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On the irreversibility of electrostatic plasma turbulence

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A possible definition of entropy similar to the classical kinetic theory of gases is considered and Boltzmann H theorem is extended to the electrostatic weak plasma turbulence. It is shown that in a weakly turbulent plasma entropy must increase and therefore it is irreversible in contrast to the stable Vlasov plasma

1. INTRODUCTION

The concept of entropy and Boltzmann H theorem is of fundamental importance in the kinetic theory of gases and irreversible thermodynamics (Chapman & Cowling 1952) According to H theorem any kinetic equation which is applicable to a real system must have some property H which is the negative of entropy of the system whose time rate of change dH/dt must be negative or zero for all the time It is also well known that the equations of motion for a system expressed microscopically are time reversible. Therefore the irreversibility of the system is related to the collective behaviour of the elements of the system.

For low density plasma the appropriate kinetic equation is the Vlasov equations. (Krall & Trivelpie 1973)

$$\frac{\partial f_\alpha}{\partial t} + \boldsymbol{v}\cdot\frac{\partial f_\alpha}{\partial \boldsymbol{x}} + \frac{q_\alpha}{m_\alpha}\left(\boldsymbol{E} + \frac{\boldsymbol{v}\times\boldsymbol{B}}{C}\right)\cdot\frac{\partial f_\alpha}{\partial \boldsymbol{v}} = \quad \ldots \quad (1)$$

where f_α is the probability density function of electron ($\alpha = 1$) and ion ($\alpha = 2$), $\boldsymbol{x}$ and $\boldsymbol{v}$ are the position and velocity vectors, respectively. $\boldsymbol{E}$ and $\boldsymbol{B}$ are the electric field and magnetic induction vector, q_α and m_α are respectively the charge and the mass of the electron and ion.

It is well known that the Vlasov equation is reversible and therefore the entropy of such system remains constant

In the present work the theory of weak electrostatic turbulence is considered An H function as well as entropy similar to the classical kinetic theory is defined It is shown that the time rate of change H is negative or zero for all the time and therefore the entropy of the weakly turbulent plasma must increase and therefore it is irreversible.

2 Boltzmann H Theorem in a Turbulent Plasma

Turbulence has been the subject of a large number of investigations (Tsytovich 1972, David 1972). The equation governing the ensemble average of the distribution function of the electrons

$$F_1(\boldsymbol{x}, \boldsymbol{v}, t) = \langle f_1(\boldsymbol{x}, \boldsymbol{v}, t) \rangle^* \qquad \ldots \quad (2)$$

is given by a Fokker Planck equation (Grey & Paul 1971, Gray & Schmidt 1971)

$$\frac{DF_1}{Dt} = \frac{\partial}{\partial \boldsymbol{v}} \cdot \left[\underset{\approx}{\boldsymbol{D}} \cdot \frac{\partial F_1}{\partial \boldsymbol{v}} \right] \qquad \ldots \quad (3)$$

where D/Dt is the total time derivative along the zeroth trajectory of the electron, i.e.,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \boldsymbol{v} \cdot \frac{\partial}{\partial \boldsymbol{x}} + \frac{q_1}{m_1} \left(\boldsymbol{E}_0 + \frac{\boldsymbol{v} \times \boldsymbol{B}_0}{c} \right) \cdot \frac{\partial}{\partial \boldsymbol{v}} \qquad \ldots \quad (4)$$

The diffusivity tensor $\underset{\approx}{\boldsymbol{D}}$ is related to the energy spectrum of the random electrostatic potential $S(\boldsymbol{k}, \omega)$ i.e.,

$$\underset{\approx}{\boldsymbol{D}} = [q_1^2/(2\pi)^4 m_1^2] \int d^3k \int d\omega\, \boldsymbol{k}\boldsymbol{k} S(\boldsymbol{k}, \omega)$$

$$\int_{-\infty}^{t} dt' \exp\{i[\boldsymbol{k} \cdot (\boldsymbol{x}' - \boldsymbol{x}) - \omega(t' - t)]\} \qquad \ldots \quad (5)$$

where $\boldsymbol{k}$ is the wave vector and ω is the frequency. We assume that for the heavy ions

$$F_2 = f_2 \qquad \ldots \quad (6)$$

We now define an H function similar to the classical theory (Nakayama 1967)

$$H_\alpha = \int d\boldsymbol{x} \int d\boldsymbol{v} F_\alpha \ln F_\alpha. \qquad \ldots \quad (7)$$

Note that this expression resembles that of the classical theory (Nakayama 1967) in reference but we considered the ensemble average of the distribution function and not the instantaneous one. The entropy may now be defined as

$$S = -\Sigma_\alpha H_\alpha. \qquad \ldots \quad (8)$$

The time derivative of H becomes

$$\frac{dH_\alpha}{dt} = \int d\boldsymbol{x}\, d\boldsymbol{v} \int (1 + \ln F_\alpha) \frac{DF_\alpha}{Dt}. \qquad \ldots \quad (9)$$

* Angular brackets stand for the ensemble average.

It is now obvious that for a stable plasma $DF_2/Dt = 0$ and hence $dH_2/dt = 0$. Therefore a stable Vlasov plasma is reversible.

For a turbulent plasma DF_1/Dt is given by eq (3). Direct substitution of eq. (3) in eq. (9) after integration by part gives

$$\frac{dH_1}{dt} = -\int d\boldsymbol{x} \int d\boldsymbol{v} \, \frac{1}{F_1} \frac{\partial F_1}{\partial \boldsymbol{v}} \cdot D \cdot \frac{\partial F_1}{\partial \boldsymbol{v}} . \quad \ldots (10)$$

From eq. (3) it is clear that D is a positive definite symmetric second order tensor. Therefore the integral on the right hand side is positive or zero at all the time. Thus

$$\frac{dH_1}{dt} \leqslant 0 \qquad \text{for all } t \quad (11)$$

The equality holds when $\partial F/\partial \boldsymbol{v} = 0$

For the ions with the assumption (6) we find

$$\frac{dH_2}{dt} = 0 \quad \ldots (12)$$

In term of entropy we find

$$\frac{dS}{dt} \geqslant 0, \quad \text{for all } t \quad \ldots (13)$$

Therefore we conclude that although a stable Vlasov plasma is reversible the unstable plasma which is weakly turbulent becomes irreversible.

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