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Compressed sensing and undersampling k-space

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COMPRESSED SENSING AND UNDERSAMPLING K-SPACE

by

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COMPRESSED SENSING AND UNDERSAMPLING K-SPACE

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Abstract

In the field of medical imaging, one of the most important concepts consists of the creation of the image from an obtained signal. The creation of the image is broken down into a subset of tasks. The first is the basic concept of isolating the element crucial to creating an image. One example is the isolation of different atoms in different modalities, for example PET or SPECT. Second, is using the intrinsic properties of these atoms to create a signal that can be recorded, this is done by magnets, gradients, coils, and other technological advances specific to other imaging modalities. Third, is the method used to record the signal. This can be done in many different ways, including but not limited to, radon space and k-space. Last but not least is the transform of the data in their respective spaces into images that are read by technologists. What is described here is, a very simple explanation for the process that different modalities go through in order to create an image. This review paper will be focused mainly on k-space acquisition and the different ways that the acquisition of k-space and image creation can be accelerated to improve patient time spent in the machine.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF ABBREVIATIONS

CS..... Compressed Sensing

MRI.....Magnetic Resonance Imaging

SPIRiT ..... Iterative self-consistent parallel imaging reconstruction

INTRODUCTION

What is Magnetic Resonance Imaging (MRI)?

MRI stands for magnetic resonance imaging. The detected signal in MRI is obtained from the protons, most often the hydrogen protons, in the various tissues within the body. These protons create a net magnetization when they are subjected to a strong static magnetic field. The static magnetic field will align protons parallel and anti-parallel to the magnetic field. Once the net magnetization is achieved, a radio frequency pulse that is emitted to the body through the coils in the magnet, will tip the protons into the transverse plane. The recovery and decay of the net magnetization is eventually observed and is the basis for the signal collected. Simply then there is resonance which the nuclei of the hydrogen atoms within the water molecules are precessing at the Larmor frequency followed by relaxation, exponential recovery (contrast is T_1) and exponential decay (contrast is T_2), and last but not least spatial encoding and reading k-space lines.

What is Fourier Transform?

Fourier transform is an integral transform that in this case transforms a time function and expresses it as a function of frequency. An inverse Fourier transform represents the frequency function in time. The concept of Fourier transform becomes important in the study of complicated waves. Complex waves can be represented by the summation of simpler and basic waves such as cosine and sine functions. This is better viewed as an image, as shown in Figure 1. Of particular interest is the fact that in MRI there is frequency and phase encoded spatial information that are used to create an image by changing the phase and

frequency along the gradients. These frequencies are then collected in k-space, which is inversely transformed into an image by the Fourier transform.

Fourier Transforms: $x(f) = \int_{\infty}^{\infty} x(t)e^{2\pi itf}\, dt$ this is the equation for a Fourier transform.

Fourier Series: In the Fourier series a wave like function is created by using combination of simple sine and cosine waves. Arguably the most beneficial aspect of a Fourier series is that it is possible to deconstruct a complex wave into a smaller series of sine and cosine waves.

Finite summation: $x_N(f) = \frac{A_0}{2} + \sum_{n=1}^{N} A_n \cdot \sin\left(\frac{2\pi nf}{p} + \phi n\right)$, for integers $N \geq 1$

Fast-Fourier Transform: A way to compute discrete Fourier transform and its inverse. Let $x_0, \ldots, x_{N-1}$ be a complex number, the Discrete Fourier transform is defined as: $X_k = \sum_{n=0}^{N-1} x_n e^{-i2\pi k\frac{n}{N}}$ k=0,….,N-1

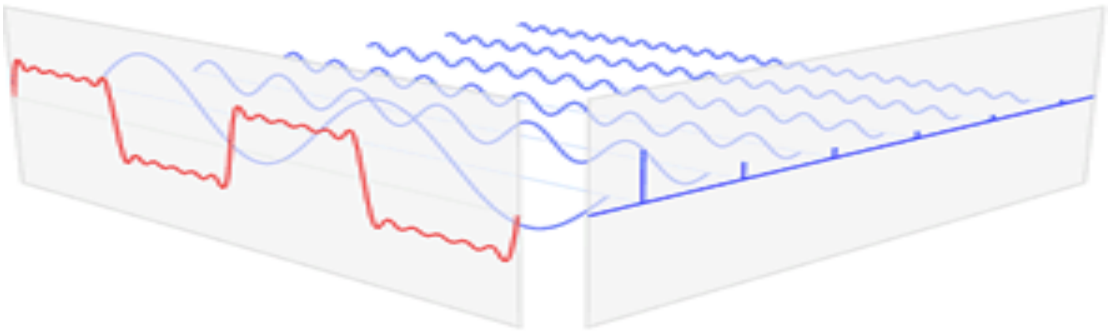


Figure 1: Fourier Transform- A simple transform breaking a complex wave (shown in red) into simpler sine waves (shown in blue) in relation to time. Time (red) is transformed to frequency domain (blue).

http://en.wikipedia.org/wiki/File:Fourier_transform_time_and_frequency_domains_(small).gif

Source: Wikipedia (Barbosa)

What is K-space?

The main purpose of k-space is to maintain the data acquired during a scan prior to processing the image. In the clinical world, k-space is one level in the creation of the image. It is not seen or stored in regular scans. In the research world often you have a choice in keeping the k-pace, for later processing, or changing the parameters in which it is acquired. Creating an MRI image requires the collection of acquired data. Each acquisition within a new transverse magnetization is created and sampled in k-space in a particular order. K-space

is a two or three dimensional entity where there exist phase encoding directions along y or z and a frequency encoded direction along x. Changes to the acquisition method are possible and will be discussed later. These changes can affect the scanning and acquisition time, although potentially leading to artifacts. MRI acquires an image by selecting three different gradients, dedicated to spatial encoding, phase encoding, and frequency encoding.

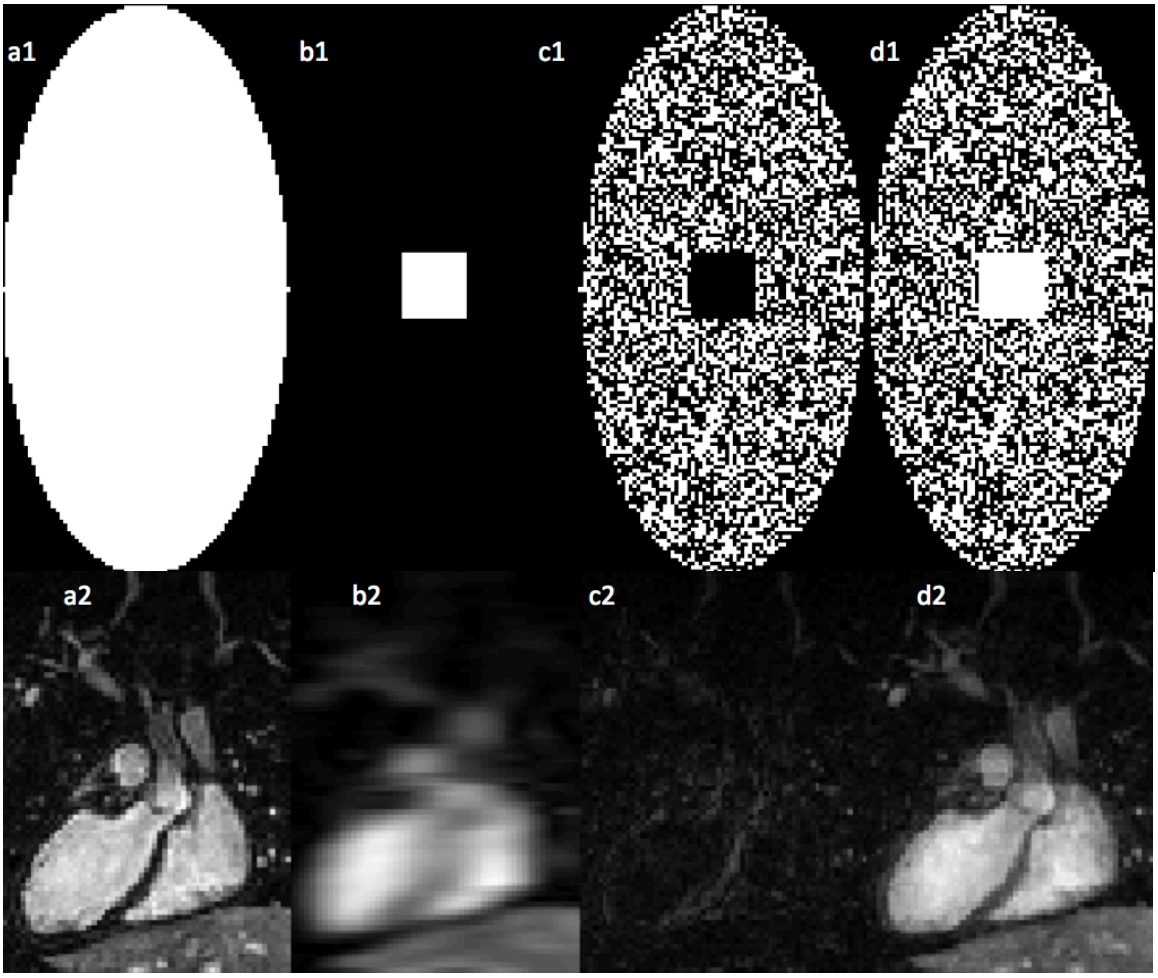


Figure 2: Image resolution is determined by the extent of the k-space that is covered. Top row represents the pattern chosen for image reconstruction (a1,b1,c1,d1): a1 represents the fully sampled k-space which is standard throughout all Phillips machine; a2 is the achieved cardiac image when the corresponding k-

space is reconstructed; b1 represents the k-space that is only sampled in the center; b2 is the reconstructed image and it can be seen that the center is extremely important in the contrast of the images; c1 represents the peripheral portion of the k-space sampled, as seen in c2 the peripheral of k-space contributes to the resolution of the image; d1 represents the uniformly undersampled k-space data which will be later reconstructed with a reconstruction algorithm, the original reconstruction will be d2.

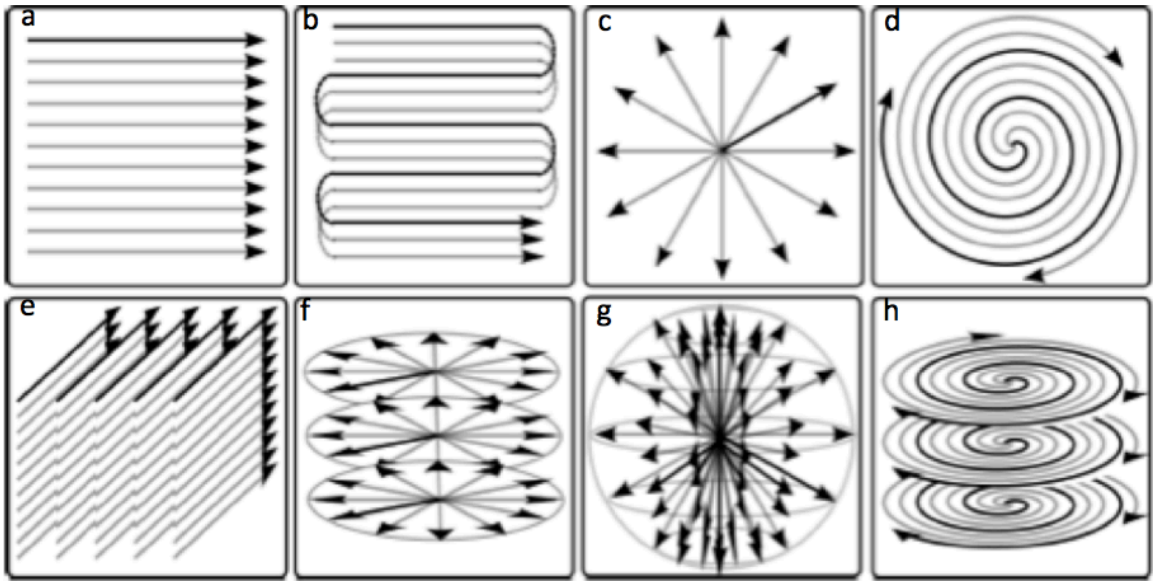


Figure 3: Various k-space acquisition from the a) linear b) echo planar imaging c) radial d) spiral; from e-h these are in 3D space, e) linear f) stack of radial g) radial h) stack of spiral (Lustig, 2008).

Figure 2 displays k-space sampling trajectories and actual undersampling pattern. As it can be seen, each value represents a portion of the image and selectively picking areas, for sample the middle, you would have a blurred image with lower resolution. As shown in

Figure2 undersampling the k-space would not have a significant effect on quality unless certain essential parts of k-space are undersampled.

Altering the acquisitions of k-space will result in different images and artifacts,(Lustig,2008). Also there are different trajectories (Figure 3) that k-space can be sampled to decrease scanning time and avoid certain artifacts like aliasing where there will be a misrepresentation of frequency instead of the actual. One of the most important concepts of k-space is the how the center of k-space contributes to contrast while the edges contribute to resolution.

What is Compressed Sensing?

Compressed Sensing (CS) is a way to reconstruct an image using fewer acquisition points. Often times in MRI, acquired signals are repeated. In theory a few selected points in MRI can be sampled and the rest can be estimated to cut the scan time while preserving the image quality. The ability to acquire the image faster is a great benefit in MR. Not only it will cut down on the imaging time but also increase the efficiency at which images are constructed. A successful application of CS requires 3 important aspects:

Transform Sparsity: The desired image must have a sparse representation in a known transform domain.

Incoherence of Undersampling Artifacts: The aliasing artifacts in a linear reconstruction caused by k-space undersampling must be incoherent (noise-like) in the sparsifying transform domain.

Nonlinear Reconstruction: The image must be reconstructed by a non-linear method which enforces both sparsity of the image representation and consistency of the reconstruction with the acquired samples. (Lustig,2007)

The goal for applying CS in MRI is to be able to decrease the scan time by sampling less of the k-space, this decrease in scan time will be beneficial in certain scans which require faster acquisition. For example in cardiac imaging the acquisition of the image is timed with respiration and the heart rate (Otazo,2010).

Methods

In general, the purpose of undersampling k-space is to reduce the scan time without sacrificing image quality. At first glance, it may seem that improving data collection would also decrease the scan time. One of the main issues that researchers are faced with in data collection is the constraint imposed upon by the MRI machine. This constraint comes from the physical (gradient amplitude and slew rate) and physiological (nerve stimulation) components (Lustig,2007). In MRI we look at a special case of CS, where the sampled linear combinations are simply individual Fourier coefficients (k-space samples). In that setting, CS is claimed to be able to make accurate reconstructions from a small subset of k-space rather than an entire k-space grid (Lustig,2007). This demonstrates that because certain scans that are sparse in pixel representations, it is possible to achieve the same resolution image by undersampling the data.

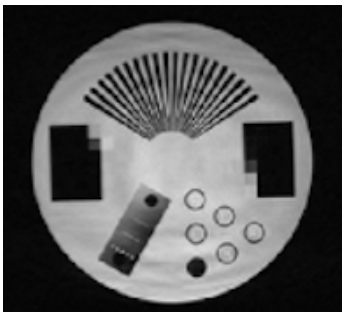


Figure 4: A phantom that has been fully sampled and reconstructed, this data is then retrospectively used to test sharpness and visual quality at different under sampling rates.

Let's look at some examples of an undersampled k-space. Figure 4 represents a phantom that has a fully sampled k-space with no alterations in the acquisition.

If we imagine the raw data as a matrix (200X85) and the values as 0 or 1 (not actual values but one that we have generated), the image will have a matrix that is all ones in the x and the y direction. The dark blocks to either side of the center designate the contrast of the phantom, whereas the resolution is represented by the radiating striations appearing from the center. In Figure 5 below, k-space has been randomly sampled, also known as "uniform random sampling", by MATLAB®. The images were produced by first identifying a certain shape to represent the center of the image. Without the center identified, the image would not have the contrast it needed. Then the matrix, excluding the center, was randomly sampled. The purpose of this test was to show how the center and the undersampling rate would influence our image. In Figure 5, the phantom images each have a center of k-space with a different shape but the same calculated area, each at 5%. The k-space that was undersampled for the outside only contains 25%.

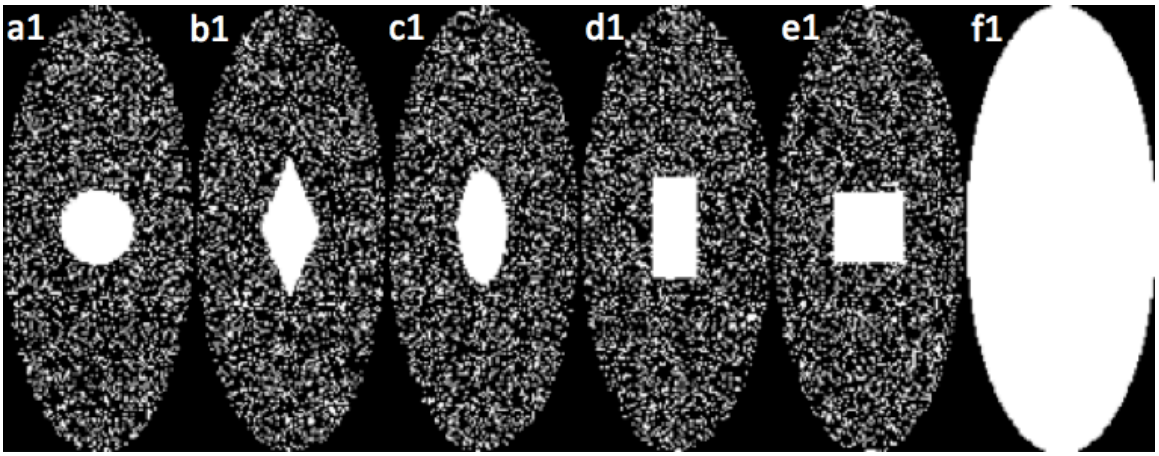


Figure 5a: K-space undersampling pattern at approximately 30% from left to right each has different shape with the same center size and rate. Throughout the process of creating the k-space data the center sizes at each rate was kept constant.

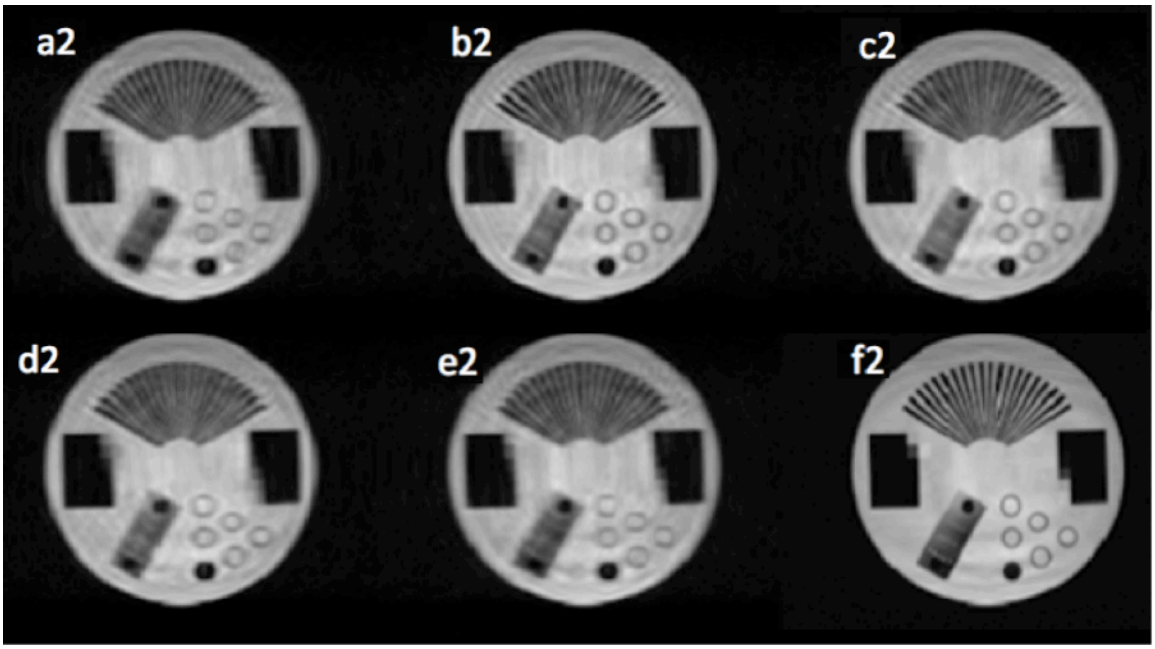


Figure 5b: Sampling of the k-space with different shapes. Left to right upper images k-space shapes- a2) circle, b2) diamond, c2) ellipse, lower images k-space shapes- d2) rectangle, e2) square, and f2) fully sampled. All phantoms shown have the same rate of undersampling and same shape for sampling the middle of k-space. With

these images and the corresponding k-space pattern we can visually conclude that rectangle would not be a good choice while diamond and square give us the best results.

The first image Figure 5a was made with the k-space containing a circle as the center while the second image contains a diamond, these are the zero filled images which are created by undersampling k-space and zero filling. The remainder of the images utilizes different shapes to model the center. Once these shapes and reconstructions are achieved, they have to be compared to the original image.

All the images of the phantom were obtained retrospectively. The reason for the retrospective acquisition is because we want to identify the best rate of undersampling and center size. In this process a fully sampled phantom image is obtained by a Phillips machine, the coil used was a 32-channel coil. Once the raw data is obtained it is Fast Fourier Transformed into an image corresponding to the fully sampled k-space. Now retrospectively we go back and choose five simple shapes, for our purposes we chose a circle, diamond, ellipse, rectangle, and a square. For each of the chosen shapes we will have 5 different sizes of the center, .5%, 2.5%, 5%, 10%, and 15%, the sizes that have been chosen are proportional to our k-space pattern size, i.e. if the k-space pattern area is 100 pixels, at 15% we would have a shape with an area that corresponds to 15 pixels of the center. The next step is to choose the uniform undersampling rate for our pattern, which are from 15% to 50% with increments of 5%, in total of eight different rates. Each set of images are then repeated 10 times for an accurate sample of data for the phantom.

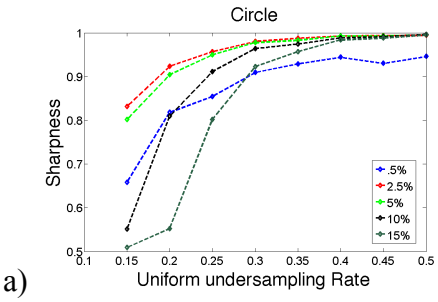
In the process of creating the final images we have three different types of image sets, the first set is the original fully sampled image, the second is the set of five center shapes with eight different rates of the zero-filled image, and the last set includes all of the previous images reconstructed from the k-space.

With all the images now reconstructed, we will choose one type and shape and one rate which can give us the highest acceleration factor, which in turn provides us with the fastest scanning time with the best image closest to the fully sampled k-space. When the certain rate and shape is chosen, we will be able to retrospectively reconstruct an in vivo data set.

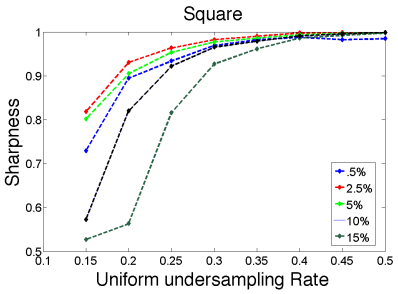
The in vivo data set is one that is obtained using a fully sampled k-space on the Phillips machine. After the in vivo data is collected it will be retrospectively used and sampled with the shape and rate that have been chosen.

Results

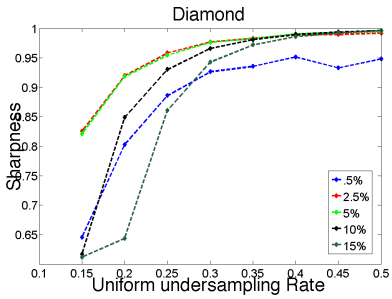
The rationale behind choosing the k-space pattern comes from a visual inspection of each of the images and the sharpness average that was obtained from the ten trials ran for each rate and shape. The first step to choose the right rate was to plot the sharpness of each shape between different center sizes of the same shape. As shown in Figure 6a-e,



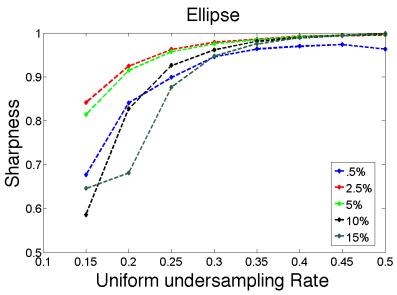
a)



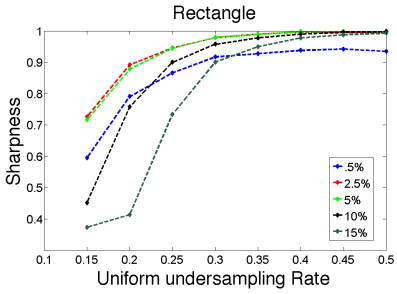
b)



c)



d)

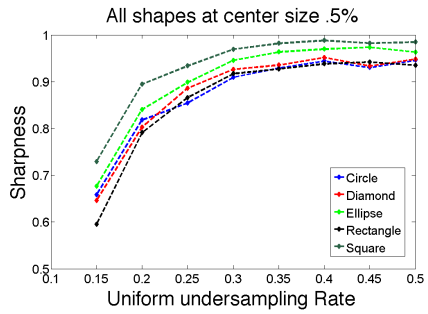


e)

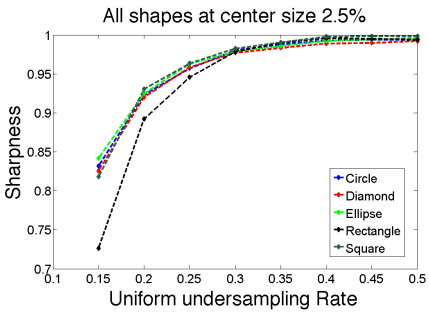
Figure6a-e. 5 graphs showing sharpness of, A) Circle B) Square C) Diamond; D) Ellipse, and E) Rectangle. From graphs of the sharpness we can see a trend in which at center size of 2.5 the overall sharpness is higher than other sizes.

Figure 6a-e gives us a good quantitative measure and visual representation on which center size is ideal for each center shape. Each shape represented by a)circle b)square c) diamond d) ellipse e) rectangle represents how the center of the k-space is sampled while the outside is uniformly sampled at the given rate. We can conclude that the sharpness is best at 2.5% center size of the k-space. The sharpness was calculated by indicating the slices in which the complete phantom was seen, the slices where then run through a Tenengrad Variance, which

is the sum of the intensity gradient over a neighborhood of the surrounding pixels. Since the 2.5% center size was chosen another set of graphs were made Figure 7a-e.



a)



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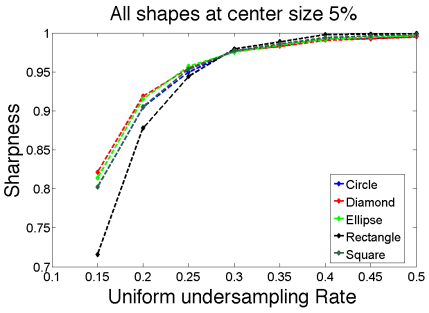

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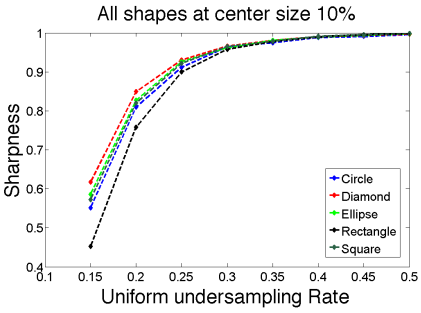

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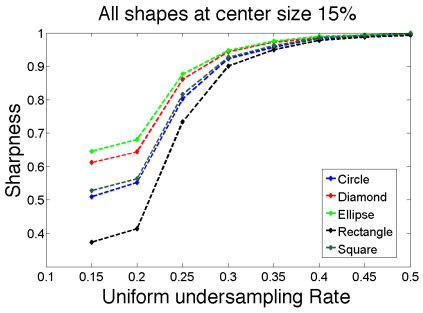

e)

Figure7a-e. 5 graphs showing sharpness of different center sizes of each individual shapes, A) .5% B) 2.5% C) 5% D) 10% and E) 15%. From graphs of the sharpness we can see a trend in which at center size of .5% and 15% there is a separation of each shape, while at 2.5% each shape gets much closer while the square has the best sharpness.

In Figure 7a-e we can visually see each shapes and how their center size compares to each other, which will visually illustrate the best center size to give the highest accuracy. In this

figure we can differentiate between the sharpness at specific rate in for each shape, this is highly beneficial because it provides with a visual representation of numerical sharpness values. The visual representation will provide us with a better understanding of how the sharpness is affected after reconstruction and before the reconstruction step. As shown in Figure 8a and 8b:

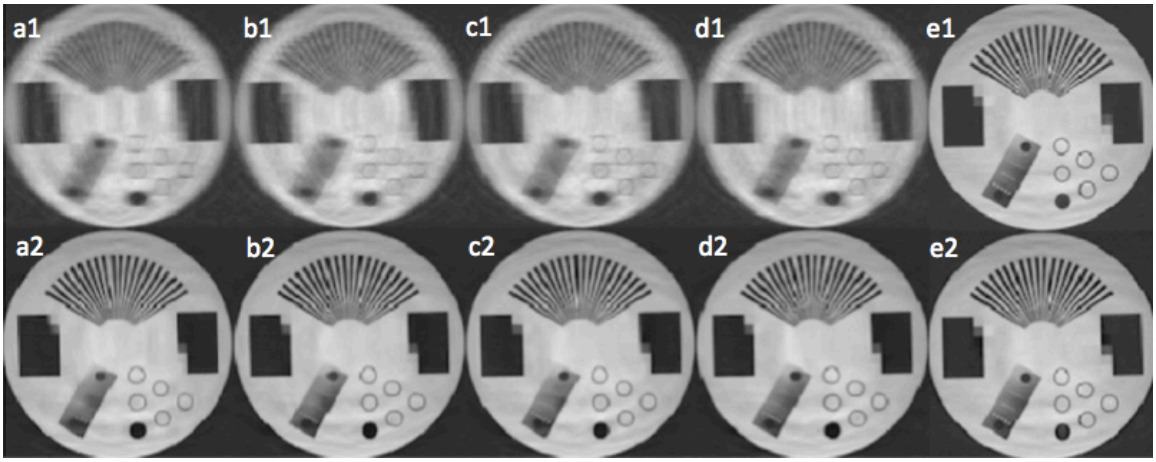

Figure8a) A set of four images with the center shape of square and a center size of 2.5% of k-space and with the addition of a fully sampled data. a1-d1 represent the undersampled data while a2-d2 is the reconstructed data from the k-space pattern and rates of 15% to 30% with increments of 5%.

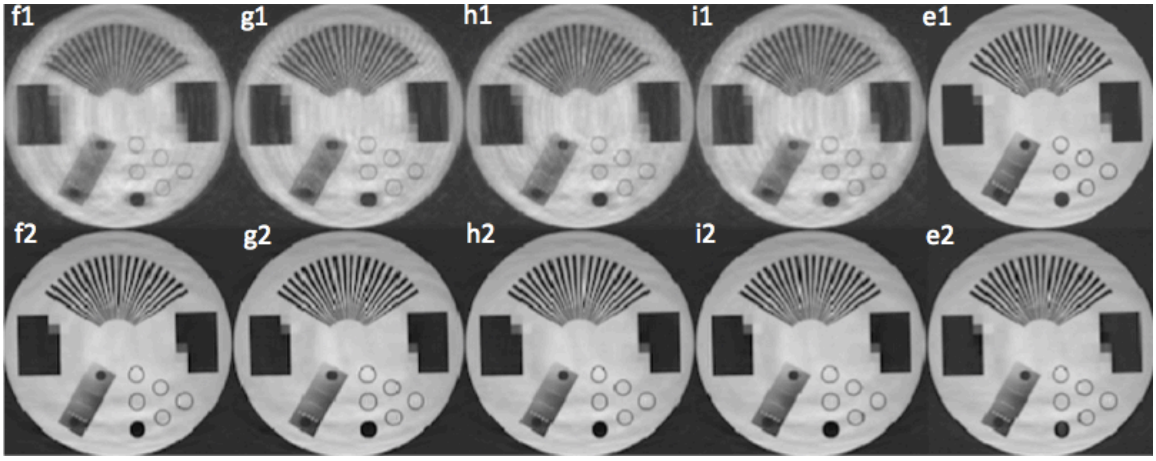

Figure8b) A set of four images with the center shape of square and a center size of 2.5% of k-space and the addition of a fully sampled data e1 and e2. f1-i1 represent the undersampled data while f2-i2 is the reconstructed data from the k-space pattern and rates of 35% to 50% with increments of 5%. Both figure 8a and 8b help show the progression of the image quality as the rate gradually increases, in Figure 8a and 8b each shape can be compared to the fully sample shape e1 and e2.

We can visually see the change that occurs from the lowest rate to the fully sampled image, in Figure8a and 8b the upper images represent the undersampled images without reconstruction while the lower images are reconstructed. From the images we can decide that the best rate and shape is 2.5% center size with a square shape and a rate of 25%, 25% rate represents 2.5% for center + 22.5% for the outer portion of k-space.

Next step is now to go back to an in vivo study and retrospectively reconstruct the images with the chosen center size.

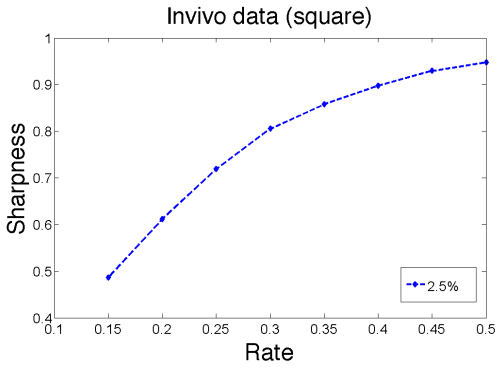

Figure 9. Represents the gradual increase of sharpness in the in vivo data as the rate increases.

As shown in Figure 9, the sharpness gradually increases as rate increases but this is not a true visual inspection and the in vivo images must be seen before a decision can be made which rate is the minimum we can chose that will provide us with the quality and speed required. Once the minimum amount of sampling chosen, we can translate the code to a language that can be read by the machine and implemented for subjects. When this code is run during the study we can see the results without the need of retrospective reconstruction of the data as Illustrated in the Figure 9a and 9b.

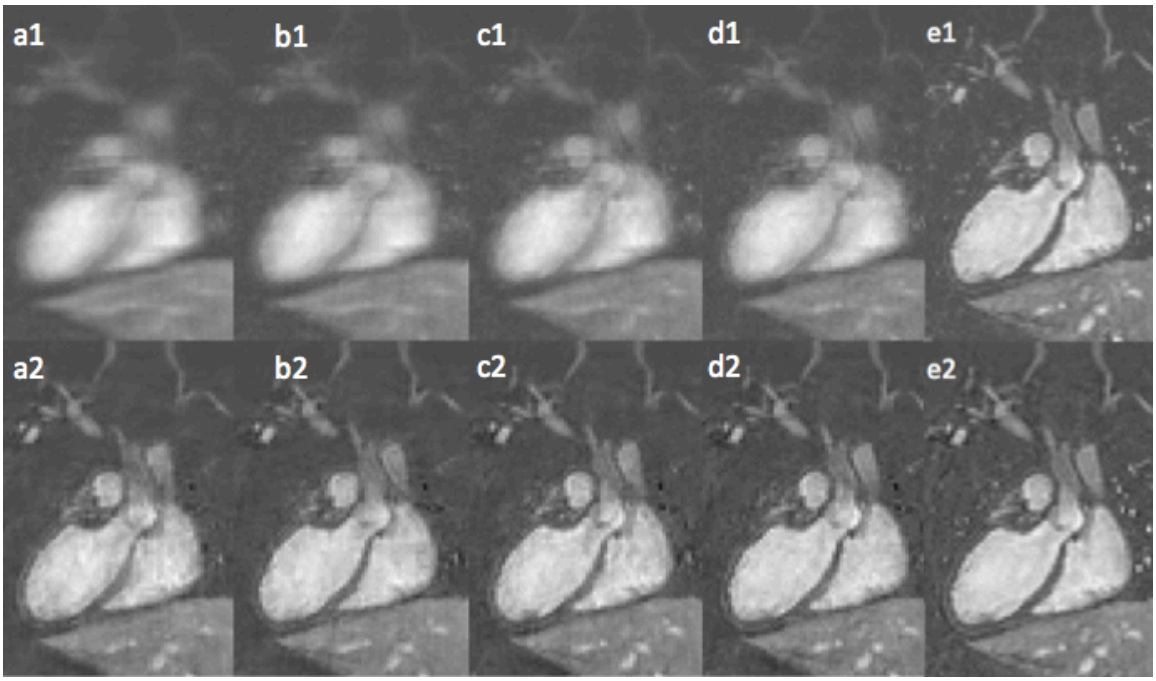

Figure9a) A set of four images with the addition of a fully sampled data. a1-d1 represent the undersampled data while a2-d2 is the reconstructed data from the k-space pattern and rates of 15% to 30% with increments of 5%. It can be seen how at lower rates, while the images looks to be sharp, in actuality it is hard for the cardiologist to see the differences required for a diagnosis, although from 20% and up we can observe little difference between the images and the fully sampled data.

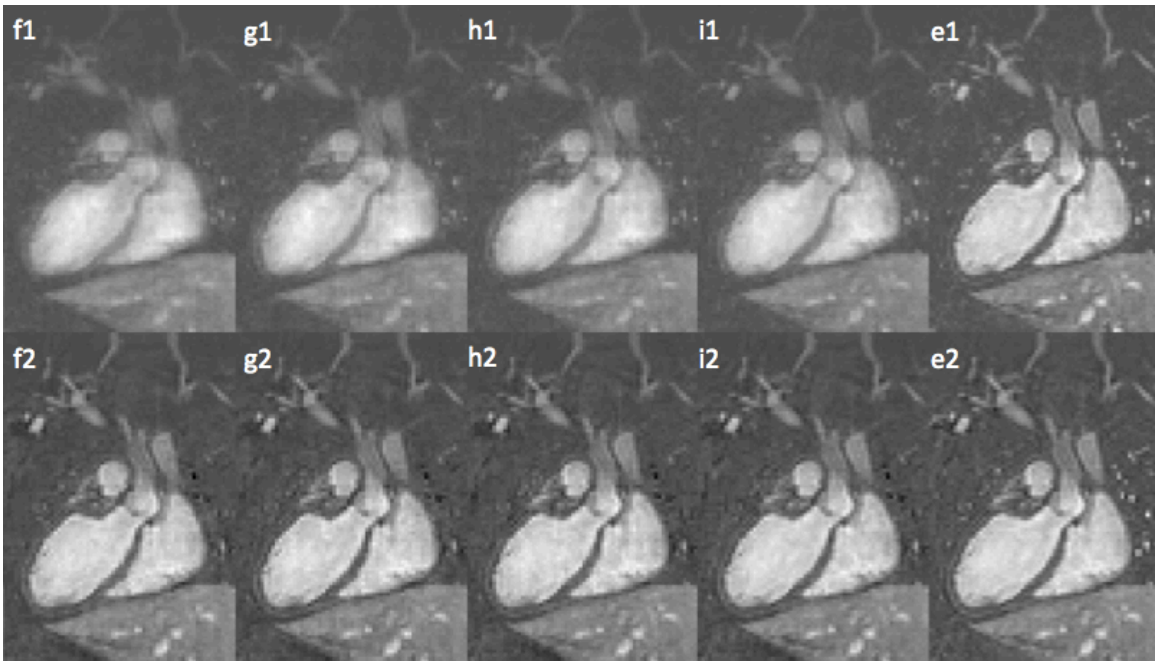

Figure9b) A set of four images with the addition of a fully sampled data. f1-i1 represent the undersampled data while f2-i2 is the reconstructed data from the k-space pattern and rates of 35% to 50% with increments of 5%. It can be seen how the image quality has improved and has become comparable to the original data.

In Figure9a and 9b there is little difference in undersampling rate of 20% to 50% while at the very low rates, lower than 20%, we can visually see the cardiac anatomy is no longer distinguishable from each other. Overall between the rate of 20% and 50% we see little to no difference in the anatomy and very low amounts of degradation in the anatomy.

Discussion

The objective of this process is to lower the total scan time of cardiac MRI scans. One of the benefits of cardiac scans is the fact that the signal can be sparse, the sparse signal allows us to undersampled the image without losing the image quality. As discussed earlier,

the central part of k-space contributes to the contrast of the image while the outer portions add to the resolution, this basic principle is why it is crucial for the trials to be run with different center size of k-space. One of the benefits of MRI is the ability to see anatomy without any invasive procedures, unfortunately in MRI movement can degrade the image quality. For cardiac scans, different forms of gating are used and often times the scan duration is highly dependent on the heart rate. What we want to do is lower scan time, by sampling less of the k-space, but hold on to the quality of the image in which a radiologist and the cardiologist can easily read the images.

The phantom images in Figure 8a and 8b compared to the in vivo cardiac images in Figures 9a and 9bhave one main difference, other than being scans of different objects. The phantom was done with a 32 channel coil while the in vivo cardiac image is done with a five channel coil, this means that using the chosen center size of 2.5% and a total rate of 25% will give us the fastest scan times for the cardiac image while we can reduce the rate much more for the phantom. Thus we can come to the conclusion that we can easily increase the acceleration factor of the cardiac scans with a coil that contains more than five channels.

Albeit in specific cases, CS and k-space undersampling has shown benefits in the research setting as well as in the clinic, i.e. angiography (Lustig, 2007), cardiac scans (Otazo, 2010), or brain scans (Lustig, 2007). The reason why only certain scans will benefit from this type of acquisition is because of sparse pixel representations and time sensitive behavior of the organ. While CS has its advantages; it also has its own set of disadvantages in which the loss of some resolution is sacrificed for time.

Simply using uniform sampling, we can accelerate the scan for the phantoms by a factor of five while with a five channel coil we can accelerate the cardiac scan up to 4. The acceleration factor for the cardiac scan is significant enough that you can take a 20 minute cardiac MRI, run the scan with the uniform sampling rate of 22.5 and a center size of 2.5%, and achieve a scan in five minutes with the quality close to a scan done in 20.

Conclusion

Acceleration of MRI is highly beneficial in scans that have a lot of motion such as cardiac imaging. By being able to speed up scans at such a rate we can possibly eliminate errors like motion artifacts, respiratory drifts, scans that take longer because of cardiac behavior, and possible discomfort which could lead to termination of scan before all the data is collected. Furthermore the increase in speed can have a secondary benefit in which more scans can be done in the allotted time and information that can be collected which otherwise would not have been collected.

The process of using shapes and uniform undersampling is in itself the simplest ways to under sample k-space. To further this research one must look into variable density under sampling of k-space. Variable density undersampling might expand on the already collected data not only by making reconstruction sharper and faster but also providing another possible way to reduce the time needed to acquire a certain image. While the phantom was ran through ten trials for each shape and rate the in vivo data also will be required to be ran at least ten times. Future goals are not only to implement variable densities but create a comparison between uniformly sampled and variable density sampled k-space.

Another future goal would be to have over ten subjects that can be scanned and the images can be retrospectively reconstructed, with this reconstruction a rate can be chosen and coded into the machine for prospectively testing the accelerated scanning times.

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VITA

Yashar Rahimpour was born in 1989 in Tehran, Iran and spent the first 11 years in Iran. Yashar Rahimpour attended Arcadia High School from 2004 to 2008. Yashar Rahimpour attended university at California State University Long beach and his interest in sciences grew. Yashar graduated in 2013 with a Bachelor of Science in Biology and a minor in Chemistry. Yashar's interest in Bioimaging brought him to Boston University, were he was able to better his knowledge on the subject of imaging and the human body. The intricacies of the human body and ways to view the human body by imaging interests Yashar and he hopes to be able add to the already vast knowledge base in the subject. Yashar plans to apply his knowledge not only in the subject field but also life.