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Integrating Transmission and Energy Markets Mitigates Market Power

Karsten Neuhoff

The
Cambridge-MIT
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Massachusetts Institute of Technology
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Karsten Neuhoff*

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Abstract

Transmission constraints fragment electricity markets and enhance regional market power of electricity generators. In continental Europe rights to access transmission between countries are auctioned to traders, which arbitrage *separate* energy spot markets of these countries. In Scandinavia the system operator *integrates* these markets and simultaneously clears energy spot markets of several countries and decides on optimal energy transmission. In any unconstrained or partially constrained network integration mitigates market power of strategic generators and avoids inefficient production decisions. A testable prediction for both effects is applied to the Dutch-German and Norway-Sweden interconnection and supports the theory. In meshed networks integration also mitigates market power when constraints are permanently binding. Le Chatelier's principle extends to electricity networks in the presence of market power. Demand is more responsive to price changes and aggregate output increases if markets are integrated.

1 Introduction

Strategic trade theory suggests that quotas are usually dominated by tariffs because the latter can be made to replicate the effect of quotas, while also providing both a means of dealing with uncertainty (Newbery and Stiglitz, 1981) and an incentive for competition (Bhagwati, 1965).[1] Yet

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[1] Weitzman (1974) assess the efficiency of tarrifs vs. quantity constraints on output to implement policy goals e.g on emissions. Uncertainty creates a second order effect and the ranking of both options depends on the curvature of cost and benefit functions.

the use of quotas remains widespread, chiefly because they help push forward political agendas, such as protecting local industries or determining $C0_2$ emission targets.

This paper explores the case of electricity markets, where transmission constraints in networks act effectively as quotas, and assesses whether the design of the allocation of quotas affects efficiency and competitiveness. Either transmission rights to use constrained links are auctioned and traders can then arbitrage *separate* energy spot markets, or transmission and energy markets are *integrated* through a system operator, who clears several energy spot markets using the available transmission capacity between these markets. The designs are compared for unconstrained, partially constrained and constrained links. First, in the case of an unconstrained link, data from the German-Dutch interconnector confirms that separation results in inefficient flow patterns and a proof is provided that strategic generators reduce output. Second, in the case of a partially constrained link, a model illustrates that strategic generators continue to reduce output. A testable prediction of the model is applied to the Dutch-German and Norway-Sweden interconnections and supports the theory. Third, in the case of a permanently constrained link, separation has no effect in a simple network with only one transmission link. This is not the case in networks with more than one link (meshed networks) with permanently binding constraint(s). Separation of transmission and energy markets creates an additional artificial constraint. According to Le Chatelier's principle additional constraints applied to a system reduce the system's compensating reaction to changes; in this case separation reduces responsiveness of net demand to output decisions of strategic generators. In this case, strategic generators located at one or two nodes of any meshed network reduce aggregate output.

The paper assumes the typical model of electricity liberalisation. Vertically integrated electricity companies are unbundelled into generation, transmission, distribution and supply companies. Transmission and distribution networks remain regulated monopolies. Generation companies compete in the wholesale electricity market to sell to larger customers and supply companies. This is intended to increase efficiency, reduce costs and reward innovation. However, limited transmission capacity can fragment the wholesale electricity market into smaller regional markets, where generation companies have large market shares and face low short-term demand elasticities.

Figure 1, using the example of the European countries, shows that cross border constraints provide an excellent environment for the exercise of market power.[2] During times of peak demand, except for Switzerland, the largest generator's production is a multiple of spare capacity even if all other generators supply at full capacity. He can therefore create scarcity and extreme

[2] Source: UCTE Power Balance Forecasts 2002-2004, ETSO's NTC publications 2001/2002, ICF Consulting, annual reports and presentations.

price spikes by withholding a fraction of his output, while more withholding is required to create scarcity at off peak times. Imports or the threat of imports reduce the dominance of the largest generator, as Figure 1 illustrates. Effective allocation of scarce transmission capacity is therefore crucial to mitigate market power in European countries.

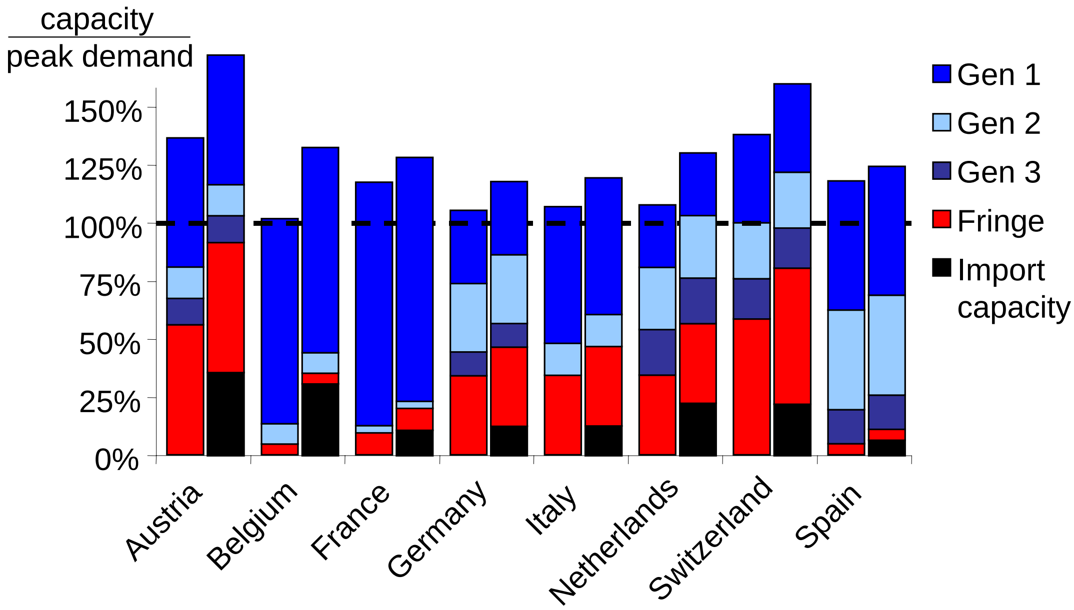

Figure 1: Capacity of generator as a multiple of peak demand during winter peak. In most countries demand is not satisfied if the largest generator withholds a fraction of output during peak demand (left columns), even if available import capacity is utilised (right columns).[1]

Integrated transmission and energy markets have evolved from a pricing mechanism introduced by Bohn, Caramanis and Schweppe (1984) to deal with transmission constraints between different nodes of a network. Generation companies, large electricity customers, and supply companies submit bids and offers to a system operator, which specify the price, location and quantity they want to buy or sell at. The system operator determines a separate price for each node at which accepted bids pay and offers must be paid for - a system referred to as nodal pricing. If all bids are competitive, nodal pricing implements the welfare maximising dispatch, subject to the transmission constraints of the system. Zonal pricing and market splitting simplify nodal pricing by aggregating several nodes into one zone at the cost of reduced efficiency and increased possibilities for the exercise of market power (Harvey and Hogan, 2000). This paper assumes nodal pricing and does not address inefficiencies due to zonal aggregation, but provides insights into zonal pricing as long as intrazonal constraints are limited. Hogan (1992) supplemented nodal pricing with tradable congestion contracts (TCC), auctioned by the transmission operator to allow hedging and provide long-term information to guide investment decisions.[3]

An alternative approach, which I will refer to as *separate* transmission and energy markets, is frequently supported because it seems not to require centralised institutions. Property rights are defined for scarce transmission capacity, which can be traded to match energy flows. Chao and Peck (1996) proved that the concept achieves a social optimum and therefore it coincides with nodal pricing complemented by TCCs in the presence of complete and competitive markets with no uncertainty and complete information.

In reality not all these conditions are satisfied and recent academic discussions have identified the following advantages of integrated energy and transmission markets: Integrated markets save the transaction costs of trading physical transmission contracts, in potentially illiquid markets, for each half hour and each location to match all energy transmissions. Bushnell (1999) showed that generators can exercise market power by withholding physical transmission contracts; however 'use-it-or-lose-it' provisions are now frequently implemented and can prevent withholding, at least partially as Joskow and Tirole (2000) argue. Smeers and Jing-Yuan (1997) show that if only a limited number of traders arbitrage prices between the nodes, then they exercise market power and distort the dispatch. Harvey, Hogan and Pope (1996) argue that competitive generators and traders face uncertainty about the prices in the energy market when deciding on their bids for transmission markets, and might therefore buy an inappropriate amount of transmission rights. In section 3.4 I provide empirical evidence from the German-Dutch interconnector to confirm this theory. Increasing contribution of intermittent (wind) energy will increase uncertainty and make arbitrage even less efficient. Furthermore, if separation of transmission and energy markets prevents a flexible use of the transmission network, then costs of integrating wind power are increased and revenues for wind generators are decreased.

This paper shows that market power of generation companies provides a further reason for the integration of energy and transmission markets. The market power is reduced by the integration of transmission and energy markets due to two different effects, dependent on whether the transmission constraint is relaxed or binding.

First, assume transmission is unconstrained. The mechanism to deal with transmission constraints is still required because volatile demand and supply can result in constraints at other hours of the day or year. At the unconstrained times, designs with integrated energy and transmission markets, e.g. nodal pricing, calculate one market clearing price for the integrated market and determine the amount of energy to be transmitted. Separate transmission and energy markets differ, as can be illustrated in the German-Dutch example. Traders buy transmission rights from Germany to the Netherlands in the day ahead auction. All potentially beneficial

[3] O'Neill e.a. (2000) present a joint energy and transmission rights auction with similar attributes.

contracts are allocated, because without scarcity the price drops to zero. Traders then decide how many of these contracts to use to arbitrage the German day ahead electricity spot market, LPX, and the Dutch day ahead electricity spot market, APX. They cannot condition their trade volume on spot prices in these markets because bids have to be submitted before results in either market are announced. Therefore, traders price their buy bid in one market very high and their offer in the other market very low to avoid exposure to imbalance fees if only one bid is accepted. As a result, transmitted energy is independent of price and therefore independent of the realised output choice of generators. Generators only compete against local generators and face the local demand slope - therefore it is profitable to withhold more output than with integrated energy markets. Assuming complete information, traders anticipate generators output choice and arbitrage the markets, but the price level is higher price than with integrated energy and transmission markets. Transmission constraints only matter in this case, because two separate markets were defined to be prepared for possible constraints. Nodal pricing combines these two markets at times when no constraint is binding thereby reducing market power of strategic generators.

Are there other approaches to resolve inefficiencies of separate markets? First, decentralised trading in energy and an iterative market for transmission could integrate demand of both markets. This would decrease the demand slope and reduce market power. In practice, electricity and transmission rights seem to be too complex to ensure liquidity in short term markets for each half-hour, and for each location is sufficient to ensure prices do not change significantly as a result of any one trade. Second, transmission constraints could be ignored, as in England and Wales, and generators and traders would only contract for energy and then submit a dispatch schedule leaving the system operator to resolve constraints. If constraints are significant, this approach results in inefficient dispatch and perverse incentives for the location of new generators (Kamat and Oren, 2000, Neuhoff, 2002) and creates additional opportunities for generators to exercise market power (Harvey and Hogan, 2000). Third, traders could sign option contracts for energy in one energy market, allowing them to submit bids conditional on moderate prices in the energy market of the other node. Traders would benefit from a share of the savings if inefficient transmissions are avoided and reduce the demand slope and therefore the market power of generators.

Can we test for increased exercise of market power as a result of separation of transmission and energy markets? The available data did not allow for such a test in the case of permanently unconstrained transmission, but did show that arbitrage is inefficient. If the output choices of strategic generators determines whether the transmission line is constrained, then an empirical test is feasible. Northern Norway, part of the market splitting regime of Nordpool, exhibits the characteristics predicted for nodal pricing while the Netherlands, connected with separate transmission and energy markets to Germany, does not.

What happens if the transmission lines are constrained for all strategic output choices of generators? In the two node model the transmitted energy is constant, independent of the market design, and therefore separation of energy and transmission markets does not matter. However, in meshed networks separation matters, and constitutes a major disadvantage of the current proposals for a coordinated auction to govern electricity trade between continental European countries.[4] If energy and transmission markets are integrated under nodal pricing, transmission capacity will be allocated as a function of the energy bids submitted by generators. Strategic generators will, as Hogan (1997) has shown, anticipate the reaction of flow patterns on their output decisions when choosing their bids in a nodal pricing regime. If energy and transmission markets are separated, then traders determine with their bids in the transmission auction how scarce transmission capacity is allocated, e.g. how scarce capacity in the network is split between exports from node A and node B to node C.[5] This split is fixed at the subsequent energy spot markets, which is an artificial constraint due to market design. I show that Le Chatelier's principle is applicable: Adding an additional constraint weakly reduces responsiveness of choice variables.

Because of reduced demand responsiveness, generators face a steeper net demand curve and will exercise more market power. E.g. exports from node A and B are independent of the response of demand at other nodes. Strategic generators therefore act as oligopolists in their local market.[6] Traders only arbitrage the markets and buy transmission capacity in expectation of the monopoly output choice of generators. In contrast, nodal pricing combines in the energy spot market demand slope from different nodes, and maintains competition between generators at different nodes. The first result is that if strategic generators are located at one node of a meshed network, integration increases output and welfare.

What if strategic generators are located at two nodes of an arbitrary meshed network, without cross holding of generation assets? Network effects imply that output at these nodes is either a 'local substitute', when an output increase at one node reduces prices at the other node, or 'local complements', when an output increase at one node increases price at the other node (Joskow and Tirole 2000, Oren, 1997, Chao and Peck, 1996). Integrating transmission and

[4] European Transmission System Operators, Coordinated use of PX for Congestion Management, 5/03/01

[5] In a three node model this problem could be solved with a flow-gate design, difficulties arise in complex networks (Ruff, 2001).

[6] Smeers and Wei (1997) and Kamat and Oren (2002) model a similar separation in a spatial oligopolistic model. Generators assume that transmission prices do not change when they calculate the profits from deviating from their output choice.

energy markets results in an increased output at both nodes, if these nodes are ‘local substitutes’ and results in lower prices at both nodes, if these nodes are ‘local complements’. In both cases total output is increased.

If strategic generators are located at more than two nodes of a meshed network, these nodes are either ‘local complements’ or ‘local substitutes’ to each other. This suggests that total output will once again be increased if energy and transmission markets are integrated, but this requires confirmation in future research.

Further research is required to assess the impact of generators holding assets at several nodes, like in Cardell, Hitt and Hogan (1997). The effects might be similar to ‘strategic complements’ and ‘strategic substitutes’ as described by Bulow, Geanakoplos and Klemperer (1985) where network interaction rather than changing marginal production costs provides the link between generation at different locations.

2 Literature Review

Market power of electricity generators has been modelled and the supply function approach has been applied to the electricity industry to analyse the UK market by Green and Newbery (1992). Borenstein and Bushnell (1999) use a Cournot approach to analyse the potential for market power in California. Joskow and Kahn (2002) show, using the example of California’s summer of 2000, that simulated competitive benchmark prices are below observed prices, even if high NO_x permit prices are considered. Harvey and Hogan (2002) repeat the simulations and run sensitivity analysis on the parameter choices. For some of their parameter combinations, simulated prices reach observed prices. Assessing generation output, Joskow and Kahn (2002) calculate that unilateral withholding of output to push up the wholesale price would have been profitable for portfolio generators, and indeed observe that ”either the units [of portfolio generators] were suffering from unusual operational problems or they were being withheld from the market to increase prices.”

Green (1992) first noted that in a simple two-period model, generators that have contracted all their energy in the forward market have no incentive to distort the spot price, and will therefore bid competitively. Joskow and Kahn (2002) confirm this theory by their observation that ”the one supplier for which we do not find any significant evidence of withholding had apparently contracted most of the output of its capacity forward.” If forward contracts mitigate market power, then the question is how to make generators sell their electricity in the forward market. Allaz and Vila’s (1993) paper shows that, given infinite trading periods, generators sell additional contracts for energy in each period, eventually committing all their output. While generators might indeed have perceived summer 2000 as a one-shot game, one might expect spot prices to be determined with an eye to their impact on the long-term contract price. Newbery (1998) suggests that the spot price could be set by incumbents such that entry is prevented for newcomers. Incumbents then sell the amount of forward contracts which commits them to this spot price.

Forward or long-term contracts have directly influence spot prices, but are ignored in this paper to simplify the analysis of transmission. One might justify this simplification by interpreting the subsequent analysis as an analysis of the residual, non-contracted energy market. The results continue to be relevant for the entire energy sold, because forward prices largely follow from expected spot prices.

Extending the market power analysis to electricity networks Borenstein, Bushnell and Stoft (2000) show that it can be profitable for generators to withhold output in order to constrain a transmission line that would not have been constrained under perfect competition. Borenstein et al. (1996) cite empirical evidence from Northern California to this effect. Oren (1997) presents an alternative scenario with the transmission constraint located between two strategic generators in a three-node network. Stoft (1998) solves the corresponding Cournot game and Joskow and Tirole (2000) give the interpretation: the transmission configuration can turn output of generators at two different nodes into 'local complements', thereby increasing the incentive for a generator to withhold output, as this constrains the output of the other generator and increases price levels. Cardell, Hitt and Hogan (1997) show if strategic generators own generation assets at node A and B of a three-node network, they might increase output at node A relative to a competitive scenario if this reduces the total energy delivered to node B due to loop flows and therefore increases prices at node B.

Transmission contracts can enhance market power of generators and provide financial incentives to change output decisions of generators even as transmission constraints are and stay constrained. This was first addressed by Hogan (1997). Joskow and Tirole (2000) show that physical and financial transmission rights have almost identical properties. However, in real networks, a complete set of physical transmission contracts is too complex and designs were developed to aggregate and simplify property rights for each individual link. Joskow and Tirole discuss different approaches and point out the need for rights to be obligations to transmit rather than just options to use the network, to ensure an efficient use of meshed networks.

Abstracting from effects of separation of energy and transmission markets, as discussed in this paper, Joskow and Tirole (2000) show how transmission contacts can provide financial incentives that enhance market power of generators. Monopoly generators will buy such contracts in a discriminatory price auction or inherit them. Gilbert, Neuhoff, Newbery (2002) extend the analysis to oligopolies and show for complete information that uniform price auctions, in contrast to discriminatory auctions, only allocated transmission contracts that weakly mitigate market power. However, given asymmetric information and uncertainty, this clear result no longer holds. Furthermore generators do in general not sell market power enhancing contracts in secondary markets. Therefore, guidelines are suggested to exclude generators from purchase of contracts that enhance market power.

3 Two nodes with partially constrained transmission

Integration of the energy and transmission market reduces market power in a two-node network as long as transmission constraints are not permanently binding. The output choice of strategic generators is calculated assuming energy markets are integrated (3.1) and assuming transmission and energy markets are separate (3.2). Comparison of both designs shows an increase in market power and provides a theorem: if the transmission line is unconstrained, separation of transmission and energy markets reduces output and increases prices. This might look puzzling, because with complete information traders can perfectly arbitrage the separate energy markets using available transmission capacity. However, to avoid imbalance fees when the energy bid in only one market is accepted, the traders have to submit high buy and low sell bids.[7] Therefore the amount of energy transmitted between the markets is independent of prices. The realised output choice of a strategic generator does not influence the amount of transmitted energy; therefore the generator will withhold more output to push up price. This is anticipated by traders bidding into the spot markets, therefore markets are arbitraged, but at a higher price level than if both markets were integrated. In 3.3, propositions are presented that allow for empirical tests in 3.4.

3.1 Integrated energy and transmission markets

In an integrated energy and transmission markets (nodal pricing) demand and supply submit their bids to the system operator. He determines market clearing prices which incorporats transmission constraints and make optimal use of transmission capacity. Traders are not needed to arbitrage the markets and therefore not present in this model.

Two strategic generators are located at each of the two nodes in Figure 2. All four generators are assumed to be symmetric, each with total costs as a function of output q:[8]

$$C(q) = \frac{1}{2}\beta q^2. \tag{1}$$

[7] Traders with option contracts for energy in one market could submit bids conditional on price in the other market. Traders would not benefit from mitigating market power, but could increase trading profitability by avoiding losses in the energy market. Inefficiencies in Figure 8 indicate that such contracts are not typical.

[8] Fixed production costs can be ignored because they do not influence output decisions, and the constant component of variable costs can be normalised to 0 by redefining A.

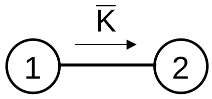

Figure 2: Two nodes interconnected by a transmission line of capacity $\overline{K}$ utilised at K.

Transmission capacity between the nodes is constrained $\overline{K}$. Residual demand at the exporting node one D_1 and importing node two D_2 is linear:

$$D_1(p) = A - p, \qquad D_2(p) = A + D - p. \tag{2}$$

I use the Cournot assumption, that generators submit quantity bids. Competitive demand, which can contain negative contributions from fringe generators, submits a price-quantity schedule according to demand schedule (2).

For small demand differences D between the nodes the link will be unconstrained (3.1.1) while very large differences result in a constrained link (3.1.2). Between these two 'extremes' is an interval of demand differences, for which no pure strategy output choices exist (3.1.3) and for the strategic generators at both nodes a mixed strategy equilibrium is calculated (3.1.4). In some cases, e.g. when all strategic generators are located at the same node and their marginal cost curves are not to seep, the interval is empty and a pure strategy equilibrium exists for all demand differences.

3.1.1 Unconstrained link

If the link is unconstrained, then both local markets are integrated and four strategic generators bid to supply to demand $D_1 + D_2$. Each generator chooses output q (without index) to maximise profits π_u taking output choice of the three other generators q_u (with index) as given.

$$\pi_u = \frac{2A + D - 3q_u - q}{2} q - \frac{1}{2}\beta q^2.$$

The first order condition gives

$$q = \frac{2A + D - 3q_u}{2\,(1+\beta)}.$$

Solving for symmetric generators $q = q_u$ gives output and price:

$$q_u = \frac{2A + D}{5 + 2\beta}, \qquad p_u = \frac{1+\beta}{5+2\beta}\frac{2A + D}{2}. \tag{3}$$

3.1.2 Constrained link

If generators anticipate that the markets will be separated, then they anticipate the system operator will use all transmission capacity $\overline{K}$ to transmit energy between the markets. Generators at each node i chose output q and q' to maximise profit $\pi_{i,c}$ given the output choice of the fellow generator at their node $q_{i,c}$.

$$\pi_{1,c} = \left(A + \overline{K} - q_{1,c} - q\right) q - \frac{1}{2}\beta q^2, \quad \pi_{2,c} = \left(A + D - \overline{K} - q_{2,c} - q'\right) q_2 - \frac{1}{2}\beta \left(q'\right)^2.$$

The first order conditions and symmetry between generators at the same node $q_{1,c} = q$ and $q_{2,c} = q'$ allows to calculate the equilibrium output choices and prices.

$$\begin{aligned} q_{1,c} &= \frac{A + \overline{K}}{3 + \beta}, & q_{2,c} &= \frac{A + D - \overline{K}}{3 + \beta}, \\ p_1 &= \frac{1 + \beta}{3 + \beta}\left(A + \overline{K}\right), & p_2 &= \frac{1 + \beta}{3 + \beta}\left(A + D - \overline{K}\right). \end{aligned} \tag{4}$$

3.1.3 Why is a pure strategy equilibrium sometimes not feasible?

First order conditions are not sufficient to determine the optimal output choice if boundary conditions create non-convexities. Output choices of individual generators can determine whether a transmission line is constrained or unconstrained. Therefore, price is not necessarily a convex function of output choice of a generator.

The implications are illustrated for the parameter choice $A = 10$, $\overline{K} = 2$, $\beta = 0$ and $D = 4$. First order condition (3) suggest a hypothetical equilibrium with an unconstrained link and output $q_u = 4.8$, price $p = 2.4$ and profit per generator $\pi_u = 11.52$. The system operator would schedule energy transmission of 2 units which still satisfies the transmission limit. The demand difference $D = 4$ is the upper bound to an interval of Ds which allow for a hypothetical unconstrained equilibrium. Profit functions are continuous in output choice, therefore subsequent argumentation extends to this interval, which can be easily calculated to be $[3.2, 4]$. Assume strategic generators would submit energy bids according to the hypothetical equilibrium. Would it be profitable for one generator at the importing node to deviate and submit a different bid? He would chose q_q such that the transmission link is constrained and the system operator calculates separate prices for both nodes. His profit function would be:

$$\pi_q\left(q_q\right) = \left(A + D - \overline{K} - q_{2,u} - q_q\right) q_q - \frac{1}{2}\beta q_q^2.$$

The first order condition gives optimal output choice of $q_q = 3.6$, prices $p_{1,q} = 2.4$, $p_{2,q} = 3.6$ and profits of $\pi_q = 12.96$. Deviation is profitable $\pi_q > \pi_u$ and the hypothetical equilibrium is not a Nash equilibrium.

For the same parameters first order condition (4) suggests a hypothetical equilibrium with a constrained link, output $q_{1,c} = q_{2,c} = 4$, prices $p_1 = p_2 = 4$ and profits $\pi_{1,c} = \pi_{2,c} = 16$. The system operator schedules 2 units of transmission so the condition for a constrained equilibrium is satisfied. Continuity of the profit functions implies that the following results also apply to an interval to the right of D, which can be calculated as $D \in [4, 6.5]$. Assuming strategic generators submit bids according to the hypothetical equilibrium with a constrained link, would it be profitable for a generator at the importing node to deviate? Yes - he would submit a bid for a larger q_d such that the link is no longer constrained and his profit function would be:

$$\pi_d = \frac{2A + D - 2q_{1,c} - q_{2,c} - q_d}{2} q_d - \frac{1}{2}\beta q_d^2.$$

The first order condition gives output $q_d = 6$ and nodal price $p_d = 3$ providing for profits $\pi_d = 18 > \pi_c = 16$. Therefore, $(q_{1,c}, q_{2,c})$ is not a Nash equilibrium.

For $D \in [3.2, 6.5]$ neither an equilibrium with a constrained link nor an equilibrium with an unconstrained link exist.[9] Appendix A shows that this interval is typically empty and a pure strategy equilibrium exists if strategic generators are only located at one instead of both nodes. The policy implications still follow and the empirical test is still applicable.

3.1.4 Mixed strategy equilibrium

Returning to the case with two strategic generators located at each nodes I describe one possible mixed strategy equilibrium for parameter choices D which does not allow for a pure strategy equilibrium. The two generators at the exporting node submit a bid for the quantity q_1. The two generators at the importing node each play a mixed strategy as to what bid they will submit to the energy spot market. They chose independently, with probability ρ a high output quantity q_H and with probability $(1 - \rho)$ a low output quantity q_L. Fellow generators cannot observe the bid, but only the distribution of bids, before the auction results are announced and can therefore not react to the bid. The link will be constrained with probability $(1 - \rho)^2$ if both generators choose a low output quantity. The strategies can only represent a Cournot equilibrium, if q_1 maximises expected profits for generators at the exporting node and both q_L and q_H maximise profits of the generator at the importing node.

Figure 3 illustrates how non-convex net demand allows for two different output choices q_L and q_H to be equally profit maximising. For simplicity only one strategic generator is located at the importing node. If he chooses a low output quantity, then the transmission link will be

[9] Stoft (1998) shows a different channel which can make a pure strategy equilibrium unfeasible. In his three-node network the output of two generators is complementary, because flows of electric energy can be superimposed and therefore cancel each other. The high-cost generator's output relieves the transmission constraint which the low-cost generator faces. This allows the high-cost generator to limit the low-cost generator's output and, by creating a constraint, receive higher prices for his output under a nodal pricing design. The high-cost generator reduces output to keep the constraint binding and to obtain high prices, while the low-cost generator would like to reduce output to relax the constraint and obtain high prices. The low-cost generator can only do so by mixing output strategies to ensure that the high cost generator does not constrain his output all of the time.

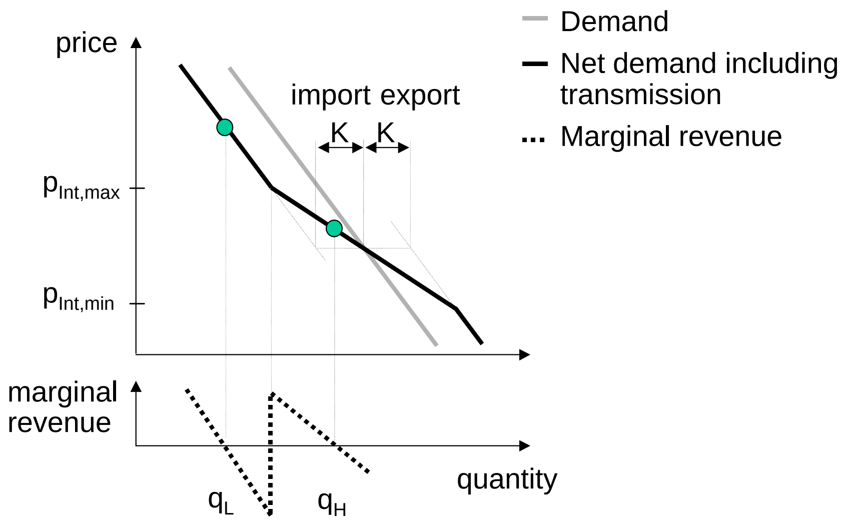

Figure 3: Non-convexity in effective demand results in two profit maximising output choices q_L and q_H.

constrained and price will be above price $p_{\text{int max}}$ at the exporting node. Residual (local) demand is shifted to the left according to total transmission capacity $\overline{K}$. If the generator at the importing node increases output, then price falls. Both nodal markets will be integrated if the price falls below $p_{\text{int max}}$. The system operator will now determine the amount of transmitted energy such that nodal prices are equal. The result is a flatter section of net demand curve. This produces a non-convexity. Increasing output starting from q_L marginal revenue is first negative because q_L is a local maximum. However, if output is increased beyond the kink, then marginal revenue is positive, because of the flatter demand curve, until the second profit maximising output choice q_H is reached.

I will now give three first order conditions and one equality that together determine the output choices q_1, q_L, q_H and mixing probability ρ. I will continue to use q, without subscript, as the variable a generator is optimising to maximise profits.

First, determine the high output choice q_H of a generator at the importing node. If the generator chooses the high output, then the link will always be unconstrained and the markets of both nodes will be integrated $D_1 + D_2$. Expected output of the fellow generator at the importing node follows from the mixing probability of high and low output choices $(1-\rho)\,q_L + \rho q_H$ and generators at the importing node chose q_1. Generators are assumed to be risk neutral, therefore they maximise expected profits, which are as a function of output choice q:

$$E\left[\pi_H\left(q\right)\right] = \frac{2A + D - (1-\rho)\,q_L - \rho q_H - q - 2q_1}{2} q - \frac{1}{2}\beta q^2.$$

Using FOC and symmetry $q = q_H$ gives the optimal output choice:

$$q_H = \frac{2A + D - q_L + \rho q_L - 2q_1}{\rho + 2 + 2\beta}. \tag{5}$$

Second, the low output choice of a generator at the importing node is slightly more complicated to calculate. If the fellow generator at the importing node chooses q_H, then the nodal markets will continue to be integrated whereas if the fellow generator at the importing node chooses q_L then nodal markets are separated and the generator only face demand of the importing node minus the constant imports at full capacity. Forming the expectation over both cases the profits for output choice q are:

$$E\left[\pi_L\left(q\right)\right] = \left(\left(1-\rho\right)\left(A + D - \overline{K} - q_L - q\right) + \rho\frac{1}{2}\left(2A + D - q_H - q - 2q_1\right)\right) q - \frac{1}{2}\beta q^2.$$

The optimal q follows from the FOC and symmetry $q_L = q$:

$$q_L = \frac{1}{2}\frac{2A + 2D - 2\overline{K} - \rho D + 2\rho\overline{K} - \rho q_H - 2\rho q_1}{3 - 2\rho + \beta}. \tag{6}$$

Third, generators at node one choose the optimal output quantity to maximise expected profits, which are obtained by aggregating all combinations of output choices for generators at node two:

$$E\left[\pi\left(q_1\right)\right] = \left(\begin{array}{c} \left(1 - 2\rho + \rho^2\right)\left(A + \overline{K} - q_1 - q\right) + \frac{\left(2\rho - 2\rho^2\right)}{2}\left(2A + D - q_1 - q_H - q_L - q\right) \\ +\frac{\rho^2}{2}\left(2A + D - q_1 - 2q_H - q\right) - \frac{1}{2}\beta q \end{array}\right) q.$$

The FOC with respect to q and symmetry $q_1 = q$ gives:

$$q_1 = \frac{2A + 2\overline{K} - 4\rho\overline{K} + 2\rho^2\overline{K} + 2\rho D - 2\rho q_H - 2\rho q_L - \rho^2 D + 2\rho^2 q_L}{6 - 6\rho + 3\rho^2 + 2\beta}. \tag{7}$$

Finally, each generator at node two chooses the probability ρ of mixing between q_L and q_H to ensure that the fellow generator at node two is indifferent between choosing q_L and q_H and therefore the mixed strategy equilibrium will be maintained. To simplify notation define $\pi_{i,j}$ as profits of the fellow generator at node two from choosing output $i \in \{L, H\}$ if the generator chooses output $j \in \{L, H\}$:

$$(1-\rho)\pi_{L,L} + \rho\pi_{L,H} = (1-\rho)\pi_{H,L} + \rho\pi_{H,H}.$$

Substituting from above gives:

$$\begin{aligned} & (1-\rho)(A + D - \overline{K} - 2q_L - \frac{1}{2}\beta q_L)q_L + \rho\left(\frac{2A + D - q_L - q_H - 2q_1}{2} - \frac{1}{2}\beta q_L\right) q_L \qquad (8) \\ = \; & (1-\rho)\left(\frac{2A + D - q_L - q_H - 2q_1}{2} - \frac{1}{2}\beta q_H\right) q_H + \rho\left(\frac{2A + D - 2q_H - 2q_1}{2} - \frac{1}{2}\beta q_H\right) q_H. \end{aligned}$$

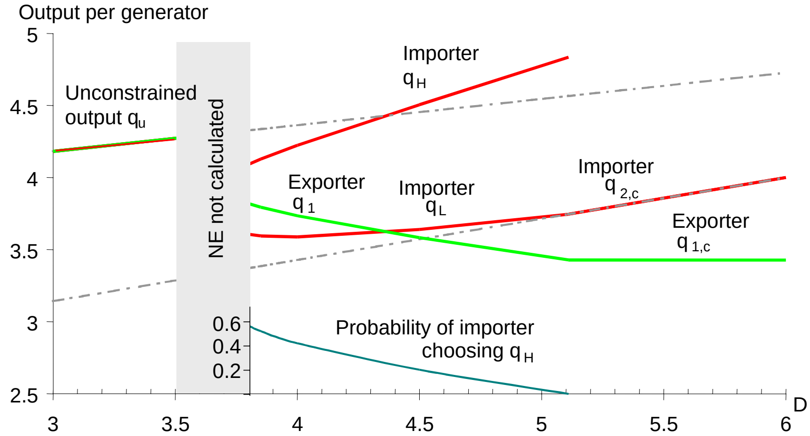

Figure 4: Output of individual generators as function of demand difference D.

The four equations (6), (5), (7) and (8) allow to solve for the four unknowns q_1, q_L, q_H and ρ. Unfortunately, I did not find an analytical solution; therefore, I only provide the numerical results.

Figure 4 presents the output of individual generators for demand parameter $A = 10$, transmission capacity $\overline{K} = 2$ and increasing marginal costs $\beta = 0.5$. For demand differences $D < 3.5$ generators chose output q_u of the unconstrained scenario. For $3.5 < D < 3.8$, a mixed strategy equilibrium in continuous output choices must exist but I could only find a mixed strategy equilibrium for $D \geq 3.8$. For example for $D = 3.8$ generators in the importing node will chose q_H with probability $\rho = 0.57$ and otherwise chose q_L. The constrained is only binding if both generators chose q_L which happens with probability $p = (1 - \rho)^2 = (1 - 0.57)^2$. Because of the positive probability of a binding constraint, the expected net-demand slope at the exporting node is steeper and generators will therefore reduce output to q_1. With increasing D the constraint is binding more frequently until at $D = 5.1$ the constraint is permanently binding ($\rho = 0$). Output at the exporting node q_1 and the low output choice q_L at the importing node coincide with previously calculated pure strategy constrained output choices $q_1 = q_{1,c}|_{D=5.1}$ and $q_L = q_{2,c}|_{D=5.1}$.

Figure 5, left, summarises the results, giving total expected output for constant marginal costs $\beta = 0$. Pure strategy equilibria exist for $D < 3.2$ (unconstrained) and $D > 6.5$ (constrained). In the intermediate domain a mixed strategy equilibrium is guaranteed by Glicksberg's Theorem (1952): Output choices are in a compact set (positive and never exceed total demand) and payoff functions (profits) are continuous in output choices (Ritzberger 2002). Continuous payoff functions require that demand be elastic at all nodes, otherwise prices could jump with a small variation of output choice. Figure 5, left, for constant marginal costs $\beta = 0$ suggests that

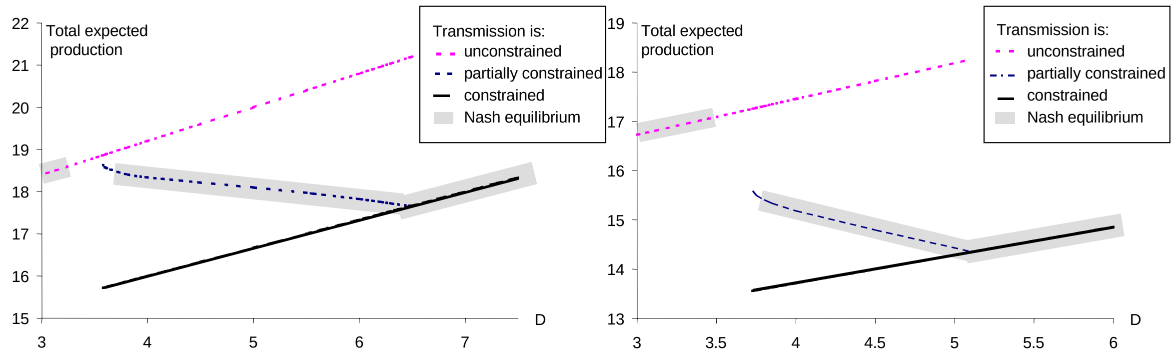

Figure 5: Expected total productionas function of demand difference D between symmetric nodes. When strategic generators play a mixed strategy, then transmission is partially constrained. Marginal costs are constant in the left graph and increasing in the right graph.

a linear interpolation between the constraint and unconstraint border-cases provides for a good approximation of the mixed-strategy output. Unfortunately, this assumption breaks down with increasing marginal costs, e.g. $\beta = 0.5$. (Figure 5, right).

One interesting effect of the transmission constraint is, that expected total output is decreasing with increasing demand in the domain of mixed strategy. This is due, first, to the decreasing probability ρ at which generators chose q_H and, second, to the increasing probability of a constrained link $p = (1-\rho)^2$ inducing generators at the exporting node to reduce output as expected demand curve is steeper.

3.2 Separate energy and transmission markets

A benefit of electricity liberalisation is closer coordination of electricity dispatch between neighboring regions to exchange flexible generation capacity, average out volatility of intermittent generation and to balance hydro and demand cycles. However, electricity markets are still separated in different regions or European countries, mainly because liberalisation replicated existing structures. These regional markets are then arbitraged by traders buying and selling electricity and transmission rights. In the simplified model this historic evolution explaining and potentially justifying such a market design is obviously not represented. The difference to the integrated design is therefore that traders, instead of the centralised system operator, arbitrage markets. I will now assess the implications for competition among generators.

To simplify the model I assume, in contrast to Smeers and Jing-Yuan (1997), that traders make on expectation zero profits either because the market is contestable or because the number of traders is large. A second simplification is, that only traders, and not generators, buy transmission contracts. If generators buy transmission contracts, then they experience financial incentives that influence their dispatch decisions, as shown by Joskow and Tirole (2000). The paper therefore complements Gilbert, Neuhoff, Newbery (2002) which evaluates policy guidelines for generators' access to transmission contracts while ignoring the effects resulting from separate energy and transmission markets.

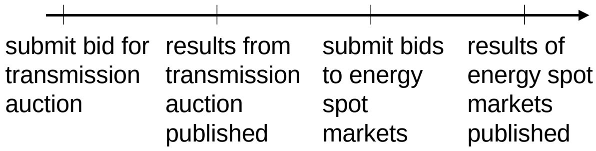

Figure 6: Timeline of day ahead market for separate energy and transmission markets.

As illustrated in Figure 6 the system operator first auctions transmission contracts to traders. I assume complete information and perfect arbitrage, therefore traders pay the marginal value for transmission contracts, both in a uniform price and discriminatory price auction. The marginal value can be zero if transmission supply exceeds demand. In step two, assuming n symmetric traders each obtains $\overline{K}/n$ units of transmission capacity.

In step three generators and traders submit their bids to the energy spot market. Following the Cournot assumption strategic generators at node i submit a quantity bid q_i to the energy spot market at their node. Each trader has to decide what quantity k of his transmission rights $\overline{K}/n$ to use for energy transmission and then submits a quantity bid to buy k units of energy at the exporting node and sell k units at the importing node. The assumption that traders submit quantity bids which are price independent is based on the model of continental power exchanges as currently implemented in Germany and the Netherlands. In these power exchanges all bids have to pay the market clearing price. I assume that traders will submit a very high priced buy bid for k in the exporting country and a very low priced sell bid for k in the importing country. This ensures that both bids will be accepted and corresponds to the Cournot model. It is important that either both or no bid is accepted, otherwise traders have an open energy position and are exposed to high imbalance fees.[10] At the end of the section I discuss bids conditional on the market clearing price at the other node, which would imply that traders no longer submit pure quantity bids.

The effect of traders submitting quantity bids is, that the amount of energy transmitted between the markets K is not directly influenced by output decision of strategic generators but by the aggregate bid $nk \leq \overline{K}$ traders submit to both energy spot markets. Setting $\overline{K} = nk$ in

[10] A trader with energy contracts in only one zone might still try to use bilateral negotiations to provide for the second energy contract. However, he is under significant time-pressure and therefore in a bad bargaining position, and is likely to obtain an unsatisfactory price.

(4) gives the spot price difference resulting from generators' decisions:

$$\Delta p = p_2 - p_1 = \frac{1+\beta}{3+\beta} (D - 2nk) . \tag{9}$$

Generators' strategic output choices only indirectly influences the amount of energy submitted between the nodes: Traders anticipate the output choice of strategic generators, and will accordingly chose the amount of energy bids k they submit to arbitrage the markets $\Delta p = 0$. This is obviously only possible as long as the transmission limit is not violated:

$$K = nk = \min(\overline{K}, \frac{D}{2}). \tag{10}$$

If the transmission limit is binding and traders obtain positive revenue $k\Delta p$ from arbitraging the two markets, then transmission capacity is scarce in the initial auction and competitive bidders will pay the marginal value Δp for the transmission capacity such that they make zero profits.

Substituting the amount of energy traded (10) in the strategic output choice of generators (4) gives individual and total production:

$$q_{1,c} = \frac{A + \min(\overline{K}, \frac{D}{2})}{3+\beta}, \quad q_{2,c} = \frac{A + D - \min(\overline{K}, \frac{D}{2})}{3+\beta}, \quad Q = 2\,(q_{1,c} + q_{2,c}) = \frac{4A + 2D}{3+\beta}.$$

Figure 7 compares total expected production under integrated transmission and energy markets with separate markets, assuming constant marginal costs: The separation of markets for transmission rights and energy enhances generators' market power and results in lower output quantities. Only, if demand difference between the two nodes exceeds $D > 6.5$ and the transmission constraint is permanently binding, then the output choices do not differ between the two market designs. In the separate markets the entire transmission capacity will already be used for $D > 4$, this does not show up in total output, because separation of energy spot markets has the same effect as if the link were always constrained.

The model is built on the assumption that generators cannot condition their bids at one node on the market clearing price at the other node. This assumption could be questioned along three different ways: First, simultaneously clearing markets at both nodes could allow traders to submit bids conditional on the market clearing price in the neighboring node. However, if such a close cooperation is feasible, why not go for an integrated market which ensures that arbitrage is perfect. Second, energy spot markets can close sequentially. Indeed the German spot markets opens later than the Dutch. If Dutch results are, as frequently the case, announced before closure of the German spot market, then this should allow traders to still submit corresponding bids in the German market. Such a conditioning might improve the situation in the two node case, however, if the auctions are expanded to several countries then sequential energy spot markets are difficult to implement. The third approach towards conditional bids is continuous trading.

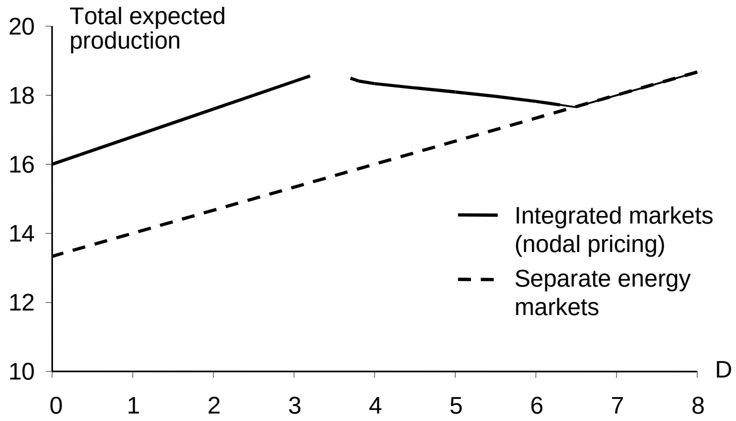

Figure 7: Total production of strategic generators as a function of demand difference between nodes.

Continuous trading allows traders in theory to continuously adapt their positions. However, liquidity is typically low in very short term energy markets and therefore, at least according to UK reports, traders are reluctant to go with a big position into this market.[11]

Given the limited alternatives the Cournot approximation seems to provide some insights into the difficulties of separate energy markets. It might be questionable to trust the quantitative results, but the qualitative outcomes seem reasonable as long as demand elasticity is finite at all nodes. In the described markets this is guaranteed, because fringe generators supply to the market dependent on the market-clearing price. Therefore net demand facing strategic generators is price responsive even if demand is rather price inelastic. The following theorem proves the benefit of integrating energy and transmission markets if the transmission line is permanently unconstrained ($D < 3.2$). The example in Figure 7 shows integration also has a positive effect if the transmission is partially constrained in a mixed strategy equilibrium. ($3.7 < D < 6.5$). Integration has no effect for $D > 6.5$.

Theorem 1 *If the transmission line between two nodes is unconstrained, integration of nodal energy markets which are only arbitraged by traders increases output and decreases prices (assuming variable costs are convex and demand is elastic at both nodes).*

Proof. *Traders determine the amount of transmission such that prices at both nodes coincide. Let $P_i(Q)$ characterise net demand faced by generators at each node and Q_i equilibrium production by generators at node $i = 1, 2$. We assess now on output at node one, but the argument equally applies to node two.*

If energy and transmission markets are separated, then output quantity q_m of generator m at

[11] Stated at Regulation Initiative Workshop, "How well is NETA doing?", LBS, October 17, 2001

node one equals the choice q that maximises profits $\pi_m(q) = P_1(Q_1 - q_m + q)q - C_m(q)$*; therefore, the FOC and* $q = q_m$ *give the result that marginal revenue equals marginal costs:*

$$P_1(Q_1)' q_m + P_1(Q_1) = C'_m(q_m)$$

Assume that generators would choose the same output quantities in the integrated market with total net demand $P_{int}(Q)$*. From* $P_1(Q_1) = P_2(Q_2)$ *it follows that* $P_{int}(Q_1 + Q_2) = P_1(Q_1)$ *and elastic demand at both nodes implies* $P'_{int}(Q_1 + Q_2) > P'_1(Q_1)$*. Therefore, marginal revenue exceeds marginal costs:*

$$P_{int}(Q_1 + Q_2)' q_m + P_{int}(Q_1) > P_1(Q_1)' q_m + P_1(Q_1) = C'_m(q_m),$$

and generators with convex costs increase output q_m *to maximise profits. This reduces prices in the integrated transmission and energy market.* ■

3.3 Testable implications

Figure 7 illustrated that the transmission link is less frequently constrained if energy markets are integrated. To understand the reason, assume the constraint is binding in the design with separate markets. Would generators bid the same output into an integrated market? Not if the price difference between the two markets is small. Then generators at the importing node could increase output such that price would fall and the markets were integrated. Generators would do so, if profits on additional sales exceed lost revenue from price decrease on existing sales. In this case the constrained situation no longer represents a Nash equilibrium.

Assume two nodes are of different size and in a design with separate markets the constrained binds. If the small node is import constrained and the markets are integrated, then a strategic generator at the small node would incur large gains from expanding output towards the big market while only loosing little revenue on his small existing sales. In contrast, if the large node were import constrained and the markets are integrated, then a strategic generators would not gain much from the additional sales in a small market while loosing more revenue on his existing output. Therefore, an import constraint into the small market is only a Nash equilibrium for much bigger prices differences between the nodes than an import constraint into the large node.[12]

To show this effect, I use the change of net demand from day to day. Assume that changes to demand and competitive supply are in first order random between the same hour of consecutive

[12]This would imply, that in an integrated market prices between nodes are either equal or differ significantly, while small price difference between the markets should not exist. This interval should differ for import and export constraints. In contrast, price differences between separated energy markets should not exhibit such a gap. I did not explore this route, but used a method derived from this initial idea.

days. If the transmission constraint is binding, then price changes at each node are also random, and if demand is linear and marginal costs are constant, then price changes at each node are independent of the price level. Therefore, the change of price difference between the nodes is also random and independent of the direction in which the transmission link is constrained.[13]

If today the small node is import constrained with price difference Δp, then a small change in prices might suffice to reach the level at which a constrained equilibrium does no longer exist. It is therefore likely that the import constrained will vanish by the same hour tomorrow. If instead the large node is import constrained, with the same price difference Δp, then a larger change in prices is required to reach the level at which a constrained equilibrium does no longer exist. It is therefore less likely that the import constrained will vanish by the same hour tomorrow. Obviously, the large node importing implies the small node exporting. Therefore, the probability of an import constrained into the small country to be resolved by the same hour next day should be higher than the probability of an export constrained to be resolved.

To formalise the results I require that demand curve is steeper in the smaller market, which is the case if demand elasticities and prices at both nodes are equal $\frac{\partial D_1}{\partial P}\frac{P}{D_1} = \frac{\partial D_2}{\partial P}\frac{P}{D_2}$. Then the smaller market $D_1 < D_2$ has a steeper demand curve $1/\frac{\partial D_1}{\partial P} < 1/\frac{\partial D_2}{\partial P}$.[14]

Proposition 2 *Transmission and energy markets are integrated and two generators with constant marginal costs are located at each of two nodes, demand is linear and demand shifts between the same hour of consecutive days are distributed symmetrically around* 0*. The slope of demand of the larger market exceeds* $\frac{1+\sqrt{5}}{2}$ *slope of demand at the smaller market. Then the probability of price difference* Δp *from an import constraint into the smaller market to vanish by the same hour next day is higher than the probability of an inverse price difference* $-\Delta p$*, caused by an export constraint, to vanish by the same hour next day. (Proof in appendix B)*

This asymmetry is also present if generators at only one node have market power. It still requires differing slopes of the demand curves at both nodes.

Proposition 3 *Transmission and energy markets are integrated and two generators with constant marginal costs* α *are located at one node with demand* $A + D - p$ *which is interconnected with a link of transmission capacity* $\overline{K}$ *to a competitive market with net demand* $A - rp$ *with* $0 < r$ *and* $2 - \sqrt{(1+r)} < \frac{3\alpha}{A+\overline{K}}r$*. The probability of price difference* Δp*, caused by an import*

[13] I ignore the link through the scarcity value of hydro. If transmission is likely to be unconstrained in the future then future trade opportunities influence todays prices.

[14] The requirement demand curve is steeper in the smaller market is easily satisfied. Assume demand elasticities and prices at both nodes are equal $\frac{\partial D_1}{\partial P}\frac{P}{D_1} = \frac{\partial D_2}{\partial P}\frac{P}{D_2}$. Then the smaller market $D_1 < D_2$ has a steeper demand curve $1/\frac{\partial D_1}{\partial P} < 1/\frac{\partial D_2}{\partial P}$.

constraint to the node were the oligopoly is located, to vanish by the same hour next day is higher than the probability that a price difference $-\Delta p$, caused by an export constraint, to vanish by the same hour of the next day. (Proof in appendix B)

The asymmetry was caused by strategic generators at the import constrained node increasing output to face the larger integrated market. In a competitive market generators submit bids at marginal prices and will therefore not increase output therefore symmetry is maintained.

If the two energy markets at both nodes are separated, then the amount of energy transmitted between the nodes is only indirectly determined by the output choice of strategic generators. Therefore, constrained Nash equilibria exist for all price differences between the nodes:

Proposition 4 *Assume transmission and energy markets are separated, costs are quadratic, demand is linear and demand shifts between the same hour of consecutive days are distributed symmetrically around* 0*. Then the probability of price difference Δp, caused by an import constraint, to vanish by the same hour next day equals the probability of price difference $-\Delta p$, caused by an export constraint, to vanish. (Proof in appendix B)*

The result for separate energy and transmission markets coincides for competitive market with the integrated markets, confirming the implication of Chao and Peck (1996) that both designs are identical in competitive markets.

3.4 Empirical evidence

Data from the German-Dutch interconnector shows that a separation of transmission and energy markets only allows traders to arbitrage on expectation, and not for each realisation (3.4.1). Traders frequently pay higher prices in the transmission auction from Germany to the Netherlands than the price difference in the energy spot markets which close later on the same morning would justify. Arbitrage is only profitable on expectation. The observed effect does not result in inefficient production decisions, but is a strong indication that the wrong amount of transmission is selected at times of unconstrained links. In 3.4.2 propositions 2 to 4 are tested by comparing the German-Dutch interconnector with the interconnection between Sweden and Northern Norway. The results do not contradict the hypothesis that integrating energy and transmission markets mitigates market power.

3.4.1 Incomplete arbitrage at the German-Dutch interconnector

Transmission rights to the interconnector are auctioned in annual and monthly auctions for the entire time span and in day-ahead auctions for each hour separately. I focus on this day-ahead auction, where traders must submit their bids by 8.30am and receive confirmation of the results by 9am.[15] Traders then submit bids to the Dutch power exchange APX by 10.30am and to the German power exchange LPX, which now includes the EEX, by 12 noon. The Dutch power exchange commits itself to publish the results by at least 12 noon, implying that the markets effectively clear simultaneously.[16]

In Figure 8 the spot price difference between the Netherlands and Germany is depicted as a function of the (positive) day-ahead auction prices for each hour in the period January 2001 to June 2002. For all the observations left of the dashed line, the price paid in the transmission auction exceeded the revenues subsequently obtained in the energy markets. The large variation

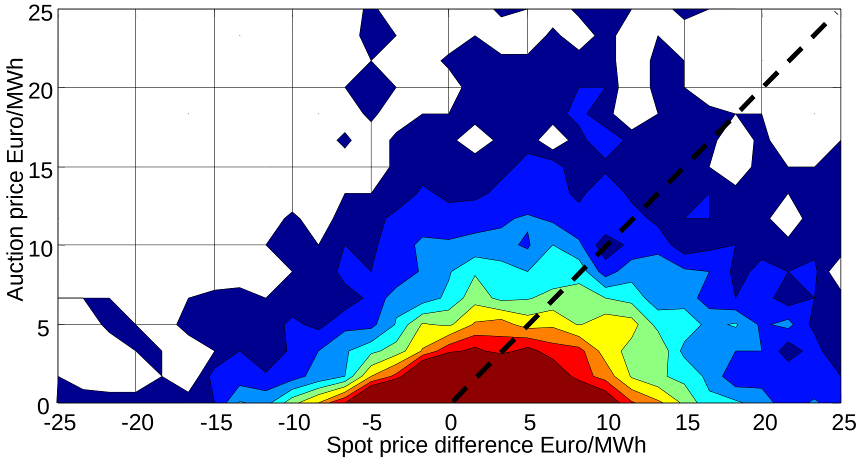

Figure 8: Spot price difference (Netherlands-Germany) observed after realisation of positive day-ahead auction prices in the period January 2001 to June 2002. Colour coding corresponds to number of hourly observations in 1Euro/MWh.

of the spot price difference for any one price paid for transmission rights shows that arbitrage

[15]Transmission rights to and from the Netherlands can be obtained in two separate auctions, starting in the grid of two neighbouring German utilities RWE or EON. The analysis is based on the average of both, because so far both rights are perfect substitutes, as traders are not exposed to transmission constraints within Germany. Rights are auctioned separately for both directions in monthly and daily auctions. To avoid abuse of transmission rights they have to be used or returned to the auction to allow for re-use.

[16]However, traders report that the Dutch power exchange frequently clears earlier. This would allow them to condition their bids to the German power exchange on the Dutch results. This represents a potential integration of transmission and energy markets and would therefore result in higher probability of the import constraint into The Netherlands being resolved, which is not observed.

The continuous trading platform Xetra of LPX is not included in the analysis, first, because trading volume is only 10-15% of total day-ahead trading volume at LPX and, second, because trading closes at 12 noon. Additional trading opportunities would only improve the situation in the period 9am to 10.30am because the bid to APX must be submitted after that time.

is only based on the expected prices. If traders could anticipate the real price difference, then transmission prices would never exceed the price difference between the two markets.

The figure only represents prices below 25 Euro/MWh, falsely creating the impression that traders lose more than profit from trading. Including all observations with positive transmission prices shows that traders' average profits from the combined interaction in transmission and energy market equal 1.56 Euro/MWh plus 0.5 times the price paid in the transmission auction. This indicates insufficient competition among traders, allowing them to bid low in the day-ahead auction to secure capacity at below its arbitrage value and thereby increase trading profits. In 2001, a very unsophisticated strategy of using all transmission contracts bought for a positive price in the auction to transmit energy from the German spot market to the Dutch one created arbitrage profits of 30.6m Euros. These high profits must have attracted additional traders and increased competition, reducing profits to 1.2m Euros for the first six months of 2002. This is a lower limit for the transfers from consumers and generators to traders, and could be higher, if traders used more sophisticated trading strategies. Borenstein e.a. (2001) observe a similar delay of "no more than a couple of months", during which price differences between the (day-ahead) future energy market and the spot energy market persisted, until traders learnt how to deal with a rule change.

Even if the markets are arbitraged on expectation, the main disadvantage of the separation of transmission and energy markets still remains. In all the hours which are represented on the left hand half of Figure 8, traders paid a positive price in the transmission auction at 9am and therefore probably bid later in the morning on the energy spot markets to trade energy from Germany to the Netherlands. However, the spot price in the Netherlands turns out to be lower than in Germany. Assuming the spot markets are efficient and represent variable costs of the marginal generator, this implies that low-cost generators in the Netherlands are replaced by higher-cost generators in Germany. This effect did not change with improved arbitrage on expectation; Figure 8 does not differ from a separate plot of 2001 or 2002.

The reason for this inefficiency is that traders cannot predict the spot prices because of uncertainty and because private information is only aggregated in the spot market. Usually, spot markets are specifically introduced to reveal private information; it is therefore inconsistent to introduce a decentralised mechanism for decisions on energy transmission, which can only work efficiently if traders correctly predict spot market prices.

Generation from wind and solar and CHP have output which is not predictable long-term, and information is only aggregated in the spot market. Therefore, a higher contribution by these energy sources will increase the inefficiency of the separate energy and transmission market. The separation is also biased against intermittent generation: imagine that traders anticipate low generation in the Netherlands and therefore schedule imports. If the spot market reveals high (renewable) generation, the price will fall below the German price and renewables will receive low revenues. If transmission and energy markets were integrated, exports would be scheduled instead of imports and Dutch renewables would receive the higher German electricity price (assuming transmission is not constrained).

3.4.2 Comparison with Nordpool

I test the theoretical claim that integrating energy and transmission markets reduces market power, using hourly data from January 2000 to November 2001. If true, in an integrated energy and transmission market like Nordpool, the probability that an import constraint into the small country with a larger demand slope will be resolved by the same hour next day should be higher than the probability of an export constraint being resolved. Under separate markets, e.g. the German-Dutch interconnector, both probabilities should be identical.

Figure 9 shows the member countries of Nordpool. Sweden and Finland each constituting one zone in the initial market splitting, while Norway and Denmark are split up into several zones to address internal transmission constraints. Discussion of market power in Sweden goes back to Andersson and Bergman (1995). Johnsen, Verma and Wolfram (1999) identify market power in Norway if transmission constraints are binding. I will focus on Northern Norway NO2 because

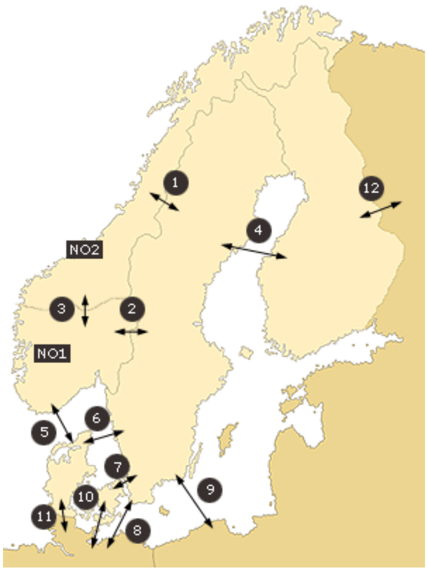

Figure 9: Different zones of Norpool to which market splitting is applied. Connection 1 between Northern Norway NO2 and Sweden represents the two-node model and is frequently constrained.

it represents a two-node model with the major interconnection to Sweden by transmission links

(1) with capacity of more than 1000MW. Interconnection (3) towards Southern Norway NO1 is comparatively weak, at only 300MW, and exhibits almost identical behaviour because Southern Norway is well-integrated with the Swedish market. Northern Norway is sometimes split up in two separately priced zones, Tromsø and Trondheim, but prices in both zones behave almost identically; therefore, only results for Trondheim are presented. Concentration in Northern Norway is high, with Statkraft owning 3002MW of 6287MW installed capacity.[17]

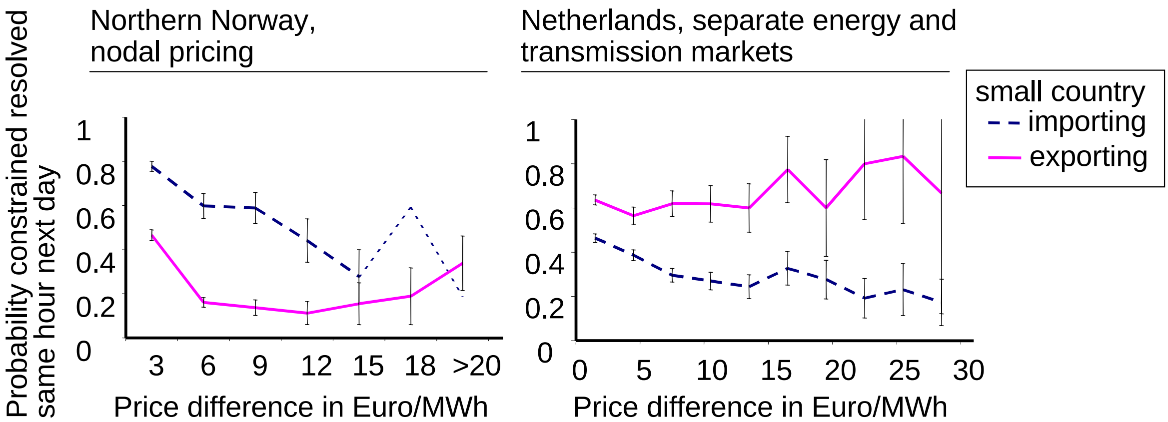

Figure 10: Observed probabilities that a price difference $|\Delta p|$ does vanish by the same hour next day, for the small market importing and exporting. (Northern Norway and Netherlands)

Figure 10 shows that, in Northern Norway, the probability is higher that an import constraint is resolved by the same hour of next day than an export constraint for all price differences $|\Delta p|$. The observation confirms Proposition 3 assuming the Swedish market is competitive or Proposition 2 assuming the Swedish market is oligopolistic. This confirms the model, which predicts that integrating markets mitigates market power.

Figure 10, right, shows that, in The Netherlands, the probability is lower that an import constraint is resolved than that an export constraint is resolved. Propositions 3 and 2 predict the opposite result for the case of an integrated market while Proposition 4 suggests equal probabilities for the case of separate markets. The result is closer to separate markets.

The difference could be attributed to two reasons: first, traders and energy companies usually schedule imports into the Netherlands in addition to the trades on the spot market. These imports are cancelled if a negative APX-LPX price difference occurred the day before, pushing towards a reversal of the price. Second, Dutch generators have sufficient market power to coordinate using the German electricity price as a lower limit, and adjust their bids upward if

[17] Norwegian Competition Authority 2002, published in context of enquiry into acquisition of Trondheim Energiverk (TEV) by Statkraft.

they observe that the Dutch price is lower. The second effect would also be present in Northern Norway but seems to be dominated by the incentives to deviate from an import constrained situation in an integrated transmission and energy market.

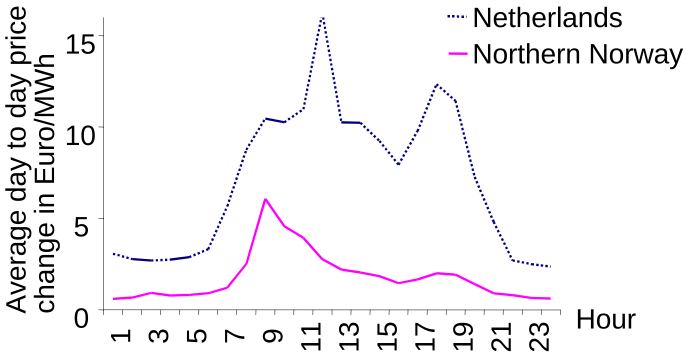

Figure 11:

I use the asymmetry between import- and export-constrained situations to eliminate other effects that might distort the results. These other effects can be observed when comparing deviations from the export-constraint scenario in Northern Norway with deviations in the Netherlands. Significantly, the higher probability of all deviations in the Netherlands can be explained by higher price volatility in the Netherlands, illustrated in Figure 11 (For a systematic comparison see Bower, 2002). Nordpool prices are generally more stable, because their main determinant is the water level in hydro storage, which evolves slowly, as Johnsen, Verma and Wolfram (1999) argue.

One effect that one might expect to distort the analysis is the different generation pattern. Northern Norway generates electricity exclusively from hydro power. Production is sometimes constrained by generation capacity and sometimes by the energy stored in the dams. The Netherlands mainly use coal and gas. However, the analysis compares the same hour of consecutive days and should therefore not 'pick up' different peaking behaviour in the first place. Using the asymmetry between deviation from imports and exports finally ensures that generation technology and demand patterns that are independent from power flows on the interconnector are filtered.

3.5 Conclusion of partially binding transmission constraints

Output choice of strategic generators in a two-node network has been calculated as a function of demand difference between the nodes. If strategic generators are located at both nodes, small demand differences result in an integrated market. If demand difference increases, a mixed strategy equilibrium with partially-binding transmission constraint exists. For large demand differences, both markets are separated. In comparison, output of strategic generators under separate transmission and energy markets is lower, which can be interpreted as reduced welfare. Empirical evidence from the German-Dutch interconnector shows that the separation of energy and transmission markets prevents traders from arbitraging the interconnector in realisation, and allows them at most to arbitrage at expectation. In the second step, the German-Dutch interconnector is compared with the interconnection between Northern Norway and Sweden. Theory suggests, and empirical evidence confirms, that it is more likely that the import constraint into the small zone of Northern Norway is resolved by next day than that the export constraint out of Northern Norway is resolved. This is because it is more profitable for generators in the small country to deviate from an import constraint towards an unconstrained equilibrium in order to face the large market than it is for generators in the large market to deviate from an import constraint situation to obtain a small additional benefit in the small market. In contrast, theory suggests that a constraint into and out of the 'small' country, the Netherlands, should be resolved with equal probability, because deviation does not change flows and does not, therefore, change the constraint. Empirical evidence even shows that the probabilities are not only equal, but even inverted, which can be explained by longer term transmission and energy contracts.

4 Constrained transmission in meshed networks

The two node model of the previous section captures some of the economics of electricity networks and can be used in small, mainly linear transmission networks. Large electricity grids typically exhibit several different constraints and their treatment requires representation of the meshed nature of the network. The effects discussed for the two node network still apply in the meshed network. In addition, as will be shown in this section, the allocation of transmission in meshed network even matters if constraints are permanently binding.

In electricity networks, Kirchhoff's laws start to bite: a fraction of the energy transmitted in an electricity network between any two nodes passes through virtually every link of the network. For an efficient dispatch, the challenge is therefore not only to schedule the right amount of energy transmission between two nodes, but also to use scarce transmission capacity on links for transmissions between nodes where it provides the highest value.

With nodal pricing, market participants submit bids or bid schedules to the system operator (SO) which specify the node, price and quantity at which they would like to supply or obtain energy. The SO calculates prices for each node, such that total surplus is maximised implying optimal allocation of transmission capacity while transmission constraints are satisfied and total demand matches supply.[18] Lower-priced supply bids and higher-priced demand bids at each node

[18] The price determination is based on the assumption that bids are cost reflective. If sufficient information

are accepted and paid for at this price.

Separate transmission markets define physical property rights for scarce transmission capacity which have to be present when scheduling transmission between different nodes of the network. Trade of the rights results in an efficient usage of scarce capacity and the design coincides with nodal pricing in the presence of complete and competitive markets.

What is the effect of separation of transmission and energy markets in the presence of market power? Separate markets imply that the configuration of transmission rights determines which energy flows traders have to schedule. Flows are therefore no longer a function of changing bids of generators in the energy spot market.[19] Integrating the markets therefore reduces the slope of the demand curve (4.1). Figure 12, left, illustrates that integration with changing demand slopes does not change outcome of competitive markets because output is only determined by intercept of demand and marginal costs not the slope. In contrast, generators with market power determine their output based on demand slope, and a decrease in demand slope result in higher output (4.2). Figure 12, right, illustrates that generators could continue to produce at the previous output level, but increase output towards the competitive choice because $\pi_I > \pi_S$.

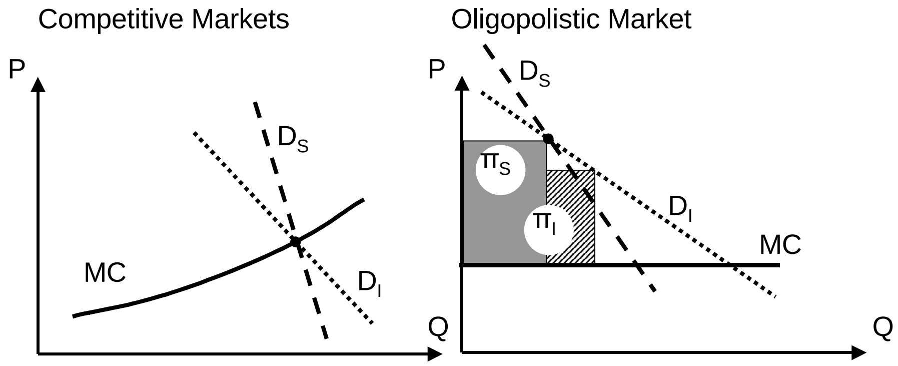

Figure 12: If separate (S) markets are integrated (I) demand slope decreases, with no effect in competitive markets but increased output of a monopolist.

The analysis assumes that the same transmission constraints are always binding, whereas the effects described in the first part required generators to relax transmission constraints on their outputs.

about generators with market power and their location is available, the algorithm determining nodal prices can be changed to mitigate market power (See DAE Mimeo Gilbert, Neuhoff, Newbery 2001 for an example in a three-node network).

[19]Flowgate rights would in theory allow bilateral trading to allow for reconfiguration of energy flows to match changing output decisions of generators. In practice with complex congestion patterns flowgate rights are frequently considered to be too complex for implementation.

4.1 Integration of energy and transmission markets increases demand elasticity

" *While the change in an x with respect to its own parameter is always negative regardless of the number of constraints, it is most negative if there are no constraints, only less so when there is a single constraint, and so forth ...* " Samuelson (1947).

I show that Le-Chatelier Samuelson principle is also applicable for electricity networks. This is not directly apparent, and traditional proofs do not apply, because the conditions required for the application of these proofs are not satisfied, e.g. Samuelson (1947), Kusumoto (1976), Fujimoto (1980), Milgrom and Roberts (1996). The proofs do not cover the twofold appearance of prices in transmission trading, first prices clear local markets and therefore determine net-exports from every node and second price differences between nodes are linked to price differences between other nodes by the scarcity value of constraint transmission links.

The allocation of transmission capacity is based on supply and demand bids by generators either to the system operator or to energy spot markets. Bids can be either quantity bids as in a Cournot game with the market clearing price determined by the intersection with demand, or bids can be supply functions as in a supply function equilibrium (Green and Newbery, 1992). The system operator does not differentiate between competitive or any kind of strategic bids and always applies the same transmission allocation mechanism. Neither do competitive traders differentiate between the bids when arbitraging the markets. The calculation of network flows and prices according to Bohn, Caramanis and Schweppe (1984) is therefore applicable both for competitive and strategic bids. The algorithm to define nodal pricing can be summarised as: The system operator allocates transmission capacity as if energy bids were competitive and he wanted to maximise welfare. The calculation is based on a DC approximation. This allows for linear treatment of all constraints while maintaining a sufficiently accurate representation of the underlying physical reality. In appendix D the effect of relaxing a constraint set by separation of energy and transmission markets is determined and the following result is calculated both in an intuitive way for one binding constraint and for several binding constraints:

Theorem 5 *Generators at any node of a meshed network face a weakly flatter effective demand curve under nodal pricing than under separate transmission and energy markets. (Proof in appendix D.)*

4.2 Increased demand elasticity reduces market power

If market power is present at only one node, then increased demand elasticity at the node has the typical effect of increasing output and reducing prices of strategic generators at the node.

This section addresses the question of the network interactions.

In the DC approximation of the network transmission constraints produce linear constraints on flows. A set of constraints defines a hyperplane restricting the larger space of possible flow patterns. As long as the same constraints stay binding the flow patterns can therefore be represented by linear equations. If this assumption, that the same constraints stay binding independent of strategic output choices of generators, is not satisfied, then the subsequent analysis does not necessarily apply and the number of possible combination of binding transmission constraints requires a numerical treatment. As long as the binding transmission constraints do not change, price can be expressed as linear function of total output Q_i of oligopolists at two different nodes $i = 1, 2$:

$$p_1 = \overline{P}_1 + \frac{\overline{Q}_1 - Q_1}{\alpha_1 + \delta\lambda_1} + \delta\frac{Q_2 - \overline{Q}_2}{\eta_1} \qquad p_2 = \overline{P}_2 + \frac{\overline{Q}_2 - Q_2}{\alpha_2 + \delta\lambda_2} + \delta\frac{Q_1 - \overline{Q}_1}{\eta_2}. \tag{11}$$

The indirect demand functions are defined such that they pivot around equilibrium price $\overline{P}_i$ and total production $\overline{Q}_i$ in the case of separate energy and transmission markets. The pair $(\overline{P}_i, \overline{Q}_i)$ obviously represents a feasible solution, even so, as previously illustrated in Figure 12, it might not be profit maximising depending on the slope of the demand curve.

If δ takes the value $\delta = 1$ then (11) represents a design with integrated transmission and energy markets which allow for flexible allocation of transmission capacity. For $\delta = 0$ transmission and energy markets are separate and transmission capacity and energy flows are therefore predetermined at the time of the energy spot markets. According to theorem 5 demand elasticity is weakly increased if energy and transmission markets are integrated, therefore $\lambda_i \geq 0$. The sign of η_i depends on whether output at the two nodes is a 'local substitute' or 'local complement'. Following Joskow and Tirole (2000), output is a 'local substitute' ($\eta_i > 0$) if an output increase at one node reduces the price at the other node. Integrating energy and transmission markets then reduces prices at both nodes, which implies that total demand is increased, and therefore total output must have been increased. It is not guaranteed that output at each node is increased, since it could also have been substituted between the nodes. Output is a 'local complement' ($\eta_i < 0$) if output increase at one node reduces prices at the other node. Local complements are due to the effect that electric energy flows in opposite directions on a link are superimposed: A transmission line with a transmission limit of 1 unit can therefore transmit 2 units of energy if 1 unit is directed in the opposite direction.

Assume n generators are located at node one and m generators are located at node two of the meshed network. Costs of generators are symmetric at each node but can differ between the nodes $i = 1, 2$ and are a function of individual output q_i:

$$C_i(q_i) = \frac{\beta_i}{2} q_i^2 - c_i q_i.$$

The profit function of a generator at node one (and similar at node two) is:

$$\pi_1(q) = q\left(\overline{P}_1 + \frac{\overline{Q}_1 - (n-1)\,q_1 - q}{\alpha_1 + \delta\lambda_1} + \delta\frac{mq_2 - \overline{Q}_2}{\eta_1}\right) - \frac{\beta_1}{2}q^2 - c_1 q.$$

The first order condition and symmetry between generators $q_1 = q$ gives:

$$q_1 = \frac{\overline{P}_1 + \frac{\overline{Q}_1}{\alpha_1+\delta\lambda_1} + \delta\frac{mq_2-\overline{Q}_2}{\eta_1} - c_1}{\frac{n+1}{\alpha_1+\delta\lambda_1} + \beta_1}, \qquad q_2 = \frac{\overline{P}_2 + \frac{\overline{Q}_2}{\alpha_2+\delta\lambda_2} + \delta\frac{nq_1-\overline{Q}_1}{\eta 2} - c_2}{\frac{m+1}{\alpha_2+\delta\lambda_2} + \beta_2}.$$

Combining both equations allows expression of q_1 as a function of parameters:

$$q_1 = \frac{\overline{P}_1 - c_1 + \frac{\overline{Q}_1}{\alpha_1+\delta\lambda_1} + \delta\frac{m\frac{\overline{P}_2 - c_2 + \frac{\overline{Q}_2}{\alpha_2+\delta\lambda_2} - \delta\frac{\overline{Q}_1}{\eta 2}}{\frac{m+1}{\alpha_2+\delta\lambda_2}+\beta_2} - \overline{Q}_2}{\eta_1}}{\frac{n+1}{\alpha_1+\delta\lambda_1} + \beta_1 - \delta\frac{m\frac{\delta\frac{n}{\eta 2}}{\frac{m+1}{\alpha_2+\delta\lambda_2}+\beta_2}}{\eta_1}}. \tag{12}$$

Setting $\delta = 0$ and $Q_1 = nq_1$ ($Q_2 = mq_2$) gives the equilibrium output choice with separate energy and transmission markets:

$$\begin{aligned} Q_1 &= n\frac{\alpha_1}{1+\beta_1\alpha_1}\left(\overline{P}_1 - c_1\right), \quad Q_2 = m\frac{\alpha_2}{1+\beta_2\alpha_2}\left(\overline{P}_2 - c_2\right), \\ P_1 &= \frac{Q_1/n * (1+\beta_1\alpha_1) + c_1\alpha_1}{\alpha_1}. \end{aligned}$$

Now assume that the demand slope has been defined by $(\overline{Q}_i, \overline{P}_i) = (Q_i, P_i)$, such that Q_i can substituted in the (12):

$$\begin{aligned} q_1 &= (P_1 - c_1)\frac{\frac{(1+\beta_1\alpha_1)(\alpha_1+\delta\lambda_1)+n\alpha_1}{(1+\beta_1\alpha_1)(\alpha_1+\delta\lambda_1)} - \delta^2\frac{m}{\eta_1}\frac{n}{\eta 2}\frac{\alpha_1}{(1+\beta_1\alpha_1)\left(\frac{m+1}{\alpha_2+\delta\lambda_2}+\beta_2\right)}}{\frac{n+1}{\alpha_1+\delta\lambda_1} + \beta_1 - \delta^2\frac{m}{\eta_1}\frac{n}{\eta 2}\frac{1}{\frac{m+1}{\alpha_2+\delta\lambda_2}+\beta_2}} \\ &\quad + \delta\frac{m}{\eta_1}(P_2 - c_2)\frac{\frac{(1+\beta_2\alpha_2)(\alpha_2+\delta\lambda_2)+m\alpha_2}{(1+\beta_2\alpha_2)(m+1+\beta_2(\alpha_2+\delta\lambda_2))} - \frac{\alpha_2}{1+\beta_2\alpha_2}}{\frac{n+1}{\alpha_1+\delta\lambda_1} + \beta_1 - \delta^2\frac{m}{\eta_1}\frac{n}{\eta 2}\frac{1}{\frac{m+1}{\alpha_2+\delta\lambda_2}+\beta_2}}. \end{aligned}$$

The change in output as result of integration of energy and transmission markets can now easily be calculated:

$$\begin{aligned} \Delta q_1 &= q_1(\delta=1) - q_1(\delta=0) \\ &= \left(\frac{m+1}{\alpha_2+\lambda_2} + \beta_2\right)\frac{(P_1 - c_1)\frac{\lambda_1}{(1+\beta_1\alpha_1)(\alpha_1+\lambda_1)} + (P_2 - c_2)\frac{m}{\eta_1}\frac{\lambda_2}{(1+\beta_2\alpha_2)(m+1+\beta_2(\alpha_2+\lambda_2))}}{\left(\frac{n+1}{\alpha_1+\lambda_1} + \beta_1\right)\left(\frac{m+1}{\alpha_2+\lambda_2} + \beta_2\right) - \frac{m}{\eta_1}\frac{n}{\eta 2}}. \end{aligned} \tag{13}$$

All components of (13) will be positive if output at both nodes is a 'local complement', allowing the general proposition:

Proposition 6 *If constraints are binding permanently and output at both nodes is a 'local complement', output will increase at both nodes after integration of energy and transmission markets.*

Proof. If output of both nodes is a 'local complement' $\eta_1 > 0$, therefore the nominator of (13) is positive and the denominator is positive, according to Lemma 17 (Appendix C). ■

However, if output at both nodes is a 'local substitute' then $\eta_1 < 0$ and Δq_1 can potentially turn negative. Instead, price changes at both nodes are assessed:

Proposition 7 *If constraints are binding permanently and output at both nodes is a 'local substitute', prices will decrease at both nodes after integration of energy and transmission markets.*

Proof. Prices (11) change with changes in output of strategic generators by

$$\Delta p_1 = \frac{-n}{\alpha_1 + \lambda_1}\Delta q_1 + \frac{m}{\eta_1}\Delta q_2.$$

Substituting Δq_i from (13) gives

$$\begin{aligned}\Delta p_1 \quad &= \quad \frac{\lambda_1 n \frac{P_1 - c_1}{(1+\beta_1\alpha_1)(\alpha_1+\lambda_1)}\left(\frac{m}{\eta_1\eta_2} - \frac{m+1+(\alpha_2+\lambda_2)\beta_2}{(\alpha_2+\lambda_2)(\alpha_1+\lambda_1)}\right)}{\left(\frac{n+1}{\alpha_1+\lambda_1} + \beta_1\right)\left(\frac{m+1}{\alpha_2+\lambda_2} + \beta_2\right) - \frac{m}{\eta_1}\frac{n}{\eta 2}} \\ &\quad + \frac{\lambda_2 \frac{P_2 - c_2}{(1+\beta_2\alpha_2)}\frac{m}{\eta_1}\left(\frac{1+(\alpha_1+\lambda_1)\beta_1}{(\alpha_1+\lambda_1)(\alpha_2+\lambda_2)}\right)}{\left(\frac{n+1}{\alpha_1+\lambda_1} + \beta_1\right)\left(\frac{m+1}{\alpha_2+\lambda_2} + \beta_2\right) - \frac{m}{\eta_1}\frac{n}{\eta 2}}.\end{aligned}$$

The denominator is positive according to Lemma 17 (Appendix C). Using Lemma 16 (Appendix C), the bracket in the denominator of the first term is $-\frac{1+(\alpha_2+\lambda_2)\beta_2}{\eta_1\eta_2}$, and the first term is therefore negative. In the second term, $\eta_1 < 0$ for the substitute, and therefore the second term is negative as well, implying that $\Delta p_1 < 0$. ■

4.3 Conclusion for constrained meshed networks

Integration of energy and transmission markets for meshed networks increases demand elasticity which generators face at each node. If strategic generators are located at one node, output will be increased and welfare improved. If strategic generators are located at two nodes, without cross holding of ownership, total output by strategic generators will be increased. If output at the nodes at which strategic generators are located is 'local substitute', prices at both nodes will decrease. If output is a 'local complement', output of strategic generators at both nodes will increase. The results suggest that integration of transmission and energy market mitigates market power in meshed networks.

5 Conclusion

The paper assumes all physical transmission contracts are acquired by traders and cannot provide financial incentives for generators to alter their energy bids as in Joskow and Tirole (2000).

Does nodal pricing with integrated energy-and-transmission markets then differ from physical transmission contracts, with subsequent separate energy markets?

The question is first asked for the case that a transmission line between two nodes is sometimes constrained, and then for the case of one or several permanently constrained transmission lines in a meshed network where the dispatch of individual generators does not change the selection of lines that is constrained while it can change flow patterns.

In a two-node model, I calculate strategies of generation companies for integrated and separated energy and transmission markets, including a mixed-strategy equilibrium with four generators located at two nodes. The analysed case illustrates that output is larger in the integrated market with nodal pricing and a general proof is provided for the case of an unconstrained transmission line: integration of transmission and energy markets mitigates market power and increases output.

The theory can be tested with an indirect effect using the asymmetry between probability of the price difference caused by an import and an export constraint being annulled by the same hour of the next day. Using the asymmetry allows comparison over different markets, while reducing the impact of country-specific effects of generation mix and demand profile. The comparison between the German-Dutch interconnector - with separate transmission and energy markets - and the integrated market between Sweden and Northern Norway, does not lead to the rejection of the hypothesis that integration mitigates market power.

In a meshed electricity network, integration allows flexible allocation of transmission capacity. I show that Le Chatelier Samuelson's principle is applicable in this case where transmission prices and local energy prices are linked. Integration of transmission and energy markets increases demand elasticity in meshed networks if the same transmission constraints continue to be binding. If generators with market power are located at one node of any meshed network, this increases welfare. If generators with market power are located at two of the nodes, without cross holding, total output will be increased either if the nodes are 'local substitutes' or if they are 'local complements'. Further research is required to ensure that the results hold for market power at additional nodes, and to assess the impact of generators' cross holding of assets at several nodes.

The empirical evidence furthermore supports Hogan (1997) that separate energy and transmission markets are inefficient in the presence of uncertainty. Usually, spot markets are specifically introduced to reveal private information. It is therefore inconsistent to introduce a sequential mechanism for decisions on energy transmission, which can only work efficiently if traders correctly predict spot market prices. Generation from wind, solar and CHP have output which is not predictable long-term, and information is only aggregated in the spot market. Therefore, a higher contribution by these energy sources will increase the inefficiency of the separate energy and transmission market. The separation is also biased against intermittent generation because prices will be excessively low at times of unexpected high generation.

This paper should be of relevance for enhancing competition and integrating European electricity markets, suggests some changes to the currently proposed joint transmission auctions, and might contribute to FERC discussions on standard market design. A further application might be in strategic trade theory. Extensive discussion has focused on the different effects between tariffs and quotas, whereas this paper suggests that quotas covering several product categories or countries increase elasticity of demand or supply relative to narrowly-defined quotas. This should increase competitiveness of markets in the presence of quotas.

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A Existence of pure strategy equilibria

If strategic generators are located at only one instead of both nodes as previously discussed, then the interval in which a pure strategy equilibrium does note exist is typically empty and a pure strategy equilibrium will always exist.

An intuitive explanation for the existence of a pure strategy equilibrium is, that if strategic generators are located only at the exporting node then they do not face a non-convexity in net demand when the export constraint is relaxed. If strategic generators are only located at the importing node, then they face a non-convexity. However, it is not as strong as if strategic generators are located at both nodes because the following effect is missing, that facilitates deviation from a hypothetical equilibrium:

If the hypothetical equilibrium implies a constrained transmission link, then production of generators at the exporting node is lower and generators at the importing node need to increase output and reduce prices less to face the integrated market.

If the hypothetical equilibrium implies an unconstrained transmission link, then production of all generators is bigger and generators at the importing node have to decrease output by less to separate the markets and face the higher demand slope of one node only. The results can be summarized in the following theorem:

Theorem 8 *In a two-node network with elastic demand at both nodes and symmetric generators located at one node, a pure strategy Nash equilibrium always exists if (but not iff)* $\beta \leq \frac{(n+1)D_1}{D_2 b_1 - D_1 b_2}$, *with* β *slope of marginal costs and demand including maximum possible transmission at node i of:* $D_i - b_i p_i$.[20]

Before going to the proof, I define *local Nash equilibria* as a set of output choices $\{q_i\}$ and a parameter ε, such that $\{q_i\}$ is a Nash equilibrium if for each generator i output choice q is restricted to $q \in [q_i - \varepsilon, q_i + \varepsilon]$. To facilitate notation, demand is formulated as net demand with transmission capacity fully utilised. Demand intercept therefore includes expected transmission K, from the exporting node $D_1 = A + K$, and to the importing node $D_2 = A + D - K$. Cost of generation is as before $C(q) = \frac{\beta}{2} q^2$, with constant marginal costs eliminated by transformation of the overall price level.

Proof. If the generators are located at node one (export) then Figure 3 shows that they face convex demand as long as they do not deviate towards an import-constrained situation, which is covered by the discussion of generators located at node 2 (import). Convex demand guarantees that the local optimal output decision (FOC) is globally optimal; therefore, a pure strategy Cournot equilibrium exists.

Generators are located at node two (import). If they assume that the transmission constraint is binding, they choose output q_c, taking output decision $\overline{q}$ of other $(n-1)$ generators as given

[20] The right-hand side will not be negative if zone two is import-constrained. Assuming otherwise would imply $\frac{D_2}{b_2} < \frac{D_1}{b_1}$; that is, without production of strategic generators only using transmission capacity for transports from node one to two, the price in the importing region two is lower than in the exporting region one, violating the assumption of a binding import constraint.

to maximise profits in their local market with constant imports:

$$\pi(q_c) = \frac{D_2 - (n-1)\overline{q} - q_c}{b_2} q_c - \frac{\beta}{2} q_c^2, \tag{14}$$

and, using symmetry among generators, choose output:

$$q_c = \frac{D_2}{n+1+b_2\beta}. \tag{15}$$

If generators assume that the transmission constraint will not be binding, they anticipate facing the demands of the integrated market and maximise profits:

$$\pi(q_u) = \frac{D_1 + D_2 - (n-1)\overline{q} - q_u}{b_1 + b_2} q_u - \frac{\beta}{2} q_u^2, \tag{16}$$

resulting in output choice:

$$q_u = \frac{D_1 + D_2}{n+1+(b_1+b_2)\beta}. \tag{17}$$

Is the transmission link is constrained (unconstrained) if all generators choose output q_c (q_u)?

Using q_u (17) and q_c (15) the condition $q_c < q_u$ is satisfied for:

$$\beta < \frac{(n+1)D_1}{D_2 b_1 - D_1 b_2}. \tag{18}$$

Assume the transmission link is constrained if generators choose q_u, then $q_c < q_u$ guarantees that it is also constrained for q_c; q_c therefore represents a local constrained equilibrium. On the other hand, an unconstrained link after a choice of q_c and $q_c < q_u$ ensures the existence of a local unconstrained equilibrium with output choice q_u. If a local Nash equilibrium exists, then it is either a Nash equilibrium or deviations are possible. If deviations are possible, Lemmas 9 and 10 guarantee the existence of a Nash equilibrium. ■

Lemma 9 *If deviations from a local Nash equilibrium with constrained transmission link are profitable, a local Nash equilibrium with unconstrained transmission link exists (I), and is a Nash equilibrium (II).*

Proof. (I) Assume the profitable deviator from a local Nash equilibrium with constrained transmission link (constrained local Nash equilibrium) reduces output. Profitable deviation is only possible because of the non-convexity in demand when the transmission constraint is relaxed. Therefore, aggregate production after deviation must result in a relaxed transmission constraint.

During the deviation, the profits of the deviator only are maximised; he chooses output according to the additional demand of the integrated market as if he were a monopolist. If all generators anticipate the deviation, they will choose an oligopolist output, which is more competitive and therefore bigger. This effect is enhanced because of higher demand elasticity of the integrated market. Therefore, the import constraint stays relaxed and a unconstrained local Nash equilibrium exists.

(II) Deviation from the constrained local Nash equilibrium is profitable; therefore, a generator anticipating a constrained link makes lower profits than a generator anticipating an unconstrained link. In the move from the constrained towards the unconstrained local Nash equilibrium, output Q_r of the remaining generators increases. According to Lemma 11, an increase in Q_r reduces profits of a generator anticipating a constrained link more than it reduces profits of a generator anticipating an unconstrained link. Therefore, in the unconstrained local Nash equilibrium, profits of a generator anticipating a constrained link are lower than profits of a generator anticipating an unconstrained link, deviation is not profitable, and the unconstrained local Nash equilibrium is a Nash equilibrium. ■

Lemma 10 *If deviations from a local Nash equilibrium with unconstrained transmission link are profitable and generators are symmetric with marginal cost curve satisfying* $\beta \leq \frac{(n+1)D_1}{D_2 b_1 - D_1 b_2}$*, a local Nash equilibrium with constrained transmission link exists (I) and is a Nash equilibrium (II).*

Proof. (I) If a deviation from the unconstrained equilibrium is to be profitable, the interconnector must be constrained if $(n-1)$ choose q_u and one generator reduces output q_d to constrain the interconnection: $q_d < q_u$. If the remaining generators reduce output q_c even further

$q_c < q_d$ when they anticipate the constraint, a local constrained equilibrium exists.

Substituting q_u from (17) for $\overline{q}$ in the profit function (16) and calculating the FOC gives the deviating output choice:

$$q_d = \frac{D_2 - (n-1)\frac{D_1 + D_2}{n+1+(b_1+b_2)\beta}}{2 + b_2\beta}. \tag{19}$$

Substituting (17),(15) and (19) in $nq_c < (n-1)q_u + q_d$ gives:

$$\beta < \frac{(n+1)\,D_1}{b_1 D_2 - b_2 D_1}.$$

(II) Given $nq_c < (n-1)q_u + q_d$ and $q_d < q_u$ it follows that $q_c < q_u$. Output of fellow generators Q_r is reduced when they anticipate a constrained rather than unconstrained transmission link. According to proposition 11, the profits from choosing output resulting in a constrained link are thereby increased more than profits from choosing an output resulting in an unconstrained link. $\Delta\pi_{constrained} > \Delta\pi_{unconstrained}$. Given that choosing output that results in a constrained link was more profitable at the start: $\pi_{constrained} > \pi_{unconstrained}$, it is even more so after fellow generators adopted their output choice:

$$\pi_{constrained} + \Delta\pi_{constrained} > \pi_{unconstrained} + \Delta\pi_{unconstrained},$$

and therefore the local constrained equilibrium is a Nash equilibrium. ■

Lemma 11 *Profits of a generator π decrease more slowly with output increase of remaining generators Q_r if he chooses the output such that the link stays unconstrained than if he constrains the link:* $\frac{\partial \pi_{constrained}}{\partial Q_r} < \frac{\partial \pi_{unconstrained}}{\partial Q_r}$.

Proof. A generator anticipates the remaining generators will produce Q_r. If he intends to constrain the transmission link, he maximises (14) choosing output $q = \frac{D_2 - Q_r}{2 + b_2 \beta}$ and makes profits

$$\pi_{constrained} = \frac{(D_2 - Q_r)^2}{2b_2 (2 + b_2 \beta)}.$$

If, alternatively, he intends not to constrain the transmission link, then by maximising (16) he makes profits

$$\pi_{unconstrained} = \frac{(D_2 + D_1 - Q_r)^2}{2 (b_1 + b_2) (2 + (b_1 + b_2) \beta)}.$$

Using these functions $\frac{\partial \pi_{constrained}}{\partial Q_r} < \frac{\partial \pi_{unconstrained}}{\partial Q_r}$ can be written as $\frac{Q_r - D_2}{b_2(2 + b_2 \beta)} < \frac{Q_r - D_2 - D_1}{(b_1 + b_2)(2 + (b_1 + b_2)\beta)}$ or $\frac{D_2 + D_1 - Q_r}{(b_1 + b_2)(2 + (b_1 + b_2)\beta)} < \frac{D_2 - Q_r}{b_2(2 + b_2 \beta)}$ or $\frac{p_{intercept\_uncon}}{2 + b_2 \beta + b_1 \beta} < \frac{p_{intercept\_const}}{2 + b_2 \beta}$. The last inequality is satisfied because $p_{intercept\_uncon} < p_{intercept\_const}$. The constrained demand function intercepts with production Q_r of remaining generators above the intercept of unconstrained demand function with Q_r. Otherwise, the generator could not influence the constraint's status (and the constrained equilibrium would be a global Nash equilibrium). ■

B Proofs of propositions required for empirical test

Proof of proposition 2: *Transmission and energy markets are integrated and two generators with constant marginal costs are located at each of two nodes, demand is linear and demand shifts between the same hour of consecutive days are distributed symmetrically around 0. The slope of demand of the larger market exceeds $\frac{1+\sqrt{5}}{2}$ slope of demand at the smaller market. Then the probability of price difference Δp from an import constraint into the smaller market to vanish by the same hour next day is higher than the probability of an inverse price difference $-\Delta p$, caused by an export constraint, to vanish by the same hour next day.*

Proof. Demand at the node with steeper demand curve (smaller market) is $A + D - p$ where D is the parameter which is shifted to represent demand changes (Note that ΔD, not D, is distributed symmetrically around 0), and demand at the other node is $A - rp$ with $r > 1$. Lemma 13 shows that, if the smaller market is importing and price difference between the nodes is Δp, D must be decreased by

$$\Delta D_s = \frac{\left(6 + 4r - 5r\sqrt{(1 + r)} - 2r^2\right)}{2r (3 + 4r)} \left(A + \overline{K}\right) + 3\Delta p$$

to ensure that the unconstrained situation is more profitable. Lemma 12 shows that, if the larger market is importing, D must be increased by:

$$\Delta D_l = \frac{1-\sqrt{(1+r)}}{2r}\left(A+\overline{K}\right)+3\Delta p,$$

to make deviations from the import constraint profitable and hence remove the constrained equilibrium. The changes in D are again symmetrical and linear in Δp, but the small country requires smaller changes in D to allow profitable deviation. Now make sure that, in the change towards an unconstrained equilibrium from constraint, the small importing country is even smaller than the change to make an importing, large country deviate from a constraint situation:

$$\Delta D_s < \Delta D_l.$$

The inequality is satisfied if $r > \frac{1+\sqrt{5}}{2}$ and parameters $A, \overline{K}$ are positive - implying that the two nodes have to be sufficiently asymmetric to satisfy the strong conditions set.

The same argumentation can be applied to demand changes at the large node (replace A by $A-D$) and the results are replicated. The model is linear in demand changes; therefore, demand changes at both nodes result in a linear superposition, and the result is therefore general. ■

Lemma 12 *If the smaller market is exporting and price difference between the nodes is Δp, D must be increased by*

$$\Delta D_l = \frac{1+2r-2\sqrt{(1+r)\,r}}{r}\left(A-\overline{K}\right)-3\Delta p,$$

to make deviation profitable.

Proof. In a Cournot game, generators in the import constraint large node choose output q_n, resulting in prices p_n and profits π_n:

$$q_n = \frac{A-\overline{K}}{3}, \quad p_n = \frac{A-\overline{K}}{3r}, \quad \pi_n = \frac{1}{r}\left(\frac{A-\overline{K}}{3}\right)^2. \tag{20}$$

If an importing generator deviates towards an unconstrained situation, optimal output choice would be q_d, resulting in profits π_d:

$$q_d = \frac{1}{6}\left(3A+D-\overline{K}\right), \quad p_d = \frac{1}{6}\frac{3A+D-\overline{K}}{1+r}, \quad \pi_d = \frac{1}{1+r}\left(\frac{3A+D-\overline{K}}{6}\right)^2. \tag{21}$$

If $\pi_d > \pi_n$, deviation is not only profitable but the transmission constraint is no longer binding either, and deviation is therefore feasible (See Figure 3). Substituting (20) and (21) in $\pi_d > \pi_n$ allows calculation of marginal value D_m, above which deviations are profitable:

$$D_m = \overline{K} - 3A + 2\left(A-\overline{K}\right)\sqrt{\frac{1+r}{r}}.$$

If the constraint is binding, price at the exporting node p_e, and therefore price difference Δp to importing node p_n (20), is:

$$p_e = \frac{A + D + \overline{K}}{3} \qquad \Delta p = \frac{2A - \left(A + \overline{K}\right)(1 + r) - Dr}{3r}.$$

Inverting Δp gives $D(\Delta p)$:

$$D(\Delta p) = -\frac{3\Delta pr - A + Ar + \overline{K} + \overline{K}r}{r},$$

and allows for calculation of the shift in demand required to facilitate deviation from the import constraint $\Delta D_l = D(\Delta p) - D_m$. ■

Lemma 13 *If the smaller market is importing and price difference between the nodes is Δp, D must be increased by:*

$$\Delta D_s = \frac{\left(A + \overline{K}\right)\left(6 + 4r - 5r\sqrt{(1 + r)} - 2r^2\right)}{2r\,(3 + 4r)} + 3\Delta p,$$

to make the unconstrained situation an equilibrium.

Proof. Output and profit in an integrated market are:

$$q_i = \frac{2A + D}{5}, \qquad \pi_i = \frac{1}{1 + r}\left(\frac{2A + D}{5}\right)^2.$$

A generator at the importing node reducing output to constrain the link chooses:

$$q_d = \frac{3A + 4D - 5\overline{K}}{10}, \qquad \pi_d = \left(\frac{3A + 4D - 5\overline{K}}{10}\right)^2.$$

Deviation is profitable and feasible if $\pi_d > \pi_i$, which implies that:

$$D > D_m = \frac{(1 + r)\left(10\overline{K} - 2A\right) - 4Ar + 5\sqrt{(1 + r)}\left(A + \overline{K}\right)}{6 + 8r}.$$

If the constraint is binding, price at the exporting node is p_e, at the importing node p_i and price difference is Δp:

$$p_e = \frac{A + \overline{K}}{3r} \qquad p_i = \frac{A - \overline{K} + D}{3} \qquad \Delta p = \frac{2Ar + Dr - \left(A + \overline{K}\right)(1 + r)}{3r}$$

Inverting Δp gives $D(\Delta p)$:

$$D(\Delta p) = -\frac{-3\Delta pr + Ar - A - \overline{K} - \overline{K}r}{r},$$

and allows for calculation of the shift in demand required to facilitate deviation from the import constraint $\Delta D_s = D(\Delta p) - D_m$. ■

Proof of proposition one-node market power:3 *Transmission and energy markets are integrated and two generators with constant marginal costs* α *are located at one node with demand* $A+D-p$ *which is interconnected with a link of transmission capacity* $\overline{K}$ *to a competitive market with net demand* $A-rp$ *with* $0 < r$ *and* $2-\sqrt{(1+r)} < \frac{3\alpha}{A+\overline{K}}r$*. The probability of price difference* Δp*, caused by an import constraint to the node were the oligopoly is located, to vanish by the same hour next day is higher than the probability that a price difference* $-\Delta p$*, caused by an export constraint, to vanish by the same hour of the next day.*

Proof. Analogue to Proposition 2 it must be shown that: $\Delta D_I < \Delta D_E$. Substituting from Lemma 14 and 15, this gives the condition $2-\sqrt{(1+r)} < \frac{3\alpha}{A+\overline{K}}r$. ■

Lemma 14 *If the duopoly market is exporting and price difference between the nodes is* Δp*,* D *must be increased by*

$$\Delta D_E = 3\Delta p,$$

to resolve the transmission constraint.

Proof. If the transmission constraint is binding, in the oligopoly model prices in exporting node p_e and importing node p_i result in price difference Δp:

$$p_e = \frac{A+D+\overline{K}+2\alpha}{3} \qquad p_i = \frac{A-\overline{K}}{r} \quad \Delta p = \frac{A-\overline{K}}{r} - \frac{A+D+\overline{K}+2\alpha}{3}$$

Inverting the equation for Δp gives the demand change required to resolve the transmission constraint. ■

Lemma 15 *If the duopoly market is importing and price difference between the nodes is* Δp*,* D *must be decreased by*

$$\Delta D_I = 3\Delta p - 3\alpha + \frac{2-\sqrt{(1+r)}}{r}\left(A+\overline{K}\right),$$

to resolve the transmission constraint.

Proof. Output choice and profits in the import constraint situation

$$q = \frac{A+D-\overline{K}-\alpha}{3} \qquad \pi_c = \left(\frac{A+D-\overline{K}-\alpha}{3}\right)^2.$$

When markets are integrated, output choice and profits are

$$q = \frac{2A+D-\alpha}{3} \qquad \pi_u = (1+r)\left(\frac{2A+D-\alpha}{3+3r}\right)^2.$$

The unconstrained equilibrium is preferred if $\pi_u > \pi_c$, with the marginal demand difference D_m of

$$D_m = \frac{-Ar+\overline{K}r+\alpha r+\left(1+1\sqrt{(1+r)}\right)\left(A+\overline{K}\right)}{r}.$$

Assuming constraint is binding, the price difference between the nodes is:

$$\Delta p = \frac{A + D - K + 2\alpha}{3} - \frac{A + \overline{K}}{r}.$$

Inverting gives $D(\Delta p)$ and allows for calculation of $\Delta D = D(\Delta p) - D_m$. ■

Proof of proposition 4: *Assume transmission and energy markets are separated, costs are quadratic, demand is linear and demand shifts between the same hour of consecutive days are distributed symmetrically around* 0. *Then the probability of price difference* Δp*, caused by an import constraint, to vanish by the same hour next day equals the probability of price difference* $-\Delta p$*, caused by an export constraint, to vanish.*

Proof. Assume two nodes $i = 1, 2$ with n_i generators, slope of marginal cost curve β_i and demand $A_i - b_i p_i$ and transmission capacity $\overline{K}$; therefore, Cournot prices in the case of binding transmission constraint are:

$$p_i = \frac{(1 + b_i\beta_i)\left(A_i \pm \overline{K}\right)}{b_i\left(n_i + 1 + b_i\beta_i\right)}.$$

If the transmission is constrained, nodal prices differ $\Delta p = p_1 - p_2$. If the transmission constraint is to be resolved, demand intercept A_i at one or two of the nodes must shift by at least $\Delta A_1, \Delta A_2$, such that

$$\Delta p \geq \frac{(1 + b_2\beta_2)}{b_2\left(n_2 + 1 + b_2\beta_2\right)}\Delta A_2 - \frac{1 + b_1\beta_1}{b_1\left(n_1 + 1 + b_1\beta_1\right)}\Delta A_1.$$

The equation is linear in ΔA_1 and ΔA_2 as long as demand is linear. The demand shift required to resolve a constraint resolving a price difference Δp is opposite to the demand shift required to resolve a price difference $-\Delta p$. ■

C Lemmas regarding output choice

Lemma 16 $(\alpha_1 + \lambda_1)(\alpha_2 + \lambda_2) = \eta_1\eta_2$.

Proof. If the markets are integrated, an output change Δq_1 results in price change $\Delta p_2 = \frac{\Delta q_1}{\eta_2}$. This price change corresponds to the price change which would be brought about by an output change at node two of $\Delta q_2 = (\alpha_2 + \lambda_2)\Delta p_2 = (\alpha_2 + \lambda_2)\frac{\Delta q_1}{\eta_2}$. Such an output change would have resulted in a change in demand at node one of $\Delta q_1 = (\alpha_1 + \lambda_1)\frac{\Delta q_2}{\eta_1} = (\alpha_1 + \lambda_1)\frac{1}{\eta_1}(\alpha_2 + \lambda_2)\frac{\Delta q_1}{\eta_2}$. ■

Lemma 17 *The denominator is positive*

Proof. Lemma 16 shows that $(\alpha_1 + \lambda_1)(\alpha_2 + \lambda_2) = \eta_1\eta_2$ and therefore

$$\begin{aligned}\left(\frac{n+1}{\alpha_1 + \lambda_1} + \beta_1\right)\left(\frac{m+1}{\alpha_2 + \lambda_2} + \beta_2\right) - \frac{m}{\eta_1}\frac{n}{\eta 2} &> \\ \frac{n+1}{\alpha_1 + \lambda_1}\frac{m+1}{\alpha_2 + \lambda_2} - \frac{m}{\eta_1}\frac{n}{\eta 2} &> \frac{m+n+1}{\eta_{1\eta 2}} > 0.\end{aligned}$$

■

D Integration increases slope of net demand

This section first gives an introduction to flow calculation according to Bohn, Caramanis and Schweppe (1984) and then provides the proof for theorem 5. Net demand P_i at any node i equals demand D_i minus supply $Y_{i,j}$ by all generators j located at the node:

$$P_i = D_i - \sum_j Y_{i,j}.$$

Given the net demand at each node, power flows on the network are determined entirely by the physical characteristics of the network. These characteristics can be represented by a transfer admittance matrix $\mathbf{H}$, which is a constant in the DC approximation. It is a function of the resistance of the links and allows the calculation of the vector $\overrightarrow{Z}$ of flows on all links as a function of net-demand $\overrightarrow{P}$ on the nodes:

$$\overrightarrow{Z} = \mathbf{H}\overrightarrow{P}. \tag{22}$$

The law of energy conservation implies that the difference between inserted energy and withdrawn energy equals network losses which I will set to zero to simplify subsequent calculations.

$$\sum_{i,j} Y_{i,j} - \sum_i D_i = -\sum_i P_i = L. \tag{23}$$

The system represented by (22) and (23) is overdetermined and one equation can be dropped by rewriting transfer matrix $\mathbf{H}$, such that row i only contains 0. Node i is called the swing bus and I assume, without loss of generality, that the swing bus is node one $i = 1$. Changes of P_1 will not directly influence $\overrightarrow{Z}$ in (22), but according to (23), they accompany changes of P_j, $j \neq 1$, and thereby 'indirectly' influence $\overrightarrow{Z}$ in (22).

The transfer admittance matrix (22) allows calculation of the effect of demand changes on link flows:

$$\frac{\partial Z_k}{\partial D_i} = h_{k,i} \qquad \frac{\partial^2 Z_k}{\partial D_i \partial D_j} = 0 \;\; \forall i,j,k.$$

The system operator determines nodal prices as if all energy bids were competitively priced and he were to maximise social surplus. This is implemented by maximising the sum of short run value added functions for customers F_i at all nodes, while satisfying the capacity constraints of generators $Y_{i,j} \leq \overline{Y}_{i,j}$ and transmission links $\overline{-Z_k} < Z_k < \overline{Z_k}$. Effectively the system operator maximises the following Lagrangian (24):

$$\begin{aligned} \pounds \;\; = & \;\; \sum_i F(D_i) - \sum_{i,j} \lambda_{i,j}\left(Y_{i,j} - c_{i,j}(Y_{i,j})\right) + \Theta\left(\sum_{i,j} Y_{i,j} - L - \sum_i D_i\right) \\ & - \sum_{i,j} \mu_{i,j}\left(Y_{i,j} - \overline{Y}_{i,j}\right) - \sum_k \eta_{k,+}\left(Z_k - \overline{Z_k}\right) + \sum_k \eta_{k,-}\left(Z_k - \overline{-Z_k}\right) \end{aligned} \tag{24}$$

The Lagrange parameters can be interpreted as marginal production costs of all generators, $\lambda_{i,j}$, energy prices at the swing bus, Θ, scarcity premiums of generators, $\mu_{i,j}$, and scarcity rent of transmission lines in either direction, $\eta_{k,+}$ and $\eta_{k,-}$. the maximisation of the. A transmission line can only be constrained in one direction, therefore define $\eta_k = \eta_{k,+} - \eta_{k,-}$. Demand D_i at any node i is assumed to be a function of local prices p_i, differentiable, and convex in p_i.

$$\frac{\partial D_i}{\partial p_i} = -\alpha_i(p_i), \quad \frac{\partial D_i}{\partial p_j} = 0 \text{ for } i \neq j. \tag{25}$$

The FOC of (24) with respect to D_i shows that the local price p_i equals the energy price at the swing bus, scaled by the local losses, plus the marginal constraint costs:

$$p_i = \frac{\partial F(D_i)}{\partial Di} = \Theta\left(1 + \frac{\partial L}{\partial D_i}\right) + \sum_k \eta_k \frac{\partial Z_k}{\partial D_i}, \qquad \forall i \tag{26}$$

where $\frac{\partial Z_k}{\partial D_i} = h_{k,i}$. The FOC $\frac{\partial \pounds}{\partial \eta_{k,+}} = 0$ or $\frac{\partial \pounds}{\eta_{k,-}} = 0$ give in the case of a binding constraint:

$$Z_k = \overline{Z}_k \text{ if } \eta_{k,+} > 0, \quad Z_k = -\overline{Z}_k \text{ if } \eta_{k,-} > 0. \tag{27}$$

Effective net demand facing generator j at node i equals $N_i = D_i - \sum_{m \neq j} Y_{i,m}$ and for node $l \neq i$ is $N_l = P_l = D_l - \sum_m Y_{l,m}$. In Cournot competition, the output of other generators is price inelastic, and is therefore included in N. Competitive generators or generators submitting a supply function have a price elastic output. Their local supply slope is added to demand slope $\alpha_i(p_i)$ as defined in (25). Output increase at higher prices is equivalent to demand decrease at higher prices. The y-axis intercept of the linear approximation of generator supply function or marginal cost curve is added to N_i.[21]

To simplify subsequent calculations, assume that the generator whose output decision is analysed is located at the swing bus $i = 1$. Notation is simplified by setting $y_1 \equiv Y_{1,j}$. Using (23) and $L = 0$ gives,

$$\sum N_i = y_1. \tag{28}$$

Assume that only constraint $\overline{Z_k}$ is binding, therefore $\eta_i = 0$ for $i \neq k$ and $\eta_k \neq 0$, so $p_i = \Theta + \eta_k h_{k,i}$ from (26). Notation is further simplified by defining $h_i \equiv h_{k,i}$ and $\overline{Z} = \overline{Z}_k$. Equations (22) and (27) give

$$\overline{Z} = \sum_i h_i N_i - h_1 y_1 = \sum_i h_i N_i, \tag{29}$$

where the second equality follows from $h_1 = 0$, as node 1 is the swing bus. Using (26) which now simplifies to $p_i = \Theta + \eta_k h_i$ gives for $i = 1$ that $p_1 = \Theta$ and allows elimination of η_k:

$$(p_2 - p_1)\, h_i = (p_i - p_1)\, h_2 \quad \forall i = 3, .., n. \tag{30}$$

[21] We do not require a linear supply function or linear increasing cost curve of competitive generators. As we only need the first order conditions, a linear approximation provides for an exact representation of the local behaviour.

The previous equations allow the calculation of the effective demand elasticity which a generator at node one faces,

Differentiating (29) with respect to y_1 and using (25) gives

$$\sum_{i=1}^{n} h_i \alpha_i \frac{dp_i}{dy_1} = 0. \tag{31}$$

Differentiating (28) with respect to y_1 and using (25) gives

$$\sum_{i=1}^{n} \alpha_i \frac{dp_i}{dy_1} = -1. \tag{32}$$

Finally, differentiating (30) with respect to y_1 gives:

$$h_2 \frac{dp_i}{dy_1} = (h_2 - h_i) \frac{dp_1}{dy_1} + h_i \frac{dp_2}{dy_1} \tag{33}$$

Prices at any nodes i can change with output of a generator at, for example, the swing bus 1. As price differences between nodes determine transportation charges, this represents Hogan's observation that "transportation prices are both endogenous and not taken as given by the Cournot participants" (1997). Inserting (33) in (31) gives

$$\frac{dp_1}{dy_1} \sum_{i=3}^{n} h_i \alpha_i \frac{h_2 - h_i}{h_2} + \frac{dp_2}{dy_1} \left(h_2 \alpha_2 + \sum_{i=3}^{n} h_i \alpha_i \frac{h_i}{h_2} \right) = 0. \tag{34}$$

Inserting (33) in (32)

$$\frac{dp_1}{dy_1} \left(\alpha_1 + \sum_{i=3}^{n} \alpha_i \frac{h_2 - h_i}{h_2} \right) + \frac{dp_2}{dy_1} \left(\alpha_2 + \sum_{i=3}^{n} \alpha_i \frac{h_i}{h_2} \right) = -1 \tag{35}$$

Combining (34) and (35) gives an analytic expression for the effective demand slope which a generator faces at node 1. Expand to give the net demand elasticity $\varepsilon = -\frac{p_1}{y_1} \frac{dy_1}{dp_1}$:

$$\begin{aligned} \varepsilon &= \frac{p_1}{y_1} \left(\alpha_1 + \sum_{i=3}^{n} \alpha_i \left(1 - \frac{h_i}{h_2} \right) - \left(\alpha_2 + \sum_{m=3}^{n} \alpha_m \frac{h_m}{h_2} \right) \frac{\sum_{i=3}^{n} h_i \alpha_i \left(1 - \frac{h_i}{h_2} \right)}{h_2 \alpha_2 + \sum_{i=3}^{n} h_i \alpha_i \frac{h_i}{h_2}} \right) \\ &= \frac{p_1}{y_1} \left(\alpha_1 + \frac{X}{h_2^2 \alpha_2 + \sum_{i=3}^{n} h_2 h_i \alpha_i \frac{h_i}{h_2}} \right) = \frac{p_1}{y_1} \alpha_1'. \end{aligned} \tag{36}$$

In order to assess whether the effective elasticity is greater or smaller than the demand elasticity the generator would face in a setting with fixed allocation of transmission capacity, the sign of X in (36) must be determined:

$$\begin{aligned} X &= \sum_{i=3}^{n} \alpha_i (h_2 - h_i) \left(\alpha_2 (h_2 - h_i) h_2 + \sum_{m=3}^{n} \alpha_m (h_m - h_i) h_m \right) \\ &= \sum_{i=3}^{n} \alpha_i (h_2 - h_i) \left(\sum_{m=2}^{n} \alpha_m (h_m - h_i) h_m \right) \end{aligned} \tag{37}$$

To allow for a complete matrix operation, the sum from $i=3,..,n$ is expanded to $i=2,..,n$. This does not change the result because the first term cancels out.

$$X=\sum_{i=1+1}^{n}\alpha_i\left(h_i-h_1\right)\sum_{m=2}^{n}\alpha_m\left(h_i-h_m\right)h_m. \tag{38}$$

Now defining $\beta_i=h_i-\frac{1}{2}h_2$ (which corresponds to $h_i=\beta_i+\beta_2$) gives:

$$X=\sum_{i=2}^{n}\sum_{m=2}^{n}\alpha_i\left(\beta_i-\beta_2\right)\alpha_m\left(\beta_m+\beta_2\right)\left(\beta_i-\beta_m\right)=\sum_{i=2}^{n}\sum_{m=2}^{n}\Psi_{im}. \tag{39}$$

The diagonal elements, $i=m$, of Ψ_{im} are zero, because $\beta_i-\beta_m=0$, and the sum of two off-diagonal elements with $i,m>1$ is positive:

$$\begin{aligned}&\Psi_{im}+\Psi_{mi}=\alpha_i\alpha_m\beta_2\left(\beta_i-\beta_m\right)^2\\=\;&\frac{1}{2}\alpha_i\alpha_m h_2\left(h_i-h_m\right)^2\geq 0.\end{aligned} \tag{40}$$

Furthermore, for all $i=1$ or $j=1$: $\Psi_{ij}=0$. Therefore, X is a sum of weakly positive terms implying $X\geqslant 0$. In all networks exist i,j such that $h_i\neq h_j$ and if demand slope is finite ($\alpha_i>0$ and $\alpha_j>0$) at the nodes i,j, then $X>0$.

Returning to (36), the denominator $h_2^2\alpha_2+\sum_{i=3}h_2h_i\alpha_i\frac{h_i}{h_2}$ is positive and therefore $\alpha_1'\geqslant\alpha_1$ and usually $\alpha_1'>\alpha_1$. This proofs theorem 5 that slope of net demand is increased if energy and transmission markets are integrated for the case of one binding constraint.

Proof of theorem 5 for the case of several binding constraints:

Effective demand elasticity at every node is slightly higher under nodal pricing than if transmission and energy markets are separated.

Proof. The generator whose output decision is analysed is located at the swing bus $i=1$. Using previous notation $y_1\equiv Y_{1,j}$, (23) and $L=0$ give energy conservation for bus injections P_i at all nodes which equal net-demand N_i at all nodes minus production of generator y_1:

$$\sum P_i=\sum N_i-y_1=\sum P_i=0. \tag{41}$$

Using notation of Bohn e.a. excludes the prices and net demand at node one from the vector notation. This implies that subsequently $\overrightarrow{p}=(p_2,..,p_N)$ and similar adjustments to the transfer admittance matrix. Assume the only the first R out of M links are constraint, and redefine admittance matrix $\mathbf{H}_{R,N}$ to represent only the first R rows of the full matrix $\mathbf{H}_{M,N}$. Flows on constraint links are:

$$\overline{Z}_k=\mathbf{H}_{R,N}\overrightarrow{N}. \tag{42}$$

The shadow constraint prices for links $i>R$ are $\eta_i=0$. Expressing (26) in matrix notation and defining $\overrightarrow{\eta}=(\eta_1,..,\eta_R)$ gives $\overrightarrow{p}=p_s\binom{1}{1}_N+\mathbf{H}'_{R,N}\overrightarrow{\eta}$, with p_s price at the swing. Define the first

R rows of $\mathbf{H}_{R,N}$ as $\mathbf{H}_{R,R}$, and the first R components of the price vector $\overrightarrow{p}$ as $\overrightarrow{p_R}$ to obtain:

$$\mathbf{H}_{RR}\overrightarrow{\eta_R} = \overrightarrow{p_R} - \begin{pmatrix}1\\1\end{pmatrix}_R p_s. \tag{43}$$

$\mathbf{H}_{N,M}$ has the form $\mathbf{\Omega A}\left(A'\Omega A\right)^{-1}$ with Ω a $[M{*}M]$ diagonal matrix with admittances of links and $\mathbf{A}$ the $[M*(N-1)]$ network incidence matrix consisting of $-1,0,1$ for network interconnections (See appendix of Bohn e.a.). The inverse of $\mathbf{H}_{N,M}$ for multiplication from the left is A'. The existence of A' can be easily understood from the law of local energy conservation. Given the flows on all links, the residual of inflows and outflows of links towards a node is the net energy demand at the node $\overrightarrow{P} = \mathbf{A}'\overrightarrow{Z}$.

Invertability of $\mathbf{H}_{N,M}$ does not imply invertability of a sub-matrix $\mathbf{H}_{R,R}$. However, usually R can be chosen out of the $(N-1)$ nodes, such that shadow prices on R constraint links $\overrightarrow{\eta_R}$ follow from the prices at the R nodes and at the swing bus. Therefore, subsequently assume that $\mathbf{H}^{-1}_{R,R}$ exists and calculate $\overrightarrow{\eta_R} = \mathbf{H}^{-1}_{RR}\left(\overrightarrow{p_R} - \binom{1}{1}_R p_1\right)$ to obtain from (26):

$$\overrightarrow{p} = \begin{pmatrix}1\\1\end{pmatrix}_N p_s + \mathbf{H}_{N,R}\mathbf{H}^{-1}_{RR}\left(\overrightarrow{p_R} - \begin{pmatrix}1\\1\end{pmatrix}_R p_s\right). \tag{44}$$

This expression allows for expression of $\overrightarrow{p}$ purely as a function of the $R+1$ variables $\overrightarrow{p_R}$ and p_s.

Now replicate the calculations previously performed in the case of one binding constraint for the matrix formulation for several binding constraints. Differentiating (42) with respect to y_1 and defining $\boldsymbol{\alpha}$ as a matrix with diagonal elements consisting of net demand slopes for all but the swing bus, according to (25), gives:

$$\mathbf{H}_{R,N}\boldsymbol{\alpha}\frac{d\overrightarrow{p}}{dy_1} = \overrightarrow{0}_R. \tag{45}$$

Differentiating (42) with respect to y_1 gives:

$$\begin{pmatrix}1\\1\end{pmatrix}'_N \boldsymbol{\alpha}\frac{d\overrightarrow{p}}{dy_1} + \alpha_s\frac{dp_s}{dy_1} = -1. \tag{46}$$

Finally, differentiating (44) with respect to y_1 gives:

$$\begin{aligned}\frac{d\overrightarrow{p}}{dy_1} &= \begin{pmatrix}1\\1\end{pmatrix}_N \frac{dp_s}{dy_1} + \mathbf{H}_{N,R}\mathbf{H}^{-1}_{RR}\left(\frac{d\overrightarrow{p_R}}{dy_1} - \begin{pmatrix}1\\1\end{pmatrix}_R \frac{dp_s}{dy_1}\right)\\ &= \left(\begin{pmatrix}1\\1\end{pmatrix}_N - \mathbf{H}_{N,R}\mathbf{H}^{-1}_{RR}\begin{pmatrix}1\\1\end{pmatrix}_R\right)\frac{dp_s}{dy_1} + \mathbf{H}_{N,R}\mathbf{H}^{-1}_{RR}\frac{d\overrightarrow{p_R}}{dy_1}.\end{aligned} \tag{47}$$

Inserting (47) in (45) gives:

$$\frac{d\overrightarrow{p_R}}{dy_1} = -\left(\mathbf{H}_{R,N}\boldsymbol{\alpha}\mathbf{H}_{N,R}\mathbf{H}^{-1}_{RR}\right)^{-1}\mathbf{H}_{R,N}\boldsymbol{\alpha}\left(\begin{pmatrix}1\\1\end{pmatrix}_N - \mathbf{H}_{N,R}\mathbf{H}^{-1}_{RR}\begin{pmatrix}1\\1\end{pmatrix}_R\right)\frac{dp_s}{dy_1}. \tag{48}$$

Inserting (47) in (46) gives:

$$\left(\begin{pmatrix}1\\1\end{pmatrix}'_N \boldsymbol{\alpha}\left(\begin{pmatrix}1\\1\end{pmatrix}_N - \mathbf{H}_{N,R}\mathbf{H}^{-1}_{RR}\begin{pmatrix}1\\1\end{pmatrix}_R\right) + \alpha_s\right)\frac{dp_s}{dy_1} + \begin{pmatrix}1\\1\end{pmatrix}'_N \boldsymbol{\alpha}\mathbf{H}_{N,R}\mathbf{H}^{-1}_{RR}\frac{d\overrightarrow{p_R}}{dy_1} = -1. \tag{49}$$

Inserting (48) into (49) gives:

$$\begin{aligned}
\frac{1}{-\frac{dp_1}{dy_1}} &= \binom{1}{1}_N' \boldsymbol{\alpha}\left(\binom{1}{1}_N - \mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\binom{1}{1}_R\right) \\
&\quad -\binom{1}{1}_N' \boldsymbol{\alpha}\mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\left(\mathbf{H}_{R,N}\boldsymbol{\alpha}\mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\right)^{-1}\mathbf{H}_{R,N}\boldsymbol{\alpha}\left(\binom{1}{1}_N - \mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\binom{1}{1}_R\right) + \alpha_s \\
&= \binom{1}{1}_N' \left(\boldsymbol{\alpha} - \boldsymbol{\alpha}\mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\left(\mathbf{H}_{R,N}\boldsymbol{\alpha}\mathbf{H}_{N,R}\mathbf{H}_{RR}^{-1}\right)^{-1}\mathbf{H}_{R,N}\boldsymbol{\alpha}\right)\binom{1}{1}_N + \alpha_s \\
&= \binom{1}{1}_N' \left(\boldsymbol{\alpha} - \boldsymbol{\alpha}\mathbf{H}_{N,R}\left(\mathbf{H}_{R,N}\boldsymbol{\alpha}\mathbf{H}_{N,R}\right)^{-1}\mathbf{H}_{R,N}\boldsymbol{\alpha}\right)\binom{1}{1}_N + \alpha_s.
\end{aligned}$$

$\boldsymbol{\alpha}$ is positive, semi-definite and diagonal, therefore define $\boldsymbol{\beta}$ such that $\boldsymbol{\beta}\boldsymbol{\beta} = \boldsymbol{\alpha}$ to obtain:

$$\frac{1}{-\frac{dp_1}{dy_1}} = \binom{1}{1}_N' \boldsymbol{\beta}\left(\mathbf{1}_{NN} - \boldsymbol{\beta}\mathbf{H}_{N,R}\left(\mathbf{H}_{R,N}\boldsymbol{\beta}\boldsymbol{\beta}\mathbf{H}_{N,R}\right)^{-1}\mathbf{H}_{R,N}\boldsymbol{\beta}\right)\boldsymbol{\beta}\binom{1}{1}_N + \alpha_s.$$

Defining $X = \boldsymbol{\beta}\mathbf{H}_{N,R}$ gives:

$$\frac{1}{-\frac{dp_1}{dy_1}} = \binom{1}{1}_N' \boldsymbol{\beta}\left(\mathbf{1}_{NN} - \mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\right)\boldsymbol{\beta}\binom{1}{1}_N + \alpha_s.$$

$\mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'$ projects $\boldsymbol{\beta}\binom{1}{1}_N$ to a subspace of R^N, therefore $\mathbf{1}_{NN} - \mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'$ gives the components orthogonal to this subspace. $\binom{1}{1}_N'\boldsymbol{\beta}\left(\mathbf{1}_{NN} - \mathbf{X}\left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\right)\boldsymbol{\beta}\binom{1}{1}_N$ gives the length of the component of $\boldsymbol{\alpha}\binom{1}{1}_N$ orthogonal to the space spanned by $\mathbf{X} = \boldsymbol{\beta}\mathbf{H}_{N,R}$, a semi-positive number. The result is that the slope of net demand facing the generator is always weakly increased by the network, relative to a setting where no network exists or where inflexible allocation of transmission capacity fix the amount of net-exports. ■