An Equivalent-Circuit Model of Miniaturized Split-Ring Resonator

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***Abstract*—In this paper, we present a new miniaturized split-ring resonator (SRR) to be easily employed in the practical realization of SRRs and derive its equivalent circuit model to accurately predict the resonance frequency. The results of the resonance frequency obtained by the developed equivalent circuit model are in very good agreement with the simulated results.**

***Keywords*—Conductor rod; equivalent circuit; miniaturization; split-ring resonators.**

I. Introduction

In the last few decades, high performance and small size devices have attracted a high level of interests in modern wireless communication systems. It is expected that these devices in such systems will offer cost-effective and low profile properties. Engineered materials, which are loaded in devices, can improve the device qualities. In order to use such materials efficiently, the array that gathered with high number of unit cell structure is required and to minimize total array size that should provide the same electrical properties for small size devices, miniaturized unit cell may be used. In this frame, to reduce the size of the unit cell, spiral resonator [1], multiple split-ring resonator [1], labyrinth resonator [1], lumped elements embedded resonator [2] or more complex geometries (like fractals or asymmetric inclusions) [3] can be exploited. In this study, a commonly known square SRR is miniaturized by additional conductor rods. The proposed model is presented with its equivalent circuit model having a good agreement when compared to the commercial numerical software.

II. Proposed Model and Its Equivalent Circuit Model

Resonance frequency of SRR (Fig. 1.a) can be controlled by tuning the capacitance and/or inductance values with the relevant equation $f_o = 1/(2\pi\sqrt{L_{SRR}C_{SRR}})$ where C_{SRR} and L_{SRR} represent the equivalent capacitance and inductance values (Fig.1.c), respectively. Those values can be calculated from the studies in [4][5]. In such structures, C_{SRR} is a virtue of the gap of the ring and coupling between adjacent cells, and L_{SRR} consist of single turn metal ring. Inclusion of conductor rods, which will achieve the miniaturization, will affect the equivalent capacitance and inductance values in addition to C_{SRR} and L_{SRR}. The geometry and corresponding equivalent circuit of the SRR with the conductor rods are depicted in Fig 1.b and Fig 1.d.

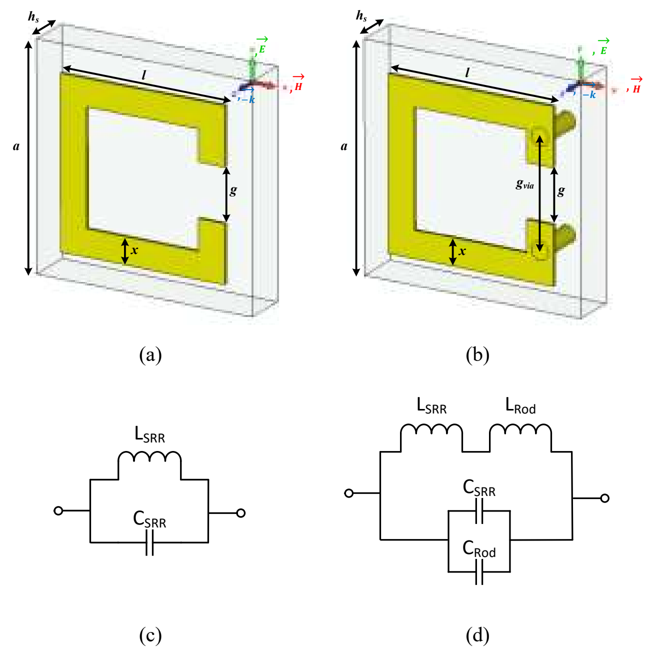

Fig. 1. (a) The conventional square SRR. (b) Proposed miniaturized square SRR. (c) Equivalent circuit of conventional square SRR. (d) Equivalent circuit of miniaturized square SRR.

As seen from the equivalent circuit model, the self-inductance value of the rod L_{Rod} are serially connected to the inductance value of the L_{SRR}. In addition to the inductance values, capacitance values are also considered in the equivalent circuit model, as well. When physically evaluated, it is expected to obtain a self-impedance of two rods which can be calculated by Equation (1);

$$L_{Rod} \approx \frac{\mu_0 \mu_r}{\pi} cosh^{-1}\left(\frac{g_{via}}{2r_o}\right).h_s \qquad (1)$$

where μ_0 is the free space permeability value, μ_r is the substrate permeability value, g_{via} is the distance between center of rods, r_o is the radius of the rod, and h_s is the thickness of the substrate. In the equation above, L_{Rod} is the total self-inductance value that is resulted from inserting the rods. The material used in order to design the SRR has a permittivity value of 3.38. Additional capacitance value, which is occurred due to the usage of the conductor rods, can be calculated via Equation (2).

$$C_{Rod} = \frac{\varepsilon_0 \varepsilon_r A}{g_{via}} \qquad (2)$$

where ε_0 and ε_r are the free space and substrate permittivity values, respectively. A is the area of a rectangular cross section of the conductor rod, which has a value of $2r_o h_s$.

III. Results

The analysis of the structure is conducted with both CST Microwave Studio and the equivalent circuit model; and the obtained results are compared. Since the self-inductance value of two rods is a function of the distance between the rods, the length of the rods, the material properties and the radius of the rods. Several parametric analyses are conducted and compared with the equivalent circuit model. The length of the conductor rods and the material properties are not changed since there is a fixed SRR parameters of a=2.5 mm, l=2.3 mm, x=0.3 mm, g=0.6 mm h_s=0.508 mm, ε_r=3.38, μ_r=1. S_{21} resonance variation due to the change of the distance between the conductor rods is presented in Fig. 3.

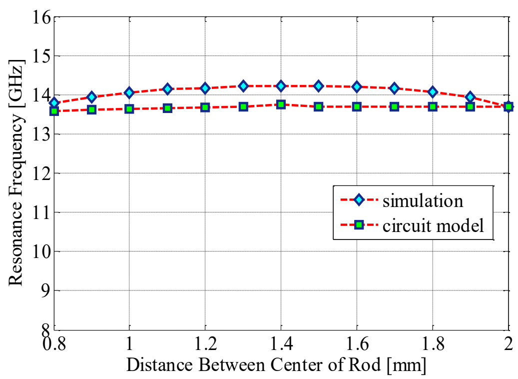

Fig. 2. Resonance frequencies obtained from CST simulations and from the equivalent circuit model for different g_{via} values (r_o= 0.1 mm h_s= 0.508 mm).

In miniaturized SRR model for g_{via}=0.8 mm, the calculated values are L_{Rod} =0.448 nH and C_{Rod}=4.06 fF. The S_{21} resonance for g_{via}=0.8 mm is 13.789 GHz for the simulation and 13.58GHz. The equivalent circuit model achieved to have good agreement with an error of 1.52%. S_{21} resonance variation due to the change of the distance between the conductor rods is presented in Fig.3.

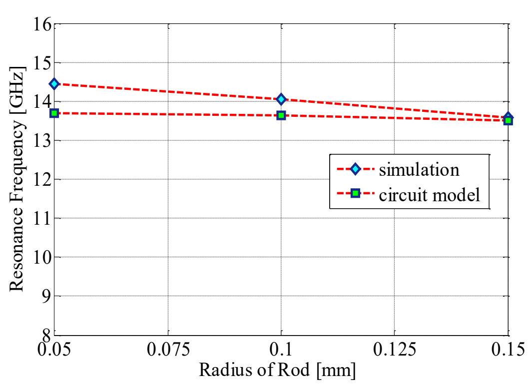

Fig. 3. Resonance frequencies obtained from CST simulations and from the equivalent circuit model for different radius of rod values (g_{via}=1 mm, h_s= 0.508 mm).

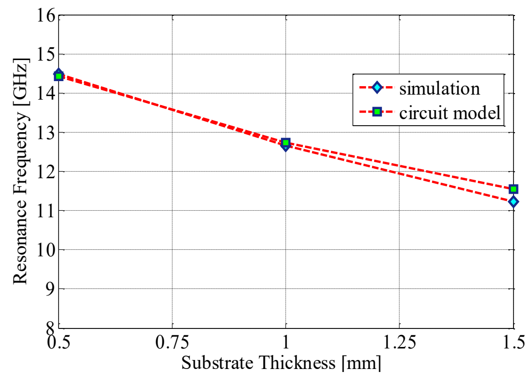

Fig. 4. Resonance frequencies obtained from CST simulations and from the equivalent circuit model for different substrate thickness values (g_{via}=1 mm, r_o=0.05 mm).

Impact of radius and length of the conductor rod are evaluated and presented in Fig. 3 and Fig. 4, respectively. The increment of the conductor rod radius causes a decrement in inductance and increment in capacitance values so that there is not a remarkable difference in frequency for different radius values. The substrate thickness value, which is also the same with the conductor rod length, increases both inductance and capacitance values. This impact results as a significant decrement in frequency value.

IV. Conclusion

The focus of the study is miniaturization of the SRR and modelling of the equivalent circuit model of it. With the proposed SRR structure, it is possible to design a compact, low profile and high efficient modern wireless communication devices. Additionally, equivalent circuit model is proposed to predict the impacts of the structure parameters and verified by simulations with satisfactory agreement.

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