rotas de recolha eficientes. Este problema de optimização pode ser modelado como um problema de planeamento de rotas de veículos com capacidade limitada (CVRP). Neste estudo, foram analisadas duas abordagens para a resolução do CVRP. A primeira começa por resolver o problema do caixeiro viajante em grafos assimétricos (ATSP) — ignorando as restrições de capacidade — e, subsequentemente, divide o circuito obtido em rotas que respeitem as restrições de capacidade dos veículos. Esta técnica chama-se *route-first-cluster-second*. A segunda abordagem para resolver o CVRP é baseada numa heurística construtiva, por Clarke e Wright.

Relativamente ao problema de optimização do problema do caixeiro viajante em grafos assimétricos, foram comparadas várias técnicas: duas heurísticas construtivas — *gulosa* e *vizinho mais próximo repetitivo* — e três meta-heurísticas — *subir-a-colina*, *algoritmos genéticos* e um sistema de formigas chamado *MAX-MIN ant system* (MMAS). Além da comparação dos vários algoritmos entre si, foi também feita uma análise de sensibilidade aos parâmetros do MMAS.

Os resultados mostram que o MMAS calcula rotas mais eficientes quando o número de formigas é mais elevado, apesar de levar a um aumento no tempo de execução do algoritmo. Quando aplicado a um cenário em que o tempo de execução disponível é limitado, é recomendado que se utilize um número reduzido de formigas. Também foi possível mostrar que o algoritmo é bastante sensível a variações no parâmetro β, que decide se uma formiga deve dar mais importância à distância entre dois vértices ou à quantidade de feromonas existente nesse arco. Esta análise mostrou que o parâmetro β deve tomar valores perto de 20.

A análise do desempenho dos vários algoritmos, relativamente ao problema de planeamento de rotas de veículos de capacidade limitada, mostrou que o sistema de formigas obtém, em média, rotas mais eficientes.

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Hugo “Wiki” Peixoto

"In computing, elegance is not a dispensable luxury but a quality that decides between success and failure"

Edsger W. Dijkstra

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Abbreviations

1-SCP	1-Skip Collection Problem
ACO	Ant Colony Optimization
ACVRP	Asymmetric Capacitated Vehicle Routing Problem
ARP	Arc Routing Problem
AS	Ant System
ATSP	Asymmetric Traveling Salesman Problem
AVRP	Asymmetric Vehicle Routing Problem
BPP	Bin Packing Problem
CARP	Capacitated Arc Routing Problem
CGRP-m	see MCGRP
CPP	Chinese Postman Problem
CVRP	Capacitated Vehicle Routing Problem
GA	Genetic Algorithm
GTSP	Graphical Traveling Salesman Problem
GRP	General Routing Problem
HC	Hill Climbing
JSON	JavaScript Object Notation
MMAS	MAX-MIN Ant System
MCGRP	Mixed-graph Capacitated General Routing Problem
M-RRVRP	Multiple Rollon-Rolloff Vehicle Routing Problem
MSW	Municipal Solid Waste
RPP	Rural Postman Problem
RRVRP	Rollon-Rolloff Vehicle Routing Problem
SGTSP	Steiner Graphical Traveling Salesman Problem
TSP	Traveling Salesman Problem
VRP	Vehicle Routing Problem
XML	Extensible Markup Language
WDF	Waste Disposal Facility

Chapter 1

Introduction

This chapter introduces this work, by presenting its context and motivation. Finally, it presents the project's objectives and the document structure.

1.1 Context and motivation

Municipal solid waste (MSW) production has been increasing in the last few years, along with economic growth [McC94]. This has led to the need — and subsequent development — of efficient waste management solutions. Waste management involves not only the collection, but also the transportation, recycling and disposal of generated waste.

According to [Bha96], municipal solid waste collection and disposal represents up to 85% of some cities' waste management budget. With this in mind, route optimization is as an important field of study regarding the improvement of MSW management processes.

Many cities around the world have studied and applied optimization techniques to their collection scenarios, which are usually very different from each other. In Portugal, however, there seems to be little research regarding this subject. [Pá03] describes the waste management scenario in Portugal from 1996 and 2002, a period in which several improvements were made due to changes in the legislation. In 2006, [MDS06] further report the legislation trends and present some statistics on the average MSW generation rate. Concerning collection routing, [TAdS04] describes a study for optimizing the collection of urban recyclable waste in the center-littoral region of Portugal.

1.2 Fill status monitoring

The Municipality of Porto, Portugal, is working together with the Fraunhofer Portugal Research Center for Assistive Information and Communication Solutions (FhP-AICOS) in order to implement a platform that allows the real-time measurement of waste containers' fill status. This

requires the deployment of low cost sensors in each container and the development of a communication system to gather information. On top of this platform, several applications will then be possible. As an example, this will allow researchers to study waste generation behaviors on a more detailed level. These monitoring systems have been studied in places, such as Pudong New Area, in Shanghai, China [RXV+09, VGR+09] and Sweden [Joh06].

Another application for this system is the optimization of waste collection routes.

1.3 Optimization of waste collection routes

The problem of optimizing waste collection routes involves deciding, for example, which streets must each garbage truck follow, which containers should each one of them collect and how many trucks should a fleet for a given city have.

One of the first articles regarding this subject was done in 1974 [Bel74], and it was applied to both New York and Washington D.C., United States of America. Since then, other cities have tried to minimize the costs by optimizing collection routes: Trabon, Turkey [AG07]; Barcelona, Spain [BP04]; Athens, Greece [Kar05]; Hanoi, Vietnam [TP00]; Porto Alegre, Brazil [LBM08] and many others.

However, many of these studies do not have real-time information of the containers' fill status. Usually, they are either based on statistical data (surveys), or they ignore the containers' fill status and simply collect the waste in every container.

Combining these techniques with the *Fill Status Monitoring* platform described in section 1.2, the municipality of Porto, Portugal might reduce even further the collection costs.

1.4 Objectives

With the deployment of a fill status monitoring solution in the municipality of Porto, there is the opportunity to develop an optimization framework for the waste collection routes.

The first challenge is to devise and implement an architecture to store and retrieve, when necessary, the information obtained from the container fill sensors. This includes specifying the information workflow, the database schema and formats to exchange data between modules.

The second goal is to analyze and compare different algorithms for the optimization of waste collection routes so that an efficient itinerary can be calculated within a time-frame of two hours, given the containers' fill status and the collection vehicles' capacities.

1.5 Document structure

The next chapter on this document, chapter 2, provides an overview of waste collection route optimization approaches. First, an informal description of each scenario is given. Then, each one of the scenarios is exposed as a mathematical formulation, followed by possible techniques that can be applied to them. Chapter 3 summarizes the problem statement, using the definitions presented by chapter 2.

Chapter 4 introduces the architecture proposal for the management of waste containers' fill status information — modules, workflow and information interchange formats. It also presents the technologies used to implement this system. Finally, this chapter presents the metrics and datasets used for both algorithm validation and analysis.

Chapter 5 describes the algorithms used to optimize collection routes and shows their validation results. Chapter 6 presents a comparative analysis of the chosen algorithms' performance. Chapter 7 finalizes this document by presenting the conclusions of this project, along with possible further developments and future work.

Chapter 2

State of the art

This chapter describes several studies regarding the optimization of solid waste collection routes. It reviews the mathematical models used in these studies and provides alternative optimization techniques that can be applied to each one.

In section 2.1, the waste collection problem will be detailed. Section 2.2 provides a background on the categorization of routing problems. As a final introductory section, section 2.1.1 exposes the basic data structure used in waste collection problems.

Sections 2.3.1, 2.3.2 and 2.3.3 describe each one of the three possible scenarios. Each section presents a mathematical formulation of the problem, along with techniques for solving it.

As seen in section 2.3.2, waste collection in a commercial scenario can be divided into two common subproblems; one of them is modeled as the *Traveling Salesman Problem* (TSP).

2.1 General problem description

Authors of [GAW01] divided waste collection routing problems into three main categories. First, the commercial collection contemplates waste collection from businesses and organizations like malls, factories and such. Second, the residential collection problem involves collecting household generated waste, usually stored in containers along the streets of a city. Comparatively, the number of containers in the commercial problem is significantly smaller than in the residential case.

In both of these two variants, each waste collection vehicle travels to a container, loads its contents into its hopper and moves on to the next container. As soon as the hopper is full, the vehicle travels to a disposal facility (such as a landfill, or a recycling/treatment facility) and deposits the collected waste.

The third variant — named Rollon-Rolloff, which is described in [BMBB00] — involves large containers, which must be transported to the disposal facilities and replaced by empty ones. Their dimensions impose that each vehicle can only transport a single container (either full or empty) at a time.

In the Rollon-Rolloff original description, each waste collection vehicle can perform four basic operations:

- **Insertion trip** — the vehicle brings an empty container from the waste disposal facility (WDF) to a new location;
- **Removal trip** — the vehicle picks up a full container from a location and leaves it at the WDF;
- **Round trip** — the vehicle picks up a container, brings it to a waste disposal facility (WDF) for emptying and returns it to its original location;
- **Exchange trip** — the vehicle leaves the WDF with an empty container, travels to a location with a full one and switches them, bringing it back to the WDF.

Although round and exchange trips could be considered as being composed by insertion and removal operations, the authors considered them as separate actions to avoid the need for modeling extra constraints. For example, in round trips the same container is brought back to its original place, while minimizing the time that location stays without a container. In exchange trips, the target location may not have room for two containers, so an extra restriction would have to be added, forcing vehicles to pick up the full one before delivering the new, empty container.

In each of these three main variants, extra parameters can vary. For example, consider a scenario with either a single or multiple waste disposal facilities. In the multiple facilities case, an extra restriction may be to balance the waste disposed at each location.

2.1.1 City graph

All three variants described in section 2.1 have a common base for their mathematical models — waste collection vehicles that travel along the streets of a given city. To model a city, the common approach is to define a graph in which each street is represented by one or two arcs (depending if the street is one or both ways), with street intersections being the vertices. Each arc may have several associated weights, reflecting the street distance, the time it takes to transverse it, or some other metric. As such, a city graph is defined as the ordered tuple $G = (V, E \cup A)$, where V represent the vertices, E the (undirected) edges and A the (directed) arcs.

This definition alone does not suffice to realistically represent the possible vehicle routes, as there are extra restrictions which must be considered, specially regarding traffic signs. For example: at a given intersection, it may not be possible to take a left turn if you are arriving from a specific street. U-turns may also be forbidden, in certain intersections.

These restrictions can be specified by defining a cost for every pair of arcs that intersect at a given vertex. $t_{ava'}$ is denoted as being the cost to go from the arc a to arc a' by making a turn at v. If the arcs do not intersect at that vertex, or if the traffic signs forbid such a turn, set $t_{ava'} = +\infty$. Further details on this approach can be seen in [CMMS02].

For now, consider the case in which there is a depot facility, located in vertex $v_0 \in V$, and that all waste collection vehicles start and finish their routes there. Consider that there are K vehicles available, and that each of the K routes can be defined as a set of round trips, each starting at the depot. The number of round trips in each vehicle route may or may not be limited. Furthermore, consider vehicles being limited to an amount Q of waste that they can carry at any given time. This can be either measured in weight or in volume.

2.2 Overview of routing problems

Before presenting the mathematical formulations that can be used to model our three waste collecting scenarios, this section provides a background on routing problems. This is useful to understand the concepts applied in the following sections. Routing problems are an important field in optimization, with many different applications. Its first formulation is usually attributed to Euler's article on the briges of Königsberg [Eul36].

The usual underlying structure for these problems is a strongly connected mixed graph. A mixed graph, $G = (V, E \cup A)$, contains both undirected and directed links (edges and arcs, respectively). Being strongly connected means that there is a directed path between every pair of vertices in V. With each link, there is an associated value that represents the cost of transversing it.

The *Mixed-graph Capacitated General Routing Problem* (CGRP-m, or MCGRP), defined in [CGSH02, PM95], aims to find a set of K routes that start and end in a specific vertex $v_d \in V$, which is called the *depot*. Additionally, these routes must respect the following set of constraints. Consider the following subsets $E_R \subseteq E$, $A_R \subseteq A$ and $V_R \subseteq V$. With each element of these subsets — which shall be called *requests* — there is an associated positive demand (q_a, q_e or q_v). These requests must all be *serviced* by one and just one of the K routes exactly once. A request being serviced implies that a route, at some point, visits it. Note that while each request may only be serviced once, its associated element (vertex or link) may be visited multiple times; transversing a link without servicing it is called *deadheading*. Furthermore, for each one of the K routes, the sum of the serviced requests' demands should not exceed a fixed capacity Q — hence the designation of *capacitated*.

In order for this problem to have feasible solutions, the service demands must fulfill two conditions. First, each demand must not be superior to the capacity Q. Second, there must be a way to partition the requests into no more than K subsets, such that the total demand for each subset does not exceed Q. These two conditions can be specified by the expressions (2.1) and (2.2).

$$\forall s \in E_R \cup A_R \cup V_R : \quad q_s \leq Q \tag{2.1}$$

$$\begin{aligned} \exists P : \quad & \bigcup P = E_R \cup A_R \cup V_R \wedge |P| \leq K \\ & \forall A, B \in P : A \cap B = \emptyset \wedge \\ & \forall A \in P : \sum_{s \in A} q_s \leq Q \end{aligned} \tag{2.2}$$

The CGRP-m, as its name states, is a general routing problem. This means that it is a generalization of several known and analyzed routing problems. A short list of CGRP-m specializations is now presented. In each of them, the previously stated conditions must always be respected.

These specializations can be divided into two main categories: *capacitated arc routing problems* (CARP), with $V_R = \emptyset$, and node routing problems, with $E_R = A_R = \emptyset$.

In the CARP category, the special case where $K = 1$ is called the *Rural Postman Problem* (RPP). In addition, if $A_R \cup E_R = E \cup A$, the *Chinese Postman Problem* (CPP) is obtained [PM95]. They can be classified as *Directed* (DRPP and DCPP), *Undirected* (URPP and UCPP) or *Mixed* (MRPP and MCPP), if $E = \emptyset$, $A = \emptyset$ or $A \neq \emptyset \wedge E \neq \emptyset$, respectively. DCPP and UCPP have been proven to be solvable in polynomial time by [Jac73], using a matching algorithm. Mixed CPP and all RPP are part of the NP-complete complexity class [PM95].

The second category represents node routing problems. Letting $K = 1$ yields the *Steiner Graphical Traveling Salesman Problem* (SGTSP) [Let99]. When a SGTSP also satisfies the condition $V_R = V$, it is called the *Graphical Traveling Salesman Problem* (GTSP). Finally, if the underlying graph is complete, the problem becomes the classical *Traveling Salesman Problem* (TSP).

On the other hand, the instances with $V_R = V$ form a widely studied variant — the *Capacitated Vehicle Routing Problem* (CVRP). This category, along with several of its variants, is described in [TV01b].

2.3 Waste collection scenarios

2.3.1 Residential scenario

In the residential scenario, due to the container density per street, it is usual to consider that vehicles should serve arcs instead of individual nodes. Oppositely, in the commercial scenario dealt with in the next section, vehicles will serve vertices.

2.3.1.1 Mathematical Formulation

Since each vehicle is also limited in its capacity, it is considered to be a problem in the CARP category. More specifically, it can be modeled as a *Mixed Capacitated Arc Routing Problem* (MCARP).

A recent survey on CARP can be found in [San08]. The problem was first suggested and modeled, in its undirected variant, by [Gol81], with Belenguer et al. providing a different mathematical model, as well as an algorithm for determining lower and upper bounds [BB98, BB03].

With respect to CARP on mixed graphs (MCARP), Belenguer et al. [BBLP06] provide a relaxed linear formulation, lower bounds for it and several heuristics. Recently, Gouveia et al. [GMP10] provided a valid linear formulation for the MCARP and presented benchmarks on large datasets. This formulation will be now described.

Start by considering a graph $G' = (N, A')$, where $A' = A \cup \{(i,j),(j,i) : (i,j) \in E\}$, and let $A'_R = A_R \cup \{(i,j),(j,i) : (i,j) \in E_R\}$ and $P = \{1,...,K\}$. A solution is given by the variables (x,y,f), in which:

$$x_{ij}^p = \begin{cases} 1 & \text{if vehicle } p \in O \text{ serves arc } (i,j) \in A'_R \\ 0 & \text{otherwise} \end{cases}$$

$y_{ij}^p =$ number of times vehicle $p \in O$ visits $(i,j) \in A'$ without servicing it

$f_{ij}^p =$ remaining demand serviced by vehicle $p \in P$, after transversing $(i,j) \in A'$

Now, consider d_{ij} and c_{ij} to be the costs of transversing arc $(i,j) \in A'$, with and without servicing it, respectively. One can specify MCARP as the following linear programming formulation:

$$\text{Minimize} \quad \sum_{p \in P} \left[\sum_{(i,j) \in A'_R} c_{ij} x_{ij}^p + \sum_{(i,j) \in A'_R} d_{ij} y_{ij}^p \right] \tag{2.3}$$

subject to

$$\forall p \in P \quad \forall v \in V \quad \sum_{(v,j) \in A'} y_{vj}^p + \sum_{(v,j) \in A'_R} x_{vj}^p = \sum_{(j,v) \in A'} y_{jv}^p + \sum_{(j,v) \in A'_R} x_{jv}^p \tag{2.4}$$

$$\forall (i,j) \in A_R \quad \sum_{p \in P} x_{ij}^p = 1 \tag{2.5}$$

$$\forall (i,j) \in E_R \quad \sum_{p \in P} x_{ij}^p + x_{ji}^p = 1 \tag{2.6}$$

$$\forall p \in P \quad \sum_{(v_0,j) \in A'} y_{v_0 j}^p + \sum_{(v_0,j) \in A'_R} x_{v_0 j}^p \leq 1 \tag{2.7}$$

$$\forall p \in P \quad \forall v \in V \setminus \{v_0\} \quad \sum_{(i,v) \in A'} f_{iv}^p - \sum_{(v,i) \in A'} f_{vi}^p = \sum_{(i,v) \in A'_R} q_{iv} x_{iv}^p \tag{2.8}$$

$$\forall p \in P \quad \sum_{(v_0,i) \in A'} f_{v_0 i}^p = \sum_{(i,j) \in A'_R} q_{ij} x_{ij}^p \tag{2.9}$$

$$\forall p \in P \quad \sum_{(i,v_0) \in A'} f_{iv_0}^p = \sum_{(i,v_0) \in A'_R} q_{iv_0} x_{iv_0}^p \tag{2.10}$$

$$\forall p \in P \quad \forall (i,j) \in A' \quad f_{ij}^p \leq Q(y_{ij}^p + x_{ij}^p) \tag{2.11}$$

$$\forall p \in P \quad \forall (i,j) \in A \quad f_{ij}^p \geq 0 \tag{2.12}$$

$$\forall p \in P \quad \forall (i,j) \in A \quad x_{ij}^p \in \{0,1\} \tag{2.13}$$

$$\forall p \in P \quad \forall (i,j) \in A \quad y_{ij}^p \in \mathbb{Z}_{\geq 0} \tag{2.14}$$

Constraints (2.4) state that the number of vehicles that enters a vertex must be the same as the number of vehicles that leave it. (2.5) and (2.6) force every arc and edge to be serviced. (2.7) ensures that each vehicle only leaves the depot once. Flow constraints (2.8), (2.9) and (2.10) force, together with the linking constraints (2.11), that each vehicle performs a connected trip. Constraints (2.11) also handle the capacity limit for each vehicle, when in conjugation with (2.9).

Further details regarding the theory of arc routing problems, along with possible approaches for solving them, can be found in [Mos00, AG95].

2.3.1.2 Solving approaches

In [BBLP06], the authors describe three fast heuristics, adapted from studies on the undirected version of CARP: path scanning, augment-merge and Ulusoy's heuristic.

Path scanning

this heuristic builds routes sequentially. In each step, the current route is extended (until the capacity constraints are violated) by adding the arcs that lead to the closest arc $v \in A'_R$ which needs to be serviced. In case of ties, the heuristic may use randomly one of five criteria:

- F_1 maximize the cost to return to the depot;
- F_2 minimize the cost to return to the depot;
- F_3 maximize the ratio q_v/c_v;
- F_4 minimize the ratio q_v/c_v;
- F_5 if the vehicle is less than half-full, use F_2 and otherwise use F_1.

Augment-merge

A route is created for each required link $e \in A_R \cup E_R$, minimizing the deadheading cost (This can easily be done using shortest path algorithms). The next step, called *Augment phase*, verifies if there are routes which include required links in their deadheading arcs. When it happens, they are merged together, if the capacity constraints hold.

The second phase, *Merge*, takes every pair of routes (r_0, r_1) and checks if their concatenation yields a better result (while respecting the capacity constraints), and merges them. The concatenation process is done by finding the shortest path between the last served arc in r_0 to the first served arc in r_1.

Ulusoy's heuristic

This heuristic builds a single route containing all arcs in A'_R, ignoring the capacity constraints, and subsequently divides it into several feasible routes.

The first step does not need to produce an optimal route, so the authors suggest the usage of the path-scanning heuristic and disregarding the capacity limit. This route defines the order in which the arcs will be serviced, so let $pos(a)$ be the arc's position on this route.

Next, the route is subdivided. This step can be done using a simple dynamic programming technique.

In the same study, the authors propose new linear relaxed formulation which can is used together with a cutting plane algorithm to determine a lower bound for MCARP instances.

Belenguer et al. also mention several metaheuristics applied to UCARP that solve a majority of instances optimally [BBLP06, BB03]. These could be adapted to solve MCARP.

2.3.2 Commercial scenario

When comparing the commercial to the residential collection problem, one expects that the number of containers is significantly lower in the first case. This allows us to model the problem as a node routing problem. Applications of this model to the waste collection problem are described in [TP00] and [KKS06].

2.3.2.1 Mathematical Formulation

Generally, the city graph is transformed into $G' = (V', A')$, in which each vertex represents either a waste container or the depot. The arcs represent the associated cost of traveling between each pair of vertices – which can be obtained through a shortest path algorithm applied to G. This way, the problem becomes finding a set of routes such that each vertex is visited exactly once. This is the *Capacitated Vehicle Routing Problem*, which was described in section 2.2.

Since the graph is directed, this problem is usually named *Asymmetric Capacitated Vehicle Routing Problem* (ACVRP). The most common mathematical model [TV01a] defines a solution as a set of binary variables x_{ij}, one for each arc (i, j). To model the ACVRP, first consider $r(S), S \subseteq V \setminus \{v_0\}$ as the minimum number of vehicles to serve all vertices in S. This value, that can be determined by solving the *Bin Packing Problem* (BPP), has a trivial lower bound:

$$\forall_{S \subseteq V' \setminus \{v_0\}} \quad r(S) \geq \lceil \frac{1}{Q} \sum_{v \in V'} d_v \rceil \tag{2.15}$$

These definitions allows us to define ACVRP as the following integer programming formulation:

$$\text{Minimize} \sum_{i \in V'} \sum_{j \in V} c_{ij} x_{ij} \tag{2.16}$$

subject to

$$\forall_{j \in V \setminus \{v_0\}} \quad \sum_{i \in V} x_{ij} = 1 \tag{2.17}$$

$$\sum_{i \in V} x_{v_0 i} = K \tag{2.18}$$

$$\forall_{i \in V} \quad \sum_{j \in V} x_{ij} = \sum_{j \in V} x_{ji} \tag{2.19}$$

$$\forall_{S \subseteq V \setminus \{v_0\}, S \neq \emptyset} \quad \sum_{i \notin S} \sum_{i \in S} x_i j \geq r(S) \tag{2.20}$$

$$\forall_{i,j \in V} \quad x_{ij} \in \{0, 1\} \tag{2.21}$$

Constraints (2.17) impose that each vertex is visited exactly once, while constraints (2.18) force that exactly K vehicles leave the disposal facility and (2.19) forces that every vehicle that visits a node must leave it. Finally, (2.20) ensures that each subset of vertices is serviced by

at least the minimum number of vehicles required to fulfill their demands, thus respecting the capacity constraints.

2.3.2.2 Solving approaches

In the book by Toth and Vigo [TV01b], CVRP is described with great detail. They divide methods into the following categories:

- Branch-and-bound
- Branch-and-cut
- Set-covering based algorithms
- Heuristics
- Metaheuristics

The first three categories are exact methods, based on relaxations of linear formulations and the determination of lower bounds. Some of these approaches have been used to solve optimally problem instances with up to 135 nodes [NR01]. These involve the study of polytopes defined by linear formulations and cutting plane algorithms.

In the heuristics field, there are three subcategories [LS01]. *Constructive heuristics* gradually build a feasible solution, minimizing the cost in a greedy fashion. *Two-phase heuristics* divide the problem into two: clustering the vertices in K clusters and constructing a route from each cluster. Some methods do clustering first (*cluster-first-route-second*), while others build a single tour — containing all vertices and ignoring capacity constraints — and consequently divide it into vehicle routes (*route-first-cluster-second*) [TB]. The latter approach is similar to Ulusoy's heuristic, described in section 2.3.1.2. The third category, containing *Improvement methods*, is applied over the two first categories. Improvement methods can be applied either by changing the order in which vertices are visited in a vehicle route or by switching vertices between routes. In the first case, this technique is the same as optimizing a single route in a subgraph — resulting in the *Traveling Salesman Problem*.

The last category of methods encompasses metaheuristics, which are general optimization methods. Metaheuristics are said to produce better results than the previous set of methods [GLP01]. These techniques are no more than *Improvement methods*.

Examples of metaheuristics applied to VRP, are *Simulated Annealing*, *Deterministic Annealing*, *Tabu Search*, *Genetic Algorithms* and *Ant Systems*. The first three approaches start by building an initial solution (using a heuristic) and search its neighborhood for a better solution. The neighborhood is usually defined by some swap operator; for example, exchanging a subset of serviced vertices between routes.

Genetic Algorithms start by generating a set of initial solutions, and proceeds to the next step by combining solutions between them, and discarding the worst. Ant systems work by applying

concepts based on the ants' pheromone system for marking paths [CDM91]. These two approaches will be described further in chapter 5.

2.3.3 Rollon-Rolloff scenario

Regarding the Rollon-Rolloff case, some of the earlier studies found on the literature are [G.94, dML97, BMBB00]. According to Bodin et al., the first three papers assume that it is known beforehand which trip type (see section 2.1) must serve each container. This problem is named *Rolloff-Rollon Vehicle Routing Problem* (RRVRP).

2.3.3.1 Mathematical Formulation

Each given trip $t \in T$ is defined by its type and by a tuple of vertices to visit, in a specific order. The problem of assigning trips to vehicles can be formulated as an *Asymmetric Vehicle Routing Problem* (AVRP). AVRP is similar to the ACVRP without the vehicle capacity constraints, but differs in that the number of vehicles is not given — it is a value which must be minimized. Usually, the number of vehicles must be bounded by a given interval $[L,U]$.

In order to model the RRVRP as a AVRP, admit a new graph, $G' = (V',A')$. One vertex represents the disposal facility, while the others represent trips that need to be serviced. Each arc $(i,j) \in A'$ represents the transition between trip i and trip j. When $i = 0$, consider this transition to be the start of a route; when $j = 0$, consider it as the end. The costs of going from the end location of a given trip to the start location of another one are given by $c_a, a \in A'$.

The AVRP formulation for the RRVRP can now be defined as:

$$\text{Minimize} \quad K_A \sum_{i \in V'} \sum_{j \in V} c_{ij} x_{ij} + K_B \sum_{j \in V} x_{v_0 j} \tag{2.22}$$

subject to

$$\forall_{j \in V \setminus \{v_0\}} \quad \sum_{i \in V} x_{ij} = 1 \tag{2.23}$$

$$\forall_{i \in V} \quad \sum_{j \in V} x_{ij} = \sum_{j \in V} x_{ji} \tag{2.24}$$

$$\sum_{j \in V} x_{v_0 j} \leq U \tag{2.25}$$

$$\sum_{j \in V} x_{v_0 j} \geq L \tag{2.26}$$

$$\forall_{i,j \in V} \quad x_{ij} \in \{0,1\} \tag{2.27}$$

Another formulation is provided in [BBM06]. This paper does not assume, as the previous ones, that the trips to be serviced are predefined. Furthermore, it addresses the problem of having multiple disposal facilities and multiple inventory locations (where empty containers are stored). The authors named this as the *Multiple Rollon-Rolloff Vehicle Routing Problem* (M-RRVRP). The

M-RRVRP can be converted to a *Vehicle Routing Problem with Time Windows* (VRPTW). This makes it possible to model the problem as a *Set Partitioning* (SP) formulation.

A similar problem is defined by [Arc05], named *1-Skip (container) Collection Problem* (1-SCP). Here, multiple disposal facilities are available, and there are compatibility constraints between the facilities and the containers. The trips that each vehicle can perform are different from the four types defined in the RRVRP. Each vehicle starts its tour from a depot with an empty container and travels to a location where there is a full one. The two containers are then exchanged, and the vehicle proceeds with the full container to a compatible disposal facility. It then proceeds to another trip, with a new empty container. This model also considers time windows for picking up and emptying containers.

The authors of [ABMN04] also present a rather interesting alternative for the Rollon-Rolloff model. They start by considering that there is a finite number of available empty containers at the depot, K_C, and that compatibility constraints defined in the 1-SCP model are also present. Each request for collection $i \in I, I = 1,...,n$ is characterized by its location (γ_i), container type (β_i) and waste material type (μ_i).

The graph model $G = (V,A)$ defines its vertices as representations of collection requests. For each request i, there are two nodes e_i and f_i in V, that represent the full container to be collected and an empty container to be delivered. As usual, vertex v_0 represents the depot from where the vehicles must begin and end their tours. There are also K_C vertices (the set D'), representing the possible pick up of an empty container at the depot, and K_C vertices (the set D'') representing their delivery. No nodes will be added to represent the disposal facilities; this information will be embed in the graph arcs.

Let $E = \{e_i : i \in I\}$, $F = \{f_i : i \in I\}$. Defining the set of vertices as $V = \{v_0\} \cup E \cup F \cup D' \cup D''$, we now need to specify the arcs connecting them.

There is an arc between $f_i \in F$ and $e_j \in E$ if both services have the same container type ($\beta_i = \beta_j$). These arcs correspond to picking up a full container at γ_i, taking it to a disposal facility of type μ_i for emptying and deploying it at location γ_j.

Between every $e_i \in E$ and $f \in F$ there is also an arc. It represents deploying an empty container at location γ_i and travelling to γ_j to pick up its full container.

At the start of a tour, a vehicle starts at the depot node, v_0, unloaded. From here, there are two alternatives: it travels to a location with a full container or it picks up an empty container. These operations are represented by the arcs $(v_0, f), f \in F$ and $(v_0, d'), \in D'$. Analogously, there are arcs that represent unloading an empty container and finishing the tour. These arcs are defined by $(e, v_0), e \in E$ and $(d'', v_0), d'' \in D''$.

Picking up an empty container from the depot and deploying it is represented by the arcs $(d', e), d' \in D' \wedge e \in E$. Picking up a full container, emptying it at a disposal facility and dropping it at the depot is modeled by the arcs $(f, d''), f \in F \wedge d'' \in D''$. Finally, there are cases where a vehicle drops an empty container at the depot to pick up another empty one, of another type, or a full one: $(d'', d'), d'' \in D'' \wedge d' \in D'$ and $(d'', f), d'' \in D'' \wedge f \in F$.

For clarity's sake, an example graph is provided in figure 2.1, with two requests and an empty container at the depot, all with the same type. Removing v_0, the graph should become bipartite, as a vehicle can only go from nodes from which it leaves loaded to a node from which he leaves unloaded.

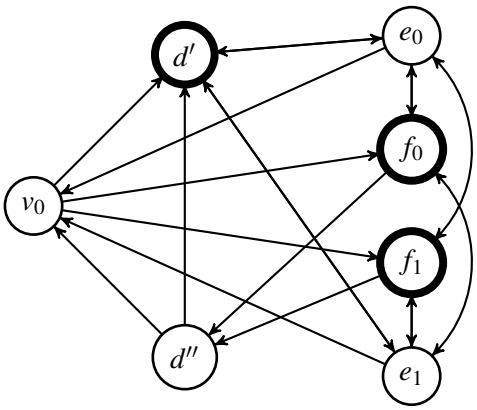


Figure 2.1: Graph for the Rollon-Rolloff problem. Thick vertices (except v_0) represent nodes from which the vehicle leaves loaded. Thin vertices are the ones from which it leaves unloaded.

This can now be modeled as an instance of AVRP. The costs c_{ij} associated with each arc are further detailed in [ABMN04]. $\delta^+(v)$ is defined as the out-neighborhood of v and $\delta^-(v)$ as its in-neighborhood. Without the time constraints, the following formulation is obtained:

$$\text{Minimize} \quad K_A \sum_{(i,j)\in A} c_{ij}x_{ij} + K_B \sum_{j\in\delta^-(v_0)} x_{v_0 j} \tag{2.28}$$

subject to

$$\forall_{i\in F\cup E} \quad \sum_{j\in\delta^+(i)} x_{ij} = 1 \tag{2.29}$$

$$\forall_{i\in V} \quad \sum_{j\in\delta^+(i)} x_{ij} = \sum_{j\in\delta^-(i)} x_{ji} \tag{2.30}$$

$$\sum_{j\in V} x_{0j} \leq U \tag{2.31}$$

$$\sum_{j\in V} x_{0j} \geq L \tag{2.32}$$

$$\forall_{i\in D'\cup D''} \quad \sum_{j\in\delta^+(i)} x_{ij} \leq 1 \tag{2.33}$$

$$\forall_{i,j\in V} \quad x_{ij} \in \{0,1\} \tag{2.34}$$

Although the graph is not complete, the missing arcs can be added with a arbitrarily large value. If the restrictions described in section 2.1 need to be enforced, it is enough to limit the arcs on the specified request. Say that the container from f_0 must return to e_0. Removing the ingoing

edges to e_0 and the outgoing edges from f_0 (except the one that connects f_0 to e_0) is enough to ensure this constraint.

2.3.3.2 Solving approaches

Solving RRVRP instances, as modeled in the previous section, can be done using the algorithms described in section 2.3.2.2.

2.4 Chapter summary

This chapter presented some background information regarding waste collection problems. It followed by giving a general overview of the modelation of route optimization problems.

Finally, section 2.3 presents the application of route optimization models to waste collection vehicle route optimization problem. Three different scenarios were introduced.

In the *residential* scenario, there are several approaches applied to the directed CARP. Although some have been adapted to the undirected variant, further study could be made regarding the remaining algorithms — namely, the usage of metaheuristics.

The *Rollon-rolloff* scenario was formulated as a AVRP instance. Although the authors provided preliminary computational results using an heuristic, further computations could be made, comparing several other heuristics, metaheuristics and exact methods.

The *commercial* scenario is modeled as a *Capacitated Vehicle Routing Problem*. This model is used whenever the density of waste containers per street is low.

The next chapter, chapter 3, uses the definitions introduced in these sections to explicitly state the goals of this study.

Chapter 3

Problem description

This chapter provides a description of the problem being addressed.

3.1 Waste collection in Porto, Portugal

The municipal council of the city of Porto is interested in implementing a platform that measures the fill status of the containers in real-time, so that the evolution of waste can be monitored, the quality and efficiency of the collection can be increased, and the sums paid to subcontractor companies by amount of km traveled each month reduced. The system should work as follows: containers send an alarm when they are full, and everyday in the evening the waste collection routes are calculated with the static values available at a certain time.

This project's scope is to build a framework that stores container fill status and that uses that information to calculate efficient collection routes. Routes have to be provided every day, based on recent fill information; although there is no need to calculate routes in real-time, a solution must be provided within 1 to 2 hours. This constraint requires that an efficient solution is found within that time frame for graphs of large cities, which are composed of several thousand vertices. To decide which algorithm should be used to calculate the routes, it was necessary to do a comparative study to evaluate the performance of several techniques.

As stated in the previous chapter, a residential waste collection scenario is modeled as a *Capacitated Arc Routing Problem* (CARP), due to high number of containers per street. CARP consists of, in summary, finding a set of vehicle routes that visit a subset of a given graph's arcs. With each arc, there is an associated demand, which must be serviced by exactly one vehicle. The sum of the demands that a given vehicle services must not exceed a fixed limit.

Although residential waste collection scenarios are usually modeled as a CARP, it must be taken into account that in Porto, containers service several homes and that containers fill status will be monitored. With this in mind, it becomes more natural to use a *Asymmetric Capacitated Vehicle Routing Problem* (ACVRP) model. This model is similar to that of CARP, with a simple difference: instead of servicing arcs, in ACVRP vehicles must service vertices instead.

3.2 Asymmetric capacitated vehicle routing problem

As exposed in the previous chapter, there are three non-exact methods to address the *Asymmetric Capacitated Vehicle Routing Problem*. One of them, *route-first-cluster-second*, first solves the associated *Asymmetric Traveling Salesman Problem* (ATSP); this implies finding the shortest tour that visits every vertice exactly once. This process is followed by the partitioning of the tour into routes that respect vehicle capacity constraints (henceforth called clustering).

Another method, *cluster-first-route-second*, first clusters the containers and then attempts to find an optimum route for each cluster. This method usually requires that one predefines the number of vehicles to use.

The third method to address the ACVRP is to use constructive heuristics, that build all the vehicle routes in parallel.

This work will focus on the comparison of the first and third methods, using different approaches to solve the ATSP. Additionally, special focus will be given to the *MAX-MIN ant system* (MMAS) — which is described in section 5.4.2 — and its sensibility to certain parameters.

3.3 Chapter summary

This chapter summarized the motivation, goals and approach of this project. The next chapter will focus on the definition of the architecure for the waste sensor framework. It also presents the evaluation metrics and validation datasets to analyse the optimization process.

Chapter 4

Solution architecture

This chapter describes the proposed architecture for managing waste containers fill status and calculating efficient routes. It starts by specifying, in section 4.1, all the modules and the information workflow of the framework. Section 4.2 presents the technologies used to implement each module, while section 4.3 specifies the data formats in which information is exchanged between modules.

Section 4.4 defines the metrics chosen to evaluate the optimization algorithms, so that they may be quantified and properly compared.

Section 4.5 introduces the datasets used throughout this study. Implemented algorithms will be applied to these datasets, and the resulting routes will be evaluated using the metrics previously defined in section 4.4.

A subset of these datasets, defined in section 4.5.1, will be used to validate the algorithms' implementations. These datasets are widely used throughout the literature to benchmark algorithms, and their optimal solutions has usually already been determined.

Section 4.5.2 presents a second subset of datasets, whose properties are similar to those of the datasets obtained when the monitoring system is deployed. These will be used to compare the algorithms' performance.

4.1 Framework workflow

This project is structured in a modular way; this chapter enumerates and describes each one of the modules, defining what is required and produced in each step.

To understand the following architecture and underlying information flow, one must have in mind that this project has two major components. First, several optimization algorithms must be compared, using both real and fabricated scenarios. In second place, the system must be ready to be integrated with the fill status monitoring solution being developed at Fraunhofer Portugal AICOS.

First, the optimization module itself must receive a dataset describing the collection scenario, which should contain:

- city topology
- waste containers' location
- vehicle starting point
- minimum and maximum number of vehicles

With this information, it shall produce a single file describing one route per used vehicle. This file can then be fed to the evaluation module, which determines the solution's performance, according to several metrics, described later on in section 4.4. This performance evaluation mechanism will be used mainly during the comparison phase of this project, while the optimization module will be used in both phases.

Dataset information is gathered from several different places: map retrieved from GIS sources, as described in section 4.5; containers' fill status can be stochastically generated or obtained from the monitoring system's database. These data are then normalized and merged to form a single dataset file. The full information flow can be seen in figure 4.1.

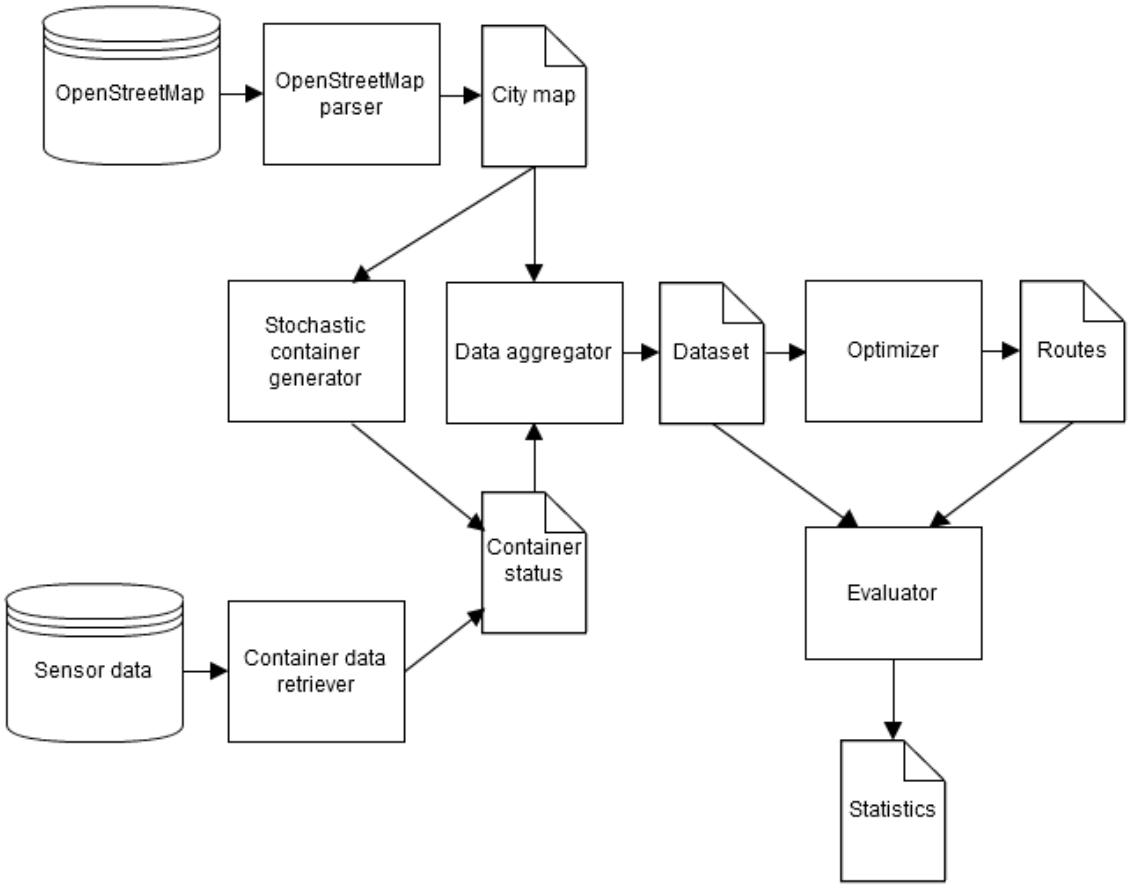


Figure 4.1: Project architecture

Note that there is the need to manage independent city and containers' status files before creating a normalized dataset. This happens because the stochastic container generator depends on the city file to create valid scenarios.

4.2 Implementation details

To implement this framework, there was the need to decide which technologies to use, regarding several modules.

Figure 4.1 shows the necessity of a database management system (DBMS), to maintain information about the containers' fill status. Due to the simple nature of the information model stored in the database, the choice of which DBMS to use is not critical. Additionally, the load of the DBMS will not be too high, as the optimization process is only run once per day. With this in mind, it was decided to use the MySQL DBMS, as it is both free and widely used in high profile projects, such as the *Wikimedia Foundation* and *Yahoo! Finance* [BD08].

Modules such as the *data aggregator*, *OpenStreetMap parser*, *Container data retriever* and *Stochastic container generator* were developed using a combination of C++ and bash scripting. C++ was used to apply complex transformations to the data, which involved handling large files. Bash scripting was used as an utility to invoke MySQL queries and convert the retrieved data into a more convenient format.

All the algorithms in the optimization module were implemented in C++. As the algorithm implementations need to be efficient (both regarding running time and memory), the usage of interpreted languages, such as Ruby or Python, was discarded. C++ was chosen due to the author's familiarity with the language, thus accelerating the development phase.

The evaluation module was also implemented in C++, so that its code could be shared with the optimization module.

4.3 Data formats

As seen in figure 4.1, there are five different files that carry information from one module to the next. To ease implementation, all five formats were specified using the same notation.

Examples of common data interchange formats are XML (Extensible Markup Language) and JSON (JavaScript Object Notation). Advantages of such standards are, for example, the ready availability of several parsers for a great number of languages [Cro09b, Rob09], their openness and simplicity.

Between these two notations, it was decided to use the JSON format. It was chosen because it is simpler than XML and because it was specifically designed to be a lightweight computer data interchange format, whereas XML was designed to be a document interchange format [Cro09b].

JSON is based on two universal data structures: ordered lists and keyed lists (from here on out called *objects*). The latter structure is also known in various programming languages as a

dictionary, *hash table*, *associative array* and others. There are also primitive types available, such as strings, numbers and three constants: `true`, `false` and `null`.

While objects' keys must be strings, their values can be of any type, either primitive or not. The same is true for ordered lists' elements. Further details regarding JSON syntax can be found in the RFC 4627 [Cro09a].

The following sections in this chapter will describe each one of the five formats present in this system's architecture.

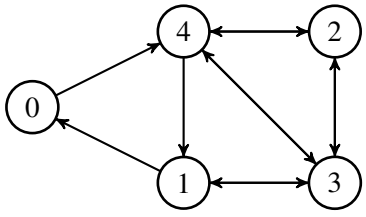
4.3.1 City map format

To simulate the waste generation and collection, there is the need to provide a topological city map. This topological map must be represented as a directed weighted graph, so that routing algorithms may be applied. A city is usually a sparse graph — each vertex, representing a street intersection, has a reduced number of arcs. This leads to the conclusion that a adjacency list representation should be used.

A city map file is composed of a single JSON object (**`city_map`**). This contains two pairs of key/values; one of them, whose key is `name`, has a string as its value and represents the city map name, being used as human-readable metadata. Its second pair has the key `graph` and its value represents the city topology, described as an adjacency list — a data structure commonly used to describe graphs. This adjacency list is defined as an ordered list, where each element represents a vertex.

Each vertex is represented by a JSON object, containing a key/value pair for its latitude (`lat`) and its longitude (`lon`), with both values being represented as JSON numbers. A third key/value pair is present; its key is `roads`, and its value defines the vertex's neighbors. This neighbor list is encoded as a JSON ordered list, with each element identifying the neighbor by its index in the main adjacency list.

A sample graph and its corresponding **`city_map`** object can be seen in figure 4.2.




```
{"name": "Sample city",
 "graph": [
  {"lat": 0, "lon": 0, "roads": [4]},
  {"lat": 2, "lon": -1, "roads": [0,3]},
  {"lat": 4, "lon": 1, "roads": [3,4]},
  {"lat": 4, "lon": -1, "roads": [1,2,4]},
  {"lat": 2, "lon": 1, "roads": [1,2,3]}
]}
```

Figure 4.2: An example city topology map and its JSON equivalent.

4.3.2 Containers status format

The containers' fill status file contains, for each container, its location — latitude and longitude — and the current amount of waste stored in it. This is represented by a JSON ordered list, with an element per container. Each container element is a JSON object and contains three key/value pairs: one for its latitude (`lat`), one for its longitude (`lon`) and one for its current fill (`fill`). An example JSON object is available in figure 4.3.

```
[
 {"lat": 0.0, "lon": 0.0, "fill": 43.01},
 {"lat": 4.1, "lon": -0.9, "fill": 18.47}
]
```

Figure 4.3: An example containers' fill status JSON file.

4.3.3 Dataset format

After obtaining both the city map file and the containers' status file, they must be merged together to form a single dataset file. This action is performed by the *normalizer* module, shown previously in figure 4.1.

This format contains both features regarding the city topology and the containers' fill status. Each container geographical position is matched against the graph's vertices and its fill status information is added to the closest vertex.

The dataset schema is similar to the one presented in section 4.3.1, with a few changes. First, for each vertex in which a waste container is present, there is an additional key/value pair representing its fill status, as defined in section 4.3.2. Second, two new key/value pairs must be added to the main JSON object; one regarding the maximum number of trucks to use (`max_trucks`) and one regarding the starting/ending node (`depot`). These two extra parameters must be defined when running the normalization process, and are not present in any of the previous data file formats.

Taking the examples presented in the two previous sections, they could be normalized into the dataset file present in figure 4.4.

4.3.4 Routes format

The optimization module must process a single dataset file and determine a near-optimal set of routes — one for each vehicle. Each vehicle route can be defined as an ordered list of references to the city vertices.

When dealing with commercial scenarios, each vertex there should be paired with a boolean indicator that tells if the vehicle must empty the waste container present at that location. In the Rollon-Rolloff scenario, this additional flag indicates if the vehicle should load/unload a container at that location.

```
{"name": "Sample city",
 "max_trucks": 3,
 "depot": 1,
 "graph": [
  {"lat": 0, "lon": 0, "roads": [4], "fill": 43.01},
  {"lat": 2, "lon": -1, "roads": [0,3]},
  {"lat": 4, "lon": 1, "roads": [3,4]},
  {"lat": 4, "lon": -1, "roads": [1,2,4], 18.47},
  {"lat": 2, "lon": 1, "roads": [1,2,3]}
]}
```

Figure 4.4: Complete dataset example object, constructed from previous examples.

The set of routes can be defined as a JSON ordered list, with each element defining a single route. Each route can also be represented by an ordered list of vertex references. These are themselves defined as ordered lists of two values; the first value represents the vertex index, while the second is a boolean value, determining if the vehicle should or should not act on that specific location.

Figure 4.5 shows an example of a two-vehicle routes file, in the Rollon-Rolloff scenario, after applying an optimization technique to the previously shown dataset. Each vehicle starts by loading an empty container at vertex 1, moves to a location with a full container and switches the empty with the full one. Then, both vehicles proceed back to the depot location, where they deposit the picked up containers.

```
[
 [[1, true], [4, true], [4, true], [0, false], [1, true]],
 [[1, true], [3, true], [3, true], [1, true]]
]
```

Figure 4.5: Rollon-Rolloff set of routes example, representing two vehicles.

4.3.5 Statistics format

The fifth data format regards the statistics obtained when evaluating the routes obtained by a given optimization module. These metrics are calculated based on the routes and the dataset files. The output object is represented as a JSON object, where each key/value pair represents a different metric. This representation is left open, as metrics may be added and/or removed during the algorithm comparison phase.

4.4 Evaluation metrics

Performance evaluation of the implemented optimization framework requires the definition of quantifiable metrics. As the goal is to reduce collection costs, an important metric is the total number of kilometers that vehicles travel during waste collection. The lower the number of kilometers is, the better.

Although the total distance is the main evaluation metric, there are additional metrics that may be used to evaluate a solution. The average ratio between a vehicle's capacity and the total urban waste it collects may be important to evaluate collection efficiency.

4.5 Datasets

This section will present datasets used for algorithm validation. This will be followed by the description of the methods used to obtain large datasets that share the same properties as the ones to be used in production, when the monitoring system is deployed.

4.5.1 Validation datasets

Routing problems have been widely studied over the last decades; this has led to the establishment of standard datasets for benchmarking algorithms and implementations, one of them being the TSPLIB[Rei91]. There are datasets for the *Capacitated Vehicle Routing Problem*, both *Symmetric* and *Asymmetric Traveling Salesman Problem* and other related routing problems. ATSP instances have been solved optimally; as such, they are accompanied by their respective optimum route cost.

This library can be used to validate the algorithm implementations described in chapter 5, as it allows one to measure the performance gap between a given solution and the optimal route.

Table 4.1 shows the ATSP validation instances, including the number of nodes and the optimum route cost, while table 4.2 shows the CVRP validation instances. In the latter case, an optimum route cost is not available. This is due to the fact that these datasets can be used to formulate several problems. For example, one might consider the number of vehicles specified in the dataset as a fixed, minimum or maximum value. It might also be possible to disregard the number of vehicles specified by the dataset. As such, table 4.2 only presents reference values obtained when considering a fixed number of vehicles.

Table 4.1: Asymmetric Traveling Salesman Problem datasets, presenting the number of vertices and the optimum route cost.

Name	Number of vertices	Optimum route cost
br17	17	39
ftv33	33	1286
ftv35	35	1473
ftv38	38	1530
p43	43	5620
ftv44	44	1613
ftv47	47	1776
ry48p	48	14422
ft53	53	6905
ftv55	55	1608
ftv64	64	1839
ftv70	70	1950
ft70	70	38673
kro124p	124	36230
ftv170	170	2755

Table 4.2: Capacitated Vehicle Routing Problem datasets, presenting the number of vertices and a reference route cost.

Name	Number of vertices	Reference route cost
eil13	13	247
eil22	22	375
eil23	23	569
eil30	30	534
eil31	31	379
eil33	33	835
eil51	51	521
eilA76	76	682
eilB76	76	735
eilC76	76	830
eilD76	76	1021
eilA101	101	815
eilB101	101	1071
gil262	262	6119

4.5.2 Realistic city datasets

In order to properly evaluate the developed algorithms, there was the need to obtain realistic datasets that represented the street topology of real cities. These datasets must be similar to those to be obtained by the monitoring system, so that results are as reliable as possible.

4.5.2.1 OpenStreetMap.org

To obtain realistic city maps, a tool to extract and convert topological maps from *OpenStreetMap.org* was developed. *OpenStreetMap.org* is an collaborative and open source initiative which aims to create a free editable map of the world [Hak08]. Users may add information by editing the world map manually — using the web editor — or by submitting data from GPS devices.

Figure 4.6 shows the topological map of the city of Porto, Portugal, which was imported from *OpenStreetMap.org*. Unfortunately, both the connectivity and the structure of the city of Porto are highly inaccurate. This leads to a strongly disconnected graph, in which several adjacent streets are not connected. This happens for several reasons. First, some of the information of the streets direction is outdated. Second, users contributing to this project may have had little attention to detail regarding the graph connectivity, aiming only to add visual information of the city streets. Users may also have been deprived of the necessary tools to provide accurate information regarding street connectivity.

Figure 4.6: The city of Porto, Portugal, retrieved from OpenStreetMap.org and loaded onto the current framework viewer.

4.5.2.2 Waste generation

Having no current access to real waste containers location and fill status — as the monitoring platform is not yet deployed — there was the need to generate this information artificially, using

a stochastic approach. Waste containers were scattered in street intersections following different patterns according to the following process:

The algorithm starts by selecting k road intersections (represented by a graph vertex) on the city map as cluster centers. A number d_i in the range $[0, 1]$ is assigned to each cluster, representing the cluster's waste container density. Then, until all intersections belong to a cluster, k vertices are chosen arbitrarily from each of the clusters' neighborhood and added to the respective cluster. When a vertex is added to a cluster it is decided, with probability d_i, if it should contain a full waste container.

4.5.2.3 Generated datasets

This approach was applied to three different city topological maps — Leeds, Lisbon and London — using different clustering parameter configurations. This yielded fifteen large datasets, whose number of vertices ranges from 518 to 7628. Table 4.3 shows information about these datasets — both the number of nodes and the cities in which their topology was based.

Table 4.3: Large realistic Capacitated Vehicle Routing Problem datasets

Name	Number of vertices	City
hp518	518	Leeds
hp841	841	Leeds
hp904	904	Leeds
hp1287	1287	Leeds
hp1175	1175	London
hp1849	1849	London
hp2038	2038	London
hp2206	2206	London
hp2561	2561	Lisbon
hp3481	3481	Lisbon
hp3859	3859	Lisbon
hp4109	4109	Lisbon
hp4628	4628	Lisbon
hp6247	6247	Lisbon
hp7628	7628	Lisbon

4.6 Chapter summary

This chapter presented the design for the fill status monitoring framework — the information workflow, data interchange formats and the technologies have been specified. Additionally, sections 4.4 and 4.5 specified the metrics and datasets with which to validate the waste collection route optimization approaches.

The following chapter will introduce the optimization algorithms used throughout this study.

Chapter 5

Algorithms

As seen in section 2.3.2.2, one way to tackle the *Asymmetric Capacitated Vehicle Routing Problem* is to first build a tour over graph G and to divide it into routes that respect the vehicle capacity constraints. The problem of finding a tour in a directed graph is called *Asymmetric Traveling Salesman Problem*.

The first sections in this chapter describe several algorithms used for solving *Asymmetric Traveling Salesman Problem* instances. A comparative analysis of their complexity is shown in section 5.5. Section 5.6 presents an algorithm for dividing a tour — a route that visits all vertices in a graph — into several routes that respect vehicle capacity constraints.

Section 5.9 explains the process with which the implemented algorithms were validated. It reports the results regarding the benchmarks done using standard *Asymmetric Traveling Salesman Problem* and *Capacitated Vehicle Routing Problem* instances.

This chapter shows the algorithms' characterization, by using asymptotic notation (also known as Bachmann-Landau notation) to represent bounds on time complexities. For a description of asymptotic notation used throughout this chapter, see [Knu76].

5.1 Construction heuristics for the ATSP

Two classical construction heuristics for the ATSP are the *nearest neighbor* and the *greedy* heuristics [GPB+02].

Nearest neighbor (NN) starts by picking an arbitrary vertex, usually chosen at random. It proceeds by advancing to the nearest unvisited neighbor, until all vertices are visited. A tour is then completed by returning to the first visited vertex. When applied to a complete graph — where $|E| = |V|^2$ — this heuristic has a time complexity of $O(|V|^2)$. Instead of simply picking an arbitrary vertex at first, one can start NN from every vertex and return the best final tour. This variation is called *repetitive nearest neighbor* (RNN) [GPB+02]. Algorithm 1 describes the *repetitive nearest neighbor* heuristic. Lines 4 through 15 apply the nearest neighbor heuristic starting in vertex v.

Algorithm 1 Repetitive nearest neighbor heuristic

Input: A complete graph $G = (V, E)$
Output: An ATSP route

```
1:  minimum ← +∞
2:  best_route ← nil
3:  for all v ∈ V do // calculate the NN for each vertex
4:      route ← []
5:      current ← v
6:      while |route| ≠ |V| do
7:          route.append(current)
8:          closest ← nil
9:          for all w ∈ V do // determine the closest non-visited vertex
10:             if w ∉ route ∧ (closest = nil ∨ cost(v,w) < cost(v,closest)) then
11:                 closest ← w
12:             end if
13:         end for
14:         current ← closest
15:     end while
16:     if best_route = nil ∨ cost(route) < cost(best_route) then
17:         best_route ← route
18:     end if
19: end for
20: return best_route
```

The *greedy* heuristic (GR) works in a similar way than that of Kruskal's minimum spanning tree algorithm [Kru56]. It starts by creating an auxiliary directed graph with no arcs, $G' = (V, \{\})$ and sorts all the arcs in G by their weight, which are then iterated in ascending order. Each arc is added to G' if and only if its vertices are not already connected in G' and both the outdegree (number of outgoing arcs) of the source vertex and the indegree (number of incoming arcs) of the destination vertex are exactly 0. This process will yield a spanning tree which can be converted to a tour by adding the arc that connects the only two vertices whose degree is less than 2. Its time complexity is equal to that of Kruskal's algorithm: $\Theta(|E|log|E|)$. When dealing with a complete graph, where $|E| = |V|^2$, the time complexity can be expressed as $\Theta(|V|^2 log(|V|^2)) = \Theta(|V|^2 log|V|)$.

The pseudo-code for this heuristic is presented in algorithm 2. Two arrays, *prev* and *next*, are used to maintain information about each vertex's incoming and outgoing connections. The *first* and *last* arrays serve to determine if two vertices are already connected or not.

Algorithm 2 Greedy heuristic

Input: A complete graph $G = (V, E)$
Output: An ATSP route

1: $prev \leftarrow []$, $next \leftarrow []$
2: $first \leftarrow []$, $last \leftarrow []$
3: $visited = 0$
4: **for** $i = 0$ to $|V| - 1$ **do** // initialization
5: $next$.append(-1),
6: $prev$.append(-1)
7: $first$.append(i)
8: $last$.append(i)
9: **end for**
10: **for each** $(v, w) \in E$, in ascending $cost(v, w)$ order **do** // create the spanning tree
11: **if** $visited < |V| - 1 \wedge next[v] < 0 \wedge prev[w] < 0 \wedge last[w] \neq v \wedge first[v] \neq w$ **then**
12: $next[v] \leftarrow w$
13: $prev[w] \leftarrow v$
14: $first[last[w]] \leftarrow first[v]$
15: $last[first[v]] \leftarrow last[w]$
16: $visited \leftarrow visited + 1$
17: **end if**
18: **end for**
19: **for all** $v \in V$ **do** // retrieve the calculated route
20: **if** $prev[v] < 0$ **then**
21: $route \leftarrow []$
22: **while** $v \geq 0$ **do**
23: $route.append(v)$
24: $v \leftarrow next[v]$
25: **end while**
26: **return** $route$
27: **end if**
28: **end for**
29: **return** nil

5.2 Hill climbing for the ATSP

Hill climbing [RN03] is a greedy local search technique. It picks an initial solution and iterates through its neighborhood, looking for a solution with higher value. This process is repeated until a local minima /maxima is reached or the specified time limit is exceeded. *first choice hill climbing* moves to the first solution whose value is higher, while *steepest ascent hill climbing* evaluates all neighbors and moves to the one that provides greater improvement [RN03].

When applying hill climbing to TSP instances, a solution's neighborhood is commonly defined by dividing the current tour into k segments and recombining them in all possible ways that yield a valid tour. This transformation is called a $k-cut$, and the process of applying it in a hill-climbing fashion is called the $k-opt$. The most common versions are 2-opt and 3-opt [JM97] and an

adaptive generalization that chooses, at each iteration, how many segments to form — the Lin-Kernighan heuristic [JM97]. The bigger the k, the greater the neighborhood. Given a route r, its neighborhood size $|N(r,k)|$ growth factor can be defined as $|N(r,k)| \in O(|V|^k)$.

Segment recombination may include the reversal of one or more segments. In STSP instances, the cost of transversing a segment is the same as the cost of transversing its reversed form. This means that calculating the total cost of a recombined tour has an average time complexity of $\Theta(k)$, since one just needs to take into account k edges that were removed and k new edges that were added.

In asymmetric instances, though, transversing a segment does not cost the same as transversing its reversed form. Calculating the total cost of a single recombined tour, in this scenario, has a worst time complexity of $\Theta(|V|)$. This would mean that when calculating the cost of all neighbors, time complexity would be $O(|V|^k) \cdot \Theta(|V|) = O(|V|^{k+1})$. However, the calculation of reversed segments' costs can be reused between neighbors. It is also possible to apply a dynamic programming technique to a given route r that, after a pre-calculation that runs in $\Theta(|V|)$, allows us to calculate the cost of any reversed segment in r in $\Theta(1)$. This reduces the time complexity upper bound from $O(|V|^{k+1})$ to $O(|V|^k + |V|) = O(|V|^k)$, which is significantly lower. As city graphs used in the next chapters have a number of vertices in the range $[500, 8000]$, it was decided to use $k = 2$.

The pseudo-code for this approach is presented in algorithm 3. In each iteration (lines 2 through 18), all possible 2-cuts are generated (lines 4 through 12) and the one which yields the minimum cost is compared to the current route (lines 8 through 10 and 13 through 17). If there is no improvement, the algorithm halts (lines 15 through 17).

Algorithm 3 Steep ascent hill climbing using a 2-cut neighborhood (2-opt algorithm)

Input: A complete graph $G = (V, E)$
Output: An ATSP route

```
1:  route = heuristic(G) // heuristic can either refer to the RNN or the greedy heuristics
2:  loop
3:      best ← route
4:      for i = 0 to |V| − 1 do // cut edge number i
5:          for k = 2 to |V| − 1 do // cut edge number (i + k)%|V|
6:              r ← route + route
7:              n ← reverse(r[i, i + k]) + r[i + k + 1, i + |V|]
8:              if cost(n) < cost(best) then
9:                  best ← n
10:             end if
11:         end for
12:     end for
13:     if cost(best) < cost(route) then
14:         route ← best
15:     else // when a local minima is reached, stop the algorithm
16:         return route
17:     end if
18: end loop
```

5.3 Genetic algorithm for the ATSP

A genetic algorithm is a search technique based on evolutionary biology [DM91]. This technique starts by defining an initial population of random solutions, which is considered as the first generation. Then, the evolution process follows. In this process, every individual in the current generation is evaluated and multiple individuals are stochastically selected according to their fitness and possibly modified to form the next generation. Usually, the best individuals from a generation, known as the elite, are moved to the next one directly, without suffering any modification [RN03]. The evolution process continues until a predefined termination condition has been satisfied. Modifications of two types can be applied to the selected individuals: crossover and mutation. The crossover operator takes two or more individuals from the current population and combines their attributes to form an offspring. The mutation operator modifies some attributes of a single individual.

There are several opinions on which modification operator — crossover or mutation — is more important [NS92]. Some consider crossover as the primary operator, with mutations serving only to prevent early convergence [Hol92]. Others believe crossover is unnecessary and mutation alone is enough to develop more efficient searches [FA90].

Several approaches of applying genetic algorithms to TSP have been studied, with different solution representations and modification operators. The genetic algorithm used throughout this work, presented in [CCL96], is a mutation-only approach whose representation is simple (each solution is encoded by a vertex permutation) and whose modification operator is based on the Lin-Kernighan heuristic.

First generation elements are initialized by using the *nearest neighbor* heuristic. Then, for each solution in a generation, a number k_i is chosen, following the discrete geometric distribution $P[X = k] = p^{k-1}(1-p)$, with $p = 0.35$. Modifications to the solution are made by applying a random, uniformly distributed, (k_i+1)-cut. This method was originally applied to symmetric TSP instances, where recalculating a route's cost is faster (as explained in section 5.2). In this work, its performance when facing asymmetric datasets will be evaluated.

To evaluate the time complexity of this algorithm, consider a single evolution iteration, with a population of m individuals. Each individual suffers a $k-cut$ operation, whose running time is $\Theta(|V|+k)$, as explained below. All m routes' costs have to be recalculated, so the total iteration running time is given by $\Theta(|V|+k+m|V|) = \Theta(m|V|+k)$. The expected value of k is given by $E(X)+1 = \frac{1}{p}+1$. With $p = 0.35$, $E(X)+1 \approx 3.86$; this means that the average running time complexity of a single genetic algorithm iteration is $\Theta(m|V|)$.

The mutation pseudo-code is presented in algorithm 4. Its inputs are parameter p and the route to be mutated, *route*. It starts by determining the number of cuts to apply, using the geometric distribution $P[X = k]$. Then, it determines the places to apply the k cuts, selecting k distinct edges from the total $|V|$ edges of the route (lines 7 through 17). This selection is done using an algorithm that runs in $\Theta(|V|)$, where each element has the same probability of being chosen.

The route segments, obtained when applying the cuts, are shuffled to obtain a random recombination. This shuffling procedure, known as the Fisher-Yates shuffle, runs in $\Theta(k)$ (lines 18 through 20) and yields an unbiased permutation [Knu97]. Finally, when recombining all the segments, each one of them has a 50% probability of being reversed (lines 24 through 30). This last step has a time complexity of $\Theta(|V|+k)$. As k has an average value of $E(X)+1$, which is a constant, one can consider the mutation total running time as $\Theta(|V|+E(X)+1+(|V|+E(X)+1))$ = $\Theta(|V|)$.

Algorithm 4 Mutation operator used in the ATSP genetic algorithm

Input: A complete graph $G=(V,E)$, an ATSP *route* and the parameter p
Output: A mutated route

1: $k = 2$
2: $v = 1 - p$
3: $r = random()$ // random real value in the range $[0,1[$
4: **while** $v < r$ **do** // determine k according to a geometric distribution
5: $k \leftarrow k+1$, $r \leftarrow r-v$, $v \leftarrow v \cdot p$
6: **end while**
7: $cuts \leftarrow []$
8: $order \leftarrow []$
9: $available \leftarrow |V|$, $needed \leftarrow k$
10: **while** $needed > 0$ **do** // choose k distinct integer values, representing the cuts
11: **if** $random(available) < needed$ **then** // random integer number in $[0, available[$
12: $cuts.append(|V| - available)$
13: $order.append(k - needed)$
14: $needed \leftarrow needed - 1$
15: **end if**
16: $available \leftarrow available - 1$
17: **end while**
18: **for** $i = k$ to 2 **do** // shuffle the cut order using Fisher-Yates algorithm
19: $swap(order[i-1], order[random(i)])$
20: **end for**
21: $cuts.append(cuts[0]+n)$
22: $order.append(order[0])$
23: $mutated \leftarrow []$
24: **for** $i = 0$ to $k-1$ **do**
25: **if** random() < 0.5 **then** // flip the cut with 50% probability
26: $mutated \leftarrow mutated + reverse(route[cuts[order[i]], cuts[order[i]+1]-1])$
27: **else** // don't flip
28: $mutated \leftarrow mutated + route[cuts[order[i]], cuts[order[i]+1]-1]$
29: **end if**
30: **end for**
31: **return** mutated

The genetic algorithm procedure is described in algorithm 5. It depends on two parameters: the population size and the number of generations. The initial population is generated by applying the nearest neighbor heuristic to yield multiple solutions (lines 1 to 4), with a time complexity of $\Theta(m|V|^2)$. Then, throughout each generation, solutions with higher cost are discarded (lines 6 through 15). This selection procedure runs in $\Theta(|V|)$. The final step of each iteration involves mutating each individual from the population, according to algorithm 4 (lines 16 through 18). Mutating all elements takes $\Theta(m \cdot (|V|)$ time. This results in a total of $\Theta(|V| + m \cdot |V|) = \Theta(|V|)$ running time per generation. Considering that the algorithm runs for R generations, the total genetic algorithm time complexity is given by $\Theta(m \cdot |V|^2 + R \cdot |V|)$.

Algorithm 5 ATSP genetic algorithm

Input: A complete graph $G = (V, E)$, population size *popsize* and number of generations *gens*.
Output: An ATSP route

```
1:  population ← []
2:  for i = 0 to popsize − 1 do // create the first generation
3:      population.append(nearest_neighbor(G))
4:  end for
5:  for i = 0 to gens − 1 do // improve for a fixed number of generations
6:      for j = 0 to popsize − 1 do // selection
7:          eb ← random(popsize)
8:          ew ← random(popsize)
9:          if cost(population[eb]) > cost(population[ew]) then
10:             swap(eb, ew)
11:         end if
12:         if random() < 0.75 then // 75% probability of the worst being discarded
13:             population[ew] ← population[eb]
14:         end if
15:     end for
16:     for j = 0 to popsize − 1 do // mutation
17:         population[j] ← mutation(population[j])
18:         reevaluate(population[j])
19:     end for
20: end for
21: best ← population[0]
22: for i = 1 to popsize − 1 do // return the best
23:     if cost(population[i]) < cost(best) then
24:         best ← population[i]
25:     end if
26: end for
27: return best
```

5.4 Ant colony algorithms

Ant colony optimization algorithms, were first proposed by Dorigo [Dor92, DMC96]. They are inspired in ants' communication model, which is based on pheromone secretion [GADP89]. Ants have the need to minimize path lengths between food sources and their colony, so that harvesting becomes more efficient. To do so, each ant secretes pheromones along their trail to influence other ants' behavior. Ants have a higher probability of choosing a path whose pheromone intensity is stronger. Those who choose shorter paths travel from the food source to the colony more often, increasing these paths' pheromone intensity. This, together with the fact that pheromones evaporate over time, makes shorter paths more likely to be chosen, thus optimizing the food harvesting process [GADP89].

5.4.1 Ant system

Ant system (AS), developed by Dorigo et al. [DGG96], is an ant colony optimization algorithm applied to TSP. It starts by defining an initial pheromone intensity value for each of the graph's edges. Then, the following iterative process is repeated until the time limit is reached, a satisfactory solution is obtained or the algorithm reaches stagnation.

Each iteration t starts by randomly placing m ants, which are simple agents, on the graph's vertices. Each ant chooses the next vertex to visit using a probability function, defined in equation 5.1, that depends on the distance between the current and next vertices and on the pheromone trail on the respective edge:

$$p_{ij}^k(t) = \begin{cases} \dfrac{[\tau_{ij}(t)]^\alpha \cdot [\eta_{ij}]^\beta}{\sum [\tau_{ij}(t)]^\alpha \cdot [\eta_{ij}]^\beta} & \text{if } j \notin \text{visited}^k \\ 0 & \text{otherwise} \end{cases} \tag{5.1}$$

$p_{ij}^k(t)$ represents the probability of ant k traveling to vertex j, from its current position i. It depends on several factors:

- whether or not j has already been visited by ant k in the current iteration;
- the edge's pheromone trail $\tau_{ij}(t)$ and
- a distance heuristic $\eta_{ij} = \dfrac{1}{d_{ij}}$, with d_{ij} being the distance between i and j.

α and β are parameters that determine the relative influence between the two heuristics. Ants keep advancing until they visit every vertex. At this point, every ant k has an associated tour, represented as a sequence $r^k(t)$. An edge $(i, j) \in E$ is said to belong to $r^k(t)$ if and only if i is immediately followed by j in sequence $r^k(t)$ or if i is the last and j is the first element of $r^k(t)$.

At this point, pheromone trail intensities are updated according to the following formula:

$$\tau_{ij}(t+1) = \rho \cdot \tau_{ij}(t) + \sum_{k=1}^{m} \Delta\tau_{ij}^k(t) \tag{5.2}$$

This expresses both the evaporation process — with $\rho \in [0,1]$ — and the pheromone secretion, represented by $\Delta\tau_{ij}^{k}(t)$. This amount depends on the tour's length $L_k(t)$ and Q, a parameter:

$$\Delta\tau_{ij}^{k}(t) = \begin{cases} \dfrac{Q}{L_k(t)} & if\,(i,j) \in r^k(t) \\ 0 & \text{otherwise} \end{cases} \tag{5.3}$$

The algorithm's resulting tour is the best one found by any of the m ants throughout the iterations.

To calculate the time complexity of ant colony system, consider each iteration. The initialization step — placing m ants on arbitrary vertices — has a complexity of $\Theta(m)$. Since every ant has to advance $|V|-1$ times and each ant movement requires the calculation of $p_{ij}^{k}(t)$ for every j, the total ant tour construction time complexity is given by $\Theta(m|V|^2)$.

The time complexity of updating pheromone intensities, with a basic implementation, would be $\Theta(|V|^2 + m|V|)$, as every edge needs to be updated and each ant contributes to the increment of $|V|$ edges' trails. However, the calculation of pheromone intensity updates — given by the second summand of equation 5.2 — can be done during the tour construction phase without rising its time complexity. This reduces the updating phase's time complexity to $\Theta(|V|^2)$. The overall time complexity of an AS iteration can now be given by $\Theta(m + m|V|^2 + |V|^2) = \Theta(m|V|^2)$.

For large problem instances, this complexity can be quite high. It can be reduced by introducing the concept of *candidate lists* [DGG96]. A candidate list contains, for every vertex i, an ordered list of the c closest vertices. In the tour construction phase, each ant only considers vertices from this list when choosing its next destination (unless they are all visited, in which case it considers all the vertices). Given that $0 < c < |V|$, the time complexity of an AS iteration is now characterized by $\Omega(mc|V|)$ and $O(m|V|^2)$.

5.4.2 MAX-MIN ant system

A variation of *ant system*, proposed by [SH97], is called *MAX-MIN ant system* (MMAS). The main difference between MMAS and AS is regarding the update of pheromone intensities. At each iteration, only the ant with the shortest tour is taken into account. Additionally, every value $\tau_{ij}(t)$ is bounded both below and above. This leads to the rewrite of equation 5.2:

$$k_{best} = \arg\min_k L_k(t) \tag{5.4}$$

$$\tau_{ij}(t+1) = max(\tau_{min}, min(\tau_{max}, \rho \cdot \tau_{ij}(t) + \Delta\tau_{ij}^{k_{best}}(t))) \tag{5.5}$$

This bounding technique helps to alleviate the problems of early stagnation [SH97] — a problem that occurs when some edges' pheromone trail intensities rise so high that all ants end up converging too soon to the same tour.

Additionally, MMAS is stated to be one of the best performing ant colony algorithms [SH00] and its convergence has been proved [SD02]. This led to the choosing of MMAS for further analysis and comparison with other approaches.

The pseudo-code for the algorithm used when deciding which vertex an ant should move to is given in algorithm 6. It starts by gathering the first c vertices that are to be visited, while calculating the probability of each one being chosen (lines 3 through 12). This probability depends on both distance and pheromone intensity of the arc, as well as on two parameters that determine the influence of each heuristic: α and β. Lines 13 through 19 implement the selection based on the probabilities. The time complexity for the vertex selection is given by the lower bound $\Omega(c)$ and the upper bound $O(|V|)$.

Algorithm 6 Ant vertex selection used in max-min ant system

Input: A graph $G = (V,E)$, pheromone levels *pheromones*, ant's visited vertices *route* and parameters α and β
Output: The next vertex to be visited by the ant

```
1:  candidates ← [], odds ← []
2:  total ← 0, current ← route[|route| − 1]
3:  for all v ∈ V do
4:      if v ∉ route then
5:          candidates.append(v)
6:          r ← cost(current, v)^α · (pheromones[current][v])^β
7:          total ← total + r
8:          odds.append(r)
9:      end if
10:     if |candidates| == c then
10:         break
11:     end if
12: end for
13: r ← random() · total
14: for k = 0 to |odds| − 1 do
15:     r ← r − odds[k]
16:     if r ≤ 0 then
17:         return candidates[k]
18:     end if
19: end for
```

The pseudo-code for MMAS is given by algorithm 7. First, it initializes the pheromone values for each edge, each represented by a pair of vertices, in $\Theta(|V|^2)$ time (lines 2 through 7). Then, in each iteration, m ants are placed in randomly chosen vertices (lines 10 through 12), in $\Theta(m)$ time. Each ant then procedes to move to the next vertex (using algorithm 6) until all vertices are visited. The vertex selection method is executed $\Theta(m|V|)$ times, yielding a total time complexity described by the bounds $\Omega(mc|V|)$ and $O(m|V|^2)$. This is followed by the evaporation process (lines 18 through 22), that runs in $\Theta(|V|^2)$. The final step regards pheromone update using the lowest cost route obtained in this iteration (lines 24 through 32), and it runs in $\Theta(|V|+m)$.

Considering that *MAX-MIN ant system* runs for R iterations, using m ants, a candidate list of size c and that the heuristic used in line 1 runs in $\Theta(H)$ time, the total running time is given by the bounds $\Omega(H + (|V|^2 + mc|V|) \cdot R)$ and $O(H + m|V|^2 \cdot R)$.

Algorithm 7 Max-min ant system

Input: A complete graph $G = (V, E)$
Output: An ATSP route

1: $best \leftarrow heuristic(G)$
2: $pheromones \leftarrow []$
3: **for all** $v \in V$ **do** // pheromone initialization
4: **for all** $w \in V$ **do**
5: $pheromones[v][w] \leftarrow \tau_{min}$
6: **end for**
7: **end for**
8: **for** $r = 1$ to $iterations$ **do**
9: $routes \leftarrow []$
10: **for** $i = 0$ to $m - 1$ **do** // ant initialization, starting in a random vertex
11: $routes.append([random(|V|)])$
12: **end for**
13: **for** $i = 1$ to $|V| - 1$ **do** // each ant must advance $|V| - 1$ times
14: **for** $j = 0$ to $m - 1$ **do**
15: $routes[j].append(ant\_selection(G, pheromones, routes[j], alpha, beta))$
16: **end for**
17: **end for**
18: **for all** $v \in V$ **do** // pheromone evaporation
19: **for all** $w \in V$ **do**
20: $pheromones[v][w] \leftarrow max(\tau_{min}, pheromones[v][w] \cdot \rho)$
21: **end for**
22: **end for**
23: $route \leftarrow routes[0]$
24: **for** $i = 1$ to $m - 1$ **do**
25: **if** $cost(routes[i]) < cost(route)$ **then**
26: $route \leftarrow routes[i]$
27: **end if**
28: **end for**
29: **for** $i = 0$ to $|V| - 1$ **do** // pheromone update
30: $v \leftarrow route[i]$, $w \leftarrow route[(i+1)\%|V|]$
31: $pheromone[v][w] \leftarrow min(\tau_{max}, pheromones[v][w] + cost(route)^{-1})$
32: **end for**
33: **if** $cost(route) < cost(best)$ **then**
34: $best \leftarrow route$
35: **end if**
36: **end for**
37: **return** best

5.5 Complexity overview of ATSP algorithms

All of the previously presented algorithms aim to solve the *Asymmetric Traveling Salesman Problem*. Table 5.1 presents comparative information regarding their time complexities.

Table 5.1: Bounds for ATSP algorithms. There is a tight bound whenever both the lower and upper bounds are the same. R represents the number of iterations of a given algorithm. H is the time complexity for a chosen heuristic, such as the repetitive nearest neighbor or the greedy algorithm

Algorithm	Lower bound	Upper bound	Tight bound
Greedy	–	–	$\Theta(\|V\|^2 log\|V\|)$
Nearest neighbor	–	–	$\Theta(\|V\|^2)$
Repetitive nearest neighbor	–	–	$\Theta(\|V\|^3)$
Hill climbing (2-opt)	–	$O(H+\|V\|^2 \cdot R)$	–
Genetic algorithm	–	–	$\Theta(m\|V\|^2 + m\|V\| \cdot R)$
MAX-MIN ant system	$\Omega(H+(\|V\|^2 + mc\|V\|) \cdot R)$	$O(H+m\|V\|^2 \cdot R)$	–

5.6 Split clustering algorithm

The *route-first-cluster-second* approach to the *Asymmetric Capacitated Vehicle Routing Problem* starts by building a tour — a route that visits all vertices on a graph — on graph G by relaxing vehicle capacity constraints. Then, this tour is split into several feasible routes, such that each one does not exceed the maximum vehicle capacity limit.

Consider that a tour always starts and ends at the depot location. This makes it possible for one to define a tour as a sequence of vertices that represent waste containers. The clustering algorithm described in [Bea83] — referred to as *Split*, from now on — creates a set of routes such that the order by which the vertices are visited in any route is a contiguous subsequence of the generated tour. For example, a possible tour representation could be $t = \langle a,b,c,d,e,f \rangle$, with possible resulting routes being $\langle a,b \rangle$ and $\langle d,e,f \rangle$, but never $\langle a,c,d \rangle$ or $\langle e,f,a \rangle$. Possible routes may be represented by the indices of both the starting and finishing vertices in the tour. Two sample routes are $t_{1,3} = \langle a,b,c \rangle$ and $t_{3,6} = \langle c,d,e,f \rangle$.

The algorithm starts by creating an auxiliary graph, $H = (V', A')$, with $V' = 0, 1, \cdots, |V|$. Then, for every route $t_{i,j}$ an arc is added to H from vertices $i-1$ to j if and only if $t_{i,j}$ respects vehicle capacity constraints. The weight associated to every arc $(i-1, j) \in A'$ is given by the cost of going from the depot to vertex i, visiting every vertex in $t_{i,j}$ and going back to the depot. The resulting graph H is acyclic, as there are no arcs connecting vertices u and v if $u > v$. This means that H is a *directed acyclic graph*.

The problem of finding the optimal cluster of routes such that each one is a contiguous subsequence of the tour is equivalent to finding the shortest path between vertices 0 and $|V|$ in graph H [Bea83]. Finding the shortest path between two vertices in a *directed acyclic graph* can be done in $\Theta(|V'| + |A'|) = \Theta(|V'|^2) = \Theta((|V|+1)^2) = \Theta(|V|^2)$, as explained in [Man89].

It is important to note that although this algorithm yields an optimal solution based on the visiting sequence of any given tour, applying it to an optimal ATSP solution does not yield a minimal ACVRP set of routes [Bea83].

The *split* algorithm can be implemented using the pseudo-code available in algorithm 8. The initialization process (lines 1 through 5) prepares auxiliary information to allow the calculation of the length and load of any subroute in constant time. This step is done in $\Theta(|V|)$ time.

The algorithm then proceeds to calculate the shortest path (lines 6 through 16), without the need to explicitly create the auxiliary graph. As explained above, this process runs in $\Theta(|V|^2)$. The final step of the clustering algorithm uses the information obtained through the shortest path procedure to build the optimal set of routes (lines 19 through 26), with a running time of $\Theta(|V|)$. This yields the total running time of $\Theta(|V|^2)$.

Algorithm 8 ACVRP clustering algorithm

Input: A complete graph $G = (V, E)$ and a route $route$ to cluster
Output: An set of ACVRP routes

1: $load \leftarrow [0]$, $previous \leftarrow [0]$, $distance \leftarrow [0]$, $length \leftarrow [0]$
2: **for all** $i = 1$ to $|route| - 1$ **do** // initialization
3: $load.append(load.last + demand(route[i]))$
4: $length.append(length.last + cost(route[i-1], route[i]))$
5: **end for**
6: **for** $i = 0$ to $|route| - 1$ **do** // shortest path algorithm
7: **for** $j = i + 1$ to $|route| - 1$ **do**
8: $v\_load \leftarrow load[j] - load[i+1] + demand(route[i+1])$
9: $v\_length \leftarrow length[j] - length[i+1] + cost(depot, route[i+1]) + cost(route[j], depot)$
10: $dist \leftarrow distance[i] + v\_length$
11: **if** $v\_load < capacity \wedge dist < distance[j]$ **then**
12: $distance[j] \leftarrow dist$
13: $previous[j] \leftarrow i$
14: **end if**
15: **end for**
16: **end for**
17: $routes \leftarrow []$
18: $i \leftarrow |route| - 1$
19: **while** $previous[i] \neq i$ **do** // route construction
20: $r \leftarrow [depot]$
21: **for** $j = previous[i]$ to i **do**
22: $r.append(route[j])$
23: **end for**
24: $routes.append(r)$
25: $i \leftarrow previous[i]$
26: **end while**
27: **return** routes

5.7 Clarke-Wright savings heuristic for the ACVRP

The Clarke-Wright heuristic [CW64], one of the most known heuristic for the CVRP [LS01], is based on the notion of *savings*. It builds a vehicle route for every container and follows by merging pairs of routes. When merging two routes in the form $\langle depot, \ldots, i, depot \rangle$ and $\langle depot, j, \ldots, depot \rangle$, the cost reduction — or saving — is given by the expression in equation 5.6:

$$s_{ij} = cost(i, depot) + cost(depot, j) - cost(i, j) \tag{5.6}$$

The heuristic proceeds by merging every pair of routes in descending order of s_{ij}, as long as the merge operation does not violate any of the following conditions:

- s_{ij} is greater of equal to 0;
- the merged route does not violate the capacity constraint;
- vertex i is the first of its route;
- vertex j is the last of its route;
- vertices i and j do not belong to the same route.

Even if a saving is 0, merging the two routes reduces the number of vehicles by one, so only negative savings are forbidden. As savings between vertices does not change and a merge between two vertices i and j can only be done once, one only needs to initially calculate all savings and sort them.

Creating a route for each vertex has a time complexity of $\Theta(|V|)$. Calculating savings and sorting them can be done in $\Theta(|V|^2 log|V|)$. With proper data structures, the merging operation can be done in constant time. This results in a total running time complexity of $\Theta(|V|^2 log|V|)$.

The pseudo-code for the Clarke-Wright heuristic is available in algorithm 9. To allow merging in constant time, each vertex contains a pointer to the previous and following vertices, as well as pointers to the first and last vertices of its route.

The initialization process (lines 1 through 12) creates the initial routes and calculates all the savings, in $\Theta(|V|^2)$ time. Then, the savings are sorted in descending order, with a time complexity of $\Theta(|V|^2 log|V|)$ (line 13). Then, savings are iterated, and if all the conditions are verified, its respective merging is applied (lines 14 to 27). As there are $|V|^2$ steps, and each merge is executed in constant time, this step runs with a time complexity of $\Theta(|V|^2)$. Finally, routes are constructed in $\Theta(|V|)$ (lines 29 through 38).

Algorithm 9 Clarke-Wright savings heuristic for the ACVRP

Input: A complete graph $G = (V, E)$
Output: An set of ACVRP routes

1: $next \leftarrow []$, $prev \leftarrow []$
2: $first \leftarrow []$, $last \leftarrow []$
3: $load \leftarrow [], savings \leftarrow []$
4: **for all** $v \in V \setminus depot$ **do** // initialization
5: $next[v] \leftarrow prev[v] \leftarrow depot$
6: $first[v] \leftarrow last[v] \leftarrow v$
7: $load[v] \leftarrow demand(v)$
8: **for all** $w \in V \setminus depot$ **do**
9: $c \leftarrow cost(v, depot) + cost(depot, w) - cost(v, w)$
10: $savings.append((c, v, w))$
11: **end for**
12: **end for**
13: sort(savings) // sort by s_{ij} in descending order
14: **for all** $(c, v, w) \in savings$ **do** // visit savings in descending order, merging routes
15: **if** $s_{vw} \geq 0$ **then**
16: **if** $next[v] = depot \wedge prev[w] = depot \wedge last[w] \neq v \wedge first[v] \neq w$ **then**
17: **if** $load[v] + load[w] \leq capacity$ **then**
18: $next[v] \leftarrow w$
19: $prev[w] \leftarrow v$
20: $first[last[w]] \leftarrow first[v]$
21: $last[first[v]] \leftarrow last[w]$
22: $load[first[v]] \leftarrow load[last[w]] \leftarrow load[v] + load[w]$
23: **end if**
24: **end if**
25: **else**
25: **break**
26: **end if**
27: **end for**
28: $routes \leftarrow []$
29: **for all** $v \in V$ **do** // route construction
30: **if** $v \neq depot \wedge prev[v] = depot$ **then**
31: $route \leftarrow [depot]$
32: **while** $v \neq depot$ **do**
33: $route.append(v)$
34: $v \leftarrow next[v]$
35: **end while**
36: $routes.append(route)$
37: **end if**
38: **end for**
39: **return** routes

5.8 Approach overview

The previous sections in this chapter presented several algorithms. First, sections 5.1 to 5.4.2 introduced six known techniques to address the ATSP. Section 5.6 described an algorithm to divide a route for the ATSP into a feasible set of routes for a ACVRP on the same graph. The process of first calculating an ATSP route and then dividing it into a ACVRP solution is known as a *route-first-cluster-second* approach.

Section 5.7 presented a technique to address the ACVRP directly, without the need to first find an optimized ATSP solution. This method will be compared to the six *route-first-cluster-second* approaches previously described.

Changing the order by which a vehicle visits the containers does not invalidate a solution, as the total waste to be collected does not change — maintaining the validity of the capacity constraints. This means that further optimization may be achieved by permuting the vertices of each route. This is equivalent to solving the ATSP within a sub-graph that only contains the route vertices.

This final optimization step will be applied to the six *route-first-cluster-second* approaches and to the Clarke-Wright approach. To optimize the routes obtained through the savings algorithm, two ATSP techniques were used: 2-opt and MMAS. On the other hand, routes obtained using the six *route-first-cluster-second* approaches were optimized using 2-opt.

Although it would be interesting to obtain data for all six ATSP techniques, it would not be possible to execute all the algorithms in time for this project's deadline. Table 5.2 summarizes all the ACVRP techniques evaluated during this study.

Table 5.2: Summary of the ACVRP techniques evaluated during this study. RNN: *repetitive nearest neighbor*; GA: *genetic algorithm*; MMAS: *MAX-MIN ant system*

ATSP algorithm	**Clustering**	**Route improvement**
Greedy	*Split*	*2-opt*
RNN	*Split*	*2-opt*
RNN + 2-opt	*Split*	*2-opt*
Greedy + 2-opt	*Split*	*2-opt*
GA	*Split*	*2-opt*
RNN + MMAS	*Split*	*2-opt*
ACVRP algorithm		**Route improvement**
Savings		*2-opt*
Savings		*RNN + MMAS*

5.9 Validation of algorithm implementations

After implementing the algorithms described in this chapter, there was the need to validate their results. Validation is done by applying these algorithms to standard *TSPLIB* datasets and comparing the results to either the optimum route cost (when known) or to the best route found so far.

5.9.1 ATSP validation

First, construction heuristics — *greedy* and *repetitive nearest neighbor* —, *hill climbing*, the *genetic algorithm* and *MAX-MIN ant system* were validated against ATSP instances. This allows one to verify if these techniques successfully build a near-optimal tour.

It is important to note that, as *hill climbing* and *MAX-MIN ant system* both depend on the *repetitive nearest neighbor* technique, their resulting routes are always better or equal to the one obtained by this construction heuristic.

Results are presented in table 5.3. All parameters used to obtain these values are according to the original authors of each algorithm. Both GA and MMAS were executed for 1000 iterations. GA was run with a population of 100 and $p = 0.35$. MMAS was run with 200 ants and $\beta = 5$.

Table 5.3: Performance of our heuristic and meta-heuristic implementations using the datasets from TSPLIB. Performance is given in the form of the ratio between the heuristic's route average cost and the dataset optimum (minimum) cost.

Dataset	Vertices	Optimum	Greedy	RNN	Greedy + 2-opt	RNN + 2-opt	GA	MMAS
br17	17	39	1.97	1.43	1.00	1.00	1.00	1.00
ftv33	33	1286	1.16	1.23	1.14	1.17	1.17	1.00
ftv35	35	1473	1.30	1.13	1.31	1.13	1.13	1.04
ftv38	38	1530	1.25	1.14	1.24	1.08	1.08	1.07
p43	43	5620	1.03	1.01	1.00	1.00	1.00	1.00
ftv44	44	1613	1.23	1.14	1.24	1.14	1.09	1.03
ftv47	47	1776	1.27	1.22	1.27	1.20	1.15	1.11
ry48p	48	14422	1.32	1.07	1.04	1.02	1.05	1.02
ft53	53	6905	1.77	1.24	1.54	1.18	1.19	1.15
ftv55	55	1608	1.40	1.21	1.34	1.21	1.13	1.07
ftv64	64	1839	1.36	1.19	1.29	1.17	1.16	1.04
ftv70	70	1950	1.30	1.17	1.26	1.17	1.16	1.09
ft70	70	38673	1.14	1.08	1.11	1.06	1.05	1.04
kro124p	124	36230	1.21	1.19	1.15	1.17	1.15	1.08
ftv170	170	2755	1.43	1.30	1.40	1.28	1.24	1.18

Table 5.3 shows that 2-opt, GA and MMAS yield significantly better results than the heuristics. MMAS, using only 1000 iterations, produces results within 5% of the optimal solution for half of the datasets.

5.9.2 ACVRP validation

Having validated the six ATSP techniques, it is necessary to validate the eight ACVRP approaches described in section 5.8 (table 5.2). These approaches were applied to the CVRP datasets available in *TSPLIB* to obtain the benchmarks in table 5.4.

Table 5.4: Performance of our heuristic and meta-heuristic implementations using the datasets from TSPLIB. Performance is given in the form of the ratio between the heuristic's route average cost and the dataset reference solution's cost.

Vertices	Greedy	RNN	Greedy + 2-opt	RNN + 2-opt	GA	MMAS	Savings + 2-opt	Savings + MMAS
13	1.223	1.198	1.198	1.193	1.223	1.190	1.174	1.174
22	1.204	1.122	1.068	1.046	1.012	1.062	1.037	1.037
23	1.172	1.066	1.012	1.030	1.066	1.007	1.013	0.999
30	0.961	0.961	0.954	0.979	0.946	0.946	1.001	0.952
31	1.472	1.631	1.760	1.548	1.451	1.310	1.657	1.617
33	1.066	1.042	1.034	1.037	1.033	1.033	1.010	1.009
51	1.148	1.137	1.139	1.110	1.087	1.095	1.136	1.125
76	1.408	1.346	1.346	1.342	1.319	1.348	1.332	1.306
76	1.593	1.573	1.515	1.498	1.518	1.491	1.471	1.471
76	1.053	0.994	1.031	0.990	0.977	0.989	0.951	0.935
76	0.772	0.759	0.780	0.746	0.732	0.757	0.747	0.719
101	1.217	1.167	1.168	1.123	1.070	1.093	1.090	1.076
101	1.176	1.136	1.141	1.120	1.130	1.072	1.076	1.066
262	1.019	1.013	1.036	0.998	0.986	0.996	0.952	0.945

It is important to remember that the reference values for these datasets represent solutions for a fixed number of vehicles, specified in the dataset. When applied to this specific problem, the number of vehicles is a decision variable, and not a fixed number. This explains why some algorithms yield solutions with ratios below 1.0, as it may be possible to obtain routes that are more efficient by using a different number of vehicles.

This values show that the *route-first-cluster-second* approach, using any of the six ATSP techniques, is surpassed by the *savings* heuristic in most of the CVRP datasets. However, there are two facts that must be taken into account. First, all of *TSPLIB*'s CVRP datasets are symmetric. Second, these datasets only go up to 262 vertices. Datasets used in chapter 6 are asymmetric and contain a much higher number of vertices. With this in mind, the values in table 5.4 are insufficient to determine if the *savings* approach is better or not than the *route-first-cluster-second* approaches.

5.10 Chapter summary

This chapter described a set of algorithms that can be used to address the *Asymmetric Capacitated Vehicle Routing Problem*. Most of these start by solving the associated *Asymmetric Traveling Salesman Problem*, by ignoring capacity constraints, and subsequently split the resulting tour into vehicle routes using the algorithm described in section 5.6. Additionally, a heuristic for the CVRP was also described — the Clarke-Wright *savings* heuristic. This technique does not start by solving the ATSP, and therefore it is not a *route-first-cluster-second* approach.

The following chapter presents the results for a sensitivity analysis on *MAX-MIN ant system*'s parameters, to maximize its performance. This is followed by an exposition of the results obtained when applying the eight techniques for the ACVRP to the large datasets. Although this chapter already provided some results regarding their comparative performance, the validation datasets are quite different from the ones to be used when the optimization framework is deployed.

Chapter 6

Results

This chapter reports several benchmarks analyzing the performance of several algorithms, as well as benchmarks to analyze the influence of several parameters of the *MAX-MIN ant system*.

The first section, section 6.1, presents the sensitivity analysis on *MAX-MIN ant system*'s parameters. It provides information that allows one to balance the trade-off between route efficiency and running time.

Section 6.2 presents the benchmark results that compare the techniques described in the previous chapter when applied to the large *Asymmetric Capacitated Vehicle Routing Problem* datasets.

Unless specified otherwise, all benchmarks shown are obtained through averaging ten runs for each algorithm/dataset. All running times were obtained by running the algorithms on the same hardware and platform. The CPU was an Intel® Xeon®, running at 2.33 GHz with 1 GB of RAM, running on the Ubuntu operating system, a Linux distribution.

6.1 Sensitivity of MAX-MIN ant system to algorithm parameters

To understand how MMAS parameters influence the performance of the resulting routes, there was the need to execute a series of benchmarks and comparisons. These were done by varying the target parameter's value and by using the validation datasets. This analysis will aid in configuring the parameters for the application of this technique to larger datasets.

6.1.1 Parameter β sensitivity

The first parameter whose influence was evaluated was β, a parameter that measures the relative weight between an given arc distance and pheromone intensity (Equation 5.1). The higher the β, the more ants tend to choose shorter arcs and ignore pheromones. If β is too high, the pheromone intensity will be negligible, degenerating into the *nearest neighbor* heuristic.

To verify whether a correlation between the best β and number of nodes exists or not, this analysis was made using datasets of 35, 70 and 170 vertices, available in the *TSPLIB* library.

The number of ants to use in each dataset is equal to the number of nodes, as suggested in [SH97]. MMAS was ran for 200 iterations in each dataset with β values ranging from 0 to 150. The resulting average route cost can be seen in Figure 6.1.

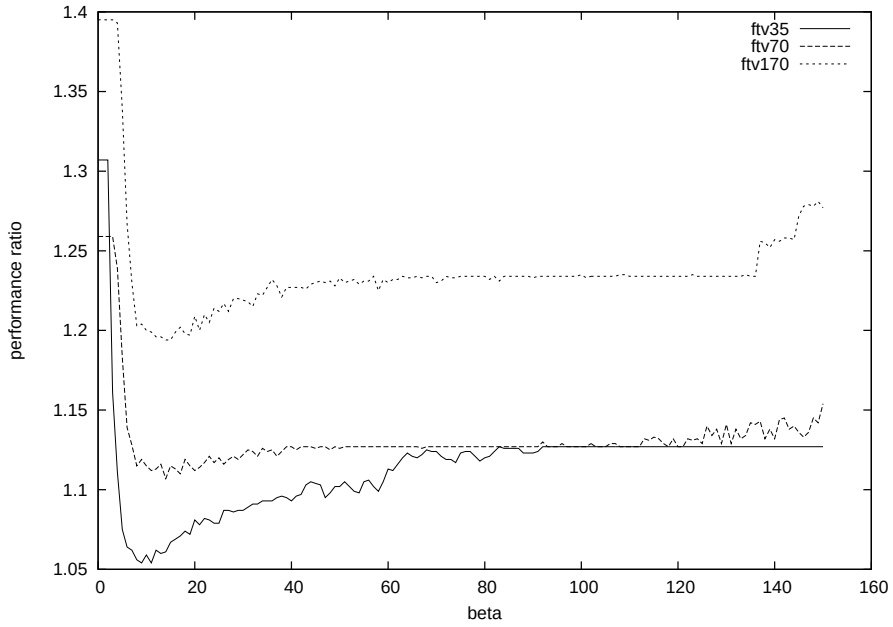


Figure 6.1: Variations in final route performance when varying beta in the MAX-MIN ant system for datasets ftv35, ftv70 and ftv170. Performance is given in the form of the ratio between the heuristic's route average cost and the dataset optimum (minimum) cost.

We can observe that the best result for all datasets occurs for, approximately, within the range $[5,20]$. This is a good indicator that the optimum value for β is independent from the number of vertices in the graph.

The curve is steeper for β values between 0 and 5. As such it was decided to use $\beta = 20$ in all subsequent executions, to prevent against possible oscillations on the curve when dealing with larger graphs.

6.1.2 Trade-off between ants and iterations

Having obtained a good reference for parameter β, the importance of the number of ants and iterations was evaluated. Both of these parameters highly influence the running time of the algorithm and the resulting route efficiency. As such, it is important to determine what are the values for these two parameters that yield more efficient routes within a limited time frame.

The first benchmark was done to verify if increasing the number of ants and iterations produces more efficient routes. MMAS was applied to the larger of the previous three datasets, *ftv170*, with a varying number of ants for 10000 iterations and with $\beta = 20$. Figure 6.2 shows the results.

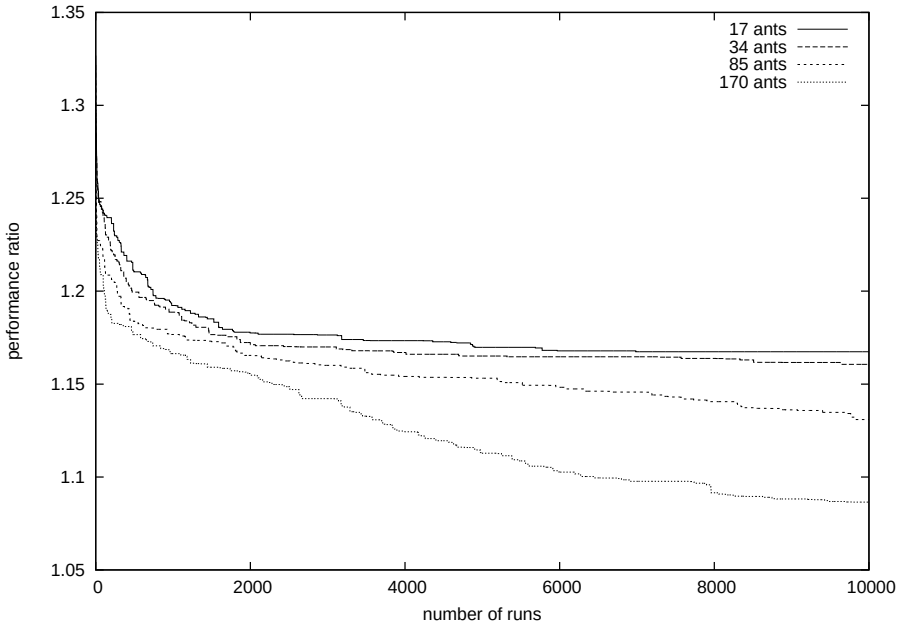


Figure 6.2: Evolution of final route performance when varying the number of ants and number of iterations for the MAX-MIN ant system on dataset ftv170. Performance is given in the form of the ratio between the heuristic's route average cost and the dataset optimum (minimum) cost.

It becomes clear from the results that increasing both the number of ants and the number of iterations — as had been suggested by the original authors — improves the performance of the meta-heuristic. It is important to verify if the behavior presented in figure 6.2 changes when the number of nodes in the dataset increases. This was done by running additional benchmarks with large datasets. Figure 6.3 shows the route cost evolution on a dataset with 1287 vertices, while figure 6.4 shows this evolution on an even larger dataset, with 6247 vertices.

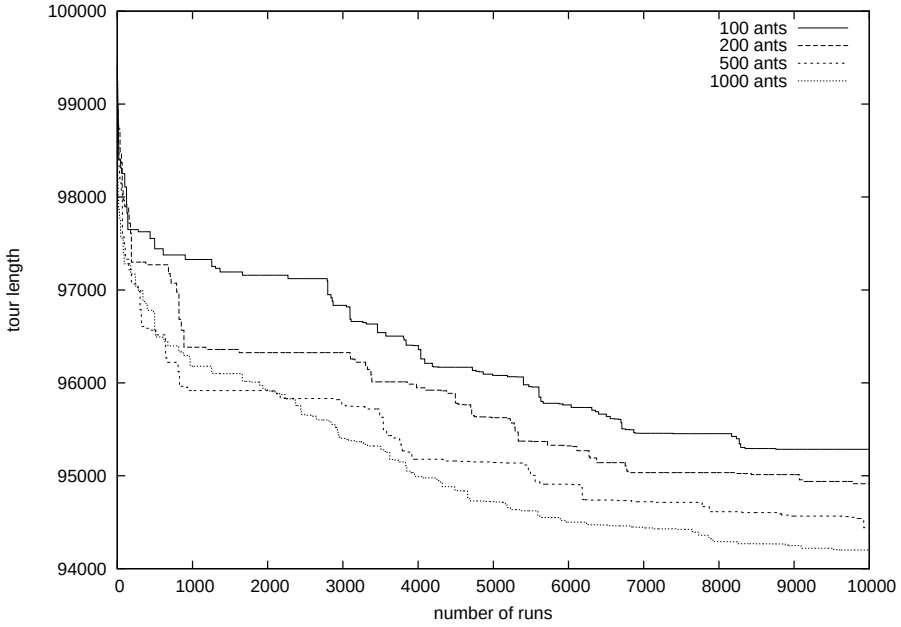


Figure 6.3: Evolution of final tour length when varying the number of ants and number of iterations for the MAX-MIN ant system a dataset with 1287 vertices.

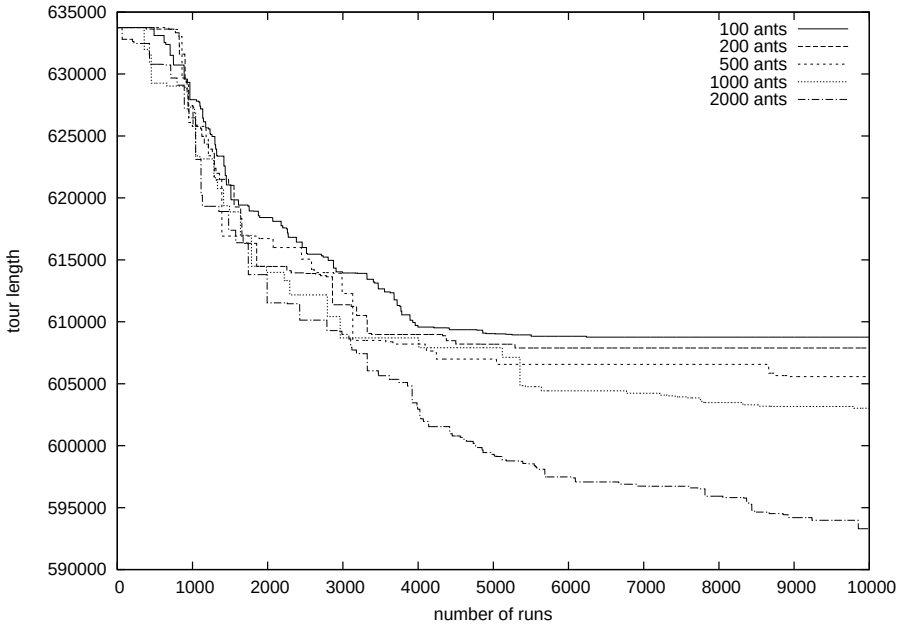


Figure 6.4: Evolution of final tour length when varying the number of ants and number of iterations for the MAX-MIN ant system a dataset with 6247 vertices.

The two figures 6.3 and 6.4 show that the behavior that occurred in figure 6.2 occurs, independently of the number of vertices in the graph. The difference in the route efficiency, when varying the number of ants, is more prominent for large number of iterations. The larger the dataset, the higher the number of iterations has to be in order for this difference to be noticeable.

It is worthy to notice that there is a trade-off between the computation time and the efficiency of the obtained solution. Although all runs using the dataset with 1287 vertices finished executing before a two hour limit, the same was not true for the 6247 vertices dataset. Table 6.1 presents the average route efficiency when varying the number of ants in dataset hp6247, after two hours of runtime.

Table 6.1: Trade-off between running time and route efficiency, when varying the number of ants in dataset hp6247.

Number of ants	Average route cost	Average number of iterations
100	609056	6963
200	608310	4423
500	616919	1775
1000	629026	841
2000	632468	355

Although figure 6.4 shows that a higher number of ants yields better results in the long run, table 6.1 shows that when given a limited time frame of two hours, it pays off to use a lower number of ants. Figure 6.4 also shows that the average stagnation point of the 100 ants curve occurs before the allowed number of iterations within a two hour time-frame, so that the execution of the last runs will bring no significant improvement. This explains why table 6.1 reports that runs with 200 ants achieve, in average, more efficient routes.

As a result of the sensibility analysis done in this section, it was concluded that parameter β should have a value around 20. It was also shown that when using small datasets — or when the allowed running time is high — it pays off to use a higher number of ants. However, when applying MMAS to large graphs within a limited time-frame, a lower number of ants achieves better results.

6.2 Algorithm performance on large ACVRP instances

To analyze the performance of the implemented algorithms when dealing with large graphs, the seven techniques described in section 5.8 were applied to the large datasets presented in section 4.5.2.

Five of these seven techniques are *route-first-cluster-second* approaches, obtained through the application of one of the five ATSP methods — *greedy* and *repetitive nearest neighbor* heuristics, *hill climbing* (2-opt), a *genetic algorithm* and *MAX-MIN ant system* — followed by clustering using the *split* algorithm. After clustering, each of the resulting routes is subjected to the 2-opt algorithm for further optimization.

The other two techniques are obtained through the application of the Clarke-Wright's *savings* heuristic, where each of the resulting routes is optimized either by applying the 2-opt algorithm or MMAS.

The parameters for MMAS were chosen according to the sensibility analysis presented in the previous section. All datasets were run with $\beta = 20$ and 200 ants for 3000 iterations. Additionally, the length of the candidate lists was set to 15. The GA was executed with a population of 200 for 10000 generations, so that its running time is close to the one of MMAS (see the complexity analysis on section 5.5). As explained in section 5.3, this technique relies in mutations only.

All seven approaches can be divided into two main steps. The first step involves finding a feasible CVRP solution, while the second step improves each of the resulting routes by applying ATSP solutions. Furthermore, five of the seven proposed approaches address the first step by further subdividing it into two phases: finding an efficient ATSP solution for the whole graph (disregarding capacity constraints) and clustering it into a feasible set of routes.

This section will present information regarding all of the three stages. As two of the approaches do not rely on finding an initial ATSP solution, they will only be analyzed on the two latter stages — finding a CVRP solution and optimizing each of its routes.

6.2.1 ATSP solutions

Table 6.2 presents the average performance of the ATSP approaches when applied to large datasets. The first four techniques — *greedy*, *repetitive nearest neighbor* and both 2-opt algorithms — are deterministic, so only one run of each was executed, as the outcome would be always the same.

Table 6.2: Performance of ATSP heuristic and meta-heuristic implementations using realistically large datasets. Minimum values for each dataset are highlighted.

Vertices	Greedy	RNN	RNN + 2-opt	Greedy + 2-opt	GA	MMAS
518	64657	60967	58781	57911	58697	**53853**
841	83008	80964	78516	78226	77871	**76368**
904	93281	90415	87507	88594	87621	**80721**
1175	151028	157984	152446	139663	152172	**138363**
1287	105806	99520	96348	97940	96468	**93179**
1849	193105	187885	183085	181541	182960	**170317**
2038	221044	218138	211112	205540	211402	**205131**
2206	210941	208363	202071	196331	202471	**192239**
2561	489241	507214	495844	465692	495874	**454568**
3481	514938	505256	492537	479361	492732	**471214**
3859	513733	519371	505310	482014	505000	**479331**
4109	531572	537033	528873	506615	529007	**504745**
4628	586891	576868	563820	555597	564614	**537557**
6247	639826	633747	623122	600162	623118	**598762**
7628	699255	690216	676043	653948	676144	**650564**

Table 6.2 shows that MMAS achieves better results when solving the ATSP for all of the large datasets. The proposed GA has a performance similar to that of RNN 2-opt. In some datasets,

2-opt applied to the greedy heuristic yields routes almost as efficient as MMAS. Additionally, it is possible to calculate that the average improvement percentage of MMAS over all 15 datasets is of 2.6%, with a standard deviation of 2.5%.

To aid the comparison of the techniques, table 6.3 shows the values from table 6.2 after a normalization process. Normalization is done by dividing each value by the minimum value of its dataset.

Table 6.3: Normalized performance of ATSP techniques. Each value expressed as the percentage excess relative to the minimum value for its dataset.

	Greedy	RNN	Greedy + 2-opt	RNN + 2-opt	GA	MMAS
	20.1 %	13.2 %	7.5 %	9.2 %	9.0 %	0.0 %
	8.7 %	6.0 %	2.4 %	2.8 %	2.0 %	0.0 %
	15.6 %	12.0 %	9.8 %	8.4 %	8.5 %	0.0 %
	9.2 %	14.2 %	0.9 %	10.2 %	10.0 %	0.0 %
	13.6 %	6.8 %	5.1 %	3.4 %	3.5 %	0.0 %
	13.4 %	0.3 %	6.6 %	7.5 %	7.4 %	0.0 %
	7.8 %	6.3 %	0.2 %	2.9 %	3.1 %	0.0 %
	9.7 %	8.4 %	2.1 %	5.1 %	5.3 %	0.0 %
	7.6 %	11.6 %	2.4 %	9.1 %	9.1 %	0.0 %
	9.3 %	7.2 %	1.7 %	4.5 %	4.6 %	0.0 %
	7.2 %	8.4 %	0.6 %	5.4 %	5.4 %	0.0 %
	5.3 %	6.4 %	0.4 %	4.8 %	4.8 %	0.0 %
	9.2 %	7.3 %	3.4 %	4.9 %	5.0 %	0.0 %
	6.9 %	5.8 %	0.2 %	4.1 %	4.1 %	0.0 %
	7.5 %	6.1 %	0.5 %	3.9 %	3.9 %	0.0 %
μ	10.1 %	8.7 %	2.9 %	5.7 %	5.7 %	0.0 %
σ	3.9 %	2.8 %	3.0 %	2.5 %	2.5 %	0.0 %

Through the normalized values in table 6.3, it is easy to see that the 2-opt algorithm with the greedy heuristic is the second best approach to address the ATSP. These data also confirm that both the *genetic algorithm* and RNN 2-opt are similar.

6.2.2 ACVRP solutions

Although MMAS outperforms the other algorithms when solving the ATSP, it is not guaranteed that it will outperform them when solving the ACVRP. To verify this, the tours obtained in the previous sections were subjected to the clustering algorithm described in section 5.6. Additionally, the *savings* heuristic was also executed, so that *route-first-cluster-second* approaches could be compared to a constructive heuristic for the CVRP.

The efficiency of each solution is given by the sum of each vehicle's route length. The results for this run are in table 6.4. The *savings* heuristic is deterministic, so there was not the need to run

it more than once. The *genetic algorithm* and *max-min ant system* had to be run multiple times, due to their stochastic nature.

Table 6.4: Performance of heuristic and meta-heuristic implementations using realistically large datasets. Minimum values for each dataset are highlighted.

Vertices	Greedy	RNN	Greedy + 2-opt	RNN + 2-opt	GA	MMAS	Savings
518	78072	77564	71521	75933	75448	**70640**	93759
841	114308	110422	109654	111192	107092	**104913**	115576
904	120087	120124	**112396**	117111	116067	113603	124092
1175	223817	234530	224408	232428	232154	212875	**194312**
1287	133382	132784	**125682**	129607	129669	126473	152229
1849	263830	273410	257318	263879	263935	249708	**238101**
2038	332029	333665	316529	317786	318345	304314	**281031**
2206	319361	325752	303652	313078	319353	304754	**291317**
2561	766226	777766	745094	767076	765974	**733627**	831986
3481	903800	905700	**865213**	894672	894868	876834	994299
3859	910598	917740	880713	904655	903134	**879900**	1040089
4109	1062410	1071682	**1038768**	1062764	1062898	1044653	1193248
4628	1099846	1104546	**1067747**	1091359	1095330	1070835	1245284
6247	1381520	1379607	**1339356**	1369770	1369637	1353391	1639008
7628	1579516	1595006	**1537125**	1577184	1577286	1556051	1912985

Table 6.4 shows that there is no direct relationship between the ATSP solution efficiency and its respective ACVRP efficiency. Nevertheless, MMAS yields more efficient routes in some of the datasets. The other two techniques that achieve the efficient routes for some datasets are the 2-opt algorithm, when used with the *greedy* heuristic, and the *savings* construction heuristic.

Another important conclusion that the results on table 6.4 allow one to make is that although the *greedy* heuristic has the lowest performance on the ATSP, this gap is greatly reduced after applying the clustering algorithm.

This is better described by quantifying the difference between algorithms when transforming ATSP solutions to ACVRP solutions. Table 6.5 shows, for each algorithm, the average excess when compared to the best solution found for each dataset.

Table 6.5: Average route efficiency excess when comparing each algorithm to the best solution found for each dataset.

	Greedy	RNN	Greedy + 2-opt	RNN + 2-opt	GA	MMAS
ATSP	10.1 %	8.7 %	2.9 %	5.7 %	5.7 %	0.0 %
ACVRP	7.3 %	8.1 %	3.2 %	6.1 %	5.9 %	2.2 %

Table 6.5 show that the *greedy* has the highest improvement, even surpassing the *repetitive nearest neighbor* heuristic. MMAS no longer dominates all the other algorithms (as shown in table 6.4), but it is the algorithm with the lowest gap.

6.2.3 Route improvement

At this stage, each algorithm has already produced a feasible ACVRP solution, composed of several routes. Each of the routes may be further optimized using the previously mentioned ATSP techniques.

As shown in table 5.2, the six *route-first-cluster-second* approaches' solutions will be optimized using the 2-opt improvement technique. On the other hand, the *savings* constructive heuristic will be optimized using two techniques: 2-opt and MMAS. Table 6.6 shows the final solution costs for all techniques.

Table 6.6: Final ACVRP solutions' performance, after the application of the 2-opt improvement technique. Minimum values for each dataset are highlighted.

2-opt improvement							MMAS
Greedy	RNN	Greedy + 2opt	RNN + 2-opt	GA	MMAS	Savings	Savings
70534	74469	70588	74917	74422	**70161**	91899	79032
107288	103128	107304	106233	**102727**	103291	112972	108669
112511	111993	**109614**	111479	113266	110986	120648	120223
212389	225613	221932	230153	229544	208737	**189620**	190846
124957	128623	**124510**	128624	128700	125161	149926	140379
248255	264648	253844	259096	259068	247085	**233170**	236821
309866	323190	310721	312768	312855	300796	**276362**	280927
301713	312762	300303	306855	313087	301897	**284023**	284310
740730	764905	740815	763068	761945	**729603**	823621	832580
865570	887982	**859474**	885902	885972	873044	975839	965098
874926	896603	**871511**	895631	893888	871832	1026658	987353
1030661	1056671	1033678	1053607	1053741	1041138	1179891	1154823
1058717	1088214	**1057246**	1085945	1090055	1064012	1212899	1216222
1332414	1360244	**1328719**	1360440	1360441	1344851	1620740	1582320
1518843	1565735	1520317	1562099	1562200	1544687	1894903	1840384

The information on table 6.6 shows that the minimum values for each dataset are now scattered throughout the eight approaches. To help with the analysis between the approaches, table 6.7 presents a normalization of these results.

Table 6.7: Final ACVRP solutions' performance, after applying the final route optimization technique. Values are normalized by calculating the excess between a given algorithm's performance and the best value found for its dataset.

	2-opt improvement							MMAS
	Greedy	RNN	Greedy + 2opt	RNN + 2-opt	GA	MMAS	Savings	Savings
	0.5 %	6.1 %	0.6 %	6.8 %	6.1 %	0.0 %	31.0 %	12.6 %
	4.4 %	0.4 %	4.5 %	3.4 %	0.0 %	0.5 %	10.0 %	5.8 %
	2.6 %	2.2 %	0.0 %	1.7 %	3.3 %	1.3 %	10.1 %	9.7 %
	12.0 %	19.0 %	17.0 %	21.4 %	21.1 %	10.1 %	0.0 %	0.6 %
	0.4 %	3.3 %	0.0 %	3.3 %	3.4 %	0.5 %	20.4 %	12.7 %
	6.5 %	13.5 %	8.9 %	11.1 %	11.1 %	6.0 %	0.0 %	1.6 %
	12.1 %	16.9 %	12.4 %	13.2 %	13.2 %	8.8 %	0.0 %	1.7 %
	6.2 %	10.1 %	5.7 %	8.0 %	10.2 %	6.3 %	0.0 %	0.1 %
	1.5 %	4.8 %	1.5 %	4.6 %	4.4 %	0.0 %	12.9 %	14.1 %
	0.7 %	3.3 %	0.0 %	3.1 %	3.1 %	1.6 %	13.5 %	12.3 %
	0.4 %	2.9 %	0.0 %	2.8 %	2.6 %	0.0 %	17.8 %	13.3 %
	0.0 %	2.5 %	0.3 %	2.2 %	2.2 %	1.0 %	14.5 %	12.0 %
	0.1 %	2.9 %	0.0 %	2.7 %	3.1 %	0.6 %	14.7 %	15.0 %
	0.3 %	2.4 %	0.0 %	2.4 %	2.4 %	1.2 %	22.0 %	19.1 %
	0.0 %	3.1 %	0.1 %	2.8 %	2.9 %	1.7 %	24.8 %	21.2 %
μ	3.2 %	6.2 %	3.4 %	6.0 %	5.9 %	2.6 %	12.8 %	10.1 %
σ	4.2 %	5.8 %	5.4 %	5.5 %	5.6 %	3.4 %	9.7 %	6.7 %

From table 6.7, one can see that although it does not achieve the minimum for many datasets, MMAS has the highest performance ratio (2.6%). It also has the smallest standard deviation (3.4%). This makes it the most stable algorithm — of those studied — to solve the ACVRP on the sample datasets.

Although the *savings* heuristic followed by a 2-opt improvement produces efficient routes for some of the datasets, it has a poor average performance ratio. The *savings* heuristic followed by MMAS has a better performance, with a lower average excess gap and reduced standard deviation.

The second best algorithm, when taking these metrics into consideration, is the *greedy heuristic*. This shows that the CVRP performance of an algorithm does not strictly depend on its performance on the ATSP. Table 6.8 summarizes the average performance for each algorithm, in each of the three steps.

Table 6.8: Summary of the several techniques' performance, over the three optimization stages.

	Greedy	RNN	Greedy + 2-opt	RNN + 2-opt	GA	MMAS	Savings	Savings + MMAS
ATSP	10.1 %	8.7 %	2.9 %	5.7 %	5.7 %	0.0 %	-	-
ACVRP	7.3 %	8.1 %	3.2 %	6.1 %	5.9 %	2.2 %	13.3 %	13.3 %
Final	3.2 %	6.2 %	3.4 %	6.0 %	5.9 %	2.6 %	12.8 %	10.1 %

6.3 Chapter summary

This chapter presented the sensitivity analysis for the *MAX-MIN ant system*, regarding three parameters: β, number of ants and number of iterations. This chapter also exposed results regarding the application of the eight ACVRP techniques to large datasets.

Next chapter finalizes this document by presenting an overview of all the conclusions made throughout this project — both related to the algorithm comparison and to the waste collection optimization framework.

Chapter 7

Conclusions

This final chapter presents a review of the relevant information obtained from this study and an exposition of further research that could be done in this area. Section 7.1 presents the sensibility analysis results regarding the *MAX-MIN ant system* and describes results on the application of the *route-first-cluster-second* approach to the waste collection scenario.

Section 7.2 exposes some thoughts on how to further extend the study on both the *Asymmetric Traveling Salesman Problem* and the *Capacitated Vehicle Routing Problem*. It also finalizes this document by presenting the next stages of implementing the waste collectionwaste collection optimization framework.

7.1 Conclusions

This document presented an architecture proposal for the storage and retrieval of containers' status, together with formats for information interchange. These formats are built on top of JSON, an open standard that can be used to represent simple data structures. An alternative would be to use XML; however, JSON was preferred over XML due to the following reasons:

- JSON has a simpler notation than that of XML;
- JSON was designed to serve as a data interchange format, whereas XML was designed to be a document interchange format.

The second goal of this work is to provide insight on existing algorithms to calculate efficient routes for waste collection. This problem is modeled as an *Asymmetric Capacitated Vehicle Routing Problem* (ACVRP). Several methods have been proposed, in the literature, on how to approach the ACVRP.

To solve the ACVRP, there are construction heuristics (such as the one by Clarke-Wright). These yield a set of vehicle routes, which can be further optimized by applying ATSP solving techniques.

Another method to address the ACVRP involves solving the associated *Asymmetric Traveling Salesman Problem* (ATSP) and dividing the resulting tour into routes that respect the vehicle capacity constraints. This is called a *route-first-cluster-second* approach.

To solve the ATSP, several techniques were compared: two construction heuristics — *greedy* and *repetitive nearest neighbor* (RNN) — and three meta-heuristics — *hill climbing* (2-opt), *genetic algorithms* (GA) and *MAX-MIN ant system* (MMAS). Apart from their comparison regarding the efficiency of resulting routes, MMAS was also targeted for a parameter sensibility analysis.

7.1.1 Sensitivity analysis of MMAS

As a result of the parameter sensitivity analysis for the *MAX-MIN ant system*, it was shown that, for the given datasets, the optimum value for parameter β lies in the interval $[5, 20]$. Although the optimum value appears to be located closed to the lower side of this interval, it is recommended that higher values are used. The rationale for this is that there is a significantly steep ascent for values close to $\beta = 5$. Thus, it was chosen to use $\beta = 20$ during this study. This helps to prevent against possible oscillations in the β optimization curve, when applying MMAS to larger graphs.

Regarding the number of ants and iterations, it was shown that increasing both parameters increases the resulting route efficiency. When applying this algorithm with a limited time frame, though, it is preferable to have a low number of ants and a high number of iterations. However, one must take into consideration that using MMAS with a low number of ants may lead to convergence too soon. Early convergence causes the algorithm not to take full advantage of the total available running time.

7.1.2 Comparison of methods for the ACVRP

When comparing the five methods to solve the ATSP, MMAS obtained more efficient routes than the other four methods. In average, the improvement of MMAS over the best route obtained using the other methods is of 2.6%.

Although MMAS outperforms the other techniques in solving the ATSP, the same needs not to hold for the ACVRP solutions. Immediately after clustering, the hill climbing approach applied to the *greedy* heuristic produces more efficient routes for half of the fifteen large datasets. Nevertheless, MMAS only exceeds the best solution for each dataset, in average, by 2.2%.

The third step of the optimization consists of applying an ATSP technique to each of the vehicle routes obtained from the previous step. After this step, MMAS is still the approach that yields better results, with an average excess of 2.6%. This technique also has the lowest standard deviation ($\sigma = 3.4\%$), which means that it is the most stable algorithm.

In terms of solution efficency, MMAS is followed by the *greedy* heuristic, which has an average excess of 3.2% ($\sigma = 4.2\%$). This shows that the initial ATSP solution performance does not directly influence the efficiency of the final ACVRP set of routes.

It was also shown that the *savings* heuristic performs better when followed by a MMAS optimization, rather than the 2-opt technique. This is consistent with the fact that MMAS had performed better than 2-opt when applied to ATSP instances. This leads to the conclusion that it would be possible to further reduce the route costs by applying MMAS at the final optimization step, instead of 2-opt.

A 12-page article summarizing the results of this study was submitted to an international workshop on algorithmic approaches for transportation modeling. The preliminary version can be seen in appendix A.

7.2 Future work

Regarding route optimization, this study analyzed a constructive heuristic and several *route-first-cluster-second* approaches for solving the *Capacitated Vehicle Routing Problem*. It would be possible to apply *cluster-first-route-second* methods to these approaches, although most of them rely on predefining a fixed number of vehicles to use.

It would also be interesting to apply MMAS and other techniques at the last optimization step, instead of simply using $2-opt$. This task was not done due to the time it would take to run all benchmarks.

Having designed the architecture for information management regarding waste containers' fill status, the next step is to implement a web service application layer between the central server and the sensors. This would consist of sending the container's fill status using through an HTTP request. Depending on the sensors' network architecture — currently being studied in Fraunhofer Portugal Research Center — authorization policies and mechanisms might have to be designed.

References

[ABMN04] Roberto Aringhieri, Maurizio Bruglieri, Federico Malucelli, and Maddalena Nonato. An asymmetric vehicle routing problem arising in the collection and disposal of special waste. *Electronic Notes in Discrete Mathematics*, 17:41 – 47, 2004. Workshop on Graphs and Combinatorial Optimization.

[AG95] Arjang A. Assad and Bruce L. Golden. Chapter 5 arc routing methods and applications. In C.L. Monma M.O. Ball, T.L. Magnanti and G.L. Nemhauser, editors, *Network Routing*, volume 8 of *Handbooks in Operations Research and Management Science*, pages 375 – 483. Elsevier, 1995.

[AG07] O. Apaydin and M.T. Gonullu. Route optimization for solid waste collection: Trabzon (Turkey) case study. *Global Nest Journal*, 9:6 – 11, 2007.

[Arc05] M.G. Speranza Archetti. Collection of waste with single load trucks: a real case. In B. Fleischmann and A. Klose, editors, *Distribution Logistics: Advanced Solutions to Practical Problems*, pages 105–119, Berlin Heidelberg, 2005. Springer-Verlag.

[BB98] J. M. Belenguer and E. Benavent. The Capacitated Arc Routing Problem: Valid Inequalities and Facets. *Comput. Optim. Appl.*, 10(2):165–187, 1998.

[BB03] José M. Belenguer and Enrique Benavent. A cutting plane algorithm for the capacitated arc routing problem. *Computers & Operations Research*, 30(5):705 – 728, 2003.

[BBLP06] José-Manuel Belenguer, Enrique Benavent, Philippe Lacomme, and Christian Prins. Lower and upper bounds for the mixed capacitated arc routing problem. *Computers & Operations Research*, 33(12):3363 – 3383, 2006. Part Special Issue: Recent Algorithmic Advances for Arc Routing Problems.

[BBM06] Roberto Baldacci, Lawrence Bodin, and Aristide Mingozzi. The multiple disposal facilities and multiple inventory locations rollon-rolloff vehicle routing problem. *Comput. Oper. Res.*, 33(9):2667–2702, 2006.

[BD08] Emilian Balanescu and Cristian Darie. *Beginning PHP and MySQL E-Commerce: From Novice to Professional, Second Edition (Beginners / Beginning Guide)*. APress, 2008.

[Bea83] JE Beasley. Route first–cluster second methods for vehicle routing. *Omega*, 11(4):403–408, 1983.

[Bel74] Beltrami, E.J., Bodin, L.D. Networks and Vehicle Routing for Municipal Waste Collection. *Networks*, 4(1):65–94, 1974.

REFERENCES

[Bha96] Vasanthakumar N. Bhat. A Model for the Optimal Allocation of Trucks for Solid Waste Management. *Waste Management Research*, 14(1):87–96, 1996.

[BMBB00] Lawrence Bodin, Aristide Mingozzi, Roberto Baldacci, and Michael Ball. The Rollon-Rolloff Vehicle Routing Problem. *TRANSPORTATION SCIENCE*, 34(3):271–288, 2000.

[BP04] Joaquín Bautista and Jordi Pereira. Ant Algorithms for Urban Waste Collection Routing. In *ANTS Workshop*, pages 302–309, 2004.

[CCL96] Sangit Chatterjee, Cecilia Carrera, and Lucy A. Lynch. Genetic algorithms and traveling salesman problems. *European Journal of Operational Research*, 93(3):490 – 510, 1996.

[CDM91] Alberto Colorni, Marco Dorigo, and Vittorio Maniezzo. Distributed optimization by ant colonies. In *European Conference on Artificial Life*, pages 134–142, 1991.

[CGSH02] Julio César, Angel Gutiérrez, David Soler, and Antonio Hervás. The capacitated general routing problem on mixed graphs. *Investigaciones Operacionales*, 23:15–26, 2002.

[CMMS02] Angel Corberán, Rafael Martí, Eulalia Martínez, and David Soler. The Rural Postman Problem on mixed graphs with turn penalties. *Computers & Operations Research*, 29(7):887 – 903, 2002.

[Cro09a] Douglas Crockford. The application/json Media Type for JavaScript Object Notation (JSON). http://www.ietf.org/rfc/rfc4627.txt?number=4627, March 2009. Last accessed on 20th July 2010.

[Cro09b] Douglas Crockford. Introducing JSON. http://json.org/, March 2009. Last accessed on 20th July 2010.

[CW64] G. Clarke and J. W. Wright. Scheduling of vehicles from a central depot to a number of delivery points. *OPERATIONS RESEARCH*, 12(4):568–581, July 1964.

[DGG96] Marco Dorigo, L.M. Gambardella, and Luca Maria Gambardella. Ant colony system: A cooperative learning approach to the traveling salesman problem. 1996.

[DM91] L. D. Davis and Melanie Mitchell. Handbook of genetic algorithms. *Van Nostrand Reinhold*, 1991.

[DMC96] Marco Dorigo, Vittorio Maniezzo, and Alberto Colorni. The ant system: Optimization by a colony of cooperating agents. *IEEE Transactions on Systems, Man, and Cybernetics-Part B*, 26:29–41, 1996.

[dML97] de Meulemeester L. Optimal sequencing of skip collections and deliveries. *Journal of the Operational Research Society*, 48:57–64(8), 1 January 1997.

[Dor92] M. Dorigo. *Optimization, Learning and Natural Algorithms*. PhD thesis, Politecnico di Milano, Italy, 1992.

[Eul36] Leonhard Euler. Solutio problematis ad geometriam situs pertinentis. *Comment. Academiae Sci. J. Petropolitanae*, 8:128–140, 1736.

[FA90] D. B. Fogel and J.W. Atmar. Comparing genetic operators with gaussian mutations in simulated evolutionary processes using linear systems. *Biological Cybernetics*, 63:111–114, 1990.

[G.94] Cristallo G. Optimisation de Tournées de Véhicules de Transport Container. *Mémoire de license en Sciences Economique et Sociales, Facultés Universitaires Notre-Dame de la Paix*, 1994.

[GADP89] S. Goss, S. Aron, J. Deneubourg, and J. Pasteels. Self-organized shortcuts in the argentine ant. *Naturwissenschaften*, 76(12):579–581, 1989.

[GAW01] B. L. Golden, A. A. Assad, and E. A. Wasil. *Routing vehicles in the real world: applications in the solid waste, beverage, food, dairy, and newspaper industries*, pages 245–286. In [TV01b], 2001.

[GLP01] M. Gendreau, G. Laporte, and J.-Y. Potvin. Metaheuristics for the capacitated VRP. [TV01b], pages 129–154.

[GMP10] Luís Gouveia, Maria Cândida Mourão, and Leonor Santiago Pinto. Lower bounds for the mixed capacitated arc routing problem. *Computers & Operations Research*, 37(4):692 – 699, 2010.

[Gol81] Wong Richard T. Golden, Bruce L. Capacitated arc routing problems. *Networks*, 11(3):305–315, 1981.

[GPB+02] Gregory Gutin, Abraham Punnen, Alexander Barvinok, Edward Kh. Gimadi, and Anatoliy I. Serdyukov. *The Traveling Salesman Problem and Its Variations*. 2002.

[Hak08] Mordechai (Muki) Haklay. Openstreetmap: User-generated street maps. *IEEE Pervasive Computing*, 7:12–18, 2008.

[Hol92] John H. Holland. *Adaptation in natural and artificial systems*. MIT Press, Cambridge, MA, USA, 1992.

[Jac73] Jack Edmonds. Matching, Euler tours and the Chinese postman. *Mathematical Programming*, 5:88–124, 1973.

[JM97] David S. Johnson and Lyle A. Mcgeoch. *The Traveling Salesman Problem: A Case Study in Local Optimization*. 1997.

[Joh06] Ola M. Johansson. The effect of dynamic scheduling and routing in a solid waste management system. *Waste Management*, 26(8):875 – 885, 2006.

[Kar05] Karadimas, N.V., Kouzas, G., Anagnostopoulos, I. and Loumos, V. Urban Solid Waste Collection and Routing: the Ant Colony Strategic Approach. *International Journal of Simulation: Systems, Science & Technology*, 6:45–53, 2005.

[KKS06] Byung-In Kim, Seongbae Kim, and Surya Sahoo. Waste collection vehicle routing problem with time windows. *Computers & Operations Research*, 33(12):3624 – 3642, 2006. Part Special Issue: Recent Algorithmic Advances for Arc Routing Problems.

[Knu76] Donald E. Knuth. Big omicron and big omega and big theta. *SIGACT News*, 8(2):18–24, 1976.

[Knu97] Donald E. Knuth. *The art of computer programming, volume 2 (3rd ed.): seminumerical algorithms*. Addison-Wesley Longman Publishing Co., Inc., 1997.

[Kru56] Joseph B. Kruskal. On the Shortest Spanning Subtree of a Graph and the Traveling Salesman Problem. *Proceedings of the American Mathematical Society*, 7(1):48–50, February 1956.

[LBM08] Jing-Quan Li, Denis Borenstein, and Pitu B. Mirchandani. Truck scheduling for solid waste collection in the City of Porto Alegre, Brazil. *Omega*, 36(6):1133–1149, 2008.

[Let99] Adam N. Letchford. The general routing polyhedron: A unifying framework. *European Journal of Operational Research*, 112(1):122 – 133, 1999.

[LS01] G. Laporte and F. Semet. *Classical heuristics for the capacitated VRP*, pages 109–128. In [TV01b], 2001.

[Man89] Udi Manber. *Introduction to Algorithms: A Creative Approach*. Addison-Wesley Longman Publishing Co., Inc., Boston, MA, USA, 1989.

[McC94] J.E. McCarthy. The municipal solid waste problem in the main industrialized countries. In Curzio, A.Q., Prosperetti, L., Zoboli, R., editor, *The Management of Municipal Solid Waste in Europe: Economic, Technological and Environmental Perspectives*, Amsterdam, 1994.

[MDS06] Alexandre Magrinho, Filipe Didelet, and Viriato Semiao. Municipal solid waste disposal in Portugal. *Waste Management*, 26(12):1477 – 1489, 2006.

[Mos00] Moshe Dror, editor. *Arc Routing: Theory, Solutions and Applications*. 2000.

[NR01] D. Naddef and G. Rinaldi. *Branch-and-cut algorithms for the capacitated VRP*, pages 53–84. In [TV01b], 2001.

[NS92] William Spears Navy and William M. Spears. Crossover or mutation? In *Foundations of Genetic Algorithms 2*, pages 221–237. Morgan Kaufmann, 1992.

[PM95] Ram Pandit and B. Muralidharan. A capacitated general routing problem on mixed networks. *Computers & Operations Research*, 22(5):465 – 478, 1995.

[Pá03] D. A. Pássaro. Report: waste management in Portugal between 1996 and 2002. *Waste Management*, 23(1):97 – 99, 2003.

[Rei91] Gerhard Reinelt. TSPLIB–A Traveling Salesman Problem Library. *INFORMS JOURNAL ON COMPUTING*, 3(4):376–384, 1991.

[RN03] Stuart J. Russell and Peter Norvig. *Artificial Intelligence: A Modern Approach*. Pearson Education, 2003.

[Rob09] Robin Cover. XML Applications and Initiatives. `http://xml.coverpages.org/xmlApplications.html`, March 2009. Last accessed on 15th March 2010.

[RXV+09] Alberto Rovetta, Fan Xiumin, Federico Vicentini, Zhu Minghua, Alessandro Giusti, and He Qichang. Early detection and evaluation of waste through sensorized containers for a collection monitoring application. *Waste Management*, 29(12):2939 – 2949, 2009.

[San08] Sanne Wøhlk. *A Decade of Capacitated Arc Routing*. 2008.

[SD02] Thomas Stutzle and Marco Dorigo. A short convergence proof for a class of ant colony optimization algorithms. *IEEE Transactions on Evolutionary Computation*, 2002:358–365, 2002.

[SH97] T. Stutzle and H. Hoos. Max-min ant system and local search for the traveling salesman problem. *Evolutionary Computation, 1997., IEEE International Conference on*, pages 309–314, 1997.

[SH00] Thomas Stützle and Holger H. Hoos. Max-min ant system. *Future Gener. Comput. Syst.*, 16(9):889–914, 2000.

[TAdS04] Joao Teixeira, Antonio Pais Antunes, and Jorge Pinho de Sousa. Recyclable waste collection planning–a case study. *European Journal of Operational Research*, 158(3):543–554, November 2004.

[TB] A. Tucker and L. Bodin. *A model for municipal street sweeping operations*, pages 107–150.

[TP00] Dang Vu Tung and Anulark Pinnoi. Vehicle routing-scheduling for waste collection in Hanoi. *European Journal of Operational Research*, 125(3):449–468, 2000.

[TV01a] P. Toth and D. Vigo. *An overview of vehicle routing problems*, pages 1–26. In [TV01b], 2001.

[TV01b] P. Toth and D. Vigo. *The vehicle routing problem*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 2001.

[VGR+09] F. Vicentini, A. Giusti, A. Rovetta, X. Fan, Q. He, M. Zhu, and B. Liu. Sensorized waste collection container for content estimation and collection optimization. *Waste Management*, 29(5):1467 – 1472, 2009. First international conference on environmental management, engineering, planning and economics.

REFERENCES

Appendix A

Submitted article

Metaheuristics for the ATSP Problem Applied to Urban Waste Collection

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Abstract. Real-time fill status of the waste containers can be used to reduce collection costs if this data is used to calculate efficient waste collection routes. This problem can be modeled as a dynamic stochastic vehicle routing and scheduling problem. One of the ways to tackle this problem is to use a route-first cluster-second approach.
This article contains a comparative analysis of several route optimization algorithms, with a focus on the sensitivity analysis of the MAX-MIN ant system. Our results show that usage of the MAX-MIN ant system provides competitive results on large datasets and also suggests there is room for further improvement.

Keywords: asymmetric traveling salesman problem, ant systems, urban waste collection

1 Introduction

Municipal solid waste (MSW) production has constantly increased since the 1960s as a consequence of economic growth [13], leading to the need for efficient waste management solutions. These involve collection, transportation, recycling and disposal of solid urban waste waste. A study by Johansson [7], shows that cost reductions of up to 20% can be achieved by taking into account the real-time fill status of the waste containers and defining collection policies appropriate for each specific case. In that study, calculating efficient waste collection routes is modeled as a dynamic stochastic vehicle routing and scheduling problem, assuming that the waste collection is done during the day, as need arises for waste containers to be emptied, signaled by level sensors in the containers.

In the specific case of Porto, Portugal, the residential solid urban waste collection is done once a day in the evening, with the vehicles picking up trash regularly from containers, based on experience-based values for trash generation. In this scenario, sometimes some full containers wait one day or more for being picked up, and sometimes partially full containers are picked up, both situation being inefficient. The city hall is interested in implementing a platform that measures the actual fill status of the containers, so that the evolution of trash can be monitored, the quality and efficiency of the collection can be increased, and the sums

paid to subcontractor companies by amount of km traveled each month reduced. The system should wok as follows: containers send an alarm when they are full, and everyday in the evening the waste collection routes are calculated with the static values available at a certain time. The problem that has to be solved every day within 1 to 2 hours is a classical vehicle routing and scheduling problem, as defined and classified in [2], and differs from the dynamic stochastic vehicle routing problem above. The 1 to 2 hours available to find the solution, though not imposing a real-time constraint, do require an efficient solution that can be found within that time frame for graphs of large cities, with several thousands of nodes.

We use a Capacitated Vehicle Routing Problem (CVRP) model for the problem described and solve it by solving the associated Asymmetric Traveling Salesman Problem (ATSP) first, followed by partitioning the resulting route into the tours for each truck subject to truck capacity (henceforth called clustering), being aware that the results of the serialized problem is suboptimal. Since the second problem can be solved optimally, we will focus on improving the approximate solutions for the ATSP. Using real city maps and Monte Carlo simulations, we study the application to the ATSP problem of the max-min ant system proposed by Stuetzle [17], showing results of a parameter sensitivity analysis. Moreover, we compare the results obtained with commonly used heuristics and meta-heuristics for the ATSP problem, evaluating them in comparable scenarios. Our results show that the max-min ant system always delivers the best performance among the meta-heuristics studied on the datasets of the TSPLIB, but the improvement never not exceeds 10%. Moreover, the performance of the MMAS on large datasets typical for current cities does not show significant improvement compared to other metaheuristics, although our parameter sensitivity analysis shows that further improvements could be achieved using other algorithm parameters at the cost of higher computation cost.

In the next section, we present related work and justify the choice of the CVRP model for the problem, in Section 3 we elaborate on our approach to solve it and we briefly describe the details of the meta-heuristics studied with stronger focus on the ant colony algorithm used. This is followed by the validation of our algorithm implementations in Section 4. In Section 5 we describe the framework and datasets for the Monte Carlo simulations used in the evaluation. Section 6 shows the results of the sensitivity analysis for the ant colony algorithm chosen, Section 7 shows the results and discussion of the comparative evaluation of the meta-heuristics and we conclude the paper in Section 8.

2 Related Work

The work by Johansson [7] studies the scenario of routing vehicles with real-time data from the containers, changing the routes when vehicles are already under way. They model the systems as a dynamic/stochastic routing problem and focus the study not only on heuristics for optimizing the result, but also on the influence of system parameters on the result of applying different policies. For

example, the best choice between collecting only full containers or also partially full containers depends on the total number of containers in the area covered.

The problem we are faced with is not dynamic, as the data for calculating the collection routes will be taken from a snapshot of the system at a certain time of the day. Hence, we are dealing with a static vehicle scheduling routing problem as in [2] and our goal is to compare heuristics that minimize the sum of the distances of all collection vehicle tours needed to collect the trash from full bins per day, using recently developed meta-heuristics which, to our best knowledge, have not been comparatively evaluated in other studies. We focus specifically on the sensitivity analysis of an ant colony algorithm, the MAX-MIN Ant System which will be described in the next section, and its performance on large graphs, such as those of cities.

Waste collecting can be seen a Vehicle Routing Problem[19], where each container is represented by a vertex that needs to be serviced, with it's current fill status representing the vertex demand. Each arc between vertices represents the distance, respecting the city's roads and traffic rules, between two containers. As several waste collection vehicles are available, and as they have a limit on how much waste it can carry, this problem is modeled as a Capacitated Vehicle Routing Problem (CVRP).

Two common approaches to solve a Capacitated Vehicle Routing Problem are to either cluster the customers and then calculate an optimized route for each one (cluster-first route-second) or to determine a single route passing through all the costumers and then divide it into feasible subroutes (route-first cluster-second) [19]. In the first scenario, clustering can be done, for example, by solving a Generalized Assignment Problem, while building each individual route can be done by solving a Traveling Salesman Problem (TSP) instance. In the second scenario, the tour passing all nodes can be constructed by solving a TSP instance, while its optimal clustering can be done using a standard dynamic programming approach.

3 Our Approach

We started by comparing route-first cluster-second approaches. The first stage — building a single tour — is equivalent to solve a TSP instance where each graph's vertex represents a waste container or the depot location and where each graph's arc weight represents the cost of traveling from one location to another. Since arc transversal represents the transversal of multiple city roads, it follows that the graph weight matrix may be asymmetric, as we may have one way streets along the way. This leads to the conclusion that these specific TSP instances are actually Asymmetric Traveling Salesman Problem (ATSP) instances, which are harder to solve than the symmetric version [20].

Having obtained a single tour that visits every of the graph's vertices, clustering it into several routes in which vertex visiting order is not altered and where capacity constraints (due to the limited truck capacity) are satisfied can be done by applying a shortest path algorithm on an auxiliary directed acyclic

graph, where each arc represents a feasible route composed by a subsequence of the tour, as proposed by Beasley [1]. Since this clustering method is both optimal and polynomially bounded — its time complexity is $O(|V|^2)$, with V being the set of vertices in the graph — we focus on solving ATSP instances. It is important to note that while the clustering algorithm is optimal, it does not mean that solving the ATSP optimally and applying this method yields an optimal CVRP solution[12].

There are two main methods to solve TSP instances. First, there are route construction heuristics, which are usually greedy in some fashion and may be randomized. Then, there are improvement techniques which can be used on top of these heuristics to further improve their results. Two commonly known route construction heuristics are the *nearest neighbor* and the *greedy* heuristic.

The *nearest neighbor* heuristic (NN) starts by picking a (usually random) vertex. It follows by choosing the closest unvisited vertex, until all vertices are visited. Applied to a complete graph, this heuristic runs in $O(|V|^2)$. Instead of simply picking the first vertex randomly, we decided to apply the heuristic to every starting vertex and return the best route. This creates a longer running time of $O(|V|^3)$, but yields better results, as will be shown in Section 7. We will refer to this heuristic as the *Best NN* heuristic.

The *greedy* heuristic works in a similar way to Kruskal's minimum spanning tree algorithm [9]. It starts by creating an auxiliary directed graph with no arcs, $G' = (V, \{\})$, and then sorts all the arcs by their weight, which are then iterated in ascending order. For each arc, we add it to G' if and only if its vertices are not already connected in G' and both the outdegree (number of outgoing arcs) of the source vertex and the indegree (number of incoming arcs) of the destination vertex are exactly 0. This process will yield a spanning tree which can be converted to a tour by adding the arc that connects the only two vertices whose degree is less than 2.

Metaheuristics such as *hill climbing*, *genetic algorithms* and *ant systems* can be applied to the ATSP problem to improve the results obtained by the heuristics above.

Hill climbing Hill climbing[16] is a greedy local search technique. It picks an initial solution and iterates through its neighborhood, looking for a solution that yields a better value. As soon as it finds one, it halts its search and moves on to the newly found solution. This process is repeated until a local minima/maxima is reached or the specified time limit is exceeded.

When applying hill climbing to TSP instances, a solution's neighborhood is commonly defined by dividing the current route into k segments and recombining them in all possible ways that yield a tour. This is called the $k - opt$ technique. The most common versions are 2-opt and 3-opt [8] and an adaptive generalization that chooses, at each iteration, how many segments to form — the Lin-Kernighan heuristic[8]. The bigger the k, the greater the neighborhood and the algorithm running time. In asymmetric instances, the neighborhood evaluation takes longer than in symmetric, hence we decided to use 2-opt techniques, to keep a low running time.

Genetic algorithm A genetic algorithm is a search technique based on evolutionary biology [3]. This technique starts by defining an initial population of random solutions, which is considered as the first generation. Then, the evolution process follows. In this process, every individual in the current generation is evaluated and multiple individuals are stochastically selected according to their fitness and possibly modified to form the next generation. Usually, the best individuals from a generation, known as the elite, are moved to the next one directly, without suffering any modification[16]. The evolution process continues until a predefined termination condition has been satisfied. Modifications of two types can be applied to the selected inviduals: crossover and mutation. The crossover operator takes two or more individuals from the current population and combines their attributes to form an offspring. The mutation operator randomly modifies attributes of any individual. In this project, we simply use the mutation operator, which is defined by selecting two vertices in the route at random and switching them.

MAX-MIN Ant System Ant systems, first proposed by Dorigo[4, 5], are based in ant colony communications and pheromone trail techniques. Ants are able to find short paths between food sources and their colony by leaving pheromone marks on their trails. Ants tend to follow paths where the pheromone intensity is higher, and the more ants follow a path, more pheromones are laid on it. These concepts were adapted into a class of probabilistic techniques for solving problems of combinatorial optimization [10, 11, 14]. Several ant systems have been proposed over the last years and we chose MAX-MIN Ant System (MMAS) [17], as it shows better results than the original ant system proposed by Dorigo and its convergence has been proved [18].

To apply MMAS to ATSP, a fixed amount of pheromones is placed in each of the graph's edges. Then, the following procedure is applied iteratively, until it converges or reaches a maximum number of steps or time limit. Each iteration starts by placing K ants in random vertices. Then, each ant starts constructing a tour by choosing the next node probabilistically according to two factors: pheromone strength in the arc and its distance. The probability is 0 if the node is already visited. Otherwise, it is proportional to:

$$p_{ij} \sim \tau_{ij}^{\alpha} \eta_{ij}^{\beta}, \tag{1}$$

where τ_{ij} is the pheromone strength in the arc and η_{ij} is the inverse of the distance between i and j. α and β are parameters that determine the relative influence of τ_{ij} and η_{ij}. When all ants finish constructing their routes, the one with the shortest tour lays down pheromones along its path, which is equivalent to updating the pheromones along the trail, according to [17]:

$$\tau_{ij}^{new} = \rho \cdot \tau_{ij}^{old} + \Delta\tau_{ij} \tag{2}$$

$\Delta\tau_{ij}$ equals to 0 if the arc from i to j does not belong to the tour of the best ant, or a fixed positive amount if it does. This amount is inversely proportional to the ant's tour length. Additionally, MMAS bounds pheromone values in each

arc so that pheromone trails do not get too low nor too high, to reduce the problem of early stagnation [17].

Finally, for large datasets, the authors of MMAS propose the use of candidate sets to speed up the algorithm's running time. Each ant, when choosing the next node, only chooses from the C closest vertices. This way, the overall running time gets reduced from $O(K \cdot |V|^2)$ to $O(C \cdot K \cdot |V|)$ [17].

4 Validation

Routing problems have been widely studied over the last decades and this has led to the establishment of standard datasets for benchmarking algorithms and implementations, one of them being the TSPLIB[15]. There are datasets for the Capacitated Vehicle Routing Problem, both Symmetric and Asymmetric Traveling Salesman Problem and other related routing problems. ATSP instances are accompanied by their respective optimum route cost. We use this library to validate our implementations of the heuristics described in the previous section, achieving the performances seen in Table 1.

Table 1. Performance of our heuristic and meta-heuristic implementations using the datasets from TSPLIB. Performance is given in the form of the ratio between the heuristic's route average cost and the dataset optimum (minimum) cost.

Dataset	Nodes	Optimum	Greedy	Best NN	2-opt	GA	MMAS
br17	17	39	1.97	1.43	1.00	1.00	1.00
ftv33	33	1286	1.16	1.23	1.17	1.17	1.00
ftv35	35	1473	1.30	1.13	1.13	1.13	1.04
ftv38	38	1530	1.25	1.14	1.08	1.08	1.07
p43	43	5620	1.03	1.01	1.00	1.00	1.00
ftv44	44	1613	1.23	1.14	1.14	1.09	1.03
ftv47	47	1776	1.27	1.22	1.20	1.15	1.11
ry48p	48	14422	1.32	1.07	1.02	1.05	1.02
ft53	53	6905	1.77	1.24	1.18	1.19	1.15
ftv55	55	1608	1.40	1.21	1.21	1.13	1.07
ftv64	64	1839	1.36	1.19	1.17	1.16	1.04
ftv70	70	1950	1.30	1.17	1.17	1.16	1.09
ft70	70	38673	1.14	1.08	1.06	1.05	1.04
kro124p	124	36230	1.21	1.19	1.17	1.15	1.08
ftv170	170	2755	1.43	1.30	1.28	1.24	1.18

5 Simulation Framework

During this project, we developed a framework to obtain optimized waste collection routes. Waste containers will be equipped with sensors that trigger an alarm

whenever they are full. This information is stored in a centralized database, ready to be used when the route optimization process starts.

Every day, two to three hours before collection starts, urrent fill status data of all of the city's containers is retrieved from the database, along with their location. This is then merged with the city's topological map — which can be obtained in *OpenStreetMaps*, for example — to form a standard CVRP dataset. Finally, meta-heuristics are applied to this dataset, yielding an optimized set of routes. This set can then be visualized and distributed to collection trucks' operators.

Realistic Graph Datasets While TSPLIB instances can be useful to provide a fast validation of implemented algorithms, available instances are limited in the number of vertices: ATSP instances only go up to 443 vertices and CVRP instances only go up to 101. Since instances corresponding to actual cities will have thousands of nodes, to compare the performance of the optimization heuristics in realistic scenarios we generated additional datasets based on the topology of cities such as Leeds, London and Lisbon. This information was obtained from the collaborative and open source initiative *OpenStreetMap* which aims to create a free editable map of the world [6]. We obtained 15 new CVRP datasets ranging from 518 to 7628 vertices. A sample dataset can be seen in figure 1.

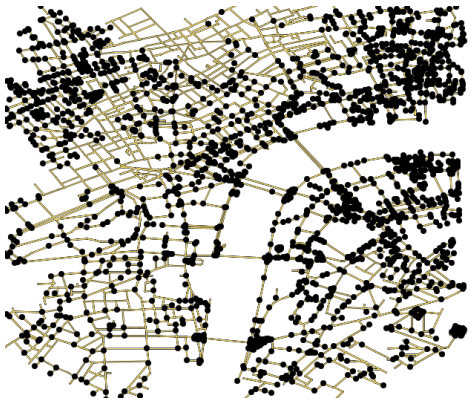

Fig. 1. Part of London with scattered waste containers. Yellow lines represent roads while black dots represent full waste containers. Road direction is not differentiated, in this image.

Waste Generation Having no access to real waste containers location and fill status, we also had to generate this information artificially, using a stochastic approach. Waste containers were scattered in street intersections following different patterns according to the following process. The algorithm starts by selecting k road intersections (represented by a graph vertex) on the city map as cluster

centers. A number d_i in the range $[0, 1]$ is assigned to each cluster, representing the cluster's waste container density. Then, until all intersections belong to a cluster, k vertices are chosen arbitrarily from each of the clusters' neighborhood and added to the respective cluster. When a vertex is added to a cluster it is decided, with probability d_i, if it should contain a full waste container.

6 Sensitivity of MAX-MIN Ant System to Algorithm Parameters

To understand the influence of the algorithm parameters described in the original MMAS proposal, several benchmarks were done, using the validation datasets. This will guide parameter configuration when applying this technique to larger datasets later on.

First, we evaluate the influence of β, the parameter that sets the relative weight between an given arc distance and pheromone intensity (Equation 1). The higher the β, the more ants tend to choose shorter arcs and ignore pheromones. We used datasets with 35, 70 and 170 nodes to verify whether a correlation between the best β and number of nodes existed.

The number of ants to use in each dataset is equal to the number of nodes, as suggested in [17]. MMAS was ran for 200 iterations in each dataset with β values ranging from 0 to 150. The resulting route cost averaged over 10 runs can be seen in Figure 2. We can observe that the best result for all datasets occurs for $\beta = 15$, indicating an optimal value for the parameter.

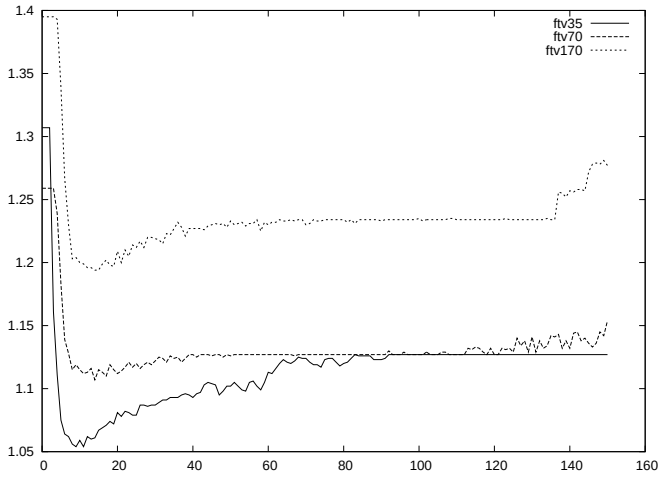

Fig. 2. Variations in final route performance when varying beta in the Max-Min Ant System for datasets ftv35, ftv70 and ftv170. Performance is given in the form of the ratio between the heuristic's route average cost and the dataset optimum (minimum) cost.

After obtaining a good reference for parameter β, the importance of the number of ants and runs was evaluated. MMAS was applied to the larger of the previous three datasets, *ftv170*, with a varying number of ants for 10000 iterations. This procedure was executed 10 times and the resulting route cost was averaged. Figure 3 shows the average results over 10 runs. It becomes clear from the results that increasing both the number of ants, as had been suggested by the original authors, and the number of runs improves the performance of the meta-heuristic. Of course, here there is a trade-off with the computation time[3].

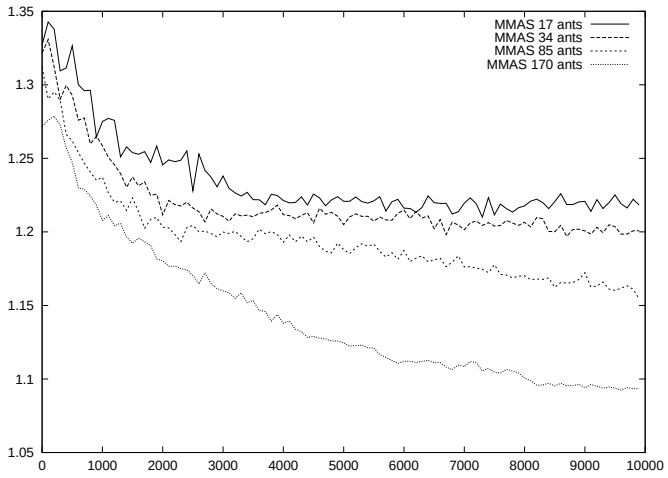

Fig. 3. Evolution of final route performance when varying the number of ants and number of runs for the Max-Min Ant System on dataset ftv170. Performance is given in the form of the ratio between the heuristic's route average cost and the dataset optimum (minimum) cost.

7 Results for ATSP Solution on Large Graphs

For the results in this section, we used the large datasets based on the topology of London, Leeds and Lisbon[4]. Table 2 presents the average performance of each algorithm when applied to our large datasets.

[3] We will show a plot of the computation times in the final paper.

[4] We did not use a map for Porto because the city map in OpenStreetMaps is strongly disconnected, hence not fitting our purposes.

Table 2. Performance of our heuristic and meta-heuristic implementations using realistically large datasets.

Nodes	Greedy	NN	2-opt	GA	MMAS
518	67149	63615	62539	60070	**55800**
841	92764	86929	84088	83235	**79233**
904	93654	92157	92721	89001	**84101**
1175	168990	152895	161411	155626	**148149**
1287	102083	105643	99297	96861	**96373**
1849	203706	192104	197998	183128	**181522**
2038	227762	226894	217183	**211612**	214072
2206	212276	210710	208236	**200794**	202899
2561	533846	497394	506316	500458	**493682**
3481	527011	518891	510918	**497069**	498720
3859	528571	520890	520404	505068	**503184**
4109	541793	538067	544977	**529847**	531881
4628	588111	570004	574073	**565019**	567795
6247	656096	642878	626662	**626587**	633663
7628	701309	690086	682962	**680691**	684773

In the final paper, we will include plots of results shown for better visualization, and a more detailed discussion of the results obtained. Moreover, we will show results for the MMAS with larger number of ants and runs, which we could not finish now due to lack of time.

8 Conclusions

We performed a sensitivity analysis of the max-min ant system meta-heuristic for the ATSP on the TSPLIB datasets, and we can conclude that there is an optimum value of the algorithm parameter β at around 15 for the datasets studied. Furthermore, we can conclude that the max-min ant system meta-heuristic always delivers the best performance on the datasets of the TSPLIB, but the improvement never exceeds 10%.

When applied to large datasets typical for current cities, the performance of the MMAS does not show significant improvement compared to other meta-heuristics. However, the meta-heuristic parameterization was not optimal to keep the computation time acceptable according to our system constraints. Nevertheless, the parameter sensitivity analysis that we carried out shows that larger improvements could be achieved using other algorithm parameters at the cost of higher computation cost.

References

1. JE Beasley. Route first–cluster second methods for vehicle routing. *Omega*, 11(4):403–408, 1983.
2. Bodin, L. Classification in Vehicle Routing and Scheduling. *Networks*, 11(2):97–108, 1981.
3. L. D. Davis and Melanie Mitchell. Handbook of genetic algorithms. *Van Nostrand Reinhold*, 1991.
4. M. Dorigo. *Optimization, Learning and Natural Algorithms*. PhD thesis, Politecnico di Milano, Italy, 1992.
5. Marco Dorigo, Vittorio Maniezzo, and Alberto Colorni. The ant system: Optimization by a colony of cooperating agents. *IEEE Transactions on Systems, Man, and Cybernetics-Part B*, 26:29–41, 1996.
6. Mordechai (Muki) Haklay. Openstreetmap: User-generated street maps. *IEEE Pervasive Computing*, 7:12–18, 2008.
7. Ola M. Johansson. The effect of dynamic scheduling and routing in a solid waste management system. *Waste Management*, 26(8):875 – 885, 2006.
8. David S. Johnson and Lyle A. Mcgeoch. *The Traveling Salesman Problem: A Case Study in Local Optimization*. 1997.
9. Joseph B. Kruskal. On the Shortest Spanning Subtree of a Graph and the Traveling Salesman Problem. *Proceedings of the American Mathematical Society*, 7(1):48–50, February 1956.
10. G. Leguizamon and Z. Michalewicz. A new version of ant system for subset problems. *Proceedings of the 1999 Congress on Evolutionary Computation*, 2:1458–1464, 1999.
11. Helena R. Loureno and Daniel Serra. Adaptive search heuristics for the generalized assignment problem. *Mathware and Soft Computing*, 9:209–234, 2002.
12. M. Haimovich. Bounds and Heuristics for Capacitated Routing Problems. *Mathematics of Operations Research*, 10(4):527–542, 1985.
13. J.E. McCarthy. The municipal solid waste problem in the main industrialized countries. In Curzio, A.Q., Prosperetti, L., Zoboli, R., editor, *The Management of Municipal Solid Waste in Europe: Economic, Technological and Environmental Perspectives*, Amsterdam, 1994.
14. S. Meshoul and M. Batouche. Ant colony system with extremal dynamics for point matching and pose estimation. In *ICPR '02: Proceedings of the 16 th International Conference on Pattern Recognition (ICPR'02) Volume 3*, page 30823, Washington, DC, USA, 2002. IEEE Computer Society.
15. Gerhard Reinelt. TSPLIB–A Traveling Salesman Problem Library. *INFORMS JOURNAL ON COMPUTING*, 3(4):376–384, 1991.
16. Stuart J. Russell and Peter Norvig. *Artificial Intelligence: A Modern Approach*. Pearson Education, 2003.
17. T. Stutzle and H. Hoos. Max-min ant system and local search for the traveling salesman problem. *Evolutionary Computation, 1997., IEEE International Conference on*, pages 309–314, 1997.
18. Thomas Stutzle and Marco Dorigo. A short convergence proof for a class of ant colony optimization algorithms. *IEEE Transactions on Evolutionary Computation*, 2002:358–365, 2002.
19. P. Toth and D. Vigo. *The vehicle routing problem*. Society for Industrial and Applied Mathematics, Philadelphia, PA, USA, 2001.
20. David P. Williamson. "Analysis of the Held-Karp lower bound for the asymmetric TSP". *Operations Research Letters*, 12(2):83 – 88, 1992.