Indian J. Phys. **51A, 343-349 (1977)**

Excitation of the cylindrical plasma columns with the ring of magnetic currents

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(*Received* 18 *March* 1976, *revised* 9 *August* 1976)

A cylindrical geometry having a conductor (radius a_1), ring source (radius a), air column (radius b), gaseous plasma (radius c), solid state semiconductor plasma (radius d) and gaseous plasma column (radius e) is studied. This multilayered system gives radiation peaks for particular values of b, c, d and e while the source of excitation being the ring of magnetic currents. The amplitude of these radiation peaks gets affected with the change in these parameters. Almost in all the cases radiation pattern happens to be of the same nature in the directions from 71° to 90°.

1. INTRODUCTION

Excitation of leaky waves on plasma columns was studied by Tamir *et al* (1962, 1963). They considered the excitation of leaky waves on plasma surfaces. Gupta *et al* (1967) discussed the radiation pattern due to ring source while plasma column was assumed to be cylindrical shaped and extending upto infinity in axial direction. They also interpretted this type of pattern due to leaky waves. Welenhanced peaks before and near critical angle were obtained by Dhani Ram *et al* (1972) when they included a central conductor along the z-axis of the cylindrical coordinates. They assumed the source to be immersed in plasma which unfortunately for experiments poses a serious problem. To be safe on the experimental side Sharma *et al* (1975) discussed the problem assuming the ring source situated outside the plasma column. In continuance of the above cited work here we have studied the plasma columns with the help of ring of magnetic currents. Plasma columns are assumed to be homogeneous, lossless, isotropic and in-compressible in nature while the semiconductor plasma column being lossy in nature. The columns are assumed to be extending upto infinity along z-axis and the source is placed in $z = 0$ plane.

2. ANALYSIS

Stating from the Maxwell's equations (Dhani Ram *et al* 1972, and Sharma *et al* 1975) one can expand the inhomogeneous eqn. in cylindrical coordinates for the first asymmetrical mode and can come across with the solution in the form of bessel's and hankel's functions which can be given as,

$$h_1 = A_1 J_1(v_0\rho) + B_1 Y_1(v_0\rho) \qquad a_1 < \rho < a \tag{1a}$$

$$h_2 = A_2 J_1(v_0\rho) + B_2 Y_1(v_0\rho) \qquad a < \rho < b \tag{1b}$$

$$h_3 = A_3 J_1(v_1\rho) + B_3 Y_1(v_1\rho) \qquad b < \rho < c \tag{1c}$$

$$h_4 = A_4 J_1(v_2\rho) + B_4 Y_1(v_2\rho) \qquad c < \rho < d \tag{1d}$$

$$h_5 = A_5 J_1(v_1\rho) + B_5 Y_1(v_1\rho) \qquad d < \rho < e \tag{1e}$$

$$A_6 H_1^{(1)}(v_0\rho) \qquad e < \rho \tag{1f}$$

where h is Fourier transform of H_ϕ given by

$$h(\rho, k_z) = \int_{-\infty}^{\infty} H_\phi(\rho, z)e^{jk_z z}\, dk_z$$

and the inverse transform is given by

$$H_\phi(\rho, z) = \int_{-\infty}^{\infty} h(\rho, z)e^{-jk_z z}\, dk_z$$

J_n, Y_n, and $H_n^{(1)}$ are respectively bassel function of first kind, 2nd **kind and** handel's function of first kind of n-th order

$$v_0^2 = w^2\mu_0\epsilon_0 - k_z^2, \quad v_1^2 = w^2\mu_0\epsilon_0\epsilon_1 - k_z^2 \text{ and } v_2^2 = w^2\mu_0\epsilon_0\epsilon_2 - k_z^2 \quad .$$

where w, μ_0, ϵ_0, ϵ_1, ϵ_2 and k_z are respectively the source frequency free space permeability, permittivity, relative permittivity of gaseous plasma, semiconductor plasma and propagation vector.

Applying the proper boundary conditions (Dhani Ram *et al* 1972) and substituting $k_z = k_0 \sin\theta$, the magnitude of the radiation field can be made known with the help of steepest descent method of saddle point integration (Sharma *et al* 1975). Now $|A_6(k_z = k_0 \sin\theta)|$ will give the magnitude of the radiation mode of the structure analysed here. The expression for A_6 consists of bessel's functions, hankel's functions, v_0, v_1, v_2 etc. and happens to be lengthy. The expression for A_6 reads

$$|A_6| = \left|\frac{R_{79}}{R_{80}}\right|$$

$$R_{80} = R_{78}R_{75} - R_{77}R_{76} \qquad R_{79} = R_{74}(R_{78} - R_{76})$$

$$R_{78} = R_{73} - R_{72}\frac{R_{35}}{R_{3}} \qquad R_{77} = \frac{R_{72}R_{33}}{R_{34}}$$

$$R_{76} = R_{73} \quad \frac{R_{31}R_{72}}{R_{3}} \qquad R_{75} = \frac{R_{32}R_{72}}{R_{30}}$$

$$R_{74} = R_{63}R_{69}, \qquad R_{73} = R_{71}R_{62}+R_{69}R_{65}$$

$$R_{72} = R_{70}R_{62}+R_{64}R_{69}. \qquad R_{71} = R_{27}R_{22}-R_{25}R_{28}$$

$$R_{70} = R_{26}R_{22}-R_{24}R_{28}, \qquad R_{69} = R_{23}R_{28}-R_{29}R_{22}$$

$$R_{65} = R_{25}R_{59}, \qquad R_{64} = R_{59}R_{24}$$

$$R_{63} = R_{61}R_{22}, \qquad R_{62} = R_{23}R_{50}-R_{60}R_{22}$$

$$R_{61} = R_{58}R_{51}. \qquad R_{60} = R_{50}R_{51}-R_{63}R_{37}$$

$$R_{59} = R_{55}R_{51}-R_{52}R_{57}, \qquad R_{58} = \frac{R_{54}R_{44}}{R_{40}}$$

$$R_{57} = \frac{R_{54}R_{59}}{R_{49}}, \qquad R_{56} = R_{10}R_{15}-R_{21}R_{17}$$

$$R_{55} = R_{18}R_{15}-R_{16}R_{21}. \qquad R_{54} = R_{14}R_{21}-R_{10}R_{15}$$

$$R_{53} = R_{17}R_{20}-R_{19}R_{14}, \qquad R_{52} = R_{16}R_{20}-R_{18}R_{14}$$

$$R_{51} = R_{15}R_{20}-R_{21}R_{14}, \qquad R_{50} = R_{43}+\frac{R_{45}R_{47}}{R_{48}}$$

$$R_{49} = R_{42}+\frac{R_{45}R_{46}}{R_{48}}. \qquad R_{48} = R_{7}R_{11}-R_{10}R_{11}$$

$$R_{47} = R_{6}R_{11}-R_{8}R_{13}. \qquad R_{46} = R_{5}R_{11}-R_{12}R_{8}$$

$$R_{45} = R_{8}R_{39}-R_{40}R_{7}, \qquad R_{44} = R_{41}R_{8}$$

$$R_{43} = R_{6}R_{40} \qquad R_{42} = R_{5}R_{40}$$

$$R_{41} = RR_{8}R_{36}-aR_{38} \qquad R_{40} = R_{38}R_{4}-R_{36}R_{37}$$

$$R_{39} = R_{3}R_{38}. \qquad R_{38} = R_{9}R_{3}-R_{4}RR_{8}$$

$$R_{37} = R_{4}RR_{8}-R_{9}R_{3}, \qquad R_{36} = \frac{R_{2}R_{3}}{R_{1}}-R_{4}$$

$$R_{35} = v_1Y_0(v_1e), \qquad R_{34} = v_1J_0(v_1e), \qquad R_{33} = \epsilon_1v_0H_0(v_0e)$$

$$R_{32} = H_1(v_0e), \qquad R_{31} = Y_1(v_1c), \qquad R_{30} = J_1(v_1e)$$

$$R_{29} = \epsilon_1v_2Y_0(v_2d). \qquad R_{28} = \epsilon_1v_2J_1(v_2d). \qquad R_{27} = \epsilon_2v_1Y_0(v_1d)$$

$$R_{26} = \epsilon_2v_1J_0(v_1d), \qquad R_{25} = Y_1(v_1d), \qquad R_{24} = J_1(v_1d)$$

$$R_{23} = Y_1(v_2d), \qquad R_{22} = J_1(v_2d), \qquad R_{21} = \epsilon_2v_1Y_0(v_1c)$$

$$R_{20} = \epsilon_2v_1J_0(v_1c), \qquad R_{19} = \epsilon_1v_2Y_0(v_2c), \qquad R_{18} = \epsilon_1v_2J_0(v_2c)$$

$$R_{17} = Y_1(v_2c), \qquad R_{16} = J_1(v_2c), \qquad R_{15} = Y_1(v_1c)$$

$$R_{14} = J_1(v_1 c), \quad R_{13} = v_1 Y_0(v_1 b), \quad R_{12} = v_1 J_0(v_1 b)$$

$$R_{11} = \epsilon_1 v_0 Y_0(v_0 b), \quad R_{10} = \epsilon_1 v_0 J_0(v_0 b), \quad R_9 = Y_1(v_0 a)$$

$$RR_8 = J_1(v_0 a), \quad R_8 = Y_1(v_0 b), \quad R_7 = J_1(v_0 b)$$

$$R_6 = Y_1(v_1 b), \quad R_5 = J_1(v_1 b), \quad R_4 = v_0 a Y_0(v_0 a) - Y_1(v_0 a)$$

$$R_3 = v_0 a J_0(v_0 a) - J_1(v_0 a). \quad R_2 = Y_0(v_0 a_1), \quad R_1 = J_0(v_0 a_1).$$

$|A_6|$ has been computed with the help of IBM-1130 computer for the different values of the various parameters e.g. a_1, a, b, c, d and e.

3. Results and Conclusion

Theoretical computed values of $|A_6|$ i.e. for the magnitude of the radiation field have been given for numerical values of the various parameters given as $\omega = 2\pi \times 10^{10}$ rad sec^{-1}, $k_0 = 2\pi/3$ cm^{-1}, ω_{p1} (gaseous plasma electron frequency $= 2\times 10^{10}$ rad sec^{-1}, $\omega_{p2} = 1{\cdot}57 \times 10^{14}$ rad sec^{-1} (for Ga-As type semiconductor) and ν_1 (collision frequency of the semiconductor plasma column) $= 10\times 10^{12}$ sec^{-1} The various features of the radiation pattern are following :

3.1 *Effect of radii of conductor and ring source*

The change in a_1 and a does not cause any important variation in the field pattern. The pattern contains the few peaks and almost the same intensity of the radiation field beyond 72°. One plot for $k_0 a = 4{\cdot}188$ has been given in the figure with dotted line. This plot gives the general behaviour of the field amplitude with $k_0 a$ i.e. a. The same kind of variation was computed in the radiation field when the change in a_1 occurs. This is not reported here.

3.2 *Effect of the radius of the air column*

One plot is given for $k_0 b = 6{\cdot}178$ with dot and cross in the figure. This shows the general nature of the field pattern with change in b.

Table 1. Effect of change in b on radiation pattern

Values of the various parameters

$k_0 a_1 = 0{\cdot}209$, $k_0 a = 3{\cdot}874$, $k_0 b = 5{\cdot}235$

$k_0 c = 7{\cdot}330$, $k_0 d = 9{\cdot}424$ and $k_0 e = 11{\cdot}519$

Direction of radiation peak (in degrees)	Magnitude of the radiation field (in rel. units)		
	$k_0 b = 5.55$	$k_0 b = 6.178$	$k_0 b = 6.80$
61	0.276743	0.310689	1.306506
62	0.397805	0.312761	1.100239
64	0.140932	1.637926	4.336998
71	2.465507	0.058649	0.003632

The peak occurs at 64°. Also in 71° to 74° there occurs no radiation. Beyond 74° the magnitude of the field almost attains the constant value upto 90°. Few more data are also given in table 1 for change in b. These data show that peaks appear and diminish with change in b. The maximum intense peak appears in the direction of 71° for k_0b -- 5·55

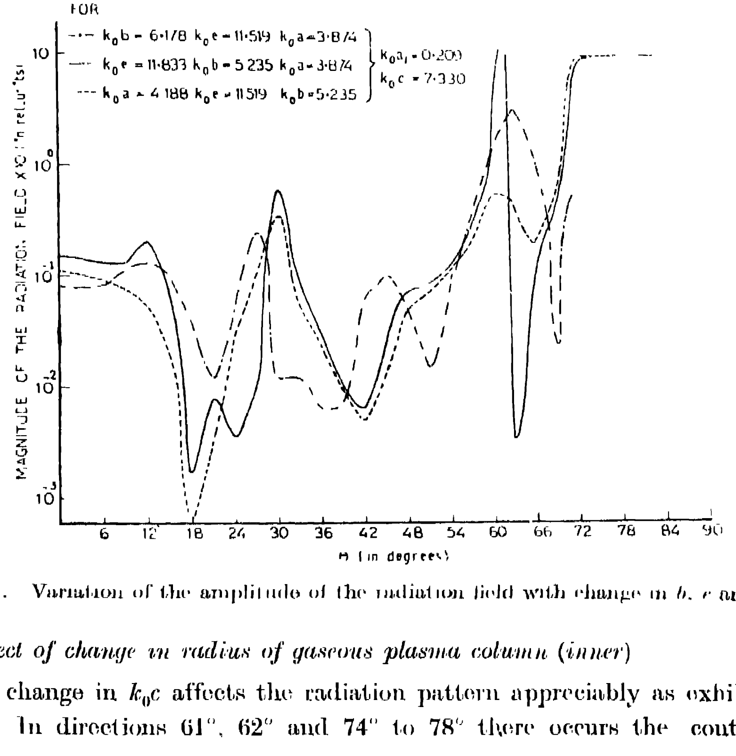

Fig. 1. Variation of the amplitude of the radiation field with change in b, e and a.

3.3 *Effect of change in radius of gaseous plasma column (inner)*

The change in k_0c affects the radiation pattern appreciably as exhibited in table 2. In directions 61°, 62° and 74° to 78° there occurs the continuous

Table 2. Effect of change in c on radiation pattern

Direction of radiation peak (in degrees)	Magnitude of the radiation field (in rel. units)		
	$k_0c = 8.377$	$k_0c = 9.424$	$k_0c = 10.471$
61	0.947127	0.162135	0.076136
62	5.271645	0.139839	0.062010
71	0.187475	0.360985	0.198519
72	0.533763	1.000000	1.000000
74 to 78	1.000000	No radiation	No radiation

decrease in the amplitude of the radiation field when increase in c is caused. In the region viewed through 74° to 78° the decrease in the amplitude occurs so drastically that for $k_0c = 9{\cdot}424$ and $k_0c = 10{\cdot}471$ the directions (74° to 78°) have no radiation field at all. The strongest peak happens to be in the direction of 62° for $k_0c = 8{\cdot}377$.

3.4 *Effect of change in radius of the semiconductor plasma* (d)

This is shown in table 3. No regular feature can be pointed out by viewing this table. For some directions, the increase in 'd' causes the decrease in amplitude of the field while for some other ones, increase in that. The maximum intense radiation peak occurs at 57° for $k_0d = 10{\cdot}995$.

Table 3. Effect of change in d on radiation pattern

Direction of radiation peak (in degrees)	Magnitude of the radiation field (in rel. units)		
	$k_0d = 9.738$	$k_0d = 10.053$	$k_0d = 10.995$
61	1.260337	0.0952770	0.288057
62	3.058328	0.0179776	0.042661
57	0.107812	0.0530960	18.441030
58	0.159423	10.8036100	0.218746
70	0.482713	2.5968930	0.520023
71	1.751459	0.0939700	0.027993

3.5 *Effect of change in radius of the outer gaseous plasma column C*

For $k_0e = 11{\cdot}833$ the plot is given with solid line The peak occur at 62° for this value of k_0e. In 61°, 62°, 71° and 72° directions the increase in e gives the continuous increase in the magnitude of the radiation field while no change in magnitude is found at 72°. Another important feature of the pattern is the occurrence of vanishing amplitude of the field in the region viewed through 74° to 79° directions.

Table 4. Effect of change in e on radiation pattern

Direction of radiation peak (in degrees)	Magnitude of the radiation field (in rel. units)		
	$k_0e = 11.833$	$k_0e = 12.147$	$k_0e = 12.461$
61	0.562195	0.679040	0.775576
62	1.553001	2.231304	2.887645
71	0.172574	0.175839	0.180503
72	1.000000	1.000000	1.000000
74 to 79	No radiation	No radiation	No radiation

Having a glance at the above results one can conclude that the theoretically calculated radiation pattern is appreciably affected with the change in *b*, *c*, *d* and *e*. For particular values of these parameters there occurs certain peaks of radiation which may happen to be useful while sending the signal in some preferred directions. The geometry analysed here comes into existence while a radiating antenna having a central conductor, semiconductor column and ring source (placed in between them) encounters the ionospheric plasma which constitutes the inner and outer layer of the gaseous plasma leaving the ring in the air The physical explanation can be borrowed from Tamir *et al* (1962 1963) and Dhani Ram *et al* (1972) to excite such geometries with sources.

ACKNOWLEDGMENT

The author (S.C.S.) expresses his sincere thanks to CSIR (New Delhi) for awarding him research fellowship. The timely help from the IPC staff of the Institute is also acknowledged with thanks. Thanks are also due to the referee for pointing out few errors in the original manuscript.

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