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群馬大学博士論文

Study on design methods for simple repetitive control systems with specified input-output frequency characteristic

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Contents

Chapter 1

Introduction

1.1 A trend of a study for repetitive control

Periodic signals are very common in engineering. They are associated with magnet power supply of synchrotron, engines, electrical motors and generators, converters, machines that perform a cyclic task and many other things [1]∼[6]. To handle them, repetitive control was originated by Inoue et al. That is, the repetitive control is the method to track a periodic reference input without steady state error and reject periodic disturbances effectively [1]∼[16]. The repetitive control system has a simple structure and is easily designed. Because the repetitive control is a very practical and effective way for a system to track a periodic reference and reject periodic disturbances and the structure is simple, the repetitive control is applied to many applications such as trajectory control of the robot manipulator [4, 5] and reducing rotational unevenness in motors[6].

1.1.1 Repetitive control

Firstly we will look at the trend of a study for the repetitive control. Requirements of repetitive control system can be divided roughly into the following two. One is input-output characteristic and the other is stability of the control system.

Input-output characteristic means that the output follows the periodic reference input without a steady state error. In order for the output to follow the reference input without a steady state error, from internal model principle [17, 18], an internal model that has same poles of reference input must be included in the controller. To obtain the internal model for the periodic reference input, we assume that the reference input $r(t)$ is a periodic function with period $T > 0$ written as

$$r(t + T) \;=\; r(t). \tag{1.1}$$

When we define a period of r(t) as

$$r_0(t) \;=\; r(t) \quad (0 \leq t \leq T), \tag{1.2}$$

the periodic reference input $r(t)$ can be rewritten as

$$r(t) \;=\; r_0(t - nT), \tag{1.3}$$

where n is a non-negative integer satisfying $0 \leq t - nT < T$. When we define Laplace transformation of $r(t)$ and $r_0(t)$ as $r(s)$ and $r_0(s)$ respectively, $r(s)$ is written by

$$r(s) \;=\; \int_0^\infty r(t)e^{-sT}dt$$

$$= \sum_{n=0}^{\infty} e^{-nsT} r_0(s)$$
$$= \frac{1}{1-e^{-sT}} r_0(s). \quad (1.4)$$

From (1.4), the structure of an internal model for the periodic reference input is shown in Fig. 1.1 [1, 2, 3]. The controller C(s) that has the internal model for the periodic reference input

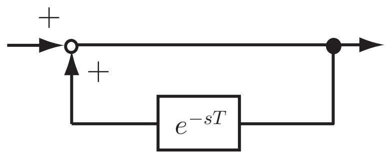

Fig. 1.1: The internal model for the periodic reference input with period T

$r(t)$ with period T in Fig. 1.1 is called the repetitive controller, and a control system using the repetitive controllers is called the repetitive control system.

1.1.2 Modified repetitive control

Under the internal model in Fig. 1.1 , we will describe the stability of repetitive control system. Because the repetitive controller has an internal model in Fig. 1.1 , the repetitive controller has infinite number of poles on imaginary axis as shown in Fig. 1.2 . This type of a system

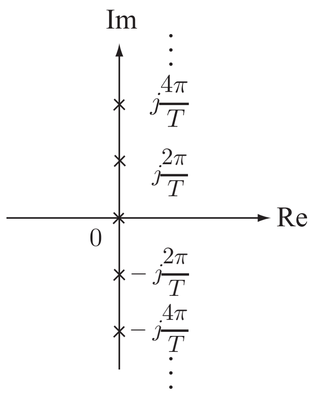

Fig. 1.2: Poles of the internal model for the periodic reference input with period T

is called the neutral type of time-delay system. Because we must stabilize infinite number of poles by feedback control, it is difficult to design stabilizing controllers for the plant [12]. To design a repetitive control system that follows any periodic reference input without steady state error, the plant needs to be biproper [3, 4, 7, 8, 9, 10, 11, 12]. Ikeda and Takano [13, 14] pointed out that it has physical difficulty that the output follows any periodic reference input without steady state error. In addition they showed that the repetitive control system is L_2 stable for periodic signal that does not include infinite frequency signals if the relative degree of controller is one. However, the actually control system is strictly proper and has any relative degree. Therefore, many design methods for repetitive control systems for strictly proper plants have been given [3, 4, 7, 8, 9, 10, 11, 12]. These systems are divided into two types. One type uses an attenuator [12] and the other type uses a low-pass filter [3, 4, 7, 8, 9, 10, 11].

The method using an attenuator was proposed in [12]. This method makes the internal model includes an attenuator α $(0 < \alpha < 1)$ as shown in Fig. 1.3 and examines a design method for repetitive control systems for strictly proper plants using the internal model in Fig. 1.3 . When the internal model includes an attenuator, poles of the internal model in Fig. 1.3

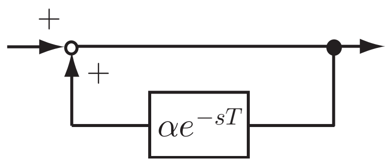

Fig. 1.3: The internal model for the reference input with period T using the method in [12]

are $s = \frac{\log_e \alpha}{T} + j\frac{2\pi}{T}k$ $(k = 0, \pm 1, \pm 2, \ldots)$. Therefore, all poles of internal model are restricted to open left half plane as shown in Fig. 1.4 . The system that has finite number of poles on

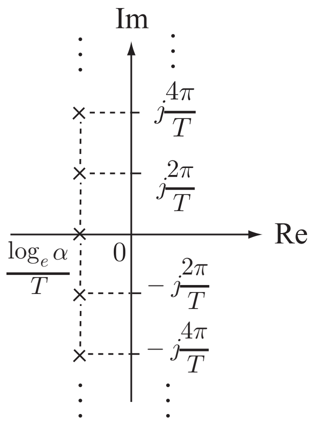

Fig. 1.4: Poles of the internal model using the method in [12]

imaginary axis and has other infinite number of poles in open left half plane, is called the delay type of time-delay system, and is known that stabilization is relatively easy. The reference in [12] achieves stabilization using partial pole placement method that is one of the stabilization methods for the delay type of time-delay system. That is, the method in [12] is a method to convert the neutral type of time-delay system into the delay type of time-delay system, and achieve stabilization using partial pole placement method [12].

The method using a low-pass filter was proposed in [3, 4, 7, 8, 9, 10, 11]. This method notes that it is impossible to follow all frequency components of the periodic reference input and examines a design method for repetitive control systems for strictly proper plants. To stabilize the repetitive control system, it must not occur unstable pole-zero cancellation between the plant and the controller. However, unstable pole-zero cancellation occurs between the zero at infinity included in the plant and the pole at infinity included in the repetitive controller. Therefore, it is impossible to follow all frequency components of the periodic reference input without steady state error for strictly proper plants. From this, using the idea that it permits tracking error for high-frequency components and follows with high precision for low-frequency components, the method using a low-pass filter was proposed [3, 4, 7, 8, 9, 10, 11]. This method makes the internal model includes a low-pass filter $q(s)$ as shown in Fig. 1.5 . When the internal model includes a low-pass filter, a pole of internal model exists in the origin on the imaginary axis, but all other poles exist in open left half plane as shown in Fig. 1.6 . Therefore the repetitive control system becomes the delay type of time-delay system and stabilization is easy.

Here we compare two methods. The former type of system is difficult to design because it uses a state-variable time-delay in the repetitive controller [12]. The latter has a simple

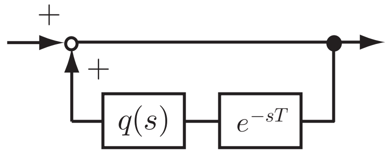

Fig. 1.5: The internal model of the modified repetitive controller using the low-pass filter

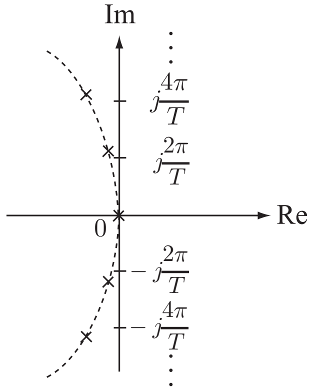

Fig. 1.6: Poles of the internal model of the modified repetitive controller using the low-pass filter

structure and is easily designed. Therefore, the former type of repetitive control system is called the modified repetitive control system [3, 4, 7, 8, 9, 10, 11].

In this way, the input-output characteristic and the stability of the repetitive control system have been examined.

1.1.3 The history of the expansion of modified repetitive control

In this subsection, how the modified repetitive control system has been researched is shown. When the control system is designed, the control problem that should be examined is different according to the class of the plant and the control performance to be achieved. Therefore, it is necessary to think about the control problem individually for the class of the plant and the control performance to be achieved.

Here, problem to time-delay system, robust stability problem, problem of disturbance attenuation characteristic, and the parameterization problem are shown.

1. Problem to time-delay system

   In an actual mechanism, there is a device that the delay is caused by the delay of the operation etc. in the transmission of the signal. The control performance decreases remarkably to take time from the change of the instrumental variable to the appearance of the influence to the control variable. When we define $u(t)$ is the input, $y(t)$ is the output and $T > 0$ is the time-delay, then the input-output relation is written by

   $$y(t) = u(t - T). \tag{1.5}$$

   When we perform Laplace transformation using (1.5), we have

   $$Y(s) = e^{-sT}U(s). \tag{1.6}$$

   Element e^{-sT} that causes the delay of the signal is called a dead time component, and the control system including dead time component e^{-sT} is called a time-delay system. Because the time-delay system includes the dead time component e^{-sT}, the system has infinite number of poles on imaginary axis. As it was previously mentioned, such a system is the neutral type of time-delay system, and stabilization is difficult [12]. The design method of repetitive control for time-delay system was examined in [15, 16]. The reference in [15] gives a design method of repetitive control system for input time-delay system using an attenuator and partial pole placement method. The method in [15] can design the repetitive control system that achieves small steady state error for large time-delay. The reference in [16] gives a design method of repetitive control system for input time-delay system using the modified repetitive controller and a state predictor, without using a state-variable time-delay in the repetitive controller. That is, this method notes that it is difficult to stabilizes the time-delay system using only modified repetitive controller and clarifies that the stabilization problem of modified repetitive control for time-delay system can be came to a same stabilization problem of control system for non-time-delay system [7, 8] by using the modified repetitive controller and a state predictor.

2. Robust stability problem

   When the modified repetitive controller is applied to real systems, the influence of uncertainties in the plant must be considered, because many real plants include the uncertainty. In some cases, the uncertainty makes the control system unstable. The stability problem with the uncertainty is known as the robust stability problem [19]∼[25]. The robust stability problem of modified repetitive control systems was considered by Hara et al. [26]. The robust stability condition for modified repetitive control systems was reduced to the μ synthesis problem [26], but the μ synthesis problem cannot be solved analytically. That is, in order to solve the μ synthesis problem, we must solve an H_∞ problem iteratively using the $D - K$ iteration method. Furthermore, the convergence of iterative methods to solve the μ synthesis problem is not guaranteed. Yamada et al. tackled this problem and proposed a design method for robust repetitive control systems without solving the μ synthesis problem [27].

3. Problem of disturbance attenuation characteristic

   When the modified repetitive controller is applied to real systems, the disturbance in the plant must be attenuated to achieve desired action. The disturbance attenuation characteristic of modified repetitive control system was examined in [5, 28, 29, 30, 31, 32]. Gotou et al. [28] notes that the ratio of the disturbance attenuation characteristic of modified repetitive control system to the disturbance attenuation characteristic of control system without modified repetitive controller satisfies $1 - q(s)e^{-sT}$, and examined the disturbance attenuation characteristic of modified repetitive control system. The gain plot of $1-q(s)e^{-sT}$ when $q(s) = 1/(1+0.05s)$ and $T = 2\pi/5$ is shown in Fig. 1.7 . Figure 1.7 shows that at certain frequencies, the disturbance is amplified as twice, because the maximum value of the gain of $1 - q(s)e^{-sT}$ is 2. Gotou et al. [28] overcame this problem by proposing a multi-period repetitive control system that uses an internal model shown in Fig. 1.8 . From Fig. 1.8 , we can construe the multi-period repetitive controller as a controller using information not only before one period but also before N period. When the multi-period repetitive controller is used, the fact that the disturbance attenuation characteristic can be improved is confirmed as follows: The gain plot of $1-\sum_{i=1}^{3} q_i(s)e^{-sT_i}$ when $q_i(s) = 1/(1 + 0.05s)(i = 1, 2, 3)$ and $T_i = 2\pi k/5(k = 1, 2, 3)$ is shown in Fig. 1.9 . From Fig. 1.9 , it is clear that the disturbance attenuation characteristic can be improved. However, the phase angle of the low-pass filter in a multi-period repetitive controller has a bad effect on the disturbance attenuation characteristics [31, 32]. Yamada et al. overcame this problem and proposed a design method for multi-period repetitive controllers to attenuate disturbances effectively [33, 34] using the time advance compensation described

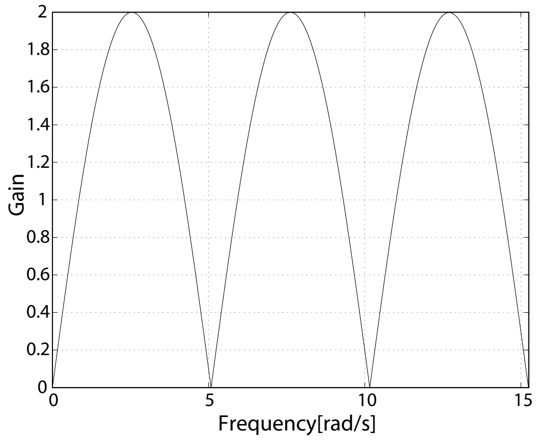

Fig. 1.7: The gain plot of $1 - q(s)e^{-sT}$

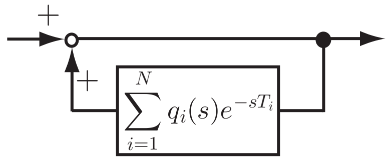

Fig. 1.8: The internal model of the multi-period repetitive controller using the method in [28]

in [31, 32, 35]. Using this multi-period repetitive control structure, Steinbuch proposed a design method for repetitive control systems with uncertain period time [36].

4. Parameterization problem

   There exists one of important control problems to find all stabilizing controllers named the parameterization problem [37, 38, 39, 40, 41]. Using the parameterization, at first the stability of control system is guaranteed by choosing a controller from the parameterization. The parameterization includes free parameter that can be chosen freely. We can satisfy specifications except the stability by using the flexibility of this free parameter. That is, we can design a control system with two phases to satisfy stability and other specifications. So, we can easily design stabilizing controllers. Therefore, it is a important control problem to obtain the parameterization. At first, the parameterization of all

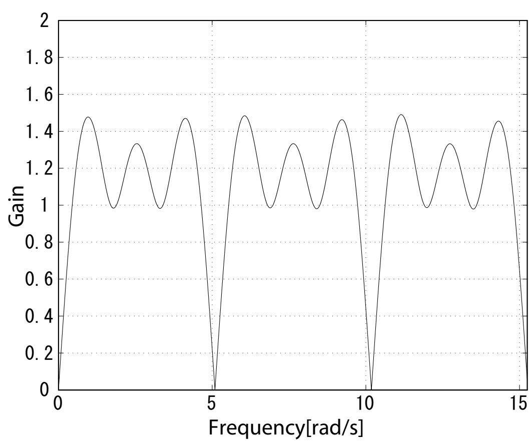

Fig. 1.9: The gain plot of $1 - \sum_{i=1}^{3} q_i(s)e^{-sT_i}$

stabilizing modified repetitive controllers was studied by Hara and Yamamoto [8]. In [8], since the stability sufficient condition of repetitive control system is decided as H_∞ norm problem, the parameterization of all stabilizing modified repetitive controllers is given by resolving into the interpolation problem of Nevanlinna-Pick. Katoh and Funahashi gave the parameterization of all stabilizing modified repetitive controllers for minimum phase systems by solving exactly Bezout equation [42]. In [42], since the parameterization is not given based on stability sufficient condition that the modified repetitive control system is internally stable, this result is important in the sense that the class of modified repetitive controllers is extensive than a class of modified repetitive controllers given in [8]. However, in [42], the plant is assumed to be stable or be stabilized by local feedback control. This implies that the reference in [42] gave a parameterization of all stabilizing modified repetitive controllers for a stable and minimum phase plant. That is, the reference in [42] did not give the exact parameterization for minimum phase systems. Yamada and Okuyama overcame this problem and gave the parameterization of all stabilizing modified repetitive controllers for minimum phase systems those are not necessarily stable [47]. Yamada et al. [43] expanded the result in [47] and gave the parameterization of all stabilizing modified repetitive controllers for a certain class of non-minimum phase systems using the idea of parallel compensation technique and the solution of Bezout equation. Yamada et al. gave the parameterization of all stabilizing modified repetitive controllers for non-minimum phase systems [44]. The parameterization of all stabilizing multi-period repetitive controllers was solved in [45, 46].

1.1.4 Simple repetitive control

Using modified repetitive controllers [3, 4, 7, 8, 9, 10, 11], even if the plant does not include time delays, transfer functions from the periodic reference input to the output and from the disturbance to the output have infinite numbers of poles. This makes it difficult to specify the input–output characteristic and the disturbance attenuation characteristic. From a practical point of view, it is desirable that these characteristics should be easy to specify. Therefore, these transfer functions should have finite numbers of poles. To overcome this problem, Yamada et al. proposed simple repetitive control systems such that the controller works as a modified repetitive controller, and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles [48]. In addition, Yamada et al. clarified the parameterization of all stabilizing simple repetitive controllers.

1.2 A trend of a study for simple repetitive control

In this section, how simple repetitive control system has been researched is shown. When the control system is designed, the control problem that should be examined is different according to the class of the plant and the control performance to be achieved. Therefore, it is necessary to think about the control problem individually for the class of the plant and the control performance to be achieved. The design methods for simple repetitive control systems hitherto examined are as follows:

1. Simple multi-period repetitive control

   Using multi-period repetitive control structure in [28, 33, 34, 45, 46], it is possible to design a control system to attenuate periodic disturbances effectively than the simple repetitive control. Yamada and Takenaga proposed simple multi-period repetitive control systems such that the controller works as a multi-period repetitive controller, and the transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles [49]. In addition, they clarified the parameterization of all stabilizing simple multi-period repetitive controllers.

2. Robust stabilization

   The stability problem with uncertainty is known as the robust stability problem [19]∼[25]. When the simple repetitive controller in [48] is applied to the real control system, the influence of uncertainty must be considered. The parameterization of all robust stabilizing controllers for the plant with uncertainty is obtained using H_∞ control theory based on the Riccati equation [19, 20] and the Linear Matrix Inequality (LMI) [21, 22]. Using this parameterization, Yamada et al. proposed the parameterization of all robust stabilizing simple repetitive controllers [50]. Sakanushi et al. proposed the parameterization of all robust stabilizing simple multi-period repetitive controllers [51].

3. Time-delay system

   The method in [50] cannot be applied to time-delay plants with uncertainty. Since many real systems include time-delays and uncertainties, the problem to obtain the parametrization of all stabilizing simple repetitive controllers for time-delay plants with uncertainty is one of important problem to solve. Yamada et al. proposed the parametrization of all robust stabilizing simple repetitive controllers for time-delay plants with uncertainty [52] and that of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with uncertainty [53]

4. Multiple-input/multiple-output plants

   For multiple-input/multiple-output plants, Sakanushi et al. [54] proposed a design method for stabilizing simple multi-period repetitive controllers. This design method is based on the doubly coprime factorization.

5. Two-degree-of-freedom control

   Simple repetitive controllers in [48] cannot specify the input-output characteristic and the disturbance attenuation characteristic separately, although it is desirable to be able to do so in practice. To solve this problem, Yamada et al. [55] adopted the two-degree-of-freedom control structure shown in Fig. 1.10 and clarified the parameterization of all stabilizing two-degree-of-freedom simple repetitive controllers that can specify these characteristics separately by using two controllers. Sakanushi et al. [56] proposed a design

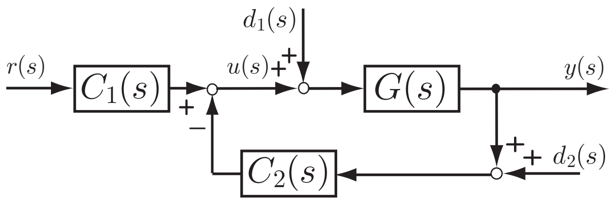

Fig. 1.10: Two-degree-of-freedom control system

   method for two-degree-of-freedom simple repetitive control systems using the parameterization in [55] and demonstrated its application in a motor control experiment.

1.3 The purpose and contents of this study

The modified repetitive control system is a type of servomechanism for a periodic reference input. In other words, the repetitive control system follows a periodic reference input with small steady-state error, even if a periodic disturbance or uncertainty exists in the plant [3, 4, 7, 8, 9, 10, 11]. Using modified repetitive controllers, even if the plant does not include time delays, transfer functions from the periodic reference input to the output and from the disturbance to the output have infinite numbers of poles. This makes it difficult to specify the input–output characteristic and the disturbance attenuation characteristic. From a practical point of view, it is desirable that these characteristics should be easy to specify. Therefore, these transfer functions should have finite numbers of poles. To overcome this problem, Yamada et al. proposed simple repetitive control systems such that the controller works as a modified repetitive controller, and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles [48]. In addition, Yamada et al. clarified the parameterization of all stabilizing simple repetitive controllers.

In recent years, many design methods for simple repetitive control systems with considering an uncertainty, useless time and disturbances, etc. have been proposed [49, 50, 51, 52, 53, 54, 55, 56]. However, using these methods, it is not easy to specify the low-pass filter in the internal model for the periodic reference input that specifies the input–output characteristic, because the low-pass filter is related to more than two free parameters. When we design a simple repetitive controller, if the low-pass filter in the internal model for the periodic reference input is set beforehand, we can specify the input–output characteristic more easily than the conventional simple repetitive control systems. This is achieved by parameterizing all stabilizing simple repetitive controllers with the specified input–output characteristic, which is the parameterization when the low-pass filter is set beforehand. However, no paper has considered the problem of obtaining the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic. In addition, the parameterization is useful to design stabilizing controllers [37, 38, 40, 41]. In this paper, in order to make specifying the input–output characteristic easier, we propose parameterizations of all stabilizing simple repetitive controllers with specified input–output characteristic with the low-pass filter specified beforehand.

This paper is organized as follows:

In Chapter 2., we propose the parameterization of all stabilizing simple repetitive controllers with the specified input-output characteristic such that low-pass filter in the internal model for the periodic reference input are settled beforehand, the controller works as a stabilizing modified repetitive controller and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles. In addition, we propose a design method for a control system using the parameterization. A numerical example is presented to illustrate the effectiveness of the proposed design method. Finally, to demonstrate the effectiveness of the parameterization for real plants, we present an application for the reduction of rotational unevenness in motors.

In Chapter 3., we adopt multi-period repetitive control structure and propose the parameterization of all stabilizing simple multi-period repetitive controllers with the specified input-output characteristic such that low-pass filters in the internal model for the periodic reference input are settled beforehand, the controller works as a stabilizing multi-period repetitive controller and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles. In addition, we propose a design method for a control system using the parameterization. A numerical example is presented to illustrate the effectiveness of the proposed design method.

In Chapter 4., we propose the parameterization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with the specified input-output characteristic such that the low-pass filters in the internal model for the periodic reference input are settled beforehand, the controller works as a robust stabilizing multi-period repetitive controller for time-delay plants and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles when the uncertainty does not exist. The basic idea of designing a robust stabilizing simple multi-period repetitive controllers for time-delay plants with the specified input-output characteristic is very simple. For a certain class of time-delay plants with uncertainty, using state predictive control, the problem to design a robust stabilizing controller is reduced to that for the plant without a time delay [59]. That is, if the simple multi-period repetitive control system is robustly stable for the time-delay plant with uncertainty, then the simple multi-period repetitive controller must satisfy the robust stability condition for system without time delay. This implies that if the simple multi-period repetitive control system is robustly stable, then the simple multi-period repetitive controller is included in the parameterization of all robust stabilizing controllers for the plant with uncertainty. The parameterization of all robust stabilizing controllers for the plant with uncertainty is obtained by employing H_∞ control theory based on the Riccati equation [19, 20]. The robust stabilizing controller for plants with uncertainty contains free parameter that is designed to achieve desirable control characteristic. When the free parameter of the parameterization of all robust stabilizing controllers is appropriately chosen, then the controller works as robust stabilizing simple multi-period repetitive controller. A numerical example is presented to illustrate the effectiveness of the proposed design method.

Chapter 5. summarizes the result of the present study by the conclusion.

Notations

R	the set of real numbers.
R_+	$R \cup \{\infty\}$.
$R(s)$	the set of real rational function with s.
RH_∞	the set of stable proper real rational functions.
H_∞	the set of stable causal functions.
$D^\perp$	orthogonal complement of D, i.e., $\begin{bmatrix} D & D^\perp \end{bmatrix}$ or $\begin{bmatrix} D \\ D^\perp \end{bmatrix}$ is unitary.
A^T	transpose of A.
$A^\dagger$	pseudo inverse of A.
$\rho(\{\cdot\})$	spectral radius of $\{\cdot\}$.
$\\|\{\cdot\}\\|_\infty$	H_∞ norm of $\{\cdot\}$.
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	represents the state space description $C(sI-A)^{-1}B+D$.
$\mathcal{L}\{\cdot\}$	the Laplace transformation of $\{\cdot\}$.
$\mathcal{L}^{-1}\{\cdot\}$	the inverse Laplace transformation of $\{\cdot\}$.

Chapter 2

A design method for simple repetitive controllers with specified input–output characteristic

2.1 Introduction

The simple repetitive control system proposed by Yamada et al. is a type of servomechanism for the periodic reference input [48]. That is, the simple repetitive control system follows the periodic reference input with small steady state error, even if a periodic disturbance or uncertainty exists in the plant. In addition, simple repetitive control systems make transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles. Yamada et al. clarified the parameterization of all stabilizing simple repetitive controllers [48].

According to Yamada et al., the parameterization of all stabilizing simple repetitive controllers includes two free parameters. One specifies the disturbance attenuation characteristic. The other specifies the low-pass filter in the internal model for the periodic reference input that specifies the input–output characteristic. However, when employing the method of Yamada et al., it is complex to specify the low-pass filter in the internal model for the periodic reference input. When we design a simple repetitive controller, if the low-pass filter in the internal model for the periodic reference input is set beforehand, we can specify the input–output characteristic more easily than in the method employed in [48]. This is achieved by parameterizing all stabilizing simple repetitive controllers with the specified input–output characteristic, which is the parameterization when the low-pass filter is set beforehand. However, no paper has considered the problem of obtaining the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic. In addition, the parameterization is useful to design stabilizing controllers [37, 38, 39, 40, 41].

In this chapter, we propose the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic and demonstrate the effectiveness of the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic. First, we give the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic. Next, we clarify control characteristics using the parameterization. In addition, a design procedure using the parameterization is presented. A numerical example is presented to illustrate the effectiveness of the proposed design method. Finally, to demonstrate the effectiveness of the parameterization for real plants, we present an application for the reduction of rotational unevenness in motors.

2.2 Problem formulation

Consider the unity feedback control system given by

$$\begin{cases} y(s) & = G(s)u(s) + d(s) \\ u(s) & = C(s)(r(s) - y(s)) \end{cases}, \tag{2.1}$$

where $G(s) \in R(s)$ is the strictly proper plant, $C(s)$ is the controller, $u(s) \in R(s)$ is the control input, $y(s) \in R(s)$ is the output, $d(s) \in R(s)$ is the disturbance and $r(s) \in R(s)$ is the periodic reference input with period $T > 0$ satisfying

$$r(t + T) = r(t) \quad (\forall t \geq 0). \tag{2.2}$$

According to [3, 4, 7, 8, 9, 10, 11, 12], the modified repetitive controller $C(s)$ is written in the form

$$C(s) = C_1(s) + C_2(s)C_r(s), \tag{2.3}$$

where $C_1(s) \in R(s)$ and $C_2(s) \neq 0 \in R(s)$. $C_r(s)$ is an internal model for the periodic reference input $r(s)$ with period T and is written as

$$C_r(s) = \frac{e^{-sT}}{1 - q(s)e^{-sT}}, \tag{2.4}$$

where $q(s) \in R(s)$ is a proper low-pass filter satisfying $q(0) = 1$.

Using the modified repetitive controller $C(s)$ in (2.3), transfer functions from the periodic reference input $r(s)$ to the output $y(s)$ and from the disturbance $d(s)$ to the output $y(s)$ in (2.1) are written as

$$\begin{aligned} \frac{y(s)}{r(s)} &= \frac{C(s)G(s)}{1 + C(s)G(s)} \\ &= \frac{\left\{C_1(s) - (C_1(s)q(s) - C_2(s))e^{-sT}\right\} G(s)}{1 + C_1(s)G(s) - \left\{(1 + C_1(s)G(s))q(s) - C_2(s)G(s)\right\} e^{-sT}} \end{aligned} \tag{2.5}$$

and

$$\begin{aligned} \frac{y(s)}{d(s)} &= \frac{1}{1 + C(s)G(s)} \\ &= \frac{1 - q(s)e^{-sT}}{1 + C_1(s)G(s) - \left\{(1 + C_1(s)G(s))q(s) - C_2(s)G(s)\right\} e^{-sT}}, \end{aligned} \tag{2.6}$$

respectively. Generally, transfer functions from the periodic reference input $r(s)$ to the output $y(s)$ in (2.5) and from the disturbance $d(s)$ to the output $y(s)$ in (2.6) have infinite numbers of poles. When transfer functions from the periodic reference input $r(s)$ to the output $y(s)$ and from the disturbance $d(s)$ to the output $y(s)$ have infinite numbers of poles, it is difficult to specify the input–output characteristic and the disturbance attenuation characteristic. From a practical point of view, it is desirable that the input–output characteristic and the disturbance attenuation characteristic are easily specified. To specify the input–output characteristic and the disturbance attenuation characteristic easily, it is desirable for transfer functions from the periodic reference input $r(s)$ to the output $y(s)$ and from the disturbance $d(s)$ to the output $y(s)$ to have finite numbers of poles. To achieve this, Yamada et al. proposed simple repetitive control systems such that the controller works as a modified repetitive controller, and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles [48]. In addition, Yamada et al. clarified the parameterization of all stabilizing simple repetitive controllers.

On the other hand, according to [3, 4, 7, 8, 9, 10, 11, 12], if the low-pass filter $q(s)$ satisfies

$$1 - q(j\omega_i) \simeq 0 \quad (\forall i = 0, \ldots, N_{max}), \tag{2.7}$$

where ω_i is the frequency component of the periodic reference input $r(s)$ written by

$$\omega_i = \frac{2\pi}{T} i \quad (i = 0, \ldots, N_{max}) \tag{2.8}$$

and $\omega_{N_{max}}$ is the maximum frequency component of the periodic reference input $r(s)$, then the output $y(s)$ in (2.1) follows the periodic reference input $r(s)$ with small steady-state error. Using the result in [48], for $q(s)$ to satisfy (2.7) in a wide frequency range, we must design $q(s)$ to be stable and of minimum phase. If we obtain the parameterization of all stabilizing simple repetitive controllers such that $q(s)$ in (2.4) is set beforehand, we can design the simple repetitive controller satisfying (2.7) more easily than in the method in [48].

From the above practical requirement, we propose the concept of the simple repetitive controller with the specified input–output characteristic as follows.

Definition 1 *(Simple repetitive controller with the specified input–output characteristic)*
We call the controller $C(s)$ a "simple repetitive controller with the specified input–output characteristic" if the following expressions hold true.

1. *The low-pass filter $q(s) \in RH_\infty$ in (2.4) is set beforehand. That is, the input–output characteristic is set beforehand.*
2. *The controller $C(s)$ works as a modified repetitive controller. That is, the controller $C(s)$ is written as (2.3), where $C_1(s) \in R(s)$, $C_2(s) \neq 0 \in R(s)$ and $C_r(s)$ is written as (2.4).*
3. *The controller $C(s)$ ensures transfer functions from the periodic reference input $r(s)$ to the output $y(s)$ in (2.1) and from the disturbance $d(s)$ to the output $y(s)$ in (2.1) have finite numbers of poles.*

The problem considered in this paper is to propose the parameterization of all stabilizing simple repetitive controllers with the specified input-output characteristic and to propose a design method for a control system using the parameterization.

2.3 The parameterization of all stabilizing simple repetitive controllers with the specified input-output characteristic

In this section, we clarify the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic defined in Definition 1.

In order to obtain the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic, $q(s) \in RH_\infty$ is assumed to be settled beforehand. The parameterization of all stabilizing simple repetitive controllers with the specified input-output characteristic is summarized in the following theorem.

Theorem 1 *There exists a stabilizing simple repetitive controller with the specified input–output characteristic if and only if the low-pass filter $q(s) \in RH_\infty$ in (2.4) takes the form:*

$$q(s) = N(s)\bar{q}(s). \tag{2.9}$$

Here, $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \frac{N(s)}{D(s)} \tag{2.10}$$

and $\bar{q}(s) \neq 0 \in RH_\infty$ is any function. When the low-pass filter $q(s) \in RH_\infty$ in (2.4) satisfies (2.9), the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic is given by

$$C(s) = \frac{X(s) + D(s)Q(s) + D(s)\left(Y(s) - N(s)Q(s)\right)\bar{q}(s)e^{-sT}}{Y(s) - N(s)Q(s) - N(s)\left(Y(s) - N(s)Q(s)\right)\bar{q}(s)e^{-sT}}. \tag{2.11}$$

Here, $X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are functions satisfying

$$X(s)N(s) + Y(s)D(s) = 1 \tag{2.12}$$

and $Q(s) \in RH_\infty$ is any function.

Proof of this theorem requires the following lemma.

Lemma 1 *The unity feedback control system in (2.1) is internally stable if and only if $C(s)$ is written as*

$$C(s) = \frac{X(s) + D(s)Q(s)}{Y(s) - N(s)Q(s)}, \tag{2.13}$$

where $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying (2.10), $X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are functions satisfying (2.12) and $Q(s) \in RH_\infty$ is any function [41].

Using Lemma 1, we present the proof of Theorem 1.

(Proof) First, the necessity is shown. That is, we show that if the controller $C(s)$ in (2.3) stabilizes the control system in (2.1) and ensures that the transfer function from the periodic reference input $r(s)$ to the output $y(s)$ of the control system in (2.1) has a finite number of poles, then the low-pass filter $q(s)$ must take the form (2.9). From the assumption that the controller $C(s)$ in (2.3) ensures that the transfer function from the periodic reference input $r(s)$ to the output $y(s)$ of the control system in (2.1) has a finite number of poles, we know that

$$\frac{G(s)C(s)}{1 + G(s)C(s)} = \frac{\left\{C_1(s) - (C_1(s)q(s) - C_2(s))e^{-sT}\right\}G(s)}{1 + G(s)C_1(s) - \left\{(1 + G(s)C_1(s))q(s) - C_2(s)G(s)\right\}e^{-sT}} \tag{2.14}$$

has a finite number of poles. This implies that

$$C_2(s) = \frac{(1 + G(s)C_1(s))q(s)}{G(s)} \tag{2.15}$$

is satisfied; that is, $C(s)$ is necessarily

$$C(s) = \frac{G(s)C_1(s) + q(s)e^{-sT}}{G(s)\left(1 - q(s)e^{-sT}\right)}. \tag{2.16}$$

From the assumption that $C(s)$ in (2.3) stabilizes the control system in (2.1), we know that $G(s)C(s)/(1 + G(s)C(s))$, $C(s)/(1 + G(s)C(s))$, $G(s)/(1 + G(s)C(s))$ and $1/(1 + G(s)C(s))$ are stable. From simple manipulation and (2.16), we have

$$\frac{G(s)C(s)}{1 + G(s)C(s)} = \frac{G(s)C_1(s) + q(s)e^{-sT}}{1 + G(s)C_1(s)}, \tag{2.17}$$

$$\frac{C(s)}{1+G(s)C(s)} = \frac{G(s)C_1(s)+q(s)e^{-sT}}{(1+G(s)C_1(s))G(s)}, \tag{2.18}$$

$$\frac{G(s)}{1+G(s)C(s)} = \frac{(1-q(s)e^{-sT})G(s)}{1+G(s)C_1(s)} \tag{2.19}$$

and

$$\frac{1}{1+G(s)C(s)} = \frac{1-q(s)e^{-sT}}{1+G(s)C_1(s)}. \tag{2.20}$$

From the assumption that all transfer functions in (2.17), (2.18), (2.19) and (2.20) are stable, we know that $G(s)C_1(s)/(1+G(s)C_1(s))$, $C_1(s)/(1+G(s)C_1(s))$, $G(s)/(1+G(s)C_1(s))$ and $1/(1+G(s)C_1(s))$ are stable. This means that $C_1(s)$ is an internally stabilizing controller for $G(s)$. From Lemma 1, $C_1(s)$ must take the form:

$$C_1(s) = \frac{X(s)+D(s)Q(s)}{Y(s)-N(s)Q(s)}, \tag{2.21}$$

where $Q(s) \in RH_\infty$. From the assumption that the transfer function in (2.18) is stable, we know that

$$\frac{q(s)}{G(s)\left(1+G(s)C_1(s)\right)} = \frac{\left(Y(s)-N(s)Q(s)\right)D^2(s)q(s)}{N(s)} \tag{2.22}$$

is stable. This implies that $q(s)$ must take the form:

$$q(s) = N(s)\bar{q}(s), \tag{2.23}$$

where $\bar{q}(s) \neq 0 \in RH_\infty$ is any function. In this way, it is shown that if there exists a stabilizing simple repetitive controller with the specified input–output characteristic, then the low-pass filter $q(s)$ must take the form (2.9).

Next, we show that if (2.9) holds true, then $C(s)$ is written as (2.11). Substituting (2.15), (2.21) and (2.23) into (2.3), we have (2.11). Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, it is shown that if $q(s)$ and $C(s)$ take the form (2.9) and (2.11), respectively, then the controller $C(s)$ stabilizes the control system in (2.1), ensures that the transfer functions from $r(s)$ and $d(s)$ to $y(s)$ of the control system in (2.1) have finite numbers of poles and works as a stabilizing modified repetitive controller. After simple manipulation, we have

$$\frac{G(s)C(s)}{1+G(s)C(s)} = \left\{X(s)+D(s)Q(s)+D(s)\left(Y(s)-N(s)Q(s)\right)\bar{q}(s)e^{-sT}\right\}N(s), \tag{2.24}$$

$$\frac{C(s)}{1+G(s)C(s)} = \left\{X(s)+D(s)Q(s)+D(s)\left(Y(s)-N(s)Q(s)\right)\bar{q}(s)e^{-sT}\right\}D(s), \tag{2.25}$$

$$\frac{G(s)}{1+G(s)C(s)} = \left\{Y(s)-N(s)Q(s)-N(s)\left(Y(s)-N(s)Q(s)\right)\bar{q}(s)e^{-sT}\right)N(s) \tag{2.26}$$

and

$$\frac{1}{1+G(s)C(s)} = \left\{Y(s) - N(s)Q(s) - N(s)\left(Y(s) - N(s)Q(s)\right)\bar{q}(s)e^{-sT}\right\} D(s). \tag{2.27}$$

Since $X(s) \in RH_\infty$, $Y(s) \in RH_\infty$, $N(s) \in RH_\infty$, $D(s) \in RH_\infty$, $Q(s) \in RH_\infty$ and $\bar{q}(s) \in RH_\infty$, the transfer functions in (2.24), (2.25), (2.26) and (2.27) are stable. In addition, for the same reason, transfer functions from $r(s)$ and $d(s)$ to $y(s)$ of the control system in (2.1) have finite numbers of poles.

Next, we show that the controller in (2.11) works as a modified repetitive controller. The controller in (2.11) is rewritten in the form in (2.3), where

$$C_1(s) = \frac{X(s) + D(s)Q(s)}{Y(s) - N(s)Q(s)} \tag{2.28}$$

and

$$C_2(s) = \frac{\bar{q}(s)}{(Y(s) - N(s)Q(s))}. \tag{2.29}$$

From the assumption of $\bar{q}(s) \neq 0$, $C_2(s) \neq 0$ holds true. These expressions imply that the controller $C(s)$ in (2.11) works as a modified repetitive controller. Thus, the sufficiency has been shown.

We have thus proved Theorem 1. ∎

Remark 1 *Note that from Theorem 1, when the plant $G(s)$ is of non-minimum phase, the low-pass filter $q(s)$ cannot be set to be of minimum phase.*

2.4 Control characteristics

In this section, we describe control characteristics of the control system in (2.1) using the stabilizing simple repetitive controller in (2.11).

First, we mention the input–output characteristic. The transfer function $S(s)$ from the periodic reference input $r(s)$ to the error $e(s) = r(s) - y(s)$ is written as

$$S(s) = \frac{1}{1+G(s)C(s)} = D(s)\left(Y(s) - N(s)Q(s)\right)\left(1 - q(s)e^{-sT}\right). \tag{2.30}$$

From (2.30), since $q(s)$ is set beforehand to satisfy (2.7), the output $y(s)$ follows the periodic reference input $r(s)$ with small steady-state error. That is, we find that by using the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic, the input–output characteristic can be specified beforehand.

Next, we mention the disturbance attenuation characteristic. The transfer function from the disturbance $d(s)$ to the output $y(s)$ is written as (2.30). From (2.30), for the frequency component $\omega_i (i = 0, \ldots, N_{max})$ in (2.8) of the disturbance $d(s)$ that is the same as that of the periodic reference input $r(s)$, since $S(s)$ satisfies $S(j\omega_i) \simeq 0 (\forall i = 0, \ldots, N_{max})$, the disturbance $d(s)$ is attenuated effectively. For the frequency component ω_d of the disturbance $d(s)$ that is different from that of the periodic reference input $r(s)$ (that is, $\omega_d \neq \omega_i$), even if

$$1 - q(j\omega_d) \simeq 0, \tag{2.31}$$

the disturbance $d(s)$ cannot be attenuated because

$$e^{-j\omega_d T} \neq 1 \tag{2.32}$$

and

$$1 - q(j\omega_d)e^{-j\omega_d T} \neq 0. \tag{2.33}$$

To attenuate the frequency component ω_d of the disturbance $d(s)$ that is different from that of the periodic reference input $r(s)$, we need to set $Q(s)$ satisfying

$$Y(j\omega_d) - N(j\omega_d)Q(j\omega_d) \simeq 0. \tag{2.34}$$

From the above discussion, the role of $q(s)$ is to specify the input–output characteristic for the periodic reference input $r(s)$ and it can be specified beforehand. The role of $Q(s)$ is to specify the disturbance attenuation characteristic for the frequency component of the disturbance $d(s)$ that is different from that of the periodic reference input $r(s)$.

2.5 Design procedure

In this section, a design procedure for stabilizing the simple repetitive controller with the specified input–output characteristic is presented.

A design procedure for stabilizing simple repetitive controllers satisfying Theorem 1 is summarized as follows.

Procedure

Step 1) Obtain coprime factors $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ of $G(s)$ satisfying (2.10).

Step 2) $X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are set satisfying (2.12).

Step 3) $\bar{q}(s) \in RH_\infty$ in (2.9) is set so that for the frequency component $\omega_i (i = 0, \ldots, N_{max})$ of the periodic reference input $r(s)$,

$$1 - q(j\omega_i) = 1 - N(j\omega_i)\bar{q}(j\omega_i) \simeq 0. \tag{2.35}$$

To satisfy $1 - N(j\omega_i)\bar{q}(j\omega_i) \simeq 0$, $\bar{q}(s) \in RH_\infty$ is set according to

$$\bar{q}(s) = \frac{1}{N_o(s)}\bar{q}_r(s), \tag{2.36}$$

where $N_o(s) \in RH_\infty$ is an outer function of $N(s)$ satisfying

$$N(s) = N_i(s)N_o(s), \tag{2.37}$$

$N_i(s) \in RH_\infty$ is an inner function satisfying $N_i(0) = 1$ and $|N_i(j\omega)| = 1(\forall\omega \in R_+)$, $\bar{q}_r(s)$ is a low-pass filter satisfying $\bar{q}_r(0) = 1$, as

$$\bar{q}_r(s) = \frac{1}{(1 + s\tau_r)^{\alpha_r}} \tag{2.38}$$

is valid, α_r is an arbitrary positive integer that ensures $\bar{q}_r(s)/N_o(s)$ is proper and $\tau_r \in R$ is any positive real number satisfying

$$1 - N_i(j\omega_i)\frac{1}{(1 + j\omega_i\tau_r)^{\alpha_r}} \simeq 0(\forall i = 0, \ldots, N_{max}). \tag{2.39}$$

Step 4) $Q(s) \in RH_\infty$ is set so that for the frequency component ω_d of the disturbance $d(s)$, $Y(j\omega_d) - N(j\omega_d)Q(j\omega_d) \simeq 0$ is satisfied. To design $Q(s)$ to hold $Y(j\omega_d) - N(j\omega_d)Q(j\omega_d) \simeq 0$, $Q(s)$ is set according to

$$Q(s) = \frac{Y(s)}{N_o(s)}\bar{q}_d(s), \tag{2.40}$$

where $\bar{q}_d(s)$ is a low-pass filter satisfying $\bar{q}_d(0) = 1$, as

$$\bar{q}_d(s) = \frac{1}{(1 + s\tau_d)^{\alpha_d}} \tag{2.41}$$

is valid, α_d is an arbitrary positive integer that ensures $\bar{q}_d(s)/N_o(s)$ is proper and $\tau_d \in R$ is any positive real number satisfying

$$1 - N_i(j\omega_d)\frac{1}{(1 + j\omega_d\tau_d)^{\alpha_d}} \simeq 0. \tag{2.42}$$

2.6 Numerical example

In this section, a numerical example is presented to illustrate the effectiveness of the proposed method.

We consider the problem of obtaining the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic for the plant $G(s)$ written as

$$G(s) = \frac{s - 50}{(s + 1)(s - 1)} \tag{2.43}$$

that follows the periodic reference input $r(t)$ with period $T = 2$[s].

A pair of coprime factors $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ of $G(s)$ in (2.43) satisfying (2.10) is given by

$$N(s) = \frac{s - 50}{(s + 30)(s + 40)} \tag{2.44}$$

and

$$D(s) = \frac{(s + 1)(s - 1)}{(s + 30)(s + 40)}. \tag{2.45}$$

$q(s)$ is set according to

$$\begin{aligned} q(s) &= N_i(s)\bar{q}_r(s) \\ &= \frac{-s + 50}{s + 50} \cdot \frac{1}{0.001s + 1}, \end{aligned} \tag{2.46}$$

where

$$N_i(s) = \frac{-s + 50}{s + 50} \tag{2.47}$$

and

$$\bar{q}_r(s) = \frac{1}{0.001s + 1}. \tag{2.48}$$

$X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ satisfying (2.12) are derived as

$$X(s) = -\frac{3943s + 29024}{(s+30)(s+40)} \tag{2.49}$$

and

$$Y(s) = \frac{s^2 + 140s + 11244}{(s+30)(s+40)}. \tag{2.50}$$

From Theorem 1, the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic for $G(s)$ in (2.43) is given by (2.11), where $Q(s) \in RH_\infty$ in (2.11) is any function. So that the disturbance

$$d(t) = \sin\left(\frac{\pi t}{2}\right) \tag{2.51}$$

can be attenuated effectively, $Q(s)$ is set by (2.40), where

$$\bar{q}_d(s) = \frac{1}{0.001s + 1} \tag{2.52}$$

and

$$N_o(s) = \frac{-s - 50}{(s+30)(s+40)}. \tag{2.53}$$

Using the abovementioned parameters, we have a stabilizing simple repetitive controller with the specified input–output characteristic.

Using the designed stabilizing simple repetitive controller with the specified input–output characteristic, the response of the error $e(t) = r(t) - y(t)$ in (2.1) for the periodic reference input $r(t) = \sin(\pi t)$ is shown in Fig. 2.1 . Here, the dotted line shows the response of the periodic

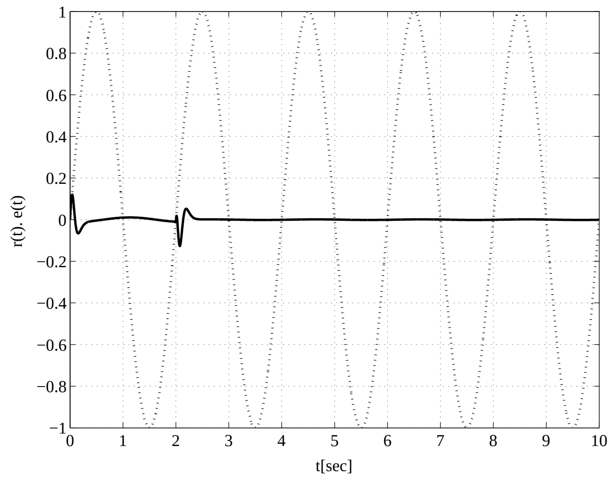

Fig. 2.1: Response of the error $e(t) = r(t) - y(t)$ for the periodic reference input $r(t) = \sin(\pi t)$

reference input $r(t) = \sin(\pi t)$ and the solid line shows that of the error $e(t) = r(t) - y(t)$. Figure 2.1 shows that the output $y(t)$ follows the periodic reference input $r(t)$ with a small steady-state error.

Next, using the designed simple repetitive controller with the specified input–output characteristic $C(s)$, the disturbance attenuation characteristic is shown. The response of the output $y(t)$ for the disturbance $d(t) = \sin(2\pi t)$ of which the frequency component is equivalent to that

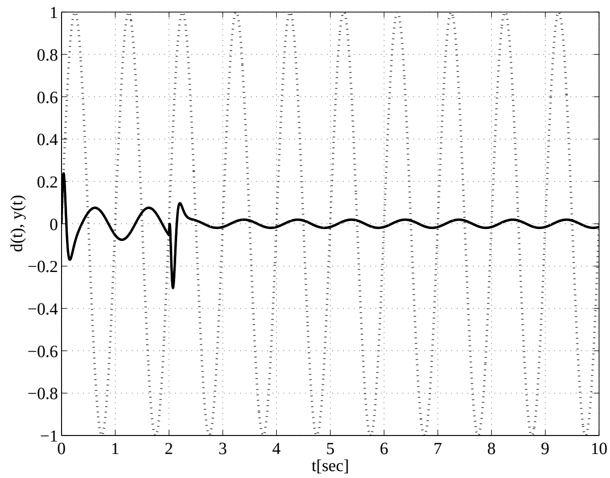

Fig. 2.2: Response of the output $y(t)$ for the disturbance $d(t) = \sin(2\pi t)$

of the periodic reference input $r(t)$ is shown in Fig. 2.2 . Here, the dotted line shows the response of the disturbance $d(t) = \sin(2\pi t)$ and the solid line shows that of the output $y(t)$. Figure 2.2 shows that the disturbance $d(t) = \sin(2\pi t)$ is attenuated effectively. Finally, the response of the output $y(t)$ for the disturbance $d(t)$ in (2.51) of which the frequency component is different from that of the periodic reference input $r(t)$ is shown in Fig. 2.3 . Here, the dotted

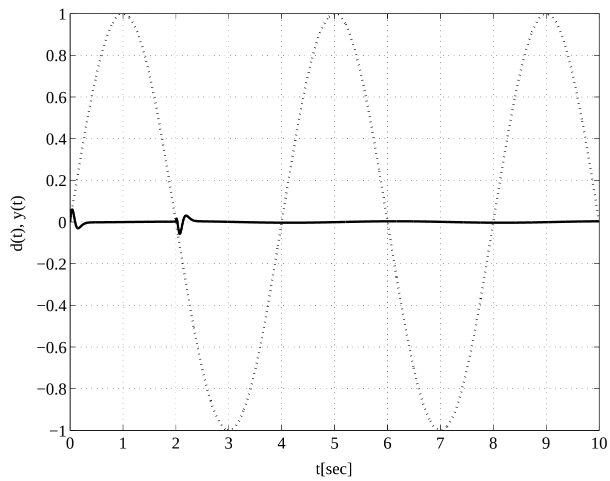

Fig. 2.3: Response of the output $y(t)$ for the disturbance $d(t) = \sin\left(\frac{\pi t}{2}\right)$

line shows the response of the disturbance $d(t)$ in (2.51) and the solid line shows that of the output $y(t)$. Figure 2.3 shows that the disturbance $d(t)$ in (2.51) is attenuated effectively.

A stabilizing simple repetitive controller with the specified input–output characteristic can be easily designed in the way shown here.

2.7 Application of reducing rotational unevenness in motors

In this section, to demonstrate the effectiveness of the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic for real plants, we present an application of reducing rotational unevenness in motors.

2.7.1 Motor control experiment and problem description

A motor control experiment is illustrated in Fig. 2.4 . The motor control experiment consists

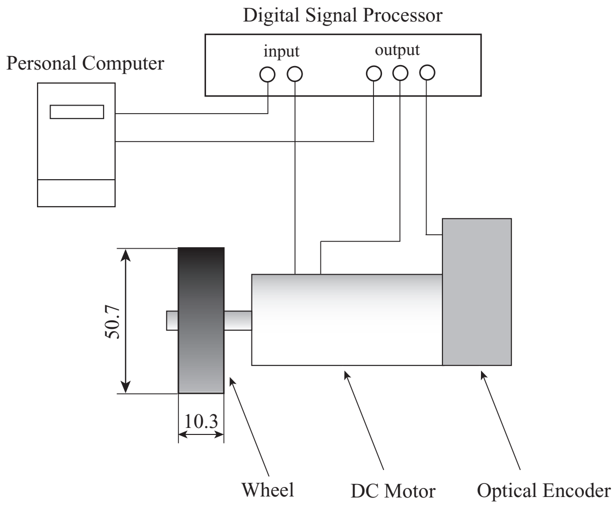

Fig. 2.4: Illustrated motor control experiment

of a direct-current motor with an optical encoder of 1000[counts/revolution] and a wheel that has a diameter of 50.7[mm], a width of 10.3[mm] and mass of 72.5[g] attached to the motor. We denote with T_v[rad/s] the estimated value of the angular velocity of the wheel calculated from the measurement of the angle of the wheel. V_m denotes a control input for the direct-current motor, and the available voltage of V_m is $-24[\text{V}] \le V_m \le 24[\text{V}]$. When we set $V_m = 2.1[\text{V}]$, the response of T_v, which is the angular velocity of the wheel, is shown in Fig. 2.5 and Fig. 2.6 . Figure 2.5 and Fig. 2.6 show disturbances including rotational unevenness in the motor. Since the rotational unevenness in the motor depends on the angle of the motor, the disturbance is considered a periodic disturbance.

The problem considered in this experiment is to design a control system to attenuate periodic disturbances including the rotational unevenness in the motor by parameterizing all stabilizing simple repetitive controllers with the specified input–output characteristic, which is an effective compensator for attenuating periodic disturbances effectively.

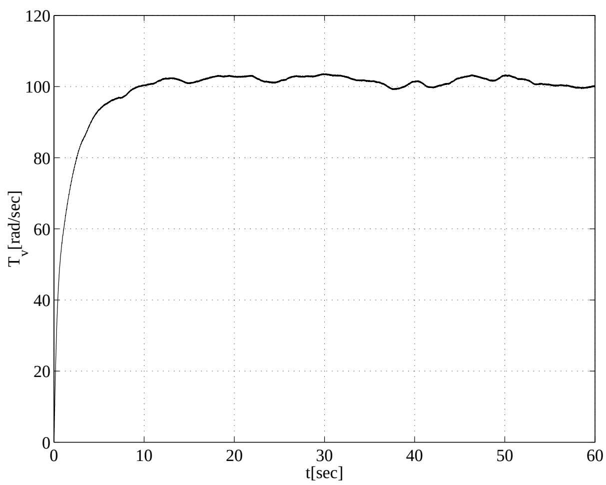

Fig. 2.5: Response of T_v when $V_m = 2.1[\text{V}]$

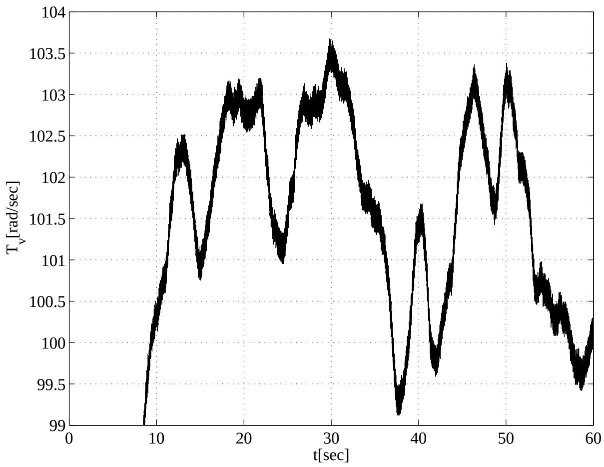

Fig. 2.6: Magnified plot of Fig. 2.5 between 99[rad/s] and 104[rad/s]

2.7.2 Experimental result

In this subsection, we present experimental results of controlling the angular velocity in the motor control experiment in Fig. 2.4 using the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic.

From Fig. 2.5 , we find that the transfer function from V_m to T_v, which is the angular velocity of the wheel, is

$$T_v = \frac{48}{1+1.31s}V_m. \tag{2.54}$$

T_v and V_m are considered as the output $y(s)$ and the control input $u(s)$ in the control system. $G(s)$ is then written as

$$G(s) = \frac{48}{1+1.31s} \in RH_\infty. \tag{2.55}$$

The reference input $r(s)$ is set as $r(t) = v_r = 100$[rad/s]. The period T of the disturbance $d(t)$ is

$$T = \frac{2\pi}{v_r} = \frac{2\pi}{100}. \tag{2.56}$$

To attenuate the periodic disturbance $d(t)$ with period T, we design a simple repetitive controller with the specified input–output characteristic $C(s)$ in (2.11). Coprime factors $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ of the plant $G(s)$ in (2.55) on RH_∞ are given by

$$N(s) = \frac{114.2857}{s+1} \tag{2.57}$$

and

$$D(s) = \frac{s+2.381}{s+1}. \tag{2.58}$$

A pair of $X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ satisfying $N(s)X(s) + D(s)Y(s) = 1$ is written as

$$X(s) = \frac{0.0167}{s+1} \tag{2.59}$$

and

$$Y(s) = \frac{s - 0.381}{s + 1}. \tag{2.60}$$

$q(s)$ is set according to

$$q(s) \quad = \quad N_i(s)\bar{q}_r(s) = \frac{1}{0.2s + 1}, \tag{2.61}$$

where

$$N_i(s) = 1 \tag{2.62}$$

and

$$\bar{q}_r(s) = \frac{1}{0.2s + 1}. \tag{2.63}$$

Using the abovementioned parameters, the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic for $G(s)$ in (2.55) is given by (2.11), where $Q(s) \in RH_\infty$ in (2.11) is any function.

$Q(s)$ is set by (2.40), where

$$\bar{q}_d(s) = \frac{1}{0.03s + 1} \tag{2.64}$$

and

$$N_o(s) = N(s). \tag{2.65}$$

Substitution of $Q(s)$ into (2.11) gives a stabilizing simple repetitive controller with the specified input–output characteristic $C(s)$.

Using the designed simple repetitive controller with the specified input–output characteristic $C(s)$, the response of the output $y(t)$, which is the angular velocity of the wheel T_v, for the reference input $r(t) = 100$[rad/s], is shown in Fig. 2.7 and Fig. 2.8 . Figure 2.7 and Fig. 2.8

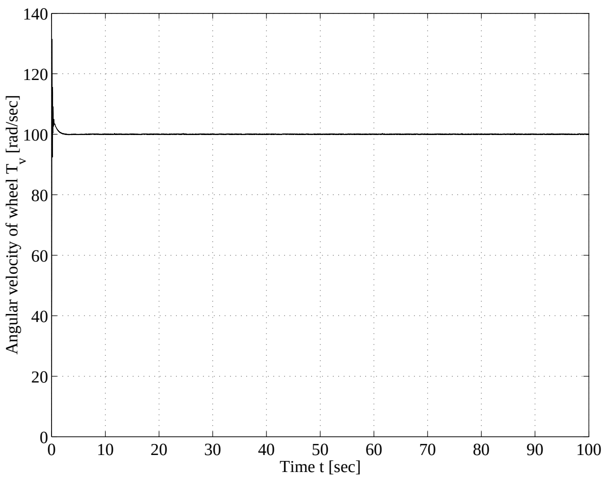

Fig. 2.7: Response of the output $y(t)$, which is the angular velocity of the wheel T_v, for the reference input $r(t) = 100$[rad/s] using the simple repetitive controller with the specified input–output characteristic

show that the output $y(t)$, which is the angular velocity of the wheel T_v, follows the reference

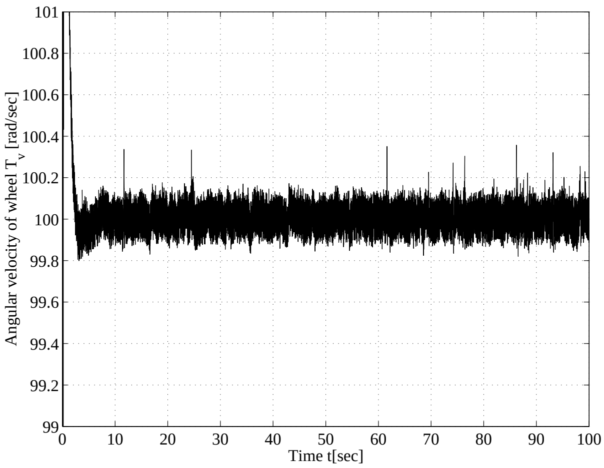

Fig. 2.8: Magnified plot of Fig. 2.7 between 99[rad/s] and 101[rad/s]

input $r(t) = 100$[rad/s] with small steady-state error. In addition, the disturbance $d(t)$ that includes the rotational unevenness in the motor is attenuated effectively.

To demonstrate the effectiveness of the simple repetitive controller with the specified input–output characteristic, a comparison was made with the response when using the parameterization of all stabilizing modified repetitive controllers with the specified input–output characteristic in [57] written as

$$C(s) = \frac{X(s) + D(s)\hat{Q}(s)}{Y(s) - N(s)\hat{Q}(s)}, \tag{2.66}$$

where

$$\hat{Q}(s) = \frac{Q_n(s) + \left(Y(s)\bar{Q}(s) - Q_n(s)\right) q(s)e^{-sT}}{Q_d(s) + \left(N(s)\bar{Q}(s) - Q_d(s)\right) q(s)e^{-sT}} \in H_\infty. \tag{2.67}$$

Here, $Q_n(s) \in RH_\infty$, $\bar{Q}(s) \neq 0 \in RH_\infty$ and $Q_d(s) \neq 0 \in RH_\infty$ are any functions. $N(s) \in RH_\infty$, $D(s) \in RH_\infty$, $X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are given by (2.57), (2.58), (2.59) and (2.60), respectively. $q(s)$ is a low-pass filter that satisfies $q(0) = 1$ and specifies the input–output characteristic for the periodic reference input $r(s)$ and the disturbance attenuation characteristic for the frequency component of the disturbance $d(s)$ that is the same as that of the periodic reference input $r(s)$. To compare the simple repetitive controller and the modified repetitive controller fairly, $q(s)$ in (2.67) is set as that of the simple repetitive controller; that is, $q(s)$ is set by (2.61). Using the abovementioned parameters, the parameterization of all stabilizing modified repetitive controllers with the specified input–output characteristic $C(s)$ is written as (2.66) with (2.67).

For $\hat{Q}(s)$ to satisfy $\hat{Q}(s) \in H_\infty$, $Q_d \in RH_\infty$ and $\bar{Q}(s) \in RH_\infty$ are set according to

$$Q_d(s) = \frac{2s + 100}{s + 0.1} \tag{2.68}$$

and

$$\bar{Q}(s) = \frac{5(s^2 + s + 1)}{3\left(10s^2 + s + 2\right)}, \tag{2.69}$$

respectively. $Q_n(s)$ in (2.67) is set according to

$$Q_n(s) = \frac{Y(s)Q_d(s)}{N(s)}\bar{q}_d(s), \tag{2.70}$$

where $\bar{q}_d(s)$ is given by (2.64). Substitution of $Q_n(s)$, $Q_d(s)$ and $\bar{Q}(s)$ into (2.67) gives a stabilizing modified repetitive controller $C(s)$.

Using the obtained modified repetitive controller $C(s)$, the response of the output $y(t)$, which is the angular velocity of the wheel T_v, for the reference input $r(t) = 100$[rad/s] is shown in Fig. 2.9 and Fig. 2.10 . Figure 2.9 and Fig. 2.10 show that the output $y(t)$, which is

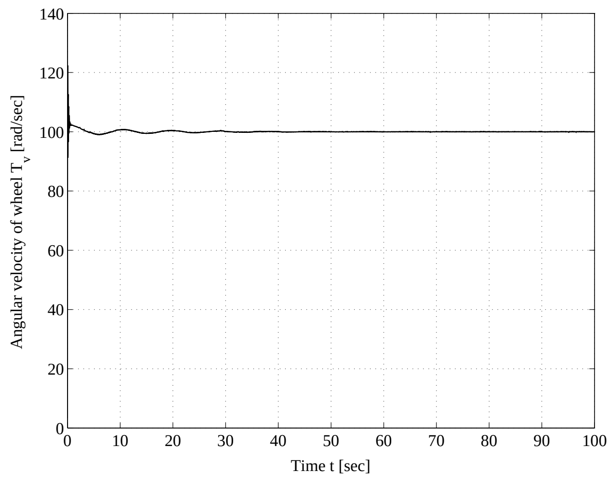

Fig. 2.9: Response of the output $y(t)$, which is the angular velocity of the wheel T_v, for the reference input $r(t) = 100$[rad/s] using the modified repetitive controller with the specified input–output characteristic

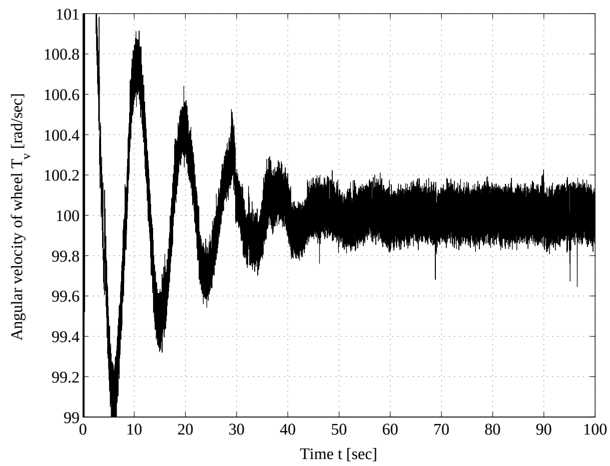

Fig. 2.10: Magnified plot of Fig. 2.9 between 99[rad/s] and 101[rad/s]

the angular velocity of the wheel T_v, follows the reference input $r(t) = 100$[rad/s] with small steady-state error. In addition, the disturbance $d(t)$ that includes the rotational unevenness of the motor is attenuated effectively.

The comparison of Fig. 2.8 with Fig. 2.10 shows that the convergence of the simple repetitive control system is faster than that of the modified repetitive control system. In addition, the simple repetitive control system attenuates the disturbance that includes the rotational unevenness in the motor more effectively than the modified repetitive control system. The simple repetitive control system has merits such as the transfer functions from the periodic reference input to the output having finite numbers of poles and the system being easy to design. This result illustrates that the simple repetitive control system is more effective for the reduction of rotational unevenness in motors than the modified repetitive control system.

In this way, the effectiveness of the control system employing the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic in (2.11) for real plants has been shown.

2.8 Conclusion

In this chapter, we proposed the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic such that the low-pass filter in the internal model for the periodic reference input is set beforehand, the controller works as a stabilizing modified repetitive controller, and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles. In addition, we demonstrated the effectiveness of the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic. Control characteristics of a simple repetitive control system were presented, as well as a design procedure for a simple repetitive controller with the specified input–output characteristic. Finally, a numerical example and an application for the reduction of rotational unevenness in motors were presented to illustrate the effectiveness of the proposed method.

Chapter 3

A design method for simple multi-period repetitive controllers with the specified input-output characteristic

3.1 Introduction

A modified repetitive control system is a type of servomechanism for a periodic reference input, i.e., it follows a periodic reference input with small steady state error, even when there exists a periodic disturbance or an uncertainty of a plant [3, 4, 7, 8, 9, 10, 11].

However, the modified repetitive control system has a bad effect on the disturbance attenuation characteristic [28], in that at certain frequencies, the sensitivity to disturbances of a control system with a modified repetitive controller becomes twice as worse as that of a control system without a modified repetitive controller. Gotou et al. overcame this problem by proposing a multi-period repetitive control system [28]. However, the phase angle of the low-pass filter in a multi-period repetitive controller has a bad effect on the disturbance attenuation characteristics [31, 32]. Yamada et al. overcame this problem and proposed a design method for multi-period repetitive controllers to attenuate disturbances effectively [33, 34] using the time advance compensation described in [31, 32, 35]. Using this multi-period repetitive control structure, Steinbuch proposed a design method for repetitive control systems with uncertain period time [36].

On the other hand, there exists an important control problem of finding all stabilizing controllers, named the parameterization problem [37, 38, 39, 40, 41]. The parameterization of all stabilizing multi-period repetitive controllers was solved in [45, 46].

Using the multi-period repetitive controllers in [28, 33, 34, 45, 46], even if the plant does not include time delays, the transfer function from the periodic reference input to the output and that from the disturbance to the output have infinite numbers of poles. In this situation, it is difficult to specify the input-output characteristic and the disturbance attenuation characteristic. From a practical point of view, it is desirable that the input-output characteristic and the disturbance attenuation characteristic are easy to determine. To do this, the transfer function from the periodic reference input to the output and that from the disturbance to the output should have finite numbers of poles. If we can design multi-period repetitive control systems where these transfer functions have finite numbers of poles, then they will become more widely used controller structures, like the Smith predictor [58] for time-delay plants. From this viewpoint, Yamada and Takenaga [49] proposed such multi-period repetitive controller, named simple multi-period repetitive controller, and clarified the parameterization of all stabilizing simple multi-period repetitive controllers. According to Yamada and Takenaga, the parameterization of all stabilizing simple multi-period repetitive controllers includes two kinds of free-parameters. One has the role to specify the disturbance attenuation characteristic. The others have the role to specify low-pass filters in the internal model for the periodic reference input of which the role is to specify the input-output characteristic. However, using the method by Yamada and Takenaga, it is complex to specify low-pass filters in the internal model for the periodic reference input. When we design a simple multi-period repetitive controller, if low-pass filters in the internal model for the periodic reference input are settled beforehand, we can specify the input-output characteristic more easily than the method in [49]. This problem is solved by obtaining the parameterization of all stabilizing simple multi-period repetitive controllers with the specified input-output characteristic, which is the parameterization when low-pass filters are settled beforehand. However, no paper has considered the problem to obtain the parameterization of all stabilizing simple multi-period repetitive controllers with the specified input-output characteristic. In addition, the parameterization is useful to design stabilizing controllers [37, 38, 39, 40, 41]. Therefore, the problem of obtaining the parameterization of all stabilizing simple multi-period repetitive controllers with the specified input-output characteristic is important to solve.

In this chapter, we propose the parameterization of all stabilizing simple multi-period repetitive controllers with the specified input-output characteristic such that low-pass filters in the internal model for the periodic reference input are settled beforehand, the controller works as a stabilizing multi-period repetitive controller and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles.

3.2 Problem formulation

Consider the unity feedback control system given by

$$\begin{cases} y(s) &= G(s)u(s) + d(s) \\ u(s) &= C(s)(r(s) - y(s)) \end{cases}, \tag{3.1}$$

where $G(s) \in R(s)$ is the strictly proper plant, $C(s)$ is the controller, $u(s) \in R$ is the control input, $y(s) \in R$ is the output, $d(s) \in R$ is the disturbance and $r(s) \in R$ is the periodic reference input with period $T > 0$ satisfying

$$r(t+T) = r(t) \quad (\forall t \geq 0). \tag{3.2}$$

According to [28, 33, 34, 45, 46], the multi-period repetitive controller $C(s)$ is written by the form in

$$C(s) \quad = \quad C_0(s) + \left(\sum_{i=1}^{N} C_i(s)e^{-sT_i}\right) C_r(s), \tag{3.3}$$

where $C_0(s) \in R(s)$, $C_i(s) \in R(s)(i = 1, \ldots, N)$ and N is an arbitrary positive integer. $C_r(s)$ is an internal model for the periodic reference input with period T written by

$$C_r(s) \quad = \quad \frac{1}{1 - \displaystyle\sum_{i=1}^{N} q_i(s)e^{-sT_i}}, \tag{3.4}$$

where $q_i(s) \in RH_\infty(i = 1, \ldots, N)$ are low-pass filters satisfying $\sum_{i=1}^{N} q_i(0) = 1$ and $T_i \in R > 0$ $(i = 1, \ldots, N)$. Without loss of generality, it is assumed to hold $C_i(s) \neq 0$ $(\forall i = 1, \ldots, N)$ and $q_i(s) \neq 0$ $(\forall i = 1, \ldots, N)$. That is, the general form of the multi-period repetitive controller

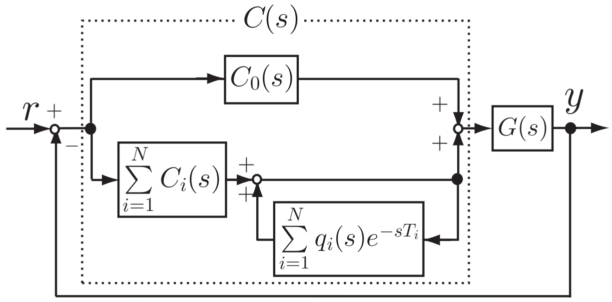

Fig. 3.1: Structure of a multi-period repetitive control system

$C(s)$ is shown in Fig. 3.1 . Gotou et al. [28] proposed a design of time-delays T_i in the multi-period repetitive controller in (3.3) as

$$T_i = T \cdot i \;\; (i = 1, \ldots, N). \tag{3.5}$$

On the other hand, Yamada et al. [34] proposed the design method for multi-period repetitive controller such that T_i $(i = 1, \ldots, N)$ do not necessarily satisfy (3.5). Therefore, in this chapter, we attach importance to the generality and assume that T_i $(i = 1, \ldots, N)$ do not necessarily satisfy (3.5).

Using the multi-period repetitive controller $C(s)$ in (3.3), the transfer function from the periodic reference input $r(s)$ to the output $y(s)$ and that from the disturbance $d(s)$ to the output $y(s)$ in (3.1) are written as

$$\begin{aligned}\frac{y(s)}{r(s)} &= \frac{G(s)C(s)}{1+G(s)C(s)} \\ &= \frac{\left\{C_0(s) - \sum_{i=1}^{N} \left(C_0(s)q_i(s) - C_i(s)\right)e^{-sT_i}\right\}G(s)}{1 + G(s)C_0(s) - \sum_{i=1}^{N}\left\{\left(1+G(s)C_0(s)\right)q_i(s) - G(s)C_i(s)\right\}e^{-sT_i}}\end{aligned} \tag{3.6}$$

and

$$\begin{aligned}\frac{y(s)}{d(s)} &= \frac{1}{1+C(s)G(s)} \\ &= \frac{1 - \sum_{i=1}^{N} q_i(s)e^{-sT_i}}{1 + G(s)C_0(s) - \sum_{i=1}^{N}\left\{\left(1+G(s)C_0(s)\right)q_i(s) - G(s)C_i(s)\right\}e^{-sT_i}},\end{aligned} \tag{3.7}$$

respectively. Generally, transfer functions from the periodic reference input $r(s)$ to the output $y(s)$ in (3.6) and from the disturbance $d(s)$ to the output $y(s)$ in (3.7) have infinite numbers of poles. When transfer functions from the periodic reference input $r(s)$ to the output $y(s)$ and from the disturbance $d(s)$ to the output $y(s)$ have infinite numbers of poles, it is difficult to specify the input-output characteristic and the disturbance attenuation characteristic. From the practical point of view, it is desirable that the input-output characteristic and the disturbance attenuation characteristic are easily specified. In order to specify the input-output characteristic and the disturbance attenuation characteristic easily, transfer functions from the periodic reference input $r(s)$ to the output $y(s)$ and from the disturbance $d(s)$ to the output

$y(s)$ are desirable to have finite numbers of poles. To overcome this problem, Yamada and Takenaga proposed simple multi-period repetitive control systems such that the controller works as a multi-period repetitive controller and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles [49]. In addition, Yamada and Takenaga clarified the parameterization of all stabilizing simple multi-period repetitive controllers.

On the other hand, according to [28, 33, 34, 45, 46], it is note that if low-pass filters $q_i(s)$ $(i = 1, \ldots, N)$ satisfy

$$1 - \sum_{i=1}^{N} q_i(j\omega_k) e^{-j\omega_k T_i} \simeq 0 \quad (\forall k = 0, 1, \ldots, N_{max}), \tag{3.8}$$

where $\omega_k (k = 0, 1, \ldots, N_{max})$ are frequency components of the periodic reference input $r(s)$ written by

$$\omega_k = \frac{2\pi}{T} k \quad (k = 0, 1, \ldots, N_{max}), \tag{3.9}$$

and $\omega_{N_{max}}$ is the maximum frequency component of the periodic reference input $r(s)$, then the output $y(s)$ in (3.1) follows the periodic reference input $r(s)$ with small steady state error. That is, to specify the low-pass filters $q_i(s)$ $(i = 1, \ldots, N)$ means to specify the input-output characteristic. Using the result in [49], in order for $q_i(s)$ $(i = 1, \ldots, N)$ to satisfy (3.8) in wide frequency range, we must design $q_i(s)$ $(i = 1, \ldots, N)$ to be stable and of minimum phase. If we obtain the parameterization of all stabilizing simple multi-period repetitive controllers such that $q_i(s)$ $(i = 1, \ldots, N)$ in (3.4) are settled beforehand, we can design the simple multi-period repetitive controller satisfying (3.8) more easily than the method in [49].

From above practical requirement, we propose the concept of the simple multi-period repetitive controller with the specified input-output characteristic as follows:

Definition 2 *(simple multi-period repetitive controller with the specified input-output characteristic)*
We call the controller $C(s)$ a "simple multi-period repetitive controller with the specified input-output characteristic", if following expressions hold true:

1. *Low-pass filters $q_i(s) \in RH_\infty$ $(i = 1, \ldots, N)$ in (3.4) are settled beforehand to satisfy (3.8). That is, the input-output characteristic is settled beforehand.*

2. *The controller $C(s)$ works as a multi-period repetitive controller. That is, the controller $C(s)$ is written by (3.3), where $C_0(s) \in R(s)$, $C_i(s) \neq 0 \in R(s) (i = 1, \ldots, N)$, $C_r(s)$ is written by (3.4) and $q_i(s) \in RH_\infty$ $(i = 1, \ldots, N)$ satisfy $\sum_{i=1}^{N} q_i(0) = 1$ and $q_i(s) \neq 0 (\forall i = 1, \ldots, N)$.*

3. *The controller $C(s)$ makes transfer functions from the periodic reference input $r(s)$ to the output $y(s)$ in (3.1) and from the disturbance $d(s)$ to the output $y(s)$ in (3.1) have finite numbers of poles.*

The problem considered in this chapter is to propose the parameterization of all stabilizing simple multi-period repetitive controllers with the specified input-output characteristic.

3.3 The parameterization of all stabilizing simple multi-period repetitive controllers with the specified input-output characteristic

In this section, we clarify the parameterization of all stabilizing simple multi-period repetitive controllers with the specified input-output characteristic defined in Definition 2.

In order to obtain the parameterization of all stabilizing simple multi-period repetitive controllers with the specified input-output characteristic, $q_i(s) \in RH_\infty$ $(i = 1, \ldots, N)$ are assumed to be settled beforehand. Under this assumption, the parameterization of all stabilizing simple multi-period repetitive controllers with the specified input-output characteristic is summarized in the following theorem.

Theorem 2 *There exists a stabilizing simple multi-period repetitive controller with the specified input-output characteristic if and only if low-pass filters $q_i(s) \in RH_\infty$ $(i = 1, \ldots, N)$ in (3.4) take the form:*

$$q_i(s) = N(s)\bar{q}_i(s) \ \ (i = 1, \ldots, N). \tag{3.10}$$

Here, $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \frac{N(s)}{D(s)} \tag{3.11}$$

and $\bar{q}_i(s) \neq 0 \in RH_\infty$ $(i = 1, \ldots, N)$ are any functions. When low-pass filters $q_i(s) \in RH_\infty$ $(i = 1, \ldots, N)$ in (3.4) satisfy (3.10), the controller C(s) is a stabilizing simple multi-period repetitive controller with the specified input-output characteristic if and only if C(s) is written by

$$C(s) = \frac{X(s) + D(s)Q(s) + D(s)\left(Y(s) - N(s)Q(s)\right)\sum_{i=1}^{N}\bar{q}_i(s)e^{-sT_i}}{Y(s) - N(s)Q(s) - N(s)\left(Y(s) - N(s)Q(s)\right)\sum_{i=1}^{N}\bar{q}_i(s)e^{-sT_i}}. \tag{3.12}$$

Here, $X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are functions satisfying

$$X(s)N(s) + Y(s)D(s) = 1 \tag{3.13}$$

and $Q(s) \in RH_\infty$ is any function.

Proof of this theorem requires following lemma.

Lemma 2 *Unity feedback control system in (3.1) is internally stable if and only if $C(s)$ is written by*

$$C(s) = \frac{X(s) + D(s)Q(s)}{Y(s) - N(s)Q(s)}, \tag{3.14}$$

where $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying (3.11), $X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are functions satisfying (3.13) and $Q(s) \in RH_\infty$ is any function [41].

Using Lemma 2, we shall show the proof of Theorem 2.

(Proof) First, the necessity is shown. That is, we show that if the controller $C(s)$ in (3.3) makes the control system in (3.1) stable and makes the transfer function from the periodic reference input $r(s)$ to the output $y(s)$ of the control system in (3.1) have finite numbers of poles, then low-pass filters $q_i(s)(i = 1, \ldots, N)$ must take the form (3.10). From the assumption that the controller $C(s)$ in (3.3) makes the transfer function from the periodic reference input $r(s)$ to the output $y(s)$ of the control system in (3.1) have finite numbers of poles,

$$\frac{G(s)C(s)}{1 + G(s)C(s)} = \frac{\left\{C_0(s) - \sum_{i=1}^{N}\left(C_0(s)q_i(s) - C_i(s)\right)e^{-sT_i}\right\}G(s)}{1 + G(s)C_0(s) - \sum_{i=1}^{N}\left\{\left(1 + G(s)C_0(s)\right)q_i(s) - G(s)C_i(s)\right\}e^{-sT_i}} \tag{3.15}$$

has finite numbers of poles. This implies that

$$C_i(s) = \frac{(1+G(s)C_0(s))q_i(s)}{G(s)} \quad (i=1,\dots,N) \tag{3.16}$$

is satisfied, that is, $C(s)$ is necessarily

$$C(s) = \frac{G(s)C_0(s) + \sum_{i=1}^{N} q_i(s)e^{-sT_i}}{G(s)\left(1 - \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right)}. \tag{3.17}$$

From the assumption that $C(s)$ in (3.3) makes the control system in (3.1) stable, $G(s)C(s)/(1+G(s)C(s))$, $C(s)/(1+G(s)C(s))$, $G(s)/(1+G(s)C(s))$ and $1/(1+G(s)C(s))$ are stable. From simple manipulations and (3.17), we have

$$\frac{G(s)C(s)}{1+G(s)C(s)} = \frac{G(s)C_0(s) + \sum_{i=1}^{N} q_i(s)e^{-sT_i}}{1+G(s)C_0(s)}, \tag{3.18}$$

$$\frac{C(s)}{1+G(s)C(s)} = \frac{G(s)C_0(s) + \sum_{i=1}^{N} q_i(s)e^{-sT_i}}{G(s)(1+G(s)C_0(s))}, \tag{3.19}$$

$$\frac{G(s)}{1+G(s)C(s)} = \frac{G(s)\left(1 - \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right)}{1+G(s)C_0(s)} \tag{3.20}$$

and

$$\frac{1}{1+G(s)C(s)} = \frac{1 - \sum_{i=1}^{N} q_i(s)e^{-sT_i}}{1+G(s)C_0(s)}. \tag{3.21}$$

From the assumption that all transfer functions in (3.18), (3.19), (3.20) and (3.21) are stable, $G(s)C_0(s)/(1+G(s)C_0(s))$, $C_0(s)/(1+G(s)C_0(s))$, $G(s)/(1+G(s)C_0(s))$ and $1/(1+G(s)C_0(s))$ are stable. This means that $C_0(s)$ is an internally stabilizing controller for $G(s)$. From Lemma 2, $C_0(s)$ must take the form:

$$C_0(s) = \frac{X(s)+D(s)Q(s)}{Y(s)-N(s)Q(s)}, \tag{3.22}$$

where $Q(s) \in RH_\infty$. From the assumption that the transfer function in (3.19) is stable,

$$\frac{q_i(s)}{G(s)\left(1+G(s)C_0(s)\right)} = \frac{D^2(s)\left(Y(s)-N(s)Q(s)\right)q_i(s)}{N(s)} \quad (i=1,\dots,N) \tag{3.23}$$

are stable. This implies that $q_i(s)(i=1,\dots,N)$ must take the form

$$q_i(s) = N(s)\bar{q}_i(s) \quad (i=1,\dots,N), \tag{3.24}$$

where $\bar{q}_i(s) \neq 0 \in RH_\infty (i = 1, \ldots, N)$ are any functions, because $q_i(s) \neq 0 (i = 1, \ldots, N)$. In this way, it is shown that if there exists a stabilizing simple multi-period repetitive controller with the specified input-output characteristic, then the low-pass filers $q_i(s) (i = 1, \ldots, N)$ must take the form (3.10).

Next, we show that if (3.10) holds true, then $C(s)$ is written by (3.12). Substituting (3.16), (3.22) and (3.24) into (3.3), we have (3.12). Thus, the necessity has been shown.

Next, the sufficiency is shown. That is, it is shown that if $q_i(s) (i = 1, \ldots, N)$ and $C(s)$ take the form (3.10) and (3.12), respectively, then the controller $C(s)$ makes the control system in (3.1) stable, makes transfer functions from $r(s)$ and $d(s)$ to $y(s)$ of the control system in (3.1) have finite numbers of poles and works as a stabilizing multi-period repetitive controller. After simple manipulations, we have

$$\frac{G(s)C(s)}{1 + G(s)C(s)} = \left\{ X(s) + D(s)Q(s) + D(s) \left(Y(s) - N(s)Q(s) \right) \sum_{i=1}^{N} \bar{q}_i(s) e^{-sT_i} \right\} N(s), \tag{3.25}$$

$$\frac{C(s)}{1 + G(s)C(s)} = \left\{ X(s) + D(s)Q(s) + D(s) \left(Y(s) - N(s)Q(s) \right) \sum_{i=1}^{N} \bar{q}_i(s) e^{-sT_i} \right\} D(s), \tag{3.26}$$

$$\frac{G(s)}{1 + G(s)C(s)} = \left\{ Y(s) - N(s)Q(s) - N(s) \left(Y(s) - N(s)Q(s) \right) \sum_{i=1}^{N} \bar{q}_i(s) e^{-sT_i} \right\} N(s) \tag{3.27}$$

and

$$\frac{1}{1 + G(s)C(s)} = \left\{ Y(s) - N(s)Q(s) - N(s) \left(Y(s) - N(s)Q(s) \right) \sum_{i=1}^{N} \bar{q}_i(s) e^{-sT_i} \right\} D(s). \tag{3.28}$$

Since $X(s) \in RH_\infty$, $Y(s) \in RH_\infty$, $N(s) \in RH_\infty$, $D(s) \in RH_\infty$, $Q(s) \in RH_\infty$ and $\bar{q}_i(s) \in RH_\infty(1, \ldots, N)$, the transfer functions in (3.25), (3.26), (3.27) and (3.28) are stable. In addition, from the same reason, the transfer function from $r(s)$ and $d(s)$ to $y(s)$ of the control system in (3.1) have finite numbers of poles.

Next we show that the controller in (3.12) works as a multi-period repetitive controller. The controller in (3.12) is rewritten by the form in (3.3), where

$$C_0(s) = \frac{X(s) + D(s)Q(s)}{Y(s) - N(s)Q(s)} \tag{3.29}$$

and

$$C_i(s) = \frac{\bar{q}_i(s)}{Y(s) - N(s)Q(s)} \quad (i = 1, \ldots, N). \tag{3.30}$$

From the assumption of $\bar{q}_i(s) \neq 0$ $(i = 1, \ldots, N)$, $C_i(s) \neq 0$ $(\forall i = 1, \ldots, N)$ hold true. These expressions imply that the feedback controller $C(s)$ in (3.12) works as a multi-period repetitive controller. Thus, the sufficiency has been shown.

We have thus proved Theorem 2. ∎

Remark 2 *Note that from Theorem 2, when the plant $G(s)$ is of non-minimum-phase, low-pass filters $q_i(s)$ $(i = 1, \ldots, N)$ cannot be settled to be of minimum-phase.*

3.4 Control characteristics

In this section, we describe control characteristics of control system in (3.1) using the stabilizing simple multi-period repetitive controller with the specified input-output characteristic in (3.12).

First, we mention the input-output characteristic. The transfer function $S(s)$ from the periodic reference input $r(s)$ to the error $e(s) = r(s) - y(s)$ is written by

$$\begin{aligned} S(s) &= \frac{1}{1+G(s)C(s)} \\ &= D(s)\left(Y(s) - N(s)Q(s)\right)\left(1 - \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right). \end{aligned} \tag{3.31}$$

From (3.31), since $q_i(s)$ $(i = 1, \ldots, N)$ are settled beforehand to satisfy (3.8), the output $y(s)$ follows the periodic reference input $r(s)$ with small steady state error.

Next, we mention the disturbance attenuation characteristics. The transfer function from the disturbance $d(s)$ to the output $y(s)$ is written by (3.31). From (3.31), for the frequency components ω_k $(k = 0, 1, \ldots, N_{max})$ in (3.8) of the disturbance $d(s)$ those are same to those of the periodic reference input $r(s)$, since $S(s)$ satisfies $S(j\omega_k) \simeq 0$, the disturbance $d(s)$ is attenuated effectively. For the frequency component ω_d of the disturbance $d(s)$ that is different from that of the periodic reference input $r(s)$, that is $\omega_d \neq \omega_k$ $(k = 0, 1, \ldots, N_{max})$, even if

$$1 - \sum_{i=1}^{N} q_i(j\omega_d) \simeq 0, \tag{3.32}$$

the disturbance $d(s)$ cannot be attenuated, because

$$e^{-j\omega_d T_i} \neq 1 \ \ (i = 1, \ldots, N) \tag{3.33}$$

and

$$1 - \sum_{i=1}^{N} q_i(j\omega_d)e^{-j\omega_d T_i} \neq 0. \tag{3.34}$$

In order to attenuate the frequency component ω_d of the disturbance $d(s)$ that is different from that of the periodic reference input $r(s)$, we need to settle $Q(s)$ satisfying

$$Y(j\omega_d) - N(j\omega_d)Q(j\omega_d) \simeq 0. \tag{3.35}$$

From above discussion, the role of $q_i(s)$ $(i = 1, \ldots, N)$ is to specify the input-output characteristic for the periodic reference input $r(s)$ and to specify the disturbance attenuation characteristic with same frequency components of the periodic reference input, and that of $Q(s)$ is to specify the disturbance attenuation characteristic for the frequency component of the disturbance $d(s)$ that is different from that of the periodic reference input $r(s)$.

3.5 Design procedure

In this section, a design procedure of stabilizing simple multi-period repetitive controller with the specified input-output characteristic is presented.

A design procedure of stabilizing simple multi-period repetitive controllers satisfying Theorem 2 is summarized as follows:

Procedure

Step 1) Obtain coprime factors $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ of $G(s)$ satisfying (3.11).

Step 2) $X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are settled satisfying (3.13).

Step 3) $\bar{q}_i(s) \in RH_\infty$ $(i = 1, \ldots, N)$ in (3.10) are settled so that for the frequency components ω_k $(k = 0, 1, \ldots, N_{max})$ of the periodic reference input $r(s)$,

$$\begin{aligned} 1 - \sum_{i=1}^{N} q_i(j\omega_k)e^{-j\omega_k T_i} &= 1 - \sum_{i=1}^{N} N(j\omega_k)\bar{q}_i(j\omega_k)e^{-j\omega_k T_i} \\ &\simeq 0 \ \ (\forall k = 0, 1, \ldots, N_{max}). \end{aligned} \tag{3.36}$$

In order to satisfy $1 - \sum_{i=1}^{N} N(j\omega_k)\bar{q}_i(j\omega_k)e^{-j\omega_k T_i} \simeq 0$, $\bar{q}_i(s) \in RH_\infty$ $(i = 1, \ldots, N)$ in (3.10) are settled by

$$\bar{q}_i(s) = \frac{1}{N_o(s)}\bar{q}_{ri}(s) \ \ (i = 1, \ldots, N), \tag{3.37}$$

where $N_o(s) \in RH_\infty$ is an outer function of $N(s)$ satisfying

$$N(s) = N_i(s)N_o(s), \tag{3.38}$$

$N_i(s) \in RH_\infty$ is an inner function satisfying $N_i(0) = 1$ and $|N_i(j\omega)| = 1(\forall \omega \in R_+)$, $\bar{q}_{ri}(s)$ $(i = 1, \ldots, N)$ are low-pass filters satisfying $\sum_{i=1}^{N} \bar{q}_{ri}(0) = 1$ and

$$1 - N_i(j\omega_k)\sum_{i=1}^{N} \bar{q}_{ri}(j\omega_k)e^{-j\omega_k T_i} \ \simeq \ 0 \ \ (\forall k = 0, 1, \ldots, N_{max}). \tag{3.39}$$

That is, using above mentioned parameters, $q_i(s)$ $(i = 1, \ldots, N)$ in (3.10) are set as

$$q_i(s) = N_i(s)\bar{q}_{ri}(s) \ \ (i = 1, \ldots, N). \tag{3.40}$$

Step 4) $Q(s) \in RH_\infty$ is settled so that for the frequency component ω_d of the disturbance d, $Y(j\omega_d) - N(j\omega_d)Q(j\omega_d) \simeq 0$ is satisfied. In order to design $Q(s)$ to hold $Y(j\omega_d) - N(j\omega_d)Q(j\omega_d) \simeq 0$, $Q(s)$ is settled by

$$Q(s) = \frac{Y(s)}{N_o(s)}\bar{q}_d(s), \tag{3.41}$$

where $\bar{q}_d(s)$ is a low-pass filter satisfying $\bar{q}_d(0) = 1$, as

$$\bar{q}_d(s) = \frac{1}{(1 + s\tau_d)^{\alpha_d}} \tag{3.42}$$

is valid, α_d is an arbitrary positive integer to make $\bar{q}_d(s)/N_o(s)$ proper and $\tau_d \in R$ is any positive real number satisfying

$$1 - N_i(j\omega_d)\frac{1}{(1 + j\omega_d\tau_d)^{\alpha_d}} \simeq 0. \tag{3.43}$$

3.6 Numerical example

In this section, a numerical example is shown to illustrate the effectiveness of the proposed method.

Consider the problem of obtaining the parameterization of all stabilizing simple multi-period repetitive controllers with the specified input-output characteristic for the plant $G(s)$ written by

$$G(s) = \frac{s-150}{(s+2)(s-2)} \tag{3.44}$$

that follows the periodic reference input $r(t)$ with period $T = 4$[sec]. N in (3.3) and $T_i(i = 1,2,3)$ are chosen as $N = 3$ and

$$T_i = T \cdot i \ \ (i = 1,2,3), \tag{3.45}$$

respectively.

A pair of coprime factors $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ of $G(s)$ in (3.44) satisfying (3.11) is given by

$$N(s) = \frac{s-150}{(s+10)(s+15)} \tag{3.46}$$

and

$$D(s) = \frac{(s+2)(s-2)}{(s+10)(s+15)}. \tag{3.47}$$

$q_i(s)$ $(i = 1,2,3)$ in (3.10) are settled by (3.40), where

$$N_i(s) = \frac{-s+150}{s+150} \tag{3.48}$$

and

$$\bar{q}_{ri}(s) = \frac{1}{3\,(0.001s+1)} \ (i = 1,2,3). \tag{3.49}$$

The gain plot of $1 - N_i(s)\sum_{i=1}^{N} \bar{q}_{ri}(s)e^{-sT_i}$ is shown in Fig. 3.2 . For comparison, the gain plot of $1 - \sum_{i=1}^{N} \bar{q}_{ri}(s)e^{-sT_i}$ is shown in Fig. 3.3 . From the comparison of Fig. 3.2 with Fig.

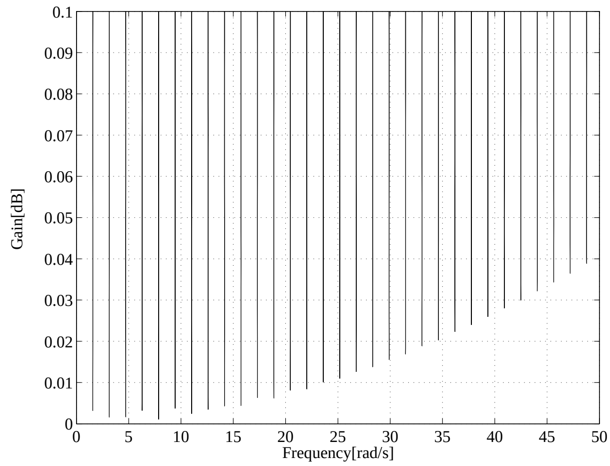

Fig. 3.2: The gain plot of $1 - N_i(s)\sum_{i=1}^{N} \bar{q}_{ri}(s)e^{-sT_i}$

3.3 , we can confirm that when the low-pass filters are settled to be of non-minimum-phase, frequency range which can satisfy (3.8) is not so wide.

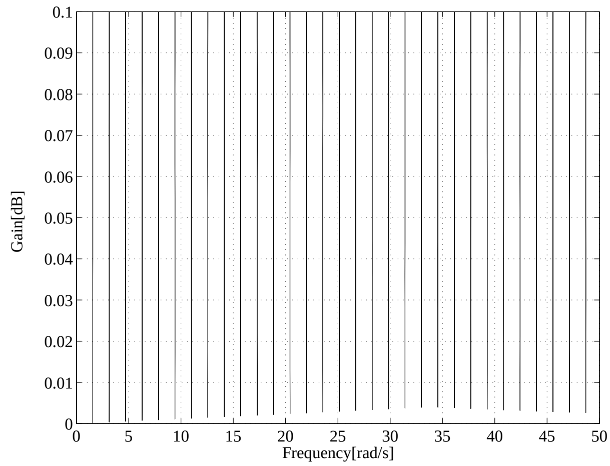

Fig. 3.3: The gain plot of $1 - \sum_{i=1}^{N} \bar{q}_{ri}(s)e^{-sT_i}$

$X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ satisfying (3.13) are derived as

$$X(s) = -\frac{-32.35s + 56.41}{s^2 + 25s + 33.87} \tag{3.50}$$

and

$$Y(s) = \frac{s^2 + 50s + 845.2}{s^2 + 25s + 33.87}. \tag{3.51}$$

From Theorem 2, the parameterization of all stabilizing simple multi-period repetitive controllers with the specified input-output characteristic for $G(s)$ in (3.44) is given by

$$C(s) = \frac{X(s) + D(s)Q(s) + D(s)\left(Y(s) - N(s)Q(s)\right)\sum_{i=1}^{N} \bar{q}_i(s)e^{-sT_i}}{Y(s) - N(s)Q(s) - N(s)\left(Y(s) - N(s)Q(s)\right)\sum_{i=1}^{N} \bar{q}_i(s)e^{-sT_i}}, \tag{3.52}$$

where $Q(s) \in RH_\infty$ is any function.

So that the disturbance

$$d(t) = \sin\left(\frac{\pi t}{8}\right) + \sin\left(\frac{\pi t}{4}\right) + \sin\left(\frac{3\pi t}{8}\right) \tag{3.53}$$

can be attenuated effectively, $Q(s)$ is settled by (3.41), where

$$\bar{q}_d(s) = \frac{1}{0.001s + 1}, \tag{3.54}$$

and

$$N_o(s) = \frac{-s - 150}{(s + 10)(s + 15)}. \tag{3.55}$$

Using above-mentioned parameters, we have a stabilizing simple multi-period repetitive controller with the specified input-output characteristic.

Using the designed stabilizing simple multi-period repetitive controller with the specified input-output characteristic, the response of the error $e(t) = r(t) - y(t)$ in (3.1) for the periodic reference input

$$r(t) = \sin\left(\frac{\pi}{2}t\right) + \sin(\pi t) + \sin\left(\frac{3\pi}{2}t\right) \tag{3.56}$$

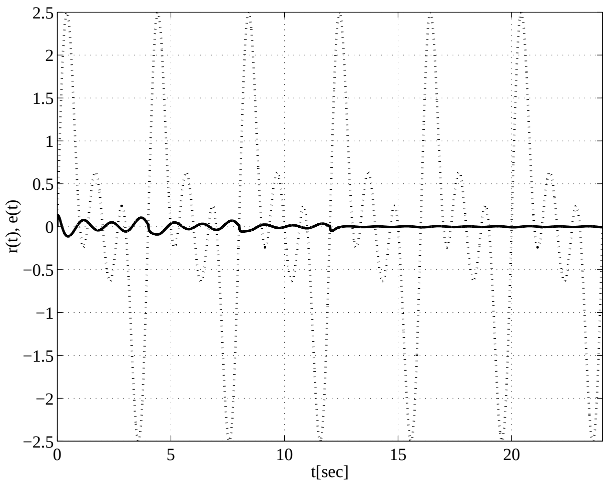

Fig. 3.4: The response of the error $e(t) = r(t) - y(t)$ for the periodic reference input $r(t)$ in (3.56)

is shown in Fig. 3.4 . Here, the dotted line shows the response of the periodic reference input $r(t)$ in (3.56) and the solid line shows that of the error $e(t) = r(t) - y(t)$. Figure 3.4 shows that the output $y(t)$ follows the periodic reference input $r(t)$ with a small steady state error.

Next, using the designed simple multi-period repetitive controller with the specified input-output characteristic $C(s)$, disturbance attenuation characteristics are shown. The response of the output $y(t)$ for the disturbance

$$d(t) = \sin(\pi t) + \sin(2\pi t) + \sin(3\pi t) \tag{3.57}$$

of which frequency components are equivalent to those of the periodic reference input $r(t)$ is shown in Fig. 3.5 . Here, the dotted line shows the response of the disturbance $d(t)$ in (3.57)

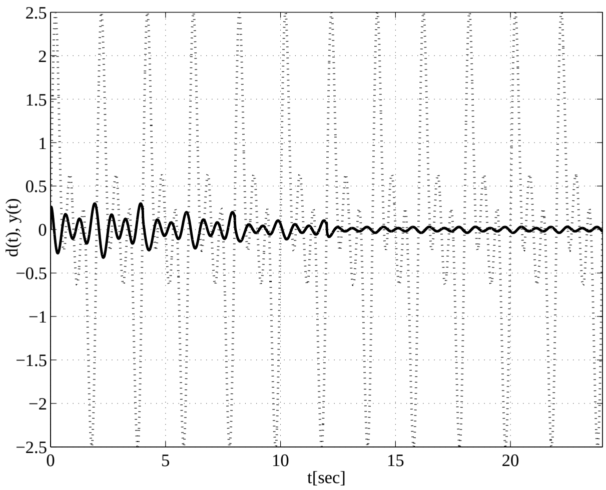

Fig. 3.5: The response of the output $y(t)$ for the disturbance $d(t)$ in (3.57)

and the solid line shows that of the output $y(t)$. Figure 3.5 shows that the disturbance $d(t)$ in (3.57) is attenuated effectively. Finally, the response of the output $y(t)$ for the disturbance $d(t)$ in (3.53) of which frequency components are different from those of the periodic reference input $r(t)$ is shown in Fig. 3.6 . Here, the dotted line shows the response of the disturbance $d(t)$ in (3.53) and the solid line shows that of the output $y(t)$. Figure 3.6 shows that the disturbance $d(t)$ in (3.53) is attenuated effectively.

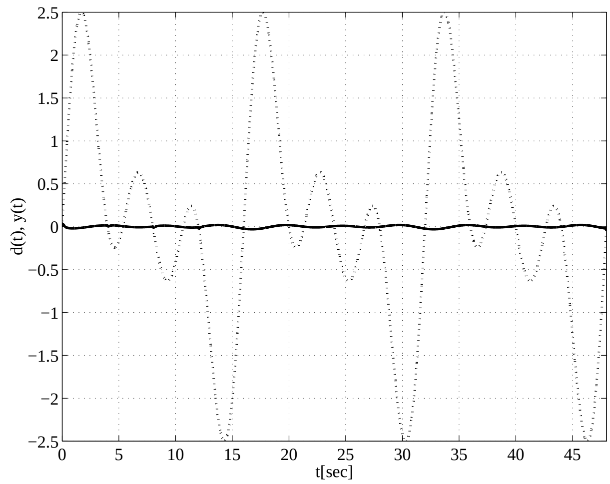

Fig. 3.6: The response of the output $y(t)$ for the disturbance $d(t)$ in (3.53)

A stabilizing simple multi-period repetitive controller with the specified input-output characteristic can be easily designed in the way shown here.

3.7 Conclusion

We have proposed the parameterization of all stabilizing simple multi-period repetitive controllers with the specified input-output characteristic such that low-pass filters in the internal model for the periodic reference input are settled beforehand, the controller works as a stabilizing multi-period repetitive controller and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles. Control characteristics of a simple multi-period repetitive control system were presented, as well as a design procedure for a simple multi-period repetitive controller with the specified input-output characteristic. Finally, a numerical example illustrated the effectiveness of the proposed method.

Chapter 4

A design method for robust stabilizing simple multi-period repetitive controllers for time-delay plants with the specified input-output characteristic

4.1 Introduction

In this chapter, we propose the parameterization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with the specified input-output characteristic.

Simple multi-period repetitive controllers in [48] cannot guarantee the stability of control system for time-delay plants with uncertainties. Almost all real plants include uncertainties and many plants have time-delays. Yamada et al. proposed the parameterization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with uncertainties [53]. However, using the method in [53], it is complex to specify the low-pass filters in the internal model for the periodic reference input of which the role is to specify the input-output characteristic. Because, the low-pass filters are related to three kinds of free parameters in the parameterization by Yamada et al. When we design a robust stabilizing simple multi-period repetitive controller, if the low-pass filters in the internal model for the periodic reference input are settled beforehand, we can specify the input-output characteristic more easily than the method in [53]. This problem is solved by obtaining the parameterization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with the specified input-output characteristic, which is the parameterization when the low-pass filters are settled beforehand. However, no paper has considered the problem to obtain the parameterization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with the specified input-output characteristic. In addition, the parameterization is useful to design stabilizing controllers [37, 38, 39, 40, 41]. Therefore, the problem of obtaining the parameterization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with the specified input-output characteristic is important to solve.

In this chapter, we propose the parameterization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with the specified input-output characteristic such that the low-pass filters in the internal model for the periodic reference input are settled beforehand, the controller works as a robust stabilizing multi-period repetitive controller for time-delay plants and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles when the uncertainty does not exist.

4.2 Problem formulation

Consider the unity feedback control system in

$$\begin{cases} y &= G(s)e^{-sL}u + d \\ u &= C(s)(r - y) \end{cases}, \tag{4.1}$$

where $G(s)e^{-sL}$ is the time-delay plant, $L > 0$ is the time-delay, $G(s) \in R(s)$, $C(s)$ is the controller, $u \in R$ is the control input, $d \in R$ is the disturbance, $y \in R$ is the output and $r \in R$ is the periodic reference input with period T satisfying

$$r(t+T) = r(t) \quad (\forall t \geq 0). \tag{4.2}$$

The nominal time-delay plant of $G(s)e^{-sL}$ is denoted by $G_m(s)e^{-sL_m}$. Both $G(s)$ and $G_m(s)$ are assumed to have no zero or pole on the imaginary axis. In addition, it is assumed that the number of poles of $G(s)$ in the closed right half plane is equal to that of $G_m(s)$. The relation between the time-delay plant $G(s)e^{-sL}$ and the nominal time-delay plant $G_m(s)e^{-sL_m}$ is written as

$$G(s)e^{-sL} = G_m(s)(e^{-sL_m} + \Delta(s)), \tag{4.3}$$

where $\Delta(s)$ is an uncertainty. The set of $\Delta(s)$ is all functions satisfying

$$|\Delta(j\omega)| < |W_T(j\omega)| \quad (\forall \omega \in R_+), \tag{4.4}$$

where $W_T(s)$ is a stable rational function.

The robust stability condition for the time-delay plant $G(s)e^{-sL}$ with uncertainty $\Delta(s)$ satisfying (4.4) is given by

$$\|T(s)W_T(s)\|_\infty < 1, \tag{4.5}$$

where $T(s)$ is given by

$$T(s) = \frac{C(s)G_m(s)}{1 + C(s)G_m(s)e^{-sL_m}}. \tag{4.6}$$

According to [28, 33, 34], in order for the output y to follow the periodic reference input r with period T in (4.1) with small steady state error, the controller $C(s)$ must have the following structure

$$C(s) = C_0(s) + \left(\sum_{i=1}^{N} C_i(s)q_i(s)e^{-sT_i}\right) C_r(s), \tag{4.7}$$

where $C_0(s) \in R(s)$, $C_i(s) \in R(s)$ $(i = 1, \ldots, N)$ and N is an arbitrary positive integer. $C_r(s)$ is an internal model for the periodic reference input with period T written by

$$C_r(s) = \frac{1}{1 - \displaystyle\sum_{i=1}^{N} q_i(s)e^{-sT_i}}, \tag{4.8}$$

where $q_i(s) \in R(s)$ $(i = 1, \ldots, N)$ are low-pass filters satisfying $\sum_{i=1}^{N} q_i(0) = 1$ and $T_i \in R > 0$ $(i = 1, \ldots, N)$. Without loss of generality, it is assumed to hold $C_i(s) \neq 0$ $(\forall i = 1, \ldots, N)$ and

$q_i(s) \neq 0$ ($\forall i = 1, \ldots, N$). The controller written by the form in (4.7) is called the multi-period repetitive controller [28, 33, 34]. Gotou et al. [28] proposed the design method for multi-period repetitive controller as

$$T_i = T \cdot i \quad (i = 1, \ldots, N). \tag{4.9}$$

On the other hand, Yamada et al. [34] proposed the design method for multi-period repetitive controller such that T_i ($i = 1, \ldots, N$) do not necessarily satisfy (4.9). Therefore, in this chapter, we attach importance to the generality and assume that T_i ($i = 1, \ldots, N$) do not necessarily satisfy (4.9).

Using the multi-period repetitive controller $C(s)$ in (4.7), transfer functions from the periodic reference input r to the output y and from the disturbance d to the output y in (4.1) are written as

$$\begin{aligned} \frac{y}{r} &= \frac{C(s)G(s)e^{-sL}}{1 + C(s)G(s)e^{-sL}} \\ &= \frac{C_0(s)G_m(s)\left(e^{-sL} + \Delta(s)\right) - \sum_{i=1}^{N}\left(C_0(s) - C_i(s)\right)q_i(s)e^{-sT_i}G_m(s)\left(e^{-sL} + \Delta(s)\right)}{1 + C_0(s)G_m(s)\left(e^{-sL} + \Delta(s)\right) - \sum_{i=1}^{N}\left\{1 + C_0(s)G_m(s)\left(e^{-sL} + \Delta(s)\right) - C_i(s)G_m(s)\left(e^{-sL} + \Delta(s)\right)\right\}q_i(s)e^{-sT_i}} \end{aligned} \tag{4.10}$$

and

$$\begin{aligned} \frac{y}{d} &= \frac{1}{1 + C(s)G(s)e^{-sL}} \\ &= \frac{1 - \sum_{i=1}^{N} q_i(s)e^{-sT_i}}{1 + C_0(s)G_m(s)\left(e^{-sL} + \Delta(s)\right) - \sum_{i=1}^{N}\left\{1 + C_0(s)G_m(s)\left(e^{-sL} + \Delta(s)\right) - C_i(s)G_m(s)\left(e^{-sL} + \Delta(s)\right)\right\}q_i(s)e^{-sT_i}}, \end{aligned} \tag{4.11}$$

respectively. Generally, transfer functions from the periodic reference input r to the output y in (4.10) and from the disturbance d to the output y in (4.11) have infinite numbers of poles, even if $\Delta(s) = 0$. When transfer functions from the periodic reference input r to the output y and from the disturbance d to the output y have infinite numbers of poles, it is difficult to specify the input-output characteristic and the disturbance attenuation characteristic. From the practical point of view, it is desirable that the input-output characteristic and the disturbance attenuation characteristic are easily specified. In order to specify the input-output characteristic and the disturbance attenuation characteristic easily, transfer functions from the periodic reference input r to the output y and from the disturbance d to the output y are desirable to have finite numbers of poles. To overcome this problem, Yamada et al. proposed robust stabilizing simple multi-period repetitive control systems such that the controller works as a robust stabilizing multi-period repetitive controller for time-delay plants and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles when the uncertainty does not exist [53]. In addition, Yamada et al. clarified the parameterization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants.

On the other hand, according to [28, 33, 34], if the low-pass filters $q_i(s)$ $(i = 1, \ldots, N)$ satisfy

$$1 - \sum_{i=1}^{N} q_i(j\omega_k)e^{-j\omega_k T_i} \simeq 0 \quad (k = 0, 1, \ldots, N_{max}), \tag{4.12}$$

where ω_k are frequency components of the periodic reference input r written by

$$\omega_k = \frac{2\pi}{T}k \quad (k = 0, 1, \ldots, N_{max}) \tag{4.13}$$

and $\omega_{N_{max}}$ is the maximum frequency component of the periodic reference input r, then the output y in (4.1) follows the periodic reference input r with small steady state error. Using the result in [53], in order for $q_i(s)$ $(i = 1, \ldots, N)$ to satisfy (4.12) in wide frequency range, we must design $q_i(s)$ $(i = 1, \ldots, N)$ to be stable and of minimum phase. If we obtain the parameterization of all robust stabilizing simple multi-period repetitive controllers such that $q_i(s)$ $(i = 1, \ldots, N)$ in (4.7) are settled beforehand, we can design a robust stabilizing simple multi-period repetitive controller satisfying (4.12) more easily than the method in [53].

From above practical requirement, we define a robust stabilizing simple multi-period repetitive controller for time-delay plants with the specified input-output characteristic as Definition 3 and clarify the parameterization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with the specified input-output characteristic.

Definition 3 *(robust stabilizing simple multi-period repetitive controller for time-delay plants with the specified input-output characteristic)*
We call the controller $C(s)$ a "robust stabilizing simple multi-period repetitive controller for time-delay plants with the specified input-output characteristic", if following expressions hold true:

1. *The low-pass filters $q_i(s) \in RH_\infty$ $(i = 1, \ldots, N)$ in (4.7) are settled beforehand. That is, the input-output characteristic is settled beforehand.*
2. *The controller $C(s)$ works as a multi-period repetitive controller. That is, the controller $C(s)$ is written by (4.7), where $C_0(s) \in R(s)$, $C_i(s) \neq 0 \in R(s)$ $(\forall i = 1, \ldots, N)$ and $q_i(s) \neq 0 \in RH_\infty$ $(i = 1, \ldots, N)$ satisfy $\sum_{i=1}^{N} q_i(0) = 1$.*
3. *When $\Delta(s) = 0$, the controller $C(s)$ makes transfer functions from the periodic reference input r to the output y in (4.1) and from the disturbance d to the output y in (4.1) have finite numbers of poles.*
4. *The controller $C(s)$ satisfies the robust stability condition in (4.5).*

4.3 The parameterization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with the specified input-output characteristic

In this section, we clarify the parameterization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with the specified input-output characteristic defined in Definition 3.

In order to obtain the parameterization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with the specified input-output characteristic, we must see that controllers $C(s)$ satisfying (4.5). The problem of obtaining the controller $C(s)$, which

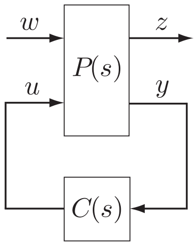

Fig. 4.1: Block diagram of H_∞ control problem

is not necessarily a simple multi-period repetitive controller, satisfying (4.5) is equivalent to the following H_∞ control problem. In order to obtain the controller $C(s)$ satisfying (4.5), we consider the control system shown in Fig. 4.1 . $P(s)$ is selected such that the transfer function from w to z in Fig. 4.1 is equal to $T(s)W_T(s)$. The state space description of $P(s)$ is, in general,

$$\left\{\begin{array}{rclll} \dot{x}(t) & = & Ax(t) & +B_1w(t) & +B_2u(t-L_m) \\ z(t) & = & C_1x(t) & & +D_{12}u(t) \\ y(t) & = & C_2x(t) & +D_{21}w(t) & \end{array}\right. , \tag{4.14}$$

where $A \in R^{n\times n}$, $B_1 \in R^n$, $B_2 \in R^n$, $C_1 \in R^{1\times n}$, $C_2 \in R^{1\times n}$, $D_{12} \in R$, $D_{21} \in R$. $P(s)$ is called the generalized plant. $P(s)$ is assumed to satisfy the following assumptions:

1. (C_2, A) is detectable, (A, B_2) is stabilizable.
2. $D_{12} \neq 0$, $D_{21} \neq 0$.
3. $\text{rank}\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} = n+1 \quad (\forall\omega \in R_+)$,
   $\text{rank}\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} = n+1 \quad (\forall\omega \in R_+)$.
4. $C_1A^iB_2 = 0 \quad (i = 0, 1, 2, \ldots)$.

Under these assumptions, from [59], the following lemma holds true.

Lemma 3 *There exists an H_∞ controller $C(s)$ for the generalized plant $P(s)$ in (4.14) if and only if there exists an H_∞ controller $C(s)$ for the generalized plant $\tilde{P}(s)$ written by*

$$\left\{\begin{array}{rclll} \dot{q}(t) & = & Aq(t) & +B_1w(t) & +\tilde{B}_2u(t) \\ \tilde{z}(t) & = & C_1q(t) & & +D_{12}u(t) \\ \tilde{y}(t) & = & C_2q(t) & +D_{21}w(t) & \end{array}\right. , \tag{4.15}$$

where $\tilde{B}_2 = e^{-AL_m}B_2$. When $u(s) = C(s)\tilde{y}(s)$ is an H_∞ control input for the generalized plant $\tilde{P}(s)$ in (4.15),

$$u(t) = \mathcal{L}^{-1}\left\{C(s)\tilde{y}(s)\right\} \tag{4.16}$$

is an H_∞ control input for the generalized plant $P(s)$ in (4.14), where

$$\tilde{y}(s) = \mathcal{L}\left\{y(t) + C_2\int_{-L_m}^{0} e^{-A(\tau+L_m)}B_2u(t+\tau)d\tau\right\}. \tag{4.17}$$

From Lemma 3 and [19], the following lemma holds true.

Lemma 4 *If controllers satisfying (4.5) exist, both*

$$X\left(A-\tilde{B}_2D_{12}^{\dagger}C_1\right)+\left(A-\tilde{B}_2D_{12}^{\dagger}C_1\right)^TX$$
$$+X\left\{B_1B_1^T-\tilde{B}_2\left(D_{12}^TD_{12}\right)^{-1}\tilde{B}_2^T\right\}X+\left(D_{12}^{\perp}C_1\right)^TD_{12}^{\perp}C_1=0 \tag{4.18}$$

and

$$Y\left(A-B_1D_{21}^{\dagger}C_2\right)^T+\left(A-B_1D_{21}^{\dagger}C_2\right)Y$$
$$+Y\left\{C_1^TC_1-C_2^T\left(D_{21}D_{21}^T\right)^{-1}C_2\right\}Y+B_1D_{21}^{\perp}\left(B_1D_{21}^{\perp}\right)^T=0 \tag{4.19}$$

have solutions $X\geq 0$ *and* $Y\geq 0$ *such that*

$$\rho\left(XY\right)<1 \tag{4.20}$$

and both

$$A-\tilde{B}_2D_{12}^{\dagger}C_1+\left\{B_1B_1^T-\tilde{B}_2\left(D_{12}^TD_{12}\right)^{-1}\tilde{B}_2^T\right\}X \tag{4.21}$$

and

$$A-B_1D_{21}^{\dagger}C_2+Y\left\{C_1^TC_1-C_2^T\left(D_{21}D_{21}^T\right)^{-1}C_2\right\} \tag{4.22}$$

have no eigenvalue in the closed right half plane. Using X *and* Y*, the parameterization of all controllers satisfying (4.5) is given by*

$$C(s) \;=\; C_{11}(s)+C_{12}(s)Q(s)\left(1-C_{22}(s)Q(s)\right)^{-1}C_{21}(s), \tag{4.23}$$

where

$$\left[\begin{array}{cc} C_{11}(s) & C_{12}(s) \\ C_{21}(s) & C_{22}(s)\end{array}\right]=\left[\begin{array}{c|cc} A_c & B_{c1} & B_{c2} \\ \hline C_{c1} & D_{c11} & D_{c12} \\ C_{c2} & D_{c21} & D_{c22}\end{array}\right], \tag{4.24}$$

$$\begin{aligned} A_c \;=\;& A+B_1B_1^TX-\tilde{B}_2\left(D_{12}^{\dagger}C_1+E_{12}^{-1}\tilde{B}_2^TX\right)\\ &-(I-YX)^{-1}\left(B_1D_{21}^{\dagger}+YC_2^TE_{21}^{-1}\right)\left(C_2+D_{21}B_1^TX\right),\end{aligned}$$

$$B_{c1}=(I-YX)^{-1}\left(B_1D_{21}^{\dagger}+YC_2^TE_{21}^{-1}\right),$$
$$B_{c2}=(I-YX)^{-1}\left(\tilde{B}_2+YC_1^TD_{12}\right)E_{12}^{-1/2},$$

$$C_{c1}=-D_{12}^{\dagger}C_1-E_{12}^{-1}\tilde{B}_2^TX,\;\; C_{c2}=-E_{21}^{-1/2}\left(C_2+D_{21}B_1^TX\right),$$

$$D_{c11}=0,\;\; D_{c12}=E_{12}^{-1/2},\;\; D_{c21}=E_{21}^{-1/2},\;\; D_{c22}=0,\;\; E_{12}=D_{12}^TD_{12},\;\; E_{21}=D_{21}D_{21}^T$$

and $Q(s)\in H_\infty$ *is any function satisfying* $\|Q(s)\|_\infty<1$ *[19].*

Using Lemma 3 and Lemma 4, the parameterization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with the specified input-output characteristic is given by following theorem.

Theorem 3 *If simple multi-period repetitive controllers satisfying (4.5) exist, both (4.18) and (4.19) have solutions $X \geq 0$ and $Y \geq 0$ such that (4.20) and both $A - \tilde{B}_2 D_{12}^{\dagger} C_1 + \{B_1 B_1^T - \tilde{B}_2 (D_{12}^T D_{12})^{-1} \tilde{B}_2^T\} X$ and $A - B_1 D_{21}^{\dagger} C_2 + Y\{C_1^T C_1 - C_2^T (D_{21} D_{21}^T)^{-1} C_2\}$ have no eigenvalue in the closed right half plane. Using X and Y, the parameterization of all robust stabilizing simple multi-period repetitive control laws with the specified input-output characteristic satisfying (4.5) is given by*

$$u(t) \;=\; \mathcal{L}^{-1}\left\{C(s)\tilde{y}(s)\right\}, \tag{4.25}$$

where

$$\tilde{y}(s) = \mathcal{L}\left\{y(t) + C_2 \int_{-L_m}^{0} e^{-A(\tau+L_m)} B_2 u(t+\tau) d\tau\right\} \tag{4.26}$$

and

$$C(s) \;=\; C_{11}(s) + C_{12}(s)Q(s)\left(1 - C_{22}(s)Q(s)\right)^{-1} C_{21}(s), \tag{4.27}$$

where $C_{ij}(s)(i = 1, 2; j = 1, 2)$ are given by (4.24) and $Q(s) \in H_\infty$ is any function satisfying $\|Q(s)\|_\infty < 1$ and written by

$$Q(s) = \frac{Q_{n0}(s) + \sum_{i=1}^{N} Q_{ni}(s) q_i(s) e^{-sT_i}}{Q_{d0}(s) + \sum_{i=1}^{N} Q_{di}(s) q_i(s) e^{-sT_i}}, \tag{4.28}$$

$$Q_{ni}(s) = \tilde{G}_d(s)\bar{Q}_i(s) \;\; (i = 1, \ldots, N) \tag{4.29}$$

and

$$Q_{di}(s) \;=\; -\frac{1}{1 + C_{11}(s)\tilde{G}_m(s)} \tilde{G}_n(s)\bar{Q}_i(s) \;\; (i = 1, \ldots, N). \tag{4.30}$$

Here, $\tilde{G}_n(s) \in RH_\infty$ and $\tilde{G}_d(s) \in RH_\infty$ are coprime factors of $-C_{22}(s) + (C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s))\tilde{G}_m(s)$ on RH_∞ satisfying

$$\frac{\tilde{G}_n(s)}{\tilde{G}_d(s)} \;=\; -C_{22}(s) + \left(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)\right)\tilde{G}_m(s), \tag{4.31}$$

where $\tilde{G}_m(s) = C_2(sI - A)^{-1}\tilde{B}_2$. $Q_{n0}(s) \in RH_\infty$, $Q_{d0}(s) \in RH_\infty$ and $\bar{Q}_i(s) \neq 0 \in RH_\infty$ $(i = 1, \ldots, N)$ are any functions satisfying

$$C_{11}(s)\left(Q_{d0}(s) + Q_{di}(s)\right) + \left(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)\right)\left(Q_{n0}(s) + Q_{ni}(s)\right) \neq 0 \quad (i = 1, \ldots, N). \tag{4.32}$$

(Proof) First, the necessity is shown. That is, we show that when the low-pass filters $q_i(s)$ $(i = 1, \ldots, N)$ in (4.7) are settled beforehand, if the multi-period repetitive controller written by (4.7) stabilizes the control system in (4.1) robustly and makes transfer functions from the periodic reference input r to the output y in (4.10) and from the disturbance d to the output y in (4.11) have finite numbers of poles, when $\Delta(s) = 0$, then $C(s)$ and $Q(s)$ are written by (4.27) and (4.28), respectively. From Lemma 4, the parameterization of all robust stabilizing controllers $C(s)$ for $G(s)e^{-sL}$ is written by (4.27), where $\|Q(s)\|_\infty < 1$. In order to prove the necessity, we will show that if the controller $C(s)$ written by (4.27) works as a multi-period

repetitive controller, then $Q(s)$ in (4.27) is written by (4.28). Substituting $C(s)$ in (4.7) into (4.27), we have (4.28), where

$$Q_{n0}(s) = \bar{C}_d(s)C_{12d}(s)C_{21d}(s)C_{22d}(s)\left(C_{0n}(s)C_{11d}(s) - C_{0d}(s)C_{11n}(s)\right), \tag{4.33}$$

$$\begin{aligned} Q_{ni}(s) = & C_{0d}(s)\bar{C}_{in}(s)C_{11d}(s)C_{12d}(s)C_{21d}(s)C_{22d}(s) \\ & -\bar{C}_d(s)C_{12d}(s)C_{21d}(s)C_{22d}(s)\left(C_{0n}(s)C_{11d}(s) - C_{0d}(s)C_{11n}(s)\right) \quad (i = 1, \ldots, N), \end{aligned} \tag{4.34}$$

$$\begin{aligned} Q_{d0}(s) = & \bar{C}_d(s)\left(C_{0d}(s)C_{11d}(s)C_{12n}(s)C_{21n}(s)C_{22d}(s)\right. \\ & -C_{0d}(s)C_{11n}(s)C_{12d}(s)C_{21d}(s)C_{22n}(s) \\ & \left.+C_{0n}(s)C_{11d}(s)C_{12d}(s)C_{21d}(s)C_{22n}(s)\right) \end{aligned} \tag{4.35}$$

and

$$\begin{aligned} Q_{di}(s) = & C_{0d}(s)\bar{C}_{in}(s)C_{11d}(s)C_{12d}(s)C_{21d}(s)C_{22n}(s) \\ & -\bar{C}_d(s)\left(C_{0d}(s)C_{11d}(s)C_{12n}(s)C_{21n}(s)C_{22d}(s)\right. \\ & -C_{0d}(s)C_{11n}(s)C_{12d}(s)C_{21d}(s)C_{22n}(s) \\ & \left.+C_{0n}(s)C_{11d}(s)C_{12d}(s)C_{21d}(s)C_{22n}(s)\right) \quad (i = 1, \ldots, N). \end{aligned} \tag{4.36}$$

Here, $C_{0n}(s) \in RH_\infty$, $C_{0d}(s) \in RH_\infty$, $C_{ijn}(s) \in RH_\infty$ $(i = 1, 2; j = 1, 2)$ and $C_{ijd}(s) \in RH_\infty$ $(i = 1, 2; j = 1, 2)$ are coprime factors satisfying

$$C_0(s) = \frac{C_{0n}(s)}{C_{0d}(s)} \tag{4.37}$$

and

$$C_{ij}(s) = \frac{C_{ijn}(s)}{C_{ijd}(s)} \quad (i = 1, 2; j = 1, 2). \tag{4.38}$$

$\bar{C}_{in}(s) \in RH_\infty$ $(i = 1, \ldots, N)$ and $\bar{C}_d(s) \in RH_\infty$ are defined by

$$\bar{C}_{in}(s) = C_{in}(s)\prod_{j=1}^{i-1} C_{jd}(s) \prod_{j=i+1}^{N} C_{jd}(s) \quad (i = 1, \ldots, N) \tag{4.39}$$

and

$$\bar{C}_d(s) = \prod_{i=1}^{N} C_{id}(s), \tag{4.40}$$

respectively. Here, $C_{in}(s) \in RH_\infty$ $(i = 1, \ldots, N)$ and $C_{id}(s) \in RH_\infty$ $(i = 1, \ldots, N)$ are coprime factors satisfying

$$C_i(s) = \frac{C_{in}(s)}{C_{id}(s)} \quad (i = 1, \ldots, N). \tag{4.41}$$

From (4.33) $\sim$ (4.41), all of $Q_{ni}(s)$ $(i = 0, \ldots, N)$ and $Q_{di}(s)$ $(i = 0, \ldots, N)$ are included in RH_∞. Thus, we have shown that if $C(s)$ written by (4.7) stabilizes the control system in (4.1) robustly, $Q(s)$ in (4.27) is written by (4.28). From the assumption of $C_i(s) \neq 0$ $(i = 1, \ldots, N)$, (4.32) is satisfied.

The rest to prove the necessity is to show that when $\Delta(s) = 0$, if $C(s)$ in (4.7) makes transfer functions from the periodic reference input r to the output y and from the disturbance d to the output y have finite numbers of poles, then $Q_{ni}(s)$ $(i = 1, \ldots, N)$ and $Q_{di}(s)$ $(i = 1, \ldots, N)$ are written by (4.29) and (4.30), respectively. From (4.28), when $\Delta(s) = 0$, transfer functions from the periodic reference input r to the output y and from the disturbance d to the output y are written by

$$\frac{y}{r} = \frac{G_{ryn}(s)}{G_{ryd}(s)} \tag{4.42}$$

and

$$\frac{y}{d} = \frac{G_{dyn}(s)}{G_{dyd}(s)}, \tag{4.43}$$

respectively, where

$$\begin{aligned} G_{ryn}(s) &= \Big[\{C_{11}(s)Q_{d0}(s) + (C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s))\,Q_{n0}(s)\} \\ &\quad + \sum_{i=1}^{N}\{C_{11}(s)Q_{di}(s) + (C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s))\,Q_{ni}(s)\} \\ &\quad \cdot q_i(s)e^{-sT_i}\Big]\tilde{G}_m(s), \end{aligned} \tag{4.44}$$

$$\begin{aligned} G_{ryd}(s) &= (Q_{d0}(s) - C_{22}(s)Q_{n0}(s)) + \{C_{11}(s)Q_{d0}(s) + (C_{12}(s)C_{21}(s) \\ &\quad - C_{11}(s)C_{22}(s))\,Q_{n0}(s)\}\,\tilde{G}_m(s) + \sum_{i=1}^{N}\Big[Q_{di}(s) - C_{22}(s)Q_{ni}(s) \\ &\quad + \{C_{11}(s)Q_{di}(s) + (C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s))\,Q_{ni}(s)\} \\ &\quad \cdot\tilde{G}_m(s)\Big]\,q_i(s)e^{-sT_i}, \end{aligned} \tag{4.45}$$

$$G_{dyn}(s) = \left(1 - \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right)(Q_{d0}(s) - C_{22}(s)Q_{n0}(s)) \tag{4.46}$$

and

$$\begin{aligned} G_{dyd}(s) &= (Q_{d0}(s) - C_{22}(s)Q_{n0}(s)) + \{C_{11}(s)Q_{d0}(s) + (C_{12}(s)C_{21}(s) \\ &\quad - C_{11}(s)C_{22}(s))\,Q_{n0}(s)\}\,\tilde{G}_m(s) + \sum_{i=1}^{N}\Big[Q_{di}(s) - C_{22}(s)Q_{ni}(s) \\ &\quad + \{C_{11}(s)Q_{di}(s) + (C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s))\,Q_{ni}(s)\} \\ &\quad \cdot\tilde{G}_m(s)\Big]\,q_i(s)e^{-sT_i}. \end{aligned} \tag{4.47}$$

From the assumption that transfer functions from the periodic reference input r to the output y in (4.42) and from the disturbance d to the output y in (4.43) have finite numbers of poles, (4.45) and (4.47),

$$\begin{aligned} &Q_{di}(s) - C_{22}(s)Q_{ni}(s) + \{C_{11}(s)Q_{di}(s) + (C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s))\,Q_{ni}(s)\}\,G_m(s) \\ &\quad = 0 \quad (i = 1, \ldots, N) \end{aligned} \tag{4.48}$$

is satisfied. Using (4.31), this equation is rewritten by

$$Q_{di}(s) = -\frac{1}{1 + C_{11}(s)\tilde{G}_m(s)}\frac{\tilde{G}_n(s)}{\tilde{G}_d(s)}Q_{ni}(s) \quad (i = 1, \ldots, N). \tag{4.49}$$

Since $Q_{ni}(s) \in RH_\infty$ $(i = 1, \dots, N)$ and $Q_{di}(s) \in RH_\infty$ $(i = 1, \dots, N)$, $Q_{ni}(s)$ $(i = 1, \dots, N)$ and $Q_{di}(s)$ $(i = 1, \dots, N)$ are written by (4.29) and (4.30), respectively, where $\bar{Q}_i(s) \in RH_\infty$ $(i = 1, \dots, N)$. From the assumption that $C_i(s) \neq 0$ $(i = 1, \dots, N)$ and from (4.34) and (4.36), $\bar{Q}_i(s) \neq 0$ $(\forall i = 1, \dots, N)$ holds true. We have thus proved the necessity.

Next, the sufficiency is shown. That is, it is shown that if $C(s)$ and $Q(s) \in H_\infty$ are settled by (4.27) and (4.28), respectively, then the controller $C(s)$ is written by the form in (4.7) and transfer functions from the periodic reference input r to the output y and from the disturbance d to the output y have finite numbers of poles. Substituting (4.28) into (4.27), we have (4.7), where $C_0(s)$ and $C_i(s)$ $(i = 1, \dots, N)$ are denoted by

$$C_0(s) = \frac{C_{11}(s)Q_{d0}(s) + (C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s))Q_{n0}(s)}{Q_{d0}(s) - C_{22}(s)Q_{n0}(s)} \tag{4.50}$$

and

$$C_i(s) = \frac{C_{11}(s)\left(Q_{d0}(s) + Q_{di}(s)\right) + \left(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)\right)\left(Q_{n0}(s) + Q_{ni}(s)\right)}{Q_{d0}(s) - C_{22}(s)Q_{n0}(s)} \quad (i = 1, \dots, N). \tag{4.51}$$

We find that if $C(s)$ and $Q(s)$ are settled by (4.27) and (4.28), respectively, then the controller $C(s)$ is written by the form in (4.7). From $\bar{Q}_i(s) \neq 0$ $(i = 1, \dots, N)$ and (4.51), $C_i(s) \neq 0$ $(i = 1, \dots, N)$ holds true. In addition, from (4.29) and (4.30) and easy manipulation, we can confirm that when $\Delta(s) = 0$, transfer functions from the periodic reference input r to the output y and from the disturbance d to the output y have finite numbers of poles.

We have thus proved Theorem 3. ∎

4.4 Control characteristics

In this section, we describe control characteristics of the control system in (4.1) using the robust stabilizing simple multi-period repetitive controller $C(s)$ in (4.27).

From Theorem 3, $Q(s)$ in (4.28) must be included in H_∞. Since $Q_{n0}(s) \in RH_\infty$, $Q_{ni}(s) \in RH_\infty$ $(i = 1, \dots, N)$ and $q_i(s) \in RH_\infty$ $(i = 1, \dots, N)$ in (4.28), if

$$\left\{Q_{d0}(s) + \sum_{i=1}^{N} Q_{di}(s)q_i(s)e^{-sT_i}\right\}^{-1} \in H_\infty,$$

then $Q(s)$ satisfies $Q(s) \in H_\infty$. That is, the role of $Q_{d0}(s)$ and $\bar{Q}_i(s)$ $(i = 1, \dots, N)$ in (4.30) is to assure the stability of the control system in (4.1).

Next, we mention the input-output characteristic. The transfer function $S(s)$ from the periodic reference input r to the error $e = r - y$ is written by

$$S(s) = \frac{1}{1 + C(s)G(s)e^{-sL}} = \frac{\left(1 - \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right)\left(Q_{d0}(s) - C_{22}(s)Q_{n0}(s)\right)}{S_d(s)}, \tag{4.52}$$

where

$$\begin{aligned} S_d(s) &= Q_{d0}(s) - C_{22}(s)Q_{n0}(s) + \left\{C_{11}(s)Q_{d0}(s) + \left(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)\right)Q_{n0}(s)\right\} \\ &\quad \cdot G(s)e^{-sL} + \sum_{i=1}^{N}\Big[Q_{di}(s) - C_{22}(s)Q_{ni}(s) + \{C_{11}(s)Q_{di}(s) + (C_{12}(s)C_{21}(s) \\ &\quad - C_{11}(s)C_{22}(s))\,Q_{ni}(s)\}\,G(s)e^{-sL}\Big]\,q_i(s)e^{-sT_i}. \end{aligned} \tag{4.53}$$

From (4.52), for frequency components ω_k ($k = 0, 1, \ldots, N_{max}$) in (4.13) of the periodic reference input r, since $q_i(s) \in RH_\infty$ ($i = 1, \ldots, N$) are settled beforehand satisfying (4.12), the output y follows the periodic reference input r with a small steady state error. That is, the role of $q_i(s)$ ($i = 1, \ldots, N$) is to specify the input-output characteristic for the periodic reference input r.

Finally, we mention the disturbance attenuation characteristic. The transfer function $S(s)$ from the disturbance d to the output y is written by (4.52) and (4.53). From (4.52), for the frequency components ω_k ($k = 0, 1, \ldots, N_{max}$) in (4.13) of the disturbance d those are same to those of the periodic reference input r, since $S(s)$ satisfies $S(j\omega_k) \simeq 0$ ($\forall k = 0, 1, \ldots, N_{max}$), the disturbance d is attenuated effectively. For the frequency component ω_d of the disturbance d that is different from that of the periodic reference input r, that is $\omega_d \neq \omega_k$ ($\forall k = 0, 1, \ldots, N_{max}$), even if

$$1 - \sum_{i=1}^{N} q_i(j\omega_d) \simeq 0, \tag{4.54}$$

the disturbance d cannot be attenuated, because

$$e^{-j\omega_d T_i} \neq 1 \tag{4.55}$$

and

$$1 - \sum_{i=1}^{N} q_i(j\omega_d) e^{-j\omega_d T_i} \neq 0. \tag{4.56}$$

In order to attenuate this frequency component, we must find $Q_{n0}(s)$ that satisfies

$$Q_{d0}(j\omega_d) - C_{22}(j\omega_d) Q_{n0}(j\omega_d) \simeq 0. \tag{4.57}$$

That is, the role of $Q_{n0}(s)$ is to specify the disturbance attenuation characteristic for the disturbance d with frequency components $\omega_d \neq \omega_k$ ($\forall k = 0, 1, \ldots, N_{max}$).

From above discussion, the role of $Q_{d0}(s)$ and $\bar{Q}_i(s)$ ($i = 1, \ldots, N$) is to assure the stability of the control system in (4.1) by satisfying $Q(s) \in H_\infty$. The role of $q_i(s)(i = 1, \ldots, N)$ are to specify the input-output characteristic for the periodic reference input r and to specify the disturbance attenuation characteristic for the disturbance d with same frequency components ω_k ($k = 0, 1, \ldots, N_{max}$) of the periodic reference input r. The role of $Q_{n0}(s)$ is to specify the disturbance attenuation characteristic for the disturbance d with frequency components $\omega_d \neq \omega_k$ ($\forall k = 0, 1, \ldots, N_{max}$).

4.5 Design procedure

In this section, a design procedure of robust stabilizing simple multi-period repetitive controller for time-delay plants with the specified input-output characteristic is presented.

A design procedure of robust stabilizing simple multi-period repetitive controller $C(s)$ satisfying Theorem 3 is summarized as follows:

<u>Procedure</u>

Step 1) Obtain $C_{11}(s)$, $C_{12}(s)$, $C_{21}(s)$ and $C_{22}(s)$ by solving the robust stability problem using the Riccati equation based H_∞ control as Theorem 3.

Step 2) $q_i(s) \in RH_\infty$ $(i = 1, \ldots, N)$ and T_i $(i = 1, \ldots, N)$ in (4.28) are settled so that for the frequency components ω_k $(k = 0, 1, \ldots, N_{max})$ of the periodic reference input $r(s)$,

$$1 - \sum_{i=1}^{N} q_i(j\omega_k)e^{-j\omega_k T_i} \simeq 0 \qquad (\forall k = 0, 1, \ldots, N_{max}) \tag{4.58}$$

is satisfied. When T_i $(i = 1, \ldots, N)$ are given by (4.9), in order to satisfy (4.58), for example, $q_i(s)$ $(i = 1, \ldots, N)$ are designed by

$$q_i(s) = \frac{1}{N(1 + s\tau_r)^{\alpha_r}}, \tag{4.59}$$

where α_r is an arbitrary positive integer and $\tau_r \in R$ is an arbitrary positive real number satisfying

$$1 - \sum_{i=1}^{N} \frac{1}{N(1 + j\omega_k\tau_r)^{\alpha_r}} = 1 - \frac{1}{(1 + j\omega_k\tau_r)^{\alpha_r}} \simeq 0 \quad (k = 0, 1, \ldots, N_{max}). \tag{4.60}$$

On the other hand, when T_i are not given by (4.9), $q_i(s)$ $(i = 1, \ldots, N)$ and T_i $(i = 1, \ldots, N)$ satisfying (4.58) for $k = 1, \ldots, N$ can be designed using the method in [34].

Step 3) $Q_{d0}(s) \in RH_\infty$ and $\bar{Q}_i(s) \in RH_\infty$ $(i = 1, \ldots, N)$ in (4.29) and (4.30) are settled so that $Q(s)$ in (4.28) is included in H_∞.

Step 4) $Q_{n0}(s) \in RH_\infty$ is designed so that for the frequency component ω_d of the disturbance d, $|Q_{d0}(j\omega_d) - C_{22}(j\omega_d)Q_{n0}(j\omega_d)|$ is effectively small. To achieve this, $Q_{n0}(s)$ is designed according to

$$Q_{n0}(s) = \frac{Q_{d0}(s)}{C_{22o}(s)}\bar{q}_d(s), \tag{4.61}$$

where $C_{22o}(s) \in RH_\infty$ is an outer function of $C_{22}(s)$ satisfying

$$C_{22}(s) = C_{22i}(s)C_{22o}(s), \tag{4.62}$$

$C_{22i}(s) \in RH_\infty$ is an inner function satisfying $C_{22i}(0) = 1$ and $\bar{q}_d(s)$ is a low-pass filter satisfying $\bar{q}_d(0) = 1$, as

$$\bar{q}_d(s) = \frac{1}{(1 + s\tau_d)^{\alpha_d}} \tag{4.63}$$

is valid, α_d is an arbitrary positive integer to make $\bar{q}_d(s)/C_{22o}(s)$ proper and $\tau_d \in R$ is any positive real number satisfying

$$1 - C_{22i}(j\omega_d)\frac{1}{(1 + j\omega_d\tau_d)^{\alpha_d}} \simeq 0. \tag{4.64}$$

When $Q_{n0}(s)$ is settled by (4.61), $Q_{d0}(j\omega_d) - C_{22}(j\omega_d)Q_{n0}(j\omega_d)$ satisfies

$$\begin{aligned} Q_{d0}(j\omega_d) - C_{22}(j\omega_d)Q_{n0}(j\omega_d) &= Q_{d0}(j\omega_d)\left(1 - C_{22i}(j\omega_d)\frac{1}{(1 + j\omega_d\tau_d)^{\alpha_d}}\right) \\ &\simeq 0. \end{aligned} \tag{4.65}$$

That is, if τ_d is adequately chosen to satisfy (4.64) for the frequency range ω_d, then the disturbance d is attenuated effectively.

4.6 Numerical example

In this section, a numerical example is shown to illustrate the effectiveness of the proposed parameterization.

Consider the problem to obtain the parameterization of all robust stabilizing simple multi-period repetitive controllers with the specified input-output characteristic for time-delay plant $G(s)e^{-sL}$ written by

$$G(s)e^{-sL} = G_m(s)(e^{-sL_m} + \Delta(s)). \tag{4.66}$$

The nominal time-delay plant of $G(s)e^{-sL}$ and the upper bound $W_T(s)$ of the set of $\Delta(s)$ are given by

$$G_m(s)e^{-sL_m} = \frac{1}{(s+3)(s+4)}e^{-0.5s} \tag{4.67}$$

and

$$W_T(s) = \frac{3s+2}{s+10}, \tag{4.68}$$

where $G_m(s) = 1/\{(s+3)(s+4)\}$ and $L_m = 0.5$[sec]. The period T of the periodic reference input r in (3.2) is $T = 20$[sec]. Solving the robust stability problem using Riccati equation based H_∞ control as Theorem 3, the parameterization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with the specified input-output characteristic is obtained as (4.27), where N is selected as $N = 3$ and T_i $(i = 1, 2, 3)$ are set as $T_i = T \cdot i$ $(i = 1, 2, 3)$. Here, $C_{ij}(s)(i = 1, 2; j = 1, 2)$ are given by

$$C_{11}(s) = 0, \tag{4.69}$$

$$C_{12}(s) = 1, \tag{4.70}$$

$$C_{21}(s) = \frac{10^6 \cdot (s^3 + 17s^2 + 82s + 120)}{s^3 + 17s^2 + 3 \cdot 10^6 s + 2 \cdot 10^6} \tag{4.71}$$

and

$$C_{22}(s) = \frac{10^6 \cdot (2.91s^2 + 33.3s + 42.4)}{s^3 + 17s^2 + 3 \cdot 10^6 s + 2 \cdot 10^6}. \tag{4.72}$$

Low-pass filters $q_i(s) \in RH_\infty (i = 1, 2, 3)$ are settled by

$$q_i(s) = \frac{1}{3\,(0.01s + 1)} \in RH_\infty \quad (i = 1, 2, 3). \tag{4.73}$$

In order to hold $Q(s) \in H_\infty$ in (4.28), $Q_{d0}(s) \in RH_\infty$ in (4.28) and $\bar{Q}_i(s) \in RH_\infty$ $(1, 2, 3)$ in (4.29) and (4.30) are settled by

$$Q_{d0}(s) = 200 \tag{4.74}$$

and

$$\bar{Q}_i(s) = 0.01 \quad (i = 1, 2, 3). \tag{4.75}$$

When $Q_{d0}(s)$ and $\bar{Q}_i(s)$ $(i = 1, 2, 3)$ are set as (4.74) and (4.75), the fact that $Q(s) \in H_\infty$ in (4.28) is confirmed as follows: Since $Q_{n0}(s) \in RH_\infty$, $Q_{ni}(s) \in RH_\infty$ $(i = 1, 2, 3)$ and

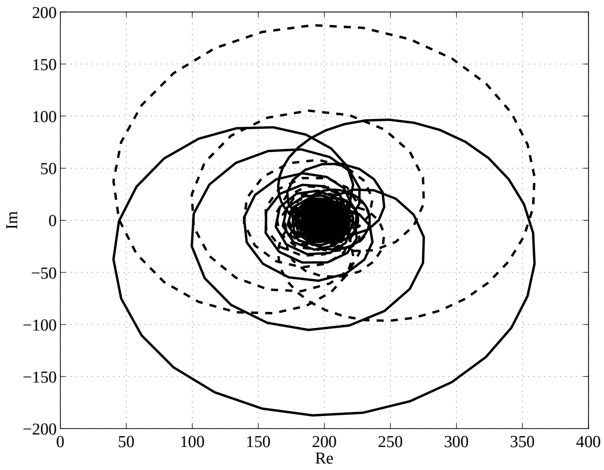

Fig. 4.2: The Nyquist plot of $Q_{d0}(s) + \sum_{i=1}^{N} Q_{di}(s)q_i(s)e^{-sT_i}$

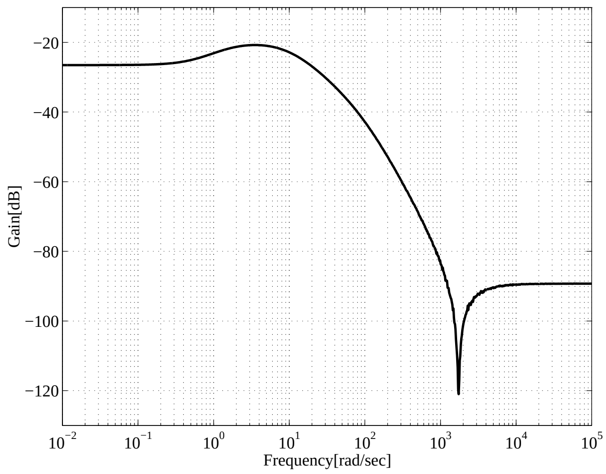

Fig. 4.3: The gain plot of $Q(s)$ in (4.28)

$q_i(s) \in RH_\infty$ (1, 2, 3), if the Nyquist plot of $Q_{d0}(s) + \sum_{i=1}^{N} Q_{di}(s)q_i(s)e^{-sT_i}$ does not encircle the origin, then $Q(s) \in H_\infty$ holds true. The Nyquist plot of $Q_{d0}(s) + \sum_{i=1}^{N} Q_{di}(s)q_i(s)e^{-sT_i}$ is shown in Fig. 4.2 . From Fig. 4.2 , since the Nyquist plot of $Q_{d0}(s) + \sum_{i=1}^{N} Q_{di}(s)q_i(s)e^{-sT_i}$ does not encircle the origin, we find that $Q(s) \in H_\infty$ holds true. The rest to show that $Q(s)$ in (4.28) satisfies $|Q(j\omega)| < 1$ $(\forall\omega \in R_+)$. The gain plot of $Q(s)$ in (4.28) is shown in Fig. 4.3 . Figure 4.3 shows that the designed $Q(s)$ satisfies $\|Q(s)\|_\infty < 1$.

In order for the disturbance

$$d(t) = \sin(0.05\pi t) \tag{4.76}$$

to be attenuated effectively, $Q_{n0}(s) \in RH_\infty$ is designed using (4.61), where

$$C_{22o}(s) = C_{22}(s) \in RH_\infty \tag{4.77}$$

and

$$\bar{q}_d(s) = \frac{1}{0.01s + 1} \in RH_\infty. \tag{4.78}$$

When $\Delta(s)$ is given by

$$\Delta(s) = \frac{2s + 1}{s + 10}, \tag{4.79}$$

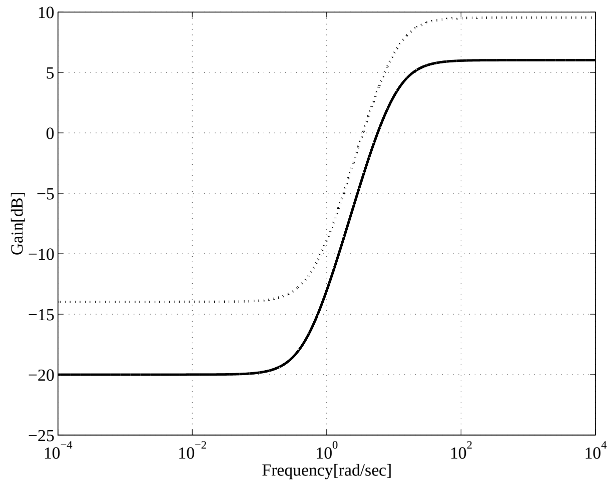

Fig. 4.4: The gain plot of $\Delta(s)$ and $W_T(s)$

the gain plot of $\Delta(s)$ and $W_T(s)$ are shown in Fig. 4.4 . Here, the dotted line shows the gain plot of $W_T(s)$ and the solid line shows that of $\Delta(s)$. Figure 4.4 shows that the uncertainty $\Delta(s)$ satisfies (4.4).

Using above-mentioned parameters, we have a robust stabilizing simple multi-period repetitive controller for time-delay plant with the specified input-output characteristic. When the designed robust stabilizing simple multi-period repetitive controller $C(s)$ is used, the response of the output $y(t)$ in (4.1) for the periodic reference input $r(t) = \sin(0.1\pi t - L_m)$ is shown in Fig. 4.5 . Here, the dotted line shows the response of the periodic reference input

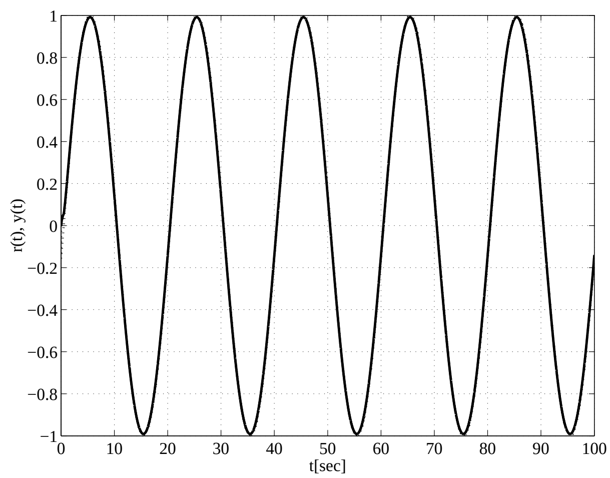

Fig. 4.5: The response of the output $y(t)$ for the periodic reference input $r(t) = \sin(0.1\pi t - L_m)$

$r(t) = \sin(0.1\pi t - L_m)$ and the solid line shows that of the output $y(t)$. Figure 4.5 shows that the output $y(t)$ follows the periodic reference input $r(t)$ with a small steady state error, even if the time-delay plant has uncertainty $\Delta(s)$.

Next, using the designed robust stabilizing simple multi-period repetitive controller for time-delay plant with the specified input-output characteristic, the disturbance attenuation characteristic is shown. The response of the output $y(t)$ for the disturbance $d(t) = \sin(0.2\pi t)$ of which the frequency component is equivalent to that of the periodic reference input $r(t)$ is shown in Fig. 4.6 . Here, the dotted line shows the response of the disturbance $d(t) = \sin(0.2\pi t)$ and the solid line shows that of the output $y(t)$. Figure 4.6 shows that the disturbance $d(t)$ is

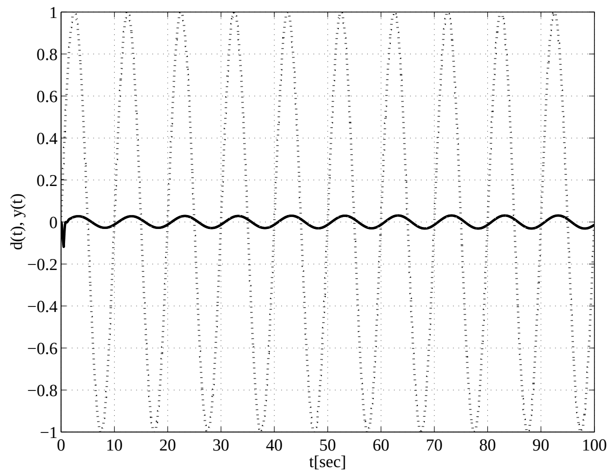

Fig. 4.6: The response of the output $y(t)$ for the disturbance $d(t) = \sin(0.2\pi t)$

attenuated effectively. Finally, the response of the output $y(t)$ for the disturbance $d(t)$ in (4.76) of which the frequency component is different from that of the periodic reference input $r(t)$ is shown in Fig. 4.7 . Here, the dotted line shows the response of the disturbance $d(t)$ in (4.76)

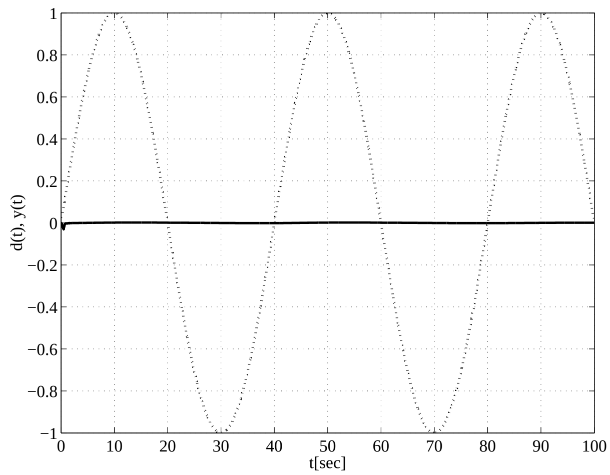

Fig. 4.7: The response of the output $y(t)$ for the disturbance $d(t) = \sin(0.05\pi t)$

and the solid line shows that of the output $y(t)$. Figure 4.7 shows that the disturbance $d(t)$ in (4.76) is attenuated effectively.

In this way, we find that we can easily design a robust stabilizing simple multi-period repetitive controller using Theorem 3.

4.7 Conclusion

In this chapter, we proposed the parameterization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with the specified input-output characteristic such that the low-pass filters in the internal model for the periodic reference input are settled beforehand, the controller works as a robust stabilizing multi-period repetitive controller for time-delay plants and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles, when the uncertainty does not exist. Control characteristics of a robust stabilizing simple multi-period repetitive control system are presented, as well as a design procedure for a robust stabilizing simple multi-period repetitive controller for time-delay plants with the specified input-output characteristic. Finally, a numerical example was illustrated to show the effectiveness of the proposed method.

Chapter 5

Conclusions

In this study, we proposed design methods for simple repetitive control systems with the specified input-output characteristic such that the low-pass filter in the internal model for the periodic reference input can be set beforehand. Results of this paper are summarized as follows:

In Chapter 2., we proposed the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic such that the low-pass filter in the internal model for the periodic reference input is set beforehand, the controller works as a stabilizing modified repetitive controller, and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles. In addition, we demonstrated the effectiveness of the parameterization of all stabilizing simple repetitive controllers with the specified input–output characteristic. Control characteristics of a simple repetitive control system were presented, as well as a design procedure for a simple repetitive controller with the specified input–output characteristic. An application for the reduction of rotational unevenness in motors was presented to illustrate the effectiveness of the proposed method.

In Chapter 3., We have proposed the parameterization of all stabilizing simple multi-period repetitive controllers with the specified input-output characteristic such that low-pass filters in the internal model for the periodic reference input are settled beforehand, the controller works as a stabilizing multi-period repetitive controller and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles. Control characteristics of a simple multi-period repetitive control system were presented, as well as a design procedure for a simple multi-period repetitive controller with the specified input-output characteristic.

In Chapter 4., we proposed the parameterization of all robust stabilizing simple multi-period repetitive controllers for time-delay plants with the specified input-output characteristic such that the low-pass filters in the internal model for the periodic reference input are settled beforehand, the controller works as a robust stabilizing multi-period repetitive controller for time-delay plants and transfer functions from the periodic reference input to the output and from the disturbance to the output have finite numbers of poles, when the uncertainty does not exist. Control characteristics of a robust stabilizing simple multi-period repetitive control system are presented, as well as a design procedure for a robust stabilizing simple multi-period repetitive controller for time-delay plants with the specified input-output characteristic.

Advantages of control systems using the proposed design methods are that its input-output characteristic is easily specified than in the method employed in [48, 49, **?**]. These simple repetitive control systems are expected to have practical applications in, for example, engines, electrical motors and generators, converters, and other machines that perform cyclic tasks.

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