Analysis and Design of Robust Stabilizing Modified Repetitive Control Systems

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March 2013

ABSTRACT

In control system practice, high precision tracking or attenuation for periodic signals is an important issue. Repetitive control is known as an effective approach for such control problems. The internal model principle shows that the repetitive control system which contains a periodic generator in the closed-loop can achieve zero steady-state error for reference input or completely attenuate disturbance. Due to its simple structure and high control precision, repetitive control has been widely applied in many systems. To improve existing results on repetitive control theory, this thesis presents theoretical results in analysis and design repetitive control system. The main work and innovations are listed as follows:

We propose a design method of robust stabilizing modified repetitive controllers for multiple-input/multiple-output plants with uncertainties. The parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output plant with uncertainty is obtained by employing H_∞ control theory based on the Riccati equation. The robust stabilizing controller contains free parameters that are designed to achieve desirable control characteristic. In addition, the bandwidth of low-pass filter has been analyzed. In order to simplify the design process and avoid the wrong results obtained by graphical method, the robust stability conditions are converted to LMIs-constraint conditions by employing the delay-dependent bounded real lemma. When the free parameters of the parameterization of all robust stabilizing controllers is adequately chosen, then the controller works as robust stabilizing modified repetitive controller.

For a time-varying periodic disturbances, we give an design method of an optimal robust stabilizing modified repetitive controller for a strictly proper plant with time-varying uncertainties. A modified repetitive controller with time-varying delay structure, inserted by a low-pass filter and an adjustable parameter, is developed for this class of system. Two linear matrix inequalities LMIs-based robust stability conditions of the closed-loop system with time-varying state delay are derived for fixed parameters. One is a delay-dependent robust stability condition that is derived based on the free-weight matrix. The other robust stability condition is obtained based on the H_∞ control problem by introducing a linear unitary operator. To obtain the desired controller, the design problems are converted to two LMI-constrained optimization problems by reformulating the LMIs given in the robust stability conditions. The validity of the proposed method is verified through a numerical example.

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Chapter 1

Introduction

People often completely master a new learned skill through repetition. By repeating the same action, a person gradually comes to understand the essential points, and achieves a significant efficiency and precision. This process, with self-learning and gradual progress, is a repetitive task. An investigation of the process reveals two main characteristics:

1. the same action is performed;

2. the action currently being performed is based on the action performed in the previous repetition.

These two characteristics imply that it is a periodic repetitive task.

Manufacturing and industrial applications often have plants that perform repetitive tasks. In these situations, exploiting the periodic properties of the design problem is an important part in maximizing performance. Inoue et al. [1, 2] devised a new control strategy called repetitive control that adds a human-like learning capability to a control system. The new type of control system for periodic repetitive task is named as repetitive control system. A repetitive control system is different from other types of control systems in that it possesses a self-learning capability. For example, Inoue et al. [1] designed a Single-Input/Single-Output repetitive control system for supplying power for the magnet of a proton synchrotron that tracks a desired periodic reference input, namely excitation current. After self-learning for 16 periods, the relative tracking precision reached 10^{-4}. This high precision was unobtainable by any other control method at that time. As a result, the theory and design methods of the repetitive control system immediately received a great deal of attention, and it is now widely used in many fields from aerospace to public welfare systems.

1.1 Background

In practical applications, tracking and/or rejecting the periodic reference input and/or disturbance signals are of great significance. For example, in industrial manipulators executing operations of picking, placing or painting, machine tools and magnetic disk or CD drives, the control systems are usually required to track or reject periodic exogenous signals with high control precision. The repetitive control theory provides an achievable and practicable theoretical foundation and solution.

At present, repetitive control has been widely applied in various high-precision control systems. As a simple learning control method, repetitive control has many advantages such as simple algorithm, insensitivity of the system performance to parameters, small online computation, high-precision, suitability for fast motion control and so on. All these characteristics are required for many control problems with periodic exogenous signals. With the improvement of technical level of modern industry, the requirement for the design of repetitive control system is higher than ever. For example, in many servo systems, the requirements are not only high steady state accuracy, but also good transient characteristics. That means the design of repetitive control system should optimize the steady state performance and transient characteristics [3]. For the plants with uncertainties, such as SPWM inverter requires the design method satisfying the robust stability [4, 5, 6]. Some plants require the variable to learn in an iteration-independent manner like the robot motion control [7]. That has high demands on the adjusting function of parameters. In addition, the repetitive control system with independently regulate and control manner should be established.

As the development of the application of the repetitive control method, the formation of new problem in the practical control system leads to further developed and improved of the repetitive control system theory and design methods. Therefore, study on design method of repetitive controllers and deeply reveal the nature of the repetitive control systems have an important theoretical and practical significance.

1.2 Repetitive control principle

Repetitive control is a control scheme applied to plants that must track a periodic trajectory or reject a periodic disturbance with the explicit use of the periodic feature of the trajectory or disturbance.

It was first introduced by Inoue et al. and applied to the control of a power supply for a proton synchrotron [1] and a contouring servo system [2] . Since then, repetitive control has been applied to many problems, including:

- power supply systems ([1, 8]),
- robotic manipulators ([9, 10]),
- computer disk drives ([11, 12]),
- CD tracking ([13, 94]),
- motor control ([15, 16]),
- thickness control in sheet metal rolling ([17]),
- peristaltic pump ([18]),
- cold rolling process control ([19]),
- navicular machining ([20]),

- vibration attenuation ([21, 22]) and
- distributed solar collector ([23, 24, 25]).

The Internal Model Principle (IMP) proposed by Francis and Wonham [26] plays an important role in repetitive control system. The IMP states that if a reference or disturbance signal can be regarded as the output of an autonomous system, including this system in a stable feedback loop guarantees asymptotically perfect tracking or rejecting performance. Figure 1.1 shows the more frequently used generator of periodic signals with a period-time L[s]. In this figure, a finite length input $u(t)$ from

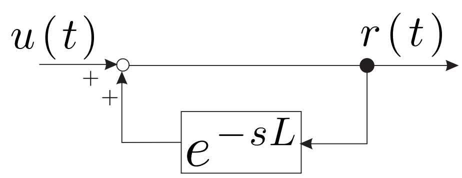

Figure 1.1: Generator of periodic signals with period-time L[s]

$t = 0$ to $t = L$ yields an output $r(t)$ that is a periodic, i.e.,

$$r(t) = \begin{cases} u(t), & 0 \leq t \leq L \\ r(t-L), & L \leq t \end{cases} . \tag{1.1}$$

An IMP-based repetitive controller incorporates this generator in a control loop as shown in Figure 1.2. In this control system, we want the control output $y(t)$ to track

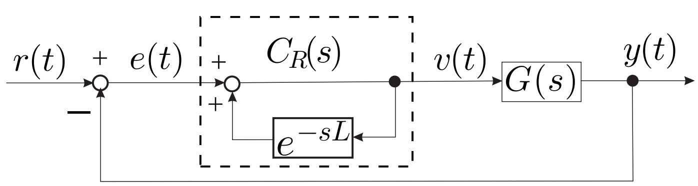

Figure 1.2: Conventional repetitive control system

a desired periodic reference input $r(t)$ with zero-steady state error. The transfer function of the repetitive controller $C_R(s)$(dotted line) is described as

$$C_R(s) = \frac{1}{1 - e^{-sL}}, \tag{1.2}$$

where L is a constant equal to the period-time of the reference input, $r(t)$. This period time is known or accurately measured. Since

$$C_R(j\omega_k) = \frac{1}{1 - e^{-j\omega_k L}} = \infty, \quad \omega_k = \frac{2k\pi}{L}, \quad k = 0, 1, 2, \cdots, \tag{1.3}$$

the gain of the repetitive controller is infinite at the angular frequencies of the fundamental and harmonic waves of a signal with period-time L[s]. Note that the tracking error of the repetitive control system in Figure 1.2 is given by

$$E(s) = S_R(s)R(s), \tag{1.4}$$

and

$$S_R(s) = \frac{1}{1 + C_R(s)G(s)} = \frac{1}{C_R(s)} \frac{1}{\frac{1}{C_R(s)} + G(s)}, \tag{1.5}$$

where $S_R(s)$ is the sensitivity function of the system. Clearly, including the internal model as a repetitive controller results in an infinite loop gain and hence, a zero closed-loop sensitivity at the angular frequencies of the fundamental and harmonic waves. Consequently, the periodic signal with a period-time L[s] can be perfectly tracked or rejected by this closed-loop system called as periodic performance. Hence, when a control system contains repetitive controller $C_R(s)$, it tracks the periodic reference input with high control accuracy.

However, it is impossible to design stabilizing repetitive controller for strictly proper plant, because the repetitive control system is a neutral type of time-delay system. To design a repetitive control system that follows any periodic reference input without steady state error, the plant must be biproper.

1.3 Modified repetitive controller

The nonexistence of a repetitive controller for a strictly proper plant has been detailed by Hara et al. in [27]. According to the servo theory, it is well-known that output regulation is possible only when plant zeros do not cancel the poles of the reference signal generator. Applying this principle to the present situation, although it is nonclassical, we see that this principle is not satisfied for a strictly proper plant $G(s)$, for $G(s)$ has infinity as its zero, whereas the generator of the periodic signal has a pole of arbitrarily high frequency. To put it differently, if $G(s)$ is strictly proper, then it integrates the input at least once, and hence the output will be smoothed out to some extent, thereby making it impossible to track a signal with an infinity sharp edge, i.e., a signal contain arbitrarily high-frequency modes.

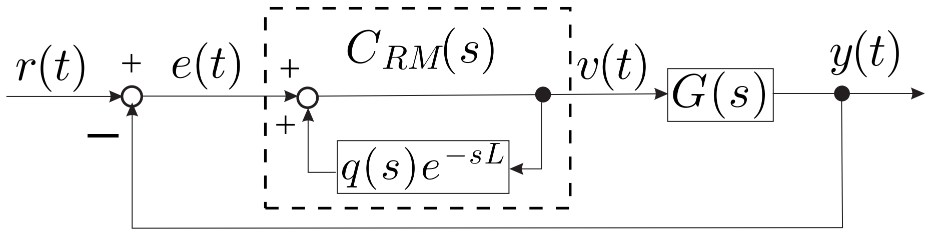

Figure 1.3: Conventional modified repetitive control system

However, the actually control plant is strictly proper and has any relative degree. This is unfortunate, but not entirely irreconcilable since this is caused by the apparently unrealistic demand of tracking any periodic signal, which contains arbitrarily high-frequency modes. It is therefore natural to expect that the stability condition can be relaxed by reducing the loop-gain of the repetitive compensator in a higher frequency range. This leads to the idea of a modified repetitive control system [27, 28] shown in Figure 1.3. In this control system, the delay element e^{-sL} is replaced by $q(s)e^{-sL}$ for a suitable proper stable rational $q(s)$, namely low-pass filter with following frequency characteristics:

i) $q(j\omega) \simeq 1$ for $|\omega| \leq \omega_c$,

ii) $|q(j\omega)| \leq \rho < 1$ for $|\omega| > \omega_c$,

where ω_c is a suitable cutoff frequency. Generally, this low-pass filter may be realized by a simple first-order system

$$q(s) = \frac{1}{1+\tau s}, \quad \tau > 0 \tag{1.6}$$

or

$$q(s) = \frac{1+\tau_2\, s}{1+\tau_1\, s}, \quad \tau_1 > \tau_2 > 0. \tag{1.7}$$

Then, the transfer function of the modified repetitive controller is

$$C_{RM}(s) = \frac{1}{1-q(s)e^{-sL}} \tag{1.8}$$

and the corresponding sensitivity function of the modified repetitive control system becomes

$$S_{RM}(s) = \frac{1}{C_{RM}(s)} \frac{1}{\dfrac{1}{C_{RM}(s)} + G(s)}. \tag{1.9}$$

The utilization of the low-pass filter, $q(s)$, changes the tracking characteristics. For example, consider the widely used low-pass filter $q(s) = 1/(\tau\, s + 1)$. Bode plots of repetitive controller $C_R(s)$ and modified repetitive controller $C_{RM}(s)$ for $L = 2\pi\, s$ in Figure 1.4 show that, when $\tau = 0.001$s, the gains of $C_{RM}(s)$ at the angular frequencies of the fundamental and second harmonic drop from infinity to 56.57[dB] and 67.85[dB], respectively. Therefore, a steady-state tracking error arises when $C_{RM}(s)$ is employed in a modified repetitive-control system. Furthermore, if the cutoff angular frequency, $\omega_c = 1/\tau$, of low-pass filter is made 100 times smaller, i.e., $\tau = 0.1$s, then the gains just mentioned decrease dramatically to 45.97[dB] and 34.44[dB]. This greatly increases the steady state tracking error. From above discussion, it is clear that, by introducing the low-pass filter, the robustness stability of repetitive control systems was guaranteed, but at cost of degrading performances at high frequencies.

Therefore, in order to obtain good tracking precision or disturbance attenuation performance, the cutoff angular frequency of the low-pass filter must be as high as possible. However, the investigation of the stability of modified repetitive control system reveals that the restriction of frequency band to be tracked is imposed only

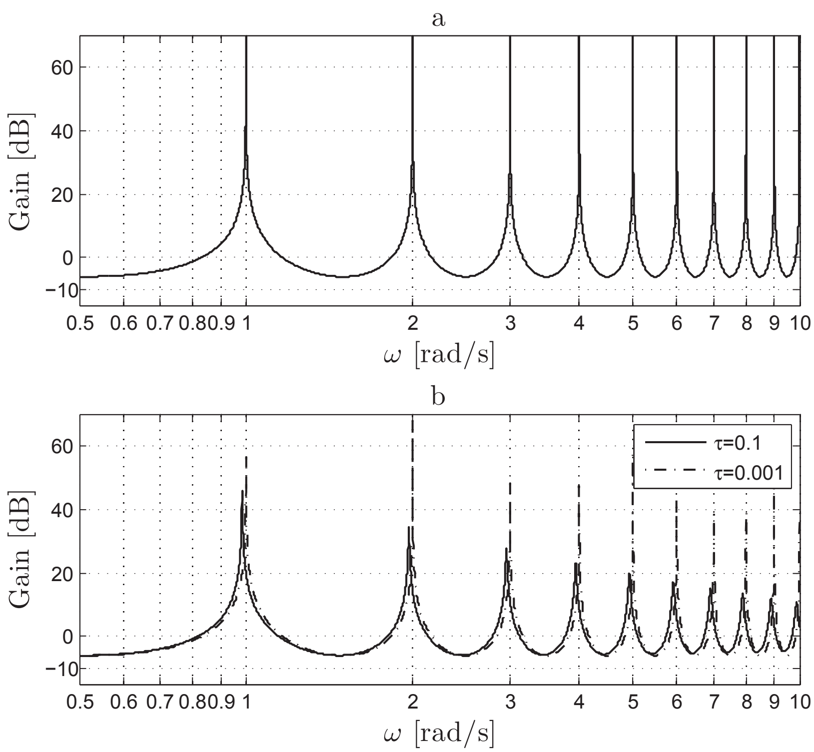

Figure 1.4: Bode plots of repetitive controller $C_R(s)$ (a) and modified repetitive controller $C_{RM}(s)$ (b)

for non-minimal phase plants [29]. This results in the tradeoff problem among steady state accuracy, robustness and transient response of the control system.

1.4 Review on modified repetitive control system design

The design problems of modified repetitive control systems are mainly to choose and optimize the dynamic compensator and the low-pass filter. The selection of parameters involves robustness stability, tracking performance, attenuation performance and tradeoff problem. Since introduction of repetitive control to the control community, a great deal of research effort has been devoted to the design methods for modified repetitive control system. What's more, various structures and algorithms have been proposed in existing literature. In this section, a detailed review of the main work on the design methods for modified repetitive control system is specified.

1.4.1 Frequency domain analysis-based design method

The frequency domain analysis and synthesis method is a main approach for modified repetitive control system design. In [1, 30], some general design guidelines were developed. Srinivasan et al. [31] analyzed the single-input/single-output continuous time repetitive control system using the regeneration spectrum. It has been proved that shaping the regeneration spectrum is an effective way to alter the relative stability and transient response of system. A modified repetitive control scheme by shaping both regeneration spectrum and the sensitivity function was proposed in [32]. To achieve a specified level of nominal performance, Srinivasan et al. [33] used the Nevanlinna-Pick interpolation method to modify repetitive controller by optimizing a measure of stability robustness (a weighting function on the complementary sensitivity function). To offer an ease of multi-objective design, Guvenc [34] described a graphical repetitive controller design procedure, which is based on mapping frequency domain performance specifications of sensitivity function magnitude and regeneration spectrum to the controller parameter space. Moon et al. [35] designed of a repetitive controller by a graphical technique based on the frequency domain analysis of a linear interval system.

Another frequency domain analysis method is to make the magnitude of system sensitivity function in the middle of two adjacent harmonics as an optimization objective to design modified repetitive control system [36]. In order to improve the tracking or attenuation performance at the high frequencies for reference input or disturbance, Kim and Tsao modified the structure of low-pass filter to make the sensitivity approximately squared by comparing with original modified repetitive control system. This method improves the tracking or attenuation performance of system at harmonics.

From Figure 1.4, we can find that the introduction of the low-pass filter $q(s)$, while improving the system stability, also introduce phase lag which shifts the frequency at which the gain of the repetitive controller reaches the maximum value. To compensate the phase lag induced by $q(s)$, Sugimoto et al. [37] proposed to modify the dead time term so that the maximum gain is exactly situated at the fundamental frequency of the periodic signal. Extending this work, Chen and Lin [38] introduced a lead compensator to widen the bandwidth of low-pass filter and improve system gain at high frequencies. The optimal modified repetitive controller is obtained by solving two optimization problems.

1.4.2 Linear matrix inequality-based design method

Linear matrix inequality (LMI), regarded as an effective tool to deal with system and control problem, has been applied to design and analysis for modified repetitive control systems. One of the first papers to consider the design of the repetitive controller as a convex optimization problem was [39]. In that paper, LMI-conditions are derived to design the low-pass filter associated with the repetitive structure. It is important to point out that in this work the authors only presented conditions to verify whether a priory fixed cutoff frequency of the low-pass filter results in a feasible solution and not focus in the design of the stabilizing controller. Latter, this work was extended and analyzed in [13, 40, 41, 42, 43] and references therein. A simultaneous optimization of the low-pass filter and state-feedback controller was design by She et al. [43] based on LMIs. They proposed an iterative algorithm to obtain the best combination of low-pass filter and state-feedback controller.

For linear systems with time-varying state-delay [44] or input delay [41], the robustness stability criterion is derived in the form of LMI and the design problem of modified repetitive controller is transformed to an LMI-constrained optimization problem. However, the control parameter obtained by solving LMI feasible problem has some conservation that restricts the control performance. To reduce the conservatism, free weighting matrices and descriptor model transformation are usually introduced to derive robustness stability conditions. In [45], considering single-input/single-output systems for the presence of control saturation, a modified state-space repetitive control structure is designed and conditions in a "quasi" LMI form are proposed. Flores et al. [46] generalized the results in [45] to consider multiple-input/multiple-output systems.

1.4.3 H_∞ robust design method

In fact, the H_∞ control approach has been also widely used in many modified repetitive control system design [47, 48]. It is mainly used to for solving robustness and optimization problems, and provides a kind of design method of state-feedback controller. For instance, Wang et al. [49] proposed a three-step design method for state-feedback controller. Wang and Tsao [50, 51] basing on H_∞ control approach, designed a robust stabilizing modified repetitive controller for time-varying periodic signals. Li and Tsao [52] viewed the time-delay element in the internal model as an uncertainty and employed the H_∞ control approach to obtain the robust stability condition and robustness performance. Using the same method, She et al. [53] proposed simultaneous optimization design method by introducing the state-feedback gains. The design problem of modified repetitive controller is converted into convex optimization problem in the form of LMI. Designed a iterative algorithm to calculate the cutoff frequency and state-feedback gains.

To some extent, H_∞ control method can improve the robust performance of repetitive control system, but the design of state-feedback controller is still independent of repetitive controller. This may result in some conservatism and affects the trade-off problem between robustness stability and robust performance.

1.4.4 Two-degree-of-freedom structure design method

To relieve the burden of the system stability on one controller, a two-degree-of-freedom structure in Figure 1.5 can also be considered, where $C(s)$ is a modified repetitive controller. It contains a feed forward compensator and feedback compensator. Some synthesis procedure for this class of repetitive control system, such as the state-space approach, coprime factorization approach, H_∞ optimal design approach and sliding mode variable structure control approach can be found in [2, 4, 28, 29, 54, 55, 56, 57], where the main task is to design the stabilizing controllers.

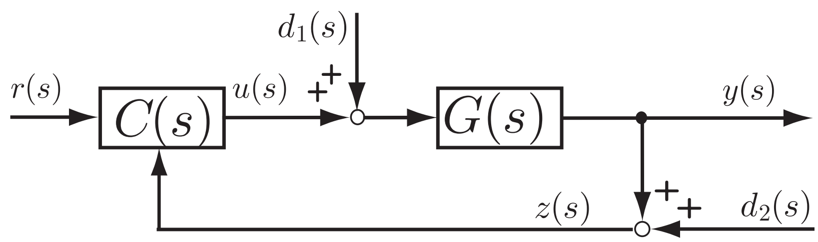

Figure 1.5: Two-degree-of-freedom modified repetitive control system

Peery et al. [54] proposed a two-degree-of-freedom H_∞ optimal respective control structure with fixed low-pass filter for Single-Input/Single-Output system. Chen et al. [56, 57] established a two-degree-of freedom modified repetitive controller for the rejection for disturbance and guaranteed the robustness of the system with actuator saturation uncertainties. Dong et al. [55] studied the design method of two-degree-of-freedom modified repetitive controller based on the factorization approach. Yamada et al. [58] designed a modified repetitive control system with feed-forward controller and feedback controller. Sakanushi et al. [59] proposed a design method for two-degree-of-freedom simple repetitive control systems for multiple-input/multiple-output plants. The design method based on two-degree-of-freedom eliminates the influence of the unstable poles to improve stability and robustness. However, there is no systematical approach for selecting the parameters of controllers. The state-space-based synthesis procedure relying on some indirect specifications of performance, for example, in the form of noise covariance and weighting matrices, involves currently much trial and error.

1.4.5 Two-dimensional-based design method

A close examination of repetitive control shows that it actually involves two independent types of actions:

- continuous control within each repetition period and
- discrete learning between periods.

From the standpoint of system design, it is difficult to stabilize a repetitive-control system, and all design methods are developed to focus mainly on stability. That is, they do not accurately describe what actually happens, or they do not thoroughly investigate the essence of the control and learning actions with only considering the overall results in the time domain. As a result, researchers impose not only very strict requirements on the plant, but also a limit on how much control performance can be improved [60, 13, 61].

From the repetitive compensator $C_R(s)$ shown in Figure 1.2, the control output $v(t)$ can be represented in time domain as

$$v(t) = \begin{cases} e(t) & 0 \leq t < L \\ v(t-L) + e(t) & t \geq L \end{cases} \tag{1.10}$$

where L is a delay element, i.e., the period of reference input $r(t)$, $e(t) = r(t) - y(t)$ is track error of the closed-loop system.

Setting the state-space description of plant is

$$\begin{cases} \dot{x}(t) & = Ax(t) + Bu(t), \\ y(t) & = Cx(t) + Du(t). \end{cases} \tag{1.11}$$

where $x(t) \in R^n$ is the state of plant, $u(t) \in R^m$ is control input, and $y(t) \in R^p$ is control output. Without losing generality, set the system matrices (A, B, C, D) are controllable and observable. For convenient, set the $m = p = 1$, i.e., Single-Input/Single-Output system.

The design problem of repetitive control system is design a controller containing $v(t)$ such that the closed-loop system is stable and the tracking error is convergent to 0 for any reference input with given period L. As a matter of fact, the stable vector $x(t)$ and control input $u(t)$ when closed-loop system is stable for any given reference input $r(t)$. However, the difference of state vector $\Delta x(t)$ and the difference of control input $\Delta u(t)$ between two adjacent periods are convergent to 0. From this aspect, consider the variation of these differences. Setting variable $\xi(t)(\xi \in \{x, y, u, e\})$ is equal to 0 as $t < 0$ and

$$\Delta\xi(t) = \xi(t) - \xi(t - L) \tag{1.12}$$

then

$$\Delta\dot{x}(t) = A\Delta x(t) + B\Delta u(t) \tag{1.13}$$

$$e(t) = e(t - L) - C\Delta x(t) - D\Delta u(t) \tag{1.14}$$

Equation (1.13) and (1.14) demonstrate the control and learn process of the repetitive control process. We divide the infinite interval $[0, +\infty)$ into an infinite number of finite intervals, $[kL, (k + 1)L)(k = 0, 1, \cdots)$. Then, for any $t \in [0, +\infty)$, there exists an interval $[kL, (k + 1)L)$ such that

$$t = kL + \tau, \tau \in [0, L)$$

This allows us to write the variable $\xi(t)$ in the time domain as

$$\xi(t) = \xi(kL + \tau) := \xi(k, \tau),$$

and

$$\Delta\xi(t) := \triangle\xi(k, \tau) = \xi(k, \tau) - \xi(k - 1, \tau)$$

Then, the equations (1.13) and (1.14) are converted into

$$\Delta\dot{x}(k,\tau) = A\Delta x(k,\tau) + B\Delta u(k,\tau) \tag{1.15}$$

$$e(k,\tau) - e(k-1,\tau) = -C\Delta x(k,\tau) - D\Delta u(k,\tau) \tag{1.16}$$

Equations (1.15) and (1.16) are represented by vector as

$$\begin{bmatrix} \Delta\dot{x}(k,\tau) \\ e(k,\tau)) \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 1 \end{bmatrix} \begin{bmatrix} \Delta x(k,\tau) \\ e(k-1,\tau) \end{bmatrix} + \begin{bmatrix} B \\ -D \end{bmatrix} \Delta u(k,\tau) \tag{1.17}$$

If we can design a two-dimensional controller such that $\Delta u(k,\tau)$ make the continuous/discrete two-dimensional system (1.17) is asymptotically stable, the corresponding repetitive control system (1.11) is asymptotically stable and convergent to zero. From these, the design problem of repetitive control system (1.11) is equivalent to stabilization problem of the continuous/discrete two-dimensional system (1.17).

Wu et al. [62] presented a design method of modified repetitive control system for a class of linear system based on two-dimensional continuous/discrete hybrid model. The design problem for the modified repetitive controller is converted in a state-feedback design problem for a continuous-discrete two-dimensional system. And then the design problem is solved by combing two-dimensional Lyapunov theory with LMIs approach. Later, wu et la. [63] proposed a guaranteed cost design method of modified repetitive control system based on two-dimensional hybrid model. Then Zhang et al. [64] designed a modified repetitive control system by using state feedback hybrid model based on two-dimensional hybrid model. This result can be extended to handle a plant with a time-varying uncertainty. Zhou et al. [65] presented a robust modified repetitive control system based on both LMI and two-dimensional hybrid model. It can adjust the control and learning actions individually by adjusting the parameters contained in the LMI.

1.4.6 Parameterization design method

Parameterization is a very common method used for dealing with the design problem of control system. This method is based on factorization theory. For parameterization-based design of modified repetitive controlled, Yamada et al have done a lot of work.

1. Minimum phase

   A parameterization of all modified repetitive controllers for the strictly proper plants is given by Yamada and Okuyama [66] . Yamada et al. [67] proposed a design method for robust stabilizing modified repetitive controllers without solving the μ synthesis problem. This method is effective for minimum phase plants.

2. Non-minimum phase

   Based on [67], Yamada et al. [68, 69] clarified the parameterization of all stabilizing mollified repetitive controllers for non-minimum phase systems. And then, Yamada et al. [70] gave a design method for robust stabilizing modified repetitive controllers for non-minimum phase plants such that the frequency range in which the output follows the periodic reference input is not restricted. In [71], the parameterization of all robust stabilizing modified repetitive controllers is given by extending the result in[70].

3. Robust stabilization

   Yamada et al. [72] proposed a parameterization of all robust stabilizing simple repetitive controllers such that the controller work as a robust stabilizing modified repetitive controller. Chen [73] solved the robust stabilizing problem for the modified repetitive control system with multiple-input/multiple-output plants. Extending this work, the robust stabilizing modified repetitive controller for multiple-input/multiple-output plants is proposed with specified input-output frequency characteristic [74].

4. Time-delay

   The design method of all stabilizing modified repetitive controllers for time-delay systems with the specified input-output frequency characteristics has been studied by Satoh et al.[75]. Referencing to [68, 69], the parameterization of all robust stabilizing controllers for time-delay plants is obtained [76]. This design method includes a free parameter which is designed to achieve desirable control characteristics.

5. Multiple-input/multiple-output plants

   For the multiple-input/multiple-output plants, Yamada et al. [77] have design the modified repetitive controllers based on the references. Chen et al. [78] obtained the stabilizing modified repetitive controller by using the free parameter in the parameterization.

6. Two-degree-of-freedom structure for single-input/single-output plants

   Yamada et al. [79] proposed the parameterization of all stabilizing two-degree-of-freedom modified repetitive controllers those can specify the input-output characteristic and the feedback characteristic separately. The design of control system with multi-period structure for single-input/single-output plants has been solved in [80].

7. Two-degree-of-freedom structure for multiple-input/multiple-output plants

   Based on existing literature, the problem of obtaining the parameterization of all stabilizing two-degree-of-freedom modified repetitive controller for multiple-input/multiple-output plants has been solved [81]. In this paper, the input-output characteristic and the feedback characteristic are specified separately. In order to specify the input-output characteristic and the disturbance attenuation characteristic, Chen et al. [82] proposed a design method for two-degree-of-freedom multi-period repetitive controllers for multiple-input/multiple-output systems. The input-output characteristic can be specified independent from the disturbance attenuation characteristic.

All above design methods of modified repetitive controllers are based on the coprime factorization. When control plant has an uncertainty or time-delay, the H_∞ control approach will be introduced to simplify the design problem.

1.4.7 Existing problems

The repetitive control technique has been widely applied in many areas since it was proposed. That fully displays its extensive engineering application value, and it has been proven to be an effective control strategy for the control problem of external periodic excitation signal. With the deeply research on the repetitive control theory and widely practicing in diverse areas, the above achievements promote the development of repetitive control, whereas, there are some issues existing:

- In the case of designing the multiple-input/multiple-output modified repetitive control system, the relationship between inputs and outputs should be coordinated to guarantee good control performance. Particularly, for multiple-input/multiple-output plants with uncertainties, the robust stability conditions and the simple design method are indispensable.

- In practical, it is inevitably to deal with the position-dependent (time-varying) or uncertain periodic signals. For example, to track the time-varying periodic signals, generally transform the linear control plant in the time domain into a nonlinear control plant in the spatial domain, or combine the adaptive control approach. This makes the design problem more complicated. For the periodic signal with uncertain period-time, the perfect performance only can be guaranteed by using the high-order repetitive controller for a small variation. There is no effective method for this situation.

1.5 Organization of the thesis

Most designs of modified repetitive control systems are based on the use of a design model. The relationship between models and the reality they represent is subtle and complex. A mathematical model provides a map from inputs to outputs. The quality of a model depends on how closely its responses match those of the true plant. There is no single fixed model that can respond exactly like the true plant. Hence, we need, at the very least, a set of maps. The term uncertainty refers to the differences or errors between models and reality, and whatever mechanism is used to express these errors will be called a representation of uncertainty. To be practical, consider the problem of bounding the magnitude of the effect of some uncertainty on the nominal plant. In the simplest case, this power spectrum is assumed to be independent of the input. This is equivalent to assuming that the uncertainty is generated by an additive noise signal with a bounded power spectrum; the uncertainty is represented as additive noise. Of course, no physical system is linear with additive noise, but some aspects of physical behavior are approximated quite well using this model. With uncertainties, the design problem of modified repetitive control should consider the robustness of control system. By this reason, this thesis is organized as follows:

In Chapter 2, we propose a design method of robust stabilizing modified repetitive controllers for multiple-input/multiple-output plants. The basic idea of robust stabilizing modified repetitive controller is very simple. If the modified repetitive control system is robustly stable for the multiple-input/multiple-output plant with uncertainty, then the modified repetitive controller must satisfy the robustness stability condition. The parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output plant with uncertainty is obtained by employing H_∞ control theory based on the Riccati equation. The robust stabilizing controller contains free parameters that are designed to achieve desirable control characteristic. When the free parameters of the parameterization of all robust stabilizing controllers is adequately chosen, then the controller works as robust stabilizing modified repetitive controller. In this chapter, the bandwidth of low-pass filter has been analyzed. In order to simplify the design process and avoid the wrong results obtained by graphical method, the robust stability conditions are converted to LMIs-constraint conditions by employing the delay-dependent bounded real lemma. The effectiveness of this proposed method is illustrated by a numerical example.

In Chapter 3, we address the problem of designing an optimal robust stabilizing modified repetitive controller for a strictly proper plant with time-varying uncertainties. This repetitive control system is used to reject position-dependent (time-varying) periodic disturbances. A modified repetitive controller with time-varying delay structure, inserted by a low-pass filter and an adjustable parameter, is developed for this class of system. Two linear matrix inequalities (LMIs)-based robust stability conditions of the closed-loop system with time-varying state delay are derived for fixed parameters. One is a delay-dependent robust stability condition that is derived based on the free-weight matrix. The other robust stability condition is obtained based on the H_∞ control problem by introducing a linear unitary operator. To obtain the desired controller, the design problems are converted to two LMI-constrained optimization problems by reformulating the LMIs given in the robust stability conditions. The validity of the proposed method is verified through a numerical example.

Chapter 4 summarizes the result of the present study by the conclusion and states the future work of the modified repetitive control system.

Notation

R	the set of real numbers.
R_+	$R \cup \{\infty\}$.
$R(s)$	the set of real rational functions with s.
RH_∞	the set of stable proper real rational functions.
H_∞	the set of stable causal functions.
$D^\perp$	orthogonal complement of D, i.e., $\begin{bmatrix} D & D^\perp \end{bmatrix}$ or $\begin{bmatrix} D \\ D^\perp \end{bmatrix}$ is unitary.
A^T	transpose of A.
$A^\dagger$	pseudo inverse of A.
$\rho(\{\cdot\})$	spectral radius of $\{\cdot\}$.
$\bar{\sigma}(\{\cdot\})$	largest singular value of $\{\cdot\}$.
$\\|\{\cdot\}\\|_\infty$	H_∞ norm of $\{\cdot\}$.
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	represents the state space description $C(sI-A)^{-1}B+D$.
$\mathbb{R}^n$	the n-dimensional Euclidean space.
$\mathbb{R}^{n\times n}$	the set of all $n \times n$ real matrices.
I	the identity matrix.
$L_2[0,t_f]$	the set of function $f(t)$ satisfies $\int_0^{t_f} f(t)f(t)dt < \infty$.
$*$	the symmetric terms in a symmetric matrix as $\begin{bmatrix} A & B \\ * & C \end{bmatrix} = \begin{bmatrix} A & B \\ B^T & C \end{bmatrix}$.

Chapter 2

Robust Stabilizing Problem for Multiple-Input/Multiple-Output Plants

2.1 Introduction

In this chapter, we examine a design method for robust stabilizing modified repetitive controllers using the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output plants. A repetitive control system is a type of servomechanism for periodic reference inputs. That is, the repetitive control system follows the periodic reference input without steady state error, even if a periodic disturbance or uncertainty exists in the plant [8, 83, 84, 85, 86, 28, 89, 87, 88, 9, 90, 91, 92]. It is difficult to design stabilizing controllers for the strictly proper plant, because a repetitive control system that follows any periodic reference input without steady state error is a neutral type of time-delay control system [90]. To design a repetitive control system that follows any periodic reference input without steady state error, the plant must be biproper [84, 85, 86, 28, 89, 87, 88, 9, 90]. Ikeda and Takano [91] pointed out that it is physically difficult for the output to follow any periodic reference input without steady state error. In addition, they showed that the repetitive control system is L_2 stable for periodic signals that do not include infinite frequency signals if the relative degree of the controller is one. In practice, the plant is strictly proper and has two or more relative degree. Many design methods for repetitive control systems for strictly proper plants those have any relative degree have been given [84, 85, 86, 28, 89, 87, 88, 9, 90]. These studies are divided into two types. One uses a low-pass filter [84, 85, 86, 28, 89, 87, 88, 9] and the other uses an attenuator [90]. The latter is difficult to design because it uses a state variable time-delay in the repetitive controller [90]. The former has a simple structure and is easily designed. Therefore, the former type of repetitive control system is called the modified repetitive control system [84, 85, 86, 28, 89, 87, 88, 9].

When modified repetitive control design methods are applied to real systems, the influence of uncertainties in the plant must be considered. In some cases, uncertainties in the plant make the modified repetitive control system unstable, even though the controller was designed to stabilize the nominal plant. The stability problem with uncertainty is known as the robust stability problem [93]. The robust stability problem of modified repetitive control systems was considered by Hara et al. [87]. The robust stability condition for modified repetitive control systems was reduced to the μ synthesis problem [87], but the μ synthesis problem cannot be solved analytically. That is, in order to solve the μ synthesis problem, we must solve an H_∞ problem iteratively using the $D-K$ iteration method. Furthermore, the convergence of iterative methods to solve the μ synthesis problem is not guaranteed. Yamada et al. tackled this problem and proposed a design method for robust repetitive control systems without solving the μ synthesis problem [67]. In this way, several design methods of robust stabilizing modified repetitive controllers have been considered.

On the other hand, there exists an important control problem to find all stabilizing controllers named the parameterization problem [96, 97, 95, 98, 99]. Yamada and

Satoh clarified the parameterization of all robust stabilizing modified repetitive controllers [100]. However, the method by Yamada and Satoh [100] cannot be applied to multiple-input/multiple-output systems. Because, the method by Yamada and Satoh [100] uses the characteristic of single-input/single-output system. Many real plants include multiple-input and multiple-output. In addition, the parameterization is useful to design stabilizing controllers [96, 97, 98, 99]. Therefore, the problem of obtaining the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output plants is important. Chen et al. examined this problem and clarified the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output plants [73]. However, in [73], complete proof of the theorem for the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output plants was omitted on account of limiting space. In addition, using the obtained parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output plants, control characteristics are not examined. Furthermore, a design method for robust stabilizing modified repetitive control system for multiple-input/multiple-output plants are not described. Therefore, we cannot find whether or not the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output plants in [73] is valid.

In this chapter, we give a complete proof of the theorem for the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output plants omitted in [73] and show effectiveness of the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output plants. First, we give a complete proof of the theorem for the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output plants omitted in [73]. Next, we clarify control characteristics using the parameterization in [73]. The generalized design method for free parameters has been proposed. Furthermore, the bandwidth limitation of cutoff frequency of low-pass filter which is used to specify disturbance attenuation characteristic is obtained by analyzing the robust stability condition. In order to simplify the design process and avoid the wrong results obtained by graphical method, the robust stability conditions are converted into LMIs-constraint conditions by employing the delay-dependent bounded real lemma. In addition, a design procedure using the parameterization is presented. Finally a numerical example is illustrated to show the effectiveness of the proposed method.

2.2 Problem Formulation

Consider the modified repetitive control system in Figure 2.1

$$\begin{cases} y &= G(s)u + d \\ u &= C(s)(r - y) \end{cases}, \tag{2.1}$$

where $G(s) \in R^{p\times p}(s)$ is the multiple-input/multiple-output plant, $G(s)$ is assumed

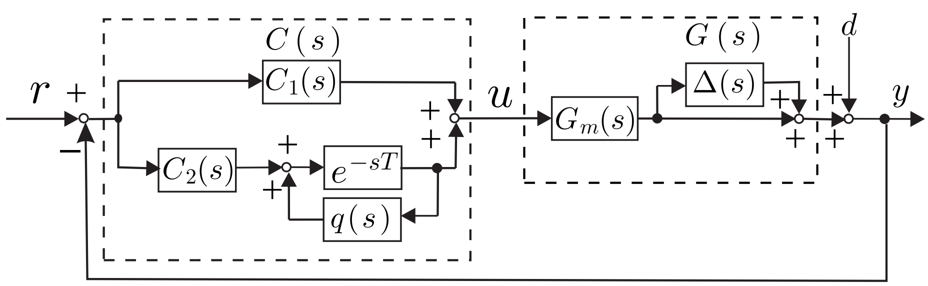

Figure 2.1: Modified repetitive control system with uncertainty

to be coprime. $C(s) \in R^{p\times p}(s)$ is the modified repetitive controller defined later, $u \in R^p$ is the control input, $y \in R^p$ is the output and $r \in R^p$ is the periodic reference input with period $T > 0$ satisfying

$$r(t + T) = r(t) \ \ (\forall t \geq 0). \tag{2.2}$$

The nominal plant of $G(s)$ is denoted by $G_m(s) \in R^{p\times p}(s)$. Both $G(s)$ and $G_m(s)$ are assumed to have no zero or pole on the imaginary axis. In addition, it is assumed that the number of poles of $G(s)$ in the closed right half plane is equal to that of $G_m(s)$. The relation between the plant $G(s)$ and the nominal plant $G_m(s)$ is written as

$$G(s) = (I + \Delta(s))G_m(s), \tag{2.3}$$

where $\Delta(s)$ is an uncertainty. The set of $\Delta(s)$ is all rational functions satisfying

$$\bar{\sigma}\left\{\Delta(j\omega)\right\} < |W_T(j\omega)| \quad (\forall\omega \in R_+), \tag{2.4}$$

where $W_T(s)$ is a stable rational function.

The robust stability condition for the plant $G(s)$ with uncertainty $\Delta(s)$ satisfying (2.4) is given by

$$\|T(s)W_T(s)\|_\infty < 1, \tag{2.5}$$

where $T(s)$ is the complementary sensitivity function given by

$$T(s) = (I + G_m(s)C(s))^{-1} G_m(s)C(s). \tag{2.6}$$

According to [84, 85, 86, 28, 89, 87, 88, 9], in order for the output $y(s)$ to follow the periodic reference input $r(s)$ with period T in (2.1) with small steady state error, the controller $C(s)$ must have the following structure

$$C(s) \;=\; C_1(s) + C_2(s)e^{-sT}\left(I - q(s)e^{-sT}\right)^{-1}, \tag{2.7}$$

where $q(s) \in R^{p\times p}(s)$ is a low-pass filter satisfying $q(0) = I$ and $\operatorname{rank} q(s) = p$, $C_1(s) \in R^{p\times p}(s)$ and $C_2(s) \in R^{p\times p}(s)$ satisfying $\operatorname{rank} C_2(s) = p$. In the following, $e^{-sT}(I - q(s)e^{-sT})^{-1}$ defines the internal model for the periodic signal with period T. According to [84, 85, 86, 28, 89, 87, 88, 9], if the low-pass filter $q(s)$ satisfy

$$\bar{\sigma}\left\{I - q(j\omega_i)\right\} \simeq 0 \quad (i = 0, 1, \ldots, \hbar), \tag{2.8}$$

where ω_i are the frequency components of the periodic reference input $r(s)$ written by

$$\omega_i = \frac{2\pi}{T} i \quad (i = 0, 1, \ldots, \hbar), \tag{2.9}$$

and $\omega_\hbar$ is the maximum frequency component, then the output $y(s)$ in (2.1) follows the periodic reference input $r(s)$ with small steady state error. The controller written by (2.7) is called the modified repetitive controller [84, 85, 86, 28, 89, 87, 88, 9].

The problem considered in this paper is to obtain the parameterization of all robust stabilizing modified repetitive controllers $C(s)$ in (2.7) satisfying (2.5) for multiple-input/multiple-output plant in (2.3) with any uncertainty $\Delta(s)$ satisfying (2.4).

2.3 The parameterization of all robust stabilizing modified repetitive controllers for MIMO plants

In this section, we give the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output plants.

In order to obtain the parameterization of all robust stabilizing modified repetitive controllers, we must see that controllers $C(s)$ hold (2.5). The problem of obtaining the controller $C(s)$, which is not necessarily a modified repetitive controller, satisfying (2.5) is equivalent to the following H_∞ control problem. In order to obtain the controller $C(s)$ satisfying (2.5), we consider the control system shown in Figure 2.2. $P(s)$ is selected such that the transfer function from w to z in Figure 2.2 is equal to $T(s)W_T(s)$. The state space description of $P(s)$ is, in general,

$$\left\{ \begin{array}{rclll} \dot{x}(t) & = & Ax(t) & +B_1w(t) & +B_2u(t) \\ z(t) & = & C_1x(t) & & +D_{12}u(t) \\ y(t) & = & C_2x(t) & +D_{21}w(t) & \end{array} \right. , \tag{2.10}$$

where $A \in R^{n\times n}$, $B_1 \in R^{n\times m}$, $B_2 \in R^{n\times p}$, $C_1 \in R^{m\times n}$, $C_2 \in R^{m\times n}$, $D_{12} \in R^{m\times p}$, $D_{21} \in R^{m\times m}$, $x(t) \in R^n$, $w(t) \in R^m$, $z(t) \in R^m$, $u(t) \in R^p$ and $y(t) \in R^m$. $P(s)$ is called as the generalized plant. $P(s)$ is assumed to satisfy the following assumptions [93]:

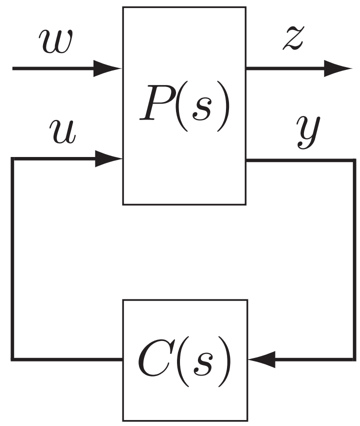

Figure 2.2: Block diagram of H_∞ control problem

1. (C_2, A) is detectable, (A, B_2) is stabilizable.
2. D_{12} has full column rank, and D_{21} has full row rank.
3. rank $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} = n + p \quad (\forall \omega \in R_+)$ and
4. rank $\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} = n + m \quad (\forall \omega \in R_+)$.

Under these assumptions, according to [93], following lemma holds true.

Lemma 2.1. *If controllers satisfying* (2.5) *exist, both*

$$X\left(A - B_2 D_{12}^{\dagger} C_1\right) + \left(A - B_2 D_{12}^{\dagger} C_1\right)^T X \\ +X\left\{B_1 B_1^T - B_2\left(D_{12}^T D_{12}\right)^{-1} B_2^T\right\} X + \left(D_{12}^{\perp} C_1\right)^T D_{12}^{\perp} C_1 = 0 \quad (2.11)$$

and

$$Y\left(A - B_1 D_{21}^{\dagger} C_2\right)^T + \left(A - B_1 D_{21}^{\dagger} C_2\right) Y \\ +Y\left\{C_1^T C_1 - C_2^T\left(D_{21} D_{21}^T\right)^{-1} C_2\right\} Y + B_1 D_{21}^{\perp}\left(B_1 D_{21}^{\perp}\right)^T = 0 \quad (2.12)$$

have solutions $X \geq 0$ *and* $Y \geq 0$ *such that*

$$\rho(XY) < 1 \quad (2.13)$$

and both

$$A - B_2 D_{12}^{\dagger} C_1 + \left\{ B_1 B_1^T - B_2 \left(D_{12}^T D_{12} \right)^{-1} B_2^T \right\} X \tag{2.14}$$

and

$$A - B_1 D_{21}^{\dagger} C_2 + Y \left\{ C_1^T C_1 - C_2^T \left(D_{21} D_{21}^T \right)^{-1} C_2 \right\} \tag{2.15}$$

have no eigenvalue in the closed right half plane. Using X and Y, the parameterization of all controllers satisfying (2.5) is given by

$$C(s) = C_{11}(s) + C_{12}(s)Q(s)(I - C_{22}(s)Q(s))^{-1}C_{21}(s), \tag{2.16}$$

where

$$\begin{bmatrix} C_{11}(s) & C_{12}(s) \\ C_{21}(s) & C_{22}(s) \end{bmatrix} = \left[\begin{array}{c|cc} A_c & B_{c1} & B_{c2} \\ \hline C_{c1} & D_{c11} & D_{c12} \\ C_{c2} & D_{c21} & D_{c22} \end{array} \right], \tag{2.17}$$

$$\begin{aligned} A_c &= A + B_1 B_1^T X - B_2 \left(D_{12}^{\dagger} C_1 + E_{12}^{-1} B_2^T X \right) \\ &\quad - (I - YX)^{-1} \left(B_1 D_{21}^{\dagger} + Y C_2^T E_{21}^{-1} \right) \left(C_2 + D_{21} B_1^T X \right), \end{aligned}$$

$$\begin{aligned} B_{c1} &= (I - YX)^{-1} \left(B_1 D_{21}^{\dagger} + Y C_2^T E_{21}^{-1} \right), \\ B_{c2} &= (I - YX)^{-1} \left(B_2 + Y C_1^T D_{12} \right) E_{12}^{-1/2}, \end{aligned}$$

$$\begin{aligned} C_{c1} &= -D_{12}^{\dagger} C_1 - E_{12}^{-1} B_2^T X, \\ C_{c2} &= -E_{21}^{-1/2} \left(C_2 + D_{21} B_1^T X \right), \end{aligned}$$

$$D_{c11} = 0, \quad D_{c12} = E_{12}^{-1/2}, \quad D_{c21} = E_{21}^{-1/2}, \quad D_{c22} = 0,$$

$$E_{12} = D_{12}^T D_{12}, \quad E_{21} = D_{21} D_{21}^T$$

and $Q(s) \in H_\infty$ *is any function satisfying* $\|Q(s)\|_\infty < 1$. $C(s)$ *in* (2.16) *is written using Linear Fractional Transformation(LFT). Using homogeneous transformation,* (2.16) *is rewritten by*

$$\begin{aligned} C(s) &= \left(Z_{11}(s)Q(s) + Z_{12}(s)\right)\left(Z_{21}(s)Q(s) + Z_{22}(s)\right)^{-1} \\ &= \left(Q(s)\tilde{Z}_{21}(s) + \tilde{Z}_{22}(s)\right)^{-1}\left(Q(s)\tilde{Z}_{11}(s) + \tilde{Z}_{12}(s)\right), \end{aligned} \tag{2.18}$$

where $Z_{ij}(s)(i = 1, 2; j = 1, 2)$ *and* $\tilde{Z}_{ij}(s)(i = 1, 2; j = 1, 2)$ *are defined by*

$$\begin{bmatrix} Z_{11}(s) & Z_{12}(s) \\ Z_{21}(s) & Z_{22}(s) \end{bmatrix} = \begin{bmatrix} C_{12}(s) - C_{11}(s)C_{21}^{-1}(s)C_{22}(s) & C_{11}(s)C_{21}^{-1}(s) \\ -C_{21}^{-1}(s)C_{22}(s) & C_{21}^{-1}(s) \end{bmatrix} \tag{2.19}$$

and

$$\begin{bmatrix} \tilde{Z}_{11}(s) & \tilde{Z}_{12}(s) \\ \tilde{Z}_{21}(s) & \tilde{Z}_{22}(s) \end{bmatrix} = \begin{bmatrix} C_{21}(s) - C_{22}(s)C_{12}^{-1}(s)C_{11}(s) & C_{12}^{-1}(s)C_{11}(s) \\ -C_{22}(s)C_{12}^{-1}(s) & C_{12}^{-1}(s) \end{bmatrix} \tag{2.20}$$

and satisfying

$$\begin{aligned} &\begin{bmatrix} \tilde{Z}_{22}(s) & \tilde{Z}_{12}(s) \\ \tilde{Z}_{21}(s) & \tilde{Z}_{11}(s) \end{bmatrix} \begin{bmatrix} Z_{11}(s) & -Z_{12}(s) \\ -Z_{21}(s) & Z_{22}(s) \end{bmatrix} = I \\ &= \begin{bmatrix} Z_{11}(s) & -Z_{12}(s) \\ -Z_{21}(s) & Z_{22}(s) \end{bmatrix} \begin{bmatrix} \tilde{Z}_{22}(s) & \tilde{Z}_{12}(s) \\ \tilde{Z}_{21}(s) & \tilde{Z}_{11}(s) \end{bmatrix}. \end{aligned} \tag{2.21}$$

Using Lemma 2.1, the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output plants is given by following theorem.

Theorem 2.1. *If modified repetitive controllers satisfying* (2.5) *exist, both* (2.11) *and* (2.12) *have solutions* $X \geq 0$ *and* $Y \geq 0$ *such that* (2.13) *and both* (2.14) *and* (2.15) *have no eigenvalue in the closed right half plane. Using* X *and* Y*, the parameterization of all robust stabilizing modified repetitive controllers satisfying* (2.5) *is given by*

$$\begin{aligned} C(s) &= \left(Z_{11}(s)Q(s) + Z_{12}(s)\right)\left(Z_{21}(s)Q(s) + Z_{22}(s)\right)^{-1} \\ &= \left(Q(s)\tilde{Z}_{21}(s) + \tilde{Z}_{22}(s)\right)^{-1}\left(Q(s)\tilde{Z}_{11}(s) + \tilde{Z}_{12}(s)\right), \end{aligned} \tag{2.22}$$

where $Z_{ij}(s)(i = 1,2; j = 1,2)$ *and* $\tilde{Z}_{ij}(s)(i = 1,2; j = 1,2)$ *are defined by* (2.19) *and* (2.20) *and satisfying* (2.21), $C_{ij}(s)(i = 1,2; j = 1,2)$ *are given by* (2.17) *and* $Q(s) \in H_\infty^{p\times p}$ *is any function satisfying* $\|Q(s)\|_\infty < 1$ *and written by*

$$Q(s) = \left(Q_{n1}(s) + Q_{n2}(s)e^{-sT}\right)\left(Q_{d1}(s) + Q_{d2}(s)e^{-sT}\right)^{-1}, \tag{2.23}$$

$Q_{n1}(s) \in RH_\infty^{p\times p}$, $Q_{d1}(s) \in RH_\infty^{p\times p}$, $Q_{n1}(s) \in RH_\infty^{p\times p}$ *and* $Q_{d2}(s) \in RH_\infty^{p\times p}$ *are any functions satisfying*

$$\bar{\sigma}\left\{Z_{22}(0)\left(Q_{d1}(0) + Q_{d2}(0)\right) + Z_{21}(0)\left(Q_{n1}(0) + Q_{n2}(0)\right)\right\} = 0 \tag{2.24}$$

and

$$\text{rank }\left(Q_{n2}(s) - Q_{n1}(s)Q_{d1}^{-1}(s)Q_{d2}(s)\right) = p. \tag{2.25}$$

Proof. First, the necessity is shown. That is, if the robust stabilizing modified repetitive controller $C(s)$ written by (2.7) stabilizes the control system in (2.1), then $C(s)$ and $Q(s)$ are written by (2.22) and (2.23), respectively. From Lemma 2.1, the parameterization of all robust stabilizing controllers $C(s)$ is written by (2.22), where $\|Q(s)\|_\infty < 1$. In order to prove the necessity, we will show that if $C(s)$ written by (2.7) stabilizes the control system in (2.1), then $Q(s)$ in (2.22) is written by (2.23). Substituting $C(s)$ in (2.7) for (2.22), we have

$$Q_{n1}(s) = N_{1n}(s)N_{2d}(s), \tag{2.26}$$

$$Q_{n2}(s) = N_{2n}(s), \tag{2.27}$$

$$Q_{d1}(s) = D_{1n}(s)D_{2d}(s)N_{1d}(s)N_{2d}(s) \tag{2.28}$$

and

$$Q_{d2}(s) = D_{2n}(s)N_{1d}(s)N_{2d}(s). \tag{2.29}$$

Here, $N_{1n}(s) \in RH_\infty^{p\times p}$, $N_{1d}(s) \in RH_\infty^{p\times p}$, $N_{2n}(s) \in RH_\infty^{p\times p}$, $N_{2d}(s) \in RH_\infty^{p\times p}$, $D_{1n}(s) \in RH_\infty^{p\times p}$, $D_{1d}(s) \in RH_\infty^{p\times p}$, $D_{1n}(s) \in RH_\infty^{p\times p}$, $D_{1d}(s) \in RH_\infty^{p\times p}$ are coprime factors satisfying

$$-\tilde{Z}_{11}(s) + \tilde{Z}_{21}(s)C_1(s) \quad = \quad D_{1n}(s)D_{1d}^{-1}(s), \tag{2.30}$$

$$\left(\tilde{Z}_{21}(s)C_2(s) + \tilde{Z}_{11}(s)q(s) - \tilde{Z}_{21}(s)C_1(s)q(s)\right) D_{1d}(s) = D_{2n}(s)D_{2d}^{-1}(s), \tag{2.31}$$

$$\left(\tilde{Z}_{12}(s) - \tilde{Z}_{22}(s)C_1(s)\right) D_{1d}(s)D_{2d}(s) = N_{1n}(s)N_{1d}^{-1}(s) \tag{2.32}$$

and

$$\begin{aligned}\left(-\tilde{Z}_{22}(s)C_2(s) - \tilde{Z}_{12}(s)q(s) + \tilde{Z}_{22}(s)C_1(s)q(s)\right)& \\ D_{1d}(s)D_{2d}(s)N_{1d}(s) = N_{2n}(s)N_{2d}^{-1}(s).&\end{aligned} \tag{2.33}$$

From (2.26)∼(2.29), all of $Q_{n1}(s)$, $Q_{n2}(s)$, $Q_{d1}(s)$ and $Q_{d2}(s)$ are included in RH_∞. Thus, we have shown that if $C(s)$ written by (2.7) stabilize the control system in (2.1) robustly, $Q(s)$ in (2.22) is written by (2.23). Since $q(0) = I$, from (2.26)∼(2.29) and (2.21), (2.24) holds true. In addition, from the assumption of rank $C_2(s) = p$ and from (2.31) and (2.33),

$$\text{rank } D_{2n}(s) = p \tag{2.34}$$

and

$$\text{rank } N_{2n}(s) = p \tag{2.35}$$

hold true. From (2.34), (2.35), (2.27) and (2.29), (2.25) is satisfied. We have thus proved the necessity.

Next, the sufficiency is shown. That is, it is shown that if $C(s)$ and $Q(s) \in H_\infty$ are settled by (2.22) and (2.23), respectively, then the controller $C(s)$ is written by the form in (2.7), $q(0) = I$ and rank $C_2(s) = p$ hold true. Substituting (2.23) into (2.22), we have (2.7), where, $C_1(s)$, $C_2(s)$ and $q(s)$ are denoted by

$$C_1(s) = \left(Z_{11}(s)Q_{n1}(s) + Z_{12}(s)Q_{d1}(s)\right)\left(Z_{21}(s)Q_{n1}(s) + Z_{22}(s)Q_{d1}(s)\right)^{-1}, \quad (2.36)$$

$$C_2(s) = \left(Q_{n1}(s)Q_{d1}^{-1}(s)\tilde{Z}_{21}(s) + \tilde{Z}_{22}(s)\right)^{-1}\left(Q_{n2}(s) - Q_{n1}(s)Q_{d1}^{-1}(s)Q_{d2}(s)\right) \left(Z_{21}(s)Q_{n1}(s) + Z_{22}(s)Q_{d1}(s)\right)^{-1} \quad (2.37)$$

and

$$q(s) = -\left(Z_{21}(s)Q_{n2}(s) + Z_{22}(s)Q_{d2}(s)\right)\left(Z_{21}(s)Q_{n1}(s) + Z_{22}(s)Q_{d1}(s)\right)^{-1}. \quad (2.38)$$

We find that if $C(s)$ and $Q(s)$ are settled by (2.22) and (2.23), respectively, then the controller $C(s)$ is written by the form in (2.7). Substituting (2.24) into (2.38), we have $q(0) = I$. In addition, from (2.25) and (2.37),

$$\text{rank } C_2(s) = p \quad (2.39)$$

holds true.

We have thus proved Theorem 2.1 $\square$

2.4 Control characteristics

In this section, we explain control characteristics of the control system in (2.1) using the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output plants. In addition, roles of $Q_{n1}(s)$, $Q_{n2}(s)$, $Q_{d1}(s)$ and $Q_{d2}(s)$ in (2.23) are clarified.

From Theorem 2.1, $Q(s)$ in (2.23) must be included in H_∞. Since $Q_{n1}(s) \in RH_\infty$ and $Q_{n2}(s) \in RH_\infty$ in (2.23), if $\left(Q_{d1}(s) + Q_{d2}(s)e^{-sT}\right)^{-1} \in H_\infty$, then $Q(s)$ satisfies $Q(s) \in H_\infty$. That is, the role of $Q_{d1}(s)$ and $Q_{d2}(s)$ is to assure $Q(s) \in H_\infty$, and the role of $Q_{n1}(s)$ and $Q_{n2}(s)$ is to guarantee $\|Q(s)\|_\infty < 1$.

Next, the input-output characteristic of the control system in (2.1) is shown. The transfer function $S(s)$ from the periodic reference input $r(s)$ to the error $e(s) = r(s) - y(s)$ of the control system in (2.1) is written by

$$S(s) = S_n(s)S_d^{-1}(s), \tag{2.40}$$

where

$$\begin{aligned} S_n(s) &= C_{21}^{-1}(s)\left\{I + (-C_{22}(s)Q_{n2}(s) + Q_{d2}(s))\left(-C_{22}(s)Q_{n1}(s) + Q_{d1}(s)\right)^{-1} e^{-sT}\right\} \\ &\quad (-C_{22}(s)Q_{n1}(s) + Q_{d1}(s)) \end{aligned} \tag{2.41}$$

and

$$\begin{aligned} S_d(s) &= Z_{21}(s)Q_{n1}(s) + Z_{22}(s)Q_{d1}(s) + (Z_{21}(s)Q_{n2}(s) + Z_{22}(s)Q_{d2}(s))\, e^{-sT} \\ &\quad + G(s)\left\{Z_{11}(s)Q_{n1}(s) + Z_{12}(s)Q_{d1}(s) + (Z_{11}(s)Q_{n2}(s) + Z_{12}(s)Q_{d2}(s))\, e^{-sT}\right\}. \end{aligned} \tag{2.42}$$

From (2.19), (2.20) and (2.38), the low-pass filter can be represented as

$$q(s) = -C_{21}^{-1}(s)\left(-C_{22}(s)Q_{n2}(s) + Q_{d2}(s)\right)\left(-C_{22}(s)Q_{n1}(s) + Q_{d1}(s)\right)^{-1} C_{21}(s), \tag{2.43}$$

and the function $S_n(s)$ is written by

$$S_n(s) = \left\{I - q(s)e^{-sT}\right\} C_{21}^{-1}(s)\left(-C_{22}(s)Q_{n1}(s) + Q_{d1}(s)\right) \tag{2.44}$$

According to the (2.44) and (2.43), if $Q_{n1}(s)$, $Q_{d1}(s)$, $Q_{n2}(s)$ and $Q_{d2}(s)$ are selected satisfying (2.8), then

$$\bar{\sigma}\left\{S_n(j\omega_i)\right\} \leq \bar{\sigma}\left\{I - q(j\omega_i)\right\}\bar{\sigma}\left\{C_{21}^{-1}(j\omega_i)\right\}\bar{\sigma}\left\{\left(-C_{22}(j\omega_i)Q_{n1}(j\omega_i) + Q_{d1}(j\omega_i)\right)\right\} \simeq 0, \tag{2.45}$$

the output $y(s)$ follows the periodic reference input $r(s)$ with frequency components

$$\omega_i = \frac{2\pi}{T} i \ \ (i = 0, 1, \dots, \hbar) \tag{2.46}$$

with a small steady state error.

Next, the disturbance attenuation characteristic of the control system in (2.1) is shown. The transfer function from the disturbance $d(s)$ to the output $y(s)$ of the control system in (2.1) is written by (2.40). From (2.40), for $\omega_i(i = 0, 1, \ldots, \hbar)$ in (2.8) of the frequency component of the disturbance $d(s)$ that is same as that of the periodic reference input $r(s)$, if (2.45) holds, then the disturbance $d(s)$ is attenuated effectively. This implies that the disturbance with same frequency component $\omega_i(i = 0, 1, \ldots, \hbar)$ of the periodic reference input $r(s)$ is attenuated effectively. That is, the role of $Q_{n2}(s)$ and $Q_{d2}(s)$ is to specify the disturbance attenuation characteristic for the disturbance with same frequency component $\omega_i(i = 0, 1, \ldots, \hbar)$ of the periodic reference input $r(s)$. When the frequency components of disturbance $d(s)$, $\bar{\omega}_k(k = 0, 1, \cdots, h)$, are not equal to $\omega_i(i = 0, 1, \cdots, \hbar)$, even if

$$\bar{\sigma}\left\{I - q(j\bar{\omega}_k)\right\} \simeq 0, \tag{2.47}$$

the disturbance $d(s)$ cannot be attenuated, because

$$e^{-j\bar{\omega}_k T} \neq 1 \tag{2.48}$$

and

$$\bar{\sigma}\left\{I - q(j\bar{\omega}_k)e^{-j\bar{\omega}_k T}\right\} \not\simeq 0. \tag{2.49}$$

In order to attenuate the frequency components $\bar{\omega}_k(k = 0, 1, \cdots, h)$ of the disturbance $d(s)$, we need to satisfy

$$\bar{\sigma}\left\{-C_{22}(j\bar{\omega}_k)Q_{n1}(j\bar{\omega}_k) + Q_{d1}(j\bar{\omega}_k)\right\} \simeq 0. \tag{2.50}$$

This implies that the disturbance $d(s)$ with frequency components $\bar{\omega}_k \neq \omega_i(i = 0, 1, \ldots, \hbar, k = 0, 1, \cdots, h)$ is attenuated effectively. That is, the role of $Q_{n1}(s)$ and $Q_{d1}(s)$ is to specify disturbance attenuation characteristics for disturbance of frequency $\omega_d \neq \omega_i(i = 0, 1, \ldots, \hbar)$.

From above discussion, the role of $Q_{n2}(s)$ and $Q_{d2}(s)$ is to specify the input-output characteristic for the periodic reference input $r(s)$ and to specify for the disturbance

$d(s)$ of which the frequency component is equivalent to that of the periodic reference input $r(s)$. The role of $Q_{n1}(s)$ and $Q_{d1}(s)$ is to specify for the disturbance $d(s)$ of which the frequency component is different from that of the periodic reference input $r(s)$.

2.5 Design parameters

Generally, the design of the free parameters are using Nyquist stability criterion by manual examination, which is very difficult and inefficient. To settle this problem, it is essential to establish an efficiently and easily method for the parameters. In this section, an efficient design method will be presented for the free parameters based on the control characteristics and H_∞ control approach.

2.5.1 Design parameters for control performance

The objective of this chapter is to develop an efficient design method so that the closed-loop system in Figure 2.1 is robust stable and has high control precision for reference input and/or disturbance. Hence, the free parameters $Q_{n1}(s)$, $Q_{d1}(s)$, $Q_{n2}(s)$ and $Q_{d2}(s)$ should be designed after the control characteristics.

First, in order to track the reference input with small steady-state error for frequency components $\omega_i(0, 1, \cdots, \hbar)$, $Q_{n1}(s)$,$Q_{d1}(s)$, $Q_{n2}(s)$ and $Q_{d2}(s)$ should be settled to satisfy (2.43). That is equal to

$$\bar{\sigma}\left\{I + \left(-C_{22}(j\omega_i)Q_{n2}(j\omega_i) + Q_{d2}(j\omega_i)\right)\left(-C_{22}(j\omega_i)Q_{n1}(j\omega_i) + Q_{d1}(j\omega_i)\right)^{-1}\right\} \simeq 0 \tag{2.51}$$

for all frequency components $\omega_i(0, 1, \cdots, \hbar)$. Since $C_{22}(s)$ is strictly proper, there exists $\tilde{C}(s)$ to satisfy

$$I - C_{22}(0)\tilde{C}(0) = I \tag{2.52}$$

and

$$\bar{\sigma}\left[I - (I - C_{22}(j\omega_i)\tilde{C}(j\omega_i))\hat{q}(j\omega_i)\right] \simeq 0, \tag{2.53}$$

where

$$\hat{q}(s) = \text{diag}\left\{\frac{1}{(1+s\tau_{r1})}, \cdots, \frac{1}{(1+s\tau_{rp})}\right\}, \tag{2.54}$$

$$Q_{n2}(s) = \tilde{C}(s)Q_{d2}(s) \in RH_\infty \tag{2.55}$$

and

$$Q_{d2}(s) = -\hat{q}(s)\left\{-C_{22}(s)Q_{n1}(s) + Q_{d1}(s)\right\}. \tag{2.56}$$

Then, the low-pass filter $q(s)$ can be written as

$$q(s) = C_{21}^{-1}\left(I - C_{22}(s)\tilde{C}(s)\right)\hat{q}(s)C_{21}(s). \tag{2.57}$$

Obviously, $\tilde{C}(s) = 0$ satisfies (2.52), (2.53) and (2.55), and $q(s) = \hat{q}(s)$.

On the other hand, to attenuate the frequency components $\bar{\omega}_k(0, 1, \cdots, h)$ effectively, $Q_{n1}(s)$ is settled by

$$Q_{n1}(s) = C_{22o}^{-1}(s)Q_{d1}(s)\bar{q}_d(s), \tag{2.58}$$

where $C_{22o}(s) \in RH_\infty$ is an outer function of $C_{22}(s)$ satisfying

$$C_{22}(s) = C_{22i}(s)C_{22o}(s), \tag{2.59}$$

$C_{22i}(s) \in RH_\infty$ is an inner function satisfying $C_{22i}(0) = I$ and $\bar{\sigma}\left\{C_{22i}(j\omega)\right\} = 1(\forall\omega \in R_+)$, $\bar{q}_d(s)$ is a low-pass filter satisfying $\bar{q}_d(0) = I$, as

$$\bar{q}_d(s) = \text{diag}\left\{\frac{1}{(1+s\tau_{d1})^{\alpha_{d1}}}, \cdots, \frac{1}{(1+s\tau_{dp})^{\alpha_{dp}}}\right\} \tag{2.60}$$

is valid, $\alpha_{di}(i = 1, \ldots, p)$ are arbitrary positive integers to make $C_{22o}^{-1}(s)\bar{q}_d(s)$ proper and $\tau_{dk} \in R(k = 1, \ldots, p)$ are any positive real numbers satisfying

$$\bar{\sigma}\left\{I - C_{22i}(j\bar{\omega}_k)\bar{q}_d(j\bar{\omega}_k)\right\} \simeq 0 \tag{2.61}$$

for $\bar{\omega}_k(0, 1, \cdots, h)$.

2.5.2 Design parameters for robust stability conditions

From Theorem 2.1, the free parameters $Q_{n1}(s)$, $Q_{d1}(s)$, $Q_{n2}(s)$ and $Q_{d2}(s)$ are required to satisfy the robust stability conditions $Q(s) \in H_\infty$ and $\|Q(s)\|_\infty < 1$. For convenience, choose the $\tilde{C}(s) = 0$ and $Q_{d1}(s) \in RH_\infty$ and substitute (2.58), (2.56) and (2.55) into (2.23) leading to

$$Q(s) = C_{22o}^{-1}(s)\bar{q}_d(s)\left\{I - \hat{q}(s)\bar{q}(s)e^{-sT}\right\}^{-1}, \tag{2.62}$$

where

$$\bar{q}(s) = I - C_{22i}(s)\bar{q}_d(s). \tag{2.63}$$

Note that

$$\|\bar{q}(s)\|_\infty < 1, \tag{2.64}$$

since $C_{22i}(s)\bar{q}_d(s)$ works as low-pass filter. According to H_∞ control approach, the conditions $Q(s) \in H_\infty$ and $\|Q(s)\|_\infty < 1$ are the robust stability conditions of closed-loop system in Figure 2.3, where $\hat{\Delta}(s)$ is an uncertainty satisfying $\|\hat{\Delta}(s)\|_\infty = 1$.

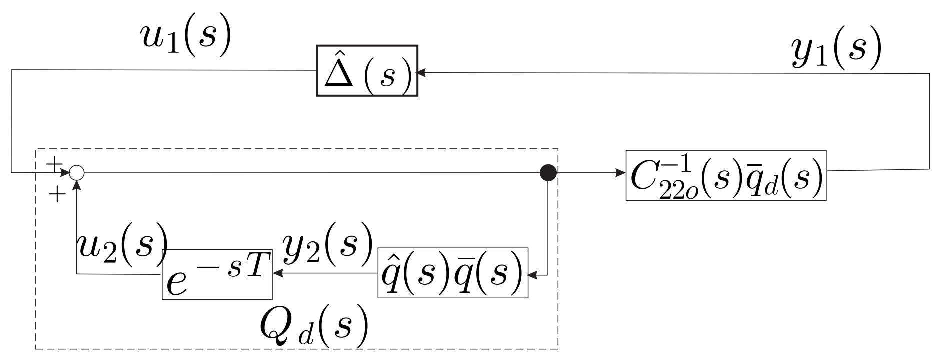

Figure 2.3: Closed-loop system for $Q(s) \in H_\infty$ and $\|Q(s)\|_\infty < 1$

Since $C_{22o}^{-1}(s)\bar{q}_d(s)$, $\hat{q}(s)$ and $\bar{q}(s)$ are RH_∞, if $(I + \hat{q}(s)\bar{q}(s)e^{-sT})^{-1} \in H_\infty$, then $Q(s) \in H_\infty$ holds. In fact, the $(I + \hat{q}(s)\bar{q}(s)e^{-sT})^{-1} \in H_\infty$ is equivalent that the closed-loop system $Q_d(s)$ is stable. Due to $\|e^{-sT}\|_\infty \leq 1$, according to the small gain theorem, the stability condition is

$$\|\hat{q}(s)\bar{q}(s)\|_\infty < 1. \tag{2.65}$$

Because of $\|\hat{q}(s)\|_\infty \leq 1$ and $\|\bar{q}(s)\|_\infty < 1$, the stability condition for $Q_d(s)$ is satisfied. That means $Q(s) \in H_\infty$.

From the Figure 2.3, the system can be represented as

$$\begin{cases} y(s) &= T(s)u(s) \\ u(s) &= \Lambda(s)y(s) \end{cases}, \tag{2.66}$$

where $y(s) = \begin{bmatrix} y_1^T(s) & y_2^T(s) \end{bmatrix}^T$, $u(s) = \begin{bmatrix} u_1^T(s) & u_2^T(s) \end{bmatrix}^T$, the transfer function $T(s)$ is written by

$$T(s) = \begin{bmatrix} C_{22o}^{-1}(s)\bar{q}_d(s) & C_{22o}^{-1}(s)\bar{q}_d(s) \\ \hat{q}(s)\bar{q}(s) & \hat{q}(s)\bar{q}(s) \end{bmatrix} \tag{2.67}$$

and the uncertainties $\Lambda(s)$ is written as

$$\Lambda(s) = \begin{bmatrix} \hat{\Delta} & 0 \\ 0 & e^{-sT} \end{bmatrix}. \tag{2.68}$$

Note that $\|\Lambda(s)\|_\infty \leq 1$, then the robust stability condition $\|Q(s)\|_\infty < 1$ is equivalent to the condition

$$\left\| \begin{bmatrix} C_{22o}^{-1}(s)\bar{q}_d(s) & C_{22o}^{-1}(s)\bar{q}_d(s) \\ \hat{q}(s)\bar{q}(s) & \hat{q}(s)\bar{q}(s) \end{bmatrix} \right\|_\infty < 1, \tag{2.69}$$

and further it as

$$\left\| \begin{bmatrix} C_{22o}^{-1}(s)\bar{q}_d(s) & 0 \\ 0 & \hat{q}(s)\bar{q}(s) \end{bmatrix} \right\|_\infty < \frac{1}{2}. \tag{2.70}$$

This condition shows that, for given $C_{22o}(s)$, we should chose suitable low-pass filters $\hat{q}(s)$ and $q_d(s)$ to satisfy robust stability conditions.

According to the design method for $C_{22i}(s)$ and $C_{22o}(s)$ in [102], $C_{22o}(s)$ must be strictly proper. We found that there exist the bandwidth restrictions for $\bar{q}_d(s)$ and

$\hat{q}(s)$ in (2.70). Without loss of generality, to explain this problem clearly and simply, we set $C_{22o}(s)$ as

$$C_{22o}(s) = \frac{\beta}{1+\alpha\, s}, \quad \alpha > 0. \tag{2.71}$$

Since $C_{22o}^{-1}(s)\bar{q}_d(s)$ is proper, we will analysis it in two cases for

$$\bar{q}_d(s) = \frac{1}{1+\tau_d\, s} \tag{2.72}$$

and

$$\bar{q}_d(s) = \frac{1}{(1+\tau_d\, s)^2}, \tag{2.73}$$

respectively.

Case 1 The low-pass filter $\bar{q}_d(s)$ is selected in (2.72), the infinity norm of $C_{22o}^{-1}(s)\bar{q}_d(s)$ is

$$\|C_{22o}^{-1}(s)\bar{q}_d(s)\|_\infty = \max_{\omega\in R}\left|\frac{1+j\,\alpha\,\omega}{\beta\,(1+j\,\tau_d\,\omega)}\right| = \max_{\omega\in R}\left\{\frac{1}{|\beta|}\sqrt{\frac{1+\alpha^2\,\omega^2}{1+\tau_d^2\,\omega^2}}\right\}. \tag{2.74}$$

Then, according to (2.70), when we choose $\alpha < \tau_d$, the inequality

$$\frac{1}{|\beta|} < \frac{1}{2} \tag{2.75}$$

must be satisfied, or when we choose $\alpha > \tau_d$, the inequality

$$\frac{\alpha}{\tau_d} < \frac{|\beta|}{2} \tag{2.76}$$

must be satisfied. These two conditions means that the restriction of the cutoff frequency ω_{dc} for low-pass filter $\bar{q}_d(s)$ is that

$$\omega_{dc} = \frac{1}{\tau_d} < \max\left\{\frac{1}{\alpha}, \frac{|\beta|}{2\,\alpha}\right\}. \tag{2.77}$$

Case 2 The low-pass filter $\bar{q}_d(s)$ is selected in (2.73), the infinity norm of $C_{22o}^{-1}(s)\bar{q}_d(s)$ is

$$\|C_{22o}^{-1}(s)\bar{q}_d(s)\|_\infty = \max_{\omega\in R}\left|\frac{1+j\,\alpha\,\omega}{\beta\,(1+j\,\tau_d\,\omega)^2}\right| = \max_{\omega\in R}\left\{\frac{1}{|\beta|}\frac{\sqrt{1+\alpha^2\,\omega^2}}{1+\tau_d^2\,\omega^2}\right\}. \tag{2.78}$$

Then, according to (2.70), when we choose $\frac{\sqrt{2}}{2}\alpha < \tau_d$, the inequality

$$\frac{1}{|\beta|} < \frac{1}{2} \tag{2.79}$$

must be satisfied, or when we choose $\frac{\sqrt{2}}{2}\alpha > \tau_d$ and the condition (2.79) holds, the inequality

$$\frac{\sqrt{2}\,\alpha}{2}\sqrt{\frac{\beta^2 - \sqrt{\beta^4 - 4\,\beta^2}}{\beta^2}} < \tau_d < \frac{\sqrt{2}\,\alpha}{2}\sqrt{\frac{\beta^2 + \sqrt{\beta^4 - 4\,\beta^2}}{\beta^2}} \tag{2.80}$$

must be stratified. Obviously, there is no low-pass filter $\bar{q}_d(s)$ in the form of (2.60) for

$$\frac{1}{|\beta|} \geq \frac{1}{2} \tag{2.81}$$

in this case. From above discussion, the restriction of the cutoff frequency ω_{dc} for low-pass filter $\bar{q}_d(s)$ is that

$$\omega_{dc} = \frac{1}{\tau_d} < \max\left\{\frac{\sqrt{2}}{\alpha}, \left(\frac{\sqrt{2}\,\alpha}{2}\sqrt{\frac{\beta^2 - \sqrt{\beta^4 - 4\,\beta^2}}{\beta^2}}\right)^{-1}\right\}. \tag{2.82}$$

with (2.79) in this case.

The results in these two cases show that we would better to choose the $\bar{q}_d(s)$ to make $C_{22o}^{-1}(s)\bar{q}_d(s)$ biproper. Furthermore, there exists the bandwidth restriction for the low-pass filter $\bar{q}_d(s)$.

On the other hand, the restriction of the cutoff frequency ω_c for the low-pass filter $\hat{q}(s)$ has been detailed in [29]. This subsection demonstrates that when choose the low-pass filter for the perfect control performance, the robust stability condition may be not guaranteed. That results in tradeoff problem for modified repetitive control system design.

2.5.3 Robust stability condition based on linear matrix inequalities (LMIs)

The computation of the infinity norm is complicated and requires a search. Control engineering interpretation of the infinity norm is the distance in the complex plane form the origin to the farthest point on the Nyquist plot, and it also appears as the peak value on the Bode magnitude plot. However, the graphical method can lead to a wrong answer for a lightly damped system if the frequency gride is not sufficiently dense [103]. Moreover, the robust stability condition (2.70) has conservativeness. To compute the infinity norm easily and reduce the conservativeness, the Bounded Real Lemma (BRL) [104] based on linear matrix inequality is employed.

Assume the sate-space description of $C_{22o}^{-1}(s)\bar{q}_d(s)$ and $\hat{q}(s)\bar{q}(s)$ are

$$C_{22o}^{-1}(s)\bar{q}_d(s) = \left[\begin{array}{c|c} A_o & B_o \\ \hline C_o & D_o \end{array}\right] \tag{2.83}$$

and

$$\hat{q}(s)\bar{q}(s) = \left[\begin{array}{c|c} A_h & B_h \\ \hline C_h & 0 \end{array}\right]. \tag{2.84}$$

Then, from Figure 2.3, the the state-space description of $Q(s)$ can be achieved as

$$\begin{cases} \dot{x}_q(s) &= A_q x_q(t) + A_d x_q(t-T)(t) + B_q u_1(t) \\ y_1(t) &= C_q x_q(t) + C_d x_q(t-T) + D_q u_1(t) \end{cases}, \tag{2.85}$$

where

$$A_q = \begin{bmatrix} A_o & 0 \\ 0 & A_h \end{bmatrix}, \quad A_d = \begin{bmatrix} 0 & B_o C_h \\ 0 & B_h C_h \end{bmatrix}, \quad B_q = \begin{bmatrix} B_o \\ B_h \end{bmatrix},$$

$$C_q = \begin{bmatrix} C_o & 0 \end{bmatrix}, \quad C_d = \begin{bmatrix} 0 & D_o C_h \end{bmatrix} \text{ and } D_q = D_o.$$

To obtain the stability condition for $\|Q(s)\|_\infty < 1$, the following result is required.

Consider a nominal system $G_n(s)$ with time-varying delay given by

$$G_n(s): \begin{cases} \dot{x}(t) &= Ax(t) + A_d x(t-\tau(t)) + B_\varpi \varpi(t) \\ z(t) &= Cx(t) + C_d x(t-\tau(t)) + D_\varpi \varpi(t) \\ x(t) &= \phi(t), \quad t \in [\bar{\tau}, 0] \end{cases}, \tag{2.86}$$

where the initial condition, $\phi(t)$, is a continuous vector-valued initial function of $t \in [\bar{\tau}, 0]$, $\tau(t)$ is a time delay and satisfying

$$0 \le \tau(t) \le \bar{\tau}, \quad |\dot{\tau}(t)| \le d < 1, \quad t \ge 0. \tag{2.87}$$

The H_∞ performance of $G_n(s)$, i.e.,

$$\|G_n(s)\|_\infty < \gamma, \quad \gamma > 0 \tag{2.88}$$

is obtained by solving the following feasible problem.

Lemma 2.2 (BRL [104])**.** *Given scalars $\bar{\tau}$, γ and $d > 0$, if there exist matrices Z, S, M, Q_{22} and $P_{11} > 0$, Q_{11} and $P_{22} \ge 0$, and any matrices Q_{12} and P_{12} with appropriate dimensions such that the following LMIs hold*

$$\Omega_1 = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & C^T & \bar{\tau}A^T Q_{22} & dP_{12} & 0 \\ * & \Omega_{22} & \Omega_{23} & 0 & C_d^T & \bar{\tau}A_d^T Q_{22} & 0 & dP_{22} \\ * & * & \Omega_{33} & \Omega_{34} & 0 & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & D_\varpi^T & \bar{\tau}B_\varpi^T Q_{22} & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 \\ * & * & * & * & * & -Q_{22} & 0 & 0 \\ * & * & * & * & * & * & -dS & 0 \\ * & * & * & * & * & * & * & -dZ \end{bmatrix} < 0, \tag{2.89}$$

$$\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_2 \end{bmatrix} \ge 0 \tag{2.90}$$

and

$$\begin{bmatrix} P_{11} & P_{12} \\ * & P_{22} \end{bmatrix} \ge 0, \tag{2.91}$$

where

$$\Omega_{11} = M - Q_{22} + \bar{\tau}^2\left(Q_{11} + Q_{12}A + A^TQ_{12}^T\right) + P_{12} + P_{12}^T + P_{11}A + A^TP_{11}^T,$$

$$\Omega_{12} = Q_{22} + \bar{\tau}^2 Q_{12}A_d + P_{11}A_d - P_{12},$$

$$\Omega_{13} = A^TP_{12} + P_{22}^T - Q_{12}^T,$$

$$\Omega_{14} = P_{11}B_{\varpi} + \bar{\tau}^2 Q_{12}B_{\varpi},$$

$$\Omega_{22} = dS - (1-d)M - Q_{22},$$

$$\Omega_{23} = A_d^TP_{12} - P_{22} + Q_{12}^T,$$

$$\Omega_{33} = dZ - Q_{11}$$

and

$$\Omega_{34} = P_{12}^TB_{\varpi},$$

then system (2.86) *is asymptotically stable.*

Applying this Lemma to the system (2.85) with $d = 0$ and $\bar{\tau} = T$, the robust stability conditions $Q(s) \in H_\infty$ and $\|Q(s)\|_\infty < 1$ are given in form of LMIs.

Theorem 2.2. *Given scalars T and $\gamma = 1$, if there exist matrices M, Q_{22} and $P_{11} > 0$, Q_{11} and $P_{22} \geq 0$, and any matrices Q_{12} and P_{12} with appropriate dimensions such that* (2.90), (2.91) *and the following LMI hold*

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & C^T & TA_q^TQ_{22} \\ * & \Xi_{22} & \Xi_{23} & 0 & C_d^T & TA_d^TQ_{22} \\ * & * & -Q_{11} & P_{12}^TB_q & 0 & 0 \\ * & * & * & -\gamma^2 I & D_q^T & TB_q^TQ_{22} \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -Q_{22} \end{bmatrix} < 0, \qquad (2.92)$$

where

$$\Xi_{11} = M - Q_{22} + T^2\left(Q_{11} + Q_{12}A_q + A_d^TQ_{12}^T\right) + P_{12} + P_{12}^T + P_{11}A_q + A_q^TP_{11}^T,$$

$$\Xi_{12} = Q_{22} + T^2 Q_{12} A_d + P_{11} A_d - P_{12},$$

$$\Xi_{13} = A_q^T P_{12} + P_{22}^T - Q_{12}^T,$$

$$\Xi_{14} = P_{11} B_q + T^2 Q_{12} B_q,$$

$$\Xi_{22} = -M - Q_{22}$$

and

$$\Xi_{23} = A_d^T P_{12} - P_{22} + Q_{12}^T,$$

then system (2.85) *is asymptotically stable.*

Finally, according to above discussions, a design procedure of robust stabilizing modified repetitive controller $C(s)$ satisfying Theorem 2.1 is summarized as follows:

<u>Procedure</u>

1. Obtain $C_{11}(s)$, $C_{12}(s)$, $C_{21}(s)$ and $C_{22}(s)$ by solving the robust stability problem using the Riccati equation based H_∞ control.

2. Settle the free parameters $Q_{n1}(s)$, $Q_{d1}(s)$, $Q_{n2}(s)$ and $Q_{d2}(s)$ as shown in Subsection 2.5.1.

3. According to Subsection 2.5.2, choose appropriate parameters τ_{ri}, τ_{di} and $\alpha_{di}(i = 1, \cdots, p)$ for the low-pass filters $\hat{q}(s)$ and $\bar{q}_d(s)$ to satisfy (2.51), (2.61) and Theorem 2.2.

2.6 Numerical example

In this section, numerical examples are made to illustrate the validity of the proposed approach. Consider the problem to obtain the parameterization of all robust stabilizing modified repetitive controllers for the set of plants $G(s)$ written by (2.3), where

$$G_m(s) = \begin{bmatrix} \dfrac{s+3}{(s-2)(s+9)} & \dfrac{2}{(s-2)(s+9)} \\ \dfrac{s+3}{(s-2)(s+9)} & \dfrac{s+4}{(s-2)(s+9)} \end{bmatrix} \tag{2.93}$$

and

$$W_T(s) = \frac{s+400}{550}. \tag{2.94}$$

The period T of the periodic reference input r is given by $T = 10$[sec].

Solving the robust stability problem using Riccati equation based H_∞ control as Theorem 2.1, the parameterization of all robust stabilizing controllers $C(s)$ is obtained. In addition, we find that $C_{22}(s)$ is of minimum phase as

$$C_{22}(s) = \begin{bmatrix} \dfrac{-434}{s+408} & \dfrac{347}{s+408} \\ \dfrac{347}{s+408} & \dfrac{434}{s+408} \end{bmatrix}. \tag{2.95}$$

Since $C_{22}(s)$ is of minimum phase, we set $Q_{n1}(s)$, $Q_{n2}(s)$, $Q_{d1}(s)$ and $Q_{d2}(s)$ in (2.23) as

$$Q_{d1}(s) = I \in RH_\infty, \tag{2.96}$$

$$Q_{n1}(s) = C_{22}^{-1}(s)\bar{q}_d(s) \in RH_\infty, \tag{2.97}$$

$$Q_{n2}(s) = 0 \in RH_\infty, \tag{2.98}$$

and

$$Q_{d2}(s) = -\hat{q}(s)\left(I - \bar{q}_d(s)\right) \in RH_\infty, \tag{2.99}$$

where $\bar{q}(s)$ and $\hat{q}(s)$ are written by

$$\bar{q}_d(s) = \begin{bmatrix} \dfrac{1}{0.002\,s+1} & 0 \\ 0 & \dfrac{1}{0.002\,s+1} \end{bmatrix} \tag{2.100}$$

and

$$\hat{q}(s) = \begin{bmatrix} \dfrac{1}{0.002\,s+1} & 0 \\ 0 & \dfrac{1}{0.002\,s+1} \end{bmatrix}. \tag{2.101}$$

Using $Q_{d1}(s)$ in (2.96) and $Q_{d2}(s)$ in (2.99), Theorem 2.2 has feasible solutions and the H_∞ performance index γ is 0.904, i.e., $\|Q(s)\|_\infty < 0.904$. To verify this result,

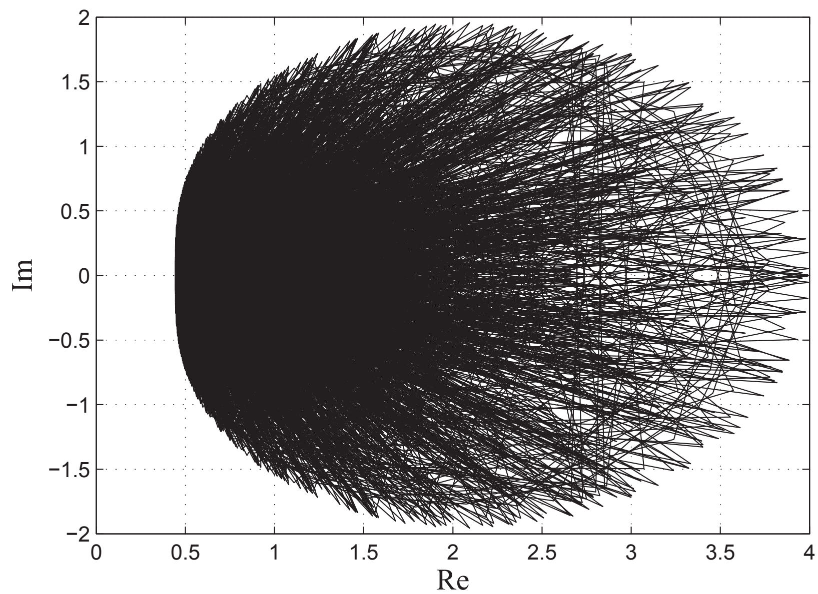

Figure 2.4: The nyquist plot of $\det(Q_{d1}(s) + Q_{d2}(s)e^{-sT})$

the Nyquist plot of $\det(Q_{d1}(s) + Q_{d2}(s)e^{-sT})$ and the largest singular value plot of $Q(s)$ are shown in Figure 2.4 and Figure 2.5, respectively. Since the Nyquist plot of $\det(Q_{d1}(s) + Q_{d2}(s)e^{-sT})$ does not encircle the origin, we find that $Q(s)$ in (2.23) is included in H_∞. Figure 2.5 illustrates $\bar{\sigma}\{Q(j\omega)\} \simeq 0.9 < 1 (\forall \omega \in R)$, i.e., $\|Q(s)\|_\infty \simeq 0.9 < 1$.

According to analysis result in Subsection 2.5.2, there exists the bandwidth limitation for low-pass filter $\bar{q}_d(s)$. To verify this result, we draw the Nyquist plot of $\det(Q_{d1}(s)+Q_{d2}(s)e^{-sT})$ and the largest singular value plot of $Q(s)$ when $\tau_{d\,1,2} = 0.001$ in Figure 2.6 and Figure 2.7. There is no feasible solution for LMIs-constraint con-

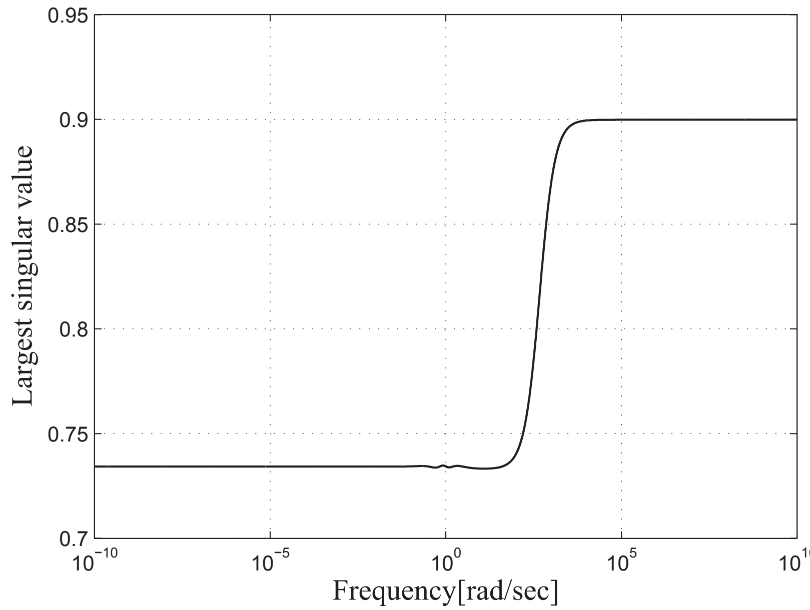

Figure 2.5: Largest singular value plot of $Q(s)$

ditions in Theorem 2.2 and the H_∞ performance index γ is 1.825, which means $\|Q(s)\|_\infty < 1.825$. And Figure 2.6 shows that $Q(s) \in H_\infty$. This verifies that the design method make $Q(s)$ belong to H_∞ for arbitrary low-pass filters $\hat{q}(s)$ and $\bar{q}_d(s)$. However, Fiure 2.7 shows that $\bar{\sigma}\{Q(j\omega)\} \simeq 1.8 > 1 (\forall\omega \in R)$, i.e., $\|Q(s)\|_\infty > 1$. This result demonstrates that there exists some restriction on the bandwidth. Therefore, when choose the low-pass filters to obtain high control precision, the robust stability must be guaranteed.

Using above-mentioned parameters, we have a robust stabilizing modified repetitive controller. When $\Delta(s)$ is given by

$$\Delta(s) \;\; = \;\; \begin{bmatrix} \dfrac{s-100}{s+500} & \dfrac{-100}{s+500} \\ \dfrac{-200}{s+500} & \dfrac{s-100}{s+500} \end{bmatrix}, \qquad (2.102)$$

in order to confirm that $\Delta(s)$ satisfies (2.4), the largest singular value plot of $\Delta(s)$ and the gain plot of $W_T(s)$ are shown in Figure 2.8. Here, the solid line shows the gain plot of $W_T(s)$ and the dashed line shows that of $\Delta(s)$. Figure 2.8 shows that the

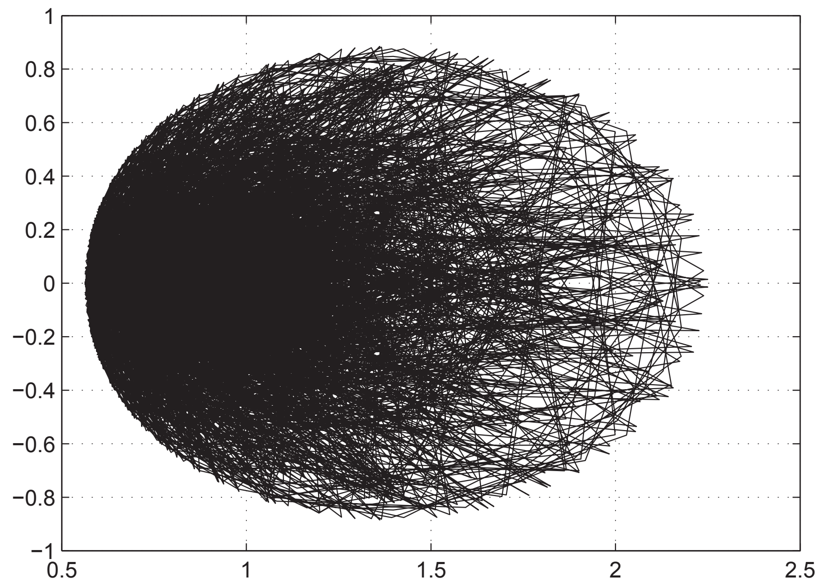

Figure 2.6: The nyquist plot of $\det(Q_{d1}(s) + Q_{d2}(s)e^{-sT})$ when $\tau_{d\,1,2} = 0.001$

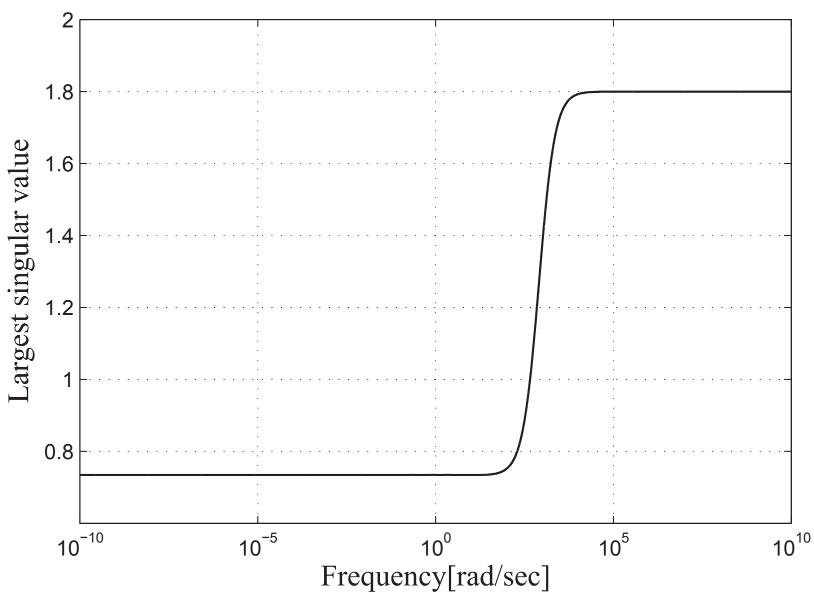

Figure 2.7: Largest singular value plot of $Q(s)$ when $\tau_{d\,1,2} = 0.001$

uncertainty $\Delta(s)$ in (2.102) satisfies (2.4).

When the designed robust stabilizing modified repetitive controller $C(s)$ is used,

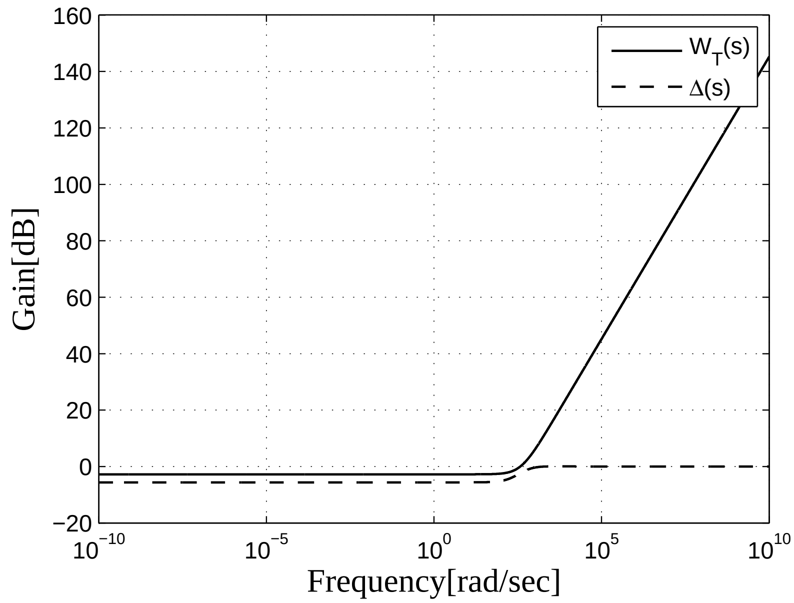

Figure 2.8: Largest singular value plot of $\Delta(s)$ and gain plot of $W_T(s)$

the response of the error $e(t) = r(t) - y(t)$ in (2.1) written by

$$e(t) = \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} = \begin{bmatrix} r_1(t) - y_1(t) \\ r_2(t) - y_2(t) \end{bmatrix} \tag{2.103}$$

for the periodic reference input r

$$r(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} = \begin{bmatrix} \sin\left(\frac{2\pi}{T}t\right) \\ 2\sin\left(\frac{2\pi}{T}t\right) \end{bmatrix} \tag{2.104}$$

is shown in Figure 2.9. Here, the broken line shows the response of the periodic reference input $r_1(t)$, the dotted line shows that of the periodic reference input $r_2(t)$, the solid line shows that of the error $e_1(t)$, and the dotted and broken line shows that of the error $e_2(t)$. Figure 2.9 shows that the output $y(t)$ follows the periodic reference input $r(t)$ with small steady state error.

Next, using the designed the robust stabilizing modified repetitive controller $C(s)$, the disturbance attenuation characteristic is shown. The response of the output $y(t)$

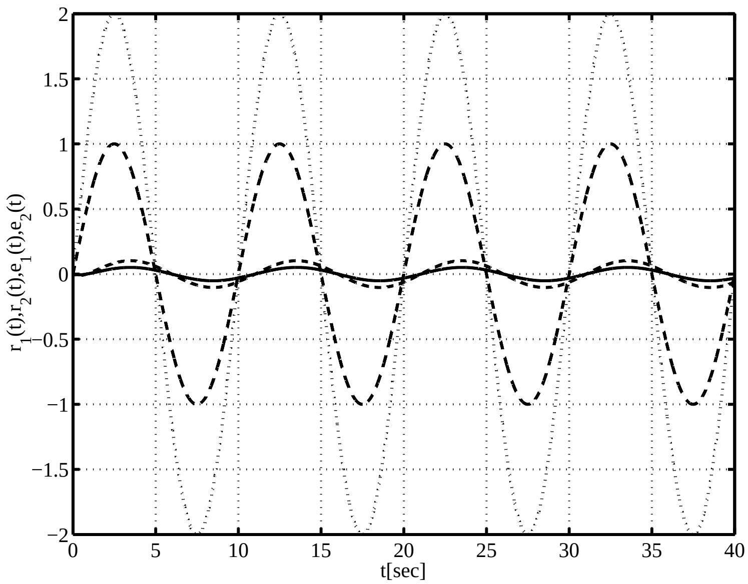

Figure 2.9: Response of the error $e(t)$ for the reference input $r(t)$

written by

$$y(t) \;=\; \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} \tag{2.105}$$

for the disturbance $\bar{d}(t)$ of which the frequency component is equivalent to that of the periodic reference input $r(t)$

$$\bar{d}(t) = \begin{bmatrix} \bar{d}_1(t) \\ \bar{d}_2(t) \end{bmatrix} = \begin{bmatrix} \sin\left(\frac{2\pi}{T}t\right) \\ 2\sin\left(\frac{2\pi}{T}t\right) \end{bmatrix} \tag{2.106}$$

is shown in Figure 2.10. Here, the broken line shows the response of the disturbance $\bar{d}_1(t)$, the dotted line shows that of the disturbance $\bar{d}_2(t)$, the solid line shows that of the output $y_1(t)$ and the dotted and broken line shows that of the output $y_2(t)$. Figure 2.10 shows that the disturbance $\bar{d}(t)$ is attenuated effectively. Finally, the response of the output $y(t)$ for the disturbance $\tilde{d}(t)$ of which the frequency component is different

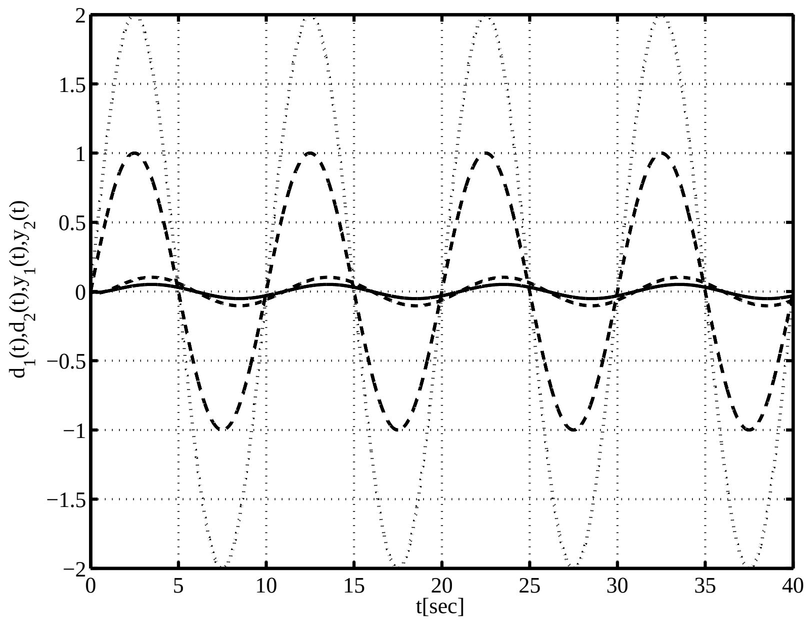

Figure 2.10: Response of the output $y(t)$ for the disturbance $\bar{d}(t)$

from that of the periodic reference input $r(t)$

$$\tilde{d}(t) = \begin{bmatrix} \tilde{d}_1(t) \\ \tilde{d}_2(t) \end{bmatrix} = \begin{bmatrix} \sin\left(\frac{\pi}{T}t\right) \\ 2\sin\left(\frac{\pi}{T}t\right) \end{bmatrix} \tag{2.107}$$

is shown in Figure 2.11. Here, the broken line shows the response of the disturbance $\tilde{d}_1(t)$, the dotted line shows that of the disturbance $\tilde{d}_2(t)$, the solid line shows that of the output $y_1(t)$ and the dotted and broken line shows that of the output $y_2(t)$. Figure 2.11 shows that the disturbance $\tilde{d}(t)$ is attenuated effectively.

A robust stabilizing modified repetitive controllers can be easily designed in the way shown here. The design method proposed in this chapter simplifies the design process which does not need to draw the Nyquist plot and the largest singular value plot of free parameter $Q(s)$ simultaneously. The simulation result show that this modified repetitive control system can be used to track or attenuate the signals with different period time.

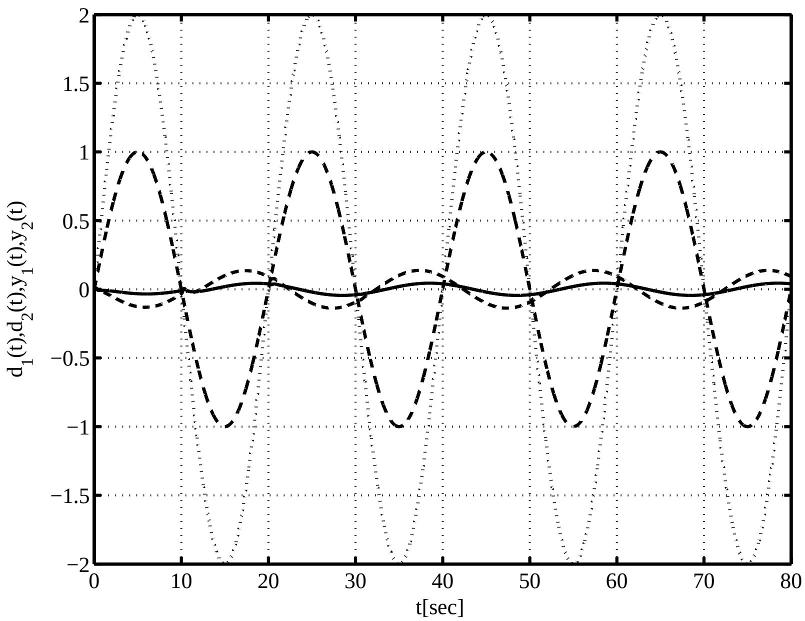

Figure 2.11: Response of the output y for the disturbance $\tilde{d}(t)$

2.7 Conclusions

In this chapter, we gave a complete proof of the theorem for the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output plants omitted in [73] and showed effectiveness of the parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output plants. We clarified control characteristics using the parameterization in [73]. The design of free parameters guarantees the perfect tracking performance and/or good disturbance attenuation characteristics for different period-time. Viewing the time-delay element as an uncertainty and applying H_∞ control approach, there exists the bandwidth limitation of low-pass filter for specify the disturbance characteristic for both minimum phase and non-minimum phase control plant. In order to simplify the design process and avoid the wrong results obtained by graphical method, the robust stability conditions are converted into LMIs-constraint conditions by employing the delay-dependent bounded real lemma. This work can be extended to solve the tradeoff problem. In addition, a design procedure using the parameterization was presented. Finally a numerical example was illustrated to show the effectiveness of the proposed method. Using the result in this paper, we can easily design a robust stabilizing modified repetitive controller.

Chapter 3

Robust Stabilizing Problem for Time-varying periodic Signals

3.1 Introduction

In practical applications, many control systems must deal with periodic reference and/or disturbance signals, for example industrial robots, computer disk drives, CD player tracking control, machine tool motion control, and vibration attenuation of engineering structures. One control system that can deal with periodic reference and/or disturbance signals is a repetitive control system, as proposed by Hara et al. [28]. A disadvantage of typical repetitive controllers is that they are based on the constant period of the external signal. This means that in practical applications, either the period must be constant ($\pm 0.1\%$) or an accurate measurement of the periodicity is necessary.

However, in practice, rotary motion systems have found applications in various industry products. For most applications, the systems are required to operate at variable speeds while following repetitive trajectories and/or rejecting disturbances, such as the brushless DC electric motor in a typical laser printer described by Chen et al. [101]. In general, the periods of reference signals and/or disturbances are mostly

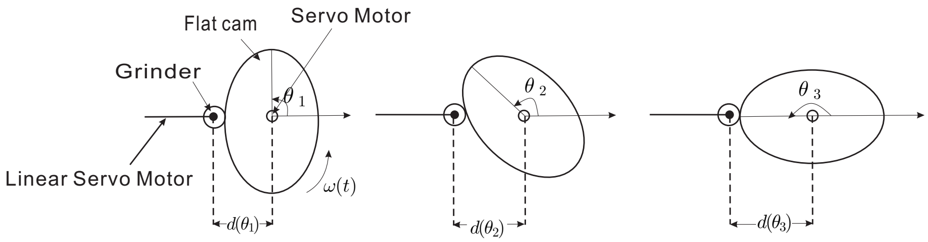

Figure 3.1: Flat cam grinding system

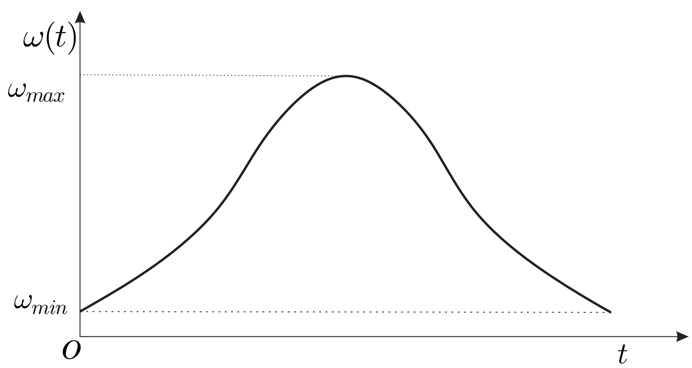

Figure 3.2: Rotation speed of servomotor

time varying in such systems. For instance, consider the flat cam grinding system in Figure 3.1, which requires the control system to track a time-varying periodic reference signal. This system uses noncircular grinding and the cam is machined by utilizing a profile copier controlled by a linear servomotor. In the traditional grinding system, the cam rotates at a constant speed, which means the cam is machined at a varying tangent velocity. This leads to different metal-removal rates and the cam may not meet the requirements. Therefore, to achieve the required machining conditions, the cam is controlled by a servomotor that is required to rotate at a varying speed $\omega(t)$, as shown in Figure 3.2, and this means that the reference input signal, the distance $d(\theta)$ between the circle centers of the grinder and flat cam, is a time-varying periodic signal, i.e., a position-dependent periodic signal. Hence, it is necessary to design a controller for the linear servomotor to track the position-dependent reference input signal $d(\theta)$. Because it is periodic with respect to angular position, but not necessarily with respect to time, the conventional repetitive control technique is not directly applicable in this case. A very common design method for this class of system is to transform a linear system from the time domain into a spatial domain.

Recently, several studies have considered the problem of rejecting and/or tracking spatially periodic disturbances and/or reference inputs for rotary motion systems using a spatial-based repetitive controller [105, 106, 107, 108, 109, 110]. Nakano et al. [105] eliminated the angular position-dependent disturbances in constant-speed rotation control systems by transforming all signals defined in the time domain to the spatial domain, and obtained a stabilizing controller using coprime factorization. To track spatially periodic reference inputs, Mahawan and Luo [106] proposed a repetitive controller design method using operator-theoretic approaches. Sun [107] addressed the tracking or rejecting problem for position-dependent signals by converting the continuous-time system into a discrete spatial system. A more advanced design based on linearization using H_∞ robust control was proposed by Chen and Allebach [108]. Chen and Chiu [109] proved that the reformulated nonlinear plant model could be cast into a quasilinear parameter-varying system that can be used to address spatially periodic disturbances. In particular, a method of designing a spatial-based repetitive control system for rotary motion systems subject to position-dependent disturbances based on adaptive feedback linearization was presented by Chen and Yang [110].

With the domain transformation, the linear system in the time domain is cast into a nonlinear system in the spatial domain. Before designing the repetitive controller, it is necessary to linearize the nonlinear control system, which makes the design of the repetitive controller more complicated and difficult. In particular, for the control of plants with uncertainties or time-varying state delay, there exists a trade-off problem between robust stability and control performance in the design of repetitive control systems, and spatial-based design methods do not provide a satisfactory solution to this trade-off. Hence, there is a clear need to develop an efficient design method for repetitive control systems that track or reject the position-dependent signals.

In this chapter, the position-dependent signal will be converted into a time-varying periodic signal. Inspired by the structure of the repetitive controller[28] and the structure of the optimal repetitive controller[38], we propose a new modified repetitive controller for position-dependent signals. Compared with the conventional modified repetitive controller, the constant time-delay element is replaced by a time-varying operator in our new controller. Moreover, an adjustable parameter is introduced in the new structure to adjust the convergence rate of the closed-loop system and improve the control precision. This controller is plugged into the closed-loop system for a strictly proper plant with uncertainties to reject position-dependent disturbances. The control performance of this repetitive control system then depends heavily on the cutoff frequency of the low-pass filter and the adjustable parameter that represent the trade-off between system robust stability and rejection performance. To achieve the optimal performance and guarantee robust stability, the design problem considered in this paper is converted into a robustly stabilizing problem based on linear matrix inequalities (LMIs). Two LMI-based robust stability conditions of the closed-loop system with time-varying state delay are derived for fixed parameters. One is a delay-dependent robust stability condition that is derived based on the free-weight matrix. The other robust stability condition is based on the H_∞ control approach and introduces a linear unitary operator. The optimal values of the cutoff frequency of the low-pass filter and the adjustable parameter can be obtained by solving the optimization problems with LMI-constrained conditions. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed design method.

3.2 Problem statement and preliminaries

In this section, we will transform the position-dependent signal into a time-varying periodic signal and establish a new structure of modified repetitive controller.

First, we convert the position-dependent reference into a time-varying periodic signal, in contrast with the conventional processing method, which transforms a linear system in the time domain into a nonlinear system in the spatial domain. The position-dependent disturbance $d(t)$ is given by

$$d(t) := \tilde{d}(\theta) = \tilde{d}(\theta - T_\theta), \tag{3.1}$$

where $\tilde{d}(\theta)$ is the position-dependent disturbance, T_θ is the period, and the rotational angle $\theta(t)$ is defined as:

$$\begin{cases} \theta(t) & := & f(t) = \int_0^t \omega(s)ds \\ \omega(t) & = & \dfrac{d\theta}{dt} > 0 \ \ \forall\ t > 0 \end{cases}, \tag{3.2}$$

where $\omega(t)$ is the rotational speed and guarantees that $\theta(t)$ is strictly monotonic such that $t = f^{-1}(\theta)$ exists. Thus, for a large enough t, there exist a $t_\theta > 0$ such that $f(t_\theta) = f(t) - T_\theta$. We define a time-varying function $\tau(t)$ as

$$\tau(t) := \begin{cases} t_0 \ \ 0 < t < t_0 \\ t - f^{-1}(f(t) - T_\theta) = t - t_\theta \ \ t \geq t_0 \end{cases}, \tag{3.3}$$

where $t_0 = f^{-1}(T_\theta)$ satisfies $T_\theta = f(t_0) - f(0)$. Then by Lagrange's mean value theorem, there exists at least one point $\xi \in (t_\theta, t)$ such that

$$T_\theta = f(t) - f(t_\theta) = f'(\xi)\tau(t) = \omega(\xi)\tau(t). \tag{3.4}$$

Then

$$\tau(t) = \frac{T_\theta}{\omega(\xi)} \leq \frac{T_\theta}{\omega_{min}}. \tag{3.5}$$

From the inverse function theorem, the derivative of function $\tau(t)$ is

$$\dot{\tau}(t) = \begin{cases} 0 \ \ 0 < t < t_0 \\ 1 - \dfrac{\omega(t)}{\omega(t_\theta)} \leq 1 - \frac{\omega_{min}}{\omega_{max}} \ \ t \geq t_0 \end{cases}. \tag{3.6}$$

From the equations (3.5) and (3.6), there exist positive scalars $\bar{\tau}$ and μ such that $\tau(t)$ satisfies

$$0 < \tau(t) \leq \bar{\tau}, \ \ \dot{\tau}(t) \leq \mu, \ \ 0 \leq \mu < 1. \tag{3.7}$$

Then, the position-dependent disturbance signal can be transformed into a time-varying periodic signal as

$$d(t) = \begin{cases} \tilde{d}(\theta(t)) & 0 < t < t_0 \\ d(t - \tau(t)) & t \geq t_0 \end{cases}, \tag{3.8}$$

where $\tau(t)$ is the period defined in (3.3) and satisfying (3.7).

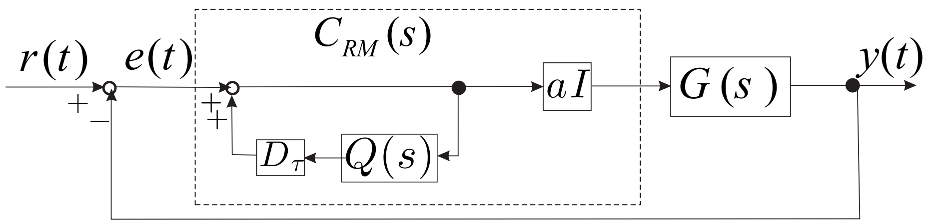

Figure 3.3: The new repetitive control system

Given the time-varying period and inspired by the structure of repetitive controllers[28] and optimal repetitive controllers[38], we establish a new repetitive controller, shown in Figure 3.3, for time-varying periodic signals. Compared with the conventional repetitive controller, the constant time-delay element is replaced by the time-varying operator D_τ defined as

$$D_\tau(v(t)) := v(t - \tau(t)), \tag{3.9}$$

where $\tau(t)$ is the period of the disturbance $d(t)$ in (3.8).

It is well known that the performance of a repetitive control system depends strongly on the cutoff frequency of the included low-pass filter, which represents the trade-off between system stability and control precision. However, it is hard to determine the optimal bandwidth in practice because of the plant uncertainty and system stability. To overcome this problem, we modify the system gain by introducing an adjustable parameter a into the repetitive controller. From the structure of the repetitive controller in Figure 3.3, the gain of $C_{RM}(s)$ is always proportional to the adjustable parameter a; thus, the performance of the repetitive control system is strongly dependent on both the cutoff frequency ω_c and the adjustable parameter a. Hence, it is clear that the cutoff frequency ω_c and the adjustable parameter a should be as high as possible to obtain good rejection.

We consider the design problem of the modified repetitive control system shown in Figure 3.4 that rejects signals that are periodic in the spatial domain while the rotational speed varies in real-time. The strictly proper plant with uncertainties is

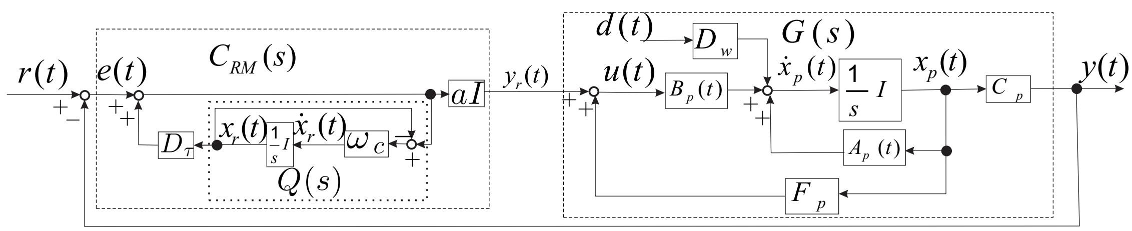

Figure 3.4: The repetitive control system with uncertainties

described as

$$\begin{cases} \dot{x}_p(t) &= A_p(t)x_p(t) + B_p(t)u(t) + D_w d(t) \\ y(t) &= C_p x_p(t) \end{cases}, \tag{3.10}$$

where $x_p(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, and $y(t) \in \mathbb{R}^m$ are the state, input, and output signals, respectively, $A_p(t) \in \mathbb{R}^{n\times n}$, $B_p(t) \in \mathbb{R}^{n\times m}$, $C_p \in \mathbb{R}^{m\times n}$, and $D_w \in \mathbb{R}^{n\times m}$. $d(t) \in \mathbb{R}^m$ is an input disturbance that is periodic in the spatial domain and belongs to $L_2[0, t_f]$. Assume that the uncertainties of the plant are given by

$$\begin{cases} \begin{bmatrix} A_p(t) & B_p(t) \end{bmatrix} &= \begin{bmatrix} A_p + \Delta A_p(t) & B_p + \Delta B_p(t) \end{bmatrix} \\ \begin{bmatrix} \Delta A_p(t) & \Delta B_p(t) \end{bmatrix} &= \Phi_p \Gamma(t) \begin{bmatrix} \Psi_A & \Psi_B \end{bmatrix} \end{cases}, \tag{3.11}$$

where $A_p \in \mathbb{R}^{n\times n}$, $B_p \in \mathbb{R}^{n\times m}$, Φ_p, Ψ_A, and Ψ_B are known constant matrices, and $\Gamma(t) \in \mathbb{R}^{n\times n}$ is an unknown real and possibly time-varying matrix with Lebesgue-measurable entries satisfying

$$\Gamma^T(t)\Gamma(t) \le I, \quad \forall t \ge 0. \tag{3.12}$$

$Q(s)$, given by

$$Q(s) = \frac{\omega_c}{s + \omega_c} I \in \mathbb{R}^{m\times m}, \tag{3.13}$$

is the low-pass filter of the repetitive controller $C_{RM}(s)$, where ω_c is the cutoff frequency of the low-pass filter $Q(s)$, and a is an adjustable parameter.

The problem that should be addressed first is to design a feedback controller of the form

$$u(t) = F_p x_p(t) + e(t) \tag{3.14}$$

such that the closed-loop system, without the modified repetitive controller, is stabilized. Applying the control law (3.14) to (3.10) with $r(t) \equiv 0$ yields the closed-loop system

$$\begin{cases} \dot{x}(t) = \{A_p(t) - B_p(t)C_p + B_p(t)F_p\}\, x_p(t) + D_w d(t) \\ y(t) = C_p x_p(t) \end{cases} . \tag{3.15}$$

The following lemma presents a rate-dependent state-feedback controller to stabilize (3.15) robustly with a prescribed H_∞ norm-bound specification.

Lemma 3.1. *[111] For a prescribed scalar $\gamma > 0$, the closed-loop system* (3.15) *is robustly stable and satisfies $\|y(t)\|_2 < \gamma\|d(t)\|_2$, if there exist a matrix $P^T = P > 0$, a scalar $\lambda > 0$, and an arbitrary matrix W with appropriate dimensions satisfying*

$$\begin{bmatrix} \Lambda_1 & D_w & PC_p^T & \Lambda_2 & \lambda\Phi \\ * & -I & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -\lambda I & 0 \\ * & * & * & * & -\lambda I \end{bmatrix} < 0, \tag{3.16}$$

$$\Lambda_1 := (A_p - B_pC_p)P + B_pW + W^TB_p^T + P(A_p - B_pC_p)^T, \tag{3.17}$$

and

$$\Lambda_2 := P\Psi_A^T - PC_p^T\Psi_B^T + W^T\Psi_B^T. \tag{3.18}$$

Then the H_∞ state-feedback controller is given by $F_p = WP^{-1}$.

The scalar γ can be regarded as a disturbance performance index. The problem of robust stabilization, to find a state-feedback controller such that the closed-loop system is stable with disturbance attenuation γ, can easily be obtained by solving the above feasible problem for the given γ.

We next present an efficient method to find the optimal values of the cutoff frequency ω_c and the adjustable parameter a.

3.3 Robust stability conditions

In this section, we describe a design method to find the optimal values of the cutoff frequency of the low-pass filter and the adjustable parameter.

As shown in Figure 3.4, the state-space description of the repetitive controller is

$$\begin{cases} \dot{x}_r(t) = -\omega_c x_r(t) + \omega_c x_r(t-\tau(t)) + \omega_c e(t) \\ y_r(t) = ae(t) + ax_r(t-\tau(t)) \end{cases} . \tag{3.19}$$

By using the augmented state vector $x := [x_p^T, x_r^T]^T$, we combine (3.19) and (3.10) with $r(t) \equiv 0$, $d(t) \equiv 0$ and

$$u(t) = F_p x_p + y_r(t) \tag{3.20}$$

to yield the closed-loop system

$$\dot{x}(t) = (A + \Delta A(t))\, x(t) + (A_1 + \Delta A_1(t))\, x(t-\tau(t)), \tag{3.21}$$

where

$$A = \begin{bmatrix} A_p + B_pF_p - aB_pC_p & 0 \\ -\omega_c C_p & -\omega_c I \end{bmatrix},\ A_1 = \begin{bmatrix} 0 & aB_p \\ 0 & \omega_c I \end{bmatrix},$$

$$\Delta A(t) = \Phi\Gamma(t)E_1,\ \Delta A_1(t) = \Phi\Gamma(t)E_2,\ \Phi = \begin{bmatrix} \Phi_p^T & 0 \end{bmatrix}^T,$$

$$E_1 = \begin{bmatrix} \Psi_A + \Psi_B F_p - a\Psi_B C_p & 0 \end{bmatrix},\ \text{and}\ \ E_2 = \begin{bmatrix} 0 & a\Psi_B \end{bmatrix}.$$

To establish the design method, the following lemmas are required.

Lemma 3.2 (Schur complement [112]). *For a real matrix $\Sigma = \Sigma^T$, the following assertions are equivalent:*

1. $\Sigma := \begin{bmatrix} \Sigma_{11} & \Sigma_{12} \\ * & \Sigma_{22} \end{bmatrix} > 0.$

2. $\Sigma_{11} > 0$, *and* $\Sigma_{22} - \Sigma_{12}^T \Sigma_{11}^{-1} \Sigma_{12} > 0.$

3. $\Sigma_{22} > 0$, *and* $\Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{12}^T > 0.$

Lemma 3.3 (BRL [113])**.** *For the system*

$$\begin{cases} \dot{x}(t) = Ax(t) + Bw(t) \\ z(t) = Cx(t) + Dw(t) \end{cases}, \tag{3.22}$$

the following assertions are equivalent:

1. *A is stable; and the* H_∞ *norm of the transfer function,* $G_{zw}(s)$*, from* $w(t)$ *to* $z(t)$ *satisfying* $\|G_{zw}\|_\infty < 1$.

2. *There exists a symmetric matrix* $P > 0$ *such that*

$$\begin{bmatrix} PA + A^T P & PB & C^T \\ * & -I & D^T \\ * & * & -I \end{bmatrix} < 0 \tag{3.23}$$

   holds.

Lemma 3.4. *[114] Given the matrices* $Q = Q^T$*,* H*,* E*, and* $R = R^T > 0$ *of appropriate dimensions,*

$$Q + HFE + E^T F^T H^T < 0$$

for all F *satisfying* $F^T F \leq R$*, if and only if there exists some* $\lambda > 0$ *such that*

$$Q + \lambda H H^T + \lambda^{-1} E^T R E < 0.$$

Lemma 3.5. *[115] Consider a nominal system with time-varying delay given by*

$$\begin{cases} \dot{x}(t) = Ax(t) + A_1 x(t - \tau(t)), & t > 0 \\ x(t) = \phi(t), & t \in [\bar{\tau}, 0] \end{cases}, \tag{3.24}$$

where the initial condition, $\phi(t)$, is a continuous vector-valued initial function of $t \in [\bar{\tau}, 0]$. Then, for given scalars $\bar{\tau}$ and μ, the system (3.24) *is globally asymptotically stable for any time delay satisfying* (3.7)*, if there exist symmetric positive definite matrices P, Q, and Z, symmetric matrices X_{11} and X_{22}, and arbitrary matrices X_{12}, Y, and T with appropriate dimensions such that the following LMIs are true.*

$$\begin{bmatrix} X_{11} & X_{12} \\ * & X_{22} \end{bmatrix} \geq 0, \tag{3.25}$$

$$\begin{bmatrix} X_{11} & X_{12} & Y \\ * & X_{22} & T \\ * & * & Z \end{bmatrix} \geq 0 \tag{3.26}$$

and

$$\Sigma := \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \bar{\tau} A^T Z \\ * & \Sigma_{22} & \bar{\tau} A_1^T Z \\ * & * & -\bar{\tau} Z \end{bmatrix} < 0, \tag{3.27}$$

where

$$\Sigma_{11} = PA + A^T P + Y^T + Y + Q + \bar{\tau} X_{11},$$

$$\Sigma_{12} = PA_1 - Y + T^T + \bar{\tau} X_{12},$$

and

$$\Sigma_{22} = -T^T - T - (1 - \mu) Q + \bar{\tau} X_{22}.$$

Now, applying these lemmas to system (3.21) yields the following theorem.

Theorem 3.1. *For given scalars $\bar{\tau}$ and μ satisfying* (3.7)*, the system* (3.21) *is robustly stable if there exist symmetric positive definite matrices P, Q, and Z, symmetric matrices X_{11} and X_{22}, a positive scalar λ, and arbitrary matrices X_{12}, Y, and T with* *appropriate dimensions such that* (3.25) $\sim$ (3.26) *and the following LMI are true.*

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \bar{\tau}A^T Z & P\Phi & \lambda E_1^T \\ * & \Sigma_{22} & \bar{\tau}A_1^T Z & 0 & \lambda E_2^T \\ * & * & -\bar{\tau}Z & \bar{\tau}Z\Phi & 0 \\ * & * & * & -\lambda I & 0 \\ * & * & * & * & -\lambda I \end{bmatrix} < 0, \tag{3.28}$$

where Σ_{11}, Σ_{12}, *and* Σ_{22} *are defined in* (3.27).

Proof. The proof follows from Lemma 3.5. Let us reconsider the matrix inequality $\Sigma < 0$ defined in (3.27). We shall replace A and A_1 with $A(t) = A + \Phi\Gamma(t)E_1$ and $A_1(t) = A_1 + \Phi\Gamma(t)E_2$, respectively, in (3.27) and rewrite the resulting inequality in the form of nominal and uncertain parts as

$$\Sigma + \Sigma_u + \Sigma_u^T < 0, \tag{3.29}$$

where Σ is defined in (3.27) and

$$\Sigma_u := \begin{bmatrix} P\Delta A(t) & P\Delta A_1(t) & 0 \\ 0 & 0 & 0 \\ Z\Delta A(t) & \bar{\tau}Z\Delta A_1(t) & 0 \end{bmatrix}. \tag{3.30}$$

We can decompose Σ_u and express it as

$$\Sigma_u = H\Gamma(t)E, \tag{3.31}$$

where $H = \begin{bmatrix} \Phi^T P & 0 & \Phi^T Z \end{bmatrix}^T$ and $E = \begin{bmatrix} E_1 & E_2 & 0 \end{bmatrix}$. For $\lambda > 0$, applying Lemma 3.4 to (3.29) results in

$$\Sigma + \lambda^{-1}HH^T + \lambda E^T E = \Sigma + \lambda^{-1}HH^T + \lambda^{-1}(\lambda E^T)(\lambda E) < 0. \tag{3.32}$$

By employing the Schur complement Lemma 3.2, the LMI given in (3.27) is obtained. Thus, system (3.21) with admissible uncertainties (3.11) satisfying (3.12) is robustly asymptotically stable.

We have thus proved this theorem. □

Because the system matrices A and A_1 contain the design parameters ω_c and a, Theorem 3.1 cannot be used directly to obtain the optimal values of the cutoff frequency and adjustable parameter. However, as we now show, (3.28) can be converted into LMIs that can be used to calculate the optimal cutoff frequency for given a.

For convenience, we represent ω_c as the sum of $\hat{\omega}_c$ and $\delta\omega_c$ that is:

$$\omega_c = \hat{\omega}_c + \delta\omega_c, \tag{3.33}$$

where $\hat{\omega}_c$ is a roughly estimated value and $\delta\omega_c$ is an unknown value to be found. The matrices A and A_1 can then be represented in the following form:

$$A = \bar{A} + \hat{A} \times \delta\omega_c \tag{3.34}$$

and

$$A_1 = \bar{A}_1 + \hat{A}_1 \times \delta\omega_c, \tag{3.35}$$

where

$$\bar{A} = \begin{bmatrix} A_p + B_pF_p - aB_pC_p & 0 \\ -\hat{\omega}_cC_p & -\hat{\omega}_cI \end{bmatrix},$$

$$\hat{A} = \begin{bmatrix} 0 & 0 \\ -C_p & -I \end{bmatrix},$$

$$\bar{A}_1 = \begin{bmatrix} 0 & aB_p \\ 0 & \hat{\omega}_cI \end{bmatrix},$$

and

$$\hat{A}_1 = \begin{bmatrix} 0 & 0 \\ 0 & I \end{bmatrix}.$$

Denote

$$Q := \bar{Q} - \hat{Q} \times \delta\omega_c > 0 \tag{3.36}$$

and

$$\lambda := \bar{\lambda} - \hat{\lambda} \times \delta\omega_c > 0, \tag{3.37}$$

where $\bar{Q}^T = \bar{Q}$, $\hat{Q}^T = \hat{Q}$, and $\bar{\lambda}$, $\hat{\lambda} \in R$. Then, the LMI (3.28) can be described by

$$\Xi + \hat{\Xi} \times \delta\omega_c < 0. \tag{3.38}$$

Ξ and $\hat{\Xi}$ are represented as

$$\Xi := \begin{bmatrix} \Xi_{11} & \Xi_{12} & \bar{\tau}\bar{A}^T Z & P\Phi & \bar{\lambda}E_1^T \\ * & \Xi_{22} & \bar{\tau}\bar{A}_1^T Z & 0 & \bar{\lambda}E_2^T \\ * & * & -\bar{\tau}Z & \bar{\tau}Z\Phi & 0 \\ * & * & * & -\bar{\lambda}I & 0 \\ * & * & * & * & -\bar{\lambda}I \end{bmatrix} \tag{3.39}$$

and

$$\hat{\Xi} := \begin{bmatrix} \hat{\Xi}_{11} & P\hat{A}_1 & \bar{\tau}\hat{A}^T Z & 0 & -\hat{\lambda}E_1^T \\ * & (1-\mu)\hat{Q} & \bar{\tau}\hat{A}_1^T Z & 0 & -\hat{\lambda}E_2^T \\ * & * & 0 & 0 & 0 \\ * & * & * & \hat{\lambda}I & 0 \\ * & * & * & * & \hat{\lambda}I \end{bmatrix}, \tag{3.40}$$

where

$$\Xi_{11} = P\bar{A} + \bar{A}^T P + Y^T + Y + \bar{Q} + \bar{\tau}X_{11},$$

$$\Xi_{12} = P\bar{A}_1 - Y + T^T + \bar{\tau}X_{12},$$

$$\Xi_{22} = -T^T - T - (1-\mu)\bar{Q} + \bar{\tau}X_{22},$$

and

$$\hat{\Xi}_{11} = P\hat{A} + \hat{A}^T P - \hat{Q}.$$

By introducing a new variable $\sigma := 1/\delta\omega_c$, then (3.36) $\sim$ (3.38) can be rewritten as

$$\hat{\Xi} < -\sigma\Xi, \quad \hat{Q} < \sigma\bar{Q}, \quad \hat{\lambda} < \sigma\bar{\lambda}. \tag{3.41}$$

This gives the following result.

Theorem 3.2. *For given a, and scalars $\bar{\tau}$ and μ satisfying* (3.7)*, if there exist the symmetric positive definite matrices P and Z, symmetric matrices, $\bar{Q}$, $\hat{Q}$, X_{11}, and X_{22}, scalars $\bar{\lambda}$ and $\hat{\lambda}$, and arbitrary matrices X_{12}, Y, and T with appropriate dimensions such that* (3.25) $\sim$ (3.26) *and* (3.41) *are true, then the cutoff frequency given by* (3.33) *guarantees the robust stability of the repetitive control system* (3.21)*.*

Proof. From Theorem 3.1 and Equations (3.33) $\sim$ (3.41), this theorem can be obtained directly. This completes the proof. □

Thus, for the given rough estimate $\hat{\omega}_c$, we can obtain the optimal cutoff frequency ω_c by solving the following LMI-constrained optimization problem

$$\min \ \sigma > 0 \ \text{ subject to } (3.25), (3.26) \text{ and } (3.41). \tag{3.42}$$

On the other hand, Mahawan and Luo [106] proved that there exists a unitary operator T such that the control system shown in Figure 3.4 is equivalent to the control system shown in Figure 3.5 with $r(t) \equiv 0$ and $d(t) \equiv 0$. The unitary operator

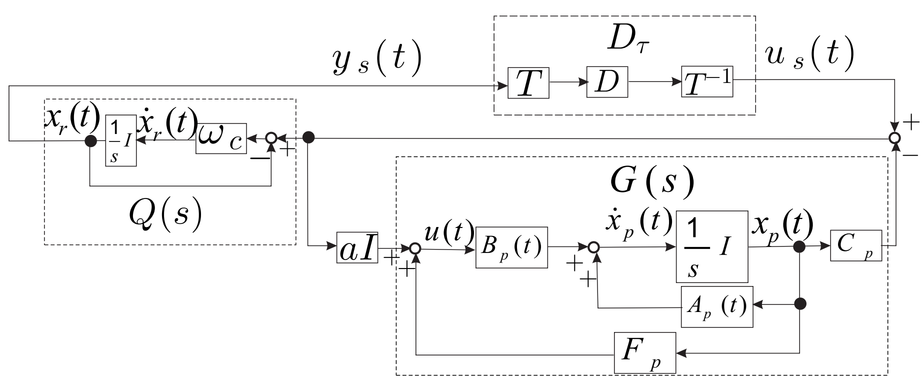

Figure 3.5: Equivalent diagram of Figure 3.4

T satisfies

$$\left\| T^{-1} D T \right\|_\infty \leq 1, \tag{3.43}$$

where the delay operator $D : L^2(0, \theta_f) \to L^2(0, \theta_f)$ is defined as

$$D\zeta(\theta) := \zeta(\theta - T_\theta) \tag{3.44}$$

and T_θ is the spatial period of the disturbances.

The transfer function $T_{y_s u_s}(s)$ from u_s to y_s is given by

$$T_{y_s u_s}(s) = Q(s)\left(I + aG(s)\right)^{-1}. \tag{3.45}$$

Then, from the small-gain theorem, the closed-loop system with the modified repetitive controller is asymptotically stable if

$$\|T_{y_s u_s}\|_\infty = \|Q(s)(I + aG(s))^{-1}\|_\infty < 1. \tag{3.46}$$

Hence, for the given $Q(s)$ and F_p, we can regulate the parameter a to the optimal value by using the H_∞ control method.

From Figure 3.5, the state space description of $T_{y_s u_s}$, in general, is given by

$$\begin{cases} \dot{x}(t) = (A_s + \Delta A_s(t))x(t) + (B_s + \Delta B_s(t))u_s(t) \\ y_s(t) = C_s x(t) \end{cases}, \tag{3.47}$$

where $x(t)$ is defined in (3.21) and
$A_s = \begin{bmatrix} A_p + B_p F_p - aB_p C_p & 0 \\ -\omega_c C_p & -\omega_c I \end{bmatrix}$, $B_s = \begin{bmatrix} aB_p \\ \omega_c I \end{bmatrix}$, $C_s = \begin{bmatrix} I & 0 \end{bmatrix}$, $\Delta A_s(t) = \Phi_s \Gamma(t) E_s$,
$\Delta B_s(t) = \Phi_s \Gamma(t) a \Psi_B$, $\Phi_s = \begin{bmatrix} \Phi_p^T & 0 \end{bmatrix}^T$ and $E_s = \begin{bmatrix} \Psi_A + \Psi_B F_p - a\Psi_B C_p & 0 \end{bmatrix}$.
Applying Lemmas 3.2 $\sim$ 3.4 to the above system yields the following result.

Theorem 3.3. *For the system* (3.47), *if a symmetric matrix* $P > 0$ *and a positive scalar* λ *exist such that the LMI*

$$\begin{bmatrix} PA_s + A_s^T P & PB_s & C^T & P\Phi_s & \lambda E_s^T \\ * & -I & 0 & 0 & a\lambda \Psi_B^T \\ * & * & -I & 0 & 0 \\ * & * & * & -\lambda I & 0 \\ * & * & * & * & -\lambda I \end{bmatrix} < 0 \tag{3.48}$$

holds, then the closed-loop system in (3.47) *is robustly stable.*

Proof. According to Lemma 3.3, a necessary and sufficient condition that guarantees both that the closed-loop system in Figure 3.5 is robustly stable and also that (3.46) holds is that there exists a symmetric matrix $P > 0$ such that the following linear matrix inequality is feasible.

$$\Pi_n + \Pi_u + \Pi_u^T < 0, \tag{3.49}$$

where

$$\Pi_n := \begin{bmatrix} PA_s + A_s^T P & PB_s & C^T \\ * & -I & 0 \\ * & * & -I \end{bmatrix} \tag{3.50}$$

and

$$\Pi_u := \begin{bmatrix} P\Phi_s \\ 0 \\ 0 \end{bmatrix} \Gamma(t) \begin{bmatrix} E_s & a\Psi_B & 0 \end{bmatrix}. \tag{3.51}$$

For a positive scalar, $\lambda > 0$, employing Lemma 3.4, we obtain

$$\Pi_u + \Pi_u^T \leq \lambda^{-1} \begin{bmatrix} P\Phi_s \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} P\Phi_s & 0 & 0 \end{bmatrix} + \lambda^{-1} \begin{bmatrix} \lambda E_s^T \\ a\lambda\Psi_B^T \\ 0 \end{bmatrix} \begin{bmatrix} \lambda E_s & a\lambda\Psi_B & 0 \end{bmatrix}. \tag{3.52}$$

Substituting (3.52) into (3.49) appropriately and applying the Schur complement Lemma 3.2, the LMI given in (3.48) is obtained.

We have thus proved this theorem. □

Hence, the problem of regulating the parameter a satisfying (3.46) is converted into the problem of regulating the parameter a satisfying the LMI condition (3.48). We now find the largest parameter a_{max} to guarantee the system stability using the result of Theorem 3.3.

Without loss of generality, represent a_{max} as the sum of a_0 and $\bar{\delta}a$

$$a_{max} = a_0 + \bar{\delta}a, \tag{3.53}$$

where a_0 is given in the theorem 3.2 and $\bar{\delta}a$ is an unknown value to be decided. Then, A_s, B_s and E_s are affinities dependent on the free parameter $\bar{\delta}a$ and are represented as the following form:

$$A_s = \bar{A}_s + \hat{A}_s \times \bar{\delta}a, \tag{3.54}$$

$$B_s = \bar{B}_s + \hat{B}_s \times \bar{\delta}a \tag{3.55}$$

and

$$E_s = \bar{E}_s + \hat{E}_s \times \bar{\delta}a, \tag{3.56}$$

where

$$\bar{A}_s = \begin{bmatrix} A_p + B_pF_p - a_0B_pC_p & 0 \\ -\omega_cC_p & -\omega_cI \end{bmatrix},$$

$$\hat{A}_s = \begin{bmatrix} -B_pC_p & 0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{B}_s = \begin{bmatrix} a_0B_p \\ \omega_cI \end{bmatrix},$$

$$\hat{B}_s = \begin{bmatrix} B_p \\ 0 \end{bmatrix},$$

$$\bar{E}_s = \begin{bmatrix} \Psi_A + \Psi_BF_p - a_0\Psi_BC_p & 0 \end{bmatrix}$$

and

$$\hat{E}_s = \begin{bmatrix} -\Psi_BC_p & 0 \end{bmatrix}.$$

In the following theorem, a modified stability condition is proposed, which is represented as an LMI.

Theorem 3.4. *For given ω_c and F_p, the adjustable parameter given by* (3.53) *guarantees the robust stability of the repetitive control system* (3.47)*, if there is a symmetric positive definite matrix P, and positive scalars λ and $\rho := \bar{\delta}a^{-1}$such that*

$$\hat{\Theta} < -\rho\Theta \tag{3.57}$$

holds with the shorthand

$$\Theta := \begin{bmatrix} P\bar{A}_s + \bar{A}_s^T P & P\bar{B}_s & C^T & P\Phi_s & \lambda\bar{E}_s^T \\ * & -I & 0 & 0 & a_0\lambda\Psi_B^T \\ * & * & -I & 0 & 0 \\ * & * & * & -\lambda I & 0 \\ * & * & * & * & -\lambda I \end{bmatrix} \tag{3.58}$$

and

$$\hat{\Theta} := \begin{bmatrix} P\hat{A}_s + \hat{A}_s^T P & P\hat{B}_s & 0 & 0 & \lambda\hat{E}_s^T \\ * & 0 & 0 & 0 & \lambda\Psi_B^T \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & 0 \\ * & * & * & * & 0 \end{bmatrix}. \tag{3.59}$$

Proof. Replacing a, A_s, B_s and E_s by (3.54) $\sim$ (3.56) in (3.48), we have:

$$\Theta + \hat{\Theta} \times \bar{\delta}a < 0. \tag{3.60}$$

By introducing the new variable $\rho := \bar{\delta}a^{-1}$ and applying it to (3.60), the LMI condition (3.57) can be obtained.

We have thus proved this theorem. □

We observe that for a given optimal cutoff frequency and a, the maximum $\bar{\delta}a$ can be obtained by solving the optimization problem

$$\min \ \rho > 0 \ \text{ subject to (3.57).} \tag{3.61}$$

The constraints in the optimization problems (3.42) and (3.61) have the standard forms of generalized eigenvalue minimization problems (GEVP) with semipositive conditions. Hence, they can be solved numerically using the bisection algorithm in YALMIP [116] or the GEVP solver in the LMI-toolbox [117].

3.4 Design procedure

In this section, we present a design procedure for a robust stabilizing modified repetitive controller with optimal performance for position-dependent disturbances.

Procedure

Step 1: Select a solution precision, ϵ, for the optimization problems and positive real scalars, γ, a, and $\hat{\omega}_c$ that are small enough.

Step 2: Solve the feasible problem (3.16) to obtain the state-feedback controller F_p with given γ for position-dependent disturbances without a repetitive controller.

Step 3: Check the feasibility of Theorem 3.1.

Step 4: If feasible, go to the next step. Otherwise, select new values for a and $\hat{\omega}_c$, and return to step 2.

Step 5: Solve the optimization problem (3.42) using a, $\hat{\omega}_c$, and F_p. If a solution exists, then set $\omega_c = \hat{\omega}_c + 1/\sigma$ and go to the next step. Otherwise, set $\omega_c = \hat{\omega}_c$ and go to the next step.

Step 6: Solve the optimization problem (3.61) using a, F_p and ω_c. If a solution exists, set $a_{max} = a + 1/\rho$ and stop. Otherwise, set $a_{max} = a$ and stop.

The design procedure proposed in this section is applicable for both single-input/single-output(SISO) linear systems and multiple-input/multiple-output(MIMO) linear systems by simply modifying the dimensions of some matrices.

3.5 Numerical example

In this section, a numerical example is shown to illustrate the effectiveness of the proposed design method.

Consider the SISO system (3.10) with

$$A_p = \begin{bmatrix} -8 & -10 \\ 1 & 0 \end{bmatrix}, \; B_p = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \; C_p = \begin{bmatrix} 1 & 1 \end{bmatrix}, \; D_w = \begin{bmatrix} 0.4 \\ 0.3 \end{bmatrix},$$

$$\Phi_p = \begin{bmatrix} 0 & 0 \\ 1 & 0.1 \end{bmatrix}, \; \Psi_A = \begin{bmatrix} 1 & 0 \\ 0 & 0.1 \end{bmatrix}, \; \Psi_B = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}$$

and

$$\Gamma(t) = \begin{bmatrix} \sin(0.1\pi t) & 0 \\ 0 & \cos(0.1\pi t) \end{bmatrix}.$$

We set $\gamma = 0.1$. Then, the state-feedback controller F_p obtained by solving the feasible problem (3.16) is

$$F_p = \begin{bmatrix} -1.308 & -21.621 \end{bmatrix}. \tag{3.62}$$

Choose $\epsilon = 10^{-3}$, $a = 1$, $\hat{\omega}_c = 30$[rad/s] and suppose that the disturbance signal, as shown in Figure 3.6, is given by

$$d(t) = \sin\left(\frac{2\pi}{5}\theta\right) + \sin\left(\frac{4\pi}{5}\theta\right) \tag{3.63}$$

and

$$\frac{d\theta}{dt} = \omega(t) = 10 + 5\cos(t). \tag{3.64}$$

Then the position-dependent disturbance is converted into a time-varying periodic signal with period $\tau(t)$, shown in Figure 3.7 and its derivative is shown in Figure 3.8. From Figs. 3.7 and 3.8, the time-varying period satisfies (3.7) and we set

$$\bar{\tau} = 1, \quad \mu = 0.4. \tag{3.65}$$

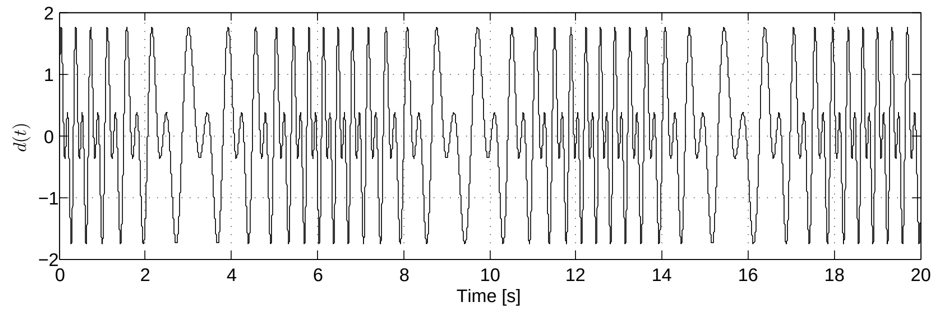

Figure 3.6: Disturbance signal used in simulations

According to the design procedures in Section 3.3 and using the above parameters, the minimum σ is obtained by solving the optimization problem (3.42) as $\sigma = 7.755\times10^{-4}$.

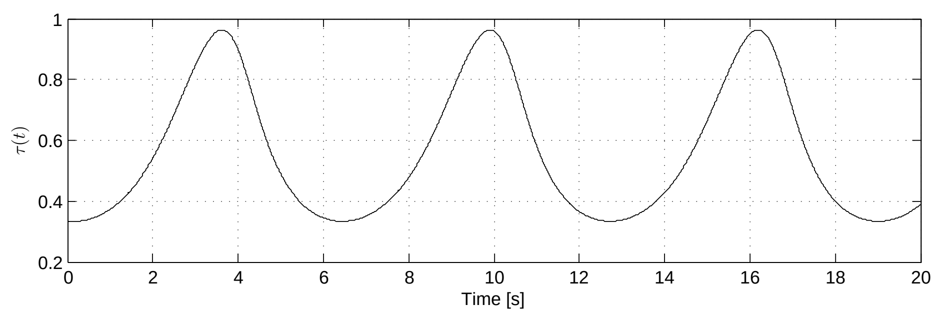

Figure 3.7: Time-varying period, $\tau(t)$

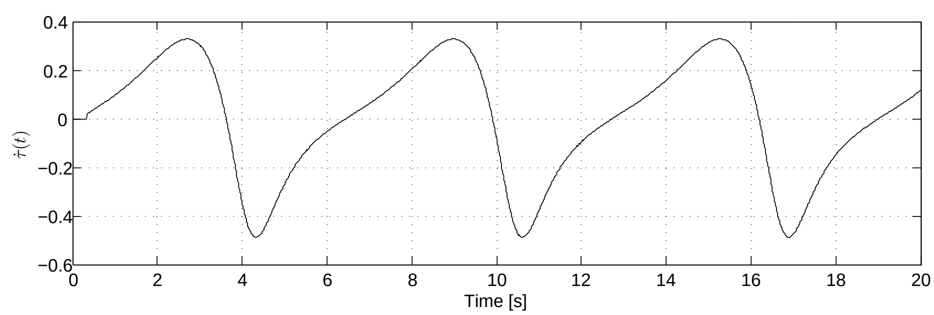

Figure 3.8: Derivative of $\tau(t)$, $\dot{\tau}(t)$

Therefore, the maximum cutoff frequency ω_c of the low-pass filter $Q(s)$ is

$$\omega_c = \hat{\omega}_c + \frac{1}{\sigma} = 1319.490 \text{ [rad/s]}. \tag{3.66}$$

After obtaining the optimal cutoff frequency ω_c, we solve the optimization problem (3.61) to obtain the largest adjustable parameter a_{max} as

$$a_{max} = 1 + \frac{1}{\rho} = 12.628 \tag{3.67}$$

with the minimum $\rho = 0.086$.

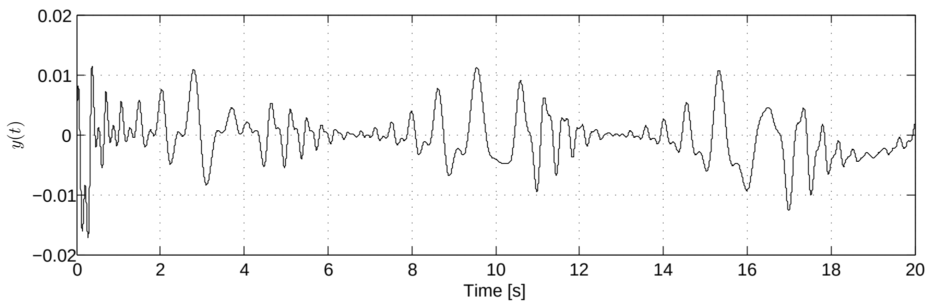

Figure 3.9: Response of the output $y(t)$ for the disturbance $d(t)$ with our repetitive controller

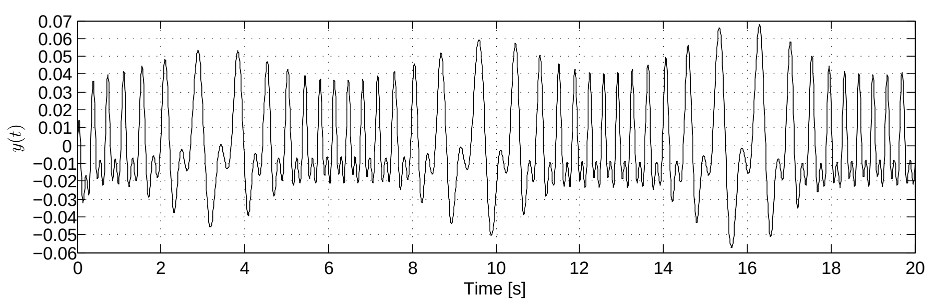

Figure 3.10: Response of the output $y(t)$ for the disturbance $d(t)$ without our repetitive controller

The simulation results in Figure 3.9 show that the system enters the steady state in the second period and that the output is 0.68% of the disturbance when considering the amplitude of the disturbance and the output after the application of the new repetitive controller. For comparison, we also simulated this control system without the repetitive controller. The simulation results in Figure 3.10 show that, without the repetitive controller, the disturbance is attenuated to about 4.00%. Clearly, better disturbance attenuation is obtained with the proposed repetitive control system than without the repetitive controller. This design procedure demonstrates that the control performance can be improved by optimizing the parameters of the new modified repetitive controller from a general disturbance attenuation control system and that robust stability can also be guaranteed. In contrast, the design methods proposed in [105, 110] achieve robust stability without considering the control precision and are required to deal with a nonlinear system in the spatial domain. Thus, an optimal modified repetitive controller can easily be designed as shown here for position-dependent disturbances.

3.6 Conclusions

In this chapter, position-dependent disturbances are converted into time-varying periodic signals and a new modified repetitive controller structure is presented. To obtain good disturbance attenuation, we proposed a design method for the optimal modified repetitive control system based on LMIs, which can be applied to rotary motion systems. We also gave a complete proof of the theorems for the design method that were omitted previously [118]. By reformulating the LMI-constrained robust stability conditions, an optimal modified repetitive control system can be obtained by solving the resulting optimization problems. A numerical example was presented to demonstrate the effectiveness of the proposed design method. The results in this paper extend the application of the repetitive control technique to systems with time-varying uncertainties, and can also be potentially applied to the systems with time-varying state delay and input delay [41].

Chapter 4

Conclusions and Future Work

Repetitive control is one of high-precision servo control methods developed from 1980s. Due to its simple structure and high-precision, this control technique captured more attention and has been widely applied in servo control system for periodic signal with high-precision requirement. Based on analysis the design method and application of modified repetitive control system, this thesis furthers the research on the robust stabilizing problems for multiple-input/multiple-output plants and time-varying periodic signals. In this chapter, we summarize the key developments in this thesis and point out areas for future research.

4.1 Conclusions

Chapter 2, provides a design method of a robust stabilizing modified repetitive controller for multiple-inupt/multiple-out plants using parameterization. The parameterization of all robust stabilizing modified repetitive controllers and the robustness stability condition are achieved by employing the H_∞ control approach and Rcaati equation. We can regulate the free parameters to guarantee the robustness stability, tracking and attenuation performance for the control system. In order to simplify the design process and avoid the wrong results obtained by graphical method, the robust stability conditions are converted into LMIs-constraint conditions by employing the delay-dependent bounded real lemma. This method has some merits such as

- the modified repetitive controller makes the control system stable for any multiple-input/multiple-output plant with uncertainty,
- this method relaxes the requirements for the actual control system because the parameterization is to find a set of this class of controllers.

Chapter 3, provides a design method of a modified repetitive controller for rejecting time-varying periodic disturbance. In this chapter, a new modified repetitive controller structure is presented. To obtain good disturbance attenuation, we proposed a design method for the optimal modified repetitive control system based on LMIs, which can be applied to rotary motion systems. Two linear matrix inequalities (LMIs)-based robust stability conditions of the closed-loop system with time-varying state delay are derived for fixed parameters. One is a delay-dependent robust stability condition that is derived based on the free-weight matrix. The other robust stability condition is obtained based on the H_∞ control problem by introducing a linear unitary operator. By reformulating the LMI-constrained robust stability conditions, an optimal modified repetitive control system can be obtained by solving the resulting optimization problems. The advantages of this design method are described as:

1. it can void solving the nonlinear system or introducing the adaptive control approach,
2. we can obtain an optimal control performance by using this design method,
3. comparing with other design methods, the design and computation of this control system are much easier.

4.2 Future Work

The following areas are recommended for future research:

1. Generally, in control system design, the requirements like tracking performance, attenuation performance and cost should be guaranteed. Some time the periodic reference input and/or disturbance are arbitrary without restriction on the frequency component. The design method which has relaxed robustness stability condition and optimal performance will attract more attentions from engineers. Chapter 2 illustrates that the low-pass filter specified the disturbance attenuation characteristics has bandwidth limitation. To obtain the largest frequency of the low-pass filter guaranteeing control precision is an important issue. Hence, it is necessary to further the research on this problem to relax the restrictions.

2. According to the analysis in Chapter 2, there exist two low-pass filters. Both these two low-pass filters have bandwidth restrictions which influence the control performance. In practice, to meet certain requirement, it is necessary to obtain the optimal performance. Therefore, optimization design method is one of the future works.

3. For the time-varying periodic signals, the research approach are based on nonlinear system and adaptive control method. Even though the optimal performance can be obtain by employing the proposed method in Chapter 3, there are some conservatism exist and the robustness stability conditions are strictly. And an important issue in the control is not solved for this class of problems, the parameterization problem.

4. Generally, most repetitive controller designs in literature suffer from two major drawbacks. One is the requirement of exact knowledge of the period-time of reference or disturbance signals [119]. This means that in practical applications, either the period-time is required to be a constant, or an accurate measurement of the periodicity is indispensable, which may be jeopardized in practice by clock error drift, jitter, measurement noise and so on. The other is due to the Bode Sensitivity Integral [120]: the perfect reduction at the harmonic frequencies is

counteracted by amplification of noise at intermediate frequencies. To address these problems, so-called high-order repetitive control has been established [80, 30, 121, 122, 123, 124, 125, 126]. When the high-order modified repetitive control structure is assigned to a closed-loop system to track or reject external signals with uncertain period-time, it is often desirable to design compensators that not only stabilize the closed-loop system but also guarantee a perfect control performance for some variations in the period-time.

Acknowledgements

I would like to express my sincere gratitude to all those who have advise me and help me through the completion of this thesis. This work would be impossible without their strong support in these years.

I'd like to express my sincere gratitude to my thesis and research advisor, Professor Kou Yamada. You passionately and patiently guided me through the entire research. Without your immeasurable help, I wouldn't even have been able to start this work.

I am grateful for the support of my family and my wife. I am deeply indebted to my wife, Jie Zeng. Throughout the years since we met, your love and support are beyond word. Thank you for living me, understanding me , believing in me and encourage me. This work is a successful result of our joint effort. Together we will have more stories of success in the rest of our journey.

I'd also like to acknowledge the financial support provided by China Scholarship Council (CSC). It made my journey possible for pursuing a Ph.D. degree after Master's.

Last, but not least, I would like to thank Dr. T. Hagiwara, Mr. T. Sakanushi and all my present and past control group colleagues for their participation in serval of my presentations, valuable suggestions, inspiring discussions with me and help in my life in Japan.

March 2013 Zhongxiang Chen

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Publications

Publications related to this thesis

Chapter 2 Z. Chen, K. Yamada, Y. Ando, I. Murakami, T. Sakanushi, N. L. T. Nguyen, S. Yamamoto, The parameterization of all robust stabilizing modified repetitive controllers for multiple-input/multiple-output plants, *ICIC Express Letters*, Vol. 5, No. 8(B), pp. 2773–2778, 2011.

Chapter 3 Z. Chen, K. Yamada, T. Sakanushi, Y. Zhao, Linear matrix inequality-based repetitive controller design for linear systems with time-varying uncertainties to reject position-dependent disturbance, *International Journal of Innovative Computing, Information and Control*, accepted for publication

Other publications

1. A Design Method for Internal Model Controllers for Minimum-Phase Unstable Plants, Z. Chen, K. Yamada, N. T. Mai, I. Murakami, Y. Ando, T. Hagiwara, T. Hoshikawa, *ICIC Express Letters*, Vol.4, No.6(A), PP.2045-2050,2010.

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