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Cosmic pion spectrum at the top of the atmosphere

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The pion spectrum in the spectral range 10–100 GeV at the top of the atmosphere has been derived from the primary nucleon spectrum of Ryan *et al* (1972) by using Bugaev model. The derived result has been compared with those expected from CKP, CE, Diffusion and Feynman scaling models of *pp* interactions. It is found that the derived results from Bugaev model is supported by the scaling hypothesis of Feynman and CE model.

1. INTRODUCTION

The study on the production spectrum of muons is of interest to cosmic ray phenomenology to get the information on the production processes. In general the experimental spectrum of primary cosmic ray nucleons at the top of the atmosphere and the ground level muon spectrum can be used to test the validity of the interaction models in the propagation of cosmic rays in space. The muon spectrum can indicate the amount of energy transferred by nucleons to muons or more particularly to their parents. The pion spectrum at the top of the atmosphere also gives information about the spectral shape of muons in the cosmic ray phenomenology and in the development of high energy physics.

The pion spectra derived from the primary cosmic ray nucleon spectrum using the scaling hypothesis of Feynman (1969), Constant Energy model (Brooke *et al* 1964), Bose type model have been presented in our earlier reports (Bhattacharyya 1972, Bhattacharyya & Basu 1975, Bhattacharyya 1975, Bhattacharyya *et al* 1975a).

In the present paper we have used the *pp* interaction model of Bugaev *et al* (1970) to derive the pion production spectrum at the top of the atmosphere from the measured nucleon spectrum of Ryan *et al* (1972). The Intersecting Storage Ring data on *pp* collisions reviewed by Erlykin *et al* (1974) have been used in the present work. The derived pion spectrum has been compared with the results obtained from CKP model (Cocconi *et al* 1972), Diffusion model (Bhattacharyya, Roy Chowdhury & Basu 1976), Scaling hypothesis of Feynman (1969), Constant Energy model (Brooke *et al* 1964).

2. Theoretical Models

(i) *Bugaev model*

The energy spectrum of pions from the spectrum of primary nucleons can be estimated by using the p–p interaction model of Bugaev *et al* (1970). The pion nucleon flux ratio $\pi(E)/N(E)$ at the energy E at the top of the atmosphere can be estimated from the following relation

$$\pi(E)/N(E) = A_\pi(\lambda_\pi/\lambda_p)[<nx^{\gamma'}>_{p\pi+} + <nx^{\gamma'}>_{p\pi-}] \quad \dots \quad (1)$$

where γ' = exponent of the integral primary nucleon spectrum,
λ_p = interaction free path of nucleons in air = 80 g-cm^{-2},
λ_π = absorption free path of pions in air = 120 g-cm^{-2},
$<nx^{\gamma'}>_{p\pi\pm}$ = the fractional energy moments for charged pions produced during p-nucleus interactions,
A_π = the kinetic constant = 0·691.

Erlykin *et al* (1974) have calculated the fractional energy moments for pions produced from the Intersecting Storage Ring experiments (p–p collisions). Table-1 shows the values of $<nx^{\gamma'}>_{p\pi+}$ and $<nx^{\gamma'}>_{p\pi-}$ for $\gamma' \approx 1{\cdot}75$.

Table 1. The values $<nx^{\gamma'}>_i$ for charged pions

Interaction process (i)	$<nx^{\gamma'}>_i$
$p+p \rightarrow \pi^+ + X(p\pi^+)$	0.0352
$p+p \rightarrow \pi^- + X(p\pi^-)$	0 0226

(ii) *CKP model*

The empirical CKP relation formulated by Cocconi *et al* (1971) for the number of pions of one sign emitted in the forward direction in the C system, $N(E_\pi)dE$ is given by

$$N(E_\pi)dE_\pi = \frac{A}{T_p}\exp(-E_\pi/T_p)dE_\pi \quad \dots \quad (2)$$

where E_π is the pion energy in L system, A is the mean multiplicity of pions of one sign emitted in the forward direction in the C-system and T_p is the mean pion energy. The parameters A, T_p and pion inelasticity $\bar{K}_\pi$ follows the relation

$$AT_p = \tfrac{1}{3}\bar{K}_\pi E_p \quad \dots \quad (3)$$

where E_p is the primary nucleon energy expressed in GeV units. The proton light nucleus interaction data obey Fermi law

$$A = aE_p^{0\cdot25} \qquad \text{where } a = 0{\cdot}45. \quad \dots \quad (4)$$

The differential pion production spectrum $P(E_\pi)dE_\pi$ at the top of the atmosphere can be expressed from the primary nucleon spectrum of the power law fit form

$$N(E_p)dE_p = BE_p^{-\gamma}dE_p \qquad \dots \quad (5)$$

by the CKP relation developed by Cocconi *et al* (1971) follows as

$$P(E_\pi)dE_\pi = \frac{2dE_\pi}{[1-(1-K_T)^{\gamma-1}]} \frac{3a^2B}{K_\pi} \int_{3E_\pi}^{\infty} E_p^{-\gamma-0.5} \exp\left(-\frac{3aE_\pi}{K_T E_p^{0.75}}\right) dE_p \quad \dots \quad (6)$$

where K_T and K_π are the nucleon inelasticity and pion inelasticity, respectively.

(iii) *Pion atmospheric diffusion model*

The usual diffusion equation described in reference (Bhattacharyya, Roy Chowdhury & Basu 1976) for the propagation of charged pions in the atmosphere accounting for loss of pions by interaction decay and gain by production in nucleon-nucleon collisions yield the differential pion spectrum at the top of the atmosphere $\pi(E)dE$ from the sea level muon spectrum $\mu(E)dE$ by the following relation

$$\pi(E)dE = \frac{\lambda_n(B_\pi r+E)\mu(E)}{\Lambda r^{\gamma-1}B_\pi r h(E) y(E)}\, dE \qquad (7)$$

where r is the energy degradation factor in $\pi \rightarrow \mu$ decay, Λ is the absorption mean free path of nucleons in air = 120 g-cm^{-2}, B_π = critical energy for pion decay = 118 GeV, λ_n is the interaction mean free path of nucleons in air = 90 g-cm^{-2}, γ is the exponent of the differential energy spectrum of primary cosmic ray nucleons expressed as a power law $\simeq 2.75$ (Ryan *et al* 1972), $h(E)$ is a function depending on $\mu - e$ decay and energy losses of muons in air and $y(E)$ is the correction factor depending on the kaon to pion ratio (K/π) at production

(iv) *Scaling hypothesis of Feynman*

From the scaling hypothesis of Feynman (1969) it is evident that the differential cross section for the production of a charged particle of a given type in high energy hadron-hadron collisions tends to a limit

$$\lim_{s\rightarrow\infty} E(d^3\sigma/d^3p) = f(x, p_T) \qquad \dots \quad (8)$$

where s is the square of the center of mass energy, E, p, p_T are energy, momentum and transverse momentum of the secondary particle in the center of mass frame and the Feynman variable $x = 2p_L/\sqrt{s}$ where p_L is the logitudinal component of p parallel to the momentum p_{inc} of the primary particle. At a very high energy $p_L \gg p_T$ the equation (8) can be written as

$$d\sigma_\pi = \pi f(x, p_T)\frac{dx}{x}\, dp_T^2. \qquad \dots \quad (9)$$

The production spectrum of charged pions $\pi(E_\pi)dE_\pi$ can be found from the following relation (Bhattacharyya 1975).

$$\pi(E_\pi)dE_\pi = \int_{E_p=E_\pi}^{\infty} BE_p^{-\gamma} C_\pm \exp(-a_\pm x^2) \frac{dE_p}{E_\pi} dE_\pi$$

$$\simeq N(E_\pi)[C_+\phi(\gamma, a_+) + C_-\phi(\gamma, a_-)] \quad \ldots \quad (10)$$

where $\phi(\gamma, a) \approx 0{\cdot}5a^{(1-\gamma)/2}\,\Gamma[(\gamma-1)/2]$.

The interaction parameters C_+, C_-, a_+, a_- have been found from the measurements of Bali *et al* (1970) and were 0·47, 0·37, 7·4 and 12·1, respectively.

(v) *Constant Energy model*

Brooke *et al* (1964) have assumed in the Constant Energy model (to be referred to as CE) that pions are emitted with equal energy in the C-system, half being in the forward direction and half in the backward direction, and the energy of the fast pions in the L system is taken as $K_\pi E_p/3A$. If one assumes the nucleon inelasticity K_T and pion inelasticity K_π as constants for each interaction at a particular energy, the production spectrum of charged pions can be found from the CE model by using the relation

$$\pi(E_\pi)dE_\pi = \frac{2}{[1-(1-K_T)^{\gamma-1}]}\,\frac{B}{(1-\alpha)}\,a^u\,(K_\pi/3)^v E_\pi^{\,w}\,dE_\pi \quad \ldots \quad (11)$$

where $u = \dfrac{(2-\gamma)}{(1-\alpha)}$, $v = \dfrac{(\gamma-\alpha-1)}{(1-\alpha)}$, $w = \dfrac{(2\alpha-\gamma)}{(1-\alpha)}$, $\alpha = 0{\cdot}25$.

3. Results and Discusion

The differential spectra of cosmic primary protons measured upto 2000 GeV by Ryan *et al* (1972) has been used in the present calculations. They used ionization spectrometers at ballon altitudes. The primary nucleon spectrum can be estimated on assuming the nucleon proton flux ratio to be 1·33 and the spectrum follows the relation

$$N(E_p)dE_p = 2{\cdot}66E_p^{-2{\cdot}75}\,dE_p \quad \ldots \quad (12)$$

where E_p is expressed in GeV and the intensity in (cm^2 sec sr GeV)$^{-1}$. The solid line in figure 1 shows the primary nucleon spectrum estimated from the Goddard Space Flight data (Ryan *et al* 1972). The relation (12) yields the values of B and γ of relation (5) are of 2·66 and 2·75, respectively. Using this nucleon spectrum, the pion nucleon flux ratio at energy E at production has been estimated by relation (1). The derived pion spectrum at the top of the atmosphere has been estimated and follows the relation

$$\pi(E_\pi)dE_\pi = 0{\cdot}158\,E_\pi^{-2{\cdot}75}\,dE_\pi. \quad \ldots \quad (13)$$

The result has been plotted in figure 1

In CKP model (relation 6) the best estimated value of Brooke *et al* (1964) on the nucleon inelasticity $K_T = 0{\cdot}47$ and pion inelasticity $K_\pi = 0{\cdot}35$ has been used in the present calculations. The pion spectrum has been estimated in the spectral range 10–100 GeV and the result is presented in figure 1

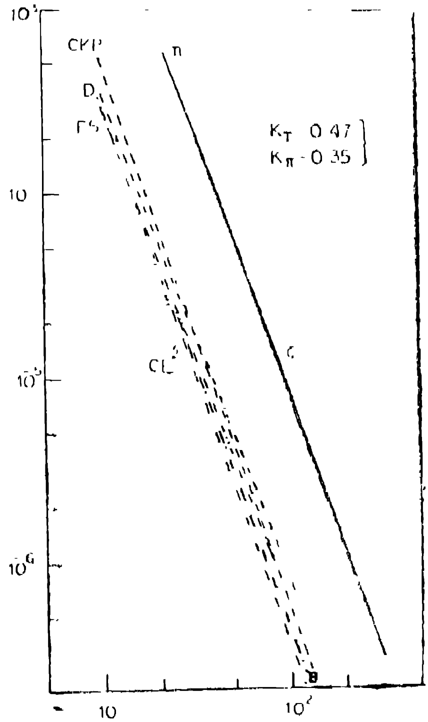

Fig 1 The full line (n) shows the primary nucleon spectrum after Ryan *et al.* (1972). The derived pion spectra at the top of the atmosphere .... (B) after Bugaov model (Bugaov *et al* 1970 , - - - - (CKP) after (Cocconi *et al* 1971); . . ... (D) after Diffusion model (Bhattacharyya *et al* 1976) —. .—. (F) after scaling model (Feynman 1969): ....— (CE) after CE model (Brooke *et al* 1964).

In the pion atmospheric diffusion model (relation 7) the experimental differential sea level muon spectrum $\mu(E)dE$ of Ayre *et al* (1975) has been used. The derived pion spectrum from the diffusion model has also been presented in figure 1.

Using the interaction parameters C_+, C_-, a_+, a_- of Bali *et al* (1971) the pion spectrum from the primary nucleon spectrum of Ryan *et al* (1972) has been estimated by Scaling model of Feynman (1969) and the result has been plotted in figure 1

Finally from the nucleon source spectrum of Ryan *et al* (1972) the pion spectrum at the top of the atmosphere has been derived by using Constant Energy model. The values of K_T and K_π being chosen from the work of Brooke *et al.* (1964) which are 0·47 and 0·35, respectively

It is evident from figure 1 that the derived pion spectrum from Bugaev model agrees with the result expected from scaling hypothesis of Feynman (1969) and the Constant Energy model of Brooke *et al* (1964)

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