Endogenous Correlation

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Endogenous Cross Correlations

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Abstract

This paper studies the links between rational herding and cross correlations in security returns. It demonstrates analytically and numerically that herding, as a temporary, fragile convergence, can indeed lead to asset dependency. Besides, the effect is most pronounced in abnormal market conditions. Overall, these imply a self-reinforcing process, where a bear market amplifies the herd effect that further exacerbates asset dependency. The simulation models with herding are found to generate results closer to the real patterns of asset dependency than the static benchmark model with non-interacting agents. The findings suggest an alternative view on the regulations towards greater transparency.

JEL Classification: D83, G11, G12.
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1. Introduction

Economists have attempted to study the intriguing nature and sources of the cross correlations in security returns, but the issues have remained controversial in the research community (e.g. Badrinath, Kale and Noe 1995; Boudoukh, Richardson and Whitelaw 1994; Conrad, Kaul and Nimalendran 1991; Mech 1993).

In conventional economic theory, fluctuations off the fundamental equilibrium path are attributable to exogenous shocks; abnormal returns are associated with exogenous characteristics specific to the event observation. This has an implication on the cross correlations in security returns that the correlations are an inevitable consequence of some *external common factors*.

The implication, attributing the empirically observed asset dependency to external common factors, is however built on the notion of market efficiency with homogeneous and perfectly rational agents. The conventional notion of economic agency neglects the interactive structure to which heterogeneous agents give rise. To gain a better understanding on the cross correlations in security returns, we need to take into account how constituent agents in the market behave.

Imitation is perhaps one most common observation among human behaviour. Imitation can lead to systematic erroneous decision-making, and convergence of behaviour across individuals, often referred to as *herding*. This paper, using both theoretical analysis and simulation experiments, studies the relations between herding (or imitation) and asset dependency. Instead of viewing the market as a static aggregation of isolated individuals, it employs the "individual-oriented" approach that develops from the basic market microstructure to explore the issue of interest. Does herding endogenously induce the cross correlations in security returns? To what extent does herding account for asset comovement? Is the pace of learning (or herding) relevant? What role does the market condition play in this possible link between herding and asset dependency? Can we identify empirically the herd effect on asset comovements, if any? These are the questions that we wish to answer.

We aware that there is a fallacy that deserves attention: is it all too obvious that herding/imitation in investment strategies leads to asset correlations? The validity of the issue of interest is not undermined by this fallacy. The logic is described as follows. The issue here concerns whether herding, as a *temporary, fragile convergence* of investment behaviour, can indeed lead to a significant level of asset dependency over a relatively long horizon. It is a danger to presume that a current winning investment strategy will remain successful in the future trading periods and continue to attract investors. The market is not static but dynamically changing, and so is the winner. Indeed, as unfolded later in this study, the market dynamics is a result of complex interaction between changing forecasts and investor ratios. Current success by no means guarantees future success. In fact, more people practicing the same strategy will give a higher incentive for a deviant to exploit the situation. This is where the winner looses its lead. Therefore, it should be noted that herding/imitation among market participants in no way automatically implies asset dependency.

The effect of herding on financial markets is traditionally investigated in the *single-asset* model, with a focus on whether the price time series exhibits the "stylised facts" of financial data, such as excess volatility, fat tails, etc. (e.g. Cont and Bouchaud 2000; Corcos et al. 2002; Lux 1995; Topol 1991). The current study extends to the *multiple-asset* model to investigate the herd effect on the cross-sectional correlations in security returns. A related work is by MacKenzie (2003), a sociologist who studies the 1998's crisis of the Long Term Capital Management and suggests that imitation among investors was the major cause to the crisis[1].

Several studies have documented the evidence of herding among mutual fund managers and financial analysts (e.g. Grinblatt, Titman and Wermers 1995; Scharfstein and Stein 1990; Welch 2000). Indeed, one of the standard features of institutional investment, at least in the UK, is to use the median fund manager as a benchmark. It is widely thought that this is systematically flawed; the Myner's Report (2001) specifically recommends that this practise be discontinued.

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[1] Imitation had developed an overlapping and unstable "superportfolio" in the markets within which the LTCM operated; triggered by an event in 1998 that LTCM itself in fact had only little exposure, the rapid unravel of the "superportfolio" lead to the crisis.

Finally, the study focuses on “rational herding[2]”, as opposed to “irrational herding”. Irrational herding is driven by factors unrelated to individuals’ well-being defined in conventional economics. Rational herding is motivated by the incentives of profits, lower search costs, or the belief that someone possesses superior knowledge. Essentially, rational herding can be considered as “imitative learning”. In the current setting, heterogeneous investors are endowed with the ability to update their investment strategies: they follow an imitative learning process and imitate the winner according to their relative *realised* trading profits. Herding results in a change in the market composition and the heterogeneous investor ratio, which in turn affects the next period’s price formation and hence the trading profits. The process repeats, and herding paves the market dynamics.

2. Asset Pricing Model with Multiple Risky Assets

We extend the classic mean-variance problem to include multiple risky assets. Consider a capital market with *S* risky assets (securities) and one riskfree asset (bond). The bond pays a fixed rate of return r_f for each time period; the gross rate of riskfree return is $R_f = 1 + r_f$. Denote by $\mathbf{P}_t$ the $S \times 1$ vector of the prices per share of the securities at time *t*. Letters in bold denote vectors. We assume that each security pays periodic dividends and denote by $\mathbf{d}_t$ the $S \times 1$ vector of the dividends paid by the securities at time *t*. We further assume that the $S \times 1$ dividend vector and also the time series dividend process of each security follow an IID process; for convenience, we write $\mathbf{d}_t \sim IID\left(\bar{\mathbf{d}}, \acute{\mathbf{O}}_d\right)$, where $\bar{\mathbf{d}}$ is an $S \times 1$ vector and $\acute{\mathbf{O}}_d$ is an $S \times S$ diagonal matrix. The vector of the gross risky payoffs from time *t* to time $t+1$ is given by $\mathbf{P}_{t+1} + \mathbf{d}_{t+1}$.

Let $E_t[\cdot]$ and $V_t[\cdot]$ denote the conditional mean and conditional (co)variance; they are the mean and (co)variance of some variable (vector), conditional on the information at time *t*. Denote by $\mathbf{Y}_t$ the $S \times 1$ vector of the number of security shares purchased by an investor at time *t*. The investor’s wealth level at time $t+1$ is given by

[2] See Devenow and Welch (1996) for a review on the studies of *rational* herding in financial economics.

$$W_{t+1} = R_f (W_t - \mathbf{Y}_t{}'\mathbf{P}_t) + \mathbf{Y}_t{}'(\mathbf{P}_{t+1} + \mathbf{d}_{t+1})\,. \qquad (1)$$

Investors are assumed to be myopic[3] mean-variance maximisers. That is, investors trade off mean and variance in a linear fashion:

$$\underset{\mathbf{Y}_t}{Max} \quad E_t[W_{t+1}] - \frac{a}{2} V_t[W_{t+1}]\,, \qquad (2)$$

where a is the risk aversion parameter. This classic mean-variance problem with multiple risky assets yields the vector of the optimal risky portfolio given by

$$\mathbf{Y}_t = \frac{1}{a}\left(V_t[\mathbf{P}_{t+1} + \mathbf{d}_{t+1}]\right)^{-1}\left(E_t[\mathbf{P}_{t+1} + \mathbf{d}_{t+1}] - R_f \mathbf{P}_t\right). \qquad (3)$$

The result states that the vector of the optimal demand for the number of risky shares can be obtained by the vector of the expected excess risky payoffs times the inverse of the conditional covariance matrix, divided by the risk aversion coefficient.

We assume heterogeneous investors and add superscript i for investor type i. Investors differ in their forecasting strategies (or beliefs) on the payoffs of the securities. However, the conditional covariance matrix of the risky payoffs is assumed to be a constant[4] diagonal matrix and equal[5] for all investor types, i.e. $V_t^i[\mathbf{P}_{t+1} + \mathbf{d}_{t+1}] = \grave{\mathbf{U}}$, where $\grave{\mathbf{U}}$ is an $S \times S$ diagonal matrix. The assumption of the diagonal covariance matrix implies no cross-asset correlation. It is crucial to isolate the ex-ante correlation factors, since our focus here is to investigate the correlation structure in security returns arising from the dynamic interaction among agents.

Let q_t^i denote the fraction of investor type i at time t, representing the popularity of strategy i at time t and satisfying $\sum_{i=1}^{N} q_t^i = 1$, where N is the number of different

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[3] The setting of myopic investors assumes single-period utility maximisation, and it has the benefit of making the computation tractable. Another approach is by the overlapping generation model (e.g. Brock and Hommes 2002). Alternatively, for the analysis of the rational expectations equilibrium in an non-myopic investor setting, see Brown and Jennings (1989) and the generalisation by He and Wang (1995).

[4] A detailed derivation that solves the conditional variance to a constant value under some distributional assumptions can be found in Hoel (1962).

[5] This is an approximation in a world where volatility forecasts are well established and agreed but mean forecasts are not; such a situation arises when there is a dominant risk management system or a implied volatility methodology that is universally accepted, see Merton (1980) who agrees that means are much harder to forecast than variances.

investor (or strategy) types. Denote by $\mathbf{Y}^S$ the $S \times 1$ vector of the supply of the security shares per investor, assumed to be constant. Market equilibrium requires

$$\sum_{i=1}^{N} q_t^i \ \mathbf{Y}_t^i = \mathbf{Y}^S . \tag{4}$$

Substituting in the optimal risky portfolio (3) with superscript *i*, the market equilibrium equation can be rewritten as

$$R_f \ \mathbf{P}_t = \sum_{i=1}^{N} q_t^i \ E_t^i[\mathbf{P}_{t+1} + \mathbf{d}_{t+1}] - a \ \grave{\mathbf{U}} \ \mathbf{Y}^s . \tag{5}$$

The term $a \ \grave{\mathbf{U}} \ \mathbf{Y}^s$ can be viewed as measuring the vector of the expected excess amount of the risky payoffs and therefore may be interpreted as the risk premium vector. In the asset pricing model with heterogeneous investors, the equilibrium price is the discounted weighted average of heterogeneous payoff expectations minus the risk premium, with the weights being the fractions of different investor types.

Therefore, market equilibrium yields the equilibrium price dependent on not only economic fundamentals, but also those factors influenced by investor psychology and emotion: the fraction (or the popularity) of investor types and their conditional forecasts. In the following sections, we will discuss how investors form their forecasts and also how the dynamic change of the investor fraction takes place.

In a conventional economic world of homogeneous, perfectly rational investors, equation (5) and the transversality condition will lead to the expression[6] known as the *fundamental value:* the equilibrium price equals the discounted sum of future dividends minus the risk premium. Further, for an IID dividend process, the $S \times 1$ fundamental price vector can be written as

$$\mathbf{P}^F = \frac{1}{r_f}\left(\bar{\mathbf{d}} - a \ \grave{\mathbf{U}} \ \mathbf{Y}^s \right). \tag{6}$$

In the world of homogeneous, perfectly rational investors, the equilibrium price is the fundamental value of the security, independent of investment behaviour. Market fluctuations are due to exogenous shocks rather than endogenous causes.

[6] For a detailed derivation, see e.g. Brock, Hommes and Wagener (2001).

3. Heterogeneous Expectations

In the asset pricing model with heterogeneous investors, the equilibrium price (5) is the discounted weighted average of heterogeneous payoff expectations minus the risk premium. In this section, we will discuss how investors form their conditional expectations.

We will mainly focus on three types[7] of investment strategies within two major classes of investors, namely, fundamentalists and technical traders. Fundamentalists believe that the price of the security should reveal its fundamental value, independent of the price and trading histories. In contrast, technical analysis involves analysing statistics generated by market activity. Technical analysis uses the price and trading histories to seek to identify patterns in price movement and to forecast future market activity. Although technical trading strategies can take many different forms, generally they are classified as trend following or contrarian. The trend following strategy buys into a rising market and sells into a falling one, while the contrarian strategy buys low, sells high, and trades against the trend signal.

We make the following assumptions about heterogeneous investors' conditional forecasts on future dividends and prices:

$$E_t^i[\mathbf{d}_{t+1}] = \bar{\mathbf{d}}\,. \tag{7}$$

$$E_t^i[\mathbf{P}_{t+1}] = (1-b^i)\mathbf{P}^F + b^i\,\mathbf{P}_t^{TA,\,i} + \mathring{\mathbf{a}}_t\,. \tag{8}$$

$\mathring{\mathbf{a}}_t$ is an $S\times 1$ vector of random noise at time *t*. $\mathbf{P}_t^{TA,\,i}$ is the $S\times 1$ vector of investor *i*'s technical forecasts made at time *t* on the next-period prices of the securities.

Equation (7) assumes, for simplicity, a common dividend expectation equal to the unconditional mean of the stochastic IID dividend process. In the price forecasts, the

[7] The attributes to investors heterogeneity can go beyond the conventional paradigm of asymmetric information to include diversity in prior beliefs. Kurz (1997) argues that the centre of individuals' disagreement lies in their diverse prior beliefs instead of information asymmetry; diverse beliefs explain why different interpretations arise given the same information. On the other hand, prior beliefs also influence information selection. Investors with different beliefs are likely to pick up dissimilar sources for their forecasts.

fundamental price vector $\mathbf{P}^F$ is assumed as common knowledge. Equation (8) expresses the price forecast as a weighted sum of the fundamental price and technical forecast, plus some common random noise. The noise $\mathbf{\mathring{a}}_t$ is to capture the effect of all other sources that may influence the price forecasts.

The weight b^i reveals the investor type. $1-b^i$ and b^i are investor *i*'s forecasting weights on the fundamental price and technical forecast respectively. Fundamentalists are considered to be using only fundamental analysis ($b^i = 0$) and technical traders to be using only technical analysis ($b^i = 1$), although the mixture of both analyses is possible.

Assumptions (7) and (8) are consistent with the asset pricing model discussed in the previous section; if all investors are fundamentalists, assumptions (7) and (8) will lead[8] to the equilibrium price being the fundamental price (6).

We propose that the formation of the technical forecast function $\mathbf{P}_t^{TA,i}$ satisfies the following properties:

(i) It is a function of past prices. More precisely, it is a function of a trend indicator, which is a function of past prices.

(ii) In order to be self-consistent, the technical forecast is considered to be either monotonically increasing or decreasing in its trend indicator.

(iii) It is bounded between two real numbers.

(iv) When the trend indicator is neutral, the technical forecast becomes the fundamental price. That is, when past prices provide no information on future price movements, the average predicted asset value by technical traders coincides with the asset's fundamental value.

Properties (ii) and (iii) make any cumulative distribution function[9] (CDF) a good choice without loss of generality. The technical forecast is defined by

$$\mathbf{P}_t^{TA,i} = \mathbf{P}^F \odot \left[CDF(h^i\, \hat{\mathbf{o}}_t^P) \odot CDF(\mathbf{0}_S)^{-1} \right], \tag{9}$$

[8] For a detailed derivation in scalars, see Yang and Satchell (2003).
[9] Alternatively, for a linear technical forecast function, see Sentana and Wadhwani (1992).

where $\odot$ denotes the element-by-element multiplication of vectors, and $\mathbf{0}_S$ is an $S \times 1$ vector of zeros. $\hat{\mathbf{o}}_t^P$ denotes the $S \times 1$ trend indicator vector at time *t*, and is given by $\hat{\mathbf{o}}_t^P = f(\mathbf{P}_{t-1}, \mathbf{P}_{t-2}, ..., \mathbf{P}_{t-M})$, a function of past prices of M lags. h^i is the sensitivity parameter to the trend indicator, and its sign varies with the investor type: for the trend following strategy, the forecast is monotonically increasing in the trend signal, i.e. $h^i > 0$; for the contrarian strategy, the forecast is monotonically decreasing in the trend signal, i.e. $h^i < 0$.

Definition (9) approximates technical forecasts using CDFs. It satisfies all the above properties. However, it makes an over-simplifying assumption of a common, dominant trend indicator function, and differentiates technical traders by asking one key question: whether they trade following or against the trend. Although far from being realistic, this design of technical forecasts makes the model tractable, free from the complication of some variables differing in a way that can grow out of control.

Note that the choices of the CDF and the trend indicator function are arbitrary. In the later simulation experiments, we will proceed with the logistic CDF and also the commonly practiced moving-average trading rule as the trend indicator:

$$\hat{\mathbf{o}}_t^P = \mathbf{P}_{t-1} - \frac{1}{M}\sum_{j=1}^{M} \mathbf{P}_{t-j} \,. \tag{10}$$

4. Imitative Learning Processes

As market equilibrium (5) suggests, the dynamics of the fractional change of investor types q_t^i influences price formation in the asset pricing model with heterogeneous investors. This section models investors' imitative learning that shapes the ratio of investor types.

An investor decides whether to update his investment strategy according to his trading performance. We use the performance criterion based on the net *realised* risky payoff. Investor i's net *realised* risky payoff from time *t* to time $t+1$ is given by

$$\eth_{t+1}^{i} = (\mathbf{Y}_t^i)' (\mathbf{P}_{t+1} + \mathbf{d}_{t+1} - \mathbf{P}_t) . \tag{11}$$

It is common that investors update their strategies based on the performance of risky investment. Riskfree investment pays a fixed anticipated return, and the gain is simply proportional to the amount invested. The criterion is not based on riskfree investment, but, given a fixed amount of capital, the riskfree share does affect the available risky share. A larger risky share, however, does not guarantee more risky payoffs. A good performance in (11) implies a good balance between risky and riskfree investments.

We consider two types of imitation: *cautious learning* (CL) and *winner takes all* (WTA). In the CL type of imitation, investors revise their strategies with caution and do not change abruptly. The WTA type of imitation corresponds to a more drastic imitation process. These two types of imitation are given in the following definitions[10].

CL: If $p_t^i \geq 0$, remain as type i.

If $p_t^i < 0$, switch to type *j* with a probability $1 - e^{-a\,\Delta p_t}$, (12)

where $\Delta p_t = p_t^j - p_t^i$ and $p_t^j = \underset{\forall k}{\mathrm{Max}}\, p_t^k$.

WTA: Always imitate type *j*, where $p_t^j = \underset{\forall k}{\mathrm{Max}}\, p_t^k$. (13)

The definition of CL (12), using a threshold in positive profitability, states that imitation may only occur in response to a net loss in the *realised* risky payoff. Besides, the loss-making investor will imitate the strategy that reaps the maximum trading gain, with a probability depending on their relative performance Δp_t. The probability function $(1 - e^{-a\,\Delta p_t})$ is bounded between 0 and 1 since $\Delta p_t \geq 0$, and is monotonically increasing in Δp_t but with a diminishing increase. Figure 1 depicts the probability function with different values of a; a greater a means a higher probability to imitate when a loss occurs.

[10] Although we consider strategy updating each period based on the performance of one-period payoff, a natural extension would be strategy updating in a longer horizon based on multiple-period payoff.

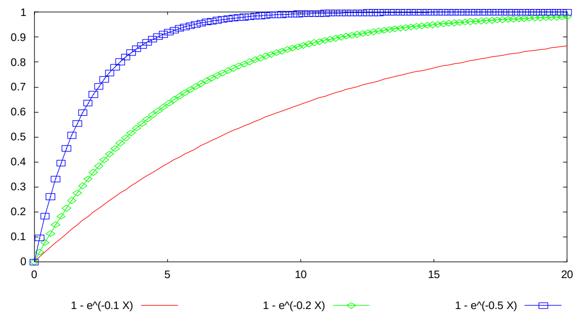


Figure 1. $1-e^{-a\Delta p}$ with different values of a.

Following the WTA rule (13), regardless of the profitability of his original strategy, the investor always makes a move as long as there exists an outperforming strategy. The design of the WTA rule is for comparison purposes and is not in line with reality. "Human behaviour, even rational human behaviour, is not to be accounted for by a handful of invariants. It is certainly not to be accounted for by assuming perfect adaptation to the environment" Simon (1979). The CL rule, with first a threshold and second a probability function, appears to be more plausible[11].

The imitative learning processes determine the dynamics of investor ratios q_t^i in the equilibrium price equation (5). The winning strategy will surely attract more market participants to put it into practice. However, more investors practising the same strategy would probably mean that a deviant may well exploit this situation. Here is when a winning strategy loses its lead. The best strategy in the current period by no means guarantees its future success. The winning investor type changes through time with market conditions and capital reallocation, and so does the investor ratio.

5. Analytical Solutions in A Simplified Model

This section solves analytically whether agents' interaction, including herding, leads to asset dependency. It considers a simplified model for tractability. The results are summarised in the following propositions.

[11] The pace of learning is influenced, though not fully determined, by institutional conditions; we consider that this influence is better reflected by the CL rule rather than the WTA rule.

Proposition 1: Assume $R_f = 1$, $\mathbf{Y}^S = \mathbf{0}_S$, and zero dividends. Consider a simple case of two investor types: fundamentalists with a fraction of $1-q_t$, and technical traders with a fraction of q_t; the $S \times 1$ vector of price forecasts made at time *t* by investor *i* is given by $\mathbf{E}_t^{TA} = \mathbf{P}^F \odot \tilde{\mathbf{A}}_t$ and $\mathbf{E}_t^F = \mathbf{P}^F$, consistent with (7), (8) and (9). Then, the market equilibrium equation (5) can be rewritten as

$$\mathbf{q}_t = q_t \mathbf{A}_t ,$$

where $\mathbf{q}_t = \mathbf{D}_F^{-1}(\mathbf{P}_t - \mathbf{P}^F)$ is the $S \times 1$ vector of *transformed* prices, and $\mathbf{A}_t = (\tilde{\mathbf{A}}_t - \mathbf{1}_S)$ is the $S \times 1$ vector of *transformed* forecast functions. $\mathbf{D}_F = \text{diag}(\mathbf{P}^F)$ is an $S \times S$ diagonal matrix with the diagonal entries from $\mathbf{P}^F$.

Proof: See Appendix 1.

Proposition 1 states that, under certain assumptions, the market equilibrium price equation (5) can be expressed in terms of the transformed prices, which are a simple product of the investor ratio and transformed forecast functions. This transformation will largely facilitate the analysis of the herd effect on asset dependency. The focus now is on the investigation of the relation between the dynamic change in the investor ratio q_t and the dependency of the transformed prices $\mathbf{q}_t$.

Proposition 2:

Following Proposition 1, let $q_t = f_q(\mathbf{q}_{t-1})$, $q : \mathbf{R}^S \to \mathbf{R}^1$; $\mathbf{A}_t = f_A(\mathbf{q}_{t-1})$, $\mathbf{A} : \mathbf{R}^S \to \mathbf{R}^S$. Assuming that the *S transformed* prices in $\mathbf{q}_{t-1}$ are independent and the Jacobian matrix of $\mathbf{A}_t$ is diagonal, we can obtain the following results:

(i) The covariance matrix of $\mathbf{A}_t$, $\text{cov}[\mathbf{A}_t]$, is diagonal. Therefore, when q_t is fixed at q, the covariance matrix of $\mathbf{q}_t$, $\text{cov}[\mathbf{q}_t]$, is also diagonal.

(ii) Let $m = \left.\dfrac{\partial q_t}{\partial \mathbf{q}_{t-1}'}\right|_{q*}$, where $\mathbf{q}^* = E[\mathbf{q}_{t-1}]$. When $m = \mathbf{0}_S$, then $\text{cov}[\mathbf{q}_t]$ is diagonal (note that this part of result is consistent with (i)). When $m \neq \mathbf{0}_S$, then $\text{cov}[\mathbf{q}_t]$ is non-diagonal.

Proof: See Appendix 2.

Here, the definitions of the investor ratio q_t and transformed forecast functions $\mathbf{A}_t$ are consistent with the discussions in the previous sections. q_t changes due to herding. Investors decide whether to imitate according to their realised trading profits, which is a result of *past prices*. Also, the technical forecast is a function of *past prices*. The only restriction here is that Proposition 2 assumes an influence of only one-period lag.

To isolate the ex-ante correlation effects, the S transformed prices in $\mathbf{q}_{t-1}$ are assumed to be uncorrelated. For a similar reason, the Jacobian matrix of $\mathbf{A}_t$ is assumed to be diagonal. The diagonal Jacobian matrix of $\mathbf{A}_t$ implies no prior belief in asset dependency in the forecast function $\mathbf{A}_t$. That is, when forecasting the future movement of one particular security, the investor uses only the information of that security, not the information of other securities.

Essentially, m measures the change in the investor ratio due to a change in the realised profits, evaluated at the average profit level. It in fact reflects herding. Imitation motivated by the comparison of realised trading profits leads to non-zero m, while a static model of no interaction among investors has a fixed investor ratio and hence $m = \mathbf{0}_S$. Proposition 2 states that, given the assumptions discussed above, a fixed investor ratio (in the absence of herding) will guarantee no asset dependency. On the other hand, a changing investor ratio, driven by profit motives, will lead to asset dependency. Overall, this section provides the analytically results that, under some general assumptions, herding that shapes the dynamics of the investor ratio is the major driving force to cross-asset comovements.

6. Asset Correlations

The focus of this chapter is to identify the association between imitative learning and asset comovement. The approach for numerical simulations is to compare the cross-sectional correlation of security returns arising from three models[12]: the model with non-adaptive (NA) investors, CL and WTA. The vector of security returns is defined by

$$\mathbf{R}_{t+1} = \mathbf{P}_t^{-1} \odot \left(\mathbf{P}_{t+1} + \mathbf{d}_{t+1}\right). \tag{14}$$

Security returns are calculated from the IID dividend process and the equilibrium price. It must be clarified that the equilibrium price is obtained from equation (5), using investors' forecasts $E_t^i[\mathbf{P}_{t+1} + \mathbf{d}_{t+1}]$ given in Section 3 and investor fractions q_t^i determined by the imitation processes given in Section 4. Imitation determines q_t^i, and hence influences the price formation and consequently the return correlation.

Let r_{NA}, r_{CL}, and r_{WTA} denote the respective correlation of security returns under the models of NA, CL and WTA. We extend to take into account market conditions. Let r^L, r^M, and r^U denote the conditional correlations of security returns; they are the correlations conditional on the downside, normal, and upside markets respectively. The market is said to be in a downside (upside) condition when the market index price is below (above) the d quantile ((1-d) quantile) of its distribution; the market condition is normal otherwise. The conditional correlations can be written as

$$\begin{aligned} r^L &= r \mid F(P_{index}) < d , \\ r^M &= r \mid d \le F(P_{index}) \le 1\text{-}d , \\ r^U &= r \mid F(P_{index}) > 1 - d , \end{aligned} \tag{15}$$

where $F(P_{index})$ is the CDF of the market index price, and $0 < d < 1$.

To investigate the association between imitative learning and asset comovement, we test the following inequalities on both unconditional and conditional correlations:

$$r_{WTA} > r_{CL} > r_{NA} . \tag{16}$$

[12] For tractability, we only consider the results of these three models. Of course, an advanced approach could be a combination, or even a regime switching, of various learning processes.

$$r_{WTA}^{U} > r_{CL}^{U} > r_{NA}^{U}.$$

$$r_{WTA}^{M} > r_{CL}^{M} > r_{NA}^{M}. \qquad (17)$$

$$r_{WTA}^{L} > r_{CL}^{L} > r_{NA}^{L}.$$

(16) implies that asset dependency can be endogenously induced by herding, and besides, hasty imitation is likely to have a significant effect. This implication can be of great relevance in understanding the making of extreme market events.

Essentially, (17) further tests the inequality (16) under each market condition. Conditional correlation bridges the possible effect of herding on asset comovement with changing market conditions. Will the herd effect on asset dependency, if any, remain unchanged across different market states? Are market conditions irrelevant when hasty imitation is present? We hope to unravel these issues using the conditional correlation results.

The cross comparison of (17) is further conducted:

$$r_{NA}^{U} > r_{NA}^{M},\ r_{NA}^{L} > r_{NA}^{M} \qquad (18\text{-}1)$$

$$r_{CL}^{U} > r_{CL}^{M},\ r_{CL}^{L} > r_{CL}^{M} \qquad (18\text{-}2)$$

$$r_{WTA}^{U} > r_{WTA}^{M},\ r_{WTA}^{L} > r_{WTA}^{M} \qquad (18\text{-}3)$$

The issue of whether cross-asset correlations tend to increase in volatile market conditions has provoked great research interest, and this pattern of non-constant correlations has been reported in many empirical studies (e.g. Silvapulle and Granger 2001). The test (18-1) is to confirm this pattern in the simple benchmark NA model. Inequalities (18-2) and (18-3) are to test how investors' learning processes interact with the market condition. The implication of (18-2) and (18-3) is that, given the same learning model, its effect on asset comovements will depend on the market condition, in a way consistent with the empirical pattern of non-constant correlations[13]. The

[13] It must be clarified that, even if (18-2) and (18-3) hold, it by no means implies that the pattern of non-constant correlations is caused by imitation; one can only conclude that the effect of imitation on asset dependency is consistent with the pattern of non-constant correlations.

implication may shed the light on how investment behaviour is associated with market conditions.

7. Simulation Results

Based on the development in the previous sections, this section carries out the numerical simulations of the unconditional and conditional cross-sectional correlation of security returns under the models of NA, CL, and WTA, and discusses the results.

There are 1250 time periods (approximately 5 years of trading days); 50 securities and hence 1225 pairs of cross correlations of security returns. The parameter values used for numerical simulations are given in Appendix 3. The summary statistics of the unconditional and conditional cross correlation results[14] are given in Table 1. Figure 2 presents the distributions of the correlation results.

From the results of the unconditional correlation coefficients, it is clear that the NA model has the lowest correlation level, with an average of 0.028 and more than 99% of r_{NA} lower than 0.1. The correlation level increases dramatically in the presence of herding, but the difference between r_{CL} and r_{WTA} is not as striking; the respective average values of r_{CL} and r_{WTA} are 0.251 and 0.319. The results also suggest that a higher level of correlation comes with a higher level of standard deviation. The unconditional correlation outcomes lead us to confirm (16).

The conditional correlations show consistent outcomes with the unconditional correlations and confirm the inequalities given by (17). The results show, across market states, persistent evidence of higher correlation levels in the presence of herding. Furthermore, the difference is more pronounced in the upside and downside markets than in the normal. Taken together, these observations have a crucial implication. They imply a situation where abnormal market conditions, such as a bear

[14] Only the results of positive correlations that correspond to asset comovement in the same direction will be reported. The study of negative correlation has a different focus. Positive and negative correlations contain separate information; taking average of them may omit useful information.

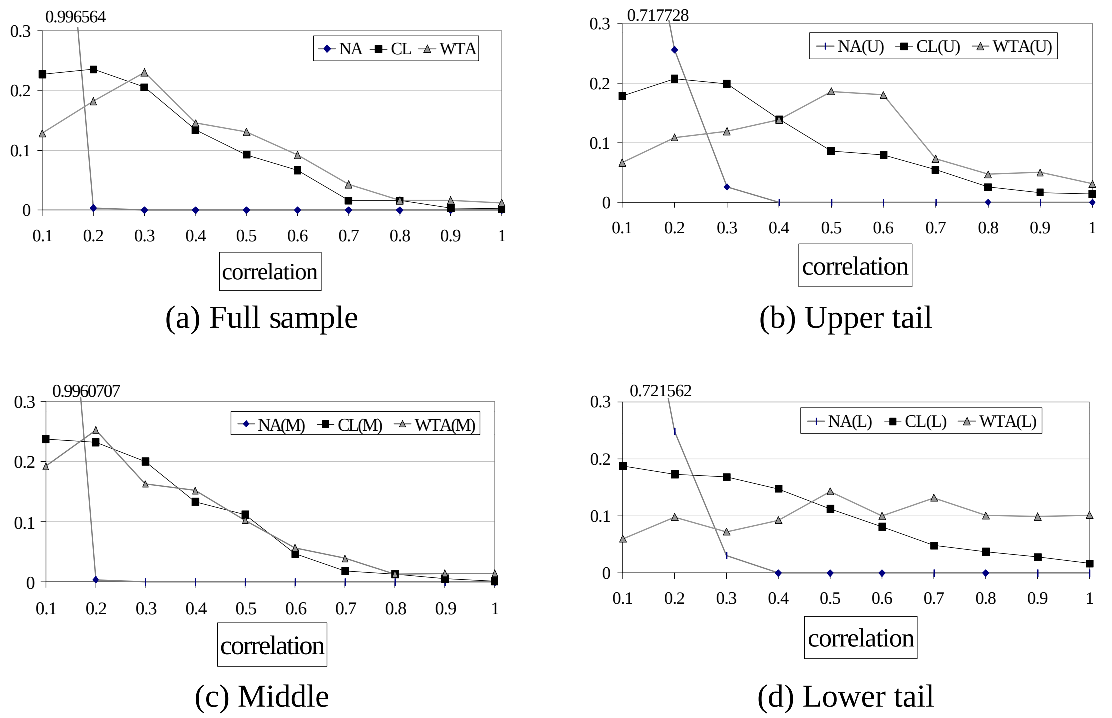
market, can amplify the herd effect, which in turn exacerbates asset dependency. The herd effect on asset dependency thus engages in a self-reinforcing process that can eventually lead to some disastrous phenomena such as crashes.

Finally, the conditional correlation results[15] show that all the inequalities given by (18) hold, suggesting that given the same learning model, its effect on asset comovement will depend on market conditions, in a way consistent with the empirical pattern of non-constant correlations.

Table 1. The summary statistics of the unconditional and conditional cross correlations of security returns under the models of NA, CL, and WTA.

Full Sample	Upper Tail	Middle	Lower Tail
Maximum Minimum Average (Standard Deviation)	Maximum Minimum Average (Standard Deviation)	Maximum Minimum Average (Standard Deviation)	Maximum Minimum Average (Standard Deviation)
r_{NA}	r_{NA}^{U}	r_{NA}^{M}	r_{NA}^{L}
0.1188459 2.70392e-05 0.0282156 (0.0216731)	0.285115 9.74418e-05 0.0733302 (0.0578932)	0.112482 9.18909e-05 0.0273755 (0.0209469)	0.292311 8.99925e-05 0.0727321 (0.0529507)
r_{CL}	r_{CL}^{U}	r_{CL}^{M}	r_{CL}^{L}
0.998841 0.00102167 0.250795 (0.178549)	0.998021 0.00189254 0.300233 (0.213958)	0.99885 5.72659e-05 0.244823 (0.176036)	0.997393 0.000291617 0.323284 (0.2297)
r_{WTA}	r_{WTA}^{U}	r_{WTA}^{M}	r_{WTA}^{L}
0.995347 0.000428903 0.31851 (0.203102)	0.998748 0.00161222 0.437796 (0.225886)	0.99495 0.00107986 0.279749 (0.205693)	0.999256 0.000335208 0.529907 (0.273672)

[15] There is also a minor observation that the level of the correlations conditional on a normal market condition is, among all the conditional ones, most close to the level of the unconditional correlations. This is not surprising, as the condition of r^{M}, on the market index price being between 10% to 90% quantiles, captures the majority of the sample distribution.



(a) Full sample

(b) Upper tail

(c) Middle

(d) Lower tail

Figure 2. Distributions[16] of unconditional and conditional cross correlations of security returns under the models of NA, CL, and WTA.

8. Empirical Study

The empirical investigation on the connection between herding and asset dependency is impeded by the difficulty in measuring the size of herding in the market, and the complexity of many interacting factors that may or may not include herding. This is a typical problem that faces many researchers in identifying the true causes of a certain phenomenon; the real world is of such complexity that mostly we are not able to study the effect that we wish to study in isolation of others.

The empirical study in this section by no means intends to identify the herd effect as a cause of asset dependency in the real world. Instead, using data from two diverse markets, namely, the UK and Taiwan equity markets, it intends to show the real patterns of cross-sectional correlation in security returns and to compare them with the simulation results given in the previous section.

[16] Note that the x axis starts from 0.1 because the labels on the x axis indicate the ending numbers of the category; for example, 0.1 indicate the category (0, 0.1).

Data are collected from the daily closing price series[17] of the top 50 FTSE 100 stocks and the top 50 Taiwan Weighted Stock Index (TWSI) from 1 November 1997 to 31 October 2002, with 1258 trading days for the FTSE and 1308 trading days for the TWSI. Note that the number of the sample trading periods is close to that in simulation. Security returns are simply computed as the one-period lag price ratio[18]. Again, there are 1225 pairs of cross correlations in security returns in each data set. Table 2 reports the unconditional and conditional correlation results, set out in the same style as Table 1. Figure 3 presents the correlation distributions.

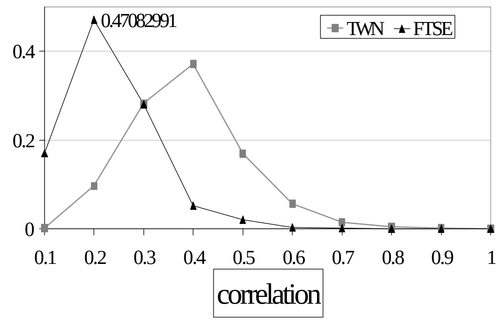
For the period concerned, the top 50 TWSI is found to have persistently higher levels of correlation than the top 50 FTSE 100; their average unconditional correlations, for instance, are 0.338 and 0.181 respectively. The correlation standard deviation of the top Taiwan equity market is nevertheless not much higher than that of the UK. Furthermore, when comparing across market conditions, both these two markets (one developed and one emerging) exhibit a common pattern of a markedly higher correlation level in the downside market condition[19].

Although it is difficult to draw any direct implication on the link between the herd effect and asset dependency using these empirical results, there are observations of some interesting patterns when comparing the simulation results with the empirical ones. First, both exhibit the patterns of non-constant correlations that are also found in the many empirical studies. Second, models with herding generate results that are closer to the real patterns of asset dependency than the static model of isolated agents.

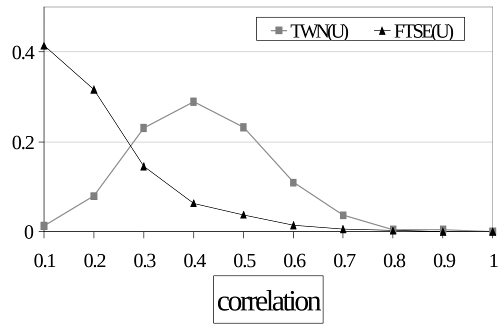
[17] The market indices of the FTSE 100 and TWSI over a slightly longer period are given in Appendix 4 for reference.
[18] Note that, here, the computation of security returns is consistent with (14) by assuming zero dividends.
[19] However, unlike the simulation results, here, we do not observe a significantly higher correlation level in the upside market.

Table 2. The summary statistics of the unconditional and conditional[20] cross correlations of security returns of the top 50 stocks of the FTSE 100 and TWSI, from 1 November 1997 to 31 October 2002.

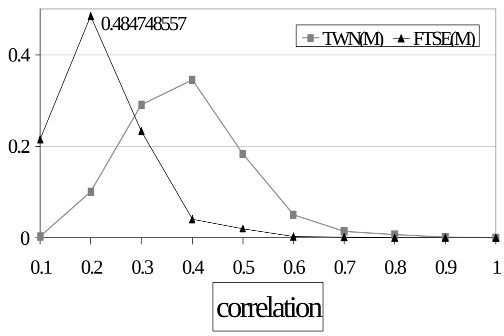
Full Sample	Upper Tail	Middle	Lower Tail
Maximum Minimum Average (Standard Deviation)	Maximum Minimum Average (Standard Deviation)	Maximum Minimum Average (Standard Deviation)	Maximum Minimum Average (Standard Deviation)
$r_{FTSE\,50}$	$r^{U}_{FTSE\,50}$	$r^{M}_{FTSE\,50}$	$r^{L}_{FTSE\,50}$
0.736083 5.87055e-3 0.180575 (0.0891813)	0.722024 4.38147e-4 0.154868 (0.1279)	0.730307 0.000404838 0.168706 (0.0889582)	0.792695 0.0438082 0.382183 (0.119405)
$r_{TaiwanTop\,50}$	$r^{U}_{TaiwanTop\,50}$	$r^{M}_{TaiwanTop\,50}$	$r^{L}_{TaiwanTop\,50}$
1 0.0944918 0.337906 (0.113236)	1 0.0470461 0.365962 (0.133239)	1 0.0777478 0.33583 (0.115286)	1 0.063036 0.43357 (0.136704)



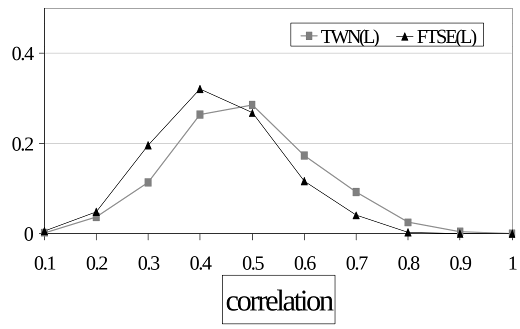
(a) Full sample



(b) Upper tail



(c) Middle



(d) Lower tail

Figure 3. Distributions of unconditional and conditional cross correlations of security returns of the top 50 stocks of the FTSE 100 and TWSI, from 1 November 1997 to 31 October 2002.

[20] The market index price for the computation of market conditions is computed as the average price of the 50 equities. Alternatively, one can use directly the published market indices that take into account the relative equity weights.

9. Concluding Remarks

This chapter explores the herd effect on asset dependency. The degrees of the cross correlations in security returns are investigated via the dynamic impact of rational imitative learning among agents with heterogeneous price forecasts. The static model with isolated non-interacting agents is set as the benchmark model for comparison. In a simple general setting, this chapter proves analytically that imitation, driven by profit motives, leads to asset dependency.

Supporting the analytical solutions, the simulation experiments show that, unlike the benchmark model that generates virtually zero cross correlations in security returns, herding endogenously induces a significant level of asset comovements. Furthermore, herding is most pronounced in abnormal market conditions. Overall, these imply a self-reinforcing process where abnormal market conditions amplify herding, which further exacerbates asset dependency. The implication can be of relevance in understanding the making of extreme market events.

The finding that the herd effect on asset dependency is most pronounced in abnormal market conditions is consistent with the finding of Sancetta and Satchell (2003). In the Sharpe's factor model, they explain that the pattern of non-constant correlations is due to one factor becoming increasing important in abnormal market conditions.

This chapter also studies the respective levels of asset comovements of the top UK and Taiwan equity markets. When comparing the empirical results with the simulation outcomes, two observations emerge. First, both exhibit the pattern of non-constant correlations found in many previous empirical studies. Second, models with herding generate results that are closer to the real patterns of asset dependency than the static benchmark model with isolated agents. Nevertheless, whether the observations have implications on the size of herding in these two markets, or the effects of other characteristics such as developed vs. emerging markets, or globalisation vs.

localness[21], requires a more detailed investigation into the markets, and could be an interesting future topic.

The financial markets are not an imperfectly insulated sphere of economic rationality, but a sphere in which the "economic" and the "social" interweave seamlessly… the key "social risks" seem to come from inside the financial markets rather than from outside, MacKenzie (2003).

Following the terminology, herding, even if rational, can be viewed as a key internal "social risk". It enters the "sphere" by reshaping the investor ecology, most of the time *uneconomically*. In the present model, imitative learning results in a temporary, fragile convergence on the outperforming investment strategy. The stability depends on the learning pace, which is in reality influenced by institutional conditions. The nature of instability and fragility is partly what brings about the internal risk.

On the other hand, convergence, even a temporary one, implies a certain degree of homogeneity, which has a counterintuitive implication on stability. The idea can be grasped by that a small error can snowball due to the lack of offsetting effects that could otherwise arise with heterogeneity. When information is incomplete and agents are boundedly rational, homogeneity can not be superior to heterogeneity in stabilising the market.

The study of this chapter is of some relevance for financial regulations. The literature tends to support the view that transparency is socially desirable in light of fairness, efficiency and the adverse selection problem (e.g. Hasbrouck 1988, 1991; Gemmill 1996; Madhavan 1996; Pagano and Roell 1996). However, regulatory changes that make investment behaviour more transparent and make individual investors more aware of other investors' actions can enhance uniformity of action and bring about the opposite of the intended purpose. A good balance needs to be obtained by taking into account both the desirable features of transparency and the potential risk in information exposure and the public's spurious response.

[21] Compared with many international corporations in the US or the UK markets, most Taiwanese companies are "local" in the sense that they are small-and-medium enterprises (SMEs).

Appendix 1

Following the market equilibrium equation (5), we assume $R_f = 1$, $\mathbf{Y}^S = \mathbf{0}_S$, and zero dividends, and consider a simple case of two investor types: fundamentalists with a fraction of $1-q_t$, and technical traders with a fraction of q_t. (5) can be rewritten as

$$\mathbf{P}_t = q_t \mathbf{E}_t^{TA} + (1-q_t)\mathbf{E}_t^F, \tag{A1}$$

where $\mathbf{E}_t^i$ denotes the $S \times 1$ vector of price forecasts made at time *t* by investor *i*, and it is defined in a way consistent with (7), (8), and (9):

$$\mathbf{E}_t^{TA} = \mathbf{P}^F \odot \tilde{\mathbf{A}}_t, \tag{A2}$$

$$\mathbf{E}_t^F = \mathbf{P}^F.$$

The equilibrium price (A1) now becomes

$$\mathbf{P}_t = q_t \mathbf{P}^F \odot \tilde{\mathbf{A}}_t + (1-q_t)\mathbf{P}^F. \tag{A3}$$

Define $\mathbf{D}_F = \text{diag}(\mathbf{P}^F)$. It is an $S \times S$ diagonal matrix whose diagonal entries are the entries of $\mathbf{P}^F$ so that $\mathbf{D}_F \mathbf{1}_S = \mathbf{P}^F$. (A3) can be rewritten as

$$\mathbf{P}_t = q_t \mathbf{D}_F \tilde{\mathbf{A}}_t + (1-q_t)\mathbf{D}_F \mathbf{1}_S. \tag{A4}$$

Multiplying both sides by $\mathbf{D}_F^{-1}$ (by definition, we know that $\mathbf{D}_F^{-1}$ exists), (A4) can be rearranged as $\mathbf{D}_F^{-1}(\mathbf{P}_t - \mathbf{P}^F) = q_t(\tilde{\mathbf{A}}_t - \mathbf{1}_S)$. (A5)

Let $\mathbf{q}_t = \mathbf{D}_F^{-1}(\mathbf{P}_t - \mathbf{P}^F)$ and $\mathbf{A}_t = (\tilde{\mathbf{A}}_t - \mathbf{1}_S)$. (A5) is then given by $\mathbf{q}_t = q_t \mathbf{A}_t$.

Appendix 2

Let $q_t = f_q(\mathbf{q}_{t-1})$, $q:\mathbf{R}^S \to \mathbf{R}^1$, and $\mathbf{A}_t = f_A(\mathbf{q}_{t-1})$, $\mathbf{A}:\mathbf{R}^S \to \mathbf{R}^S$.

To isolate the ex-ante correlation effects, we assume that the S *transformed* prices in $\mathbf{q}_{t-1}$ are independent, so that $\mathrm{cov}[\mathbf{q}_{t-1}]$ is a diagonal matrix. For convenience, write $E[\mathbf{q}_{t-1}] = \mathbf{q}^*$, $\mathrm{cov}[\mathbf{q}_{t-1}] = \Omega_q$.

Define $\mathbf{D}_S = [s_{ij}]$, where $s_{ij} = skewness(q^i_{t-1})$ for $i = j$, and $s_{ij} = 0$ for $i \neq j$. $\mathbf{D}_S$ is an $S \times S$ diagonal matrix. Similarly, $\mathbf{D}_K$ is an $S \times S$ diagonal matrix defined by $\mathbf{D}_K = [k_{ij}]$, where $k_{ij} = kurtosis(q^i_{t-1})$ for $i = j$, and $k_{ij} = 0$ for $i \neq j$.

We approximate q_t and $\mathbf{A}_t$ using the first-order Taylor's expansion.

$$q_t \cong q_t\big|_{q^*} + \left.\frac{\partial q_t}{\partial \mathbf{q}'_{t-1}}\right|_{q^*} \cdot (\mathbf{q}_{t-1} - \mathbf{q}^*) \tag{A6}$$

$$\mathbf{A}_t \cong \mathbf{A}_t\big|_{q^*} + \left.\frac{\partial \mathbf{A}_t}{\partial \mathbf{q}_{t-1}}\right|_{q^*} \cdot (\mathbf{q}_{t-1} - \mathbf{q}^*) \tag{A7}$$

Thus, $E[q_t] = q_t\big|_{q^*} = q^*$, and $E[\mathbf{A}_t] = \mathbf{A}_t\big|_{q^*} = \mathbf{A}^*$.

Let $m = \left.\frac{\partial q_t}{\partial \mathbf{q}'_{t-1}}\right|_{q^*}$ and $\mathbf{D}_m = \mathrm{diag}(m)$; $\mathbf{J} = \left.\frac{\partial \mathbf{A}_t}{\partial \mathbf{q}_{t-1}}\right|_{q^*}$.

The Jacobian matrix $\mathbf{J}$ is assumed to be diagonal, i.e. no prior belief in asset dependency in the forecast function $\mathbf{A}_t$.

The covariance of $\mathbf{A}_t$ is calculated by

$$\mathrm{cov}[\mathbf{A}_t] = E[(\mathbf{A}_t - \mathbf{A}^*)(\mathbf{A}_t - \mathbf{A}^*)'] = E[\mathbf{J}(\mathbf{q}_{t-1} - \mathbf{q}^*)(\mathbf{q}_{t-1} - \mathbf{q}^*)'\mathbf{J}'] = \mathbf{J}\Omega_q\mathbf{J}'. \tag{A8}$$

Since $\mathbf{J}$ and Ω_q are diagonal matrices, it is easy to show that $\text{cov}[\mathbf{A}_t]$ is also a diagonal matrix. Therefore, when q_t is fixed at q, the covariance matrix of $\mathbf{q}_t$, $\text{cov}[\mathbf{q}_t]$, is also a diagonal matrix.

Now we proceed to prove the result (ii) in Proposition 2.

The expectation of $\mathbf{q}_t$ is given by $E[\mathbf{q}_t] = q^* \mathbf{A}^* + \mathbf{J}\,\Omega_q m$. (A9)

The covariance of $\mathbf{q}_t$ is given by

$$\text{cov}[\mathbf{q}_t] = E\left[(\mathbf{q}_t - E[\mathbf{q}_t])(\mathbf{q}_t - E[\mathbf{q}_t])\right] = E[q_t^2 \mathbf{A}_t \mathbf{A}_t'] - E[\mathbf{q}_t]E[\mathbf{q}_t]'. \quad \text{(A10)}$$

We now calculate the first term on the right hand side (RHS) of (A10).

$$\begin{aligned} E\left[q_t^2 \mathbf{A}_t \mathbf{A}_t'\right] = {} & E\left[q_t^2\right]\mathbf{A}^*(\mathbf{A}^*)' \\ & + \mathbf{A}^* E\left[q_t^2(\mathbf{q}_{t-1} - \mathbf{q}^*)'\right]\mathbf{J}' + \mathbf{J}E\left[q_t^2(\mathbf{q}_{t-1} - \mathbf{q}^*)\right](\mathbf{A}^*)' \\ & + \mathbf{J}\,E\left[q_t^2(\mathbf{q}_{t-1} - \mathbf{q}^*)(\mathbf{q}_{t-1} - \mathbf{q}^*)'\right]\mathbf{J}' \end{aligned} \quad \text{(A11)}$$

The constituent terms in (A11) are calculated below.

$$\begin{aligned} E[q_t^2] &= E\left[(q^*)^2 + 2q^* m \cdot (\mathbf{q}_{t-1} - \mathbf{q}^*) + \left(m \cdot (\mathbf{q}_{t-1} - \mathbf{q}^*)\right)^2\right] \\ &= (q^*)^2 + E\left[\left(m \cdot (\mathbf{q}_{t-1} - \mathbf{q}^*)\right)^2\right] \\ &= (q^*)^2 + m'\Omega_q m \end{aligned} \quad \text{(A12)}$$

$$\begin{aligned} E\left[q_t^2(\mathbf{q}_{t-1} - \mathbf{q}^*)'\right] = {} & (q^*)^2 E\left[(\mathbf{q}_{t-1} - \mathbf{q}^*)'\right] \\ & + 2q^* E\left[\left(m \cdot (\mathbf{q}_{t-1} - \mathbf{q}^*)\right)(\mathbf{q}_{t-1} - \mathbf{q}^*)'\right] \\ & + E\left[\left(m \cdot (\mathbf{q}_{t-1} - \mathbf{q}^*)\right)^2(\mathbf{q}_{t-1} - \mathbf{q}^*)'\right] \end{aligned}$$

$$E\left[q_t^2(\mathbf{q}_{t-1} - \mathbf{q}^*)'\right] = 2q^* m'\Omega_q' + m'\mathbf{D}_m\mathbf{D}_s \quad \text{(A13)}$$

$$\begin{aligned} E\left[q_t^2(\mathbf{q}_{t-1} - \mathbf{q}^*)(\mathbf{q}_{t-1} - \mathbf{q}^*)'\right] = {} & (q^*)^2 E\left[(\mathbf{q}_{t-1} - \mathbf{q}^*)(\mathbf{q}_{t-1} - \mathbf{q}^*)'\right] \\ & + 2q^* E\left[\left(m \cdot (\mathbf{q}_{t-1} - \mathbf{q}^*)\right)(\mathbf{q}_{t-1} - \mathbf{q}^*)(\mathbf{q}_{t-1} - \mathbf{q}^*)'\right] \\ & + E\left[\left(m \cdot (\mathbf{q}_{t-1} - \mathbf{q}^*)\right)^2(\mathbf{q}_{t-1} - \mathbf{q}^*)(\mathbf{q}_{t-1} - \mathbf{q}^*)'\right] \end{aligned}$$

$$E\left[q_t^2(\mathbf{q}_{t-1}-\mathbf{q}^*)(\mathbf{q}_{t-1}-\mathbf{q}^*)'\right]=(q^*)^2\Omega_q+2q^*\mathbf{D}_m\mathbf{D}_s+\mathbf{D}_m^2\mathbf{D}_k \tag{A14}$$

Thus, (A11) is given by the following.

$$\begin{aligned}E\left[q_t^2\mathbf{A}_t\mathbf{A}_t'\right]=&\left((q^*)^2+m'\Omega_q m\right)\mathbf{A}^*(\mathbf{A}^*)'\\&+\mathbf{A}^*\left(2q^*m'\Omega_q'+m'\mathbf{D}_m\mathbf{D}_s\right)\mathbf{J}'+\mathbf{J}\left(2q^*\Omega_q m+\mathbf{D}_s\mathbf{D}_m m\right)(\mathbf{A}^*)'\\&+\mathbf{J}\left((q^*)^2\Omega_q+2q^*\mathbf{D}_m\mathbf{D}_s+\mathbf{D}_m^2\mathbf{D}_k\right)\mathbf{J}'\end{aligned} \tag{A15}$$

Substituting (A15) into (A10) and from $E[\mathbf{q}_t]$ given by (A9), we finally obtain $\text{cov}[\mathbf{q}_t]$.

$$\begin{aligned}\text{cov}[\mathbf{q}_t]=&m'\Omega_q m\mathbf{A}^*(\mathbf{A}^*)'-q^*A^*m'\Omega_q'\mathbf{J}'-q^*\mathbf{J}\,\Omega_q m(A^*)'-\mathbf{J}\,\Omega_q mm'\Omega_q'\mathbf{J}'\\&+2q^*\mathbf{A}^*\left(m'\Omega_q'\right)\mathbf{J}'+2q^*\mathbf{J}\Omega_q m(\mathbf{A}^*)'+\mathbf{J}\mathbf{D}_s\mathbf{D}_m m(\mathbf{A}^*)'+\mathbf{A}^*m'\mathbf{D}_m\mathbf{D}_s\mathbf{J}'\\&+(q^*)^2\mathbf{J}\Omega_q\mathbf{J}'+2q^*\mathbf{J}\mathbf{D}_m\mathbf{D}_s\mathbf{J}'+\mathbf{J}\mathbf{D}_m^2\mathbf{D}_k\mathbf{J}'\end{aligned} \tag{A16}$$

First, it is straightforward to show that the sum of the terms on the 3rd line on the RHS of (A16) is diagonal. Second, when $m=\mathbf{0}_S$, all the terms on both the 1st and 2nd lines on the RHS of (A16) vanish. Therefore, when $m=\mathbf{0}_S$, $\text{cov}[\mathbf{q}_t]$ is a diagonal matrix given by $\text{cov}[\mathbf{q}_t]=(q^*)^2\mathbf{J}\Omega_q\mathbf{J}'+2q^*\mathbf{J}\mathbf{D}_m\mathbf{D}_s\mathbf{J}'+\mathbf{J}\mathbf{D}_m^2\mathbf{D}_k\mathbf{J}'$.

When $m\neq\mathbf{0}_S$, the fourth term on the RHS of (A16) is crucial. Although symmetric, it is a non-diagonal matrix. This guarantees that when $m\neq\mathbf{0}_S$, $\text{cov}[\mathbf{q}_t]$ becomes a non-diagonal matrix.

Appendix 3

Table A1. Parameter values used for numerical simulations.

Parameter values	
$d = 0.1$	$a = 0.05$
$r_f = 0.01$	$q_{NA}^{i} = 1/3$
$a = 1$	$\mathbf{Y}^S = \mathbf{0}_S$
$M = 20$	$\mathbf{å}_t \sim N(\mathbf{0}_S, \mathbf{É}_S)$
$h = \pm 1$	$\mathbf{d}_t \sim \mathrm{U}(\mathbf{0}_S, \mathbf{1}_S)$

$\mathbf{É}_S$ is the $S \times S$ identity matrix and $\mathbf{1}_S$ is the $S \times 1$ vector of 1s. $q_{NA}^{i} = 1/3$ denotes the equal fractions of investor types in the NA model. Also, $\mathbf{d}_t \sim \mathrm{U}(\mathbf{0}_S, \mathbf{1}_S)$ means that the dividend process of each security is uniformly distributed between 0 and 1.

Appendix 4

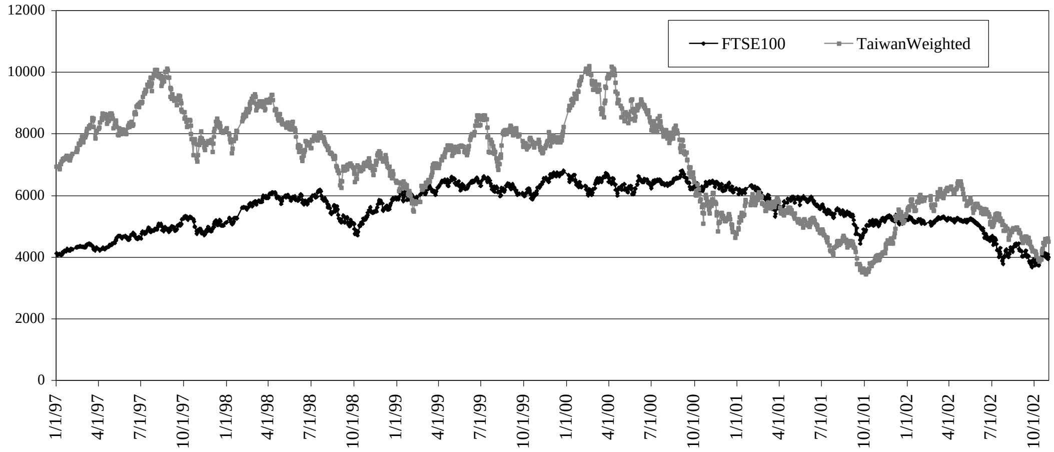


Figure A1. The market indices of the FTSE 100 and TWSI from 1 January 1997 to 31 October 2002.

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