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Method for evaluating material viscoelasticity

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A method for evaluating the viscoelasticity of materials under oscillation load is proposed. In the method, a material under test is connected to a mass, which generates an oscillating inertial force after the mass is manually struck using a hammer. A pneumatic linear bearing is used to realize linear motion with sufficiently small friction acting on the mass that is the moving part of the bearing. The inertial force acting on the mass is determined highly accurately by means of measuring the velocity of the mass using an optical interferometer. © *2004 American Institute of Physics.* [DOI: 10.1063/1.1634363]

I. INTRODUCTION

Force, which is one of the most basic mechanical quantities, is defined as the product of mass and acceleration as

$$\mathbf{F} = M\mathbf{a},$$

where **F** is the force acting on an object, M is the mass of the object, and **a** is the acceleration of the center of the gravity of the object. This means that a well-defined acceleration is required to generate force accurately and to calibrate force transducers accurately.

Acceleration due to gravity (g) is convenient and usually used for generating and/or measuring constant force. Constant force can be accurately compared using a conventional balance with a knife-edge or a hinge.

However, there are no working methods for calibrating force transducers under dynamic conditions. Only static methods, in which transducers are calibrated by static weighting under static conditions, are widely available at present. Methods for the dynamic calibration of force transducers are important for fulfilling these requirements. Therefore, it is very difficult to determine the uncertainty in measuring a varying force or dynamic force by means of force transducers.

Although the methods of dynamic calibration of force transducers are not yet well established, there have been a number of trials aiming toward the development of dynamic calibration methods for force transducers.

One method was proposed by the author and has been under development.[1–4] This method was first proposed[1] as an impulse response evaluation method for force transducers; a mass is made to collide with a force transducer and the impulse (i.e., the time integration of the impact force) is measured highly accurately as a change in momentum of the mass. To realize linear motion with sufficiently small friction acting on the mass, a pneumatic linear bearing[5,6] is used, and the velocity of the mass (i.e., the moving part of the bearing) is measured using an optical interferometer. This method was subsequently improved[2,3] as a method for determining the instantaneous value of the impact force in the impulse. In this case, the instantaneous value of impact force is measured as the inertial force acting on the mass, by means of measuring the instantaneous acceleration of the mass. The author has shown the possible applications and importance of this method in force measurement.[4]

The other method, which was proposed and has been developed by Kumme, uses the inertial force of the attached mass generated by a shaker.[7,8] In this method, dynamic force of a single frequency is generated and applied to a force transducer. This method is effective for evaluating the characteristics of force transducers under the conditions in which calibration is conducted, such as continuous vibration at a single frequency. Park *et al.* have used this method for the dynamic investigation of multicomponent force-moment sensors.[9,10] However, this method is not suitable for evaluating the impulse response of transducers, which is important particularly in the crash testing of structures, instruments, and machines. The author also proposed a method for calibrating force transducers under oscillation force by means of modifying the previous method using the mass levitation and the optical interferometer.[11]

In this article, a method for evaluating the viscoelasticity of materials is proposed.

II. EXPERIMENTAL SETUP

Figure 1 shows a schematic diagram of the experimental setup for evaluating the viscoelasticity of materials. One side of a material under test is connected to a mass. The other side of the material is attached to the base. A pneumatic linear bearing is used to realize linear motion with sufficiently small friction acting on the mass; that is, the moving part of the bearing. The mass, the material, and the base form a spring–mass system.

The inertial force of the mass is used as the standard oscillation force and is applied to the material. To excite the

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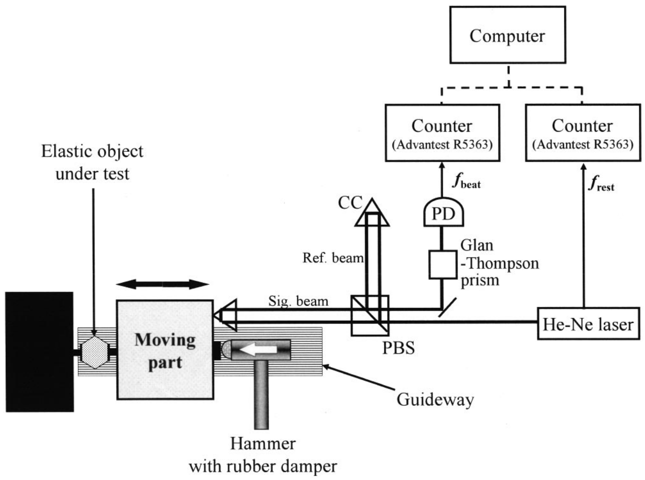

FIG. 1. Experimental setup.

oscillation of the mass–spring system, the initial kinetic energy is given by means of manually striking the mass using a plastic hammer. The inertial force acting on the mass is determined highly accurately by measuring the velocity of the mass using an optical interferometer.

To minimize the vibration of the optics of the optical interferometer, the base with which the material is connected is directly fixed to the foundation of the building. All the other parts of the experimental setup are fixed on another base, which is supported using thin rubber dampers. This guarantees the condition that the point at which the material under test is connected to the base is the quiescent point.

The force acting on the material from the mass and the force acting on the mass from the material have the same absolute value but the different signs according to the principle of action and reaction. The force acting on the moving part from the material ($\mathbf{F}$) and the inertial force of the moving part ($M\mathbf{a}$) are equal according to the law of inertia, under the condition in which other forces, such as the frictional force inside the bearing, can be ignored. The mass of the moving part as the rigid object is 4.5818 kg. In the experiment, a silicon-rubber damper is used as the material to be tested. Both sides of the damper are firmly glued to metal parts, with which it is fixed to the mass and the base.

The inertial force is measured as the product of mass and acceleration, and the acceleration is calculated by differentiating the velocity of the moving part. The position of the mass is calculated by integrating the velocity. The velocity is measured as the Doppler shift frequency of the signal beam of a laser interferometer (f_{Doppler}), which can be expressed as

$$v=\lambda_{\mathrm{air}}(f_{\mathrm{Doppler}})/2,$$

$$f_{\mathrm{Doppler}}=-(f_{\mathrm{beat}}-f_{\mathrm{rest}}),$$

where λ_{air} is the wavelength of the signal beam under the experimental conditions; f_{beat} is the beat frequency (i.e., the frequency difference between the signal beam and the reference beam); and f_{rest} is the rest frequency, which is the value of f_{beat} when the moving part is at a standstill; the direction of the coordinate system for the position, the velocity, the acceleration, and the force acting on the moving part is towards the right in Fig. 1.

A Zeeman-type two-frequency He-Ne laser is used as the light source. The frequency difference between the signal beam and the reference beam (i.e., the beat frequency f_{beat}) is measured from an interference fringe which appears at the output port of the interferometer; it varies around f_{rest}, approximately 2.7 MHz, depending on the velocity of movement. An electric frequency counter (model: R5363, manufactured by Advantest Corp., Japan) continuously measures and records the beat frequency (f_{beat}) 2000 times with a sampling interval of $T=4000/f_{\mathrm{beat}}$, and stores the values in its memory. This counter continuously measures the interval time of every 4000 periods without dead time. The sampling period of the counter is approximately 1.5 ms at a frequency of 2.7 MHz. Another electric counter (same model: R5363, manufactured by Advantest Corp., Japan) measures the rest frequency (f_{rest}) using the electric signal supplied by a photodiode embedded inside the He-Ne laser.

The pneumatic linear bearing, "Air-Slide TAAG10A-02" (NTN Co., Ltd., Japan), is attached to an adjustable tilting stage. The maximum weight of the moving part is approximately 30 kg, the thickness of the air film is approximately 8 μm, the stiffness of the air film is more than 70 N/μm, and the straightness of the guideway is better than 0.3 μm/100 mm. The frictional characteristics are determined in detail by means of the developed method.[6,7]

To minimize the moment acting on the moving part, the line that connects the center of gravity of the moving part and the point at which the material under test is connected with the moving part is set to be parallel to the moving direction of the moving part; that is, the direction of the guideway of the bearing, by means of attaching an additional metal mass to the moving part. The tilt angle of the tilting stage is adjusted so that the moving part including all the attached parts is standstill near the point at which the material under test should be connected.

III. RESULTS

Figure 2 shows the data processing procedure. In the experiment, an impulse is manually applied using a hammer at the beginning. When the impulse is applied to the mass, the beat frequency appearing at the output port of the interferometer changes steeply. After that, it begins to oscillate at a frequency of approximately 13 Hz. During the oscillation experiment, only the frequency is measured. Using the measured value of the frequency, the velocity, the position, the acceleration and the inertial force are numerically calculated using a computer.

Figure 3 shows the change in the inertial force of the mass in detail. As shown in the figure, the time range of approximately 0.92 s, in which 12 periods with 24 peaks exist, is selected for the further analysis. The first positive impulse is excluded from this time range, because the effect of the impulse manually applied by means of the hammer is observed to be included in it.

Figure 4 shows the selected data in the time range shown

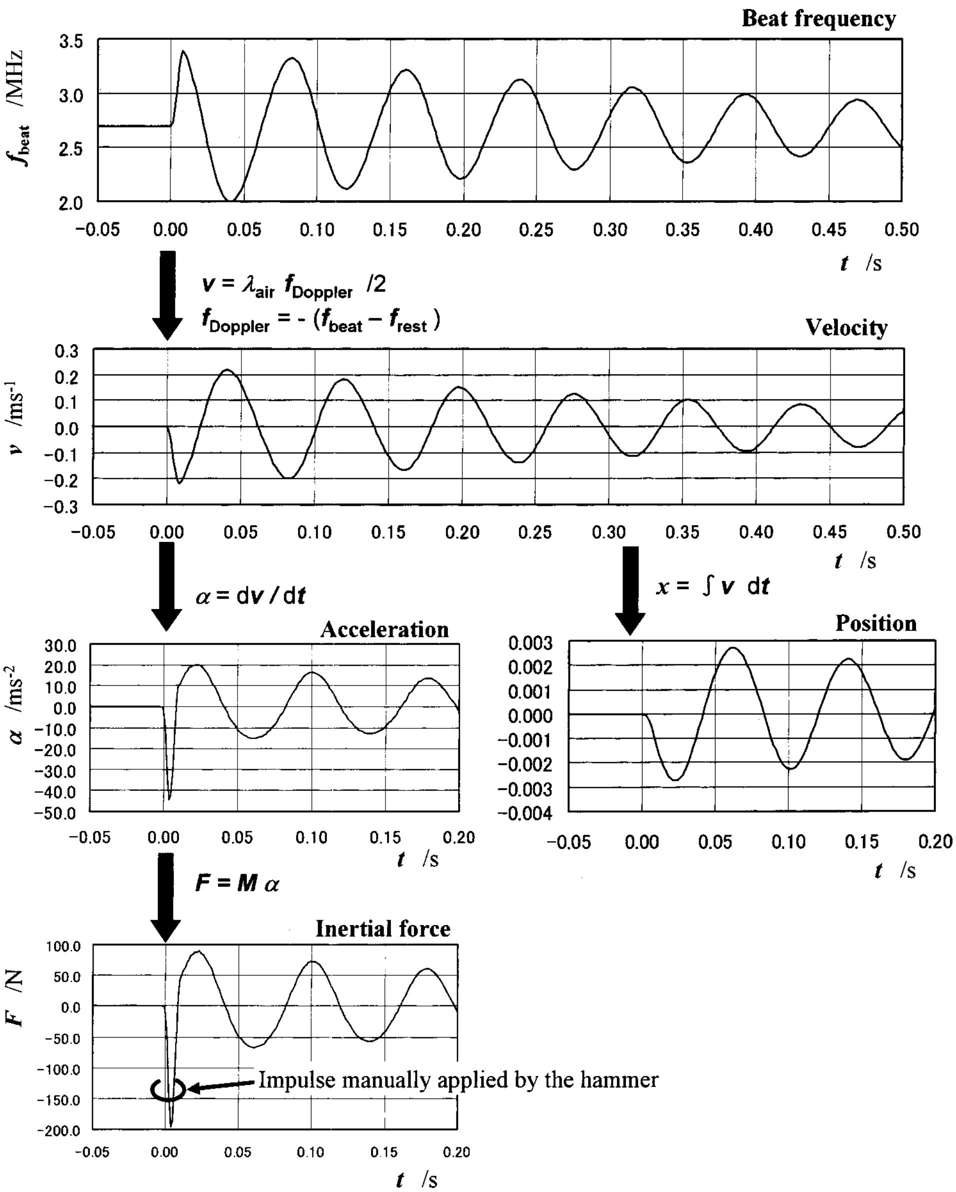

FIG. 2. Data processing procedure: Calculation of velocity, position, acceleration, and force from frequency.

in Fig. 3 in detail, but in a different manner. It shows the relationship between the inertial force and the position of the mass. The linear regression line is also drawn in the figure. The relationship between the inertial force and the position is not perfectly proportional.

Figure 5 shows the residual force, which is the difference between the measured inertial force ($\mathbf{F}_{inertial}$) and the linear regression shown in Fig. 4 ($\mathbf{F}_{regression}$). In Fig. 5, the elastic hysteresis, which is caused by the attenuation, is clearly observed.

Figure 6 shows the change in the logarithmic decrement (Λ) and the period of the oscillation (T) against the number of the peaks (i). The logarithmic decrement is defined as

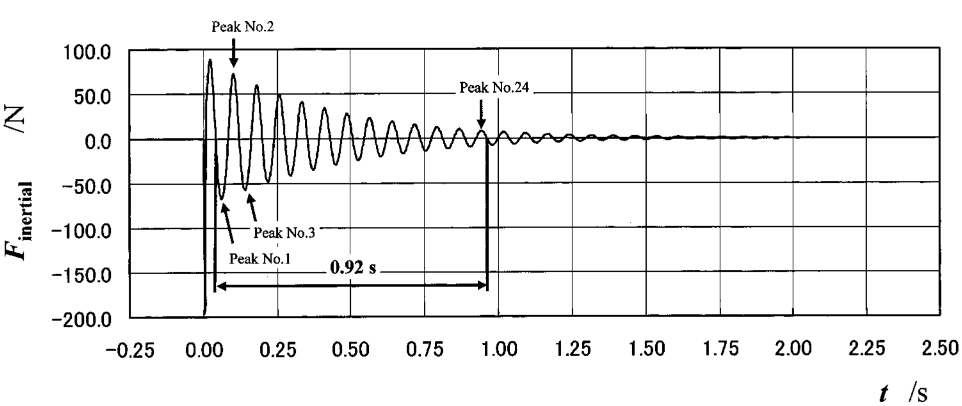

FIG. 3. Change in force in a single measurement.

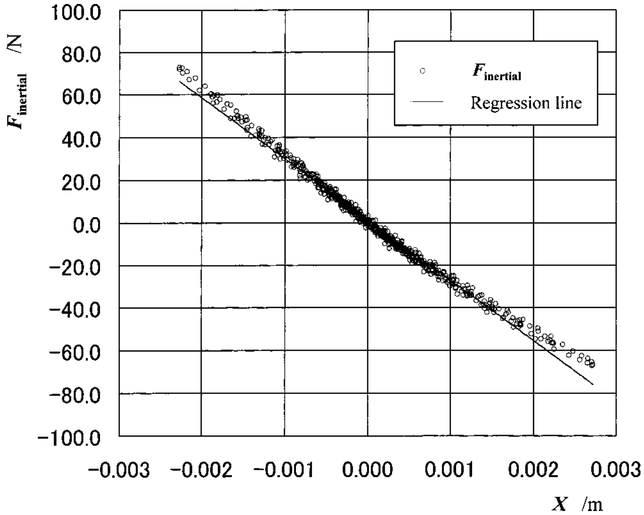

FIG. 4. Relationship between force and position.

$\Lambda = -\log(\mathbf{F}_{\mathrm{peak},i+2}/\mathbf{F}_{\mathrm{peak},i})$, using the peak value of the inertial force as shown in Fig. 3. The period of the oscillation is defined as $T = t_{\mathrm{peak},i+2} - t_{\mathrm{peak},i}$, where $t_{\mathrm{peak},i}$ is the time at which the force takes the i th peak, $\mathbf{F}_{\mathrm{peak},i}$.

The logarithmic decrement calculated using the peaks of the compressive load is smaller than that calculated using the peaks of the tensile load. The period of the oscillation gradually decreases along with the number of peaks.

Figure 7 shows the power spectrum of the inertial force. The power spectrum is calculated in the sense of the Lomb normalized periodogram. The data is the same as in Figs. 4–6. The power spectrum has the first peak at 13.1 Hz and the second peak at 26.1 Hz.

IV. UNCERTAINTY EVALUATION

The uncertainty components in the determination of the instantaneous value of the oscillation force acting on the test specimen are as follows.

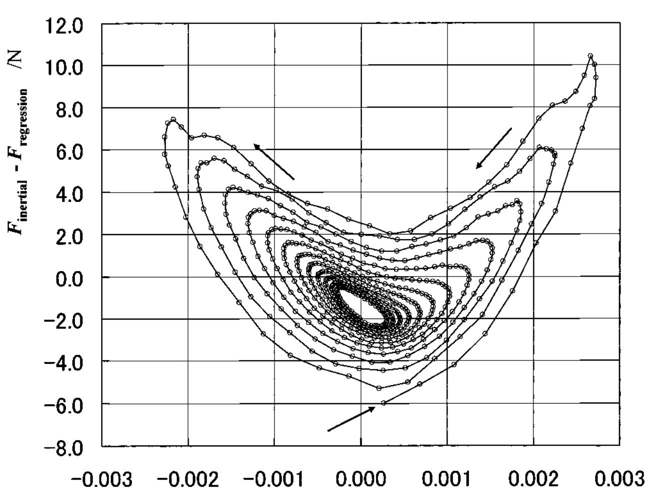

FIG. 5. Relationship between relative force and position.

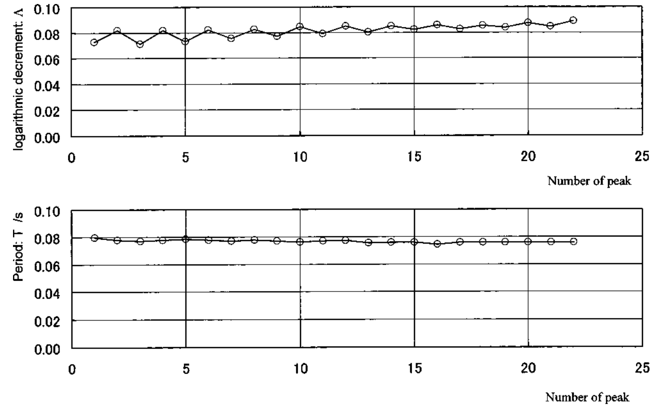

FIG. 6. Change in logarithmic decrement and period.

A. Determination of the inertial force of the moving part

1. Mechanical vibration

The mechanical vibration of the optical interferometer is significant particularly at the beginning of the measurement. The reason is thought to be that the impact force manually applied using the hammer is not perfectly parallel to the moving direction, and the force component vertical to the moving direction is transported to the base plate on which the optical interferometer is placed. However, this vibration seems to be attenuated rapidly. From the observation of the experimental result shown in Fig. 5, this vibration is less than 1 N after the impulse applied by the hammer. Therefore, the standard deviation of the vibration is estimated to be 0.3 N.

2. Electric counter (R5363)

The uncertainty originating from the electric counter R5363 with the sampling interval of $dt = 4000/f_{\mathrm{beat}}$ (s) is estimated to be approximately 100 Hz. This uncertainty of the beat frequency corresponds to the uncertainty of the velocity of the moving part of approximately 3×10^{-5} m/s, according to the relational expression, $v = -\lambda_{\mathrm{air}}(f_{\mathrm{beat}}$

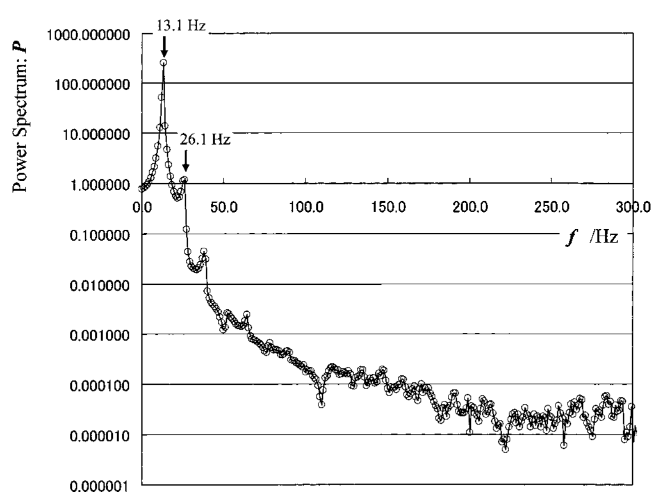

FIG. 7. Power spectrum of the inertial force.

$-f_{\mathrm{rest}})/2$. This corresponds to the uncertainty of the acceleration and force of approximately $3\times10^{-2}\,\mathrm{ms}^{-2}$ and 0.14 N, respectively.

3. Optical alignment

The major source of uncertainty in the optical alignment is the inclination of the signal beam of 1 mrad, and it results in a relative uncertainty in the velocity of approximately 5×10^{-7}, which is negligible.

4. Frequency stability

The uncertainty of the frequency difference of the laser is estimated to be 10 Hz. This corresponds to the uncertainty of the velocity of the moving part of approximately 3×10^{-6} m/s and the uncertainty of the force of approximately 0.014 N. This is negligible.

5. Mass

Mass of the moving part is calibrated with a standard uncertainty of approximately 0.1 g, which correspond the relative standard uncertainty in force determination of approximately 2×10^{-5}. This is negligible.

B. Determination of the external force

For the external force acting on the moving part, the frictional force acting inside the pneumatic linear bearing is dominant under the condition that the air film of approximately 8 μm thickness inside the bearing is not broken. The frictional characteristics of the air bearing are determined using the developed method.[5,6] The dynamic frictional force acting on the moving part ($\mathbf{F}_{\mathrm{df}}$) is estimated by

$$\mathbf{F}_{\mathrm{df}}=Av,$$

$$A=8\times10^{-2}/\mathrm{kg\,s}^{-1}.$$

This is calculated to be approximately 0.02 N at a velocity of approximately 0.2 $\mathrm{m\,s}^{-1}$, which is negligible.

Therefore, the standard uncertainty in the determination of the force acting on the transducer is estimated to be 0.3 N. This corresponds to 3×10^{-3} (0.3%) of the maximum applied force in the experiments.

V. DISCUSSION

The restoring force of the silicon rubber ($\mathbf{F}_{\mathrm{restore}}$) corresponds to $-\mathbf{F}_{\mathrm{inertial}}$ in Fig. 4. This restoring force can be approximated using a power series including higher order nonlinear components as follows:

$$\mathbf{F}_{\mathrm{restore}}=-\mathbf{F}_{\mathrm{inertial}}=k_1x+k_2x^2,$$

where, $k_1=2.92\times10^4$ and $k_2=-1.69\times10^6$ were identified using the least-squares method. This expression is appropriate in the range of $x=-0.0022$ to 0.0025 mm. Here, the hysteresis effect is neglected conveniently. This reveals that the restoring force includes characteristics of a softening spring. Accordingly, the small peak at 26.1 Hz in Fig. 7 can be regarded as the second-harmonic component of the natural frequency 13.1 Hz, which originates from the quadratic term in this restoring force ($\mathbf{F}_{\mathrm{restore}}$).

The hysteretic curve shown in Fig. 5 has a counterclockwise direction. This results in the dissipation of kinetic energy as heat, followed by the attenuation of oscillation. This is due to the viscosity of the material.

In this article, only one oscillation experiment is shown to demonstrate the performance of the proposed method. The total time spent for one measurement including the data transfer time between the electric counters and the computer is approximately 1 min. This indicates the ease in performance of the method.

In the experiment, an oscillation with a relatively low frequency of approximately 13 Hz occurs. The frequency can be adjusted by changing the mass of the moving part. In the case of evaluating a very hard material and dealing with an oscillation with a very high frequency, for example, up to 1 kHz, the acceleration distribution inside the moving part must be carefully considered, for example, by means of finite element method analysis.

In the proposed method, only the frequency is measured during the oscillation experiment, and all the other quantities, such as velocity, position, acceleration and force, are numerically calculated afterward. In addition, force is directly calculated according to its definition, that is the product of mass and acceleration. The authors consider that this simplicity is the most significant advantage of the proposed method compared with other conventional methods using a force transducer and a position sensor.

ACKNOWLEDGMENT

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