# Letters to the Editor

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## Elastic scattering of electrons with laser photons in Coulomb background

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In recent years due to development of powerful optical sources many new physical phenomenon have come into existence e.g., generation of harmonics, self focussing of wave beams, multiphoton processes (Man Mohan *et al* 1973 to 1975), multiphoton inverse bremmstrahlung processes (Bunkin & Fedorov 1966, Man Mohan 1974) etc. Here we have studied the elastic scattering of electrons with laser photons in the Coloumb background and have calculated the differential cross-section using time dependent unitary transformation method (Henneberger 1968).

In the presence of a strong field denoted by vector potential **A**, the Schrodinger equation for an electron becomes

$$\frac{1}{2m}\left[\frac{\hbar}{i}\nabla-\frac{e}{c}\boldsymbol{A}(t)\right]^2\psi(\boldsymbol{\gamma},t)+V(\boldsymbol{\gamma})\psi(\boldsymbol{\gamma},t)=i\hbar\frac{\partial\psi(\boldsymbol{\gamma},t)}{\partial t}\quad\ldots\quad(1)$$

Now under the unitary transformation

$$U=\exp\left\{\frac{i}{\hbar}\int_{-\infty}^{t}\left[\frac{ie\hbar}{mc}\boldsymbol{A}(\tau).\nabla+\frac{e^2}{2mc^2}\boldsymbol{A}(\tau)\right]d\tau\right\}\quad\ldots\quad(2)$$

Eq. (1) becomes

$$-\frac{\hbar^2}{2m}\nabla^2\psi_1(\boldsymbol{\gamma},t)+V(\boldsymbol{\gamma}+\boldsymbol{\alpha})\psi_1(\boldsymbol{\gamma},t)=i\hbar\frac{\partial\psi_1}{\partial t}(\boldsymbol{\gamma},t)\quad(3)$$

where

$$\psi_1=U\psi \text{ and } \boldsymbol{\alpha}=-\int_{-\infty}^{t}\frac{e}{mc}\boldsymbol{A}(\tau)d\tau=\boldsymbol{\alpha}_0\sin\omega t,\ \alpha_0=\frac{ea\epsilon}{mc\omega}.$$

Further eq. (3) can be written as

$$\left\{-\frac{\hbar^2}{2m}\nabla^2+W(\boldsymbol{\gamma},t)+V(\boldsymbol{\gamma})\right\}\psi_1(\boldsymbol{\gamma},t)=i\hbar\frac{\partial\psi_1(\boldsymbol{\gamma},t)}{\partial t}\quad(4)$$

where

$$W(\boldsymbol{\gamma},t)=V(\boldsymbol{\gamma}+\boldsymbol{\alpha})-V(\boldsymbol{\gamma}).$$

Clearly in the absence of radiation field $W(\boldsymbol{\gamma}, t) = 0$ and eq. (4) reduces to

$$\left\{-\frac{\hbar^2}{2m}\nabla^2 + V(\boldsymbol{\gamma})\right\}\psi^0(\boldsymbol{\gamma}, t) = i\hbar\frac{\partial\psi^0}{\partial t)}(\boldsymbol{\gamma}, t) \qquad \ldots (5)$$

where $\psi^0(\gamma, t)$ represents the unperturbed states i.e., wavefunction in the absence of e.m. field.

In eq. (4), the term $U(\boldsymbol{\gamma}, t)$ represents the effective perturbation on the electron due to radiation field which causes the transition from the initial state $\psi_i^0(\boldsymbol{\gamma}, t)$ to final state $\psi_f^0(\gamma, t)$. $\psi_i^0(\gamma, t)$ and $\psi_f^0(\gamma, t)$ are the exact solution of eq. (5) which are noting but the coloumb solutions and are written as

$$\psi_i^0(\boldsymbol{\gamma}, t) = e^{\eta\pi/2}\Gamma(1-i\eta)e^{i\boldsymbol{p}_i\cdot\boldsymbol{\gamma}}F(i\eta, 1, i(p_i\gamma - \boldsymbol{p}_i\boldsymbol{\gamma})) \qquad \ldots (6)$$

and

$$\psi_f^0(\boldsymbol{\gamma}, t) = e^{\eta'\pi/2}\Gamma(1-i\eta')e^{i\boldsymbol{p}_f\cdot\boldsymbol{\gamma}}F(i\eta', 1\ i(p_f\gamma - \boldsymbol{p}_f\cdot\boldsymbol{\gamma})) \qquad \ldots (7)$$

Thus we can write the scattering amplitude for the above mentioned process as

$$A_{i\to f} = -\frac{i}{\hbar}\int_{-v}^{+v} dt(\psi_f^0(\boldsymbol{\gamma}, t),\ W(\boldsymbol{\gamma}, t)\,\psi_i^0(\boldsymbol{\gamma}, t)). \qquad \ldots (8)$$

Putting the value of $\psi_i^0(\boldsymbol{\gamma}, t)$ and $\psi_f^0(\boldsymbol{\gamma}, t)$ from eqs. (6) and (7) in eg (8) and after simplification we ger the requisite differential cross-section in the usual way as (Landau & Lifshitz 1958)

$$\frac{d\sigma}{d\Omega} = \frac{16m^2ze^2\pi^2\eta^2e^{2\pi\eta}}{\hbar^4(e^{2\pi\eta}-1)^2}\times(2p^2-2\boldsymbol{p}_f, \boldsymbol{p}_i)^{-2}\,|\,J_0(\boldsymbol{a}_0\cdot(\boldsymbol{p}_i-\boldsymbol{p}_f)-1\,|^2$$

$$\times\left|\,F\left(i\eta, i\eta, 1, -2\frac{(p^2-\boldsymbol{p}_i-\boldsymbol{p}_f)}{(p_i-p_f)^2}\right)\right|^2. \qquad \ldots (9)$$

Clearly eq. (9) in an exact expression for the elastic scattering of electrons with laser photons. Such an exact expression should be employed for further study of interaction laser with matter where the electrons are in the coloumb background.

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