Pendulum for precision force measurement

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(Received 7 June 2005; accepted 31 January 2006; published online 24 March 2006)

A pendulum and a method for correcting the restoring force of the pendulum are proposed for realizing an instrument based on the levitation mass method without the use of pneumatic linear bearings. As an example a material tester using the pendulum, which evaluates the mechanical response of general objects against impact forces, is developed. The characteristics of the restoring force are accurately determined using the same instrument under the free-swing condition without the object under test. To demonstrate the high performance of the developed instrument, the impact response of a gel block is accurately determined. The possible applications of the developed method are discussed. © *2006 American Institute of Physics.* [DOI: 10.1063/1.2185148]

I. INTRODUCTION

At present there are no standard methods for evaluating the dynamic characteristics of force transducers. Force transducers are typically calibrated with standard methods using weights under static conditions. In material testing, the force acting on the material under test is measured using a force transducer and the displacement of the point at which the force is acting on it is measured using a displacement transducer. The lack of the dynamic force calibration method results in the two major problems concerning material testing. One is that it is difficult to evaluate the uncertainty in the measured value of the varying force. The other is that it is difficult to evaluate the uncertainty in the time at which the varying force is measured.

Although methods for the dynamic calibration of force transducers are not yet well established, there have been several attempts to develop these.[1–7] These attempts can be divided into three categories: methods for calibrating transducers using an impact force,[1,2] methods for calibrating transducers against a step force,[3] and methods for calibrating transducers against an oscillation force.[4–7]

The author has proposed a method, the levitation mass method. In the method, the inertial force of a mass levitated using a pneumatic linear bearing is used as the reference force applied to the objects under test, such as force transducers, materials, or structures. The inertial force of the levitated mass is measured using an optical interferometer. The author has modified it as methods for the dynamic force calibration of force transducers against some typical types of dynamic forces, such as impact force,[1] step force,[3] and oscillation force.[7] However, it has not been established how to apply the results of such dynamic calibration to the actual wave profile of the varying force. This difficulty mainly comes from the fact that the validity of applying the frequency response obtained from the oscillation force calibration to other types of force such as impact force and step force has not been proven and is quite unlikely.

The levitation mass method has also been applied to investigate the frictional characteristics of pneumatic linear bearings.[8] The author has also applied the levitation mass method for material testing, such as methods for evaluating material viscoelasticity under an oscillating load[9] and under an impact load[10] and a method for generating and measuring micro-Newton level forces.[11] The levitation mass method has been proven to be a very accurate and efficient method for measuring the varying force based on the definition of force, the product of mass, and acceleration.

However, developing an instrument based on the levitation mass method for use in high temperature, low temperature, or vacuum environments, usage of a pneumatic linear bearing is a serious obstacle. In addition, a substitute for expensive pneumatic linear bearings that require filtered compressed air is desirable for an inexpensive and compact instrument for practical use.

In this article, a simple pendulum and a method for correcting the large restoring force of the pendulum are proposed for realizing an instrument based on the levitation mass method without the use of a pneumatic linear bearing. Its performance is evaluated by evaluating material viscoelasticity under an impact load. It is proven that the developed instrument using the pendulum has the comparable performance as that using a pneumatic linear bearing described in Ref. 10.

II. EXPERIMENTAL SETUP

Figure 1 shows a schematic diagram of the experimental setup for evaluating mechanical response of general objects against impact forces. Figure 2 shows a schematic diagram of the mechanical part of the experimental setup. In the method, the inertial force of a mass is used as the reference force acting on the material under test. A pendulum is used to produce well-defined motion of the mass, i.e., the swinging part of the bearing. Impact force is generated and applied to the material by colliding the mass into the test measurement. The mass is suspended using a pendulum with four tungsten wires. Each wire has the diameter of approximately 0.1 mm and the length of approximately 245 mm. The pendulum mass consists of some metal blocks including the cube cor-

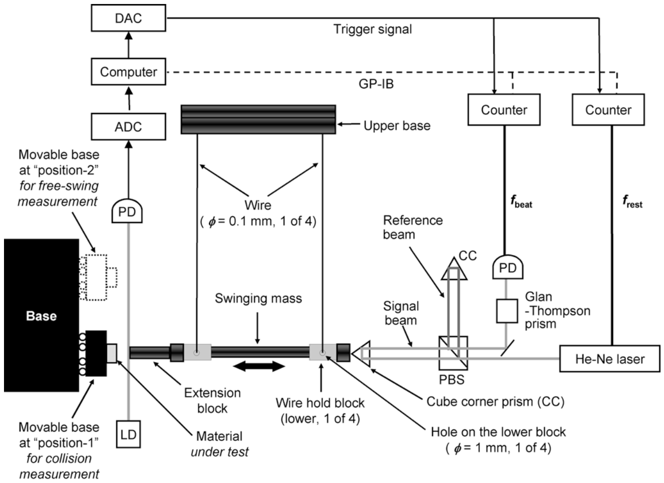

FIG. 1. Experimental setup.

ner prism for interferometry, the extension block, and the wire holder blocks. The total mass of the swinging part of the pendulum is approximately 2.0427 kg.

The velocity of the mass is measured highly accurately using an optical interferometer. The measurement procedure is as follows: at the beginning, the mass is pulled using a thin synthetic fiber (not shown in the figure) toward the right in Fig. 1, then the mass begins to swing by cutting the fiber using a cutter. The inertial force acting on the mass is calculated from the velocity of the mass.

The material under test is attached to a movable base, which enables the two modes of measurement, i.e., the mode of collision measurement and the mode of free-swing measurement. In Fig. 1, the movable base is set to be "position 1" for the mode of collision measurement. The movable base at upper position or "position 2" is also shown by the dotted line in Fig. 1. The movable base is set to be at position 2 for the mode of free-swing measurement, which is for evaluating the restoring force of the pendulum. At position 2, the material under test is set back from the moving path of the swinging mass and the swinging mass travels back and forth due to the restoring force of the pendulum.

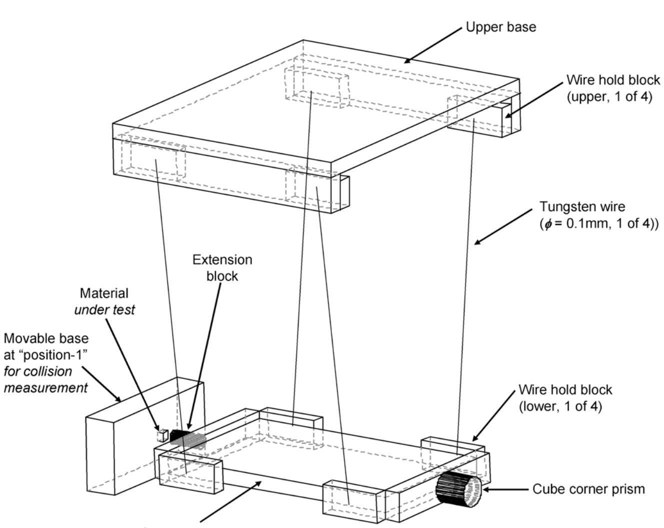

FIG. 2. Schematic of the pendulum.

The total force acting on the swinging part F_{mass} is calculated as the product of its mass M and its acceleration a. The total force acting on the swinging part F_{mass} is divided into two components, i.e., the force acting from the material under test $F_{material}$ and the restoring force of the pendulum $F_{restoring}$.

$$F_{mass} = F_{material} + F_{restoring}.$$

In the measurement, the total force F_{mass} is measured as the product of the mass and the acceleration. The acceleration is calculated from the velocity of the swinging part. The velocity is calculated from the measured value of the Doppler shift frequency of the signal beam of a laser interferometer $f_{Doppler}$, which can be expressed as

$$v = \lambda_{air}(f_{Doppler})/2,$$

$$f_{Doppler} = -(f_{beat} - f_{rest}),$$

where λ_{air} is the wavelength of the signal beam under the experimental conditions, f_{beat} is the beat frequency, i.e., the frequency difference between the signal beam and the reference beam, and f_{rest} is the rest frequency which is the value of f_{beat} when the moving part is at a standstill. The direction of the coordinate system for the velocity, the position, the acceleration, and the force is towards the right in Fig. 1.

A Zeeman-type two-frequency He–Ne laser is used as the light source. The frequency difference between the signal beam and the reference beam, i.e., the beat frequency f_{beat}, is measured from an interference fringe which appears at the output port of the interferometer; it varies around f_{rest}, approximately 2.75 MHz, depending on the velocity of movement. An electric frequency counter (model R5363, manufactured by Advantest Corp., Japan) continuously measures and records the beat frequency f_{beat} 6000 times with a sampling interval of $T=4000/f_{beat}$ and stores the values in memory. This counter continuously measures the interval time of every 4000 periods without dead time. The sampling period of the counter is approximately 1.5 ms at a frequency of 2.75 MHz. Another same-model electric counter measures the rest frequency f_{rest} using the electric signal supplied by a photodiode embedded inside the He–Ne laser.

Measurements using the two electric counters are triggered by means of a sharp trigger signal generated using a digital to analog converter. This signal is initiated by means of a light switch, a combination of a laser diode and a photodiode. The origins of the time and position axes are set to be the time and the position where the swinging part passes the bottom of the swing trajectory and the total force acting on the mass has a positive value.

In the experiment, one set of free-swing measurements and one set of collision measurements are conducted.

III. MEASUREMENT

A. Free-swing measurement

To evaluate the restoring force of the pendulum $F_{restoring}$, the free-swing measurement is conducted. In the experiment, the movable base is set to be position 2 and the swinging part is made to have a swinging motion back and forth.

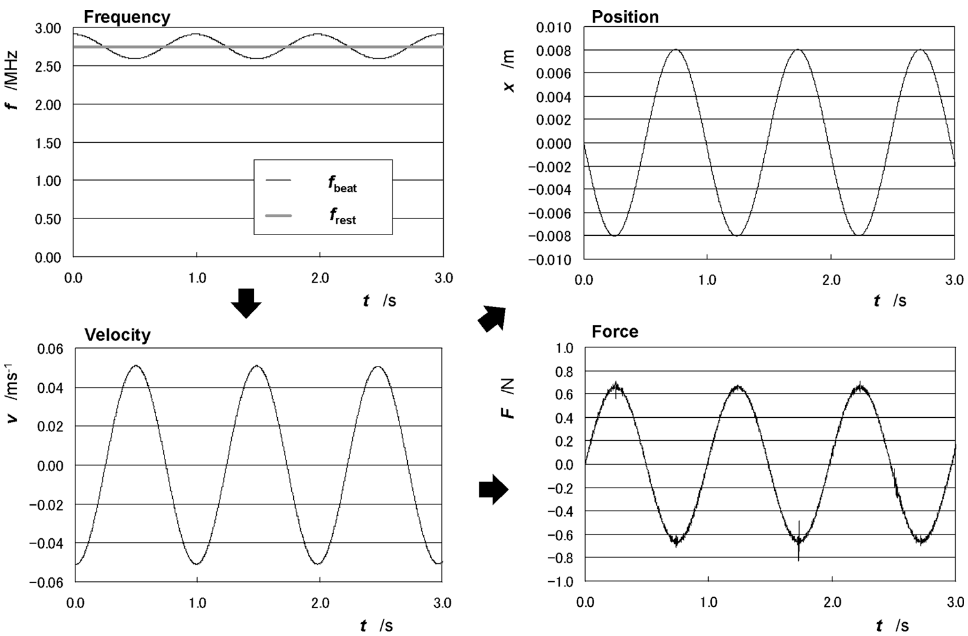

FIG. 3. Data processing procedure: Calculation of velocity, position, acceleration, and force from measured frequency.

Figure 3 shows the data processing procedures. During the free-swing measurement, only the time-varying beat frequency f_{beat} and the rest frequency f_{rest} are highly accurately measured using an optical interferometer. The Doppler frequency shift is measured as the difference between the beat frequency and the rest frequency. The velocity, the position, the acceleration, and the force are calculated from the measured motion-induced time-varying Doppler frequency. In this case, the amplitude of the swing motion is approximately 8 mm and the damping is quite small.

Figure 4 shows change in the restoring force $F_{\text{restoring}}$ against the position x during the free-swing motion. The measurement period is approximately 9 s. The linear relationship between the restoring forces $F_{\text{restoring}}$ against the position x is clearly observed. In the figure, the linear regression equation, $F_{\text{regression}}=82.3x+0.0006$, is also shown. The coefficient of determination (R^2) is approximately 0.9991. In Fig. 4, the residual force, $F_{\text{residual}}=F_{\text{restoring}}-F_{\text{regression}}$, in the same free-swing measurement is also shown. The root mean square (rms) value of F_{residual} is approximately 0.02 N. Since the position dependency of the force is not observed anymore, it can be said that the form of the regression line was adequate.

B. Collision measurement

In the collision measurement, the mass is made to collide to the material under test and the total force acting on the mass is measured as the product of mass and acceleration in the same way as the free-swing measurement.

Figure 5 shows the change in the total force acting on the mass F_{mass} against time t. During the measurement period of approximately 9 s, the swinging part collides with the material under test ten times. Figures 6–9 show the result of the same collision measurement shown in Fig. 5 but in different manner.

Figure 6 shows the change in the total force acting on the mass F_{mass} against position x. The effect of the restoring force $F_{\text{restoring}}$ is clearly observed in the figure.

Figure 7 shows the corrected force, $F_{\text{corrected}}=F_{\text{mass}}-F_{\text{regression}}$, against position. To obtain the estimated restoring force $F_{\text{restoring}}$, the linear regression equation obtained from the free-swing measurement, $F_{\text{regression}}=82.3x+0.0006$, is used. The corrected force $F_{\text{corrected}}$ is the estimated value of the force acting on the mass from the material under test

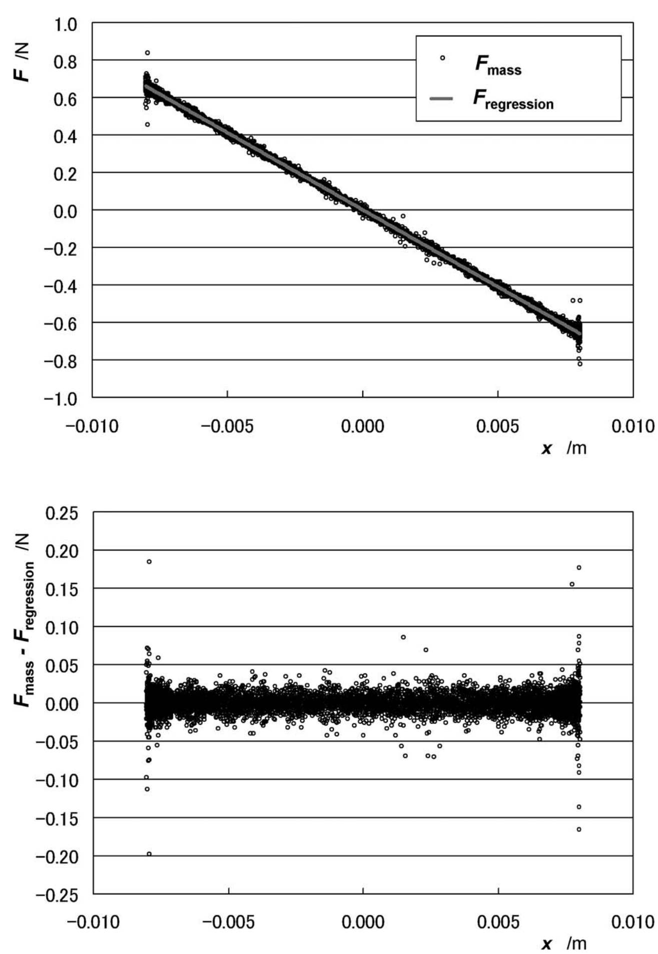

FIG. 4. Change in force against position in free-swing motion.

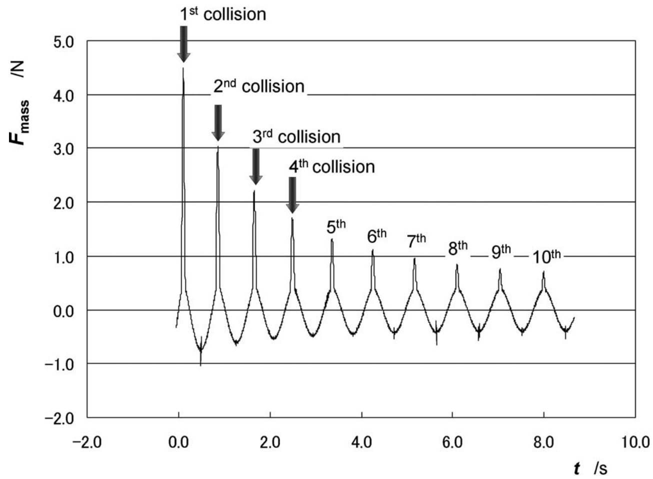

FIG. 5. Change in force against time in collision measurement.

F_{material} throughout the period of the collision measurement. The force acting on the material from the mass can be expressed as $-F_{\text{material}}$ according to the principle of action and reaction.

Figure 8 shows the relationship between the force $F_{\text{corrected}}$ and the position $x-x_0$, where x_0 is the position at which the force acting forms the material detected. The elastic hysteresis, which is caused by the viscosity of the material, is clearly observed.

Figure 9 shows the relationship between the force $F_{\text{corrected}}$ and the velocity v. The lead of force against the velocity, which is caused by the viscosity of the material, is observed. The velocity where the force has its maximum value of approximately 3.97 N, $V_{F\,\max}$, is approximately -0.015 m s^{-1} in the first collision.

IV. UNCERTAINTY EVALUATION

The uncertainty sources in determining the impact force acting on the material from the mass during collision are as follows.

A. Uncertainty sources concerning the zero point

The mean value, the standard deviation, and the rms value of the corrected force $F_{\text{corrected}}$ before and after the

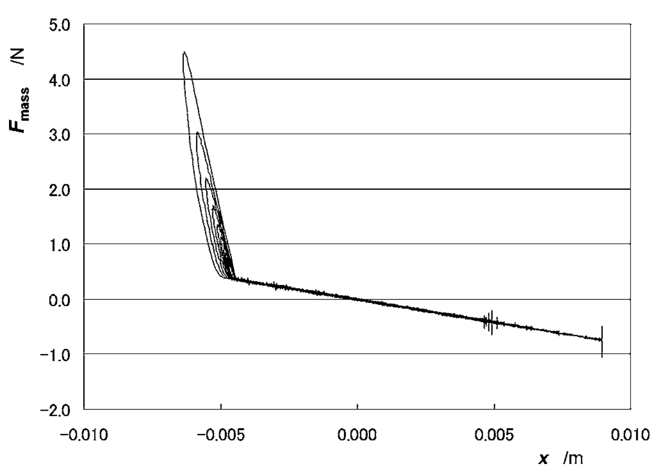

FIG. 6. Change in force against position in collision measurement.

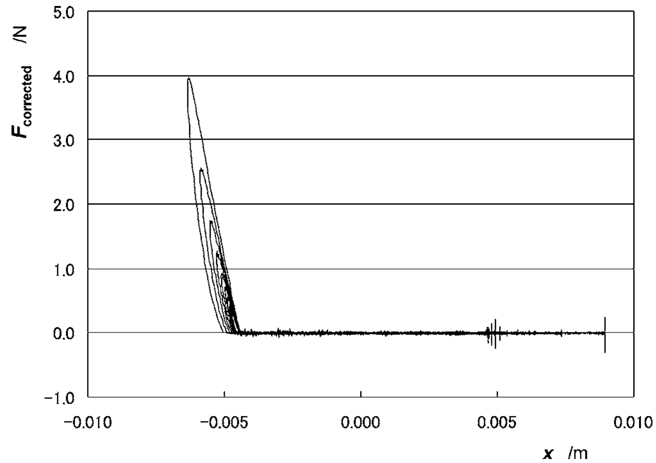

FIG. 7. Change in corrected force against position in collision measurement.

collision shown in Fig. 8 in the range of 0.1 mm$<x-x_0$ <0.3 mm are approximately -0.004, 0.014, and 0.015 N, respectively. The rms value of approximately 0.015 N is thought to correspond to the total uncertainty of the zero point including the uncertainty due to the mechanical vibration of the mass, the uncertainty in correcting the restoring force of the pendulum, the frequency stability of the laser, and the uncertainty in measuring frequencies, f_{beat} and f_{rest}, using the optical interferometer. Therefore, the standard uncertainty concerning the zero point is estimated to be 0.015 N. This corresponds to 4×10^{-3} (0.4%) of the maximum force applied to the material in the collision experiment.

B. Uncertainty sources concerning the scale

The uncertainty sources concerning the scale of the determined impact force acting on the material from the mass during collision are as follows.

1. Mass

Mass of the moving part is calibrated with a standard uncertainty of approximately 0.1 g, which correspond to the relative standard uncertainty in force determination of approximately 5×10^{-4}. This is negligible.

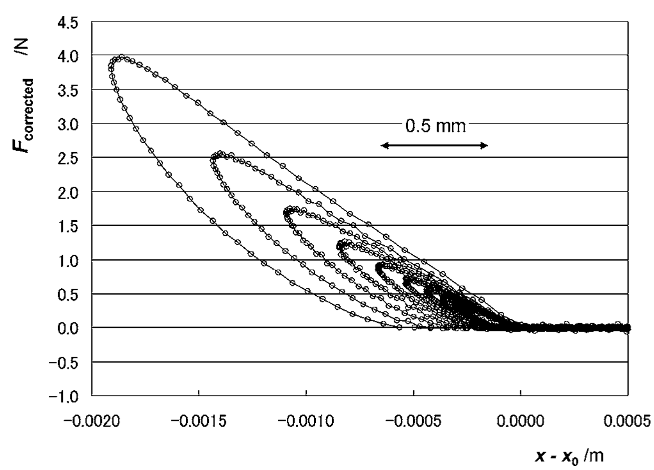

FIG. 8. Relationship between force and position of the material under impact load.

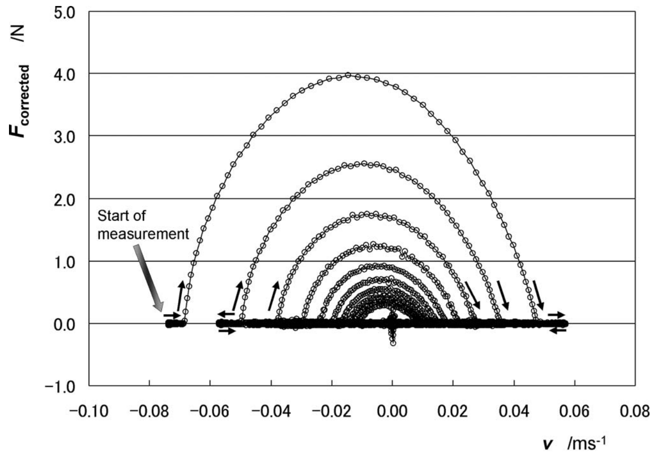

FIG. 9. Relationship between force and velocity of the material under impact load.

2. Alignment

The major source of uncertainty in the optical alignment is the inclination of the signal beam of 1 mrad, and it results in a relative uncertainty in the velocity of approximately 5×10^{-7}, which is negligible.

3. Frequency standard

The uncertainty of the frequency standard used in the experiment is approximately 1×10^{-6}, which is negligible.

C. Combined standard uncertainty

Therefore, the standard uncertainty in determining the impact force acting on the material from the mass during collision is estimated to be 0.015 N. This corresponds to 4×10^{-3} (0.4%) of the maximum force applied to the material in the collision experiment. Therefore, the developed instrument with the pendulum is as accurate as the instrument with the pneumatic linear bearing described in Ref. 10.

V. DISCUSSION

To prevent a vibration of the mass induced by the impact of the collision, there is much left to study on the configuration of wires. In this experiment, the two wires, the mass, and the upper base form a trapezoid. To restrict the motion of the mass to one degree of freedom, a triangular form is suitable. Monitoring the motion of the other five degrees of freedom will be effective to precisely evaluate the performance of the pendulum.

To develop an inexpensive portable material tester based on the levitation mass method, the following subjects should be solved in addition to the method using the pendulum described in this article.

(1) Using a laser diode instead of the Zeeman-type two-frequency He–Ne laser.
(2) Using a digitizer board instead of the frequency counters.

If the material under test is put in a constant temperature box, the temperature dependency of the material characteristics, which is especially important for high molecular weight materials, can be evaluated using the proposed method. In this case, if the extension block is long enough and made of a material, such as a ceramic, which has insignificant heat conductivity and large stiffness, the rest of the apparatus including the pneumatic linear bearing can be placed outside the constant temperature box.

Using the developed pendulum and the method for correcting the restoring force of the pendulum, many variations of the levitation mass method that had been originally proposed for instruments using pneumatic linear bearings could be modified to forms that are suitable for use under high temperature environment, low temperature environment, and/or vacuum environment.

Pendulums are sometimes used for precise measurement of force, such as the measurement of the gravitational constant G.[12] The proposed method might help researchers who require the precise generation and measurement of force, especially who consider to use a pendulum for the same kind of purpose.

ACKNOWLEDGMENT

This work was supported by a research-aid fund of the Asahi Glass Foundation.

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