# AN EFFICIENT ITERATIVE DFT-BASED CHANNEL ESTIMATION FOR MIMO-OFDM SYSTEMS ON MULTIPATH CHANNELS 

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#### Abstract

Fourier transform (DFT) -based channel estimator with good performance for multiple-input and multiple-output orthogonal frequency division multiplexing (MIMO-OFDM) systems such as IEEE 802.11 n which retain some sub-carriers as null sub-carriers (or virtual carriers) is proposed. In order to eliminate the mean-square error (MSE) floor effect existed in conventional DFT-based channel estimators, we proposed a low-complexity method to detect the significant channel impulse response (CIR) taps, which neither need any statistical channel information nor a predetermined threshold value. Analysis and simulation results show that the proposed method has much better performance than conventional DFTbased channel estimators and without MSE floor effect.


Keywords-Channel estimation; MIMO-OFDM; DFT-based; IEEE P802.11n D2.0

## I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) has been applied widely in wireless communication systems such as IEEE 802.11a and the European equivalent HIPERLAN/2 due to its high data rate transmission capability with high bandwidth efficiency and its robustness to multi-path delay. But with the development of the information technology, the demand of faster and more reliable wireless communication systems is gradually increased. Multiple-input multiple-output (MIMO) systems which employ multiple antennas at both transmitter and receiver can improve the data rate with higher bandwidth efficiency and better performance by using spatial multiplexing(SM) and space-time block coding (STBC) schemes. These benefits have made the combination of MIMOOFDM an attractive technique for future high data rate systems, such as DAB, DVB, WLAN, and WMAN [1]. The latest 802.11 n technical proposal based on MIMO-OFDM can achieve a maximum data rate of 600 Mbps [10].

Channel estimation is a challenging problem in wireless systems due to multi-path propagation, mobility, and local scatterings, and so on. In this paper, we focus on pilot-aided channel estimation techniques which are commonly used in MIMO-WLAN system. Generally speaking, the realization of pilot-aided channel estimation is based on either least squares (LS) approach or linear minimum mean square error (LMMSE) approaches. The LS estimation is the simplest channel estimation based on parallel Gaussian channel model in frequency domain, which does not use any information about channels, but the performance is not quite acceptable. The LMMSE estimation can achieve better performance by using channel statistics such as channel covariance matrix in frequency domain and average signal-to-noise ratio (SNR), but this method requires very large amount of computation, such as matrix inversing. Though there are many attempts to reduce the complexity of the LMMSE [2][3], these modified LMMSE methods still require exact channel covariance matrices. Furthermore, they have quite high-computational complexity for practical implementation.

The discrete Fourier transform (DFT) -based channel estimator give us an alternative, low-complexity choice [4]. In contrast to the frequency domain estimation, the transform domain estimation method uses the time domain properties of channels. In [5], different channel estimation techniques are analyzed and discussed, from which we can see that if the length of the channel impulse response (CIR) or the number of significant channel taps are estimated correctly, the DFT-based estimation can achieve performance very close to the ideal CIR. Most of the published work on DFT-based channel estimation assumes the information about the CIR length is known at the receiver, however, this assumption generally does not hold in practice. The CIR length can be simply assumed to be the length of the cyclic prefix ( CP ), but this assumption will result in overall performance degradation [6]. Alternatively, for more accurate results, some method of obtaining the channel length must be used. Some algorithms are also developed to estimate the number of significant channel taps [7] [8], however these
algorithms will increase the complexity of the channel estimation method obviously.

In this paper, we propose a modified iterative DFT-based channel estimation algorithm which can simply obtain the significant taps and cope with the mean-square error (MSE) floor so as to increase the estimation accuracy in MIMOOFDM systems. The novelty in our proposed method lies in how the significant channel taps are determined. On the other hand, in practical system such as IEEE802.11a, only 52 out of the total 64 sub-carriers are used, and the other 12 sub-carriers will be retained as null sub-carriers (virtual carriers), which will make the conventional DFT-based estimator not work well[9]. The IEEE802.11n system has also the same problem. Our proposed method can interpolate the frequency response of null sub-carriers by exploiting the frequency correlation of limited time excess delay channels to improve the performance of the DFT-based estimator.

This paper is organized as follows. Section II describes the MIMO-OFDM system model used. In section III, the efficient iterative DFT-based channel estimation method is discussed in detail. Section IV shows the simulation results and performance of the novel channel estimation method. Finally the conclusions are drawn in section $V$.

## II. System Model For Mimo-ofdm

We consider a MIMO-OFDM system with $N_{t x}$ transmit and $N_{r x}$ receive antennas. The information bits are mapped into data symbols depending on the modulation type and then are demultiplexed for different transmitter antennas. And we consider the channel between each transmitter receiver link is a frequency selective, Rayleigh fading channel, modeled as a multi-taps channel with the same statistics. The typical channel at time $t$ is expressed as
$h(t, \tau)=\sum_{l=0}^{L-1} \alpha_{l}(t) \delta\left(\tau-\tau_{l}\right)$
Where $L$ is the number of taps, $\alpha_{l}$ is the $l$ th complex path gain, and $\tau_{l}$ is the corresponding path delay. The path gains are Wide-Sense stationary (WSS) complex Gaussian processes. The individual paths can be correlated, and the channel can be sparse. At time $t$, the channel frequency response (CFR) of the CIR is given by,

$$
\begin{equation*}
H(t, f)=\int_{-\infty}^{+\infty} h(t, \tau) e^{-j 2 \pi f \tau} d \tau \tag{2}
\end{equation*}
$$

Compared to single antenna IEEE802.11a system, more training sequences are needed to estimate the MIMO channel, which is due to the higher number of channel parameters in the MIMO systems. To this end, each antenna transmits at least $N_{r}$ OFDM training symbols $T_{t, n}\left(\mathrm{t}, \mathrm{n}=1, \ldots, \mathrm{~N}_{\mathrm{r}}, \ldots\right)$, and these training symbols are orthogonal across antennas. The result of the channel estimation process is a $N_{t} \times N_{r}$ matrix for each of the

64 tones. The transmit diversity scheme can be achieved by space-time block code and spatial multiplexing.

## III. Proposed Channel Estimation Method

If we take the LS estimate $H_{L S}$ as the initial FD channel estimate. The $n$th estimated sample of CIR can be expressed with the LS estimation, and then we have

$$
\begin{align*}
\hat{h}_{L S}[n] & =I F F T_{N}\left\{\hat{H}_{L S}[k]\right\} \\
& =h[n]+w[n] \tag{3}
\end{align*}
$$

Where $h[\mathrm{n}]$ is the ideal CIR and $w[\mathrm{n}]$ is the noise. For practical multi-path wireless channels, there are not so many channel paths with significant energy compared to the FFT size $N$. It is mentioned that in general the CIR length is much smaller than the length of CP, that is, $L<N_{C P}$. Hence, among $N$ samples (taps) of the CIR estimate, many samples (taps) will have little or no energy at all except noise perturbation. So we can think that all information of channels is contained in the first $L$ samples that are the significant taps which values relatively have more energy or magnitude than the noise, and other samples are only noise, which can be given by
$\hat{h}_{D F T}[n]=\left\{\begin{array}{lr}h[n]+w[n], 0 \leq n \leq L-1 \\ w[n], & \text { otherwise }\end{array}\right.$

Since the noise is assumed to be AWGN in frequency domain, it is AWGN in transform domain as well. If the significant values of the transform domain signal are retained, and the non-significant ones are treated as zero, then the noise term will be eliminated significantly. The traditional DFT-based channel estimation exploits this property of OFDM systems which have the symbol period much longer than the duration of the CIR to reduce the noise power that exists in only outside of the CIR part. Hence taking the first $L$ samples only and ignoring noise-only samples, we can obtain a better performance. Expressing these processes in equations, we obtain
$\hat{h}_{\text {windowed }}[n]=\left\{\begin{array}{lr}\hat{h}_{D F T}[n], & 0 \leq n \leq L-1 \\ 0, & \text { otherwise }\end{array}\right.$

This noise reduction scheme for OFDM transmissions is also regarded as to window the channel estimate in time-domain (TD) so as to preserve the energy of the significant channel taps while reducing the energy of others. The windowed TD channel estimation is then brought back to the FD and inverted to form the noise-reduced initial taps. The information about the CIR length is important in achieving higher performance in DFT-based transform domain approaches. A CIR length taken to be smaller than the actual CIR length will eliminate the
significant taps, while a channel taken to be longer will result in less noise suppression. However, the first case is more critical than the second one, and hence in the practical DFTbased estimation, the length of CP is usually taken as the CIR length.

On the other hand, this simple method, however, will generally not work well if the original FD channel estimate can not be determined for all frequencies, as is in the case of the IEEE802.11a, which has only 52 out of the total 64 FD points defined in a symbol. To simply assign arbitrary values to the channel estimate at these frequencies(e.g. make them ' 0 ') means to create a significant TD channel estimate energy spread, which makes it impossible to window out the TD noise energy without distorting the original channel impulse response.

In [9], an efficient algorithm for single antenna IEEE 802.11a systems shows how the frequency response of null sub-carriers can be interpolated by exploiting the frequency correlation of limited time excess delay channels. The algorithm is briefly described as follows:

1. Obtain initial channel estimate (typically performed using the simple LS method).
2. Convert channel estimate to the time domain (TD) and window significant taps.
3. Convert this TD signal back to the frequency domain.
4. Replace the values of the known sub-carriers with the initial estimate in step 1 (ignore this step for the last iteration).
5. Repeat steps 2-4.

For this operation, some sort of threshold is needed to differentiate between the significant values of the signal and noise terms. Now we propose a new method to determine the significant taps, and extend this iterative estimation algorithm to MIMO systems. Consider a MIMO-OFDM system with $M$ transmit, $N$ receive antennas, and $K$ sub-carriers used. Let $C_{m, k}$ be a pilot symbol transmitted from antenna $m$ for sub-carrier $k$. The received signal at receive antenna $n$ can be modeled as:
$r_{n, k}=\sum_{i=1}^{M} H_{n, m, k} C_{m, k}+W_{n, k}$
Where $H_{n, m, k}$ is the frequency response of sub-carrier $k$ between transmit antenna m and receive antenna $n$, and $W_{n, k}$ represents additive white Gaussian noise with zero mean and variance $\sigma_{n / 2}^{2}$ per-dimension. Hence, the received symbol at each receive antenna is a linear combination of transmitted symbols that are modified by channel gains and noise. Obviously, the key difference between the MIMO and single antenna case is the superposition of transmitted symbols at the receiver.

The key to our method is how to determine the significant CIR taps. Energy detection can be used to detect signal in additive Gaussian noise [11]. Some methods have been proposed to detect energy of CIR in time domain based on the LS algorithm's result $h_{l s}$ and search the significant taps of
channel [12]. In these methods, a constant value of threshold is needed to differentiate between the significant taps and the others, but it's not practical in real OFDM system because we must preset different threshold values for different environment. For example, in [12], the taps are searched from $N$ to 1 , where $N$ is the FFT length, and for each $h_{l s}^{i}(1 \leq i \leq N)$, the decision is given by
$K_{i}=\frac{\left(\sum_{j=i-5}^{i}\left|h_{l s}^{j}\right|^{2}\right) / 5}{\frac{1}{N-i+1} \sum_{j=i+1}^{N}\left|h_{l s}^{j}\right|^{2}}$

A constant threshold value $\lambda$ is preset, and when $K_{i} \geq \lambda$, we'll get the significant taps: $h_{l s}^{s}(1 \leq s \leq i)$.


Fig. 1 the CIR information in time-domain
We propose a new Low-Complexity method to distinguish the significant taps and the others. In the iterative DFT-based estimator, we replace the values of the known sub-carriers of $H_{\text {windowed }}$ with the initial LS estimate's values, and covert it into time-domain $h_{l s}^{\prime}$, as illustrated in Fig 1, and we can use $h_{l s}^{\prime}$ to get the significant taps. We calculate the max noise power $E_{\max }$ from the initial LS estimate and take it as the threshold instead a constant value, here

$$
\begin{equation*}
E_{\max }=M A X\left(\left|h_{l s}^{j}\right|^{2}\right),(C P<j \leq N) \tag{8}
\end{equation*}
$$

In the $i$ th iterative step of our algorithm, $E_{i}$ is calculated by

$$
\begin{equation*}
E_{i}=\left(\sum_{j=i-2}^{j}\left|h_{j}^{\prime}\right|^{2}\right) / 3 \tag{9}
\end{equation*}
$$

Here, we take the average value of three taps to get a more accurate result. When $E_{i} \geq E_{\max }$, we get the significant taps. The proposed algorithm is summarized as follows:

Step 1: calculate the initial FD channel estimate $H_{L S}$ in the usual LS manner. Covert $H_{L S}$ to Time domain $h_{L S}$;

$$
h_{L S}=\operatorname{IFF} T\left(H_{L S}\right)
$$

Step 2:

$$
E_{\max }=M a x\left(\left|h_{L S}^{j}\right|^{2}\right),(C P<j \leq N)
$$

Step 3: $i=C P$;
Step 4: for the first iterative, $h^{\prime}=h_{L S}$, otherwise:

$$
\begin{aligned}
& h^{\prime}=\operatorname{IFFT}\left\{H_{\text {windowed }}^{\prime}\right\} \\
& E_{i}=\left(\sum_{j=i-2}^{j}\left|h_{j}^{\prime}\right|^{2}\right) / 3
\end{aligned}
$$

if $\left(E_{i}>E_{\max }\right)$, go to Step 7 ;

Step 5:

$$
\left.\begin{array}{l}
h_{\text {windowed }}[n]= \begin{cases}h '[n], & 0 \leq n \leq i-1 \\
0, & \text { otherwise }\end{cases} \\
H_{\text {windowed }}=F F T\left\{h_{\text {windowed }}\right\}
\end{array}\right\} \begin{aligned}
& H_{\text {windowed }}^{\prime}[k]=\left\{\begin{array}{l}
H_{L S}[k], k \notin V \\
H_{\text {windowed }}[k], k \in V
\end{array}\right.
\end{aligned}
$$

Where ' $V$ ' indicate the null sub-carries ;
Step 6: $i=i-1$, go to Step 4;
Step 7: the number of significant taps $=i$, the proposed estimate:

$$
\hat{H}=H_{\text {windowed }} \text {; }
$$

## IV. Simulation Results

In this section, we investigate the performance of the proposed channel estimation algorithm on multi-path channels. The MSE and bit-error-rate (BER) performances are examined. A 2 transmit - 2 receive antenna ( $2 \times 2$ ) MIMO-OFDM system with symbols modulated by 16QAM is simulated. We construct the preamble and data field according to the IEEE P802.11n D2.00 [10]. In this paper, we only consider the mixed mode, but this channel estimation method is also adapted to the HT Greenfield mode. The system bandwidth is 20 MHz , which is divided into 64 tones with a total symbol period of $4 \eta s$. An OFDM symbol thus consists of 80 samples, 16 of which are included in the CP. The constructed i.i.d. Rayleigh fading channel has $L$ paths determined by root mean squares (RMS) channel delay, and the amplitude of the each path varies independently with an exponential power delay profile [11]. Unit delay of channel is assumed to be the same as OFDM sample period. Thus, there is no power loss caused by nonsample spaced. We assume the channel are static over one OFDM frame, where the HT long training field is 2 OFDM symbol long and data are composed of 100 OFDM symbols as Fig.2. A new channel is generated at each simulation, and the system performance is averaged over CIR realizations. We simulate the system with three different channels shown in

Table 1. Fig. 3 shows the CFR information between the antenna pairs gotten by different estimate method, and compared with the perfect channel status information (CSI), when $\mathrm{SNR}=25$ dB.

TABLE I. Channel parameters

| $\boldsymbol{R M S}$ delay | Channel length, $L$ <br> (taps) |
| :---: | :---: |
| 20 ns | 5 |
| 30 ns | 8 |
| 50 ns | 12 |



Fig. 2 The IEEE P802.11n frame format


Fig. 3 the CFR information between the antenna pairs $(T x=2, R x=2)$, $\mathrm{SNR}=25 \mathrm{~dB}$

A 4 transmit - 4 receive antenna ( $4 \times 4$ ) MIMO-OFDM system with symbols modulated by 16QAM is also simulated. Fig.4, 5 respectively confirms the ( $2 \times 2$ ) and ( $4 \times 4$ ) MSE performance of the conventional DFT-based estimation and shows the performance improvement of the proposed DFTbased channel estimation in three common indoor i.i.d. Rayleigh fading cases: RMS delay $=20,30,50 \mathrm{~ns}$.

Finally, the BER performance of the proposed method is examined and compared with the LS estimate and conventional DFT-based method. As shown in Fig.6, when we apply the proposed algorithm, the required SNR is $1 \sim 2 \mathrm{~dB}$ lower than the conventional DFT-based estimation with 50ns RMS delay.


Fig. 4 MSE of $2 \times 2$ system with 20, 30, 50ns RMS delay, (taps $=5,8,12$ respectively).


Fig. 5 MSE of 4 x 4 system with 20, 30, 50ns RMS delay, (taps $=5,8,12$ respectively).


Fig. 6 BER performance with 50 ns RMS delay (taps $=12$ )

## V. Conclusions

An efficient iterative DFT-based channel estimation algorithm for MIMO-OFDM systems is proposed in this paper. The proposed method can simply detect the significant channel impulse response taps, which don't need any channel statistical information. The novel method calculates the max noise power $E_{\max }$ from the initial LS estimation result and takes it as the threshold instead of a constant value preset, so it is adaptive in practical system. Moreover, the proposed method can interpolate the frequency response of null subcarriers to improve the MSE performance of channel estimator by exploiting the frequency correlation of limited time excess delay channels. We evaluated the performance of the proposed algorithm by computer simulation with $2 \times 2$ and $4 \times 4$ MIMOOFDM system in 5,8,12-tap i.i.d. Rayleigh fading channels respectively. The proposed algorithm can also work well in other types of fading channels such as Rician fading channel. Furthermore, this technique can be easily extended to systems with other number of antennas. Simulation results show that the proposed scheme is very robust against variations of the propagation environment and achieves a satisfying tradeoff between performance and complexity.

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