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Electromagnetic wave propagation in the ionosphere : a general treatment

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The propagation of electromagnetic waves through the ionosphere has been investigated theoretically, taking due account of the anisotropy, spatial inhomogeneity and temporal fluctuations of the physical characteristics of the medium A generalised differential equation is derived for the electric field which determines the refractive index of the medium. This generalised differential equation is in a quite handy form and reduces to the simpler equation derived previously (Gerson, 1962a), when suitable assumptions are introduced. The physical conditions incorporated in our treatment exist in reality but have not been so far accounted for in any mathematical theory.

1. Introduction

The subject of electromagnetic wave propagation in the ionosphere has been dealt with theoretically by a large number of workers (Mitra 1955 Ratcliffe 1959, Al'pert 1960, Ginzburg 1962, Rawer 1952, Gerson 1962 etc.) the pioneering work in the field being those of Appleton (Appleton 1925) and Chapman (Chapman 1932). While several simplifying assumptions were made by many authors, Gerson treated the problem quite generally It appears, however, that certain relistic aspects, e.g. the anisotropy, and inhonogeneity of the medium, etc were not taken care of in Gerson's work

In the present communication, we treat the problem of electromagnetic wave propagation in the ionosphere, taking into account many such features as have been experimentally known to exist, but have not hitherto, been accounted for in any mathematical theory.

It might be mentioned that the problem we treat here is in many ways analogous to the situation prevailing in connection with the behaviour of solids under the action of varying fields. Thus, the results derived here are expected to bring forth useful information about the electrical and magnetic properties of solids.

The generalised aspects incorporated in the treatment are elaborated at appropriate places in the following three sections, where the mathematical theory is developed, while the salient features of the results are discussed in the last section.

Assuming the damping to be small, we can equate the frequency of oscillations of the electrons with that of the propagating electromagnetic field, which is the major cause of the oscillations. We can, thus use

$$\frac{\partial \boldsymbol{S}}{\partial t} = -j\omega \boldsymbol{S}\,; \quad \frac{\partial^2 \boldsymbol{S}}{\partial t^2} = -\omega^2 \boldsymbol{S} \qquad \dots \quad (9)$$

From (7), (8) and (9) we get :

$$\boldsymbol{E}' = -[(a)+(b)]\boldsymbol{P}' \qquad \dots \quad (10)$$

where $\quad (a) = \{m\omega^2 + j\omega(g)\}/ne^2\,; \quad (b) = -(j\omega/nec)\psi_{B'} \qquad \dots \quad (11)$

and, the dyadic,

$$\psi_{B'} = \begin{pmatrix} 0 & B'z & -B'y \\ -B'z & 0 & B'x \\ B'y & -B'x & 0 \end{pmatrix} \qquad \dots \quad (12)$$

Solutions

Equations (6) and (10) give us two relations between the electric field and the polarisation. Using two unknown dyadics ϕ_E and ϕ_P, we convert E, P to E', P' by

$$\boldsymbol{E} = \phi_E \boldsymbol{E}' \qquad \dots \quad (13a)$$

$$P = \phi_P \boldsymbol{P}' \qquad \dots \quad (13b)$$

So that $\quad (6) \equiv A_E\boldsymbol{E}' = A_P\boldsymbol{P}'$

$$(10) \equiv \boldsymbol{E}' = B_P\boldsymbol{P}'$$

Where the A's and B's are operators. From these, we get the two relations :

$$(A_E B_P - A_P)P' = 0 \qquad \dots \quad (14)$$

$$(A_E - A_P B_P^{-1})E' = 0 \qquad \dots \quad (15)$$

Our ultimate aim is to obtain expressions for the refractive index, η, defined as :

$$\eta^2 = (c/\omega E)^2 \boldsymbol{E}\nabla^2\boldsymbol{E} \qquad \dots \quad (16)$$

(Kelso, 1964). In getting η from (16) it is advantageous to use (15), which affords an easy evaluation of Div $\boldsymbol{E}'$. When written explicitly, eq. (15) becomes :

$$\{[c^2\psi_{\nabla}(\mu'^{-1})\psi_{\nabla} - j\omega]\phi_E - 4\pi\frac{\partial}{\partial t}\phi_P[(a)+(b)]^{-1}\}\boldsymbol{E}' = 0 \qquad \dots \quad (17)$$

where

$$\psi_\nabla = \begin{pmatrix} 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ -\frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} & -\frac{\partial}{\partial x} & 0 \end{pmatrix} \qquad \ldots \quad (18)$$

4. Gerson's Result

When suitably modified, our result reduces to that of Gerson. To see this we write (14) explicitly as:

$$\left\{[c^2\psi_\nabla\cdot(\mu'^{-1})\psi_\nabla - j\omega]\phi_E\cdot[(a)+(b)] - 4\pi\frac{\partial}{\partial t}\phi_P\right\}\boldsymbol{P}' = 0 \qquad \ldots \quad (19)$$

Now, the physical situations treated by Gerson would follow from our case under the following conditions:

$$(\mu) = 1, \text{ or } (\mu'^{-1}) = (1/j\omega)\cdot \qquad \ldots \quad (20)$$

$$\phi_E = 1 = \phi_P; \qquad \ldots \quad (21)$$

$$\psi_{B'} = \psi_{B^*} \qquad \ldots \quad (22)$$

and

$$(g) = m\nu \qquad \ldots \quad (23)$$

The implications of conditions (20) to (23) are as follows: (20) implies a time-invariant permeability; (21) neglects the electric field and polarisation vectors arising from ionospheric charges; the electromagnetic part of the magnetic induction is neglected in comparison to the terrestrial component through eq (22); and the tensor (g) is replaced by a scalar through eq (20); where ν is the collision frequency. When eqs. (20) to (237 are substituted in (19), we get:

$$\{(\text{Curl Curl} - \omega^2/c^2)[(m/ne^2)(\omega^2 + j\omega\nu) - (j\omega/nec)\psi_{B^*}\cdot 4\pi\omega^2/c^2\}\boldsymbol{P}' = 0 \quad (24)$$

This is Gerson's result, as the $\boldsymbol{P}'$ and $\boldsymbol{B}^*$ used here correspond to Gerson's $\boldsymbol{P}$ and $\boldsymbol{B}$. The Lorentz term is, of course, absent.

5 Discussion

As mentioned in the Introduction, the purpose behind the present work is to treat generally the problem of electromagnetic wave propagation in the ionosphere. The most significant characteristic of our treatment is the simultaneous inclusion, in the mathematical theory, of the following aspects:

(I) Anisotropy of the medium which requires μ and κ to be considered as tensors of rank 2.

(II) The inhomogeneity and the temporal fluctuations of the physical characteristics of the medium. These conditions require the components of (μ) and (κ) to be assumed as dependent on both space coordinates and time (Gerson, 1962b).

(III) Effects of fields other than those of the test e.m. wave. are taken into account, while studying the motion of ionospheric charges.

Eq (17) takes full care of these aspects, and would be fairly tractable if an attempt is made to solve it for E', the latter being the only entity which determines the refractive index via eq. (16). The incorporation of aspects (I) and (II) have further led to the introduction of the new dyadics ϕ_E, ϕ_P and ψ_B, all three of which have made it possible to derive a compact equation, (17), for E'. We hope to report, in future communications, quantitative investigations of the theory developed here.

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