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Wholesale Price Contracts for Reliable Supply

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Firms can enhance the reliability of their supply through process improvement and overproduction. In decentralized supply chains, however, these mitigating actions may be the supplier's responsibility yet are often not contractible. We show that wholesale price contracts, despite their simplicity, can perform well in inducing reliable supply, and we identify when and why they perform well. This could explain the widespread use of wholesale price contracts in business settings with unreliable supply. In particular, we investigate how the performance of wholesale price contracts depends on the interplay between the nature of supply risk and the type of procurement process. Supply risk is classified as *random capacity* when events such as labor strike disrupt the firm's ability to produce, or as *random yield* when manufacturing defects result in yield losses. The procurement process is classified as *control* when the buyer determines the production quantity, or as *delegation* when instead the supplier does. Analyzing the four possible combinations, we find that for random capacity, irrespective of the procurement process type, contract performance monotonically increases with the supplier's bargaining power; thus, wholesale price contracts perform well when the supplier is powerful. However, this monotonic trend is reversed for random yield with control: in that case, wholesale price contracts perform well when instead the buyer is powerful. For random yield with delegation, wholesale price contracts perform well when either party is powerful.

Key words: supplier reliability, random capacity, random yield, wholesale price contracts
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1. Introduction

In an era of outsourcing and globalization, reliability of supply is an increasingly important aspect of supply chain management. Supply disruptions are often classified as either *random capacity* or *random yield* (Wang et al. 2010). Random capacity disruptions affect the supplier's production capacity, as when there is a labor strike or machine breakdown. By contrast, random yield disruptions affect the supplier's production yield–for example, when firms in the semiconductor sector or in high-tech electronics suffer from manufacturing defects. A critical aspect of such disruptions is that suppliers can exert *ex ante* effort to improve their processes and thus their reliability. For instance, suppliers can proactively invest in labor relations to avoid strikes (Lexology 2010), or undertake projects to improve yield (Zhang and Armer 2013). However, such effort is often costly and not contractible. Since suppliers capture only a fraction of the overall margin, they do not fully internalize the consequences of suboptimal effort, and this leads to a *moral hazard* problem.

There is a vast literature on coordinating contracts that deals with aligning incentives in a supply chain. Indeed, we verify the conventional wisdom that the moral hazard problem just mentioned can be resolved by using contracts that incorporate a penalty for the nondelivery of goods and make the supplier the residual claimant. However, such coordinating penalty contracts can be quite difficult to implement. For instance, the supplier's limited liability may induce an upper limit on the imposed penalty, and thus impede coordination (Sappington 1983). Another problem is that suppliers may view penalties as unfair and hence resist agreeing to such contracts (LinkedIn Discussion 2011). Therefore, firms often prefer to use wholesale price contracts that are easier to implement (e.g., Kalkancı et al. 2011).

For instance, wholesale price contracts are often employed in semiconductor manufacturing, where yield issues are a serious concern. In work done jointly with managers at KLA-Tencor Corporation and based on interviews with ten "fabless" companies, Chatterjee et al. (1999) report the widespread use of wholesale price contracts for outsourcing production of integrated circuits–which remains the preferred mode of contracting nowadays in the semiconductor industry. In conversations with the authors, a leading semiconductor foundry based in East Asia acknowledged that it typically uses wholesale price contracts regardless of the buyer and duration of the interaction, despite concerns about improving yield. Wholesale price contracts are also frequently used in the agriculture industry. In floriculture, for instance, wholesalers often outsource the cultivation of flowers and plants to contract growers (MacDonald et al. 2004). Due to the nature of floriculture, contract growers experience yield issues. Nevertheless, MacDonald et al. (2004) document that wholesale price contracts are typically used.

Motivated by this evidence that wholesale price contracts are often preferred, we investigate when and why such contracts perform well–that is, result in high supply chain efficiency.[1] We examine in particular the four cases typically observed in practice, which are characterized by the combination of two types of supply risk and two types of procurement process. As described previously, the two types of supply risk correspond to *random capacity* and *random yield*; the two types of procurement process correspond to *control* and *delegation*.

The procurement process is classified as *control* if the buyer determines the production quantity or as *delegation* if the supplier does. As with the type of supply risk, the type of procurement process is often determined by the business context. For instance, the control scenario is observed in the floriculture industry. Although contract growers experience yield issues, the wholesalers provide them with the inputs of seedlings, fertilizers, and chemicals—thus effectively controlling the production quantity (MacDonald et al. 2004). By contrast, we observe the delegation scenario in

Figure 1 **Performance of Wholesale Price Contract As a Function of Supplier's Bargaining Power**

		Type of Supply Risk	
		Random Capacity	Random Yield
Procurement Process Type	Control	Increasing	Decreasing
	Delegation	Increasing	V-Shaped

the semiconductor industry. Semiconductor manufacturers, such as Samsung and TSMC, often experience severe yield issues but can identify most defects through internal testing. These manufacturers therefore inflate their production quantities beyond the buyer's order, so that they can meet customer demand without shipping any defects.

With regard to the four cases considered, we summarize our main findings in the 2×2 matrix in Figure 1. The two columns correspond to the two types of supply risk, and the two rows to the two types of procurement process. For each of the four cases, the figure indicates whether the efficiency of the wholesale price contract is increasing, decreasing, or V-shaped with respect to the supplier's bargaining power.

Specifically, in the random capacity case—irrespective of the procurement process type—the efficiency of the wholesale price contract increases monotonically with the supplier's bargaining power (and with the wholesale price).[2] The reason is that a more powerful supplier has a bigger margin and thus a greater incentive to invest effort and thereby improve efficiency. This suggests that, if the supplier is powerful, a wholesale price contract may be preferred to more complex contracts that perform better in theory but are difficult to implement in practice.

In the random yield case, however, if the buyer *controls* the production quantity decision then the monotonic trend in efficiency associated with the wholesale price contract is reversed.[3] As before, a more powerful supplier, with his bigger margin, has a greater incentive to invest in improving reliability. However, in addition to the supplier's effort, overproduction (inflating the order quantity above demand) can also mitigate yield risk—and a more powerful supplier gives the buyer less incentive to inflate the order quantity. Importantly, the buyer's order quantity has a dominating effect on the supply chain efficiency compared with the supplier's effort, because the order quantity plays a *dual role*: it directly influences proportional yield and indirectly generates incentives for the supplier to invest effort via a larger order size. As a result, the efficiency of the wholesale price contract decreases with the supplier's bargaining power, and thus the wholesale price contract may be preferred when the buyer is powerful.

If, in the random yield case, the buyer *delegates* the production quantity decision to the supplier, then efficiency exhibits a V-shaped pattern: efficiency is high when either the buyer or the supplier is powerful. Specifically, efficiency is monotonically decreasing in the supplier's bargaining power (as in the control scenario) up to a threshold value, whereafter it increases monotonically. It is intuitive that, since the supplier determines the production quantity, the buyer's order quantity no longer plays a dual role, but can provide the supplier with only an indirect incentive to exert effort. Therefore, even though the efficiency trend parallels the one observed under the control scenario up to a threshold value of the supplier's bargaining power, beyond that point it is no longer profitable for the buyer to inflate production. Then the supplier unilaterally determines both the effort and the production quantity, and thus efficiency is increasing in the supplier's bargaining power as in the random capacity case.

Overall, in the face of unreliable supply, our analysis provides guidance as to when and why the wholesale price contract is preferable to a more complex coordinating contract.

2. Related Literature

Supply reliability has received considerable attention in recent years. Early studies have focused on situations in which supply reliability is *exogenously* given, and explored buyer-led risk management strategies such as multisourcing (Babich et al. 2007, Dada et al. 2007, Federgruen and Yang 2008, 2009b, Tang and Kouvelis 2011, Tomlin and Wang 2005, Tomlin 2006, 2009); carrying inventory (Tomlin 2006); or using a backup production option (Yang et al. 2009). Building upon this research, later studies expanded the scope of attention to *endogenous* reliability. Some studies investigate how buyers can intervene directly to improve suppliers' reliability (Liu et al. 2010, Wang et al. 2010). Others examine how buyers can indirectly induce suppliers to improve either product quality (Baiman et al. 2000, Kaya and Özer 2009) or reliability—for example, by way of horizontal competition (Federgruen and Yang 2009a) or investment subsidies and/or order inflation (Tang et al. 2014).[4]

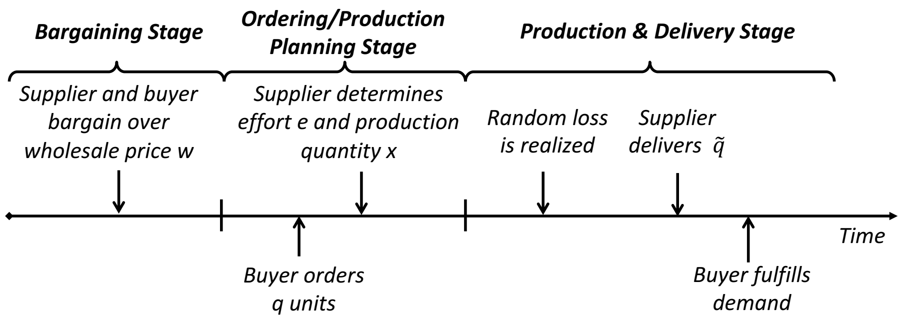
Of greater relevance to our work are papers exploring, in the case of endogenous reliability, contractual incentives as a means to induce suppliers' investment in reliability. However, the coordinating contract invariably takes on a complex form.[5] In the context of vaccine manufacturing, for instance, Dai et al. (2016) identify the coordinating contract that can induce suppliers to deliver products on time. This contract combines buy-back features with a rebate for timely delivery (in effect, a penalty for late delivery). Similarly, Ang et al. (2016) find that, when facing the risk of supply disruption, firms have to resort to coordinating contracts that impose penalties—thereby inducing their suppliers to choose appropriate upstream network partners. Yet as discussed in the introduction, such penalty features are difficult to implement in practice.

At the same time, some theoretical and behavioral studies have argued that, in practice, simple contracts are often preferred to complex coordinating contracts. In this strand of literature, the popularity of simple contracts is attributed to two transaction-cost arguments: simple contracts are (i) easier to design and negotiate (Holmstrom and Milgrom 1987, Kalkancı et al. 2011, 2014, Lariviere and Porteus 2001) and (ii) easier to enforce legally (Schwartz and Watson 2004). A few papers in economics and supply chain management have also explored the efficiency of simple contracts. The economics papers that assess the performance of simple contracts (e.g., Chu and Sappington 2007, Rogerson 2003) focus on their mathematical form, such as different combinations of a fixed price and a cost reimbursement scheme. As such, they do not consider the enabling role of operational factors. In supply chain management, Cachon (2004), Lariviere and Porteus (2001), and Perakis and Roels (2007) investigate the performance of wholesale price contracts and the role of operational factors in settings with demand uncertainty.

In this paper, we take our cue from and build upon the above literature. Indeed, as we mentioned in the introduction, we observe widespread use of wholesale price contracts even in the context of endogenous supply reliability. However, the insights obtained from the literature dealing with *demand* uncertainty we mentioned above do not carry over to the case of *supply* uncertainty. The latter setting, and hence our own model, poses two distinct challenges. First, besides investing in process improvement, firms can influence yield distribution by inflating the production quantity; inflating the order quantity does not influence the demand distribution when demand is uncertain. Second, the contract performance critically depends on the interplay between the nature of supply risk and the two types of procurement process (control versus delegation). We therefore focus on the performance of wholesale price contracts when supply is unreliable.

Our objective in this research is to focus on the widely-used and easy-to-implement wholesale price contract and understand its efficiency properties. While we do characterize the coordinating contract for the various scenarios that we study, we do so for illustrating the benchmark performance and offering an alternative to the wholesale price contract when its efficiency is low. However, to keep the focus of our manuscript, we do not consider other contracts with intermediate efficiency. A case-in-point is the two-part tariff (Feldstein 1972). This particular contract form can coordinate the supply chain in our setting. However, the coordinating contract is such that the buyer pays the supplier for delivered units, and the supplier makes a lump-sum payment to the buyer to "buy out" the business and become the *residual claimant*. Such bidirectional payment exchanges may be difficult to implement in some situations, and they are observed mainly as slotting allowances in the retail sector (Lariviere and Padmanabhan 1997). Moreover, capital-constrained suppliers cannot always afford to "buy out" the buyer. It is also worth noting that although two-part tariffs with unidirectional payment do not coordinate in our setting, they may yield greater supply chain efficiency than a wholesale price contract. For instance, for random yield with control, a two-part tariff can achieve higher efficiency if the firms agree on a low wholesale price (see Figure 5) and

Figure 2 Sequence of Events in a Decentralized Supply Chain



adjust the lump sum payment from the buyer to the supplier to allocate the profit to the firms depending on their relative bargaining power. However, as stated before, since we identify a coordinating contract for cases when wholesale price contracts perform poorly, we do not further analyze two-part tariffs in this paper.

3. Basic Model

We now describe our basic model. In §4 and §5, we show how the basic model can be applied to the cases with random capacity and random yield, respectively. We consider a supply chain with one supplier (*he*) and one buyer (*she*) who faces deterministic demand for a single selling season. Both the supplier and buyer are risk neutral. Demand D is modeled as being deterministic so that we can focus on the effect of supply uncertainty; this approach is in line with a large share of the existing literature (Deo and Corbett 2009, Dong and Tomlin 2012, Gümüş et al. 2012, Yang et al. 2009, 2012). We will model supply uncertainty with uniform distributions in §4 and §5, and we check the robustness of our results with general distributions in Appendix A.

As illustrated in Figure 2, the sequence of events is as follows. After observing the demand D, the buyer and supplier bargain over the wholesale price w. Next, the buyer orders quantity q from the supplier, and the supplier then exerts unverifiable effort e to improve his reliability and chooses production quantity x. Given the effort e, a corresponding random loss is associated with production; as a result, the supplier delivers $\tilde{q} \leq q$ units. Finally, the buyer fulfills demand at unit price p.

In principle, we would like to analyze the four cases illustrated in Figure 1: {random capacity, random yield} × {control, delegation}. Note, however, that the supplier's production quantity decision is relevant only for the case of random yield with delegation. This is because it is *not optimal* for the supplier to inflate his production quantity under random capacity and because the supplier is *not allowed* to inflate his production quantity under random yield with control. We therefore assume $x = q$ until §5.2, where we study the case of random yield with delegation.

We introduce our basic model in terms of the linear wholesale price contract, where the buyer pays a wholesale price w for each *delivered* unit.[6] The expected profits for buyer and supplier can thus be written as

$$\begin{aligned} \pi_b(q,e,w) &= pS(q,e) - wy(q,e), \\ \pi_s(q,e,w) &= wy(q,e) - c(q,e), \end{aligned} \tag{1}$$

where $y(q,e)$, $S(q,e)$, and $c(q,e)$ are the expected values for the delivered quantity, sales, and cost, respectively. In the centralized supply chain, there is no bargaining over the wholesale price w, and the order quantity is the same as the production quantity, denoted by q. All decisions are made by a central planner who seeks to maximize the expected supply chain profit, $\Pi(q,e) = pS(q,e) - c(q,e)$.

In the decentralized supply chain, the buyer and supplier first bargain over the wholesale price w. To model this process, we use the Nash bargaining model with asymmetric bargaining power (Binmore et al. 1986, Roth 1979). Let $\pi_b^*(w)$ and $\pi_s^*(w)$ be the equilibrium expected profits of the buyer and supplier, respectively, once the wholesale price w has been determined. Each firm bargains while anticipating those expected profits, and the resulting wholesale price solves the optimization problem

$$\max_w \pi_s^*(w)^\alpha \pi_b^*(w)^{1-\alpha}, \tag{2}$$

where $\alpha \in (0,1)$ is the supplier's relative bargaining power. We assume that the firms will reach an agreement provided the resulting expected profit is nonnegative (i.e., both firms' threat points are zero).[7]

Once the wholesale price is negotiated, the buyer acts as a Stackelberg leader by deciding the order quantity. However, the buyer faces a moral hazard problem because the supplier's effort is not contractible. The buyer's decision problem, which determines the firms' equilibrium profits, $\pi_b^*(w)$ and $\pi_s^*(w)$, can therefore be written as follows:

$$\begin{aligned} \max_q \quad & \pi_b(q,e,w), \\ s.t. \quad & e = \operatorname*{argmax}_{e\geq 0}\ \pi_s(q,e,w), \\ & \pi_s(q,e,w) \geq 0. \end{aligned} \tag{3}$$

The first constraint ensures incentive compatibility for the supplier—in other words, the supplier chooses the effort e that maximizes his expected profit. The second constraint ensures the supplier's participation by providing the supplier with at least his reservation profit, which we normalize to zero.

A brief discussion about this model is in order. It is prohibitively difficult to solve problem (2) analytically because this problem takes as an input the solution to problem (3). However, we can greatly simplify the analysis by showing that we can represent the supplier's relative bargaining power not only by α but also—and equivalently—by the wholesale price w. Interpreting the wholesale price w as the supplier's relative bargaining power allows us to just focus on problem (3) *without* solving problem (2). This approach suffices for our analysis, because we are interested only in how the expected profits of buyer and supplier, $\pi_b^*(w)$ and $\pi_s^*(w)$, depend on their relative bargaining power. For that purpose, we establish the following result.

PROPOSITION 1. *The optimal solution w^* to problem* (2) *is monotonically increasing in α if both $\pi_s^*(w)$ and $\pi_b^*(w)$ are continuous in w and either $\pi_s^*(w)$ is strictly increasing or $\pi_b^*(w)$ is strictly decreasing in w.*[8]

Appendix B includes additional results showing that, for all cases we consider, the requirements of Proposition 1 hold under some mild conditions. Proposition 1 suggests that a higher wholesale price w implies that the supplier has greater bargaining power. Therefore, in the rest of the paper, we will solve only problem (3) and interpret the wholesale price w as the supplier's relative bargaining power.

To avoid trivial results and simplify the exposition, we make the following assumption.

ASSUMPTION 1. *The following conditions hold:*

(i) In the centralized supply chain, it is profitable to produce a strictly positive amount even when the supplier does not exert any effort; that is, $\partial\Pi(q,e)/\partial q|_{q=0,e=0} > 0$.

(ii) If the buyer is indifferent among order quantities $Q \subset [0,D]$, then she chooses the largest quantity $q = \sup Q$.

Assumption 1(i) ensures that the optimal production quantity in the centralized supply chain will be strictly positive. Assumption 1(ii) implies that the buyer will satisfy demand provided her profit is not hurt, thus precluding outcomes that are Pareto suboptimal. We commence our analysis with the random capacity scenario.

4. Random Capacity

Here we model disruptions that destroy part or all of the supplier's capacity (Ciarallo et al. 1994, Wang et al. 2010), where the capacity loss is independent of the production quantity. Examples include labor strikes, machine breakdowns, fire, etc. In §4.1, we show how the basic model of §3 can be applied to the case of random capacity, state our assumptions, and characterize the optimal decisions in the centralized supply chain. We analyze the decentralized setup in §4.2.

4.1. Model and Centralized Supply Chain

In the random capacity case, production quantity does not affect the capacity loss. Hence the supplier has no incentive to inflate his production quantity beyond the buyer's order quantity. We therefore assume, without loss of generality, that the production quantity x is equal to the order quantity q (i.e., the scenario of random capacity with delegation converges to the one of random capacity with control). Consequently, the supplier delivers a random quantity $\tilde{q} = \min\{q, K - \xi\}$, where q is the order quantity, K is the supplier's nominal capacity, and ξ is the random capacity loss. We assume the random loss to be $\xi = \psi/(e+1)$, where ψ is uniformly distributed in $[0, K]$ and hence the support of ξ is $[0, K/(e+1)]$. The density and cumulative distribution function (CDF) of the random loss ξ—conditional on the effort e—are $g(\xi \mid e) = (e+1)/K$ and $G(\xi \mid e) = (e+1)\xi/K$, respectively.

Finally, the expected delivered quantity is $y(q,e) = E_\xi[\tilde{q}]$ and the expected sales are $S(q,e) = E_\xi[\min\{\tilde{q}, D\}]$. In Appendix A, we extend this model to the case where the random loss ξ follows a general distribution.[9]

We assume that the supplier initiates production after the random loss is realized. Hence the expected cost is $c(q,e) = cy(q,e) + v(e)$, where c is the unit production cost and $v(e)$ is the cost of effort to improve reliability. We shall also need the following technical assumption.

Assumption 2. *The following conditions hold:*

(i) The cost of effort is thrice continuously differentiable, and satisfies $v(0) = 0$, *and* $v'(e) > 0$ *and* $v''(e) \geq 0$ *for* $e > 0$.

(ii) In equilibrium, the effort level is strictly positive.

Part (i) implies that the cost of the supplier's effort is convex and increasing. Part (ii) precludes the trivial case of zero effort level, thus simplifying the exposition.

We find that if the order quantity is smaller than demand ($q \leq D$) then so is the delivered quantity $\tilde{q}$, in which case the buyer is able to sell everything. As a result, the expected sales and delivered quantities coincide ($S(q,e) = y(q,e)$) and increase with the order quantity q. Yet producing a quantity that exceeds demand does not affect the likelihood of satisfying that demand, and so does not affect expected sales, because the capacity loss is independent of the order quantity. Therefore, the expected sales function $S(q,e)$ has a kink at $q = D$. The technical properties of $S(q,e)$ and $y(q,e)$ are summarized in Lemma 1 (see Appendix B.1).

We now characterize the optimal order quantity and effort level in the centralized supply chain.

Proposition 2. *Let Assumptions 1 and 2 hold. Then, in the centralized supply chain, there exist unique optimal decisions* (q^o, e^o), *under which the optimal order quantity* q^o *is equal to demand* D.

The optimal order quantity q^o is equal to the demand D because producing more than D does not increase expected sales. Hence the only way to mitigate risk is for the supplier to exert effort. Given that effort is unverifiable, we next address the efficiency of the wholesale price contract in a decentralized supply chain.

4.2. Performance of the Wholesale Price Contract

We study the wholesale price contract and show that supply chain efficiency is generally increasing in the wholesale price, which is a proxy for the supplier's bargaining power (per Proposition 1). Therefore, a wholesale price contract may be the preferred mode of contracting for supply chains with sufficiently powerful suppliers—even if a theoretically superior (yet complex) contract exists.

PROPOSITION 3. *Let Assumptions 1 and 2 hold. Then, in the random capacity case, the efficiency of the wholesale price contract is monotonically increasing in w if $p < 3c$.*

The intuition behind this observed efficiency trend is as follows. The buyer will typically order D units for any "reasonable" size of the gross margin, $p - c$. Then, a more powerful supplier (higher wholesale price) has a bigger margin and thus a greater incentive to invest effort, and thereby improve efficiency. We illustrate the efficiency trend visually in Panel (a) of Figure 3. Note that the supplier with all the bargaining power will choose the maximum wholesale price $w = p$, and the buyer with all the bargaining power will choose the minimum wholesale price $w = c$, as follows from Proposition 1. Therefore, it suffices to use the wholesale price contract in achieving high supply chain efficiency if the supplier is powerful.

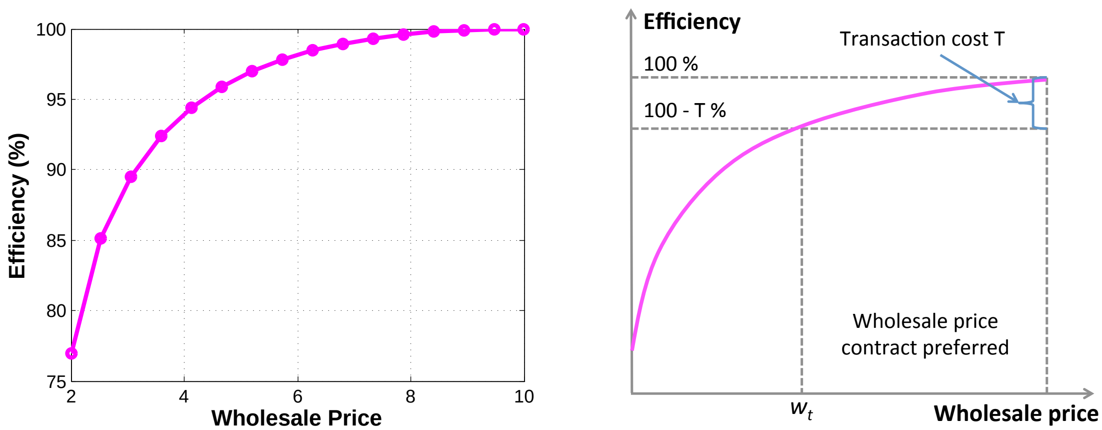
Then which contract should firms use when the buyer has greater bargaining power? It is a well-known result that under a moral hazard setting with risk-neutral players, the first-best outcome is achieved by transferring the risk to the agent and making the agent the residual claimant. We find that unit-penalty contracts coordinate the supply chain through this mechanism, while allowing for flexibility in the allocation of profits between buyer and supplier. Under such contracts, the buyer imposes a penalty z for each unit of shortage, while paying w for each unit delivered. The next result formalizes our findings.

PROPOSITION 4. *Let Assumptions 1 and 2 hold. Then, there exists $\bar{\chi} > 0$ such that the following unit penalty contracts coordinate the supply chain: $w^* = p - \chi$ and $z^* = \chi$ for $\chi \in [0, \bar{\chi})$; and the buyer's expected profit is $\pi_b = \chi D$.*

By setting the unit-penalty z equal to her unit margin $p - w$, the buyer transfers the entire risk to the supplier and thereby induces the supplier's first-best effort. Flexible profit allocation is achieved by varying the unit penalty $z^* = \chi$, where the upper bound on the penalty $\bar{\chi}$ ensures that the supplier earns nonnegative profit and that the buyer does not order more than D to exploit the high penalty fee.

Figure 3 Efficiency of Wholesale Price Contracts under Random Capacity and Contract Choice

Panel (a) depicts the supply chain efficiency (vertical axis) for random capacity when the cost of effort is θe^m and the wholesale price (horizontal axis) ranges between the unit production cost c and the unit price p; here $D = 100, K = 120, p = 10, c = 2, \theta = 100$, and $m = 2$. Panel (b) depicts the transaction cost T of the coordinating contract and illustrates when wholesale price contracts are preferred to the coordinating contract.



(a) Efficiency of wholesale price contracts

(b) Contract choice

Although the unit penalty contract coordinates in principle, there are complexities associated with its implementation that may make wholesale price contacts preferable. More specifically, unit penalty contracts have the following two sources of transaction cost.[10] First, suppliers may view penalties as unfair and may resist such contracts; this would likely increase both the length and cost of the bargaining process (LinkedIn Discussion 2011). Second, the bounded rationality and limited cognitive ability of individuals prevents them from fully optimizing the contractual parameters, even with contract forms that seem to be relatively simple.[11]

Let T represent this transaction cost, which we express as a percentage of the centralized supply chain profit. For wholesale prices w such that the efficiency of the wholesale price contract exceeds $100 - T\%$, it is optimal—from the aggregate supply chain perspective—to use the wholesale price contract. This is illustrated in Panel (b) of Figure 3, which shows the transaction cost T of the unit penalty contract as well as the wholesale price w_t for which the efficiency of the wholesale price contract is $100 - T\%$. It follows that, for any wholesale price $w \geq w_t$, it is optimal to use the wholesale price contract from an aggregate supply chain perspective.

Moreover, one can argue that the wholesale price contract is optimal not only from the aggregate supply chain's perspective but also from the individual firm's perspective. In other words: the wholesale price contract offers a Pareto improvement (with respect to the coordinating contract), for buyer and supplier both, when transaction costs are taken into account. To see this, we should compare the two contracts under the same level of supplier bargaining power α. We can then show the following: if the transaction cost T is above some threshold, then there always exists a set of wholesale prices such that the wholesale price contract achieves a Pareto improvement. The intuition is that, in the theoretically extreme case when T approaches 100%, both firms' expected profits approach zero. Since we can always find a wholesale price contract under which both firms earn strictly positive profits, it follows that wholesale price contracts can offer Pareto improvement when the transaction cost T is sufficiently high. For more details, see Proposition 15 in Appendix B.4.

Next we study how our insights change in the context of random yield.

5. Random Yield

In this section we model disruptions in which the random loss is stochastically proportional to the production quantity; that is, a larger production quantity increases the likelihood of obtaining a larger amount of usable output (Federgruen and Yang 2008, 2009a,b, Tang and Kouvelis 2011). This model applies, for example, when manufacturers of semiconductor or biotech products face uncertain yields in their manufacturing processes. The key distinguishing feature of random yield versus random capacity is that supply risk can be mitigated not only by increasing effort but also by producing more than demand.

We study two different cases that depend on the supplier's decision regarding production quantity. In §5.1 we examine the control scenario, in which the buyer's order fully determines the supplier's production quantity decision. In §5.2 we investigate the delegation scenario, in which the supplier independently determines his production quantity after receiving the buyer's order. For each case, we show how the basic model of §3 can be applied and discuss the performance of the wholesale price contract.

5.1. Control Scenario

In the control scenario, the supplier's production quantity equals the buyer's order quantity. Recall from the introduction that, in the floriculture industry, where yield issues are an important concern, wholesalers often outsource production to contract growers and provide them with seedlings, fertilizers, and chemicals; thus the wholesalers effectively control contract growers' production quantities (MacDonald et al. 2004). More generally, Federgruen and Yang (2009a) argue that the control scenario is commonly observed when the supplier cannot test the quality of the manufactured products at his site. Full inspection at the supplier's site is often impossible or impractical (e.g., Baiman et al. 2000, Balachandran and Radhakrishnan 2005). This is the case (i) when the testing technology is proprietary, in which case the buyer deliberately limits—because of intellectual property concerns—the supplier's ability to detect failures (Doucakis 2007, p. 23); or (ii) when the end product consists of multiple *interacting* components, each procured independently, which rules out testing of any component in isolation by the supplier (Li et al. 2011). In such cases, the supplier cannot find out which product will pass the buyer's quality test. Furthermore, the buyer will seldom accept a delivery in excess of her order quantity because she incurs appraisal cost. Therefore, the supplier has no choice but to deliver only the ordered quantity, and thus the supplier's production quantity must equal the buyer's order quantity.

5.1.1. Model and Centralized Supply Chain. The supplier delivers a random quantity $\tilde{q} = (1-\xi)q$, where q is the order quantity and ξ is the *random proportional loss.* To focus on the effect of the random proportional loss, we assume here that the supplier has no capacity constraints. We further assume that the random loss is $\xi = \psi/(e+1)$, where ψ is uniformly distributed in $[0,1]$ and the support of ξ is $[0, 1/(e+1)]$. Let the density and CDF of the random proportional loss be denoted $h(\xi \mid e) = e+1$ and $H(\xi \mid e) = (e+1)\psi$, respectively, and let $E[\xi] = \mu_y^e$ be the expected random loss. Then the expected delivered quantity is $y(q,e) = E_\xi[\tilde{q}]$ and expected sales are $S(q,e) = E_\xi[\min\{\tilde{q}, D\}]$. In Appendix A.2 we extend this model to the case where the random loss ξ follows a general distribution.

We assume that the supplier incurs the production cost for all q units. This is a reasonable assumption because yield and quality problems generally arise after all raw materials have been put into production. Therefore, the cost is $c(q,e) = cq + v(e)$. Assumption 2 applies also in this random yield model.

An interesting property of the random yield model with control is that, in contrast with the random capacity model, expected sales are increasing in the order quantity even when that quantity exceeds demand ($q > D$). The reason is that the random loss is stochastically proportional to the order quantity q, which means that ordering more *could* increase the supplier's likelihood of delivering D units and thus could increase expected sales. The technical properties of expected sales, $S(q,e)$, and of the expected delivered quantity, $y(q,e)$, are summarized in Lemma 2 (see Appendix B.2). We are now in a position to characterize optimal decisions in the centralized supply chain.

PROPOSITION 5. *Let Assumptions 1 and 2 hold. Then, in the centralized supply chain with random yield, there exist both an optimal order quantity q^o and an optimal effort level e^o; moreover, the optimal order quantity q^o is strictly greater than the demand D.*

The optimal decisions under random yield differ qualitatively from those, described in Proposition 2, under random capacity; it is now optimal to order (or, equivalently, to produce) more than the demand ($q^o > D$). Therefore, the decision maker can increase expected profit not only by exerting additional effort but also by ordering more. The reason is that, as we mentioned, expected sales increase with order quantity even when the order quantity exceeds demand. We now investigate how, in a decentralized setting, the need to coordinate both the buyer's order inflation and the supplier's effort results in dynamics that differ from those under random capacity.

5.1.2. Performance of the Wholesale Price Contract. Our main finding is that the efficiency of the wholesale price contract generally *decreases* with the supplier's bargaining power. Therefore, the wholesale price contract is more likely to be preferred when the buyer is powerful.[12] This result contrasts sharply with the result for random capacity, where the efficiency of the wholesale price contract *increases* with w. The reason is as follows. Just as under random capacity, a higher wholesale price increases the supplier's incentive to invest in reliability, which in turn increases efficiency. At the same time, however, increasing w also reduces efficiency by diminishing the buyer's incentive to inflate her order quantity. Moreover, the buyer's order quantity has a dominating effect on the supply chain efficiency compared with the supplier's effort, because the order quantity plays a *dual role*: it directly influences proportional yield and indirectly generates incentives (via a larger order size) for the supplier to invest effort. Note that the supplier bears no overage cost, which makes a larger order quantity more effective at inducing effort. Therefore, the supply chain efficiency—like the order quantity—decreases with the wholesale price.

We now discuss the results that lead us to conclude that efficiency is generally decreasing in the wholesale price under random yield with control.

Proposition 6. *Let Assumptions 1 and 2 hold. Then, for random yield with control, the efficiency of the wholesale price contract is monotonically decreasing in w for $w \in [\underline{w}_c, p]$ if either of these sufficient conditions holds:*

(i) $v(e) = \theta e$, *and*

$$\underline{w}_c = \frac{-(8pk^2 - 32c - 2p) + \sqrt{(8pk^2 - 32c - 2p)^2 + 4p(16k+9)(7p+32c)}}{2(16k+9)},$$

where $k = 2 - \sqrt{\frac{2\theta}{cD}}$; *or*

(ii) $\underline{w}_c = c$, $v(e) = \theta e^m$ *where* $m > 1$, *and either* θ *or* m *is sufficiently large.*

Part (i) gives *sufficient* conditions for efficiency to be monotonically decreasing when the cost of effort is linear. In particular, it provides an explicit expression for the threshold $\underline{w}_c$. Part (ii) shows that efficiency is decreasing in the entire range of wholesale prices—provided the cost of effort is sufficiently large.

Figure 4 shows how the range of wholesale prices for which the sufficient condition in Proposition 6(i) holds depends on the cost of effort θ. The solid horizontal line represents the retail price ($p = 12$). The dashed line shows the *minimum feasible wholesale price*, which is the minimum wholesale price for which the supplier's participation constraint can be satisfied; hence equilibria exist only above this line. The line drawn through the square markers is the threshold $\underline{w}_c$ given in Proposition 6(i), and it shows that the sufficient conditions hold for a fairly wide range of wholesale prices. Moreover, Proposition 6(i) gives only sufficient conditions and thus the monotonic trend

Figure 4 Threshold $\underline{w}_c$

This figure plots the threshold $\underline{w}_c$ given in Proposition 6(i) when the cost of effort θ varies from 0 to 80. Here the parameters are $D = 100, p = 12$, and $c = 6$.

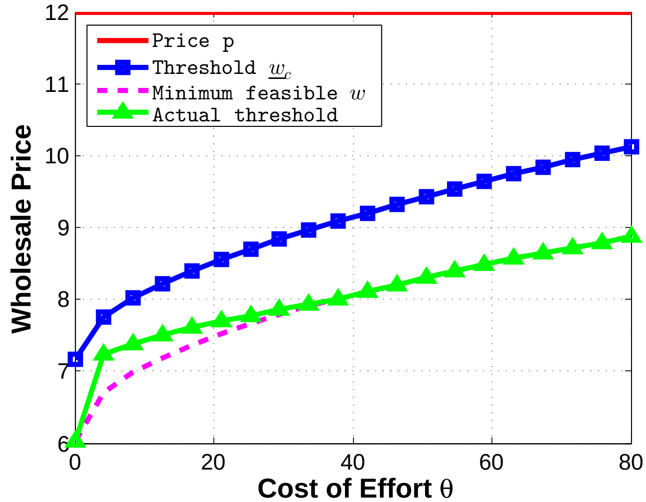
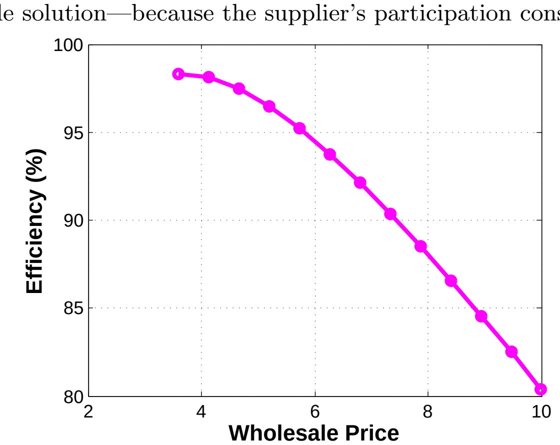


Figure 5 Efficiency of Wholesale Price Contracts under Random Yield with Control

This figure depicts the efficiency (vertical axis) under random yield with control when the wholesale price (horizontal axis) ranges between the unit production cost c and the unit price p; here the parameters are $D = 100, p = 10, c = 2, \theta = 100$, and $m = 2$. Note that, in the random yield case, if the wholesale price w is close to the unit production cost c, then there is no feasible solution—because the supplier's participation constraint cannot be satisfied.



can hold also for wholesale prices even lower than $\underline{w}_c$. Indeed, the line drawn through triangular markers plots the *actual* threshold wholesale price—above which efficiency is monotonically decreasing—and shows that the actual threshold is less than $\underline{w}_c$. We remark that we can only compute the actual threshold numerically. Finally, for costs of effort $\theta \geq 40$, the monotonic trend holds over the entire range of feasible wholesale prices; this result is consistent with Proposition 6(ii).

To verify the robustness of our analytical findings, we conduct a comprehensive numerical investigation. Toward that end, we use the following cost function: $c(q,e) = cq + \theta e^m$, where $c, \theta > 0$ and $m \geq 1$.

Figure 5 plots supply chain efficiency as a function of wholesale price for the same set of parameters used to generate Figure 3. First of all, we observe a very robust decreasing trend in efficiency. Recall that the supplier with all the bargaining power will choose the maximum wholesale price and that the buyer with all the bargaining power will choose the minimum wholesale price. Therefore, the efficiency is fairly high (about 98%) when the buyer is powerful but is relatively low (about 80%) when the supplier is powerful. We then repeat our experiment with different parameter combinations, choosing seven values each for c, θ, and m in the following ranges: $c \in [0.1, 5], \theta \in [1, 100]$, and $m \in [1, 5]$. We therefore perform our analysis with 7^3=343 different combinations of parameters. We find that supply chain efficiency is monotonically decreasing for the entire range of wholesale prices in 84.3% of all cases. The remaining 15.7% of cases exhibit a slight increase in efficiency (typically less than 0.1%) for small w only, yet otherwise the general decreasing trend prevails.[13]

Having assessed the efficiency of the wholesale price contract, we now turn to the question of which contract to use when the wholesale price contract results in low efficiency. The unit penalty contract no longer coordinates because—even though it is optimal for the buyer to inflate her order quantity above demand—the supplier shares none of the overage risk. One obvious way to share the overage cost is through a buy-back agreement. Indeed, we find that a unit-penalty with buy-back contract coordinates the supply chain while allowing for *arbitrary* profit allocation between the buyer and supplier.[14]

PROPOSITION 7. *Let Assumptions 1 and 2 hold. Then, there exists a continuum of unit-penalty with buy-back contracts that satisfy the Karush–Kuhn–Tucker (KKT) conditions at* (q^o, e^o) *in optimization problem (3), allowing for arbitrary profit allocation.*

We have established that our insights concerning random yield with control are substantially different from those in the case of random capacity. This underscores the pivotal role that the type of supply risk plays in the performance of contracts under unreliable supply. Next we investigate what happens when the buyer *delegates* the production quantity decision to the supplier under random yield.

5.2. Delegation Scenario

There are a number of contexts in which the buyer may delegate the production quantity decision to the supplier (see e.g., Chick et al. 2008 and Tang et al. 2014). In these contexts, it is possible that the buyer inflates the order quantity above demand, and the supplier further inflates the production quantity. This aspect of the scenario introduces an additional source of inefficiency into the supply chain: the supplier might inflate his production quantity, even if the buyer has already padded her order quantity to buffer against yield losses. These buffers may accumulate and thus exacerbate inefficiency. We now examine whether this is indeed the case.

5.2.1. Model and Centralized Supply Chain. The model is similar to that of the control scenario in §5.1 except that the supplier determines his own production quantity x. Therefore, we present only those parts of the model that differ from the control scenario. The supplier delivers a random quantity $\tilde{q} = \min\{q, (1-\xi)x\}$, where q is the order quantity, x is the production quantity, and ξ is the *random proportional loss* defined in §5.1. The expected delivered quantity is $y(q,x,e) = E_\xi[\tilde{q}]$ and the expected sales are $S(q,x,e) = E_\xi[\min\{\tilde{q}, D\}]$. The cost is $c(x,e) = cx + v(e)$.

The delegation scenario differs from the control scenario in that, if the order quantity is larger than the demand ($q > D$) then, as in the random capacity model, expected sales become constant. This is because, as long as $q \geq D$, the probability of receiving D units depends only on the production quantity x and the effort e. Therefore, ordering more than D does not *directly* increase expected sales, and thus $S(q,x,e)$ has a kink at $q = D$. Note, however, that a higher q does give the supplier an incentive to choose higher x and e, which would *indirectly* increase expected sales. The properties of $S(q,x,e)$ and $y(q,x,e)$ are summarized in Lemma 3 (see Appendix B.3).

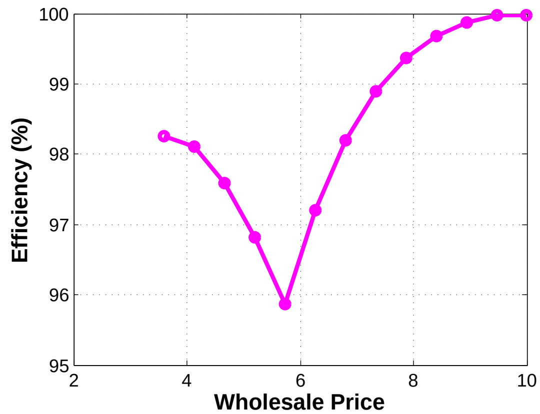
In the centralized supply chain, the order quantity is redundant, and the decision maker chooses only the production quantity x and the effort e. Therefore, the optimal decisions in the centralized supply chain are the same as for the control scenario discussed in §5.1 except that (a) we replace the order quantity q with the production quantity x in Proposition 5 and (b) we refer to the optimal production quantity as x^o. We examine the decentralized setup next.

5.2.2. Performance of the Wholesale Price Contract. We find that the efficiency associated with the wholesale price contract exhibits a V-shaped pattern as the wholesale price (supplier's bargaining power) increases. We therefore argue that if *either* party has most of the bargaining power then the wholesale price contract is likely to be preferred over more complex contracts that could theoretically offer better performance. This result contrasts with the case of random capacity and also with the case of random yield with control.

The intuition for the V-shaped pattern is as follows. Recall that in the control scenario the buyer's order quantity played a dual role: It directly influenced proportional yield and indirectly generated incentives for the supplier to invest effort via a larger order size. However, since in the delegation scenario the supplier determines the production quantity, the buyer's order no longer plays a dual role, but can only provide the supplier with an indirect incentive to exert effort. Therefore, the buyer whose profit margin is $p - w$ finds it profitable to inflate the order quantity only up to a threshold wholesale price. The efficiency trend for delegation parallels the one in the control scenario up to this threshold. Above the threshold wholesale price the buyer does not inflate at all and the supplier unilaterally determines his effort as well as the production quantity. Hence supply chain efficiency increases in the supplier's bargaining power, or equivalently, in the wholesale price (as in the case of random capacity), thus giving rise to the V-shape.

Figure 6 Efficiency of Wholesale Price Contracts under Random Yield with Delegation

This figure plots supply chain efficiency (vertical axis) under random yield with delegation when the wholesale price (horizontal axis) ranges between the unit production cost c and the unit price p. Parameters: $D = 100, p = 10, c = 2, \theta = 100$, and $m = 2$.



We now discuss the results that lead us to conclude that efficiency exhibits a V-shaped pattern. First, we analytically show that if the wholesale price w is sufficiently large, then the efficiency of the wholesale price contract monotonically *increases* with w. This trend corresponds to the right-hand side of the V-shape.

PROPOSITION 8. *Let Assumptions 1 and 2 hold. Then, there exists a* $\underline{w}_d < p$ *such that the efficiency associated with the wholesale price contract is monotonically increasing in* $w \in [\underline{w}_d, p]$.

Second, we numerically verify the results by examining the efficiency pattern of the wholesale price contract across the entire bargaining power spectrum. For this purpose, we use the same cost function as for the numerical analysis in §5.1: $c(q, e) = cq + \theta e^m$, where $c, \theta > 0$ and $m \geq 1$. Figure 6 illustrates that efficiency follows a clear V-shaped pattern as a function of the wholesale price. The lowest efficiency is 95.9%, at the bottom of the V-shape. When w is low, the efficiency rises to 98.3%, and when w is high, it rises to 100%. We analyze 343 different cases—using the same parameter combinations as in §5.1—and observe an unambiguous V-shape in 88.9% of them. In 11.1% of the cases, we again observe a prominent V-shape, but with a slight increase (typically less than 0.1%) in efficiency when w is very low, at the left extreme of the bargaining power spectrum, followed by the expected V-shaped pattern.

Our results suggest that when using incentives to improve supply reliability in a decentralized supply chain, one must consider whether the buyer controls or delegates the production quantity decision, in addition to bargaining power and the nature of supply risk. An interesting result in the delegation scenario is that as the supplier—the agent undertaking unverifiable action—earns greater margin (and therefore payoff), the efficiency trend is neither increasing (as with random capacity) nor decreasing (as with random yield with control); instead, the efficiency follows a V-shaped pattern.

To consolidate this insight, there is still one remaining loose end—namely, which contract coordinates in the delegation scenario? We find that the unit-penalty contract coordinates the supply chain by making the supplier the residual claimant, as it does under random capacity.

Proposition 9. *Let Assumptions 1 and 2 hold. Then, there exists a* $\bar{\chi} > 0$ *such that the following unit penalty contracts coordinate the supply chain:* $w^* = p - \chi$, $z^* = \chi$, *where* $0 \leq \chi \leq \bar{\chi}$. *The buyer's expected profit is then* $\pi_b = \chi D$.

The unit penalty contract coordinates the supply chain with flexible profit allocation *despite* the additional dimension of moral hazard (production quantity) as compared with the control scenario, in which coordination requires the more complex unit-penalty with buy-back contract. The intuition is that if the penalty fee is set equal to the margin (and is not too large), then the buyer does not inflate the order, because she can earn her margin through either a sale or the penalty imposed on the supplier. Therefore, the supplier faces exactly the same trade-offs as the centralized decision maker and thus chooses the first-best effort and production quantity.

6. Conclusions

Our research is motivated by the widespread prevalence of wholesale price contracts in settings with unreliable supply that could be improved by the supplier. Specifically, we investigate when and why wholesale price contracts can be used to generate efficient outcomes in a decentralized supply chain. We characterize how the performance of the wholesale price contract depends on the interplay between the type of supply risk (random capacity versus random yield) and type of procurement process (control versus delegation). In this way we demonstrate that careful appreciation of the operational features of the setting is instrumental in identifying the circumstances under which wholesale price contracts perform well.

When a supplier invests unverifiable effort (e.g., reliability investment), coarse intuition might suggest that the supplier will invest more effort if he can negotiate a higher share of the supply chain profit, thereby increasing efficiency. We find this intuition to be correct in our setting with random capacity. However, the scenario of random yield with control exhibits the exact opposite trend, and for random yield with delegation, we find that supply chain efficiency is no longer monotonic in bargaining power but instead is V-shaped.

Even though they are suboptimal in theory, wholesale price contracts may be preferred to complex contracts that come with a transaction cost—a phenomenon we refer to as a preference for "appropriate" contracts. Our findings translate into simple insights that have the potential to inform managerial decisions. To illustrate, compare the following two scenarios: (i) Nike sources garments from Sabrina Garment Manufacturing, and (ii) Apple sources semiconductor chips from

Samsung (Bloomberg Business 2015, CNN 2013). Sabrina is based in Cambodia, and has encountered labor disruptions (random capacity), whereas Samsung has to contend with random yield. Both Nike and Apple form the bulk of their respective suppliers' business, and thus have greater bargaining power in the respective relationships (The Wall Street Journal 2013). Although both firms face supply risk, our results indicate that the wholesale price contract would generate high efficiency for Apple's supply chain, but result in low efficiency for Nike's.

We believe our findings offer guidance regarding when and why to use the wholesale price contract—or instead a more complex coordinating contract—and identify the role played by the types of supply risk and procurement process in this outcome.

Appendix A: General Model

Unless otherwise specified, all the results in Appendix B (technical results), Appendix C (tables), and Appendix D (proofs of all results) are based on the general model we define in this Appendix A.

A.1. Random Capacity

We generalize the assumption that the random loss ξ is uniformly distributed. The random loss is $\xi = f_c(\psi, e)$, where ψ is a random variable that captures the underlying supply risk and f_c is a function that models the dependence of the random loss on the supplier's effort e. We make the following technical assumption, which our basic model with the uniform distribution satisfies.

ASSUMPTION 3. *The following conditions hold:*

(i) The random loss ξ has support $[0, a_c(e)]$, where both $G(\xi \mid e)$ (the CDF) and $a_c(e)$ are twice continuously differentiable with finite derivatives in $e \geq 0$ and $\xi \in [0, a_c(e)]$.

(ii) Either of the following holds:

(a) $a_c(e) = K$, $\partial G(\xi \mid e)/\partial e > 0$, and $\partial^2 G(\xi \mid e)/\partial e^2 < 0$ for $e \geq 0$ and $\xi \in (0, K)$.

(b) $a_c(0) = K$, $a_c'(e) < 0$, $\partial G(\xi \mid e)/\partial e > 0$, and $\partial^2 G(\xi \mid e)/\partial e^2 \leq 0$ for $e \geq 0$ and $\xi \in (0, a_c(e)]$.

Part (ii) implies that the effort e mitigates the random loss ξ in the sense of first-order stochastic dominance (FOSD) with decreasing returns to scale. Moreover, the supplier's effort achieves the stochastic dominance either by shifting the CDF upward while preserving the support of the random loss (as in part (a)) or by reducing the support of the random loss (as in part (b)).

With this generalized model and Assumption 3, both Proposition 2 and Proposition 4 continue to hold. However, Proposition 3 no longer holds; we have instead the following weaker results for monotonicity.

PROPOSITION 10. *Let Assumptions 1, 2, and 3 hold. Then, for random capacity, the efficiency of the wholesale price contract is monotonically increasing in w for $w \in [\underline{w}, p]$ if either of these sufficient conditions holds:*

(i) $\underline{w}$ is sufficiently close to the unit price p; or

(ii) $\underline{w} = c$ and the buyer's optimal order quantity is equal to the demand D.

In part (i) we show that the supply chain efficiency is monotonically increasing in the wholesale price (or in the supplier's bargaining power) when the wholesale price is sufficiently high. In (ii), we show that this monotonicity result holds over the entire interval, $w \in [c, p]$, provided the buyer always orders D units. We have also numerically verified that this efficiency trend continues to hold when the loss distribution is triangular or of beta type; for the sake of brevity, we omit the details.

A.2. Random Yield: Control Scenario

We generalize the assumption that the random loss ξ is uniformly distributed. The random loss is $\xi = f_y(\psi, e)$, where ψ is a random variable that captures the underlying supply risk and f_y is a function that models the dependence of the random loss on the supplier's effort e. We make the following technical assumption, which our basic model with the uniform distribution satisfies.

Assumption 4. *The following conditions hold:*

(i) The random loss ξ has support $[0, a_y(e)]$, where both $H(\xi \mid e)$ (the CDF) and $a_y(e)$ are thrice continuously differentiable with finite derivatives in $e \geq 0$ and $\xi \in [0, a_y(e)]$.

(ii) Either of the following holds:

(a) $a_y(e) = 1$, $\partial H(\xi \mid e)/\partial e > 0$, and $\partial^2 H(\xi \mid e)/\partial e^2 < 0$ for $e \geq 0$ and $\xi \in (0, 1)$.

(b) $a_y(0) = 1$, $a_y'(e) < 0$, $\partial H(\xi \mid e)/\partial e > 0$, and $\partial^2 H(\xi \mid e)/\partial e^2 \leq 0$ for $e \geq 0$ and $\xi \in (0, a_y(e)]$.

Part (ii) implies that the effort e mitigates the random loss ξ in the sense of FOSD with decreasing returns to scale. Moreover, the supplier's effort achieves the stochastic dominance either by shifting the CDF upward while preserving the support of the random loss (as in part (a)) or by reducing the support of the random loss (as in part (b)).

With this generalized model and Assumption 4, the statements in Proposition 5, Proposition 6(ii), and Proposition 7 continue to hold. Now, however, Proposition 6(i) no longer holds; instead we obtain the following weaker result.

Proposition 11. *Let Assumptions 1, 2, and 4 hold. Then, under random yield with control, the efficiency of the wholesale price contract is monotonically decreasing in w for $w \in [\underline{w}_c, p]$, provided that $\underline{w}_c$ is sufficiently close to p.*

Proposition 11 shows that, if the wholesale price is sufficiently high, then supply chain efficiency decreases with the supplier's bargaining power (or the wholesale price).

Moreover, we numerically verify the monotonicity result. Recall that our model assumes the loss distribution to have a bounded support and to exhibit first-order stochastic dominance as effort increases. The uniform distribution we use in the basic model exhibits FOSD as the support shrinks with greater effort. We consider two other loss distributions that have bounded and fixed support: a triangular distribution and a beta-type distribution. The triangular distribution exhibits FOSD as the mode approaches zero with greater effort, while the support remains fixed. For the beta-type distribution we consider, the CDF has a closed-form expression;[15] this function exhibits FOSD as the mode approaches zero while the support remains fixed (see Jones 2009). This beta-type distribution is quite flexible and encompasses a variety of bell shapes.

We conduct a comprehensive numerical analysis with the triangular and beta-type distributions, and find that the decreasing efficiency trend is robust. Specifically, we explore the triangular distribution with the same parameter combinations as used for examining the uniform distribution[16] and find that efficiency is completely monotonic in every case. For the beta-type distribution, we conduct the numerical analysis with a narrower range of parameters to ensure feasibility, and again find that efficiency is monotonic in 97.8% of the cases.[17] The remaining 2.2% of the cases exhibit a slight increase for small w, but again the general decreasing trend persists.

A.3. Random Yield: Delegation Scenario

The basic model for the uniform distribution in §5.2 introduces only those modeling elements that differ from the control scenario. That basic model can be immediately extended to the general model without any changes. We need to only make the following assumption for tractability, which our basic model with the uniform distribution satisfies.

Assumption 5. *The expected delivered quantity $y(q,x,e)$ is jointly concave in q and e, and also in x and e, in the feasible region of problem (3).*

Under Assumption 5, both Proposition 8 and Proposition 9 continue to hold. We conduct a comprehensive numerical analysis with the triangular and beta-type distributions and with the same combinations of parameters as used in Appendix A.2, finding that the V-shaped pattern is robust. With the triangular distribution, we observe an unambiguous V-shape in 85.1% of the cases; in 1.4% of the cases, we find a prominent V-shape but with a slight increase in efficiency at the left extreme of the V. In 13.5% of the cases, efficiency is just increasing. However, these are exceptional cases in which the unit production cost c is so high that feasible solutions exist only when the wholesale price w is greater than 90% of the retail price p. For the beta-type distribution, we observe an unambiguous V-shape in 97.1% of the cases. In 1.9% of the cases, we find a prominent V-shape but with a slight increase in efficiency at the left extreme of the V, and in 1% of the cases, the efficiency was just increasing.

Appendix B: Technical Results

B.1. Random Capacity

The following lemma holds under the *general* model defined in Appendix A, and it establishes both the properties of and the relationship between expected sales and expected delivered quantity.

Lemma 1. *Let Assumptions 2 and 3 hold. Then the following holds.*

(i) $S(q,e)=y(q,e)$ *if* $q\le D$, *and* $S(q,e)=y(D,e)$ *if* $q>D$.

(ii) $y(q,e)$ *is increasing and concave in both* q *and* e.[18] *Also,* $y(q,e)$ *is twice continuously differentiable in* q *and* e, *except when* $q=K-a_c(e)$, *in which case* $y(q,e)$ *is once continuously differentiable.*

The next proposition holds under our *basic* model with the uniform distribution, and—together with Proposition 1—allows us to interpret the wholesale price w as a proxy for the supplier's bargaining power α.

Proposition 12. *The buyer's and the supplier's expected profits at equilibrium,* $\pi_b^*(w)$ *and* $\pi_s^*(w)$, *satisfy the following: i)* $\pi_b^*(w)$ *and* $\pi_s^*(w)$ *are both continuous in* w *and ii)* $\pi_s^*(w)$ *is strictly increasing in* w *if* $p<3c$.

B.2. Random Yield: Control Scenario

The following lemma holds under the *general* model defined in Appendix A, and it establishes both the properties of and the relationship between expected sales and expected delivered quantity.

LEMMA 2. *Let Assumptions 2 and 4 hold. Then the following holds.*

(i) If $q \le D$, then $S(q,e) = y(q,e)$. If $q > D$, then $S(q,e) < y(q,e)$.

(ii) $y(q,e)$ and $S(q,e)$ are increasing and concave in both q and e.[19] *Also, $y(q,e)$ and $S(q,e)$ are thrice continuously differentiable in q and e, except that $S(q,e)$ is once continuously differentiable (a) in q when $q = D$, and (b) in q and e when $q = D/(1-a_y(e))$.*

The next proposition holds under the *general* model defined in Appendix A, and—together with Proposition 1—allows us to interpret the wholesale price w as a proxy for the supplier's bargaining power α.

PROPOSITION 13. *The buyer's and the supplier's expected profits at equilibrium, $\pi_b^*(w)$ and $\pi_s^*(w)$, satisfy the following: i) $\pi_b^*(w)$ and $\pi_s^*(w)$ are both continuous in w and ii) $\pi_b^*(w)$ is strictly decreasing in w.*

B.3. Random Yield: Delegation Scenario

The following lemma holds under the *general* model defined in Appendix A, and it establishes both the properties of and the relationship between expected sales and expected delivered quantity. The difference from the control scenario is that the expected sales are constant when the buyer orders at least D units. Hence, the expected sales $S(q,x,e)$ have a kink at $q = D$.

LEMMA 3. *Let Assumptions 2 and 4 hold. If the supplier determines his own production quantity, then the following holds.*

(i) If $q \le D$, then $S(q,x,e) = y(q,x,e)$. If $q > D$, then $S(q,x,e) = y(D,x,e)$.

(ii) $y(q,x,e)$ is increasing and concave in both x and e. Also, $y(q,x,e)$ is thrice continuously differentiable in x and e, except that it is once continuously differentiable (a) in x when $x = q$, and (b) in x and e when $x = q/(1-a_y(e))$.

The next proposition holds under our *basic* model with the uniform distribution, and—together with Proposition 1—allows us to interpret the wholesale price w as a proxy for the supplier's bargaining power α.

PROPOSITION 14. *The buyer's and the supplier's expected profits at equilibrium, $\pi_b^*(w)$ and $\pi_s^*(w)$, satisfy the following: i) $\pi_b^*(w)$ and $\pi_s^*(w)$ are both continuous in w and ii) $\pi_b^*(w)$ is strictly decreasing in w if $p \le 2c$ and $v(e) = \theta e$, where $\theta \ge (5/7) \cdot cD$.*

B.4. Other Results

The next lemma gives the conditions equivalent to Assumption 1(i) for each type of supply risk.

LEMMA 4. *Assumption 1(i) is equivalent to (i) $p > c$ for random capacity, and (ii) $p(1-\mu_y^0) > c$ for random yield.*

The following proposition states that if the transaction cost T of a coordinating contract is sufficiently high, then the wholesale price contract is actually Pareto efficient compared to the coordinating contract because of the high transaction cost.

Proposition 15. *There exists a threshold $T' > 0$ such that if the transaction cost of a coordinating contract T exceeds this threshold T', then there exists a set of wholesale prices for which the wholesale price contract is Pareto efficient compared to the coordinating contract.*

Appendix C: Tables

The results in these tables apply to the *general* model defined in Appendix A. We use "+" and "−" to signify "strictly positive" and "strictly negative", respectively.

Table 1 Derivatives of $y(q,e)$ under Random Capacity.

Derivatives	$0 \le q \le K - a_c(e)$	$K - a_c(e) < q \le K$
$\frac{\partial y(q,e)}{\partial q}, \frac{\partial^2 y(q,e)}{\partial q^2}$	$+, 0$	$+, -$
$\frac{\partial y(q,e)}{\partial e}, \frac{\partial^2 y(q,e)}{\partial e^2}$	$0, 0$	$+, -$

Table 2 Derivatives of $y(q,e)$ and $S(q,e)$ under Random Yield with Control.

Derivatives	$0 \le q \le D$	$D < q \le \frac{D}{1-a_y(e)}$	$\frac{D}{1-a_y(e)} < q$
$\frac{\partial y(q,e)}{\partial q}, \frac{\partial^2 y(q,e)}{\partial q^2}$	$+, 0$	$+, 0$	$+, 0$
$\frac{\partial y(q,e)}{\partial e}, \frac{\partial^2 y(q,e)}{\partial e^2}$	$+, -$	$+, -$	$+, -$
$\frac{\partial S(q,e)}{\partial q}, \frac{\partial^2 S(q,e)}{\partial q^2}$	$+, 0$	$+, -$	$0, 0$
$\frac{\partial S(q,e)}{\partial e}, \frac{\partial^2 S(q,e)}{\partial e^2}$	$+, -$	$+, -$	$0, 0$

Notes

[1] Supply chain efficiency is the ratio of the expected profit in the decentralized supply chain to the optimal expected profit in the centralized supply chain.

[2] We use the Nash bargaining model with asymmetric bargaining power to model the bargaining process, and Proposition 1 establishes that the equilibrium wholesale price increases with the supplier's bargaining power. Hence, throughout the paper, we use wholesale price as a proxy for the supplier's bargaining power.

[3] We do not directly compare the efficiencies of wholesale price contracts under random capacity and random yield on a point-by-point basis, because they are two different models. Instead, we compare only whether the efficiency under each model is increasing or decreasing with the supplier's bargaining power.

[4] There is also an emergent stream of literature that investigates the role of network configuration in determining supply reliability (see e.g., Ang et al. 2016, Bimpikis et al. 2013, Jain et al. 2014, Lim et al. 2013).

[5] Over the past two decades, coordinating contracts have received much attention in the supply chain management literature. Numerous studies have identified coordinating contracts in different business contexts (for a review, see Cachon 2003).

[6] In the context of random yield, such contracts are also referred to as quality-based incentive pricing (Q-pricing) contracts (Baiman et al. 2004).

[7] Zhou (1997) shows that the Nash bargaining model with asymmetric bargaining power can be extended to the case when the payoff set (i.e., $\{(\pi_b^*(w), \pi_s^*(w)\}$ in our case) is non-convex, as in our model, for the bargaining power

parameter $\alpha \in (0,1)$. The supplier has all the bargaining power in the limit as $\alpha \to 1$, whereas the buyer has all the bargaining power in the limit as $\alpha \to 0$.

[8]Monotonicity and convexity/concavity results are all used in the weak sense throughout the paper, unless stated otherwise.

[9]We remark that analyses that calibrate the performance of simple contracts are typically hard problems that require specific assumptions about the functional form of the uncertainty distributions involved (Chu and Sappington 2007, Rogerson 2003). Our context with endogenous yield, and our focus on the sensitivity analysis of supply chain efficiency, pose additional analytical challenges. For these reasons, we restrict our focus (in the main text) to the case where ξ is uniformly distributed and then extend the results to a general distribution in Appendix A.

[10]Coase (1960) and Dahlman (1979) argue that transaction cost consists of three components: (i) search and information costs, (ii) bargaining and decision costs, and (iii) policing and enforcement costs. In our context, the transaction cost we identify mostly belongs to the the second component (bargaining and decision costs).

[11]Ho and Zhang (2008) experimentally show that, in contrast to theoretical predictions, two-part tariffs may not improve supply chain profits over those obtained with wholesale price contracts in the typical setting of double marginalization.

[12]Moreover, using a similar argument to that in the last paragraph of Section 4, one can argue that the wholesale price contract is Pareto efficient compared to the coordinating contract when the buyer is powerful once transaction costs are considered.

[13]The reason why the decreasing monotonic trend *may* not hold when w is sufficiently close to c is that both effort and quantity exhibit diminishing marginal impact (returns to scale) on expected sales and therefore on efficiency; the relevant derivatives can be found in Table 2 (see Appendix C). This means that, as w increases, the marginal increase in efficiency due to additional effort is greater when w is close to c, and the baseline effort is small; the corresponding marginal loss in efficiency due to a reduced order quantity is less significant. It follows that the monotonic trend in efficiency could be contravened only if w were close to c, a conclusion in line with Proposition 6.

[14]Although Proposition 7 checks only the *necessary* KKT conditions, we numerically confirm that a unit-penalty with buy-back contract does coordinate the supply chain under the parameters used in our numerical analysis.

[15]Specifically, the CDF is $H(\xi \mid e) = 1-(1-\xi^a)^e$, where a is a parameter and e is the effort level.

[16]We choose seven values each for c, θ, and m in the following ranges: $c \in [0.1,5], \theta \in [1,100]$, and $m \in [1,5]$. We therefore perform our analysis with 7^3=343 different combinations of parameters.

[17]The beta-type distribution generally results in higher yield losses than other distributions do, and these losses are increasing in the parameter a in the CDF. Therefore, we use a narrower range of parameters to ensure feasible solutions. Specifically, we choose five values each for a, c, θ, and m in the following ranges: $a \in [1,3]$, $c \in [0.1,\bar{c}], \theta \in [1,\bar{\theta}]$, and $m \in [1,5]$, where i) $\bar{c}=3, \bar{\theta}=100$ for $a=1$ and 1.5; ii) $\bar{c}=2, \bar{\theta}=50$ for $a=2$ and 2.5; and iii) $\bar{c}=1, \bar{\theta}=50$ for $a=3$. These combinations result in $5^4=625$ cases.

[18]The detailed derivatives are summarized in Table 1 (see Appendix C).

[19]Note that $y(q,e)$ is linear in q. The detailed derivatives are summarized in Table 2 (see Appendix C).

References

Ang, E., D. A. Iancu, R. Swinney. 2016. Disruption risk and optimal sourcing in multitier supply networks. *Management Science* forthcoming.

Babich, V., A.N. Burnetas, P.H. Ritchken. 2007. Competition and diversification effects in supply chains with supplier default risk. *Manufacturing & Service Operations Management* **9**(2) 123–146.

Baiman, S., P.E. Fischer, M.V. Rajan. 2000. Information, contracting, and quality costs. *Management Science* **46**(6) 776–789.

Baiman, S., S. Netessine, H. Kunreuther. 2004. Procurement in supply chains when the end-product exhibits the weakest link property. *Working Paper* .

Balachandran, K.R., S. Radhakrishnan. 2005. Quality implications of warranties in a supply chain. *Management Science* **51**(8) 1266–1277.

Bimpikis, K., D. Fearing, A. Tahbaz-Salehi. 2013. Multi-sourcing and miscoordination in supply chain networks. *Working Paper* .

Binmore, Ken, Ariel Rubinstein, Asher Wolinsky. 1986. The nash bargaining solution in economic modelling. *The RAND Journal of Economics* **17**(2) 176–188.

Bloomberg Business. 2015. Samsung said to win apple a9 chip orders for next iphone. `http://www.bloomberg.com/news/articles/2015-04-03/samsung-said-to-win-apple-a9-chip-orders-for-next-iphone`.

Cachon, G. 2004. The allocation of inventory risk in a supply chain: Push, pull, and advance-purchase discount contracts. *Management Science* **50**(2) 222–238.

Cachon, G.P. 2003. Supply chain coordination with contracts. *Handbooks in Operations Research and Management Science* **11** 229–340.

Chatterjee, A., D. Gudmundsson, R. Nurani, S. Seshadri, G. Shanthikumar. 1999. Fabless-foundry partnership: models and analysis of coordination issues. *Semiconductor Manufacturing, IEEE Transactions on* **12**(1) 44–52.

Chick, S.E., H. Mamani, D. Simchi-Levi. 2008. Supply chain coordination and influenza vaccination. *Operations Research* **56**(6) 1493–1506.

Chu, Leon Yang, David EM Sappington. 2007. Simple cost-sharing contracts. *The American Economic Review* **97**(1) 419–428.

Ciarallo, F.W., R. Akella, T.E. Morton. 1994. A periodic review, production planning model with uncertain capacity and uncertain demand-optimality of extended myopic policies. *Management Science* **40**(3) 320–332.

CNN. 2013. Garment workers, police clash in cambodia. `http://edition.cnn.com/2013/05/31/world/asia/cambodia-garment-workers-strike/`.

Coase, R. H. 1960. The problem of social cost. *Journal of law and economics* **3** 1–44.

Dada, M., N.C. Petruzzi, L.B. Schwarz. 2007. A newsvendor's procurement problem when suppliers are unreliable. *Manufacturing & Service Operations Management* **9**(1) 9–32.

Dahlman, Carl J. 1979. The problem of externality. *Journal of law and economics* **22**(1) 141–162.

Dai, T., S.-H. Cho, F. Zhang. 2016. Contracting for on-time delivery in the U.S. influenza vaccine supply chain. *Manufacturing & Service Operations Management* **18**(3) 332–346.

Deo, S., C.J. Corbett. 2009. Cournot competition under yield uncertainty: The case of the us influenza vaccine market. *Manufacturing & Service Operations Management* **11**(4) 563–576.

Dong, Lingxiu, Brian Tomlin. 2012. Managing disruption risk: The interplay between operations and insurance. *Management Science* **58**(10) 1898–1915.

Doucakis, T. 2007. Study of supply chain disruptions at a high tech corporation. `http://dspace.mit.edu/bitstream/handle/1721.1/40097/184985741.pdf?...1`.

Federgruen, A., N. Yang. 2008. Selecting a portfolio of suppliers under demand and supply risks. *Operations Research* **56**(4) 916–936.

Federgruen, A., N. Yang. 2009a. Competition Under Generalized Attraction Models: Applications to Quality Competition Under Yield Uncertainty. *Management Science* **55**(12) 2028–2043.

Federgruen, A., N. Yang. 2009b. Optimal supply diversification under general supply risks. *Operations Research* **57**(6) 1451–1468.

Feldstein, Martin S. 1972. Equity and efficiency in public sector pricing: the optimal two-part tariff. *The Quarterly Journal of Economics* **86**(2) 176–187.

Gümüş, M., S. Ray, H. Gurnani. 2012. Supply-side story: Risks, guarantees, competition, and information asymmetry. *Management Science* **58**(9) 1694–1714.

Ho, Teck-Hua, Juanjuan Zhang. 2008. Designing pricing contracts for boundedly rational customers: Does the framing of the fixed fee matter? *Management Science* **54**(4) 686–700.

Holmstrom, Bengt, Paul Milgrom. 1987. Aggregation and linearity in the provision of intertemporal incentives. *Econometrica* **55**(2) 303–328.

Jain, N., K. Girotra, S. Netessine. 2014. Global sourcing: Impact of sourcing strategies on supply chain resilience .

Jones, MC. 2009. Kumaraswamys distribution: A beta-type distribution with some tractability advantages. *Statistical Methodology* **6**(1) 70–81.

Kalkancı, B., K-Y. Chen, F. Erhun. 2011. Contract complexity and performance under asymmetric demand information: An experimental evaluation. *Management science* **57**(4) 689–704.

Kalkancı, B., K-Y. Chen, F. Erhun. 2014. Complexity as a contract design factor: A human-to-human experimental study. *Production and Operations Management* **23**(2) 269–284.

Kaya, M., Ö. Özer. 2009. Quality risk in outsourcing: Noncontractible product quality and private quality cost information. *Naval Research Logistics* **56**(7) 669–685.

Lariviere, M., E. Porteus. 2001. Selling to the newsvendor: An analysis of price-only contracts. *Manufacturing & Service Operations Management* **3**(4) 293–305.

Lariviere, Martin A, V Padmanabhan. 1997. Slotting allowances and new product introductions. *Marketing Science* **16**(2) 112–128.

Lexology. 2010. Implications of recent labor unrest for mncs operating or investing in china. `http://www.lexology.com/library/detail.aspx?g=f31127fe-281f-4773-9e32-a37407977212`.

Li, Linlin, Izak Duenyas, Seyed Iravani. 2011. Group warranty contracts to coordinate assembly supply chains with non-testable components. Tech. rep., Working Paper. Department of Industrial Engineering and Management Sciences. Northwestern University.

Lim, Michael K, Achal Bassamboo, Sunil Chopra, Mark S Daskin. 2013. Facility location decisions with random disruptions and imperfect estimation. *Manufacturing & Service Operations Management* **15**(2) 239–249.

LinkedIn Discussion. 2011. True or false: Penalties improve supplier performance. `https://www.linkedin.com/groups/True-false-penalties-improve-supplier-139021.S.42247612`.

Liu, S., K.C. So, F. Zhang. 2010. Effect of supply reliability in a retail setting with joint marketing and inventory decisions. *Manufacturing & Service Operations Management* **12**(1) 19–32.

Luenberger, D.G., Y. Ye. 2008. *Linear and Nonlinear Programming*. Springer.

MacDonald, James, Janet Perry, Mary Ahearn, David Banker, William Chambers, Carolyn Dimitri, Nigel Key, Kenneth Nelson, Leland Southard. 2004. *Contracts, Markets, and Prices: Organizing the Production and Use of Agricultural Commodities*. United States Department of Agriculture, Washington DC, USA.

Mas-Colell, A., M.D. Whinston, J.R. Green. 1995. *Microeconomic theory*. Oxford university press New York.

Perakis, G., G. Roels. 2007. The price of anarchy in supply chains: Quantifying the efficiency of price-only contracts. *Management Science* **53**(8) 1249–1268.

Rogerson, William P. 2003. Simple menus of contracts in cost-based procurement and regulation. *American Economic Review* **93**(3) 919–926.

Roth, Alvin E. 1979. *Axiomatic models of bargaining*. Berlin: Springer-Verlag.

Sappington, D. 1983. Limited liability contracts between principal and agent. *Journal of Economic Theory* **29**(1) 1–21.

Schwartz, A., J. Watson. 2004. The law and economics of costly contracting. *The Journal of Law, Economics, and Organization* **20**(1) 2–31.

Tang, S., H. Gurnani, D. Gupta. 2014. Managing disruptions in decentralized supply chains with endogenous supply process reliability. *Production and Operations Management* forthcoming.

Tang, S.Y., P. Kouvelis. 2011. Supplier diversification strategies in the presence of yield uncertainty and buyer competition. *Manufacturing & Service Operations Management* **13**(4) 439–451.

The Wall Street Journal. 2013. Apple finds it difficult to divorce samsung. `http://www.wsj.com/articles/SB10001424127887324682204578513882349940500`.

Tomlin, B. 2006. On the value of mitigation and contingency strategies for managing supply chain disruption risks. *Management Science* **52**(5) 639–657.

Tomlin, B. 2009. Impact of supply learning when suppliers are unreliable. *Manufacturing & Service Operations Management* **11**(2) 192–209.

Tomlin, B., Y. Wang. 2005. On the value of mix flexibility and dual sourcing in unreliable newsvendor networks. *Manufacturing & Service Operations Management* **7**(1) 37–57.

Wang, Y., W. Gilland, B. Tomlin. 2010. Mitigating supply risk: dual sourcing or process improvement? *Manufacturing & Service Operations Management* **12**(3) 489–510.

Yang, Z.B., G. Aydın, V. Babich, D.R. Beil. 2009. Supply disruptions, asymmetric information, and a backup production option. *Management Science* **55**(2) 192–209.

Yang, Z.B., G. Aydın, V. Babich, D.R. Beil. 2012. Using a dual-sourcing option in the presence of asymmetric information about supplier reliability: Competition vs. diversification. *Manufacturing & Service Operations Management* **14**(2) 202–217.

Zhang, L., H. Armer. 2013. Fab-wide effort increases yield at cmos imager facility. `http://www.appliedmaterials.com/nanochip/nanochip-fab-solutions/december-2013/fab-wide-effort-increases-yield-at-cmos-imager-facility`.

Zhou, Lin. 1997. The nash bargaining theory with non-convex problems. *Econometrica* **65**(3) 681–685.

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Appendix D: Proofs of All Results

We prove all results using the general model defined in Appendix A, except Propositions 3, 6(i), 12, and 14, for which we only use the uniform distribution. We first provide proofs for the four lemmas in Appendix B, and then provide proofs for all propositions and lemmas in the same order as they appear in the paper, because we use the lemmas in Appendix B extensively in the proofs.

Proof of Lemma 1(i) The expected sales is $S(q,e)=E_\xi[\min\{\min\{q,K-\xi\},D\}]$. If $q\le D$, then $S(q,e)=E_\xi[\min\{q,K-\xi\}]=y(q,e)$, by definition. If $q>D$, then $S(q,e)=E_\xi[\min\{D,K-\xi\}]=y(D,e)$. □

Proof of Lemma 1(ii) We show the property with respect to q in Part (1) and with respect to e in Part (2). The monotonicity and concavity properties are used in the weak sense, and the detailed derivatives are summarized in Table 1.

(1) Property of $y(q,e)$ in q: Recall that $a_c(e)$ is the maximum random loss. For any given $e\ge 0$, let $\hat{q}(e)=K-a_c(e)$, which is the capacity that is never affected by the random loss. We consider the following two cases, $0\le q\le \hat{q}(e)$ and $\hat{q}(e)\le q\le K$, and check the continuity and differentiability at $q=\hat{q}(e)$.

First, if $0\le q\le\hat{q}(e)$, then $q\le K-a_c(e)$, and the order is never affected by the random loss. Thus, $y(q,e)=E_\xi[\min\{q,K-\xi\}]=q$. Also, $\partial y(q,e)/\partial q=1>0$ and $\partial^2 y(q,e)/\partial q^2=0$. Thus, $y(q,e)$ is twice continuously differentiable, increasing, and concave in q.

Second, if $\hat{q}(e)\le q\le K$, then $q\ge K-a_c(e)$, and a fraction of the order quantity is affected by the random loss. Hence

$$y(q,e)=E_\xi[\min\{q,K-\xi\}]=\int_0^{K-q} q g(\xi\mid e)d\xi+\int_{K-q}^{a_c(e)}(K-\xi)g(\xi\mid e)d\xi=(K-a_c(e))+\int_{K-q}^{a_c(e)}G(\xi\mid e)d\xi,$$

where the last equality is obtained from integration by parts. $y(q,e)$ is twice continuously differentiable in q by Leibniz integral rule, because $G(\xi\mid e)$ and $a_c(e)$ are twice continuously differentiable in q (which are zeros), and so is $K-q$. Therefore,

$$\frac{\partial y(q,e)}{\partial q}=0\cdot G(a_c(e)\mid e)+G(K-q\mid e)+\int_{K-q}^{a_c(e)}\frac{\partial G(\xi\mid e)}{\partial q}d\xi=G(K-q\mid e)>0.$$

Also, $\partial^2 y(q,e)/\partial q^2=-g(K-q\mid e)<0$ because $K-q$ is in the support $[0,a_c(e)]$. Hence, $y(q,e)$ is twice continuously differentiable, increasing, and concave in q.

Last, we check the differentiability at $q=\hat{q}(e)$. We have $y(\hat{q}(e),e)=E_\xi[\min\{\hat{q}(e),K-\xi\}]=\hat{q}(e)$. Also, $\lim_{q\to\hat{q}(e)^-}y(q,e)=\lim_{q\to\hat{q}(e)^-}q=\hat{q}(e)$, and

$$\lim_{q\to\hat{q}(e)^+}y(q,e)=\lim_{q\to\hat{q}(e)^+}\left[(K-a_c(e))+\int_{K-q}^{a_c(e)}G(\xi\mid e)d\xi\right]=K-a_c(e)=\hat{q}(e).$$

In addition, $\lim_{q\to\hat{q}(e)^-}\partial y(q,e)/\partial q=\lim_{q\to\hat{q}(e)^-}1=1$, and $\lim_{q\to\hat{q}(e)^+}\partial y(q,e)/\partial q=\lim_{q\to\hat{q}(e)^+}G(K-q\mid e)=G(a_c(e)\mid e)=1$. Thus, $y(q,e)$ is once continuously differentiable at $q=\hat{q}(e)$.

(2) Property of $y(q,e)$ in e: Recall that $a_c(e)$ is the maximum random loss, and $K-a_c(e)$ is the capacity that is not affected by the random loss. Assumption 3 states that either i) $a_c(e)=K$ for all $e\ge 0$ or ii)

$a_c(0) = K$ and $a_c'(e) < 0$ for all $e \geq 0$. Hence, for a given q, there may exist $e' > 0$ such that $q = K - a_c(e')$. We assume such e' exists, and consider two cases, $0 \leq e < e'$ and $e' \leq e$, and check the differentiability at $e = e'$. (The case when such e' does not exist trivially follows from this general case.)

First, if $0 \leq e < e'$, then $q > K - a_c(e)$, and we know $y(q,e) = (K - a_c(e)) + \int_{K-q}^{a_c(e)} G(\xi \mid e) d\xi$ from Part (1) of this proof. $y(q,e)$ is continuously differentiable in e by Leibniz integral rule, because $G(\xi \mid e)$ and $a_c(e)$ are continuously differentiable in e by Assumption 3, and so is $K - q$. Thus,

$$\frac{\partial y(q,e)}{\partial e} = -a_c'(e) + a_c'(e) G(a_c(e) \mid e) + \int_{K-q}^{a_c(e)} \frac{\partial G(\xi \mid e)}{\partial e} d\xi = \int_{K-q}^{a_c(e)} \frac{\partial G(\xi \mid e)}{\partial e} d\xi,$$

because $G(a_c(e) \mid e) = 1$. Since $\partial G(\xi \mid e)/\partial e > 0$ for all $\xi \in (0, a_c(e))$ by Assumption 3, and $a_c(e) > K - q$, it is obvious that $\partial y(q,e)/\partial e > 0$. Again, $\partial y(q,e)/\partial e$ is continuously differentiable in e because $\partial G(\xi \mid e)/\partial e$ and $a_c(e)$ are continuously differentiable by Assumption 3, and thus

$$\frac{\partial^2 y(q,e)}{\partial e^2} = a_c'(e) \frac{\partial G(a_c(e) \mid e)}{\partial e} + \int_{K-q}^{a_c(e)} \frac{\partial^2 G(\xi \mid e)}{\partial e^2} d\xi.$$

By Assumption 3, either i) $a_c'(e) = 0$, $\partial^2 G(\xi \mid e)/\partial e^2 < 0$ for $e \geq 0$ and $\xi \in (0, K)$, or ii) $a_c'(e) < 0$, $\partial G(\xi \mid e)/\partial e > 0$, $\partial^2 G(\xi \mid e)/\partial e^2 \leq 0$ for all $e \geq 0$ and $\xi \in (0, a_c(e)]$. Either way, we have that $\partial^2 y(q,e)/\partial e^2 < 0$. Hence, $y(q,e)$ is twice continuously differentiable, increasing, and strictly concave in e.

Second, if $e' \leq e$, then $q \leq K - a_c(e)$. In this case, $y(q,e) = q$, and $\partial y(q,e)/\partial e = \partial^2 y(q,e)/\partial e^2 = 0$. Hence, $y(q,e)$ is twice continuously differentiable, increasing, and concave in e.

Last, we check continuity and differentiability at $e = e'$. We have $y(q,e') = q$, $\lim_{e \to e'+} y(q,e) = \lim_{e \to e'+} q = q$ and

$$\lim_{e \to e'-} y(q,e) = \lim_{e \to e'-} (K - a_c(e)) + \int_{K-q}^{a_c(e)} G(\xi \mid e) d\xi = q,$$

since $a_c(e') = K - q$. Also, $\lim_{e \to e'+} \partial y(q,e)/\partial e = \lim_{e \to e'+} 0 = 0$, and

$$\lim_{e \to e'-} \frac{\partial y(q,e)}{\partial e} = \lim_{e \to e'-} \int_{K-q}^{a_c(e)} \frac{\partial G(\xi \mid e)}{\partial e} d\xi = 0,$$

because $\lim_{e \to e'-} a_c(e) = K - q$ and $\partial G(\xi \mid e)/\partial e$ is finite by Assumption 3. Thus, $y(q,e)$ is once continuously differentiable at $e = e'$. $\square$

Proof of Lemma 2(i) We look at three cases. First, if $q \leq D$, then $(1-\xi)q \leq D$, because $0 \leq \xi \leq 1$. Therefore, $S(q,e) = E_\xi[\min\{(1-\xi)q, D\} = E_\xi[(1-\xi)q] = y(q,e)$, by definition.

Second, if $q > D$ and $(1 - a_y(e))q \geq D$, then the demand can be always met even with the maximum random loss. (Recall that $\xi = a_y(e)$ is the maximum random loss.) Hence, $S(q,e) = E_\xi[\min\{(1-\xi)q, D\}] = E_\xi[D] = D$. Thus, $y(q,e) = E_\xi[(1-\xi)q] = (1 - \mu_y^e)q > (1 - a_y(e))q \geq D = S(q,e)$, since $\mu_y^e < a_y(e)$.

Third, if $q > D$ and $(1 - a_y(e))q < D$, then a fraction of the demand may not be met. Specifically, $\min\{(1-\xi)q, D\} = D$ if $0 \leq \xi \leq 1 - D/q$ and $\min\{(1-\xi)q, D\} = (1-\xi)q$ if $1 - D/q < \xi \leq a_y(e)$. Therefore,

$$S(q,e) = E_\xi[\min\{(1-\xi)q, D\}] = \int_0^{1-\frac{D}{q}} D \cdot h(\xi \mid e) d\xi + \int_{1-\frac{D}{q}}^{a_y(e)} (1-\xi)q \cdot h(\xi \mid e) d\xi.$$

Note that $y(q,e) = E_\xi[(1-\xi)q] = \int_0^{a_y(e)} (1-\xi)q \cdot h(\xi \mid e) d\xi$. Hence, $y(q,e) - S(q,e) = \int_0^{1-D/q} ((1-\xi)q - D) h(\xi \mid e) d\xi > 0$, since $(1-\xi)q > D$ when $\xi \in (0, 1 - D/q)$.

For further uses in the rest of the proofs, we simplify $S(q,e)$. Using integration by parts, we have

$$\begin{aligned} S(q,e) &= DH\left(1-\frac{D}{q}\mid e\right) + (1-a_y(e))q - D + \int_{1-\frac{D}{q}}^{a_y(e)} q\cdot H(\xi\mid e)d\xi \\ &= (1-a_y(e))q + \int_{1-\frac{D}{q}}^{a_y(e)} q\cdot H(\xi\mid e)d\xi. \end{aligned}$$

□

Proof of Lemma 2(ii) Since we are showing the properties of $y(q,e)$ and $S(q,e)$ with respect to both q and e, we divide the proof into four parts: (1) $y(q,e)$ with q, (2) $S(q,e)$ with q, (3) $y(q,e)$ with e, and (4) $S(q,e)$ with e. Note that the monotonicity and concavity properties are used in the weak sense.

(1) PROPERTY OF $y(q,e)$ IN q: For any $e\geq 0$, $y(q,e)=(1-\mu_y^e)q$, and hence $y(q,e)$ is thrice continuously differentiable in q. Also, $\partial y(q,e)/\partial q = (1-\mu_y^e) > 0$, because $0\leq \mu_y^e < 1$, and $\partial^2 y(q,e)/\partial q^2 = 0$. Thus, $y(q,e)$ is increasing and concave in q.

(2) PROPERTY OF $S(q,e)$ IN q: We consider three cases in which we have different functional forms: (a) $q<D$, (b) $D<q<D/(1-a_y(e))$, and (c) $D/(1-a_y(e))<q$. Also, we check once differentiability at the two boundaries between three cases. If $a_y(e)=1$, the third case never happens, but the result trivially follows from this more general case. Thus, we assume $a_y(e)\neq 1$.

First, if $q<D$, then $S(q,e)=y(q,e)$ from part (i) of this Lemma. Thus, $S(q,e)$ is thrice continuously differentiable, increasing, and concave in q.

Second, if $D<q<D/(1-a_y(e))$, then $S(q,e)=(1-a_y(e))q+\int_{1-D/q}^{a_y(e)} qH(\xi\mid e)d\xi$ from the proof of part (i) of this Lemma. $S(q,e)$ is thrice continuously differentiable in q by Leibniz integral rule, because $H(\xi\mid e)$ and $a_c(e)$ are thrice continuously differentiable in q (note that both are independent of q), and so is $(1-D/q)$. Thus,

$$\begin{aligned} \frac{\partial S(q,e)}{\partial q} &= (1-a_y(e)) - \frac{D}{q}H\left(1-\frac{D}{q}\mid e\right) + \int_{1-\frac{D}{q}}^{a_y(e)} H(\xi\mid e)d\xi \\ &= \int_{1-\frac{D}{q}}^{a_y(e)} \left[H(\xi\mid e) - H\left(1-\frac{D}{q}\mid e\right)\right]d\xi + (1-a_y(e))\left(1-H\left(1-\frac{D}{q}\mid e\right)\right). \end{aligned}$$

The first integral term is strictly positive, because $H(\xi\mid e)-H(1-D/q\mid e)$ is strictly positive when $1-D/q<\xi\leq a_y(e)$. Also, the second term is strictly positive, because $1-D/q<a_y(e)$ and thus $H(1-D/q\mid e)<1$, and we assumed $a_y(e)<1$. Therefore, $\partial S(q,e)/\partial q>0$. Again, using Leibniz integral rule,

$$\frac{\partial^2 S(q,e)}{\partial q^2} = \frac{D}{q^2}H\left(1-\frac{D}{q}\mid e\right) - \frac{D^2}{q^3}h\left(1-\frac{D}{q}\mid e\right) - \frac{D}{q^2}H\left(1-\frac{D}{q}\mid e\right) = -\frac{D^2}{q^3}h\left(1-\frac{D}{q}\mid e\right).$$

Therefore, $\partial^2 S(q,e)/\partial q^2<0$, because $h(1-D/q\mid e)>0$ since $1-D/q$ lies in the support $[0,a_y(e)]$. Hence, $S(q,e)$ is thrice continuously differentiable, increasing, and strictly concave in q.

Third, if $D/(1-a_y(e))<q$, then $S(q,e)=D$ from the proof of Lemma 2(i). This is obviously thrice continuously differentiable in q, increasing ($\partial S(q,e)/\partial q=0$), and concave ($\partial^2 S(q,e)/\partial q^2=0$).

Now, we show $S(q,e)$ is once continuously differentiable in q at the two boundaries. First, we look at $q=D$. We have that $S(D,e)=(1-\mu_y^e)D$, $\lim_{q\to D^-} S(q,e)=\lim_{q\to D^-}(1-\mu_y^e)q=(1-\mu_y^e)D$, and

$$\lim_{q\to D^+} S(q,e) = \lim_{q\to D^+}\left[(1-a_y(e))q + \int_{1-\frac{D}{q}}^{a_y(e)} qH(\xi\,|\,e)d\xi\right] = (1-a_y(e))D + \int_0^{a_y(e)} D\cdot H(\xi\,|\,e)d\xi$$
$$= (1-a_y(e))D + \left(a_y(e) - \int_0^{a_y(e)} \xi h(\xi\,|\,e)d\xi\right)D = \left(1 - \int_0^{a_y(e)} \xi h(\xi\,|\,e)d\xi\right)D = (1-\mu_y^e)D.$$

Therefore, $S(q,e)$ is continuous in q at $q=D$. Also, we observe that $\lim_{q\to D^-}\partial S(q,e)/\partial q = \lim_{q\to D^-}(1-\mu_y^e) = (1-\mu_y^e)$, and

$$\lim_{q\to D^+}\frac{\partial S(q,e)}{\partial q} = \lim_{q\to D^+}\left[\int_{1-\frac{D}{q}}^{a_y(e)}\left[H(\xi\,|\,e) - H\left(1-\frac{D}{q}\,|\,e\right)\right]d\xi + (1-a_y(e))\left(1-H\left(1-\frac{D}{q}\,|\,e\right)\right)\right]$$
$$= \int_0^{a_y(e)} H(\xi\,|\,e)d\xi + (1-a_y(e)) = a_y(e) - \int_0^{a_y(e)}\xi h(\xi\,|\,e)d\xi + (1-a_y(e)) = (1-\mu_y^e),$$

using integration by parts. Therefore, $S(q,e)$ is once continuously differentiable in q when $q=D$.

Second, we consider $q = D/(1-a_y(e))$. Let $\hat{q}(e) = D/(1-a_y(e))$. Then, $S(\hat{q}(e),e) = D$, $\lim_{q\to\hat{q}(e)^+} S(q,e) = D$, and

$$\lim_{q\to\hat{q}(e)^-} S(q,e) = \lim_{q\to\hat{q}(e)^-}\left[(1-a_y(e))q + \int_{1-\frac{D}{q}}^{a_y(e)} qH(\xi\,|\,e)d\xi\right] = D.$$

Hence, $S(q,e)$ is continuous in q at $q = D/(1-a_y(e))$. Also, $\lim_{q\to\hat{q}(e)^+}\partial S(q,e)/\partial q = \lim_{q\to\hat{q}(e)^+} 0 = 0$, and

$$\lim_{q\to\hat{q}(e)^-}\frac{\partial S(q,e)}{\partial q} = \lim_{q\to\hat{q}(e)^-}\left[(1-a_y(e)) - \frac{D}{q}H\left(1-\frac{D}{q}\,|\,e\right) + \int_{1-D/q}^{a_y(e)} H(\xi\,|\,e)d\xi\right]$$
$$= (1-a_y(e)) - (1-a_y(e))H(a_y(e)\,|\,e) + \int_{a_y(e)}^{a_y(e)} H(\xi\,|\,e)d\xi = 0,$$

since $H(a_y(e)\,|\,e) = 1$. Therefore, $S(q,e)$ is once continuously differentiable in q when $q = D/(1-a_y(e))$.

(3) <u>Property of $y(q,e)$ in e</u>: Note that $y(q,e) = (1-\mu_y^e)q = (1-\int_0^{a_y(e)}\xi h(\xi\mid e)d\xi)q = (1-a_y(e) + \int_0^{a_y(e)} H(\xi\mid e)d\xi)q$ by integration by parts. We find $y(q,e)$ is continuously differentiable in e by Leibniz integral rule, because $H(\xi\,|\,e)$ and $a_y(e)$ are continuously differentiable in e by Assumption 4. Therefore,

$$\frac{\partial y(q,e)}{\partial e} = \left[-a_y'(e) + a_y'(e)H(a_y(e)\,|\,e) + \int_0^{a_y(e)}\frac{\partial H(\xi\,|\,e)}{\partial e}d\xi\right]q = q\int_0^{a_y(e)}\frac{\partial H(\xi\,|\,e)}{\partial e}d\xi,$$

because $H(a_y(e)\,|\,e) = 1$. Since $\partial H(\xi\,|\,e)/\partial e > 0$ when $\xi\in(0,a_y(e))$ by Assumption 4, we get $\partial y(q,e)/\partial e > 0$ if $q>0$, and $\partial y(q,e)/\partial e = 0$ if $q=0$. In addition, $\partial y(q,e)/\partial e$ is twice more continuously differentiable in e, because $\partial H(\xi\,|\,e)/\partial e$ and $a_y(e)$ are twice continuously differentiable in e by Assumption 4. Thus,

$$\frac{\partial^2 y(q,e)}{\partial e^2} = q\left[a_y'(e)\frac{\partial H(a_y(e)\,|\,e)}{\partial e} + \int_0^{a_y(e)}\frac{\partial^2 H(\xi\,|\,e)}{\partial e^2}d\xi\right].$$

Assumption 4 states that either i) $a_y'(e) = 0$ and $\partial^2 H(\xi\,|\,e)/\partial e^2 < 0$ for $e\geq 0$ and $\xi\in(0,1)$, or ii) $a_y'(e) < 0$, $\partial H(\xi\,|\,e)/\partial e > 0$, and $\partial^2 H(\xi\,|\,e)/\partial e^2 \leq 0$ for $e\geq 0$ and $\xi\in(0,a_y(e)]$. Either way, we have that $\partial^2 y(q,e)/\partial e^2 < 0$. Therefore, $y(q,e)$ is thrice continuously differentiable, increasing, and strictly concave in e.

(4) <u>Property of $S(q,e)$ in e</u>: Following a similar structure to that of part (2), we consider three cases: i) $q\leq D$, ii) $D < q < D/(1-a_y(e))$, and iii) $D/(1-a_y(e)) < q$. However, unlike part (2), the only boundary we need to check is the one between cases ii) and iii), because the boundary between cases i) and ii) are not

determined by e. As in part (2), if $a_y(e)=1$, then the third case never occurs, but this is subsumed in the more general three case scenario, so we assume $a_y(e)\neq 1$.

First, if $q\leq D$, then $S(q,e)=y(q,e)$ by part (i) of this Lemma. Therefore, the result follows from part (3).

Second, if $D<q<D/(1-a_y(e))$, then $S(q,e)=(1-a_y(e))q+\int_{1-D/q}^{a_y(e)} qH(\xi\mid e)d\xi$ from the proof of Lemma 2(i). We find $S(q,e)$ is continuously differentiable in e by Leibniz integral rule, because $H(\xi\mid e)$ and $a_y(e)$ are continuously differentiable in e by Assumption 4, and so is $1-D/q$. Therefore,

$$\frac{\partial S(q,e)}{\partial e}=-a_y'(e)q+\left[a_y'(e)H(a_y(e)\mid e)+\int_{1-\frac{D}{q}}^{a_y(e)}\frac{\partial H(\xi\mid e)}{\partial e}d\xi\right]q=q\int_{1-\frac{D}{q}}^{a_y(e)}\frac{\partial H(\xi\mid e)}{\partial e}d\xi,$$

because $H(a_y(e)\mid e)=1$. Note that $\partial H(\xi\mid e)/\partial e>0$ when $\xi\in(0,a_y(e))$ by Assumption 4. Thus, $\partial S(q,e)/\partial e>0$ if $q>0$ and $\partial S(q,e)/\partial e=0$ if $q=0$. In addition, $\partial S(q,e)/\partial e$ is twice more continuously differentiable in e, because $\partial H(\xi\mid e)/\partial e$ and $a_y(e)$ are twice continuously differentiable in e by Assumption 4. Thus,

$$\frac{\partial^2 S(q,e)}{\partial e^2}=q\left[a_y'(e)\frac{\partial H(a_y(e)\mid e)}{\partial e}+\int_{1-\frac{D}{q}}^{a_y(e)}\frac{\partial^2 H(\xi\mid e)}{\partial e^2}d\xi\right].$$

Assumption 4 states that either i) $a_y'(e)=0$ and $\partial^2H(\xi\mid e)/\partial e^2<0$ for $e\geq 0$ and $\xi\in(0,1)$, or ii) $a_y'(e)<0$, $\partial H(\xi\mid e)/\partial e>0$, and $\partial^2H(\xi\mid e)/\partial e^2\leq 0$ for $e\geq 0$ and $\xi\in(0,a_y(e)]$. Either way, we have that $\partial^2S(q,e)/\partial e^2<0$. Hence, $S(q,e)$ is thrice continuously differentiable, increasing, and strictly concave in e.

Third, if $D/(1-a_y(e))<q$, then $S(q,e)=D$, and hence $\partial S(q,e)/\partial e=\partial^2 S(q,e)/\partial e^2=0$. Therefore, $S(q,e)$ is thrice continuously differentiable, increasing, and concave in e.

Now, we check once differentiability at e such that $q=D/(1-a_y(e))$. Assume there exists e' such that $q=D/(1-a_y(e'))$. If $e<e'$, then $q<D/(1-a_y(e))$, and in the neighborhood of e' we fall into the second case above, and thus $S(q,e)=(1-a_y(e))q+\int_{1-D/q}^{a_y(e)} qH(\xi\mid e)d\xi$. If $e\geq e'$, then $q\geq D/(1-a_y(e))$, which is the third case above. Hence, $S(q,e)=D$. At $e=e'$, we have $S(q,e')=D$, $\lim_{e\to e'+}S(q,e)=\lim_{e\to e'+}D=D$, and

$$\lim_{e\to e'-}S(q,e)=\lim_{e\to e'-}\left[(1-a_y(e))q+\int_{1-D/q}^{a_y(e)}qH(\xi\mid e)d\xi\right]=D.$$

In addition, $\lim_{e\to e'+}\partial S(q,e)/\partial e=\lim_{e\to e'+}0=0$, and

$$\lim_{e\to e'-}\frac{\partial S(q,e)}{\partial e}=\lim_{e\to e'-}q\left(\int_{1-\frac{D}{q}}^{a_y(e)}\frac{\partial H(\xi\mid e)}{\partial e}d\xi\right)=0.$$

Therefore, $S(q,e)$ is once continuously differentiable at $e=e'$. □

Proof of Lemma 3(i) If $q\leq D$, then $S(q,x,e)=E_\xi[\min\{\tilde q,D\}]=E_\xi[\min\{q,(1-\xi)x,D\}]=E_\xi[\min\{q,(1-\xi)x\}]=y(q,x,e)$. If $q\geq D$, then $S(q,x,e)=E_\xi[\min\{q,(1-\xi)x,D\}]=E_\xi[\min\{(1-\xi)x,D\}]=y(D,x,e)$. □

Proof of Lemma 3(ii) $y(q,x,e)=E_\xi[\min\{(1-\xi)x,q\}]$ is equivalent to $S(x,e)=E_\xi[\min\{(1-\xi)x,D\}]$ if we set $q=D$. Therefore, we can obtain the stated properties of $y(q,x,e)$ from $S(x,e)$ in Lemma 2. □

Proof of Lemma 4(i) For random capacity, $\Pi(q,e)=pS(q,e)-(cy(q,e)+v(e))$. Note that, when $q\leq D$, $S(q,e)=y(q,e)$ by Lemma 1 and $\partial y(q,e)/\partial q=G(K-q\mid e)$ by the proof of Lemma 1. Therefore, when $q\leq D$,

$\partial\Pi(q,0)/\partial q = (p-c)\partial y(q,0)/\partial q = (p-c)G(K-q\,|\,0)$. Thus, $\partial\Pi(0,0)/\partial q = p-c$, and hence Assumption 1(i) is equivalent to $p > c$. □

Proof of Lemma 4(ii) For random yield, $\Pi(q,e) = pS(q,e) - (cq + v(e))$. By Lemma 2, when $q \le D$, $S(q,e) = y(q,e) = (1-\mu_y^e)q$. Therefore, when $q \le D$, $\partial\Pi(0,0)/\partial q = p(1-\mu_y^0) - c$. Hence, Assumption 1(i) is equivalent to $p(1-\mu_y^0) > c$. □

Proof of Proposition 1 The proof consists of two steps. In Step 1, we show that if either $\pi_s^*(w)$ is strictly increasing in w or $\pi_b^*(w)$ is strictly decreasing in w, then we can transform the objective function in problem (2) to a new one without affecting the solution. In Step 2, we take a log transformation of the new problem, which again does not affect the solution. Then, we show that the new objective function is supermodular. This means that the optimal solution w^* to problem (2) is increasing in α by Topkis' monotonicity theorem.

Step 1. We only show that if $\pi_s^*(w)$ is strictly increasing, then the optimal solution w^* is increasing in α. The case when $\pi_b^*(w)$ is strictly decreasing can be analogously obtained and hence we omit the proof. Let $\rho(w,\alpha) = \pi_s^*(w)^{\alpha}\pi_b^*(w)^{1-\alpha}$. Note that for all $\alpha \in (0,1)$, any w that satisfies either $\pi_s^*(w) = 0$ or $\pi_b^*(w) = 0$ cannot be optimal, because this results in $\rho(w,\alpha) = 0$ and we can always find w such that $\rho(w,\alpha) > 0$.

First, assume that $\pi_s^*(w)$ is strictly increasing. Also, assume that $\pi_b^*(w)$ is weakly decreasing in w except the region $w \in (w_1, w_2)$ in which $\pi_b^*(w)$ is strictly increasing. (The following proof can be easily extended to the cases in which $\pi_b^*(w)$ has multiple increasing regions.) Then, $\rho(w_2,\alpha) > \rho(w,\alpha)$ for all $w \in [w_1, w_2)$. Furthermore, if there exists $w_0 < w_1$ such that $\pi_b^*(w_0) = \pi_b^*(w_2)$, then $\rho(w_2,\alpha) > \rho(w,\alpha)$ for all $w \in [w_0, w_1]$, since $\pi_b^*(w)$ is continuous. Therefore, we can conclude that any $w \in [w_0, w_2)$ will never be optimal. Note that if such w_0 does not exist, then by continuity any $w \in [c, w_2)$ cannot be optimal, in which case we define $w_0 = c$.

Define $\hat{\pi}_b^*(w)$ as follows.

$$\hat{\pi}_b^*(w) = \begin{cases} \pi_b^*(w_2) & \text{if } w \in [w_0, w_2], \\ \pi_b^*(w) & \text{otherwise.} \end{cases}$$

That is, $\hat{\pi}_b^*(w)$ flattens out any trough of $\pi_b^*(w)$ with the constant $\pi_b^*(w_2)$ while maintaining all decreasing regions in such a way that $\hat{\pi}_b^*(w)$ is weakly decreasing for all w. Note that $\hat{\pi}_b^*(w)$ is still continuous. Now, we define a new optimization problem as follows:

$$\max_{w}\ \pi_s^*(w)^{\alpha}\hat{\pi}_b^*(w)^{1-\alpha}. \tag{4}$$

It is easy to see that problems (2) and (4) are equivalent in terms of the solutions they generate, because $w \in [w_0, w_2)$ can never be optimal for either.

Step 2. We take a log transformation of problem (4), which preserves the solution, as follows.

$$\max_{w}\ \hat{\rho}(w,\alpha) = \alpha\log\pi_s^*(w) + (1-\alpha)\log\hat{\pi}_b^*(w). \tag{5}$$

Let $\alpha < \alpha'$ and $w < w'$. Then, $\hat{\rho}(w,\alpha)$ is supermodular if

$$\begin{aligned}&(\alpha'\log\pi_s^*(w') + (1-\alpha')\log\hat{\pi}_b^*(w')) - (\alpha'\log\pi_s^*(w) + (1-\alpha')\log\hat{\pi}_b^*(w))\\ &\quad \ge (\alpha\log\pi_s^*(w') + (1-\alpha)\log\hat{\pi}_b^*(w')) - (\alpha\log\pi_s^*(w) + (1-\alpha)\log\hat{\pi}_b^*(w)).\end{aligned} \tag{6}$$

We can rearrange (6) and obtain

$$\log \frac{\pi_s^*(w')}{\pi_s^*(w)} \geq \log \frac{\hat{\pi}_b^*(w')}{\hat{\pi}_b^*(w)}.$$

This obviously holds because $\pi_s^*(w)$ is strictly increasing and $\hat{\pi}_b^*(w)$ is weakly decreasing. Therefore, by Topkis' monotonicity theorem, the optimal solution w^* is (weakly) increasing in α. □

Proof of Proposition 2 The proof is organized in two steps. In Step 1, we show that the optimal order (production) quantity is $q^o = D$ regardless of effort e. In Step 2, we show that e^o is uniquely obtained by the first-order condition.

Step 1: Optimal order quantity. The expected profit is $\Pi(q,e) = pS(q,e) - (cy(q,e) + v(e))$. We consider two cases: 1) $q > D$ and 2) $q < D$. If $q > D$, then $S(q,e) = y(D,e)$ by Lemma 1, and hence $\Pi(q,e) = py(D,e) - (cy(q,e) + v(e)) < py(D,e) - (cy(D,e) + v(e)) = \Pi(D,e)$, since $\partial y(q,e)/\partial q > 0$ by Lemma 1 and Table 1. If $q < D$, then $S(q,e) = y(q,e)$ by Lemma 1, and hence $\Pi(q,e) = (p-c)y(q,e) - v(e) < (p-c)y(D,e) - v(e) = \Pi(D,e)$, since $\partial y(q,e)/\partial q > 0$, and $p > c$ by Lemma 4. Therefore, the solution is $q^o = D$ regardless of e.

Step 2: Optimal effort. With $q^o = D$, we have $\Pi(D,e) = (p-c)y(D,e) - v(e)$, since $S(D,e) = y(D,e)$ by Lemma 1. $\Pi(D,e)$ is strictly concave in e, because $y(D,e)$ is strictly concave in e by Lemma 1 and Table 1 and $v(e)$ is convex by Assumption 2. Thus, the optimal effort e^o can be uniquely obtained by the the first-order condition: $\partial\Pi(D,e)/\partial e = (p-c)\partial S(D,e)/\partial e - v'(e) = 0$. (Note that we focus only on interior solutions by Assumption 2.) □

Proof of Proposition 3 We prove the result with the *uniform distribution* and use the results from the proof of Proposition 10. In Step 1, we obtain a sufficient condition under which $d\pi_b(q)/dq < 0$ for $q \geq D$ and $w \in [c,p]$. In Step 2, we show that the sufficient condition from Step 1 holds if $c > p/3$ for the uniform distribution. Note that, in the proof of Proposition 10(i), we show that the buyer always orders *at least* D units (in Step 2), and that if the buyer orders D units, the efficiency is monotonically increasing for all w (in Step 3). Therefore, if $c > p/3$, then the buyer always orders D units, and thus the efficiency is monotonically increasing for all w.

Step 1: General sufficient condition. If $q \geq D$, using the inequality (39) and the best response function $e(q)$ obtained by (38), we have that

$$\begin{aligned}
\frac{d\pi_b(q)}{dq} &\leq (p-w)\frac{y(q,e)}{\partial e} \cdot \frac{de(q)}{dq} - w\frac{\partial y(q,e)}{\partial q} \\
&= (p-w)\frac{\partial y(q,e)}{\partial e} \cdot \frac{(w-c)\frac{\partial^2 y(q,e)}{\partial e \partial q}}{v''(e) - (w-c)\frac{\partial^2 y(q,e)}{\partial e^2}} - w\frac{\partial y(q,e)}{\partial q} \\
&\leq (p-w)\frac{\partial y(q,e)}{\partial e} \cdot \frac{(w-c)\frac{\partial^2 y(q,e)}{\partial e \partial q}}{-(w-c)\frac{\partial^2 y(q,e)}{\partial e^2}} - w\frac{\partial y(q,e)}{\partial q} \\
&= \frac{1}{-\frac{\partial^2 y(q,e)}{\partial e^2}}\left[(p-w)\frac{\partial y(q,e)}{\partial e} \cdot \frac{\partial^2 y(q,e)}{\partial e \partial q} + w\frac{\partial y(q,e)}{\partial q}\frac{\partial^2 y(q,e)}{\partial e^2}\right],
\end{aligned}$$

where the third step holds since $v''(e) \geq 0$ by Assumption 2, $\partial y(q,e)/\partial e > 0$ and $\partial^2 y(q,e)/\partial e^2 < 0$ by Lemma 1 and Table 1, and $\partial^2 y(q,e)/\partial e \partial q = \partial G(K-q \mid e)/\partial e > 0$ by Assumption 3. Therefore, $d\pi_b(q)/dq < 0$ if

$$(p-w)\frac{\partial y(q,e)}{\partial e} \cdot \frac{\partial^2 y(q,e)}{\partial e \partial q} + w\frac{\partial y(q,e)}{\partial q}\frac{\partial^2 y(q,e)}{\partial e^2} < 0.$$

This always holds if

$$\frac{\frac{\partial^2 y(q,e)}{\partial e \partial q}}{\frac{\partial y(q,e)}{\partial q}} < -\frac{c}{p-c} \cdot \frac{\frac{\partial^2 y(q,e)}{\partial e^2}}{\frac{\partial y(q,e)}{\partial e}}. \tag{7}$$

Step 2: Sufficient condition for the uniform distribution. First, we show that the optimal effort satisfies that $q > K - a_c(e)$. The supplier's first-order condition is: $(w-c)\frac{\partial y(q,e)}{\partial e} - v'(e) = 0$. If $q \le K - a_c(e)$, then $\frac{\partial y(q,e)}{\partial e} = 0$ by Table 1, while $v'(e) > 0$, $e > 0$ by Assumption 2. Therefore, the optimal effort should satisfy that $q > K - a_c(e)$, since we focus only on interior solutions ($e > 0$) by Assumption 2.

Second, for the uniform distribution, we have that $G(\xi \mid e) = \frac{(e+1)\xi}{K}$ and $a_c(e) = \frac{K}{e+1}$. Therefore, when $q > K - a_c(e)$, or when $q > \frac{e}{e+1}K$ for uniform, the expected delivered quantity is

$$y(q,e) = (K - a_c(e)) + \int_{K-q}^{a_c(e)} G(\xi \mid e) d\xi = \left(1 - \frac{1}{2(e+1)}\right) K - (e+1)\frac{(K-q)^2}{2K}.$$

Hence,

$$\frac{\frac{\partial^2 y(q,e)}{\partial e \partial q}}{\frac{\partial y(q,e)}{\partial q}} = \frac{1}{e+1}, \quad \text{and} \quad -\frac{\frac{\partial^2 y(q,e)}{\partial e^2}}{\frac{\partial y(q,e)}{\partial e}} = \frac{\frac{K}{(e+1)^3}}{\frac{K}{2(e+1)^2} - \frac{(K-q)^2}{2K}} \ge \frac{\frac{K}{(e+1)^3}}{\frac{K}{2(e+1)^2}} = \frac{2}{e+1},$$

where the inequality holds because $\partial y(q,e)/\partial e > 0$ and $\partial^2 y(q,e)/\partial e^2 < 0$ by Lemma 1 and Table 1. Therefore, Condition (7) holds if

$$\frac{1}{e+1} < \frac{c}{p-c} \cdot \frac{2}{e+1},$$

or equivalently, $c > p/3$. □

Proof of Proposition 4 Under a unit-penalty contract, each firm's expected profit is as follows: $\pi_b(q,e) = pS(q,e) - wy(q,e) + z(q - y(q,e))$, and $\pi_s(q,e) = wy(q,e) - z(q - y(q,e)) - (cy(q,e) + v(e))$. We define $\bar{\chi}$ as follows:

$$\bar{\chi} = \min\left\{\frac{\Pi(q^o, e^o)}{D}, \frac{p(y(K,0) - y(D,0))}{K-D}\right\}. \tag{8}$$

The proof is organized in four steps. In Step 1, we reformulate problem (3). In Step 2 and 3, we solve the reformulated problem assuming $q \le D$ and $D \le q \le K$, respectively, because the expected sales $S(q,e)$ has a kink at $q = D$. In both Steps 2 and 3, we find that the supply chain is coordinated with $q^* = q^o = D$ and $e^* = e^o$. In Step 4, we obtain the expected profits.

Step 1: Reformulation. In problem (3), we replace the first constraint with its first-order condition, because they are equivalent. The supplier's expected profit, $\pi_s(q,e) = (p-c)y(q,e) - \chi q - v(e)$, is strictly concave in e, because $p - c > 0$ by Lemma 4, $y(q,e)$ is strictly concave in e by Lemma 1 and Table 1, and $v(e)$ is convex by Assumption 2. Also, by Assumption 2, we focus only on interior solutions. Therefore, problem (3) can be reformulated as follows:

$$\begin{aligned} \max_{q,e} \; & pS(q,e) - py(q,e) + \chi q, \\ s.t. \; & (p-c)\frac{\partial y(q,e)}{\partial e} - v'(e) = 0, \\ & (p-c)y(q,e) - \chi q - v(e) \ge 0. \end{aligned} \tag{9}$$

Step 2: Solving the problem when $\mathbf{q \le D}$. The objective function in problem (9) collapses to $\pi_b(q,e) = \chi q$, because $S(q,e) = y(q,e)$ by Lemma 1. We ignore the two constraints in problem (9), solve the problem, and check the solution satisfies the ignored constraints. Ignoring the constraints, the optimal order quantity is obviously $q^* = q^o = D$ regardless of e, since $\chi \ge 0$ and by Assumption 1. Now, at $q = D$, the first constraint is satisfied if and only if $e^* = e^o$, since $y(D,e) = S(D,e)$ by Lemma 1 and there exists a unique $e = e^o$ that satisfies $(p-c)\partial S(D,e)/\partial e = v'(e)$, which is equivalent to the first-order condition of the centralized supply chain. At (q^o, e^o), the LHS of the second constraint becomes $(p-c)y(q^o,e^o) - \chi q^o - v(e^o) = \Pi(q^o,e^o) - \chi q^o$, because $S(q^o,e^o) = y(q^o,e^o)$. The second constraint is also satisfied, because $\chi \le \Pi(q^o,e^o)/D$ since $\chi < \bar{\chi}$ where $\bar{\chi}$ is given by (8). Therefore, (q^o,e^o) is the unique solution.

Step 3: Solving the problem when $\mathbf{D \le q \le K}$. The objective function in problem (9) becomes $\pi_b(q,e) = py(D,e) - py(q,e) + \chi q$, because $S(q,e) = y(D,e)$ by Lemma 1. Again, we ignore the two constraints, solve the problem, and check the solution satisfies the ignored constraints. The objective function $\pi_b(q,e)$ is convex in q, because $y(q,e)$ is concave in q by Lemma 1. Therefore, $q = D$ is optimal regardless of e, if $\pi_b(D,e) > \pi_b(K,e)$ for any e, due to convexity of $\pi_b(q,e)$ in q.

The condition $\pi_b(D,e) > \pi_b(K,e)$ can be rewritten as $\chi < p(y(K,e) - y(D,e))/(K-D)$ with basic arithmetic calculations. It is easy to see that $y(K,e) - y(D,e)$ is increasing in e, because $\partial^2 y(q,e)/\partial q \partial e = \partial G(K - q \mid e)/\partial e > 0$ by Assumption 3. Using the inequality $\chi < \bar{\chi}$ where $\bar{\chi}$ is given by (8), we have

$$\chi < \frac{p(y(K,0) - y(D,0))}{K-D} \le \frac{p(y(K,e) - y(D,e))}{K-D},$$

for any $e \ge 0$. Therefore, $q^* = D$ is indeed optimal ignoring the two constraints. In Step 2, we already showed that the two constraints are satisfied only if $e^* = e^o$ when $q^* = D$.

Step 4: Expected profits. The buyer's expected profit is $\pi_b(q^*,e^*) = pS(q^o,e^o) - py(q^o,e^o) + \chi q^o = \chi q^o = \chi D$ and the supplier's expected profit is $\pi_s(q^*,e^*) = (p-c)y(q^o,e^o) - \chi q^o - v(e^o) = \Pi(q^o,e^o) - \chi q^o = \Pi(q^o,e^o) - \chi D$, since $S(q^o,e^o) = y(q^o,e^o)$ by Lemma 1. □

Proof of Proposition 5 The proof is organized in three steps. In Step 1, we show the existence of a solution. In Step 2, we show that a solution satisfies $D < q^o < D/(1 - a_y(e^o))$. Finally, in Step 3, we prove that optimal solutions satisfy the first-order necessary conditions.

Step 1: Existence of a solution. The expected profit of the centralized supply chain satisfies $\Pi(q,e) = pS(q,e) - c(q,e) \le pD - (cq + v(e))$, because $S(q,e)$ is bounded by D. It is easy to see that if $q > pD/c$, then $\Pi(q,e) < 0$ for any e. Also, if $e > v^{-1}(pD)$ where $v^{-1}(\cdot)$ is an inverse function of $v(e)$, then $\Pi(q,e) < 0$ for any q. Therefore, if an optimal solution were to exist, it should be in the compact set $\{(q,e) \mid q \in [0, pD/c], e \in [0, v^{-1}(pD)]\}$. Since $\Pi(q,e)$ is continuous by Lemma 2, the optimal (q^o,e^o) does exist in the compact set.

Step 2: Range of a solution. If $q \le D$, then $S(q,e) = y(q,e) = (1 - \mu_y^e)q$ by Lemma 2, and therefore $\Pi(q,e) = (p(1-\mu_y^e) - c)q - v(e)$. Since $p(1-\mu_y^e) - c > 0$ for any $e \ge 0$ by Lemma 4, we have $\partial \Pi(q,e)/\partial q = p(1-\mu_y^e) - c > 0$ for any $e \ge 0$. Therefore, $q^o > D$. Also, if $q \ge D/(1 - a_y(e))q$, then $S(q,e) = D$ by the proof of Lemma 2. Thus, $\Pi(q,e) = pD - (cq + v(e))$ and $\partial \Pi(q,e)/\partial q = -c < 0$. Hence, $q^o < D/(1 - a_y(e^o))$.

Step 3: First-order conditions. First, we obtain the first-order condition with respect to q. $\Pi(q,e) = pS(q,e) - (cq + v(e))$ is concave in q, because $S(q,e)$ is concave in q by Lemma 2, and in particular strictly concave when $D < q < D/(1-a_y(e))$ by Table 2. Hence the optimal order quantity q^o should satisfy the first-order condition, $p \cdot \partial S(q^o, e^o)/\partial q = c$, unless a corner solution is optimal. But, a corner solution cannot be optimal. Since $D < q^o < D/(1-a_y(e^o))$, the only possible corner solution is $q^o = \infty$ when $a_y(e^o) = 1$. But, $\lim_{q\to\infty} \Pi(q,e) = \lim_{q\to\infty} pS(q,e) - (cq + v(e)) = -\infty$, since $S(q,e)$ is bounded by D whereas the cost can be infinite. Hence, a corner solution cannot be optimal.

Second, we obtain the first-order condition with respect to e. $\Pi(q,e) = pS(q,e) - (cq + v(e))$ is strictly concave in e when $D < q < D/(1-a_y(e))$, because $S(q,e)$ is strictly concave in e when $D < q < D/(1-a_y(e))$ by Lemma 2 and Table 2, and $v(e)$ is convex in e by Assumption 2. Hence, the optimal effort e^o should satisfy the first-order condition, $p \cdot \partial S(q^o, e^o)/\partial e = v'(e^o)$. Note that, by Assumption 2, we focus only on interior solutions. □

Proof of Proposition 6(i) We prove the result using the *uniform distribution*. The proof is organized in four steps. In Step 1, we formulate the problem with the uniform distribution, and obtain the supplier's best response function and the buyer's first-order condition. In Step 2, we obtain the expression of $d\Pi^*(w)/dw$. In Step 3, we obtain two inequalities, which we use to find the upper bound of $d\Pi^*(w)/dw$. Finally, in Step 4, we show that $d\Pi^*(w)/dw < 0$ if $w \in [\underline{\mathrm{w}}_c, p]$.

Step 1: Formulating the problem with the uniform distribution. First, we obtain the expected delivered quantity $y(q,e)$ and sales $S(q,e)$ for the uniform distribution. Second, we obtain the supplier's best response function $e^*(q,w)$. Third, we show that the optimal order quantity always satisfies that $D \le q \le (1+\frac{1}{e})D$. Finally, we get the buyer's first-order condition, and obtain $dq^*(w)/dw$, where $q^*(w)$ is the optimal order quantity given w and $e^*(q,w)$.

First, we obtain $y(q,e)$ and $S(q,e)$ as follows using the proof of Lemma 2:

$$y(q,e) = \left(1 - \frac{1}{2(e+1)}\right) q,$$

$$S(q,e) = \begin{cases} y(q,e), & \text{if } q \le D, \\ D, & \text{if } q \ge (1+\frac{1}{e})D, \\ \left(1 - \frac{1}{2(e+1)}\right) q - (e+1)\left(1 - \frac{D}{q}\right)^2 \frac{q}{2}, & \text{otherwise.} \end{cases}$$

Second, we obtain the supplier's best response function $e^*(q,w)$. The supplier's expected profit is

$$\pi_s(q,e) = wy(q,e) - (cq + \theta e) = w\left(1 - \frac{1}{2(e+1)}\right) q - (cq + \theta e),$$

which is strictly concave in e. Hence, the optimal effort is uniquely obtained by the first-order condition, $\frac{\partial \pi_s(q,e)}{\partial e} = 0$, which produces

$$e^*(q,w) = \sqrt{\frac{wq}{2\theta}} - 1. \tag{10}$$

Third, we show that the buyer's optimal order quantity always satisfies that $D \le q \le (1+\frac{1}{e})D$. The buyer's expected profit is $\pi_b(q,e) = pS(q,e) - wy(q,e)$. Let $\pi_b(q,w) = \pi_b(q, e^*(q,w))$ be the buyer's expected profit given $e^*(q,w)$. If $q < D$, then $\pi_b(q,w) = (p-w)y(q, e^*(q,w))$ since $S(q,e) = y(q,e)$ by Lemma 2. Therefore, $\pi_b(q,w)$ increases in q because $y(q,e)$ is increasing in both q and e by Lemma 2 and Table 2, and $e^*(q,w)$ is increasing in q by (10). Hence, it is optimal to order $q \ge D$. If $q > (1+\frac{1}{e})D$, then $S(q,e) = D$ and, thus,

$\pi_b(q,w) = pD - wy(q, e^*(q,w))$, which decreases in q because of the same reason. Therefore, it is optimal to order $q \le (1+\frac{1}{e})D$.

Last, we obtain the buyer's first-order condition and $dq^*(w)/dw$. When $D \le q \le (1+\frac{1}{e})D$, the buyer's expected profit is

$$\pi_b(q,e) = p\left[\left(1 - \frac{1}{2(e+1)}\right)q - (e+1)\left(1-\frac{D}{q}\right)^2\frac{q}{2}\right] - w\left(1-\frac{1}{2(e+1)}\right)q. \quad (11)$$

By plugging the supplier's best response function (10) in the buyer's expected profit (11), we get $\pi_b(q,w) = \pi_b(q, e^*(q,w))$. Then, we can obtain the optimal order quantity $q^*(w)$ from the buyer's first-order condition as follows:

$$\frac{\partial \pi_b(q,w)}{\partial q} = \phi(q,w) = (p-w) - \frac{(p-w)}{4\sqrt{\frac{wq}{2\theta}}} - \frac{3p}{4}\sqrt{\frac{wq}{2\theta}}\left(1-\frac{D}{q}\right)^2 - p\sqrt{\frac{wq}{2\theta}}\left(1-\frac{D}{q}\right)\frac{D}{q} = 0. \quad (12)$$

Note that

$$\frac{\partial^2 \pi_b(q,w)}{\partial q^2} = \frac{\partial \phi(q,w)}{\partial q} = \left(\frac{p-w}{8(e+1)q} - \frac{3p(e+1)}{8q}\right) - \frac{p(e+1)D}{4q^2} - \frac{3p(e+1)D^2}{8q^3} < 0, \quad (13)$$

where $e = \sqrt{\frac{wq}{2\theta}} - 1$, because the term in the bracket is negative for all $e \ge 0$. Therefore, $\pi_b(q,w)$ is strictly concave in q, and the optimal order quantity $q^*(w)$ is uniquely obtained from the first-order condition (12). Using the implicit function theorem, we have that

$$\frac{q^*(w)}{dw} = -\frac{\partial \phi(q,w)}{\partial w}\left(\frac{\partial \phi(q,w)}{\partial q}\right)^{-1}. \quad (14)$$

Step 2: $\mathbf{d\Pi^*(w)/dw}$ for the uniform distribution. In this step, we obtain the expression for $d\Pi^*(w)/dw$. Let $\Pi^*(q,w) = \pi_b(q,w) + \pi_s(q,w)$ be the expected profit of the total supply chain given the supplier's best response function $e^*(q,w)$. Also, let $\Pi^*(w) = \Pi^*(q^*(w), w)$, where $q^*(w)$ is the buyer's optimal order quantity. Then, using (14),

$$\frac{d\Pi^*(w)}{dw} = \frac{\partial \Pi^*(q,w)}{\partial w} + \frac{\partial \Pi^*(q,w)}{\partial q}\frac{dq^*(w)}{dw} = \frac{1}{\frac{\partial \phi(q,w)}{\partial q}}\left(\frac{\partial \Pi^*(q,w)}{\partial w}\frac{\partial \phi(q,w)}{\partial q} - \frac{\partial \Pi^*(q,w)}{\partial q}\frac{\partial \phi(q,w)}{\partial w}\right).$$

Note that $\frac{\partial \phi(q,w)}{\partial q} < 0$ by (13). Therefore, $d\Pi^*(w)/dw < 0$ if

$$\frac{\partial \Pi^*(q,w)}{\partial w}\frac{\partial \phi(q,w)}{\partial q} - \frac{\partial \Pi^*(q,w)}{\partial q}\frac{\partial \phi(q,w)}{\partial w} > 0. \quad (15)$$

We can obtain the following expressions with some basic calculations:

$$\frac{\partial \phi(q,w)}{\partial w} = -\frac{1}{4(e+1)}\left(\frac{p(2e+1)}{w} + 2(e+1)\right) < 0, \quad (16)$$

$$\frac{\partial \Pi^*(q,w)}{\partial q} = \frac{\partial \pi_b(q,w)}{\partial q} + \frac{\partial \pi_s(q,w)}{\partial q} = w\left(1 - \frac{1}{2(e+1)}\right) - c > 0, \quad (17)$$

$$\frac{\partial \Pi^*(q,w)}{\partial w} = \frac{\partial \pi_b(q,w)}{\partial w} + \frac{\partial \pi_s(q,w)}{\partial w} = \frac{qp}{(e+1)4w}\left(\frac{p-w}{p} - (e+1)^2\left(1-\frac{D}{q}\right)^2\right), \quad (18)$$

noting that $e = \sqrt{\frac{wq}{2\theta}} - 1$. Also note that $\frac{\partial \Pi^*(q,w)}{\partial q} > 0$ because $\frac{\partial \Pi^*(q,w)}{\partial q} = \frac{\partial \pi_s(q,e)}{\partial q} = \frac{\pi_s(q,e)+\theta e}{q} > 0$ since the supplier's participation constraint is satisfied.

Step 3: Inequalities to obtain the upper bound of $\mathbf{d\Pi^*(w)/dw}$. In this step, we obtain the lower bound of $\frac{\partial \phi(q,w)}{\partial q}$ and the upper bound of $\frac{\partial \Pi^*(q,w)}{\partial w}$.

First, we obtain the lower bound of $\frac{\partial\phi(q,w)}{\partial q}$. We showed that the buyer always orders $q \geq D$ in Step 1. Therefore, using (10) we have that $(e+1)^2 = \frac{wq}{2\theta} \geq \frac{wD}{2\theta}$. Using this inequality, (10), and (13), we have

$$\begin{aligned}
\frac{\partial\phi(q,w)}{\partial q} &= \frac{p-w}{8(e+1)q} - \frac{3p(e+1)}{8q} - \frac{p(e+1)D}{4q^2} - \frac{3p(e+1)D^2}{8q^3} \\
&= \frac{w}{16\theta(e+1)}\left[\frac{p-w}{(e+1)^2} - 3p - \frac{pwD}{\theta(e+1)^2} - \frac{3pw^2D^2}{4\theta^2(e+1)^4}\right] \\
&\geq \frac{w}{16\theta(e+1)}\left[\frac{p-w}{(e+1)^2} - 3p - \frac{pwD}{\theta}\cdot\frac{2\theta}{wD} - \frac{3pw^2D^2}{4\theta^2}\cdot\frac{4\theta^2}{w^2D^2}\right] \\
&= \frac{w}{16\theta(e+1)}\left[\frac{p-w}{(e+1)^2} - 8p\right]. \qquad (19)
\end{aligned}$$

Second, we obtain the upper bound of $\frac{\partial\Pi^*(q,w)}{\partial w}$. Using (10), the buyer's first-order condition (12) can be written as follows:

$$(p-w) - \frac{p-w}{4(e+1)} - \frac{3p}{4}(e+1)\left(1-\frac{D}{q}\right)^2 - p(e+1)\left(1-\frac{D}{q}\right)\frac{D}{q} = 0.$$

Rearranging this first-order condition, we get

$$\left(1-\frac{D}{q}\right)\left[\frac{3p}{4}\left(1-\frac{D}{q}\right) + p\frac{D}{q}\right] = (p-w)\left[\frac{1}{e+1} - \frac{1}{4(e+1)^2}\right].$$

Therefore,

$$\begin{aligned}
\left(1-\frac{D}{q}\right) &= \frac{4q}{3q+D}\left(1-\frac{w}{p}\right)\left[\frac{1}{e+1} - \frac{1}{4(e+1)^2}\right] \\
&\geq \frac{4q}{3q+q}\left(1-\frac{w}{p}\right)\left[\frac{1}{e+1} - \frac{1}{4(e+1)^2}\right] \\
&= \left(1-\frac{w}{p}\right)\frac{1}{(e+1)}\left[1 - \frac{1}{4(e+1)}\right] \\
&\geq \frac{3}{4}\left(1-\frac{w}{p}\right)\frac{1}{(e+1)},
\end{aligned}$$

where the second step holds because $q \geq D$ and the last step holds because $e \geq 0$.

Now using this result, (10), and (18), we have

$$\begin{aligned}
\frac{\partial\Pi^*(q,w)}{\partial w} &= \frac{p}{(e+1)4w}\cdot\frac{2\theta(e+1)^2}{w}\left[\left(1-\frac{w}{p}\right) - (e+1)^2\left(1-\frac{D}{q}\right)^2\right] \\
&= \frac{p\theta(e+1)}{2w^2}\left[\left(1-\frac{w}{p}\right) - (e+1)^2\left(1-\frac{D}{q}\right)^2\right] \\
&\leq \frac{p\theta(e+1)}{2w^2}\left[\left(1-\frac{w}{p}\right) - (e+1)^2\frac{9}{16}\left(1-\frac{w}{p}\right)^2\frac{1}{(e+1)^2}\right] \\
&= \frac{\theta(e+1)(p-w)}{2w^2}\left(\frac{7p+9w}{16p}\right). \qquad (20)
\end{aligned}$$

Step 4: Monotonicity. In this step, we show that $d\Pi^*(w)/dw < 0$ if $w \in [\underline{w}_c, p]$. In Step 2, we already showed that $d\Pi^*(w)/dw < 0$ if (15) holds. Note that if $\frac{\partial\Pi^*(q,w)}{\partial w} < 0$, then (15) always holds, because we have $\frac{\partial\phi(q,w)}{\partial q} < 0$, $\frac{\partial\Pi^*(q,w)}{\partial q} > 0$, and $\frac{\partial\phi(q,w)}{\partial w} < 0$ from Steps 1 and 2.

If $\frac{\partial \Pi^*(q,w)}{\partial w} > 0$, then using the lower bound of $\frac{\partial \phi(q,w)}{\partial q}$, (19), and the upper bound of $\frac{\partial \Pi^*(q,w)}{\partial w}$, (20), along with (16) and (17), we have that

$$\begin{aligned}
\frac{\partial \Pi}{\partial w}\frac{\partial \phi}{\partial q} - \frac{\partial \Pi}{\partial q}\frac{\partial \phi}{\partial w} &\geq \frac{\theta(e+1)(p-w)}{2w^2} \cdot \frac{7p+9w}{16p} \cdot \frac{w}{16\theta(e+1)}\left(\frac{p-w}{(e+1)^2} - 8p\right) \\
&\quad + \left[w\left(1 - \frac{1}{2(e+1)}\right) - c\right] \cdot \left[\frac{1}{4(e+1)}\left(\frac{p(2e+1)}{w} + 2(e+1)\right)\right] \\
&= \frac{(p-w)^2(7p+9w)}{512wp(e+1)^2} - \frac{(p-w)(7p+9w)}{64w} \\
&\quad + \frac{p}{8}\left(\frac{2e+1}{e+1}\right)^2 + \frac{w}{4}\left(\frac{2e+1}{e+1}\right) - \frac{cp}{4w}\left(\frac{2e+1}{e+1}\right) - \frac{c}{2} \\
&\geq -\frac{(p-w)(7p+9w)}{64w} + \frac{p}{8}\left(\frac{2e+1}{e+1}\right)^2 + \frac{w}{4}\left(\frac{2e+1}{e+1}\right) - \frac{cp}{4w}\left(\frac{2e+1}{e+1}\right) - \frac{c}{2}.
\end{aligned}$$

We have that $e+1 = \sqrt{\frac{wq}{2\theta}} \geq \sqrt{\frac{cD}{2\theta}}$, because $w \geq c$ and $q \geq D$. Therefore,

$$2 - \sqrt{\frac{2\theta}{cD}} \leq \frac{2e+1}{e+1} = 2 - \frac{1}{e+1} < 2.$$

Let $k = 2 - \sqrt{\frac{2\theta}{cD}}$. Then, we have that

$$\begin{aligned}
\frac{\partial \Pi}{\partial w}\frac{\partial \phi}{\partial q} - \frac{\partial \Pi}{\partial q}\frac{\partial \phi}{\partial w} &\geq -\frac{(p-w)(7p+9w)}{64w} + \frac{p}{8}\left(\frac{2e+1}{e+1}\right)^2 + \frac{w}{4}\left(\frac{2e+1}{e+1}\right) - \frac{cp}{4w}\left(\frac{2e+1}{e+1}\right) - \frac{c}{2} \\
&> -\frac{(p-w)(7p+9w)}{64w} + \frac{p}{8}k^2 + \frac{w}{4}k - \frac{pc}{2w} - \frac{c}{2} \\
&= \frac{1}{64w}\left[(16k+9)w^2 + (8pk^2 - 32c - 2p)w - p(7p+32c)\right].
\end{aligned}$$

Therefore, using the solution of the quadratic equation in the square bracket, $\frac{\partial \Pi}{\partial w}\frac{\partial \phi}{\partial q} - \frac{\partial \Pi}{\partial q}\frac{\partial \phi}{\partial w} > 0$ if

$$w \geq \underline{w}_c = \frac{-(8pk^2 - 32c - 2p) + \sqrt{(8pk^2 - 32c - 2p)^2 + 4p(16k+9)(7p+32c)}}{2(16k+9)}.$$

□

Proof of Proposition 6(ii) In this proof, we use the results from the proof of Proposition 11. The proof is organized in four steps. In Step 1, we show some properties of the cost of effort $v(e) = \theta e^m$ with respect to θ and m. In Step 2, we show some properties of the supplier's best response function $e^*(q,w)$ with respect to θ and m. In Step 3, we obtain the upper bound of $dq^*(w)/dw$ using the properties from the previous steps. Finally, in Step 4, we show that the efficiency is decreasing if either θ or m is large enough.

Step 1: Properties of the cost of effort. We show the following four properties of $v(e)$ at the equilibrium effort e^*: $\lim_{\theta\to\infty} v''(e^*) = \infty$, $\lim_{m\to\infty} v''(e^*) = \infty$, $\lim_{\theta\to\infty} \frac{v'''(e^*)}{(v''(e^*))^3} = 0$, and $\lim_{m\to\infty} \frac{v'''(e^*)}{(v''(e^*))^3} = 0$.

First, we show that $\lim_{\theta\to\infty} v''(e^*) = \infty$. The supplier's first-order condition is the following.

$$\frac{\partial \pi_s(q^*,e^*)}{\partial e} = w\frac{\partial y(q^*,e^*)}{\partial e} - v'(e^*) = w\frac{\partial y(q^*,e^*)}{\partial e} - \theta m e^{*m-1} = 0. \tag{21}$$

Therefore, $\theta = \frac{w}{m}\frac{\partial y(q^*,e^*)}{\partial e} e^{*1-m}$. Note that $\frac{\partial y(q,e)}{\partial e}$ is decreasing in e since $\frac{\partial^2 y(q,e)}{\partial e^2} < 0$ by Lemma 2 and Table 2 and finite by Assumption 4, and also e^{1-m} is decreasing in e since $m > 1$. Hence, it is easy to see that $\lim_{\theta\to\infty} e^* = 0$. Thus, using (21),

$$\lim_{\theta\to\infty} v''(e^*) = \lim_{\theta\to\infty} \theta m(m-1)e^{*m-2} = \lim_{e^*\to 0} w\frac{\partial y(q^*,e^*)}{\partial e}(m-1)\frac{1}{e^*} = \infty, \tag{22}$$

since $\frac{\partial y(q,e)}{\partial e} > 0, e \geq 0$ by Lemma 2 and Table 2.

Second, we show that $\lim_{m\to\infty} v''(e^*) = \infty$. Note that $\lim_{m\to\infty} e^* < \infty$, because the first-order condition (21) cannot hold otherwise because of the following reason: $w\frac{\partial y(q^*,e^*)}{\partial e}$ is finite by Assumption 4, but $\lim_{m\to\infty} \theta m e^{*m-1} = \infty$ if $\lim_{m\to\infty} e^* = \infty$. Therefore,

$$\lim_{m\to\infty} v''(e^*) = \lim_{m\to\infty} w\frac{\partial y(q^*,e^*)}{\partial e}(m-1)\frac{1}{e^*} = \infty. \tag{23}$$

Third, we show that $\lim_{\theta\to\infty} \frac{v'''(e^*)}{(v''(e^*))^3} = 0$. Using the first-order condition (21),

$$\lim_{\theta\to\infty} \frac{v'''(e^*)}{(v''(e^*))^3} = \lim_{\theta\to\infty} \frac{\theta m(m-1)(m-2)e^{*m-3}}{\theta^3 m^3 (m-1)^3 e^{*3m-6}} = \lim_{\theta\to\infty} \frac{(m-2)}{\left(w\frac{\partial y(q^*,e^*)}{\partial e}\right)^2 (m-1)^2} e^* = 0, \tag{24}$$

because $\frac{\partial y(q,e)}{\partial e} > 0, e \geq 0$ by Lemma 2 and Table 2, and $\lim_{\theta\to\infty} e^* = 0$.

Last, we show that $\lim_{m\to\infty} \frac{v'''(e^*)}{(v''(e^*))^3} = 0$.

$$\lim_{m\to\infty} \frac{v'''(e^*)}{(v''(e^*))^3} = \lim_{m\to\infty} \frac{(m-2)}{\left(w\frac{\partial y(q^*,e^*)}{\partial e}\right)^2 (m-1)^2} e^* = 0, \tag{25}$$

since $\lim_{m\to\infty} e^* < \infty$.

Step 2: Properties of the best response function. We show the supplier's best response function $e^*(q^*,w)$ satisfies the following properties: $\lim_{\theta\to\infty} \frac{\partial e^*(q^*,w)}{\partial q} = \lim_{m\to\infty} \frac{\partial e^*(q^*,w)}{\partial q} = 0$, $\lim_{\theta\to\infty} \frac{\partial e^*(q^*,w)}{\partial w} = \lim_{m\to\infty} \frac{\partial e^*(q^*,w)}{\partial w} = 0$, $\lim_{\theta\to\infty} \frac{\partial^2 e^*(q^*,w)}{\partial q^2} = \lim_{m\to\infty} \frac{\partial^2 e^*(q^*,w)}{\partial q^2} = 0$, and $\lim_{\theta\to\infty} \frac{\partial^2 e^*(q^*,w)}{\partial q \partial w} = \lim_{m\to\infty} \frac{\partial^2 e^*(q^*,w)}{\partial q \partial w} = 0$.

First, we show that $\lim_{\theta\to\infty} \frac{\partial e^*(q^*,w)}{\partial q} = \lim_{m\to\infty} \frac{\partial e^*(q^*,w)}{\partial q} = 0$. We can obtain $\frac{\partial e^*(q^*,w)}{\partial q}$ by differentiating the first-order condition (21) with respect to q as follows:

$$w\frac{\partial^2 y(q^*,e^*)}{\partial e \partial q} + \left(w\frac{\partial^2 y(q^*,e^*)}{\partial e^2} - v''(e^*)\right)\frac{\partial e^*(q^*,w)}{\partial q} = 0. \tag{26}$$

Therefore,

$$\frac{\partial e^*(q^*,w)}{\partial q} = -\frac{w\frac{\partial^2 y(q^*,e^*)}{\partial e \partial q}}{w\frac{\partial^2 y(q^*,e^*)}{\partial e^2} - v''(e^*)}. \tag{27}$$

Thus, $\lim_{\theta\to\infty} \frac{\partial e^*(q^*,w)}{\partial q} = \lim_{m\to\infty} \frac{\partial e^*(q^*,w)}{\partial q} = 0$ using (22) and (23), because $y(q,e)$ has finite derivatives regardless of e by Assumption 4.

Second, we show that $\lim_{\theta\to\infty} \frac{\partial e^*(q^*,w)}{\partial w} = \lim_{m\to\infty} \frac{\partial e^*(q^*,w)}{\partial w} = 0$. Using the first-order condition (21) and the implicit function theorem, we have

$$\frac{\partial e^*(q^*,w)}{\partial w} = -\frac{\frac{\partial y(q^*,e^*)}{\partial e}}{w\frac{\partial^2 y(q^*,e^*)}{\partial e^2} - v''(e^*)}. \tag{28}$$

Therefore, $\lim_{\theta\to\infty} \frac{\partial e^*(q^*,w)}{\partial w} = \lim_{m\to\infty} \frac{\partial e^*(q^*,w)}{\partial w} = 0$ using (22) and (23), because $y(q,e)$ has finite derivatives regardless of e by Assumption 4. Note that we can observe the following relationship between (27) and (28):

$$\frac{\partial e^*(q^*,w)}{\partial w} = \frac{q}{w}\frac{\partial e^*(q^*,w)}{\partial q}, \tag{29}$$

where $\frac{\partial^2 y(q,e)}{\partial e \partial q} = \frac{\partial y(q,e)}{\partial e} \cdot \frac{1}{q}$ because $y(q,e)$ is linear in q.

Third, we show that $\lim_{\theta\to\infty}\frac{\partial^2 e^*(q^*,w)}{\partial q^2}=\lim_{m\to\infty}\frac{\partial^2 e^*(q^*,w)}{\partial q^2}=0$. We can obtain $\frac{\partial^2 e^*(q^*,w)}{\partial q^2}$ by differentiating (26) with respect to q as follows.

$$w\cdot\frac{\partial^3 y(q^*,e^*)}{\partial e^2\partial q}\frac{\partial e^*(q^*,w)}{\partial q}+\left[\left(w\frac{\partial^3 y(q^*,e^*)}{\partial e^3}-v'''(e^*)\right)\frac{\partial e^*(q^*,w)}{\partial q}+w\frac{\partial^3 y(q^*,e^*)}{\partial e^2\partial q}\right]\frac{\partial e^*(q^*,w)}{\partial q}$$
$$+\left(w\frac{\partial^2 y(q^*,e^*)}{\partial e^2}-v''(e^*)\right)\frac{\partial^2 e^*(q^*,w)}{\partial q^2}=0,$$

where $\frac{\partial^3 y(q^*,e^*)}{\partial e\partial q^2}=0$, because $y(q,e)$ is linear in q. Therefore,

$$\frac{\partial^2 e^*(q^*,w)}{\partial q^2}=-\frac{2w\frac{\partial^3 y(q^*,e^*)}{\partial e^2\partial q}+\left(w\frac{\partial^3 y(q^*,e^*)}{\partial e^3}-v'''(e^*)\right)\frac{\partial e^*(q^*,w)}{\partial q}}{w\frac{\partial^2 y(q^*,e^*)}{\partial e^2}-v''(e^*)}\cdot\frac{\partial e^*(q^*,w)}{\partial q}$$
$$=-\frac{2w\frac{\partial^2 y(q^*,e^*)}{\partial e^2}\cdot\frac{1}{q^*}}{w\frac{\partial^2 y(q^*,e^*)}{\partial e^2}-v''(e^*)}\cdot\frac{\partial e^*(q^*,w)}{\partial q}-\frac{w\frac{\partial^3 y(q^*,e^*)}{\partial e^3}-v'''(e^*)}{\left(w\frac{\partial^2 y(q^*,e^*)}{\partial e^2}-v''(e^*)\right)^3}\cdot\left(w\frac{\partial^2 y(q^*,e^*)}{\partial e\partial q}\right)^2,$$

where $\frac{\partial^3 y(q,e)}{\partial e^2\partial q}=\frac{\partial^2 y(q,e)}{\partial e^2}\frac{1}{q}$ because $y(q,e)$ is linear in q, and also we use (27). Therefore, we can see that $\lim_{\theta\to\infty}\frac{\partial^2 e^*(q^*,w)}{\partial q^2}=0$ and $\lim_{m\to\infty}\frac{\partial^2 e^*(q^*,w)}{\partial q^2}=0$ using (22), (23), (24), (25), and that $\lim_{\theta\to\infty}\frac{\partial e^*(q^*,w)}{\partial q}=\lim_{m\to\infty}\frac{\partial e^*(q^*,w)}{\partial q}=0$. Note that $y(q,e)$ has finite derivatives regardless of e by Assumption 4.

Finally, we show that $\lim_{\theta\to\infty}\frac{\partial^2 e^*(q^*,w)}{\partial q\partial w}=\lim_{m\to\infty}\frac{\partial^2 e^*(q^*,w)}{\partial q\partial w}=0$. We can obtain $\frac{\partial^2 e^*(q^*,w)}{\partial q\partial w}$ by differentiating (26) with respect to w as follows.

$$\left[\frac{\partial^2 y(q^*,e^*)}{\partial e^2}+\left(w\frac{\partial^3 y(q^*,e^*)}{\partial e^3}-v'''(e^*)\right)\frac{e^*(q^*,w)}{\partial w}\right]\frac{\partial e^*(q^*,w)}{\partial q}$$
$$+\left(w\frac{\partial^2 y(q^*,e^*)}{\partial e^2}-v''(e^*)\right)\frac{\partial^2 e^*(q^*,w)}{\partial q\partial w}+\left(\frac{\partial^2 y(q^*,e^*)}{\partial e\partial q}+w\frac{\partial^3 y(q^*,e^*)}{\partial e^2\partial q}\frac{\partial e^*(q^*,w)}{\partial w}\right)=0.$$

Therefore,

$$\frac{\partial^2 e^*(q^*,w)}{\partial q\partial w}=-\frac{\frac{\partial^2 y(q^*,e^*)}{\partial e\partial q}+w\frac{\partial^3 y(q^*,e^*)}{\partial e^2\partial q}\frac{\partial e^*(q^*,w)}{\partial w}+\left[\frac{\partial^2 y(q^*,e^*)}{\partial e^2}+\left(w\frac{\partial^3 y(q^*,e^*)}{\partial e^3}-v'''(e^*)\right)\frac{\partial e^*(q^*,w)}{\partial w}\right]\frac{\partial e^*(q^*,w)}{\partial q}}{w\frac{\partial^2 y(q^*,e^*)}{\partial e^2}-v''(e^*)}$$
$$=-\frac{\frac{\partial^2 y(q^*,e^*)}{\partial e\partial q}+2\frac{\partial^2 y(q^*,e^*)}{\partial e^2}\frac{\partial e^*(q^*,w)}{\partial q}}{w\frac{\partial^2 y(q^*,e^*)}{\partial e^2}-v''(e^*)}-\frac{\frac{q^*}{w}\left(w\frac{\partial^3 y(q^*,e^*)}{\partial e^3}-v'''(e^*)\right)\left(w\frac{\partial^2 y(q^*,e^*)}{\partial e\partial q}\right)^2}{\left(w\frac{\partial^2 y(q^*,e^*)}{\partial e^2}-v''(e^*)\right)^3},$$

using (27) and (29), and that $\frac{\partial^3 y(q,e)}{\partial e^2\partial q}=\frac{\partial^2 y(q,e)}{\partial e^2}\cdot\frac{1}{q}$ because $y(q,e)$ is linear in q. Again, we can observe that $\lim_{\theta\to\infty}\frac{\partial^2 e^*(q^*,w)}{\partial q\partial w}=\lim_{m\to\infty}\frac{\partial^2 e^*(q^*,w)}{\partial q\partial w}=0$, using (22), (23), (24), (25), and that $\lim_{\theta\to\infty}\frac{\partial e^*(q^*,w)}{\partial q}=\lim_{m\to\infty}\frac{\partial e^*(q^*,w)}{\partial q}=0$. Note that $y(q,e)$ has finite derivatives regardless of e by Assumption 4.

Step 3: Upper bound of $\mathbf{dq^*(w)/dw}$. Using the results on limits of $e^*(q^*,w)$ from Step 2, and using (45) and (46), we have

$$\lim_{\theta\to\infty}\frac{\partial^2\pi_b(q^*,w)}{\partial q\partial w}=\lim_{m\to\infty}\frac{\partial^2\pi_b(q^*,w)}{\partial q\partial w}=-\frac{\partial y(q^*,e^*)}{\partial q},$$
$$\lim_{\theta\to\infty}\frac{\partial^2\pi_b(q^*,w)}{\partial q^2}=\lim_{m\to\infty}\frac{\partial^2\pi_b(q^*,w)}{\partial q^2}=p\frac{\partial^2 S(q^*,e^*)}{\partial q^2}.$$

Hence, for any arbitrarily small $\epsilon>0$, there exists $\theta'>0$ such that for all $\theta>\theta'$, we have that

$$-\frac{\partial y(q^*,e^*)}{\partial q}-\epsilon<\frac{\partial^2\pi_b(q^*,w)}{\partial q\partial w}<-\frac{\partial y(q^*,e^*)}{\partial q}+\epsilon,$$
$$p\frac{\partial^2 S(q^*,e^*)}{\partial q^2}-\epsilon<\frac{\partial^2\pi_b(q^*,w)}{\partial q^2}<p\frac{\partial^2 S(q^*,e^*)}{\partial q^2}+\epsilon.$$

Also, for any arbitrarily small $\epsilon > 0$, there exists $m' > 1$ such that for all $m > m'$, the same holds. Then, using (44), we can obtain the following upper bound for $dq^*(w)/dw$:

$$\frac{dq^*(w)}{dw} = -\left(\frac{\partial^2 \pi_b(q^*,w)}{\partial q \partial w}\right)\left(\frac{\partial^2 \pi_b(q^*,w)}{\partial q^2}\right)^{-1} < \frac{\frac{\partial y(q^*,e^*)}{\partial q} - \epsilon}{p\frac{\partial^2 S(q^*,e^*)}{\partial q^2} - \epsilon},$$

where $\frac{\partial y(q^*,e^*)}{\partial q} > 0$ and $\frac{\partial^2 S(q^*,e^*)}{\partial q^2} < 0$ by Lemma 2 and Table 2.

Step 4: Decreasing efficiency. From equations (42) and (43), we have

$$\frac{d\Pi^*(w)}{dw} = \frac{d\pi_b^*(w)}{dw} + \frac{d\pi_s^*(w)}{dw} = \lambda\frac{\partial y(q^*,e^*)}{\partial e} + \left(w\frac{\partial y(q^*,e^*)}{\partial q} - c\right)\frac{dq^*(w)}{dw}.$$

Using the results from Step 3 and the second equation in (41) for λ, for any $\epsilon > 0$ if θ or $m > 1$ is large enough, we get the following inequality.

$$\begin{aligned}
\frac{d\Pi^*(w)}{dw} &= \lambda\frac{\partial y(q^*,e^*)}{\partial e} + \left(w\frac{\partial y(q^*,e^*)}{\partial q} - c\right)\frac{dq^*(w)}{dw} \\
&= -\frac{p\frac{\partial S(q^*,e^*)}{\partial e} - w\frac{\partial y(q^*,e^*)}{\partial e}}{w\frac{\partial^2 y(q^*,e^*)}{\partial e^2} - v''(e)} \cdot \frac{\partial y(q^*,e^*)}{\partial e} + \left(w\frac{\partial y(q^*,e^*)}{\partial q} - c\right)\frac{dq^*(w)}{dw} \\
&< -\frac{p\frac{\partial S(q^*,e^*)}{\partial e} - w\frac{\partial y(q^*,e^*)}{\partial e}}{w\frac{\partial^2 y(q^*,e^*)}{\partial e^2} - v''(e^*)} \cdot \frac{\partial y(q^*,e^*)}{\partial e} + \left(w\frac{\partial y(q^*,e^*)}{\partial q} - c\right)\frac{\frac{\partial y(q^*,e^*)}{\partial q} - \epsilon}{p\frac{\partial^2 S(q^*,e^*)}{\partial q^2} - \epsilon},
\end{aligned} \tag{30}$$

where $w\frac{\partial y(q^*,e^*)}{\partial q} - c = (wy(q^*,e^*) - cq)\frac{1}{q} = (\pi_s(q^*,e^*) + v(e^*))\frac{1}{q} > 0$, since $y(q,e)$ is linear in q and the supplier's participation constraint is satisfied.

Note that the first term in (30) tends to zero as $\theta \to \infty$ or as $m \to \infty$, because $\frac{\partial S(q,e)}{\partial e}$, $\frac{\partial y(q,e)}{\partial e}$, and $\frac{\partial^2 y(q,e)}{\partial e^2}$ have finite bounds regardless of e by Assumption 4, and $\lim_{\theta\to\infty} v''(e^*) = \lim_{m\to\infty} v''(e^*) = \infty$ by (22) and (23). Also, we can find a sufficiently small $\epsilon > 0$ such that the second term is negative, because

$$\left(w\frac{\partial y(q^*,e^*)}{\partial q} - c\right)\frac{\frac{\partial y(q^*,e^*)}{\partial q}}{p\frac{\partial^2 S(q^*,e^*)}{\partial q^2}} < 0,$$

where $\frac{\partial y(q^*,e^*)}{\partial q} > 0$ and $\frac{\partial^2 S(q^*,e^*)}{\partial q^2} < 0$ by Lemma 2 and Table 2. Therefore, 1) there exists θ' such that for all $\theta > \theta'$, $\frac{d\Pi^*(w)}{dw} < 0$ for all w, and 2) there exists $m' > 1$ such that for all $m > m'$, $\frac{d\Pi^*(w)}{dw} < 0$ for all w. □

Proof of Proposition 7 Under a unit-penalty with buy-back contract (w,z,b), each firm's expected profit is as follows:

$$\pi_b(q,e,w,z,b) = pS(q,e) - wy(q,e) + z(q - y(q,e)) + b(y(q,e) - S(q,e)),$$
$$\pi_s(q,e,w,z,b) = wy(q,e) - z(q - y(q,e)) - b(y(q,e) - S(q,e)) - c(q,e).$$

For $\chi \in \left[0, \frac{\Pi(q^o,e^o)}{\Pi(q^o,e^o)+v(e^o)}\right]$, the coordinating contract parameters are $w^* = p(1-\chi) + \chi(\mu_y^{e^o}M + c)$, $z^* = \chi((1-\mu_y^{e^o})M - c)$, and $b^* = p(1-\chi)$, where $M = v'(e^o)/(\partial y(q^o,e^o)/\partial e)$.

The proof is organized in two steps. In Step 1, we reformulate problem (3). In Step 2, we show that the contract (w^*,z^*,b^*) satisfies the KKT conditions at (q^o,e^o).

Step 1: Reformulation. In problem (3), we can replace the first constraint with its first-order condition. The supplier's expected profit is $\pi_s(q,e,w^*,z^*,b^*) = (w^* + z^* - b^*)y(q,e) + b^*S(q,e) - z^*q - (cq + v(e))$, and this is strictly concave in e because of the following reason. First, $w^* + z^* - b^* = \chi M = \chi v'(e^o)/(\partial y(q^o,e^o)/\partial e) \geq 0$, because $\chi \geq 0$, $v'(e^o) > 0$ by Assumption 2, and $\partial y(q^o, e^o)/\partial e > 0$ by Lemma 2 and Table 2. Second, $y(q,e)$ and $S(q,e)$ are both strictly concave in e by Lemma 2 and Table 2. Third, $v(e)$ is convex by Assumption 2. Also, by Assumption 2, we focus only on the interior solutions. Therefore, problem (3) can be reformulated as

$$\begin{aligned} \max_{q,e} \ & \pi_b(q, e, w^*, z^*, b^*), \\ s.t. \ & \frac{\partial \pi_s(q, e, w^*, z^*, b^*)}{\partial e} = 0, \\ & \pi_s(q, e, w^*, z^*, b^*) \geq 0. \end{aligned} \tag{31}$$

Step 2: KKT conditions. Assume (q^o, e^o) is a solution to problem (31). Then, there exist $\lambda, \mu \in \mathbb{R}$ such that

$$\begin{aligned} \frac{\partial \pi_b(q^o, e^o, w^*, z^*, b^*)}{\partial q} + \lambda \frac{\partial^2 \pi_s(q^o, e^o, w^*, z^*, b^*)}{\partial e \partial q} + \mu \frac{\partial \pi_s(q^o, e^o, w^*, z^*, b^*)}{\partial q} = 0, \\ \frac{\partial \pi_b(q^o, e^o, w^*, z^*, b^*)}{\partial e} + \lambda \frac{\partial^2 \pi_s(q^o, e^o, w^*, z^*, b^*)}{\partial e^2} + \mu \frac{\partial \pi_s(q^o, e^o, w^*, z^*, b^*)}{\partial e} = 0, \end{aligned}$$

where $\mu \cdot \pi_s(q^o, e^o, w^*, z^*, b^*) = 0$ and $\mu \geq 0$. Also, (q^o, e^o) satisfies the two constraints.

Note that, under the contract (w^*, z^*, b^*), each firm's expected profit is the following:

$$\begin{aligned} \pi_s(q, e, w^*, z^*, b^*) &= \chi M y(q,e) + p(1-\chi) S(q,e) - \chi((1-\mu_y^{e^o})M - c)q - (cq + v(e)), \\ \pi_b(q, e, w^*, z^*, b^*) &= p\chi S(q,e) - \chi M y(q,e) + \chi((1-\mu_y^{e^o})M - c)q, \end{aligned}$$

where $M = v'(e^o) \cdot (\partial y(q^o, e^o)/\partial e)^{-1}$.

From the first KKT condition, we have

$$\begin{aligned} &\left[p\chi \frac{\partial S(q^o, e^o)}{\partial q} - \chi M \frac{\partial y(q^o, e^o)}{\partial q} + \chi\left((1-\mu_y^{e^o})M - c\right) \right] + \lambda \left[\chi M \frac{\partial^2 y(q^o, e^o)}{\partial e \partial q} + p(1-\chi) \frac{\partial^2 S(q^o, e^o)}{\partial e \partial q} \right] \\ &+ \mu \left[\chi M \frac{\partial y(q^o, e^o)}{\partial q} + p(1-\chi) \frac{\partial S(q^o, e^o)}{\partial q} - \chi\left((1-\mu_y^{e^o})M - c\right) - c \right] = 0. \end{aligned}$$

Simple arithmetic calculations reveal that the first and the third square brackets are zero, because $\partial y(q^o, e^o)/\partial q = (1-\mu_y^{e^o})$, and also $p\partial S(q^o, e^o)/\partial q = c$ by the first-order condition of the centralized supply chain. In addition, for the second square bracket, note that $M\partial^2 y(q^o, e^o)/\partial e \partial q = M(\partial y(q^o, e^o)/\partial e) \cdot (1/q^o) = v'(e^o)/q^o$, because $y(q,e) = (1-\mu_y^e)q$. Therefore, we have

$$\lambda \left[\chi v'(e^o) \frac{1}{q^o} + p(1-\chi) \frac{\partial^2 S(q^o, e^o)}{\partial e \partial q} \right] = 0.$$

Let $\lambda = 0$. Then, the first condition is always satisfied. From the second KKT condition, we have

$$\begin{aligned} &\left[p\chi \frac{\partial S(q^o, e^o)}{\partial e} - \chi M \frac{\partial y(q^o, e^o)}{\partial e} \right] + \lambda \left[\chi M \frac{\partial^2 y(q^o, e^o)}{\partial e^2} + p(1-\chi) \frac{\partial^2 S(q^o, e^o)}{\partial e^2} - v''(e^o) \right] \\ &+ \mu \left[\chi M \frac{\partial y(q^o, e^o)}{\partial e} + p(1-\chi) \frac{\partial S(q^o, e^o)}{\partial e} - v'(e^o) \right] = 0. \end{aligned}$$

The second square bracket disappears, because $\lambda = 0$. Simple arithmetic calculations reveal that the first and the third square brackets are zero, because $M \cdot \partial y(q^o, e^o)/\partial e = v'(e^o)$, and also $p\partial S(q^o, e^o)/\partial e = v'(e^o)$ by the first-order condition of the centralized supply chain. Therefore, the second KKT condition holds regardless of μ, and thus $\mu \cdot \pi_s(q^o, e^o, w^*, z^*, b^*) = 0$ is also satisfied. (We can simply set $\mu = 0$ if the second constraint does not bind.) Thus, the second constraint may or may not bind. Also, it is easy to check that the two constraints are satisfied at (q^o, e^o). Therefore, KKT conditions are satisfied at (q^o, e^o).

The buyer's expected profit is $\pi_b(q^*, e^*, w^*, z^*, b^*) = \chi(\Pi(q^o, e^o) + v(e^o))$, and the supplier's expected profit is $\pi_s(q^*, e^*, w^*, z^*, b^*) = (1-\chi)\Pi(q^o, e^o) - \chi v(e^o)$. □

Proof of Proposition 8 The proof is organized as follows. From Step 1 to 4, we ignore the participation constraint in problem (3), but in Step 5, we show that the participation constraint is always satisfied if w is above some threshold.

- Step 1: The supplier's optimal production quantity x^* and effort e^* always satisfy $0 < 1 - q/x^* < a_y(e^*)$ for any q.
- Step 2: The supplier's best response functions $x(q)$ and $e(q)$ are once continuously differentiable with $x'(q) > 0$ and $e'(q) > 0$.
- Step 3: The buyer's optimal order quantity q^* satisfies $q^* \geq D$ regardless of the wholesale price.
- Step 4: There exists $w_1 < p$ such that the optimal order quantity is $q^* = D$ if $w \in [w_1, p]$.
- Step 5: There exists $w_2 < p$ such that if $w \in [w_2, p]$, then the participation constraint is always satisfied.
- Step 6: If $q^* = D$, the efficiency of the supply chain strictly increases in w.

Then, if we set $\underline{w}_d = \max\{w_1, w_2\}$, the statement holds.

Step 1: Feasible region. First, we show that $x^* > q$ by contradiction. If $x^* \leq q$, then $y(q, x^*, e^*) = (1 - \mu_y^{e^*})x^*$. The participation constraint is satisfied if $\pi_s(q, x^*, e^*) = w(1-\mu_y^{e^*})x^* - cx^* - v(e^*) \geq 0$, or $w(1 - \mu_y^{e^*}) - c \geq v(e^*)/x^* > 0$. However, this means that the first-order condition can never be satisfied because $\partial \pi_s(q, x, e)/\partial x = w(1-\mu_y^e) - c > 0$. Hence, $x^* > q$, or $0 < 1 - q/x^*$.

Second, we show that $1 - q/x^* < a_y(e^*)$ by contradiction. When $x > q$, we have $\partial y(q, x, e)/\partial x = (1 - a_y(e)) + \int_{1-q/x}^{a_y(e)} H(\xi \mid e) d\xi - \frac{q}{x} H(1 - q/x \mid e)$. If $1 - q/x^* \geq a_y(e^*)$, then $\partial y(q, x^*, e^*)/\partial x = 0$ because $H(a_y(e^*) \mid e^*) = 1$. Therefore, the first-order condition cannot be satisfied, and thus $0 < 1 - q/x^* < a_y(e^*)$.

Step 2: Best response functions. The supplier's best response functions are obtained by the following first-order conditions:

$$w \frac{\partial y(q, x, e)}{\partial x} - c = 0, \qquad w \frac{\partial y(q, x, e)}{\partial e} - v'(e) = 0. \tag{32}$$

Note that the second equation may not hold for any $e \geq 0$ if q is small, in which case $e(q) = 0$ and

$$x'(q) = -\frac{w \partial^2 y(q, x, e)/\partial x \partial q}{w \partial^2 y(q, x, e)/\partial x^2},$$

by the implicit function theorem. Hence, $x'(q) > 0$ because $\frac{\partial^2 y(q,x,e)}{\partial x \partial q} > 0$ and $\frac{\partial^2 y(q,x,e)}{\partial x^2} < 0$ (which we show later in this step). We assume that the second equation in (32) holds in equilibrium when $q \geq D$, which is our Assumption 2 of focusing on interior solutions.

Now, assume both first-order conditions hold. The determinant of the Jacobian of the two first-order conditions is strictly positive, that is, $w^2 \left(\frac{\partial^2 y}{\partial x^2} \frac{\partial^2 y}{\partial e^2} - (\frac{\partial^2 y}{\partial x \partial e})^2 \right) - w \frac{\partial^2 y}{\partial x^2} v''(e) > 0$, with a slight abuse of notation using y for $y(q, x, e)$. This is because $y(q, x, e)$ is jointly concave in x and e by Assumption 5, and thus

$\frac{\partial^2 y}{\partial x^2}\frac{\partial^2 y}{\partial e^2}-(\frac{\partial^2 y}{\partial x\partial e})^2>0$, and also $\partial^2 y/\partial x^2\le 0$ by Lemma 3 and $v''(e)\ge 0$ by Assumption 2. Therefore, by the implicit function theorem, $x(q)$ and $e(q)$ are once continuously differentiable. (Also note that $y(q,x,e)$ is thrice continuously differentiable in x and e by Lemma 3, and $v(e)$ is thrice continuously differentiable by Assumption 2.)

Hence,

$$\begin{bmatrix} x'(q)\\ e'(q)\end{bmatrix} = -\begin{bmatrix} w\frac{\partial^2 y}{\partial x^2}, & w\frac{\partial^2 y}{\partial x\partial e}\\ w\frac{\partial^2 y}{\partial e\partial x}, & w\frac{\partial^2 y}{\partial e^2}-v''(e)\end{bmatrix}^{-1}\cdot\begin{bmatrix} w\frac{\partial^2 y}{\partial x\partial q}\\ w\frac{\partial^2 y}{\partial e\partial q}\end{bmatrix}$$
$$= -\frac{1}{w^2\left(\frac{\partial^2 y}{\partial x^2}\frac{\partial^2 y}{\partial e^2}-(\frac{\partial^2 y}{\partial x\partial e})^2\right)-w\frac{\partial^2 y}{\partial x^2}v''(e)}\cdot\begin{bmatrix} w\frac{\partial^2 y}{\partial x\partial q}\left(w\frac{\partial^2 y}{\partial e^2}-v''(e)\right)-w^2\frac{\partial^2 y}{\partial x\partial e}\frac{\partial^2 y}{\partial e\partial q}\\ w^2\left(\frac{\partial^2 y}{\partial x^2}\frac{\partial^2 y}{\partial e\partial q}-\frac{\partial^2 y}{\partial x\partial e}\frac{\partial^2 y}{\partial x\partial q}\right)\end{bmatrix}. \quad (33)$$

Because we already know the sign of the determinant of the Jacobian, we only need to check the signs of the two components in the matrix in (33). But before we proceed, we need to obtain the derivatives of $y(q,x,e)$ and find some important relationships that we use in checking the signs of $x'(q)$ and $e'(q)$. We have $y(q,x,e)=[(1-a_y(e))+\int_{1-q/x}^{a_y(e)}H(\xi\mid e)d\xi]x$. We obtain the following derivatives:

$$\frac{\partial y(q,x,e)}{\partial e}=x\int_{1-q/x}^{a_y(e)}\frac{\partial H(\xi\mid e)}{\partial e}d\xi>0,\quad \frac{\partial^2 y(q,x,e)}{\partial e\partial q}=\frac{\partial H(1-\frac{q}{x}\mid e)}{\partial e}>0,\quad \frac{\partial^2 y(q,x,e)}{\partial x\partial q}=\frac{q}{x^2}h\left(1-\frac{q}{x}\mid e\right)>0,$$
$$\frac{\partial^2 y(q,x,e)}{\partial q^2}=-\frac{1}{x}h\left(1-\frac{q}{x}\mid e\right)<0,\quad \frac{\partial^2 y(q,x,e)}{\partial x\partial e}=\int_{1-q/x}^{a_y(e)}\frac{\partial H(\xi\mid e)}{\partial e}d\xi-\frac{q}{x}\frac{\partial H(1-\frac{q}{x}\mid e)}{\partial e},$$
$$\frac{\partial^2 y(q,x,e)}{\partial x^2}=-\frac{q^2}{x^3}h\left(1-\frac{q}{x}\mid e\right)<0,$$

where we can obtain the signs because $h(\xi\mid e)>0$ and $\partial H(\xi\mid e)/\partial e>0$ when $\xi\in(0,a_y(e))$ by Assumption 4. Therefore, we can identify the following two relationships between the derivatives:

$$\frac{\partial^2 y(q,x,e)}{\partial x\partial e}=\frac{1}{x}\frac{\partial y(q,x,e)}{\partial e}-\frac{q}{x}\frac{\partial^2 y(q,x,e)}{\partial e\partial q},\quad\text{and}\quad \frac{\partial^2 y(q,x,e)}{\partial x\partial q}=-\frac{q}{x}\frac{\partial^2 y(q,x,e)}{\partial q^2}. \quad (34)$$

Now, using the equations (34), we first find that $x'(q)>0$, because

$$w\frac{\partial^2 y}{\partial x\partial q}\left(w\frac{\partial^2 y}{\partial e^2}-v''(e)\right)-w^2\frac{\partial^2 y}{\partial x\partial e}\frac{\partial^2 y}{\partial e\partial q}\le w^2\left[\frac{\partial^2 y}{\partial x\partial q}\frac{\partial^2 y}{\partial e^2}-\frac{\partial^2 y}{\partial x\partial e}\frac{\partial^2 y}{\partial e\partial q}\right]$$
$$=w^2\left[-\frac{q}{x}\left(\frac{\partial^2 y}{\partial q^2}\frac{\partial^2 y}{\partial e^2}-\left(\frac{\partial^2 y}{\partial e\partial q}\right)^2\right)-\frac{1}{x}\frac{\partial y}{\partial e}\frac{\partial^2 y}{\partial e\partial q}\right]<0,$$

where the first step holds since $\partial^2 y(q,x,e)/\partial x\partial q>0$ and $v''(e)\ge 0$ by Assumption 2, and the second step holds by equations (34). The last step holds because $y(q,x,e)$ is jointly concave in q and e in the feasible region by Assumption 5, and thus $\frac{\partial^2 y}{\partial q^2}\frac{\partial^2 y}{\partial e^2}-\left(\frac{\partial^2 y}{\partial e\partial q}\right)^2>0$, and also $\partial y/\partial e>0$ and $\partial^2 y/\partial e\partial q>0$.

Second, we find that $e'(q)>0$, because

$$\frac{\partial^2 y}{\partial x^2}\frac{\partial^2 y}{\partial e\partial q}-\frac{\partial^2 y}{\partial x\partial e}\frac{\partial^2 y}{\partial x\partial q}=-\frac{q}{x^2}h\left(1-\frac{q}{x}\mid e\right)\int_{1-q/x}^{a_y(e)}\frac{\partial H(\xi\mid e)}{\partial e}d\xi<0,$$

since $h(1-q/x\mid e)>0$ and $\partial H(\xi\mid e)/\partial e>0$ by Assumption 4.

Step 3: Lower bound for optimal order quantity. Let $\pi_b(q)$ be the buyer's expected profit given the best response functions $x(q)$ and $e(q)$ of the supplier. When $q\le D$, the buyer's expected profit is $\pi_b(q)=(p-w)y(q,x(q),e(q))$, because $S(q,x,e)=y(q,x,e)$ by Lemma 3. Hence,

$$\frac{d\pi_b(q)}{dq}=(p-w)\left[\frac{\partial y(q,x,e)}{\partial q}+\frac{\partial y(q,x,e)}{\partial x}\frac{dx(q)}{dq}+\frac{\partial y(q,x,e)}{\partial e}\frac{de(q)}{dq}\right]>0,$$

because $\partial y(q,x,e)/\partial x, \partial y(q,x,e)/\partial e > 0$ by Lemma 3, and $x'(q) > 0, e'(q) \geq 0$ by Step 2, and $\partial y(q,x,e)/\partial q = H(1-q/x \mid e) > 0$. Therefore, it is optimal to order at least D units.

Step 4: Optimal order quantity. We show that there exists $w_1 < p$ such that, if $w \in [w_1, p]$, then $\pi_b(q)$ is strictly decreasing in q when $q > D$. This means that the optimal order quantity is $q^* = D$ when $w \in [w_1, p]$.

The buyer's expected profit when $q > D$ is $\pi_b(q) = py(D, x(q), e(q)) - wy(q, x(q), e(q))$, because $S(q,x,e) = y(D,x,e)$ by Lemma 3. Hence,

$$\frac{d\pi_b(q)}{dq} = -w\frac{\partial y(q,x,e)}{\partial q} + \left(p\frac{\partial y(D,x,e)}{\partial x} - w\frac{\partial y(q,x,e)}{\partial x}\right)\frac{dx(q)}{dq} + \left(p\frac{\partial y(D,x,e)}{\partial e} - w\frac{\partial y(q,x,e)}{\partial e}\right)\frac{de(q)}{dq}.$$

When $w = p$, $d\pi_b(q)/dq < -w\frac{\partial y(q,x,e)}{\partial q} < 0$ for any $q > D$, because $\partial^2 y(q,x,e)/\partial x \partial q = (q/x^2)h(1-q/x \mid e) > 0$, $\partial^2 y(q,x,e)/\partial e \partial q = \frac{\partial H(1-q/x|e)}{\partial e} > 0$ by Assumption 4, and $x'(q), e'(q) > 0$ by Step 2. Since $d\pi_b(q)/dq$ is continuous in w, and $x'(q)$ and $e'(q)$ are finite by Step 2, there exists $w_1 < p$ such that, when $w \in [w_1, p]$, $d\pi_b(q)/dq < 0$ for all $q > D$.

Step 5: Participation constraint. We know that there exists $w_1 < p$ such that the optimal order quantity is $q^* = D$ when $w \in [w_1, p]$. When $q^* = D$, the supplier's expected profit is $\pi_s(D,x,e) = wy(D,x,e) - (cx + v(e))$. Let $\pi_s^*(w) = \max_{x,e \geq 0} \pi_s(D,x,e)$ be the supplier's expected profit at the equilibrium given a wholesale price $w \in [w_1, p]$. If $w = p$, then $\pi_s(D,x,e) = \Pi(x,e)$, because $y(D,x,e) = E_\xi[D, (1-\xi)x]$ in the delegation scenario is equivalent to $S(x,e) = [(1-\xi)x, D]$ in the control scenario with demand D. Therefore, the supply chain is coordinated with $\pi_s^*(p) = \Pi(x^o, e^o)$. If $w < p$, then by the envelope theorem, $d\pi_s^*(w)/dw = y(D,x,e)$, which is finite. Therefore, there exists $w_2 < p$ such that if $w \in [w_2, p]$ then $\pi_s^*(w) \geq 0$ by continuity, and thus the participation constraint is satisfied.

Step 6: Increasing efficiency. We show that, when $q = D$, the efficiency is monotonically increasing in w. The supplier's expected profit is $\pi_s(D,x,e) = wy(D,x,e) - (cx + v(e))$. Let $x(w)$ and $e(w)$ be the supplier's optimal production quantity and effort as functions of w when $q = D$. Then, the supplier's optimal expected profit is $\pi_s^*(w) = wy(D, x(w), e(w)) - (cx(w) + v(e(w)))$. By the envelope theorem, $d\pi_s^*(w)/dw = \partial \pi_s(D,x,e)/\partial w = y(D,x,e)$. Also, the buyer's expected profit at the equilibrium is $\pi_b^*(w) = (p-w)y(D, x(w), e(w))$, since $S(D,x,e) = y(D,x,e)$ by Lemma 3. Hence,

$$\frac{d\pi_b^*(w)}{dw} = -y(D,x,e) + (p-w)\left[\frac{\partial y(D,x,e)}{\partial x}\frac{dx(w)}{dw} + \frac{\partial y(D,x,e)}{\partial e}\frac{de(w)}{dw}\right].$$

Let $\Pi^*(w) = \pi_b^*(w) + \pi_s^*(w)$. Then,

$$\frac{d\Pi^*(w)}{dw} = \frac{d\pi_b^*(w)}{dw} + \frac{d\pi_s^*(w)}{dw} = (p-w)\left[\frac{\partial y(D,x,e)}{\partial x}\frac{dx(w)}{dw} + \frac{\partial y(D,x,e)}{\partial e}\frac{de(w)}{dw}\right]. \tag{35}$$

Both functions $x(w)$ and $e(w)$ can be jointly obtained by the following two first-order conditions:

$$w\frac{\partial y(D,x,e)}{\partial x} - c = 0, \qquad w\frac{\partial y(D,x,e)}{\partial e} - v'(e) = 0.$$

With a slight abuse of notation using $y = y(D,x,e)$, we can apply the implicit function theorem as follows.

$$\begin{aligned}\begin{bmatrix} x'(w) \\ e'(w) \end{bmatrix} &= -\begin{bmatrix} w\frac{\partial^2 y}{\partial x^2}, & w\frac{\partial^2 y}{\partial x \partial e} \\ w\frac{\partial^2 y}{\partial e \partial x}, & w\frac{\partial^2 y}{\partial e^2} - v''(e) \end{bmatrix}^{-1} \cdot \begin{bmatrix} \frac{\partial y}{\partial x} \\ \frac{\partial y}{\partial e} \end{bmatrix} \\ &= -\frac{1}{w^2\left(\frac{\partial^2 y}{\partial x^2}\frac{\partial^2 y}{\partial e^2} - (\frac{\partial^2 y}{\partial x \partial e})^2\right) - w\frac{\partial^2 y}{\partial x^2}v''(e)} \cdot \begin{bmatrix} \frac{\partial y}{\partial x}\left(w\frac{\partial^2 y}{\partial e^2} - v''(e)\right) - w\frac{\partial^2 y}{\partial x \partial e}\frac{\partial y}{\partial e} \\ w\left(\frac{\partial^2 y}{\partial x^2}\frac{\partial y}{\partial e} - \frac{\partial^2 y}{\partial x \partial e}\frac{\partial y}{\partial x}\right) \end{bmatrix}.\end{aligned} \tag{36}$$

Let $m(x,e) = w^2\left(\frac{\partial^2 y}{\partial x^2}\frac{\partial^2 y}{\partial e^2} - (\frac{\partial^2 y}{\partial x \partial e})^2\right) - w\frac{\partial^2 y}{\partial x^2}v''(e)$. Since $y(q,x,e)$ is jointly concave in x and e by Assumption 5, we know that $m(x,e) > -w\frac{\partial^2 y}{\partial x^2}v''(e) \geq 0$, because $\partial^2 y/\partial x^2 \leq 0$ by Lemma 3 and $v''(e) \geq 0$ by Assumption 2.

Now, we can rewrite (35) using (36) as follows.

$$\begin{aligned}\frac{d\Pi^*(w)}{dw} &= -\frac{p-w}{m(x,e)}\left[w\frac{\partial^2 y}{\partial e^2}\left(\frac{\partial y}{\partial x}\right)^2 + w\frac{\partial^2 y}{\partial x^2}\left(\frac{\partial y}{\partial e}\right)^2 - 2w\frac{\partial^2 y}{\partial x \partial e}\frac{\partial y}{\partial e}\frac{\partial y}{\partial x} - v''(e)\left(\frac{\partial y}{\partial x}\right)^2\right] \\ &= -\frac{p-w}{m(x,e)}\left[w\frac{\partial^2 y}{\partial e^2}\left(\frac{\partial y}{\partial x} - \frac{\frac{\partial^2 y}{\partial x \partial e}\frac{\partial y}{\partial e}}{\frac{\partial^2 y}{\partial e^2}}\right)^2 + w\frac{\left(\frac{\partial y}{\partial e}\right)^2}{\frac{\partial^2 y}{\partial e^2}}\left(\frac{\partial^2 y}{\partial x^2}\frac{\partial^2 y}{\partial e^2} - \left(\frac{\partial^2 y}{\partial x \partial e}\right)^2\right) - v''(e)\left(\frac{\partial y}{\partial x}\right)^2\right].\end{aligned}$$

Note that $\frac{\partial^2 y}{\partial x^2}\frac{\partial^2 y}{\partial e^2} - \left(\frac{\partial^2 y}{\partial x \partial e}\right)^2 > 0$ due to joint concavity of $y(q,x,e)$. In addition, $\partial^2 y/\partial e^2 < 0$ by Lemma 3 and Table 2. Also, $v''(e) \geq 0$ by Assumption 2. Therefore, a simple sign check reveals that $d\Pi^*(w)/dw > 0$, and thus the efficiency is monotonically increasing in w. □

Proof of Proposition 9 We relax problem (3) by ignoring the participation constraint, solve the problem, and show that there exists $\bar{\chi} > 0$ such that if $0 \leq \chi \leq \bar{\chi}$, then the given penalty contract coordinates the supply chain, and also satisfies the participation constraint.

The proof is organized in three steps. In Step 1, we show that the supplier's optimal production quantity x^* and effort e^* satisfy $0 < 1 - q/x^* < a_y(e^*)$ for any $q \geq 0$. In Step 2, we show that the supplier's best response functions, $x(q)$ and $e(q)$, are once continuously differentiable and satisfy $x'(q) > 0, e'(q) \geq 0$ for all $q \geq 0$. In Step 3, we show that there exists $\bar{\chi} > 0$ such that if $0 \leq \chi \leq \bar{\chi}$, then the given penalty contract coordinates the supply chain and satisfy the supplier's participation constraint.

Step 1: Feasible region. With the given contract, $w^* = p - \chi$ and $z^* = \chi$, the supplier's expected profit is $\pi_s(q,x,e) = py(q,x,e) - (cx + v(e)) - \chi q$. The first-order condition for x^* is: $\partial\pi_s(q,x^*,e^*)/\partial x = p\partial y(q,x^*,e^*)/\partial x - c = 0$. First, we show that $x^* > q$ by contradiction. If $x^* \leq q$, then $y(q,x^*,e^*) = (1-\mu_y^{e^*})x^*$, and thus $\partial y(q,x^*,e^*)/\partial x = (1-\mu_y^{e^*})$ by the proof of Lemma 3. But, we know that $p(1-\mu_y^{e^*}) - c > p(1-\mu_y^0) - c > 0$ by Lemma 4. Therefore, the first-order condition cannot be satisfied, and hence $x^* > q$, which is equivalent to $0 < 1 - q/x^*$.

Second, we show that $1 - q/x^* < a_y(e^*)$ by contradiction as well. When $x > q$, we have $\partial y(q,x,e)/\partial x = (1 - a_y(e)) + \int_{1-q/x}^{a_y(e)} H(\xi \mid e)d\xi - \frac{q}{x}H(1-q/x \mid e)$ by the proof of Lemma 3. If $1 - q/x^* \geq a_y(e^*)$, then $\partial y(q,x^*,e^*)/\partial x = 0$ because $H(a_y(e^*) \mid e^*) = 1$. Therefore, the first-order condition cannot be satisfied, and thus $0 < 1 - q/x^* < a_y(e^*)$.

Step 2: Best response functions. The supplier's best response functions are obtained by the following first-order conditions:

$$p\frac{\partial y(q,x,e)}{\partial x} - c = 0, \qquad p\frac{\partial y(q,x,e)}{\partial e} - v'(e) = 0. \tag{37}$$

Note that these first-order conditions are the same as (32) under a wholesale-price contract if we set $w = p$. In Step 2 in the proof of Proposition 8, we have shown that $x(q)$ and $e(q)$ obtained from these two first-order conditions are once continuously differentiable and $x'(q) > 0, e'(q) \geq 0$ regardless of w.

Step 3: Coordination of penalty contracts. First, we show that the buyer's expected profit, given the supplier's best response functions, is strictly increasing in q when $q < D$. Second, we show that there exists

$\bar{\chi}' > 0$ such that if $0 \le \chi \le \bar{\chi}'$, then the buyer's expected profit is strictly decreasing in q when $q > D$. Then, we can conclude that the buyer's optimal order quantity is $q = D$ if $0 \le \chi \le \bar{\chi}'$. Last, we show that the supplier chooses the optimal production quantity x^o and effort e^o when $q = D$. In addition, if $0 \le \chi \le \bar{\chi} = \min\{\bar{\chi}', \Pi(x^o, e^o)/D\}$, then the participation constraint also holds. Therefore, if $0 \le \chi \le \bar{\chi}$, the supply chain is coordinated.

First, with the given contract, $w^* = p - \chi$ and $z^* = \chi$, and the supplier's best response functions, $x(q)$ and $e(q)$, the buyer's expected profit is $\pi_b(q) = p(S(q, x(q), e(q)) - y(q, x(q), e(q))) + \chi q$. If $q \le D$, then $\pi_b(q) = \chi q$, because $S(q, x, e) = y(q, x, e)$ by Lemma 3. Therefore, $\pi_b(q)$ increases in q, and thus the buyer orders at least D units (even when $\chi = 0$ by Assumption 1).

Second, when $q \ge D$, $\pi_b(q) = p(y(D, x(q), e(q)) - y(q, x(q), e(q))) + \chi q$, because $S(q, x, e) = y(D, x, e)$ by Lemma 3. Hence,

$$\begin{aligned} \frac{d\pi_b(q)}{dq} &= p\left[-\frac{\partial y(q,x,e)}{\partial q} + \left(\frac{\partial y(D,x,e)}{\partial x} - \frac{\partial y(q,x,e)}{\partial x}\right)\frac{dx(q)}{dq} + \left(\frac{\partial y(D,x,e)}{\partial e} - \frac{\partial y(q,x,e)}{\partial e}\right)\frac{de(q)}{dq}\right] + \chi \\ &< -p\frac{\partial y(q,x,e)}{\partial q} + \chi = -pH\left(1 - \frac{q}{x} \mid e\right) + \chi, \end{aligned}$$

because $x'(q) > 0, e'(q) \ge 0$ by Step 2, and $\partial^2 y(q, x, e)/\partial x \partial q > 0$, $\partial^2 y(q, x, e)/\partial e \partial q > 0$ by Step 2 in the proof of Proposition 8.

We can see that if there exists $\epsilon > 0$ such that $H(1 - q/x \mid e) > \epsilon$ for any q and we let $\bar{\chi}' = p\epsilon$, then $\pi_b(q)$ is strictly decreasing in $q > D$ when $\chi \le \bar{\chi}'$, and thus $q = D$ is optimal. We can check the existence of such $\epsilon > 0$ by contradiction using the supplier's first-order condition: $p\partial y(q, x, e)/\partial x - c = 0$. We know that $\partial y(q, x, e)/\partial x = (1 - a_y(e)) + \int_{1-q/x}^{a_y(e)} H(\xi \mid e) d\xi - \frac{q}{x} H\left(1 - \frac{q}{x} \mid e\right)$. Let $k = 1 - q/x(q)$. If $k = 1 - q/x(q) = 0$, then $\partial y(q, x, e)/\partial x = (1 - a_y(e)) + \int_0^{a_y(e)} H(\xi \mid e) dx = (1 - \mu_y^e) > c/p$ by Lemma 4. Therefore, the first-order condition cannot be satisfied, and thus there exists $\delta > 0$ such that $k > \delta$, and this condition does not depend on q. Therefore, we can find $\epsilon > 0$ such that $H(1 - q/x \mid e) > \epsilon$ for all q, because $H(\xi \mid e) > 0$ for $\xi \in (0, a_y(e)]$.

Last, when $q = D$, the supplier's expected profit is $\pi_s(D, x, e) = py(D, x, e) - (cx + v(e)) - \chi D = pS(x, e) - (cx + v(e)) - \chi D = \Pi(x, e) - \chi D$, because $y(D, x, e) = S(x, e)$ by the proof of Lemma 3. Therefore, the supplier chooses the optimal production quantity x^o and effort e^o, because χD is a constant. In addition, if $\chi \le \Pi(x^o, e^o)/D$, then $\pi_s(D, x^o, e^o) \ge 0$, and thus the supplier's participation constraint is satisfied. Therefore, if $0 \le \chi \le \bar{\chi} = \min\{\bar{\chi}', \Pi(x^o, e^o)/D\}$, then the supply chain is coordinated. □

Proof of Proposition 10(i) This proof is organized in three steps. Let $e(q)$ be the supplier's best response function. In Step 1, we show that, if $w > c$, then $e(q)$ is once continuously differentiable, has a finite derivative, and $de(q)/dq > 0$. In Step 2, we show that the buyer always orders at least D units, and if $de(q)/dq$ is finite, then there exists $\underline{w} < p$ such that for all $w \in [\underline{w}, p]$, the optimal order quantity is $q^* = D$. Finally, in Step 3, we show that, if $q = D$ is fixed, then the efficiency is strictly increasing in w.

Step 1: The supplier's best response function e(q). The supplier's expected profit is $\pi_s(q, e) = (w - c)y(q, e) - v(e)$, and this is once continuously differentiable in e by Lemma 1 and Assumption 2. The optimal effort is uniquely obtained by the first-order condition, $\partial \pi_s(q, e)/\partial e = 0$, because $\pi_s(q, e)$ is strictly concave in e, since $y(q,e)$ is strictly concave in e by Lemma 1 and Table 1 and $v(e)$ is convex by Assumption 2. (Note that we focus only on interior solutions by Assumption 2.)

Note that $\pi_s(q,e)$ is twice continuously differentiable in the neighborhood of the optimal solution. It is because $\pi_s(q,e)$ is twice continuously differentiable in both q and e if $q \neq K - a_c(e)$ by Lemma 1 and Assumption 2, and the optimal effort indeed satisfies $q \neq K - a_c(e)$. If $q = K - a_c(e)$, then $y(q,e) = q$, $\partial y(q,e)/\partial e = 0$, and thus the first-order condition cannot be satisfied. Therefore, by the implicit function theorem, $e(q)$ is once continuously differentiable (Luenberger and Ye 2008), and we have

$$\frac{de(q)}{dq} = -\frac{\partial^2 \pi_s(q,e)}{\partial e \partial q}\left(\frac{\partial^2 \pi_s(q,e)}{\partial e^2}\right)^{-1} = \frac{(w-c)\frac{\partial^2 y(q,e)}{\partial e \partial q}}{v''(e) - (w-c)\frac{\partial^2 y(q,e)}{\partial e^2}}. \tag{38}$$

First, $\partial y(q,e)/\partial q = G(K-q \mid e)$ from the proof of Lemma 1, and thus $\partial^2 y(q,e)/\partial e \partial q = \partial G(K-q \mid e)/\partial e$ is strictly positive and finite by Assumption 3. Second, if $w > c$, then $v''(e) - (w-c)\frac{\partial^2 y(q,e)}{\partial e^2} > 0$, because $v''(e) \geq 0$ by Assumption 2 and $\partial^2 y(q,e)/\partial e^2 < 0$ by Lemma 1 and Table 1. Hence if $w > c$, then $de(q)/dq$ is strictly positive and finite.

Step 2: Optimal order quantity. First, we show that we can ignore the supplier's participation constraint in problem (3). Second, we show that the buyer's expected profit $\pi_b(q)$ is always increasing in q when $q \leq D$, and there exists $\underline{w} < p$ such that if $w \geq \underline{w}$, $\pi_b(q)$ is decreasing in q when $q \geq D$. Then, we can conclude that, if $w \geq \underline{w}$, the optimal order quantity is D.

First, the supplier's participation constraint is always satisfied because of the following reason. The supplier's expected profit $\pi_s(q,e) = (w-c)y(q,e) - v(e)$ is strictly concave in e since $y(q,e)$ is strictly concave in e by Lemma 1 and Table 1, and $v(e)$ is convex in e by Assumption 2. In addition, if $w \in [c,p]$, then $\pi_s(q,0) = (w-c)y(q,0) - v(0) = (w-c)y(q,0) \geq 0$, because $v(0) = 0$ by Assumption 2. Thus, the supplier's first-order condition that generates a strictly positive effort (by Assumption 2) always produces a set of feasible solutions such that $\pi_s(q,e) \geq 0$ (due to strict concavity).

Second, we show that $\pi_b(q)$ is increasing in q when $q \leq D$, and there exists $\underline{w} < p$ such that, if $w \geq \underline{w}$, then $\pi_b(q)$ is decreasing in q when $q \geq D$. We ignore the participation constraint and write the buyer's expected profit as $\pi_b(q) = pS(q,e(q)) - wy(q,e(q))$. If $q \leq D$ then $\pi_b(q) = (p-w)y(q,e(q))$ because $S(q,e) = y(q,e)$ by Lemma 1. Thus, $\pi_b(q)$ is always increasing in q, since $e(q)$ is increasing in q by Step 1, and $y(q,e)$ is increasing in q and e by Lemma 1 and Table 1. If $q \geq D$ then $\pi_b(q) = py(D,e(q)) - wy(q,e(q))$ since $S(q,e) = y(D,e)$ by Lemma 1. Then,

$$\frac{d\pi_b(q)}{dq} = \left[p\frac{\partial y(D,e)}{\partial e} - w\frac{\partial y(q,e)}{\partial e}\right]\frac{de(q)}{dq} - w\frac{\partial y(q,e)}{\partial q} \leq (p-w)\frac{y(q,e)}{\partial e}\cdot\frac{de(q)}{dq} - w\frac{\partial y(q,e)}{\partial q}, \tag{39}$$

because $\partial y(q,e)/\partial e$ is increasing in q, since $\partial^2 y(q,e)/\partial e \partial q = \partial G(K-q \mid e)/\partial e > 0$ by Assumption 3. If $w = p$, then $d\pi_b(q)/dq < 0$ since $\partial y(q,e)/\partial q > 0$ by Lemma 1 and Table 1. Note that $\pi_b(q)$ is once continuously differentiable for all q, because $y(q,e)$ and $e(q)$ are once continuously differentiable by Lemma 1 and Step 1. Therefore, by continuity, there exists $\underline{w} < p$ such that for all $w \in [\underline{w}, p]$, $d\pi_b(q)/dq < 0$, because $de(q)/dq$ and $\partial y(q,e)/\partial e$ are finite. Therefore, for all such $w \in [\underline{w}, p]$, the optimal order quantity is $q^* = D$.

Step 3: Increasing efficiency. We show that, if we fix $q^* = q^o = D$, then the total expected profit of the supply chain $\Pi(q^*, e^*)$ is strictly increasing in $w \in [c,p]$. The proof is structured as follows. First, we show that, if we fix $q^* = D$, then $\Pi(q^*, e)$ is strictly increasing in $e \in [0, e^o]$. Second, we show that the optimal effort e^* is strictly increasing in w. Lastly, we show that $e^* = e^o$ when $w = p$.

First, we show that $\Pi(q^*, e)$ strictly increases in $e \in [0, e^o]$. The total supply chain profit $\Pi(D, e) = (p - c)y(D, e) - v(e)$ is strictly concave in e with the first-order condition satisfying at $e = e^o$, because that is when the centralized supply chain achieves the maximum profit. Hence, $\Pi(q^*, e)$ is strictly increasing in $e \in [0, e^o]$.

Second, we show that the optimal effort e^* strictly increases with w. The supplier's expected profit is $\pi_s(D, e) = (w - c)y(D, e) - v(e)$. The optimal effort e^* is obtained by the first-order condition, since $y(q, e)$ is strictly concave in e by Lemma 1 and Table 1 and $v(e)$ is convex by Assumption 2. (Note that we focus only on the interior solutions by Assumption 2.) Hence, $(w - c)\frac{\partial y(D, e^*)}{\partial e} - v'(e^*) = 0$. Note that $\partial y(D, e)/\partial e$ is strictly decreasing in e because $\partial^2 y(q, e)/\partial e^2 < 0$ by Lemma 1 and Table 1, and $v'(e)$ is increasing in e since $v''(e) \geq 0$ by Assumption 2. Therefore, it is easy to see that the equilibrium effort e^* strictly increases with w.

Lastly, we show that $e^* = e^o$ when $w = p$. When $w = p$ the supplier's expected profit is $\pi_s(D, e) = (p - c)y(D, e) - v(e)$, which is equivalent to the expected profit of the centralized supply chain with $q^o = D$. Therefore, the optimal effort is $e^* = e^o$. □

Proof of Proposition 10(ii) We have shown this in Step 3 in the proof of Proposition 10(i). □

Proof of Proposition 11 The proof is organized in five steps. The following is the overview of the proof.

- Step 1: We reformulate problem (3) under a wholesale-price contract.
- Step 2: We show the existence of a solution to problem (3).
- Step 3: We show that there exists $w_1 < p$ such that, if $w > w_1$, then the participation constraint in problem (3) does not bind, and the supplier's best response function $e^*(q, w)$ satisfies $\partial e^*(q, w)/\partial q > 0$. In addition, we show that the optimal order quantity $q^*(w)$ and the optimal effort level $e^*(q^*(w), w)$ are once continuously differentiable in w.
- Step 4: We show that, if $w > w_1$, the expected profit of the supply chain at a solution, $\Pi^*(w) = \Pi(q^*(w), e^*(q^*(w), w))$, is once continuously differentiable in w. Then, we show that $d\Pi^*(w)/dw$ is strictly negative at $w = p$ if $dq^*(w)/dw|_{w=p} < 0$.
- Step 5: We show that $dq^*(w)/dw|_{w=p} < 0$.

Then, we can conclude that $d\Pi^*(w)/dw|_{w=p} < 0$, and, since $d\Pi^*(w)/dw$ is continuous by Step 4, there exists $\underline{w} < p$ such that $\Pi^*(w)$ is decreasing in $w \in [\underline{w}, p]$.

Step 1: Reformulation. In problem (3), we can replace the first constraint with its first-order condition. The supplier's expected profit is $\pi_s(q, e) = wy(q, e) - cq - v(e)$, and this is strictly concave in e, because $y(q, e)$ is strictly concave in e by Lemma 2 and Table 2 and $v(e)$ is convex by Assumption 2. In addition, by Assumption 2, we focus only on interior solutions. Note that the buyer's expected profit is $\pi_b(q, e) = pS(q, e) - wy(q, e)$. Therefore, problem (3) can be reformulated as

$$\begin{aligned} \max_{q,e} \quad & pS(q, e) - wy(q, e), \\ s.t. \quad & w\frac{\partial y(q, e)}{\partial e} - v'(e) = 0, \\ & wy(q, e) - cq - v(e) \geq 0. \end{aligned} \tag{40}$$

Step 2: Existence of a solution. We show the existence of a solution by showing that the objective function is continuous and the feasible set is compact (i.e. closed and bounded). First, the objective function in problem (40) is continuous in q and e by Lemma 2.

Second, to show that the feasible set is compact, we temporarily add another constraint that does not affect any solution, if a solution exists, but reduces the set of feasible solutions. We know $q=0$ and $e=0$ are feasible and make the objective function zero. Hence, any set of q that makes the objective function non-negative can be added as a constraint without affecting any solution. We choose $q \le pD/((1-\mu_y^0)w)$, because, if this is violated, then the objective function satisfies $pS(q,e) - wy(q,e) \le pD - wy(q,0) = pD - w(1-\mu_y^0)q < 0$, because $S(q,e) \le D$ and $y(q,e)$ is increasing in e by Lemma 2.

Now, we check that the new set of feasible solutions is compact. First, the first constraint in problem (40) produces a closed set of feasible solutions. Second, the new temporary constraint produces a closed and bounded set of feasible solutions for q. Finally, the second constraint in problem (40) produces a closed and bounded set of feasible solutions for q and e, because, for any $q \ge 0$, the feasible set for e has an upper bound since $\lim_{e\to\infty} \pi_s(q,e) = -\infty$. Therefore, the feasible set is compact, and thus a solution exists to problem (40).

Step 3: Continuity of a solution. First, we show that there exists $w_1 < p$ such that if $w > w_1$, then we can ignore the participation constraint. Second, we show that the supplier's best response function $e^*(q,w)$ is twice continuously differentiable in q and w, and $\partial e^*(q,w)/\partial q > 0$. Last, we show that, if $w > w_1$, then the buyer's optimal order quantity $q^*(w)$ is once continuously differentiable in w. (Then, it naturally follows that $e^*(q^*(w),w)$ is once continuously differentiable in w.)

First, we show that if w is above some threshold, the participation constraint does not bind. There exists $w_1 < p$ such that if $w > w_1$, then the supplier's expected profit at $e=0$ satisfies $\pi_s(q,0) = wy(q,0) - cq = [w(1-\mu_y^0) - c]q > 0$ by Lemma 4. In addition, $\pi_s(q,e) = wy(q,e) - cq - v(e)$ is strictly concave in e, because $y(q,e)$ is strictly concave in e by Lemma 2 and Table 2 and $v(e)$ is convex by Assumption 2. Therefore, if $w > w_1$, then the supplier's first-order condition that generates an interior solution necessarily implies that $\pi_s(q,e) > 0$ (due to strict concavity). Hence, we can ignore the participation constraint.

Second, we show that the supplier's best response function $e^*(q,w)$ is twice continuously differentiable in q and w, and $\partial e^*(q,w)/\partial q > 0$. The best response function $e^*(q,w)$ is obtained by the first constraint in problem (40). Note that both $y(q,e)$ and $v(e)$ are thrice continuously differentiable in q and e by Lemma 2 and Assumption 2. Hence, by the implicit function theorem, $e^*(q,w)$ is twice continuously differentiable in q and w, and

$$\frac{\partial e^*(q,w)}{\partial q} = -\left(w\frac{\partial^2 y(q,e^*)}{\partial e \partial q}\right)\left(w\frac{\partial^2 y(q,e^*)}{\partial e^2} - v''(e^*)\right)^{-1},$$

where $e^* = e^*(q,w)$ (Luenberger and Ye 2008). Note that $w\frac{\partial^2 y(q,e^*)}{\partial e^2} - v''(e^*) < 0$, because $\partial^2 y(q,e)/\partial e^2 < 0$ by Lemma 2 and Table 2, and $v''(e) \ge 0$ by Assumption 2. Also, note that $y(q,e) = (1-\mu_y^e)q$, and thus $\partial^2 y(q,e)/\partial e \partial q = (\partial y(q,e)/\partial e)\cdot(1/q) > 0$ by Lemma 2 and Table 2. Therefore, $\partial e^*(q,w)/\partial q > 0$.

Last, we show that, if $w > w_1$, then the buyer's optimal order quantity $q^*(w)$ is once continuously differentiable in w. If $w > w_1$, we already showed that we can ignore the participation constraint, and thus the buyer's expected profit can be represented as $\pi_b(q,w) = pS(q,e^*(q,w)) - wy(q,e^*(q,w))$. The optimal order quantity $q^*(w)$ is obtained from the first-order condition $\partial\pi_b(q,w)/\partial q = 0$. If $S(q,e)$ and $y(q,e)$ are twice continuously differentiable in q and e, then, by the implicit function theorem, we can conclude that $q^*(w)$ is once continuously differentiable, because $e^*(q,w)$ is twice continuously differentiable in q and w as we have shown.

Therefore, we need to show that $S(q,e)$ and $y(q,e)$ are twice continuously differentiable in the neighborhood of any possible solution (q^*,e^*) to problem (40). Any possible solution should satisfy $D \le q^* < D/(1-a_y(e^*))$ because of the following reason. First, if $q \le D$, then $\pi_b(q,w) = (p-w)y(q,e^*(q,w))$ since $S(q,e) = y(q,e)$ by Lemma 2. If $w < p$, then $\pi_b(q,w)$ is strictly increasing in q, because $y(q,e)$ is strictly increasing in both q and e by Lemma 2 and Table 2, and $\partial e^*(q,w)/\partial q > 0$. Hence, $q \le D$ cannot be optimal. When $w = p$, the buyer is indifferent among any $q \in [0,D]$, and chooses $q^* = D$ by Assumption 1. Hence, it is always the case that $q^* \ge D$. Second, if $q \ge D/(1-a_y(e))$, then $\pi_b(q,w)$ is strictly decreasing in q, because $S(q,e)$ is constant $(= D)$ and $\partial S(q,e)/\partial q = 0$ by Table 2, but $y(q,e)$ is strictly increasing in both q and e by Lemma 2 and Table 2, and also $\partial e^*(q,w)/\partial q > 0$. Therefore, it should be the case that $D \le q^* < D/(1-a_y(e^*))$.

By Lemma 2, we know that $S(q,e)$ and $y(q,e)$ are thrice continuously differentiable in q and e if $D \le q \le D/(1-a_y(e))$. Therefore, $S(q,e)$ and $y(q,e)$ are thrice continuously differentiable in the neighborhood of any solution (q^*,e^*), and thus the optimal quantity $q^*(w)$ is once continuously differentiable.

Step 4: Derivative of $\Pi^*(w)$. Note that the total expected profit of the supply chain is $\Pi^*(w) = \pi_b^*(w) + \pi_s^*(w)$, where $\pi_b^*(w)$ and $\pi_s^*(w)$ are the expected profits of the buyer and the supplier, respectively, *at a solution* given w. First, we show that, if w is above some threshold, then $\pi_b^*(w)$ and $\pi_s^*(w)$ are once continuously differentiable in w. Second, we derive the expression for $d\pi_b^*(w)/dw$. Third, we derive the expression for $d\pi_s^*(w)/dw$. Finally, we show that $d\Pi^*(w)/dw|_{w=p} < 0$ if $dq^*(w)/dw|_{w=p} < 0$.

First, we show continuous differentiability of $\pi_b^*(w)$ and $\pi_s^*(w)$. For ease of notation, let $e^*(w) = e^*(q^*(w),w)$. Then,

$$\pi_b^*(w) = pS(q^*(w),e^*(w)) - wy(q^*(w),e^*(w)),$$
$$\pi_s^*(w) = wy(q^*(w),e^*(w)) - cq^*(w) - v(e^*(w)).$$

In Step 3, we have shown that, if $w > w_1$, then $q^*(w)$ and $e^*(w)$ are once continuously differentiable. Also, $S(q,e)$ and $y(q,e)$ are once continuously differentiable by Lemma 2. Therefore, if $w > w_1$, then $\pi_b^*(w)$ and $\pi_s^*(w)$ are once continuously differentiable in w, and so is $\Pi^*(w)$.

Second, we derive the expression for $d\pi_b^*(w)/dw$. Note that $\pi_b^*(w)$ is the objective function of problem (40) *at a solution* (q^*,e^*) as a function of w. We assume $w > w_1$ and ignore the participation constraint, as we have shown in Step 3. Then, for problem (40), there exists $\lambda \in \mathbb{R}$ such that

$$p\frac{\partial S(q^*,e^*)}{\partial q} - w\frac{\partial y(q^*,e^*)}{\partial q} + \lambda \cdot w\frac{\partial^2 y(q^*,e^*)}{\partial e \partial q} = 0, \tag{41}$$
$$p\frac{\partial S(q^*,e^*)}{\partial e} - w\frac{\partial y(q^*,e^*)}{\partial e} + \lambda\left[w\frac{\partial^2 y(q^*,e^*)}{\partial e^2} - v''(e^*)\right] = 0.$$

Then, by the envelope theorem (Mas-Colell et al. 1995),

$$\frac{d\pi_b^*(w)}{dw} = -y(q^*,e^*) + \lambda \frac{\partial y(q^*,e^*)}{\partial e} = -y(q^*,e^*) + \left(w\frac{\partial y(q^*,e^*)}{\partial q} - p\frac{\partial S(q^*,e^*)}{\partial q}\right)\frac{q^*}{w} = -\frac{p}{w}\cdot\frac{\partial S(q^*,e^*)}{\partial q}q^*, \tag{42}$$

using condition (41) and the relationships $\partial y(q,e)/\partial q = y(q,e)/q$ and $\partial^2 y(q,e)/\partial e\partial q = (\partial y(q,e)/\partial e)\cdot(1/q)$ (because $y(q,e) = (1-\mu_y^e)q$).

Third, we derive the expression for $d\pi_s^*(w)/dw$. Given w and the optimal order quantity $q^*(w)$, the supplier's optimal expected profit is $\pi_s^*(w) = \max_e wy(q^*(w),e) - cq^*(w) - v(e)$. Therefore, by the envelope theorem,

$$\frac{d\pi_s^*(w)}{dw} = y(q^*,e^*) + \left(w\frac{\partial y(q^*,e^*)}{\partial q} - c\right)\frac{dq^*(w)}{dw}. \tag{43}$$

Thus,

$$\frac{d\Pi^*(w)}{dw} = \frac{d\pi_b^*(w)}{dw} + \frac{d\pi_s^*(w)}{dw} = -\frac{p}{w}\cdot\frac{\partial S(q^*,e^*)}{\partial q}q^* + y(q^*,e^*) + \left(w\frac{\partial y(q^*,e^*)}{\partial q} - c\right)\frac{dq^*(w)}{dw}.$$

Finally, we show that $d\Pi^*(w)/\partial w|_{w=p} < 0$ if $dq^*(w)/dw|_{w=p} < 0$. Note that $q^*(p) = D$ from Step 3, and $y(q,e) = S(q,e)$ when $q \le D$ and both $y(q,e)$ and $S(q,e)$ are once continuously differentiable by Lemma 2. Thus, $\partial S(q^*(w),e^*)/\partial q|_{w=p} = \partial y(q^*(w),e^*)/\partial q|_{w=p}$. Therefore,

$$\left.\frac{d\Pi^*(w)}{dw}\right|_{w=p} = \left(p\frac{\partial y(q^*,e^*)}{\partial q} - c\right)\cdot\left.\frac{dq^*(w)}{dw}\right|_{w=p},$$

because $(\partial S(D,e)/\partial q)D = (\partial y(D,e)/\partial q)D = y(D,e)$, since $y(q,e) = (1-\mu_y^e)q$.

We have $p\partial y(q^*,e^*)/\partial q - c = p(1-\mu_y^{e^*}) - c > p(1-\mu_y^0) - c > 0$ by Lemma 4. Hence, $d\Pi^*(w)/dw|_{w=p} < 0$ if $dq^*(w)/dw|_{w=p} < 0$.

Step 5: Derivative of $q^*(w)$ at $w = p$. Recall that the buyer's expected profit is $\pi_b(q,w) = pS(q,e^*(q,w)) - wy(q,e^*(q,w))$. The buyer's optimal order quantity q^* satisfies

$$\frac{\partial \pi_b(q^*,w)}{\partial q} = \left[p\frac{\partial S(q^*,e^*)}{\partial q} - w\frac{\partial y(q^*,e^*)}{\partial q}\right] + \frac{\partial e^*(q^*,w)}{\partial q}\left[p\frac{\partial S(q^*,e^*)}{\partial e} - w\frac{\partial y(q^*,e^*)}{\partial e}\right] = 0,$$

where $e^* = e^*(q^*,w)$. By the implicit function theorem,

$$\frac{dq^*(w)}{dw} = -\left(\frac{\partial^2\pi_b(q^*,w)}{\partial q\partial w}\right)\left(\frac{\partial^2\pi_b(q^*,w)}{\partial q^2}\right)^{-1}, \tag{44}$$

where

$$\begin{aligned}
\frac{\partial^2\pi_b(q^*,w)}{\partial q\partial w} &= p\frac{\partial^2 S(q^*,e^*)}{\partial q\partial e}\frac{\partial e^*(q^*,w)}{\partial w} - \left(\frac{\partial y(q^*,e^*)}{\partial q} + w\frac{\partial^2 y(q^*,e^*)}{\partial q\partial e}\frac{\partial e^*(q^*,w)}{\partial w}\right)\\
&+ \frac{\partial^2 e^*(q^*,w)}{\partial q\partial w}\left(p\frac{\partial S(q^*,e^*)}{\partial e} - w\frac{\partial y(q^*,e^*)}{\partial e}\right)\\
&+ \frac{\partial e^*(q^*,w)}{\partial q}\left[p\frac{\partial^2 S(q^*,e^*)}{\partial e^2}\frac{\partial e^*(q^*,w)}{\partial w} - \left(\frac{\partial y(q^*,e^*)}{\partial e} + w\frac{\partial^2 y(q^*,e^*)}{\partial e^2}\frac{\partial e^*(q^*,w)}{\partial w}\right)\right],
\end{aligned} \tag{45}$$

$$\begin{aligned}
\frac{\partial^2\pi_b(q^*,w)}{\partial q^2} &= p\left(\frac{\partial^2 S(q^*,e^*)}{\partial q^2} + \frac{\partial^2 S(q^*,e^*)}{\partial q\partial e}\frac{\partial e^*(q^*,w)}{\partial q}\right) - w\frac{\partial^2 y(q^*,e^*)}{\partial q\partial e}\frac{\partial e^*(q^*,w)}{\partial q}\\
&+ \frac{\partial^2 e^*(q^*,w)}{\partial q^2}\left(p\frac{\partial S(q^*,e^*)}{\partial e} - w\frac{\partial y(q^*,e^*)}{\partial e}\right)\\
&+ \frac{\partial e^*(q^*,w)}{\partial q}\left[p\left(\frac{\partial^2 S(q^*,e^*)}{\partial e\partial q} + \frac{\partial^2 S(q^*,e^*)}{\partial e^2}\frac{\partial e^*(q^*,w)}{\partial q}\right) - w\left(\frac{\partial^2 y(q^*,e^*)}{\partial e\partial q} + \frac{\partial^2 y(q^*,e^*)}{\partial e^2}\frac{\partial e^*(q^*,w)}{\partial q}\right)\right].
\end{aligned} \tag{46}$$

Note that, in the expressions above, we use the relationship $\partial^2 y(q^*,e^*)/\partial q^2 = 0$, which holds because $y(q,e) = (1-\mu_y^e)q$. Also, note that $\partial^2 \pi_b(q^*,w)/\partial q^2 \neq 0$.

Now, to evaluate the derivatives at $w = p$, we use the following relationships: $\partial S(q,e)/\partial e|_{q=D} = \partial y(q,e)/\partial e|_{q=D}$, $\partial^2 S(q,e)/\partial e^2|_{q=D} = \partial^2 y(q,e)/\partial e^2|_{q=D}$, and $\partial^2 S(q,e)/\partial e \partial q|_{q=D} = \partial^2 y(q,e)/\partial e \partial q|_{q=D}$. These relationships hold, because, at $q = D$, $S(q,e)$ and $y(q,e)$ are thrice continuously differentiable in e and once continuously differentiable in q, and also $S(q,e) = y(q,e)$ for $0 \leq q \leq D$ by Lemma 2. Therefore,

$$\lim_{w \to p^-} \frac{\partial^2 \pi_b(q^*,w)}{\partial q \partial w} = -\frac{\partial y(q^*,e^*)}{\partial q} - \frac{\partial e^*(q^*,w)}{\partial q}\frac{\partial y(q^*,e^*)}{\partial e},$$
$$\lim_{w \to p^-} \frac{\partial^2 \pi_b(q^*,w)}{\partial q^2} = \lim_{w \to p^-} p\frac{\partial^2 S(q^*,e^*)}{\partial q^2}.$$

We have shown that $\partial e^*(q^*,w)/\partial q > 0$ in Step 3. Therefore, $\lim_{w\to p^-} \frac{\partial^2 \pi_b(q^*,w)}{\partial q \partial w} < 0$, because $\partial y(q^*,e^*)/\partial q > 0$ and $\partial y(q^*,e^*)/\partial e > 0$ by Lemma 2 and Table 2. In addition, $\partial^2 S(q^*,e^*)/\partial q^2 \leq 0$ by Lemma 2, and thus $\lim_{w\to p^-} \frac{\partial^2 \pi_b(q^*,w)}{\partial q^2} \leq 0$. Therefore, $dq^*(w)/dw|_{w=p} < 0$. (It is possible that $\lim_{w\to p^-} \partial^2 S(q^*,e^*)/\partial q^2 = 0$, which is the denominator of $dq^*(w)/dw|_{w=p}$, but the result still holds.) □

Proof of Proposition 12 We prove the result using the *uniform distribution*. We already showed that the optimal order quantity is D if $p < 3c$ in the proof of Proposition 3. Therefore, the supplier's optimal expected profit is obtained by solving the following problem: $\pi_s^*(w) = \max_{e \geq 0}(w-c)y(D,e) - v(e)$. By the envelope theorem, we have that

$$\frac{d\pi_s^*(w)}{dw} = y(D,e^*) > 0.$$

Furthermore, since the optimal solutions q^* and e^* are continuous in w, both firms' expected profits at equilibrium, $\pi_b^*(w)$ and $\pi_s^*(w)$ are continuous in w too. □

Proof of Proposition 13 Note that we only need to consider the case when $\pi_s^*(w) > 0$, since $\alpha \in (0,1)$ (see Step 1 in the proof of Proposition 1). We showed in the proof of Proposition 11 with the general functional form that the optimal solution satisfies $D \leq q^* < D/(1-a_y(e^*))$ and both firms' expected profits at equilibrium, $\pi_b^*(w)$ and $\pi_s^*(w)$ are continuous when $\pi_s^*(w) > 0$. Furthermore, we showed that

$$\frac{d\pi_b^*(w)}{dw} = -\frac{p}{w} \cdot \frac{\partial S(q^*,e^*)}{\partial q} q^* < 0,$$

since $\partial S(q^*,e^*)/\partial q > 0$. □

Proof of Proposition 14 We prove the result using the *uniform distribution*. As we have shown in the proof of Proposition 8, the buyer's expected profit has a kink at $q = D$ and the optimal order quantity always satisfies $q^* \geq D$. Therefore, we divide the proof into two parts. Specifically, we consider the case when $q^* > D$ in Part (i) and $q^* = D$ in Part (ii). In both cases, we show that $d\pi_b^*(w)/dw < 0$. Note that we only need to consider the case when $\pi_s^*(w) > 0$, since $\alpha \in (0,1)$ (see Step 1 in the proof of Proposition 1).

Part (i): $\mathbf{q^* > D}$. We can reformulate problem (3) as following by i) replacing the first constraint with the first-order conditions, and ii) ignoring the second constraint since we focus on the case when $\pi_s^*(w) > 0$.

$$\begin{aligned} \max_{q,x,e} \quad & pS(q,x,e) - wy(q,x,e), \\ s.t. \quad & w\frac{\partial y(q,x,e)}{\partial x} - c = 0, \\ & w\frac{\partial y(q,x,e)}{\partial e} - v'(e) = 0. \end{aligned} \tag{47}$$

Note that (47) is a more general problem than (3) because the supplier's profit may not be unimodal. In (47), the objective function and constraints are once continuously differentiable by Lemma 3 and Assumptions 2. Then, there exist $\lambda_1, \lambda_2 \in \mathbb{R}$ such that, at equilibrium (q^*, x^*, e^*),

$$-w\frac{\partial y(q^*,x^*,e^*)}{\partial q} + \lambda_1 \cdot w\frac{\partial^2 y(q^*,x^*,e^*)}{\partial x \partial q} + \lambda_2 \cdot w\frac{\partial^2 y(q^*,x^*,e^*)}{\partial e \partial q} = 0, \tag{48}$$

$$p\frac{\partial S(q^*,x^*,e^*)}{\partial x} - w\frac{\partial y(q^*,x^*,e^*)}{\partial x} + \lambda_1 \cdot w\frac{\partial^2 y(q^*,x^*,e^*)}{\partial x^2} + \lambda_2 \cdot w\frac{\partial^2 y(q^*,x^*,e^*)}{\partial e \partial x} = 0, \tag{49}$$

$$p\frac{\partial S(q^*,x^*,e^*)}{\partial e} - w\frac{\partial y(q^*,x^*,e^*)}{\partial e} + \lambda_1 \cdot w\frac{\partial^2 y(q^*,x^*,e^*)}{\partial x \partial e} + \lambda_2 \left(w\frac{\partial^2 y(q^*,x^*,e^*)}{\partial e^2} - v''(e^*) \right) = 0. \tag{50}$$

Note that $\partial S(q,x,e)/\partial q = 0$ when $q \geq D$ by Lemma 3. Hereafter we simplify the notations by using $y = y(q,x,e)$ and $S = S(q,x,e)$. Note that $y(q,x,e) = [(1 - a_y(e)) + \int_{1-q/x}^{a_y(e)} H(\xi \mid e) d\xi]x$ and, with some calculus, we can show that the following relationships hold:

$$\frac{\partial y^2}{\partial x \partial e} = \frac{1}{x} \cdot \frac{\partial y}{\partial e} - \frac{q}{x} \cdot \frac{\partial^2 y}{\partial e \partial q}, \qquad \frac{\partial^2 y}{\partial q^2} = \frac{x^2}{q^2} \cdot \frac{\partial^2 y}{\partial x^2}, \qquad \frac{\partial^2 y}{\partial x \partial q} = -\frac{q}{x} \cdot \frac{\partial^2 y}{\partial q^2} = -\frac{x}{q} \cdot \frac{\partial^2 y}{\partial x^2}, \qquad \frac{\partial y}{\partial x} = \frac{y}{x} - \frac{q}{x} \cdot \frac{\partial y}{\partial q}.$$

Using the above relationships and conditions (48) and (49), we can obtain

$$\lambda_1 = \frac{\frac{\partial y}{\partial q}\frac{\partial y}{\partial e} - \frac{\partial^2 y}{\partial e \partial q}\left(y - p\frac{x}{w}\frac{\partial S}{\partial x}\right)}{\frac{\partial^2 y}{\partial x \partial q}\frac{\partial y}{\partial e}}, \qquad \lambda_2 = \frac{y - p\frac{x}{w}\frac{\partial S}{\partial x}}{\frac{\partial y}{\partial e}}.$$

By the envelope theorem,

$$\begin{aligned} \frac{d\pi_b^*(w)}{dw} &= -y(q^*,x^*,e^*) + \lambda_1 \frac{\partial y(q^*,x^*,e^*)}{\partial x} + \lambda_2 \frac{\partial y(q^*,x^*,e^*)}{\partial e} \\ &= \frac{1}{\frac{\partial^2 y^*}{\partial x \partial q}\frac{\partial y^*}{\partial e}} \left[\frac{\partial y^*}{\partial x}\left(\frac{\partial y^*}{\partial e}\frac{\partial y^*}{\partial q} - y^* \frac{\partial^2 y^*}{\partial e \partial q} \right) + p\frac{x^*}{w}\frac{\partial S^*}{\partial x}\left(\frac{\partial y^*}{\partial x}\frac{\partial^2 y^*}{\partial e \partial q} - \frac{\partial^2 y^*}{\partial x \partial q}\frac{\partial y^*}{\partial e} \right) \right], \end{aligned}$$

where $y^* = y(q^*,x^*,e^*)$. We know that $\partial y/\partial x > 0$, $\partial y/\partial e > 0$, and $\partial S/\partial x > 0$ by Lemma 3. Also, $\partial^2 y/\partial x \partial q = \frac{q}{x^2} h\left(1 - \frac{q}{x} \mid e\right) > 0$. Furthermore, with the uniform distribution,

$$\left(\frac{\partial y^*}{\partial e}\frac{\partial y^*}{\partial q} - y^* \frac{\partial^2 y^*}{\partial e \partial q} \right) = \frac{e^*(q^* - x^*)}{1 + e^*} < 0, \qquad \left(\frac{\partial y^*}{\partial x}\frac{\partial^2 y^*}{\partial e \partial q} - \frac{\partial^2 y^*}{\partial x \partial q}\frac{\partial y^*}{\partial e} \right) = -\frac{(q^* + e^* q^* - e^* x^*)^2}{2(e^* + 1)x^{*2}} < 0,$$

since $x^* > q^*$, which we showed in the proof of Proposition 8. Therefore, $d\pi_b^*(w)/dw < 0$.

Part (ii): $\mathbf{q^* = D}$. The proof consists of three steps. In Step 1, we simplify the expression for $d\pi_b^*(w)/dw$. In Step 2, we derive a sufficient condition under which $d\pi_b^*(w)/dw < 0$. Finally, in Step 3, we show that this condition always holds if $p \leq 2c$ and $\theta \geq (5/7) \cdot cD$.

Step 1: Simplifying the expression for $\mathbf{d\pi_b^*(w)/dw}$. Let $x(w)$ and $e(w)$ be the supplier's optimal production quantity and effort as functions of w when $q = D$, which can be jointly obtained by the following two first-order conditions:

$$w\frac{\partial y(D,x,e)}{\partial x} - c = 0, \qquad w\frac{\partial y(D,x,e)}{\partial e} - v'(e) = 0. \tag{51}$$

Using the notation $y = y(D,x,e)$, we can apply the implicit function theorem as follows.

$$\begin{bmatrix} x'(w) \\ e'(w) \end{bmatrix} = -\begin{bmatrix} w\frac{\partial^2 y}{\partial x^2}, & w\frac{\partial^2 y}{\partial x \partial e} \\ w\frac{\partial^2 y}{\partial e \partial x}, & w\frac{\partial^2 y}{\partial e^2} - v''(e) \end{bmatrix}^{-1} \cdot \begin{bmatrix} \frac{\partial y}{\partial x} \\ \frac{\partial y}{\partial e} \end{bmatrix} = -\frac{1}{m(x,e)} \cdot \begin{bmatrix} \frac{\partial y}{\partial x}\left(w\frac{\partial^2 y}{\partial e^2} - v''(e)\right) - w\frac{\partial^2 y}{\partial x \partial e}\frac{\partial y}{\partial e} \\ w\left(\frac{\partial^2 y}{\partial x^2}\frac{\partial y}{\partial e} - \frac{\partial^2 y}{\partial x \partial e}\frac{\partial y}{\partial x}\right) \end{bmatrix},$$

where $m(x,e) = w^2\left(\frac{\partial^2 y}{\partial x^2}\frac{\partial^2 y}{\partial e^2} - (\frac{\partial^2 y}{\partial x \partial e})^2\right) - w\frac{\partial^2 y}{\partial x^2}v''(e)$. Since $y(q,x,e)$ is jointly concave in x and e by Assumption 5, we know that $m(x,e) > -w\frac{\partial^2 y}{\partial x^2}v''(e) \geq 0$, because $\partial^2 y/\partial x^2 \leq 0$ by Lemma 3 and $v''(e) \geq 0$ by Assumption 2.

The buyer's expected profit at the equilibrium is $\pi_b^*(w) = (p-w)y(D,x(w),e(w))$, since $S(D,x,e) = y(D,x,e)$ by Lemma 3. Hence, using the notation $y^* = y(D,x^*,e^*)$,

$$\begin{aligned} \frac{d\pi_b^*(w)}{dw} &= -y^* + (p-w)\left[\frac{\partial y^*}{\partial x}\frac{dx(w)}{dw} + \frac{\partial y^*}{\partial e}\frac{de(w)}{dw}\right]. \\ &= -\frac{1}{m(x^*,e^*)}\left[-v''(e^*)\left(y^* \cdot w\frac{\partial^2 y^*}{\partial x^2} + (p-w)\left(\frac{\partial y^*}{\partial x}\right)^2\right) + y^* \cdot w^2\left(\frac{\partial^2 y^*}{\partial x^2}\frac{\partial^2 y^*}{\partial e^2} - \left(\frac{\partial^2 y^*}{\partial x \partial e}\right)^2\right)\right. \\ &\qquad \left. + (p-w)w\left(\frac{\partial^2 y^*}{\partial e^2}\left(\frac{\partial y^*}{\partial x}\right)^2 - 2\frac{\partial^2 y^*}{\partial x \partial e}\frac{\partial y^*}{\partial x}\frac{\partial y^*}{\partial e} + \frac{\partial^2 y^*}{\partial x^2}\left(\frac{\partial y^*}{\partial e}\right)^2\right)\right]. \end{aligned}$$

The optimal solution always satisfies that $q < x^* < (1+1/e^*)q$, which we have shown in the proof of Proposition 8. When $q < x < (1+1/e)q$, we have

$$y(q,x,e) = \left[1 - \frac{1}{2(e+1)} - \frac{e+1}{2}\left(1 - \frac{q}{x}\right)^2\right]x.$$

We calculate the derivatives of $y = y(q,x,e)$ and represent y as follows, representing some of them using the variable $k = \partial y/\partial x$. At equilibrium, by the first-order condition (51), $k = \partial y/\partial x = c/w \in [c/p,1]$.

$$\begin{aligned} &\frac{\partial y}{\partial x} = \frac{(e+1)^2q^2 - e^2x^2}{2(e+1)x^2} = k, \qquad \frac{\partial y^2}{\partial x^2} = -\frac{(e+1)q^2}{x^3} = -\frac{e^2 + 2k(e+1)}{(e+1)x}, \qquad \frac{\partial^2 y}{\partial e^2} = -\frac{x}{(e+1)^3}, \\ &\frac{\partial y}{\partial e} = \left(\frac{1}{2(e+1)^2} - \frac{(x-q)^2}{2x^2}\right)x = (q-x) + \frac{(1-k)x}{(e+1)}, \qquad \frac{\partial^2 y}{\partial x \partial e} = \frac{(e+1)^2q^2 - e(e+2)x^2}{2(e+1)^2x^2} = \frac{k(e+1)-e}{(e+1)^2}, \\ &y = \frac{e}{e+1}x + (e+1)\frac{\partial y}{\partial e}. \end{aligned}$$

Using the above expressions and $v''(e) = 0$, we can simplify $d\pi_b^*(w)/dw$ as follows. To simplify the notation for the rest of the proof, we omit asterisks and regard all q, x, and e as equilibrium values.

$$\frac{d\pi_b^*(w)}{dw} = -\frac{1}{m(x,e)} \cdot \frac{w}{(e+1)^3} \cdot (A - B),$$

where

$$\begin{aligned} A &= kw(k-2)(-(e+1)^2q + (e^2+k+ek)x), \\ B &= (p-w)\left[k^2x + \frac{(e^2+2k+2ek)((1+e)q-(e+k)x)^2}{x} - 2k(e(k-1)+k)((e+k)x-(e+1)q)\right]. \end{aligned}$$

Step 2: Deriving a sufficient condition for $\mathbf{d\pi_b^*(w)/dw < 0}$. We obtain a lower bound of $A - B$ and show that it is strictly positive. This means that $d\pi_b^*(w)/dw < 0$, since $m(x,e) > 0$. An important property we use in the rest of the proof is that, at equilibrium, $(e+1)q > (e+k)x$, because $((e+1)q)^2 - ((e+k)x)^2 = k(2-k)x^2 > 0$ (since $k \in [c/p,1]$) using the relationship $k = \partial y/\partial x$.

First, to obtain a lower bound of A, we observe that the following inequality holds:

$$\begin{aligned}-(e+1)^2q+(e^2+k+ek)x &= -(2(e+1)k+e^2)x\cdot\frac{x}{q}+(e^2+k+ek)x\\ &< -(2(e+1)k+e^2)x+(e^2+k+ek)x=-k(e+1)x<0,\end{aligned}$$

where we use the relationships $k=\partial y/\partial x$ in the first step and $q<x$ in the second step. Therefore, $A>0$ (since $k\in[c/p,1]$) and

$$A > kw(k-2)(-k(e+1)x) = (2-k)(e+1)k^2wx. \tag{52}$$

Second, we obtain a upper bound of B as follows.

$$\begin{aligned}B &= (p-w)\left[k^2x+\left(\frac{1}{x}(e^2+2k+2ek)((1+e)q-(e+k)x)+2k(e(k-1)+k)\right)((1+e)q-(e+k)x)\right]\\ &< (p-w)\left[k^2x+\left(\frac{1}{x}(e^2+2k+2ek)(1-k)x+2k(e(k-1)+k)\right)((1+e)q-(e+k)x)\right]\\ &= (p-w)\left[k^2x+(e^2(1-k)+2k)((1+e)q-(e+k)x)\right]\\ &< (p-w)\left[k^2x+(e^2(1-k)+2k)(1-k)x\right] = (p-w)x\left[e^2(1-k)^2+2k-k^2\right],\end{aligned} \tag{53}$$

where we use $q<x$ for both inequalities in (53).

Using (52) and (53), we have

$$A-B > (2-k)(e+1)k^2wx-(p-w)x\left[e^2(1-k)^2+2k-k^2\right].$$

Therefore, $A-B>0$ if

$$p \le \left(1+\frac{(2-k)(e+1)k^2}{e^2(1-k)^2+2k-k^2}\right)w \tag{54}$$

Step 3: Showing that condition (54) **holds if $\mathbf{p\le 2c}$ and $\theta\ge(\mathbf{5/7})\cdot\mathbf{cD}$.** We first show that the optimal effort satisfies $e\le 9/5$, and then show that the inequality (54) always holds.

First, we show that $e\le 9/5$. To do so, we first obtain the upper bound of x. We already know that $(e+1)q>(e+k)x$. Furthermore, it is easy to verify that $(e+1)/(e+k)$ is decreasing in e. Therefore,

$$x<\frac{e+1}{e+k}\cdot q\le\frac{1}{k}\cdot q\le\frac{p}{c}\cdot q=\frac{pD}{c}.$$

Using the first-order condition (51) with respect to e and the expression for $\partial y/\partial e$ in Step 1, we have that

$$(q-x)+\frac{(1-k)x}{(e+1)}=\frac{\theta}{w}\quad\Longrightarrow\quad\frac{(1-k)x}{(e+1)}>\frac{\theta}{w},$$

because $q<x$. Hence,

$$e<\frac{w(1-k)x}{\theta}-1\le\frac{(p-c)pD}{c\theta}-1\le\frac{c\cdot 2c\cdot D}{c\theta}-1=\frac{2cD}{\theta}-1\le\frac{9}{5},$$

where the second step uses $w\le p$, $k\ge c/p$, and $x\le pD/c$, the third step uses $p\le 2c$, and the final step uses $\theta\ge\frac{5cD}{7}$.

Second, we show that the inequality (54) always holds. We observe the following inequality.

$$\begin{aligned}\left(1+\frac{(2-k)(e+1)k^2}{e^2(1-k)^2+2k-k^2}\right)w &= \left(1+\frac{(2-k)(e+1)k^2}{(e+1)(e-1)(1-k)^2+1}\right)w \\ &\geq \left(1+\frac{(2-k)(e+1)k^2}{(e+1)(e-1)(1-k)^2+e+1}\right)w \\ &= w+\frac{(2-k)}{(e-1)(1-k)^2+1}\cdot\frac{c^2}{w} \geq 2c\cdot\sqrt{\frac{2-k}{(e-1)(1-k)^2+1}}, \qquad (55)\end{aligned}$$

where the last step uses the fact that $\left(\sqrt{w}-\sqrt{\frac{2-k}{(e-1)(1-k)^2+1}}\cdot\frac{c}{\sqrt{w}}\right)^2 \geq 0$. Let $\gamma = \frac{2-k}{(e-1)(1-k)^2+1}$. It is easy to verify that $d\gamma/dk \leq 0$ if $p \leq 2c$ (which means $k = c/w \geq c/p \geq 1/2$) and $e \leq 9/5$, because

$$\frac{d\gamma}{dk} = \frac{(e-1)(1-k)(3-k)-1}{((e-1)(1-k)^2+1)^2}.$$

Hence, γ achieves the minimum when $k=1$, and therefore the inequality (55) can be simplified as follows.

$$\left(1+\frac{(2-k)(e+1)k^2}{e^2(1-k)^2+2k-k^2}\right)w \geq 2c.$$

Thus, (54) always holds and $A-B>0$, concluding that $d\pi_b^*(w)/dw < 0$. □

Proof of Proposition 15 Let Π_{max} be the expected profit of the entire supply chain under a coordinating contract. Since a coordinating contract has a transaction cost $T \in [0,1]$, expressed as a fraction of Π_{max}, firms will be effectively splitting the total profit of $(1-T)\Pi_{max}$. We are only interested in the expected profit of each firm (not the contract parameters), and therefore we can formulate the bargaining problem with a coordinating contract as follows:

$$\max_x \quad x^{\alpha}((1-T)\Pi_{max}-x)^{1-\alpha},$$

where $x \in [0,(1-T)\Pi_{max}]$ and $((1-T)\Pi_{max}-x)$ are the expected profits of the supplier and buyer, respectively. It is easy to see that, given α, the optimal expected profits of the supplier and buyer are $x = \alpha(1-T)\Pi_{max}$ and $(1-T)\Pi_{max}-x = (1-\alpha)(1-T)\Pi_{max}$, respectively.

As $T \to 1$, both firms' expected profits approach zero. Since we can always find a wholesale price such that the wholesale price contract generates strictly positive profits for both firms, we can conclude that there exists $T' > 0$ such that for all $T > T'$, the wholesale price contract is Pareto efficient compared to the coordinating contract.