群馬大学博士論文

修正PID補償器の設計法に関する研究

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群馬大学大学院工学研究科
先端生産システム工学領域

萩原　隆明

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Chapter 1

Introduction

1.1 A trend of a study for PID control

In the process industry such as a petroleum, a chemistry, a steel and a food, etc., various systems are used to convert the raw materials into the products by a chemical change and a physical change of the material. The process control to the temperature, pressure and the flowing quantity of the device is done. There is Proportional-Integral-Derivative (PID) control in one of these process controls. Though the modern control theory develops and it is maintained, the PID control is used to the chemical plant of actual 50% or more. The PID control structure is the most widely used one in industrial applications [1, 2, 3, 4, 5].

Recently, in a large-scale control system, the PID control is used to control the temperature, pressure and flow. From this viewpoint, the PID control is important and a practical indispensable control scheme. The action of the Proportional parameter, the Integral parameter and the Derivative parameter is easy to understand intuitively. In addition, the method with good control performance is developed by improving the control system. It is a reason why the PID control is still applied to a lot of practical control systems [3].

The research on the PID control is advanced in the shape that sticks to the site. The control method that developed PID control is proposed.

1.1.1 PID control

Consider a unity feedback control system shown in Fig. 1.1 , where $G(s)$ is the plant, $C(s)$ is

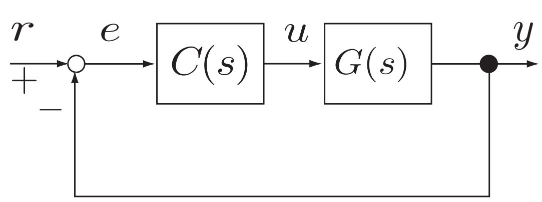

Figure 1.1: A unity feedback control system

the controller, $r \in R$ is the reference input, $u \in R$ is the control input and $y \in R$ is the output.

When the controller has the form written by

$$C(s) = a_P + \frac{a_I}{s} + a_D s = a_P \left(1 + \frac{1}{T_I s} + T_D s\right), \tag{1.1}$$

$$a_I = a_P/T_I, \quad a_D = a_P T_D, \tag{1.2}$$

then the controller is called a PID controller. a_P is the Proportional-parameter, a_I is the Integral-parameter, a_D is the Derivative-parameter, T_I is the integral time and T_D is the derivative time. The role of Proportional, Integral and Derivative are as follows.

1. Proportional Action

   The proportional control action outputs the control input proportional to the size of error, and reduces the steady-state error.

2. Integral Action

   The main function of the integral action is to make sure that the process output agrees with the setpoint in steady state. With proportional control, there is normally a control error in steady state. With integral action, a small positive error will always lead to an increasing control signal, and a negative error will give a decreasing control signal no matter how small the error is.

3. Derivative Action

   The purpose of the derivative action is to improve the closed-loop stability. The derivative parameter is proportional to the time derivative of the control error. This term allows prediction of the future error.

Next, the characteristic of the PID control system in Fig. 1.1 is described. PID control system is simplicity structure and the tuning is easy. From (1.1), it only has to decide the value of three kinds of parameters of P-parameter a_P I-parameter a_I and D-parameter a_D (or, proportional gain a_P, integral time T_I and derivative time T_I). Moreover, a physical meaning of three kinds of parameters is understood. If vibrating response is caused, the proportional gain is smaller. If it doesn't approach the reference value easily, the integral time is reduced. To suppress the vibration of the response and to increase stability, the derivative time is enlarged. It is possible to correspond at once on the site.

Next, it is show that the PID control has an enough control performances. The gain amends and the phase amends can be appropriately done by giving three kinds of parameters of P-parameter a_P, I-parameter a_P and D-parameter a_P. Therefore, even if other complex control schemes are not adopted, an enough control performance can be obtained by using the PID control. Moreover, if a few corrections and devices are given, it can meet a target control specification enough.

Finally, it can be said that the PID control is a practicality and a dependable control method. The idea of the PID control is seen in thesis of Minorsky in 1922 [1], and the prototype of the PID conditioner appears in theses of Callender in 1936 [2]. As for the history of the PID control, a lot of researches are performed long, and the device in practicable respect has been done actively. Therefore, a lot of knowledge is accumulated, and it can be said that reliability and the practicality are high. Also, as a feedback control system, which has the following characteristics. The stability of the control system can be improved. The influence of disturbance on the control system can be attenuated. The transfer function from the reference input to the output is made a desired characteristic. The robustness of the closed-loop transfer function of the system can be improved for the uncertainty of the open-loop transfer function of the system.

As mentioned above, it can be said that the generality of the PID control that can bring the control performance worth practical use enough is a reason that has been used for a lot of things.

1.1.2 Tuning method of PID control system

Typical tuning methods of PID control are shown as follows.

Tuning methods based on the response characteristic of closed-loop were proposed [7, 8]. The method in [7] is controlled only by the proportional control, and the tuning based on the stability of the control system and information on the attenuation characteristic. When the proportional gain is enlarged by the proportional control, the response of the output to the reference input or disturbance vibrates gradually. The response oscillates when the limit with the gain is exceeded. Then, it pays attention to the proportional gain, when the control system exists in the stability limit, that is, the output causes the persistent oscillation of the constant amplitude. In addition, from the result of an experiment, the parameter of the PID control is related at the proportional gain and the cycle of the vibration. Moreover, it is not desirable to generate the persistent oscillation in the stability limit in practical plants. In [8], the tuning method put into the state of the 1/4 attenuation vibration instead of the stability limit is proposed.

The tuning method based on the shape of the step response of the plant was proposed [9, 10, 11, 12, 13, 14, 15, 16]. In [9, 10, 11, 12, 13, 14, 15, 16], the input of the unit step function is added to the plant in the state of the open loop, and the step response of the plant is requested. The tangent is pulled in the point where the inclination of the step response is the most sudden. This inclination is assumed to call time when the rapidity of response and the tangent intersect with the time axis delay time. In addition, it is a tuning method that decides the parameter of the PID control from three parameters that put a constant value together. The method in [9] requests the PID parameter that minimizes the integration of the absolute value of error into the step change in disturbance by the numerical calculation. In [10, 11], only the PI control is taken up. It aims minimizing the integration of the second power of the error of the output for the disturbance that joins the output side of the controlled system. In [12], it explains only the PI control. The integration of deflection is assumed to be a criterion. In order to cause the overshoot of the response, a real root and an imaginary number part of the characteristic equation only has to be equal to the real number part of the smallest complex root. This method in [13] searches for the parameter by the numerical calculation within the range to meet this requirement. In [13], the system at a delay and by the first useless time is examined. The combination by four kinds of in total when the amount of excess is assumed to be 0 and when it is assumed 20% for a step change disturbance and reference value each other is examined. It has aimed to assume the time to reaching to a constant value by the output to be minimum. Here, the amount of excess in case of turbulence is a pull of a regular value from the output. Moreover, time until the output passes a regular value for the first time is assumed to be arrival time, and the parameter that minimizes arrival time by the simulation is requested for the amount of excess of 20%. It thinks about the step turbulence of the system at a delay and by the first useless time, and a basic specification of reference in [14] is 1/4 attenuation of the complex root with the smallest imaginary number part. It is a method of requesting a dimensionless parameter that fills this. After meeting this requirement, the degree of freedom of the adjustment still remains in the PD control, the PI control, and the PID control including more than two kinds of operation. The steady-state deviation is minimum, and the PI control exists about a suitable combination at integration and the period of vibration of error and the PID controls a critical braking, and assumes the PD control to choose the combination of parameters dimensionless, that the proportion gain becomes the maximum. The criterion of reference literature in [15, 16] is an integral quantity of error of the system at a delay and by the first useless time to the step turbulence. A dimensionless parameter that minimizes the criterion is requested by the simulation and the optimum seeking method.

The tuning method based on the moment of the step response of the plant was proposed [17, 18]. In [17, 18], it is proposed to tune the coefficient of the PID control. When this tuning method is used, the first clause several coefficients of the Maclaurin series the plant in is needed. In a word, if the transfer function of the plant is already known, it is a tuning method that can be easily requested. Especially, when the plant is a rational function of Laplace operator s, the denominator polynomial is divided by a molecular polynomial, and clause several of the start is requested. Moreover, it is possible to request it from the moment though the coefficient pulled the constant value from the step response of the plant. Therefore, it can be said the tuning method based on the moment of low order of the step response of the plant.

The tuning method that used the response characteristic of closed-loop and the characteristic of the plant in combination was proposed [19]. The stability and the transient characteristic of control system greatly influence the character of the bandwidth of the intersection neighborhood of the phase of the plant. The limit sensitivity method by Ziegler and Nichols [7] is a tuning method based on the limit cycle and the limit sensitivity, and a good point aimed at. However, even if the value of the PID parameter is decided by the method in [7], the response is large overshoot and vibrates. That is, a satisfying response cannot necessarily be obtained. It is thought that there is impossibility in the dependence for the characteristic of the plant on two parameters of the limit cycle and the limit sensitivity, and needs the readjustment of the response. On the other hand, it can be said that the method in [17, 18] to which the value of the PID parameter is decided by using the transfer function of the plant, is a tuning method that has the generality that can correspond to various plants. However, it is difficult to identify an accurate and reliable transfer function model in the field of the process control. From such a viewpoint, the improved limit sensitivity method that used the method in [7] with the method in [17, 18] was proposed [7].

Thus, several papers on tuning methods for PID parameters have been established.

1.1.3 Improvement of structure of PID control system

Recently, it is pointed out that the characteristic of the PID control system can improve. The structure of the control system is changed, and the feedforward element and the set point filter are added. Typical structures are shown as follows.

1. PI-D control [3, 20]

   In a PID control, it thinks about the case where the reference input changes like the step function. The derivative of the step function, that is, the impulse function will be included in the amount of the operation for the derivative action. Therefore, pulsed sharp signal will be included in the control input, and it is not desirable. Then, it is stopped to derivative the reference input. The derivative action made it work only the output feedback. This is called PI-D control. In the transfer function from the target input to the output, both the proportional control action, the integral action and the derivative action are included in a PID control in the form of the serial compensation. On the other hand, the proportional action and the integral action are included in PI-D control and the serial compensation and the derivative action will be included in the form of the parallel compensation. As a result, a rapid change of a needless control input by the derivative action when the step of the reference input changes can be suppressed.

2. I-PD control [18, 21]

There is the case that it is not preferable for a step function to be included in a control input in practical application. Then, to avoid the step function of the control input, the composition in which it is made to work is thought only by the output to which not only the derivative action but also the proportional action is feedback. This is called proportion and PI-D control. The proportional action and the derivative action influence the control input. Only the integral action influences the error. As a result, the change in the control input for the set point change can be eased.

3. Partial model matching method [21]

   The model matching is one of the ideas of matching the transfer function of the entire system that adds the control system to the plant to the transfer function of hope. The partial model matching method ignores the high term of the degree of the whole trensfer function and makes it agree mainly on the low term of the degree.

   This idea is application of the method of deciding the parameter of the controller to make a closed-loop system the characteristic of hope to the PID controller. In [21], it is shown that the partial compensation is useful when it will design referring to the shape of the step response when the control system is designed. Greatly it influences and the coefficient with a high degree hardly influences the shape of the curve of the step response from the shape of the step response of the simulation in the coefficient with a low degree. It is shown that it is important that it make amends for the coefficient with a low degree from this in the method of reference literature [21]. The reference in [21] shows that it is important that it make amends for the coefficient with a low degree.

4. Feedforward PID control [3, 20]

   It is not the constitution that is simple like a control system of Fig. 1.1 . From the practical standpoint, it is the PID control that it adds various functions, and aimed at the advancement of the control performance. The feedback control is a control that does the correction operation from the result of the output. Therefore, there is a strong point that can be corrected to the uncertainty of the plant and disturbance that cannot be measured. On the other hand, and shape corrected after the influence appears for a change and already-known disturbance of the reference input. Because the amount of the operation in which the influence is denied can be requested, this is added directly to the plant for the factor that is already-known. In addition, the feedback control is done in preparation for unknown factor. For the reference input and disturbance to become the response characteristic of hope, the setpoint of the feedforward loop and the transfer function to disturbance are decided. After it makes amends to PID for the reference input by feedback, it becomes a control system that denies disturbance by feedforward.

5. Internal model control method（Internal Model Control; IMC） [20]

   The internal model control is a design method of the controller based on a process model. The name internal model controller derives from the fact that the controller contains a model of the process internally. This model is connected in parallel with the process. The internal model principle is a general method for design of control systems that can be applied to PID control.

1.1.4 Method of considering characteristic of plant

Here, the method of considering the characteristic of the plant is shown.

1. Robustness stabilization problem

   Many of practical palnt include the uncertainty. If the control system is designed disregarding this uncertainty, the control system become unstable. The stabilization problem to the control system with the uncertainty is known as robustness stabilization problem [41], and is a important problem. H_∞ control theory is completed as a design theory of the robustness control system to the uncertainty, and the utility is admitted widely through the applied research to the real system. Because an practical plant includes the uncertainty, it can be said that it is important to design robust stabilizing PID controller to the plant with the uncertainty. The design method of robust stabilizing PID controller is examined by a lot of papers [31, 32, 33, 34, 35, 36, 37, 38, 39]. In [31], the parameter space method that gives the solution set of the robustness sensitivity minimization problem in the class of the PID controller is given. The method of reference in [31] can be requested by the math calculation with an easy sets of parameters that fill the stability condition of the control system and sets of parameters that meet the frequency requirement of the sensitivity function and the complementary sensitivity function. In [32], the parameter space planning method of the PID controller that fills H_∞control problem is given. Stability is guaranteed by requesting admissible sets of PID parameters that satisfy H_∞ control. The method of reference in [32] proposes the method of requesting the method of the direct solving of the frequency of the controller in case of the frequency area condition the set by using a general solution of H_∞control problem.

2. Problem to time-delay system

   In an actual mechanism, there is a device that the delay is caused by the delay of the operation etc. in the transmission of the signal. The control performance decreases remarkably to take time from the change of the instrumental variable to the appearance of the influence to the control variable. $u(t)$ is the input, $y(t)$ is the output, $T > 0$ is the time-delay, then the input-output relation is written by

   $$y(t) = u(t - T). \tag{1.3}$$

   When you Laplace transform expression (1.3),

   $$Y(s) = e^{-sT}U(s). \tag{1.4}$$

   Element e^{-sT} where the delay of the signal is caused is called a dead time component, and the control system including dead time component e^{-sT} says the useless time system. In general, to contain dead time component e^{-sT} the useless time system has the pole of infinity piece. Therefore, there is a problem that the control becomes difficult.

   When the plant includes time-delay, the predictive control system of the target to follow is proposed. It is called the Smith predictive control from proposer's name [63].

   The controller that builds the PID control into Smith predictor is Smith-PID control [64]. From the transfer function from the reference input to the output of Smith-PID control, the response of the output is delayed, but it is understood not to receive other influences. However, time-delay cannot be completely controlled for the influence of the model error. When the Smith predictive control is used, it is important to construct the exact model.

3. Problem of obtaining admissible sets of PID parameters that guarantee the stability of control system

Several papers on tuning methods for PID parameters have been considered [7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22]. However the method in [7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22] do not guarantee the stability of closed-loop system. The reference in [25, 26, 27, 28] propose design methods of PID controllers to guarantee the stability of closed-loop system. However, using the method in [25, 26, 27, 28], it is difficult to tune PID parameters to meet control specifications. If admissible sets of PID parameters to guarantee the stability of closed-loop system are obtained, we can easily design stabilizing PID controllers to meet control specifications.

Moreover, when the parameter is adjusted, the stability of the feedback control system might be demanded from the safety problem according to the controlled system. The problem to obtain admissible sets of PID parameters to guarantee the stability of closed-loop system is known as a parametrization problem [6, 29, 30]. If there exists a stabilizing PID controller, the parametrization of all stabilizing PID controller is considered in [6, 29, 30]. However the method in [6, 29, 30] remains a difficulty. The admissible sets of P-parameter, I-parameter and D-parameter in [6, 29, 30] are related each other. That is, if P-parameter is changed, then the admissible sets of I-parameter and D-parameter change. From practical point of view, it is desirable that the admissible sets of P-parameter, I-parameter and D-parameter are independent from each other. Yamada and Moki initially tackle this problem and propose a design method for modified PI controllers for any minimum phase systems such that the admissible sets of P-parameter and I-parameter are independent from each other [45]. Yamada expand the result in [45] and propose a design method for modified PID controllers for minimum phase plant such that the admissible sets of P-parameter, I-parameter and D-parameter are independent from each other [46].

1.2 A trend of a study for modified PID control system

In this section, how modified PID control system has been researched is shown. When the control system is designed, the control problem that should be examined is different according to the class of the plant and the control performance to be achieved. Therefore, it is necessary to think about the control problem individually for the class of the plant and the control performance to be achieved. If we can construct the control system that has simplicity and characteristics similar to the PID control for the plant that cannot be stabilized by the PID control, the knowledge of the PID control can be used. Therefore, the area where the PID control is used extends and it is useful. From this viewpoint, Yamada et al. proposed a design method of PID controller by using the parameterization of all stabilizing controllers. Here, the parameterization is described. The parameterization problem is problem of finding of all stabilizing controllers that stabilizes the control system, and it is known as one of the important problem[**?**, 62]. The PID controller designed by using the parameterization is called modified PID controller. The design method of modified PID controllers proposed by Yamada et al. is shown.

1. Minimum phase plant

   Yamada and Moki proposed a design method for modified PI controllers for any minimum phase system such that modified PI controllers can stabilize any plant and admissible sets of P-parameter and I-parameter are independent from each other [45]. Yamada expanded the result in [45] and proposed a design method for modified PID controllers for minimum phase plants [46].

2. Non-minimum phase plant

   Yamada et al. proposed a design method for modified PID controllers for any non-minimum phase system such that modified PID controllers can stabilize any plant and admissible sets of P-parameter and I-parameter are independent from each other [47].

3. Stable plant

   Yamada et al. expand the result in [45, 46, 47] and propose a design method for modified PID controllers such that modified PID controller makes the closed-loop system stable for any stable plants and the admissible sets of P-parameter, I-parameter and D-parameter to guarantee the stability of closed-loop system are independent from each other [48, 49].

4. Unstable plant

   Yamada and Hagiwara gave a design method of modified PID controllers to make the closed-loop system sstable for any unstable plants [50].

5. Plant with uncertainty

   The stability problem with uncertainty is known as the robust stability problem [41]. When the modified PID controller is applied to the real control system, the influence of uncertainty must be considered. The parametrization of all robust stabilizing controllers for the plant with uncertainty is obtained using H_∞ control theory based on the Riccati equation [41, 42] and the Linear Matrix Inequality (LMI) [43, 44].

6. Time-delay system Yamada et al. expand the results in [45, 46, 47] and propose a method for designing modified PID controllers such that the controller makes the feedback control system stable for any stable and/or minimum-phase time-delay plant and the admissible sets of P-, I- and D-parameters are independent [49]. Proposed method adopted the parameterization of all stabilizing modified Smith predictors for any stable and/or minimum-phase time-delay plant in [59].

7. Multiple-input/multiple-output

   Hagiwara and Yamada expand the result in [45, 46, 50] and propose a design method of modified PID controllers such that the modified PID controller makes the closed-loop system stable for any multiple-input/multiple-output plants and the admissible sets of P-parameter, I-parameter and D-parameter to guarantee the stability of closed-loop system are independent from each other. In order to apply any multiple-input/multiple-output plants, the parametrization of all stabilizing controllers for multiple-input/multiple-output plants in [62] is used.

Thus, a design method of modified PID controllers has been examined. As mentioned above, the study on a modified PID controller is summarized in Table 1.1 . It means × in Table 1.1 is a problem that has not been examined.

1.3 The perpose and contents of this study

Proportional-Integral-Derivative (PID) controller is most widely used controller structure in industrial applications [3, 4, 6]. Its structural simplicity and sufficient ability of solving many practical control problems have contributed to this wide acceptance.

Table 1.1: The past studies on the design method of modified PID controllers

plant	design method for modified PID controllers
minimum phase	Yamada, Moki [45], Yamada [46]
non-minimum phase	Yamada, Moki, Hai [47]
stable	Yamada, Matsushima, Hagiwara [48, 49]
unstable	Yamada, Hagiwara [50]
plant with uncertainty	Yamada, Hagiwara, Shimizu [51]
time-delay	Yamada, Hagiwara, Shimizu [49, 52, 53]
time-delay plant with uncertainty	Hagiwara, Yamada, Murakami, Ando, Sakanushi [56]
multiple-input/multiple-output	Hagiwara, Yamada [54]
multiple-input/multiple-output with uncertainty	Hagiwara, Yamada, Murakami, Ando, Sakanushi [55]
multiple-input/multiple-output time-delay	×
multiple-input/multiple-output time-delay with uncertainty	×
attenuate unknown disturbance	Hagiwara, Yamada, Murakami, Ando, Matsuura [58], Hagiwara, Yamada, Murakami, Ando, Matsuura, Aoyama [57]

Several papers on tuning methods for PID parameters have been considered [7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22]. However the method in [7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22] do not guarantee the stability of closed-loop system. The reference in [25, 26, 27, 28] propose design methods of PID controllers to guarantee the stability of closed-loop system. However, using the method in [25, 26, 27, 28], it is difficult to tune PID parameters to meet control specifications. If admissible sets of PID parameters to guarantee the stability of closed-loop system are obtained, we can easily design stabilizing PID controllers to meet control specifications.

The problem to obtain admissible sets of PID parameters to guarantee the stability of closed-loop system is known as a parametrization problem [6, 29, 30]. If there exists a stabilizing PID controller, the parametrization of all stabilizing PID controller is considered in [6, 29, 30]. However the method in [6, 29, 30] remains a difficulty. The admissible sets of P-parameter, I-parameter and D-parameter in [6, 29, 30] are related each other. That is, if P-parameter is changed, then the admissible sets of I-parameter and D-parameter change. From practical point of view, it is desirable that the admissible sets of P-parameter, I-parameter and D-parameter are independent from each other. Yamada and Moki initially tackle this problem and propose a design method for modified PI controllers for any minimum phase systems such that the admissible sets of P-parameter and I-parameter are independent from each other [45]. Yamada expand the result in [45] and propose a design method for modified PID controllers for minimum phase plant such that the admissible sets of P-parameter, I-parameter and D-parameter are independent from each other [46]. For stable plants, a design method of modified PID controllers was considered in [48, 49]. For unstable plant, Yamada and Hagiwara gave a design method for modified PID controllers [50]. In this way, the modified PID controller that can be stabilize the control system has been established. However, modified PID controllers in [45, 46, 48, 49, 50] cannot be applied to a practical control system. In a practical control system, it is necessary to consider an uncertainty, useless time and disturbance, etc. In this paper, in order to solve these problems, we expand the results in [45, 46, 48, 49, 50] and propose a design method for modified PID controllers such that the controller makes the feedback control system stable for plants with uncertainty, for time-delay plants with uncertainty and the admissible sets of P-parameter, I-parameter and D-parameter to guarantee the stability of control system are independent from each other. In addition, we propose a design method for modified PID control systems to attenuate unknown disturbances and their applications.

This paper is organized as follows:

In Chapter 2., we propose a design method of robust stabilizing modified PID controllers for plants with uncertainty. The basic idea of robust stabilizing modified PID controller is very simple. If the modified PID control system is robustly stable for the plant with uncertainty, then the modified PID controller must satisfy the robust stability condition. This implies that if the modified PID control system is robustly stable, then the modified PID controller is included in the parametrization of all robust stabilizing controllers for the plant with uncertainty. The parametrization of all robust stabilizing controllers for the plant with uncertainty is obtained using H_∞ control theory based on the Riccati equation [41, 42] and the Linear Matrix Inequality (LMI) [43, 44]. Robust stabilizing controllers for the plant with uncertainty include a free parameter, which is designed to achieve desirable control characteristics. When the free parameter of the parametrization of all robust stabilizing controllers is adequately chosen, then the controller works as a robust stabilizing modified PID controller.

In Chapter 3., we propose a design method for robust stabilizing modified PID controllers for time-delay plants with uncertainty. The basic idea of designing a robust stabilizing modified PID controller for any time-delay plant with uncertainty is very simple. For a certain class of time-delay plants with uncertainty, using state preview control, the problem to design a robust stabilizing controller is reduced to that for the plant without a time delay [40]. That is, if the modified PID control system is robustly stable for the time-delay plant with uncertainty, then the modified PID controller must satisfy the robust stability condition for system without time delay. This implies that if the modified PID control system is robustly stable, then the modified PID controller is included in the parameterization of all robust stabilizing controllers for the plant with uncertainty. The parameterization of all robust stabilizing controllers for the plant with uncertainty is obtained using H_∞ control theory based on the Riccati equation [41, 42] and the linear matrix inequality (LMI) [43, 44]. Robust stabilizing controllers for plants with uncertainty include a free-parameter, which is designed to achieve desirable control characteristics. When the free-parameter of the parameterization of all robust stabilizing controllers is appropriately chosen, then the controller works as a robust stabilizing modified PID controller.

In Chapter 4., we propose a design method for modified PID control systems to attenuate unknown disturbances. The modified PID controller that can stabilize the control system has been established till now. However, the modified PID controller in [45, 46, 48, 49, 50, 51] remains two difficulties. One is that the modified PID control system in [45, 46, 48, 49, 50, 51] cannot specify the input-output characteristic and the disturbance attenuation characteristic separately. From the practical point of view, it is desirable that the input-output characteristic and the disturbance attenuation characteristic can be specified separately. The other is that the modified PID control system in [45, 46, 48, 49, 50, 51] cannot attenuate unknown disturbances. In many cases, the disturbance in the plant is unknown. It is comparatively easy to attenuate known disturbance, but it is difficult to attenuate unknown disturbances. However, no paper examines a design method for modified PID control systems to specify the input-output characteristic and to attenuate unknown disturbances. In Chapter 4., in order to solve these problems, we propose a design method for modified PID control systems to specify the input-output characteristic and the disturbance attenuation characteristic separately and to attenuate unknown disturbances effectively.

In Chapter 5., we propose an application of the modified PID control system for Heat Flow Experiment. In Chapter 4., a design method for modified PID control system to attenuate unknown disturbances was proposed [56]. In addition, the control system in [56] has desirable control characteristic such that the input-output characteristic and the disturbance attenuation characteristic can be specified separately. Therefore, the method in [56] may be an effective control design method for practical plants. However, an application of the modified PID control system to attenuate unknown disturbances for plants with any disturbance in [56] is not examined. Therefore, the effectiveness of the method in [56] for controlling practical systems is not confirmed. In Chapter 5., we apply the modified PID control system to attenuate unknown disturbances for plants with any disturbance in [56] for temperature control for heat flow experiment and show the effectiveness of the modified PID control systems to attenuate unknown disturbances for plants with any disturbance in [56].

Chapter 6. summarizes the result of the present study by the conclusion.

Notations

R	the set of real numbers.
R_+	$R \cup \{\infty\}$.
$R(s)$	the set of real rational function with s.
RH_∞	the set of stable proper real rational functions.
H_∞	the set of stable causal functions.
$\mathcal{U}$	the set of unimodular functions on RH_∞. That is, $U(s) \in \mathcal{U}$ implies both $U(s) \in RH_\infty$ and $U^{-1}(s) \in RH_\infty$.
$D^\perp$	orthogonal complement of D, i.e., $\begin{bmatrix} D & D^\perp \end{bmatrix}$ or $\begin{bmatrix} D \\ D^\perp \end{bmatrix}$ is unitary.
A^T	transpose of A.
$A^\dagger$	pseudo inverse of A.
$\rho(\{\cdot\})$	spectral radius of $\{\cdot\}$.
$\bar{\sigma}(\{\cdot\})$	maximum singular value of $\{\cdot\}$.
$\\|\{\cdot\}\\|_\infty$	H_∞ norm of $\{\cdot\}$.
$\left[\begin{array}{c\|c} A & B \\ \hline C & D \end{array}\right]$	represents the state space description $C(sI - A)^{-1}B + D$.

Chapter 2

A Design Method of Robust Stabilizing Modified PID Controllers

2.1 Introduction

PID (Proportional-Integral-Derivative) controller is most widely used controller structure in industrial applications [3, 4, 6]. Its structural simplicity and sufficient ability of solving many practical control problems have contributed to this wide acceptance.

Several papers on tuning methods for PID parameters have been considered [7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22]. However the method in [7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22] do not guarantee the stability of closed-loop system. The references in [25, 26, 27, 28, 29, 30] propose design methods of PID controllers to guarantee the stability of closed-loop system. However, plants to which the method in [25, 26, 27, 28, 29, 30] are restricted. Yamada and Hagiwara gave a design method of modified PID controllers to make the closed-loop system stable for any unstable plants [50]. However the method in [50] cannot apply for plants with uncertainty. The stability problem with uncertainty is known as the robust stability problem [41]. Since almost all practical plants include uncertainty, the problem to design robust stabilizing modified PID controllers for any plants with uncertainty is important. Several papers on design methods of robust stabilizing PID controllers have been considered [32, 33, 34, 35, 36, 37, 38, 39]. However, no design method of modified PID controllers has been published to guarantee the robust stability of PID control system for any plants with uncertainty.

In this paper, we propose a design method of robust stabilizing modified PID controllers such that modified PID controller makes the closed-loop system stable for any plants with uncertainty. The basic idea of robust stabilizing modified PID controller is very simple. If the modified PID control system is robustly stable for the plant with uncertainty, then the modified PID controller must satisfy the robust stability condition. This implies that if the modified PID control system is robustly stable, then the modified PID controller is included in the parametrization of all robust stabilizing controllers for the plant with uncertainty. The parametrization of all robust stabilizing controllers for the plant with uncertainty is obtained using H_∞ control theory based on the Riccati equation [41, 42] and the Linear Matrix Inequality (LMI) [43, 44]. Robust stabilizing controllers for the plant with uncertainty include a free parameter, which is designed to achieve desirable control characteristics. When the free parameter of the parametrization of all robust stabilizing controllers is adequately chosen, then the controller works as a robust stabilizing modified PID controller. A numerical example is illustrated to show the effectiveness of the proposed method.

2.2 Problem formulation

Consider the closed-loop system written by

$$\begin{cases} y &= G(s)u \\ u &= C(s)\,(r-y) \end{cases}, \tag{2.1}$$

where $G(s) \in R(s)$ is the plant, $C(s) \in R(s)$ is the controller, $r \in R$ is the reference input, $u \in R$ is the control input and $y \in R$ is the output. The nominal plant of $G(s)$ is denoted by $G_m(s) \in R(s)$. Both $G(s)$ and $G_m(s)$ are assumed to have no zero or pole on the imaginary axis. In addition, it is assumed that the number of poles of $G(s)$ in the closed right half plane is equal to the number of poles of $G_m(s)$ in the closed right half plane. The relation between the plant $G(s)$ and the nominal plant $G_m(s)$ is written as

$$G(s) = G_m(s)(1 + \Delta(s)), \tag{2.2}$$

where $\Delta(s) \in R(s)$ is the uncertainty. The set of $\Delta(s)$ is all rational functions satisfying

$$|\Delta(j\omega)| < |W_T(j\omega)| \quad (\forall \omega \in R_+), \tag{2.3}$$

where $W_T(s)$ is an asymptotically stable rational function. Under these assumption, the robust stability condition for the plant $G(s)$ with uncertainty $\Delta(s)$ satisfying (2.3) is given by

$$\|T(s)W_T(s)\|_\infty < 1, \tag{2.4}$$

where $T(s)$ is the complementary sensitivity function given by

$$T(s) = \frac{G_m(s)C(s)}{1 + G_m(s)C(s)}. \tag{2.5}$$

When the controller $C(s)$ has the form written by

$$C(s) = a_P + \frac{a_I}{s} + a_D s, \tag{2.6}$$

then the controller $C(s)$ is called PID controller [6], where $a_P \in R$ is the P-parameter, $a_I \in R$ is the I-parameter and $a_D \in R$ is the D-parameter. a_P, a_I and a_D are settled so that the closed-loop system in (2.1) has desirable control characteristics such as steady state characteristic and transient characteristic. For easy explanation, we call $C(s)$ in (2.6) the conventional PID controller.

The purpose of this paper is to propose a design method of robust stabilizing modified PID controllers $C(s)$ to make the closed-loop system in (2.1) stable for any plant $G(s)$ in (2.2) with uncertainty $\Delta(s)$ satisfying (2.3).

2.3 The basic idea

In this section, we describe the basic idea to design of robust stabilizing modified PID controllers $C(s)$ to make the closed-loop system in (2.1) stable for the plant $G(s)$ with uncertainty $\Delta(s)$.

In order to design robust stabilizing modified PID controllers $C(s)$ that can be applied to any plant $G(s)$ with uncertainty $\Delta(s)$, we must see that the robust stabilizing controllers hold (2.4). The problem of obtaining the controller $C(s)$, which is not necessarily a PID controller,

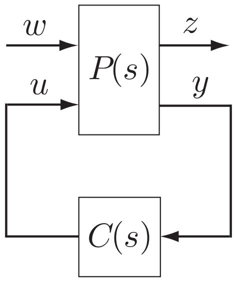

Figure 2.1: Block diagram of H_∞ control problem

satisfying (2.4) is equivalent to the following H_∞ problem. In order to obtain the controller $C(s)$ satisfying (2.4), we consider the control system shown in Fig. 2.1 . $P(s)$ is selected such that the transfer function from w to z in Fig. 2.1 is equal to $T(s)W_T(s)$. The state space description of $P(s)$ is, in general,

$$\left\{\begin{array}{rcllll} \dot{x}(t) & = & Ax(t) & +B_1w(t) & +B_2u(t) \\ z(t) & = & C_1x(t) & & +D_{12}u(t) \\ y(t) & = & C_2x(t) & +D_{21}w(t) & \end{array}\right. , \tag{2.7}$$

where $A \in R^{n\times n}$, $B_1 \in R^n$, $B_2 \in R^n$, $C_1 \in R^{1\times n}$, $C_2 \in R^{1\times n}$, $D_{12} \in R$, $D_{21} \in R$. $P(s)$ is called the generalized plant [41]. $P(s)$ is assumed to satisfy the following standard assumptions in [41, 42]:

1) (A, B_2) is stabilizable and (C_2, A) is detectable;

2) D_{12} has full column rank and D_{21} has full row rank;

3) $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full column rank for all ω and $\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}$ has full row rank for all ω.

Under these assumptions, according to [41, 42], the parametrization of all robust stabilizing controllers $C(s)$ is written by

$$C(s) = C_{11}(s) + C_{12}(s)Q(s)\left(I - C_{22}(s)Q(s)\right)^{-1} C_{21}(s), \tag{2.8}$$

where

$$\begin{bmatrix} C_{11}(s) & C_{12}(s) \\ C_{21}(s) & C_{22}(s) \end{bmatrix} = \left[\begin{array}{c|cc} A_c & B_{c1} & B_{c2} \\ \hline C_{c1} & D_{c11} & D_{c12} \\ C_{c2} & D_{c21} & D_{c22} \end{array}\right] \tag{2.9}$$

$$\begin{aligned} A_c &= A + B_1B_1^TX - B_2\left(D_{12}^\dagger C_1 + E_{12}^{-1}B_2^TX\right) \\ &\quad - (I - XY)^{-1}\left(B_1D_{21}^\dagger + YC_2^TE_{21}^{-1}\right)\left(C_2 + D_{21}B_1^TX\right) \end{aligned}$$

$$B_{c1} = (I - XY)^{-1}\left(B_1D_{21}^\dagger + YC_2^TE_{21}^{-1}\right), \quad B_{c2} = (I - XY)^{-1}\left(B_2 + YC_1^TD_{12}\right)E_{12}^{-1/2},$$

$$C_{c1} = -D_{12}^{\dagger}C_1 - E_{12}^{-1}B_2^T X,\ C_{c2} = -E_{21}^{-1/2}\left(C_2 + D_{21}B_1^T X\right)$$

$$D_{c11} = 0,\ \ D_{c12} = E_{12}^{-1/2},\ \ D_{c21} = E_{21}^{-1/2},\ \ D_{c22} = 0,$$

$$E_{12} = D_{12}^T D_{12}, E_{21} = D_{21}D_{21}^T,$$

$X \geq 0$ and $Y \geq 0$ are solutions of

$$\begin{aligned}&X\left(A - B_2D_{12}^{\dagger}C_1\right) + \left(A - B_2D_{12}^{\dagger}C_1\right)^T X\\&+X\left(B_1B_1^T - B_2\left(D_{12}^TD_{12}\right)^{-1}B_2^T\right)X + \left(D_{12}^{\perp}C_1^T\right)^T D_{12}^{\perp}C_1^T = 0\end{aligned} \tag{2.10}$$

and

$$\begin{aligned}&Y\left(A - B_1D_{21}^{\dagger}C_2\right)^T + \left(A - B_1D_{21}^{\dagger}C_2\right)Y\\&+Y\left(C_1^TC_1 - C_2^T\left(D_{21}D_{21}^T\right)^{-1}C_2\right)Y + B_1D_{21}^{\perp}\left(B_1D_{21}^{\perp}\right)^T = 0\end{aligned} \tag{2.11}$$

such that

$$\rho\,(XY) < 1 \tag{2.12}$$

and both $A-B_2D_{12}^{\dagger}C_1+\left(B_1B_1^T - B_2\left(D_{12}^TD_{12}\right)^{-1}B_2^T\right)X$ and $A-B_1D_{21}^{\dagger}C_2+Y\left(C_1^TC_1 - C_2\left(D_{21}D_{21}^T\right)^{-1}C_2\right)$ have no eigenvalue in the closed right half plane and the free parameter $Q(s) \in RH_\infty$ is any function satisfying $\|Q(s)\|_\infty < 1$.

On the parametrization of all robust stabilizing controllers $C(s)$ in (2.8) for $G(s)$, the controller $C(s)$ in (2.8) includes free-parameter $Q(s)$. Using free-parameter $Q(s)$ in (2.8), we propose a design method of robust stabilizing modified PID controllers $C(s)$ to make the closed-loop system in (2.1) stable. In order to design the robust stabilizing modified PID controllers $C(s)$, the free parameter $Q(s)$ in (2.8) is settled for $C(s)$ in (2.8) to have the same characteristics to conventional PID controller $C(s)$ in (2.6). Therefore, next, we describe the role of conventional PID controller $C(s)$ in (2.6) in order to clarify the condition that the modified PID controller $C(s)$ must be satisfied. From (2.6), using $C(s)$, the P-parameter a_P, the I-parameter a_I and the D-parameter a_D are decided by

$$a_P = \lim_{s\to\infty}\left\{-s^2\frac{d}{ds}\left(\frac{1}{s}C(s)\right)\right\}, \tag{2.13}$$

$$a_I = \lim_{s\to 0}\{sC(s)\} \tag{2.14}$$

and

$$a_D = \lim_{s\to\infty}\frac{d}{ds}\{C(s)\}, \tag{2.15}$$

respectively. Therefore, if the controller $C(s)$ holds (2.13), (2.14) and (2.15), the role of controller $C(s)$ is equivalent to the conventional PID controller $C(s)$ in (2.8). That is, we can design robust stabilizing modified PID controllers such that the role of controller $C(s)$ (2.8) is equivalent to the conventional PID controller $C(s)$ in (2.6).

In the next section, using the idea described in this section, we propose a design method of robust stabilizing modified PID controllers that satisfies (2.13), (2.14) and (2.15).

2.4 Robust Stabilizing Modified PID controller

In this section, we propose a design method of robust stabilizing modified PID controllers.

2.4.1 Robust Stabilizing Modified P controller

The robust stabilizing modified P controller $C(s)$ satisfying (2.13) is written by (2.8), where

$$Q(s) = \frac{a_P}{\lim_{s\to\infty}\left(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)\right)}. \tag{2.16}$$

If a_P satisfies

$$-\left|\lim_{s\to\infty}\left(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)\right)\right| < a_P < \left|\lim_{s\to\infty}\left(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)\right)\right|, \tag{2.17}$$

$Q(s)$ in (2.16) satisfies $\|Q(s)\|_\infty < 1$. This implies that when (2.17) holds true, the controller $C(s)$ in (2.8) with (2.16) makes the closed-loop system in (2.1) stable for the plant $G(s)$ with uncertainty $\Delta(s)$.

2.4.2 Robust Stabilizing Modified I controller

The robust stabilizing modified I controller $C(s)$ satisfying (2.14) is written by (2.8), where

$$Q(s) = \frac{q_0 + q_1 s}{\tau_0 + \tau_1 s}, \tag{2.18}$$

$$q_0 = \frac{\tau_0}{C_{22}(0)}, \tag{2.19}$$

$$q_1 = \frac{\tau_1}{C_{22}(0)} - \frac{\tau_0}{a_I C_{22}^2(0)}\left\{\left.\frac{d}{ds}\left(C_{22}(s)\right)\right|_{s=0} a_I + C_{12}(0)C_{21}(0)\right\}, \tag{2.20}$$

$\tau_i \in R > 0$ $(i = 0, 1)$. If

$$|C_{22}(0)| < 0 \tag{2.21}$$

and

$$-1 < \frac{1}{C_{22}(0)} - \frac{\tau_0}{\tau_1 a_I C_{22}^2(0)}\left\{\left.\frac{d}{ds}\left(C_{22}(s)\right)\right|_{s=0} a_I + C_{12}(0)C_{21}(0)\right\} < 1 \tag{2.22}$$

hold true, then $Q(s)$ in (2.18) satisfies $\|Q(s)\|_\infty < 1$. This implies that when (2.21) and (2.22) hold true, the controller $C(s)$ in (2.8) with (2.18) makes the closed-loop system in (2.1) stable for the plant $G(s)$ with uncertainty $\Delta(s)$.

2.4.3 Robust Stabilizing Modified D controller

The robust stabilizing modified D controller $C(s)$ satisfying (2.15) is written by (2.8), where

$$Q(s) = \frac{a_D}{\lim_{s\to\infty}\left(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)\right) + a_D \lim_{s\to\infty}\left(sC_{22}(s)\right)} s. \tag{2.23}$$

Since $Q(s)$ in (2.23) is improper, $Q(s)$ in (2.23) is not included in RH_∞. In order for $Q(s)$ to be included in RH_∞, (2.23) is modified as

$$Q(s) = \frac{a_D}{\lim_{s\to\infty}(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)) + a_D \lim_{s\to\infty}(sC_{22}(s))} \frac{s}{1+\tau_D s}, \tag{2.24}$$

where $\tau_D \in R > 0$. From $\tau_D > 0$ in (2.24), $Q(s)$ in (2.24) is included in RH_∞. If

$$-1 < \frac{a_D}{\tau_D \left\{ \lim_{s\to\infty}(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)) + a_D \lim_{s\to\infty}(sC_{22}(s)) \right\}} < 1 \tag{2.25}$$

is satisfied, then $Q(s)$ in (2.24) satisfies $\|Q(s)\|_\infty < 1$. This implies that when (2.25) is satisfied, the controller $C(s)$ in (2.8) with (2.24) makes the closed-loop system in (2.1) stable for the plant $G(s)$ with uncertainty $\Delta(s)$.

2.4.4 Robust Stabilizing Modified PI controller

The robust stabilizing modified PI controller $C(s)$ satisfying (2.13) and (2.14) is written by (2.8), where

$$Q(s) = \frac{q_0 + q_1 s + q_2 s^2}{\tau_0 + \tau_1 s + \tau_2 s^2}, \tag{2.26}$$

$$q_0 = \frac{\tau_0}{C_{22}(0)}, \tag{2.27}$$

$$q_1 = \frac{\tau_1}{C_{22}(0)} - \frac{\tau_0}{a_I C_{22}^2(0)} \left\{ \frac{d}{ds}(C_{22}(s))\bigg|_{s=0} a_I + C_{12}(0)C_{21}(0) \right\}, \tag{2.28}$$

$$q_2 = \frac{\tau_2 a_P}{\lim_{s\to\infty}(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s))} \tag{2.29}$$

and $\tau_i \in R > 0$ $(i = 0, 1, 2)$. From $\tau_i > 0$ $(i = 0, 1, 2)$, $Q(s)$ in (2.26) is included in RH_∞. If a_P and a_I are settled to make $Q(s)$ in (2.26) satisfy $\|Q(s)\|_\infty < 1$, then the controller $C(s)$ in (2.8) with (2.26) makes the closed-loop system in (2.1) stable for the plant $G(s)$ with uncertainty $\Delta(s)$.

2.4.5 Robust Stabilizing Modified PD controller

The robust stabilizing modified PD controller $C(s)$ satisfying (2.13) and (2.15) is written by (2.8), where

$$Q(s) = q_0 + q_1 s, \tag{2.30}$$

$$q_1 = \frac{a_D}{\lim_{s\to\infty}(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)) + a_D \lim_{s\to\infty}\{sC_{22}(s)\}} \tag{2.31}$$

and

$$q_0 = \frac{\left\{1-\lim_{s\to\infty}(sC_{22}(s))\,q_1\right\}^2 a_P}{\lim_{s\to\infty}(C_{12}(s)C_{21}(s)-C_{11}(s)C_{22}(s))\left\{1-\lim_{s\to\infty}(sC_{22}(s))\,q_1-\lim_{s\to\infty}\left(s^2\frac{d}{ds}(C_{22}(s))\,q_1\right)\right\}}$$
$$+\frac{1}{\lim_{s\to\infty}(C_{12}(s)C_{21}(s)-C_{11}(s)C_{22}(s))\left\{1-\lim_{s\to\infty}(sC_{22}(s))\,q_1-\lim_{s\to\infty}\left(s^2\frac{d}{ds}(C_{22}(s))\,q_1\right)\right\}}$$
$$\cdot\left[\lim_{s\to\infty}\left(s^2\frac{d}{ds}(C_{12}(s)C_{21}(s)-C_{11}(s)C_{22}(s))\right)\left\{q_1-\lim_{s\to\infty}(sC_{22}(s))\,q_1^2\right\}\right.$$
$$\left.+\lim_{s\to\infty}(C_{12}(s)C_{21}(s)-C_{11}(s)C_{22}(s))\left\{\lim_{s\to\infty}\left(s^3\frac{d}{ds}(C_{22}(s))\right)+\lim_{s\to\infty}\left(s^2C_{22}(s)\right)\right\}q_1^2\right]. \tag{2.32}$$

Since $Q(s)$ in (2.30) is improper, $Q(s)$ in (2.30) is not included in RH_∞. In order for $Q(s)$ to be included in RH_∞, (2.30) is modified as

$$Q(s) = q_0 + \frac{q_1 s}{1+\tau_D s}, \tag{2.33}$$

where $\tau_D \in R > 0$. From $\tau_D > 0$ in (2.33), $Q(s)$ in (2.33) is included in RH_∞. If

$$\left|\frac{\left\{1-\lim_{s\to\infty}(sC_{22}(s))\,q_1\right\}^2 a_P}{\lim_{s\to\infty}(C_{12}(s)C_{21}(s)-C_{11}(s)C_{22}(s))\left\{1-\lim_{s\to\infty}(sC_{22}(s))\,q_1-\lim_{s\to\infty}\left(s^2\frac{d}{ds}(C_{22}(s))\,q_1\right)\right\}}\right.$$
$$+\frac{1}{\lim_{s\to\infty}(C_{12}(s)C_{21}(s)-C_{11}(s)C_{22}(s))\left\{1-\lim_{s\to\infty}(sC_{22}(s))\,q_1-\lim_{s\to\infty}\left(s^2\frac{d}{ds}(C_{22}(s))\,q_1\right)\right\}}$$
$$\cdot\left[\lim_{s\to\infty}\left(s^2\frac{d}{ds}(C_{12}(s)C_{21}(s)-C_{11}(s)C_{22}(s))\right)\left\{q_1-\lim_{s\to\infty}(sC_{22}(s))\,q_1^2\right\}\right.$$
$$\left.\left.+\lim_{s\to\infty}(C_{12}(s)C_{21}(s)-C_{11}(s)C_{22}(s))\left\{\lim_{s\to\infty}\left(s^3\frac{d}{ds}(C_{22}(s))\right)+\lim_{s\to\infty}\left(s^2C_{22}(s)\right)\right\}q_1^2\right]\right|<1 \tag{2.34}$$

and

$$\left|\frac{\left\{1-\lim_{s\to\infty}(sC_{22}(s))\,q_1\right\}^2 a_P}{\lim_{s\to\infty}(C_{12}(s)C_{21}(s)-C_{11}(s)C_{22}(s))\left\{1-\lim_{s\to\infty}(sC_{22}(s))\,q_1-\lim_{s\to\infty}\left(s^2\frac{d}{ds}(C_{22}(s))\,q_1\right)\right\}}\right.$$
$$+\frac{1}{\lim_{s\to\infty}(C_{12}(s)C_{21}(s)-C_{11}(s)C_{22}(s))\left\{1-\lim_{s\to\infty}(sC_{22}(s))\,q_1-\lim_{s\to\infty}\left(s^2\frac{d}{ds}(C_{22}(s))\,q_1\right)\right\}}$$
$$\cdot\left[\lim_{s\to\infty}\left(s^2\frac{d}{ds}(C_{12}(s)C_{21}(s)-C_{11}(s)C_{22}(s))\right)\left\{q_1-\lim_{s\to\infty}(sC_{22}(s))\,q_1^2\right\}\right.$$
$$\left.+\lim_{s\to\infty}(C_{12}(s)C_{21}(s)-C_{11}(s)C_{22}(s))\left\{\lim_{s\to\infty}\left(s^3\frac{d}{ds}(C_{22}(s))\right)+\lim_{s\to\infty}\left(s^2C_{22}(s)\right)\right\}q_1^2\right]$$
$$\left.+\frac{a_D}{\tau_D\left\{\lim_{s\to\infty}(C_{12}(s)C_{21}(s)-C_{11}(s)C_{22}(s))+a_D\lim_{s\to\infty}\{sC_{22}(s)\}\right\}}\right|<1 \tag{2.35}$$

hold true, then $Q(s)$ in (2.33) satisfy $\|Q(s)\|_\infty < 1$. This implies that if (2.34) and (2.35) hold true, then the controller $C(s)$ in (2.8) with (2.33) makes the closed-loop system in (2.1) stable for the plant $G(s)$ with uncertainty $\Delta(s)$.

2.4.6 Robust Stabilizing Modified PID controller

The robust stabilizing modified PID controller $C(s)$ satisfying (2.13), (2.14) and (2.15) is written by (2.8), where

$$Q(s) = \frac{q_0 + q_1 s + q_2 s^2}{\tau_0 + \tau_1 s + \tau_2 s^2} + q_3 s, \tag{2.36}$$

$$q_0 = \frac{\tau_0}{C_{22}(0)}, \tag{2.37}$$

$$q_1 = \frac{\tau_1}{C_{22}(0)} - q_3\tau_0 - \frac{\tau_0}{a_I C_{22}^2(0)}\left(\frac{d}{ds}\left(C_{22}(s)\right)\Big|_{s=0} a_I + C_{12}(0)C_{21}(0)\right), \tag{2.38}$$

$$\begin{aligned} q_2 = {} & \frac{\left\{1 - \lim_{s\to\infty}\left(sC_{22}(s)\right) q_3\right\}^2 \tau_2 a_P}{\lim_{s\to\infty}\left(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)\right)\left\{1 - \lim_{s\to\infty}\left(sC_{22}(s)\right) q_3 - \lim_{s\to\infty}\left(s^2\frac{d}{ds}\left(C_{22}(s)\right)\right) q_3\right\}} \\ & + \frac{\tau_2}{\lim_{s\to\infty}\left(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)\right)\left\{1 - \lim_{s\to\infty}\left(sC_{22}(s)\right) q_3 - \lim_{s\to\infty}\left(s^2\frac{d}{ds}\left(C_{22}(s)\right)\right) q_3\right\}} \\ & \cdot \left[\lim_{s\to\infty}\left(s^2\frac{d}{ds}\left(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)\right)\right)\left\{q_3 - \lim_{s\to\infty}\left(sC_{22}(s)\right) q_3^2\right\}\right. \\ & \left. + \lim_{s\to\infty}\left(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)\right)\left\{\lim_{s\to\infty}\left(s^3\frac{d}{ds}\left(C_{22}(s)\right)\right) + \lim_{s\to\infty}\left(s^2 C_{22}(s)\right)\right\} q_3^2\right], \end{aligned} \tag{2.39}$$

$$q_3 = \frac{a_D}{\lim_{s\to\infty}\left(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)\right) + a_D \lim_{s\to\infty}\left(sC_{22}(s)\right)} \tag{2.40}$$

and $\tau_i \in R > 0$ $(i = 0, 1, 2)$. Since $Q(s)$ in (2.36) is improper, $Q(s)$ in (2.36) is not included in RH_∞. In order for $Q(s)$ to be included in RH_∞, (2.36) is modified as

$$Q(s) = \frac{q_0 + q_1 s + q_2 s^2}{\tau_0 + \tau_1 s + \tau_2 s^2} + \frac{q_3 s}{1 + \tau_D s}, \tag{2.41}$$

where $\tau_D \in R > 0$. From $\tau_D > 0$ and $\tau_i > 0$ $(i = 0, 1, 2)$ in (2.41), $Q(s)$ in (2.41) is included in RH_∞. If a_P, a_I and a_D are settled to make $Q(s)$ in (2.41) satisfy $\|Q(s)\|_\infty < 1$, then the controller $C(s)$ in (2.8) with (2.41) makes the closed-loop system in (2.1) stable for the plant $G(s)$ with uncertainty $\Delta(s)$.

2.4.7 Controller structure

In this subsection, we explain the structure of modified PID controller $C(s)$ in (2.8) with (2.36).

The structure of modified PID controller $C(s)$ in (2.8) with (2.36) is shown in Fig. 2.2 . Figure 2.2 shows that in order for the controller in (2.8) with (2.41) to specify (2.4) and to stabilize any plant $G(s)$, Fig. 2.2 is complex than the structure of the conventional PID controller $C(s)$ in (2.6). That is, the order of the conventional PID controller is 2, but the order of the modified PID controller is $3n + 6$, which is greater than that of the conventional PID controller.

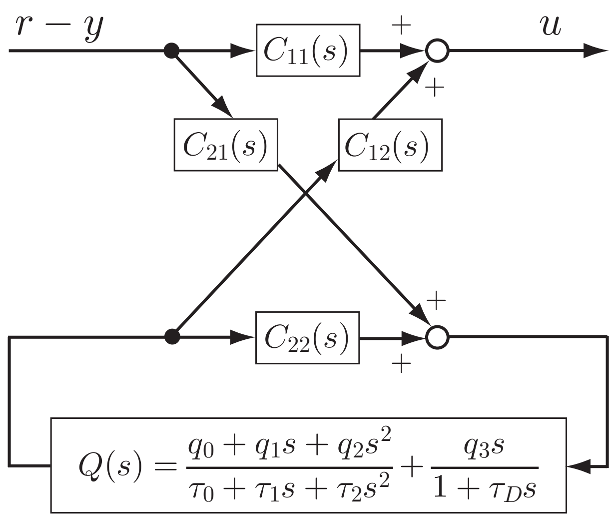

Figure 2.2: Structure of modified PID controller

2.5 Numerical example

In this section, we illustrate a numerical example to show the effectiveness of the proposed method.

Consider the problem to design a robust stabilizing modified PID controller $C(s)$ for the plant $G(s)$ in (2.2) with uncertainty $\Delta(s)$, where the nominal plant $G_m(s)$ and the upper bound $W_T(s)$ of the set of $\Delta(s)$ are given by

$$G_m(s) = \frac{11}{s^3 - s^2 - 3s - 5} \tag{2.42}$$

and

$$W_T(s) = \frac{(s+2)(s+10)(s+50)}{2 \times 10^4}, \tag{2.43}$$

respectively. Note that there exists no stabilizing conventional PID controllers for the nominal plant $G_m(s)$ in (2.42). Therefore, methods in [32, 33, 34, 35, 36, 37, 38, 39] cannot make the stabilizing PID controller.

Using the method in 2.3, the parametrization of all robust stabilizing controllers $C(s)$ in (2.8) is obtained. $Q(s)$ in (2.8) is designed as (2.36), where

$$\begin{cases} a_P &= 10 \\ a_I &= 100 \\ a_D &= 1 \end{cases}, \tag{2.44}$$

$$\begin{cases} \tau_0 &= 50.41 \\ \tau_1 &= 14.2 \\ \tau_2 &= 1 \end{cases} \tag{2.45}$$

and τ_D is selected by $\tau_D = 0.1$.

From the discussion in 2.4.6, designed $Q(s)$ in (2.8) must hold $\|Q(s)\|_\infty < 1$. Next, we confirm that designed $Q(s)$ satisfies $\|Q(s)\|_\infty < 1$. The gain plot of designed $Q(s)$ is shown in Fig. 2.3 . Figure 2.3 shows that designed $Q(s)$ satisfies $\|Q(s)\|_\infty < 1$.

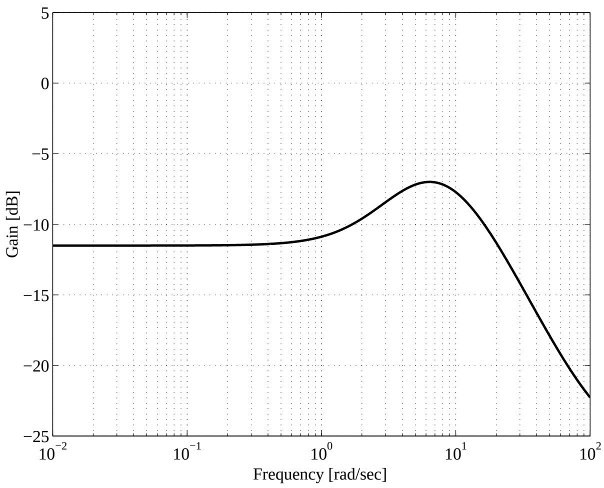

Figure 2.3: Gain plot of the free parameter $Q(s)$

When $\Delta(s)$ is given by

$$\Delta(s) = \frac{s+2}{500}, \tag{2.46}$$

the response of the output y of the closed-loop system in (2.1) for the step reference input r using the robust stabilizing modified PID controller $C(s)$ is shown in Fig. 2.4 . Figure 2.4 shows that the robust stabilizing modified PID controller $C(s)$ makes the closed-loop system stable.

On the other hand, using conventional PID controller in (2.6) with (2.44), response of the output y of the closed-loop system in (2.1) for the step reference input r is shown in Fig. 2.5 . Figure 2.5 shows that the conventional PID control system is unstable.

Next, when a_P, a_I and a_D in the robust stabilizing modified PID controller are varied, the comparison of step responses is examined. First, the comparison of step responses for various a_P as $a_P = 1$, $a_P = 50$ and $a_P = 100$ is shown in Fig. 2.6 . Here, the solid line, the dotted line and the broken line show the step response of the robust stabilizing modified control system using $a_P = 1$, $a_P = 50$ and $a_P = 100$, respectively. Figure 2.6 shows that as the value of a_P increased, the overshoot is larger and the rise time is shorten. Since this characteristic is equivalent to the conventional PID controller, the role of P-parameter a_P in the robust stabilizing modified PID controller is equivalent to that of the conventional PID controller. Secondly, the comparison of step responses for various a_I as $a_I = 0.2$, $a_I = 0.3$ and $a_I = 1$ is shown in Fig. 2.7 . Here the solid line, the dotted line and the broken line show the step response of the robust stabilizing modified PID control system using $a_I = 0.2$, $a_I = 0.3$ and $a_I = 1$, respectively. Figure 2.7 shows that as the value of a_I increased, the overshoot is smaller and the convergence speed is faster. Since this characteristic is equivalent to the conventional PID controller, the role of I-parameter a_I in the robust stabilizing modified PID controller is equivalent to that of the conventional PID controller. Thirdly, the comparison of step responses for various a_D as $a_D = 10$, $a_D = 50$ and

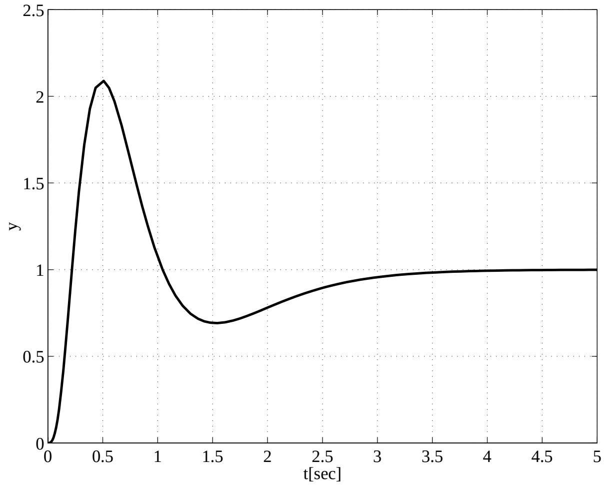

Figure 2.4: Step response of the closed-loop system using the robust stabilizing modified PID controller

$a_D = 100$ is shown in Fig. 2.8 . Here, the solid line, the dotted line and the broken line show the step response of the robust stabilizing modified PID control system using $a_D = 10$, $a_D = 50$ and $a_D = 100$, respectively. Figure 2.8 shows that as the value of a_D increased, the response is smoothly. Since this characteristic is equivalent to the conventional PID controller, the role of D-parameter a_D in the robust stabilizing modified PID controller is equivalent to that of the conventional PID controller. Since these characteristics are equivalent to the conventional PID controller, the role of P-parameter a_P, I-parameter a_I and D-parameter a_D in the robust stabilizing modified PID controller is equivalent to that of the conventional PID controller.

In this way, it is shown that we can easily design a robust stabilizing modified PID controller for the plant $G(s)$ in (2.2) with uncertainty $\Delta(s)$, which has same characteristic to conventional PID controller, and guarantee the stability of the closed-loop system.

2.6 Conclusion

In this paper, we proposed a design method of robust stabilizing modified PID controllers such that modified PID controller makes the closed-loop system stable for any plants with uncertainty. Proposed modified PID controllers lose the advantage of the conventional PID controllers such as

1. the control structure is simple.
2. the order of the controller is 1.

but have following advantages:

1. The modified PID controller makes the control system stable for any plant $G(s)$ with uncertainty.
2. The roles of P-parameter a_P, I-parameter a_I and D-parameter a_D in the robust stabilizing modified PID controller are equivalent to that of the conventional PID controller. That

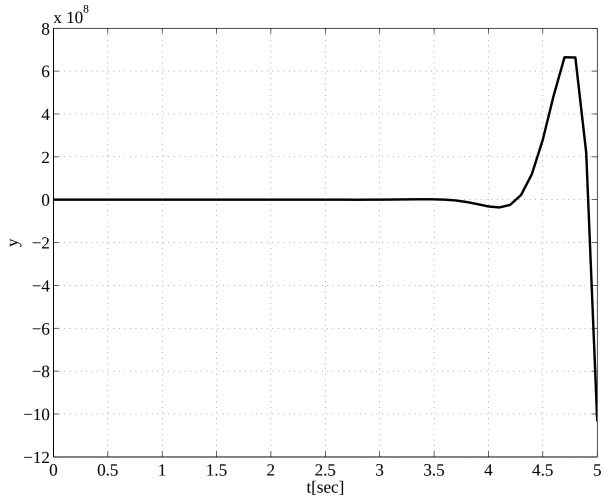

Figure 2.5: Step response of the closed-loop system using conventional PID controller

is, P-parameter a_P, I-parameter a_I and D-parameter a_D in the robust stabilizing modified PID controller can be tuned using previously proposed methods in [7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22].

A numerical example was shown to illustrate the effectiveness of the proposed method.

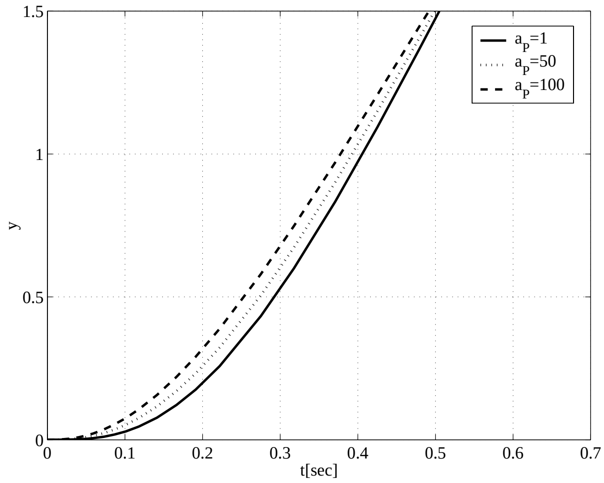

Figure 2.6: Step response using the robust stabilizing modified P controller with $a_P = 1, 50, 100$

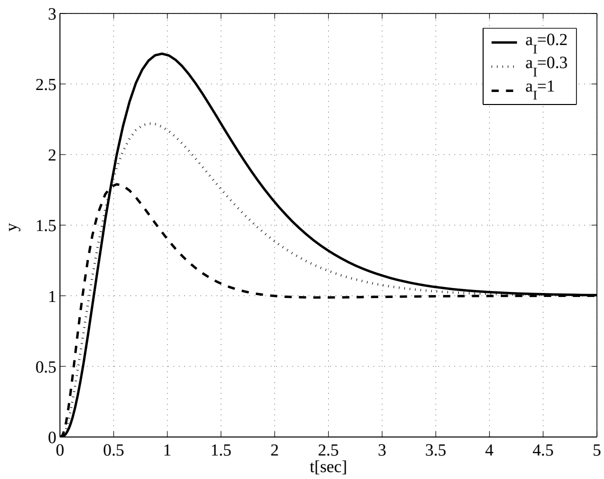

Figure 2.7: Step response using the robust stabilizing modified I controller with $a_I = 0.2, 0.3, 1$

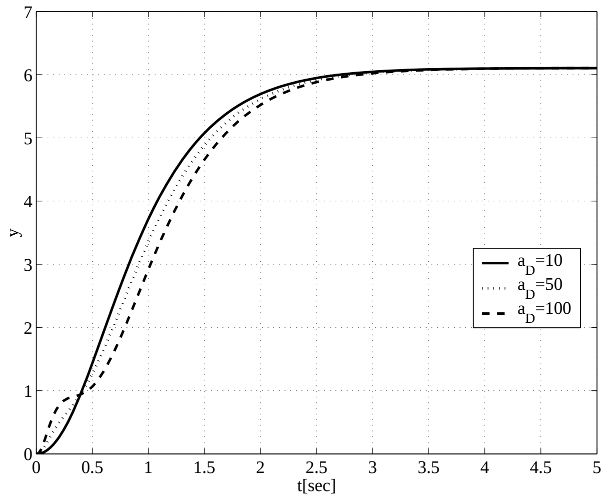

Figure 2.8: Step response using the robust stabilizing modified D controller with $a_D = 10, 50, 100$

Chapter 3

A Design Method for Robust Stabilizing Modified PID Controllers for Time-delay Plants with Uncertainty

3.1 Introduction

The proportional–integral–derivative (PID) controller is the most widely used controller structure in industrial applications [3, 4, 6]. Its structural simplicity and ability to solve many practical control problems have contributed to this wide acceptance.

Several papers on tuning methods for PID parameters have been presented [7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24]. However, the methods in [7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24] do not guarantee the stability of the closed-loop system. Design methods for PID controllers that guarantee the stability of the closed-loop system were proposed in [25, 26, 27, 28, 29, 30]. However, the plants to which these methods can be applied are restricted. To stabilize any plant using a PID controller, Yamada and Hagiwara gave a design method for modified PID controllers to make the closed-loop system stable for any unstable plant [50].

When we apply a PID controller in a practical application, we must consider the influence of uncertainty in the plant. In some cases, even if a PID controller stabilizes the nominal plant, the uncertainty makes the closed-loop system unstable. The stability problem with uncertainty is known as the robust stability problem [41]. Because almost all practical plants include uncertainty, the problem of designing robust stabilizing modified PID controllers for any plant with uncertainty is important. Several papers on design methods for robust stabilizing PID controllers have been presented [32, 33, 34, 35, 36, 37, 38, 39]. However, no design method for modified PID controllers has been published to guarantee the robust stability of PID control system for any plant with uncertainty. To overcome this problem, Yamada, Hagiwara and Shimizu gave a design method for robust stabilizing modified PID controllers to make the closed-loop system stable for any plant with uncertainty [**?**]. However, their method cannot be applied to time-delay plants with uncertainty. Almost all real plants include uncertainties and many plants have time delays. In addition, the PID controller is useful to design closed-loop systems for real plants [6]. The problem of designing robust stabilizing modified PID controllers to make the closed-loop system stable for any plant with uncertainty is therefore important.

We expand the result in [**?**] and propose a design method for robust stabilizing modified PID controllers such that the modified PID controller makes the closed-loop system stable for any time-delay plant with uncertainty. The basic idea of designing a robust stabilizing modified PID controller for any time-delay plant with uncertainty is very simple. For a certain class of time-delay plants with uncertainty, using state preview control, the problem to design a robust stabilizing controller is reduced to that for the plant without a time delay [40]. That is, if the modified PID control system is robustly stable for the time-delay plant with uncertainty, then the modified PID controller must satisfy the robust stability condition for system without time delay. This implies that if the modified PID control system is robustly stable, then the modified PID controller is included in the parameterization of all robust stabilizing controllers for the plant with uncertainty. The parameterization of all robust stabilizing controllers for the plant with uncertainty is obtained using H_∞ control theory based on the Riccati equation [41, 42] and the linear matrix inequality (LMI) [43, 44]. Robust stabilizing controllers for plants with uncertainty include a free-parameter, which is designed to achieve desirable control characteristics. When the free-parameter of the parameterization of all robust stabilizing controllers is appropriately chosen, then the controller works as a robust stabilizing modified PID controller.

3.2 Problem formulation

Consider the closed-loop system described by:

$$\left\{ \begin{array}{rcl} y & = & G(s)e^{-sT}u \\ u & = & C(s)\left(r-y\right) \end{array} \right. , \tag{3.1}$$

where $G(s)e^{-sT}$ is the single-input/single-output time-delay plant; $G(s) \in R(s)$ is assumed to be strictly proper and to be coprime. $T > 0$ is the time delay. $C(s)$ is the controller, $r \in R$ is the reference input, $u \in R$ is the control input and $y \in R$ is the output. The nominal plant of $G(s)e^{-sT}$ is denoted by $G_m(s)e^{-sT_m}$. Both $G(s)$ and $G_m(s)$ are assumed to have no zero or pole on the imaginary axis. In addition, it is assumed that the number of poles of $G(s)$ in the closed right half plane is equal to that of $G_m(s)$ in the closed right half plane. The relation between the plant $G(s)e^{-sT}$ and the nominal plant $G_m(s)e^{-sT_m}$ is written as:

$$G(s)e^{-sT} = G_m(s)\left(e^{-sT_m} + \Delta(s)\right), \tag{3.2}$$

where $\Delta(s) \in R(s)$ is the uncertainty. The set of $\Delta(s)$ is all functions satisfying:

$$|\Delta(j\omega)| < |W_T(j\omega)| \quad (\forall \omega \in R_+), \tag{3.3}$$

where $W_T(s)$ is a stable rational function. Under these assumptions, the robust stability condition for the plant $G(s)e^{-sT}$ with uncertainty $\Delta(s)$ satisfying (3.3) is given by:

$$\|T(s)W_T(s)\|_\infty < 1, \tag{3.4}$$

where $T(s)$ is given by:

$$T(s) = \frac{G_m(s)C(s)}{1+G_m(s)e^{-sT_m}C(s)}. \tag{3.5}$$

When the controller $C(s)$ has the form:

$$C(s) = a_P + \frac{a_I}{s} + a_D s, \tag{3.6}$$

then the controller $C(s)$ is called a PID controller [6], where $a_P \in R$ is the P-parameter, $a_I \in R$ is the I-parameter and $a_D \in R$ is the D-parameter. a_P, a_I and a_D are defined so that the closed-loop system in (3.1) has desirable control characteristics such as steady state and transient characteristics. For easy explanation, we call $C(s)$ in (3.6) the conventional PID controller.

The purpose of this paper is to propose a design method for robust stabilizing modified PID controllers $C(s)$ to make the closed-loop system in (3.1) stable for any time-delay plant $G(s)e^{-sT}$ in (3.2) with uncertainty $\Delta(s)$ satisfying (3.3).

3.3 The basic idea

In this section, we describe the basic idea for designing robust stabilizing modified PID controllers $C(s)$ to make the closed-loop system in (3.1) stable for any time-delay plant $G(s)e^{-sT}$ with uncertainty $\Delta(s)$.

To design such controllers that can be applied to any time-delay plant $G(s)e^{-sT}$ with uncertainty $\Delta(s)$, we must see that the robust stabilizing controllers conform to (3.4). The problem of obtaining the controller $C(s)$, which is not necessarily a PID controller, satisfying (3.4) is equivalent to the following H_∞ problem. To obtain the controller $C(s)$ satisfying (3.4), we consider the closed-loop system shown in Fig. 3.1 . $P(s)$ is selected such that the transfer function

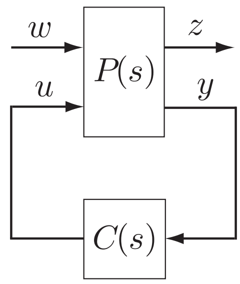

Figure 3.1: Block diagram of H_∞ control problem

from w to z in Fig. 3.1 is equal to $T(s)W_T(s)$. $P(s)$ is called the generalized plant [41]. In general, the state space description of the generalized plant $P(s)$ is defined by:

$$\left\{\begin{array}{rclll} \dot{x}(t) & = & Ax(t) & +B_1w(t) & +B_2u(t-T_m) \\ z(t) & = & C_1x(t) & & +D_{12}u(t) \\ y(t) & = & C_2x(t) & +D_{21}w(t) & \end{array}\right. , \tag{3.7}$$

where $A \in R^{n\times n}$, $B_1 \in R^n$, $B_2 \in R^n$, $C_1 \in R^{1\times n}$, $C_2 \in R^{1\times n}$, $D_{12} \in R$, $D_{21} \in R$. $P(s)$ is assumed to satisfy the following.

1. (C_2, A) is detectable. (A, B_2) is stabilizable.

2.

$$\text{rank } D_{12} = 1; \tag{3.8}$$

$$\text{rank } D_{21} = 1. \tag{3.9}$$

3.

$$\text{rank} \begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix} = n + 1 \ \ \forall \omega \in R; \tag{3.10}$$

$$\text{rank} \begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix} = n + 1 \ \ \forall \omega \in R. \tag{3.11}$$

4.

$$C_1 A^i B_2 = 0. \quad (i = 0, 1, 2, \dots) \tag{3.12}$$

According to [40], under these assumptions, there exists a controller $C(s)$ satisfying (3.4) if and only if there exists an H_∞ controller $C(s)$ for the generalized plant $\tilde{P}(s)$ defined by:

$$\left\{ \begin{array}{rcllll} \dot{q}(t) & = & Aq(t) & +B_1 w(t) & +\tilde{B}_2 u(t) \\ z(t) & = & C_1 q(t) & & +D_{12} u(t) \\ \tilde{y}(t) & = & C_2 q(t) & +D_{21} w(t) & \end{array} \right. , \tag{3.13}$$

where $\tilde{B}_2 = e^{-AT_m} B_2$. When $u(s) = C(s)\tilde{y}(s)$ is an H_∞ controller for (3.13),

$$u(t) \ \ = \ \ L^{-1}\left\{C(s)\tilde{y}(s)\right\} \tag{3.14}$$

is an H_∞ control input for (3.7), where

$$\tilde{y}(s) = L\left\{ y(t) + C_2 \int_{-T_m}^{0} e^{-A(\tau + T_m)} B_2 u(t+\tau) d\tau \right\}. \tag{3.15}$$

From (3.14), (3.15) and the references in [41, 42], all control laws satisfying (3.4) are defined by:

$$u(t) \ \ = \ \ L^{-1}\left\{C(s)\tilde{y}(s)\right\}, \tag{3.16}$$

where:

$$\tilde{y}(s) = L\left\{ y(t) + C_2 \int_{-T_m}^{0} e^{-A(\tau + T_m)} B_2 u(t+\tau) d\tau \right\}, \tag{3.17}$$

$$C(s) = C_{11}(s) + C_{12}(s) Q(s) \left(I - C_{22}(s) Q(s)\right)^{-1} C_{21}(s), \tag{3.18}$$

$$\begin{bmatrix} C_{11}(s) & C_{12}(s) \\ C_{21}(s) & C_{22}(s) \end{bmatrix} = \left[\begin{array}{c|cc} A_c & B_{c1} & B_{c2} \\ \hline C_{c1} & D_{c11} & D_{c12} \\ C_{c2} & D_{c21} & D_{c22} \end{array} \right], \tag{3.19}$$

$$\begin{aligned} A_c \ \ = \ \ & A + B_1 B_1^T X - \tilde{B}_2 \left(D_{12}^{\dagger} C_1 + E_{12}^{-1} \tilde{B}_2^T X \right) \\ & - (I - XY)^{-1} \left(B_1 D_{21}^{\dagger} + Y C_2^T E_{21}^{-1} \right) \left(C_2 + D_{21} B_1^T X \right), \end{aligned}$$

$$\begin{aligned} B_{c1} &= (I - XY)^{-1}\left(B_1 D_{21}^{\dagger} + Y C_2^T E_{21}^{-1}\right), \\ B_{c2} &= (I - XY)^{-1}\left(\tilde{B}_2 + Y C_1^T D_{12}\right) E_{12}^{-1/2}, \end{aligned}$$

$$C_{c1} = -D_{12}^{\dagger} C_1 - E_{12}^{-1} \tilde{B}_2^T X, \ \ C_{c2} = -E_{21}^{-1/2}\left(C_2 + D_{21} B_1^T X\right),$$

$$D_{c11} = 0, \ \ D_{c12} = E_{12}^{-1/2}, \ \ D_{c21} = E_{21}^{-1/2}, \ \ D_{c22} = 0,$$

$$E_{12} = D_{12}^T D_{12}, \ \ E_{21} = D_{21} D_{21}^T,$$

$X \geq 0$ and $Y \geq 0$ are solutions of:

$$\begin{aligned} & X\left(A - \tilde{B}_2 D_{12}^{\dagger} C_1\right) + \left(A - \tilde{B}_2 D_{12}^{\dagger} C_1\right)^T \\ & + X\left(B_1 B_1^T - \tilde{B}_2\left(D_{12}^T D_{12}\right)^{-1} \tilde{B}_2^T\right) X + \left(D_{12}^{\perp} C_1^T\right)^T D_{12}^{\perp} C_1^T = 0 \end{aligned} \tag{3.20}$$

and:

$$\begin{aligned} & Y\left(A - B_1 D_{21}^{\dagger} C_2\right)^T + \left(A - B_1 D_{21}^{\dagger} C_2\right) Y \\ & + Y\left(C_1^T C_1 - C_2^T\left(D_{21} D_{21}^T\right)^{-1} C_2\right) Y + B_1 D_{21}^{\perp}\left(B_1 D_{21}^{\perp}\right)^T = 0 \end{aligned} \tag{3.21}$$

such that:

$$\rho\left(XY\right) < 1, \tag{3.22}$$

and neither

$$A - \tilde{B}_2 D_{12}^{\dagger} C_1 + \left(B_1 B_1^T - \tilde{B}_2\left(D_{12}^T D_{12}\right)^{-1} \tilde{B}_2^T\right) X$$

nor

$$A - B_1 D_{21}^{\dagger} C_2 + Y\left(C_1^T C_1 - C_2\left(D_{21} D_{21}^T\right)^{-1} C_2\right)$$

have an eigenvalue in the closed right half plane and the free-parameter $Q(s) \in RH_\infty$ is any function satisfying $\|Q(s)\|_\infty < 1$.

The controller $C(s)$ in (3.18) includes the free-parameter $Q(s)$. Using this free-parameter, we propose a design method for robust stabilizing modified PID controllers $C(s)$ to make the closed-loop system in (3.1) stable. To design the controllers $C(s)$, the free-parameter $Q(s)$ in (3.18) is chosen so that $C(s)$ in (3.18) has the same characteristics as the conventional PID controller $C(s)$ in (3.6). Therefore, we next describe the role of the conventional PID controller $C(s)$ in (3.6) to clarify the condition that the modified PID controller $C(s)$ must satisfy. From (3.6), using $C(s)$, the P-parameter a_P, the I-parameter a_I and the D-parameter a_D are defined as:

$$a_P = \lim_{s\to\infty}\left\{-s^2 \frac{d}{ds}\left(\frac{1}{s} C(s)\right)\right\}, \tag{3.23}$$

$$a_I = \lim_{s \to 0} \{sC(s)\} \tag{3.24}$$

and:

$$a_D = \lim_{s \to \infty} \frac{d}{ds} \{C(s)\}, \tag{3.25}$$

respectively. Therefore, if the controller $C(s)$ in (3.18) conforms to (3.23), (3.24) and (3.25), the role of controller $C(s)$ is equivalent to that of the conventional PID controller $C(s)$ in (3.6). That is, we can design robust stabilizing modified PID controllers such that the role of the controller $C(s)$ in (3.18) is equivalent to that of the conventional PID controller $C(s)$ in (3.6).

In 3.4, we use the ideas discussed above to describe a method for designing the modified PID controller $C(s)$ in (3.18) that works as a modified PID controller. In the following, we call $C(s)$:

1. the modified P controller if $C(s)$ in (3.18) satisfies (3.23),
2. the modified I controller if $C(s)$ in (3.18) satisfies (3.24),
3. the modified D controller if $C(s)$ in (3.18) satisfies (3.25),
4. the modified PI controller if $C(s)$ in (3.18) satisfies (3.23) and (3.24),
5. the modified PD controller if $C(s)$ in (3.18) satisfies (3.23) and (3.25), and
6. the modified PID controller if $C(s)$ in (3.18) satisfies (3.23), (3.24) and (3.25).

3.4 Robust stabilizing modified PID controller

In this section, we propose a design method for robust stabilizing modified PID controllers.

3.4.1 Robust stabilizing modified P controller

In this subsection, we present a design method for a robust stabilizing modified P controller $C(s)$ that conforms to (3.23), makes the closed-loop system in (3.1) stable and can be applied to the time-delay plant $G(s)e^{-sT}$ with uncertainty $\Delta(s)$.

The controller is defined by (3.18), where:

$$Q(s) = \frac{a_P}{\lim_{s \to \infty} \left(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)\right)}. \tag{3.26}$$

Because $Q(s)$ in (3.26) is constant, $Q(s)$ is included in RH_∞. If a_P is chosen to make $Q(s)$ in (3.26) satisfy $\|Q(s)\|_\infty < 1$, then the controller $C(s)$ in (3.18) with (3.26) makes the closed-loop system in (3.1) stable for the time-delay plant $G(s)e^{-sT}$ with uncertainty $\Delta(s)$.

3.4.2 Robust stabilizing modified I controller

In this subsection, we present a design method for a robust stabilizing modified I controller $C(s)$ that conforms to (3.24), makes the closed-loop system in (3.1) stable and can be applied to the time-delay plant $G(s)e^{-sT}$ with uncertainty $\Delta(s)$.

This controller is defined by (3.18), where:

$$Q(s) = \frac{q_0 + q_1 s}{\tau_0 + \tau_1 s}, \tag{3.27}$$

$$q_0 = \frac{\tau_0}{C_{22}(0)}, \tag{3.28}$$

$$q_1 = \frac{\tau_1}{C_{22}(0)} - \frac{\tau_0}{a_I C_{22}^2(0)} \left\{ \frac{d}{ds} \{C_{22}(s)\} \bigg|_{s=0} a_I + C_{12}(0)C_{21}(0) \right\} \tag{3.29}$$

and $\tau_i \in R > 0$ $(i = 0, 1)$. From $\tau_i > 0$ $(i = 0, 1)$, $Q(s)$ in (3.27) is included in RH_∞. If a_I is chosen to make $Q(s)$ in (3.27) satisfy $\|Q(s)\|_\infty < 1$, then the controller $C(s)$ in (3.18) with (3.27) makes the closed-loop system in (3.1) stable for the time-delay plant $G(s)e^{-sT}$ with uncertainty $\Delta(s)$.

3.4.3 Robust stabilizing modified D controller

In this subsection, we present a design method for a robust stabilizing modified D controller $C(s)$ that conforms to (3.25), makes the closed-loop system in (3.1) stable and can be applied to the time-delay plant $G(s)e^{-sT}$ with uncertainty $\Delta(s)$.

This controller is defined by (3.18), where:

$$Q(s) = \frac{a_D}{\lim_{s\to\infty} (C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)) + a_D \lim_{s\to\infty} (sC_{22}(s))} s. \tag{3.30}$$

Because $Q(s)$ in (3.30) is improper, $Q(s)$ in (3.30) is not included in RH_∞. For $Q(s)$ to be included in RH_∞, (3.30) is modified as:

$$Q(s) = \frac{a_D}{\lim_{s\to\infty} (C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)) + a_D \lim_{s\to\infty} (sC_{22}(s))} \frac{s}{1 + \tau_D s}, \tag{3.31}$$

where $\tau_D \in R > 0$. From $\tau_D > 0$ in (3.31), $Q(s)$ in (3.31) is included in RH_∞. If a_D is chosen to make $Q(s)$ in (3.31) satisfy $\|Q(s)\|_\infty < 1$, then the controller $C(s)$ in (3.18) with (3.31) makes the closed-loop system in (3.1) stable for the time-delay plant $G(s)e^{-sT}$ with uncertainty $\Delta(s)$.

3.4.4 Robust stabilizing modified PI controller

In this subsection, we present a design method for a robust stabilizing modified PI controller $C(s)$ that conforms to (3.23) and (3.24), makes the closed-loop system in (3.1) stable and can be applied to the time-delay plant $G(s)e^{-sT}$ with uncertainty $\Delta(s)$.

This controller is defined by (3.18), where:

$$Q(s) = \frac{q_0 + q_1 s + q_2 s^2}{\tau_0 + \tau_1 s + \tau_2 s^2}, \tag{3.32}$$

$$q_0 = \frac{\tau_0}{C_{22}(0)}, \tag{3.33}$$

$$q_1 = \frac{\tau_1}{C_{22}(0)} - \frac{\tau_0}{a_I C_{22}^2(0)} \left\{ \frac{d}{ds} \{C_{22}(s)\} \bigg|_{s=0} a_I + C_{12}(0)C_{21}(0) \right\}, \tag{3.34}$$

$$q_2 = \frac{\tau_2 a_P}{\lim_{s\to\infty} (C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s))} \tag{3.35}$$

and $\tau_i \in R > 0$ $(i = 0, 1, 2)$. From $\tau_i > 0$ $(i = 0, 1, 2)$, $Q(s)$ in (3.32) is included in RH_∞. If a_P and a_I are chosen to make $Q(s)$ in (3.32) satisfy $\|Q(s)\|_\infty < 1$, then the controller $C(s)$ in (3.18) with (3.32) makes the closed-loop system in (3.1) stable for the time-delay plant $G(s)e^{-sT}$ with uncertainty $\Delta(s)$.

3.4.5 Robust stabilizing modified PD controller

In this subsection, we present a design method for a robust stabilizing modified PD controller $C(s)$ that conforms to (3.23) and (3.25), makes the closed-loop system in (3.1) stable and can be applied to the time-delay plant $G(s)e^{-sT}$ with uncertainty $\Delta(s)$.

This controller is defined by (3.18), where:

$$Q(s) = q_0 + q_1 s, \tag{3.36}$$

$$\begin{aligned} q_0 &= \frac{\left\{1 - \lim_{s\to\infty}(sC_{22}(s))\, q_1\right\}^2 a_P}{\lim_{s\to\infty}(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s))\left\{1 - \lim_{s\to\infty}(sC_{22}(s))\, q_1 - \lim_{s\to\infty}\left(s^2 \frac{d}{ds}\{C_{22}(s)\}\, q_1\right)\right\}} \\ &+ \frac{1}{\lim_{s\to\infty}(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s))\left\{1 - \lim_{s\to\infty}(sC_{22}(s))\, q_1 - \lim_{s\to\infty}\left(s^2 \frac{d}{ds}\{C_{22}(s)\}\, q_1\right)\right\}} \\ &\cdot \left[\lim_{s\to\infty}\left(s^2 \frac{d}{ds}\{C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)\}\right)\left\{q_1 - \lim_{s\to\infty}(sC_{22}(s))\, q_1^2\right\}\right. \\ &\left. + \lim_{s\to\infty}(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s))\left\{\lim_{s\to\infty}\left(s^3 \frac{d}{ds}\{C_{22}(s)\}\right) + \lim_{s\to\infty}\left(s^2 C_{22}(s)\right)\right\} q_1^2\right] \end{aligned} \tag{3.37}$$

and:

$$q_1 = \frac{a_D}{\lim_{s\to\infty}(C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)) + a_D \lim_{s\to\infty}\{sC_{22}(s)\}}. \tag{3.38}$$

Because $Q(s)$ in (3.36) is improper, $Q(s)$ in (3.36) is not included in RH_∞. For $Q(s)$ to be included in RH_∞, (3.36) is modified as:

$$Q(s) = q_0 + \frac{q_1 s}{1 + \tau_D s}, \tag{3.39}$$

where $\tau_D \in R > 0$. From $\tau_D > 0$ in (3.39), $Q(s)$ in (3.39) is included in RH_∞. If a_P and a_D are chosen to make $Q(s)$ in (3.39) satisfy $\|Q(s)\|_\infty < 1$, then the controller $C(s)$ in (3.18) with (3.39) makes the closed-loop system in (3.1) stable for the time-delay plant $G(s)e^{-sT}$ with uncertainty $\Delta(s)$.

3.4.6 Robust stabilizing modified PID controller

In this subsection, we present a design method for a robust stabilizing modified PID controller $C(s)$ that conforms to (3.23), (3.24) and (3.25), makes the closed-loop system in (3.1) stable and can be applied to the time-delay plant $G(s)e^{-sT}$ with uncertainty $\Delta(s)$.

This controller is defined by (3.18), where:

$$Q(s) = \frac{q_0 + q_1 s + q_2 s^2}{\tau_0 + \tau_1 s + \tau_2 s^2} + q_3 s, \tag{3.40}$$

$$q_0 = \frac{\tau_0}{C_{22}(0)}, \tag{3.41}$$

$$q_1 = \frac{\tau_1}{C_{22}(0)} - q_3\tau_0 - \frac{\tau_0}{a_I C_{22}^2(0)} \left(\frac{d}{ds} \{C_{22}(s)\}\Big|_{s=0} a_I + C_{12}(0)C_{21}(0) \right), \tag{3.42}$$

$$\begin{aligned} q_2 &= \frac{\left\{1 - \lim_{s\to\infty} (sC_{22}(s))\, q_3\right\}^2 \tau_2 a_P}{\lim_{s\to\infty} (C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)) \left\{1 - \lim_{s\to\infty} (sC_{22}(s))\, q_3 - \lim_{s\to\infty} \left(s^2 \frac{d}{ds} \{C_{22}(s)\}\right) q_3\right\}} \\ &+ \frac{\tau_2}{\lim_{s\to\infty} (C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)) \left\{1 - \lim_{s\to\infty} (sC_{22}(s))\, q_3 - \lim_{s\to\infty} \left(s^2 \frac{d}{ds} \{C_{22}(s)\}\right) q_3\right\}} \\ &\cdot \left[\lim_{s\to\infty} \left(s^2 \frac{d}{ds} \{C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)\}\right) \left\{q_3 - \lim_{s\to\infty} (sC_{22}(s))\, q_3^2\right\} \right. \\ &\left. + \lim_{s\to\infty} (C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)) \left\{\lim_{s\to\infty} \left(s^3 \frac{d}{ds} \{C_{22}(s)\}\right) + \lim_{s\to\infty} \left(s^2 C_{22}(s)\right)\right\} q_3^2\right], \end{aligned} \tag{3.43}$$

$$q_3 = \frac{a_D}{\lim_{s\to\infty} (C_{12}(s)C_{21}(s) - C_{11}(s)C_{22}(s)) + a_D \lim_{s\to\infty} (sC_{22}(s))} \tag{3.44}$$

and $\tau_i \in R > 0$ $(i = 0, 1, 2)$. Because $Q(s)$ in (3.40) is improper, $Q(s)$ in (3.40) is not included in RH_∞. For $Q(s)$ to be included in RH_∞, (3.40) is modified as:

$$Q(s) = \frac{q_0 + q_1 s + q_2 s^2}{\tau_0 + \tau_1 s + \tau_2 s^2} + \frac{q_3 s}{1 + \tau_D s}, \tag{3.45}$$

where $\tau_D \in R > 0$. From $\tau_D > 0$ and $\tau_i > 0$ $(i = 0, 1, 2)$ in (3.45), $Q(s)$ in (3.45) is included in RH_∞. If a_P, a_I and a_D are chosen to make $Q(s)$ in (3.45) satisfy $\|Q(s)\|_\infty < 1$, then the controller $C(s)$ in (3.18) with (3.45) makes the closed-loop system in (3.1) stable for the time-delay plant $G(s)e^{-sT}$ with uncertainty $\Delta(s)$.

3.5 Numerical example

In this section, we illustrate a numerical example to show the effectiveness of the proposed method.

Consider the problem to design a robust stabilizing modified PID controller $C(s)$ for the plant $G(s)$ in (3.2) with uncertainty $\Delta(s)$, where the nominal plant $G_m(s)$ and the upper bound $W_T(s)$ of the set of $\Delta(s)$ are given by

$$G_m(s) = \frac{1}{s^2 + 5s + 6} e^{-0.3s} \tag{3.46}$$

and

$$W_T(s) = \frac{s + 1}{14s + 28}, \tag{3.47}$$

respectively. Note that there exists no stabilizing conventional PID controllers for the nominal plant $G_m(s)$ in (3.46). Therefore, methods in [32, 33, 34, 35, 36, 37, 38, 39] cannot make the stabilizing PID controller.

Using the method in 3.3, the parametrization of all robust stabilizing controllers $C(s)$ in (3.18) is obtained. $Q(s)$ in (3.18) is designed as (3.40), where

$$\left\{ \begin{array}{rcl} a_P &=& 10 \\ a_I &=& 100 \\ a_D &=& 0.1 \end{array} \right. , \tag{3.48}$$

$$\left\{ \begin{array}{rcl} \tau_0 &=& 50.41 \\ \tau_1 &=& 14.2 \\ \tau_2 &=& 1 \end{array} \right. \tag{3.49}$$

and τ_D is selected by $\tau_D = 0.1$.

From the discussion in 3.4.6, designed $Q(s)$ in (3.18) must hold $\|Q(s)\|_\infty < 1$. Next, we confirm that designed $Q(s)$ satisfies $\|Q(s)\|_\infty < 1$. The gain plot of designed $Q(s)$ is shown in Fig. 3.2 . Figure 3.2 shows that designed $Q(s)$ satisfies $\|Q(s)\|_\infty < 1$.

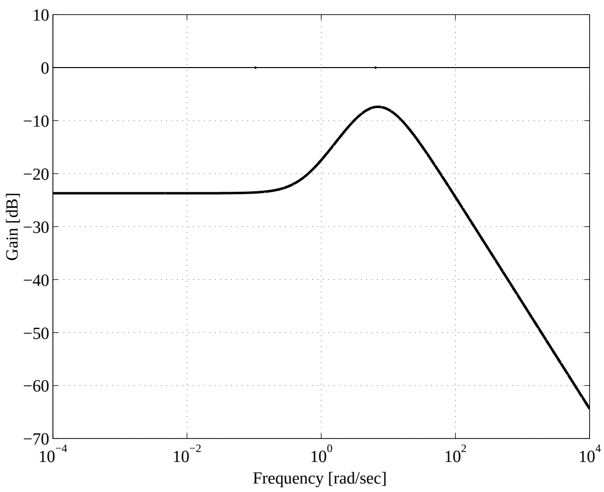

Figure 3.2: Gain plot of the free parameter $Q(s)$

When $\Delta(s)$ is given by

$$\Delta(s) = \frac{s+1}{15s+60}, \tag{3.50}$$

the response of the output y of the closed-loop system in (3.1) for the step reference input r using the robust stabilizing modified PID controller $C(s)$ is shown in Fig. 3.3 . Figure 3.3 shows that the robust stabilizing modified PID controller $C(s)$ makes the closed-loop system stable.

On the other hand, using conventional PID controller in (3.6) with (3.48), response of the output y of the closed-loop system in (3.1) for the step reference input r is shown in Fig. 3.4 . Figure 3.4 shows that the conventional PID control system is unstable.

Next, when a_P, a_I and a_D in the robust stabilizing modified PID controller are varied, the comparison of step responses is examined. First, the comparison of step responses for various a_P as $a_P = 30$, $a_P = 40$ and $a_P = 50$ is shown in Fig. 3.5 . Here, the solid line, the dotted line and

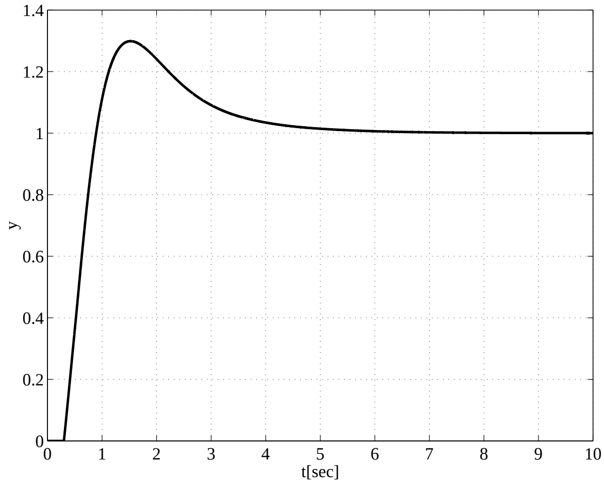

Figure 3.3: Step response of the closed-loop system using the robust stabilizing modified PID controller

the broken line show the step response of the robust stabilizing modified control system using $a_P = 30$, $a_P = 40$ and $a_P = 50$, respectively. Figure 3.5 shows that as the value of a_P increased, the overshoot became larger and the rise time became shorter. Since this characteristic is equivalent to the conventional PID controller, the role of P-parameter a_P in the robust stabilizing modified PID controller is equivalent to that of the conventional PID controller. Secondly, the comparison of step responses for various a_I as $a_I = 0.0001$, $a_I = 0.0003$ and $a_I = 0.0005$ is shown in Fig. 3.6 . Here the solid line, the dotted line and the broken line show the step response of the robust stabilizing modified PID control system using $a_I = 0.0001$, $a_I = 0.0003$ and $a_I = 0.0005$, respectively. Figure 3.6 shows that as the value of a_I increased, the overshoot became smaller and the convergence became more rapid. Since this characteristic is equivalent to the conventional PID controller, the role of I-parameter a_I in the robust stabilizing modified PID controller is equivalent to that of the conventional PID controller. Thirdly, the comparison of step responses for various a_D as $a_D = 10$, $a_D = 30$ and $a_D = 50$ is shown in Fig. 3.7 . Here, the solid line, the dotted line and the broken line show the step response of the robust stabilizing modified PID control system using $a_D = 10$, $a_D = 30$ and $a_D = 50$, respectively. Figure 3.7 shows that as the value of a_D increased, the response was smoothed. Since this characteristic is equivalent to the conventional PID controller, the role of D-parameter a_D in the robust stabilizing modified PID controller is equivalent to that of the conventional PID controller. Since these characteristics are equivalent to the conventional PID controller, the role of P-parameter a_P, I-parameter a_I and D-parameter a_D in the robust stabilizing modified PID controller is equivalent to that of the conventional PID controller.

In this way, it is shown that we can easily design a robust stabilizing modified PID controller for the time-delay plant $G(s)$ in (3.2) with uncertainty $\Delta(s)$, which has same characteristic to conventional PID controller, and guarantee the stability of the closed-loop system.

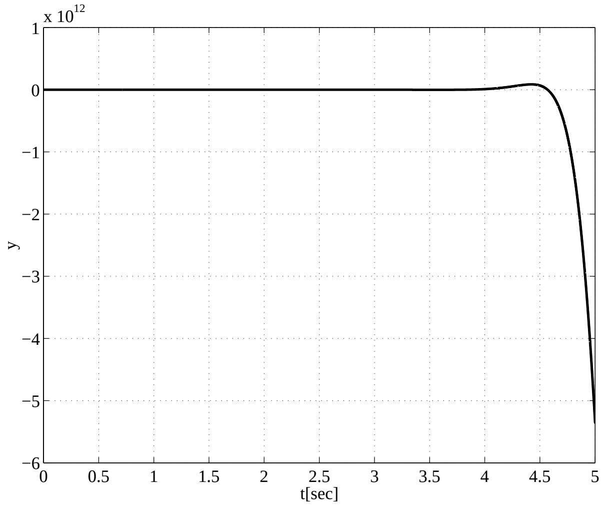

Figure 3.4: Step response of the closed-loop system using conventional PID controller

3.6 Conclusion

We have proposed a design method for a robust stabilizing modified PID controller that makes the closed-loop system stable for any time-delay plant with uncertainty. The proposed modified PID controllers do not have the advantages of the conventional PID controllers in previous papers [6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30] such as:

1. the control structure is simple.

2. the order of the controller is 1.

However, they have the following advantages:

1. The modified PID controller makes the closed-loop system stable for any time-delay plant $G(s)e^{-sT}$ with uncertainty. This implies that plants that cannot be stabilized by the methods in [6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30] can be stabilized using the proposed method.

2. The roles of the P-parameter a_P, I-parameter a_I and D-parameter a_D in the robust stabilizing modified PID controller are equivalent to those of the conventional PID controller. That is, the parameters can be tuned using methods proposed in [7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24].

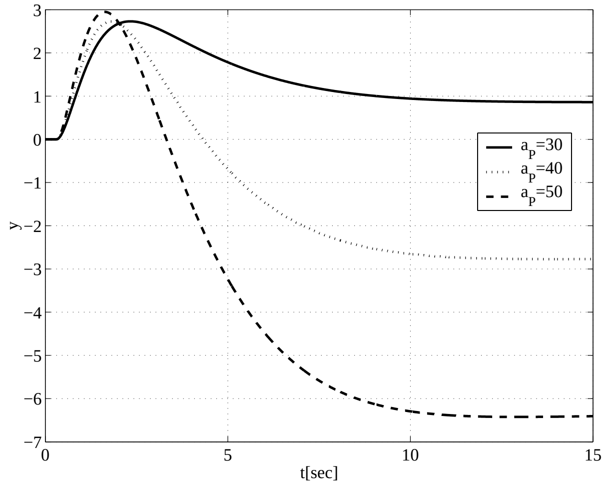

Figure 3.5: Step response using the robust stabilizing modified P controller with $a_P = 30, 40, 50$

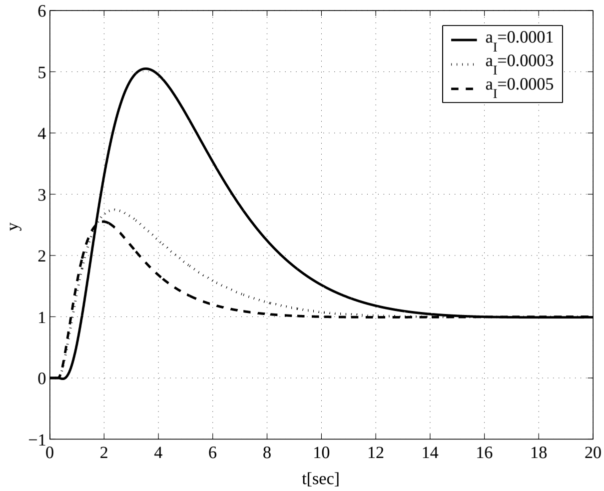

Figure 3.6: Step response using the robust stabilizing modified I controller with $a_I = 0.0001, 0.0003, 0.0005$

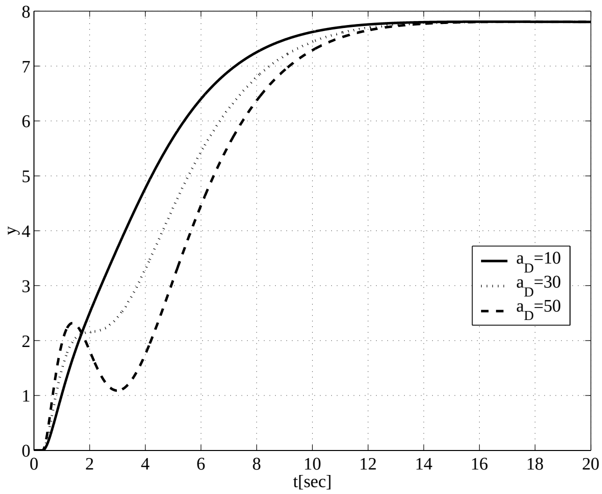

Figure 3.7: Step response using the robust stabilizing modified D controller with $a_D = 10, 30, 50$

Chapter 4

A Design Method for Modified PID Control Systems to Attenuate Unknown Disturbances

4.1 Introduction

In this paper, we consider a design method for modified Proportional-Integral-Derivative (PID) control systems to attenuate unknown disturbances. PID controller structure is the most widely used one in industrial applications [3, 4]. Its structual simplicity and sufficient ability of solving many practical control problems have contributed to this wide acceptance [6].

Several papers on tuning methods for PID parameters have been published [7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22], but these methods do not guarantee the stability of a control system. However using the method in [7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22], the PID control system is not necessarily stable. If the admissible sets of PID parameters that would guarantee the stability of a control system can be determined, we can easily design stabilizing PID controllers to meet control specifications.

The problem to obtain admissible sets of PID parameters to guarantee the stability of the control system is known as a parameterization problem [6, 29, 30]. If there exists a stabilizing PID controller, the parameterization of all stabilizing PID controller is considered in [6, 29, 30]. However, these methods in [6, 29, 30] remains a difficulty. The admissible sets of P-parameter, I-parameter and D-parameter in [6, 29, 30] are related each other. That is, if the P-parameter changes, then the admissible sets of I-parameter and D-parameter also change. From a practical point of view, it is desirable that the admissible sets of P-parameter, I-parameter and D-parameter are independent from each other. Yamada and Moki initially approached this problem and proposed a design method for modified PI controllers for any minimum phase system such that the admissible sets of P-parameter and I-parameter are independent from each other [45]. Yamada expanded the results in [45] and proposed a design method for modified PID controllers for minimum phase plants [46]. For stable plants, Yamada et al. considered a design method for modified PID controllers [48, 49]. For unstable plants, Yamada and Hagiwara gave a design method for modified PID controllers [50]. In addition, Yamada, Hagiwara and Shimizu proposed a design method for robust stabilizing modified PID controllers such that the modified PID controller makes the control system stable for any plant with uncertainty [51]. In this way, the modified PID controller that can stabilize the control system has been established. However, the modified PID controller remains in [45, 46, 48, 49, 50, 51] two difficulties. One is that the modified PID control system in [45, 46, 48, 49, 50, 51] cannot specify the input-output characteristic and the disturbance attenuation characteristic separately. The other is that the modified PID control system in [45, 46, 48, 49, 50, 51] cannot attenuate unknown disturbances. It is desirable to specify the input-output characteristic and the disturbance attenuation characteristic separately and to attenuate unknown disturbances. However, no paper examines a design method for modified PID control systems to specify the input-output characteristic and to attenuate unknown disturbances.

In this paper, we propose a design method for modified PID control systems to specify the input-output characteristic and the disturbance attenuation characteristic separately and to attenuate unknown disturbances effectively.

4.2 Modified PID controller and problem formulation

Consider the control system written by

$$\begin{cases} y(s) &= G(s)u(s) + d(s) \\ u(s) &= C(s)(r(s) - y(s)) \end{cases}, \tag{4.1}$$

where $G(s) \in R(s)$ is the plant, $C(s) \in R(s)$ is the controller, $r(s) \in R(s)$ is the reference input, $u(s) \in R(s)$ is the control input, $y(s) \in R(s)$ is the output and $d(s) \in R(s)$ is the disturbance. It is assumed that $d(s)$ is unknown.

When the controller $C(s)$ has the form written by

$$C(s) = a_P + \frac{a_I}{s} + a_D s, \tag{4.2}$$

then the controller $C(s)$ is called the PID controller [3, 4, 6, 29, 30], where $a_P \in R$ is the P-parameter, $a_I \in R$ is the I-parameter and $a_D \in R$ is the D-parameter. a_P, a_I and a_D are settled so that the control system in (4.1) has desirable control characteristics such as the steady state characteristic and the transient characteristic. For easy explanation, we call the controller $C(s)$ in (4.2) the conventional PID controller.

Using the conventional PID controller $C(s)$ in (4.2), the transfer function from the reference input $r(s)$ to the output $y(s)$ in (4.1) is written by

$$y(s) = \frac{G(s)\left(a_P + \frac{a_I}{s} + a_D s\right)}{1 + G(s)\left(a_P + \frac{a_I}{s} + a_D s\right)} r(s). \tag{4.3}$$

It is obvious that when P-parameter a_P, I-parameter a_I and D-parameter a_D are settled at random, the stability of the control system in (4.1) is not guaranteed. In addition, there exists the plant $G(s)$ that cannot be stabilized using the conventional PID controller $C(s)$ in (4.2). In order to overcome these problems, Yamada et al. proposed the modified PID controller such that the modified PID controller can stabilize the plant which cannot be stabilized using conventional PID controller $C(s)$ in (4.2), and admissible sets of P-parameter a_P, I-parameter a_I and D-parameter a_D are independent from each other [50]. According to [50], the modified PID controller, which can stabilize any plant, is written by

$$C(s) = \frac{X(s) + D(s)Q(s)}{Y(s) - N(s)Q(s)}, \tag{4.4}$$

where $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \frac{N(s)}{D(s)}, \tag{4.5}$$

$X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are functions satisfying

$$X(s)N(s) + Y(s)D(s) = 1, \tag{4.6}$$

$$Q(s) = \frac{q_0 + q_1 s + q_2 s^2}{\tau_0 + \tau_1 s + \tau_2 s^2} + \frac{q_3 s}{1 + \tau_D s}, \tag{4.7}$$

$$q_0 = \frac{Y(0)}{N(0)}\tau_0, \tag{4.8}$$

$$\begin{aligned} q_1 = {} & \frac{\tau_0}{a_I N(0)} \left[a_I \left\{ \lim_{s \to 0} \left(\frac{d}{ds}\{Y(s)\} \right) \right.\right. \\ & \left.\left. - \lim_{s \to 0} \left(\frac{d}{ds}\{N(s)\} \right) \frac{q_0}{\tau_0} - N(0) \left(-\frac{q_0 \tau_1}{\tau_0^2} + q_3 \right) \right\} - X(0) - D(0)\frac{q_0}{\tau_0} \right], \end{aligned} \tag{4.9}$$

$$q_2 = \frac{\left\{ \lim_{s\to\infty} Y(s) - \lim_{s\to\infty} (sN(s))\, q_3 \right\}^2 a_P \tau_2}{\dfrac{\lim_{s\to\infty} D(s) \left[\lim_{s\to\infty} Y(s) - \left\{ \lim_{s\to\infty} (sN(s)) + \lim_{s\to\infty} \left(s^2 \frac{d}{ds}\{N(s)\} \right) \right\} q_3 \right] + \tau_2 \left[\lim_{s\to\infty} \left(s^2 \frac{d}{ds}\{D(s)\} \right) \left\{ \lim_{s\to\infty} Y(s) - \lim_{s\to\infty} (sN(s))\, q_3 \right\} q_3 \right.}{\left. - \lim_{s\to\infty} D(s) \left\{ \lim_{s\to\infty} \left(s^2 \frac{d}{ds}\{Y(s)\} \right) - \lim_{s\to\infty} \left(s^3 \frac{d}{ds}\{N(s)\} + s^2 N(s) \right) q_3 \right\} q_3 \right]}}, \tag{4.10}$$

$$q_3 = \frac{\lim_{s\to\infty} Y(s) a_D}{\lim_{s\to\infty} D(s) + a_D \lim_{s\to\infty} (sN(s))}, \tag{4.11}$$

$\tau_i \in R > 0 (i = 0, 1, 2)$ and $\tau_D \in R > 0$.

However, using the modified PID controller in (4.4), the input-output characteristic and the disturbance attenuation characteristic cannot be specified separately. In addition, the modified PID controller $C(s)$ in (4.4) of the control system in (4.1) can attenuate the step disturbance, but cannot attenuate unknown disturbances.

The problem considered in this paper is to propose a design method for modified PID control systems to specify the input-output characteristic and the disturbance attenuation characteristic separately and to attenuate unknown disturbances effectively.

4.3 Modified PID control systems to attenuate unknown disturbances

In this section, we propose a modified PID control system to specify the input-output characteristic and the disturbance attenuation characteristic separately and to attenuate unknown disturbances effectively.

In order to specify the input-output characteristic and the disturbance attenuation characteristic separately and to attenuate unknown disturbances effectively, we propose the modified PID control system shown in Fig. 4.1 . Here, $C(s) \in R(s)$ is the modified PID controller in (4.4),

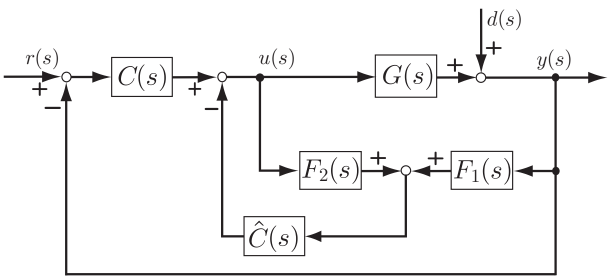

Figure 4.1: Modified PID control system to specify the input-output characteristic and the disturbance attenuation characteristic separately and to attenuate unknown disturbances

$\hat{C}(s) \in R(s)$ is the controller to attenuate unknown disturbance, $F_1(s) \in R(s)$ and $F_2(s) \in R(s)$ are written by

$$F_1(s) = D(s) + \tilde{Q}(s)D(s), \tag{4.12}$$

$$F_2(s) = -N(s) - \tilde{Q}(s)N(s) \tag{4.13}$$

and $\tilde{Q}(s) \in RH_\infty$ is any function.

Next, we clarify control characteristics of the modified PID control system in Fig. 4.1 . First, the input-output characteristic of the control system in Fig. 4.1 is shown. Transfer functions from the reference input $r(s)$ to the output $y(s)$ and from the reference input $r(s)$ to the error $e(s) = r(s) - y(s)$ are written by

$$y(s) = \frac{G(s)C(s)}{1 + G(s)C(s)} r(s) \tag{4.14}$$

and

$$e(s) = r(s) - y(s) = \frac{1}{1 + G(s)C(s)} r(s), \tag{4.15}$$

respectively. From (4.14) and (4.15), the role of the modified PID controller $C(s)$ in (4.4) of the control system in Fig. 4.1 is to specify the input-output characteristic.

Next, the disturbance attenuation characteristic of the control system in Fig. 4.1 is shown. The transfer function from the disturbance $d(s)$ to the output $y(s)$ is written by

$$y(s) = \frac{1 + \hat{C}(s)F_2(s)}{1 + G(s)C(s) + \hat{C}(s)\left(F_1(s)G(s) + F_2(s)\right)} d(s)$$

$$= \frac{1 - N(s)\hat{C}(s)\left(1 + \tilde{Q}(s)\right)}{1 + G(s)C(s)} d(s). \tag{4.16}$$

From (4.16), the role of the controller $\hat{C}(s)$ of the control system in Fig. 4.1 is to specify the disturbance attenuation characteristic. Therefore, even if the disturbance is unknown, the disturbance $d(s)$ is attenuated effectively if

$$1 - N(j\omega)\hat{C}(j\omega)\left(1 + \tilde{Q}(j\omega)\right) = 0, \tag{4.17}$$

where ω is the frequency component of the disturbance $d(s)$. This implies that when the controller $\hat{C}(s)$ is designed to satisfy (5.19), the control system in Fig. 4.1 can attenuate unknown disturbances $d(s)$ effectively.

From (4.14) and (4.16), the role of the controller $C(s)$ is different from that of $\hat{C}(s)$. The role of the modified PID controller $C(s)$ is to specify the input-output characteristic. The role of the controller $\hat{C}(s)$ is to specify the disturbance attenuation characteristic.

Finally, the condition that the control system in Fig. 4.1 is stable is clarified. From (4.14) and (4.16), it is obvious that the control system in Fig. 4.1 is stable if and only if following expressions hold.

1. The modified PID controller $C(s)$ makes the control system in (4.1) stable.
2. $\hat{C}(s) \in RH_\infty$.

4.4 Controller design

In this section, we describe a design method for the controller $\hat{C}(s)$ to specify the disturbance attenuation characteristic of the control system in Fig. 4.1 is shown.

A design method is summarized as follows: $\tilde{Q}(s) \in RH_\infty$ in (4.12), (4.13) is settled to satisfy $1 + \tilde{Q}(s) \in \mathcal{U}$. Using the method in [61], there exists $N_r(s) \in RH_\infty$ satisfying

$$N(s)N_r(s) = \frac{1}{(1+\tau s)^\alpha} N_i(s), \tag{4.18}$$

where $N_i(s) \in RH_\infty$ is an inner function of $N(s)$ satisfying

$$N(s) = N_i(s)N_o(s) \tag{4.19}$$

and $N_i(0) = 1$, $N_o(s) \in RH_\infty$ is an outer function, $\tau \in R$, α is an arbitrary positive integer to make $N_r(s)$ proper. Using $N_r(s)$, if the controller $\hat{C}(s)$ is selected as

$$\hat{C}(s) = \frac{N_r(s)}{1 + \tilde{Q}(s)}, \tag{4.20}$$

even if the disturbance $d(s)$ is unknown, then the disturbance $d(s)$ in the frequency range ω satisfying

$$1 - N(j\omega)\hat{C}(j\omega)\left(1 + \tilde{Q}(j\omega)\right) = 1 - \frac{1}{(1+\tau j\omega)^\alpha} N_i(j\omega) \simeq 0 \tag{4.21}$$

is attenuated effectively.

4.5 Numerical example

In this section, a numerical example is shown to illustrate the effectiveness of the proposed method.

Consider the problem to design a modified PID control system in Fig. 4.1 to attenuate unknown disturbances effectively for the plant $G(s)$ written by

$$G(s) = \frac{s+1}{s^4 - 4s^3 - s^2 + 16s - 12}. \tag{4.22}$$

First, we design a modified PID controller $C(s)$ in Fig. 4.1 . a_P, a_I and a_D are settled by

$$\begin{cases} a_P = 100 \\ a_I = 1000 \\ a_D = 100 \end{cases}. \tag{4.23}$$

$N(s)$, $D(s)$, $X(s)$, $Y(s)$, $\tau_i(i = 0, 1, 2)$ and τ_D are set as

$$N(s) = \frac{s+1}{s^4 + 50s^3 + 875s^2 + 6250s + 1.5 \times 10^4}, \tag{4.24}$$

$$D(s) = \frac{s^4 - 4s^3 - s^2 + 16s - 12}{s^4 + 50s^3 + 875s^2 + 6250s + 1.5 \times 10^4}, \tag{4.25}$$

$$X(s) = \frac{10^6 \left(5.605s^3 - 7.949s^2 + 67.71s + 180.5\right)}{s^4 + 50s^3 + 875s^2 + 6250s + 1.5 \times 10^4}, \tag{4.26}$$

$$Y(s) = \frac{s^4 + 104s^3 + 4667s^2 + 1.188 \times 10^5 s - 3.707 \times 10^6}{s^4 + 50s^3 + 875s^2 + 6250s + 1.5 \times 10^4}, \tag{4.27}$$

$$\begin{cases} \tau_0 = 100 \\ \tau_1 = 77.5 \\ \tau_2 = 1 \end{cases} \tag{4.28}$$

and $\tau_D = 1$. Using above mentioned parameters, the modified PID controller $C(s)$ is designed by (4.4) with (4.7).

Next, we design a controller $\hat{C}(s)$ in Fig. 4.1 . A controller $\hat{C}(s)$ is set as (4.20), where

$$N_r(s) = \frac{s^4 + 50s^3 + 875s^2 + 6250s + 1.5 \times 10^4}{10^{-12}s^4 + 3 \cdot 10^{-8}s^3 + 0.0003s^2 + s + 1}, \tag{4.29}$$

$$N_i(s) = 1, \tag{4.30}$$

$$\tilde{Q}(s) = 0, \tag{4.31}$$

$\tau = 0.0001$ and $\alpha = 3$.

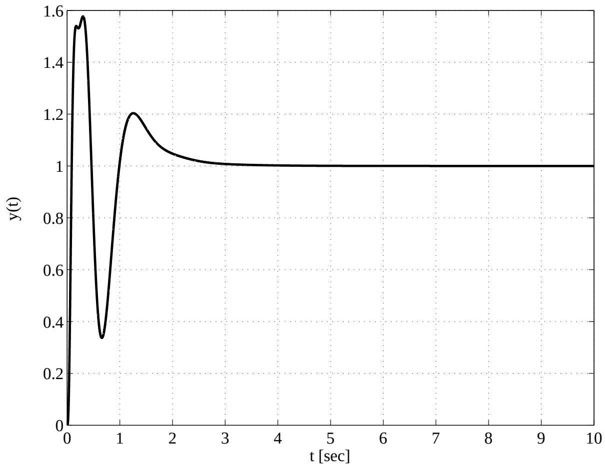

Figure 4.2: Response of the control system in Fig. 4.1

Using designed modified PID controller $C(s)$ and controller $\hat{C}(s)$, the response of the control system in Fig. 4.1 is shown in Fig. 4.2 . Figure 4.2 shows that the control system in Fig. 4.1 is stable.

When the disturbance $d(t)$ is given by

$$d(t) = \sin 10t, \tag{4.32}$$

the response of the output $y(t)$ in Fig. 4.1 is shown in Fig. 4.3 . Here, the solid line shows the response of the output $y(t)$ and the dotted line shows that of the disturbance $d(t)$. Figure 4.3 shows that the disturbance $d(t)$ is attenuated effectively.

On the other hand, using the modified PID controller $C(s)$, the response of the output $y(t)$ in (4.1) is shown in Fig. 4.4 . Here, the solid line shows the response of the output $y(t)$ and the dotted line shows that of the disturbance $d(t)$. Figure 4.4 shows that the disturbance $d(t)$ cannot be attenuated.

In this way, it is shown that using proposed method, even if the disturbance $d(t)$ is unknown, we can easily design control system to specify the input-output characteristic and the disturbance attenustion characteristic separately and to attenuate unknown disturbance $d(t)$ effectively.

4.6 Conclusion

In this paper, we proposed a design method for modified PID control systems to specify the input-output characteristic and the disturbance attenuation characteristic separately and to attenuate unknown disturbances. The results in this paper are summarized as follows:

1. Modified PID control system was proposed as Fig. 4.1 .

2. Control characteristics of the modified PID control system in Fig. 4.1 were clarified. We find that proposed modified PID control system in Fig. 4.1 can specify the input-output characteristic and the disturbance attenuation characteristic separately. The role of the modified PID controller $C(s)$ is to specify the input-output characteristic. The role of the controller $\hat{C}(s)$ is to specify the disturbance attenuation characteristic.

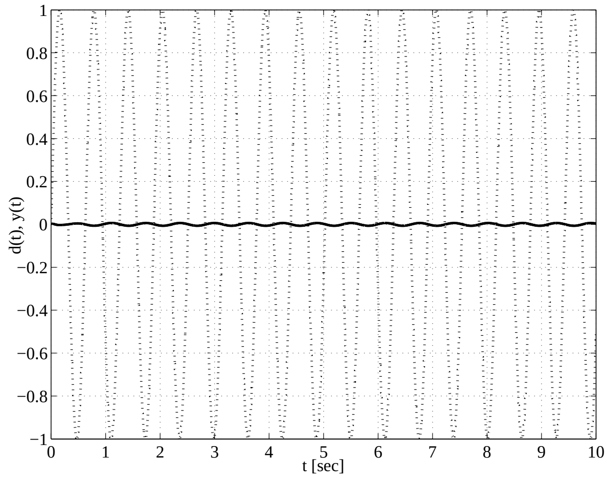

Figure 4.3: Response of the output $y(t)$ for the disturbance $d(t)$

3. We present a design method of the controller $\hat{C}(s)$ to attenuate unknown disturbances effectively.

4. A numerical example is shown to illustrate the effectiveness of the proposed method.

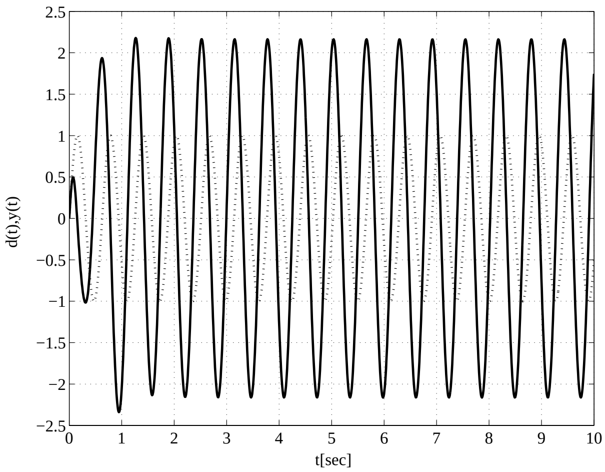

Figure 4.4: Response of the output $y(t)$ of (4.1) for the disturbance $d(t)$

Chapter 5

An Application of the Modified PID Control System for Heat Flow Experiment

5.1 Introduction

PID (Proportional-Integral-Derivative) controller structure is the most widely used one in industrial applications [3, 4, 6]. Its structural simplicity and sufficient ability of solving many practical control problems have contributed to this wide acceptance.

Several papers on tuning methods for PID parameters have been considered [7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22], but these methods do not guarantee the stability of a control system. The reference in [25, 26, 27, 28] proposed design methods of PID controllers to guarantee the stability of a control system. However, using methods in [25, 26, 27, 28], it is difficult to tune PID parameters, since methods in [25, 26, 27, 28] do not obtain admissible sets of PID parameters. If admissible sets of PID parameters that would guarantee the stability of a control system can be determined, we can easily design stabilizing PID controllers and tune PID parameters to meet desirable control specifications.

Recently, the problem to obtain admissible sets of PID parameters to guarantee the stability of a control system that is known as a parameterization problem is obtained [6, 29, 30]. However, these methods in [6, 29, 30] remain a difficulty. Admissible sets of P-parameter, I-parameter and D-parameter in [6, 29, 30] are related each other. That is, if P-parameter is changed, then admissible sets of I-parameter and D-parameter also change. From a practical point of view, it is desirable that admissible sets of P-parameter, I-parameter and D-parameter are independent from each other. Yamada and Moki initially tackled this problem and proposed a design method for modified PI controllers for any minimum phase system such that modified PI controllers can stabilize any plant and admissible sets of P-parameter and I-parameter are independent from each other [45]. Yamada expanded the result in [45] and proposed a design method for modified PID controllers for minimum phase plants [46]. For stable plants, Yamada et al. considered a design method for modified PID controllers [48, 49]. For unstable plants, Yamada and Hagiwara gave a design method for modified PID controllers [50]. In addition, Yamada, Hagiwara and Shimizu proposed a design method for robust stabilizing modified PID controllers such that the modified PID controller makes the control system stable for any plant with uncertainty [51]. In this way, the modified PID controller such that the modified PID controller can stabilize any plant and admissible sets of P-parameter, I-parameter and D-parameter are independent from each other has been established. However, the modified PID controller in [45, 46, 48, 49, 50, 51] remains two difficulties. One is that the modified PID control system in [45, 46, 48, 49, 50, 51] cannot specify the input-output characteristic and the disturbance attenuation characteristic separately. From the practical point of view, it is desirable that the input-output characteristic and the disturbance attenuation characteristic can be specified separately. The other is that the modified PID control system in [45, 46, 48, 49, 50, 51] cannot attenuate unknown disturbances. In many cases, the disturbance in the plant is unknown. It is comparatively easy to attenuate known disturbance, but it is difficult to attenuate unknown disturbances. From this viewpoint, a design method for modified PID control system to attenuate unknown disturbances was proposed [56]. In addition, the control system in [56] has desirable control characteristic such that the input-output characteristic and the disturbance attenuation characteristic can be specified separately. Therefore, the method in [56] may be an effective control design method for practical plants. However, an application of the modified PID control system to attenuate unknown disturbances for plants with any disturbance in [56] is not examined. Therefore, the effectiveness of the method in [56] for controlling practical systems is not confirmed.

In this paper, we apply the modified PID control system to attenuate unknown disturbances for plants with any disturbance in [56] for temperature control for heat flow experiment and show the effectiveness of the modified PID control systems to attenuate unknown disturbances for plants with any disturbance in [56]. This paper is organized as follows: In Section 5.2, we introduce heat flow experiment and show that unknown disturbances for heat flow experiment exist. In addition, the problem considered in this paper is described. Section 5.3 introduce the method in [56] to attenuate unknown disturbance for heat flow experiment. In Section 5.4, we show the experimental result for temperature control for heat flow experiment using the modified PID control system described in Section 5.3. Section 5.5 gives concluding remarks.

5.2 Heat Flow Experiment and Problem Description

The heat flow apparatus is shown in Fig. 5.1 . The heat flow apparatus consists of a duct

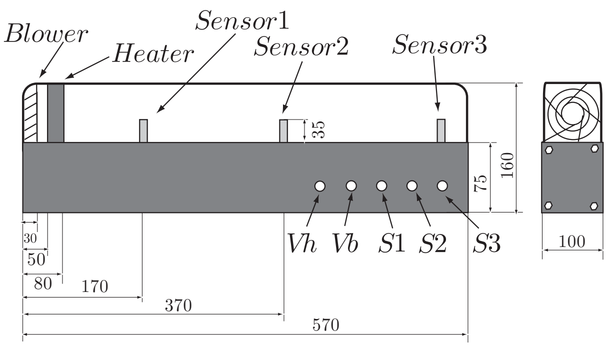

Figure 5.1: Heat flow apparatus.

equipped with a heater and a blower at one end and three temperature sensors located along the duct as shown in Fig. 5.1 . V_h and V_b denote the voltage to heater and that to blower, respectively. S_1, S_2 and S_3 are terminals for measurement of temperature at Sensor 1, Sensor 2 and Sensor 3. We denote T_i deg the measurement of temperature at Sensor $i(i = 1, 2, 3)$. V_b is constant as $V_b = 5$ V, and V_h is considered as a control input and an available voltage of V_h is

$0 \leq V_h \leq 5$ V.

When we settle $V_h = 5$ V, the response of T_1, which is the temperature at Sensor 1, is shown in Fig. 5.2 . Specially magnified detail drawing showing between 53 deg and 57 deg of Fig. 5.2

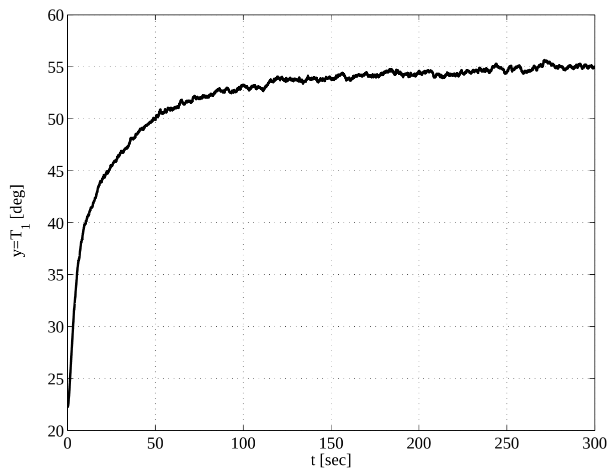

Figure 5.2: Response of the temperature T_1, when $V_h = 5$ V.

is shown in Fig. 5.3 . Fig. 5.2 and Fig. 5.3 show that unknown disturbances of which the

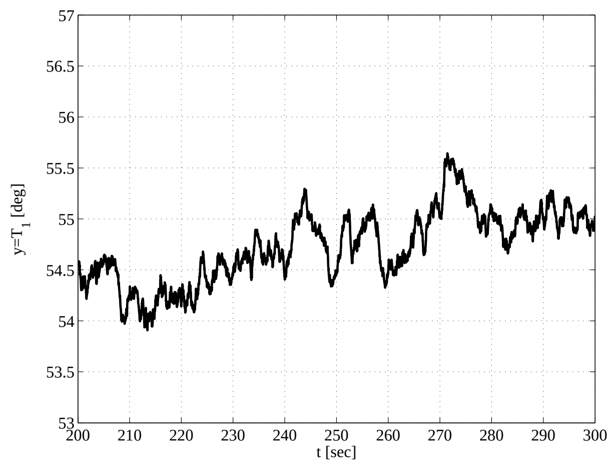

Figure 5.3: Specially magnified detail drawing showing between 53 deg and 57 deg of Fig. 5.2 .

maximum gain will be 0.7 deg exist. From the practical point of view, this unknown disturbance needs to be attenuated.

The problem considered in this paper is to design a modified PID control system to attenuate unknown disturbances for plants with any disturbance in [56] as described in Section 5.3 to make T_1, which is the temperature at Sensor 1, 40 deg steadily and to attenuate unknown disturbances effectively.

5.3 Modified PID Control System to Attenuate Unknown Disturbances

In this section, we briefly introduce the modified PID control system to specify the input-output characteristic and the disturbance attenuation characteristic separately and to attenuate unknown disturbances effectively for plants with any disturbance proposed in [56].

According to [56], the modified PID control system to specify the input-output characteristic and the disturbance attenuation characteristic separately and to attenuate unknown disturbances effectively for plants with any disturbance is shown in Fig. 5.4 . Here, $r(s) \in R$ is

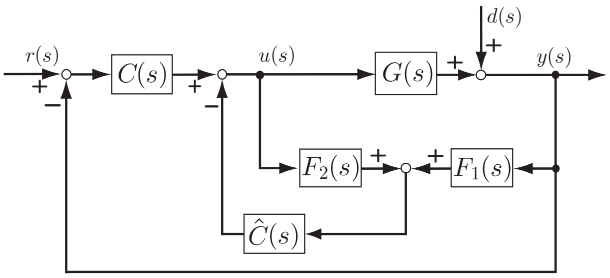

Figure 5.4: Modified PID control system to attenuate disturbances.

the reference input, $u(s) \in R$ is the control input, $y(s) \in R$ is the output, $d(s) \in R$ is the unknown disturbance, $G(s) \in R(s)$ is the plant, $C(s) \in R(s)$ is the modified PID controller described later, $\hat{C}(s) \in R(s)$ is the controller to attenuate unknown disturbance, $F_1(s) \in R(s)$ and $F_2(s) \in R(s)$, which have a role to estimate the unknown disturbance, are given by

$$F_1(s) = D(s) + \tilde{Q}(s)D(s) \tag{5.1}$$

and

$$F_2(s) = -N(s) - \tilde{Q}(s)N(s), \tag{5.2}$$

respectively, where $N(s) \in RH_\infty$ and $D(s) \in RH_\infty$ are coprime factors of $G(s)$ on RH_∞ satisfying

$$G(s) = \frac{N(s)}{D(s)} \tag{5.3}$$

and $\tilde{Q}(s) \in RH_\infty$ is any function.

Next, we described the modified PID controller $C(s)$ and the controller $\hat{C}(s)$ in Fig. 5.4 . First, the modified PID controller $C(s)$ is shown. According to [50], the modified PID controller $C(s)$ is written by

$$C(s) = \frac{X(s) + D(s)Q(s)}{Y(s) - N(s)Q(s)}, \tag{5.4}$$

where $X(s) \in RH_\infty$ and $Y(s) \in RH_\infty$ are functions satisfying

$$X(s)N(s) + Y(s)D(s) = 1, \tag{5.5}$$

$$Q(s) = \frac{q_0 + q_1 s + q_2 s^2}{\tau_0 + \tau_1 s + \tau_2 s^2} + \frac{q_3 s}{1 + \tau_D s}, \tag{5.6}$$

$$q_0 = \frac{Y(0)}{N(0)}\tau_0, \tag{5.7}$$

$$\begin{aligned} q_1 = & \frac{\tau_0}{a_I N(0)}\left[a_I\left\{\frac{d}{ds}\{Y(s)\}|_{s=0}\right.\right. \\ & -\frac{d}{ds}\{N(s)\}|_{s=0}\frac{q_0}{\tau_0} \\ & \left.+N(0)\left(\frac{q_0\tau_1}{\tau_0^2} - q_3\right)\right\} \\ & \left.-X(0) - D(0)\frac{q_0}{\tau_0}\right], \end{aligned} \tag{5.8}$$

$$q_2 = \frac{q_{2n}}{q_{2d}}, \tag{5.9}$$

$$q_3 = \frac{\lim_{s\to\infty} Y(s) a_D}{\lim_{s\to\infty} D(s) + a_D \lim_{s\to\infty}(sN(s))}, \tag{5.10}$$

$\tau_i \in R > 0 (i = 0, 1, 2)$ and $\tau_D \in R > 0$. Here, q_{2n} and q_{2d} are

$$\begin{aligned} q_{2n} = & \left\{\lim_{s\to\infty} Y(s) - \lim_{s\to\infty}(sN(s))q_3\right\}^2 a_P \tau_2 \\ & +\tau_2\left[\lim_{s\to\infty}\left(s^2\frac{d}{ds}\{D(s)\}\right)\right. \\ & \left\{\lim_{s\to\infty} Y(s) - \lim_{s\to\infty}(sN(s))q_3\right\} q_3 \\ & -\lim_{s\to\infty} D(s)\left\{\lim_{s\to\infty}\left(s^2\frac{d}{ds}\{Y(s)\}\right)\right. \\ & \left.\left.-\lim_{s\to\infty}\left(s^3\frac{d}{ds}\{N(s)\} + s^2 N(s)\right) q_3\right\} q_3\right] \end{aligned} \tag{5.11}$$

and

$$\begin{aligned} q_{2d} = & \lim_{s\to\infty} D(s)\left[\lim_{s\to\infty} Y(s) - \left\{\lim_{s\to\infty}(sN(s))\right.\right. \\ & \left.\left.+\lim_{s\to\infty}\left(s^2\frac{d}{ds}\{N(s)\}\right)\right\} q_3\right], \end{aligned} \tag{5.12}$$

respectively. Here, $a_P \in R$ is the P (proportional) parameter, $a_I \in R$ is the I (integral) parameter and $a_D \in R$ is the D (derivative) parameter. Since $Q(s)$ in (5.6) satisfies $Q(s) \in RH_\infty$ independent from the P parameter a_P, the I parameter a_I and the D parameter a_D. In addition, according to [62], if $Q(s) \in RH_\infty$, the controller $C(s)$ in (5.4) stabilizes $G(s)$. This implies that the modified PID controller $C(s)$ in (5.4) with (5.6) makes the control system stable for any plant independent from P-parameter a_P, I-parameter a_I and D-parameter a_D.

Next, a design method for the controller $\hat{C}(s)$ to attenuate disturbance in Fig. 5.4 is shown. According to [56], the controller $\hat{C}(s)$ to attenuate disturbance is given by

$$\hat{C}(s) = \frac{N_r(s)}{1+\tilde{Q}(s)}, \tag{5.13}$$

where $N_r(s) \in RH_\infty$ is a function satisfying

$$N(s)N_r(s) = \frac{1}{(1+\tau s)^\alpha} N_i(s), \tag{5.14}$$

where $N_i(s) \in RH_\infty$ is an inner function of $N(s)$ satisfying

$$N(s) = N_i(s)N_o(s) \tag{5.15}$$

and $N_i(0) = 1$, $N_o(s) \in RH_\infty$ is an outer function, $\tau \in R$ and α is an arbitrary positive integer to make $N_r(s)$ proper. $\tilde{Q}(s) \in RH_\infty$ is settled to satisfy $1 + \tilde{Q}(s) \in \mathcal{U}$.

Next, we summarize control characteristics of the control system in Fig. 5.4 . First, the input-output characteristic of the control system in Fig. 5.4 is described. Transfer functions from the reference input $r(s)$ to the output $y(s)$ and from the reference input $r(s)$ to the error $e(s) = r(s) - y(s)$ are written by

$$y(s) = \frac{G(s)C(s)}{1+G(s)C(s)} r(s) \tag{5.16}$$

and

$$\begin{aligned} e(s) &= r(s) - y(s) \\ &= \frac{1}{1+G(s)C(s)} r(s), \end{aligned} \tag{5.17}$$

respectively. From (5.16) and (5.17), the role of the modified PID controller $C(s)$ in (5.4) of the control system in Fig. 5.4 is to specify the input-output characteristic. In addition, we find that from (5.16) and (5.17), the input-output characteristic is independent from $\hat{C}(s)$, $F_1(s)$ and $F_2(s)$.

Next, the disturbance attenuation characteristic of the control system in Fig. 5.4 is described. The transfer function from the disturbance $d(s)$ to the output $y(s)$ is written by

$$\begin{aligned} & y(s) \\ &= \frac{1+\hat{C}(s)F_2(s)}{1+G(s)C(s)+\hat{C}(s)\left(F_1(s)G(s)+F_2(s)\right)} d(s) \\ &= \frac{1-N(s)\hat{C}(s)\left(1+\tilde{Q}(s)\right)}{1+G(s)C(s)} d(s). \end{aligned} \tag{5.18}$$

From (5.18), the role of the controller $\hat{C}(s)$ of the control system Fig. 5.4 is to specify the disturbance attenuation characteristic. Therefore, even if the disturbance is unknown, the disturbance $d(s)$ is attenuated effectively if

$$\begin{aligned} & 1 - N(j\omega)\hat{C}(j\omega)\left(1+\tilde{Q}(j\omega)\right) \\ &= 1 - \frac{1}{(1+\tau j\omega)^\alpha} N_i(j\omega) \\ &\simeq 0, \end{aligned} \tag{5.19}$$

where ω is the frequency component of the disturbance $d(s)$. This implies that when the controller $\hat{C}(s)$ is designed to satisfy (5.19), the control system in Fig. 5.4 can attenuate unknown disturbances $d(s)$ effectively.

From (5.16) and (5.18), the role of the controller $C(s)$ is different from that of $\hat{C}(s)$. The role of the modified PID controller $C(s)$ is to specify the input-output characteristic. The role of the controller $\hat{C}(s)$ is to specify the disturbance attenuation characteristic.

Finally, the condition that the control system in Fig. 5.4 is stable is described. From (5.16) and (5.18), it is obvious that the control system in Fig. 5.4 is stable if and only if following expressions hold.

1. The modified PID controller $C(s)$ makes the control system stable.
2. $\hat{C}(s) \in RH_\infty$.

In the next section, we apply the modified PID control system in Fig. 5.4 for heat flow experiment in Fig. 5.1 and illustrated the effectiveness of the modified PID control system in Fig. 5.4 .

5.4 Experimental Result

In this section, we show the experimental result for temperature control for heat flow experiment in Fig. 5.1 using the modified PID control system in Fig. 5.4 .

From Fig. 5.2 , we find that the transfer function from V_h to T_1, which is temperature at Sensor 1, is written by

$$T_1 = \frac{6.58}{1+22.13s} V_h. \tag{5.20}$$

T_1 and V_h are considered as the output $y(s)$ and the control input $u(s)$ in the modified PID control system in Fig. 5.4 . Then, from (5.20), $G(s)$ in Fig. 5.4 is written by

$$G(s) = \frac{6.58}{1+22.13s} \in RH_\infty. \tag{5.21}$$

The reference input $r(s)$ in Fig. 5.4 is settled as $r(t) = 40$ deg.

For the plant $G(s)$ in (5.21), we design the control system in Fig. 5.4 . First, we design the controller $C(s)$ in Fig. 5.4 . $N(s)$, $D(s)$, $X(s)$ and $Y(s)$ in (5.4) satisfying (5.3) and (5.5) are set as

$$N(s) = \frac{0.2973}{s+10} \in RH_\infty, \tag{5.22}$$

$$D(s) = \frac{s+0.0452}{s+10} \in RH_\infty, \tag{5.23}$$

$$X(s) = \frac{333.2895}{s+10} \in RH_\infty \tag{5.24}$$

and

$$Y(s) = \frac{s+19.9548}{s+10} \in RH_\infty, \tag{5.25}$$

respectively. a_P, a_I and a_D are settled by

$$\begin{cases} a_P &= 1 \\ a_I &= 0.1 \\ a_D &= 0.01 \end{cases}. \tag{5.26}$$

Using above mentioned parameters, the modified PID controller $C(s)$ is designed by (5.4) with (5.6), where $q_i(i = 0, \ldots, 3)$ are determined by (5.7), (5.8), (5.9) and (5.10),

$$\begin{cases} \tau_0 &= 15 \\ \tau_1 &= 25 \\ \tau_2 &= 1 \end{cases} \tag{5.27}$$

and $\tau_D = 0.01$.

Next, we design $F_1(s)$ and $F_2(s)$ in Fig. 5.4 . $F_1(s)$ and $F_2(s)$ are set as (5.1) and (5.2), respectively, where $N(s)$, $D(s)$ and $\tilde{Q}(s)$ are set as (5.22), (5.23) and

$$\tilde{Q}(s) = \frac{-s^2 - s}{s^2 + s + 0.01}. \tag{5.28}$$

The controller $\hat{C}(s)$ in Fig. 5.4 to attenuate disturbance is designed by (5.13) with (5.14), where

$$N_r(s) = \frac{1}{G(s)} \frac{1}{(1 + \tau s)^\alpha} \in RH_\infty \tag{5.29}$$

and

$$\begin{cases} \tau &= 0.05 \\ \alpha &= 3 \end{cases}. \tag{5.30}$$

Using above-mentioned parameters, we have the modified PID control system in Fig. 5.4 . Using designed modified PID control system in Fig. 5.4 , the response of the output $y(t)$, which is the temperature T_1, is shown in Fig. 5.5 . Specially magnified detail drawing showing between 38 deg and 42 deg of Fig. 5.5 is shown in Fig. 5.6 . Fig. 5.5 and Fig. 5.6 show that the output $y(t)$, which is the temperature T_1, follows the reference input $r(t) = 40$ deg with small steady state error. In addition, Fig. 5.6 show that the maximum gain of disturbance will be reduced 0.2 deg.

In order to show that the proposed modified PID control system in Fig. 5.4 attenuates unknown disturbances effectively, the difference is clarified by comparison with the response using the conventional PID controller. When the conventional PID controller is used, the response of the output $y(t)$, which is the temperature T_1, is shown in Fig. 5.7 . Specially magnified detail drawing showing between 38 deg and 42 deg of Fig. 5.7 is shown in Fig. 5.8 . Fig. 5.7 and Fig. 5.8 show that the output $y(t)$, which is the temperature T_1, follows the reference input $r(t) = 40$ deg with small steady state error and the convergence speed is similar to that of the control system in Fig. 5.4 . In addition, Fig. 5.8 show that the maximum gain of disturbance will be reduced 0.4 deg, but it is less effective than that of the control system in Fig. 5.4 .

In addition, in order to show that the proposed modified PID control system in Fig. 5.4 attenuates unknown disturbances effectively, the difference is clarified by comparison with the response using only the modified PID controller. That is, we show the difference between

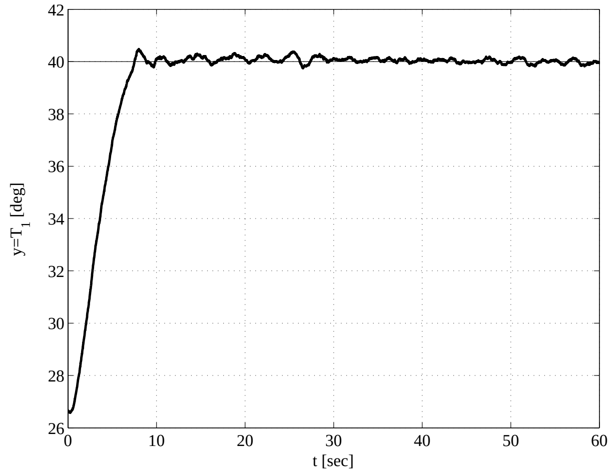

Figure 5.5: Experimental step response using the proposed modified PID control system.

proposed modified PID control system in Fig. 5.4 and the modified PID control system in Fig. 5.4 with $\hat{C}(s) = 0$. When $\hat{C}(s) = 0$ in Fig. 5.4 , an experimental result of temperature control for heat flow experiment in Fig. 5.1 is shown. Using the modified PID control system in Fig. 5.4 with $\hat{C}(s) = 0$, the response of the output $y(t)$, which is the temperature T_1, is shown in Fig. 5.9 . Specially magnified detail drawing showing between 38 deg and 42 deg of Fig. 5.9 is shown in Fig. 5.10 . Fig. 5.9 and Fig. 5.10 show that the output $y(t)$, which is the temperature T_1, follows the reference input $r(t) = 40$ deg with small steady state error. Fig. 5.10 show that the maximum gain of disturbance will be reduced 0.4 deg. The comparison of Fig. 5.5 with Fig. 5.2 , Fig. 5.7 and Fig. 5.9 shows that using proposed modified PID control system in Fig. 5.4 with the modified PID controller $C(s)$ and the controller $\hat{C}(s)$ can attenuate unknown disturbances effectively. In addition, the convergence speed of T_1 to 40 deg in Fig. 5.5 is faster than that in Fig. 5.9 . From the theoretical result that if the transfer function from $r(s)$ to $y(s)$ in Fig. 5.4 is equal to that in Fig. 5.4 with $\hat{C}(s) = 0$, the reason why the convergence speed of T_1 to 40 deg in Fig. 5.5 is faster than that in Fig. 5.9 is influence of the disturbance $d(s)$.

In this way, it is shown that the modified PID control system in Fig. 5.4 is more effective for temperature control for heat flow experiment in Fig. 5.1 than the conventional PID control system and the modified PID control system in Fig. 5.4 with $\hat{C}(s) = 0$.

Next, when τ in the controller $\hat{C}(s)$ in (5.13) is varied, the comparison of the responses is examined. The comparison of responses y for various τ as $\tau = 0.01$, $\tau = 0.05$ and $\tau = 0.4$ are shown in Fig. 5.11 , Fig. 5.12 and Fig. 5.13 , respectively. From Fig. 5.11 , Fig. 5.12 and Fig. 5.13 , as τ decreases, the maximum gain decreases.

5.5 Conclusions

In this paper, we apply the modified PID control system in Fig. 5.4 to attenuate unknown disturbances in [56] for temperature control heat flow experiment and show the effectiveness of the control system to attenuate unknown disturbances in [56]. Results of this paper are summarized as follows:

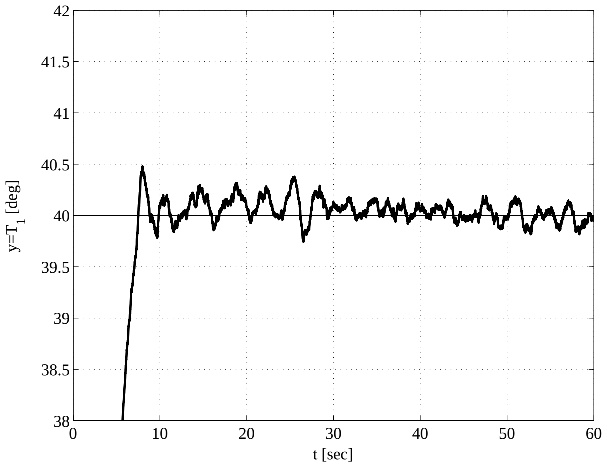

Figure 5.6: Specially magnified detail drawing showing between 38 deg and 42 deg of Fig. 5.5 .

1. Proposed modified PID control system in Fig. 5.4 is effective temperature control for heat flow experiment in Fig. 5.1 .

2. Proposed modified PID control system in Fig. 5.4 for temperature control for heat flow experiment in Fig. 5.1 attenuates unknown disturbance effectively.

3. The convergence speed of the conventional PID control system is similar to that of the control system in Fig. 5.4 with $\hat{C}(s) \neq 0$, but the disturbance attenuation characteristic is less effective than that of proposed modified PID control system in Fig. 5.4 .

4. When $\hat{C}(s) = 0$, the convergence speed of the control system in Fig. 5.4 is similar to that of the control system in Fig. 5.4 with $\hat{C}(s) \neq 0$. That is, we found that the convergence speed is independent from $\hat{C}(s)$. However, the disturbance attenuation characteristic of the control system in Fig. 5.4 with $\hat{C}(s) = 0$ is less effective than that of proposed modified PID control system in Fig. 5.4 with $\hat{C}(s) \neq 0$.

5. From Fig. 5.11 , Fig. 5.12 and Fig. 5.13 , the value of τ in the controller $\hat{C}(s)$ diminished, the vibration of the response can be attenuated.

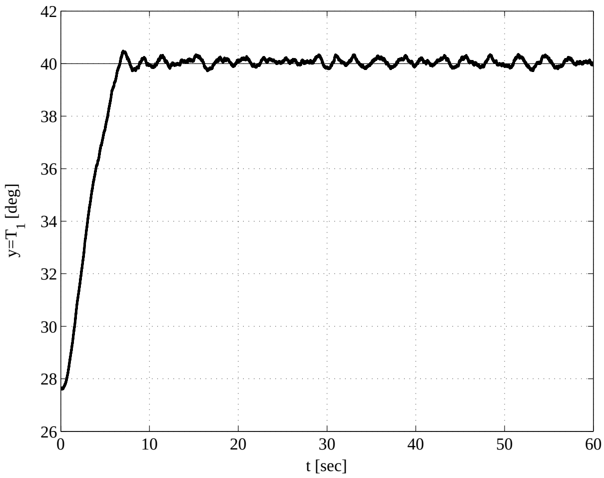

Figure 5.7: Experimental step response using the conventional PID control system.

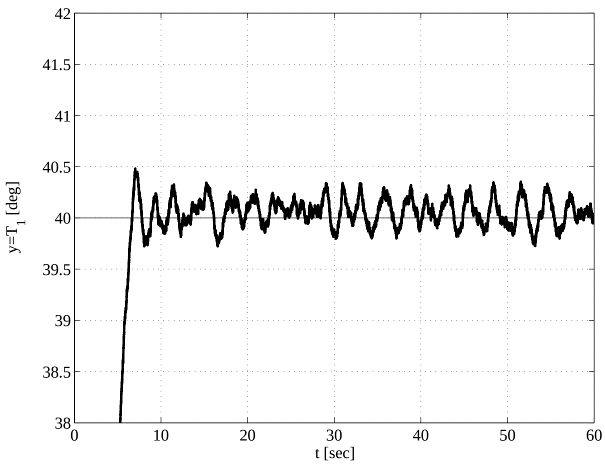

Figure 5.8: Specially magnified detail drawing showing between 38 deg and 42 deg of Fig. 5.7 .

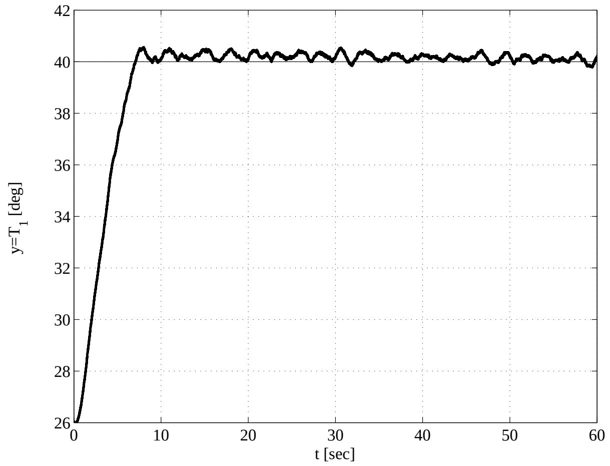

Figure 5.9: When $\hat{C}(s) = 0$, experimental step response.

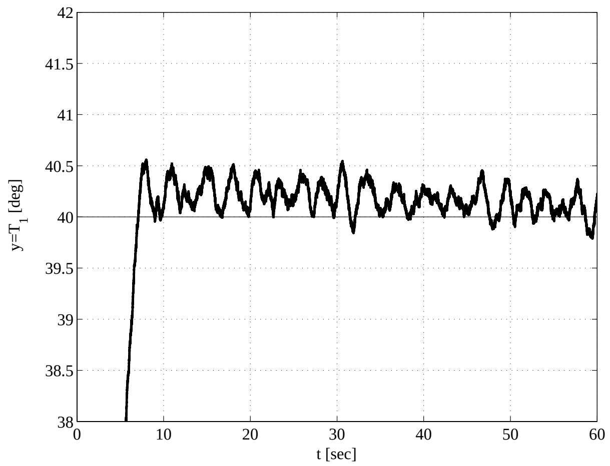

Figure 5.10: Specially magnified detail drawing showing between 38 deg and 42 deg of Fig. 5.9
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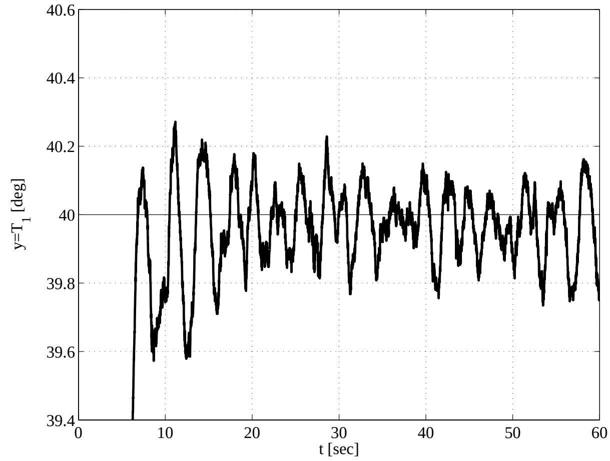

Figure 5.11: The response of the output y, when $\tau = 0.01$.

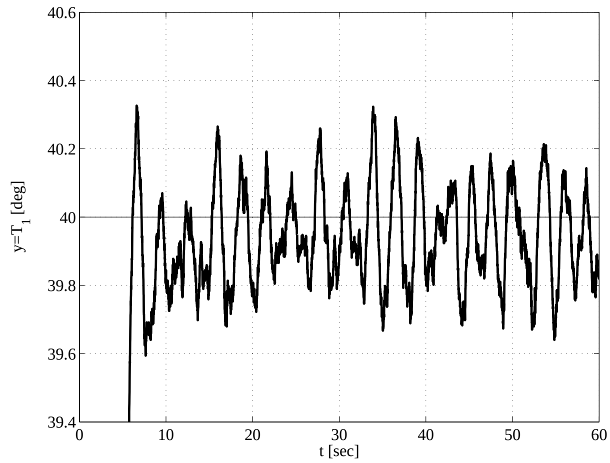

Figure 5.12: The response of the output y, when $\tau = 0.05$.

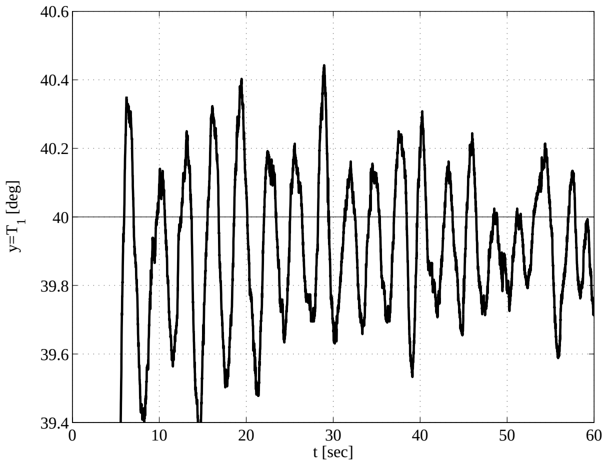

Figure 5.13: The response of the output y, when $\tau = 0.4$.

Chapter 6

Conclusion

In this paper, we proposed a design method for modified PID controllers such that modified PID controller makes the closed-loop system stable. Results of this paper are summarized as follows:

In Chapter 2., we proposed a design method of robust stabilizing modified PID controllers such that modified PID controller makes the closed-loop system stable for any plants with uncertainty. Proposed modified PID controllers lose the advantage of the conventional PID controllers such as

1. the control structure is simple.
2. the order of the controller is 1.

but have following advantages:

1. The modified PID controller makes the control system stable for any plant $G(s)$ with uncertainty.
2. The roles of P-parameter a_P, I-parameter a_I and D-parameter a_D in the robust stabilizing modified PID controller are equivalent to that of the conventional PID controller. That is, P-parameter a_P, I-parameter a_I and D-parameter a_D in the robust stabilizing modified PID controller can be tuned using previously proposed methods in [7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22].

In Chapter 3., we have proposed a design method for a robust stabilizing modified PID controller that makes the closed-loop system stable for any time-delay plant with uncertainty. The proposed modified PID controllers do not have the advantages of the conventional PID controllers in previous papers [6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30] such as:

1. the control structure is simple.
2. the order of the controller is 1.

However, they have the following advantages:

1. The modified PID controller makes the closed-loop system stable for any time-delay plant $G(s)e^{-sT}$ with uncertainty. This implies that plants that cannot be stabilized by the methods in [6, 7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24, 25, 26, 27, 28, 29, 30] can be stabilized using the proposed method.

2. The roles of the P-parameter a_P, I-parameter a_I and D-parameter a_D in the robust stabilizing modified PID controller are equivalent to those of the conventional PID controller. That is, the parameters can be tuned using methods proposed in [7, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 24].

In Chapter 4., we proposed a design method for modified PID control systems to specify the input-output characteristic and the disturbance attenuation characteristic separately and to attenuate unknown disturbances. The results in this paper are summarized as follows:

1. Modified PID control system was proposed as Fig. 4.1 .
2. Control characteristics of the modified PID control system in Fig. 4.1 were clarified. We find that proposed modified PID control system in Fig. 4.1 can specify the input-output characteristic and the disturbance attenuation characteristic separately. The role of the modified PID controller $C(s)$ is to specify the input-output characteristic. The role of the controller $\hat{C}(s)$ is to specify the disturbance attenuation characteristic.
3. We present a design method of the controller $\hat{C}(s)$ to attenuate unknown disturbances effectively.

In Chapter 5., we apply the modified PID control system in Fig. 5.4 to attenuate unknown disturbances in [56] for temperature control heat flow experiment and show the effectiveness of the control system to attenuate unknown disturbances in [56]. Results of this paper are summarized as follows:

1. Proposed modified PID control system in Fig. 5.4 is effective temperature control for heat flow experiment in Fig. 5.1 .
2. Proposed modified PID control system in Fig. 5.4 for temperature control for heat flow experiment in Fig. 5.1 attenuates unknown disturbance effectively.
3. The convergence speed of the conventional PID control system is similar to that of the control system in Fig. 5.4 with $\hat{C}(s) \neq 0$, but the disturbance attenuation characteristic is less effective than that of proposed modified PID control system in Fig. 5.4 .
4. When $\hat{C}(s) = 0$, the convergence speed of the control system in Fig. 5.4 is similar to that of the control system in Fig. 5.4 with $\hat{C}(s) \neq 0$. That is, we found that the convergence speed is independent from $\hat{C}(s)$. However, the disturbance attenuation characteristic of the control system in Fig. 5.4 with $\hat{C}(s) = 0$ is less effective than that of proposed modified PID control system in Fig. 5.4 with $\hat{C}(s) \neq 0$.
5. From Fig. 5.11 , Fig. 5.12 and Fig. 5.13 , the value of τ in the controller $\hat{C}(s)$ diminished, the vibration of the response can be attenuated.

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Bibliography

[1] N.Minorsky, “Directional stability of automatically steered bodies”, J. Amer. Coc. , Naval Engineers,42-2, pp.280–309, 1922.

[2] A. Callender, D.R. Hartree and A. Porter, “Time-lag in a control system”, Phil. Trans. Roy. Soc. London, A235-756, 1932.

[3] N. Suda, “PID contro”, Asakura Shoten, 1992 (in Japanese).

[4] K. Astrom and T. Hagglund, “PID controllers: Theory design, and tuning”, *Instrument Society of America*, North Carolina, 1995.

[5] H. Takatsu and T. Ito, “Report of the control technology survey in Jpanese industry”, *Journal of the Society of the Instrument and Congtrol Engineers*, 36-4, pp.238–244, 1997 (in Japanese).

[6] A. Datta, M.Z. Ho, and S.P. Bhattacharyya, “Structure and Synthesis of PID Controllers”, Springer-Velag, London, 2000.

[7] J.G Zieglae and N.B. Nicholes, “Optimum settings for automatic controllers”, *Trans. ASME*, 64, pp.759–768, 1942.

[8] Taylor Instrument Companies: Instructions for Transcope Controller, Bull. 1B404, 1961.

[9] Y. Takahashi, “Automatic control”, *Kagakugijyutsusha*, 1949(in Japanese).

[10] P.Hazebroek and B.L. van der Warden, “The Optimal Adjustment of Regulators”, *Trans. ASME*, 72, pp.317–332, 1950.

[11] P. Hazebroek and B.L. van der Warden, “Theoretical Considerations on the Optimal Adjustment of Regurators”, *Trans. ASME*, 72, pp.309–315, 1950.

[12] W.A. Wolf, “Controller Setting for Optimum Control”, *Trans. ASME*, 73, pp.413–418, 1951.

[13] K.L. Chien, J.A. Hrones, and J.B. Reswick, “On the Automatic Control of Generalized Passive Systems”, *Trans. ASME*, 74, pp.175–185, 1952.

[14] G.H. Cohen and G.A. Coon, “Theoretical Consideration of Retaeded Control”, *Trans. ASME*, 75, pp.857–834, 1953.

[15] A.M. Lopez, J.A. Miller, C.L. Smith, and P.W. Murrill, “Tuning Controllers with Error-Integral Criteria”, *Instrumentation Technology*, 14, pp.52–62, 1967.

[16] J.A. Miller, A.M. Lopez, C.L. Smith, and P.W. Murrill, "A Comparison of Controller Tuning Techniques", *Controll Engineering*, 14, pp.72–75, 1967.

[17] T. Kitamori, "A method of control system design based upon partial knowledge about controlled process", *Transactions of the Society of Instrument and Control Engineers*, 15-4, pp.549–555, 1979 (in Japanese).

[18] T. Kitamori, "Design method for PID control systems", *Journal of the Society of the Instrument and Congtrol Engineers*, 19-4, pp.382–391, 1980 (in Japanese).

[19] R. Kuwata, "An imprived ultimate sensitivity method and PID; I-PD control characteristics", *Transactions of the Society of Instrument and Control Engineers*, 15-4, pp.232–239, 1987 (in Japanese).

[20] I. Hashimoto, S. Hasebe and M, Kanou, "Process control", *Asakura Shoten*, 2002 (in Japanese).

[21] T. Kitamori, "A theory of I-PD control system and design method", pp.7–17, 1998 (in Japanese).

[22] P. Cominos and N. Munro, "PID Controllers: Recent Tuning Methods and Design to Specification", *IEE Proceedings*, 149, pp.46–53, 2002.

[23] Q. Xiong, W.J. Cai and M. He, "A practical decentralized PID auto-tuning method for TITO systems under closed-loop control", *Int. J. Innovative Computing, Information and Control*, vol.2, no.2, pp.305–322, 2005.

[24] Q. Xiong and G. Jin, "Iterative decentralized PID tuning based on gain and phase margins for TITO systems", *Int. J. Innovative Computing, Information and Control*, vol.3, no.2, pp.385–396, 1996.

[25] F. Zheng, Q.G. Wang, and T.H. Lee, "On the Design of Multivariable PID Controllers via LMI Approach", *Automatica*, 38-3, pp.517–526, 2002.

[26] C. Lin, Q.G. Wang, and T.H. Lee, "An Improvement on Multivariable PID Controller Design via Iterative LMI Approach", *Automatica*, 40-3, pp.519–525, 2004.

[27] N. Viorel, M. Constantin, A. Dorel, and C. Emil, "Aspects of Pole Placement Technique in Symmetrical Optimum Method for PID Controller Design", *Preprints of the 16th IFAC World Congress DVD-ROM*, 2005.

[28] K. Tamura and K. Shimizu, "Eigenvalue Assignment Method by PID Control for MIMO System", *Transactions of The Institute of Systems, Control and Information Engineers*, 19-5, pp.193–202, 2006 (in Japanese).

[29] J. Yang, "Parameter Plane Control Design for a Two-tank Chemical Reactor Systems", *Journal of the Franklin Institute*, 331B-1, pp.61–76, 1994.

[30] M.T. Ho, A. Datta, and S.P. Bhattacharyya, "A linear programming characterization of all stabilizing PID controllers", *Proceedings of the American Control Conference 1997*, pp.3922–3928, 1997.

[31] M. Saeki and D. Hirayama, "Parameter Space Design Method of PID Controller for Robust Sensitivity Minimization Problem", *The Society of Instrument and Control Engineers*, Vol.32, No.12, pp.1612–1619, 1996.

[32] M. Saeki, J. Kimura, H. Kurata and S. Taiji, "Parameter space design method of PID controller for H^{∞} control problem", *Trans. Institute of System, Control and Information, Engineers*, vol.11, pp.35–40, 1998.

[33] J. Bao, J.F. Forbes and P.J. McLellan, "Robust multiloop PID controller design: a successive semidefinite programming approach", *Ind. Eng. Chem. Res.*, vol.38, pp.3407–3419, 1999.

[34] Q.G. Wang, C.C. Hang, Y. Yang and J.B. He, "Quantitative robust stability analysis and PID controller design", *IEE Proceedings of Control Theory and Application*, vol.149, pp.3–7, 2002.

[35] J. Jairo, E. Oviedo, T. Boelen and P.V. Overschee, "Robust advanced PID control", *IEEE Control Systems Magazine*, vol.26, no.1, pp.15–19, 2006.

[36] S. Hara, T. Iwasaki and D. Shiokata, "Robust PID using generalized KYP synthesis: direct open-loop shaping in multiple frequency ranges", *Control System Magazine*, vol.26, no.1, pp.80–91, 2006.

[37] M. Shamsuzzoha and M. Lee, "Design of robust PID controllers for unstable processes", *Proceedings of International Joint Conference SICE-ICASE 2006*, pp.3324–3329, 2006.

[38] E.N. Gongalves, R.M. Palhares and R.H.C. Takahashi, "H_2/H_∞ Robust PID synthesis for uncertain systems", *Proceedings of 45th IEEE Conference on Decision and Control*, San Diego, U.S.A., pp.4375–4380, 2006.

[39] R. Namba, T. Yamamoto and M. Kaneda, "Robust PID controller and its application", *Proceedings of 1997 IEEE International Conference on Systems, Man, and Cybernetics*, Orlando, U.S.A., pp.3636–3641, 2007.

[40] N. Abe and A. Kojima, "Control in time-delay and distributed parameter systems", *Corona Publishing*, 2007.

[41] J.C. Doyle, K. Glover, P.P. Khargonekar and B.A. Francis, "State-space solution to standard H_2 and H_∞ control problem", *IEEE Trans. on Automatic Control*, vol. AC34, pp.831–847, 1989.

[42] K. Glover and J.C. Doyle, "State space formulae for stabilizing controllers that satisfy a ∞ bound and relations to risk sensitivity", *System & Control Letters*, vol.11, pp.167–172, 1988.

[43] T. Iwasaki and R.E. Skelton, "All controllers for the general H_∞ control problem LMI existence conditions and state space formulas", *Automatica*, vol.30, pp.1307–1317, 1994.

[44] P. Gahinet and P. Apkarian, "A linear matrix inequality approach to H_∞ control", *Int. J. Robust and Nonlinear Control*, vol.4, pp.421–448, 1994.

[45] K. Yamada and T. Moki, “A design method for PI control for minimum phase systems”, *Intelligent Engineering Systems Through Artificial Neural Networks*, Vol.13, pp.571–576, 2003.

[46] K. Yamada, “Modified PID controllers for minimum phase systems and their practical application”, *Proceedings of The 2005 Electrical Engineering/Electronics, Computer, Telecommunication, and Information Technology (ECTI) International Conference*, Volume II of II, pp.457–460, 2005.

[47] K. Yamada, T. Moki, and Z. Hai-rong, “Modified PI control using model driven control for certain class of non-minimum phase systems”, *Transactions of the Japan Society of Mechanical Engineers*, 71-707(C), pp.2203–2209, 2005 (in Japanese).

[48] K. Yamada, N. Matsushima, and T. Hagiwara, “A design method for modified PID controllers for stable plants”, *Proceedings of the 2006 Electrical Engineering/Electronics, Computer, Telecommunication, and Information Technology (ECTI) International Conference*, Vol.I of II, pp.123–126, Ubon Ratchathani, Thailand, 2006.

[49] K. Yamada, N. Matsushima, and T. Hagiwara, “A design method for modified PID controllers for stable plants”, *ECTI Transactions on Electrical Eng., Electronics, and Communications*, Vol.5-1, pp.31–40, 2007.

[50] K. Yamada and T. Hagiwara, “A design method of modified PID controller for unstable plants”, *Intelligent Engineering System Through Artificial Neural Networks*, Vol.16, pp.753–758, 2006.

[51] K. Yamada, T. Hagiwara, and Y. Shimizu, “A design method of robust stabilizing modified PID controllers”, *Theoretical and Applied Mechanics*, Volume 56, pp. 123-134, 2007.

[52] K. Yamada, T. Hagiwara and Y. Shimizu, “A design method for modified PID controllers for time-delay plants and their application”, *ECTI Transactions on Electrical Eng., Electronics, and Communications*, Vol.6, No.1, pp.53–63. 2008.

[53] K. Yamada, T. Hagiwara and Y. Shimizu, “Modified PID controller for non-minimum phase time-delay plants”, *Intelligent Engineering System Through Artificial Neural Networks*, Vol.17, pp.537–542, 2007.

[54] T. Hagiwara, K. Yamada, “A design method of modified PID controllers for multiple-input/multiple-output plants”, *Proceedings of the 17th World Congress The International Federation of Automatic Control, Seoul, Korea*, 2008.

[55] T. Hagiwara, K. Yamada, I. Murakami, Y. Ando, and T. Sakanushi, “A design method for robust stabilizing modified PID controllers for multiple-input/multiple-output plants”, *Intelligent Engineering Systems Through Artficial Neural Networks*, Vol.18, pp.601–607, 2008.

[56] T. Hagiwara, K. Yamada, I. Murakami, Y. Ando, and T. Sakanushi, “A design method for robust stabilizing modified PID controllers for time-delay plants with uncertainty”, *International Journal of Innovative Computing, Information and Control*, Vol.5, No.10(B), pp.3553–3563, 2009.

[57] T. Hagiwara, K. Yamada, I. Murakami, Y. Ando, and S. Matsuura, and S. Aoyama, "An application of the modified PID control system for heat flow experiment", *International Journal of Automation Technology*, Vol.4, No.5, pp.469–476, 2010.

[58] T. Hagiwara, K. Yamada, I. Murakami, Y. Ando, and S. Matsuura, "A design method for modified PID control systems to attenuate unknown disturbances", *Key Engineering Materials*, Vol.459, pp.211–220, 2011.

[59] K. Yamada and N. Matsushima, "A design method for Smith predictors for minimum-phase time-delay plants", *ECTI Transactions on Computer and Information Technology*, Vol.2-2, pp.100–107, 2005.

[60] K. Yamada, I. Murakami, Y. Ando, T. Hagiwara, Y. Imai, and M. Kobayashi, "The parametrization of all disturbance observers", *ICIC Express Letters, An International Journal of Research and Surveys*, Vol.2, No.4, pp. 421-426, 2008.

[61] K. Yamada and W. Kinoshita, "New state space design method of stable filtered inverse systems and their application", Transactions of the Institute of Systems, Control and Information Engineers, Vol.16, No.2, pp.85–93, 2003.

[62] M. Vidyasagar, "Control System Synthesis-A factorization approach-", MIT Press, 1985.

[63] O.J.M. Smith, "A controller to overcome dead-time", *ISA Journal*, Vol.6, pp.28–33, 1959.

[64] M. Araki, R. Kuwata and T. Fujinaka, "PID/I-PD control and discretization optimizing control for time-delay system", *System and Control*, 28-5, pp.278–289, 1984 (in Japanese).

[65] M. Morari and E. Zafiriou, "Robust process control", PTR Prentice Hall, New Jersey, 1989.

[66] H. Kimura, "Is the Model a Good Controller? Perspectives on Brain Motor Control", Proceedings of CDC, 2000.

[67] K. Yamada, N.T. Mai, Y. Ando, T. Hagiwara, I. Murakami and T. Hoshikawa, "A design method for stabilizing modified Smith predictors for multiple-input/multiple-output time-delay plants", submitted.

Publication papers

Chapter 2 ○ Kou Yamada, Takaaki Hagiwara and Yosuke Shimizu, A Design Method of Robust Stabilizing Modified PID Controllers, *Theoretical and Applied Mechanics Japan*, Vol. 56, (2008), pp.123–134.

Chapter 3 ○ Takaaki Hagiwara, Kou Yamada, Iwanori Murakami, Yoshinori Ando and Tatsuya Sakanushi, A Design Method for Robust Stabilizing Modified PID Controllers for Time-delay Plants with Uncertainty, *International Journal of Innovative Computing, Information and Control*, Vol.5, No.10(B), (2009), pp.3553–3563.

Chapter 4 ○ Takaaki Hagiwara, Kou Yamada, Iwanori Murakami, Yoshinori Ando, and Shun Matsuura, A design method for modified PID control systems to attenuate unknown disturbances, *Key Engineering Materials*, Vol.459, (2011), pp.211–220.

Chapter 5 ○ Takaaki Hagiwara, Kou Yamada, Iwanori Murakami, Yoshinori Ando, Shun Matsuura, and Satoshi Aoyama, An Application of the Modified PID Control System for Heat Flow Experiment, *International Journal of Automation Technology*, Vol.4, No.5, (2010), pp.469–476.