AN EMPIRICAL RELATION FOR ALPHA DISINTEGRATION ENERGIES FOR MEDIUM-HEAVY ELEMENTS

M. RAMA RAO

SAHA INSTITUTE OF NUCLEAR PHYSICS, CALCUTTA, INDIA

(*Received, December* 27, 1962)

ABSTRACT It has been observed that for elements with $N > 82$ and $Z < 82$, the function $(Q-0.624Z)$ has a regular variation with mass number A, where Q is the alpha disintegration energy MeV A plot of $(Q-0.624Z)$ vs. mass number A is found to yield a linear graph which takes up the form $(Q-0.624Z) = -0.21A-5.5$. This equation has been utilised in calculating the decay energies of alpha emitters of the medium-heavy elements. The agreement between observed values and those calculated with the help of this formula lies within an average of ± 5 per cent.

INTRODUCTION

To the experimentalist engaged in the search for new alpha activities, a beforehand knowledge of the decay energy of such alpha emitters is a desirable thing This information together with a reasonable estimate of the half-lives of the unknown alpha emitters as can be deduced from Bethe equation [cf. Segrè (1959)] relating half-life T to the decay energy E and charge number Z, is quite useful and serves as a checkup on the discrepancies in experimental results

Bethe-Weizäcker (1936) semi-empirical mass formula derived on the basis of the liquid drop model of the nucleus has been used by a number of workers in the calculation of the alpha-decay energies.

Pryce (1950) calculated the alpha-decay energies from Bethe-Weizsäcker formula and found that for nuclei $Z \geqslant 84$, $N \geqslant 128$, the difference between experimental and calculated values of the energy is a smoothly varying function of the mass number A and is expressed by the relation .

$$Q(\text{obs})-Q\,(\text{calc}) = 4\epsilon-4a(A-210.5) \qquad \ldots \quad (1)$$

where ϵ and a are constants and A is the mass number. Das (1950) has also calculated the alpha-decay energies and showed that it satisfactorily described the observed trend in alpha-decay energies as a function of mass number.

Bethe—Weizacker mass formula has been refined by Stern (1949) who by adding a correction term could obtain more correct values of energies for nuclei $A \geqslant 208$. Jha and Dube (1952) calculated the alpha-decay energies for isotopes $Z = 83$, 84 and 85 with the help of B-W formula after accounting for the Stern correction factor and found satisfactory results. Dube and Singh (1945) have suggested other types of correction terms to the mass formula to close up the differences between the observed and calculated energy values.

Another Semi-empirical formula, which included among other refinements, empirical corrections in binding energy due to shell effects has been presented by Cameron (1957). Alpha—decay energies were calculated using this formula by Macfarlane (1959) for elements between Sn and Pb and compared with the values obtained from mass data and in a number of cases the disagreement between observed and calculated values is very large.

When the alpha decay energy of heavy elements was closely observed it was found that for mass numbers except in the neighbourhood of $Z = 82$, where the closed shell effects come in, there exists a systematic dependence of the alpha decay energy on the number of neutrons and protons. Roger (1955) has suggested a formula :

$$Q \text{ (in Mev)} = 9 - \frac{N-128}{6} \cdot \frac{102-Z}{6} \qquad \ldots \quad (2)$$

for nuclei beyond $Z = 84$ and $N = 128$.

Utilising the same observation Varshni (1956) developed an empirical relation relating the number of protons and neutrons to the alpha decay energy for heavy elements .

$$Q \text{ (in Mev)} = 0.4Z - 1.795(N-132)^{1/2} - 26.208 \qquad \ldots \quad (3)$$

for nuclei $N > 128$.

Varshni found that the energies calculated with the help of this formula agreed with the observed values to within an average of ± 3.6 per cent for the heavy elements.

In so far as it has been mentioned in the earlier paragraphs a brief outline has been given regarding the systematics of alpha-decay energies in relation to the heavier elements. Prior to 1947 not much was known about alpha radioactive decay nuclides with mass numbers below 210. In the following years several investigations were reported in the rare earth region with light nuclides of the elements from Cerium to Holmium ($Z = 58-67$).

Using the same formula as proposed by Varshni, Ramaswamy (1956) has redetermined the empirical constants and established a similar equation for the rare earth alpha emitters :

$$Q = 0.36Z - 1.19(N-82)^{1/2} - 17.89 \qquad \ldots \quad (4)$$

for $N > 82$, $Z < 82$.

The energies calculated with the help of this formula agreed with the observed values to within an average of ± 5.3 per cent for 15 nuclides in the rarre earth region.

More recently Varshni and Bhargava (1961) starting from a different approach have suggested another empirical formula valid for the region $Z \geqslant 84$ and $N \geqslant 128$.

TABLE I

Nuclide	Z	A	Q(obs) (Mev)	Ref.	Q(calc) (Mev)	Q(obs) $-Q$(calc)	Percentage error
Nd	60	144	1.90	a	1.70	0.20	+10.0
		147	1.04	a	1.07	0.03	− 2.8
Pm	61	145	2.31	b	2.11	0.20	+ 9.0
		147	1.56	a	1.69	0.13	− 8.0
Sm	62	146	2.62	c	2.52	0.10	+ 3.8
		147	2.26	a	2.31	0.05	− 2.2
		148	2.22	d	2.10	0.12	+ 5.4
		149	1.90	d	1.89	0.01	+ 0.5
Eu	63	147	3.00	c	2.94	0.06	+ 2.0
Gd	64	148	3.27	a	3.35	0.08	− 2.1
		149	3.10	a	3.14	0.04	− 1.3
		150	2.80	a	2.93	0.13	− 4.6
		152	2.21	c	2.51	0.30	−13.0
Tb	65	149	4.08	a	3.77	0.31	+ 5.0
		151	3.56	a	3.35	0.21	+ 5.9
Dy	66	150	4.35	a	4.18	0.17	+ 3.9
		151	4.20	a	3.97	0.23	+ 5.7
		152	3.73	a	3.76	0.03	− 0.8
		153	3.57	c	3.55	0.02	+ 0.6
		154	3.44	e	3.34	0.10	+ 3.0
Ho	67	151	4.60	f	4.59	0.01	+ 0.2
		152	4.52	f	4.38	0.14	+ 3.1
		153	4.45	f	4.17	0.28	+ 6.3
		154	4.23	f	4.04	0.19	+ 4.5
		155	4.05	f	3.75	0.30	+ 7.4
Hf	72	174	2.56	c	2.88	0.32	−12.5
W(?)	74	178	3.15	g	3.19	0.04	− 1.3
Pt	78	190	3.2	c	3.27	0.07	− 2.2
		192	2.66	h	2.85	0.19	− 7.1

a—Ramaswamy (1956); b—Nurmia (1962); c—Taagepera and Nurmia (1961); d—Karras, M., (1960); e—Toth and Rasmussen (1960); f—Macfarlane and Griffioen (1961); g—Porschen and Riezler (1953); h—Porschen and Riezler (1956).

It has been shown that $(Q-0.625Z)$ is a smooth function of the mass number A, where Q is the alpha disintegration energy. A reasonable representation of the plot between $(Q-0.625Z)$ and mass number is incorporated in the analytical expression of the form .

$$(Q-0.625Z) = -364.1233 + 0.8025944A + 6779630/A^2$$

$$\text{for } Z \geqslant 84 \text{ and } N \geqslant 128. \qquad \ldots \quad (5)$$

Using this formula, the agreement between observed and calculated values is within an average per cent error of ± 2.45.

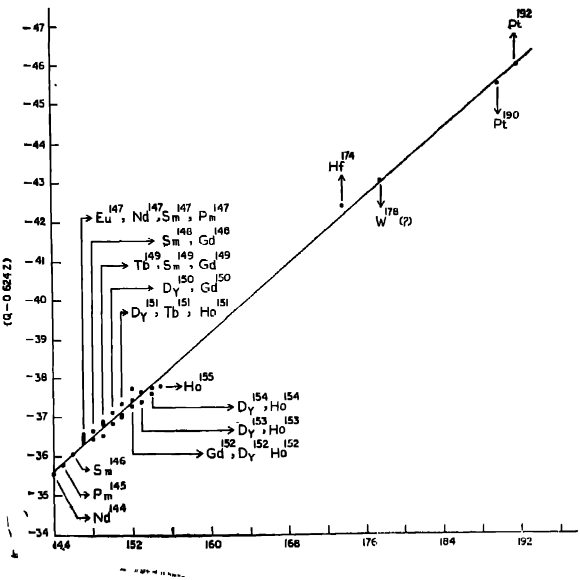
RESULTS

In the present work an empirical formula is presented for the alpha disintegration energies for the medium-heavy elements. The formula is as follows :

$$Q \text{ (in Mev)} = 0.624Z - 0.21A - 5.5 \qquad \ldots \quad (6)$$

for $N > 82$, $Z < 82$,

where Q = the disintegration energy in Mev,
A = the mass number,
Z = the charge number.



$\rightarrow A$

Fig. 1. The function $(Q-0.624Z)$ *vs.* the mass number A.

This is derived from the simple consideration that the function $(Q\text{-}0.624Z)$ has a regular variation with mass number A showing that $(Q-0.624Z)$ decreases in a linear fashion with increasing mass number A. A plot of the points between $(Q-0.624Z)$ and mass number A for all available data between $Z = 60$ (Nd) and $Z = 78$(Pt) yields a straight line graph and is represented by an equation of the form

$$(Q-0.624Z) = -0.21A-5.5$$

The data used and their sources are presented in Table 1.

DISCUSSIONS

The average per-cent error is ± 4.6 The agreement between observed values and those calculated by the present formula can be considered satisfactory. Whereas the present formula works quite well in the region of rare earth elements. its scope is extended to include other medium—heavy elements like Hf^{174}, W^{184}, Pt^{190} and Pt^{192}.

ACKNOWLEDGMENTS

The author is thankful to Prof B. D. Nagchaudhuri, Director of the Institute and Prof. D N. Kundu for their keen interest and valuable suggestions

REFERENCES

Bethe, A. H. and Bacher, R. F., 1936, *Rev Mod Phys*, **8**, 82
Cameron, A G. W, 1957, *Can. J Phys*, **35**, 1021
Das, R. K., 1950, *Ind J. Phys.*, **24**, 523.
Dube, G. P. and Singh, L. S., 1954, *Ind. J. Phys.*, **28**, 177.
Jha, S. and Dube, G. P. 1952, *Ind. J. Phys.*, **26**, 15.
Karras, M, 1960, *Ann. Acad Scient. Fen A.*, **6**, 65
Macfarlane, R. D., (1959) Thesis (Unpublished), Carnegie Institute of Technology, Pittsburg, U S A.
Macfarlane, R D. and Griffioen, R D., 1961, *Bull. Am. Phys. Soc. Ser.* II, **6**, 287
Nurmia, M., 1962, *Phys Rev.*, **127**, 943.
Porschen, W. and Riezler, W., 1953, *Zeit. f Natwef.*, **8a**, 502
Porschen, W and Riezler, Z, 1956, *Z. Naturforsch.*, **11a**, 143.
Pryce, M. H. L, 1950, *Proc Phys. Soc.*, **63**, 692.
Ramaswamy, M. K, 1956, *Nuovo cimento*, **3**, 1148.
Roger, F., 1955, *Compt. Rend Acad Sci.*, **240**, 858.
Segre, E, 1959, Experimental Nuclear Physics, III, Sec 2 B 2, John Wiley & Sons.
Stern, M. O, 1949, *Rev Mod Phys*, **21**, 316.
Taagepera, R. and Nurmia, M., 1961, *Ann. Acad. Scient. Fen. A.*, **6**, 78.
Toth, K. S and Rasmussen, J. O., 1960, *Nuclear Physics*, **16**, 474.
Varshni, Y. P. 1956, *Nuovo Cimento*, **3**, 1148.
Varshni, Y. P. and Bhargava, R. N., 1961, *Nuovo Cimento*, **22**, 450.