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Effect of the ponderomotive force on ion-acoustic waves

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In general, it is shown that the nonlinear effects due to the ponderomotive force become important whenever the applied field or the fields generated internally exceed a certain limit. The redistribution of the particle can lead to an effective pressure analogous to kinetic pressure in plasma. The propagation of ion acoustic waves, in such a media is studied.

1. INTRODUCTION

Among the nonlinear problems of a plasma obeying the Vlasov equations, one of the most interesting is the distribution of plasma particles in the equilibrium. Moreover, the nonlinear effects are entirely diverse and it is important to study their nature in detail in the realistic situations. One of these is the ponderomotive force which changes the effective wave particle interaction (Tsytovich 1970, Hasegawa 1975, Schmidt 1966), wave conversion (Fichet 1976), and important in the laser fusion (Hora 1971).

In this paper, we have studied the distribution of plasma due to the ponderomotive force and have shown that it can alter substantially the equilibrium distribution, and the plasma dispersion relations. Further more, the despersion relations derived with the assumption of infinitesimally small perturbations may not be true for the plasmas, where fields of finite amplitudes either self generated or due to external sources exist. In Section 2, we derive the ponderomotive force on the charged particle in the presence of uniform magnetic field and in the homogeneous plasma. We also obtain the distribution of the particles in the absence of magnetic field and its equivalent pressure term. The effect of this on the ion acoustic waves is studied. The Section 3 discusses briefly the importance of the present study and the need for the inclusion of the ponderomotive force on the wave particle interaction is stressed.

2. THE DERIVATION OF PONDEROMOTIVE FORCE AND THE DISPERSION RELATION FOR THE ION ACOUSTIC WAVES.

The motion of a charged particle in presence of the electric field $\boldsymbol{E}$ and the magnetic field $\boldsymbol{B}$ is given by the equation

$$m_j \frac{d\boldsymbol{v}_j}{dt} = e_j \left(\boldsymbol{E} + \frac{\boldsymbol{v}_j \times \boldsymbol{B}}{c} \right) \quad \ldots \quad (1)$$

where m_j, e_j, $\boldsymbol{v}_j$ are the mass, charge and velocity of the j-th particle. c is the velocity of light. When the fields $\boldsymbol{E}$ and $\boldsymbol{B}$ are specified, with appropriate initial conditions, this equation describes the trajectory of the particle. In the realistic situations, the particle trajectories are entirely nonlinear, therefore often the perturbation methods are used. This procedure simplifies the picture and gives an approximate estimate of the wave particle interactions. We also assume here the absence of zeroth order streaming of particles, and study the non-linear effects using the perturbation method, in the plasma which is homogeneous and is immersed in the ambient uniform magnetic field, $\boldsymbol{B}_0$, along x axis of the right hand coordinate system. The zeroth order electric field $\boldsymbol{E}_0$ is absent. We write for perturbations as $\boldsymbol{v}_j^{(1)}$, $\boldsymbol{E}_j^{(1)}$, and $B^{(1)}$ and seek solutions for which all the perturbed quantities have a dependence $\exp i(kx-\omega t)$. In this case, to the first order approximation the solution for $\boldsymbol{v}_j^{(1)}$ can be shown to be

$$\boldsymbol{v}_j^{(1)} = \frac{-e_j/m_j}{(\Omega_j^2-\omega^2)}\left\{i\omega\boldsymbol{E}^{(1)}+\boldsymbol{\Omega}_j\times\boldsymbol{E}^{(1)}-\frac{i\boldsymbol{\Omega}_j\boldsymbol{\Omega}_j.\boldsymbol{E}^{(1)}}{\omega}\right\} \quad \ldots \quad (2)$$

where $\Omega_j = e_jB_0/m_jc$ is the gyrofrequency of the j-th particle and ω is the frequency of the oscillating field. It is easy to see that the average $\boldsymbol{v}_j^{(1)}$ over oscillating periods vanishes. Therefore, it is valid in the perturbation theory dealing with resonances, to neglect the effect of first order motion on the zero order motion. However, the second order effect averaged over fast varying time periods become important, as shown below.

The neglected term in eq. (1) is the nonlinear part which can be estimated by

$$\widetilde{f} = (\boldsymbol{v}_j^{(1)}.\nabla)\boldsymbol{v}_j^{(1)} - \frac{\partial\boldsymbol{v}_j^{(1)}}{\partial t} + \frac{e_j}{m_j}\left(\boldsymbol{E}+\frac{\boldsymbol{v}_j^{(1)}\times\boldsymbol{B}}{c}\right) \quad \ldots \quad (4)$$

This term, on the average, is finite and the particle motions are altered. This is the term, which leads to ponderomotive force, which is often unaccounted, because, it turns to be very small for all phenomena where the finite but negligible amplitude fields are involved. The general expression for this term can be written as

$$<\widetilde{f}> = 1/4\delta_{ik}\nabla(E_i{}^*{}_kE_k) \quad \ldots \quad (5)$$

where * denotes the complex conjugate and i and k denoted the components along i and k coordinates respectively. From eq. (4) the components of δ_{ik} can be shown to be

$$\delta_{ik} = -\frac{e^2}{m_j\omega^2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & \dfrac{\omega_2}{\omega^2-\Omega_j^2} & \dfrac{i\omega\Omega}{\omega^2-\Omega_j^2} \\ 0 & \dfrac{-i\omega\Omega_j}{\omega^2-\Omega_j^2} & \dfrac{\omega^2}{\omega^2-\Omega_j^2} \end{pmatrix} \quad \ldots \quad (6)$$

This is equivalent to the result obtained by Gurevich & Pitaovskii (1964). The derivation of eq. (5) is general one, and we will be discussing only a spatial case, where there is no magnetic field. In this case, the field components do get decoupled and lead to ponderomotive force (F) term

$$F = -\frac{e^2}{4m_j\omega^2}\nabla E^2 \quad \dots (7)$$

which is often discussed in the literature. This force is important and will be discussed in the following few paragraphs.

In this paper for clarity, we discuss only one case. For example in the presence of the high frequency electrostatic fields such that $\omega > \omega p_i$, (ωp_i is ion plasma frequency), the force acting on the ions of mass M is M/m times smaller than that on the electron of mass m and therefore the ions can be treated as a neutralising background. Hence we study only the electron redistribution.

It is also important to note that the above force equation is true (7) when the adiabaticity is valid, i.e. the field amplitude does not vary very much in a distance R such that $R \gg V_T/\omega$ (V_T is the thermal velocity, (2) The scale length L is such that $L \gg cE/m\omega^2$ which limits the field amplitude. Under these conditions in the absence of any ambient fields, the constant of motion can be obtained and the equilibrium distribution of the electrons, g, can be written as

$$g = g\left(\tfrac{1}{2}mv^2 + \frac{e^2E^2}{4m\omega^2}\right) \quad \dots (8)$$

where E^2 is the magnitude of the wave field.

Using this distribution the average energy of the particle can be obtained and then the equivalent pressure term becomes

$$P = nT_e + \frac{ne^2E^2}{4m\omega^2} \quad \dots (9)$$

where P is the constant; T_e is the thermal energy of the electrons, n is the density of electrons. This is nothing but the total pressure which reduces to the kinetic pressure consistent with the observation in the absence of the field E. The neglect of the term $e^2E^2/4m\omega^2$ is valid provided, the ratio $e^2E^2/4m\omega^2T_e$ is very much less than unity, i.e. in the region of weak turbulence or when the applied fields are very small In this case, the propagation characters of ion acoustic waves are unaltered. However, when the finite fields are present, those wave propagations are altered substantially. The ion acoustic waves are of low frequency and therefore the electrons can easily follow them and maintain the charge neutrality. In this case the equation of motion

$$\rho\frac{d\boldsymbol{v}}{dt} = -\Delta P_T \quad \dots (10a)$$

and the continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \boldsymbol{v} = 0 \qquad \ldots \quad (10b)$$

describe its propagation characteristics. Where ρ is the density $n_i + n_e m_e$, i and e represent ions and electrons respectively. Since the charge neutrality assures $n_i = n_e$, and masses $m_e \ll m_i$, the density ρ reduces to ion mass density. The pressure P_T is the total pressure due to both the ions and the electrons. Often, the pressure due to ions turn out be smaller compared to electrons because the electron temperature T_e is very much greater than ion temperature T_i. Therefore the total pressure is nothing but the electron pressure which in the present case is the expression (9). In the case of zero ion streaming, using the first order perturbation theory, and with the help of eqs (10a) and (10b), one can write the dispersion relation for ion acoustic waves as

$$\frac{\omega_s^2}{k_s^2} = \frac{T_e}{m_i} + \frac{e^2 E^2}{4 M m_e \omega^2} \qquad \ldots \quad (11)$$

where ω_s, k_s denote the ion acoustic wave frequency and wave numbers. This shows that due to the ponderomotive force the phase velocity increases, and the waves propagate much faster than those in the absence of ponderomotive force.

3. Discussion

This is interesting to note that as in the case of cold ions and hot electrons, where ion acoustic waves propagate because of the electrons which help them, similarly the ponderomotive force on electrons also assists in their propagation. Moreover, these waves propagate faster than those in the absence of electrostatic wave fluctuations. Therefore, the effect of turbulence may alter effectively the damping of ion acoustic waves.

In general, based on the above study, we see that the ponderomotive force changes the nature of wave particle interactions drastically. These forces may arise due to internal sources or externally. It is also seen that the low frequency pressure dependent waves, like ion acoustic waves can feel the effect of the ponderomotive forces. Therefore, it is important to study the nonlinear effects arising due to the ponderomotive forces.

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