SCATTERING OF LOW ENERGY NEUTRONS BY CARBON

CHHAYA GANGULY AND N. C. SIL

DEPARTMENT OF THEORETICAL PHYSICS, INDIAN ASSOCIATION FOR THE CULTIVATION OF SCIENCE, JADAVPUR, CALCUTTA-32

(*Received March* 26, 1963)

ABSTRACT. The cross sections of scattering of low energy neutrons by carbon have been calculated by the phase shift method, the complex nuclear potential is taken as of Woods-Saxon form and the phases are calculated with the help of a relation given by Brysk. The theoretical values of the differential cross sections for 2 7 and 4.1 Mev neutrons agree fairly well compared with the experimental findings for the same given by Little *et al*, and Walt & Beyster respectively.

INTRODUCTION

A number of investigations (Feshbach *et al.* 1954; Feshbach, 1958) on neutron—nucleus interaction have been carried out with considerable success in terms of an optical model of the nucleus. In this model the interaction due to the target nucleus consisting of many individual nucleons is replaced by an average potential well of complex nature which can be used to determine the differential cross section for elastic scattering, its imaginary portion takes care of the absorption due to the formation of the compound nucleus. Most of the calculations of this type have been done with a square well potential which, though, has the advantage of mathematical simplicity and of giving fairly reasonable result, is however unsatisfactory in predicting much larger values of scattering cross section for larger angles than what are experimentally found. The cause of the discrepancy is due to the sharp edge of the potential well and as such the model has been modified by Woods-Saxon (1954) who has replaced the sharp edge by a diffuse one. Various results obtained with this model having a smoothly rounded edge agree reasonably well with experimental findings.

The object of the present paper is to calculate by the phase-shift method the cross section of scattering of low energy neutrons by carbon; the complex nuclear potential is taken to be of Woods-Saxon (1954) type and the phases are calculated with the help of a relation given by Brysk (1962). Brysk's formula differs from the Born's relation by a factor in the denominator which for high energy becomes 1. The claim of Brysk is that his relation gives more exact values of phase angles for energies lower than those where Born method is valid. At low energy of the incident particle the contribution to the scattering cross section will come mostly from the first few phase-shifts. The calculation of phase shift is generally difficult and analytical solutions can be obtained only in few special cases. It is doubtful whether the Born approximation is valid for energies below 100 Mev, the method of calculating the individual phases by the Born method may push the limit of validity a little lower than 100 Mev. But Brysk claims that the calculation of the phase angles by his method brings down the limit of validity to as low as 5 Mev particle energy for spherical well potential with range of the order of $\hbar/mv$ i.e. $\lambda/2\pi$. His method is not very useful for long range potential —a limitation imposed by certain approximations introduced in his calculations. The Woods-Saxon potential being of short range nature will not offer any difficulty in Brysk method.

RESULTS AND DISCUSSIONS

The scattering amplitude is expressed by

$$f(\theta) = (2ik)^{-1}\sum_{l=0}^{\infty} (2l+1)(e^{2i\delta_l} -1)P_l(\cos\theta) \qquad \ldots \quad (1)$$

We calculate the phases δ_l by Brysk's relation

$$\tan\,\delta_l = \frac{-\dfrac{2\mu k}{\hbar^2}\displaystyle\int_0^\infty r^2 \mathrm{d}r j_l^2(kr)V(r)}{1-\dfrac{2\mu k}{\hbar^2}\displaystyle\int_0^\infty r^2 dr j_l(kr)y_l(kr)V(r)}$$

where j_l, y_l are spherical Bessel and Neumann functions respectively,

$$k = \frac{\sqrt{2\mu E}}{\hbar}$$

μ being the reduced mass and E, the energy in the centre of mass system, the nuclear potential $V(r)$ is of the form as given by Woods and Saxon viz.,

$$V(r) = -\frac{(V+iW)}{1+e^{(r-R)/a}} \qquad \ldots \quad (3)$$

where R is the nuclear radius and a is the diffusivity parameter.

The integrals for s-wave phase-shift have been calculated analytically. With the substitution $x = \dfrac{r-R}{a}$ the integral in the numerator of the right hand side of eqn. (2)

$$\int^\infty \frac{\sin^2 kr}{1+e^{(r-R)/a}}\,dr$$

takes the form $\quad 1/2\,[\,L(0)-L(2k)]$

where $$L(p)=a\int_{-R/a}^{0}\frac{\cos p(R+ax)}{1+e^{x}}\,dx+a\int_{0}^{\infty}\frac{\cos p(R+ax)}{1+e^{x}}\,dx$$

Now $$\frac{1}{1+e^{x}}=\sum^{\infty}e^{nx}(-)^{n}\quad\text{for}\quad r<R$$

and $$\frac{1}{1+e^{x}}=\sum_{n=1}^{\infty}e^{-nx}(-)^{n+1}\quad\text{for}\quad r>R$$

Hence $$L(p)=a\sum_{n=0}^{\infty}(-)^{n}\int_{-R/a}^{0}\cos p(R+ax)\,e^{nx}dx$$

$$+a\sum_{n=1}^{\infty}(-)^{n+1}\int_{0}^{\infty}\cos p(R+ax)e^{-nx}dx$$

The integrals are elementary and can be evaluated easily.

Making use of the relation (c.f. Bromwich, 1947)

$$1+2a^{2}\sum_{n=1}^{\infty}\frac{(-)^{n}}{n^{2}+a^{2}}=\frac{a\pi}{\sin h a\,\pi}$$

we may finally write

$$L(p)=\frac{2\sin pR}{p}-\frac{a\pi\sin pR}{\sin h a\pi}-a\sum_{n=1}^{\infty}\frac{(-)^{n}ne^{-nR/a}}{n^{2}+p^{2}a^{2}}$$

Similarly, the integral in the denominator can be evaluated and we have

$$\int_{0}^{\infty}\frac{\sin 2kr}{1+e^{(r-R)/a}}\,dr=-\frac{\cos 2kR}{k}+\frac{1}{2k}+\frac{a\pi\cos 2kR}{\sin h\,2ak\,\pi}+a\sum_{n=1}^{\infty}\frac{(-)^{n}2ake^{-nR/a}}{n^{2}+4k^{2}a^{2}}$$

The infinite integrals for p- wave and d-wave phase-shifts in the numerator have been split up as

$$\int_{0}^{R}r^{2}dr\,j_{l}^{2}(kr)-\int_{0}^{R}\left(1-\frac{1}{1+e^{(r-R)/a}}\right)j_{l}^{2}(kr)r^{2}dr+\int_{R}^{\infty}\frac{1}{1+e^{(r-R)/a}}\,j_{l}^{2}(kr)r^{2}dr$$

The first integral is evaluated analytically and last two integrals have been calculated numerically. The integrals in the denominator have been calculated in a similar way.

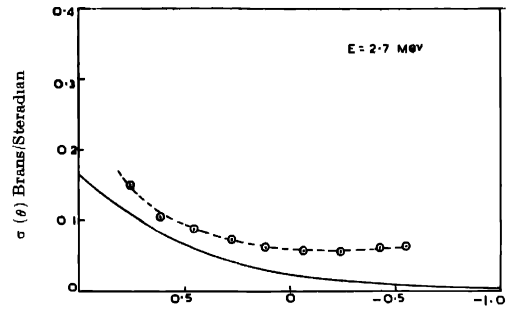

Fig. Differential cross section of scattering of 2.7 Mev neutrons by carbon
- - - - - - experimental curve
——— theoretical curve

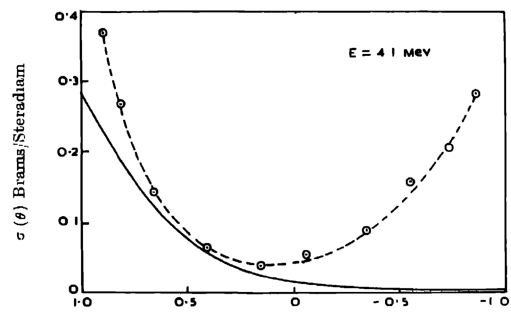

Fig 2 Differential cross section of scattering of 4.1 Mev neutrons by carbon
- - - - - - experimental curve
——— theoretical curve

We have to calculate the differential cross sections in the c.m. system for the incident neutron energies corresponding to 2.7 Mev and 4.1 Mev in the laboratory system. We use the values of the parameters as given by Feshbach *et al.* (1958)

$$R = (1.15A^{1/3}+.4)\times 10^{-13} \text{ cm}$$

$$a = 0.57\times 10^{-13}$$

$$V = 52 \text{ Mev}$$

$$W = 3.12 \text{ Mev}.$$

Results of our calculation for tan δ_l in the c.m. system have been given in the following Table for the elastic scattering of 2.7 Mev and 4.1 Mev neutrons by carbon. The differential cross sections for these incident energies have been compared with the experimental values given by R. N. Little *et al* (1955) and Walt and Beyster (1955) respectively.

Energy in lab system	tan δ_0	tan δ_1	tan δ_2
2.7 Mev	$-0.6930+0.0059\,i$	$-.2194+.0033\,i$	$-.0340+.0012\,i$
4.1 Mev	$-0.9838+0.0099\,i$	$-.3736+.0061\,i$	$-.1092+.0036\,i$

It is found that the experimental values of scattering cross sections for 2.7 Mev particles show a decrease with increasing angle till 90° and after that a little tendency to increase, whereas for 4.1 Mev particles the cross section decreases with angle upto 90° and thereafter goes on increasing, the experimental curve for the differential cross section of 4.1 Mev particles is almost symmetric about 90°. Our theoretical values agree fairly well with the experimental results upto 90° but after that the theory fails to show the increase which is presumably due to the preponderance of *d* scattering brought in by the deformation of the nucleus. In the energy region considered we have appreciable compound elastic scattering and it is very difficult to separate this from the total elastic scattering observed experimentally. The theoretical curves which give only the shape elastic part of the elastic scattering are necessarily below the experimental ones which include both the shape elastic and the compound elastic scattering.

ACKNOWLEDGMENTS

We wish to thank Professor D. Basu, for his kind interest in the problem and for many helpful discussions during the progress of the work

REFERENCES

Bromwich, 1947, Introduction to the theory of Infinite Series, 2nd Edition.

Brysk, Henry, 1962, *Phys. Rev.*, **126**, No 4, 1589.

Feshbach, H., Porter, C. E. and Weisskopf, V. F., 1954, *Phys Rev.*, **96**, 448.

Feshbach, H., 1958, *Annual Rev. of Nuclear Science*, **8**, 88.

Little, R. N., Jr., Leonard, B P., Jr., Prud'homme, J T., and Vincent, L D., 1955, *Phys Rev*, **98**, 634.

Walt, M. and Beyster, J. R., 1955, *Phys Rev*, **98**, 677.

Woods, R. D. and Saxon, D. S., 1954, *Phys. Rev.*, **95**, 577.