"On the Evolution of Regional Asymmetries"

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August 9, 1999

Abstract

We model the evolution of regional asymmetries, and the associated intra-distribution mobility, as the outcome of adoption decisions taken by firms at different locations. We characterise the following equilibria: *Persistent asymmetry* where nobody adopts, *Leapfrogging* where low quality firms adopt, *Forging ahead* where high quality firms adopt, and *Catching up* where everybody adopts. We study the impact of economic integration on these equilibria and the evolution of technological asymmetries as well as intra-distribution mobility. In equilibrium the regional intra-distribution mobility is described by fluctuations between periods of Leapfrogging, Forging ahead and Catching up.

Keywords: *Leapfrogging, Convergence, Local Interaction, Technological Adoption, Intra-Distribution Mobility.*

JEL: L13, R12, O31

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*I would like to thank, Jayasri Dutta, Robert Evans, Rupert Gatti, Sandeep Kapur, Emanuela Sciubba and Giancarlo Spagnolo for their help in previous drafts of this paper. The usual disclaimer applies.

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1 Introduction

1.1 Stylized facts

By looking at the regional distribution of per-capita incomes in Europe one can trace some clear spatial patterns along which richer and poorer regions are lined. One such patterns has been identified as the *"blue banana":* a slice of the continent where the most prosperous regions across different countries are lined up. Often European regions display stronger per-capita income similarities with their cross-border neighbours than with other regions of the same country.

Dunford [5] analysed the temporal evolution of regional inequalities, within European countries for the period 1977 -1989. He found:

- A positive relation between regional rates of growth of per capita GDP with end[1] of period GDP levels in the U.K., where Surrey, Sussex, Cumbria and Greater London had the highest rates of growth and highest GDP levels, while other regions, Leicester and Northampton, Dorset and Somerset and Hereford, had high growth rates and lower GDP levels, and South Yorkshire, Cleveland and Meyerside had both low levels and low growth rates.

- In Italy, some of the poorest regions: Basilicata, Calabria and Campania, showed the lowest growth rates while regions with average levels of per capita GDP such as: Molise, Abruzzi, Marche and Veneto achieved the highest growth rates. The gap dividing these fast growing average income regions from the richest, Lombardia and Emilia Romagna, remained however wide.

- In Spain, regions with average per capita incomes had high dispersion in growth rates: the highest growth rates were in Canarias, Ceuta y Melilla and Baleares, the lowest in Cantabria, Asturia, Galicia and Rioja. Regions with higher per capita GDP levels like Madrid, Cataluña,

[1] In the economic literature the initial GDP level has generally been considered, rather than the final period.

Navarra and Pais Vasco, as well as the poorer Andalucia and Extremadura, showed close to average growth rates.

- Germany's laenders showed no clear relation between levels and growth rates.
- In France the Île de France, with the highest income level, had the second highest growth rate after the Midi-Pyrenees, a much poorer region.

Regional income dynamics in Europe show national specificities and rich intra-distribution mobility. Relative ranking effects, non monotonic relations between level and growth rates, neighbouring effects, regional market characteristics, infrastructure and cost conditions might all be elements composing the picture of inequality and its evolution in space and time. The relevance of these elements has been emphasized by Quah [13, page 953]:

> "...a region's economic well-being can be predicted by that in the surrounding regions and the host state... . More than does the host country's economic performance, that of surrounding regions helps us understand the inequalities across European regions".

Quah [13] found that the standard deviation of the per-capita income distribution in the European regions in 1989 was reduced by 28% when normalized for the effects of neighbouring regions. Even more interestingly, the intra-distribution dynamics, expressed via a discretized transition probability matrix, shows that neighbours-conditioned transition probabilities of exiting the poorest income class (below 74% of the average European income) are much higher than without conditioning. Armstrong [2], noticed that the convergence of European region per capita incomes has been a slow and highly variable process showing strong spatial auto correlation, with clustered fast growing regions separated by slow growth ones.

1.2 Modelling intra-distribution mobility

Models analyzing intra-distribution mobility features - such as *leapfrogging*, where a lagging nation overtakes a leading one, or *forging ahead* where the existing gap is instead increased- focused on two main aspects:

- the different roles played by invention and imitation of new technologies, and
- the geographical extent of technological spillovers in an endogenous growth context.

Leaders and followers are often described as having different abilities or costs of technological upgrading. If an innovation is a local public good so that its externality is contained within a single country or region, long run patterns of growth across countries will show hysteresis. Convergence[2] is, on the contrary, obtained through international imitation[3] of the state of the art technology [see Grossman and Helpman [9]].

Another intra-distribution feature, *Club convergence*, takes place when initial conditions select one of the possibly multiple steady state paths. In this case polarization, persistent poverty and clustering may take place, [see Quah [13] and [14]]. A typical case of such multiplicity arises when imitation, or knowledge spillovers, are locally delimited, in this setting Durlauf [6] showed how localized productivity spillovers may generate, depending on the strength of the neighboring linkages, multiple stochastic steady states [For the implications of different theoretical models on the convergence issue see Galor [7]].

We model the evolution of regional asymmetries, and the regions intra-distribution mobility, as the equilibrium outcome of a technological adoption[4] game played at different geographical locations. Firms have different initial technological conditions. Assume that a quality improving innovation is ready for adoption at a fixed cost: for example a new computer processor is on sale, some of the firm-locations have the last period state of the art processor, while others have a more obsolete one. Firms first decide whether to adopt the new technology and then compete in price against their immediate neighbours.

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[2]For a major contribution on the hypothesis of regional convergence, see Sala-i-Martin[15].

[3]Imitation is often modeled as a cheaper way to adopt the state of the art technology. In this case the growth rate of the lagging economies can be higher than that of the leading ones and leapfrogging may occur. If instead the cost differential between innovative and imitative research is a decreasing function of the technological gap, catching up is observed.

[4]We study the incentives for technological adoption as more evidence, see Jovanovich [10], is suggesting its relevance in explaining growth performances compared to technological invention.

We characterize the geographical adoption patterns forming subgame perfect equilibria of the price and adoption game played by each firm against its nearest neighbours. If the equilibria display partial adoption rates, we look at the identity of the adopters[5]. Will adoption take place in the previously leading locations, or in the lagging ones? Symmetric adoption patterns emerge either when nobody adopts, and the initial technological asymmetry is preserved or when every location adopts. In this last case the initial technological asymmetries disappear.

We define the following equilibria:

- *Persistent asymmetry,* as the equilibrium where nobody adopts,
- *Leapfrogging,* as the equilibrium where only low quality firms adopt,
- *Forging ahead,* as the equilibrium where only high quality firms adopt, and
- *Catching up,* as the equilibrium where everybody adopts.

At each date one of these equilibria is determined by the existing product-quality differences and the innovation available for adoption, given the model's parameters. These are: transport costs, the level of which is inversely related to the degree of market integration, consumers' preference for quality and adoption costs,

Brezis, Tsiddon and Krugman [4] explained historical cases of leapfrogging as the result of different learning capabilities about new technologies between leading and following nations, and Brezis and Krugman [3] described the leapfrogging of leading cities by upstart metropolitan areas, by assuming localized learning by doing and, again, the reluctance of the established cities to learn about the new technology. Differently from this literature we assume no asymmetric learning abilities between leaders and followers. The different adoption behaviour is an endogenous equilibrium feature of the game played.

Pre-existing quality leads can be increased, maintained, cancelled and reversed in equilibrium. Whenever technology adoption costs do not exceed transport costs, when integration, expressed by low transport costs, is not

[5]Findings from different innovation surveys point out at the co-existence in time of innovating and non-innovating firms and at a highly concentrated geographical distribution of the innovative activity.[1]

too strong, there is an upper limit to the amount of neighbouring regions asymmetry.

We study the effects of economic integration on the evolution of regional asymmetries. Forging ahead, the equilibrium leading to increased local asymmetries, is more likely with higher integration while Catching up is more difficult to be achieved. The effects of integration on Persistent asymmetry, the equilibrium where nobody adopts, and Leapfrogging, the equilibrium reverting pre-existing asymmetries, depend on the initial quality differences. When these are large, economic integration helps maintaining the Persistent asymmetry while for lower levels of initial quality differences, integration increases the chances of Leapfrogging.

Motta, Thisse and Cabrales [11] analysed the effects of trade between two countries having productive structures of different qualities. They characterized two equilibrium outcomes: one in which the quality leader maintains its position and a second where leapfrogging occurs. This last outcome only occurs when there are small initial quality asymmetries between the two countries.

Contrary to Motta, Thisse and Cabrales [11], we find that the Leapfrogging equilibrium is more likely with large initial asymmetry. On the other hand wider initial asymmetry reduces the chances of both Forging ahead and Persistent asymmetry.

Furthermore the maximum equilibrium asymmetry, between neighbouring regions, depends, negatively, on integration and consumers preferences for quality, and positively, on adoption costs.

We move on describing the evolution of the state of the art technology as generated by an exogenous stochastic process and we derive the associated evolution of local technological asymmetries as well as the regional intra-distribution mobility. We do this by deriving the transition probability matrices for both, local asymmetries and intra-distribution dynamics and we show that both, these matrices, have a unique invariant distribution. The economy, therefore, once settled on its equilibrium distribution will be fluctuating between periods of leapfrogging, forging ahead and catching up with time frequencies converging towards the values of the invariant distribution.

The remainder of the paper is organized as follows: in Section 2 we describe the model. Section 3 looks at the two stage game equilibria under asymmetric initial conditions. In Section 4 we derive the transition probability matrices generating both regional asymmetry and intra-distribution dynamics. Section 5 concludes the paper. Appendix 1 contains results from numerical simulations and studies the effects of some parameters changes on the frequency distributions of local asymmetries. Appendix 2 considers an extension to a generalized state space for the innovation shock, and proves existence and convergence of the derived Markov process on local asymmetries. All the proofs are in Appendix 3.

2 The Model

2.1 Market geography

Let I firms be exogenously located at regular intervals along a circle. A market is defined by the geographical space between two neighbouring firms. Each firm can charge different prices for the same quality commodity in the two adjacent markets where it operates .

Firms produce commodities which differ along two dimensions:

- Location, assumed as given, and
- quality, endogenously determined in the model.

2.2 Demand

Consumers, uniformly distributed on each market[6], buy one unit of the commodity from one of the surrounding firms[7], and face linear transportation costs, t.

Consider the market between two firms i and $i+1$[8] which produce goods of quality n_i and n_{i+1} and charge prices $p_{i,l}$ and $p_{i+1,r}$ respectively($p_{i,l}$ is the price that firm i charges on the market defined on its left side and $p_{i+1,r}$ is the price charged by firm $i+1$ on the market at its right side where it competes with firm i). Then the left market demand function for firm i is given by[9]:

[6] The mass of the consumers located between any two firms is set to be equal to one.

[7] They can only buy in the market defined by the two firms located around them.

[8] We consider integers modulo I, i.e. if $i = I$, then $i + 1 = 1$.

[9] For each price-quality configuration we obtain a cut-off location $\tilde{y}_{i,l}$. A consumer located at $\tilde{y}_{i,l}$ will be indifferent between buying from either of the two firms: i, or $i+1$.

$$\tilde{y}_{i,l} = \begin{cases} 0 & if \ p_{i,l} - p_{i+1,r} \geq vd_{i,l} + t \\ \frac{1}{2t}\left[vd_{i,l} + t + p_{i+1,r} - p_{i,l}\right] & if \ p_{i,l} - p_{i+1,r} \in (vd_{i,l} - t, vd_{i,l} + t) \\ 1 & if \ p_{i,l} - p_{i+1,r} \leq vd_{i,l} - t \ , \end{cases} \quad (1)$$

where v is the surplus associated with the consumption of one unit of good of quality $n = 1$, t is the linear transportation cost, and $d_{i,l} \equiv n_i - n_{i+1}$ is the quality difference between the products of firms i and $i+1$. The right market demand for firm $i+1$ is:

$$\tilde{y}_{i+1,r} = 1 - \tilde{y}_{i,l}. \quad (2)$$

2.3 The two stages of the game

The game, describing the adoptions of the new technology, is as follows:

- In the first stage all firms simultaneously decide whether to adopt, at a given fixed cost q, a quality improving[10] innovation,
- in the second stage each firm competes, in separate markets, against its two immediate neighbours by choosing a pair of prices, for its own commodity, one for each market.

3 Prices and profits in the second stage

We assume constant and identical marginal costs of production and we set them without loss of generality equal to zero. Given the symmetry of the pricing problems[11], in the following Lemma we consider the equilibrium prices of a market between firm i and firm $i+1$.

Lemma 1

[10] We model quality as follows: one unit of good of quality n is equivalent to n times a unit of a numeraire good of quality 1.

[11] Remember that each firm has the ability of charging different prices in different markets.

Given a distribution of product qualities, $\{n_i, i \in I\}$, equilibrium prices of the second stage of the game are:

$$p^*_{i,l} = \begin{cases} 0 & \text{if } d_{i,l} \leq -\dfrac{3t}{v} \\ t + \dfrac{1}{3} v d_{i,l} & \text{if -}\dfrac{3t}{v} < d_{i,l} < \dfrac{3t}{v} \\ v d_{i,l} - t & \text{if } d_{i,l} \geq \dfrac{3t}{v}, \end{cases} \tag{3}$$

and

$$p^*_{i+1,r} = \begin{cases} 0 & \text{if } d_{i,l} \geq \dfrac{3t}{v} \\ t - \dfrac{1}{3} v d_{i,l} & \text{if -}\dfrac{3t}{v} < d_{i,l} < \dfrac{3t}{v} \\ -v d_{i,l} - t & \text{if } d_{i,l} \leq -\dfrac{3t}{v}. \end{cases} \tag{4}$$

and,

Corollary 1

Equilibrium profits for firm i are given by the sum of profits in the right market where it competes against firm $i-1$,

$$\pi^*_{i,r} = \begin{cases} 0 & \text{if } d_{i,r} \leq -\dfrac{3t}{v} \\ \dfrac{\left[\dfrac{1}{3} v d_{i,r} + t\right]^2}{2t} & \text{if -}\dfrac{3t}{v} < d_{i,r} < \dfrac{3t}{v} \\ v d_{i,r} - t & \text{if } d_{i,r} \geq \dfrac{3t}{v}, \end{cases} \tag{5}$$

and the profits in its left market where firm i competes against firm $i+1$,

$$\pi^*_{i,l} = \begin{cases} 0 & \text{if } d_{i,l} \leq -\dfrac{3t}{v} \\ \dfrac{\left[\dfrac{1}{3} v d_{i,l} + t\right]^2}{2t} & \text{if -}\dfrac{3t}{v} < d_{i,l} < \dfrac{3t}{v} \\ v d_{i,l} - t & \text{if } d_{i,l} \geq \dfrac{3t}{v}. \end{cases}$$

4 The first stage: Adoption under asymmetric initial conditions

Since we want to study the conditions governing the evolution of regional technological asymmetries, and evaluate the effects of economic integration and the level of preexisting asymmetry on such process, we assume asymmetric initial conditions:

Assumption 3.1

- *There are high quality firms and low quality ones:*

$$n_i \in \{n_h, n_l\} \text{ and } n_h > n_l,$$

and

- *each low quality firm is surrounded*[12] *by two high quality ones and vice versa:*

$$n_i = n_h \Rightarrow n_{i+1} = n_{i-1,} = n_l, \text{ and}$$
$$n_i = n_l \Rightarrow n_{i+1} = n_{i-1,} = n_h.$$

Let n^* be the new product quality available for adoption and define the innovation size, x^*, as the difference between the *state of the art* quality, available for adoption, and the existing high quality of the product of the leaders:

$$x^* \equiv n^* - n_h.$$

Any firm can adopt this innovation by incurring a fixed cost, q.

Finally let firm i's decision, whether to adopt the new technology, be represented by the binary variable:

$$\omega_i \in \{0, 1\}.$$

[12] *Given this alternating technology pattern, the existing product quality asymmetry between one location and its two neighbours coincide:*

$$d_{i,l} = d_{i,r} = d_i = n_i - n_{i\pm1}.$$

After the adoption decision, the product quality of a previously lagging firm, is represented by:

$$\hat{n}_{i,l} = (1 - \omega_{i,l})\, n_{i,l} + \omega_{i,l} n^*,$$

while the product quality of a previously leading (high quality) firm is:

$$\hat{n}_{i,h} = (1 - \omega_{i,h})\, n_{i,h} + \omega_{i,h} n^*.$$

The associated adoption cost for both previously high and low quality firms are:

$$\hat{q}_i = q\omega_i.$$

We consider adoption strategies decisions $\{\omega_{i,l}^*, \omega_{i,h}^*\}$ which form a subgame perfect equilibrium of the two stage game defined by the payoffs:

$$\pi_{i,l}^* \left(\hat{n}_{i,l}, \hat{n}_{i+1,h}, \hat{n}_{i-1,h}\right) - \hat{q}_i;$$

and

$$\pi_{i,h}^* \left(\hat{n}_{i,h}, \hat{n}_{i+1,l}, \hat{n}_{i-1,l}\right) - \hat{q}_i;$$

and by the strategies

$$\omega_{i,l}, \omega_{i,h}, \in \{0, 1\}, i \in I.$$

We intend to capture the intra-distribution dynamics of different locations by characterizing the following pure strategies equilibria:

- *Persistent asymmetry*, where nobody adopts and asymmetry remains unchanged: $\omega_{i,h} = \omega_{i,l} = 0$;
- *Leapfrogging*, where the lagging firms adopt while the high quality ones do not adopt: $\omega_{i,l} = 1$, and $\omega_{i,h} = 0$;
- *Forging ahead,* where the leading firms adopt while the low quality ones do not adopt: $\omega_{i,l} = 0$ and $\omega_{i,h} = 1$; and,
- *Catching up,* where everybody adopts: $\omega_{i,h} = \omega_{i,l} = 1$.

In studying the equilibria of this game we maintain, unless otherwise stated, the following assumption:

Assumption 3.2.

a) The initial quality difference between neighbouring firms is bounded from above:

$$d \equiv n_h - n_l < \frac{3t}{v}$$

(which implies that all firms make positive profits given the initial conditions) and,

b) adoption costs are lower than or equal to unit transportation costs: $q \leq t$.

We first want to ask:

"*under what conditions will the existing asymmetry be preserved?*"

We find that, given the parameters representing: adoption costs, q, transport costs, t and consumers' preference for quality, v, the persistence of existing asymmetry will jointly depend upon the initial quality difference between leaders and followers, d, and the innovation size x^*.

Proposition 1: Persistent asymmetry

Let assumptions (3.1) *and* (3.2) *hold, then Persistent Asymmetry is an equilibrium if and only if :*

$$x^* < \frac{\sqrt{9qt + (3t - dv)^2} - 3t}{v} \quad \text{and } 0 \leq \ d < \frac{3t - 3\sqrt{t^2 - tq}}{v} \tag{6}$$

□

It is easy to see[13] that the Persistent asymmetry equilibrium region is decreasing in the amount of preexisting asymmetry: with higher d , it is less likely that nobody will adopt the new available technology providing the superior quality n^*.

The effects of economic integration on the Persistent asymmetry equilibrium region, expressed by a reduction in transport costs, t, are not linear; the sign of their effect depends on the degree of the preexisting asymmetry, d. If

[13]The partial derivatives are available from the Author on request.

$$0 \leq d < \frac{3q}{4v},$$

decreasing transport costs reduce the chances of observing the Persistent asymmetry equilibrium. If, on the contrary,

$$\frac{3q}{4v} \leq d < \frac{3t - 3\sqrt{t^2 - tq}}{v},$$

a reduction in transport costs will increase the Persistent asymmetry equilibrium region.

When finally existing asymmetry belongs to the interval

$$\frac{3t - 3\sqrt{t^2 - tq}}{v} \leq d < \frac{3t}{v}$$

Persistent asymmetry ceases to be an equilibrium.

The next proposition studies the conditions under which a spatial adoption pattern where only the lagging locations adopt the new technology, and by so doing leapfrog the leading ones, is an equilibrium.

Proposition 2: Leapfrogging

Let assumptions (3.1) *and* (3.2) *hold, then Leapfrogging is an equilibrium if and only if:*

$$\frac{\sqrt{9qt + (3t - dv)^2} - 3t}{v} \leq x^* < \frac{3t - 3\sqrt{t^2 - tq}}{v} \tag{7}$$

□

Leapfrogging becomes an equilibrium as soon as the innovation, x^*, overtakes the upper bound of Persistent asymmetry. From (7) it is easy to see that the initial quality difference, d, has a positive effect on the size of the Leapfrogging equilibrium region.

The effects of economic integration, on the lower bound of this equilibrium region, are again nonlinear: for small values of the preexisting asymmetry, increasing transport costs raise the lower bound of the leapfrogging equilibrium region, for intermediate values of d, higher transport costs reduce the lower bound. Finally, for even higher values of the preexisting asymmetry, the lower bound for leapfrogging goes to zero independently of transport costs.

The upper bound of the leapfrogging region is independent of d, and decreasing in transport costs. For higher values of the innovation size, x^*, economic integration has the effect of increasing the chances of a reversal of existing asymmetry via leapfrogging.

Next we look at the conditions leading to a spatial pattern of adoptions where only the leading locations adopt the new technology (thereby increasing the preexisting technological asymmetry).

Proposition 3: Forging ahead

Let assumption (3.1) *and* (3.2) *hold, then Forging ahead is an equilibrium if and only if :*

$$\frac{\sqrt{9qt+(3t+dv)^2}-3t}{v}-d\leq x^*<\frac{3t-3\sqrt{t^2-tq}}{v}-d. \quad (8)$$

□

First notice that, by comparing (7)and (8), when Forging ahead is an equilibrium Leapfrogging is an equilibrium too.

The amount of preexisting technological asymmetry, d, reduces the set of values of the innovation size, x^*, for which Forging ahead is an equilibrium [14]. Indeed asymmetry cumulation through forging ahead cannot go on beyond the upper bound of the equilibrium region given by:

$$d=\frac{3t-3\sqrt{t^2-tq}}{v}, \quad (9)$$

[14] Notice that for $d=0$ Forging Ahead and Leapfrogging equilibrium regions coincide. This makes sense, given that for $d=0$ there are neither leaders nor followers, and the two equilibria concepts coincide, describing a state that, from symmetric initial conditions, leads to a new alternating pattern of adoptions in space.

For a full analysis of the symmetric initial conditions case see Giovannetti [8].

after which Forging ahead can longer be an equilibrium. Notice that (9) also defines the maximum technological asymmetry between neighbouring locations, [15] and no adoption pattern may lead to higher asymmetry[16].

Finally, economic integration, by lowering transport costs, t,raises the possibility of Forging ahead being an equilibrium.

The last configuration of spatial adoption decisions that we analyse is one in which there is global adoption of the new technology. In the following proposition we derive the necessary and sufficient conditions for Catching up to be an equilibrium of the game.

Proposition 4: Catching up

Let assumption (3.1) *and* (3.2) *hold, then Catching Up is an equilibrium if and only if:*

$$x^* \geq \frac{3t - 3\sqrt{t^2 - tq}}{v} \qquad (10)$$

□

The Catching up equilibrium region[17] is independent of the previous level of asymmetry, d. This implies that, even under symmetric initial conditions, if the innovation size, x^*, belongs to (10) every location will adopt the new technology.

Integration, by reducing transport costs, raises the lower bound of Catching up and, consequently, the maximum potential level of asymmetry. This is due to an increase in the strength of competition caused by lower transport costs which makes the global adoption of the new technology more unlikely. Higher adoption costs also increase the potential maximum level of asymmetry and lower the chances of global adoption.

Finally consumers' preference for quality also play an important role. The more consumers value quality, the higher is v, the lower is the maximum level of asymmetry and the more likely is Catching Up.

[15] In facts it coincides with the maximum innovation size leading to leapfrogging and it defines the level of existing asymmetry after which Forging ahead disappears.

[16] Moreover such maximum asymmetry falls within the limit assumed in assumption (3.2).

[17] Notice that the Catching up lower bound (10) also coincide with the maximum level of technological asymmetry in the system, discussed in equation (9).

In concluding this section notice that if the effects of integration are so strong that transport costs are below the level of adoption costs, $q > t$, if assumption (3.2) does not hold, Catching up is never an equilibrium and asymmetry is self replicating: starting from asymmetric initial conditions, there is no possible equilibrium outcome where all the firms upgrade their product quality.

5 The evolution of regional asymmetries

In this section we focus on the temporal evolution of the regional technological asymmetries. We consider an exogenous process driving the evolution of the state of the art technology. This process, together with the previous period asymmetries, determines today's quality differences between neighbouring locations, as described in the equilibria analysed in the previous section. We derive the evolution of local technological asymmetries and the intra distribution mobility of the single firms-locations by characterizing the transition probability matrices generating such processes.

Let the evolution of the "*state of the art*" technology be described by the additive process:

$$n_t^* = n_{t-1}^* + x_t^*$$

where x_t^* represents the innovation size at period t[18].

Given that initial conditions are described by an alternating pattern of low and high quality levels where:

$$n_{i,t} = n_{i\pm 2,t}$$

every equilibrium studied in the previous section maintains such an alternating pattern of low and high qualities[19]. This implies that the evolution of the local asymmetries is captured by the evolution of the product quality asymmetry between neighbouring firms, d_t, where $d_t = |n_{i,t} - n_{1\pm 1,t}|$.

In the following we focus on sequences of innovations which lead to, at least partial, adoption of the state of the art technology ruling out small

[18] At time $t-1$ each firm, i,in the industry is characterized by a product quality corresponding to the highest available quality at the date in which the firm last adopted.

[19] In the Catching up equilibrium, both low and high quality coincide with the state of the art technology.

innovation leading to the Persistent asymmetry equilibrium region for any given value of the existing asymmetry, d[20]. Furthermore, having seen in the previous section that Forging ahead (8) and the Leapfrogging (7)equilibrium conditions overlap, we assume that, when multiplicity arises, Forging ahead prevails due to extra-model factors.

Assumption 4.1

- *The innovation process* $x_t^* = \{n_t^* - n_{t-1}^*\}$ *has a lower bound*[21] *given by:*

$$x_t^* \geq \frac{3\sqrt{t\,(q+t)} - 3t}{v}. \tag{11}$$

- *Leapfrogging only occurs in the parameters' region where it is the unique equilibrium.*

Consider the transition law

$$g : D_{t-1} \times X_t \rightarrow D_t, \tag{12}$$

which maps a pre-existing asymmetry d_{t-1} and the actual innovation shock, x_t^*, into the present asymmetry level , d_t.

- Assumption (4.1) excludes the possibility of *Persistent asymmetry* being an equilibrium such that the quality asymmetry between neighbouring firms, remains unchanged.

[20] This restriction is nothing more than assuming a lower bound for the innovation shock and implies that the following analysis applies when, at each date, an innovation arises for which at least some firms in the industry adopt in equilibrium (half of them given the self replicating pattern of the industry).

[21] Notice that if the innovation process, has the lower bound given by (11) then it coincides with the difference between the latest available quality and the highest one in the industry:

$$x_t^* = n_t^* - n_{t-1}^* = n_t^* - n_{h,t-1}.$$

This is clear because n_{t-1}^* has been adopted by some firms, being x_t^* outside the persistent asymmetry region for any value of d_{t-1}.

- Under *Forging ahead* only the leaders adopt. So the new product quality difference between neighbouring firms, d_t, will be:

$$d_t = g(x_t^*, d_{t-1}) = x_t^* + d_{t-1}.$$

- Under *Leapfrogging*, only the followers adopt the new quality n_t^*. This implies a leadership reversal and a new quality difference equal to:

$$d_t = g(x_t^*, d_{t-1}) = x_t^*.$$

Finally,

- under *Catching up,* every firm adopts the new quality, n_t^*, and the quality difference between neighbouring firms disappears:

$$d_t = g(x_t^*, d_{t-1}) = 0.$$

Notice that, whenever the innovation shock falls in the Catching up equilibrium region, the game starts again from symmetric initial conditions maintaining no memory of the past asymmetry. In this case we have seen that the Forging ahead and the Leapfrogging equilibrium regions coincide producing an alternating pattern of technological qualities in space. When on the contrary there is no initial asymmetry, $d_t = 0$, and the innovation, x^*, falls within the Catching up region, initial conditions remain symmetric, and all the technological levels are upgraded.

The next step is to derive the transition probability matrices generating both these processes.

We assume that the innovation shock has a discrete support[22]. This is defined so that different values of the realization of the shock, associated with the pre existing asymmetry, will define, through the equilibrium correspondence (12), the new levels of local asymmetries.

Assumption 4.2

[22] In Appendix 1 we derive the transition probability function generating the process of evolution of local asymmetries for a continuum state space, and we prove its ergodicity.

- *The innovation shock* $\{x_t^* = n_t^* - n_{t-1}^*\}_{t=0}^{\infty}$ *is generated by a sequence of i.i.d. random variables defined on a discrete support:*

$$Supp\,(x_t) \equiv \{x_1, x_2, x_3\}$$

where:

$$x_1 \in \left[\frac{3\sqrt{t\,(q+t)} - 3t}{v}, \frac{3t - 3\sqrt{t^2 - tq}}{2v}\right); \tag{13}$$

$$x_2 \in \left[\frac{3t - 3\sqrt{t^2 - tq}}{2v}, \frac{3t - 3\sqrt{t^2 - tq}}{v}\right); \tag{14}$$

and

$$x_3 \in \left[\frac{3t - 3\sqrt{t^2 - tq}}{v}, \frac{3t}{v}\right). \tag{15}$$

with probabilities:

$$p\,(x_1)\,, p\,(x_2)\,, p\,(x_3)$$

Note that, given the equilibrium correspondence (12) , Assumption (4.2) implies that:

- Persistent asymmetry does not occur, given that the lower bound of x_1 is above the upper bound of the Persistent asymmetry equilibrium for any initial d.
- if $x_t = x_1$ and $d_t = x_1$ then $d_{t+1} = g\,(x_1, d_t) = 2x_1$, while if $d_t \in \{0, 2x_1, x_2\}$ then $d_{t+1} = g\,(x_1, d_t) = x_1$;
- if $x_t = x_2$ and $d_t \in \{0, x_1, 2x_1, x_2\}$ then $d_{t+1} = g\,(x_2, d_t) = x_2$, and
- if $x_t = x_3$ and $d_t \in \{0, x_1, 2x_1, x_2\}$ then $d_{t+1} = g\,(x_3, d_t) = 0$.

From assumption (4.2) and the equilibrium correspondence (12) it is easy to derive the transition probability matrix for the local asymmetries of the discretized process:

$\frac{d_{t+1}}{d_t}$	0	x_1	$2x_1$	x_2
0	$p\,(x_3)$	$p\,(x_1)$	0	$p\,(x_2)$
x_1	$p\,(x_3)$	0	$p\,(x_1)$	$p\,(x_2)$
$2x_1$	$p\,(x_3)$	$p\,(x_1)$	0	$p\,(x_2)$
x_2	$p\,(x_3)$	$p\,(x_1)$	0	$p\,(x_2)$

(16)

This matrix maps probability distributions on local technological asymmetries, d_t, from one period into the next one, d_{t+1}, and summarizes the interaction between the exogenous innovation process and the two stage game of adoption described before. These distributional changes represent the period by period, short-term evolution of technological asymmetries between neighbouring firms-locations.

Once the transition matrix has been derived, it is possible to analyse the long run behaviour of the distribution of local technological asymmetries. In the next proposition we see that this can be described by an invariant distribution of the process generated by the transition probability matrix(16).

Proposition 5

The transition probability matrix (16) *has a unique invariant distribution.*

□

We now focus on intra-distribution dynamics, or the probabilistic evolution of the ranking of a single firm-location conditional on its present location and product-quality. We want to characterise the long-term probabilities by which individual firms, with a given geographical location, interchange their position of leaders or followers.

Let us define the set of states describing what can happen to the single firms. Let:

- IA_1 be the state where, starting from symmetric initial conditions, half of the firms adopt the innovation x_1;
- IA_2 be the state where, from symmetric initial conditions, half of the firms adopt the innovation x_2;
- FA be the state where, given the initial local asymmetry $d_t = x_1$, only the leaders adopt the innovation $x_{1;}$
- LF_1 be the state where, given the initial local asymmetry $d_t \in \{x_2, 2x_1\}$ only the followers adopt the innovation x_1;
- LF_2 be the state where, given the initial local asymmetry $d_t \in \{x_1, x_2, 2x_1\}$ only the followers adopt the innovation x_2; and finally,

- CU be the state where, given any initial value of local asymmetry, everybody adopts the innovation x_3.

  The intra-distribution mobility transition matrix is now easily derived:

$\frac{t+1}{t}$	IA_1	IA_2	FA	LF_1	LF_2	CU
IA_1	0	0	$p(x_1)$	0	$p(x_2)$	$p(x_3)$
IA_2	0	0	0	$p(x_1)$	$p(x_2)$	$p(x_3)$
FA	0	0	0	$p(x_1)$	$p(x_2)$	$p(x_3)$
LF_1	0	0	$p(x_1)$	0	$p(x_2)$	$p(x_3)$
LF_2	0	0	0	$p(x_1)$	$p(x_2)$	$p(x_3)$
CU	$p(x_1)$	$p(x_2)$	0	0	0	$p(x_3)$

(17)

Proposition 6

The transition probability matrix (17) *has a unique invariant distribution.*

□

Discussion

The previous proposition implies that in the long term the industry will be fluctuating between periods of Leapfrogging, Forging Ahead and Catching up, with frequencies described by the probabilities of the invariant distribution. Once the parameters of the exogenous innovation process are specified, one can study the effects of their changes both on the equilibrium distribution and on the transition probability matrices. Moreover the ergodicity of both: the local asymmetry and the intra-distribution mobility processes reinforce the meaning of numerical simulations performed in Appendix 1.

6 Conclusions

In this paper we studied the transitions of a cross-section distribution of product qualities.

We have seen that every transition can be supported as a subgame perfect equilibrium, depending on the values of the relevant parameters of the economy. These are: the pre-existing level of neighbours asymmetry, the size of the innovation shock, transport costs, preferences for quality and adoption costs.

The resulting local asymmetries refer to a geography composed by a sequence of spatially linked markets where all the firms have the same access to the state of the art technology. In this framework Forging ahead, the equilibrium where only the leaders adopt the new technology, stops after the level of cumulated asymmetry has reached a given threshold. Leapfrogging, the equilibrium with leadership reversal, on the contrary, can go on forever.

Catching up, when global adoption takes place, which would not occur under pure, undifferentiated, Bertrand competition, stops being an equilibrium when transport costs, expressing the degree of horizontal differentiation, fall below adoption costs.

We then analised the effects of economic integration on the evolution of regional asymmetries. Forging ahead becomes more likely with higher integration while Catching up becomes more difficult. The effects of integration on Persistent asymmetry and Leapfrogging depend on the amount of preexisting asymmetry: for intermediate initial asymmetry levels economic integration helps maintaining it, for lower levels it increases the chances of Leapfrogging.

By assuming an exogenous discrete probability distribution describing the evolution of the state of the art technology, we derived the transition probability matrices describing both the evolution of the local asymmetries and intra-distribution mobility.

Having proved that such matrices have a unique invariant distribution, provides a description of a long term stochastic equilibrium characterized by intra-distribution mobility.

7 Appendices

7.1 Appendix 1

In this Appendix we display the results derived from numerical simulations[23] of the process generating the sequence of local asymmetries: $\{d_t\}$. The innovation process follows a uniform distribution on a bounded interval, the size of which is specified in the different simulations [24].

We start by studying statistics of the frequency distribution of local asymmetries, d_t, and analyse the effects of some relevant parameter changes.

We start from a base case with the following values of the parameters:
transport costs, $t = 4$,
adoption costs, $q = 3$,
consumers' preference for quality, $v = 2$,
the innovation shock is uniformly distributed on the interval:

$$x \in X = [0, 6]$$

In this case more than 70% of the times local asymmetries disappear. The following table gives: mean, standard deviation and maximum value of the local asymmetries for this base case.

$mean(d_t)$	0.54
$std\,(d_t)$	0.99
$\max\,(d_t)$	2.99

Next we consider the effects of lowering the value of the consumers' preference for quality, v, while maintaining the other parameters unchanged. Let:

$$t = 4, q = 3, v = 1,$$

and

$$x \in X = [0, 6]$$

An increase in the preference for quality, v, leads to higher local asymmetries. Under these parametric conditions we observe local asymmetries equal

[23]Numerical simulations have been performed with Simulink, a simulation package of Matlab.

[24]We allowed the equilibrium of persistent asymmetry, i.e., we did not assume the lower bound introduced before for analytical convenience.

to zero in less than 2% of the simulation runs. The following table gives again mean, standard deviation and maximum value of the local asymmetries, reflecting the increment in local asymmetries d_t.

$mean\,(d_t)$	3.77
$std\,(d_t)$	1.64
$\max\,(d_t)$	5.99

Consider now the effects of higher adoption costs, q, on the frequency distribution of local asymmetries, d_t. Let:

$$t = 4, q = 4, v = 2,$$

and

$$x \in X = [0, 6]\,.$$

Higher local asymmetries are now much more frequent than before. This is reflected in the mean of local asymmetries given in the following table together with the standard deviation and maximum value:

$mean(d_t)$	3.95
$std\,(d_t)$	1.48
$\max\,(d_t)$	5.99

Finally we consider the effects of an higher support for the innovation shock on the distribution of local asymmetries, for the case in which adoption and transport costs were equal to transport costs. Let:

$$t = 4, q = 4, v = 2,$$

and

$$x \in X = [0, 12]\,.$$

Doubling the upper bound of the innovation shock leads to higher symmetry. The following table gives mean, standard deviation and maximum value of the local asymmetries:

$mean(d_t)$	1.31
$std\,(d_t)$	2.05
$\max\,(d_t)$	5.99

.

7.2 Appendix 2

7.2.1 The transition probability function

In this appendix we study the existence and the asymptotic behavior of the stochastic process of equilibrium sequences of neighbouring firms quality asymmetries, generated by the interaction of the equilibrium correspondence (12) and an exogenous Markov process on the innovation size.

Assumption 7.1

- *The sequence* $\{x_t^* = n_t^* - n_{t-1}^*\}_{t=0}^{\infty}$ *of innovation shocks follows a Markov process generated by a stationary transition probability:* $Q(x, A)$*. Where* $x \in X$ *defined in* (11) *and* $A \in B\ (X)$*, the Borel* $\sigma-$*algebra on* X.[25]

Assumption 7.2 (Equilibrium selection)

- *We consider the selection of the equilibrium correspondence such that if:*

$$g\left(x_t, d_{t-1}\right) \in \{x_t, d_{t-1} + x_t\}$$
$$g\left(x_t, d_{t-1}\right) = d_{t-1} + x_t.$$

is selected.

We are now ready to derive the transition probability function for the endogenous state variable,

$$d_t \in D = \left[0, \frac{3t - 3\sqrt{t^2 - tq}}{v}\right].$$

Lemma A 1

For any $d_t \in D$ *and* $B \in B(D)$ *the transition probability function on* d_t *is given by:*

$$P\left(d_t, B\right) = Q\left(x_t, \{x_{t+1} \in X : g\left(x_{t+1}, d_t\right) \in B\}\right) \qquad (18)$$

[25] *This means that for every* $x \in X$ *and every* $A \in B\ (X)$ *the value of the transition probability* $Q(x, A) \in [0, 1]$ *gives the probability that the innovation shock* x_{t+1}^**, belongs to the set* A *at date* $t+1$ *if its value at date* t *is* x_t^**. By fixing* $x \in X$*, the transition probability function,* $Q\left(x, .\right)$ *defines a probability measure on* $B\ (X)$ *while given* $A \in B\left(X\right), Q\left(., A\right)$ *defines a measurable function of* X *into* $[0, 1]$.

where g is the equilibrium selection and $Q(x_t, A)$ is the transition probability function for the exogenous innovation shock. The sequence of period equilibrium asymmetries in the industry

$$\{d_t, t \geq 0\}$$

follows a Markov process generated by the transition probability function (18).

7.2.2 Asymptotic behaviour

In this section we study the asymptotic behaviour of the process generated by the transition probability function (18). The questions we want to answer are:

- *will the limit of the iteration of the transition probability function, starting from any initial distribution over the sate space D, settle down to a fixed point, invariant probability distribution, over the same state space ?*
- *will this limit be unique? and*
- *at which rate will the process converge to such a limit if it exists?*

To study the period transitions over the set of possible local asymmetries we need to define the following operator:

Definition

For any probability measure λ *defined on* $(D, B(D))$, *define* $T^*\lambda$ *by*

$$(T^*\lambda)(A) = \int P(d_t, A)\,\lambda(\mathbf{d}d_t),\forall A \in B(D)$$

$T^*\lambda$ is a mapping from the space of probability measures on $(D, B(D))$ into itself and represents the probability that the equilibrium asymmetry, d_{t+1}, will be in a set A, if the current equilibrium asymmetry , d_t, is drawn accordingly to the probability measure λ.

In the following we will focus on the convergence[26] of the sequence of probability measures over the state variable induced by the iteration of $T^*\lambda$.

[26] We are looking at *Strong Convergence,* formally we have that:

Proposition A1

Given a sequence $\{x_t^* = n_t^* - n_{t-1}^*\}_{t=0}^{\infty}$ *of innovation shocks which follows a Markov process generated by a stationary, regular transition probability:* $Q(x, A)$, *where*

$$x \in X = \left[\frac{3\sqrt{t(q+t)} - 3t}{v}, \infty\right)$$

and $A \in B(X)$, *the Borel* $\sigma-$*algebra on* X, *if* $q \leq t$, *and* λ_0 *is an initial probability distribution on*

$$d_0 \in \left[0, \frac{3t}{v}\right)$$

there exists a unique invariant probability measure, λ^*,*on*

$$D = \left[0, \frac{3t - 3\sqrt{t^2 - tq}}{v}\right]$$

such that the sequence of probability measures $\lambda_{t+1} = T^*(\lambda_t)$ *generated by the operator* T^* *associated to the transition probability function* (18), *strongly converges to* λ^* *at a geometric rate independently of the initial probability measure* λ_0.

□

Proof of Proposition A1

The proof is in two steps: first we state a necessary and sufficient condition on Markov transition functions to generate a process which converges

---

The sequence of probability measures $\{\lambda_t\}$ converges in the total variation norm to the probability measure λ if

$$\lim_{t\to\infty} \|\lambda_t - \lambda\| = 0$$

Let $(D, B(D))$ be the measurable space for the process on local asymmetries and let $\{\lambda_t\}$ and λ be measures on $(D, B(D))$. Then we say that $\{\lambda_t\}$ converges strongly to λ if

$$\lim_{t\to\infty} \int f\mathbf{d}\boldsymbol{\lambda}_t = \int f\mathbf{d}\boldsymbol{\lambda}, \forall f \in B(D, B(D))$$

for all f such that $\|f\| = \sup_{d\in D} |f(d)| \leq 1$

strongly, second we show that the transition probability function (18) satisfies this condition.

□

Definition(condition M) [27]

There exists $\epsilon > 0$ and an integer $N \geq 1$ such that for any $A \in B(D)$, either $P^N(d, A) \geq \epsilon$, or $P^N(d, A^c) \geq \epsilon$, all $d \in B(D)$.

Lemma A.2

If $q \leq t$, and $Q(x_t, .)$, the transition probability for the shock x_{t+1}, is a regular Markov transition probability, the transition probability function (18) *satisfies condition M.*

□

Proof

Consider the point in the state space where $d_{t+1} = \{0\}$. The transition probability function gives the probability of reaching $\{0\}$ from a given d_t as the probability of the set defined as the anti-image of the equilibrium selection when it takes value $\{0\}$:

$$P^1(d_t, \{0\}) = Q\left(x_t, \{x_{t+1} \in X : g(x_{t+1}, d_t) = \{0\}\}\right) = \mu\left(g_{d_t}^{-1}(\{0\})\right)$$

from (12) we know that if $q \leq t$, for any given d_t, the probability of having the shock driving the system to the catching up region, where $d_{t+1} = 0$, is always positive, greater than ϵ.

Furthermore $\forall A \in B(D)$ either $A \supset \{0\}$ or $A^c \supset \{0\}$ which implies that either $P^N(d, A) \geq \epsilon$, or $P^N(d, A^c) \geq \epsilon$, all $d \in B(D)$.

Having proved that (18) satisfies condition M, we refer to the theorem proving that this condition is a necessary and sufficient for the strong convergence of a Markov process[28], i.e. for the existence of a unique invariant probability distribution generated by the iteration of the transition probability function defined in (18).

□

[27] See Stokey Lucas[16, pages 348-349].

[28] See Stokey Lucas [16, pages 349-350]. They show that if condition M holds the adjoint operator T^* associated with the transition function P, is a contraction mapping on the space of probability measures in $(D, B(D))$ and that the converse is also true.

Corollary A.1

The time average of the realization of the process of quality asymmetries, generated by the transition function (18) *converges to the invariant distribution of the process*

$$\lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} T^{*n} \lambda_0 = \lambda^*,$$

$$\forall \lambda_0 \in \Lambda \left(D, B \left(D \right) \right)$$

□

Proof of Corollary A.1

Stokey and Lucas [16] refer to Neveu [12, sec.V.3] for the Proof of the Corollary.

□

7.3 Appendix 3

Proofs

Proof of Lemma 1

From (1) we can see that firm i best reply on its left side market to $p_{i+1,r}$ will lie in the interval:

$$p_{i,l} \in \left[p_{i+1,r} + vd_{i,l} - t,\ p_{i+1,r} + vd_{i,l} + t\right].$$

This is the case because a price higher than the upper bound, $p_{i+1,r} + vd_{i,l} + t$, induces firms' i demand to zero. Similarly firm i captures the entire market at a price equal to: $p_{i+1,r} + vd_{i,l} - t$ which defines the lower bound. By the same reasoning, firm $i+1$ chooses $p_{i+1,r}$ in the interval:

$$p_{i+1,r} \in \left[p_{i,l} - vd_{i,l} - t,\ p_{i,l} - vd_{i,l} + t\right]$$

The problem faced by each firm in the second stage of the game is:

$$\max_{p_{i,l},\, p_{i,r}} \left\{ p_{i,l}\frac{1}{2t}\left[vd_{i,l} + t + p_{i+1,r} - p_{i,l}\right] + \right. \\ \left. + p_{i,r}\frac{1}{2t}\left[vd_{i,r} + t + p_{i-1,l} - p_{i,r}\right] \right\} \tag{19}$$

subject to:

$$\begin{gathered} p_{i,l} \in \left[p_{i+1,r} + vd_{i,l} - t,\ p_{i+1,r} + vd_{i,l} + t\right] \\ \textit{and} \\ p_{i,r} \in \left[p_{i-1,l} + vd_{i,r} - t,\ p_{i-1,l} + vd_{i,r} + t\right] \end{gathered}$$

The Kuhn-Tucker, necessary and sufficient conditions for a solution of (19) give us the best reply function for firm i on its left market in which it competes against firm $i+1$:

$$p_{i,l}\left(p_{i+1,r}\right) = \begin{cases} 0 & \text{if } p_{i+1,r} \leq -vd_{i,l} - t \\ \dfrac{1}{2}\left(vd_{i,l} + t + p_{i+1,r}\right) & \text{if } p_{i+1,r} \in \left(-vd_{i,l} - t\ ,\ -vd_{i,l} + 3t\right) \\ p_{i+1,r} + vd_{i,l} - t & \text{if } p_{i+1,r} \geq -vd_{i,l} + 3t \end{cases} \tag{20}$$

In a similar way we obtain the reaction function for firm $i+1$ on its right market.

$$p_{i+1,r}\left(p_{i,l}\right)=\begin{cases} 0 & \text{if } p_{i,l}\leq vd_{i,l}-t \\ \dfrac{1}{2}\left(-vd_{i,l}+t+p_{i+1,r}\ \right) & \text{if } p_{i,l}\in\left(vd_{i,l}-t,+vd_{i,l}+3t\ \right) \\ p_{i,l}-vd_{i,l}-t & \text{if } p_{i,l}\geq vd_{i,l}+3t \end{cases} \tag{21}$$

From (20) and (21) we obtain the equilibrium prices on the market (between firms i and $i+1$) as the solution of the system of reaction functions of two adjacent firms.

□

Proof of Corollary 1

Equilibrium profits are obtained by substitution.

□

Proof of Propositions 1, 2, 3 and 4.

In the first stage of the game each firm chooses a best reply to the adoption decisions of its neighbours.

These will depend on whether the firm is originally a low or a high quality one. In the next Lemma B1, that we use to prove Propositions 1, 2, 3 and 4, we show the parameter regions for which different adoption decisions are best replies to their local environment.

Let us define:

- $\omega_{i,l}^{*}\left(\omega_{i\pm1,h}=0\right)$, as the best reply for a low quality firm i, surrounded by two high quality firms, which decided not to adopt the new technology,
- $\omega_{i,l}^{*}\left(\omega_{i\pm1,h}=1\right)$,as the best reply for a low quality firm i, surrounded by two high quality firms, which decided to adopt the new technology,
- $\omega_{i,h}^{*}\left(\omega_{i\pm1,l}=0\right)$,as the best reply for a high quality firm i, surrounded by two low quality firms, which decided not to adopt the new technology, and finally
- $\omega_{i,h}^{*}\left(\omega_{i\pm1,l}=1\right)$,as the best reply for a high quality firm i, surrounded by two low quality firms, which decided to adopt the new technology.

Then:

LemmaB1

Let assumptions (3.1) *and* (3.2) *hold, then:*

$$\omega_{i,l}^{*}\left(\omega_{i\pm1,h}=0\right) = \begin{cases} 0 & x^{*}<\dfrac{\sqrt{9qt+\left(3t-dv\right)^{2}}-3t}{v} \quad \text{and } 0\leq d<\dfrac{3t-3\sqrt{t^{2}-tq}}{v} \\ 1 & \text{otherwise} \end{cases} \tag{22}$$

$$\omega_{i,l}^{*}\left(\omega_{i\pm1,h}=1\right) = \begin{cases} 0 & x^{*}<\dfrac{3t-3\sqrt{t^{2}-tq}}{v}-d \quad \text{and } d<\dfrac{3t-3\sqrt{t^{2}-tq}}{v} \\ 1 & \text{otherwise} \end{cases} \tag{23}$$

$$\omega_{i,h}^{*}\left(\omega_{i\pm1,l}=0\right) = \begin{cases} 0 & x^{*}<\dfrac{\sqrt{9qt+\left(3t+dv\right)^{2}}-3t}{v}-d \quad \text{and } 0\leq\ d\leq\dfrac{-3t+3\sqrt{4t^{2}-tq}}{v} \\ & \text{or} \\ 0 & x^{*}<\dfrac{vd^{2}}{18t}+\dfrac{3t}{2v}-\dfrac{2}{3}d+\dfrac{q}{2v} \quad \text{and } \dfrac{-3t+3\sqrt{4t^{2}-tq}}{v}<d<\dfrac{3t}{v} \\ 1 & \text{otherwise} \end{cases} \tag{24}$$

$$\omega_{i,h}^{*}\left(\omega_{i\pm1,l}=1\right)=\begin{cases} 0 & x^{*}<\dfrac{3t-3\sqrt{t^{2}-tq}}{v} \\ 1 & \text{otherwise} \end{cases} \tag{25}$$

□

Proof of Lemma B1

- To prove (22) we need to look at the two following cases:
  - For $x^{*}<\dfrac{3t}{v}$, from Corollary 1 the follower does not adopt, given that the leader does not adopt if and only if:

$$\frac{\left[-\frac{v}{3}d+t\right]^{2}}{t}>\frac{\left[\frac{v}{3}x^{*}+t\right]^{2}}{t}-q. \tag{26}$$

or:

$$x^* < \frac{\sqrt{9qt + (3t - dv)^2} - 3t}{v} \tag{27}$$

– For $x^* \geq \frac{3t}{v}$ the follower, by adopting when the leader is not adopting, will capture the entire market therefore,from Corollary 1 we know that it does not adopt if and only if:

$$\frac{\left[-\frac{v}{3}d + t\right]^2}{t} > 2\left[vx^* - t\right] - q \tag{28}$$

or

$$x^* < \frac{vd^2}{18t} + \frac{3t}{2v} - \frac{d}{3} + \frac{q}{2v}. \tag{29}$$

Condition (27) is binding, i.e.

$$\frac{\sqrt{9qt + (3t - dv)^2} - 3t}{v} \leq \frac{3t}{v} \tag{30}$$

for any $d \geq 0$

in facts (30) holds if and only if:

$$d > \frac{3t - 3\sqrt{4t^2 - tq}}{v}$$

but given that $q \leq t$ we have that

$$\frac{3t - 3\sqrt{4t^2 - tq}}{v} < 0$$

Furthermore

$$\frac{\sqrt{9qt + (3t - dv)^2} - 3t}{v} \geq 0$$

if and only if

$$d < \frac{3t - 3\sqrt{t^2 - tq}}{v}.$$

Condition (29) is never binding, in facts:

$$\frac{vd^2}{18t} + \frac{3t}{2v} - \frac{d}{3} + \frac{q}{2v} \geq \frac{3t}{v}$$

if and only if:

$$d < \frac{3t - 3\sqrt{4t^2 - tq}}{v} < 0$$

which proves (22).

□

- To prove (23) we look at the incentives to adoption for the follower when the leader is adopting.

  – For $x^* < \frac{3t}{v} - d$, the follower does not adopt if and only if:

$$\frac{\left[-\frac{v}{3}\left(d + x^*\right) + t\right]^2}{t} > t - q \tag{31}$$

or

$$x^* < \frac{3t - 3\sqrt{t^2 - tq}}{v} - d. \tag{32}$$

  – For $x^* \geq \frac{3t}{v} - d$ the follower will not adopt if and only if

$$0 > t - q$$

which is never binding under assumptions (3.1) and (3.2).

□

- To prove (24) we look at the incentives to adoption for the leader when the follower is not adopting:

– For $x^* < \frac{3t}{v} - d$, we know from Corollary 1 that the leader does not adopt if and only if

$$\frac{\left[\frac{v}{3}d + t\right]^2}{t} > \frac{\left[\frac{v}{3}(x^* + d) + t\right]^2}{t} - q \tag{33}$$

or

$$x^* < \frac{\sqrt{9qt + (3t + dv)^2} - 3t}{v} - d. \tag{34}$$

and condition (34) is binding, i.e.

$$\frac{\sqrt{9qt + (3t + dv)^2} - 3t}{v} - d < \frac{3t}{v} - d,$$

for $d < \frac{-3t + 3\sqrt{4t^2 - tq}}{v}$.

– For $x^* \geq \frac{3t}{v} - d$ the leader, by adopting when the follower does not, monopolizes the market because the quality difference becomes: $x^* + d \geq \frac{3t}{v}$. We know from Corollary 1 that he does not adopt if and only if:

$$\frac{\left[\frac{v}{3}d + t\right]^2}{t} > 2\left[v(x^* + d) - t\right] - q \tag{35}$$

or

$$x^* < \frac{vd^2}{18t} + \frac{3t}{2v} - \frac{2}{3}d + \frac{q}{2v} \tag{36}$$

and condition (36) is binding, i.e.

$$\frac{vd^2}{18t} + \frac{3t}{2v} - \frac{2}{3}d + \frac{q}{2v} \geq \frac{3t}{v} - d$$

for $d \geq \frac{-3t + 3\sqrt{4t^2 - tq}}{v}$.

□

- Finally to prove (25) we look at the incentives to adoption for the leader when the follower is adopting, we need to consider two cases:

  - For $x^* < \frac{3t}{v}$, we know from Corollary 1 that the leader does not adopt if and only if:

$$\frac{\left[-\frac{v}{3}x^* + t\right]^2}{t} > t - q \tag{37}$$

or

$$x^* < \frac{3t - 3\sqrt{t^2 - tq}}{v} \tag{38}$$

  - For $x^* \geq \frac{3t}{v}$ by non adopting the leader will lose the entire market, so he does not adopt if and only if:

$$0 > t - q$$

Clearly, given assumptions (3.1) , and (3.2) (38) gives us the condition for adoption to be the best reply for the leader when the follower adopts.

□

Propositions 1,2,3 and 4 are just proved by combining the equilibrium regions of their best replies characterized in Lemma B1.

Proof of Proposition 5

It is easy to see that the states of the process are all part of a communicating class and this is a necessary and sufficient condition for existence and uniqueness of the invariant distribution, given by the normalized eigenvector associated to its first eigenvalue.

□

Proof of Proposition 6

As before it is easy to see that the states of the process are all part of a communicating class and this is a necessary and sufficient condition for existence and uniqueness of the invariant distribution, given by the normalized eigenvector associated to its first eigenvalue.

□

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